TECHNICAL NOTES.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

No. 23.

HORIZONTAL BUOYANCY IN WIND TUNNELS.

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November, 1920.
The following report on "Horizontal Buoyancy in Wind Tunnels" was submitted to the National Advisory Committee for Aeronautics by Dr. A. F. Zahm, Member, Committee on Aerodynamics.

Distinction of relative flow. - From the earliest study of hydro-mechanics it has been assumed that the reaction between a stagnant fluid and a body moving through it is the same as for the fluid passing the fixed body with the same relative motion. So many experiments however seemed to disprove the assumption that at one time some prominent investigators questioned its validity. For example, Dubuat and Duchemin found experimentally that a plate fixed in a stream of water has about thirty per cent more resistance than when moving through still water at the same speed. This phenomenon was called "Dubuat's paradox". But when due precaution is taken to make the relative motion the same in both cases, as was done with an apparatus developed by Joukovski and Kouznetzoff, the assumption is completely vindicated. The reader may find a description of this apparatus in Joukovski's Aerodynamique, Chapter II.

But such equivalence of relative motion cannot always be realized. When for example, a body moves through an extended homogeneous still fluid the relative velocity of all distant parts of the fluid is uniform, and the undisturbed horizontal pressure gradient is zero; but when the fluid moves past the body usually all the distant parts have not the same velocity, usually there is some turbulence, and the pressure gradient is not zero. And if the gradient is not zero there is a horizontal buoyancy to be applied as a correction to make the observed resistance equal that for the still fluid.

Static pressure gradient. - In untapering wind tunnels, for instance, there is a material fall of static pressure down stream within the walled working portion. If the tunnel is well designed and unobstructed, the down-stream pressure gradient is constant for any one wind speed, and varies nearly as the square of the wind velocity. If the tunnel is partly obstructed with apparatus, the gradient may, at any fixed speed, vary in amount from point to point along stream.
Point pressure and pressure drag. - In general the pressural drag on a wind tunnel model, apart from the frictional drag, equals the surface integral of the horizontal component \( p_x \) of the point pressure \( p \). Now this latter may be written \( p = p_1 + p_2 + p_3 \), where \( p_1 \) is the constant barometric pressure at the reference section \( A \) of the tunnel, say where the nose of the model is to be, and before insertion of the model; \( p_2 \) is the pressure drop down stream from \( A \) due to the static gradient in the unobstructed tunnel; \( p_3 \) is the kinetic pressure, or departure from \( p_1 \), due to the presence of the model in the stream. It is assumed that the tunnel is too large for material wall effect.

Obviously the drag due to the constant barometric pressure \( p_1 \) is zero. The drag due to \( p_3 \) would be zero for a frictionless fluid, but for natural fluids is a chief element of resistance, especially for bodies of blunt form. Its discussion is irrelevant to the present treatment. The drag due to \( p_2 \) may here be formulated for various conditions.

Computation of horizontal buoyancy. - The component of horizontal buoyancy at any element of the model's surface is \( l \ p_2 \ d S \), in which \( l \) is the direction cosine, referred to the tunnel axis, of the normal to the surface element \( d S \). To find the surface integral of this component analytically one must know the equation to \( S \); also to the pressure drop \( p_2 \), in terms of the distance along stream. The method of integrating is too familiar to require treatment here.

In wind tunnel practice \( S \) and \( p_2 \) are usually given graphically, and may require graphical integration to find the drag due to pressure drop. For example, \( S \) may be a surface of revolution, such as a balloon hull; or a cylinder, such as a uniform stream-line strut. The static pressure gradient may be given by a curved diagram, in which \( p_2 \) is plotted against the down stream distance from the model's nose; or, as a particularly interesting case, the gradient may be constant. These special examples will be briefly treated in turn.

The pressure-drop drag on a surface of revolution with its axis along stream is

\[
R = \int p_2 \cdot 2 \pi r \ d r = \pi \int p_2 \cdot d(r^2), \quad (1)
\]

where \( p_2 \) is the pressure drop at radius \( r \), and the integral extends over the whole surface. The integral is found graphically as the area of the curve obtained by plotting \( p_2 \) against \( r^2 \), both of which are assumed to be given for various distances along stream. This method was given in Report No. 107 of the British Advisory Committee for Aeronautics.

The pressure-drop drag on a cylinder held transverse to the stream, and having a plane of symmetry parallel thereto, is, per unit length,

\[
R = 2 \int p_2 \cdot d \ r, \quad (2)
\]
where \( r \) is the semi-thickness at any point along stream, and the integral extends from front to rear. The solution is found graphically as the area of the curve of \( \rho_2 \) plotted against \( r \).

If the pressure gradient is a constant, \( d \rho_2/dx = a \), the pressure-drop drag on a body of whatever shape is

\[
R = a \, v, \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (3)
\]

where \( v \) is the volume of the body. This formula is the same as that for the vertical buoyancy of a body immersed in a liquid, and has been known from time immemorial.

Methods of computation in practical use. - Formula (3) has been used in the reports from the Aerodynamical Laboratory, Washington Navy Yard, since July, 1919, when it was first shown by accurate measurement that the pressure gradient is constant along the working portion of the 8' x 8' tunnel freed of obstructions. In some of the earlier reports graphical integration had been employed because measurements, too hurried and insufficiently checked, had indicated a slightly variable pressure gradient. This mistake in measuring the gradient entailed an error of the order of one or two per cent in finding the total drag on the models tested.

A detailed account of the calibration of the 8' x 8' tunnel, for pressure gradient, is given in the laboratory's Report No.148.

For streamline bodies of considerable bulk the pressure-drop correction is a good percentage of the whole resistance. If, therefore, the tunnel must have a material pressure gradient, it is fortunate when this is so nearly constant that formula (3) can be used to compute the pressure-drop correction. The value of \( a \) need be determined but once for a given tunnel, and the value of \( v \) for a model of any shape can easily be found by immersion in water.