EXPERIMENTAL DETERMINATION OF THERMAL CONDUCTIVITY
OF LOW-DENSITY ICE

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SUMMARY

The thermal conductivity of low-density ice has been computed from data obtained in an experimental investigation of the heat transfer and mass transfer by sublimation for an iced surface on a flat plate in a high-velocity tangential air stream.

The results are compared with data from several sources on the thermal conductivity of packed snow and solid glaze ice. The results show good agreement with the equations for the thermal conductivity of packed snow as a function of snow density. The agreement of the curves for packed snow near the solid ice regime with the values of thermal conductivity of ice indicates that the curves are applicable over the entire ice-density range.

INTRODUCTION

Removal of ice formations which accrete on aircraft surfaces from the impingement and freezing of cloud droplets has been the subject of a considerable amount of research in recent years. The accumulation of ice on aircraft surfaces occurs over a range of air temperatures from 32°F to as low as -40°F (ref. 1). The ice formations may be nearly clear glaze ice with little porosity, such as those formed at air temperatures near the freezing point, or they may be of the very porous type characterized by frost or ice formed at low temperatures. Some factors which affect the nature of the ice formations are the speed of the aircraft and the cloud droplet size and droplet size distribution. Very few data are available on the physical properties of ice formations having a density other than that of clear, solid ice. Since much of the de-icing of aircraft is currently accomplished by the application of heat to the surface to be de-iced, knowledge of the thermal conductivity of ice is of importance in determining the heat lost through the ice. A similar problem is encountered in the determination of heat-transfer
rates for frosted refrigeration equipment surfaces. A study of the mass transfer by sublimation and the heat transfer for an iced surface in a high-velocity air stream (ref. 2) required the determination of ice density, ice surface temperature, and rate of heat flow through the ice. From these data, the thermal conductivity of low-density ice has been determined for several values of ice density. The results obtained are presented herein and are combined with information from several sources to make possible a more complete evaluation of the thermal conductivity of ice over the entire range of ice density. The investigation was conducted in a 3.84- by 10-inch tunnel at the NACA Lewis laboratory.

METHOD, APPARATUS, AND PROCEDURE

The thermal conductivity of a substance is usually determined by means of standard laboratory procedures and equipment. Few such determinations have been made for low-density ice, however; and since the investigation reported in reference 2 supplied all the data necessary for the determination of thermal conductivity, the computations were made and are presented herein.

The ice formations to be studied were formed on a section of the upper surface of a flat-plate model which was mounted in the test section of the 3.84- by 10-inch tunnel. The ice which formed on the surface was then maintained at constant thickness (no net transfer of mass by sublimation or condensation) while the heat-flow rate and the temperature were measured. The condition of constant ice thickness was necessary for two reasons: (1) The stream-side surface temperature could be readily obtained only for the condition for which the surface temperature was equal to the temperature of saturation of the air stream, and (2) the thermal conductivity must be determined for steady-state conditions for a constant ice thickness.

The flat-plate model (shown in fig. 1) was 18 inches long, 0.75 inch thick, and 3.84 inches wide and was made of wood to minimize heat conduction through the model. One section of the upper surface of the model consisted of a 3.84- by 5.75-inch copper plate 1/16 inch thick set flush in the surface and located with its leading edge 4 7/16 inches from the leading edge of the model. The copper plate was the upper surface of a multipass copper box through which cold alcohol could be pumped. Five thermocouples were located in the surface of the copper plate spaced at 1 7/8-inch intervals along the center line. The thermocouple leads passed through tubes in the alcohol chamber, and the thermocouple junctions were made flush with the outer plate surface. The tubes were soldered to the copper plate and to the bottom of the
copper box. Additional thermocouples were located in the tunnel plenum chamber to measure the total temperature of the air stream and in the alcohol inlet and outlet lines to the copper box. A schematic diagram of the alcohol and thermocouple system is included in figure 1.

A supply of refrigerated air, initially at approximately -20°F and with a specific humidity of approximately 5.0x10^{-4} pounds of water per pound of dry air, was conditioned to provide the desired temperatures and humidities at the tunnel test section. The humidity of the air stream was controlled by means of steam injected at a point sufficiently far upstream to insure thorough mixing at the test section.

The mass-transfer investigation reported in reference 2 placed certain restrictions on the method of forming the ice which were best met by causing the water vapor in the humidity-controlled air stream to condense directly to the solid state on the surface of the cold copper plate. The plate was cooled to a temperature below that of the frost point of the air stream by the flow of cold alcohol through the copper box. The alcohol was cooled in an alcohol-dry ice bath heat exchanger. The resultant ice formation thus took the form of a dense frost-like formation as shown in figure 2. Ice formations were deposited on the refrigerated surface of the copper plate at nearly constant values of 5000 feet pressure altitude and 45°F total temperature of the air stream for Mach numbers of 0.4, 0.6, and 0.8. The thickness of the ice formation, which is dependent upon the humidity, velocity, and static pressure of the air stream, was controlled through regulation of the steam supply used for humidification of the air stream. The ice was allowed to form until a thickness of approximately 0.20 centimeter had been obtained. Measurements of the ice thickness while the tunnel was in operation were made through a window in the tunnel wall by the use of a short-focal-length telescope mounted on a vernier carriage. The telescope was focused on the pointer (fig. 2) which was directly above the center line of the copper plate. The depth of focus of the telescope was less than 3/4 inch; therefore, with the pointer in focus the ice surface viewed was within the range of approximately 3/8 inch from the center line. The mean height of the ice surface was used in order to account for roughness projections and depressions.

To obtain measurements of ice density, a 2- by 4-inch sheet of brass 0.002 inch thick was wetted on one side and frozen to the copper plate. Ice was then formed by condensation on the thin brass sheet for each Mach number condition. The ice thickness was measured with the telescope, and then the brass sheet and the ice were lifted from the cold plate and removed from the tunnel to determine the ice weight and the average ice density. The density of the ice was assumed to be uniform throughout its thickness and at all points on the surface.
The heat-transfer rate through the ice could not be measured directly from the heat gained by the alcohol in flowing through the copper box, as there was some contribution to the total heat gained by the alcohol from conduction through the wood model. The total heat transferred to the alcohol was determined from measurements of the alcohol flow rate, alcohol inlet temperature, and change in temperature between inlet and outlet to the copper box. The Dewar flasks and stirrer shown in figure 1 ensured uniform temperature of the alcohol across the tubing cross section at the points of temperature measurement. The alcohol flow rate was measured immediately following the temperature measurements by diverting the flow and obtaining volume measurements over a 45-second time interval. Values of the specific gravity of the alcohol at the temperatures used were determined experimentally, and values of the specific heat of the alcohol for the temperature and water content conditions existing during the investigation were obtained from reference 3.

It was necessary to determine the heat gained by conduction through the model, which must be subtracted from the total heat gained by the alcohol to determine the heat flow through the ice. For this purpose an insulating material (cork board) of known thermal conductivity was cemented to the surface of the copper plate. The total heat gained by the alcohol with the cork in place is the sum of the conduction heat flow through the model and through the cork. For a good insulating material such as cork, the surface temperature on the stream side is approximately equal to the adiabatic wall temperature; thus the heat transfer through the cork can be calculated from the adiabatic wall temperature, the measured plate surface temperature beneath the cork, and the conductivity of cork board (ref. 4 and the manufacturer's data). The use of the adiabatic wall temperature (computed from measurements of the total temperature and the total and static pressure) for the surface temperature of the cork during heat transfer through the cork results in an error of approximately 5 percent of the heat flow through the cork. However, because the heat conducted through the cork is a relatively small proportion of the total heat gained by conduction, the resulting error in the determination of the heat conducted through the wood model is approximately 1 percent. Values of the heat gained by conduction were determined for the various Mach numbers, pressure altitudes, and alcohol bulk temperatures. A conduction correction factor for the heat gained through the model was determined which is a function of Reynolds number, as the heat conducted through the wood was first transferred to the wood by convection from the air stream. By use of the conduction correction factor, values of the heat gained by conduction through the wood were obtained and were subtracted from the total heat gain measured with ice on the surface to determine the rate of heat transfer through the ice.
In order that the heat transfer to the alcohol should not include any heat required for the transfer of mass either to or from the ice surface, the humidity of the air stream was adjusted until the ice remained at constant thickness. With constant ice thickness there is no net mass transfer; that is, sublimation of the ice and condensation of the vapor from the air on the ice surface occur at the same rate. Under these conditions, the surface temperature of the ice is the temperature of saturation of the air stream (frost-point temperature). The frost-point temperature was measured with a sensitive dew-point meter from continuous samples of the air from the plenum chamber, expanded to the static pressure existing over the ice surface, and was thus a measurement of the ice surface temperature. Computations of the thermal conductivity of the ice formation could thus be made from the equation

\[ k = \frac{q x}{A (t_{\text{ice}} - t_{\text{plate}})} \]

where

- \( q \): corrected rate of heat flow, Btu/hr
- \( x \): ice thickness, in.
- \( A \): ice surface area, sq ft
- \( t_{\text{ice}} \): temperature of stream-side ice surface, °F
- \( t_{\text{plate}} \): temperature of plate surface beneath the ice, °F

and \( k \) has the units Btu/(hr)(sq ft)(°F/in.).

From an examination of the accuracy of the various measurements required for the determination of the thermal conductivity of the ice, an estimate of the precision of the results may be obtained.

The temperature rise of the alcohol in flowing through the copper box and the rate of alcohol flow were measured with equipment which ensured a high degree of accuracy. The accuracy of these measurements combined with the application of the conduction correction factor to the total heat gained by the alcohol results in a precision of the order of ±5 percent for the determinations of the corrected rate of heat flow through the ice.

Measurements of the ice thickness were limited to the area in the field of view of the telescope. The telescope and its vernier carriage were mounted on a vertical shaft, and by rotating the telescope and carriage about the shaft a somewhat larger viewing area could be covered. A limited number of observations made in this manner indicated that the
ice thickness was essentially constant. Any gradient in thickness from the leading edge to trailing edge was averaged since the ice thickness was measured near the center of the plate. From the accuracy of repetition of measurements with the telescope, the determination of the ice thickness was estimated to be within ±2 percent of the average ice thickness over the entire plate.

The measurements of the average ice surface temperature (frost-point temperature) were estimated to be in error by less than ±1°F.

The plate surface temperature beneath the ice showed a temperature profile in the stream direction which was low near the center of the plate by approximately 10°F. However, the average of these temperatures was estimated to be within ±2°F of the average temperature of the plate surface, and the combined error in the plate surface temperature and the ice surface temperature measurements would be of the order of ±7.5 percent.

From the combined errors in the measurements, the accuracy of the determination of the thermal conductivity of the ice was indicated to be within ±15 percent.

RESULTS AND DISCUSSION

The density of the ice formed by the condensation of the vapor from the air stream at constant values of total temperature and pressure altitude varied as a function of the Mach number at which the ice was formed, as shown in figure 3. Average values of the ice density for three or more ice samples were determined for each Mach number.

Values of the thermal conductivity of ice \((\text{Btu/(hr)(sq ft)(°F/in.))})\) were obtained for ice formed at Mach numbers of 0.4 to 0.8 and are presented in figure 4 as a function of the density (obtained from fig. 3) of the ice formation. The average temperature of the ice as determined from the average of the stream-side ice-surface temperature and the model-side plate-surface temperature was in the temperature range from 3°F to -13°F. The values of thermal conductivity obtained showed no consistent variation with temperature for the temperature range used in this investigation. Included in figure 4 are values obtained by several investigators as reported in references 4 to 6 for solid glaze ice and for packed snow. All the data points shown in figure 4 for densities less than 0.9 grams per cubic centimeter are for packed snow except for the results of the present investigation which are for ice formed by the condensation method. The equations for the conductivity of packed snow determined by van Dusen (ref. 6)
k = 0.21 + 4.2\rho + 21.6\rho^3 \text{ milliwatt/(cm)(°C)}

and by Devaux (ref. 7), for 0.1 < \rho < 0.6,

k = 0.29 (1 + 100\rho^2) \text{ milliwatt/(cm)(°C)}

have been computed over the range of density from \rho = 0.1 to that of solid ice (approximately 0.92) and are shown as dashed lines on the figure. The ranges of thermal conductivity of solid, glaze ice for temperatures from 0° to 90° C as given in reference 6 from computations of van Dusen's equations (k = 20.9 (1 - 0.0017t) \text{ milliwatt/(cm)(°C)} and by Jakob and Erk (ref. 6) are also presented. van Dusen shows a relatively small increase in thermal-conductivity values with decrease in ice temperature, while the work of Jakob and Erk, which is more recent, shows a considerably greater change with temperature. The equations of van Dusen and Devaux for packed snow with a density near that of solid ice show good agreement with all the results presented for solid ice except for the low-temperature results of Jakob and Erk. The values of thermal conductivity determined in the present investigation are, for the most part, bounded by the van Dusen and Devaux curves. The spread in the data is of the order of the error indicated by the analysis of the precision of the measurements.

The values of thermal conductivity of snow at densities of 0.45 and 0.5 gram per cubic centimeter, as taken from the International Critical Tables from the results of Jansson and from Ingersoll and Zobel, appear low when compared with the value Jansson reported at a density of 0.11 gram per cubic centimeter and the other data of figure 4. The data and curves shown in figure 4 indicate that either the Devaux or the van Dusen curves will probably give good approximation of the thermal conductivity of ice over the whole range of ice densities; it appears probable that the use of the van Dusen curve may result in values of thermal conductivity which are too low for ice at very low temperatures.

CONCLUDING REMARKS

Values of the thermal conductivity of low-density ice formed by a condensation process on a cold surface have been determined. The data show good agreement with the equations of van Dusen and Devaux for the thermal conductivity of packed snow as a function of the snow density.
REFERENCES


Figure 1. Schematic diagram of flat-plate model and alcohol system.
Figure 3. - Density of ice formed by condensation from air stream as a function of Mach number.
Figure 4. - Thermal conductivity of ice and packed snow as function of density.