A COMPARATIVE STUDY OF TWO METHODS USED IN THE TEACHING OF ARITHMETIC IN THE SEVENTH GRADE

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CHAPTER I

NATURE AND PURPOSE OF THE STUDY

The Problem

This comparative study of methods used in teaching arithmetic was carried out in the La Vega Public School, McLennan County, Texas, during the 1941-1942 school term. Two seventh-grade classes under the direction of the same teacher were used as experimental and control groups. In the experimental group the rules to be used in solving the problems were discovered and formulated by the pupils from their relationships to steps and processes already learned, thereby making the meaning clear before learning the operations. In the control group the rules to be used were presented by the teacher before the work was begun. The pupils studied these rules and applied them in the solution of examples and problems. To lead the children to formulate their own rules takes much more time, and the purpose of this study was to discover, experimentally, whether or not this expenditure of time is justifiable.

Limitation of the Study

This report was based on the work done by two seventh-grade classes consisting of twenty pupils each during a

class period of forty-five minutes each day for a period of approximately six months, beginning September 15, 1941, and ending March 11, 1942. The study included the following phases of seventh-grade arithmetic and the application of each:

1. Addition of decimals.
2. Subtraction of decimals.
3. Multiplication of decimals.
4. Division of decimals.
5. Finding the per cent of a number.
6. Finding what per cent one number is of another.
7. Finding the number when the per cent of it is known.

Explanation of Terms

Throughout this report the following terms will be used:

Ar.A. -- Educational Age in Arithmetic.

C. A. -- Chronological Age.

M. A. -- Mental Age.

I. Q. -- Intelligence Quotient.

E -- Experimental Group.

C -- Control Group.

Plan of the Report

Chapter I gives the nature and purpose of the study and its limitations.

Chapter II contains an explanation of the procedure followed in equating the groups and the methods of teaching.

Chapter III gives a comparison of the members of each match with one another and a comparison of the two groups, based on both daily work and standardized test results.

Chapter IV outlines the conclusions and recommendations set forth on the basis of data compiled.

CHAPTER II

METHOD OF PROCEDURE

Equating the Groups

At the beginning of this study each pupil was given Form A of Analytical Scales in Arithmetic.[1] The educational age in arithmetic (Ar.A.) was found from this test. The I. Q. was taken from the record of the Otis Group Intelligence Test[2] in the school files, and the mental age (M.A.) was brought to date, using the formula, I.Q. x C.A. = M.A. A separate card was written for each pupil, the card stating his educational age in arithmetic, chronological age, mental age, and I.Q. These cards were arranged in pairs, placing the two highest Ar.A.'s as match one, and so on down the list. The mean Ar.A., C.A., M.A., and I.Q. were then found, and the cards were rearranged in order that mean scores on all tests would be brought nearer together. This division of pupils is shown in Table 1.

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[1]Martha Kellogg, L. J. Brueckner, and M. J. Van Wagenen, Analytical Scales of Attainment, Division 3, Form A.

[2]Arthur S. Otis, Otis Group Intelligence Test, Form A, Grades 4-8.

TABLE 1

ARITHMETIC AGE, CHRONOLOGICAL AGE, MENTAL AGE, AND INTELLIGENCE QUOTIENT OF THE FORTY STUDENTS THAT WERE PAIRED

Pupil	Ar.A.		C.A.		M.A.		I.Q.	
	E	C	E	C	E	C	E	C
1	12-6	12-8	11-8	13-0	11-1	13-8	95	105
2	12-4	12-3	13-11	12-11	14-6	14-3	104	110
3	12-2	12-2	11-6	11-7	12-11	13-4	112	115
4	11-4	11-9	11-11	11-3	16-0	14-9	134	131
5	11-4	11-11	11-1	11-4	13-8	14-3	123	126
6	11-10	11-2	13-3	13-8	15-4	15-0	116	110
7	11-9	11-1	12-7	11-6	12-9	12-1	101	105
8	11-7	11-6	11-7	11-3	10-4	10-10	89	96
9	11-6	11-5	11-6	11-2	12-11	12-9	112	114
10	11-3	11-3	13-1	12-0	12-10	13-5	98	112
11	11-1	11-10	15-0	14-11	15-2	15-2	101	102
12	10-7	10-6	14-7	13-1	15-4	13-6	105	103
13	10-6	10-6	11-6	11-4	12-4	12-6	107	110
14	10-2	10-1	12-9	11-8	13-0	11-9	102	101
15	10-1	10-1	12-0	12-8	12-6	12-3	104	97
16	10-7	9-8	14-4	12-5	11-4	10-10	79	87
17	9-7	9-9	13-1	13-4	14-3	14-0	109	105
18	9-5	9-7	11-10	12-4	12-7	12-3	106	99
19	9-6	9-6	11-6	11-1	11-7	11-0	101	99
20	9-2	9-2	14-1	14-9	11-5	11-6	81	78
Median	11-2	11-2	12-4	12-2	12-11	12-11	104	105

Method of Teaching the Experimental Group

The first phase of arithmetic included in this study was addition of decimals. In the experimental group this was introduced by showing the analogy to common fractions. Each pupil worked on paper as the teacher wrote the problems on

the board. The pupils knew how to read and write decimals using tenths, hundredths, thousandths, and ten-thousandths.

We began,

3/10 and 4/10 are 7/10, or
.3 and .4 are .7, or

$$\begin{array}{r} .3 \\ .4 \\ \hline .7 \end{array}$$

6/10 and 7/10 are 1 3/10, or
.6 and .7 are 1.3, or

$$\begin{array}{r} .6 \\ .7 \\ \hline 1.3 \end{array}$$

Likewise,

17/100 and 24/100 are 41/100,
or .17 and .24 are .41, or

$$\begin{array}{r} .17 \\ .24 \\ \hline .41 \end{array}$$

4/100 and 5/100 are 9/100,
or .04 and .05 are .09, or

$$\begin{array}{r} .04 \\ .05 \\ \hline .09 \end{array}$$

At this point one of the pupils suggested that the decimal point was kept in a line in all of these problems. For further study the following problems were used:

8/10 and 4/100 are 84/100,
or .8 and .04 are .84, or

$$\begin{array}{r} .8 \\ .04 \\ \hline .84 \end{array}$$

3/100 and 16/1000 are 46/1000,
or .03 and .016 are .046, or

$$\begin{array}{r} .03 \\ .016 \\ \hline .046 \end{array}$$

The following rules were worked out by suggestions from several of the pupils as we solved other similar problems:

1. Arrange the numbers so that the decimal points are in a line one under the other.

2. Add in one place at a time.

3. Place the decimal point in the answer directly under the line of decimal points in the numbers added.

The rules were written on the board as the pupils wrote them on paper. Following this procedure, eight problems in addition of decimals were given the pupils to solve in class.

The process of subtraction of decimals was introduced in about the same way as addition. The problems were written first as common fractions and then changed to decimal fractions.

9/10 - 2/10 is 7/10, or
.9 - .2 is .7, or

$$\begin{array}{r} .9 \\ -\ .2 \\ \hline .7 \end{array}$$

19/100 - 4/100 is 15/100, or
.19 - .04 is .15, or

$$\begin{array}{r} .19 \\ -\ .04 \\ \hline .15 \end{array}$$

After several of these problems were worked and discussed, one pupil said, "It works just like addition, doesn't it?" Next, problems requiring the annexing of zeros were given.

19/100 - 14/100 is 5/100, or
.19 - .14 is .05, or

$$\begin{array}{r} .19 \\ -\ .14 \\ \hline .05 \end{array}$$

The last type problem to be introduced was:

$$\begin{array}{r} 4.23 \\ -\ .741 \end{array} \quad \text{or} \quad \begin{array}{r} 4.23 \\ \hline \end{array} \qquad \text{and} \qquad 12 - .56 \text{ or } \begin{array}{r} 12. \\ -\ .56 \end{array}$$

From these problems the pupils worked out the following rules to be applied in subtraction of decimals:

1. Place the decimal point in the subtrahend directly below the decimal point in the minuend.
2. Place the decimal point in the answer directly below the other two points.

The pupils were again given eight problems to solve. After several days of class discussion and solving problems involving addition and subtraction of decimals, eight sentence problems were given the pupils.

The next unit to be studied consisted of multiplication and division of decimals. In introducing multiplication to the experimental group, the examples were grouped into three main classes which corresponded to the three major classes in multiplication with common fractions:

1. Multiplication of a decimal by an integer.
2. Multiplication of an integer by a decimal.
3. Multiplication of a decimal by a decimal.

The first problems used were:

4/10 x 2 = 8/10, or .4 x 2 = .8, or

$$\begin{array}{r} .4 \\ \times\ 2 \\ \hline .8 \end{array}$$

9/10 x 5 = 4 5/10, or .9 x 5 = 4.5, or

$$\begin{array}{r} .9 \\ \times\ 5 \\ \hline 4.5 \end{array}$$

From examples of this kind the pupils saw that if tenths were multiplied by an integer, the product was tenths, but

if there were more than 10 tenths, the product would be a decimal mixed number.

In multiplying an integer by a decimal, these problems were solved:

3 x 3/10 = 9/10, or 3 x .3 = .9, or

$$\begin{array}{r} 3 \\ \times\ .3 \\ \hline .9 \end{array}$$

6 x 4/10 = 2 4/10, or 6 x .4 = 2.4, or

$$\begin{array}{r} 6 \\ \times\ .4 \\ \hline 2.4 \end{array}$$

From these examples the pupils worked out the rule, "The number of decimal places in the product is the same as the number in the multiplicand or multiplier."

The examples which follow were used as an approach to multiplication of a decimal by a decimal.

3/10 x 7/10 = 21/100, or
.3 x .7 = .21, or

$$\begin{array}{r} .3 \\ \times\ .7 \\ \hline .21 \end{array}$$

3/10 x 21/100 = 63/1000, or
.3 x .21 = .063, or

$$\begin{array}{r} .21 \\ \times\ .3 \\ \hline .063 \end{array}$$

12/100 x 24/100 = 288/10000, or
.12 x .24 = .0288, or

$$\begin{array}{r} .24 \\ \times\ .12 \\ \hline .0288 \end{array}$$

Problems similar to these were used until the pupils understood that when a decimal is multiplied by a decimal, the number of decimal places in the product is equal to the number in the multiplicand plus the number in the multiplier.

After the rule was written and discussed, eight problems involving the three groups of problems in multiplication of decimals were given the pupils.

In introducing division of decimals, the following procedure was used:

6 apples divided by 3 = ? apples

10 books divided by 2 = ? books

12 pencils divided by 4 = ? pencils

9 tenths divided by 3 = ? tenths

$.9 \div 3 = .3$ $\qquad 3 \overline{)\,.9}$ $\quad .3$

Common fractions were then used:

$9/10 \div 3 = 3/10$ (same as .3)

The pupils discovered from these problems that when the divisor is a whole number there are just as many decimal places in the quotient as in the dividend.

In dividing whole numbers by decimal fractions, we began with the problem $.2 \overline{)\,6}$ and solved by common fractions.

$.2 \overline{)\,6} = 6 \div 2/10 = 30$; therefore, $.2 \overline{)\,6}$ (30)

Other problems used were:

$.7 \overline{)\,28} = 28 \div 7/10 = 40$ or $.7 \overline{)\,28}$ (40)

$.3 \overline{)\,87} = 87 \div 3/10 = 290$ or $.3 \overline{)\,87}$ (290)

$.13 \overline{)\,26} = 26 \div 13/100 = 200$ or $.13 \overline{)\,26}$ (200)

From these and similar problems the pupils discovered that when a whole number is divided by tenths, one zero must be annexed to the quotient; that when a whole number is divided by hundredths, two zeros must be annexed to the quotient; etc. The pupils also recognized from these problems that when the divisor is less than one, the quotient is larger than the dividend.

After using decimal mixed numbers as divisors, problems were used which yielded remainders.

$.2\overline{)\,7} = 7 \div 2/10 = 35$ or $\begin{array}{r} 35 \\ .2\overline{)\,7.0} \end{array}$

After several examples of this kind were given, the pupils discovered that they could place a decimal point after the last digit of the dividend and annex as many zeros as there are decimal places in the divisor without having decimal places in the quotient, but that for every additional zero annexed to the dividend there would be a decimal place in the quotient. Examples with two or three digits in either divisor or dividend or both were used for further practice.

The last type of division problem used involved division of a decimal by a decimal.

$.2\overline{)\,.4} = 4/10 \div 2/10 = 2$ or $\begin{array}{r} 2 \\ .2\overline{)\,.4} \end{array}$

$.2\overline{)\,.04} = 4/100 \div 2/10 = 2/10$ or $\begin{array}{r} .2 \\ .2\overline{)\,.04} \end{array}$

$.2 \overline{)\,.004} = 4/1000 \div 2/10 = 2/100$ or $.2 \overline{)\,.004}$ (.02)

$.02 \overline{)\,.004} = 4/1000 \div 2/100 = 2/10$ or $.02 \overline{)\,.004}$ (.2)

Similar examples were continued until the pupils saw that the number of decimal places in the quotient was the number in the dividend less the number in the divisor. The difficulty of the examples was gradually increased.

After this explanation, the pupils were given eight problems to solve, and after solving problems involving multiplication and division of decimals, eight sentence problems were given.

The next unit of work included the three type problems of percentage and the application of each. In introducing these problems to the experimental group the close relationship between percentage and common or decimal fractions was stressed.

The first type problem, finding a per cent of a number, was introduced by this problem:

> James, who sold vegetables for his father, was allowed to keep 25% of the money. One week James sold $4.80 worth of vegetables. How much money should James keep?

One pupil suggested that 25% meant 1/4, and explained that James would keep 1/4 of $4.80 or $1.20. After several problems of this type were solved, one was given in which the per cent was not an aliquot part of 100. The pupils discovered that if the per cent in former problems was

written as a decimal they would get the same answer. The following rule was worked out: "To find a per cent of a number, first write the per cent as a common fraction or as a decimal fraction and multiply."

The second type problem, finding what per cent one number is of another, was introduced by use of the following problem:

> We have an enrollment in our room of 35 pupils. 28 pupils are present today. What per cent of our pupils are present?

The pupils were first asked, "What part of the pupils are present?" The answer was 4/5, and they knew that meant 80 per cent. Several problems were solved, each resulting in a fraction that the pupils could write as per cent. Later a problem was given resulting in the fraction 4/7, one that the pupils did not know as per cent. This called for a method of work to express the fraction as per cent. The pupils applied a rule previously learned: "To change a common fraction to a decimal fraction, divide the numerator by the denominator." After the fraction was expressed as a decimal, the transition to per cent was easy.

The third type problem, finding a number when a per cent of it is known, was introduced by using the pupils' knowledge of fractions. The first problems given were:

6 is 50% of _____

4 is 25% of _____

8 is 33 1/3% of _____

In each case the per cent was expressed as a common fraction and the pupils gave the answers. When problems were given in which the per cent was not an aliquot part of 100, the first problems were again discussed. The pupils worked out the solution in problem 1. They found that by dividing 6 by 1/2 and later by .5, they got the same answer. From these and similar problems, this rule was worked out: "To find the number when a per cent of it is known, divide the number by the per cent, written as a common fraction or decimal fraction."

As the three problems in per cent were studied, meaningful applications of these cases to business affiars with which the pupils were acquainted, were given. After each type problem was introduced, eight problems were given the pupils, and, later, after applications were made, they were given eight sentence problems to solve.

Method of Teaching the Control Group

In presenting this work to the control group, the rules which the experimental group had formulated were presented and illustrated by the teacher. The pupils studied these rules and applied them to the solving of problems. The same problems were given the two groups as a check on the type of work done.

Ways of Evaluating the Work

Immediately following the introduction of each phase of new work, the pupils in each group were given eight problems to solve. Another group of eight problems was given each group after the method and its applications were studied.

At the conclusion of the seven units of work, Form B of the same standardized test which was used at the beginning of the study was given the pupils in each group.

CHAPTER III

TREATMENT OF DATA

Individual Comparison

The comparison of individuals comprising the 20 matches was based on the results obtained from the twelve sets of daily papers and the 2 standardized tests, Forms A and B. The pupils were numbered from 1 to 20 in each group. The pupils in the experimental group will be referred to as 1E, 2E, 3E, etc., and those in the control group as 1C, 2C, 3C, etc. The data for the following comparisons are found in Tables 1 to 16, inclusive.

In Match 1, Pupil 1E made 80.5 points and Pupil 1C made 20.5 points of improvement as measured by the standardized tests given. The greatest amount of superiority in favor of 1E came in fundamental operations. On the daily papers 1E made a total of 4 errors with no repetitions, and 1C made 10 errors, consisting of 6 in computation, 2 in method, and 2 in placing decimals. Ninety per cent of the mistakes made by 1C were made in solving problems involving applications of the methods studied. There was little difference in Arithmetic Age (Ar.A.) of these two pupils. Pupil 1C was one year and four months

older and had an I.Q. 10 points higher than 1E. It seemed that the difference would have been in favor of 1C.

The pupils in Match 2 made about the same improvement as shown on the standardized tests. Pupil 2E made 40.5 points; 2C made 34 points of improvement. On the 12 daily papers, 2E made 11 errors, consisting of 6 in computation, 2 in method, and 3 in the meaning of per cent; 2C made 25 errors, consisting of 14 in computation, 9 in placing decimal points correctly, and 2 in method used. There was very little difference in Ar.A. and C.A. of the two pupils. 2C had an I.Q. of 110, which was 6 points higher than that of 2E.

The pupils in Match 3 were well matched according to Ar.A., C.A., and I.Q. There was not enough difference to expect a better type of work from either pupil. Pupil 3E made 35 points of improvement on the standardized test, and 3C made 51.5 points of improvement. Pupil 3C showed a superiority in problem solving. On the daily papers 3E made a total of 21 errors. Of these, 9 were in computation, 6 in placing decimal points correctly, 2 in the meaning of per cent, and 4 in method used. Ten of these errors were made on problems that applied the method studied. Pupil 3C made 42 errors, including 19 in computation, 11 in placing decimal points correctly, 6 in the meaning of per cent, and 6 in method used. Of these errors, 21 were made on application of methods.

In Match 4, pupil 4C had an Ar.A. of 11-9, which was 5 months higher than that of 4E. This match was composed of the two pupils with the highest I.Q. in the groups, both being above 130. The work for the two pupils showed very little difference throughout the study. Both pupils were conscientious workers and attended school regularly. The improvement as measured by the standardized tests differed by only .5 point. Pupil 4E made 62.5 points, and Pupil 4C made 62 points. In errors tabulated from the daily papers, 4E made 17 and 4C made 18. The only noticeable difference of any value for this match was the fact that 47 per cent of the errors made by 4E and 72 per cent of the errors made by 4C were on problems involving application of methods.

The classification in Match 5 was slightly in favor of 5C. Ar.A. was 11-4 for 5E and 11-11 for 5C; C.A. was 11.1 for 5E and 11-4 for 5C; I.Q. was 123 for 5E and 126 for 5C. However, the difference was not great enough to explain the difference shown in the work of these two pupils. On the standardized test 5C surpassed 5E by 34 points in improvement. Pupil 5E made 25 errors on the daily papers, consisting of 7 in computation, 8 in knowledge of decimals, and 4 in the meaning of per cent. Five errors were made in method used, and 1 problem was omitted. Pupil 5C made only 15 errors, consisting of 8 in computation, 3 in decimals, 1 in per cent, and 3 in choosing correct method.

Fourteen of 5E's errors and 7 of 5C's errors were on application of methods.

In Match 6 there was a difference of 8 months in Ar.A. and 6 points in I.Q. in favor of 6E. The total improvement made, as shown by the standardized test, was 27.5 points for 6E and 19.5 points for 6C. In errors compiled from daily papers, 6E made 20 errors, consisting of 5 in computation, 7 in use of decimals, 1 in meaning of per cent, and 4 in method used, with one problem omitted. Of these errors, 12 were made on application of methods. Pupil 6C made a total of 75 errors, consisting of 16 in computation, 18 in use of decimals, 17 in meaning of per cent, and 23 in method chosen, with 1 problem omitted. Thirty-five of these errors were made on application of methods studied.

In Match 7, Pupil 7E showed a superiority of 8 months in Ar.A., while 7C had an I.Q. that was 4 points higher than 7E's. Pupil 7C made 13 points more than 7E on the standardized test, although 7E made a few less errors on the daily papers. Pupil 7E made 26 errors, consisting of 4 in computation, 8 in use of decimals, 4 in meaning of per cent, and 10 in methods used. Pupil 7C made 30 errors, consisting of 6 in computation, 1 in use of decimals, 11 in meaning of per cent, and 12 in method used. There was no noticeable difference in errors made by the two pupils on problems involving application of method used.

The Ar.A. of the members of Match 8 differed only 1 month. Pupil 8E was four months older chronologically; 8C had 7 points higher I.Q. than did 8E. Pupil 8E made 65.5 points of improvement, as shown by the standardized test given, and 8C made only 34 points. The superiority of 8E was shown in quantitative relationship and vocabulary. Pupil 8E also surpassed 8C in number of errors made on the daily papers, making only 9 errors, as compared with 41 errors for 8C. Of the 9 errors made by 8E, 6 were in computation, 1 in use of decimals, and 2 in methods used. Pupil 8C made 8 errors in computation, 20 in use of decimals, 3 in meaning of per cent, and 9 in method, with 1 problem omitted. Again 8E showed a superior type of work by having 7 perfect papers from the 12 tests, while 8C had none.

The pupils in Match 9 were closely matched according to Ar.A., C.A., and I.Q. There was not enough difference to merit any noticeable superiority in work. Pupil 9E made 44 points of improvement on the standardized test, and 9C made 50 points of improvement. There was a greater difference in work according to errors tabulated. Pupil 9E made 23 errors and 9C made 43 errors. This was probably due to the fact that 9C had a somewhat careless attitude toward his school work and seemed capable of much better work than he attempted to do. The 23 errors made by 9E

consisted of 9 in computation, 4 in use of decimals, 4 in the meaning of per cent, and 6 in method. The 43 errors made by 9C consisted of 14 in computation, 14 in use of decimals, and 13 in method used, with 2 problems omitted. Pupil 9E made 10 errors in problems involving application of methods used.

The two pupils composing Match 10 had the same Ar.A. Pupil 10E was about a year older chronologically and 10C had an I.Q. 14 per cent higher than that of 10E. They were not as closely matched as the other pairs; however, there was no great amount of difference shown in their work. Pupil 10E made 42.5 points of improvement on the standardized test as compared with 38 points for 10C. Pupil 10E made 32 errors, consisting of 12 in computation, 9 in use of decimals, 6 in meaning of per cent, and 5 in method used. Pupil 10C made 24 errors, consisting of 7 in computation, 4 in meaning of per cent, and 8 in methods, with 1 problem omitted.

In Match 11, there was a difference in Ar.A. of 9 months in favor of 11C. In C.A. and I.Q. the two pupils ranked about the same. There was a difference of 28.5 points in improvement in favor of 11E as measured by the standardized test. Pupil 11E made 40 errors, consisting of 14 in computation, 4 in use of decimals, 8 in meaning of per cent, and 14 in methods used. Pupil 11C made 35

errors, consisting of 10 in computation, 8 in use of decimals, 10 in meaning of per cent, and 7 in methods used.

In Match 12 the pupils had about the same Ar.A. and I.Q. Pupil 12E was one year and six months older chronologically. On the standardized tests 12E made 42.5 points of improvement and 12C made 30.5 points. The gain was not uniform on the different parts of the test. Pupil 12E led in quantitative relationships and problem solving; 12C led in vocabulary and fundamental operations. On the daily papers, 12E again showed superiority by making 42 errors, as compared with 69 errors for 12C.

In Match 13, there was not enough difference in Ar.A., C.A., and I.Q. to explain any difference in the type of work done. However, 13E showed marked superiority in points of improvement on the standardized test, making 81 points, as compared with 32 points for 13C. A consistent lead was shown throughout the test. Again, the work of 13E was superior as shown by daily work. Pupil 13E made 32 errors, consisting of 18 in computation, 4 in the use of decimals, 6 in meaning of per cent, and 4 in methods used. Pupil 13C made 79 errors, consisting of 22 in computation, 19 in use of decimals, 18 in per cent, and 18 in method, with 2 problems omitted.

The only difference of any value in the classification of Match 14 was in C.A. Pupil 14E was one year and one

month older than 14C. The improvement as measured by the standardized tests was practically the same, 57.5 points for 14E and 66 points for 14C. There was a greater variation in the daily work of the two pupils. Pupil 14E made 31 errors, consisting of 16 in computation, 2 in use of decimals, 5 in meaning of per cent, and 8 in method. Pupil 14C made 72 errors, consisting of 24 in computation, 18 in use of decimals, 10 in meaning of per cent, and 15 in method, with 5 problems omitted.

The two pupils in Match 15 each had an Ar.A. of 10-1; pupil 15C was eight months older chronologically and 15E had an I.Q. 7 per cent higher than 15C. These differences were not great enough, however, to explain the superiority of the work done by 15E. This pupil made a gain of 89.5 points on the standardized test, the greatest gain made by any pupil in the group. Pupil 15C made only 24.5 points improvement. Pupil 15E also led in daily work, making only 16 errors, as compared with 83 errors for 15C.

Match 16 was composed of two pupils with low I.Q.'s. Pupil 16E had an I.Q. of 79, and 16C had an I.Q. of 87. There was a difference of 11 months in Ar.A. in favor of 16E. On the standardized test 16E made 36.5 points of improvement, and 16C made 68.5 points. The greatest difference came in quantitative relationships and vocabulary. In daily work, 16E made fewer errors than 16C. Although 16C

led by 32 points on the standardized test as a whole, Pupil 16E led in problem solving and fundamental operations, the only phases which corresponded to the problems given on the daily papers. Pupil 16E made 47 errors, consisting of 16 in computation, 10 in use of decimals, 10 in meaning of per cent, and 11 in methods. Pupil 16C made 70 errors, consisting of 24 in computation, 17 in decimals, 14 in per cent, and 13 in method, with 2 problems omitted.

In Match 17 the pupils ranked about the same in Ar.A., C.A., and I.Q. The work of the two pupils showed a superiority throughout the study for 17E. On the standardized test 17E made 62 points of improvement, and 17C made only 27 points. A large part of this gain was made in quantitative relationships. Pupil 17E made 43 errors, consisting of 14 in computation, 8 in use of decimals, 9 in meaning of per cent, and 11 in method, with 1 problem omitted. Pupil 17C made 92 errors, consisting of 19 in computation, 34 in use of decimals, 13 in meaning of per cent, and 26 in method.

In Match 18, Pupil 18E had an Ar.A. of 9-5, as compared to 9-7 for 18C. Pupil 18C was six months older, chronologically, and 18E had an I.Q. 7 points higher. The differences were not in favor of either individual to the extent that their work varied. Pupil 18E made 68.5 points of improvement on the standardized test, as compared with

49 points for 18C. The greatest amount of superiority for 18E came in quantitative relationships and fundamental operations. Pupil 18E also showed better work on the daily papers, making a total of 36 errors, consisting of 17 in computation, 9 in decimals, 2 in per cent, and 4 in method used, with 4 problems omitted. Pupil 18C made 71 errors, consisting of 14 in computation, 16 in use of decimals, 22 in meaning of per cent, and 17 in method, with 2 problems omitted.

The two pupils in Match 19 were matched closely according to Ar.A., C.A., and I.Q. There was no great amount of difference in the work of the pupils, although the work of Pupil 19E ranked a little higher than that of 19C. On the standardized test, 19E made 84 points of improvement, and 19C made 77 points. The total number of errors on the daily papers was 41 for 19E and 57 for 19C. Of these, 19E made only 4 errors in use of decimals, as compared to 19 errors for 19C.

Match 20 was composed of the two pupils with the lowest I.Q.'s that were in the group, 81 for 20E and 78 for 20C. They also were lowest in Ar.A., each ranking 9-2. Pupil 20C had a C.A. of 14-9, eight months higher than 20E. On the standardized test Pupil 20E made 84 points of improvement, and 20C made 43 points. Pupil 20E also ranked higher on daily work, having a total of 38 errors, consisting of 6 in computation, 11 in decimals, 5 in per cent, and 16 in method.

Pupil 20C made 60 errors, consisting of 9 in computation, 16 in decimals, 13 in per cent, and 16 in method, with 6 problems omitted.

Group Comparison

In equating the groups for this study the individual pupils were matched according to Ar.A., C.A., M.A., and I.Q. The mean for the groups was checked in order to keep the E and C groups as evenly matched as possible. In the final classification of the pupils, the average difference was .15 month in Ar.A., 3.3 months in C.A., 1.6 months in M.A., and 1.3 points in I.Q. These differences, with the exception of I.Q., were in favor of the E group. Expressed as nearest months, the average Ar.A. was 10-11 for each group; C.A. was 12-7 for the E group and 12-4 for the C group; M.A. was 13-1 for the E group and 13-0 for the C group; I.Q. was 104 for the E group and 105.3 for the C group.

The improvement over the six months period as measured by the two standardized tests given showed a superiority in favor of the E group. The total improvement in C-Scores was 1137 for the E group, as compared with 879 for the C group. Although the E group led in all divisions of the test, this improvement was not uniform throughout. The greatest difference came in quantitative relationships, with

the E group more than doubling the C group. In fundamental operations the E group made 478.5 points, and the C group made 390.5 points. There was very little variation in the work of the two groups in the problem-solving division of the test; the E group made 245.5 points and the C group made 212 points. The least difference came in vocabulary, with a difference of only one point for the two groups.

On the twelve sets of daily papers studied, the E group again showed superior work. This group made a total of 554 errors on all the papers, as compared with 1011 errors for the C group. The difference here was greater than on the standardized test, due in part to the fact that the daily checks were made soon after the methods were presented whereby the pupils had experienced more drill and practice before taking the last standardized test. The superiority of the E group, as shown by the number of errors made, was greatest in the use of decimals. This group made a total of 112 errors, as compared with 261 errors for the C group. The least difference in the number of errors made came in computation of whole numbers. Of these errors, 40.7 per cent were made by the E group, and 59.3 per cent by the C group. In all the papers studied, the E group omitted 7 problems. The C group omitted 50 problems. Of the errors made, the E group made 302 errors on problems involving application of methods studied, as compared with 536 errors for the C group.

TABLE 2

C-SCORES OF EACH PUPIL IN ARITHMETIC TAKEN FROM ANALYTICAL SCALES OF ATTAINMENT, FORM A, SEPTEMBER 15, 1941

Pupil	Quantitative Relationships		Problems		Vocabulary		Fundamental Operations	
	E	C	E	C	E	C	E	C
1	86.5	79.5	84.5	88	85	85	61.5	80.5
2	83	81.5	84.5	84.5	88	82	61.5	67.5
3	84.5	86.5	81	79	94	100.5	58.5	53.5
4	75	77.5	77	81	92.5	92.5	61.5	58.5
5	77.5	67	69	81	78	88	61.5	69.5
6	79.5	79.5	82.5	69	75	76.5	67.5	58.5
7	77.5	77.5	81	77	44 -	71	67.5	61.5
8	75	75	81	79	85	78	53.5	53.5
9	71	81.5	79	69	88	82	58.5	58.5
10	71	83	81	69	75	76.5	58.5	61.5
11	51 -	51 -	69	82.5	76.5	83.5	67.5	67.5
12	62	77.5	72	72	75	64.5	58.5	43 -
13	75	67	75	69	60.5	82	43 -	53.5
14	62	67	66	66	73	60.5	53.5	43 -
15	81.5	62	69	66	44 -	60.5	53.5	43 -
16	67	51 -	72	66	75	55.5	58.5	53.5
17	51 -	51 -	75	75	68.5	76.5	43 -	43 -
18	51 -	51 -	69	72	73	44 -	43 -	61.5
19	67	67	79	79	44 -	44.5 -	43 -	43 -
20	51 -	51 -	66	61	44 -	44 -	53.5	58.5

TABLE 3

C-SCORES OF EACH PUPIL IN ARITHMETIC TAKEN FROM ANALYTICAL SCALES OF ATTAINMENT, FORM B, MARCH 11, 1941

Pupil	Quantitative Relationships		Problems		Vocabulary		Fundamental Operations	
	E	C	E	C	E	C	E	C
1	93	75	107.5	105	94	82	103.5	91.5
2	91.5	84.5	101	97	82	82	84	86
3	91.5	81.5	86	103	89.5	100.5	86	86
4	90	91.5	95	97	97.5	100.5	86	82.5
5	77.5	88.5	93.5	107.5	80	95.5	78.5	91.5
6	83	81.5	91.5	79	75	78	82.5	64.5
7	79.5	81.5	82.5	77	55.5	85	76.5	80.5
8	88.5	67	93.5	93.5	94	76.5	84	82.5
9	86.5	86.5	82.5	82.5	91	89.5	80.5	82.5
10	77.5	79.5	90	86	78	82	82.5	80.5
11	79.5	71	86	86	82	86.5	82.5	78.5
12	71	51 -	84	72	76.5	91	78.5	73.5
13	84.5	67	88	77	85	88	76.5	71.5
14	81.5	79.5	90	81	71	75	69.5	67.5
15	83	62	86	72	88	68.5	80.5	53.5
16	75	79.5	84	77	76.5	76.5	73.5	61.5
17	71	51 -	81	82.5	78	78	69.5	61.5
18	67	51 -	79	79	83.5	71	75	76.5
19	83	75	86	82.5	73	76.5	75	76.5
20	77.5	77.5	72	61	68.5	44 -	80.5	75

TABLE 4

THE IMPROVEMENT IN C-SCORES IN ARITHMETIC OVER A PERIOD OF SIX MONTHS AS MEASURED BY ANALYTICAL SCALES OF ATTAINMENT

Pupil	Quantitative Relationships		Problems		Vocabulary		Fundamental Operations		Total	
	E	C	E	C	E	C	E	C	E	C
1	6.5	-4.5	23	17	9	-3	42	11	80.5	20.5
2	8.5	3	15.5	12.5	-6	0	22.5	18.5	40.5	34
3	7	-5	5	24	-4.5	0	27.5	32.5	35	51.5
4	15	14	18	16	5	8	24.5	24	62.5	62
5	0	21.5	24.5	26.5	2	7.5	17	22	43.5	77.5
6	3.5	2	9	10	0	1.5	15	6	27.5	19.5
7	2	4	1.5	0	11.5	14	9	19	24	37
8	13.5	-8	12.5	14.5	9	-1.5	30.5	29	65.5	34
9	15.5	5	3.5	13.5	3	7.5	22	24	44	50
10	6.5	-3.5	9	17	3	5.5	24	19	42.5	38
11	28.5	20	17	3.5	5.5	3	15	11	66	37.5
12	9	-26.5	12	0	1.5	26.5	20	30.5	42.5	30.5
13	10	0	13	8	24	6	34	18	81	32
14	19.5	12.5	24	15	-2	14.5	16	24	57.5	66
15	1.5	0	17	6	44	8	27	10.5	89.5	24.5
16	8	28.5	12	11	1.5	21	15	8	36.5	68.5
17	20	0	6	7	9.5	1.5	26.5	18.5	62	27
18	16	0	10	7	10.5	27	32	15	68.5	49
19	16	8	7	3.5	29	32	32	33.5	84	77
20	26.5	26.5	6	0	24.5	0	27	16.5	84	43
Total	233	97.5	245.5	212	180	179	478.5	390.5	1137	879

TABLE 5

NATURE AND FREQUENCY OF ERRORS, IN ARITHMETIC, IN ADDITION OF DECIMALS (COMPUTATION)

Pupil	Misplaced decimal in sum		Copied number wrong		Failed to carry number in addition		Wrong sum in adding whole numbers		Wrote decimals wrong for adding		Total	
	E	C	E	C	E	C	E	C	E	C	E	C
1	1			1							1	1
2			1	1							1	1
3					3					5	3	5
4							1				1	0
5	1										1	0
6				1							0	1
7									1		1	0
8										2	0	2
9			1		1			2			2	2
10			2				1		1		4	0
11					1		2				3	0
12		1						2	1	1	1	4
13	1	2		1					2	4	3	7
14		1	1							6	1	7
15	1									1	1	1
16			1	1		2	1				2	3
17		2					1		1		2	2
18	1		1	1	2					1	4	2
19	1	1			2					1	3	2
20								1			0	1
Total	6	7	7	6	9	2	6	5	6	21	34	41

TABLE 6

NATURE AND FREQUENCY OF ERRORS, IN ARITHMETIC, IN SUBTRACTION OF DECIMALS (COMPUTATION)

Pupil	Copied number wrong		Wrong remainder in subtracting whole numbers		Did not keep decimals straight in subtracting		Omitted decimal in answer		Total	
	E	C	E	C	E	C	E	C	E	C
1	1								1	0
2		1	2	3					2	4
3						1			0	1
4									0	0
5			1		1				2	0
6				1	3	1			3	2
7									0	0
8	1		3			1		1	4	2
9		1	2						2	1
10				1					0	1
11		1							0	1
12			1	2		2			1	4
13		1	1		2	1			3	2
14						1			0	1
15				1		2			0	3
16	1			1	2	2			3	3
17				2		1			0	3
18			2			1			2	1
19	1		3	1				1	4	2
20									0	0
Total	4	4	15	12	8	13		2	27	31

TABLE 7

NATURE AND FREQUENCY OF ERRORS, IN ARITHMETIC, IN ADDITION AND SUBTRACTION OF DECIMALS (PROBLEM SOLVING)

Pupil	Wrong sum in adding whole numbers		Wrong remainder in subtracting whole numbers		Wrong product in multiplying whole numbers		Copied number wrong		Omitted number in problem		Misplaced decimal in writing dollars and cents		Misplaced decimal in answer		Omitted decimal in answer		Did not keep decimal straight in addition and subtraction		Omitted the problem		Method		Total	
	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C
1		1						1										1					0	1
2	1					1		1								2	1					1	1	3
3	1	1		1				1									1						2	6
4		1	1	2													1						2	3
5			2																		1		3	0
6			1	1							1	1						1				1	2	4
7							1			1	1												2	1
8				1								1										2	0	4
9				1																	1	1	1	2
10												1					1						1	1
11							1				1											2	2	2
12			1								1			1			1	1				1	3	3
13		1	2														2	3				1	4	5
14		1				1								1			1	3			2		3	6
15	1			2			1			1			1			1	1	3		1		1	1	7
16		1					1					1	1			1	1	1				4	3	8
17	1			3			1											1				1	2	5
18	1			1												1		1		1	2	3	3	7
19	1			1														1			2		3	2
20											1							1			3	3	4	4
Total	6	6	7	13	0	2	4	2	0	2	5	4	1	2	0	4	8	16	0	2	11	21	42	74

TABLE 8

NATURE AND FREQUENCY OF ERRORS, IN ARITHMETIC, IN MULTIPLICATION OF DECIMALS (COMPUTATION)

Pupil	Misplaced decimal in the product		Set product too many places to the left when multiplying by two numbers		Wrong sum in adding whole numbers		Failed to carry number in multiplying		Wrong product in multiplying whole numbers		Wrote two partial products on the same line		Omitted decimal in product		Copied number wrong		Total	
	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C
1																	0	0
2																1	0	1
3	2	1	1			1				2							3	4
4	1	1			2					1							3	2
5										2						1	0	3
6		1					1			4							1	5
7																	0	0
8		6															0	6
9		7					1			1							1	3
10				1		1			1								1	2
11									1								1	0
12		4								1							0	5
13		6					1			1							0	8
14	1	5			2	1			1	3						1	4	10
15		3							1	3							1	6
16						1				1							0	2
17	2					1										1	2	2
18		4							3		4						7	4
19										2							0	2
20		1				1							1				1	2
Total	6	39	1	1	4	6	2	1	7	21	4	0	1	0	0	4	25	72

TABLE 9

NATURE AND FREQUENCY OF ERRORS, IN ARITHMETIC, IN DIVISION OF DECIMALS (COMPUTATION)

Pupil	Omitted decimal in quotient		Annexed zeros to the dividend without placing decimal before		Misplaced decimal in the quotient		Wrong quotient in dividing whole numbers		Omitted zero in quotient		Placed two decimals in quotient		Total	
	E	C	E	C	E	C	E	C	E	C	E	C	E	C
1													0	0
2		1		1									0	2
3	1							1					1	1
4	1		1										2	0
5	1					1							1	1
6				1		1							0	2
7	1		1										2	0
8					1	6							1	6
9						1							0	1
10	1		1			1		1					2	2
11						1	1						1	1
12		1			1		1	1					2	2
13						1							0	1
14						3							0	3
15		1		1		4				1			0	7
16		1		1	2	1	2						4	3
17		1			1	4	1	1				1	2	7
18				1	1	2	1						2	3
19	1					7	2		1				4	7
20	1				1	1	1						3	1
Total	7	5	3	5	7	34	9	4	1	1	0	1	27	50

TABLE 10

NATURE AND FREQUENCY OF ERRORS, IN ARITHMETIC, MULTIPLICATION AND DIVISION OF DECIMALS (PROBLEM SOLVING)

Pupil	Wrong sum in adding whole numbers		Wrong remainder in subtracting whole numbers		Wrong product in multiplying whole numbers		Wrong quotient in dividing whole numbers		Misplaced decimal in the product		Misplaced decimal in the quotient		Wrote mills wrong when expressing them as decimals		Misplaced decimal when annexing zeros to dividend		Misplaced numbers in the quotient		Method		Copied number wrong		Omitted the problem		Total	
	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C
1						2														1					0	3
2								1		3															0	4
3						1	1	1		1			1	1											2	4
4						1			1				1	1					1						3	2
5					2	1		1		1	1														3	3
6					1	1				1			1						1	1	1				4	3
7									1				1						1	1					3	1
8					1			1		1				1					1	1				1	2	5
9							1	1		2				1					1						2	4
10			1		1	1			1	2	1		2						1	1					7	4
11					1								1			1			3	2	1	1		1	6	5
12						1		1		1	1		1	1					1	1					3	5
13				1	1		1	1		1		1	2	1		1				2					4	6
14				1						3			1	1					2	2					3	7
15					1	1		2		2				2				1		1				1	1	10
16	1	2		1					2	2		2		2					2					1	5	9
17						1	1	2	1	1		1	1						1	1		1			4	7
18										2			1	2						1		1	1		2	6
19					1			1		2		1	1	3					2	2		1			4	10
20									1	1	2			1					4	1	1			3	8	6
Total	1	2	1	3	9	10	4	12	7	26	5	5	14	17	0	2	0	1	21	18	3	4	1	6	66	106

TABLE 11

NATURE AND FREQUENCY OF ERRORS, IN ARITHMETIC, IN FINDING A PER CENT OF A NUMBER (COMPUTATION)

Pupil	Wrong sum in adding whole numbers		Wrong product in multiplying whole numbers		Wrong quotient in dividing whole numbers		Wrong fraction when writing % as decimal fraction		Wrong fraction when writing % as a common fraction		Wrong product when multiplying fraction by whole no.		Wrong product when multiplying mixed no. by whole no.		Misplaced decimal in product		Omitted decimal in the product		Method		Omitted the problem		Copied number wrong		Total	
	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C
1															1										1	0
2			2					1																	2	1
3			1	2				2							1									1	2	5
4				2			1															1			1	3
5			1			1	1	1							1	1									3	3
6				4				4						1	1	1			1						2	10
7				1			1	1																	1	2
8			1	1				2								3									1	6
9							1													5		1	1		2	6
10				1			1			1					2										3	2
11							2									1									2	1
12			1	1			3	3	1						1	1				1		1			6	7
13			2				1	7	1		2					1									6	8
14			2	2			1	1						1		1									3	5
15				4			1	3								1						1			1	9
16		1	1	1			2	1							1										4	3
17	1							6			1				1	2		5			1				4	13
18		1		1			1									1							1		2	3
19			1	1			1																		2	1
20	1			1			2									1						1			3	3
Total	2	2	12	22	0	1	19	32	2	1	3	0	0	2	9	14	0	5	1	6	1	5	2	1	51	91

TABLE 12

NATURE AND FREQUENCY OF ERRORS, IN ARITHMETIC, IN FINDING A PER CENT OF A NUMBER (PROBLEM SOLVING)

Pupil	Wrong sum in adding whole numbers		Wrong product in multiplying whole numbers		Wrong quotient in dividing whole numbers		Wrong fraction in writing % as a decimal fraction		Wrong fraction in writing % as a common fraction		Wrong answer in reducing fraction to lowest terms		Wrong product in multiplying whole no. by mixed no.		Wrong product in multiplying three fractions		Misplaced decimal in the product		Copied number wrong		Method		Omitted the problem		Total	
	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C
1												1													0	1
2							1									1				1	1				2	2
3	1	1	1				1	1				1		1		1					2	1			5	6
4															1							2		2	1	4
5				1							1						1				2				4	1
6			1	1				1		1											1	1	1	1	3	5
7						1	2	1		1							1				2				5	3
8																						2			0	2
9																					2			1	2	1
10		1					2	1	1												2	1	1		5	4
11				2			1	1				1				1					5	2			6	7
12							1						1		1	1		2			3	2		3	6	8
13			1				2	1													1	2		1	4	4
14	1						1	1			2							1			1	2		2	5	6
15				1			1														2			7	3	8
16			1	1			1									1		1			2	2		1	4	6
17							1														5	8			6	8
18		1	1					1						1								3	3		4	6
19		1				1									1						4	3		1	5	6
20			1				1	1		2								2		1	3	3		1	5	10
Total	2	4	6	6	0	2	15	9	1	4	3	3	1	2	3	5	2	6	0	2	38	34	4	21	75	98

TABLE 13

NATURE AND FREQUENCY OF ERRORS, IN ARITHMETIC, IN FINDING WHAT PER CENT ONE NUMBER IS OF ANOTHER (COMPUTATION)

Pupil	Wrong remainder in subtracting whole numbers		Wrong quotient in dividing whole numbers		Wrong quotient in dividing fraction by a fraction		Wrong answer in reducing fraction to lowest terms		Changed common fraction to wrong per cent		Changed decimal fraction to wrong per cent		Changed whole number to wrong per cent		Misplaced decimal in the quotient		Copied number wrong		Method		Omitted the problem		Total	
	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C
1																							0	0
2								1	2			2											2	3
3								1				1		1									0	3
4							1																1	0
5																		1			1		1	1
6												6		1					6				0	13
7									1			4		1					4	6			5	11
8						1								1									0	2
9					1		1													2			2	2
10												1	2	1			1			3			3	5
11					1	1								6	1	4			2				4	11
12									1			6	1	1						4			2	11
13								1				4	1				1	1	1	3			3	9
14			1									1		1						3			1	5
15													1	3						7			1	10
16			2						1				1	3	1	1		1					5	5
17					1					1		1	1	3	2			1		4			4	10
18						1				1		3	1	4	2					3			3	12
19	1												1	3	1	1				7			3	11
20										1	1	2		2		1							1	6
Total	1	0	3	0	3	3	2	3	5	3	1	31	9	31	7	7	2	4	7	48	1	0	41	130

TABLE 14

NATURE AND FREQUENCY OF ERRORS, IN ARITHMETIC, FINDING WHAT PER CENT ONE NUMBER IS OF ANOTHER (PROBLEM SOLVING)

Pupil	Wrong remainder in subtracting whole numbers		Wrong product in multiplying whole numbers		Wrong quotient in dividing whole numbers		Wrong answer in reducing fraction to lowest terms		Changed common fraction to wrong per cent		Changed decimal fraction to wrong per cent		Changed whole number to wrong per cent		Misplaced decimal in the quotient		Omitted decimal in the quotient		Method		Copied number wrong		Omitted the problem		Total	
	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C
1												1				1			1	1		1			1	4
2		1				1										1			1	2					1	4
3																			1	3		1			1	4
4						1													2	3					2	4
5																			2	3					2	3
6						1						1		5		2			3	8					3	17
7						1	1					2		1		1			3	5				1	4	11
8																			1	2					1	2
9		1							1							1			2	2		1		1	3	6
10																			2	3					2	3
11			1			1		1		1			2	1				1	3	1					7	6
12											4	1	2						7	5		1			13	7
13											1			2		1		2	2	3					3	8
14	1			1								4							2	5				1	3	11
15						1					2			1					2	3				2	4	7
16				1		1		1			1			1				1	3	4	1				5	9
17						1	1					1		6		3		4	4	6					5	21
18					1							2		1			1	1	2	4					4	8
19	1						1				1	1	2	1		1		1	4	4		1			9	9
20										1		4	1			1	2		4	6	1				8	12
Total	2	2	1	2	1	8	3	2	1	2	9	17	7	19	1	11	3	10	51	73	2	5	0	5	81	156

TABLE 15

NATURE AND FREQUENCY OF ERRORS, IN ARITHMETIC, IN FINDING A NUMBER WHEN A PER CENT OF IT IS KNOWN (COMPUTATION)

Pupil	Wrong product in multiplying whole numbers		Wrong quotient in dividing whole numbers		Wrong quotient in dividing whole numbers by fractions		Wrong fraction in writing % as a common fraction		Wrong fraction in writing % as a decimal fraction		Misplaced decimal in quotient		Annexed zeros to the dividend without placing decimal before		Method		Copied wrong number		Omitted the problem		Total	
	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C
1																					0	0
2																					0	0
3				1			1			1					1						2	2
4											1										1	0
5									3												3	0
6				1					1	2	1	1		2		1					2	7
7		1									2										2	1
8												1						1			0	2
9							1				2		1					1			4	1
10			1		1																2	0
11			1		1				3						1		1				7	0
12				2					2	1								1			2	4
13						3			1	1						3					1	7
14			1		1					1		1						1			2	3
15				1					2	1					1					3	3	5
16						1			2	6		3			1	1					3	11
17		1			2		3			1					1						6	2
18				1				1		6	2	1									2	9
19						1			1	1				1	1						2	3
20			1			1			2			3									3	4
Total	0	2	4	6	5	6	5	1	17	21	8	10	1	3	6	5	1	4	0	3	47	61

TABLE 16

NATURE AND FREQUENCY OF ERRORS, IN ARITHMETIC, IN FINDING A NUMBER WHEN A PER CENT OF IT IS KNOWN (PROBLEM SOLVING)

Pupil	Wrong product in multiplying whole numbers		Wrong quotient in dividing whole numbers		Wrong product in multiplying a fraction by a fraction		Wrong quotient in dividing a fraction by a fraction		Wrong fraction when writing % as a common fraction		Wrong fraction when writing % as a decimal fraction		Misplaced decimal in the product		Misplaced decimal in the quotient		Method		Copied number wrong		Omitted the problem		Total	
	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C	E	C
1																							0	0
2																							0	0
3																		1					0	1
4																							0	0
5															2								2	0
6												1						5					0	6
7											1												1	0
8																		2					0	2
9				1					1					1	1	1		3		1		2	2	9
10	1																		1				2	0
11												1							1				1	1
12			1									2		1			2	3				3	3	9
13			1	1		1						3				2		4				1	1	12
14	1		1	1							3	1		1			1	3				2	6	8
15												3		2		1		2				2	0	10
16			1	1				1			2	3			3		3	2				1	9	8
17				1			2	1	4					4				6					6	12
18		1		1								3		1	1			3				1	1	10
19										1	1								1	1			2	2
20		1		2										1		3	2	3				1	2	11
Total	2	2	4	8	0	1	2	2	5	1	7	17	0	11	7	7	8	37	3	2	0	13	38	101

CHAPTER IV

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

A careful summary of the data collected from this comparative study of arithmetic methods in the seventh grade of La Vega School gave the following conclusive findings:

1. In 70 per cent of the matches, the pupil in the experimental group showed a superiority over the corresponding pupil in the control group.

2. The greatest amount of superiority in favor of the experimental group came in quantitative relationships; the least difference was in vocabulary, where the two groups differed by only one point.

3. The matches composed of pupils whose average I.Q. was above 100 showed practically the same per cent of superiority for the experimental group as did the matches composed of pupils whose average I.Q. was below 100.

4. The match composed of pupils with the highest average I.Q. ($132\frac{1}{2}$) showed the least difference in work; the match composed of the pupils with the lowest average I.Q. ($79\frac{1}{2}$) showed a marked superiority for the pupil in the experimental group.

5. In 85 per cent of the matches, the pupil in the experimental group made fewer errors on the daily work than his match in the control group.

6. Of errors tabulated, the greatest amount of superiority for the experimental group was shown in problems involving the use of decimals.

7. The experimental group made fewer errors on problems involving applications of methods studied.

Recommendations

On the basis of this study the following recommendations are made:

1. In the teaching of arithmetic in the seventh grade, and perhaps in all intermediate grades, it is better to let the pupils discover new methods rather than merely to accept them as presented by the teacher. This develops skill with understanding. This method is better for the low I.Q.'s as well as for the high I.Q.'s.

2. In the area of quantitative thinking it is better to let the pupils discover new steps and formulate their own rules of procedure.

3. Drill is valuable after the process has been developed and is understood. Drill was used in both groups, but, in the experimental group, it was not used until the meanings were understood.

4. In several matches the pupil in the control group was favored, although his match in the experimental group surpassed him in work. This was probably due to the fact that meanings were made clearer in the experimental group.

APPENDIX

PROBLEMS GIVEN FOR DAILY WORK

I. Addition of Decimals (November 3, 1941)

Write in columns and add:

(1)	.6	.4	.3
(2)	9.4	8.2	.6
(3)	.06	.2	.103
(4)	3.09	.8	.007
(5)	.6	.09	126.1
(6)	.52	3.6	.8
(7)	300.002	62.8	.07
(8)	6.28	.081	12

II. Subtraction of Decimals (November 5, 1941)

Write smaller number under larger number and subtract:

(1)	.6	.3
(2)	18.28	16.15
(3)	.05	.065
(4)	600.12	14.20
(5)	.012	3.1
(6)	1.24	1.246
(7)	12	.54
(8)	7.4	8.162

III. Addition and Subtraction of Decimals -- Problem Solving (November 13, 1941)

(1) A surveyor measured a piece of valuable property in a city and found it to be 48.74 feet wide and 87.4 feet deep. How much greater than its width is its depth?

(2) What is the distance around a small rectangular piece of land 47.25 feet long and 68.7 feet wide?

(3) In a recent year Denmark produced on the average 116. pounds of butter for each person. In the United States 17.7 pounds of butter were produced for each person. Find the difference.

(4) A large wholesale bakery reported that the total cost of making a loaf of bread weighing 1 pound was 6.56 cents. The cost of the materials alone was 3.49 cents. How much was the difference between the cost of materials and the total cost of the bread?

(5) In the United States each person consumes on the average 1.3 pints of milk each day. How many pints did he consume in a week?

(6) Mary sold 6.2 lb. butter on Monday, 1.48 lb. on Tues., 3.04 lb. on Wed., 7 lb. on Thurs., and 4.5 lb. on Fri. How much butter did she sell?

(7) Mr. Mauske bought a steer weighing 1025.75 lbs. and kept it 84 days. When he sold the steer it weighed 1274 lbs. What was the gain during the period of 84 days?

(8) The average monthly rainfall of Porto Rico for the first four months of the year was: Jan., 3.75 in., Feb., 1.3 in.; Mar., 2.12 in.; Apr., 2.8 in. What was the total rainfall for the four months?

IV. Multiplication of Decimals (November 18, 1941)

Multiply:

(1)	(2)	(3)	(4)
.8	7.2	.09	72
.4	.6	.08	.04
(5)	**(6)**	**(7)**	**(8)**
.68	7.5	.84	95
.8	2.4	.76	1.7

V. Division of Decimals (December 5, 1941)

Divide:

(1)	(2)	(3)	(4)
$7 \overline{)\ .49}$	$.3 \overline{)\ .6}$	$.12 \overline{)\ .24}$	$4 \overline{)\ 2}$
(5)	**(6)**	**(7)**	**(8)**
$.4 \overline{)\ .8}$	$.02 \overline{)\ .004}$	$.8 \overline{)\ .344}$	$.006 \overline{)\ 1.2}$

VI. Multiplication and Division of Decimals -- Problem Solving (December 9, 1941)

(1) What is the perimeter of a square 84.84 feet on one side?

(2) A train travelled 87.5 miles in 2.5 hours. What was the speed per hour?

(3) One person consumes 1.3 pints of milk per day. At 7¢ per pint, what would the milk cost per day for a family of 6?

(4) James estimated the cost of travel in his car amounted to 2.5¢ per mile. At this rate, what would it cost him to travel 80 miles?

(5) The seventh-grade girls took in $47.50 from the sale of tickets to their play. They sold the tickets for 25¢ each. How many tickets did they sell?

(6) How many badges, each 2.4 inches long, can be made from 36 inches of ribbon?

(7) Jack harvested 21.3 bu. of corn from an acre of fertilized ground and 14.2 bu. per acre from an unfertilized field. He got ___ times as many bu. from the fertilized as from the unfertilized.

(8) Mr. Brown harvested 42.8 bu. of grain per acre. What was the total yield from 34.5 acres?

VII. Finding the Per Cent of a Number (January 12, 1942)

(1) 62% of 87 =
(2) 5% of 304 =
(3) 125% of 162 =
(4) 37½% of 48 =
(5) 59.2% of 178 =
(6) 400% of 75 =
(7) 4.8% of 600 =
(8) ½% of 62 =

VIII. Finding the Per Cent of a Number -- Problem Solving (February 11, 1942)

(1) In a school term of 180 days, Henry lost 5% on account of illness. How many days did he lose?

(2) James had $5.00 and spent 12% of it. How much did he spend?

(3) A dealer bought 12 dozen pencils and sold 62½% of them. How many did he sell?

(4) A grocer bought eggs for $.30 a dozen and sold them for 20% more than he paid for them. How much did he receive a dozen?

(5) Find the interest on $200 for 48 days at 6%.

(6) There are 40 pupils enrolled in Miss Smith's class. If 92.5% of the pupils were present, how many were present?

(7) Mr. Smith paid a collection agency 3/4% for collecting bills. How much did he pay for the collection of a bill of $100?

(8) Last year Helen weighed 96 lbs. Her weight is now 115% of her weight of last year. How much does she now weigh?

IX. Finding What Per Cent One Number Is of Another (February 17, 1942)

(1) 5 is ___% of 40
(2) 250 is ___% of 200
(3) 12 is ___% of 48
(4) 125 is ___% of 125
(5) 7.8 is ___% of 30
(6) 7½ is ___% of 22½
(7) 3 is ___% of 100
(8) 20 is ___% of 5

X. Finding What Per Cent One Number Is of Another (February 20, 1942)

(1) Jane missed 4 out of 20 words given on a spelling test. What per cent did she miss?

(2) The regular price of butter was 48¢. The sale price was 44¢. The saving was ___% of the regular price.

(3) Mr. Brown borrowed $200 at a bank. The interest for a year amounted to $10. What was the rate of interest?

(4) A school baseball team won 16 out of 20 games played. What per cent of the games played did the team win?

(5) Ann walks 8 blocks to town. What per cent of the distance has she covered when she has walked 3 blocks?

(6) Harry's kite string was 60 feet long. He lost 20 feet. What per cent of the string remained?

(7) In a schoolroom of 40 pupils, 4 pupils were absent. What per cent was absent?

(8) A farmer bought a cow for $48 and sold her for $60. What per cent did he make?

XI. Finding a Number When the Per Cent of It Is Known (March 2, 1942)

(1) 75 is 15% of ___
(2) 18 is 66 2/3 of ___
(3) 55 is 125% of ___
(4) 64 is 8% of ___
(5) 1124 is 50% of ___
(6) $10 is 40% of ___
(7) $15 is 4% of ___
(8) 45 is 100% of ___

XII. Finding a Number When the Per Cent of It Is Known -- Problem Solving (March 6, 1942)

(1) Thomas gave away 10 marbles. If this was 33 1/3% of all the marbles he had, how many marbles did he have?

(2) If 72 pages are 25% of the number of pages in a book, how many pages does the book contain?

(3) James spent $.60. This was 12% of his money. How much money did he have?

(4) In a spelling test Alice spelled 80% of the words correctly. If she spelled 40 words correctly, how many words were there on the test?

(5) Jim weighs 144 pounds. If this was 120% of his normal weight, what was his normal weight?

(6) Mr. Brown paid $15 interest one year on money borrowed at 4% rate of interest. How much money did Mr. Brown borrow?

(7) Mr. Sims saved $10 by paying his gas bill by a certain date. This amount was 5% of the total bill. What was the amount of the bill?

(8) Our boys won 75% of all the ball games played this year. If they won 12 games, how many did they play?

ANALYTICAL SCALES of ATTAINMENT

Arithmetic

Developed by
Martha Kellogg
L. J. Brueckner
M. J. Van Wagenen

Division 3 **Grades VII-VIII** **Form B**

NAME ..... BOY or GIRL.....
Last First Middle

CITY ..... GRADE.....

DATE 19..... .... SCHOOL .....
Year Month Day

DATE of BIRTH 19..... .... TEACHER .....
Year Month Day

AGE .... C-SCORES ....
Years Months Quan. Rel. Probs. Vocab. Fund. Ops.

Published by
EDUCATIONAL TEST BUREAU, Inc.
Minneapolis - Nashville - Philadelphia

QUANTITATIVE RELATIONSHIPS

Directions: Read these two sentences carefully.

A. Milk is sold by the 1. liter 2. ounce 3. quart 4. gram 5. peck - - - - A. ...3...

B. The man who keeps a grocery store sells 1. dishes 2. books 3. stoves 4. sugar 5. overcoats - - - - - - - - - - - - - - B. .....

You see that there are five possible answers in each sentence. Only one answer is right. In the first sentence the right answer is **quart**, so a line is drawn under **quart**, and the number in front of it, 3, is put at the end of the line.

Now look at the second sentence above and listen to the next directions.

In each of the following sentences you are to find the right answer, draw a line under it and then put the number that is in front of it at the end of the line, just as in the samples above.

---

1. Which of the following does not equal 28? 1. two tens and eight ones 2. XXVIII 3. a dime, a nickel, and eight pennies 4. three tens less one two 5. four sixes and one four - 1. .....

2. It takes the least time 1. to run a mile 2. to walk a mile 3. to skate a mile 4. to ride a bicycle a mile 5. to swim a mile - - - - - - - - - - 2. .....

3. Which of these can go at the greatest speed? 1. an airplane 2. a speed boat 3. a railroad passenger train 4. a race horse 5. a racing automobile - - - - - - 3. .....

4. The contents of cans of fruits is usually given on the label 1. as quarts 2. as pints 3. as cubic inches 4. as part of a gallon 5. as pounds and ounces - - - - - - 4. .....

5. In December in the United States 1. the day is longer than the night 2. the night is longer than the day 3. the length of night and day is the same 4. daylight is the longest of the year 5. the night is the shortest of the year - - - - - 5. .....

6. The longer a person travels 1. the slower he travels 2. the faster he travels 3. the farther he travels 4. the better he travels 5. the nearer he is to home - - - - - 6. .....

7. Addition is to sum as subtraction is to 1. quotient 2. product 3. addend 4. arithmetic 5. difference - - - - - - - - - - - - - - - 7. .....

8. We need to measure most accurately the volume of 1. a coal bin 2. the earth 3. an apple 4. gold in a coin 5. a ball - - - - - - - - - - - - - 8. .....

9. The average farm is likely to be largest in 1. France 2. Texas 3. New England 4. Ireland 5. Japan - - - - - - - - - - - - - - 9. .....

10. Which of the following numbers has the greatest value? 1. 1,000,000 2. $\frac{1467}{5964}$ 3. .3764 4. 1486.593 5. .00023 - - - - - - - - - - - - - - - - - 10. .....

11. The most accurate measurements of distance may be found on a map drawn to the scale of 1. fifty miles to an inch 2. one mile to an inch 3. hundred miles to a half inch 4. five miles to an inch 5. thousand miles to four inches - - - - - - - - - - - - - 11. .....

12. If the height and width of a box are increased 1. its surface will be less 2. its volume will be less 3. its volume will be doubled 4. its volume will be the same 5. its surface will be greater - - - - - - - - - - - - - - 12. .....

13. A sandglass is sufficiently accurate to measure the time required 1. to run 100 yards 2. to work 25 examples 3. to go to New York 4. to boil an egg 5. to plow a field - - - - 13. .....

14. Sales taxes would be easiest to administer on sales of 1. gasoline 2. milk 3. butter 4. sugar 5. fruit - - - - - - - - - - - - - - - 14. .....

15. Prices of an article are likely to be highest when 1. few people have the article to sell 2. many people have the article to sell 3. only one firm has the article to sell 4. the article is sold by a peddler 5. when it is out of style - - - - - - - - - 15. .....

16. The shortest distance between two points is 1. a broken line 2. a curved line 3. any line drawn through the points 4. a straight line 5. a dotted line - - - - - - 16. .....

17. The surface with unknown dimensions whose area can be most quickly determined is a 1. triangle 2. square 3. rectangle 4. parallelogram 5. five-sided figure - - - - - - 17. .....

18. Which of these is usually drawn to a scale larger than the original? 1. a house plan 2. a map 3. a tree 4. a deer 5. a housefly - - - - - - - - - - - - 18. .....

19. The opposite edges of a sheet of writing paper 1. are parallel to each other 2. are perpendicular to each other 3. intersect 4. form a right angle with each other 5. form a trapezoid 19. .....

20. Factories that double the amount they produce 1. increase the cost of producing an article 2. are likely to raise prices 3. reduce the cost of producing an article 4. are likely to cut prices in two 5. are likely to double prices - - - - - - - - 20. .....

(Continue on next page)

21. The assessed value of a piece of property is usualy 1. more than its true value
2. about double its true value 3. equal to its true value 4. equal to about 10 per cent of its true value
5. less than its true value - - - - - - - - - - - - - - - 21. .....

22. We can sell most conveniently by the square foot 1. cloth 2. screening for windows
3. land 4. houses 5. wallpaper - - - - - - - - - - - - - - 22. .....

23. Hundred percent of a number equals 1. ten times the number 2. the number
3. half the number 4. hundred times the number 5. one tenth of the number - - - - - - 23. .....

24. Premiums on automobile insurance against accident and liability are usually lowest in
1. small towns 2. New York 3. large cities 4. rural communities 5. small cities - - - - - 24. .....

25. The least costly form of insurance for a young person to carry is 1. twenty-year limited life
2. twenty-year endowment 3. ordinary life 4. thirty-year endowment 5. endowment at age 65 - - 25. .....

26. Rate of interest is usually lowest on 1. farm mortgages 2. checking accounts
3. mortgages on big office buildings 4. savings accounts 5. mortgages on homes - - - - 26. .....

27. Through the center of a circle one can draw 1. any number of diameters
2. not more than three diameters 3. not more than six diameters 4. not more than twenty-five diameters
5. only one diameter - - - - - - - - - - - - - - - - 27. .....

28. The most satisfactory way to sell bananas is 1. by the dozen 2. according to their size
3. by the quart 4. by the bushel 5. by the pound - - - - - - - - - 28. .....

29. Triangles are used so often in designs of bridges and buildings, because
1. their shape is easily changed 2. they are beautiful 3. they vary in size
4. they do not break down under pressure 5. they contain right angles - - - - - - - 29. .....

30. The volume of gasoline weighing 200 pounds is 1. less in cold weather than in warm weather
2. more in cold weather than in warm weather 3. the same regardless of the temperature
4. the same in summer as in winter 5. equal to 200 gallons - - - - - - - - 30. .....

31. The area of a triangle having the same base and altitude as a rectangle
1. is equal to half of the area of the rectangle 2. is the same as the area of the rectangle
3. is equal to the product of the base and altitude 4. is equal to twice the area of the rectangle
5. is equal to one fourth of the area of the rectangle - - - - - - - - - - - 31. .....

32. In comparison with a refrigerator operated by ice, the temperature in an electric refrigerator 1. will vary more 2. is usually higher 3. will fluctuate more 4. can be easily regulated
5. will be more nearly constant - - - - - - - - - - - - - - - 32. .....

33. Which statement below is incorrect? Two triangles are congruent if
1. their three sides are equal 2. they have the same base and altitude
3. two sides and their included angles are equal 4. two angles and their included sides are equal
5. their three angles and sides are equal - - - - - - - - - - - 33. .....

34. Four is to nine as eight is to 1. 12 2. 72 3. 32 4. 18 5. 21 - - - - - - 34. .....

35. If a five pound shot and a one pound shot are dropped from a 50 foot tower
1. the five pound shot will fall five times as fast as the one pound shot
2. the one pound shot will fall five times as fast as the five pound shot
3. they will fall at the same rate 4. the one pound shot will take five times as long as the five pound
5. the five pound shot will take 50 seconds to reach the ground - - - - - - - - 35. .....

36. What percent of 40 is 50? 1. 125% of 40 2. 100% of 40 3. 80% of 40 4. 200% of 40 5. 40% of 40 36. .....

37. To find the cost of fencing a field, one would have to find 1. the area of the field
2. the value of the field 3. the length of one side 4. the difference between the length and width
5. the perimeter of the field - - - - - - - - - - - - - - 37. .....

38. Fire insurance premiums are likely to be lowest on a house 1. in the country
2. in the residence section of a city 3. next to an oil reservoir 4. on the outskirts of a city
5. near a gunpowder factory - - - - - - - - - - - - - - 38. .....

39. To describe precisely we need 1. a ruler 2. a dictionary 3. to measure weight
4. standard units of measurement 5. synonyms - - - - - - - - - - 39. .....

40. Specific duties are based on 1. value of goods 2. the country from which they come
3. the quantity in the shipment 4. the distance shipped 5. the size - - - - - - - - 40. .....

Number right..... .....

ARITHMETIC —PROBLEMS

Directions: Do each example as you come to it.

Do All Figuring in Blank Spaces Below
Write the answers at the right side of this page.

Answers

1. How long will it take an automobile to travel 225 miles at a rate of 25 miles an hour? *1.* .....

2. How many hours is it from 9 A. M. to 2 P. M.? *2.* .....

3. If ¼ yard of ribbon costs 10c, what is the price of a yard? *3.* .....

4. Helen bought a roast for $2.47 and a steak for $1.43. What change should she receive from a ten-dollar bill? *4.* .....

5. A boy rode 12.5 miles on his bicycle on Monday, .6 of a mile on Tuesday, and 15.0 miles on Wednesday. How far did he ride in the three days? *5.* .....

6. At 18c for ¼ pound, how much does ½ pound cost? *6.* .....

7. At 2 for 5c find the cost of 8 cakes. *7.* .....

8. How many sixths are there in 1/3? *8.* .....

9. A grocer bought 80 bushels of potatoes at $1.75 a bushel and sold them at $2.25 a bushel. What was his gross profit on the 80 bushels? *9.* .....

10. Harry worked at the drug store 1¾ hours on Wednesday, and 2½ hours on Saturday. He was paid $.40 an hour. How much did he earn for his work on the two days? *10.* .....

11. Find the area of a farm 1.4 miles long and 1.5 miles wide. *11.* .....

12. A government report states that in a recent flood the water rose 16½ inches in 3 hours. Find the average rise an hour. *12.* .....

(Continue on next page)

Answers

13. Find the reduction on a piece of cloth, regularly priced at $8, marked 25% off. *13.* .....

14. In a seventh grade class 60% of the pupils were boys. What percent were girls? *14.* .....

15. Donald needs 9 boards, each 2¼ feet long, for the floor and sides of a cage. How many of these can he cut from a 9 foot board? *15.* .....

16. How many quarter pound boxes will 5½ pounds of candy fill? *16.* .....

17. Coal selling at $16 a ton in the summer months, sells for $17.50 in the fall. How much is saved on a winter's supply of 10 tons by buying it in the summer, without considering the interest on the money used? *17.* .....

18. Alice had 4¼ yards of cloth in one piece. From this she cut off a piece ¾ of a yard long and another piece 1 ⅞ yards long. What was the length of the piece that remained? *18.* .....

19. Half of what number equals 3 x 2? *19.* .....

20. A passenger train travels 300 miles in 8 hours. At the same average rate an hour, how many miles can it travel in 12 hours? *20.* .....

21. How many degrees is it from 15 degrees below zero to 40 degrees above zero? *21.* .....

22. If 17 acres of land produce 442 bushels of grain, what is the average value of the yield an acre at $1.40 a bushel? *22.* .....

23. At 40 cents a pound for candy and 5 cents each for boxes, what is the cost of 6 half-pound boxes of candy? *23.* .....

24. A grocer bought a box of canned peaches containing 24 cans for $5.76 and sold them at $.32 a can. How much more than he paid did he receive a can? *24.* .....

25. If 1% of a number is 4, what is the number? *25.* .....

Number right.....

End of Arithmetic—Problems. Look over your work.

ARITHMETIC VOCABULARY

Directions: Look at the first line of words carefully.

A. **simple** 1. hard 2. funny 3. easy 4. busy 5. tiny - - - - - - - - A....3...

B. **guard** 1. stop 2. watch over 3. hit 4. run away 5. climb - - - - - B.....

You see there are five words after the word **simple.** One of them means the same as **simple.** Which one is it? Yes, easy, so easy has a line drawn under it and the number that is in front of it, 3, is put in the space at the end of the line.

Now look at the second sample and listen to the next directions.

In each of the following sentences you are to find the right answer, draw a line under it and then put the number that is in front of it at the end of the line, just as in the samples above.

1. **shallow** 1. hollow 2. full 3. empty 4. not deep 5. very high - - - - - - 1. .....

2. **expense** 1. cost 2. offer 3. selling price 4. profit 5. office - - - - - 2. .....

3. **mortgage** 1. endorsed chack 2. stock 3. insurance policy 4. pledge of property 5. bank draft - - - - - - - - - - - - - - 3. .....

4. **second hand** 1. new 2. clean 3. used 4. handy 5. neat - - - - - 4. .....

5. **property** 1. losses 2. debts 3. deeds 4. payments 5. possessions - - - - 5. .....

6. **lease** 1. buy 2. rent 3. sell 4. own 5. build - - - - - - - - 6. .....

7. **gradual** 1. sharp 2. steep 3. slow 4. dull 5. quick - - - - - - - 7. .....

8. **bid** 1. offer 2. mail 3. tell 4. buy 5. return - - - - - - - - 8. .....

9. **penalty** 1. figure 2. sale 3. reward 4. fine 5. purchase - - - - - - 9. .....

10. **date** 1. fruit 2. facts 3. tress 4. stone 5. time - - - - - - - 10. .....

11. **recipe** 1. cost 2. directions for making 3. size of article 4. note 5. value - - 11. .....

12. **bankrupt** means unable to 1. break 2. make 3. sell 4. pay 5. eat - - - - - 12. .....

13. **credit** 1. trust 2. spend 3. lose 4. waste 5. sell - - - - - - - - 13. .....

14. **reservoir** 1. building 2. office 3. tank 4. measure 5. law - - - - - 14. .....

15. **circulation** 1. consumption 2. construction 3. manufacture 4. ornament 5. distribution 15. .....

16. **incorporate** 1. bankrupt 2. organize 3. separate 4. uncommon 5. prepare - - - 16. .....

17. **forgery** 1. stealing 2. cashing a check 3. counterfeiting a bill 4. copying 5. signing another's name - - - - - - - - - - - 17. .....

18. **convert** 1. ask 2. change 3. sell 4. promise 5. think - - - - - - - 18. .....

19. **cwt.** 1. by the bushel 2. 160 acres 3. 100 pounds 4. ton 5. per cent - - - - - 19. .....

20. **proprietor** 1. owner 2. partner 3. agent 4. clerk 5. borrower - - - - 20. .....

(Continue on next page)

21. **approximate** 1. quite far 2. per cent 3. question 4. appeal 5. fairly close - - 21. .....

22. **protective tariff** 1. income tax 2. import duty 3. export duty 4. property tax 5. tax on bonds - - - - - - - - - - - 22. .....

23. **formula** 1. rule 2 answer 3. division 4. majority 5. multiple - - - - - 23. .....

24. **transaction** 1. construction 2. building 3. prevention 4. business 5. signature - 24. .....

25. **prepay** 1. pay afterwards 2. refuse to pay 3. pay in advance 4. pay again 5. delay paying - - - - - - - - - - - - - 25. .....

26. **extensive** 1. weak 2. broad 3. small 4. heavy 5. valuable - - - - - 26. .....

27. **drawee** 1. maker 2. buyer 3. clerk 4. payer 5. receiver - - - - - - 27. .....

28. **remuneration** 1. destruction 2. charges 3. compensation 4. inventory 5. sovereign 28. .....

29. **resources** 1. wealth 2. beginning 3. climate 4. health 5. debts - - - - - 29. .....

30. **deficiency** 1. more than enough 2. taxes 3. effect 4. lack 5. receipts - - - 30. .....

31. **pool** 1. separate 2. combine 3. sell 4. lower costs 5. lose - - - - - - 31. .....

32. **minimum** 1. least 2. gradual 3. chief 4. honorable 5. timely - - - - - 32. .....

33. **underwrite** 1. subscribe 2. beg 3. guarantee 4. question 5. certify - - - 33. .....

34. **net profit** 1. marked price less cost 2. net proceeds 3. cost less selling price 4. selling price less cost 5. list price - - - - - - - - - 34. .....

35. **remittance** 1. cheapness 2. punrchase 3. contract 4. obligation 5. payment - 35. .....

36. **coupon** is attached to 1. a bond 2. a stock 3. a bill 4. a note 5. an invoice - - - 36. .....

37. **tangent** 1. parallel 2. perpendicular 3. touching 4. horizontal 5. corresponding - - 37. .....

38. **calculation** 1. assets 2. elevation 3. collateral 4. computations 5. climate - - 38. .....

39. **compile** 1. compute 2. select 3. borrow 4. declare 5. gather - - - - - - - 39. .....

40. **enumerate** 1. answer 2. count 3. buy 4. create 5. spend - - - - - - 40. .....

Number right.....

ARITHMETIC —FUNDAMENTAL OPERATIONS

Directions: Do each example as you come to it.
Do your figuring on this paper.
Use additional blank paper of your own for figuring if necessary.
Write the answers at the right side of this page.

Illustration: *1. Subtract* 476 − 122 = 354

Answ

1. 354

1. Divide

$6\overline{).008}$

2. Multiply

$18.4 \times .26$

3. Divide

$2.8\overline{)18.96}$

4. Subtract

$2\frac{1}{6} - \frac{2}{3}$

5. Add
Find the sum of:
.8 + 4 + .135

6. Subtract

9010200
−3725962

7. Divide

$697\overline{)4.182}$

8. Add

$2\frac{2}{3} + 3\frac{4}{5} + 8\frac{7}{10}$

9. Divide

$1\frac{3}{8} \div \frac{3}{10} =$

10.

1.2 = ___ %

11. Multiply

$23 \times 18\frac{3}{5}$

12. Divide

$.795\overline{)182.85}$

13.

20 = 20% of ____

14.

1000% of 86 =

15. Add

8 gal. 2½ qts.
7 gal. 1 qt.
3 gal. 3 qts.

16.

$1\frac{7}{8}$ = ____ %

17.

$\frac{7}{8}$ of .6896 =

18.

.6% of 1258 =

19.

3.5% of 650 =

20.

231 = ____ % of 168

21.

Express .625 As a fraction.

22. Subtract

9 bu. 10 qts.
8 bu. 16 qts.

23.

144 = ____ % of 48

24.

Solve for X:

$\frac{x}{3} + 4 = 6$

25. Multiply

.697
× 40.6

Look over your work.

Number right.....

ANALYTICAL SCALES of ATTAINMENT

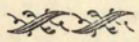
ARITHMETIC

Developed by
L. J. Brueckner
Martha Kellogg
M. J. Van Wagenen

Directions

for

ADMINISTERING AND SCORING



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GENERAL DIRECTIONS

Since the ANALYTICAL SCALES of ATTAINMENT are designed to measure the abilities and ranges of information of the pupils they should be given under the same conditions that any physical measuring instrument would be used. As far as the pupils are concerned the test should come unannounced, and as a part of the regular work of the day. The conditions under which they are administered should occasion no more rigidity or excitement in the pupils than their stepping on scales to have their weights taken.

The same care should be exercised by the teacher in giving these scales as would be taken in using any physical measuring instrument. The examiner should become acquainted with the instrument and have a thorough understanding of and familiarity with the directions for giving each scale before attempting their administration.

While the tests are being given the conditions in the classroom should be kept as normal as possible. The examiner's manner should put the children at their ease. There should be no conversation between the teacher and visitors nor should visitors be encouraged to remain in the room during the testing period.

As the time limits enable the slower pupils to do as much as they can, the more rapid workers will complete the scales before the time limits are up. The pupils who finish quickly should be encouraged to look through their work carefully for errors. When this has been done they should be permitted to read something until the end of the test period rather than sit idle.

When the children have been comfortably seated tell them to remove all books and papers from the tops of their desks and to provide themselves with two well sharpened pencils. Then pass out the test papers face up.

When each pupil has a test paper say to them, "Fill in all the blank spaces on the front page but do not turn any pages until you are told to do so."

For information on VALIDITY, RELIABILITY, NORMS, the DIFFICULTY OF THE ITEMS, and STRUCTURE OF THE SCALES see the **Directions** for the **Unit Scales** of **Attainment.**

ENTERING THE C-SCORE ON THE CLASS RECORD

Besides furnishing a record of the pupil's names and scores the **Class Record** sheet provides a means for tabulating the C-Scores on each scale by tally marks.

In tabulating the C-Scores for each pupil, put a tally mark, **1,** after the corresponding C-Score on the **Class Record** sheet, at the right of the vertical line with the name of the scale at the top. When a pupil makes a C-Score in any scale that is not printed on the **Class Record** sheet, as 75.5, put the tally mark in the upper part of the space after the 75. To facilitate counting the tally marks the fifth one after any C-Score should be drawn across the first four. All C-Scores higher or lower than those in the columns should be put in the spaces above or below the last C-Scores in the columns.

The last column with **Arithmetic Age** at the top gives the ages corresponding with the C-Scores at the left. This column may be used to transmute the C-Scores into **Arithmetic Ages** and at the same time gives the distribution of the **Arithmteic Ages** for the class.

When tally marks have been entered for all the C-Scores in each scale, the mid-C-Score and mid-age for each scale may be found by counting either up or down until the middle case is found in each column. Should there be an even number of pupils in the class, take the average of the two middle C-Scores as the mid-C-Score. For example, if 34 pupils have taken the test the mid-C-Score will be the average of the 17th and 18th from either end while the mid-C-Score for a class of 33 would be the 17th C-Score from either end.

The mid-C-Score for each scale may be designated by drawing a circle around the group of tally-marks in which the mid-C-Score or mid-age falls. A comparison of the mid-C-Scores on the several scales will show on which phases of the subject the greatest and least stress have been laid.

Directions for the Analytical Scales in Arithmetic

Time limit—two periods of 45 minutes.

First Sitting

When the pupils are comfortably seated with their desks cleared of books and papers pass out the test papers, one to each pupil. Then say to them, "Fill in the blank spaces on the first page." When this has been finished, say to them, "Turn over the page to the first test 'Quantitative Relationships.'" The directions say: "Read these two sentences carefully." Read them aloud while the pupils follow silently.

A. Milk is sold by the **1. liter 2. ounce 3. quart 4. gram 5. peck** - - - - - - A....3....

B. The man who keeps a grocery store sells **1. dishes 2. books 3. stoves 4. sugar 5. overcoats** - - - - - - - - - B.....

You see that there are five possible answers in each sentence. Only one answer is right. In the first sentence the right answer is **quart,** so a line is drawn under **quart** and the number in front of it, **3,** is put at the end of the line. Now look at the second sentence above and listen to the next directions. What is the right answer? Yes, **sugar.** What number is in front of this right answer? Yes, 4, so put 4 at the end of the line.

"In each of the following sentences you are to find the right answer, draw a line under it and then put the number that is in front of it at the end of that line, just as in the samples above."

"Begin."

At the end of 15 minutes or earlier if all the pupils have finished the Quantitative Relationships Test, say, "Stop, turn over the page to the test on Arithmetic Problems."

When all have found the place, say, "On this and the next page you have 25 problems in arithmetic. Do each problem as you come to it. Write the answers at the right side of the pages after the numbers of the problems. Do not turn the page to the next test. When you finish this test, close your book and put it on the right side of your desk ready for collection. Begin work."

The examiner should see that the pupils are working on the right test and that they are putting their answers in the proper places at the right side of the page.

Allow the slowest working pupils at least 25 minutes to complete the test. More time may be allowed if several in the class have not finished at the end of this time. As soon as the test is completed collect the test papers so that the pupils will **not** look over the next test.

Second Sitting

Pass out the test papers so that each pupil gets his own test paper. When the papers have been distributed, say to them, "Turn over the pages until you come to the test 'Arithmetic Vocabulary'":

"Look at the first line of words carefully."

A. simple—**1. hard 2. funny 3. easy 4. busy 5. tiny** - - - - - - - - - A....3....

B. guard—**1. stop 2. watch over 3. hit 4. run away 5. climb** - - - - - - B.....

"You see there are five words after the word **simple.** One of them means the same as **simple.** Which one is it? Yes, **easy,** so **easy** has a line drawn under it and the number that is in front of it, 3, is put in the space at the end of the line.

"Now look at the second sample and listen to the next directions.

"Which of the words or phrases after **guard** means the same as **guard?** Yes, **watch over,** so draw a line under **watch over** and put the number that is in front of it in the space at the end of the line.

"In each of the exercises below look at the first word in each line. Then find the word or phrase in the rest of the line that has the same meaning, draw a line under it and put the number that is in front of it at the end of the line, just as in the samples above.

"Ready. Begin."

At the end of 12 minutes or earlier, if all the pupils have finished the reading Vocabulary, say:

"Stop. Turn over the page to the test, 'Arithmetic Fundamental Operations'."

As soon as the pupils have turned the page say:

"Do each example as you come to it and write the answer after its number at the right side of the page, as is done in the sample at the top. When you have finished this test close your book and put it on the corner of your desk ready for collection."

Allow at least 28 minutes for the slowest working pupils to complete their work. If several of the pupils have not finished within this time, they may be allowed more time.

Collect the papers and tie them together.

MEANING OF THE C-SCORE*

Since the Scales are like yard sticks with a task of known value in place of the inch mark at each unit distance, the C-Scores measure amounts of abilities just as inches measure amounts of height or as pounds measure amounts of weight. The C-Score is not only expressed in terms of a consistent unit of measurement throughout the range of the scales but is an unweighted score in that it has the same meaning at every point on the scale. It shows just how difficult tasks a pupil can attempt and succeed in half of the tasks. The level of difficulty at which half of the tasks can be done was selected as the one in which to express the C-Score because at this point the error of measurement is smaller than at any other percentage of correctness.

The number right on an unscaled test not only gives no clue as to the proportion of tasks that can be done successfully by the pupil but the proportion even varies with the different scores.

It is the consistency as well as the definiteness of meaning of the C-Score that makes feasible the comparison of gains made at all points on the scales and also from one scale to another. Thus the gain made by a pupil in **ARITHMETIC** in going from a C-score of 60 to one of 66 is the same as the gain made by another pupil in going from a C-score of 80 to one of 86. The C-scores express the amounts of abilities with reference to a definite zero point that is 60 C-score units below the ability of normal children mentally ten years of age.

*Called C-score because one can see just how difficult tasks a pupil can attempt and get half of them right, just as in the case of mental ages on the Stanford and Kuhlmann revisions of the Binet-Simon Individual Intelligence Examinations.

DIRECTIONS FOR SCORING

In scoring each scale, place the answer **key** designed for that scale at the left of the pupil's answer numbers, seeing that the numbers of the tasks on the **key** match the numbers of the tasks on the page of the scale to be scored.

Also see that the **key** used has the same name at the top as the scale itself, as each scale has a different answer **key** and different table for changing the number correct into C-Scores.

Then with a red or blue pencil put a 1 after each correct answer. **When the pupil has put down a wrong answer do not put any mark after it. Also do not put any mark after an omitted answer.**

When all the correct answers have been marked with a 1, count up the number of **1's** in each scale and put it after "Number Right" at the end of the scale.

Next, look under the C-Score on the same scoring **key** for the C-Score opposite the Number right. Put this C-Score on the **Class Record** sheet after the pupil's name and under the name of the scale on which it was made.

ANALYTICAL SCALES of ATTAINMENT
ARITHMETIC

Class Analysis
(64)

Grade..... Date..... Form..... Div..... Class Mid-Score.....

Teacher ..... Room..... Previous Score.....

School ..... Class Median Pc.Av. or I.Q.....

NAMES	C-SCORES				Class Distribution ARITHMETIC C-SCORES				ARITH. AGE
	Quan. Rel.	Prob.	Arith. Vocab.	Fund Op.	Quan. Rel.	Problems	Arithmetic Vocabulary	Fund Op.	
1						108		92	17-0
									10
2						107		91	8
					105		105		6
3						106		90	4
									2
4					104	105	104	89	16-0
					103		103		10
5						104		88	8
					102		102		6
6					101	103	101	87	4
					100		100		2
7					99	102	99	86	15-0
					98	101	98	85	10
8					97		97		8
					96	100	96	84	6
9					95	99	95	83	4
					94	98	94	82	2
10					93	97	93	81	14-0
					92	96	92	80	10
11					91	95	91	79	8
					90	94	90	78	6
12					89	93	89	77	4
					88	92	88	76	2
13					87	91	87	75	13-0
					86	90	86	74	10
14					84.5	88	84.5	73	8
					83		83	72	6
15					82	86	82	71	4
					80.5	84	80.5	70	2
16					79		79	69	12-0
					78	82	78	68	10
17					76.5	80	76.5	67	8
					75		75	66	6
18					74.5	78	74.5	65	4
					72	76	72	64	2
19					71		71	63	11-0
					70	74	70	62	10
20					68.5	72	68.5	60	8
					67	70	67		6
21					65	68	65	58	4
					63	66	63		2
22					60	64	60	56	10-0
					58	62	58	54	10
23					56	60	56	52	8
					53	58	53		6
24					50	56	50	50	4
						54			2
25						52		48	9-0
						50		46	10
									8
									6

Grade VIII | Grade VII | Grade VI | Grade V | Grade IV

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