HEAT-EXCHANGER-CORE WEIGHTS FOR USE WITH HYDROGEN-EXPANSION TURBINE

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SUMMARY

An analysis of probable heat-exchanger weights for stationary and rotary regenerator heat exchangers for use in a hydrogen-expansion turbine engine is presented. Heat-exchanger-core weights alone will probably be above 48 pounds per pound of hydrogen flow per second for stationary regenerators at a turbine-inlet temperature of 2000° R. This weight is for a pressure drop through the exchanger of about 10 percent of the inlet pressure.

Lowering the turbine-inlet temperature to about 1500° R would cut the regenerator-core weight in half.

Rotary regenerators offer possibilities for a considerably lighter heat-exchanger core, possibly one-fourth that of the stationary regenerators. At the same time, lower combustor temperatures may be employed. The mechanical problems associated with this type of exchanger may be quite severe.

Calculated physical properties of rich-mixture combustion products of hydrogen and air, which are required for heat-transfer calculations, are presented.

INTRODUCTION

One engine cycle that has been considered for possible high-speed, high-altitude aircraft use (ref. 1) is illustrated by the engine cross section shown in figure 1. In this cycle, which is called Rex III in reference 1, the cruise engine is essentially a ramjet burning hydrogen fuel. In order to obtain takeoff and boost thrust, a fan is provided at the inlet. The power to drive the fan is obtained by expanding hot hydrogen gas through the turbine. The hydrogen is heated in turn in a heat exchanger by the combustion products of the hydrogen with a portion of the total engine airflow.
An evaluation of the usefulness of this engine cycle will depend in part upon an analysis of the component weights. While it is realized that many of these component weights are interdependent, this report examines only the effect of varying parameters on the required heat-transfer area and probable weight of the heat-exchanger core itself. Heat exchangers of both the stationary and rotary type are considered.

In the particular cycle considered herein, the high-temperature combustion products for heating the hydrogen are obtained by burning hydrogen rich. The effect of obtaining these high temperatures by rich-mixture combustion rather than by lean is twofold: (1) The inner combustor is smaller, since much less air is required, and (2) the physical properties of the combustion gases are such that higher heat-transfer coefficients can be obtained, which would result in a smaller heat exchanger.

Mechanical problems associated with the design of these heat exchangers are not evaluated; only the performance is considered from the heat-transfer point of view. Physical properties of hydrogen and of combustion products of hydrogen-rich air mixtures of interest in heat-transfer calculations are presented.

This report presents the results of the analysis as generalizations independent of the heat-exchanger configurations to the extent possible. The calculations of actual core weight and pressure drops, however, obviously require selection of actual configurations. The heat-exchanger cores considered were taken from reference 2.

PHYSICAL PROPERTIES

Heat-transfer coefficient data are usually presented as correlations involving the dimensionless groups, Nusselt, Stanton, Reynolds, and Prandtl numbers. Evaluation of these groups requires certain physical property data for the fluid involved. The physical properties of hydrogen and the combustion products of rich hydrogen-air mixtures required for the analysis of the heat exchangers presented herein are (1) viscosity, (2) thermal conductivity, (3) specific heat, (4) mean molecular weight, (5) combustion temperature, and (6) enthalpy.

All symbols are defined in appendix A, and the method of calculating the properties of the mixtures is presented in appendix B. The calculated values are presented in figures 2 to 8.

ENGINE CYCLE

A diagram showing the components of an engine and the conditions under consideration in the following discussion is shown in figure 9.
fuel flow rate of 6 pounds per second was chosen as a basis for presenting the calculations. This flow rate might correspond to the takeoff fuel flow rate for a 67-inch-diameter engine designed for Mach 4 at 100,000 feet. The total engine airflow would be about 300 pounds per second and the afterburner temperature about 3600° R.

Liquid hydrogen is pumped to approximately 8-atmosphere pressure and is vaporized and heated to the desired turbine-inlet temperature $T_{CO}$ by the heat exchanger. The hydrogen gas is expanded through the turbine to $\frac{1}{2}$ atmospheres and a corresponding lower temperature. The hydrogen then enters the combustor where it combines with air from a secondary fan at 700° R and at a combustor pressure of about 2.2 atmospheres to arrive at a flame temperature $T_{H1}$. This hot combustion gas enters the other side of the heat exchanger and exits from the heat exchanger at $T_{H0}$. This fuel-rich mixture then combines with the major portion of the air, and a combustion temperature of about 3600° R is achieved. The gases are discharged through a suitable exhaust nozzle.

**STATIONARY HEAT EXCHANGER**

The variables will be discussed without reference to specific heat-exchanger cores insofar as possible. Calculations will then be presented for certain specific cores.

The required heat-exchanger area is determined from the relation

$$q/t = UA \Delta T$$

For a hydrogen flow rate of 6 pounds per second and a turbine-inlet temperature of 2000° R, the required heat-exchanger capacity $q/t$ is about 150,000,000 Btu per hour. For a selected combustion temperature and heat-exchanger arrangement (i.e., counterflow, crossflow with one fluid mixed, etc.), the $\Delta T$ of equation (1) is established. It is then only necessary to evaluate the over-all coefficient $U$ to determine the heat-transfer area (and, hence, approximate heat-exchanger-core weight) required.

**Combustion Temperature**

The combustion temperature is determined by the equivalence ratio and the inlet temperatures of the fuel and air. The fuel temperature entering the combustor is assumed to be 700° R less than the heat-exchanger-outlet temperature. This temperature difference corresponds to the work required of the turbine to drive the fans.
In the following calculations, a turbine-inlet temperature of 2000° R will be assumed. The effect of varying this temperature will be considered in the section entitled Turbine-Inlet Temperature. The combustion temperature must be above 2000° R for a finite heat-exchanger area. An increase in the combustion temperature will require more air to be burned (lower equivalence ratio) since this is a rich-fuel combustor. The combustor cross-sectional area must then be increased if the same flow Mach number is to be maintained. As the equivalence ratio varies, the heat capacity and molecular weight of the combustion products vary. Table I shows the variations of some of these factors as the combustion temperature is changed from 2300° to 2900° R, while the combustor-exit Mach number is held at 0.1.

Table I also shows the increased effective temperature gradient for heat transfer $\Delta T_{zm}$ as the combustion temperature is increased. The $\Delta T_{zm}$ is the effective temperature gradient for counterflow arrangement. The relative heat-transfer area required would be decreased correspondingly as the combustion temperature is increased if the over-all transfer coefficient was not affected.

Relative combustor cross-sectional area and relative effective temperature gradient for heat transfer $\Delta T_{zm}$ for varying combustion temperatures are plotted in figure 10. For simplicity, this comparison is made by assuming counterflow arrangement. With a crossflow exchanger, a greater change in effective $\Delta T$ with changing combustion temperature would result.

Figure 10 shows that, unless the over-all heat-transfer coefficient decreases greatly as the combustion temperature increases, the smallest heat-exchanger area will be obtained by going to the highest combustion temperature compatible with structural limitations on the metal.

**Heat-Transfer Coefficients**

The final variable remaining to be determined in order to establish the heat-transfer area requirement is the over-all heat-transfer coefficient $U$. For negligible wall resistance, $U$ is related to the individual coefficients by

$$ U = \frac{1}{h_c} + \frac{1}{h_h} $$

(2)

Next to be examined are the probable values of the coefficients $h_c$ and $h_h$ and their variation with temperature.
The heat-transfer coefficient may be written as follows, from the definition of the Stanton number:

\[ h = (St)C_pV \quad (3) \]

By using the ideal equation of state \( \rho = \frac{pm}{RT} \) and the relation for the speed of sound \( V = M\sqrt{\frac{R}{pRT/m}} \), the above equation may be transformed to

\[ h = (St)\frac{C_pM}{\sqrt{T}} p\sqrt{\frac{Ygm}{R}} \quad (4) \]

Experimental heat-transfer coefficients on the heat-exchanger cores to be compared later are plotted as \((St)(Pr)^{2/3}\) against \(Re\). Equation (4) could be written

\[ h = \frac{(St)(Pr)^{2/3}}{(Pr)^{2/3}} \frac{C_pM}{\sqrt{T}} p\sqrt{\frac{Ygm}{R}} \quad (5) \]

where \(p\) and \(T\) in this equation are static pressure and static temperature, respectively. The Mach number range is low enough that static and total temperature are considered the same. Static pressure is used throughout the comparison.

The probable range of heat-transfer coefficients for the fuel and combustion products can be estimated from equation (5).

**Hydrogen.** - For a constant pressure of 8 atmospheres and an assumed value of \((St)(Pr)^{2/3}\) of 0.003, the heat-transfer coefficient of the hydrogen will have the following variation with temperature and Mach number:

<table>
<thead>
<tr>
<th>(T, \frac{\text{C}^\circ}{\text{R}})</th>
<th>(C_p)</th>
<th>(Pr)</th>
<th>(h, \text{Btu/}(\text{hr})(\text{sq ft})(\text{OF}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>600</td>
<td>3.46</td>
<td>0.68</td>
<td>8060 M</td>
</tr>
<tr>
<td>1300</td>
<td>3.55</td>
<td>.685</td>
<td>5600 M</td>
</tr>
<tr>
<td>2000</td>
<td>3.69</td>
<td>.69</td>
<td>4680 M</td>
</tr>
</tbody>
</table>

This assumed value of \((St)(Pr)^{2/3}\) is in the range to be expected for flow of the hydrogen inside small tubes, which is the situation considered herein. Probable minimum values of the hydrogen-side coefficient are shown in figure 11.

It appears that, at the highest fuel temperature, if the flow Mach number is maintained above about 0.11, the heat-transfer coefficient should be at least 500 Btu/(hr)(sq ft)(OF).
Combustion products. - For a constant pressure of 2.2 atmospheres and an assumed value of \((\text{St})(\text{Pr})^{2/3} = 0.003\), the heat-transfer coefficient of the combustion products will vary with temperature and Mach number as follows:

<table>
<thead>
<tr>
<th>(T_{\text{h}, \theta_R})</th>
<th>(m)</th>
<th>(C_p)</th>
<th>(h_{\text{R}}, \text{Btu/(hr)(sq ft)}(\theta_R))</th>
<th>Ratio, (h/h_{2900^\circ R})</th>
</tr>
</thead>
<tbody>
<tr>
<td>2300</td>
<td>7.45</td>
<td>1.06</td>
<td>811 M</td>
<td>1.38</td>
</tr>
<tr>
<td>2400</td>
<td>8.07</td>
<td>1.00</td>
<td>772 M</td>
<td>1.32</td>
</tr>
<tr>
<td>2500</td>
<td>8.7</td>
<td>.92</td>
<td>730 M</td>
<td>1.25</td>
</tr>
<tr>
<td>2700</td>
<td>10.17</td>
<td>.80</td>
<td>651 M</td>
<td>1.11</td>
</tr>
<tr>
<td>2900</td>
<td>11.46</td>
<td>.72</td>
<td>585 M</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Probable minimum values of the hot-side coefficient are plotted in figure 12.

Thus, by maintaining the Mach number above 0.17, the minimum heat-transfer coefficient to be expected on the hot side should be at least 100 Btu/(hr)(sq ft)(\(\theta_R\)), since the value of \((\text{St})(\text{Pr})^{2/3} = 0.003\) is near the minimum that will be encountered under the conditions considered. For flow over tubes of small diameter (as on the shell side of shell-tube exchangers), the value of \((\text{St})(\text{Pr})^{2/3}\) should be considerably higher than 0.003, and, hence, values of \(h\) of 100 or above may be obtained at considerably lower Mach numbers.

The last column in the previous table also shows how the heat-transfer coefficient of the combustion products varies as the temperature varies for a constant Mach number flow. The value at 2900\(^\circ\) R is taken as the reference point. These relative values are plotted in figure 13 as a function of temperature along with the relative \(\Delta T\) values from figure 10.

The heat-transfer coefficient of the combustion products, which should essentially control the over-all coefficient \(U\), does not decrease as rapidly as the effective \(\Delta T\) increases with increasing combustion temperature. Thus, it is apparent that the highest combustion temperature compatible with allowable metal wall temperatures will give the lower weight heat-exchanger cores.

Limiting Metal Temperature

The maximum equilibrium metal wall temperature will be determined by the maximum hot- and cold-gas temperatures and the ratio of the hot- to the cold-side heat-transfer coefficients:
where the turbine-inlet temperature $T_{co}$ is 2000° R.

The variation of metal temperature $T_w$ with combustion temperature $T_{hi}$ is shown for several ratios of heat-transfer coefficients in figure 14(a).

For an estimate of maximum possible allowable metal temperatures with present materials, a plot of yield strength and 1000-hour rupture stress against temperature is shown (fig. 14(b)) for a high-temperature alloy, Inconel X (ref. 3). Depending upon the stress limitations of the design, it appears that metal temperatures of 2100° to 2200° R might be allowed, which would permit combustion temperatures close to 3000° R.

A 1/4-inch-diameter tube, for example, at 8-atmosphere (120 lb/sq in.) pressure and with a wall thickness of 0.012 inch would have a hoop stress in the wall of about 1190 pounds per square inch. At 2200° R, the indicated 1000-hour rupture stress (fig. 14(b)) for Inconel X is about 2500 pounds per square inch.

**Turbine-Inlet Temperature**

Decreasing the turbine-inlet temperature lowers the heat-exchanger capacity required and gives higher temperature differences for heat transfer for the same combustion temperature. Both of these factors decrease the heat-exchanger size. At the same time, however, a lower fuel temperature into the combustor results, and the burning of more air is required to reach the same combustion temperature. Thus, the combustor would be larger. Alternatively, the combustion temperature may be reduced.

Table II shows changes in pertinent variables as the turbine-inlet temperature is changed while the combustion temperature is held constant at 2900°, 2700°, and 2500° R.

The relative UA requirements for a given turbine-inlet temperature and combustion temperature are shown in the last column of table II. Counterflow arrangement is assumed for simplicity of analysis. The UA requirement for a turbine-inlet temperature of 2000° R and a combustion temperature of 2900° R is used as the basis for comparison. If the overall coefficient $U$ is assumed to vary with combustion temperature as the heat-transfer coefficient of the combustion products, $h_h$ curve in figure 13,
the relative heat-transfer-area change may be estimated by properly combining the $h_h$ curve of figure 13 with the UA values of table II. This comparison is shown in figure 15.

From the curves of figure 15, it is apparent that decreasing the turbine-inlet temperature from 2000° to 1400° R would about cut the heat-exchanger weight in half at the combustion temperature level of 2900° R. Also, at the lower turbine-inlet temperature, the combustion temperature could be reduced several hundred degrees without greatly affecting the heat-exchanger area. At these lower turbine-inlet temperatures, the change in $h_h$ nearly counteracts the change in effective $\Delta T$, so that it would be more advantageous to go to the lower combustion temperatures.

Specific Heat-Exchanger Cores

The principal objective here was to establish a range of heat-transfer areas (and, hence, probable core weights) for a range of combustion-products-side pressure drops through the system. Since the weight was of primary concern, the minimum heat-transfer area for any allowable pressure drop was desired. For this reason, the various types of heat-exchanger cores in reference 2 were examined.

Previous discussion has pointed out that the heat-transfer coefficient on the hydrogen side of the exchanger should be easily maintained considerably higher than that expected on the combustion-products side. Further, the pressure drop on the hydrogen side could be increased, if necessary, by merely increasing the liquid pump discharge pressure. In calculating an over-all coefficient for heat transfer, the cold-side (hydrogen) coefficient was assumed equal to 500 Btu/(hr)(sq ft)(°F).

Heat exchangers of the shell-tube type, with the combustion products over the shell side, gave minimum heat-transfer-area requirements. In order to keep the pressure drop through the shell side of this exchanger in the neighborhood of 10 percent, the frontal area requirements are large, and the exchanger becomes voluminous. Without considering the practicality of such arrangements for this application, calculations are shown in table III for various cores and several inlet Mach numbers since the calculations yielded the minimum exchanger weights. The number identifying the core corresponds to the figure number of the performance curves in reference 2. The pressure drops were estimated by the method of reference 4.

The heat-transfer-area - pressure-drop relations from this table are plotted in figure 16. A curve bounding these points was drawn which essentially establishes a minimum heat-transfer area for a given pressure drop through the system. For a $\Delta p/p$ of 10 percent, a minimum heat-transfer area would appear to be about 570 square feet. If this core were made from 0.012-inch-thick stainless steel or Inconel, it would weigh about 1/2 pound per square foot, or about 285 pounds. This is approximately 48 pounds per pound of hydrogen flow per second.
ROTORV HEAT EXCHANGER

A schematic diagram of a rotary-heat-exchanger installation is shown in figure 17. The purpose of the rotary heat exchanger is exactly the same as for the stationary one. A rotary heat exchanger consists of a matrix of material which rotates and causes the matrix to pass alternately through the hot and cold fluids. Heat is thus transferred from the hot gas to the matrix, thence to the cold gas.

An additional variable to be chosen with the rotary exchanger is the capacity rate ratio \( \frac{C_r}{C_{\text{min}}} \); that is, the ratio of the capacity rate of the core to the minimum capacity rate of the two gas flows. A smaller capacity rate ratio gives a greater temperature excursion to the matrix material, a resultant lower effective \( \Delta T \) for heat transfer, and, hence, a greater area requirement and greater core weight. The rotative speed is lower for lower ratios of \( \frac{C_r}{C_{\text{min}}} \).

A summary of design theory for rotary regenerators is given in reference 5, and effectiveness - NTU curves are given in reference 2. Experimental heat-transfer and friction drop performance for several wire-screen matrices is also given in reference 2. Data on these same matrices are extended to high Reynolds numbers in reference 6.

Since the core heat-transfer and flow areas are common to both heat-transfer fluids, the independent choice of flow conditions is more limited than with the stationary heat exchanger. In the case considered here, the hydrogen flow is considerably smaller than the combustion gas flow. One compromise that has to be made is to keep the hydrogen-side Mach number as low as possible to increase the flow area on that side without decreasing the heat-transfer coefficient of the hydrogen to too low a value.

Heat-Transfer Coefficients

The wire-screen matrices from reference 2 with the smallest wire diameters were used to establish the ranges of heat-transfer coefficients to be expected. Table IV lists some of the parameters of the matrices of interest. Again the number identifying the matrix corresponds to the figure number of the performance curve in reference 2 for that particular matrix. Small wire diameters which result in high heat-transfer coefficients and large heat-transfer areas per unit weight of core are the distinctive features of these matrices.

Hydrogen. - The heat-transfer coefficients for the hydrogen at two exit conditions, an exit hydrogen velocity of 200 feet per second \( (M = 0.024) \) and 125 feet per second \( (M = 0.015) \) and a temperature of \( 2000^\circ R \), are shown in table V.
The hydrogen experiences a large temperature change through the heat exchanger, and, hence, a considerable Reynolds number change because of the change in viscosity with temperature. The viscosity of hydrogen in this temperature range is proportional to $T^{0.695}$. Heat capacity and Prandtl number are essentially constant over this temperature range. The $(St)(Pr)$ for these screen matrices was assumed to vary inversely with Reynolds number to $T^{0.375}$ (ref. 6), although this variation is slightly different than indicated on the curves of reference 2. Therefore, the heat-transfer coefficient will vary approximately directly with $T^{0.26}$.

A mean value of $h_c$ was taken to be that at an average temperature of the hydrogen of about 1000° R. These estimated mean heat-transfer coefficients are also shown in table V.

Combustion products. - The range of heat-transfer coefficients for the combustion products at inlet conditions of 2300°, 2400°, and 2500° R and at inlet Mach numbers, based on free-flow area, of 0.04 and 0.06 are shown in table VI.

The viscosity of the combustion products varies directly as $T^{0.71}$, so that the heat-transfer coefficient varies about at $T^{0.27}$. A mean value of $h_h$ was taken to be the value at the mean temperature of the combustion products. These estimated mean values are also shown in table VI.

Combustion Temperature

Some parts of the matrix material will approach the combustion temperature, so that the maximum allowable combustion temperature would be lower for the rotary than for the stationary heat exchanger.

To evaluate the heat-transfer-area requirements at various combustion temperatures, the following relations are used (ref. 5):

\[ (NTU)_o = (NTU)_c \left[ \frac{1}{1 + (hA) *} \right] \]  \hspace{1cm} (7)

\[ (NTU)_c = \frac{(hA)_c}{C} \]  \hspace{1cm} (8)

\[ (hA) * = \frac{(hA)_c}{(hA)_h} \]  \hspace{1cm} (9)

At the flow velocities assumed in estimating the heat-transfer coefficients, the flow frontal areas are established. The ratio of heat-transfer areas $A_c/A_h$ will be the same as the ratio of the flow frontal areas. Thus,
The ratio of the heat-transfer coefficients $h_c/h_h$ using the mean of values established previously is shown in table VI. The higher velocity flows of both sides are assumed together as one set of conditions and the lower velocity flows as another combination.

The variation of the flow parameters as combustion temperature varies is shown in table VII. In this table, the capacity rate ratio $C_r/C_{\text{min}}$ was set at 2.0. The effect of a variation in this ratio will be discussed later.

From the values in tables V, VI, and VII and the core properties in table IV, the heat-transfer areas, core weights, and speeds can be calculated.

The calculations of heat-transfer area, core weight, and rotational speed for the three matrices at combustion temperatures of $2300^\circ$, $2400^\circ$, and $2500^\circ$ are shown in table VIII. These values are for a turbine-inlet temperature of $2000^\circ$ and a capacity rate ratio $C_r/C_{\text{min}}$ of 2.0.

It will be noted, as with the stationary exchanger, that, as the combustion temperature is increased, the required heat-exchanger area and weight decrease. However, the required rotational speed is increased as the weight is decreased, since the matrix core capacity rate $C_r$ is constant.

The friction pressure drop for these configurations can be estimated from the relation

$$\frac{\Delta p}{p} = r \left( \frac{A}{A_f} \right) \frac{1}{2} M^2 \tag{11}$$

This relation is derived from the usual Fanning equation:

$$\Delta p = 4f \frac{L}{D_h} \frac{\rho V^2}{2g}$$

By substituting,

$$\frac{4L}{D_h} = \frac{A}{A_f}$$

and

$$\frac{\rho V^2}{2g} = \frac{1}{2} \rho M^2$$
As with the stationary heat exchanger, lowering the turbine-inlet temperature decreases the rate of heat exchange and increases the effective rate ratio for a given rate of heat exchange.

Capacity Rate Ratio

\[
\frac{\Delta T}{t} = \frac{\text{core weight}}{\text{metal weight}} \times \frac{\text{speed}}{\text{metal speed}}
\]

From the curves of temperature effectiveness against the number of transfer units (NTU), it can be seen that increasing the capacity rate ratio will decrease the (NTU) for the same effectiveness. The core weight required will decrease in direct proportion to this change in (NTU). However, when the capacity rate of the matrix is increased, the heat capacity, which comprises the matrix capacity rate, must be increased since it is the product of (weight) x (speed) x (metal heat capacity) which comprises the matrix capacity rate.

A change in the capacity rate ratio also changes the temperature excursion with the core metal takes. The range of temperature through which the metal passes is

It should again be pointed out here that no evaluation of the mechanical problems associated with the rotary exchanger is included herein. It is recognized that the additional structural complexity required for the rotary exchanger might modify these weight comparisons.

For core number 108 at a combustion temperature of 2500°F, the core weight would appear to be about 70 pounds at a pressure drop of 10 per cent. This weight is only about one-fourth the minimum estimated for the stationary-exchanger core for the same pressure drop. Furthermore, the combustion temperature is 400°F lower for the rotary exchanger in this comparison. This weight is only about one-fourth the minimum estimated for the stationary-exchanger core for the same pressure drop. Furthermore, the combustion temperature is 400°F lower for the rotary exchanger in this comparison.
temperature difference for heat transfer. Both factors tend to decrease the heat-exchanger-core weight.

However, if the capacity rate ratio \( \frac{C_r}{C_{\text{min}}} \) is held constant, the speed of rotation is increased. Since \( C_{\text{min}} \) is not a function of the quantity of heat exchanged, \( C_r \) is not changed. Hence, since core weight times speed is constant, the speed must vary inversely with the weight change.

The estimated change in heat-exchanger-core weight with changing turbine-inlet temperature is shown in figure 19 for a combustion temperature of 2500° R. It appears that the core weight could be reduced by one-half by lowering the turbine-inlet temperature to about 1500° R. This is about the same ratio as was estimated for the stationary exchanger also.

Lowering the turbine-inlet temperature will also decrease the variation in the core metal temperature, since the temperature excursion of the metal is changed in direct proportion to the rate of heat exchanged.

CONCLUSIONS

Through an analysis of the heat-transfer performance possibilities of stationary and rotary heat exchangers for use in a hydrogen-expansion turbine engine, a system called Rex III in reference 1, the following conclusions have been drawn:

1. For stationary heat exchangers, core weights alone would be above 48 pounds per pound per second of hydrogen flow at a turbine-inlet temperature of 2000° R and for a pressure drop of about 10 percent of the inlet combustion gas pressure.

2. Rotary regenerators offer possibilities for heat-exchanger-core weights one-fourth that of the stationary exchanger for the same pressure drop. At the same time, lower combustion temperatures may be used.

3. Lowering the turbine-inlet temperature to about 1500° R would cut the heat-exchanger-core weight about in half for either type of exchanger.

CONCLUDING REMARKS

While the analysis shows that the heat-exchanger-core weight for a rotary heat exchanger may be considerably smaller than that for a stationary exchanger to do the same job, it should be kept in mind that the mechanical problems with the rotary exchanger may be quite severe.
In particular, at the present state of the art, the seals necessary to prevent too high a hydrogen leakage at the high differential pressures involved are a formidable problem. A high rate of hydrogen leakage would reduce the quantity of turbine working fluid and would affect the heat-exchanger performance adversely. The high temperature levels and temperature changes in the system will cause thermal expansion problems that will aggravate the sealing problem.

Contamination of the hydrogen by carryover of combustion products trapped in the matrix is small, less than 1/2 percent for the conditions considered herein, so that the effect on the physical properties of the hydrogen would be negligible.

The analysis presented herein compares only weights of the heat-exchanger cores based on their heat-transfer performance. The additional mechanical complexity of the rotary exchanger may add more to its weight than that of a stationary exchanger. The amount cannot be estimated until some satisfactory seal designs are developed to operate at these conditions.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, August 14, 1957
APPENDIX A

SYMBOLS

A       heat-transfer area, sq ft
A_f     free-flow frontal area, sq ft
A_ij    function, defined in text
C       capacity ratio, wC_p, Btu/(hr)(°R)
C_p     heat capacity at constant pressure, Btu/(lb)(°R)
C_r     core capacity ratio, (core weight)\times(core heat capacity)\times(rpm\times60), Btu/(hr)(°R)
D       diameter, ft
f       friction factor
g       conversion factor, 32.174 ft/sec²
H       enthalpy, Btu
h       heat-transfer film coefficient, Btu/(hr)(sq ft)(°R)
(hA)²   (hA)_o/(hA)_h
k       thermal conductivity, Btu/(hr)(sq ft)(°F/ft)
L       flow length, ft
M       Mach number, V/\sqrt{gRT}/m
m       molecular weight
NTU     number of transfer units of an exchanger, AU/C_min
(NTU)_o number of over-all transfer units of an exchanger
Pr      Prandtl number, C_p\mu/k
p       pressure, lb/sq ft
q       heat transferred, Btu
R universal gas constant, 1544 ft-lb/(lb mole)\(^{\circ}R\)
Re Reynolds number, \(4 rh V \rho / \mu\)
\(4 rh\) hydraulic diameter, ft
S Sutherland constant
St Stanton number, \(h / C_p \rho V\)
T temperature, \(^{\circ}R\)
\(\Delta T_{lm}\) log mean temperature difference for heat transfer,
\[
\Delta T_{lm} = \frac{(T_{h0} - T_{c1}) - (T_{h1} - T_{c0})}{\ln \frac{T_{h0} - T_{c1}}{T_{h1} - T_{c0}}}
\]
\(T_{h0}, T_{h1}\) temperature on the hot side
\(T_{c0}, T_{c1}\) temperature on the cold side
\(t\) time, hr
U over-all heat transfer coefficient, Btu/(hr)(sq ft)(\(^{\circ}F\))
V velocity, ft/hr
w weight-flow rate, lb/hr
x mole fraction
\(\gamma\) ratio of specific heats
\(\varepsilon\) heat-transfer effectiveness,
\[
\varepsilon = \frac{C_h}{C_{min}} \left( \frac{T_{h1} - T_{h0}}{T_{h1} - T_{c1}} \right) = \frac{C_c}{C_{min}} \left( \frac{T_{c0} - T_{c1}}{T_{h1} - T_{c1}} \right)
\]
\(\mu\) viscosity, lb/(ft)(hr)
\(\rho\) density, lb/cu ft
\(\Phi_{ij}\) function, defined in text

Subscripts:
a air
c cold side
\( c_i \) cold gas in
\( c_0 \) cold gas out
\( f \) fuel
\( h \) hot side
\( h_i \) hot gas in
\( h_0 \) hot gas out
\( i \) component i
\( j \) component j
\( m \) mixture
\( \text{min} \) minimum
\( w \) wall
APPENDIX B

PHYSICAL PROPERTIES

The mixture compositions for which the physical properties are presented are those resulting from the reaction

\[ n(2H_2) + O_2 + 3.77 N_2 \rightarrow 2H_2O + 3.77 N_2 + 2(n - 1)H_2 \]

where \( n \) will vary from 1 (stoichiometric) to infinity (pure \( H_2 \)). No dissociation is assumed at any condition encountered herein, since temperatures are below \( 3000^\circ R \).

The composition parameter used for the physical property plots is mole fraction of \( H_2 \) in the mixture of combustion products. At a hydrogen concentration of zero, the mole fraction of hydrogen is zero, and water and nitrogen are in the mole ratio 2.0 to 3.77. This is the composition resulting from stoichiometric combustion \( (n = 1) \). A mole fraction equal to 1 is pure hydrogen. An equivalence ratio scale is also included on the plots for reference. Equivalence ratio is the ratio of actual fuel to that required for stoichiometric combustion.

Experimental viscosity and thermal conductivity data are available for the pure component gases (ref. 7). Heat capacities for the pure gases were taken from reference 8. Properties of normal-hydrogen gas were used throughout, since all data were not available for the para-hydrogen. The difference in properties between normal and para-hydrogen would not affect appreciably any of the results presented herein.

Physical property data for the mixtures were calculated by the methods outlined as follows. Although, as is discussed in the references cited, these equations are not considered applicable when one of the gases is highly polar (water vapor in this instance), the relations were nevertheless assumed to apply, since no experimental data for these mixtures were available.

Viscosity

The viscosities of the mixtures were calculated according to the method of reference 9:

\[ \mu_m = \sum_{i=1}^{n} \frac{\mu_i}{1 + \frac{1}{x_i} \sum_{j=1}^{n} x_j^{\phi_{ij}}} \]
Pure component gas viscosities were taken from reference 7. Values of $\varphi_{ij}$ are plotted in reference 9 for simplifying the calculations. Calculated curves are shown in figure 2. For values at intermediate temperatures, the mean temperature dependence of viscosity is approximately $\mu \propto T^{0.71}$.

**Thermal Conductivity**

The thermal conductivities of the mixtures were calculated according to a similar expression (ref. 10):

$$k_m = \sum_{i=1}^{n} \frac{k_i}{1 + \frac{1}{x_i} \sum_{j=1, j \neq i}^{n} x_j A_{ij}}$$

$$A_{ij} = \frac{1}{4} \left( 1 + \left[ \frac{\mu_i/m_i}{\mu_j/m_j} \right]^{0.75} \left( 1 + \frac{S_i}{T} \right) \right)^{1/2} \left( 1 + \frac{S_i}{T} \right) \left( 1 + \frac{S_j}{T} \right)$$

The pure component thermal conductivities were taken from reference 7.

The calculated gas-mixture thermal conductivities are shown in figure 3. For values at intermediate temperatures the temperature dependence of thermal conductivity is approximately $k \propto T^{0.77}$.

**Heat Capacity**

The heat capacities of the gas mixtures were taken as the mass average of the heat capacities of the constituents:

$$C_{pm} = \sum x_i C_{pi}$$
The pure component heat capacities were taken from reference 8. The calculated gas-mixture heat capacities are plotted in figure 4.

Flame Temperature

Rich-mixture flame temperatures were calculated for the reaction

\[ n(2H_2) + O_2 + 3.77N_2 \rightarrow 2H_2O + 3.77N_2 + 2(n - 1)H_2 \]

by assuming no dissociation of the products of combustion. The heat balance is

\[ w_m \int_{T_a}^{T_f} C_p m \, dT = (2)(2.016)(51,570) + 2n(2.016)(T_{H_2} - T_a) \]

where \( T_a \), the entering air temperature, was set at 700\(^\circ\) R and \( T_{H_2} \), the entering hydrogen temperature, was set at values of 700\(^\circ\), 900\(^\circ\), 1100\(^\circ\), and 1300\(^\circ\) R.

Curves of rich-mixture flame temperature against equivalence ratio are plotted in figure 5.

Other Calculated Values

The Prandtl number \( \mu / k \) was computed from the curves of figures 2, 3, and 4 and is shown in figure 6.

Mean molecular weights were taken as the volumetric average of the molecular weights of the constituents:

\[ m_m = \sum x_i m_i \]

Values of the mean molecular weight for a range of equivalence ratios are plotted in figure 7.

The enthalpy of hydrogen from the normal liquid state to 2000\(^\circ\) R is plotted in figure 8. Liquid hydrogen at 1 atmosphere is taken as the zero reference point.
REFERENCES


<table>
<thead>
<tr>
<th>Combustion temperature, $T_{h1}$, $\circ R$</th>
<th>Equivalence ratio</th>
<th>Mean molecular weight, w, lb/sec</th>
<th>Total hot-gas flow, $w_h$, lb/sec</th>
<th>Heat capacity of combustion products, $C_{p,h}$, Btu/(lb)(\circ R)</th>
<th>Density of combustion products, $\rho_{h1}$, lb/cu ft</th>
<th>Gas velocity at combustor exit (M=0.1), V</th>
<th>Combustor-exit area, sq ft</th>
<th>$\Delta T_h$, $\circ R$</th>
<th>Heat-exchanger exit temperature, $T_{h0}$, $\circ R$</th>
<th>Log mean temperature difference for counter-flow, $\Delta T_{lm}$, $\circ R$</th>
<th>Relative UA required (counter-flow)</th>
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*Basis of comparison.
### TABLE II. - VARIATION OF FLOW PARAMETERS OF COMBUSTOR AND STATIONARY HEAT EXCHANGER WITH VARYING TURBINE-INLET TEMPERATURE AND COMBUSTION TEMPERATURE

<table>
<thead>
<tr>
<th>Combustion inlet temperature, $T_{in}$, °R</th>
<th>Turbine-inlet temperature, $T_{co}$, °R</th>
<th>Fuel, ΔH, Btu/lb</th>
<th>Relative heat-exchange rate $^a$</th>
<th>Equivalence ratio</th>
<th>Airflow rate, $w_a$, lb/sec</th>
<th>Total hot-side flow, $w_h$, lb/sec</th>
<th>Heat capacity of combustion products, $(C_p)_h$, Btu/(lb°C)</th>
<th>Capacity rate of hot side, $(wC_p)_h$</th>
<th>$\Delta T_h$, °R</th>
<th>Heat-exchanger exit temperature, $T_{eo}$, °R</th>
<th>Log mean temperature difference for counterflow, $\Delta T_{lm}$, °R</th>
<th>Relative UA required</th>
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<td>1660</td>
<td>1225 1.0</td>
<td>1.0</td>
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<td>2005</td>
<td>1610 1.0</td>
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<td>61.0</td>
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<td>39.7</td>
<td>715</td>
<td>2185</td>
<td>1800 0.663</td>
<td>0.463</td>
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<td>39.3</td>
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$^a$Relative heat-exchange rate = $\Delta H_{fuel}/\Delta H_{fuel}$ at 2000° R.

$^b$Basis for comparison.
TABLE III. - STATIONARY-EXCHANGER-CORE HEAT-TRANSFER AREAS
AND PRESSURE RATIOS ACROSS HOT SIDE

[Turbine-inlet temperature, 2000° R; combustion
temperature, 2900° R; shell side.]

<table>
<thead>
<tr>
<th>Core number or figure number in ref. 2</th>
<th>Inlet Mach number (based on free-flow area)</th>
<th>Hydraulic diameter, $4r_h$, ft</th>
<th>Reynolds number, Re</th>
<th>$(S)/(Pr)^{2/3}$</th>
<th>Friction factor, f</th>
<th>Heat-transfer coefficient, $h_b$, Btu/(hr) (sq ft) (°F)</th>
<th>Over-all heat-transfer coefficient, U</th>
<th>Heat-transfer area, A, sq/ft</th>
<th>Pressure-drop ratio, $\Delta p/p$</th>
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<tr>
<td>44</td>
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<td>0.0166</td>
<td>2390</td>
<td>0.014</td>
<td>0.072</td>
<td>272</td>
<td>176</td>
<td>695</td>
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<tr>
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<td>1790</td>
<td>0.013</td>
<td>233</td>
<td>0.047</td>
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<td>597</td>
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<td>3590</td>
<td>0.012</td>
<td>348</td>
<td>0.069</td>
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TABLE IV. - PROPERTIES OF SOME ROTARY-HEAT-EXCHANGER MATRICES

<table>
<thead>
<tr>
<th>Matrix number or figure number in ref. 2</th>
<th>Wire diameter, in.</th>
<th>Hydraulic diameter, 4rh, ft</th>
<th>Heat-transfer area per unit volume, sq ft cu ft</th>
<th>Porosity</th>
<th>Bulk density, lb/cu ft</th>
<th>Weight per unit heat-transfer area, lb/sq ft</th>
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<td>107</td>
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<td>0.675</td>
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TABLE V. - HEAT-TRANSFER COEFFICIENTS FOR HYDROGEN IN ROTARY HEAT EXCHANGER

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<th>Matrix number or figure number in ref. 2</th>
<th>Exit velocity, ft/sec</th>
<th>Reynolds number, Re</th>
<th>(St)(Pr)</th>
<th>Friction factor, f</th>
<th>Heat-transfer coefficient, h, 2000° R</th>
<th>Heat-transfer coefficient, h, 1000° R</th>
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TABLE VI. - HEAT-TRANSFER COEFFICIENTS FOR COMBUSTION PRODUCTS IN ROTARY HEAT EXCHANGER

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<th>Combustion temperature, ( T_{h1} )</th>
<th>Matrix number or figure number in ref. 2</th>
<th>Reynolds number, Re</th>
<th>(St)(Pr)</th>
<th>Friction factor, ( f )</th>
<th>Heat-transfer coefficient, ( h_{Th1} )</th>
<th>Heat-transfer coefficient, ( h_h )</th>
<th>Heat-transfer coefficient, ( h_c )</th>
<th>Ratio of cold- to hot-side coefficient, ( h_c/h_h )</th>
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TABLE VII. - VARIATION OF FLOW PARAMETERS IN COMBUSTOR AND
ROTARY HEAT EXCHANGER WITH COMBUSTION TEMPERATURE

[Turbine-inlet temperature, 2000° R; capacity rate ratio, 
\( \frac{C_r}{C_{\text{min}}} \), 2.0.]

<table>
<thead>
<tr>
<th>Combustion temperature, ( T_{h_i}), °R</th>
<th>Equivalence ratio</th>
<th>Hot-gas flow, ( w_h ), lb/sec</th>
<th>Heat capacity of hot gas, ( (C_p)_h )</th>
<th>Capacity rate of hot side, ( (wC_p)_h )</th>
<th>Capacity rate ratio of gas flows, ( \frac{C_{\text{min}}}{C_{\text{max}}} )</th>
<th>Heat-transfer effectiveness, ( \varepsilon )</th>
<th>Over-all number of transfer units required, ( (\text{NTU})_o )</th>
<th>Heat-transfer area ratio, ( A_c/A_h )</th>
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TABLE VIII. - HEAT-TRANSFER AREAS, WEIGHTS, SPEEDS, AND PRESSURE DROPS
FOR ROTARY HEAT EXCHANGER AT VARIOUS COMBUSTION TEMPERATURES

<table>
<thead>
<tr>
<th>Combustor-inlet temperature, $T_h$, $\circ R$</th>
<th>Matrix core number or figure number of ref. 2</th>
<th>$(NTU)_o$</th>
<th>(hA)*</th>
<th>$(NTU)_c$</th>
<th>Cold-side heat-transfer area, sq ft</th>
<th>Total heat-transfer area, sq ft</th>
<th>Core weight, lb</th>
<th>Core rotational speed, rpm</th>
<th>Friction-factor hot side, f</th>
<th>Pressure drop ratio on hot side, $(\Delta P/p)_h$</th>
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Figure 2. - Calculated viscosity of hydrogen-nitrogen-water vapor mixtures. Mole ratio of nitrogen/water = 3.77/2.0; \( \mu \propto T^{0.71} \).
Figure 3. Calculated thermal conductivity of hydrogen-nitrogen-water vapor mixtures. Mole ratio of nitrogen/water = 3.77/2.0; \( k \propto T^{0.77} \).
Figure 4. - Heat capacity of hydrogen-nitrogen-water vapor mixtures. Mole ratio of nitrogen/water = 3.77/2.0.
Figure 5: Calculated rich-mixture flame temperatures for hydrogen-air mixtures. No dissociation assumed; air temperature, 7000 R.
Figure 6. - Calculated Prandtl numbers of hydrogen-nitrogen-water vapor mixtures. Mole ratio of nitrogen/water = 3.77/2.0.
Figure 7. - Average molecular weight of combustion products of hydrogen-air mixtures. Products of complete oxidation with no dissociation.
Figure 8. - Heat content of hydrogen above liquid at normal boiling point.
Figure 9. - Diagram showing relation of flow components under consideration.
Figure 10. - Effect of combustion temperature on relative combustor area and on relative temperature gradient for heat transfer.  
Constant combustor-exit Mach number, 0.1.
Figure 11. - Heat-transfer coefficient for hydrogen at 8-atmosphere pressure. \( (St)(Pr)^{2/3} = 0.003; \ h = 3600 \frac{(St)(Pr)^{2/3}}{Pr} \left( \frac{\rho c_p \sqrt{T_{gm}}}{R} \right)^{1/2} \frac{M}{\sqrt{T}}. \)
Figure 12. - Heat-transfer coefficient for hydrogen-rich mixture combustion products. Pressure, 2.2 atmospheres; \((\text{St})(\text{Pr})^{2/3} = 0.003;\
\[
h = 3600 \frac{(\text{St})(\text{Pr})^{2/3}}{(\text{Pr})^{2/3}} \left( \frac{\text{C}_p}{\text{R}} \right) \frac{M}{\sqrt{T}}.
\]
Figure 13. - Comparison of effect of combustion temperature on relative heat-transfer coefficient and relative effective temperature gradient for heat transfer.
(a) Maximum wall temperature as determined from gas temperatures and relative heat-transfer coefficients.
Hydrogen-outlet temperature, 2000° R.

(b) Yield strength - temperature relation for a high-strength alloy, Inconel X (ref. 3).

Figure 14. - Parameters determining limiting wall temperature for stationary regenerator.
Figure 15. - Change in heat-transfer-area requirement with changing turbine-inlet temperature and combustion temperature. Stationary exchanger.
Figure 16. - Heat-exchanger-area - pressure-drop relations for several tube-shell core configurations.
Figure 17. - Schematic diagram of Rex III cycle (ref. 1) engine with rotary heat exchanger.
Figure 18. - Heat-exchanger-core weight - pressure-drop relation for several rotary heat exchangers.
Figure 19. Effect of varying turbine-inlet temperature on relative weight of rotary-heat-exchanger core.