RESEARCH MEMORANDUM

AN ANALYSIS OF A NUCLEAR POWERED SUPERCRITICAL-WATER CYCLE FOR AIRCRAFT PROPULSION

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SUMMARY

An analysis to indicate the feasibility of the supercritical water compressor jet cycle for nuclear powered aircraft is presented. Performance values of the cycle are given for a range of design-point engine operating conditions at supersonic flight conditions of 1.5 flight Mach number and 50,000, 40,000, and 30,000 feet altitudes, and at subsonic flight conditions of 0.9 flight Mach number and 40,000 feet altitude.

Calculations were made at these combinations of flight conditions for various steam temperatures at the reactor outlet and steam pressures at the turbine outlet (the steam pressure at the reactor outlet was fixed at 5000 lb/sq in.) to evaluate the combination of compressor pressure ratio and Mach number of the air entering the heat exchanger which result in maximum thrust per unit engine weight (or minimum airplane gross weight). Airplane gross weight, reactor heat release, airflow rate, the engine frontal area, and heat exchanger frontal area required at these conditions for maximum thrust per unit engine weight are evaluated for a range of values of lift-drag ratio of the airplane assuming a fixed value of the ratio of airplane structural to gross weight of 0.35 and a fixed value of the sum of the reactor, shield, payload, and auxiliary weights of 150,000 pounds.

The combination of compressor pressure ratio and Mach number of air entering exchanger for minimum airplane gross weight is not the same as that for minimum reactor heat release, minimum engine frontal area, or minimum exchanger frontal area. In general, when operation is at the...
combination for minimum airplane gross weight, reasonable values of reactor heat release rates and exchanger frontal areas but high values of engine frontal areas are obtained.

The performance of the cycle at a pressure ratio and exchanger inlet Mach number which result in a 2 percent greater gross weight but about 18 percent lower engine frontal area than the minimum gross weight case, for a given set of flight and steam operating conditions, is presented in the following table. These values are representative of the performance of the cycle at the flight and altitude conditions listed.

<table>
<thead>
<tr>
<th>Sum of reactor, shield, payload, and auxiliary weights, lb</th>
<th>150,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio of airplane structure to gross weight</td>
<td>0.35</td>
</tr>
<tr>
<td>Flight Mach number</td>
<td>1.5</td>
</tr>
<tr>
<td>Altitude, ft</td>
<td>50,000</td>
</tr>
<tr>
<td>Over-all lift to drag ratio of airplane</td>
<td>5</td>
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<tr>
<td>Steam temperature at reactor outlet, °R</td>
<td>1460</td>
</tr>
<tr>
<td>Steam pressure at reactor outlet, lb/sq in.</td>
<td>5000</td>
</tr>
<tr>
<td>Steam pressure at turbine outlet, lb/sq in.</td>
<td>1250</td>
</tr>
<tr>
<td>Compressor pressure ratio</td>
<td>1.27</td>
</tr>
<tr>
<td>Mach number of air at exchanger inlet</td>
<td>0.18</td>
</tr>
<tr>
<td>Thrust per unit air flow rate, lb/(lb/sec)</td>
<td>12.37</td>
</tr>
<tr>
<td>Engine weight per unit air flow rate, lb/(lb/sec)</td>
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</tr>
<tr>
<td>Thrust per unit engine weight, lb/1b</td>
<td>0.98</td>
</tr>
<tr>
<td>Air flow rate, lb/sec</td>
<td>5440</td>
</tr>
<tr>
<td>Steam flow rate, lb/sec</td>
<td>577</td>
</tr>
<tr>
<td>Reactor heat release rate, Btu/sec</td>
<td>460,000</td>
</tr>
<tr>
<td>Airplane gross weight, lb</td>
<td>336,000</td>
</tr>
<tr>
<td>Engine weight, lb</td>
<td>68,500</td>
</tr>
<tr>
<td>Exchanger weight, lb</td>
<td>39,400</td>
</tr>
<tr>
<td>Structure weight, lb</td>
<td>117,500</td>
</tr>
<tr>
<td>Compressor frontal area, sq ft</td>
<td>570</td>
</tr>
<tr>
<td>Exchanger frontal area, sq ft</td>
<td>1195</td>
</tr>
</tbody>
</table>

In order for the exchanger to fit into the duct determined by compressor frontal area, the exchanger must be slanted at an angle of approximately 30° with the horizontal axis of the duct.

The performance of this typical case indicates that the cycle is one requiring low compressor pressure ratios, and one developing low thrust per unit air flow; consequently, it requires high air flow rates and large compressor and exchanger frontal areas.

INTRODUCTION

Analyses to determine some of the characteristics of various aircraft propulsion cycles utilizing nuclear energy are being made at the
NACA Lewis laboratory. Studies of the direct-air and liquid-metal turbojet cycles are presented in references 1 and 2, the liquid-metal turbine-propeller cycle in reference 3, and the liquid-metal ducted-fan cycle in reference 4.

A supercritical water-compressor jet cycle is described in reference 5. In this cycle, water at supercritical pressure is the reactor coolant and also serves as the moderating material in the reactor. A favorable characteristic of this cycle is that relatively low coolant temperatures out of the reactor are encountered. Also, the use of water as a coolant may relieve the corrosion problem in the reactor that exists with liquid-metal coolants. The water, however, is a poorer heat-transfer material than liquid metals so that the heat-removal problem in the reactor at high heat-release rates becomes important. In addition, the high supercritical pressure at which the coolant is maintained in the reactor creates another important reactor design problem. The water is highly pressurized in order to avoid boiling during the heating process. The sudden and extreme variation in fluid properties that accompanies boiling aggravates the heat-transfer design problem and also might further complicate reactor control. In short, this cycle is characterized by some reactor development problems that are more severe and some that are less severe than those encountered in the liquid-metal and air-cooled reactor. Inasmuch as it offers some possible advantages in the development of nuclear-powered aircraft, the cycle deserves further investigation.

Many of the important performance parameters of the supercritical water-compressor jet cycle, such as airplane gross weight and reactor heat release required, are functions of the airplane characteristics as well as of the engine characteristics. A complete cycle study requires detailed studies of all factors involved in reactor, airplane, and engine design and performance, for both design and off-design-point operation.

As a first step, an exploratory design-point analysis of this cycle has been made which utilizes many simplifying assumptions to avoid some of the detailed studies just described. These assumptions are listed and discussed in appendix A. In the analysis, the performance of the cycle is evaluated over a wide range of flight and engine design-point operating conditions in order to determine some of the effects of these operating conditions on cycle performance. Both supersonic and subsonic flight conditions are studied. The supersonic case investigated was at a flight Mach number of 1.5 and altitudes of 50,000, 40,000, and 30,000 feet. The subsonic case was at a flight Mach number of 0.9 and altitude of 40,000 feet. The engine operating conditions investigated were supercritical steam temperature at the reactor outlet, steam pressure at turbine outlet (the steam pressure at turbine inlet
being fixed), air compressor pressure ratio, and Mach number of the air entering the heat exchanger. The effect of over-all lift-drag ratio of the airplane was also studied.

Some of the parameters that are used to evaluate the performance of the cycle are airplane gross weight, reactor heat release rate, engine frontal area, and exchanger frontal area. In general, optimum operating conditions for any one of these parameters are not the optimum conditions for the other parameters. In this analysis at each set of given flight conditions and steam conditions, the values of compressor pressure ratio and Mach number of air entering the exchanger which result in maximum thrust per unit engine weight (or minimum airplane gross weight) were determined. In general, this combination of compressor pressure ratio and exchanger-inlet air Mach number results in reasonable values of the other performance parameters. Values of airplane gross weight, reactor heat release rate, air flow, and compressor frontal area at these conditions were evaluated for a range of lift-drag ratios of the airplane assuming a fixed value of airplane structure to gross weight ratio and a fixed value for the sum of reactor, shield, payload, and auxiliary weights. The effect of the variation in individual engine operating variables on the performance and sizes of the components is included in order to help give a clearer picture of the effect of these variables on over-all cycle performance.

DESCRIPTION OF CYCLE

A schematic diagram of the cycle is shown in figure 1. High-temperature steam at supercritical pressure leaves the reactor and is expanded through a turbine, producing useful work output. It then passes through a heat exchanger where it is completely condensed. The condensate is then pumped to a sufficiently high pressure to maintain the flow of the water in the closed water circuit and enters the reactor where it is heated to operating temperature.

Air, which is the propulsive fluid, enters the inlet diffuser where it is compressed. It is further compressed in the compressor and then heated in the heat exchanger by the condensing steam. The heated air is then expanded through the exhaust nozzle from which it issues as a high-velocity jet.

The steam turbine drives both the air compressor and condensate pump. The turbine-to-pump drive is assumed to be direct, whereas the turbine drives the air compressor through reduction gears.
SYMBOLS

The following symbols are used in this report:

- $A_c$: compressor frontal area, sq ft
- $A_x$: exchanger frontal area, sq ft
- $A_{x,f}$: exchanger air flow area, sq ft
- $C_v$: exhaust nozzle velocity coefficient
- $D_f$: fuselage drag, lb
- $D_n$: nacelle drag, lb
- $D_t$: tail drag, lb
- $D_{tot}$: total airplane drag, lb
- $D_w$: wing drag, lb
- $d$: hydraulic diameter of air passage
- $F$: net thrust, lb
- $f$: free flow factor in heat exchanger
- $g$: acceleration due to gravity, ft/sec$^2$
- $\Delta H_c$: compressor work, Btu
- $\Delta H_p$: condensate pump work, Btu
- $\Delta H_t$: turbine work, Btu
- $h$: heat transfer coefficient, $\frac{\text{Btu}}{\text{sec} \cdot \text{O}\text{F} \cdot \text{sq ft}}$
- $h_{pt}$: turbine horsepower
- $k$: thermal conductivity, $\frac{\text{Btu-ft}}{\text{sec} \cdot \text{O}\text{F} \cdot \text{sq ft}}$
- $L$: lift of airplane, lb
- $l$: length of heat transfer passage, ft
Mach number of air entering heat exchanger

Mach number of air entering heat exchanger

steam pressure at reactor outlet, lb/sq in.

steam pressure at reactor outlet, lb/sq in.

steam pressure at turbine outlet, lb/sq in.

steam pressure at turbine outlet, lb/sq in.

water pressure at exchanger outlet, lb/sq in.

water pressure at condensate pump outlet, lb/sq in.

water pressure at condensate pump outlet, lb/sq in.

Prandtl number

Prandtl number

total pressure of free stream air, lb/sq ft

total pressure of free stream air, lb/sq ft

total pressure of air at compressor inlet, lb/sq ft

total pressure of air at compressor inlet, lb/sq ft

total pressure of air at compressor outlet, lb/sq ft

total pressure of air at compressor outlet, lb/sq ft

total pressure of air at exchanger outlet, lb/sq ft

total pressure of air at exchanger outlet, lb/sq ft

ambient air static pressure, lb/sq ft

ambient air static pressure, lb/sq ft

reactor heat release rate, Btu/sec

reactor heat release rate, Btu/sec

heat-exchanger heat flow, Btu/sec

heat-exchanger heat flow, Btu/sec

Reynolds number

Reynolds number

total temperature of steam at reactor outlet, °R

total temperature of steam at reactor outlet, °R

total temperature of steam at turbine outlet, °R

total temperature of steam at turbine outlet, °R

total temperature of condensate at exchanger outlet, °R

total temperature of condensate at exchanger outlet, °R

total temperature of water at condensate pump outlet, °R

total temperature of water at condensate pump outlet, °R

bulk total temperature of air in the exchanger, °R

bulk total temperature of air in the exchanger, °R

saturation temperature of steam corresponding to steam pressure, P_B, °R

saturation temperature of steam corresponding to steam pressure, P_B, °R

wall temperature in exchanger, °R

wall temperature in exchanger, °R

effective wall temperature in exchanger, °R

effective wall temperature in exchanger, °R

equivalent constant wall temperature in exchanger, °R

equivalent constant wall temperature in exchanger, °R
T₁  total temperature of air at compressor inlet, °R
T₂  total temperature of air at compressor outlet, °R
T₂' total temperature of air after being heated by condensing phase of the steam, °R
T₃  total temperature of air at exchanger outlet, °R
Vₖ  jet velocity, ft/sec
V₀  airplane velocity, ft/sec
Wₗ  weight of gear box, lb
Wₖ  weight of compressor, lb
Wₑ  weight of engine, lb  Wₑ = Wₗ + Wₖ + Wₑ₊ₚ + Wₓ
Wₔ  gross weight of airplane, lb  Wₔ = Wₑ + Wₖ + Wₛ
Wₖ  weight of auxiliary group; shield, reactor, payload, and piping, lb
Wₛ  airplane structural weight, lb
Wₑ₊ₚ weight of turbine and condensate pump, lb
Wₓ  weight of heat exchanger, lb
Wₑ  air flow rate, lb/sec
Wₛ  steam flow rate, lb/sec
ηₑ  reduction gearing efficiency  
ηₑ  compressor efficiency
ηₖ  fin effectiveness
ηₚ  condensate pump efficiency
ηₜ  turbine efficiency
ANALYSIS

Equations for Performance Parameters

Some of the important parameters used to evaluate the characteristics of the cycle are airplane gross weight $W_g$, reactor heat release required $Q_r$, air flow $w_a$, compressor frontal area $A_c$, and exchanger frontal area $A_x$. The value of $L/D_{tot}$ that can be obtained at supersonic flight in general depends on $A_c$.

In reference 2 it is shown that the airplane gross weight can be expressed as

$$W_g = \frac{W_k}{l - \frac{W_s}{W_g} - \frac{D_{tot}}{L} \frac{W_e}{F}}$$  \hspace{1cm} (la)

Another form of equation (la) can be obtained by expressing the total drag as the sum of the wing drag, fuselage drag, tail drag, and nacelle drag. Thus the net thrust, which is equal to the total drag, is

$$F = D_{tot} = D_w + D_f + D_t + D_n$$

Inasmuch as the nacelle drag is often charged against the engine

$$F - D_n = D_w + D_f + D_t$$

and equation (la) can be rewritten as

$$W_g = \frac{W_k}{l - \frac{W_s}{W_g} - \frac{W_e}{F-D_n} \frac{D_w + D_f + D_t}{L}}$$  \hspace{1cm} (lb)

It is apparent from equations (la) or (lb) that for assumed values of $W_k$ and $W_s/W_g$, airplane gross weight is a minimum at those conditions giving a maximum value of

$$\frac{L}{D_{tot}} \frac{F}{W_e}$$
or its equivalent

$$\frac{L}{D_w + D_t + D_f} W_e$$

In the analysis, a constant $L/D_{tot}$ was chosen, mainly to avoid the more complicated calculations involved in the evaluation of all
the drag forces. For this constant $L/D_{tot}$, performance at maximum $F/We$ was determined. For one case, constant $L/(D_w + D_t + D_f)$ was assumed, a given airplane configuration was selected, the nacelle drag of this configuration was evaluated, and the performance at maximum $F-D_n/We$ was determined.

The net thrust per unit engine weight is the ratio of the net thrust per unit air flow rate to the engine weight per unit air flow rate

$$\frac{F}{We} = \frac{F/w_a}{We/w_a}$$

(2)

The reactor heat release can be expressed as $Q_r \equiv \frac{Q_r}{w_a} \frac{w_a}{F}$ and substituting in this identity $F = W_g \frac{D_{tot}}{L}$

$$Q_r = \frac{Q_r}{w_a} \frac{w_a}{F} \frac{D_{tot}}{L} W_g$$

(3)

The air flow

$$w_a = \frac{F}{F/w_a} = \frac{W_g}{L} \frac{F}{D_{tot} w_a}$$

(4)

The compressor frontal area

$$A_c = \frac{w_a}{w_a/A_c}$$

(5)

A value for the corrected air flow per unit compressor frontal area is assumed for this analysis (see appendix A). This value and the flight conditions determine $w_a/A_c$.

The exchanger frontal area

$$A_x = \frac{A_{x,f}}{f} = \frac{A_{x,f}}{w_a f} w_a$$

(6)

The value of $w_a/A_{x,f}$ is evaluated from the air conditions and the Mach number at the exchanger inlet, and $f$ depends on the exchanger configuration assumed.
Equations (1a), and (2) to (6) show that for any assumed fixed values of \( W_k, W_s/W_g, \) and \( L/D_{tot} \) the performance parameters \( F/W_e, W_g, Q_r, \) \( A_x, \) and \( A_c \) at any set of design operating conditions can be determined by evaluating the values of \( F/w_a, W_e/w_a, \) and \( Q_r/w_a \) at these conditions.

Evaluation of Thrust per Unit Air Flow Rate, \( F/w_a \)

The thrust per unit air flow rate is equal to \( F/w_a = \frac{1}{g} (V_j - V_0) \) so that, for a given flight speed, evaluating \( F/w_a \) consists essentially in evaluating \( V_j. \) The jet velocity is a function of the air temperature at inlet to the exhaust nozzle \( T_3, \) and the pressure ratio across the nozzle \( P_3/P_0, \) a constant value of nozzle velocity coefficient being assumed. The values of \( P_3/P_0 \) and \( T_3 \) depend on the given set of operating conditions of both the closed steam cycle and the air cycle.

Steam cycle. - All steam conditions throughout the steam cycle and energy interchanges in the various components, such as the turbine work per unit steam flow \( \Delta H_t/w_s, \) the heat exchanged in the condenser per unit steam flow for subcooling to 20° F below the saturation temperature \( Q_x/w_s, \) the condensate pump work per unit steam flow \( \Delta H_p/w_s, \) and the reactor heat release per unit steam flow \( Q_r/w_s \) are determined using steam tables (ref. 6). The following table gives a list of the combinations of design-point steam conditions investigated at various flight and altitude conditions. The minimum value of \( T_A \) is limited to 1360° R in order to avoid excessive condensation of the steam expanding through the turbine.
<table>
<thead>
<tr>
<th>Flight Mach number</th>
<th>Altitude, ft</th>
<th>( P_A, ) lb/sq in.</th>
<th>( T_A, ) °R</th>
<th>( P_B, ) lb/sq in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>50,000</td>
<td>5000</td>
<td>1660</td>
<td>2500</td>
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<td></td>
<td>1560</td>
<td>2500</td>
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<td>1250</td>
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<tr>
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<td>1460</td>
<td>2500</td>
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<td>2500</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>625</td>
</tr>
</tbody>
</table>

The total work available to drive the air compressor, with gear losses neglected, is

\[
\Delta H_C = w_s \left( \frac{\Delta H_t}{w_s} - \frac{\Delta H_p}{w_s} \right)
\]

**Air cycle.** - Entering air is compressed in the diffuser to a pressure ratio \( \frac{P_1}{P_0} \) which depends on flight conditions and diffuser efficiency. Further compression in the compressor to a given pressure ratio \( \frac{P_2}{P_1} \) requires a compressor work per unit air flow \( \Delta H_c/w_a \) and an accompanying \( T_2 \) which is evaluated from air tables (ref. 7).

Equating the total compressor work \( \Delta H_c = w_a \frac{\Delta H_c}{w_a} \) to the work available from the steam results in

\[
\frac{w_a}{w_s} = \frac{\Delta H_t}{w_s} - \frac{\Delta H_p}{w_s}
\]

(7)
This equation shows that for a given set of steam operating conditions the ratio $\frac{w_a}{w_s}$ is a function of $\frac{\Delta H_c}{w_a}$ (or $\frac{P_2}{P_1}$).

Air entering the exchanger at $T_2$ and $P_2$ acquires all the heat released by the steam so that the heat gained per unit air flow $\frac{Q_x}{w_a} = \frac{(\frac{Q_x}{w_s})}{(\frac{w_s}{w_a})}$. The air temperature at exchanger outlet $T_3$ is determined from $T_2$ and $\frac{Q_x}{w_a}$ by air tables. The air temperature $T_3$ cannot exceed $T_B$ so that any value of $\frac{P_2}{P_1}$ which results in such an ambiguous solution of the equations is not in the achievable operating range of the engine.

The method used to calculate the pressure ratio across the exchanger $\frac{P_3}{P_2}$ is described in a subsequent discussion on heat exchanger performance. The jet velocity is evaluated from $\frac{P_3}{P_0}$ and $T_3$ where $\frac{P_3}{P_0} = \frac{P_3}{P_2} \frac{P_1}{P_0}$.

**Heat-Exchanger Calculations**

An exchanger configuration as shown in figure 2 was selected in order to provide a basis for evaluating the exchanger weight, size, and pressure drop (for further discussion of exchanger configuration, see appendix A). The heat-exchanger calculations are based on the assumption that the finned passage through which the air flows is a tube having an equivalent diameter, in this case, of 0.01713 feet.

For the more general case where the steam from the turbine discharge is superheated, the cooling of the steam is divided into two stages for the purpose of calculation. The superheated steam is first considered to be cooled to dry saturated steam in the first stage and is then completely condensed and subcooled in the second stage. The exchanger calculations are based on the assumption that the heat exchanger is equivalent to two exchangers in series; in the first exchanger the air is heated from $T_2$ to an intermediate temperature $T_2'$ by the condensing steam, and in the second exchanger the air is further heated to the temperature $T_3$ by the superheated steam. Figure 3 shows this orientation of exchangers as well as the temperatures of the air and steam. The sum of the length-diameter ratio $\frac{l}{d}$ of each of these two exchangers is the $\frac{l}{d}$ of the equivalent single exchanger, and the sum of the pressure drops in the two exchangers is the over-all pressure drop of the equivalent single exchanger.
Evaluation of \( l/d \). - Heat-transfer coefficients of the air are evaluated with the equation given in reference 8.

\[
\frac{h d}{k} = 0.023 \, Re^{0.8} \left( \frac{T_b}{T_w} \right)^{0.8} \, Pr^{0.4}
\]

where \( T_b \) is the bulk temperature of the air and \( T_w \) is the wall temperature (which in this case is assumed equal to the steam temperature). The Reynolds number \( Re \) is evaluated from the velocity and density based on bulk temperature of air, while all the other properties are evaluated at the wall temperature.

For the first exchanger (in which air is heated by condensing steam) an average value of \( h \) is determined at an air bulk temperature of \( \frac{1}{2} (T_2 + T_2') \) and the corresponding wall temperature (saturated steam temperature). This value of \( h \) is used to evaluate the fin effectiveness and the effective wall temperature at the inlet and outlet of the exchanger (see sample calculations in appendix B). For the second exchanger, an average value of \( h \) is evaluated at an air bulk temperature of \( \frac{1}{2} (T_{sat} + T_B) \) and its corresponding average wall temperature of \( \frac{1}{2} (T_{sat} + T_B) \). For each exchanger the log mean temperature difference is evaluated from the temperature differences between the effective wall temperature and the air bulk temperature at both inlet and outlet of each exchanger. The \( l/d \) of each exchanger is evaluated from the equation

\[
l/d = \frac{Q_x/w_a}{4 \, h \left( \frac{A_{x,f}}{w_a} \right) (\text{log mean temperature difference})}
\]

\( Q_x/w_a \) is evaluated from \( Q_x/w_s \) which is fixed for a given set of steam cycle conditions, and \( w_a/w_s \) which is determined as previously described. The value of \( w_a/A_{x,f} \) is evaluated from the values of \( P_2 \) and \( T_2 \) of air and the given value of the Mach number at the inlet to the exchanger.

Evaluation of air pressure drop. - In each exchanger a constant equivalent wall temperature is evaluated which results in the same log mean temperature difference as previously obtained to calculate \( l/d \) from effective wall temperatures. The air pressure drops are determined from these constant equivalent wall temperatures and the
pressure-drop charts of reference 2, which are based on the method presented in reference 9. The pressure ratio across each exchanger is evaluated from the values of the constant equivalent wall temperature, the bulk air temperatures entering and leaving each exchanger and the Mach number of the air entering each exchanger. The Mach number of the air entering the second exchanger is equal to that of the air leaving the first exchanger, which is evaluated from the air flow rate per unit area and exit conditions from the first exchanger.

In order to account for additional losses in pressure due to the turning of the air flow into and out of the exchanger, the pressure drop previously evaluated is increased by 15 percent. Referring to the sample calculations presented in appendix B will help clarify the heat-exchanger calculations.

Evaluation of Engine Weight per Unit Air Flow Rate $W_e/w_a$

The engine weight consists of the weights of exchanger, compressor, reduction gears, steam turbine and condensate pump. In appendix A, equations used to evaluate the weights of these components are given.

Evaluation of Reactor Heat Release per Unit Air Flow Rate $Q_r/w_a$

The reactor heat release $Q_r/w_a$ is obtained from the values of $Q_r/w_s$ and $w_a/w_s$. The value of $Q_r/w_s$ is fixed for a given set of steam cycle conditions, and the value of $w_a/w_s$ obtained from equation (7).

A sample set of calculations illustrating the method used in evaluating cycle performance is given in appendix B.

RESULTS AND DISCUSSION

In the analysis the performance at two flight Mach numbers was investigated: a supersonic case at a flight Mach number of 1.5 and altitudes of 50,000, 40,000, and 30,000 feet; and a subsonic case at a flight Mach number of 0.9 and altitude of 40,000 feet.
Supersonic Flight Conditions

Effect of compressor pressure ratio $P_2/P_1$ and $M_2$. - In order to show the effect of $P_2/P_1$ and $M_2$ the following fixed set of flight and engine operating conditions were arbitrarily selected: altitude, 50,000 feet; reactor outlet steam temperature $T_A$, 1460° R; reactor outlet steam pressure $P_A$, 5000 pounds per square inch; and turbine outlet steam pressure $P_B$, 1250 pounds per square inch.

Figure 4 shows the effect of varying $P_2/P_1$ and $M_2$ on several performance parameters. Figure 4(a) shows the variation in thrust per unit engine weight $F/W_e$ and thrust per unit air flow rate $F/w_a$. It is apparent that there are values of both $P_2/P_1$ and $M_2$ which result in maximum $F/W_e$ (that is, minimum gross weight) for the set of fixed design conditions. In figure 4(a) a maximum value of $F/W_e$ of 1.02 is obtained at a value of $P_2/P_1$ equal to 1.214 and a value of $M_2$ equal to 0.20. The corresponding value of $F/w_a$ is 10.0 pounds thrust per pound per second. Figure 4(b) presents the corresponding values of reactor heat release rate $Q_r$, air flow rate $w_a$, and exchanger frontal area $A_x$ required, for values of $L/D_{tot}$ equal to 5, $W_k$ equal to 150,000 pounds, and $W_s/W_g$ equal to 0.35. There are also optimum values of $P_2/P_1$ and $M_2$ resulting in minimum $Q_r$, in minimum $w_a$ (or minimum $A_c$), and in minimum $A_x$ (these being functions of the values of $L/D_{tot}$, $W_s/W_g$, and $W_k$ as well as steam conditions $T_A$, $P_A$, and $P_B$). From figure 4(b) the minimum $Q_r$ of $4.34 \times 10^3$ Btu per second is obtained at a value of $M_2$ of 0.15 and a value of $P_2/P_1$ of 1.230. This $Q_r$ is about 4 percent lower than the $Q_r$ at conditions for maximum $F/W_e$. Minimum $w_a$ of 4800 pounds per second occurs at about a $P_2/P_1$ of 1.34 and $M_2$ of about 0.12. This $w_a$ is about 35 percent lower than the $w_a$ at conditions for maximum $F/W_e$. Minimum exchanger frontal area $A_x$ of 1100 square feet is obtained at a $P_2/P_1$ of about 1.30 and $M_2$ of 0.20 which is approximately 20 percent lower than the $A_x$ at conditions for maximum $F/W_e$.

Figures 5 and 6 show how the performance of the various components of the engine are interrelated and affected by variation in compressor pressure ratio and $M_2$. These help give a clearer picture of cycle performance variation with design-point operating variables. Figure 5 illustrates the effect produced by varying $P_2/P_1$ while other cycle...
conditions are held constant \((T_A, 1460^\circ R; P_A, 5000 \text{ lb/sq in.}; P_B, 1250 \text{ lb/sq in.}; M_2, 0.20)\). Figure 5(a) shows the variation in air temperature leaving the exchanger \(T_3\), exchanger \(l/d\), pressure ratio across the exchanger \(P_3/P_2\), available pressure ratio across the exhaust nozzle \(P_3/P_0\), and \(F/\omega_a\), as \(P_2/P_1\) varies. As a result of equating the net work output of the steam cycle to the compressor work input, and in the exchanger equating the heat released by the steam to the heat absorbed by the air, \(T_3\) increases as \(P_2/P_1\) is increased. This increase in \(T_3\) necessitates increases in the \(l/d\) of the exchanger, which becomes large as \(T_3\) approaches the maximum steam temperature into the exchanger \(T_B\) (which is fixed). The pressure drop in the exchanger increases with \(l/d\); hence the drop in \(P_3/P_2\) as \(P_2/P_1\) increases. The nozzle pressure ratio \(\frac{P_3}{P_0} = \frac{P_1}{P_0} \frac{P_2}{P_2}\), where \(P_1\) is fixed. At first as \(P_2/P_1\) increases, the accompanying drop in \(P_3/P_2\) is small enough so that \(P_3/P_0\) increases. This increase in \(P_3/P_0\) together with an increase in \(T_3\) results in a large increase in \(F/\omega_a\). Soon a value of \(P_2/P_1\) is reached where the drop in \(P_3/P_2\) is large enough to result in a decrease in \(P_3/P_0\), and eventually \(P_3/P_0\) is decreasing rapidly enough to cause \(F/\omega_a\) to start leveling off despite the increasing \(T_3\).

Figure 5(b) shows the effects of \(P_2/P_1\) on the weights per unit air flow of engine components and the over-all engine. The values of compressor weight \(W_c\), gearing weight \(W_b\), turbine and pump weight \(W_{t+p}\) are relatively small and their variation depends largely on the weight assumptions made. The weight of the exchanger \(W_x\) is the major item influencing engine weight \(W_e\). The exchanger weight increases almost directly with \(l/d\); hence, the values of \(W_x/\omega_a\) and \(W_e/\omega_a\) increase with \(P_2/P_1\), this increase becoming large when the increase in \(l/d\) becomes very rapid.

The variation in \(F/W_e\) (also shown in fig. 5(b)) depends on the manner in which \(F/\omega_a\) and \(W_e/\omega_a\) vary. It is apparent that if \(F/W_e\) is at first increasing with increasing \(P_2/P_1\), it will eventually reach a maximum and then decrease because \(F/\omega_a\) will eventually be leveling off whereas \(W_e/\omega_a\) will be continuously increasing.

Figure 5(c) presents the variation in airplane gross weight, reactor heat release, total thrust required, engine weight, air and steam
flow rates and the compressor and exchanger frontal areas with compressor pressure ratio. The values of these parameters are based on fixing the values of \( \frac{L}{D_{\text{tot}}} \) equal to 5, \( W_k \) equal to 150,000 pounds, and \( \frac{W_s}{W_g} \) equal to 0.35. It is to be noted from figure 5(c) that minimum gross weight, engine weight, and thrust required all occur when \( \frac{F}{W_e} \) is a maximum, and that minimum reactor heat release is required when the steam flow rate \( w_s \) is a minimum. Minimum compressor frontal area which depends directly on air flow occurs at minimum air flow.

The effect of varying \( M_2 \) on heat-exchanger performance largely determines its effect on over-all cycle performance. In figure 6, a range of values of \( M_2 \) is studied, the value of \( \frac{P_2}{P_1} \) being fixed at 1.21. Other fixed operating conditions are the same as for figure 5. For this case, fixing steam cycle conditions and \( \frac{P_2}{P_1} \) fixes \( T_3 \). Increased \( M_2 \) increases the heat-transfer coefficient and also the air flow through any exchanger flow passage, with an over-all effect that \( \frac{1}{d} \) necessary to obtain a given \( T_3 \) is very nearly proportional to \( M_2^{0.2} \). Thus, \( \frac{1}{d} \) increases slightly with \( M_2 \). Increased \( M_2 \) increases the air flow per unit flow area of the exchanger \( \frac{w_a}{A_x,f} \), which is mostly influential in decreasing \( \frac{W_x}{W_a} \) and \( A_x/A_c \). On the other hand, the pressure drop through the exchanger increases with increased \( M_2 \) resulting in a continual drop in \( \frac{F}{W_a} \). At some value of \( M_2 \), the decrease in \( \frac{F}{W_a} \) eventually overcomes the effect of decrease in \( \frac{W_e}{W_a} \), so that \( \frac{F}{W_e} \) reaches a maximum value and then drops off.

In the analysis, performance at maximum \( \frac{F}{W_e} \) was considered most interesting so that \( \frac{P_2}{P_1} \) and \( M_2 \) were selected for maximum \( \frac{F}{W_e} \). The engine performance at conditions for maximum \( \frac{F}{W_e} \) usually results in reasonable values of \( Q_r, A_c, \) and \( A_x \), as will be shown subsequently.

Effect of steam pressure at turbine outlet \( P_B \). - Maximum values of \( \frac{F}{W_e} \) and the corresponding values of \( \frac{P_2}{P_1}, M_2, \) and \( \frac{F}{W_a} \) such as those discussed for figure 4 (which was for a turbine discharge pressure \( P_B \) of 1250 lb/sq in.) are plotted for a range of values of \( P_B \) in figure 7. The maximum value of \( P_B \) was limited to 2500 pounds per square inch. It is noted from figure 7 that for the given conditions \( \frac{F}{W_e} \) increases with increased \( P_B \), the values ranging from about 0.90 at a \( P_B \) equal to 625 pounds per square inch to about 1.18 at a \( P_B \) equal to 2500 pounds per square inch. The corresponding
values of $F/W_a$ show the same trend, increasing from 8.7 to 11.5 pounds per pound per second as $P_B$ varies from 625 to 2500 pounds per square inch. Other items of interest seen from the figure are that $M_2$ varies only slightly with $P_B$, its value being close to 0.20 while $P_2/P_1$ decreases as $P_B$ increases. The values of $P_2/P_1$ range from 1.23 to 1.16, which values are readily attainable in the single-stage compressor assumed.

Figure 8 is included to indicate the influence of $P_B$ on component performance in order to show more clearly its effect on overall performance. Several curves of figure 5 ($P_B = 1250$ lb/sq in.) were cross-plotted against $T_3$ and similar sets of curves plotted for values of $P_B$ of 625 and 2500 pounds per square inch. Other fixed operating conditions are $T_A$, $1460^\circ$ R; $P_A$, 5000 pounds per square inch; and $M_2$, 0.20. Increasing $P_B$ increases the steam temperatures in the exchanger resulting in lower $l/d$ required to obtain a given $T_3$ (with consequent lower pressure drop of the air in the exchanger). On the other hand, at higher $P_B$, less turbine work per unit steam flow is obtained which results in lower $P_2/P_1$. The net effect of increased $P_B$ on $F/W_e$ depends on whether the benefits of lower pressure drop in the exchanger and lower exchanger weight overbalance the effect of lower $P_2/P_1$. From figure 8 it is apparent that for the given set of conditions, this is the case and higher $P_B$ improves $F/W_e$.

The overall performance of the cycle operating at values of $P_2/P_1$ and $M_2$ giving maximum $F/W_e$ (that is minimum $W_g$) for a range of values of $L/D_{tot}$ is presented in figure 9. Values of $W_g$ are plotted against values of $Q_r$ for values of $L/D_{tot}$ equal to 4, 5, and 6 in figure 9(a). Lines of constant $P_B$ are also located on these plots. As for most of the previous figures, the given conditions are $T_A$, $1460^\circ$ R; $P_A$, 5000 pounds per square inch; $W_k$, 150,000 pounds; and $W_s/W_g$, 0.35.

It is seen from the figure that values of $W_g$ ranging from 300,000 to 400,000 pounds and values of $Q_r$ ranging from 350,000 to 650,000 Btu per second (depending on the value of $L/D_{tot}$ assumed) are obtained for this set of conditions. It should be remembered that both $W_g$ and $Q_r$ vary directly with $W_k$. Therefore, for any value of $W_k$ other than 150,000 pounds, values of $W_g$ and $Q_r$ are obtained by multiplying the values of figure 9(a) by the factor $W_k/150,000$. 
Values of \( w_a \), \( A_c \), and \( A_x/A_c \) corresponding to the values of figure 9(a) are presented in figure 9(b). The very high values of \( w_a \) (420 to 1150 lb/sec) required are mainly due to low values of \( F/w_a \) characteristic of the cycle and the low values of \( L/D_{tot} \) encountered at the flight conditions (flight Mach number, 1.5; altitude, 50,000 ft) of figure 9. At these flight conditions the compressor handles only 9.53 pounds of air per second per square foot of frontal area with consequent huge engine frontal areas (450 to 1220 sq ft) required. It is interesting to note that the exchanger frontal area is about 2.0 times as large as the compressor frontal area. This requires slanting the exchanger so it will fit in any nacelle determined by compressor frontal area. The angle of slant will be about 30° with the horizontal, which is large enough so that no excessive pressure losses are incurred in deflecting the air stream into and out of the exchanger.

Figure 10 is included to compare the performances when operating at conditions for minimum \( W_g \) (i.e. maximum \( F/W_e \)), minimum \( Q_r \), minimum \( A_c \), or minimum \( A_x \). Values of \( W_g \), \( Q_r \), \( A_c \), and \( A_x \) as well as the optimum values of \( P_2/P_1 \) and \( M_2 \) are plotted for a range of values of \( P_B \). The steam temperature \( T_A \) is 1460° R and the value of \( L/D_{tot} \) assumed is 5. It is seen that higher values of \( P_2/P_1 \) are required for minimum \( A_c \) and \( A_x \) than for minimum \( Q_r \) and \( W_g \), and that higher values of \( M_2 \) are required for minimum \( W_g \) and \( A_x \) than for minimum \( Q_r \) and \( A_c \). It is also seen from the figure that operation at conditions for minimum \( W_g \) results in reasonable values of \( Q_r \) and \( A_x \) but somewhat high values of \( A_c \).

Variations in \( P_2/P_1 \) and \( M_2 \) from the values giving minimum gross weight will increase \( W_g \) and \( Q_r \) only slightly but can result in appreciable decreases in \( A_x \) and \( A_c \), hence resulting in a more reasonable airplane configuration. The performance of a case where \( P_2/P_1 \) and \( M_2 \) were chosen to give a such a combination of values of performance parameters is compared in the following table with the performance at minimum \( W_g \). Steam operating conditions are the same for both cases and are representative of what may be attained. The performance values presented in the table are typical of what can be expected from the cycle at a flight Mach number of 1.5 and altitude of 50,000 feet.
The table shows that varying conditions slightly from the values giving minimum $W_g$ may give somewhat more desirable performance. For the comparison shown in the table, increasing $P_2/P_1$ from 1.21 (the value for minimum $W_g$) to 1.27 and decreasing $M_2$ from 0.20 (the value for minimum $W_g$) to 0.18 result in a decrease in $w_a$ and $A_c$ of about 18 percent, and a decrease in $A_x$ of about 12 percent. Only a slight increase of 2 percent in $W_g$ and $Q_r$ is incurred.

Effect of steam temperature at reactor outlet $T_A$. - So far all performance has been discussed for a $T_A$ of 1460°F. It is interesting to see what improvement in cycle performance can be obtained by going to higher values of $T_A$, and also what penalty is incurred by going to lower values of $T_A$. In figures 11 to 13 are presented the performance of the cycle for a range of $T_A$ from 1360°F to 1660°F.

Maximum values of $F/We$ and corresponding values of $P_2/P_1$, $M_2$, and $F/w_a$ are plotted in figure 11 for a range of values of $T_A$, with $P_B$ fixed at 2500 pounds per square inch (the pressure at which highest $F/We$ was obtained). The optimum values of $P_2/P_1$ increase as $T_A$ increases, whereas the value of $M_2$ remains essentially constant between 0.19 and 0.20. As expected, increasing $T_A$ increases $F/We$ and $F/w_a$. Increasing $T_A$ from 1460°F to 1660°F R results in increased $F/We$ from 1.18 to 1.33 pounds per pound and increased $F/w_a$ from 11.5 to 13.4 pounds per pound per second. Decreasing $T_A$ from 1460°F to 1360°F R results in a decreased $F/We$ of 1.07 pounds per pound and a decreased $F/w_a$ of 10.2 pounds per pound per second.
Figure 12 is included to show again what effects a design-point variable (TA) has on component performance. At a given PB equal to 2500 pounds per square inch and M2 equal to 0.20, values of P2/P1, l/d, P3/P0, F/wa, W2/wa, We/wa, and F/We are plotted against T3 for values of TA equal to 1360°, 1460°, and 1660° R. It is seen from this figure that the main effect of increasing TA is to increase the work output per unit steam flow which results in higher P2/P1. This results in greater F/wa. Lower values of l/d are required to obtain a given T3; this improves Wx/wa and We/wa somewhat. Both effects of increased F/wa and decreased We/wa result in higher values of F/We.

Figure 13(a) presents the parameters Wg against Qr for this range of values of TA, at values of L/Dtot equal to 4, 5, and 6. The PB was fixed at 2500 pounds per square inch. The effects of TA on Wg and Qr are largely dependent on the value of L/Dtot; the effects being more pronounced the lower the value of L/Dtot. At an L/Dtot of 4, increasing TA from 1460° to 1660° R resulted in a decrease in Wg of about 5 percent, and a decrease in Qr of about 6 percent. Lowering TA to 1360° from 1460° R increased Wg about 5 percent and increased Qr almost 11 percent.

The air flows corresponding to the conditions of figure 13(a) are shown in figure 13(b). At an L/Dtot of 4, increasing the temperature TA from 1460° to 1660° R results in a reduction of about 20 percent in the air flow, whereas about a 20 percent increase in air flow results when TA is reduced from 1460° to 1360° R. The values of Ac corresponding to these air flows and the values of A/Ac required are also presented in figure 13(b).

Effect of altitude. - Figures 14 and 15 show the effect of altitude on the performance of the cycle. Altitudes considered were 50,000, 40,000, and 30,000 feet. The flight Mach number was held constant at 1.5, TA at 1460° R, and PB at 5000 pounds per square inch. Figure 14 shows the variation in F/wa, We/wa, and F/We (P2/P1 and M2 being chosen for best F/We) with altitude for a given value of PB equal to 2500 pounds per square inch. It is seen that We/wa decreases rapidly as altitude decreases. This is due mainly to the increased density at lower altitude; the result is that a smaller engine is required to handle a given air flow. On the other hand, F/wa drops off slightly as altitude decreases. The values of F/We increase appreciably as altitude decreases, indicating that the decrease in We/wa is much more effective than the decrease in F/wa.
In figure 15(a) the values of $W_g$ are plotted against $Q_r$ at the altitudes of 50,000, 40,000, and 30,000 feet. The values of $W_g$ and $Q_r$ are based on a fixed value of $L/D_{tot}$ equal to 5 and values of $W_s/W_g$ of 0.35 and $W_k$ of 150,000 pounds. Lines of constant $P_B$ of 2500, 1250, and 625 pounds per square inch are located on these plots. From this figure it is seen that $W_g$ decreases as altitude decreases (because $F/We$ increases), whereas $Q_r$ reaches a minimum value at some altitude between 30,000 and 40,000 feet.

The air flows corresponding to figure 15(a) are shown in figure 15(b). It is seen that a minimum air flow occurs at some altitude around 40,000 feet. Also shown on this figure are values of $A_c$, $A_x/A_c$, and $A_x$ required at the various altitudes. At lower altitudes, the large increase in air handling capacity per square foot of compressor frontal area due to increased air density greatly reduces the compressor area required. For the given conditions, for example, although the required air flow at 50,000 feet and 30,000 feet altitude are about the same, the compressor frontal area required at 50,000 feet is about 2.5 times as great as that required at 30,000 feet.

Performance with nacelle drag included (for case of $L/D_w + D_t$ constant). A set of calculations was made to evaluate the cycle performance for a range of constant values of lift-drag ratio of the wing and tail. In these calculations a constant nacelle configuration as described in appendix C was assumed and nacelle drag evaluated. For the configuration and supersonic flight conditions, a value of $D_{n}/W_a$ of 2.69 pounds per pound per second is evaluated. To simplify calculations, the fuselage drag $D_f$ is neglected (that is, assuming that the airplane consists of wing, tail, and nacelles) and equation (4b) is used to evaluate gross weights. From this equation it is apparent that maximum $(F - D_n)/W_e$ results in minimum $W_g$, and in the calculations $P_2/P_1$ and $M_2$ were selected to give maximum $(F - D_n)/W_e$. These values of $P_2/P_1$ and $M_2$ as well as the corresponding values of $(F - D_n)/W_e$ and $F/W_a$ are shown plotted against $P_B$ in figure 16. Other operating conditions held constant are $T_A$ equal to $1460^\circ$ R and $P_A$, 5000 pounds per square inch. Comparison with figure 7 shows that somewhat higher values of $P_2/P_1$ and lower values of $M_2$ are required where nacelle drag is considered. These conditions tend to increase $F/W_a$, which is evident from a comparison of the $F/W_a$ values of figures 16 and 7.
Figure 17(a) shows the values of $W_g$ plotted against $Q_r$ for a range of values of $L/(D_w + D_t)$. Lines of constant $P_B$ are located on these plots. Figure 17(b) shows the values of air flow rate and the values of $L/D_{tot}$ that correspond to the values of figure 17(a). The expression for $L/D_{tot}$ in terms of $L/(D_w + D_t)$ and nacelle drag is derived in appendix C.

SUBSONIC FLIGHT CONDITIONS

A set of performance figures is also presented for the case of subsonic design-point flight conditions (flight Mach number, 0.9; altitude, 40,000 ft). For this case, higher values of $L/D_{tot}$ are obtainable which improve airplane performance considerably.

Performance at various values of $P_B$. - Figure 18, similar to figure 7, presents the best value of $F/W_e$ and corresponding $P_2/P_1$, $M_2$, and $F/w_a$ for various values of $P_B$. The fixed conditions besides flight Mach number and altitude are $T_A$, 1460° R and $P_A$, 5000 pounds per square inch. For this subsonic case, it is seen that $F/W_e$ reaches a maximum value at a $P_B$ within the range investigated. For the given conditions, this value of $P_B$ is about 1250 pounds per square inch. In general, the values of $F/W_e$, $F/w_a$, $M_2$, and $P_2/P_1$ are close to those obtained for the supersonic case. Briefly, as $P_B$ ranged from 625 to 2500 pounds per square inch, values of $F/W_e$ ranged from 1.18 to 1.13 pounds per pound, $F/w_a$ varied from 12.7 to 13.5 pounds per pound per second, $M_2$ varied from 0.18 to 0.20, and $P_2/P_1$ ranged from 1.25 to 1.38. As previously described for the supersonic case, increasing $P_B$ results in operating at lowered $P_2/P_1$ but also lower values of exchanger pressure drop and exchanger weight. The resulting effect of increased $P_B$ on $F/W_e$ depends on whether benefits of lower exchanger pressure drop and weight can overbalance the lower $P_2/P_1$. For the subsonic case, mainly because of lower ram pressure ratio, the best combination of these effects occurs at a $P_B$ of about 1250 pounds per square inch, which is lower than the best $P_B$ for the supersonic case.

The gross weight and heat release rate required for operation at values of $T_A$ equal to 1460° R and $P_A$ equal to 5000 pounds per square inch are shown in figure 19(a), for values of $L/D_{tot}$ of 12, 15, and 18. Values of constant $P_B$ are also located on this plot. The higher values of $L/D_{tot}$ obtainable at subsonic flight substantially reduce
the values of \( W_g \) and \( Q_r \) from those at supersonic flight. For the range of \( L/D_{\text{tot}} \) considered, \( W_g \) of about 250,000 to 260,000 pounds and values of \( Q_r \) from 80,000 to 170,000 Btu per second are obtained. The variation in \( W_g \) is much less sensitive to variation in \( L/D_{\text{tot}} \) and \( F/W_e \) at these higher values of \( L/D_{\text{tot}} \). This can be seen from a consideration of equation (4a). The required \( w_a \) corresponding to the values of figure 19(a) are shown in figure 19(b). These values are largely dependent on the value of \( L/D_{\text{tot}} \) and for the given set of conditions range from about 1000 pounds per second at \( L/D_{\text{tot}} \) of 18 to about 1700 pounds per second at an \( L/D_{\text{tot}} \) of 12. For the subsonic design-point flight conditions considered, the \( w_a/A_c \) is 8.06 pounds per second per square foot. The values of required \( A_c \) are also shown in figure 19(b). These range from about 125 square feet to about 210 square feet. Also included on this figure are the values of \( A_x/A_c \), which vary from about 1.8 to 2.2; this again necessitates an angle of slant of the exchanger of about 30° with the horizontal.

Performance at various values of \( T_A \). - The effect of \( T_A \) on performance is shown in figure 20. Values of \( W_g \) are plotted against \( Q_r \) in figure 20(a) at values of \( L/D_{\text{tot}} \) of 12, 15, and 18 over a range of values of \( T_A \) from 1360° to 1660° R. The value of \( P_A \) is fixed at 5000 pounds per square inch, \( P_B \) is 1250 pounds per square inch, and again values of \( W_k \) equal to 150,000 pounds and \( W_b/W_g \) equal to 0.35 were assumed. The changes in \( W_g \) and \( Q_r \) due to varying \( T_A \) from 1360° to 1660° R are rather moderate. The largest variations which occur at an \( L/D_{\text{tot}} \) of 12 is about 3 percent for \( W_g \) and about 20 percent for \( Q_r \). The corresponding values of \( F/W_e, w_a, A_c, \) and \( A_x/A_c \) are presented in figure 20(b).

Effect of \( W_b/W_g \)

Throughout the analysis a value of \( W_b/W_g \) equal to 0.35 was assumed. From equation (4a) the expression for the ratio of \( W_b/W_g \) at any value of \( W_b/W_g \) to \( W_g \) at \( W_b/W_g \) equal to 0.35 can readily be derived. The same expression holds for the ratio of \( Q_r \) at any value of \( W_b/W_g \) to \( Q_r \) at a value of \( W_b/W_g \) of 0.35. In figure 21 the values of these ratios of airplane gross weights or reactor heat release rates are plotted against values of \( W_b/W_g \) for a range of values of \( (L/D_{\text{tot}})(F/W_e) \). These curves serve mainly to show the magnitude of the effect of \( W_b/W_g \) on \( W_g \) and \( Q_r \). The effect is more pronounced at lower values of \( (L/D_{\text{tot}})(F/W_e) \) as is expected from equation (4a).
# SUMMARY OF RESULTS

Performance values of the supercritical-water-compressor jet cycle determined from a preliminary cycle analysis for a range of flight and engine design-point operating conditions are listed in the following table. The performance values presented are at values of compressor pressure ratio and Mach number of air entering the heat exchanger which result in best thrust per unit engine weight (or minimum gross weight). Also included in the table is one case where the compressor pressure ratio and exchanger inlet Mach number were chosen to give a good compromise in the values of gross weight, reactor heat release rate, compressor frontal area, and exchanger frontal area. In the table, the sum of the reactor, shield, and auxiliary weights is taken as 150,000 pounds, structure to gross weight ratio is 0.35, and steam pressure at the reactor outlet is 5000 pounds per square inch.

| Flight Mach number | Altitude, ft | T_A, °R | P_B, lb/sq in | L/D, lb | W_a, lb | W_e, lb | Q_r, Btu/sec | V_a, lb/sq ft | A_c, sq ft | A_e/A_c | F/V_e | F/V_a | P_o/P_i | M_2 |
|-------------------|-------------|---------|-------------|--------|--------|--------|-------------|--------------|-----------|----------|--------|--------|--------|--------|---|
| 1.5               | 50,000      | 1460    | 2500        | 5      | 312    | 58.8   | 6460        | 7410         | 778       | 2.09    | 11.55  | 1.180  | 2.09   | 0.195 |
|                   | 1250        | 5       | 350        | 41.7   | 471    | 5610   | 4520        | 4250         | 447       | 2.09    | 11.55  | 1.180  | 2.09   | 0.195 |
|                   | 625         | 5       | 350        | 64.5   | 452    | 6260   | 695         | 9.97         | 1.050     | 0.21    | 0.200  |        |        |        |     |
|                   | 1550        | 5       | 395        | 79.5   | 460    | 8120   | 852         | 1.94         | 6.69      | 1.23    | 0.200  |        |        |        |     |
|                   | 2500        | 5       | 295        | 45.0   | 448    | 4450   | 467         | 1.97         | 13.48     | 1.33    | 0.200  |        |        |        |     |
|                   | 40,000      | 1460    | 2500        | 5      | 350    | 60.6   | 6460        | 677          | 2.14      | 10.04   | 1.059  | 1.13   | 1.19   |        |     |
|                   | 50,000      | 1460    | 2500        | 5      | 350    | 36.6   | 4950        | 322          | 2.18      | 11.60   | 1.565  | 1.16   | 1.185  |        |     |
|                   | 625         | 5       | 295        | 53.0   | 433    | 5380   | 225         | 2.35         | 10.30     | 1.845   |        | 1.14   | 1.175  |        |     |
| 0.9               | 40,000      | 1460    | 1250        | 12     | 360    | 19.0   | 137         | 1.98         | 13.50     | 1.150   | 1.35   | 0.180  |        |        |     |
|                   |             |         | 15           | 254    | 15.1   | 107    | 1250         | 1.98         | 13.50     | 1.150   | 1.35   | 0.180  |        |        |     |
|                   |             |         | 18           | 250    | 12.5   | 88     | 1300         | 1.96         | 12.70     | 1.105   | 1.58   | 0.195  |        |        |     |
|                   |             |         | 625         | 15      | 255    | 15.8   | 95          | 1340         | 1.81      | 13.12   | 1.065  | 1.25   | 0.175  |        |     |
|                   |             |         | 2500        | 15      | 255    | 15.8   | 136         | 1300         | 2.18      | 13.50   | 1.105   | 1.58   | 0.195  |        |     |
|                   |             |         | 1550        | 15      | 257    | 17.1   | 116         | 1500         | 2.09      | 11.40   | 1.065  | 1.25   | 0.175  |        |     |
|                   |             |         | 2500        | 15      | 257    | 17.1   | 116         | 1500         | 2.09      | 11.40   | 1.065  | 1.25   | 0.175  |        |     |
| 1.5               | 50,000      | 1460    | 1250        | 5      | 336    | 68.5   | 460         | 571          | 2.09      | 12.57   | 0.978  | 1.27   | 0.180  |        |     |

*Compressor pressure ratio and M_2 chosen to give good combination of \( W_e, Q_r, A_c, A_e \).*

Only a range of values of operating conditions sufficient to show the important effects of these operating conditions and to illustrate some of the performance characteristics of the cycle, are presented in the table. Some of the cycle characteristics, seen from the table are:

1. The cycle is one having low thrust per unit air flow rate (ranging from about 8 to 16 lb/(lb/sec)) and consequently requires large air flow rates and large engine frontal areas.
2. For the range of flight and operating conditions investigated, low compressor pressure ratios (ranging from about 1.1 to 1.4) are required.

3. The cycle is one capable of operating at relatively low steam temperatures out of the reactor. Reasonably good performance is obtainable at as low a temperature as $1360^\circ R$.

Other cycle performance characteristics presented by the analysis are that

4. Lower air flows and consequently lower compressor frontal areas are obtained at higher compressor pressure ratios and lower Mach numbers of air entering the exchanger than those required for minimum airplane gross weight.

5. Lower exchanger frontal areas are obtained at higher compressor pressure ratios and about the same Mach number of air entering the exchanger as those required for minimum airplane gross weight.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, January 19, 1953
ASSUMPTIONS

WEIGHTS OF ENGINE AND AIRPLANE COMPONENTS

An accurate determination of the variation in the weight of most of the airplane or engine components as a function of the design-point operating variables requires a detailed design study of each of these components. To avoid this, simplified relationships for the weights of the various airplane and engine components were used.

Compressor Weight

A characteristic of the cycle is that relatively low values of $P_2/P_1$ (about 1.1 to 1.4) are required for good cycle performance. In the analysis it was assumed that these values of $P_2/P_1$ could be obtained with a single-stage axial-flow compressor and that the weight of the compressor stage was independent of the value of $P_2/P_1$ but varies directly with $A_c$. Reference 2 presents an expression for the weight of a multistage axial-flow compressor having a pressure ratio of 1.2 per stage. From this expression, the following equation was obtained for the weight of a single stage

$$W_c/w_a = 84.8 \sqrt{T_1/P_1}$$

(8)

The frontal area of the compressor is determined by assuming that at static sea-level conditions the air flow per unit compressor frontal area is 25 (lb/sec)/sq ft.

Heat-Exchanger Weight

In the analysis a representative heat-exchanger design was assumed in order to provide a basis for the accurate evaluation of exchanger weight, size, and pressure drops. A schematic sketch of the heat exchanger considered is shown in figure 2. High-pressure steam from the turbine discharge flows through narrow rectangular passages 0.25 inch wide with walls of 0.02-inch steel (this wall thickness is assumed constant for all values of steam pressure entering the exchanger). These passages are separated by finning surfaces of 0.01-inch aluminum spaced 12 to the inch, the air flowing between these fins. The air-passage fin width was chosen as 0.75 inch. Calculations were made which indicated that this value gives close to the best combination of
The value of \( A_{x,f/w_a} \) is determined from the air temperature, pressure, and \( M_2 \).

It should be pointed out that the range of operating variables investigated includes cases where the steam temperatures into the exchanger are too high for aluminum fins to be used. Where steel fins are used, the exchanger configuration would be changed markedly (optimum fin width and spacing would vary), and would also result in heavier exchangers. For the design-point operating conditions currently being considered for this cycle (\( T_A \approx 1460^\circ R \) and \( P_B \) about 1250 lb/sq in. or lower) an exchanger with aluminum fins can be used. In order to avoid the complication involved in varying the exchanger designs as conditions varied, the design described was fixed for the range of conditions investigated, which favors the performance at higher values of \( P_B \).

**Gear Weight**

The weight of reduction gearing is largely a function of the power transmitted through the gears and the speed reduction ratio. For the cycle considered, both of these depend on the size and number of engines selected. In the analysis the effect of size and number of engines and their configuration in the airplane was not studied; instead, a value of gear weight of 0.10 lb/hp was assumed. This value is representative of what is being currently obtained for gearing rated at about 10,000 hp and about 10 to 11 speed reduction ratios.

**Turbine and Condensate Pump Weights**

The turbine and the condensate pump will be relatively small in size. Their weights were assumed to vary with the number of stages and amount of steam handled; the relationship used for the combined weights is

\[
W_{t+p}/W_s = 8.0 \log_e P_A/P_B
\]
The value of the constant is based on estimates of size, power transmitted, and some values of weights given in reference 5.

Reactor, Shield, and Auxiliary Weights

In the analysis a group of weights $W_k$ is considered to be nearly independent of the size of the airplane and design-point operating conditions. In this group are included the weights of the reactor and shield assembly, the pay load, the piping, the water used as cycle fluid, the controls, and the accessory equipment. A constant value of $W_k$ equal to 150,000 pounds has been assumed in order to evaluate $W_g$, $Q_r$, $W_a$, $A_c$, and $A_x$. This value of $W_k$ is based on a reactor core diameter of about 2.5 feet, this size assumed to remain constant as power varies (sufficiently high heat flux rates and heat-transfer surface to handle the range in reactor heat release without excessive temperatures are assumed obtainable within this volume). The performance values vary directly with $W_k$ so that they can very easily be determined at any other values of $W_k$.

Airplane Structural Weight

The ratio $W_s/W_g$ was assumed constant as the size and weight of the airplane varies. A value of $W_s/W_g$ equal to 0.35 was chosen.

PERFORMANCE OF ENGINE AND AIRPLANE COMPONENTS

Engine Component Efficiencies

The efficiencies listed are representative of the values obtained in current practice. For the components which require considerable development (such as the steam turbine and condensate pump) these efficiencies are optimistic values.

Compressor efficiency, $\eta_c$ ................. 0.88
Steam turbine efficiency, $\eta_t$ ............... 0.85
Condensate pump efficiency, $\eta_p$ ............ 0.80
Exhaust nozzle velocity coefficient, $C_v$ .......... 0.98
Inlet diffuser total pressure ratio, $P_1/P_0$
  at 0.9 flight Mach number ................. 0.975
  at 1.5 flight Mach number ................. 0.95

Gear losses were neglected.
Heat-Exchanger Performance

Both heat-transfer and pressure-drop calculations are based on the assumption that the air passages between fins are smooth tubes.

The resistance of the steam film is neglected; this assumption is more accurate for condensing steam. The resistance of the direct heating surface (which is the surface in contact with the steam) is also neglected. However, the resistance of the fins is accounted for by evaluating a fin effectiveness which in turn is used to evaluate a constant effective wall temperature around any cross section in the airflow passage.

Exchanger calculations are based on the assumption of counterflow during the heat transfer from superheated steam to air. The pressure drops evaluated are increased arbitrarily by 15 percent to account for pressure losses involved in bending of the air flow when entering and leaving the exchanger. Bending of the air stream is required because, in general, the frontal area of the exchangers is larger than that of the compressor and the exchanger must be inclined to fit behind the compressor (see fig. 1).

Aerodynamic Performance

At supersonic flight speeds, $L/D_w$ and $L/D_{tot}$ will vary with airplane size (particularly if the airplane and engine configuration depends on airplane size). In the analysis, the more simplifying assumption was used that $L/D_{tot}$ remains constant as the airplane size (which is a function of engine design point operating conditions) varies. This assumption was made for all flight conditions investigated. However, several values of $L/D_{tot}$ were chosen at each set of design flight conditions.
APPENDIX B

SAMPLE CALCULATION

To illustrate the method of calculation, the performance of the cycle operating at the following given design-point flight and engine operating conditions is determined:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flight Mach number</td>
<td>1.5</td>
</tr>
<tr>
<td>Altitude, ft</td>
<td>50,000</td>
</tr>
<tr>
<td>$T_A$, $^\circ$R</td>
<td>1460</td>
</tr>
<tr>
<td>$P_A$, lb/sq in.</td>
<td>5000</td>
</tr>
<tr>
<td>$P_B$, lb/sq in.</td>
<td>1250</td>
</tr>
<tr>
<td>$P_D$, lb/sq in.</td>
<td>5050</td>
</tr>
<tr>
<td>$P_2/P_1$</td>
<td>1.214</td>
</tr>
<tr>
<td>$M_2$</td>
<td>0.20</td>
</tr>
<tr>
<td>$\eta_c$</td>
<td>0.88</td>
</tr>
<tr>
<td>$\eta_t$</td>
<td>0.85</td>
</tr>
<tr>
<td>$\eta_p$</td>
<td>0.80</td>
</tr>
<tr>
<td>$C_V$</td>
<td>0.98</td>
</tr>
<tr>
<td>Diffuser pressure ratio</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Assume values of

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L/D_{tot}$</td>
<td>5</td>
</tr>
<tr>
<td>$W_s/W_g$</td>
<td>0.35</td>
</tr>
<tr>
<td>$W_k$, lb</td>
<td>150,000</td>
</tr>
<tr>
<td>$w_a \sqrt{T_1/519}$ lb/sec</td>
<td>25.0</td>
</tr>
</tbody>
</table>

EVALUATION OF $F/w_a$

Steam Cycle Calculations

From $P_A$, $T_A$, $P_B$, and $\eta_t$ using the steam tables (ref. 6),

$\Delta H_t/w_s = 122.6$ Btu/lb

$T_B = 1089^\circ R$
Ts at corresponding to $P_B = 1032^\circ R$

$T_C = T_{s a t} - 20 = 1012^\circ R$

From $T_B$, $T_{s a t}$, and $T_C$ using the steam tables:

Heat release during condensation = 628.5 Btu/lb

Heat release from superheat to saturated condition = 65.9 Btu/lb

Total $Q_X/\dot{w}_S = 694.4$ Btu/lb

The pump compresses the condensate to a pressure $P_D$ of 5050 pounds per square inch. From the values of $T_C$, $P_C$ (assumed the same as $P_B$), $P_D$, and $\eta_p$, using figure III of the steam tables

$\Delta H_P/\dot{w}_S = 19.1$ Btu/lb

From enthalpy values at pump outlet and reactor outlet,

$Q_T/\dot{w}_S = 797.9$ Btu/lb

The net work out of steam cycle = $\Delta H_t/\dot{w}_S - \Delta H_P/\dot{w}_S = 103.5$ Btu/lb.

Air Cycle Calculations

From flight conditions and diffuser pressure recovery

$V_0 = 1457$ ft/sec

$T_1 = 570^\circ R$

$P_1/P_0 = 3.487$

$P_1 = 845$ lb/sq ft

From $P_2/P_1$, $T_1$, and $\eta_c$ using the air tables (ref. 7)

$\Delta H_c/\dot{w}_a = 8.86$ Btu/lb

$T_2 = 607^\circ R$

$P_2 = 1026$ lb/sq ft
neglecting gear losses

\[ w_a \left( \frac{\Delta H_c}{w_a} \right) = w_s \left( \frac{\Delta H_t}{w_s} - \frac{\Delta H_p}{w_s} \right) \]

\[ \frac{w_a}{w_s} = \frac{122.6 - 19.1}{8.86} = 11.68 \]

From exchanger heat balance

\[ \frac{Q_x}{w_a} = \frac{w_s}{w_a} \frac{Q_x}{w_s} = \frac{694.4}{11.68} = 59.40 \frac{\text{Btu/sec}}{\text{lb/sec}} \]

then using \( Q_x/w_a \), and \( T_2 \), from the air tables

\[ T_3 = 852^\circ R \]

This is an attainable operating condition because \( T_3 \) is less than \( T_B \).

The determination of \( P_3 \) requires evaluating the heat-exchanger performance.

Heat-Exchanger Calculations

The exchanger is considered as made up of two exchangers in series. In the first, air enters at a Mach number of 0.20 and is heated by the condensing steam; in the second, the air is further heated by superheated steam (counterflow assumed).

The mass flow of air per unit flow area through the exchanger can be expressed as

\[ \frac{w_a}{A_{x,f}} = \sqrt{\frac{Yg}{R}} \left( \frac{M_2}{M_2} \right) \frac{P_2}{\sqrt{T_2}} \left[ 1 + \frac{Y-1}{2} (M_2)^2 \right]^{\frac{Y+1}{2(Y-1)}} \]

where:

\( Y \) ratio of specific heats of air = 1.4
\( g \) acceleration due to gravity = 32.2 ft/sec\(^2\)
\( R \) gas constant for air = 53.3 Btu/lb °F
so

\[
\frac{w_a}{A_{x,f}} = \sqrt{\frac{1.4 \times 32.2}{53.3} \left[ 1 + (0.2)^2 \left( \frac{1.4 - 1}{2} \right) \right]^3 \frac{1026}{\sqrt{607}}} = 7.48 \text{ lb/sec sq ft}
\]

Condensing exchanger calculations: evaluation of \( \frac{1}{d} \). - The temperature of the air after being heated by condensing steam can be determined from \( T_2 \), the heat transferred from the condensing steam per pound of air \( (628.5/11.68 = 53.81) \), and the air tables (ref. 7)

\[T_2' = 829^\circ R\]

The average bulk temperature of the air in the exchanger is

\[T_b = \frac{1}{2} (T_2' + T_2) = \frac{1}{2} (829 + 607) = 718^\circ R\]

The wall temperature = \( T_{sat} = 1032^\circ R \)

The following properties of air at wall temperature are evaluated from table III in the air tables (ref. 7)

- Viscosity, \( \mu \), lb/ft-sec .................. 196x10^{-7}
- Conductivity, \( k \), Btu/sec-ft-\(^\circ\)F .................. 74x10^{-7}
- Specific heat, \( c_p \), Btu/lb-\(^\circ\)F .................. 0.250
- Prandtl number, \( Pr \) .................. 0.66

Using these values in the equation

\[h = 0.023 \frac{k}{d} \left( \frac{w_a}{A_{x,f}} \frac{d}{\mu} \right)^{0.8} \left( \frac{T_b}{T_w} \right)^{0.8} (Pr)^{0.4}\]

\[h = 0.00774 \frac{\text{Btu}}{\text{sec-ft}^2 \text{-OF}}\]

In the exchanger fins there is less heat transferred per unit fin surface than through a unit of direct heating surface. The fin effectiveness is the ratio of these rates of heat flow and can be expressed as the ratio of the effective temperature difference between the fin surface and fluid to the available temperature difference between the direct heating surface and the fluid.
A factor is defined:

\[ \phi = \frac{\text{fin width}}{2} \sqrt{\frac{\text{heat transfer coefficient between fin and air}}{\text{conductivity of fin}(\text{thickness of fin})}} \]

then the fin effectiveness \( \eta_f \) is \( \frac{\tanh \phi}{\phi} \)

In the exchanger considered, aluminum fins 0.01 inch thick, 0.75 inch wide, having an average value of conductivity of 125 Btu/hr-ft-\( ^\circ \)F, were selected.

From these values and the value of \( h \) previously determined, the value \( \eta_f = 0.85 \) is obtained.

The rectangular air flow passage in the exchanger has fins on two sides and direct heating surfaces on the other two sides. The fins are 0.75 inch wide, while the direct heating surfaces are 0.0733 inch long between fins. The effective wall temperature of the passage is considered constant around the whole perimeter of the passage and is evaluated from the expression

\[ (T_{w,\text{eff}} - T_b)(0.0733 + 0.75) = (T_w - T_b) 0.0733 + (T_w - T_b) 0.75 \eta_f \]

so that

\[ T_{w,\text{eff}} - T_b = (T_w - T_b) \left( \frac{0.0733 + 0.75 \eta_f}{0.0733 + 0.75} \right) \]

In the condensing exchanger the wall temperature is considered to be equal to \( T_{\text{Sat}} \) at all points (neglecting the effect of subcooling). At the exchanger inlet where the bulk air temperature is \( T_2 \), the effective temperature difference = \( (1032 - 607) \frac{0.0733 + 0.85 \times 0.75}{0.0733 + 0.75} = 367^\circ \) F. Similarly, at the outlet where the bulk air temperature is \( T_2' \), the effective temperature difference = \( (1032 - 829) \frac{0.0733 + 0.85 \times 0.75}{0.0733 + 0.75} = 175^\circ \) F. The log mean temperature difference = \( \frac{367 - 175}{\log_e \frac{367}{175}} = 259^\circ \) F.

From this value and the average value of heat-transfer coefficient,

\[ \frac{l}{d} = \frac{\frac{w_a}{A_{x,f}}}{\frac{Q}{w_a}} \cdot \frac{7.48 \times (628.5)}{4 \times 259 \times 0.00774} = 50.4 \]
Condensing exchanger calculations: evaluation of pressure drop. - An equivalent constant wall temperature $T_{w,\text{equ}}$ is determined so that the log mean temperature difference obtained from $T_{w,\text{equ}} - T_2$ and $T_{w,\text{equ}} - T_2'$ is the same as that previously determined. Thus,

$$\frac{(T_{w,\text{equ}} - T_2) - (T_{w,\text{equ}} - T_2')}{\log_e \frac{T_{w,\text{equ}} - T_2}{T_{w,\text{equ}} - T_2'}} = \text{log mean temperature difference} = 259^\circ \text{ F}$$

from which

$$T_{w,\text{equ}} = 993^\circ \text{ R}$$

Then

$$\frac{T_2}{T_{w,\text{equ}}} = \frac{607}{993} = 0.611$$

$$\frac{T_2'}{T_{w,\text{equ}}} = \frac{829}{993} = 0.835$$

From the constant wall pressure drop chart presented in reference 2, the pressure ratio across the exchanger is equal to 0.937. This results in a pressure out of the exchanger equal to 961 lb/sq ft. In the expression for $\frac{w_a}{A_x,f}$ using this value of total pressure and values of $T_2'$ equal to 829$^\circ$ R and $w_a/A_x,f$ of 7.48 lb/(lb/sec), a value of Mach number at exchanger outlet equal to 0.255 is determined.

Superheat exchanger calculations: evaluation of $l/d$. - For the given cycle operating conditions selected, the amount of heat released by the superheat portion of the steam is slight.

From $T_2'$ and $T_3$ the average bulk temperature

$$T_b = \frac{1}{2} (829 + 852) = 840^\circ \text{ R}$$

From $T_{\text{sat}}$ and $T_B$ the average wall temperature

$$T_w = \frac{1}{2} (1032 + 1089) = 1060^\circ \text{ R}$$
The air properties evaluated at the average wall temperature are essentially the same as previous values. The mass flow of air per unit flow area remains constant so that

\[ h = 0.023 \times \frac{74 \times 10^{-7}}{0.01113} \left( \frac{7.48 \times 0.01113}{196 \times 10^{-7}} \right)^{0.8} \left( \frac{840}{1050} \right)^{0.8} (0.66)^{0.4} \]

\[ = 0.00859 \frac{\text{Btu}}{\text{sec-ft}^2 \text{OF}} \]

The fin effectiveness at this value of \( h \) becomes 0.84. The temperature difference at inlet = \((1032 - 829) \left( \frac{0.0733 + 0.75 \times 0.84}{0.0733 + 0.75} \right) = 173^\circ \text{F} \).

The temperature difference at outlet = \((1089 - 852) \left( \frac{0.0733 + 0.75 \times 0.84}{0.0733 + 0.75} \right) = 202^\circ \text{F} \).

The log mean temperature difference = \( \frac{202 - 173}{\log_e 173} = 187 \).

\[ l/d = \frac{7.48 \times 65.9}{4 \times 187 \times 0.00859} = 6.6 \]

Superheat exchanger calculations: evaluation of pressure drop. - When used with \( T_2' = 829^\circ \text{R} \) and \( T_3' = 852^\circ \text{R} \), the following value of equivalent constant wall temperature results in a log mean temperature difference of 187^\circ \text{R}:

\[ T_{w,\text{equ}} = 1029^\circ \text{R} \]

\[ \frac{T_2'}{T_{w,\text{equ}}} = \frac{829}{1029} = 0.806 \]

\[ \frac{T_3}{T_{w,\text{equ}}} = \frac{852}{1029} = 0.828 \]

The inlet Mach number of air to this exchanger is the same as the outlet Mach number of the first exchanger and is equal to 0.255.

Again by use of the pressure drop chart, a value of pressure ratio across the second exchanger of 0.989 is determined from these values.

Over-all exchanger performance. - The over-all \( l/d \) of the exchanger = 50.2 + 6.6 = 56.8.
The over-all pressure ratio is $0.989 \times 0.937 = 0.927$. An additional 15 percent of the pressure drop is added for losses in bending the airflow into and out of the exchanger.

The entire pressure drop $\Delta P/P_2 = 1.15 \times (1 - 0.927) = 0.084$. Therefore, $P_3/P_2 = 1 - 0.084 = 0.916$. The pressure ratio across the exhaust nozzle

$$\frac{P_3}{P_0} = \frac{P_1 P_2}{P_0 P_1} = \frac{3.487}{(1.214)(0.916)} = 3.877$$

From the values of $P_3/P_0$, $T_3$, and $C_v$, and from the air tables,

$$V_j = 1778 \text{ ft/sec}$$

$$\frac{f}{w_a} = \frac{1778 - 1457}{32.2} = 9.97 \frac{\text{lb}}{(\text{lb/sec})}$$

EVALUATION OF $F/W_e$

From equation (8)

$$\frac{W_c}{w_a} = 34.8 \sqrt{\frac{T_1}{P_1}} = \frac{84.8 \sqrt{570}}{845} = 2.40 \frac{\text{lb}}{(\text{lb/sec})}$$

From equation (9)

$$\frac{W_x}{w_a} = (0.595 \frac{1}{d} + 3.0) \frac{A_{x,f}}{w_a} = \frac{0.595 \times 56.8 + 3.0}{7.48} = 4.92 \frac{\text{lb}}{\text{lb/sec}}$$

From equation (10)

$$\frac{W_{t+p}}{w_a} = \frac{W_{t+p}}{w_s} \frac{w_s}{w_a} = 8.0 \log_e \frac{P_A}{P_B} = \frac{8.0}{11.68} \log_e 4 = 0.95 \frac{\text{lb}}{\text{lb/sec}}$$

Assuming the horsepower transmitted through the gears is equal to turbine horsepower output, then

$$\frac{W_b}{w_a} = 0.10 \frac{hpt}{w_a} = 0.10 \frac{\Delta H_t}{w_s} \frac{w_s}{w_a} 778 = 0.10 \times \frac{122.6}{11.68} \times 778 = 1.48 \frac{\text{lb}}{(\text{lb/sec})}$$
\[ \frac{W_e}{W_a} = 1.48 + 0.95 + 4.92 + 2.40 = 9.75 \text{ lb (lb/sec)} \]

\[ F/W_e = \frac{9.97}{9.75} = 1.023 \text{ lb lb} \]

EVALUATION OF OTHER PERFORMANCE PARAMETERS

Gross Weight

The gross weight of the airplane is determined from equation (1a)

\[ W_g = \frac{150,000}{1 - 0.35 - \frac{1}{5 \times 1.023}} = 330 \times 10^3 \text{ lb} \]

Air Flow Rate

The air flow can be expressed as

\[ w_a = \frac{F}{F/W_a} = \frac{W_g D}{F/W_a} \]

by equation (1a), \( w_a \) can be written as

\[ w_a = \frac{W_k}{L \frac{F}{D_{tot}} \frac{1 - \frac{W_s}{W_g}}{w_a}} = \frac{150,000}{5 \times 9.97 (1 - 0.35) - 9.75} = 6620 \text{ lb sec} \]

Steam Flow Rate

\[ w_s = w_a \frac{w_s}{w_a} = \frac{6620}{11.68} = 567 \text{ lb/sec} \]

Reactor Heat Release Rate

\[ Q_r = w_s \frac{Q_r}{w_s} = 797.9 \times 567 = 452 \times 10^3 \text{ Btu/sec} \]
Engine Component Weights

Compressor weight:

\[ W_C = \left( \frac{W_C}{W_a} \right) W_a = 2.40 \times 6620 = 15,900 \text{ lb} \]

Exchanger weight:

\[ W_X = \left( \frac{W_X}{W_a} \right) W_a = 4.92 \times 6620 = 32,500 \text{ lb} \]

Turbine and condensate pump weight:

\[ W_{t+p} = \left( \frac{W_{t+p}}{W_a} \right) W_a = 0.95 \times 6620 = 6300 \text{ lb} \]

Gear box weight:

\[ W_b = \frac{W_b}{W_a} W_a = 1.48 \times 6620 = 9800 \text{ lb} \]

Total engine weight:

\[ W_e = 15,900 + 32,500 + 6,300 + 9,800 = 64,500 \text{ lb} \]

Compressor Frontal Area

\[ A_c = \frac{W_a}{W_e/A_c} \]

\[ \frac{W_a}{A_c} = \left( \frac{W_a \sqrt{\frac{T_1}{519}}}{A_c \frac{P_1}{2116}} \right) \sqrt{\frac{519}{T_1}} = \frac{25.0 \sqrt{\frac{519}{570}}}{2116/845} = 9.527 \frac{\text{lb/sec}}{\text{sq ft}} \]

\[ A_c = \frac{6620}{9.527} = 695 \text{ sq ft} \]
Exchanger Area

\[ A_x = \frac{A_{x,f}}{f} = \frac{v_a}{w_a} = \frac{6620}{7.48 \times 0.65} = 1360 \text{ sq ft} \]

Turbine Power Output

\[ h_{pt} = \frac{h_{pt}}{w_s} = 122.6 \times \frac{778}{550} \times 567 = 98,300 \text{ hp} \]
APPENDIX C

EVALUATION OF NACELLE DRAG

Evaluation of $D_n/w_a$

From data given in an NACA Conference on Aircraft Propulsion Systems Research (Jan. 18-19, 1950), for the assumed configuration and flight Mach number 1.5, and altitude 50,000 feet,

Wave drag coefficient for diffuser ................. 0.0106
Friction drag coefficient ............................ 0.0025
Wave drag .......... coefficient x maximum frontal area x dynamic pressure
Friction drag .......... coefficient x surface area x dynamic pressure
Surface area of diffuser section ....................... 10.5 $A_C$
Surface area of straight section ...................... 12 $A_C$
Dynamic velocity head, lb/sq ft ...................... 382

$$\frac{\text{Drag on diffuser section}}{A_C} = 0.0106 \times 382 + 0.0025 \times 10.5 \times 382 = 14.1 \frac{\text{lb}}{\text{sq ft}}$$

$$\frac{\text{Drag on straight section}}{A_C} = 0.0025 \times 12 \times 382 = 11.5 \frac{\text{lb}}{\text{sq ft}}$$

$$\frac{\text{Total nacelle drag}}{A_C} = \frac{D_n}{A_C} = 25.6 \frac{\text{lb}}{\text{sq ft}}$$

$\frac{w_a}{A_C}$ for flight conditions = 9.53 $\frac{\text{lb/sec}}{\text{sq ft}}$

$$\frac{D_n}{w_a} = \frac{D_n}{A_C w_a} = \frac{25.6}{9.53} = 2.69 \frac{\text{lb}}{\text{lb/sec}}$$
This configuration was assumed to have a constant exhaust-nozzle area, whereas the nozzle area would actually vary with changes in operating conditions. The difference introduced in evaluating nacelle drag due to this assumption is very slight.

Expression for $L/D_{\text{tot}}$ in Terms of $L/(D_w + D_t)$

and Nacelle Drag

Neglecting fuselage drag,

$$\frac{D_{\text{tot}}}{L} = \frac{D_w + D_t}{L} + \frac{D_n}{L}$$

also,

$$D_n = \frac{D_n}{w_a} F D_{\text{tot}} \quad \text{(because } D_{\text{tot}} = F)$$

so that,

$$\frac{D_{\text{tot}}}{L} = \frac{D_w + D_t}{L} + \frac{D_{\text{tot}} D_n}{L w_a F}$$

and

$$\frac{L}{D_{\text{tot}}} = \frac{L}{D_w + D_t} \left(1 - \frac{D_n}{w_a F}ight)$$
REFERENCES


Figure 1. Schematic diagram of supercritical-water-compressor jet cycle.
Figure 2. - Heat-exchanger configuration assumed for evaluation of exchanger weight, size, and pressure drop.
Figure 3. - Equivalent exchanger arrangement assumed for evaluating exchanger length-diameter ratio l/d and pressure drop when entering steam is superheated.
Figure 4. - Performance values of cycle at various values of $P_2/P_1$ and $M_2$.
Flight Mach number, 1.5; altitude, 50,000 feet; $T_A$, 1460° R; $P_A$, 5000 pounds per square inch; $P_B$, 1550 pounds per square inch.
Figure 4. - Concluded. Performance values of cycle at various values of $P_2/P_1$ and $M_2$. Flight Mach number, 1.5; altitude, 50,000 feet; $T_A$, 1460° R; $P_A$, 5000 pounds per square inch; $P_B$, 1250 pounds per square inch.
Figure 5. - Effect of $P_2/P_1$ on engine components and over-all cycle performance. Flight Mach number, 1.5; altitude, 50,000 feet; $T_A$, 1460° R; $P_A$, 5000 pounds per square inch; $P_B$, 1250 pounds per square inch; $W_2$, 0.20.
Figure 5. - Continued. Effect of $P_2/P_1$ on engine components and over-all cycle performance. Flight Mach number, 1.5; altitude, 50,000 feet; $T_A$, 1460°F; $P_A$, 5000 pounds per square inch; $P_B$, 1250 pounds per square inch; $M_2$, 0.20.
Figure 5. - Concluded. Effect of compressor pressure ratio on engine components and over-all cycle performance. Flight Mach number, 1.5; altitude, 50,000 feet; $T_A$, 1460°R; $P_A$, 5000 pounds per square inch; $P_B$, 1250 pounds per square inch; $M_2$, 0.20.
Figure 6. - Effect of $M_2$ on engine components and over-all cycle performance.
Flight Mach number, 1.5; altitude, 50,000 feet; $T_A$, 1460° R; $P_A$, 5000 pounds per square inch; $P_B$, 1250 pounds per square inch; $P_2/P_1$, 1.21.
Figure 7. - Optimum values of $P_2/P_1$ and $M_2$ giving maximum $F/W_e$ and corresponding values of $F/W_e$ and $F/W_a$, for various values of $P_B$.

Flight Mach number, 1.5; altitude, 50,000 feet; $T_A$, 1460° R; $P_A$, 5000 pounds per square inch.
Figure 9. - Performance at optimum $P_2/P_1$ and $M_2$ for various values of $L/D_{tot}$ and $P_B$. Flight Mach number, 1.5; altitude, 50,000 feet; $T_A$, 1460° R; $P_A$, 5000 pounds per square inch; $W_e/W_g$, 0.35; $W_g$, 150x10^5 pounds.
Figure 9. - Concluded. Performance at optimum $P_2/P_1$ and $M_2$ for various values of $L/D_{tot}$ and $P_B$. Flight Mach number, 1.5; altitude, 50,000 feet; $T_A$, 1460°F; $P_A$, 5000 pounds per square inch; $W_e/W_0$, 0.35; $W_k$, 150x10^3 pounds.
Figure 10. - Comparison of performances at $P_2/P_1$ and $M_2$ selected for minimum airplane gross weight, minimum reactor heat release rate, minimum compressor frontal area, and minimum exchanger frontal area. Flight Mach number, 1.5; altitude, 50,000 feet; $T_A$, 1460° R; $P_A$, 5000 pounds per square inch; $W_s/W_g$, 0.35; $W_k$, 150,000 pounds; $L/D_{tot}$, 5.
Figure 11. - Optimum values of $P_2/P_1$ and $M_0$ and corresponding values of $F/W_e$ and $F/W_a$ at various values of $T_A$. Flight Mach number, 1.5; altitude 50,000 feet; $P_A$, 5000 pounds per square inch; $P_B$, 2500 pounds per square inch.
Figure 12. - Effect of $T_A$ on engine component performance and over-all cycle performance. Flight Mach number, 1.5; altitude, 50,000 feet; $P_A$, 5000 pounds per square inch; $P_B$, 2500 pounds per square inch; $M_2$, 0.20.
Figure 12. - Concluded. Effect of $T_A$ on engine component performance and overall cycle performance. Flight Mach number, 1.5; altitude, 50,000 feet; $P_A$, 5000 pounds per square inch; $P_B$, 2500 pounds per square inch; $M_2$, 0.20.
Figure 13. - Performance at optimum $P_0/P_1$ and $M_2$ for various values of $L/D_{tot}$ and $T_A$. Flight Mach number, 1.5; altitude, 50,000 feet; $P_A$, 5000 pounds per square inch; $P_B$, 2500 pounds per square inch; $W_g/W$, 0.35; $W_k$, 150x10^3 pounds.
Figure 13. - Concluded. Performance at optimum $P_2/P_1$ and $M_2$ for various values of $L/D_{tot}$ and $T_A$. Flight Mach number, 1.5; altitude, 50,000 feet; $P_A$, 5000 pounds per square inch; $P_B$, 2500 pounds per square inch; $W_a/W_g$, 0.35; $W_k$, 150003 pounds.
Figure 14. - Effect of altitude on the optimum values of $F/W_e$ and corresponding values of $F/W_a$ and $W_e/W_a$. Flight Mach number, 1.5; $T_A$, 14600° R; $P_A$, 5000 pounds per square inch; $P_B$, 2500 pounds per square inch.
Figure 15. - Performance at optimum values of $P_2/P_1$ and $M_2$ at various altitudes and values of $P_B$. Flight Mach number, 1.5; $T_A$, 1460° R; $P_A$, 5000 pounds per square inch; $L/D_{tot}$, 5; $W_g/W_g$, 0.35; $W_k$, 150,000 pounds.

(a) $W_g$ against $Q_r$. Reactor heat release rate, $Q_r$, Btu/lb
Figure 15. - Concluded. Performance at optimum values of $P_2/P_1$ and $M_z$ at various altitudes and values of $P_B$. Flight Mach number, 1.5; $T_A$, $1460^\circ$ R; $P_A$, 5000 pounds per square inch; $L/D_{tot}$, 5; $W_e/W_g$, 0.35; $W_k$, 150,000 pounds.
Figure 16. - Values of optimum \((F-D_n)/W_a\) and corresponding values of \(P_2/P_1, M_2,\) and \(F/w_a\) for various values of \(P_B.\) Flight Mach number, 1.5; altitude, 50,000 feet; \(T_A, 1460^\circ\ R; P_A, 5000\) pounds per square inch; \(D_n/w_a, 2.69\) pounds per pound per second.
Figure 17. - Performance at conditions for optimum \( \frac{(F-D_n)}{W_g} \) for various values of \( L/(D_w+D_t) \) and \( P_B \).

Flight Mach number, 1.5; altitude, 50,000 feet; \( T_A \), 1460° R; \( P_A \), 5000 pounds per square inch; \( D_n/W_a \), 2.69 pounds per pound per second; \( W_g/W_g \), 0.35; \( W_k \), 150x10^3 pounds.
Figure 17. - Concluded. Performance at conditions for optimum \((F-D_H)/W_0\) for various values of \(L/(D_w+D_t)\) and \(P_B\). Flight Mach number, 1.5; altitude, 50,000 feet; \(T_A\), 1460° R; \(P_A\), 5000 pounds per square inch; \(D_n/w_a\), 2.69 pounds per pound per second; \(W_S/W_g\), 0.35; \(W_K\), 150x10^3 pounds.
Figure 18. - Values of optimum $F/W_e$ and corresponding values of $P_2/P_1$, $M_2$, and $F/w_a$. Flight Mach number, 0.9; altitude, 40,000 feet; $T_A$, $1460^\circ$ R; $P_A$, 5000 pounds per square inch.
Figure 19. - Performance at conditions for optimum $F/W_g$ for various values of $L/D_{tot}$ and $P_B$. Flight Mach number, 0.9; altitude, 40,000 feet; $T_A$, 1460° R; $P_A$, 5000 pounds per square inch; $W_g/W_g$, 0.35; $W_K$, 150x10^3 pounds.
Figure 19. - Concluded. Performance at conditions for optimum $F/W_e$ for various values of $L/D_{tot}$ and $P_B$. Flight Mach number, 0.9; altitude, 40,000 feet; $T_A$, 1450° R; $P_A$, 5000 pounds per square inch; $W_s/W_g$, 0.35; $W_k$, 150x10$^3$ pounds.
Figure 20. - Performance at conditions for optimum \( F/W_g \) for various values of \( L/D_{tot} \) and \( T_A \). Flight Mach number, 0.9; altitude, 40,000 feet; \( P_A \), 5000 pounds per square inch; \( W_g/W_g \), 0.35; \( W_k \), 150×10³ pounds.
Figure 20. - Concluded. Performance at conditions for optimum $F/W_e$ for various values of $T_A$ and $L/D_{tot}$. Flight Mach number, 0.9; altitude, 40,000 feet; $P_A$, 5000 pounds per square inch; $P_B$, 1250 pounds per square inch; $W_{s}/W_k$, 0.35; $W_k$, 150x$10^3$ pounds.
Figure 21. - Effect of $W_s/W_g$ on values of $W_g$ and $Q_T$. 

Ratio of $Q_T$ or $W_g$ at any $W_s/W_g$ to value at $W_s/W_g = 0.35$ 

Structure to gross weight ratio, $W_s/W_g$ 

F

L

$D_{tot}$

3

4

6

10

20

NACA