A COMPARISON OF DISCOUNTING PARAMETERS OBTAINED THROUGH
TWO DIFFERENT ADJUSTING PROCEDURES:

BISECTION AND UP-DOWN

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The study compared delay discounting in adult humans using two different methods of adjustments. Both methods used hypothetical choices of monetary outcomes. One involved adjustments using a fixed sequence of ascending or descending amounts, the other used a bisection algorithm in which the changes in amounts varied as a function of the subjects' choices. Two magnitudes of delayed outcomes were used: $1,000 and $10,000. A within subject design was used to compare indifference curves and discounting measures across the two adjusting procedures. Twenty four subjects were divided in two groups and exposed to the procedures in opposite order, to account for sequence effects. Results from within subject comparisons showed no systematic differences between procedures.
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INTRODUCTION

Research in an area called behavioral economics seeks to utilize the rigor and precision that characterizes experimental analyses of behavior in the study of the role of economic variables in the organization of behavior - issues typically of concern to microeconomists. In general, the goal in this line of work has been to arrange laboratory preparations to ask some important questions about the behavioral functions of economic variables, such as cost-benefit ratios, unit price, present and future value, etc. (Hursh, 1984). One area that behavioral economists have been particularly interested in studying is how the subjective value of a commodity changes as a function of delays to its delivery (e.g., Ainslie, 1974). The long term goal in this line of research is to understand how changes in subjective value may interact with other variables to organize behavior in the natural environment (see Critchfield & Kollins, 2001).

As an example, it has been shown that great health benefits (increased lifespan, reduced incidence of disease or impact of disease, etc.) accrue from maintaining a healthy diet and a regular regimen of exercise. The failure of some individuals to make healthy choices despite this knowledge is explained by suggesting that the long term benefits outlined above fail to support healthy choices because of the large delays involved in their receipt. Many other examples can be found to support the general conclusion that events at a temporal distance are less valued than events available closer to the present time (Logue, 1995).

The following section will provide a brief historical overview of the preparations used to study temporal discounting. The earliest preparations studying the effects of delays on the value of outcomes used discrete trial procedures in which subjects were
asked to choose between responses that provided a small amount of a commodity immediately and responses that produced a larger amount of that commodity delivered after a delay. In general, this research showed that delays to an outcome altered its value such that animals preferred the smaller, sooner outcome over the larger, later outcome, even at the expense of greater overall food amount or density. These procedures showed that human and nonhuman animals were sensitive to outcome delays. The procedures, however, were not useful for characterizing the precise nature of delay discounting.

A procedure introduced by Mazur (1987) served as an important early contribution in the ability to accurately describe temporal discounting. In his procedure, the delay to outcome delivery was continuously adjusted as a function of the subjects' immediately preceding choices. For example, extended preference for the smaller, sooner outcome would decrease the delay to the larger outcome in the next block of trials to make the larger outcome more attractive. Similarly, extended preference for the larger, later outcome would increase the delay to its delivery in the next block of trials to make the smaller, sooner outcome relatively more attractive. With this procedure, Mazur (1987) was able to identify the amount-delay combinations at which subjects were indifferent between a smaller, sooner outcome and a larger, later outcome. Mazur (1987) called the parameters under which choices were equally distributed between the two options “indifference points.” By measuring a large number of indifference points under various amount and delay combinations, he was also able to describe the rate at which subjects discounted the value of delayed outcomes. A considerable amount of
data now suggests that outcome value decays hyperbolically as a function of the programmed delay (Green & Myerson, 2004).

Variations of Mazur’s (1987) procedure have been used extensively with human subjects to precisely characterize loss of outcome value as a function of delays. As with the original procedure, these preparations present subjects with choices between an amount of money available immediately and a larger amount of money available after a delay (e.g., Rachlin, Raineri & Cross, 1991; Read, 2001). Human participants, however, are generally insensitive to the delays to and amounts of outcomes that are analogous to the delays and amounts used with nonhumans (see Hyten, Madden, & Field, 1994). Furthermore, amount and delay values over which humans’ choices would be sensitive are prohibitively expensive and not practically feasible. These factors served as an obstacle in our ability to generate discounting functions with human subjects.

In an effort to make the procedure more efficient it has become common practice to use hypothetical amounts of money and hypothetical delays to its delivery and to ask subjects to respond as if the amounts and delays were real. It should be noted that some authors have raised concerns regarding the use of hypothetical outcomes as equivalents to real, consumable outcomes (see Navarick, 2004 for a review). However, it also has been argued, on empirical and logical grounds, that there are no substantial differences between hypothetical and real choices (Johnson & Bickel, 2002; Lagorio & Madden, 2005). Hypothetical procedures have been successful in increasing the efficiency with which discount functions can be generated with human subjects. These methods are low-cost and fast ways to estimate discounting functions with human subjects (for one example, see Rachlin et al., 1991). The use of hypothetical amounts
and delays has increased the clinical efficacy of these procedures (see Critchfield & Kollins, 2001, for an extended discussion). Brief and economical methods are particularly important if they are to be widely used in clinical settings.

In addition to the use of hypothetical amounts and delays, there have been other procedural modifications introduced to generate discounting functions in human subjects more efficiently. In a non-exhaustive review of the literature, three distinct types of adjusting procedures based on hypothetical choices were found. Typically, the adjusting procedures were based on adjusting amounts, keeping delays constant for each condition (e.g., Rachlin et al., 1991; Read, 2001; Richards, Zhang, Mitchell, & de Wit, 1999). Additionally, two types of estimation procedures that were not based on adjusting methods (Kirby & Marakovic, 1995, 1996) were found. These 5 types of procedures are described below.

The adjusting procedure most frequently used with humans was developed by Rachlin et al. (1991) and will, hereafter, be referred to as the up-down procedure. In each condition, the smaller and larger outcome amounts were correlated with fixed delay values such that the smaller outcome was available sooner and the larger outcome was available later. Then, within a condition, the amount of the smaller-sooner outcome was adjusted in fixed increments with subjects being asked to choose between the options after each adjustment. Both ascending and descending sequences consisted of 25-30 values with small increments or decrements between successive values in the list. The adjustments were continued until the preference reliably switched from the larger to the smaller option in the ascending sequence and from the smaller to the larger option in the descending sequence. The direction of adjustments was
reversed after subject’s preference had switched from one option to the other. The indifference point, in the up-down procedure, was defined as the average of the switch points from the ascending and descending sequences (Alessi & Petry, 2003; Coelho, Hanna, & Todorov, 2003; Giordano, Bickel, Loewenstein, Jacobs, Marsch, & Badger, 2002; Odum, Madden, & Bickel, 2002; Rachlin & Green, 1972). Implementation of this procedure has typically involved writing the amounts and delays on index cards and presenting the choices to the subjects. Thus, one advantage of the procedure is the low cost of materials and reduced preparation time. A disadvantage is that this approach requires a relatively large number of choice trials before subjects’ choices reach an indifference point.

A second type of adjusting procedure, hereafter called the bisection procedure uses a bisection algorithm (Grace & McLean, 2005) to determine the parameters of choice options made available to subjects and allows discounting functions to be generated more efficiently. In general, the bisection procedure is similar to the up-down procedure except in the manner in which choice parameters are determined. Rather than following a fixed sequence of values, the bisection procedure uses the midpoint of prior accepted and rejected adjusting choices as one of the choices on subsequent trials. Suppose, for example, that the subject is asked to choose between $500 now and $1,000 in a year. If he chooses $1,000, the next choice will be between $750 now and $1,000 in a year (for an extended example, see Table 2 below). The bisection procedure first appeared in slightly different formats in two publications: Read (2001) and Du, Green & Myerson (2002), later used by Grace & McLean (2005), Green, Myerson & Macaux (2005), Holt, Green & Myerson (2003), Read & Read (2004), and
others. This procedure typically uses some type of correction for the possibility that the subject made an incorrect choice at some point, or if he changed his preference. Without the correction, one incorrect choice would make it impossible to reach the indifference point, since the adjustments are reduced in half after each choice. Read (2001) used a screen at the end of the condition asking the subject to confirm that the two options had the same value. If not, the subject could re-start the condition. Du et al. (2002) implemented a way to restart the condition before reaching the indifference point, by instructing the subject to press a key in case he wanted to change a previous choice. The indifference point in each condition is reached when the adjustment gets too small to be significant. An advantage of this procedure is the fewer number of choices trials required to reach an indifference point (e.g. 6 trials in Du et al., 2002).

A third type is the random adjusting procedure, first introduced by Richards et al. (1999). In this procedure, the subject’s choices do not precisely determine the amounts and delays presented on the upcoming choice trial. For each adjustment, a computer program randomly selects an amount from a range of amounts with set boundaries. The subject’s choices adjust the boundaries, continually narrowing the range of amounts that can be selected. The indifference point is reached when the ranges are reduced to a small size.

One important difference of the random adjusting procedure is that its adjusting nature is masked by the randomness of the adjustments. It can also reach indifference points quicker than the up-down procedures, but on average it requires more trials than bisection procedures. For instance, Richards et al. (1999) reported that his subjects
required a median of 103 trials (ranging from 74 to 148 trials) to reach 10 indifference points.

Two other procedures, developed by Kirby and Marakovic (1995; 1996), were designed to further increase the efficiency with which discount functions could be generated with human subjects. In one of these (Kirby & Marakovic, 1995), subjects directly report estimations of the present value of delayed outcomes. In this procedure, the subject’s task is to indicate the least amount of money they would be willing to receive at the end of the session in exchange for a delayed outcome. For example, subjects may see the following statement on their computer screen: “In exchange for $1,000 in 500 days, I would accept $(variable amount) but not $(variable amount minus 25¢).” The subject can increase or decrease the variable amount by clicking on up or down arrows, respectively. The fixed amounts and delays are changed on subsequent trials. Each trial produces an estimation of an indifference point allowing the derivation of discounting parameters.

In another procedure introduced by Kirby and Markovic (1996), subjects choose between smaller and larger monetary outcome available after short and long delays, respectively, in a questionnaire format. In this questionnaire format, neither delays nor amounts are adjusted and all participants simply respond through the entire set. Each pair in the questionnaire represents an indifference point calculated from a range of discounting rates. Discounting rates are estimated based on the proportion of choices consistent with each $k$ value. It is not possible to estimate independent indifference points from the subjects’ answers – rather, this procedure allows us to identify the rate
at which subjects discount monetary value by plotting subjects' choices against theoretical discount functions.

Data from all five variants have been interpreted as measuring the same general tendency to discount the value of delayed outcomes. To date, however, there have been no within-subject comparisons of these various procedures to ascertain whether procedural factors play a role in the degree of discounting observed. It has been suggested that different types of commodities are likely to be discounted at different rate (Odum & Rainaud, 2003). For example, the instances cited by Logue (1995) suggest that health outcomes are more sharply discounted than money. These data suggest that procedural details may play an important role in the degree of discounting observed.

These comparisons would also have a more immediate practical utility. Critchfield & Kollins (2001) and others have cogently argued that an understanding of value discounting is important to understanding a host of choices made by humans in their everyday lives. One way to facilitate the growth of this knowledge is to identify efficient procedures with which such questions can be asked and reliable, valid and interpretable answers generated.

As indicated above, there have been no within-subject comparisons of these various procedures to ascertain the compatibility of the data generated by them. Thus, the interchangeability of these procedures is not well established and interpretive generalizations may be premature. It is possible, for example, that some types of procedures could drive the discounting rates comparatively higher or lower. Accordingly, the purpose of this study was to systematically compare, within individual subjects, the
discount rates produced by two different procedures – the *up-down procedure* and the *bisection procedure*, both described above.
METHOD

Subjects

Twenty four undergraduate students, recruited via newspaper advertisements, posted flyers, and announcements in classrooms, were divided in two groups of 12: Group UD and Group B. Ten of the subjects (B1 to B5 and UD1 to UD5) were recruited from the University of North Texas (UNT). The remaining 14 subjects (S6 – S12 and UD6 – UD 12) were recruited from Pontifícia Universidade Católica de São Paulo (PUCSP) in São Paulo, Brazil. The subjects were 58% female and 42% male and their ages ranged from 18 to 30 years old with an average of 22.3 years. Subjects from UNT were paid $10.00 for their participation. Subjects from PUCSP received an equivalent amount in Brazilian currency (R$ 20). Payment was not contingent on performance. The participants were free to leave the session at any time but none chose to do so.

Setting and Apparatus

The sessions at UNT were run in a small room (1m x 3m) equipped with a table, two chairs, and a windows-based PC. The sessions at PUCSP were run in a small room (2m x 2m) with the same kind of equipment. Sessions were conducted using custom-developed software which presented stimuli and recorded subjects’ choices.

The program and instructions used in both countries was identical, with the only difference being that all English words were translated to Portuguese for the Brazilian participants.
Procedure

The general plan of the experiment was to compare indifference curves generated within individual subjects using two different methods of adjustments (described below). The subjects were presented with the following set of instructions prior to any exposure to the experimental procedures:

For this study you will be asked to make a series of choices, which will be displayed on this computer screen. There are no right or wrong answers. Please just answer honestly. Please click the Start button. For each choice, you will see a box on each side of the screen. Each box will contain a money amount and also whether that money would be available now or after a delay. When the computer shows the question “What do you prefer” you may use the mouse to select the alternative you prefer. Take your time to make your choice. Once you have made your choice, you will hear a beep and the computer screen will black out, then a new choice will become available. Click on “show alternatives.” As you can see, the immediate alternative is always presented on the left side, and the delayed on the right side. Please continue making your choices. I will stay here during first series of choices and make sure the procedure is understandable.

A trial began with the presentation of a box with the words “Show alternatives” inside in the center of the screen. A click on the Show Alternatives box removed the box and simultaneously presented two other boxes (14 cm wide by 10 cm high) separated by 4 cm. The text in the box on the left side of the screen presented a monetary amount and the delay with which the amount could be hypothetically obtained. The left box always presented the smaller, sooner (SS) option across all conditions. The text in the box on the right side of the screen similarly presented a monetary amount and a delay with which the amount could hypothetically be obtained. The right box always presented the larger, later (LL) option across all conditions. Subjects had to observe the alternatives for 3 seconds before making a choice. After 3 seconds passed, they were prompted to make the choice by a question “What do you
prefer” that appeared in the top center of the screen. All responses before that prompt were ineffective.

A condition within the study consisted of a series of choice trials (described above) presented with changing parameter values until the subject’s choices reflected indifference among the options. The LL amount and delay parameters were held constant within a condition and varied across conditions. Across the study, subjects were exposed to 12 different combinations of amounts and delays (i.e. 12 conditions) consisting of two monetary amounts\(^1\) ($1,000 and $10,000) and six delay durations (2, 7, 30, 180, 365 and 1095 days). These combinations defined the LL option for each condition. Subjects were exposed to amounts and delays in ascending order (see Table 1).

Within a condition, the delay associated with the SS option was held at immediate (i.e. “NOW”) and the amount was adjusted as a function of the subjects’ prior choices. For example, if a subject chose “$1000 after 2 days” over “$5 now”, the SS amount would be increased in the next trial and another choice presented to the subject. The SS amount would continue increasing in this manner until there was a shift in the subject’s preference to the SS option.

Before the experiment started, subjects were exposed to two sequences of choice trials as a means of familiarization. One sequence used the up-down procedure and required 22 to 26 trials to complete. The other used the bisection procedure and required 5 or 6 trials. The familiarization trials presented a smaller LL magnitude (i.e. $500) with a delay of 2 days. Data from these trials were not used for the analyses and will not be presented here.
The study was divided into two phases separated by a 5 minute rest period. During each phase the subject completed the 12 conditions described above (see Table 1). The only difference between the two phases was the method by which monetary amounts were adjusted across trials within a condition. Subjects in Group B were exposed to the 12 conditions using the bisection procedure (described below) first and the up-down procedure (described below) second. The order of phases was reversed for Group UD subjects.

_Bisection Procedure_

The bisection procedure consisted in continually changing the SS amount to the midpoint between the highest value that the participant judged as too low (the \textit{min}), and the lowest value that he or she judged as acceptable or too high (the \textit{max}), rounded down to the nearest multiple of $10$ (e.g., 725 would become 720). The equation to calculate the next variable amount was:

$$\text{min} + (\text{max} - \text{min})/2$$

The condition continued until an indifference point was reached. That point was reached when the difference between the \textit{min} and the \textit{max} values was less than $10$. The indifference point was then estimated as equal to:

$$(\text{min} + \text{max})/2.$$

Table 2 contains an example of a condition using the bisection procedure, with the calculation of the variables done after each choice was made.
**Up-Down Procedure**

The up-down procedure consisted in changing the variable amount following a pre-determined array of values, until the preference was reversed three times. The array used against LL amount of $1,000 was: 1, 5, 10, 20, 40, 60, 80, 100, 150, 200, 250, 300, 350, 400, 450, 500, 550, 600, 650, 700, 750, 800, 850, 900, 920, 940, 960, 980, 990 and 1,000 dollars. The conditions with LL amount of $10,000 used the same array, but each number was multiplied by 10.

Each condition started with the SS amounts presented in an ascending order from one choice trial to the next. For the conditions with the standard amount of $1,000 the first SS value was then $1, the next one $5 and so on. The SS amount was continually increased, until the preference switched from the LL amount to the SS amount. When the preferred amount was the SS, the amount at the next trial was reduced instead of increased. The indifference point was found when the preference switched for the third time, back to the SS amount. At this point, an indifference point for the ascending sequence was recorded as the midpoint between the last two SS amounts presented. The same was done next, but with SS amounts presented in a descending order, starting at the largest amount in the array. The indifference point for the complete condition was calculated as the midpoint between the indifference points recorded for the ascending and descending sequences. Table 3 contains an example of the choice trials from an ascending sequence.
RESULTS

The median session duration was 63 minutes and ranged from 52 to 103 minutes across subjects. As expected, the up-down conditions were much longer than bisection conditions, since they required a greater number of choice trials. The median duration of the entire set of conditions was 43 minutes for the up-down (range = 38 to 62 minutes) and 14 minutes for the bisection procedure (range = 9 to 27 minutes). An average of 25 choice trials (range = 17 to 32) was required to reach an indifference point in up-down condition compared to eight choice trials (range = 5 to 10) in the bisection condition.

Discounting curves were obtained for each combination of the planned conditions: small or large magnitudes ($1,000 or $10,000) and the up-down or bisection procedures. The estimated data for these combinations are shown in Figures 1 to 4. Each graph presents two indifference curves per subject. The curves in each graph are for the same magnitudes, but different procedures. Subjective values of hypothetical money are presented on the y-axis as ratios calculated by dividing the delayed amount by the immediate amount at the indifference point. For example, if the delayed amount is $1,000 and the discounted amount is $610, then the subjective value is 0.61. Transforming indifference points to ratios allows direct comparisons of results from the two magnitude conditions. Since the immediate amount could never be larger than the delayed, all ratios ranged from 0 to 1. The obtained indifference points are represented by symbols in the figures -- open symbols represents values obtained in the bisection procedure; solid symbols represent values obtained in the up-down procedure. In general, these graphs show that there were no systematic differences, across subjects,
introduced as a function of the procedure by which choice-trial parameters were adjusted.

A broad consensus in the literature on temporal discounting suggests that hyperbolic equations provide the best overall fit to obtained data. The hyperbolic model was empirically validated for both human (e.g., Rachlin et al., 1991; Rodriguez & Logue, 1988) and nonhuman animals (e.g., Mazur, 1987; Richards, Mitchell, De Wit, & Seiden, 1997). Equation 1 below describes the hyperbolic model, proposed by Mazur (1987):

\[ V = \frac{A}{1 + kD} \]  

(Equation 1: hyperbolic)

in which \( V \) is the subjective value of the outcome and \( A \) is the amount of the outcome to be delivered, \( D \) is the delay to delivery of the outcome and \( k \) is a free parameter that represents the rate at which the value of delayed outcomes is discounted.

Hyperbolic functions were fitted\(^2\) to the obtained data and evaluated for the proportion of variance in the data accounted for by the equations. The curves going through the symbols represent fitted hyperbolic functions, using nonlinear regression. In general, the hyperbolic functions fit the obtained data very well -- the median \( R^2 \) value was 0.90 for both procedures (interquartile range = 0.77 to 0.96).

Figure 5 presents the median indifference curves from the two different procedures for the two different magnitudes. The medians for all subjects (top of the figure) show that the bisection procedure curves are slightly less discounted than up-down for the small condition, and slightly more discounted for the large condition. The median indifference points are fairly similar for each magnitude condition. Taken
together, these graphs show that there was no difference in the observed rates of
dISCOUNTING AS A FUNCTION OF THE PROCEDURE.

The same data separated for Groups B and UD (Figure 5, center and lower
Graphs) show that less discounting was observed with Group B subjects in the up down
Procedure, particularly at the larger delay values. Note, however, more discounting was
observed with Group UD subjects in the same procedure. Furthermore, the apparent
Procedure effects seen for Group B subjects’ data (in Figures 1 and 2) are not
Supported by an analysis of individual curves, and data for Group UD (in Figures 3 and
4) also do not support systematic procedure effects. Three subjects (B5, B8 and B9)
Discounted more steeply in the bisection procedure but three others (B1, B2 and B4)
Discounted more steeply in the up down procedure. The remaining six subjects from
Group B show opposite effects for small and large conditions. Three subjects (B3, B6
and B12) discounted more steeply with the bisection procedure for the small condition
only, while three other subjects (B7, B10 and B11) did so only in the large condition.
The effect seen in the median curves is partly produced by outliers, such as the large
differences for subjects B5 and B8. An opposite large effect occurs for B1, but only for
the small condition. For many of the subjects (such as B2, B4, B7, B9, B11 and B12)
the differences between indifference curves for each procedure are small or show much
Overlap.

Procedure effects for individual subjects of Group UD (Figures 3 and 4) are also
Varied, but a small majority of subjects shows more discounting with the up-down
Procedure. Seven subjects (UD1, UD2, UD6, UD8, UD9, UD11 and UD12,), for
Example, discounted more steeply in the small up-down condition. However, Subject
UD7 discounted more steeply in the bisection procedure. Also note that those effects are frequently small (see UD1, UD2 and UD11 in Figures 3 and 4). Four subjects (UD3, UD4, UD5 and UD10) discounted equally the value of the dollar amounts under both procedures. In the large condition, five subjects (UD1, UD6, UD8, UD10 and UD12) discounted more with the up-down procedure. The opposite effect was found for 2 subjects (UD2, UD9) and five subjects (UD3, UD4, UD5, UD7 and UD11) discounted the values equally or almost equally with both procedures.

Values of $k$ parameters estimated from the fitted functions were log transformed to allow small changes in the values to be discerned and are presented in each graph in Figures 1 to 4. The log $k$ data are also plotted in Figure 6. These figures show that the obtained range of $k$ values varied widely from the extreme discounting of $-0.7 \log k$ (see B5’s data in Figure 1) to almost no discounting of $-5.1 \log k$ (see UD3’s data in Figure 4). The hyperbolic function could not be fitted for two subjects who showed exclusive preferences for the delayed (Subject UD4) or the immediate option (Subject UD5). In general, $k$ values were very similar across procedures for most subjects. Procedure effects measured by within-subject differences of $k$ values for each group and for each outcome magnitude were not statistically significant ($p > 0.05$).

Although the effects of procedure appeared to be negligible, other aspects of the procedure could have introduced systematic differences in subjects’ choices. A few subjects (B1, B8, B9, B11, UD1 and UD2), for example, had reported trying to be consistent with previous choices during debriefing. Thus, the variability found in procedure comparisons could have been a function of subjects’ attempts to match
previous choices. However, no systematic sequence effects were found for either group.

On the other hand, the effect of the magnitude of the outcome on discounting is substantial for the 2 groups. To describe that effect, Figure 7 shows the same median discounting functions shown in Figure 5, this time comparing the effect of outcome magnitudes within procedures. The curves show that the subjects discounted the smaller outcome ($1,000) more steeply than the larger outcome ($10,000). This magnitude effect is supported by an analysis of the individual data. For both discounting procedures, most subjects (B1, B2, B5, B6, B7, B8, B9, B11, UD1, UD2, UD6, UD7, UD8 and UD11) consistently discounted more the values of the smaller outcomes. B10 is the only subject that discounted more the value of the larger outcome with both procedures. Also, three other subjects discounted more the larger outcome with only one of the discounting procedures. For the up-down procedure, B3 and UD10 discounted more the larger outcome. With the bisection procedure, only UD9 discounted the larger outcome more. These magnitude effects (more discounting of small outcomes than of large outcomes) measured by within subject differences in log \( k \) values were statistically significant for both groups with both procedures (\( p < 0.01 \)).
DISCUSSION

Economists have long noted that the value of money in the future is not as great as money available immediately. The observation is at the basis of the distinction drawn by economists between the present and future value of money. The data from this study replicated this observation in that most subjects chose small monetary amounts delivered immediately when the alternative choice would have produced a large amount of money but after a delay. In the parlance of the temporal discounting literature, the value of monetary amounts was discounted as a function of the delay to its delivery. Furthermore, and consistent with a large experimental literature, the degree of discounting observed in the current study was well described by a single-parameter hyperbolic decay function.

The literature on temporal discounting with human subjects is largely based on five different procedures in which subjects are presented with choices between hypothetical monetary amounts delivered after hypothetical delays. Although the results from these procedures are considered directly comparable, there have been no direct investigations on the role of procedural differences for individual subjects. In the current study the effects of two procedures were compared directly in a within-subject experimental design. The data presented here suggest that there were no systematic and persistent effects of the method by which choice-trial parameters are adjusted on the degree of discounting observed within-subject.

The literature on human temporal discounting has also shown a general sensitivity to magnitude effects with subjects discounting small monetary amounts more steeply than larger monetary amounts. The indifference points generated in both
procedures were generally sensitive to outcome magnitudes in a manner consistent with data reported by other laboratories. In general, the subjects, like others, tended to discount the value of smaller outcomes more steeply than larger outcomes (see Green, Myerson, Holt, Slevin, & Estle, 2004; Raineri & Rachlin, 1993).

The procedures used in this study produced the discounting characteristically seen in studies of this sort in both procedures. Consistent with most data in the literature, the degree of discounting observed in both procedures was well described by single-parameter hyperbolic decay functions. Also consistent with the existing literature, subjects’ choices in both procedures were sensitive to the magnitude of outcomes. These similarities are important because they suggest that the two procedures appear to measure the same phenomena.

These data are important for theoretical as well as practical reasons. Theoretically speaking, the results of temporal discounting procedures have been used to provide a behavior analytic explanation for a host of problems that plague individual lives and societal functioning. It is important, if the notion is going to be put to such great use, to be sure that these findings are indeed general. The data from this experiment suggest generality of the phenomenon of discounting.

Practically speaking, these results are important because they could lead to greater use of the more efficient procedure in clinical and research settings. Earlier procedures would have been contraindicated in a clinical context because of the great deal of time and energy it would take to identify individual discounting functions. Several developments in the last few years have made the procedures increasingly efficient and easy to administer. The identification of efficient procedures to generate discounting
functions has taken on even more importance in the last few years with the growth of neuro-imaging experiments cast in behavior analytic research design molds. For example, Yarkoni, Braver, Gray, & Green (2005) have shown that localization of brain regions active during delay discounting tasks is possible. One obstacle to this research, however, has been the long session durations typically required to generate discounting functions using the traditional up-down procedure.

Unfortunately, there are no standard measures independent of arbitrary procedural differences to compare with the indifference point estimations. If finding systematic procedure differences are found, how can one decide which one is better? Of course, the best estimations will be the ones that more accurately correlate with choices from the subjects in their everyday lives. Also, a limitation of this study is the small sample of subjects, which suggests some caution for interpreting the results. Further research comparing procedures should try to replicate the current comparison of up-down and bisection procedures with larger samples and also include other types of procedures.

In sum, studies designed to investigate directly the comparability of different procedures claiming to measure the same things is an important exercise. The compatibility of up-down and bisection procedures found in this study suggests that other untested methods may also be compatible, especially those with similar procedural characteristics. For instance, in both the random adjusting procedures previously described and in the bisection procedures, amount adjustments can go upwards and downwards and gradually narrow to a small range. The main difference in the random adjusting type is in the masking of the adjustments by the random
selections of amounts within a certain range. If the masking aspect is irrelevant, these procedures would be compatible.

Once it is discovered that multiple procedures are able to reproduce results from established methods, then the procedure that is more convenient for specific purposes may be chosen. The bisection procedure, for instance, has the advantage of requiring fewer choices than the up-down procedure, thus requiring less time to estimate discounting curves. Other procedures could possibly be even faster than the bisection. One example is the procedure used by Kirby & Marakovic (1995) in which the subject directly reports his estimation of indifference points. In this study, some subjects (B1, B2, B4, B6, B9, B12, UD8, UD10, UD11 and UD12) reported deciding on such point (or limit) in the beginning of the conditions, even before adjustments took place. Future research could test if a direct report approach yields results compatible with adjusting procedures.
ENDNOTES

1The monetary unit was different for UNT and PUC-SP subjects. The currency used with UNT subjects was United States Dollars (US$) and with PUC-SP subjects it was Brazilian Reais (R$).

2Prism (2005) statistical software was used to fit the functions using non-linear regression.
Table 1

Order of Presentation of LL Conditions

<table>
<thead>
<tr>
<th>Amounts</th>
<th>Delay (days)</th>
<th>2</th>
<th>7</th>
<th>30</th>
<th>180</th>
<th>365</th>
<th>1095</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1,000</td>
<td>1\textsuperscript{st}</td>
<td>2\textsuperscript{nd}</td>
<td>3\textsuperscript{rd}</td>
<td>4\textsuperscript{th}</td>
<td>5\textsuperscript{th}</td>
<td>6\textsuperscript{th}</td>
<td></td>
</tr>
<tr>
<td>$10,000</td>
<td>7\textsuperscript{th}</td>
<td>8\textsuperscript{th}</td>
<td>9\textsuperscript{th}</td>
<td>10\textsuperscript{th}</td>
<td>11\textsuperscript{th}</td>
<td>12\textsuperscript{th}</td>
<td></td>
</tr>
</tbody>
</table>
Table 2

Example of a Bisection Adjusting Procedure and Calculation of Variables for Subject B6

<table>
<thead>
<tr>
<th>Choice Trials</th>
<th>Calculation of the next SS amount</th>
<th>Value of variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>$500 now</td>
<td>$1000 in 7 days</td>
<td>$500 + (1000 - 500)/2</td>
</tr>
<tr>
<td>$750 now</td>
<td>$1000 in 7 days</td>
<td>$500 + (750 - 500)/2</td>
</tr>
<tr>
<td>$620 now</td>
<td>$1000 in 7 days</td>
<td>$620 + (750 - 620)/2</td>
</tr>
<tr>
<td>$680 now</td>
<td>$1000 in 7 days</td>
<td>$680 + (750 - 680)/2</td>
</tr>
<tr>
<td>$720 now</td>
<td>$1000 in 7 days</td>
<td>$680 + (720 - 680)/2</td>
</tr>
<tr>
<td>$700 now</td>
<td>$1000 in 7 days</td>
<td>$680 + (700 - 680)/2</td>
</tr>
<tr>
<td>$690 now</td>
<td>$1000 in 7 days</td>
<td>$690 + (700 - 690)/2</td>
</tr>
</tbody>
</table>

Indifference point = (690 + 700)/2 = 695
Table 3

*Example of an Ascending Sequence of the Up-Down Adjusting Procedure for Subject B6*

<table>
<thead>
<tr>
<th>Choice Trials</th>
<th>Array of SS Amounts</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 now</td>
<td>$1000 in 365 days</td>
</tr>
<tr>
<td>$5 now</td>
<td>$1000 in 365 days</td>
</tr>
<tr>
<td>$10 now</td>
<td>$1000 in 365 days</td>
</tr>
<tr>
<td>$20 now</td>
<td>$1000 in 365 days</td>
</tr>
<tr>
<td>$40 now</td>
<td>$1000 in 365 days</td>
</tr>
<tr>
<td>$60 now</td>
<td>$1000 in 365 days</td>
</tr>
<tr>
<td>$80 now</td>
<td>$1000 in 365 days</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$60 now</td>
<td>$1000 in 365 days</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$80 now</td>
<td>$1000 in 365 days</td>
</tr>
</tbody>
</table>

Indifference point of the sequence: 
\[(60 + 80)/2 = 70\]
Figure 1. Indifference curves for Group B for the small condition.
Figure 2. Indifference curves for Group B for the large condition.
Figure 3. Indifference curves for Group UD for the small condition.
Figure 4. Indifference curves for Group UD for the large condition.
Figure 5. Median indifference curves and procedure effects. On top median for all subjects, below medians for Group B and for Group UD. Error bars represent the interquartile range. Data for bisection procedure was nudged left to avoid overlapping the error bars.
Figure 6. Log k value comparison by procedure.

Figure 7. Median indifference curves and magnitude effects. Error bars represent the interquartile range. Data for small condition was nudged left to avoid overlapping the error bars.
REFERENCES


Prism (version 4.0) [computer program] (2005).


