THE EFFECT OF ROUGHNESS ELEMENTS
ON THE MAGNUS CHARACTERISTICS
OF
ROTATING SPHERICAL PROJECTILES

THESIS

Presented to the Graduate Council of the
North Texas State University in Partial
Fulfillment of Requirements

For the Degree of

MASTER OF SCIENCE

by

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Thirty trials of each of three roughness conditions were examined. The first condition consisted of a baseball pitched so that two of the roughness elements opposed the flow. The second condition consisted of a pitched baseball with four of the roughness elements opposing the flow. The third consisted of a pitched uniformly rough sphere. The conclusions were that roughness elements increase horizontal flight deviations when a baseball rotates about a vertical axis; roughness elements on the surface of a baseball may cause a decrease in the encountered drag forces; linear velocity has a dominating effect on the trajectory of a spinning baseball; previously developed mathematical models do not adequately predict flight deviations.
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CHAPTER I

INTRODUCTION

A projected object which is both translating and rotating will deviate from a hypothetical parabolic pathway due to the presence of air resistance (Magnus, 1853). The deviation in flight is caused by the spin of the object and is called the Magnus or Robins effect (Magnus, 1853; Robins, 1842; Jacobsen, 1973). Robins (1842) noted that the spin imparted to cannon balls resulted in accuracy losses. Magnus (1853) successfully demonstrated, with rotating bodies, the presence of pressure inequalities and a resulting eccentric force.

One of the factors influencing the motion of a solid object through a fluid is the nature of the boundary layer of the fluid surrounding the object. There exists five forms of boundary layer flow; i.e., creeping flow, laminar flow, turbulent flow, partially turbulent flow and supersonic flow.

Creeping flow occurs with a non-viscous fluid and is characterized by no boundary layer separation. Martin (1955) found that the Magnus effect for creeping flow was caused by an assymetrical fluid boundary layer. That is, the portion of a rotating body that is moving in relative
opposition to the incoming fluid carries with it a thicker fluid boundary layer than does the portion which is moving in the same direction as the incoming flow. This, in effect, creates a new body on which the point of application of the resultant fluid drag force is eccentric to the center of gravity. The resultant eccentricity of the force causes the object to deviate from a normal path.

Laminar flow exists when the boundary layer separation point(s) occurs in a region of the object not presented to the flow. Briggs (1959) studied lateral deflections of baseballs and smooth spheres caused by the Magnus effect in primarily laminar flow. He stated, that in the case of the baseballs, the spin imparted to the ball causes an inequality of pressures and affects the general flow field around the body. This, in accordance with the Bernoulli principle, causes a flight deviation from a hypothetical parabolic path. Briggs found a negative Magnus effect occurred for smooth spheres. This effect, to date, has not been completely explained.

In turbulent flow, boundary layer separation occurs in the region of the object presented to the flow. It is achieved when the Reynolds number exceeds an experimentally determined critical value. This critical value is achieved under conditions of high fluid velocity and/or motion of the object through a highly viscous fluid (Daily & Harleman, 1966).
Partially turbulent flow occurs when an immersed body is asymmetrical or a symmetrical body is rotating in a fluid. Until a critical Reynolds number is reached, the boundary layer around a nonspinning cylinder separates at approximately 82 degrees from the forward stagnation point (Jacobsen, 1973; Krahan, 1956). The separation point moves downstream to about 130 degrees as the Reynolds number increases and the fluid flow becomes turbulent. The location of the separation points are affected when the Reynolds number approaches a critical value and an immersed cylinder begins to rotate. On the portion of the cylinder opposing the flow, the separation point moves toward the forward stagnation point. The separation point of the side moving in the same direction remains located at 82 degrees, and the fluid flow in that portion of the body remains laminar. This causes a resultant lift force in the opposite direction to the classical Magnus effect (Jacobsen, 1973).

Supersonic flow differs from turbulent flow in that shock waves are created. As the critical Mach number is exceeded, the flow in the region of the shock waves becomes non-isentropic. As this critical Mach number is exceeded, drag forces increase (Hughes & Brighton, 1967). Efforts to reduce drag forces at supersonic speeds include the "swept-wing" design of aircraft (Daily & Harleman, 1966). The separation points and fluid flow conditions are as illustrated in Figure 1.
Fig. 1—Boundary layer conditions

- Creeping Flow
- Partially Turbulent Flow
- Laminar Flow
- Turbulent Flow
Of the five fluid flow conditions only laminar, turbulent and partially turbulent flow are likely to be encountered in sports situations. Turbulent flow primarily occurs in ballistic activities. For example, Jacobsen (1973) reviewed the literature pertaining to ballistics and concluded that the dynamic stability of a bullet is dependent upon the location of the center of gravity, shape and angular velocity about the longitudinal axis. Borg (1980) discussed the potential use of an immersed revolving cylinder to aid in the steering of large ocean vessels. The literature pertaining to partially turbulent flow is scarce. Partially turbulent flow is rarely encountered although its presence has been recorded (Briggs, 1959).

In the majority of sport settings, the fluid flow is laminar. In addition, in many activities spin is purposely imparted to a ball to cause flight deviations.

Barnaby (1978) discussed the importance of spin in tennis and stated that

On any surface, the flight of the ball through the air can and must be controlled by spin whenever the ball is played vigorously enough so that gravity will not do the job. That is why topspin should be the first new skill acquired by an intermediate.

Gray (1974), in discussing the experimental results of Plagenhoef (1970), claimed that if no spin was imparted to a typical served ball, the ball would land well beyond the opponent's baseline. In the game of soccer, a predominance of sidespin is often used to project the ball around
obstacles. This "bending ball" has been described by Miller (1979). In the game of golf, the inclination of the clubface (i.e., its loft) causes the ball to be projected with a backspin (Hay, 1978; Davies, 1949). The magnitude of the spin can approach 8000 revolutions per minute and cause an increase in the range of the ball up to 50 percent (Chase, 1981; Davies, 1949). Both desirable and undesirable slices or hooks also result from the Magnus effect. Chase (1981) explains that when the spin imparted to the golfball has an axis of rotation deviating from horizontal, deviation in flight will occur to an amount which is dependent upon the deviation of the axis.

In baseball, the ability of a pitcher is perhaps the most important factor in determining the success of a team. Reiff (1971) claims that 65 to 85 percent of winning baseball depends on a pitcher's ability. The quality of a pitcher is, in turn, dependent upon an ability to both vary the linear velocity of the pitches and to deviate the ball from normal projectile flight. In most situations, this deviation is accomplished by imparting spin to the ball to create a Magnus effect. An indication of the amount that a ball can deviate from normal projectile flight has been reported by Briggs (1959). Lateral deflections ranging from 6.1 inches to 26.0 inches for a spinning baseball in a six foot drop across windtunnel streams were reported. Selin (1959) reported maximum vertical deviations of 2.4
feet and horizontal deviations of 2.1 feet for a variety of spinning pitches. Attempts have been made to mathematically model a pitched baseball. Kriegbaum and Hunt (1978) developed a mathematical model to simulate pitches of various linear and angular velocities. The results of their approximated deviations from normal projectile flight yielded maximum values more than 50 percent lower than those reported by either Selin (1959) or Briggs (1959). A factor not considered in the Kriegbaum and Hunt model was the roughened surface of the ball. Hay (1978) claims that the variables effecting the drag force on a projectile are the velocity of the flow, the surface area of the body, the characteristics of the fluid involved, the cross-sectional area presented to the flow, and the smoothness of the surface of the body. The effects of the smoothness of the body on drag forces have been examined by Watts and Sawyer (1975). In discussing the aerodynamics of a knuckleball, the authors stated:

There are two possible mechanisms for the erratic lateral force that causes the fluttering flight of the knuckleball. A fluctuating lateral force can result from a portion of the strings being located just at the point where the boundary layer separation occurs. A far more likely situation is that the ball spins very slowly, changing the location of the roughness elements (strings), and thereby causing a nonsymmetric velocity distribution and a shifting of the wake.

The differences between the reported mathematical and experimental data indicate that a need exists to further
examine the Magnus effect for rotating spheres and to in-
clude a consideration of the roughness of the spheres.

**Purpose**

The purpose of this study was to examine the effect of
roughness elements on the Magnus characteristics of ro-
tating spheres.

**Delimitations of the Study**

The delimitations in the analysis of experimental
pitching performances included the following.

1. A baseball pitching machine was used to approxi-
mate the motion of a pitched baseball.

2. Data extracted off the film records of thirty
trials for each of three conditions were analyzed:

   a. A baseball rotating with four roughness ele-
   ments opposing the flow

   b. A baseball rotating with two roughness ele-
   ments opposing the flow

   c. A rotating sphere of uniform roughness

**Limitations of the Study**

1. Normal cinematographical and data extraction
limitations were recognized.

2. The limitations of numerical approximation sam-
pling rates of a 64 Kilobyte Tektronics 4052
computer were recognized.
3. Angular velocity was assumed to remain constant throughout the flight of the ball.
4. Laminar fluid flow was assumed.

Definitions of Terms and Symbols
Angle of attack - The angle between the velocity vector and the angular velocity vector
Numerical Approximation - A mathematical technique used to estimate a solution of unsolvable differential equations which involves a solution for the integral of an equation using geometric means. A sufficiently small time increment, together with known constants and initial values for variables are used in the equations. The solutions represent new initial values for the variables. This process is repeated, usually by computer, using these new initial values for the variables. The accuracy of such approximations is determined by the sampling frequency, or how many times the process is repeated.

\( r \) = radius of the ball
\( \mu \) = kinematic viscosity of the fluid
\( \rho \) = density of the fluid
\( m \) = mass of the ball
\( t \) = time
\( \Delta \) = change in any variable (i.e., \( \Delta t \) = change in time)
\( |V| \) = magnitude of velocity
\[ R = \text{Reynolds number} = \frac{\rho |\mathbf{V}| r}{\mu} \]

\[ \mathbf{V} = \text{velocity vector (subscripts indicate direction)} \]

\[ \Omega = \text{angular velocity vector (subscripts indicate direction)} \]

\[ \mathbf{a} = \text{acceleration vector (subscripts indicate direction)} \]

\[ d = \text{any distance} \]

\[ \theta = \text{angle of attack} \]

\[ \mathbf{F}_L = \text{lift force} \]

\[ \mathbf{F}_D = \text{drag force} \]

\[ \mathbf{F}_G = \text{force due to gravity} \]

\[ \mathbf{F} = \mathbf{F}_L + \mathbf{F}_D + \mathbf{F}_G + \text{total force} \]
CHAPTER BIBLIOGRAPHY


CHAPTER II

REVIEW OF LITERATURE

In many sports activities the performer and/or sport implement is projected into the air. According to Hay (1978), the factors affecting projectile flight are the velocity of the flow, the surface area of the body, the characteristics of the fluid involved, the cross-sectional area presented to the flow, and the smoothness of the surface of the projectile. The fluid resistance forces may be of sufficient magnitude to cause appreciable deviations from a parabolic flight path. The observed airborne motion of a badminton shuttlecock provides an obvious example of a non-parabolic trajectory (Hay, 1978). In some activities the sport implements are projected in such a way to purposely cause flight deviations. These flight deviations commonly result from imparted rotations. For example, in the game of soccer, the sidespin of the ball permits it to travel around obstacles (Miller, 1979; Hay, 1978).

One of the major factors influencing the fluid force acting on a projectile is the nature of the boundary layer of fluid surrounding the object. There exists five forms
of boundary layer flow; i.e., creeping flow, laminar flow, turbulent flow, partially turbulent flow and supersonic flow.

Creeping flow is characterized by no boundary layer separation. It occurs only in a non-viscous fluid or at extremely low velocities (Jacobsen, 1973). Martin (1955) found that the Magnus effect for creeping flow was caused by an assymetrical fluid boundary layer. That is, the portion of a rotating body that is moving in relative opposition to the flow carries with it a thicker fluid boundary layer than does the portion moving in the same direction as the flow. This creates a new body upon which the point of application of the resultant fluid drag force is eccentric to the center of gravity of the body. This eccentricity of the resultant force causes the object to deviate from a hypothetical parabolic path.

Laminar flow exists when the boundary layer separation occurs in a region of the object not presented to the flow. Briggs (1959) studied lateral deflections of baseballs and smooth spheres caused by the Magnus effect in primarily laminar flow. He stated, that in the case of the baseballs, the spin imparted to the ball causes an inequality in pressures and affects the general flow field around the body. This, in accordance with the Bernoulli principle, causes the deflection from a hypothetical parabolic path.
Briggs, however, found that an unexplained negative Magnus effect occurred for smooth spheres.

In turbulent flow, boundary layer separation occurs in the region of the object presented to the flow. For a sphere this separation occurs at an angle of 130 degrees from the forward stagnation point (Jacobsen, 1973). Turbulent flow is achieved when the so-called Reynolds number exceeds a critical value. This critical Reynolds number is achieved under conditions of high fluid velocity and/or motion of the object through viscous fluid (Daily & Harleman, 1966).

Partially turbulent flow occurs only when the immersed body is asymmetrical or a symmetrical body is rotating in a fluid. Under laminar flow conditions, boundary layer separation occurs at 82 degrees from the forward stagnation point. When the Reynolds number approaches the critical value and a symmetrical body begins to spin, the boundary layer separation in the portion of the body moving in relative opposition to the flow moves downstream to 130 degrees. The portion of the body moving in the same direction as the flow remains laminar with boundary layer separation occurring at 82 degrees from the forward stagnation point. This causes a resultant lift force in the opposite direction to the classical Magnus effect (Jacobsen, 1973). This force is commonly referred to as the negative Magnus effect (Briggs, 1959; Jacobsen, 1973).
When the velocity of the flow becomes extremely high, the flow can become supersonic. Flow becomes supersonic when a critical Mach number is exceeded. As this Mach number is exceeded, shock waves are created within the flow which causes the flow to become non-isentropic. In supersonic flow, fluid drag increases due to the large frictional forces (Hughes & Brighton, 1967). Daily and Harleman (1966) discussed efforts in aviation to reduce drag forces at supersonic speeds by the use of "swept-wing" designs.

Of the five fluid flow conditions only laminar, turbulent and partially turbulent flow are likely to be encountered in sports situations. Turbulent flow primarily occurs in ballistic activities. For example, Jacobsen (1973) reviewed the literature pertaining to ballistics and concluded that the dynamic stability of a bullet is dependent upon the location of the center of gravity, shape and angular velocity about the longitudinal axis. Borg (1980) discussed the potential use of an immersed revolving cylinder to aid in the steering of large ocean vessels. The literature pertaining to partially turbulent flow is scarce. Partially turbulent flow is rarely encountered although its presence has been recorded (Briggs, 1959). In the majority of sport settings, the fluid form is laminar. Moreover in many activities spin is purposely imparted to a ball to cause flight deviations.
Barnaby (1978) discussed the importance of spin in tennis and stated that

On any surface, the flight of the ball through the air can and must be controlled by spin whenever the ball is played vigorously enough so that gravity will not do the job. That is why topspin should be the first new skill acquired by an intermediate.

Gray (1974), in discussing the experimental results of Plagenhoef (1970), claimed that if no spin was imparted to a typical served ball, the ball would land well beyond the opponent's baseline. In the game of soccer, a predominance of imparted sidespin is often used to project the ball around obstacles. This so-called "bending ball" has been described by Miller (1979). In the game of golf, the inclination of the clubface (i.e., its loft) causes the ball to be projected with a backspin (Hay, 1978; Davies, 1949). The magnitude of the spin can approach 8000 revolutions per minute and cause an increase in the range of the ball up to 50 percent (Chase, 1981; Davies, 1949). Both desirable and undesirable slices or hooks also result from the Magnus effect. Chase (1981) explains that when the spin imparted to the golf ball has an axis of rotation deviating from horizontal, deviation in flight will occur to an amount which is dependent upon the deviation of the axis.

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There are two possible mechanisms for the erratic lateral force that causes the fluttering flight of the knuckleball. A fluctuating lateral force can result from a portion of the strings being located just at the point where the boundary layer separation occurs. A far more likely situation is that the ball spins very slowly, changing the location of the roughness elements (strings), and thereby causing a non-symmetric velocity distribution and a shifting of the wake.

Attempts have been made to mathematically model both the path of a pitched baseball and the forces acting on a sphere in a fluid. Kriehbaum and Hunt (1978) developed a
mathematical model for a pitched baseball. Using the parameters described by Selin (1959) the following model was constructed:

\[
\begin{align*}
\frac{dV_x}{dt} &= K C_d V_x + C_1 (V_y \cos B + V_z \sin A \sin B) \\
\frac{dV_y}{dt} &= K C_d V_y + C_1 (V_z \sin B \cos A - V_x \cos B) \\
\frac{dV_z}{dt} &= K C_d V_z + C_1 (V_x \sin B \sin A - V_y \sin B \cos A)
\end{align*}
\]

where \( K = \frac{pV_A \Delta t}{2m} \).

The displacement in each direction during the interval \( dt \) was given as \( dS_j = (V_j + dV_j) \Delta t \) where \( j = x, y \) or \( z \).

The sum of the \( dS_j \) terms over the time of flight of the ball was given as the total displacement in the direction of \( j \). The model took the following factors into account:

- **A** = horizontal angle - defined as the "angle between the direction of the pitch and the axis of rotation measured in a counterclockwise direction from the direction of the pitch to the axis of rotation" (Kriegbaum & Hunt, 1978).
- **B** = vertical angle - defined as the "angle between a vertical line to the center of the ball and the axis of rotation measured from the upward-directed vertical to the axis of rotation" (Kriegbaum & Hunt, 1978).

\( C_d \) = coefficient of drag  
\( C_1 \) = coefficient of lift  
\( F_d \) = drag force  
\( F_l \) = lift force
\( F_g \) = gravitational force
\( \frac{\text{d}t}{\text{d}t} \) = change in time
\( m \) = mass of the ball
\( \bar{V} \) = velocity (subscripts indicate direction)
\( A \) = area presented to the flow
\( p \) = density of the air
\( g \) = acceleration due to gravity

This model yields deviations over 50 percent less than those reported by either Selin (1959) or Briggs (1959).

Two possible causes for the difference between actual and hypothetical values for flight deviation are the exclusion of a roughness factor in the model and no consideration for the magnitude of angular velocity. The necessity for including a roughness factor has been implied by Hay (1978).

Oseen, as referenced by Rubinow and Keller (1963), developed a formula for the drag force acting on a totally immersed translating sphere. The developed equation was

\[
F_d = -6\pi r \mu \bar{V}(1 + 3R/8)
\]

where \( F_d \) is the force of drag, \( r \) is the radius of the sphere, \( \mu \) is viscosity, \( \bar{V} \) is the velocity vector and \( R \) is the Reynolds number \( \rho |\bar{V}|r/\mu \) where \( \rho \) is the density of the air.

In the above equation the first term \( -6\pi r \mu \bar{V} \) was calculated by Stokes, as referenced by Rubinow and Keller (1963). From this information, Rubinow and Keller (1963) developed a mathematical model for the forces acting on a rotating
and translating sphere. Their results yielded the following formula:

\[ F_t = F_d + F_1 = -6\pi r \mu \bar{V}(1 + 3R/8) + \pi r^3 \rho (\bar{\omega} \times \bar{V}) \]

where \( \bar{\omega} \) is the angular velocity vector. These relationships were confirmed by Hess (1968). This model differs from that of Kriegbaum and Hunt (1978) in that it provides a more comprehensive consideration of the factors affecting the nature of the fluid and, in particular, it takes into account the magnitude of angular velocity.

In summary, a review of literature revealed that very few scientific studies have been concerned with the fluid forces acting on a pitched baseball. Attempts have been made to measure flight deviations caused by the Magnus effect and to model the forces acting on spheres and baseballs travelling through a fluid. Discrepancies between experimentally and mathematically obtained deviations indicate a need to include a roughness factor in the models.


CHAPTER III

PROCEDURES

The purpose of this study was to examine the effects of roughness elements on the Magnus characteristics of rotating spheres.

Instrumentation

Cinematographical Instrumentation

A high-speed 16mm motion picture camera (Teledyne Camera Systems, Model DBM-54) was used to obtain film records of each trial. The camera was positioned 2.2 meters above the level at which the ball was to be pitched. Appropriate levelling techniques were used to ensure that the optical axis of the camera corresponded with a vertical axis. The operating speed of the camera was 500 frames per second. Temporal scales were obtained by means of a timing light generator used in conjunction with the motion picture camera. In order to make it possible to determine the release angle of the ball, a mirror was placed thirty centimeters to the right, and at the same vertical height, of the anticipated initial flight of the ball. The mirror was oriented at approximately 0.8 radians. One number coded card was included within the field of view of the
camera and recorded on film for each trial. The number was subsequently used to identify the trial.

**Pitching Machine**

An automatic pitching machine (Jugs Curveball Pitcher, JoPaul Industries, Inc., Tualatin, Oregon) was used to deliver balls towards a measurement board. The height of release of the ball was fixed at 2.59 meters which corresponded to the average height of release of a pitched ball (Williams, 1971).

**Projectiles**

The projectiles used during the testing session were ten baseballs (Rawlings R. O., St. Louis, Missouri) and a rubber sphere of uniform roughness.

**Measurement Board**

A board was anchored 17.1 meters from the release point of the ball. This distance corresponded to the distance from a pitcher's mound to home plate. A cartesian coordinate system was marked on the board. The origin of the coordinate system was located at the expected horizontal landing point of the ball neglecting all fluid and gravitational forces and at ground level. The board was coated with chalk which permitted the determination of the contact point of the ball.
Testing Procedures

All of the trials for the study were conducted during a one-day filming session. The testing location was the Men's Gymnasium of the North Texas State University (Denton, Texas). The location of the testing instruments was as shown in Figure 2. The filming session occurred at a time when all air conditioning and heating systems were inoperative and the building was closed. These conditions were necessary to minimize extraneous air turbulence.

Prior to and at thirty-minute intervals during the filming session, the temperature and barometric pressure at the testing site were determined and recorded. These recorded values were converted at a later time to approximate values for both the density and kinematic viscosity of the air.

To assist in determining angular velocity, the balls were marked with black dots approximately 0.5 centimeters in diameter. To ensure that at least one point on the ball was visible at all times, the landmarks were placed such that the largest distance between any two marks was approximately four centimeters.

Ten trials at each of three pitching machine settings for each of three roughness conditions were recorded on film. The first condition consisted of a baseball pitched in a manner that two of the roughness elements opposed the incoming flow. The second condition consisted of a pitched
baseball with four of the roughness elements opposing the flow. The third consisted of a pitched rotating uniformly rough sphere. An illustration of the roughness conditions is shown in Figure 3. Any trials in which the pitched ball did not strike the measurement board were eliminated from this study. After each trial, the location of the ball with respect to the origin of the measurement board was recorded.

Data Acquisition Procedures

For each successful trial, the initial portion of the flight of the ball was analyzed with the aid of a Lafayette 16mm Analyzer (Lafayette Instrument Co., Lafayette, Indiana) in conjunction with a Numonics Electronic Graphics Digitizer (Model 1200, Numonics Corp., North Wales, Pennsylvania), which was interfaced to a Tektronix 4052 Graphics Calculator (Tektronix Inc., Beaverton, Oregon). The motion of the ball for three frames in which the projected image in the mirror was visible was analyzed. The cartesian coordinates of each of the following landmarks on the ball were digitized and recorded for all film frames:

1. Any three landmarks on the circumference of the ball
2. Any three landmarks on the circumference of the ball's projected image in the mirror
3. One landmark on the surface of the ball.

The three landmarks on the circumference of the ball were used to determine the trajectory of the center of gravity of the ball in the X-Y plane. The three landmarks on the circumference of the ball's projected image on the mirror were used to determine the trajectory of the center of gravity of the ball in the X-Z plane. The derivative of each of the component displacements of the ball was computed to determine the initial linear velocity of the ball in the component directions. The angular velocity of the ball was computed from the displacement history of the landmark. The values of barometric pressure and temperature were used to calculate the density of the air and the kinematic viscosity of the air as prescribed by the Handbook of Chemistry and Physics (CRC, 1978). These variables were then used in a computer program which used numerical approximation techniques to give predicted values for deviation from normal projectile flight.

The formula used to compute angular velocity appears in Appendix A. A listing of the computer program used to compute the angular and linear velocity of the balls appears in Appendix B. The mathematical development of the mathematical model developed by Rubinow and Keller (1963) for numerical approximation of hypothetical landing points appears in Appendix C. A listing of the computer program
utilizing this numerical approximation to predict values for the landing points of the projectiles appears in Appendix D.

Statistical Analysis

A one way analysis of variance was used to determine if differences existed in component flight deviations between roughness conditions. A statistical analysis utilizing mixed design repeated measures analysis of variance procedures with the roughness conditions serving as the non-repeating factor was conducted to determine whether or not interactions existed between roughness conditions and the method of determining component landing points. The actual and hypothetical component landing points served as dependent variables. Pearson product-moment correlation coefficients were computed to ascertain the relationship between the component landing locations for each condition and the hypothetical landing locations. Multiple linear regression was used to determine if the component landing points could be accurately predicted using statistical techniques. The component initial and angular velocities for each condition were entered as the independent measures. Pearson product-moment correlation coefficients were computed between the component initial linear velocities for each of the roughness conditions.
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CHAPTER IV

RESULTS

The purpose of this study was to examine the effects of roughness elements on the Magnus characteristics of rotating spheres.

Environment and Projectiles

The temperature and barometric pressure were determined during the data collection session. Both remained constant and yielded values of $1.84 \times 10^{-5}$ mPl and 1.193 kg/m$^3$ for air viscosity and air density respectively. The ten baseballs used during the testing session ranged in mass from 146.5 grams to 150.75 grams ($M = 148.56$ grams) with radii of 3.689 centimeters. The rubber sphere of uniform roughness had a mass of 94.78 grams and a radius of 3.598 centimeters.

Results

The mean component flight deviations of the projectiles for each of the roughness conditions are shown in Table I. All of the component deviations are expressed in differences between the component landing points on the
measurement board and hypothetical component landing points calculated by assuming flight in the absence of air resistance.

TABLE I

MEAN X- AND Z- FLIGHT DEVIATIONS FOR EACH OF THE ROUGHNESS CONDITIONS

<table>
<thead>
<tr>
<th>Condition</th>
<th>Flight Deviation⁹</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X</td>
<td>Z</td>
</tr>
<tr>
<td>Two roughness elements</td>
<td>45.83</td>
<td>41.70</td>
</tr>
<tr>
<td>Four roughness elements</td>
<td>48.54</td>
<td>55.51</td>
</tr>
<tr>
<td>Uniformly rough sphere</td>
<td>4.82</td>
<td>146.83</td>
</tr>
</tbody>
</table>

⁹Deviation measured in centimeters

Statistical analyses revealed significant differences (p<0.05) between both of the baseball conditions and the uniformly rough sphere for both the X- and Z- flight deviations. Summary tables for the statistical analyses appear in Appendices E and F.

The flight deviations in the X- direction for the baseball conditions were due to a relatively large Magnus effect whereas for the uniformly rough sphere this effect was minimal. The mean component angular velocities for each condition are shown in Table II. The results show that the resultant axis of rotation for all conditions was primarily oriented in the Z- direction. A statistical
TABLE II
MEAN COMPONENT INITIAL ANGULAR VELOCITIES FOR EACH OF THE ROUGHNESS CONDITIONS

<table>
<thead>
<tr>
<th>Condition</th>
<th>Angular Velocity$^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Two roughness elements</td>
<td>-715.8</td>
</tr>
<tr>
<td>Four roughness elements</td>
<td>-452.4</td>
</tr>
<tr>
<td>Uniformly rough sphere</td>
<td>-236.2</td>
</tr>
</tbody>
</table>

$^a$Angular velocity measured in radians per second

analysis revealed a significant difference ($p<0.05$) between the baseball with four roughness elements opposing the flow and the uniformly rough sphere. The finding of significant differences between both baseball conditions and the uniformly rough sphere for the $X$- component of flight deviation, and the lack of a significant difference between the baseball with two roughness elements and the uniformly rough sphere for the $Z$- component of angular velocity indicates that differences in the Magnus effect were not primarily due to differences in angular velocity. A summary table for the statistical analysis of the angular velocities appear in Appendix G.

The mean magnitude of the initial linear velocity for the baseball with two roughness elements, the baseball with four roughness elements and the uniformly rough sphere were
34.93 meters per second, 36.04 meters per second and 44.28 meters per second respectively. The greater velocity magnitude for the uniformly rough sphere could have contributed to a reduced Magnus effect for this condition. However, the Reynolds number for this magnitude of velocity was approximately one-third of the critical value. This suggests that a classical Magnus effect should have been present. The reported small flight deviation in the X-direction was therefore probably not primarily due to a greater linear velocity but instead to the absence of roughness elements.

The reported flight deviations in the Z-direction can in part be attributed to differences in the masses of the baseballs and the uniformly rough sphere. That is, the drag forces acting on the relatively smaller mass of the uniformly rough sphere prolonged the time of flight and therefore extended the gravitational effects. However, if the differences in mass are normalized, the Z-component of flight deviation for the baseballs should have been 1.93 times that recorded. It is therefore possible that the roughness elements increased the turbulence of the boundary layer which decreased the fluid drag force and thus decreased the time of flight. A decrease in the time of flight would reduce the gravitational effects and therefore the flight deviation.
The mean component landing points with respect to the origin established on the measurement board for each of the roughness conditions are shown in Table III.

TABLE III
MEAN COMPONENT DISPLACEMENTS FOR EACH OF THE ROUGHNESS CONDITIONS

<table>
<thead>
<tr>
<th>Condition</th>
<th>Linear Displacementa</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Two roughness elements</td>
<td>-26.05</td>
</tr>
<tr>
<td>Four roughness elements</td>
<td>-25.12</td>
</tr>
<tr>
<td>Uniformly rough sphere</td>
<td>-2.66</td>
</tr>
</tbody>
</table>

aDisplacements measured in centimeters

The mean component landing points as calculated using the formulae developed by Rubinow and Keller (1963) for each of the roughness conditions are shown in Table IV.

A statistical analysis revealed significant differences (p<0.05) between the actual and hypothetical X-components of the landing points for the baseball with four roughness elements and for the uniformly rough sphere. A significant interaction was found between the roughness conditions and the methods for determining the Z-component of the landing point. An analysis of the simple main effects revealed a significant difference between the methods of determining the Z-component of the landing
**TABLE IV**

MEAN HYPOTHETICAL COMPONENT DISPLACEMENTS FOR EACH OF THE ROUGHNESS CONDITIONS

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Linear Displacement&lt;sup&gt;a&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Two roughness elements</td>
<td>-44.61</td>
</tr>
<tr>
<td>Four roughness elements</td>
<td>-63.25</td>
</tr>
<tr>
<td>Uniformly rough sphere</td>
<td>-42.34</td>
</tr>
</tbody>
</table>

<sup>a</sup>Displacements measured in centimeters

points for the uniformly rough sphere. This indicates that for this condition, the equations developed by Rubinow and Keller (1963) provide a poor prediction. Summary tables of the statistical analyses appear in Appendices H and I.

Pearson product-moment correlation coefficients calculated between the actual and hypothetical component landing points for all of the conditions indicate the existence of an overall positive relationship. However, the finding of some significant differences between the actual and hypothetical component displacements suggests that the predictive equations are of questionable practical value for the conditions examined in this study. There exists the possibility that a roughness factor should be included in the mathematical model. The correlation coefficients
between the actual and hypothetical component landing points are shown in Table V.

TABLE V

CORRELATION COEFFICIENTS BETWEEN ACTUAL LANDING POINTS AND HYPOTHETICAL LANDING POINTS

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Component of Landing Points</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>X</td>
<td>Z</td>
</tr>
<tr>
<td>Two roughness elements</td>
<td>28</td>
<td>.4568*</td>
<td>.4467*</td>
</tr>
<tr>
<td>Four roughness elements</td>
<td>29</td>
<td>.6404*</td>
<td>.8770*</td>
</tr>
<tr>
<td>Uniformly rough sphere</td>
<td>28</td>
<td>.7545*</td>
<td>-.3111</td>
</tr>
</tbody>
</table>

* p<0.05

Multiple linear regressions were used to determine if the component landing points could be accurately predicted using statistical techniques. The component initial linear and angular velocities for each condition were entered as the independent measures. The results of the analyses appear in Table VI. The results indicate that the regression analyses provided a better prediction of the actual landing points than did the model developed by Rubinow and Keller (1963). The order of entry of the independent variables into the regression analyses is shown in Table VII. The results show, that with the exception of the two roughness elements condition, the X-component of linear velocity provides the greatest contribution towards predicting both
TABLE VI

R AND R-SQUARED VALUES RESULTING FROM THE MULTIPLE LINEAR REGRESSION USED TO PREDICT COMPONENT LANDING POINTS FOR EACH OF THE ROUGHNESS CONDITIONS

<table>
<thead>
<tr>
<th>Condition</th>
<th>R²</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>X-direction</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Two roughness elements</td>
<td>0.46</td>
<td>0.67</td>
</tr>
<tr>
<td>Four roughness elements</td>
<td>0.79</td>
<td>0.89</td>
</tr>
<tr>
<td>Uniformly rough sphere</td>
<td>0.68</td>
<td>0.83</td>
</tr>
<tr>
<td><strong>Z-direction</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Two roughness elements</td>
<td>0.42</td>
<td>0.65</td>
</tr>
<tr>
<td>Four roughness elements</td>
<td>0.80</td>
<td>0.89</td>
</tr>
<tr>
<td>Uniformly rough sphere</td>
<td>0.46</td>
<td>0.68</td>
</tr>
</tbody>
</table>

components of the landing points. The consistent relationship between the X-component of linear velocity and the X-component of the landing points, and the relationship between the Z-component of linear velocity and the Z-component of the landing point for the baseball with two roughness elements reflect the anticipated landing locations when projectiles are released in a given direction. Pearson product-moment correlations calculated between the X-components of linear velocity and the Z- and Y-component
TABLE VII
ENTRY ORDER OF THE INDEPENDENT VARIABLES FOR EACH OF THE STEP-WISE REGRESSION ANALYSES

<table>
<thead>
<tr>
<th>Condition</th>
<th>Linear Velocity</th>
<th>Angular Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X</td>
<td>Y</td>
</tr>
<tr>
<td><strong>X-Displacement Prediction</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Two roughness elements</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Four roughness elements</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Uniformly rough sphere</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td><strong>Z-Displacement Prediction</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Two roughness elements</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>Four roughness elements</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Uniformly rough sphere</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

linear velocities revealed, for the baseball with four roughness elements that there existed concomitant increases in all of the component velocities. That is, for this condition the pitching machine consistently projected the ball either rapidly upward to the right or slowly downward to the left. For the uniformly rough sphere, a significant relationship was found between the X-component of linear velocity and the Z-component of linear velocity. That is, for this condition, the pitching machine consistently projected the ball either upward and to the right or downward
and to the left. The correlation coefficients describing the relationships between the component initial linear velocities are shown in Table VIII.

**TABLE VIII**

PEARSON PRODUCT-MOMENT CORRELATION COEFFICIENTS CALCULATED BETWEEN THE COMPONENTS OF LINEAR VELOCITY

<table>
<thead>
<tr>
<th>Component Velocity</th>
<th>Component Velocity</th>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Two Roughness Elements</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td>0.4191</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Z</td>
<td>0.5873*</td>
<td>0.3098</td>
<td></td>
</tr>
<tr>
<td><strong>Four Roughness Elements</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td>0.7336*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Z</td>
<td>0.4351*</td>
<td>0.3373</td>
<td></td>
</tr>
<tr>
<td><strong>Uniformly Rough Sphere</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td>0.2781</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Z</td>
<td>0.8248*</td>
<td>0.2203</td>
<td></td>
</tr>
</tbody>
</table>

*p<0.05*
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CHAPTER V

SUMMARY AND CONCLUSIONS

Introduction

The purpose of this study was to examine the effects of roughness elements on the Magnus characteristics of rotation spheres.

The motion of a totally immersed and translating object is affected by both gravitational and fluid forces (Hay, 1978). For an object which is also rotating, one component of the fluid forces is the lift caused by the Magnus or Robins effect (Magnus, 1953; Robins, 1842). One of the major factors influencing the fluid forces acting on a totally immersed object is the nature of the boundary layer surrounding the object. Of the various fluid flow conditions, only laminar, turbulent or partially turbulent flow are encountered in the majority of sport situations. Laminar flow exists when the boundary layer separation occurs in a region not presented to the flow (Jacobsen, 1973). In turbulent flow, boundary layer separation occurs in the region of the object presented to the flow. Partially turbulent flow, present only when the immersed object is assymetrical or rotating, is characterized by laminar flow in one region of the object and turbulent flow.
in another region. This type of flow can cause a resultant lift force in the opposite direction to the classical Magnus effect (Jacobsen, 1973).

In many sports, the performer imparts a spin to the sport implement to cause flight deviations. In baseball, the pitcher uses the Magnus effect to deviate the ball from a hypothetical parabolic pathway. Briggs (1959) reported horizontal flight deviations of up to twenty-six inches for a spinning baseball. The effects of roughness elements on the flight of a baseball with little or no angular velocity have been described by Watts and Sawyer (1975).

Rubinow and Keller (1963) and Kreighbaum and Hunt (1978) developed mathematical models to describe both the trajectory of a pitched sphere and the nature of the acting fluid forces. However, it would appear as though no attempts have been made to experimentally validate the models.

**Procedures**

Thirty trials of each of three roughness conditions were recorded on film. An appropriately oriented plane mirror ensured that both superior and lateral perspectives of each of the conditions were simultaneously recorded. The first condition consisted of a baseball pitched in such a manner that two of the roughness elements opposed
the incoming flow. The second condition consisted of a pitched baseball with four of the roughness elements opposing the flow. The third consisted of a pitched uniformly rough sphere. The initial linear velocities of the projectiles varied from 27.3 meters per second to 51.9 meters per second and angular velocity varied from 127.8 radians per minute to 11558 radians per minute. Any trials in which the pitched ball did not strike a measurement board were repeated. Any trials in which both images of the ball were not visible for three successive film frames were eliminated from the study. After each trial, the location of the landing point of the ball with respect to the origin of the measurement board was recorded. During the testing session, recordings were made of the ambient barometric pressure and temperature.

The motion of the ball for each successful trial was analyzed. The cartesian coordinates of each of the following landmarks or locations were digitized and recorded for three film frames:

1. Any three points on the circumference of the ball;
2. Any three points on the circumference of the projected image of the ball in the mirror;
3. One marked point on the surface of the ball.

The three points on the circumference of the ball were used to determine the trajectory of the center of gravity of the ball in the X - Y plane. The three points on the
circumference of the balls in the mirror were used to determine the trajectory of the ball in the Y - Z plane. The derivative of each of the component displacements of the ball were computed to determine the initial linear velocity of the ball in the component directions. The angular velocity of the ball was computed from the displacement history of the marked point on the surface of the ball. The values of the barometric pressure and temperature were used to compute the density and kinematic viscosity of the air. The components of initial linear and angular velocity, the mass and radius of the balls and the density and kinematic viscosity of the air were input into a computer program which utilized numerical approximation techniques to derive predicted values for deviation from normal projectile flight.

A one way analysis of variance was used to determine if differences existed in component flight deviations between roughness conditions. A statistical analysis utilizing mixed design repeated measures analysis of variance procedures with the roughness conditions serving as the non-repeating factor was conducted to determine whether or not interactions existed between roughness conditions and the method of determining component landing points. The actual and hypothetical component landing points served as dependent variables. Pearson product-moment correlation coefficients were computed to ascertain the relationship
between the component landing locations for each condition and the hypothetical component landing locations. Multiple linear regression was used to determine if the component landing points could be accurately predicted using statistical techniques. The component initial linear and angular velocities for each condition were entered as the independent measures. Pearson product-moment correlations coefficients were computed between the component initial linear velocities for each of the roughness conditions.

Results

Statistically significant differences were found between both of the baseball conditions and the uniformly rough sphere for both the X- and Z- flight deviations. The flight deviations in the X-direction for the baseball conditions were found to be due to the presence of a relatively large Magnus effect. The differences in flight deviations between the baseball conditions and the uniformly rough sphere could not be accounted for by differences in the Z-component of angular velocity and were therefore attributed to differences in the roughness of the surface of the conditions. The differences in the Z-component of flight deviation can be attributed, in part, to the relatively small mass of the uniformly rough sphere and an increase in boundary layer turbulence for the baseball conditions.
Statistically significant differences were found between some of the actual and hypothetical component landing points despite a predominance of significant relationships between these parameters. It would therefore appear as though the prediction formulae developed by Rubinow and Keller (1963) take into account appropriate input parameters but provide inadequate estimations of flight deviations.

A prediction of the actual component landing points using multiple linear regression techniques provided better estimations than the Rubinow and Keller model (1963) although the practical usefulness of the derived results is questionable.

Conclusions

Based on the results of the study, the following conclusions appear to be warranted.

1. The presence of roughness elements on the surface of a baseball increases the horizontal flight deviation when the baseball rotates about a vertical axis.

2. The presence of roughness elements on the surface of a baseball may increase the turbulence of the boundary layer and therefore cause a decrease in the encountered drag forces.
3. Linear velocity is the parameter which has the dominating effect on the trajectory of a spinning baseball.

4. The mathematical model developed by Rubinow and Keller (1963) does not adequately predict flight deviations.

Recommendations

Based on the results of this study, an additional examination of the effect of roughness elements on the Magnus characteristics of rotating spheres would seem appropriate. In particular, future attention should be directed towards minimizing all sources of measurement error. The recommendations for future studies are therefore made:

1. An examination of the effect of roughness elements on the Magnus characteristics of rotating spheres using a wind tunnel to control fluid velocity, a motor drive to control the angular velocity of the spheres and transducers to directly measure the lift and drag forces;

2. An examination of the effects of varying roughness configurations on flight deviations of those projectiles used in sports in which the rules permit either modifications or variations of the surface of the projectiles;
3. A determination of the optimal combinations of the linear and angular velocity components for achieving the objective of specific skill which involves the use of a projected sport implement.
APPENDICES
FORMULAS FOR THE COMPUTATION OF ANGULAR VELOCITY

Given that the equation for a plane through $P_1 = (x_1, y_1, z_1)$ with some normal vector $\langle a, b, c \rangle$ is

$$a(x-x_1) + b(y-y_1) + c(z-z_1) = 0$$

A normal vector $\langle a, b, c \rangle$ given three points $P_1 = (x_1, y_1, z_1)$, $P_2 = (x_2, y_2, z_2)$ and $P_3 = (x_3, y_3, z_3)$ can be found with the cross product of $\overrightarrow{P_1P_2}$ and $\overrightarrow{P_1P_3}$:

$$\overrightarrow{N} = \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = \begin{vmatrix}
i & j & k \\
x_2-x_1 & y_2-y_1 & z_2-z_1 \\
x_3-x_1 & y_3-y_1 & z_3-z_1
\end{vmatrix}$$

$$= [(y_2-y_1)(z_3-z_1) - (y_3-y_1)(z_2-z_1)] \hat{i}$$

$$+ [(x_3-x_1)(z_2-z_1) - (x_2-x_1)(z_3-z_1)] \hat{j}$$

$$+ [(x_2-x_1)(y_3-y_1) - (x_3-x_1)(y_2-y_1)] \hat{k}.$$

Letting $\overrightarrow{N} = \langle a, b, c \rangle$ where:

$$a = (y_2-y_1)(z_3-z_1) - (y_3-y_1)(z_2-z_1)$$

$$b = (x_3-x_1)(z_2-z_1) - (x_2-x_1)(z_3-z_1)$$

$$c = (x_2-x_1)(y_3-y_1) - (x_3-x_1)(y_2-y_1)$$

the equation for the plane containing $P_1$, $P_2$, $P_3$ becomes

$$a(x-x_1) + b(y-y_1) + c(z-z_1) = 0$$
The three points $P_1$, $P_2$ and $P_3$ also define a circle with the center at $(x_c, y_c, z_c)$. The intersection of the perpendicular bisectors, that lie in the same plane with $P_1$, $P_2$ and $P_3$, of any two chords connecting two of the points define that center $(x_c, y_c, z_c)$. The midpoints of the chords $P_1P_2$ and $P_1P_3$ are given by

$$M_2 = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$$

and

$$M_3 = \left( \frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}, \frac{z_1 + z_3}{2} \right).$$

The dot product of two vectors that are perpendicular to each other is 0. Thus, a perpendicular bisector of $P_1P_2$, $M_2C$, dotted with $P_1P_2$ is 0:

$$\left(1\right) \overline{P_1P_2} \cdot \overline{M_2C} = (x_2-x_1)(\frac{x_1+x_2}{2} - x_c) + (y_2-y_1)(\frac{y_1+y_2}{2} - y_c) + (z_2-z_1)(\frac{z_1+z_2}{2} - z_c) = 0$$

Also, $P_1P_3$ dotted with $M_3C$, a perpendicular bisector of $P_1P_3$, is 0:

$$\left(2\right) \overline{P_1P_3} \cdot \overline{M_3C} = (x_3-x_1)(\frac{x_1+x_3}{2} - x_c) + (y_3-y_1)(\frac{y_1+y_3}{2} - y_c) + (z_3-z_1)(\frac{z_1+z_3}{2} - z_c) = 0$$

where $C = (x_c, y_c, z_c)$.

If the point C lies on the same plane with $P_1, P_2$ and $P_3$

$$\left(3\right) a(x_c-x_1) + b(y_c-y_1) + c(z_c-z_1) = 0.$$
Simplifying equation (1)

\[ \frac{x_2^2-x_1^2}{2} -(x_2-x_1)x_c + \frac{y_2^2-y_1^2}{2} -(y_2-y_1)y_c \]

\[ + \frac{z_2^2-z_1^2}{2} -(z_2-z_1)z_c = 0 \]

\[-(x_2-x_1)x_c -(y_2-y_1)y_c -(z_2-z_1)z_c = -\frac{x_2^2-x_1^2}{2} -\frac{y_2^2-y_1^2}{2} -\frac{z_2^2-z_1^2}{2} \]

Multiplying by -1

\[ (x_2-x_1)x_c +(y_2-y_1)y_c +(z_2-z_1)z_c = \frac{x_2^2-x_1^2}{2} +\frac{y_2^2-y_1^2}{2} +\frac{z_2^2-z_1^2}{2} \]

\[ x_c = \frac{x_2^2-x_1^2}{x_2-x_1} +\frac{y_2^2-y_1^2}{x_2-x_1} +\frac{z_2^2-z_1^2}{x_2-x_1} -(y_2-y_1)y_c -(z_2-z_1)z_c \]

Letting \( Q_1 = \frac{x_2^2-x_1^2}{x_2-x_1} +\frac{y_2^2-y_1^2}{x_2-x_1} +\frac{z_2^2-z_1^2}{x_2-x_1} \)

we get

\[ x_c = Q_1 -(y_2-y_1)y_c -(z_2-z_1)z_c \]

Letting \( P_1 = \frac{y_2-y_1}{x_2-x_1} \) and \( R_1 = \frac{z_2-z_1}{x_2-x_1} \)

we get

\[ (4) x_c = Q_1 -P_1y_c -R_1z_c \]

Substituting (4) into (2) and simplifying we get

\[ \frac{P_1P_3}{M_3} = (x_3-x_1)[\frac{x_1+x_3}{2} -(Q_1-P_1y_c-R_1z_c)] \]

\[ + (y_3-y_1)(\frac{y_1+y_3}{2} -y_c) \]

\[ + (z_3-z_1)(\frac{z_1+z_3}{2} -z_c) \]
\[
\frac{x_3^2-x_1^2}{2} - (x_3-x_1)Q_1 + (x_3-x_1)P_1y_c + (x_3-x_1)R_1z_c
\]
\[+ \frac{y_3^2-y_1^2}{2} - (y_3-y_1)y_c + \frac{(z_3^2-z_1^2)}{2} - (z_3-z_1)z_c = 0
\]
\[
\frac{x_3^2-x_1^2}{2} - (x_3-x_1)Q_1 + \frac{y_3^2-y_1^2}{2} + \frac{z_3^2-z_1^2}{2} = (y_3-y_1)y_c - (x_3-x_1)P_1y_c
\]
\[+ (z_3-z_1)z_c - (x_3-x_1)R_1z_c
\]
\[
\frac{x_3^2-x_1^2}{2} - (x_3-x_1)Q_1 + \frac{y_3^2-y_1^2}{2} + \frac{z_3^2-z_1^2}{2} - \frac{[(z_3-z_1)-(x_3-x_1)R_1]z_c}{(y_3-y_1) - (x_3-x_1)P_1}
\]
\[= y_c
\]

Letting
\[
Q_2 = \frac{x_3^2-x_1^2}{2} - (x_3-x_1)Q_1 + \frac{y_3^2-y_1^2}{2} + \frac{z_3^2-z_1^2}{2}
\]
\[\frac{(y_3-y_1) - (x_3-x_1)P_1}{(y_3-y_1) - (x_3-x_1)P_1}
\]

and
\[
P_2 = \frac{(z_3-z_1)-(x_3-x_1)R_1}{(y_3-y_1) - (x_3-x_1)P_1}
\]

we get

(5) \[y_c = Q_2 - P_2z_c\]

Substituting (4) and (5) into (3) we get
\[a[Q_1-P_1(Q_2-P_2z_c) - R_1z_c - x_1] + b(Q_2-P_2z_c-y_1)
\]
\[+ c(z_c-z_1) = 0\]
\[ aQ_1 - aP_1Q_2 + aP_1P_2z_C - aR_1z_C - ax_1 + bQ_2 - bP_2z_C - by_1 + cz_C - cz_1 = 0 \]
\[ aQ_1 - aP_1Q_2 - ax_1 + bQ_2 - by_1 - cz_1 = (aR_1 - aP_1P_2 + bP_2 - c)z_C \]
\[ \frac{aQ_1 - aP_1Q_2 - ax_1 + bQ_2 - by_1 - cz_1}{aR_1 - aP_1P_2 + bP_2 - c} = z_C \]

Replacing \( z_C \) in (5) we get
\[ Y_C = Q_2 - P_2 \left[ \frac{aQ_1 - aP_1Q_2 - ax_1 + bQ_2 - by_1 - cz_1}{aR_1 - aP_1P_2 + bP_2 - c} \right] \]

and, replacing \( z_C \) and \( Y_C \) in (4) we get
\[ x_C = Q_1 - P_1 \left[ Q_2 - P_2 \left( \frac{aQ_1 - aP_1Q_2 - ax_1 + bQ_2 - by_1 - cz_1}{aR_1 - aP_1P_2 + bP_2 - c} \right) \right] \]
\[ -R_1 \left( \frac{aQ_1 - aP_1Q_2 - ax_1 + bQ_2 - by_1 - cz_1}{aR_1 - aP_1P_2 + bP_2 - c} \right) \]

Thus, the center of the circle containing \( P_1, P_2 \) and \( P_3 \) on its circumference is
\[ C = (x_C, y_C, z_C). \]
APPENDIX B

A BASIC PROGRAM TO COMPUTE ANGULAR AND LINEAR KINEMATICS OF A BALL IN FLIGHT

1 REM ****************************
2 REM ***** PROGRAM MENU  *****
4 GO TO 250
6 REM ***** DIGITIZE FILM DATA  *****
8 GO TO 610
10 REM***** UNIT CONVERSION  *****
12 GO TO 1400
14 REM***** DETERMINE CENTER OF GRAVITY IN THE X - Y PLANE  *****
16 GO TO 2090
18 REM***** DETERMINE CENTER OF GRAVITY IN THE Y - Z PLANE  *****
20 GO TO 2750
34 REM***** DETERMINE A POINT ON THE AXIS OF ROTATION  *****
36 GO TO 5410
38 REM***** DETERMINE LINEAR AND ANGULAR VELOCITY VECTORS  *****
40 GO TO 6670
42 REM ****************************
PROGRAM MENU

REM*****
REM***** PAGE
REM*****
REM***** PROGRAM DESCRIPTION
REM*****
REM***** KEY 1...........MENU
REM***** KEY 2...........DIGITIZE
REM***** KEY 3...........UNIT CONVERSION
REM***** KEY 4...........C of G. X-Y PLANE
REM***** KEY 5...........C of G. Y-Z PLANE
REM***** KEY 9...........POINT ON AXIS OF ROTATION
REM***** KEY 10...........LINEAR AND ANGULAR
REM***** VELOCITY AND DIRECTION
REM*****
REM***** DIGITIZING PORTION OF PROGRAM
REM*****
REM***** VARIABLE IDENTIFICATION BLOCK--DIGITIZING PORTION
REM***** P=Amount of points on subject <if any> to be
digitized.
660 REM**** S=Frames per second at which film was shot
670 REM**** I#=ID for digitized data
680 REM**** I=Frame #
690 REM**** T(I)=Time at frame I
700 REM**** N=Number of frames digitized
710 REM**** Subscripts: X and Y Row index
720 REM**** 1,2,3---Circumference points
730 REM**** 4 ---Marked Point on the Ball
735 REM**** 5 ---Marked Point on the Ball
740 REM**** 6,7,8---Mirror Circumference Points
750 REM****
760 REM*************************************************************************
770 INIT
780 PAGE
790 SET KEY
800 PRINT "This program allows you to digitize points with the numerics"
810 PRINT "digitizer. These points will be used in two different"
820 PRINT "programs. The first will calculate the axis of rotation,"
830 PRINT "the velocity of the spin and the degree of MAGNUS EFFECT"
840 PRINT "on the spinning ball. You must input your data as follows:");
850 PRINT "1) ANY 3 POINTS ON THE CIRCUMFERENCE OF THE"
860 PRINT "BALL;"
870 PRINT "2) ONE MARKED POINT ON THE BALL. NOTE-"
880 PRINT "YOU MUST USE THE SAME POINT THROUGHOUT"
890 PRINT "A GIVEN TRIAL;"
893 PRINT "3) THE SAME MARKED POINT AS SEEN IN THE MIRROR"
900 PRINT "4) ANY THREE POINTS ON THE CIRCUMFERENCE OF"
910 PRINT "THE BALL AS SEEN ON THE MIRROR;"
920 PRINT "5) ANY POINTS ON THE BODY OF THE SUBJECT"
930 PRINT "NECESSARY FOR FUTURE COMPUTATIONS;"
940 PRINT "How many points on the subject will be digitized? ");
950 INPUT P
960 PRINT "At how many frames per second was the film shot? ");
970 INPUT S
980 PRINT
990 PRINT "Input file name."
1000 INPUT I$
1010 PRINT "BEGIN DIGITIZING"
1020 DIM X(P+14,50), Y(P+14,50), T(50), M(4,50)
1030 FOR I=1 TO 50
1040 FOR J=1 TO P+8
1050 INPUT @7,32>X(J,I),Y(J,I)
1060 IF X(J,I)>60 THEN 1230
1070 IF J>3 THEN 1100
1080 PRINT "CIRCUMFERENCE POINT" "X(J,I),Y(J,I)"
1090 GO TO 1170
1100 IF J>4 THEN 1121
1110 PRINT "MARKED POINT" "X(J,I),Y(J,I)"
1120 GO TO 1170
1121 IF J>5 THEN 1130
1122 PRINT "MARKED POINT MIRROR" "X(J,I),Y(J,I)"
1123 GO TO 1170
1130 IF J>8 THEN 1160
1140 PRINT "CIRC. POINT MIRROR" "X(J,I),Y(J,I)"
1150 GO TO 1170
1160 PRINT "BODY POINT" "X(J,I),Y(J,I)"
1170 NEXT J
1180 T(I)=(I-1)/S
1190 PRINT
1200 PRINT "FRAME" ""I:" DIGITIZED"
1210 PRINT
1220 NEXT I
1230 H=I-1
1240 PRINT "IS THIS A CORRECTION? (Y OR N):"
1250 INPUT A$
1260 IF A$="H" THEN 1290
1270 I=H
1280 GO TO 1040
1290 PRINT "DIGITIZING COMPLETE. PRESS KEY #4 TO CONTINUE."
1292 RETURN
UNIT CONVERSION PORTION - CAN BE PREANSWERED

VARIABLE IDENTIFICATION:

R=Radius of the ball

D=Diameter of the ball

C=Circumference of the ball

U=Unit conversion factor

R1=Conversion of each frame to meters

NOTE: THIS SECTION FOLLOWS C OF G SECTIONS

You will now be required to input either the radius, 
diameter or circumference of the ball. Which will be:

given: radius(R), diameter(D) or circumference(C)?:

DIM R1(N)

INPUT R$

IF R$="C" THEN 1610

IF R$="D" THEN 1570

What is the radius of the ball?

INPUT R

GO TO 1640

What is the diameter of the ball? 

INPUT D

R=D/2

GO TO 1640

What is the circumference of the ball? 

INPUT C

R=C/(2*PI)

The above value was in: feet(F), inches(I), yards(Y),
1650 PRINT "millimeters(M), centimeters(C), or meters(K)?";
1660 INPUT U$
1670 IF U$="F" THEN 1820
1680 IF U$="I" THEN 1800
1690 IF U$="Y" THEN 1780
1700 IF U$="M" THEN 1760
1710 IF U$="C" THEN 1740
1720 U=39.37/12
1730 GO TO 1830
1740 U=1/(2.54*12)
1750 GO TO 1830
1760 U=1/(25.4*12)
1770 GO TO 1830
1780 U=3
1790 GO TO 1830
1800 U=1/12
1810 GO TO 1830
1820 U=1
1830 R=R*U
1840 FOR I=1 TO N
1850 R1(I)=SQR((X(3,I)-X(10,I))^2+(Y(10,I)-Y(3,I))^2)
1860 NEXT I
1870 FOR I=1 TO N
1880 FOR J=4 TO 5
1890 X(J,I)=R/R1(I)*X(J,I)
1900 Y(J,I)=R/R1(I)*Y(J,I)
1910 NEXT J
1920 NEXT I
1930 FOR I=1 TO N
1940 FOR J=9 TO 10
1950 X(J,I)=R/R1(I)*X(J,I)
1960 Y(J,I)=R/R1(I)*Y(J,I)
1970 NEXT J
1980 NEXT I
1981 DELETE R1
1982 PRINT "UNIT CONVERSION COMPLETED. PRESS KEY #9 TO CONTINUE."
1990 RETURN
2000 REM**********************************************************************
2010 REM**********************************************************************
2020 REM**
2030 REM**** DETERMINATION OF THE CENTER OF GRAVITY IN THE
2040 REM**** X-Y PLANE
2050 REM****
2060 REM**** VARIABLE IDENTIFICATION:
2070 REM**** X(10,1), Y(10,1)=C OF G. X-Y PLANE
2080 REM**** M(1), M(2)=SLOPES OF CHORDS CONNECTING POINTS
2090 REM**** OH CIRCUMFERENCE OF THE BALL
2100 REM**** X(13,1), Y(13,1)=COMPONENT LENGTHS OF ONE CHORD
2110 REM**** X(14,1), Y(14,1)=COMPONENT LENGTHS OF 2ND CHORD
2120 REM****
2130 REM**********************************************************************
2140 REM**********************************************************************
2150 REM
2160 REM
2170 PAGE
2180 PRINT "This program segment will take the data digitized from film"
2190 PRINT "and yield the center of gravity in the X-Y plane."
2200 PRINT
2210 PAGE
2220 FOR I=1 TO N
2230 FOR J=1 TO 2
2240 A=J
2250 B=I
2260 GOSUB 2560
2270 NEXT J
2280 NEXT I
2290 FOR I=1 TO N
2300 FOR J=1 TO 2
2310 IF Y(J+1,I)-Y(J,I)=0 THEN 2350
2320 IF X(J+1,I)-X(J,I)=0 THEN 2370
2330 M(J,I)=-1/((Y(J+1,I)-Y(J,I))*(X(J+1,I)-X(J,I)))
2340 GO TO 2380
2350 M(J,I)=100+99
2360 GO TO 2390
2370 M(J,I)=0
2380 NEXT J
2390 NEXT I
2400 FOR I=1 TO N
2410 X(10,I)=M(1,I)*X(13,I)-Y(13,I)+Y(14,I)-M(2,I)*X(14,I)
2420 X(10,I)=X(10,I)/((M(1,I)-M(2,I))
2430 Y(10,I)=Y(14,I)+M(2,I)*X(10,I)-X(14,I)
2440 NEXT I
2450 PRINT USING 2460: "ALL UNITS IN FEET"
2460 IMAGE 24X,18A
2470 PRINT USING 2510: "TIME", "X(10,I)", "Y(10,I)"
2480 FOR I=1 TO N
2490 PRINT
2500 PRINT USING 2520: T(I), X(10,I), Y(10,I)
2510 IMAGE 12A,12A,12A
2520 IMAGE 30.5D,4X,30.5D,4X,30.5D
2530 NEXT I
2540 PRINT "C OF G. X-Y COMPLETED. PRESS KEY #5 TO CONTINUE."
2550 RETURN
2560 REMARK ****************** SUBROUTINE - MIDPOINTS(CG) ******************
2570 IF X(A+1,B)<X(A,B) THEN 2600
2580 X(A+12,B)=X(A+1,B)-ABS((X(A+1,B)-X(A,B))/2)
2590 GO TO 2610
2600 X(A+12,B)=X(A,B)-ABS((X(A+1,B)-X(A,B))/2)
2610 IF Y(A+1,B)<Y(A,B) THEN 2640
2620 Y(A+12,B)=Y(A+1,B)-ABS((Y(A+1,B)-Y(A,B))/2)
2630 GO TO 2650
2640 Y(A+12,B)=Y(A,B)-ABS((Y(A+1,B)-Y(A,B))/2)
2650 RETURN
2660 REM*************************************************************************
2670 REM*************************************************************************
DETERMINATION OF THE CENTER OF GRAVITY IN THE Y-Z PLANE

VARIABLE IDENTIFICATION:
X(9,I), Y(9,I) = C OF G. Y-Z PLANE
M(6), M(7) = SLOPES OF CHORDS CONNECTING POINTS ON MIRROR'S IMAGE OF THE CIRC. OF THE BALL
X(11, I), Y(11, I) = COMPONENT LENGTHS OF 1ST CHORD
X(12, I), Y(12, I) = COMPONENT LENGTHS OF 2ND CHORD

0 DIM M(10, H)
10 PRINT "This program segment will take points digitized from film" 20 PRINT "and give the center of gravity in the Y-Z plane. This" 30 PRINT "information will be used to determine the initial" 40 PRINT "velocity of the ball."
50 FOR I = 1 TO H
60 FOR J = 6 TO 7
70 A = J
80 B = I
90 GOSUB 3240
100 NEXT J
110 NEXT I
120 FOR I = 1 TO H
130 FOR J = 6 TO 7
140 IF Y(J+1, I) - Y(J, I) = 0 THEN 3040
150 IF X(J+1, I) - X(J, I) = 0 THEN 3060
160 M(J, I) = -1 / ((Y(J+1, I) - Y(J, I)) / (X(J+1, I) - X(J, I)))
3030 GO TO 3070
3040 \textbf{M(J,I)}=100199
3050 \textbf{GO TO} 3070
3060 \textbf{M(J,I)}=0
3070 \textbf{NEXT} J
3080 \textbf{NEXT} I
3090 \textbf{FOR} I=1 \textbf{TO} N
3100 \textbf{X(9,I)}=M<6,I>*X<11,I>-Y<11,I>*Y<12,I>-M<7,I>*X<12,I>
3110 \textbf{X(9,I)}=X<9,I>-M<7,I>
3120 \textbf{Y<9,I)}=Y<12,I>*M<7,I>*X<9,I>-X<12,I>
3130 \textbf{NEXT} I
3140 \textbf{PRINT} \textbf{USING} 3150:\"ALL UNITS IN FEET\"
3150 \textbf{IMAGE} 24X,18A
3160 \textbf{PRINT} \textbf{USING} 3200:\"TIME\",\"Z(10,I)\",\"Y(10,I)\"
3170 \textbf{FOR} I=1 \textbf{TO} N
3180 \textbf{PRINT}
3190 \textbf{PRINT} \textbf{USING} 3210:\textbf{T(I)},X(9,I),Y<9,I>
3200 \textbf{IMAGE} 12A,12A,12A
3210 \textbf{IMAGE} 3D.5D,4X,3D.5D,4X,3D.5D
3220 \textbf{NEXT} I
3225 \textbf{PRINT} \textbf{"C of G. Y-Z COMPLETED. PRESS KEY \#3 TO CONTINUE."
3230 \textbf{RETURN}
3240 \textbf{REMARK} \textbf{********** SUBROUTINE - MIDPOINTS(CG) **********}
3250 \textbf{IF} X(A+1,B)<X(A,B) \textbf{THEN} 3280
3260 \textbf{X(A+5,B)}=X(A+1,B)-\textbf{ABS}(X(A+1,B)-X(A,B))/2
3270 \textbf{GO TO} 3290
3280 \textbf{X(A+5,B)}=X(A,B)-\textbf{ABS}(X(A+1,B)-X(A,B))/2
3290 \textbf{IF} Y(A+1,B)<Y(A,B) \textbf{THEN} 3320
3300 \textbf{Y(A+5,B)}=Y(A+1,B)-\textbf{ABS}(Y(A+1,B)-Y(A,B))/2
3310 \textbf{GO TO} 3330
3320 \textbf{Y(A+5,B)}=Y(A,B)-\textbf{ABS}(Y(A+1,B)-Y(A,B))/2
3330 \textbf{RETURN
4000 \textbf{PAGE}
5320 \textbf{REMARK} \textbf{*****************************************************************************}
5330 \textbf{REMARK} \textbf{*****************************************************************************}
5340 \textbf{REMARK} \textbf{*************}
DETERMINE POINT ON THE AXIS OF ROTATION

VARIABLE IDENTIFICATION:
X(3,I)=X COORDINATE OF THE CENTER OF GRAVITY
Y(3,I)=Y COORDINATE OF THE CENTER OF GRAVITY
X(4,I), Y(4,I)=1ST ROTATING POINT
X(5,I), Y(5,I)=MIRROR ROTATING POINT
X(6,I)=Z COORDINATE OF THE CENTER OF GRAVITY

MAKE SUBSCRIPTS COMPATIBLE WITH PROGRAM

FOR I=1 TO 3
X(3,I)=X(10,I)
Y(3,I)=Y(10,I)
X(6,I)=X(9,I)
NEXT I

DETERMINE COORDINATE DISTANCES OF ROTATING POINT FROM THE CENTER OF GRAVITY

X1=X(4,1)-X(3,1)
X2=X(4,2)-X(3,2)
5740 \(X3=X(4,3)-X(3,3)\)
5750 \(Y1=Y(4,1)-Y(3,1)\)
5760 \(Y2=Y(4,2)-Y(3,2)\)
5770 \(Y3=Y(4,3)-Y(3,3)\)
5771 \(Z1=X(5,1)-X(6,1)\)
5772 \(Z2=X(5,2)-X(6,2)\)
5773 \(Z3=X(5,3)-X(6,3)\)
5780 \(A=(Y2-Y1)*(Z3-Z1)-(Y3-Y1)*(Z2-Z1)\)
5790 \(B=(X3-X1)*(Z3-Z1)-(X2-X1)*(Z3-Z1)\)
5800 \(C=(X2-X1)*(Y3-Y1)-(X3-X1)*(Y2-Y1)\)
5810 \(Q1=((X2+2-X1+2)/2+(Y2+2-Y1+2)/2+(Z2+2-Z1+2)/2)/(X2-X1)\)
5811 \(P1=(Y2-Y1)/(X2-X1)\)
5812 \(R1=(Z2-Z1)/(X2-X1)\)
5820 \(Q2=(X3+2-X1+2)/2+(Y3+2-Y1+2)/2+(Z3+2-Z1+2)/2-(X3-X1)*Q1\)
5821 \(Q2=Q2/(Y3-Y1)-(X3-X1)*P1\)
5830 \(P2=(Z3-Z1)-(X3-X1)*R1)/(Y3-Y1-(X3-X1)*P1)\)
5840 \(C3=(A*Q1-A*P1*Q2-A*X1+B*Q2-B*Y1-C*Z1)/(A*R1-A*P1*P2+B*P2-C)\)
5850 \(C2=Q2-P2*C3\)
5860 \(C1=Q1-P1*C2-R1*C3\)
5915 DG 7(6, N)
5920 FOR I=1 TO N
5930 \(X(2, I)=C1+X(3, I)\)
5940 \(Y(2, I)=C2+Y(3, I)\)
5945 \(Z(2, I)=C3\)
5950 NEXT I
6270 FOR I=1 TO N
6280 \(X(1, I)=X(3, I)\)
6290 \(Y(1, I)=Y(3, I)\)
6300 NEXT I
6350 PRINT USING 6390: "TIME", "X( CG )", "Y( CG )", "X( AXIS )", "Y( AXIS )"
6360 FOR I=1 TO N
6370 PRINT USING 6390: T(I), X(1, I), Y(1, I), X(2, I), Y(2, I), Z(2, I)
6380 IMAGE 12A, 12A, 12A, 12A, 12A, 12A
6390 IMAGE 3D. 3D. 3D. (4X, 3D. 3D)
6400 NEXT I
VARIABLE IDENTIFICATION:

D<1,I>=DISPLACEMENT IN THE X DIRECTION
D<2,I>=DISPLACEMENT IN THE Y DIRECTION
D<3,I>=DISPLACEMENT IN THE Z DIRECTION
D<4,I>=MAGNITUDE OF DISPLACEMENT
A<1,I>=ANGLE OF RELEASE IN THE Y-Z PLANE
B<1,I>=ANGLE OF RELEASE IN THE X-Y PLANE
K=TIME INCREMENT
U=VELOCITY   [1,2,3->X,Y,Z],
U<4,I>=MAGNITUDE OF VELOCITY
M1,M2,M3=SUMMED VELOCITIES
M4=AUG. VELOCITY IN THE X DIRECTION
M5=AUG. VELOCITY IN THE Y DIRECTION
M6=AUG. VELOCITY IN THE Z DIRECTION
Z<2,I>=Z COORDINATE OF THE POINT ON THE AXIS OF ROTATION
E<1,I>=THEDA (POLAR COORDINATES)
F<1,I>=PHI (POLAR COORDINATES)
C=AVERAGE VALUE OF THEDA
J=AVERAGE VALUE OF PHI
<ω,U,Θ>=CARTESEAN COORDINATES OF POINT ON AXIS
6980 REM
6990 PAGE
7000 SET DEGREES
7010 PRINT "COMBINED C. OF G., LINEAR VELOCITY AND AXIS OF ROTATION."
7020 DELETE A,B,F,U,D,L,S,Q
7030 DIM U(4,H),D(9,H),A(H),B(H),L(6,N),S(H),Q(H)
7040 M1=0
7050 M2=0
7060 M3=0
7070 FOR I=1 TO N
7080 Z(I,1)=X(6,I)
7090 NEXT I
7100 REM
7110 REM
7120 REM**** DETERMINE COMPONENTS OF DISPLACEMENT-TIME HISTORY ****
7130 REM
7140 REM
7150 FOR I=1 TO N-1
7160 D(1,I)=X(1,I+1)-X(1,I)
7170 D(2,I)=Y(1,I+1)-Y(1,I)
7180 D(3,I)=Z(1,I+1)-Z(1,I)
7190 D(4,I)=SQR(D(1,I)**2+D(2,I)**2+D(3,I)**2)
7200 A(I)=ATH(D(3,I)/D(2,I))
7210 B(I)=ATH(D(2,I)/D(1,I))
7220 K=T(I+1)-T(I)
7230 U(1,I)=D(1,I)/K
7240 U(2,I)=D(2,I)/K
7250 U(3,I)=D(3,I)/K
7260 U(4,I)=D(4,I)/K
7270 M1=M1+U(1,I)
7280 M2=M2+U(2,I)
7290 M3=M3+U(3,I)
7300 Q(I)=T(I)+K/2
7310 NEXT I
7320 M4=M1/(N-1)
H5 = H2 / (H - 1)
H6 = H3 / (H - 1)
IF C1 < 0 THEN 7420
C1 = 1.0E-7
E = ATN(C2 / C1)
IF C2 < 0 THEN 7460
E = E + 180
GO TO 7480
IF E > 0 THEN 7480
E = E + 360
F = ACS(C3 / R)
7500 PAGE
7510 PRINT "VELOCITIES (FT/SEC)"
7520 PRINT "TIME", "VELOCITY", "ANGLE(X-Y)", "ANGLE(Y-Z)"
7530 PRINT "", "", ", "POS.=RIGHT"
7540 FOR I = 1 TO N-1
7550 PRINT T(I), V(4, I), B(I), A(I)
7570 NEXT I
7580 PRINT "VELOCITY VECTOR:"
7590 PRINT "< "H4, M5, M61 " >"
7600 PRINT "PRESS 'RETURN' TO CONTINUE"
7610 INPUT C +
7620 PAGE
7630 PRINT "ANGLES, WITH RESPECT TO A POLAR COORDINATE SYSTEM OF THE"
7640 PRINT "AXIS OF ROTATION"
7650 PRINT "R", "PHI", "THETA"
7660 PRINT R, F, E
7670 PRINT "CARTESIAN COORDINATES: "
7900 PRINT "<";C1,C2,C3;">
7910 REM**********************************************************************
7920 REM**********************************************************************
7930 REM**********************************************************************
7940 REM**** VARIABLE IDENTIFICATION:
7950 REM**** D(5,I)=X COORDINATE OF CHORD FROM AXIS POINT
7960 REM**** TO THE ROTATING POINT
7970 REM**** D(6,I)=Y COORDINATE OF SAME CHORD
7980 REM**** D(7,I)=Z COORDINATE OF SAME CHORD
7990 REM**** D(8,I)=MAGNITUDE OF SAME CHORD
8000 REM**** L(1,I)=SCALAR PRODUCT OF MAGNITUDES OF
8010 REM**** SUCCESSIVE CHORDS
8020 REM**** L(2,I)=DOT PRODUCT OF SUCCESSIVE CHORDS
8030 REM**** L(3,I)=ANGLE BETWEEN SUCCESSIVE CHORDS
8040 REM**** L(4,I)=ANGULAR VELOCITY FROM T(I) TO T(I+1)
8050 REM**** S(I)=SUM OF ANGULAR VELOCITIES TO T(I)
8060 REM**** S(H-1)=AVG. ANGULAR VELOCITY IN RADIANS
8070 REM**********************************************************************
8080 REM**********************************************************************
8090 REM**********************************************************************
8100 FOR I=1 TO H
8110 Z(4,I)=X(5,I)-X(6,I)
8120 D(5,I)=X(2,I)-X(4,I)
8130 D(6,I)=Y(2,I)-Y(4,I)
8140 D(7,I)=Z(2,I)-Z(4,I)
8150 D(8,I)=SQR(D(5,I)*2+D(6,I)*2+D(7,I)*2)
8160 NEXT I
8170 FOR I=1 TO H-1
8180 L(1,I)=D(8,I)*X(I+1)
8190 L(2,I)=(D(5,I)*D(5,I+1)+D(6,I)*D(6,I+1)+D(7,I)*D(7,I+1))/L(1,I)
8200 K=T(I+1)-T(I)
8210 L(3,I)=ACOS(L(2,I))
8220 L(4,I)=L(3,I)/K
8230 NEXT I
8240 S(I)=0
8250 FOR I=2 TO N
8260 S(I)=S(I-1)+L(4,I-1)
8270 NEXT I
8280 S(N)=S(N)/(N-1)
8290 S(N-1)=S(N-1)/360*2*PI
8300 PRINT
8310 PRINT "ANGULAR VELOCITY=";S(N-1);" RADIANS PER SECOND"
8311 C4=SQR(C1+2+C2+2+C3+2)
8320 W=C1*S(N-1)/C4
8330 U=C2*S(N-1)/C4
8340 D=C3*S(N-1)/C4
8350 PRINT "ANGULAR VELOCITY VECTOR:"
8360 PRINT "<";W,U,O;" >"
8370 PRINT "STORE IN WHAT FILE?"
8380 DELETE F
8390 INPUT F
8400 FIND F
8410 PRINT "IS THIS A NEW FILE?<Y OR N> ";
8420 INPUT A$;
8430 IF A$="Y" THEN 8460
8440 MARK 1,768
8450 FIND F
8460 WRITE 833:N4,N5,N6,W,U,O,R
8470 CLOSE
8480 PRINT "DO YOU WISH TO ANALYZE ANOTHER TRIAL? ";
8490 INPUT A$;
8500 IF A$="Y" THEN 6
8510 PRINT "PROGRAM TERMINATED."
8520 END
APPENDIX C

FORMULAS OF RUBINOW AND KELLER (1963) TO A NUMERICAL APPROXIMATION OF HYPOTHETICAL LANDING POINTS

(1) \[ F_b = -6\pi r \mu \overline{V} (1 + \frac{3\rho |\overline{V}|}{8\mu}) = -6\pi r \mu \overline{V} - \frac{9}{4} \pi r^2 \rho |\overline{V}| \overline{V} \]

(2) \( F_I = \pi r^4 \rho (\overline{\Omega} \times \overline{V}) \) and \( F_G = mg \)

where

- \( r \) = radius of the ball
- \( \overline{\Omega} \) = angular velocity
- \( \rho \) = density of the air
- \( \overline{V} \) = linear velocity
- \( \mu \) = viscosity of the air
- \( m \) = mass of the ball
- \( g \) = acceleration due to gravity = -9.8066 m/sec^2

(4)(5)(6)(7) Letting \( B = -6\pi r \mu \), \( C = (-\frac{9}{4}) (\pi \rho r^2) \), \( D = \pi r^3 \rho \)

and \( E = mg \)

(8)(9)(10) \[ F_b = B \overline{V} + C |\overline{V}| \overline{V} \quad F_I = D (\overline{\Omega} \times \overline{V}) \quad F_G = E \]

(11) \[ F_{\text{total}} = B \overline{V} + C |\overline{V}| \overline{V} + D (\overline{\Omega} \times \overline{V}) + E \]

By Newton's Second Law \( F = ma \)

(12) \[ a_{\text{total}} = \frac{B}{m} \overline{V} + \frac{C}{m} |\overline{V}| \overline{V} + \frac{D}{m} (\overline{\Omega} \times \overline{V}) + \frac{E}{m} \]

By definition \( a = \frac{\Delta V}{\Delta t} \) where \( t \) = time \( \Delta = \text{change in} \)

thus \( \Delta V = a(\Delta t) \)

\[ \Delta V = \frac{B\Delta t}{m} \overline{V} + \frac{C\Delta t}{m} |\overline{V}| \overline{V} + \frac{D\Delta t}{m} (\overline{\Omega} \times \overline{V}) + \frac{E\Delta t}{m} \]

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(13) \( \Delta V = [B \vec{V} + C |V| \vec{V} + D(\vec{a} \times \vec{V}) + E] \frac{\Delta t}{m} \)

(14) Letting velocity \( \vec{V} \) at time = \( i \) be \( \vec{V}_i \) and taking \( \Delta t \) sufficiently small

\[
\vec{V}_{i+\Delta t} = \vec{V}_i + (B \vec{V}_i + C |V| \vec{V} + D(\vec{a} \times \vec{V}) + E) \frac{\Delta t}{m}
\]

in component form

\[(15_x) \quad V_{xi+\Delta t} = V_{xi} + [B V_{xi} + C V_i |V| V_{xi} + D(\Omega_y V_z - \Omega_z V_y)] \frac{\Delta t}{m} \]

\[(15_y) \quad V_{yi+\Delta t} = V_{yi} + [B V_{yi} + C V_i |V| V_{yi} + D(\Omega_x V_z - \Omega_z V_x)] \frac{\Delta t}{m} \]

\[(15_z) \quad V_{zi+\Delta t} = V_{zi} + [B V_{zi} + C V_i |V| V_{zi} + D(\Omega_y V_x - \Omega_x V_y) + E] \frac{\Delta t}{m} \]

Letting \( S \) be displacement and \( \Delta S \) be change in displacement

(16) \( \Delta S = V_{av} \Delta t \implies \Delta S = (\frac{V_{i} + \nu_{i + \Delta t}}{2}) \Delta t \)

(17) \( S_{i+\Delta t} = S_i + \Delta S \)

Therefore at time = \( i + \Delta t \)

\[(18_x) \quad S_{xi+\Delta t} = S_{xi} + \Delta S_x \]

\[(18_y) \quad S_{yi+\Delta t} = S_{yi} + \Delta S_y \]

\[(18_z) \quad S_{zi+\Delta t} = S_{zi} + \Delta S_z \]
APPENDIX D

A BASIC PROGRAM UTILIZING NUMERICAL APPROXIMATIONS TO PREDICT
VALUES FOR THE LANDING POINTS OF PROJECTILES

10 INIT
20 PAGE
30 PRINT "NUMERICAL APPROXIMATION PROGRAM"
40 PRINT
50 PRINT "PROGRAM MENU-----------------------------"
60 PRINT ""
70 PRINT "1. MAGNUS EFFECT"
80 PRINT "2"
90 PRINT "3"
100 PRINT "4"
110 PRINT "5"
120 PRINT "6"
130 PRINT "7"
140 PRINT "8"
150 PRINT "9"
160 PRINT "10"
170 PRINT
180 PRINT "---------------------------------------------"
190 PRINT
200 PRINT "WHICH OF THE ABOVE PROGRAMS DO YOU WISH TO USE?"
210 INPUT P
220 GO TO P OF 230,540
230 INIT
240 PAGE
250 PRINT "MAGNUS EFFECT APPROXIMATIONS.------------------"
260 PRINT
270 PRINT "PLEASE ENTER THE TYPE OF BALL USED "
280 PRINT " "
290 PRINT " "
300 PRINT " "
310 PRINT " "
320 PRINT " "
330 PRINT " "
340 PRINT " OTHER.-----------------------------"
350 INPUT Q1
360 GO TO Q1 OF 370,400,430,460,490,520,550
370 R=0.036382993
380 M=0.145
390 GO TO 590
400 R=0.6985/(2*PI)
410 M=0.42525
420 GO TO 590
430 PRINT "TENNIS BALL PRESENTLY UNDEFINED."
440 PRINT
450 GO TO 550
460 PRINT "RACKETBALL PRESENTLY UNDEFINED."
470 PRINT
480 GO TO 550
490 PRINT "TABLE TENNIS BALL PRESENTLY UNDEFINED."
500 PRINT
510 GO TO 550
520 PRINT "GOLF BALL PRESENTLY UNDEFINED."
530 PRINT
540 GO TO 550
550 PRINT "ENTER THE RADIUS OF THE BALL ";
560 INPUT R
570 PRINT "ENTER THE MASS OF THE BALL ";
580 INPUT M
590 PRINT
600 PRINT "WILL YOU WANT APPROXIMATIONS OF:";
610 PRINT "1 SINGLE CONDITIONS."
620 PRINT "2 TABLE(S) OF SEVERAL CONDITIONS."
630 INPUT Q2
640 GO TO Q2 OF 670,3000
650 DELETE 10,640
660 DELETE 4000,10000
670 PAGE
680 PRINT "ENTER THE FOLLOWING VALUES:"
690 PRINT "VISCOSITY OF THE AIR"
700 INPUT U
710 PRINT "DENSITY OF THE AIR"
720 INPUT Q3
730 PRINT
740 PRI "WILL YOU BE ENTERING VELOCITIES MANUALLY(1) OR FROM FILES(2)??"
750 INPUT F1
760 IF F1=2 THEN 840
770 PRINT "ENTER LINEAR VELOCITY--VX,VY,VZ"
780 INPUT V1,V2,V3
810 PRINT "ENTER ANGULAR VELOCITY--OX,OY,OZ"
820 INPUT O1,O2,O3
830 GO TO 880
840 PRINT "ENTER FILE #"
850 INPUT F2
860 FIND F2
870 READ Q33;V1,V2,V3,O1,O2,O3,A
880 T1=1.0E-3
890 PRINT "DO YOU WANT THIS DATA STORED ON TAPE? ";
900 INPUT A$;
910 IF A$="N" THEN 1030
920 PRINT "ENTER FILE # ";
930 INPUT F3
940 FIND F3
950 PRINT "IS THIS A NEW FILE? ";
960 INPUT B$;
970 IF B$="H" THEN 1000
980 MARK 1,40000
990 FIND F3
1000 PRINT "ENTER ID FOR FILE";
1010 INPUT C$;
1020 WRITE Q33:C$;600
1030 T2=1
1040 B=-6*PI*R*U
1050 C=-9/4*PI*Q3*R+2
1060 D=PI*R*Q3
1070 D1=M*9.8066
1080 S1=0
1090 S2=0
1100 S3=2.21
1101 E=V1
1102 F=V2
1103 G=V3
1104 H=01
1105 J=02
1106 L=03
1107 W=SQR(E+2+F+2+G+2)
1108 O=SQR(H+2+J+2+L+2)
1110 WINDOW -1,1,0,3
1120 PAGE
1130 MOVE S1,S3
1140 FOR T=0 TO T2 STEP T1
1150 U4=SQR(V1+2+V2+2+V3+2)
1160 U5=V1+(B*U1+C*V1+U4+D*(02+V3-03+V2))*T1/M
1170 U6=V2+(B*U2+C*V2+U4+D*(03+V1-01+V3))*T1/M
1180 U7=V3+(B*U3+C*V3+U4+D*(01+V2-02*V1)+D1)*T1/M
1190 S5=(V1+U5)/2*T1
1200 S6=(V2+U6)/2*T1
1210 S7=(V3+U7)/2*T1
1220 S1=S1+S5
1230 S2=S2+S6
1240 S3=S3+S7
1250 V1=U5
1260 V2=U6
1270 V3=U7
1280 DRAW S1,S3
1290 IF A="N" THEN 1310
1300 WRITE @33:S1,S2,S3
1310 IF S2=>17.1 THEN 1330
1320 NEXT T
1330 HOME
1340 PRINT S1,S2,S3
1350 PRINT V1,V2,V3
1355 IF A="N" THEN 1370
1360 WRITE @33:5000,E,F,G,H,J,L,W,0
1370 GO TO 770
APPENDIX E

SUMMARY TABLE FOR THE ANALYSIS OF VARIANCE CONDUCTED ON THE X-COMPONENT OF FLIGHT DEVIATIONS

<table>
<thead>
<tr>
<th>Source(^a)</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3.4257</td>
<td>2</td>
<td>1.7128</td>
<td>18.1937*</td>
</tr>
<tr>
<td>S/A</td>
<td>7.7199</td>
<td>82</td>
<td>0.0941</td>
<td></td>
</tr>
<tr>
<td>Comp 1/2</td>
<td>0.0143</td>
<td>1</td>
<td>0.0143</td>
<td>0.1520</td>
</tr>
<tr>
<td>Comp 1/3</td>
<td>2.3546</td>
<td>1</td>
<td>2.3546</td>
<td>25.011*</td>
</tr>
<tr>
<td>Comp 2/3</td>
<td>2.7808</td>
<td>1</td>
<td>2.7808</td>
<td>29.5380*</td>
</tr>
</tbody>
</table>

\*p<0.05

\(^a\)A = Roughness conditions
S = Trials

Comp 1/2 = comparison between the baseballs with two roughness elements opposing the flow and the baseballs with four roughness elements opposing the flow

Comp 1/3 = comparison between the baseballs with two roughness elements opposing the flow and the uniformly rough sphere

Comp 2/3 = comparison between the baseballs with four roughness elements opposing the flow and the uniformly rough sphere
APPENDIX F

SUMMARY TABLE FOR THE ANALYSIS OF VARIANCE CONDUCTED ON THE Z-COMPONENT OF FLIGHT DEVIATIONS

<table>
<thead>
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<th>F</th>
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</thead>
<tbody>
<tr>
<td>A</td>
<td>18.1879</td>
<td>2</td>
<td>9.0940</td>
<td>30.980*</td>
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<tr>
<td>S/A</td>
<td>24.0706</td>
<td>82</td>
<td>0.2953</td>
<td></td>
</tr>
<tr>
<td>Comp 1/2</td>
<td>0.3153</td>
<td>1</td>
<td>0.3153</td>
<td>1.074</td>
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<tr>
<td>Comp 1/3</td>
<td>15.4739</td>
<td>1</td>
<td>15.4739</td>
<td>52.714*</td>
</tr>
<tr>
<td>Comp 2/3</td>
<td>11.6047</td>
<td>1</td>
<td>11.6047</td>
<td>39.533*</td>
</tr>
</tbody>
</table>

*p<0.05

\[ A = \text{Roughness condition} \]

\[ S = \text{Trials} \]

Comp 1/2 = comparison between the baseballs with two roughness elements opposing the flow and the baseballs with four roughness elements opposing the flow.

Comp 1/3 = comparison between the baseballs with two roughness elements opposing the flow and the uniformly rough sphere.

Comp 2/3 = comparison between the baseballs with four roughness elements opposing the flow and the uniformly rough sphere.
APPENDIX G

SUMMARY TABLE FOR THE ANALYSIS OF VARIANCE CONDUCTED ON THE Z-COMPONENT OF ANGULAR VELOCITY

<table>
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<th>Source^a</th>
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<tbody>
<tr>
<td>A</td>
<td>21738821.68</td>
<td>2</td>
<td>10869410.84</td>
<td>3.5343*</td>
</tr>
<tr>
<td>S/A</td>
<td>252185686.52</td>
<td>82</td>
<td>3075435.20</td>
<td></td>
</tr>
<tr>
<td>Comp 1/2</td>
<td>11095091.91</td>
<td>1</td>
<td>11095091.91</td>
<td>3.6080</td>
</tr>
<tr>
<td>Comp 1/3</td>
<td>1292997.25</td>
<td>1</td>
<td>1292997.25</td>
<td>0.4200</td>
</tr>
<tr>
<td>Comp 2/3</td>
<td>20052136.09</td>
<td>1</td>
<td>20052136.09</td>
<td>6.5200*</td>
</tr>
</tbody>
</table>

*p<0.05

^aA = Roughness conditions

S = Trials

Comp 1/2 = comparison between the baseballs with two roughness elements opposing the flow and the baseballs with four roughness elements opposing the flow

Comp 1/3 = comparison between the baseballs with two roughness elements opposing the flow and the uniformly rough sphere

Comp 2/3 = comparison between the baseballs with four roughness elements opposing the flow and the uniformly rough sphere

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### APPENDIX H

SUMMARY TABLE FOR THE ANALYSIS OF VARIANCE CONDUCTED ON THE X-COMPONENT OF THE LANDING POINTS

<table>
<thead>
<tr>
<th>Source</th>
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<th>F</th>
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<tbody>
<tr>
<td>A</td>
<td>14161.31</td>
<td>2</td>
<td>7080.65</td>
<td>1.19</td>
</tr>
<tr>
<td>S/A</td>
<td>480330.57</td>
<td>81</td>
<td>5930.01</td>
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</tr>
<tr>
<td>B</td>
<td>42037.54</td>
<td>1</td>
<td>42037.54</td>
<td>23.43*</td>
</tr>
<tr>
<td>AxB</td>
<td>3657.65</td>
<td>2</td>
<td>1828.82</td>
<td>1.02</td>
</tr>
<tr>
<td>BxS/A</td>
<td>145308.44</td>
<td>81</td>
<td>1793.93</td>
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</tr>
<tr>
<td>A at B1</td>
<td>10446.02</td>
<td>4</td>
<td>2611.50</td>
<td>3.36*</td>
</tr>
<tr>
<td>A at B2</td>
<td>7372.93</td>
<td>4</td>
<td>1843.23</td>
<td>0.60</td>
</tr>
<tr>
<td>B at A1</td>
<td>4819.29</td>
<td>1</td>
<td>4819.29</td>
<td>4.16</td>
</tr>
<tr>
<td>B at A2</td>
<td>18834.45</td>
<td>1</td>
<td>18834.45</td>
<td>23.78*</td>
</tr>
<tr>
<td>B at A3</td>
<td>22041.45</td>
<td>1</td>
<td>22041.45</td>
<td>6.43*</td>
</tr>
</tbody>
</table>

* *p<0.05

*a A = Roughness conditions

B = Method of determining landing points

S = Trials
APPENDIX I

SUMMARY TABLE FOR THE ANALYSIS OF VARIANCE CONDUCTED ON THE Z-COMPONENT OF THE LANDING POINTS

<table>
<thead>
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<th>Source</th>
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<th>F</th>
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<tbody>
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<td>A</td>
<td>39722.91</td>
<td>2</td>
<td>19861.46</td>
<td>3.15</td>
</tr>
<tr>
<td>S/A</td>
<td>510832.87</td>
<td>81</td>
<td>6306.58</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>73626.72</td>
<td>1</td>
<td>73626.72</td>
<td>16.30*</td>
</tr>
<tr>
<td>AxB</td>
<td>117133.52</td>
<td>2</td>
<td>58566.76</td>
<td>12.97*</td>
</tr>
<tr>
<td>BxS/A</td>
<td>365778.26</td>
<td>81</td>
<td>4515.78</td>
<td></td>
</tr>
<tr>
<td>A at B1</td>
<td>11307.88</td>
<td>4</td>
<td>2826.97</td>
<td>8.08*</td>
</tr>
<tr>
<td>A at B2</td>
<td>145548.55</td>
<td>4</td>
<td>36387.14</td>
<td>7.19*</td>
</tr>
<tr>
<td>B at A1</td>
<td>588.25</td>
<td>1</td>
<td>588.25</td>
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<tr>
<td>B at A2</td>
<td>3927.88</td>
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<td>3927.88</td>
<td>1.36</td>
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<td>B at A3</td>
<td>186244.11</td>
<td>1</td>
<td>186244.11</td>
<td>29.92*</td>
</tr>
</tbody>
</table>

*p<0.05

aA = Roughness conditions
B = Method of determining landing points
S = Trials
BIBLIOGRAPHY

Books


Unpublished Materials

Articles


