A SURVEY OF THE TWENTIETH CENTURY AMERICAN TRENDS IN SECONDARY MATHEMATICS EDUCATION

THESIS

Presented to the Graduate Council of the North Texas State University in Partial Fulfillment of the Requirements

For the Degree of

MASTER OF SCIENCE

By

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This investigation of twentieth century trends in mathematics education includes the survey of existing literature and questionnaires conducted with retired and active Texas teachers. Historical events, trends in curriculum, instruction, learning theories, and contradictions of twenty-year periods are delineated. Questionnaire responses are tabulated along the same periods and vignettes of typical classrooms are drawn from the data.

Results of the survey show the impact of societal forces on mathematics curricula, a continued downward expansion of content into lower grades and expanding knowledge of learning processes.

A unified mathematics curriculum, classroom-related learning theory research, and further development of team-teaching are postulated as future trends.

Recommendations include further examination of trends through isolation of other variables such as region and ethnicity.
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CHAPTER I

INTRODUCTION

Preface

Education has always held much promise and expectation in our society. The Puritans held the belief that education would assure spiritual salvation. Thomas Jefferson claimed that an educated citizenry was necessary for a democracy. Horace Mann challenged the society to build a publicly funded education for all its citizens. John Dewey's shift from subject-centered to a student-centered philosophy was a critical turning point in the style of teaching in this nation. In all of these postures, education has both affected and been affected by the course of our nation's history. Today a teacher must provide equitable and effective services to children from diverse social, ethnic, racial and linguistic backgrounds, deal with the social and learning problems of the physically and mentally handicapped, and correct the long years of overt and subtle educational discrimination suffered by girls and young women (4). These are radically different descriptors from those which characterized public education at the beginning of this century.
Teachers are demanding a more significant role in the course of educational research (4). Will a direction for the future evolve from the perspective of the teacher by a careful analysis of the trends in education in the course of this century? This study seeks to answer that question by categorizing the types of curriculum and instruction as well as the theories of learning reflected in them over against the external forces and events that have shaped the society in each period. The major issues and contradictions that each period held are revealed in a way that should benefit the education profession.

This study centers on mathematics education which has uniquely contributed precision to societal changes and extracted new challenges from each era. Early in the twentieth century, interest in mathematics education was on the wane. Two World Wars, economic depressions, an explosion of computer technology, space exploration and global interdependence with dwindling natural resources have radically changed the content of mathematics education. Simple reckoning with numbers has given way to such recondite studies as topology and transfinite numbers.

Early in the century, reports of the trends in mathematics education appeared regularly throughout the 1920's in the yearbooks and annual reports of the National Council
of Teachers of Mathematics (11). Other studies appeared in the 1950's after dramatic changes in curriculum were experienced during World War II (6, 12). "New Mathematics" was much publicized and discussed as a result of the shifting emphasis on scientific education needed in the wake of Russia's "Sputnik" advantage. A cry for "back to basics" is often heard today as National Assessment of Educational Progress reports continue to show declines in mathematics achievement and student progress (7). The history of mathematics education was the focus of the thirty-second yearbook of the National Council of Teachers of Mathematics covering the American experience from colonial days until the end of the fifties (5). Eight decades of mathematics education have passed. A new gestalt of the trends in mathematics education throughout the century furthers efforts to focus attention with clarity on the directions which must be taken for the next two decades.

The Problem and Its Purposes

Statement of the Problem

The problem of this study was to survey trends in mathematics education in the secondary schools of America during the twentieth century.
**Purposes of the Study**

The purposes of this study were to

1. determine the trends in curriculum and instruction of secondary mathematics throughout the century,

2. categorize the accepted theories of learning that characterized each generation,

3. provide a review of the external societal forces and events that marked each era,

4. discern the major contradictions that occasioned each shift in the style or content of teaching,

5. provide a descriptive vignette of a typical regional classroom of each generation, and

6. extrapolate from the analysis of the data a possible direction for the next twenty years.

**Sources of Data**

The trends in the development of secondary school mathematics are considered in view of reports and recommendations of the International Commission on the Teaching of Mathematics, National Council of Teachers of Mathematics, Mathematical Association of America, U. S. Bureau of Education, various state departments of education, various outstanding educators, and textbook authors.

The National Council of Teachers of Mathematics has published in the form of yearbooks reports and
recommendations concerning the study of mathematics in this country. In its first yearbook published in 1923, the Council was concerned with the creation of the College Entrance Examination Board as the purpose of high school mathematics was to satisfy requirements for entering college (11). In 1902, John Perry started a movement to discard the pure mathematics syllabus and introduce a new improved method of mathematical teaching, with the idea that usefulness must determine what subjects be taught and in what ways (10). In 1906, the National Committee of the Mathematical Association of America was appointed to plan a reorganization of mathematics for junior and senior high schools (14). A struggle existed between those who claimed that mathematics was too formal and mechanical and those who saw that this country was at least two years behind other leading countries in mathematical attainments.

Since 1930, the progressive education movement has been felt in general, and there is a growing tendency to organize instruction in activity units instead of the traditional formal classes (13). C. A. Webb, writing in the Texas Outlook in October of 1939, confirmed the trend toward activity units, as Texas educators struggled with adoptions of new textbooks and new methods of teaching (16). These and other reports are included in concise
form in "Trends in Purpose and Content of the High School Mathematics Course in Texas," a Master's thesis presented at North Texas State Teachers College in June, 1941 (3).

Other studies of the 1940's concerned themselves with historical trends through various approaches. Wylie recommended the incorporation of more pragmatic experiences in school curricula characteristic of the concerns of the 1940's (17). Other authors spoke of the needs of more practical scientific and mathematical education as a result of the experiences and demands of a world at war (6, 15).

David Bergamini was among the many who reported on the flood of federal and foundation dollars released in the wake of the national nervousness caused by Russia's "Sputnik" that issued in the revolution in the classroom known as "The New Mathematics" (2). "Mathematics Achievement: A Plus and Minus," a report by the National Assessment of Educational Progress, indicates that with all the concerted effort brought to bear, achievement in mathematics continues to decline throughout the 1970's (7).

The National Council of Teachers of Mathematics published its thirty-second yearbook, A History of Mathematics Education in the United States and Canada, in 1970 (5). This major work covered the forces and issues in a

The thirty-seventh yearbook of the National Council of Teachers of Mathematics published in 1975 includes a chapter on the directions of curricular change. One emphasis is on activity learning and the use of manipulative devices in the classroom. A sense of urgency that accompanied the many compensatory education models funded through the federal government as a treatment of societal ills has affected other changes. The use of the computer and television in the classroom is another significant arena of influence (9).
Definition of Terms, Limitations and Assumptions

Definition of Terms

An operational definition is provided for the following terms for the purpose of this study:

1. The period of one generation is considered as twenty years, the generally accepted span of time between the birth of parents and that of their offspring.

2. Historical image is defined as the descriptive category that will represent symbolically a particular era as in "The Age of Reason" or "The Depression Years."

3. Learning theory is defined as the generally accepted scientific principles of the cognitive or behavioral processes as espoused by signal educators and psychologists in a particular era.

4. Contradiction is defined as that which occasions a logical incongruity in the status of education and thus is indicative of the necessity for change.

5. Gestalt is used in this study as a schema that will hold in inclusive phrases the trends discerned in each category.

Limitations

Data collected from interviews and questionnaires are limited to those who responded and to the subjective
nature of their reflections. Analysis is governed by the subjective insight of the historians and the author.

**Basic Assumptions**

Although no specific hypotheses were tested in such a survey, it was assumed that mathematics education has changed throughout the century and will continue to change. It was also assumed that there is a correlation between the events in society at large and the educational programs of that society.

**Methods of Procedure**

**Collection of Data**

The data for this work were gathered through the review of existing literature in mathematics journals, U. S. government reports, as well as works of outstanding educators in the field. Syllabi and analyses of historical events of the century were referenced. Old textbooks and curriculum materials from each period were reviewed. Visits to museums and archives of various school districts and teachers colleges were conducted. Interviews with a list of colleagues representative of each generation were completed. Questionnaires were sent to a random sample of retired teachers and active teachers.
Responses from these questionnaires and interviews provided a sampling of the educational experiences of each time period.

**Research Design**

The research for this study was conducted in two arenas: (1) the survey of literature, materials and artifacts and (2) the collection of responses from personal interviews and questionnaires.

**Survey of Literature.** -- An ERIC computer search of existing literature was initiated. Materials were sought for twenty-year periods of mathematics education delineated in the following manner: (1) from 1901 to 1920, (2) from 1921 to 1940, (3) from 1941 to 1960, (4) from 1961 to 1980, and (5) projections and forecasts from 1981 to 2000. Research in the trends in mathematics education for each period included the topics of the existing learning theories, curriculum and instruction, historical image of each period, and the major issues and contradictions experienced.

The content of information was delimited to twentieth century American education. Educational philosophies and learning theories were covered in broad categories and not in detail. In the more recent time periods, the complexities caused by the rural-urban shift of school
populations from homogeneous to heterogeneous groupings required further subdivisions of categories.

Questionnaires and interviews. -- Current membership lists of retired teachers were secured from the Research Division and Members Services Division of the Texas State Teacher's Association through the regional membership associations. From the lists a sub-set of the retired mathematics teachers was constructed. A second master list was constructed of the active mathematics teachers from three large local urban school districts. Utilizing a TI-59 micro-computer to generate random numbers, a sample of 100 was taken from each of the two master lists. Questionnaires were mailed with a letter of transmittal and enclosed self-addressed and stamped return envelopes. A copy of the cover letter requesting the cooperative response as well as a copy of the questionnaire are included in the Appendix. The accepted number of responses from each sampling was fifty. The response from the retired teachers reflected the mathematics experience of the period from 1901 to 1940. The response from the active teachers reflected the experience from 1941 to the present. No other stratification of the sampling was attempted. The questionnaire was constructed to allow the respondents to record their earliest recollections.
as both teachers and students of mathematics. Responses were solicited by personal contact until a minimum of twenty responses was received for each twenty year period.

Tabulation of the results from the questionnaires was done manually. The most typical response or mode(s) for each item was reported and used as the basis for the development of the descriptive vignettes of typical classrooms.

The random sampling response was supplemented with data gathered from personal interviews conducted with an accessible list of colleagues from this region. A total of forty interviews was conducted by telephone, tape-recording or in person using the construct of the questionnaire and an informal conversation. The members of this list were from a range of ages representative of the four generations as well as from different ethnic, social and economic backgrounds. Tabulations of these data were done separately and the modal responses compared to the random sampling group responses to investigate a posteriori a variety of characteristics of the interview sample. The sample did not differ appreciably from the larger population. Where the mode of responses of this group was atypical, it was reported separately.

The content of data gathered for this aspect of the research was delimited to the particular regional
responses. The subjective nature of the reflections was also a delimiting factor. No generalizations are proposed and the vignettes bear a regional title.

Reporting and analyzing the data. -- Survey of the literature is reported in the body of the thesis. An inclusive chart of the topical categories is constructed in Chapter IV. Conclusions and recommendations are drawn in the form of projections in each of the categories for the next twenty years.

Tabulations of the responses to questionnaires and interviews are included in the Appendix. Vignettes of typical classrooms from each era are included in Chapter III. A hypothetical classroom of the future is the concluding vignette in Chapter IV.

Significance of the Study

This study is significant in that it provides an up-to-date and concise gestalt of the trends in mathematics education in this country. The study serves as a resource of historical background material for future research projects.

In the preparation of new teachers or in in-service programs for the teaching community, the results of this survey provide a helpful asset to understanding both the current and future status of mathematics education. A
workshop based on the information collated can provide a tool for future planning. Finally, in its projections, the study is significant in that it provides the rationale for initiation of programs or research that is indicative and imperative for teachers of mathematics.
CHAPTER BIBLIOGRAPHY


CHAPTER II

TWENTIETH CENTURY TRENDS IN MATHEMATICS EDUCATION

Introduction

This survey emphasizes the historical content, methodology and theories of learning in mathematics education existing in the United States during the course of this century. A chronology of events may serve as an end in itself. Here it is directed toward educators in order to increase the understanding of the present nature and values of the discipline and build the basis for forecasting the trends of the future. In the thirty-second yearbook of the National Council of Teachers of Mathematics (NCTM), devoted to the history of mathematics education in the United States and Canada, the authors state, "The continuing development of new mathematics, new uses of mathematics, new pedagogical devices, and changing goals for a changing society all demonstrate the need for continued change in mathematics education" (29, p. 1). The century has been divided into five twenty-year periods for purposes of this research. It is clear, however, that educational and cultural forces and issues themselves are always overlapping and intermingling.
throughout all periods of time. The report includes the following topics for each of the first four periods: a brief chronology of historical events and the historical image of the period, a description of the curriculum and instruction of secondary mathematics, the prevailing theory of learning or educational psychology of the period, and the forces and issues that raised major contradictions and were forerunners of change.

The Period from 1901 to 1920

**Historical Image: The Progressive Era**

The legacy of the nineteenth century with its remarkable achievements in industrialization, the development of the railroads, and urbanization of the nation led the American people into the twentieth century with optimistic enthusiasm. The future seemed to hold for them almost limitless potentialities for good. There was no thought that the century would bring world conflict and the threat of global destruction. In the "Progressive Era," as the period was often labeled, the challenge faced was internal and not external. Since the Civil War, the power had shifted from a broad agrarian base toward a narrow industrial oligarchy. American reformers were determined to right the wrongs that had appeared as outcomes of the increasing industrialization: the residue of
rural poverty, urban slum conditions, political graft and monopolistic corporations (22).

The period from 1901 to 1920 continued to issue in vast changes in technological development and international relationships. It was a time of emerging power. In 1904, Henry Adams wrote in his autobiography:

He could see that the new American - the child of incalculable coal power, chemical power, electric power, and radiating energy, as well as of new forces yet undetermined - must be a sort of God compared with any former creation of nature. At the rate of progress since 1800, every American who lived into the year 2000 would know how to control unlimited power. He would think in complexities unimaginable to an earlier mind. He would deal with problems altogether beyond the range of earlier society (1, pp. 496-97).

Power was indeed the mark of this era. A partial chronology of events (44) lists the firsts that were portents of things to come:

<table>
<thead>
<tr>
<th>Year</th>
<th>Event</th>
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<tbody>
<tr>
<td>1901</td>
<td>First transatlantic wireless message</td>
</tr>
<tr>
<td>1903</td>
<td>Wright Brother's flight</td>
</tr>
<tr>
<td>1904</td>
<td>Invention of the vacuum tube</td>
</tr>
<tr>
<td>1905</td>
<td>Einstein's special theory of relativity</td>
</tr>
<tr>
<td>1909</td>
<td>Ford's Model T</td>
</tr>
<tr>
<td>1920</td>
<td>First regular commercial radio broadcast</td>
</tr>
<tr>
<td>1920</td>
<td>Freud's General Introduction to Psychoanalysis</td>
</tr>
</tbody>
</table>

From the early prosperity to the end of the "Great War," reformers and progressives claimed the nation's
attention. Internally the movements for the abolition of trusts, changes in labor laws, better health care, urban slum clearance and agrarian reform had many champions. Crusades for social justice, among them the Women's Suffrage Movement, culminated in this period as changing values permeated every arena of life. All of these issues were played out against the tyranny of a changing economy and a growing nationalism in Europe that finally set the stage for a new kind of war (22).

The progressive generation felt few qualms over the use of armed force to police the American periphery, but it seemed inconceivable to a majority of Americans in 1914 when war broke out in Europe that the United States might become involved. In less than three years, the nation reluctantly embarked on the great crusade "to make the world safe for democracy." The American people mobilized their vast resources, conscripted its men into the armed forces and transformed the minds of its people into a religious fervor. The costs were staggering and the changes in power of the federal government were to be a benchmark in the nation's history (22, pp. 168-169).

President Wilson had outlined in idealistic speeches the nature of the post-war world he wished to see emerge. His dream of the League of Nations was not to be ratified by the irreconcilable Republicans or his own unbending
moralism in refusing to compromise on key issues. Wilson declared when he submitted the Versailles Treaty to the Senate, "Our isolation was ended twenty years ago. There can be no question of our ceasing to be a world power. The only question is whether we can refuse the moral leadership that is offered" (22, p. 216). The United States did refuse that leadership until another quarter of a century had elapsed and the world had gone through still another and more devastating war.

**Curriculum and Instruction: The Move Toward Utility**

At the beginning of the century the syllabi in mathematics in the American high schools were determined largely by the requirements for entering college. The College Entrance Examination Board (CEEB) was organized in 1900 resulting in local school district input and uniformity of requirements (24, p. 1).

The traditional courses of the period included the focus on retention of large amounts of abstractions. "In neither subject [algebra and geometry] did there seem to be any clear conception of the purpose of teaching mathematics in the twentieth century as distinguished from that which came into being with the rise of analysis and algebraic symbolism three hundred years ago" (24, p. 1). Indeed one of the key issues during the period was the
up-dating of the content and methods of teaching mathematics. At the beginning of the period, content and methods were much more closely related to the tenth century than the twentieth century. Universal secondary education was not yet a reality, although enrollment in secondary education more than doubled during this period. The science itself was stagnant and out of touch with the needs of the masses of people.

Much of the entirely useless and uninteresting work of elaborate multiplications and divisions of polynomials was struck out of the algebra courses during this period. Simplifying the content of operations with fractions and ratios was also accomplished. Such terms as "alternation" and "composition" were abandoned. David Eugene Smith wrote, "One of the most popular texts of twenty-five years ago had eight pages of definitions and theory before a single example was given" (65, p. 15). Other authors noted that the nineteenth century workbooks included examples of problems with numbers up to thirty-five digits and multiplication in six and eight digits (11, p. 8).

Progress in arithmetic at the junior high level was also seen in this period by a comparison of the nature of the topics as set forth in the arithmetics of 1900 and those of 1920. In 1900, arithmetic of special and unusual occupations was covered such as alligation, arbitrated
exchange, equations of payments, marine insurance and measurements of hogsheads, granaries and cisterns. By 1920, the arithmetic of daily life was more prominent in the texts. Topics included were simple bank accounts, cost of production, farm problems and transportation. In the early 1900's, cube roots, troy weights, compound numbers, Gregorian and Julian calendars and work with long and unusual fractions were commonplace. By 1920, the study of fractions was limited to those of everyday life. Decimals were related to United States money. Graphs, thrift and savings were major topics. Generally no reviews were included in the texts of 1900 except by going over the same work. All topics held equal importance. More systematic reviews of principles with new problems and emphasis on minimum essentials were more characteristic of the later part of the period (24, p. 15).

A sampling of problems taken from texts popular in 1900 show the complexities students encountered:


Reduce to ounces: 5 T., 10 cwt, 24 lb., 8 oz.

A, B, and C formed a partnership. A put in $3000 for 5 months, and then increased it $1500 for 4 months more. B put in $9000 for 4 months, and then, withdrawing half his capital continued the remainder 3 months longer. C put in $5500 for 7 months. They gained $3630. What was each partner's share of the gain?
Reduce 9,230 scruples to higher denominations. (24, pp. 17-18).

The prevalent thinking at the beginning of this period was that the subject of mathematics should be hard in order to be valuable. W. D. Reeve noted, "It was thought, the only thing we need to do is to make mathematics hard so that children will hate it. The more they hate it the better it will be for them" (55, p. 137).

An interesting example of the kinds of problems students encountered came as an atypical footnote to the historical events of the period. The Carnegie Endowment for Peace commissioned David Smith to publish a series of problems on the issues of war and peace. The examples below are a sampling of his effort.

Five superdreadnoughts of the Queen Elizabeth type are put in commission by Great Britain in 1915. The powder used in firing a single shot from one of the eight 15-inch guns costs $750. How much will it cost, for powder alone, to fire one round of the eight guns on a single ship? How much will it cost for the five ships?

In the great wars from 1790 to 1913 there were 18,552,200 men engaged, of whom 5,498,097 lost their lives. Find to the nearest tenth per cent the per cent of those who were killed (64, pp. 6-11).

The Perry Movement that had begun in Great Britain in 1901 was an example of a move toward the practical. John Perry favored education for citizenship. He noted that the fault lay in teaching mathematics as though every student were to be a teacher/mathematician himself.
Perry listed among the issues of concern; poorly taught mathematics, the concept of uselessness, and the need for more practical problems (51, p. 15).

R. D. Carmichael (14, p. 105) echoed some of these concerns in a popular journal, "It is customary to think of mathematics as separated from the usual concerns of ordinary life. As conceived by many people it is essentially a sort of monstrosity of the human mind."

Other forces that spoke to the issues of mathematics education during this period were the increase in the number of professional associations, their journal publications, and the work of several key commissions and committees. The American Mathematical Society recommended new college entrance requirements in 1902, the first of many expressions of national rather than local opinions and concerns. The International Congress of Mathematics (30) held in Rome in 1908 recommended that the function concept be made the unifying element of all mathematics leading to the calculus and statistics in the twelfth year. The report also compared United States mathematics education to other nations and noted that the United States was considerably behind other countries in secondary education. The noted German mathematician, Felix Klein, also proposed that the function concept be at the very center of instruction. Klein said, "Of all the concepts
of mathematics of the past centuries, this one plays the leading role wherever mathematical thought is used" (36, p. 4). These efforts to upgrade the quality of scholarship and unify the curriculum would continue to be heard throughout the century and re-surface particularly in the "New Mathematics" of the 1950's and 1960's. Under the auspices of the Mathematical Association of America, a report by the National Committee on Mathematical Requirements entitled, "The Reorganization of Mathematics in Secondary Education," signaled more than any other factor the call for reform (57). It was to be cited in textbooks throughout the thirties and provided the guidance for content and selection of mathematics courses to come (29, p. 198).

Theories of Learning: Learning by Transfer

The typical teacher of the period was guided by the philosophy of mental discipline. "The basis of this prevailing concept of learning and transfer was a nonempirically based psychology that smacked of Cotton Mather and Calvinism" (29, p. 156). Mental disciplinarians held that the best materials and methods were those that trained the faculties of the mind. The study of mathematics included activities to train for perception, reasoning, will and attention. It was thought that these
capacities would automatically transfer to another discipline. George A. Wentworth's Grammar School Arithmetic shows the climate of the day in the preface,

Pupils can be trained to logical habits of the mind and stimulated to a high degree of intellectual energy by solving problems adapted to their capabilities (69, p. iii).

Textbooks of the period show that the mental discipline theory was operant as little attention was given to the utility of mathematics. Subjects were compartmentalized and little effort was made to establish interrelationships of ideas. A notable exception was the work of Joseph Ray (often called the "McGuffey" of mathematics). Thirty-eight different arithmetics and nineteen algebras were published under his name (29, p. 158). His texts provided a choice; either a text directed toward vocationalism or a more general approach based on the concept of mental discipline (53).

A distinctly American pragmatism was growing in the country. Charles Sanders Peirce, and later William James, sought to bring together scientific, metaphysical and theological truths by the suggestion that the truth of a belief depended on its value for the individual believer. John Dewey, a contemporary of James, began to insist late in this period that the schools could best provide the proper environment and training for curing the ills of society. The debate which thus began is still continuing
more than a half century later (22, p. 142), Dewey proposed "learning by doing." Schools should develop not merely knowledge but also adjustment to life. Thus the mental discipline theory of learning began to shift even as the 1920's came to a close.

**Major Contradiction: Obsolescence**

In his opening address to the American Mathematical Society in March 1903, President E. H. Moore (45) expressed his concern for improving the quality of scholarship in mathematics education, the low standards as compared to the European schools and the need for a unified mathematics program.

The major contradictions of the period revolved around the appropriate ways to implement the thinking on the utility of mathematics and the needs of a growing student population. As many as forty percent of the students were failing the courses in algebra. Harvard University in 1918 had lowered its standards on admission tests to accommodate this increasing rate of failure (11, p. 14).

The growing awareness of the psychology of individual differences and the lack of an adequate psychological explanation of the transfer theory of learning were widening issues. Teacher training had not been adequate
for the times. The major structural change in school administration, the junior high school, was to play an important role in developing the purposes of mathematics education.

In all these arenas, the issue of the obsolescence of the American mathematical educational programs was primary. The spirit of the times the world over recognized that the traditional education of the nineteenth century was not suited to the present day.

The Period from 1921 to 1940

**Historical Image: Economic Collapse**

By 1920, a good bit of the progressivism that remained in the nation was on the negative side. Determined to fight for the cherished small town way of life, the progressive of this period was likely to resort to the Klu Klux Klan if the ballot failed to produce the results wanted. The war had raised the standard of living and caused some shifts in consumer demands. The "robber barons" now became the "industrial statesmen" taking credit for the rise in the standard of living. The postures of this twenty-year period made it a time of economic nationalism and economic collapse. The Republican presidents of the period championed the "business of business."

Excessive war debts and reparations kept Germany in a state of economic disaster (22).
In support of the "business of America is business" theory espoused by Coolidge, tax policies helped the rich become richer and the poor to stay where they were. Production of automobiles and other consumer goods steadily rose. As production rose, too little profits went to farmers and workers. As long as expansion of capitalization continued the economy was stimulated, but more was built than could be used at home and abroad. The government's role had served to widen the inequalities. Tariff policies kept foreign trade low. The 1929 stock market crash, when sixteen million shares were sold and losses of over forty percent of value were experienced, precipitated the great depression replacing the inflationary spiral with a deflationary one equally as hard to stop.

World depression and a tide of rising militarism around the globe marked the beginning of the thirties. When Roosevelt took office in 1932, thirteen million people were unemployed and several hundred thousand wandered homelessly. His program was pragmatic and flexible: "new economic theories would grow from it, not it from theories" (22, p. 303). Recovery programs moved steadily away from business interests to reform measures, and finally to massive governmental intervention. Public works programs, social security, minimum wages and labor relations programs evolved. Changes occurred in the federal income tax
structure, and for the first time the highest rates were at the top.

A partial chronology of events of this twenty-year period includes (16):

1925 Scopes trial
1929 Stock Market crash
1931 Urey discovers "Heavy Water"
1933 "New Deal" programs initiated
1935 Repeal of Prohibition
1935 Social Security Act
1938 Germany occupies Austria

The two depression factors of a lack of funds and an excess of leisure time operated also in education. A third of the unemployed were young people. Many went to school for lack of an alternative and high school enrollment went up a third between 1929 and 1934 (22, p. 359). Medical advances such as the development of sulfa drugs, the typhus vaccine, blood plasma and the artificial lung brought life expectancy up from fifty-six years in 1920 to sixty-four years in 1940. Science and research, often a neglected step child of the New Deal, was to become the salvation of a nation soon to be at war again (16, 22).
Although enrollments in secondary education were to triple during this period, increasing rates of failure and decreasing rates of enrollment in classical mathematics courses led to a growing criticism of instruction dominated by drill and manipulative formalism. Mathematicians and professional educators were concerned for the quality of mathematics education. Throughout the twenties and thirties, and to some extent during the post-World War II years, the mathematics education community assumed a defensive stance (29, p. 197).

"The 1923 Report," was the signal work of the Mathematics Association of America. The committee had been instructed to investigate the whole field of mathematical education from the secondary school through the college and to make recommendations looking toward a desirable reorganization of courses and the improvements of teaching (57). Advice and constructive criticism was solicited from teachers of mathematics and the committee acted as a national clearinghouse for ideas.

From the table of contents of the report the evidence of the scope of concerns was clear. Investigations conducted by the committee included questions of the discipline, theory of correlation, mathematics curricula in foreign countries, experimental courses, standardized tests,
and the training of teachers (57, p. i). The scientism of psychological research and the cult of efficiency during the twenties were forces from the business community that influenced curriculum changes. The report listed three types of aims of mathematics education: practical, disciplinary and cultural. Among the aims listed were:

**Practical Aims**

The fundamental processes of arithmetic have an immediate and undisputed utility in the life of every individual.

Of almost equal importance to every educated person is an understanding of the language of algebra and the ability to use this language intelligently and readily.

**Disciplinary Aims**

Training in functional thinking is one of the most fundamental disciplinary aims of the teaching of mathematics.

**Cultural Aims**

Appreciation of the power of mathematics and the role that mathematics and abstract thinking have played in the development of civilization (29, pp. 202-204).

Although the efforts of the NCTM to disseminate the 1923 report were thorough and extensive, immediate and significant influence of the report was not realized. Modification of junior high school curricula awaited the creation of text materials and the education of a set of teachers in the philosophy of the report.
More than any other single force, Progressive Education became the mark of this twenty-year period. The ideas implicit in the Progressive Education's credo had been in evidence since the turn of the century. The unified mathematics movement, the real-applied-problem movement, the early attempts at using laboratory teaching methods, and the scientism of the early testing movement all were well suited to progressive education and considered appropriate to secondary school mathematics in particular (29, p. 215).

The progressive movement focused attention on the issue of the aims for teaching mathematics. The mathematics educator, once more closely identified with the community of mathematicians, soon became identified even more with the community of educators. These mathematics educators were more subject to the influence of the professional reform movements in education, in psychology and in sociology (29, p. 234). As universal education was extended upward from the elementary school to grade ten and larger and different pupil populations were brought into the schools by the depression, the issue of the aims of mathematics became more complex.

Pressing for the utility of mathematics, John Milton Gross wrote in 1936, "The pupil must be led into a belief that the thing is worthwhile (26, p. 11)."
Early in this period, drill was seen as the significant aspect of learning. E. L. Thorndike spoke of the "law of exercise" which fixed the number of number-fact connections through drill. He had little use for problem solving and understanding of arithmetic meanings. Most of his emphasis was on speed and accuracy of computation. The extremes to which this notion was taken are reflected in the research of one of Knight's students. "The analysis of Knight's students of all possible two digit divisors, single digit quotients, long division examples, zeroes excepted (there were 40,095 of them) is an illustration of the extent to which the emphasis on specific connections affected the teaching of arithmetic" (28, p. 146).

By the end of the period, mathematicians were calling for the complete rejection of the drill theory. "There is little doubt that the drill theory has almost ruined the teaching of arithmetic" (28, p. 120). "We must turn away from mechanical teaching and must emphasize meaning, insight and intelligence" (28, p. 126). Wheeler advised teachers to "forget drills. Prepare your work logically and concentrate on relations. The whole purpose of arithmetic is to discover number relationships and to be able to reason with numbers. It is not to learn the tables" (70, p. 247)."
Curriculum materials of the period reflected the growing emphasis on the utility of mathematics and on practical problem solving. Words in the advertising of the school materials described the material as "easy, practical, motivating, socializing, and informal" (29, p. 233).

**Learning Theory: Activity Learning**

The growing awareness of the uses of psychology was to mark this era. The models and tools for experimental work and evaluation through standardized tests had their impact on the increasing respect for psychology. In his model for the slow learner, Thorndike's theory of human learning based on connectionism became the foundation for development of strategies to determine how much drill should be used. Not all agreed with this approach to mathematics as a tool subject. Judd attacked Thorndike's approach stating that mathematics is a "mode of thinking" and argued from an anthropological historical viewpoint that the tool subject premise denigrates mathematics as a field of endeavor (35, p. 10). Other arguments concerned the difficulty of mathematics (29). "The study of mathematics is a stumbling block to large numbers of students. Math is taught in the high school in a form too technical for students to comprehend" (29, p. 214).
It was the pragmatic psychology of Dewey and James and not the scientific psychology of connectionism that finally dominated the thought of this period (29, p. 216). John Dewey's philosophy served as the operational psychology for countless teachers. Dewey's view was that an aim of education must be built from a foundation of the intrinsic activities and the needs of the child. Putting a human being in between Thorndike's stimulus and response, Dewey focused on the interaction of people and events. Human learning was purposive (18). "Dewey caught the spirit of the times in a way that Thorndike did not, so that when differences became apparent, Dewey was chosen by educators" (41, p. 14).

Moreover, the advent of Gestalt psychology provided mathematics educators support for the unifying concepts and meaning and understanding they sought. The Gestalt theory provided the mathematics educator with a better model for explaining some of the observable learning in the classroom and fit objectives other than social utility in a more comfortable manner. In the thirty-second yearbook of the NCTM, the authors summarize the issue:

The emergence of Gestalt psychology and field theories was, in a sense, an extension of Dewey's notion of purposive behavior. By 1940, it had become perfectly clear that the controversy over learning theories was decided. Thorndike was out. Cognition was in (29, p. 217).
However, translating this concept into the schools was a slow process. "The drill theory did not die an easy death, as the analysis of the issues and programs throughout the period will show" (29, p. 217).

**Major Contradiction: Loss of Excellence**

This period has been characterized as mathematics education on the defensive. Critics of mathematics education (the editorial writers in the popular press, school superintendents, and college professors) leveled two types of criticism. The first criticism was that school mathematics never related to life. David Sneddon said that, "Algebra is nonfunctional and a nearly valueless subject for 90% of all boys and 99% of all girls and no changes in methods or content will change the situation" (66, p. 1)." The second type of criticism cited the large number of failures in secondary school mathematics.

Two highly pessimistic reports by E. T. Bell in 1935 and William Betz in 1940 gave evidence of the low status and state of defensiveness in the country reeling from the depression and becoming aware of the danger of war. Bell, writing in the *American Mathematical Monthly*, states:

American mathematics is in harnessed retreat, fighting a desperate rear-guard action to ward off annihilation. ..., Mathematics has not yet made out a compelling case for democratic support. This must be done, and immediately, if mathematics is to survive in America (7, p. 559).
The pessimistic tone is apparent in the list of factors contributing to the poor status of mathematics according to William Betz writing in The Mathematics Teacher in 1940. He lists mechanistic methods of teaching, one-sided emphasis on utility, the doctrines of progressive education, and inadequate training of teachers (8, p. 450) as significant factors. Betz also attacked the work with individual differences, the problems of mass education and incoherent curriculum revisions.

Two major curriculum reports for secondary school mathematics were published in 1940. Both were written during the period described by Bell and Betz. The Progressive Education Association Report focused on the student and his life, whereas the Joint Commission Report formulated recommendations anchored to the subject matter (29, p. 230). Both were conceived during the depression and published on the eve of war, but they were positive and forward looking statements. Each report saw a case for the study of mathematics broader than the single issue of utility as a determiner of content.

Mathematics education from 1921 to 1940 revolved around the arguments for its usefulness in society, and in this struggle over objectives, experienced a loss of excellence that was to be felt in the decade ahead. "A sense of desperation stemming from larger and different
pupil populations and the depression helped mathematics educators lose sight of the fact that neither mathematics for its own sake nor mathematics for direct and immediate use provided a sufficient rationale for students" (29, p. 234). The battle of objectives was suddenly to take secondary importance with the beginning of a world war. Teachers shifted from a preoccupation with the social well-being of students to a concern for the existence of the American way of life. The mathematics educators began to be aware that a heavy responsibility was to be theirs during the coming years. Awareness of the plight of Negro teachers and students in "separate but equal" schools as shown in a 1940 report of the NCTM was also to foreshadow the turmoil and revolutionary changes of the decades to come (29, p. 234).

The Period from 1941 to 1960

**Historical Image: Victory Without Peace**

Tentative attempts at isolation and remaining neutral proved futile and Roosevelt was elected to an unprecedented third term as the nation prepared its defenses. In the grim days following the attack on Pearl Harbor, Americans united in the task of producing armaments and training men to fight with them. Revolutionary changes for the future came out of laboratories as scientists
pooled their skill in a race against the Germans to turn basic knowledge into decisive weapons of war. The Manhattan Project culminated in the development of the atomic bomb. The awesome and frightening reality was present that global annihilation was now possible (16, 22).

Drastic changes were occasioned by the mobilization of manpower for the war effort. Almost twenty-seven million people moved during the war. Women became a significant part of the work force. The birth rate soared. Migrations of Blacks from the South to war plants and urban centers was to change forever the complexion of the cities (22).

The American people had met the demands for production and brought the Axis powers to final defeat, only to discover that they had miscalculated the intentions of the Russians. The fragile victory was to be a triumph without peace. The United Nations was inaugurated in 1945. At the same time, a "Cold War" and struggle for atomic power began. Russia had rushed into the power vacuums in Europe and Asia at the end of the war and now dropped an "iron curtain" around its arena of influence. President Truman's Marshall Plan of massive foreign aid to rebuild the war-torn cities of Europe and Asia was both humanitarian in nature and also an effort to consolidate the influence of the United States around the world (16, 22).
Against this melancholy backdrop of permanent crises in international affairs, Americans after 1945 settled into a life of precarious prosperity at home. Civil rights came to the forefront as America participated in the unsettling events of the McCarthy era. The nation engaged in a widespread hunt for traitors as it feared both the menace from without and subversion from within. The United States struggled to build a Cold War balance against the forces of Communism pressing from Eastern Europe and Communist expansion in Asia. Armed conflict broke out and the United States fought yet another war in Korea. Americans turned to General Dwight Eisenhower to get them out of Korea in the 1952 elections. They sought moderation in all things. "The middle of the road" was a favorite slogan of the times (16).

A brief chronology of events during this period includes:

1941 Pearl Harbor
1945 Atomic bomb - Hiroshima, United Nations formed.
1948 Berlin blockade
1950 Korean War
1954 Brown vs. Board of Education
1955 Polio Vaccine
1957 Sputnik
1958 National Defense Education Act

The Eisenhower years were marked by a continued series of international crises and a seeking of a nuclear "balance of terror" with Russia. World turmoil appeared in the Middle East, the Arab world, in the developing nations of Africa, and in the undiminished struggles in Asia. New nations emerged from the colonial rule of the western world and an explosion of population was felt around the globe.

Domestically, the nation experienced a growing materialism and consumerism. The American people enjoyed a living standard far beyond any they had previously known. The baby boom of the post-war years meant that housing and highway building programs were desperately needed. Scientific research continued with explorations of the globe, developments in medicine such as the polio vaccine in 1955, and the dramatic increase in air travel and television. School enrollments were up and there was a drastic shortage of teachers and classrooms (16).

A dramatic event in the courts was to be a portent of the struggles of the next twenty years. The Supreme Court in 1954 interpreted the requirement of the Fourteenth Amendment to mean that separate but equal facilities in the public schools were unconstitutional (Brown vs. Board of Education of Topeka). Justice Warren (22, p. 343) said,
reporting the unanimous opinion, "We conclude that in the field of public education the doctrine of separate but equal has no place. Separate educational facilities are inherently unequal." Reactions in the South reached their peak in 1957 as protests and ugly riots broke out in desegregation efforts.

A second dramatic event marked the end of the period, and it too was to have enormous impact on mathematics education. In October 1957, Russia launched Sputnik, the first satellite to orbit the earth. The space race was engaged.

Curriculum and Instruction: Revolution in Content

The events in the field of mathematics education during this period were sweeping. "As measured against that of any comparable period both the pace and extent of change over the past twenty years have been revolutionary" (29, p. 235). The publication of the Steelman report in 1947, the establishment of the National Science Foundation in 1950, and the founding of the School Mathematics Study Group in 1958 were among the many significant events in the revolution in school mathematics that occurred in the period from 1940 to 1960.

The era began with the urgent need for preparedness for a global war. The induction testing for World War II
presented evidence that many youth were incompetent in mathematics. Excerpts from Admiral Chester W. Nimitz's letter to the War Preparedness Committee dated November 12, 1941, show the extent of the problem:

A carefully prepared selected examination was given to 4200 freshman at 27 of the leading universities and colleges of the United States. 68% of the men taking the exam were unable to pass the arithmetical reasoning test. 62% failed the whole test...Out of 8,000 applications, all college graduates, some 3,000 had to be rejected because they had no mathematics or insufficient mathematics at college (48, p. 37).

The concern extended beyond military needs to encompass the employment and training problems of increasingly technical industries. Trends in advertisements that appeared in *The Mathematics Teacher* during the war years reflect the impact of World War II on secondary school mathematics. "The advertisement shifted from emphasizing words like easy, practical, motivating, socializing, and informal to stressing preinduction needs and competencies. Indirect measure became a significant topic"(29, p. 233). Because of the demands of the armed services and industries, the importance of utilitarian aims was again emphasized, but these "utilitarian" aims were broader in scope including new and formerly "pure" mathematics for technicians, engineers and scientists of an expanding wartime technology.

The War Preparedness Committee went far beyond the historical role of mere recommendation. The research
mathematicians and classroom teachers worked together to actually develop a textbook and put it into use in a number of classrooms with very gratifying results (5).

Early in the period some reports had appeared that alarmed those who were concerned about the quality of mathematics. In 1944, the Educational Policies Commission had reported in *Education for All American Youth* what many believed to be the natural outcome of the progressive education movement. The commission listed four divisions of learning: "Vocational Preparation," "Individual Interests," "Common Learnings" and "Health and Physical Education." It was difficult to find the place of mathematics in such a curriculum (20). Mathematics was restricted to traditional programs for the college preparatory students, to specialized minimum training in vocational courses, or to nothing at all for all others. The emphasis was clearly on the vocational, consumer-oriented programs of previous times. Many mathematics educators would see this as an emerging threat to the role of mathematics in the secondary schools. A second report during this time did little more to secure the place for mathematics in the secondary school program. In the Harvard Report (27), "appreciation" seemed to be the key word in describing the role of mathematics. Essentially the fears that these reports raised with their emphasis on
common learnings were that no compartment at all would remain for mathematics as a separate discipline.

Growing pressure from the federal government reflected an emerging national concern for excellence in education. Exploding college enrollments of more mature returning veterans, recent developments in mathematics and its applications, and critical attacks of the popular press against education in general meant that by 1950, mathematics education was ripe for a revolution.

Articles stressing the needs of the gifted were typical in the journals of the period. The articles were written from the point of view of national crises and national need. May declared, "The present crisis in mathematics is due not to any deterioration in the work of math teachers, but to an urgent national need for more and better mathematics" (43, p. 303).

The work of the University of Illinois Committee on School (UICSM) Mathematics was the first major effort in curriculum reform. It was concerned primarily with secondary school programs which left students short of minimum needs in the field of engineering. The UICSM received support from the Carnegie Foundation and later from the National Science Foundation and the United States Office of Education. This extensive financial support from private and governmental agencies became the most critical
factor in the extent of reform and the rapidity of its implementation in the schools. Max Beberman, director of the UICSM, decided that any realistic proposal for improvement would have to include classroom tested instructional materials. The success of new materials also depended in large measure on well trained teachers. Teacher training institutes, summer programs and academic year programs were first supported by private corporations such as General Electric and Shell Oil Company and later in unprecedented amounts by the federal government (29, p, 252).

The two main facets of the UICSM program were precision in language and discovery of generalizations. Features that represented clear changes from previous traditional programs included (1) integrated mathematics curricula with algebra found throughout the four year program, (2) solid geometry minimized, (3) trigonometry more generalized in the study of circular functions, (4) introduction of set terminology and quantifiers, and (5) the relocation of several topics to lower grade levels (29, p. 253).

The coalescence of national support for curriculum reform has been cited by many as the launching of the first Soviet satellite. "The historical record shows clearly that curriculum reform had already begun and with federal support well before Sputnik jolted the public
conscience" (29, p. 256). But there is little doubt that this event focused public attention and with the resultant furor in the popular press gave support for vast amounts of governmental support.

The National Defense Education Act, a culmination federally of the efforts for secondary programs for college capable students, was passed in 1958. Although it was an unhealthy force in American political life, the McCarthy hearings followed by the event of Sputnik, probably led to an attitude that made funds for education more available.

Examples of curricula projects underway in the late fifties were the Ball State Teachers College Experimental Program, the Developmental Project in Secondary Mathematics of Southern Illinois University and the University of Maryland Mathematics Project. The School Mathematics Study Group (SMSG) grew directly out of the concerns raised by these investigations and out of two conferences sponsored by the National Science Foundation.

The SMSG set forth two recommendations: to prepare a detailed syllabus for a model secondary school program and to write a series of monographs on mathematical topics of interest and value to secondary students (29, p 273).

With the appointment of the Committee of Eight from distinguished members of the American mathematical
community, the stamp of respectability had finally been placed on working with schools. Wooten noted,

For more than thirty years, the AMS had held itself aloof from the elementary and secondary school level of mathematics and had contributed very little to the teaching of it. With the appointment of the Committee of Eight, it officially expressed an interest in the mathematics curriculum of the schools (71, p. 1).

The work of the SMSG was to span over a decade and the contribution of literally hundreds of persons would be included in the products of the writing sessions that provided textbooks and curriculum materials for all levels of secondary mathematics. The SMSG published in March 1959 (61, p. 9) a progress report to the mathematicians of this country stating that, "The number of our citizens skilled in mathematics must be greatly increased; and understanding of the role of mathematics in our society is now a prerequisite for intelligent citizenship."

Each writing team developed its own style and philosophy and the committee guarded carefully against writing a single national curriculum for mathematics. Innovations included set notation, stress on the function concept and a rather careful development of the properties of real numbers. Close cooperation with the existing curriculum pilot projects developed. The list of publications included texts, teacher commentaries, programmed materials, computer-programming instruction, monographs and inservice training materials.
In 1961, the curriculum movement was officially labeled a revolution. A NCTM pamphlet, "The Revolution in School Mathematics," compared the characteristics of the work of the various curriculum projects of the late fifties. "All the programs we have discussed attempt to avoid the presentation of new material as a string of unrelated topics. Indeed, they stress unifying themes of ideas in mathematics" (58, p. 9).

Set terminology, the structure of mathematics and systems of numeration became hallmarks of the "New Mathematics." There was a renewal of interest in number theory during this period at the secondary and college level (49). These concepts and terms would be greatly discussed in homes and halls of learning in the decades to come.

Learning Theory: A Time of Discovery

The period began with the utilitarian concept of mathematics as the prevailing concept. In the course of twenty years, that emphasis on the social and consumer nature of mathematics was to give way to the development of a revolution in the mathematics curricula. The focus on the needs of the gifted student was the first impetus of the shifting pedagogical theories. G. B. Price (54) argued in 1951 that slighting the education of the gifted was slighting the need for a creative cadre of people for national defense purposes.
The UICSM addressed itself to the development of a teaching philosophy and learning theory throughout the scope of its research and implementation. Discovery teaching and learning became the key theory of the curriculum revision projects. Beberman discussed this feature of the UISCM program in excerpts from his Ingles Lecture.

We must design both exposition and exercises in such a way that the student will discover principles and end rules.

Thus the discovery method develops interest in mathematics, and power in mathematical thinking. Because of the student's independence of rote rules and routines, it also develops versatility in applying mathematics (3, p. 373).

The Commission on Mathematics of the CEEB also echoed this concern for teaching methods and recognized the importance of the discovery method. "Most if not all of the Commission members would prefer to see a developmental approach [to teaching], which would encourage the student to discover as much of the mathematical subject matter for himself as his ability and time will permit" (17, p. 59).

In the twenty-first yearbook of the NCTM, there appeared a list of elements derived from various learning theories. The list was used as a supportive rationale for the importance of guided discovery.

1. There must be a goal on the part of the student to learn.
2. All cognitive learning involved association.
3. We recognize trial and error or analysis in most learning.
4. Learning is complete to the extent to which the relationships and their implications have been understood.
5. The learner must be in action, mentally and or physically.
6. Intrinsic reward of success and awareness of progress toward a goal strengthens the learning and the motivation for further learning.
7. Discrimination of attributes (abstraction) and generalization are essential to effective learning.
8. New learning is in part a matter of transference of past learning.
9. We learn facts and skills and we also learn how to learn.
10. We also learn feelings (attitudes) (39, p. 54).

The approaches advocated by Jerome Bruner in *The Process of Education* were significant in developing and articulating the discovery method. He called for organization of a program around the structure of the subject (12). Clinchy (15) noted that discovery methods became a part of virtually all curriculum development projects. He quotes the following from a very interesting source on this point:

Teach your scholar to observe the phenomena of nature, you will soon raise his curiosity, but if you would have it grow do not be in too great a hurry to satisfy the curiosity. Put the problems before him and let him solve them himself. Let him know nothing because you have told him, but because he has learnt it for himself. If ever you substitute authority for reason, he will cease to reason, he will be a mere plaything of other people's thoughts...(29, p. 254).

The citation (60) is from Rousseau's *Emile*, published in 1762.
Major Contradiction: Individual Differences

The reform movements of the era had their roots in the technological and international events of the era responding to a global war and an unsettled peace. The first responses to the situation were called for by higher education and were thus directed at the secondary level and the college capable student only. As the era closed the needs of all the students became more apparent. John Goodlad noted as he wrote of the expanding directions of the SMSG:

With the task of creating sample curricula for grades K-12 largely finished, SMSG now plans to concentrate on three major concerns: to provide a closer connection between mathematics and the various areas in which mathematics is used; to continue research on how students learn mathematics, and to devise curriculum materials that are suitable for students whose achievement in mathematics is below average (25, p. 12).

The need for reform in mathematics education was never at issue in this period. But as the time progressed the problem of individual differences became more crucial. More and more students aspired to a college education and the new curricula did not adequately serve students with lower abilities.

Many of the major curriculum development projects turned their attention toward the elementary field. Some, reacting to criticism in the popular press, began to re-examine their materials and to modify them to remove excess
abstractness and to increase emphasis on such areas as applications and skill development. "Partly because the slow learner and the disadvantaged youth had been virtually ignored in the first round of revision and largely because new pressures in our society created a dramatic shift in national concern and federal support, several existing projects and some new projects turned their attention to these students" (29, p. 290).

The Brown vs. The Board of Education decision in 1954 was to alter forever the course of education in this nation. Special populations within the schools and the minorities, whose experience and educational opportunity had been limited, would become the major concerns of the years to come. A new revolution, or perhaps a second round in the existing revolution, began. Attention was to shift from content to methodology. Educators and mathematicians began to concern themselves more and more with questions of how mathematics is learned and how it can be taught to increase the likelihood that it will be learned (29, p. 290). What method was most appropriate and what content could be expected were the questions of the next twenty years.
The Period from 1961 to 1980

Historical Image: The Cultural Revolutions

The period from 1961 to 1980 has been a time of massive cultural upheaval. There has been a shaking of the foundations of every arena of life in the technological movements, the political shifts and the economic crises. The period encompassed a massive crusade for civil rights, space explorations, a wrenching war that nearly ripped the nation asunder, a political scandal that all but toppled the presidency, a mounting crisis in energy and spiraling inflation, and loss of prestige at home and abroad that brings Americans once again to the brink of global war. Personal and individual movements for human rights, ethnic rights and other minority rights called for new ethical systems to answer the moral dilemma of the age.

The tragically brief administration of John Kennedy set new themes for the nation. In relation to Russia it passed through a period of frightening crisis and then moved toward accommodation and a subsiding of the Cold War. The Cuban missile crisis was handled with great care averting a threat of thermo-nuclear war, and the Test Ban Treaty brightened the end of the administration. The achievements of the Kennedy Administration were not in
the rather meager legislative record but in the Kennedy style which created expectations and rekindled hope. After the assassination of John Kennedy in 1963, Lyndon Johnson accomplished the programs that were set forth in the vision of the "New Frontier," affecting more change in civil rights, health, education, the arts and sciences, poverty and aid for the cities than in any earlier era (16).

When the Elementary and Secondary Education Act (ESEA) was passed in 1965, Johnson flew to the small rural Texas schoolhouse where he had once taught to provide a dramatic context for the signing of the bill (22).

The involvement of the United States in Vietnam had steadily escalated from President Eisenhower's administration through President Johnson's, each President becoming more and more committed to a military solution. The growing student movements, with anti-war demonstrations and Black ghetto rebellions dramatized the alienation and estrangement in the country. The successful Vietcong "Tet" offensive of 1968 precipitated Johnson's decision not to run for re-election. Martin Luther King, Jr., and Robert F. Kennedy were both assassinated in the Spring of 1968 (16, 22). The 1968 Democratic convention in Chicago was played out in the midst of student demonstrations and charges of police brutality.
Events in the first ten years of this period included (16):

1963  Assassination of John Kennedy
1964  Supreme Court decision on prayer in schools
1965  War on Poverty
1966  Medicare Act passed
1967  Six Day War - Israel
1968  Martin Luther King, Jr. and Robert Kennedy assassinated
1969  U. S. landed on moon

In his first term, Nixon's contributions in international affairs were dramatic. The nation and the world celebrated the first landing of men on the moon in July of 1969. In 1962 after careful and secret preparations by Kissinger, Nixon visited China and reopened the relationships that had been closed for twenty-five years.

Tensions within the nation continued to grow. Domestically, Nixon cut back on many of the programs of Johnson's "Great Society." The issues of law and order and the end of the war in Vietnam continued. Terrorism around the globe, protests that led to violent reactions of the government and more and more demands for equity in human and economic rights wracked the nation. The 1972 elections brought yet another internal crisis, as the story of Watergate unfolded in the days following Nixon's
defeat of McGovern. The champions of "law and order," Nixon and Agnew were to both resign their offices in disgrace before the term was half over.

Gerald Ford took office in 1974 and struggled to reassure the nation. Mounting problems of inflation, unemployment and energy consumed the nation. Throughout the twenty years, the Blacks had made significant progress in civil rights but still found themselves largely on the unemployment rolls. Other minority groups organized to claim a share of the economic opportunities. The middle-income majority suffered under the higher and higher tax burdens and the dollar suffered on the world market. Crises in the Middle East intensified.

A partial chronology of events in the final decade of this period includes:

1973  Vietnam cease-fire
1974  Nixon resigns
1976  Bicentennial celebration
1978  Camp David Summit talks
1979  U. S. Embassy taken in Iran
1980  Selective Service re-enacted
       Inflation rate reaches 19%

In 1976 Carter promised to restore a moral order and turn the economy around. A reluctant Congress and the complexities of the world situation have not allowed those
promises to be fulfilled. Inflation has continued to spiral with rates expected to pass twenty percent before the end of 1980. No energy programs of conservation or new sources have emerged.

The nation finds itself with few answers domestically and again on the brink of global war in its international relationships. Old alliances and postures are called into question in Iran and other parts of the Middle East, Americans are held hostage in their own embassy, and the Russians have moved troops into neighboring Afghanistan.

**Curriculum Instruction: Reaction and Refinement**

By 1960 the new curricular programs were sufficiently underway to be fair game for analysis and criticism. Those who took issue with the new developments were objecting only to the nature of the change, not to the fact that change was occurring. Thus, the era from 1961 to 1980 can be characterized as a period of reaction to and refinement of the revolution in mathematics of the late fifties.

The Cambridge Conference on School Mathematics, called to project changes of the future, reported its work in 1963. The report urged that a student who has worked through the full twelve years of mathematics should have a level of training comparable to three years of top level
college training (13). The report stirred debate and controversy. The report was off target, moreover, with the existing changes and pressures in society. The reform movement of the late fifties was most in evidence in the curriculum for the college-capable student, but the problem of individual differences was not addressed. The new curricula did not adequately serve students with lower ability. As the problems of inner-city schools were accentuated by protest, racism and riot during this period, the aims of teaching mathematics needed to be examined again.

With increasing expenditures for education by the federal government, there came increasing questions as to the effectiveness of the funding and the quality of educational opportunity. The United States commissioner of education initiated a series of conferences to explore ways to help measure the quality of education in 1964. First begun under a private grant, and later adopted as a project of the Education Commission of the States, the project (34) was named the National Assessment of Educational Progress (NAEP). Its goals included, "making available on a continuing basis comprehensive information on the educational achievement of young Americans, measuring the change in the educational achievement, conducting special interest 'probes' into selected areas and providing
data, analyses of the data, and reports for a variety of audiences" (34, p. 5).

During the first five years of its endeavors, the NAEP interviewed, tested, or examined more than 400,000 different students. These were selected so that the levels of achievement they demonstrated would be representative of the achievement of the entire country and classified by age, race, sex, region and locale.

The fact that disparity in achievement existed was not news. Every teacher knew it; every parent suspected it. The results of the initial assessments showed the magnitude of the disparity. "Generally, those groups whose typical levels of achievement in academic matters were relatively low compared to national levels were composed of individuals who were from families with little formal schooling, who were Black, who lived in rural or low-income city neighborhoods or who lived in the southeastern part of the country" (34, p. 13). Those groups of people whose levels of achievement in academic matters were relatively high were composed of individuals from families with a better educational background or who lived in affluent suburbs of metropolitan areas (34, p. 13).

John Gardner articulated the concern for dealing with the differences that these reports made apparent. "What we want is a system in which youngsters at every level of
ability are stretched to their best performance and get the maximum education of which they are capable" (23, p. 97).

The Elementary and Secondary Education Act (ESEA) of 1965 was the nation's attempt to provide extra money to schools in low-income districts. Compensatory education became the concern of the schools, and, for a time, education's politically defined task was to be a vehicle for curing societal ills (67).

However, significant integration of schools continued to be illusory. The conflict between increased racial pride among black students and the reality of disproportionate poverty and unemployment was brought into the schools, there to be left unresolved. The busing riots of the early 1970's underscored the fragility and inadequacy of desegregation programs. Many schools also continued to face severe financial crises, exasperated by recession and by declining enrollments (20).

Mathematics educators turned to isolating several variables to test the possibility of improving the situation. The focus of attention was directed to the elementary school and early childhood education with the development of manipulative devices and programmed instruction. The use of the media and computer-assisted instruction attempted to give instructional form to a
society that was more and more bound to these devices. The issues of competency and criterion-referenced testing became a structural means for dealing with differences. The development and funding of alternative schools, magnet programs and career centers were additional efforts of the period to accommodate individual differences. Previously, the structural means that had been used to deal with individual differences was the "two-track" system. One set of courses was developed for the college bound student and another for the non-college bound student (29).

Morris Kline attacked the "New Mathematics" curriculum in his book, Why Johnny Can't Add, caricaturing the features of the new curriculum in this hypothetical conversation between father and son:

A father asked his son how he was faring. "Not so well," the boy replied. "The teacher keeps talking about associative, commutative and distributive laws. I just add and get the right answer, but she doesn't like that"(37, p. 3).

Many of the curriculum projects of the late fifties did modify their materials to remove excess abstractness and to increase emphasis on such areas as applications and skill development. Results from the first assessment of mathematics achievement that were conducted in 1973, were discouraging and many laymen and educators alike began to call for a "Back to Basics" approach to mathematics instruction.
In 1975, the NCTM devoted its yearbook to the issue of early childhood education. The major emphasis of the work was on the many productive research experiments and findings in the learning and cognitive development of children that were conducted in the last fifteen years. The effects of television programming such as "Sesame Street" and the development of the many compensatory education programs such as DISTAR (both a reading and mathematics program) and ILM (a trans-actional games approach) were also discussed. Another phenomenon spawned in part by the availability of government funds at the local level during the 1960's was felt particularly at the elementary school level, that of the "locally developed curriculum." These programs have also appeared useful at the secondary level and are often directed toward laboratory learning or behavioral objectives.

Manipulative devices, cuisennere rods, Montesorri beads, overhead projectables, film strips, and cassette tapes designed for specific mathematical concepts and operations have proliferated during this period. Lott and Dayoub reported on the recent invention of a new geometric tool called a "mira," in an article in *The Mathematics Teacher* in 1977. The Mira is a transparent plastic reflector that is useful in teaching
transformational geometry. It allows students to map the reflections and symmetries of geometrical shapes as well as perform the classic Euclidean constructions of geometry (40).

Ability grouping became a prevalent approach to dealing with individual differences during this period, and much research was conducted as to its viability as a method. E. G. Begle compiled a review of the empirical literature in 1974. He found the results of ability grouping to be mixed. "Grouping students homogeneously by IQ or general ability is not a very reliable way of improving mathematics achievement. However, grouping students by previous mathematics achievement rather than general ability was likely to show positive results in achievement" (4, p. 5).

The issue of team-teaching was another of the experiments of this period, particularly at the junior high school level. A number of the newer schools were designed with open spaces and modular units to accommodate this new concept of teaching that would deal with individual differences. Begle (5) reviewed the literature on team-teaching in mathematics in 1975. The research revealed little difference between team-teaching and traditional instruction. The lack of preparation and training for
such a cooperative approach is often cited as a cause in its demise. Many of the open classrooms of just a few years ago are now being restructured.

Another significant trend that occurred during this period was computer-assisted instruction. The computer was in use in virtually every arena of life by the end of the sixties. As a tutorial device for the mentally handicapped, the slow learner and low achieving student, early experiments with the computer were gratifying (32).

The World Conference on Computer Education held in Amsterdam in August 1970 brought the terms "informatics" and "computer literacy" into general use. The use of the word computer literacy, with its parallel connotation of illiteracy and all that is undesirable in that state, firmly implies the very basic necessity of some form of computer knowledge for meaningful existence in modern society.

Whether a school had access to particular hardware, computer terminals or data processing machines for its students use, the impact of computer science was felt. Every mathematics text of this period included sections on flow-charts, programming or computer problem solving as part of the content or enrichment activities of the texts (32).
Indeed, the declining ability in problem solving was one concern that the advocates of computer education addressed. Krulik (38, p. 649) suggested a four-step problem solving strategy and used the flow chart format illustrated in Figure 1 on the following page.

Atchison (2, p. 29) noted that use of computers in secondary schools is projected to be at approximately forty-five percent of all schools by 1984 both for administrative and instructional use. He also recommended the inclusion of a universal computer literacy course for all students.

Another significant dimension of the computer instruction issue is that of the hand-held calculator. By 1977, the cumulative calculator sales in the United States was seventy-two million (31). The "rock-bottom" price makes the pocket calculator available to almost every student. Jacobsen discussed the rational for the place of hand-held calculators in the mathematics and science classes of schools in a 1977 paper. "Many people question if the calculator should be used at the primary level, but only how it should be used at the secondary level" (31, p. 110).

The computer "board games" or strategy games currently on the market are extremely popular. The professional literature also supports use of computers. The National
A Four-Step Problem Solving Strategy

Start

Examine the "real world" problem

Form a suitable mathematical model

"Solve" the mathematical model

Translate the solution into terms of the real world problem

Does this solution solve the problem?

Yes
Stop

No

(Fig. 1)
Council of Supervisors of Mathematics, in their 1977 position paper on basic mathematical skills, stated that "the availability of computers and calculators demand a refining of the priorities for basic mathematical skills" (33, p. 91). In the February 1980 issue of The Mathematics Teacher, an article entitled "Computer Literacy - What is it?" suggests a list of cognitive and affective objectives for computer literacy (33).

The question of accountability has affected the curriculum and instruction of mathematics increasingly during the last part of the seventies. The summarized report of the National Assessment of Educational Progress showed new data on mathematics achievement:

U. S. students seem to be able to cope with the routine 'basics' of mathematics - facts, definitions, symbols and whole number computation - but overall mathematics achievement declined from the first assessment of 1973 to the second of 1978.

And rather paradoxically, mathematics educators who looked at the NAEP findings speculate that education's current "back to basics" movement may have backfired (46, p. 1).

Educators today are responsive to demands for accountability and the issues of accountability have encouraged the development of new testing procedures. Many states are now engaged in their own assessment programs and developing criterion-referenced and specific competencies tests to determine student achievement and
graduation requirements (21). Teacher competency is also of recent concern. Farrell explores some of the issues and problems in assessing the competency of teachers. She notes the NCTM's policy statement of 1977 in regard to teacher competency. "Evaluation in teacher education programs (should) be characterized by systematic assessment of all competencies over a period of time to identify consistent and effective performance" (21, p. 579).

Learning Theories: Developmental Stages of Cognition

The period was characterized by a marked increase in psychological studies and research in the theories of learning. The theorists considered both the cognitive and affective domains.

Mathematics educators strove to give practical application to Bloom's cognitive taxonomy, Gagne's hierarchy of learning types, Bruner's modes of representing content in mathematics, and Piaget's readiness stages (56). To provide diagnostic techniques for the affective domain, the emotional aspects of learning, use was made of Carl Rogers' concept of the integrated person and Maslow's hierarchy of human needs. An entirely new field of instructional technology based on specific and behavioral objectives emerged as the mathematics community and psychologists responded to the question of how learning
takes place. The more diverse student populations gave impetus to the search for diagnostic procedures (56).

The work of the imminent Swiss psychologist, Jean Piaget, in the arena of developmental readiness and articulating stages of cognitive development, has been the foremost contribution of this period. Although Piaget's research has spanned more than half a century, his work did not receive attention in the yearbooks on mathematics education until this period (52). Shulman notes, "Today it is literally impossible to discuss the psychology of instruction in mathematics without placing his contributions at center stage"(63, p. 24). Throughout the sixties and seventies, critiques and analyses of Piaget's theories have been made. Today there are fewer attempts to check the validity of the stages he proposed and more studies that apply the theory or some modification of it to concepts from the mathematics curriculum (e.g., Rosskopf, Steffe and Taback in 1971). A recent article in the January 1980 Journal for Research in Mathematics Education is illustrative of the continuing sphere of influence of Piaget. The authors argue the fine points of interpretation of Piaget's theory of cognitive development (59).

The major themes of Piaget's work are that continuous and progressive changes take place in the structures of
behavior and thought in the developing child, that these successive structures make their appearance in a fixed order, the rate of development is a function of the child's encounters with his environment, and that a close relationship exists between thought processes and properties of formal logic (63). Shulman notes the influence on mathematics education of Piaget's theory:

His characterization of the number-related concepts understood by children of different ages (e.g., one-to-one correspondence, reversibility, conversation, and the like,) has influenced our grasp of what children at different stages can be expected to learn meaningfully (63, p. 42)

Bruner, one of the prophets of discovery learning, based much of his work on Piaget. Bruner suggested that the conception of readiness be modified to include not only the child, but the subject matter as well (12).

Psychologists also argued that attitudes and emotions were directly involved in learning mathematics and that diagnostic strategies of teaching mathematics must include awareness of the student's affective domain (56). Abraham Maslow's "hierarchy of needs" published in 1962 and Carl Roger's concept of the "integrated person" were applied during this period to the student-teacher relationship in the educational setting. "Math anxiety" has been a popular topic during this period. During the seventies, a number of works appeared on this subject and colleges
throughout the country developed programs for math-anxious people (67). Many were directed specifically at women.

The growth and development of computers increased the scope and sophistication of data analyses in psychological research, and permitted the development of psychological theory by means of computer simulation of cognitive processes. As Estes observes, "computer-based information-processing systems have probably had their greatest impact as a new source of analogies and a new theoretical framework for investigations concerned with the structure and function of human memory" (10, p. 18). Many recent research projects have dealt with the specific aspects of learning mathematical structures (e.g., Mayer in 1974, Greeno in 1973 and 1976, and Branca in 1974). The implications of these works are that individual differences in mathematical abilities should be viewed in conjunction with differences in the mathematical tasks.

If any trend can be discerned in learning theories, it is a trend away from competitions between various theories and toward a more precise specification of the conditions under which a given theory seems to explain the phenomena best. Kilpatrick (32) noted in 1978 that the tides of learning theory shift rapidly and that theory-guided practice changes more slowly than do the theories
themselves. The most pressing criticisms have been about incompleteness and irrelevance to the practical classroom situation. In a 1978 article on discipline, Doyle (19, p. 24) noted that "many of the principles of learning from the research theorists were best applied to individuals and that as such were not always applicable to the classroom." Kilpatrick (32) notes that learning theories and the research studies based on them are being criticized for their failure to deal adequately with four commonplaces of educational thought: the learner, the teacher, the milieu, and the subject matter. As this period ends, the role of the teacher, the particular classroom environment and the ability to put into practical application the theories of learning are issues of immediate and future concern.

Major Contradictions: Arts of the Practical

The major contradictions facing the mathematics educator are practical ones. With the content of the mathematics curriculum revolutionized, this twenty-year period sought to build the methods to make that content accessible to a wider range of students. The questions of what was taught were shifted to who shall be taught and in what way. As noted in the previous section, the research in the theory of learning and its specific processes during this period was monumental. The paradox
is that despite an enormous amount of research on learning and teaching, we lack reliable knowledge of how to put this information into practice (47).

Most critical reviews of the "New Mathematics" articulated by teachers noted this kind of problem with theory and practice. Robert Shaw, wrote in 1974, "Content, methods and materials are often altered to reflect the views imposed by the most dominant force at a particular time. Adding to its instability as a discipline is the fact that an implementation lag from theory to practice usually exists" (62, p. 90). Boys (9) noted similar concerns among teachers in Great Britain about the bewilderment of teachers over the number of choices in books and other teaching materials that emerged during the period. The practical challenge to researchers in the next decade will be to find ways to make their own research more relevant, and the work of teachers more productive, by enlisting teachers as partners in the research enterprise (47). Action research conducted by teachers in the classroom has proved successful in individual settings and the possibility of replications of these school developed models is being experienced (42).

The need for putting theory into practice is seen in other arenas. The successes and failures of compensatory education have impressed upon the educational community
the difficult realization that there is no single best design for overcoming the school problems of disadvantaged students (20). This, and other political mandates of the period — filling the post-Sputnik need for scientific and managerial personnel, integrating Black and other minority populations into mainstream society, and breaking the "cycle of poverty" — have not as yet yielded practical results. The changes in economic, demographic and cultural structures of society have sought to use educational intervention as a means to broad social ends. Many of these forces are in conflict with each other. Declining enrollments due to a declining birth rate, an inflationary economy, increasing demands of teacher unions, and confused authority patterns present a challenge that has more to do with practice than with theory.

Underlying these issues is the question of funding. Enormous amounts of monies have been poured into education from federal, state and local agencies over the past two decades, especially for targeted populations. The results both socially and academically have been questionable. Currently, many school districts face serious funding problems. If these economic issues do not ease over the next few years, education's financial crises will swamp any of the other issues that are now considered.
Summary: 1900 to 1980

The basic interrogatives of why, what, how, to whom and when have been the questions of mathematics education during this century.

The question of "why" throughout the century revolved around the old dichotomy: Is mathematics an art or a science? Is mathematics taught because it is useful in everyday life? Is mathematics an art the appreciation of which can enrich everyone's life? Sullivan (29) called mathematics a free creation of the mind. Wigner (29, p. 579) described mathematics as the "science of skillful operations with concepts and rules invented for just this purpose."

The question of who and what mathematics should be taught changed with each new generation of mathematics education and the body of knowledge of mathematical content and application grew at an incredible rate. The question of how and when to teach mathematics evolved and was impacted by the growing knowledge of the processes of learning and the structures of the human brain.

Mathematics education throughout the century was stimulated by forces from within mathematics itself, from psychology and from the society at large. "The forces derived from within mathematics were the rapid growth of its content, the phenomenal expansion of its applications,
and the changing nature of the subject" (29, p. 580)."
The one clear and discernable effect has been to cause a continuing downward movement of content. The school program now includes parts of classical college algebra, analytics, the calculus, matrices, probability, and statistics.

Over the century there has also developed a teamwork between psychologists and mathematicians. The changing theories and research results in psychology are increasingly recognized as strong forces affecting mathematics education. Recent investigations of learning seem sometimes to return to earlier concepts of stimulus-response (as in work on programmed learning) or to rehabilitate mental discipline in a new and scientific guise.

Societal forces have had direct and immediate impact on mathematics education to one degree or another during the century. Responses to particular practical technological or business needs have been brought on by the external events of global wars and depressions. Cultural revolutions have resulted in development of programs for special target populations and diverse groups. Universal education is an accomplished fact of the twentieth century, yet the validity of the high school diploma is under attack with many graduates still functioning marginally in reading and mathematical skills. Forces from the cultural
arena have also come from youthful activists calling for relevance in content and experience. Every line of classification previously known to society has come into question: age, sex, race, ethnic and religious backgrounds, economic and physical status. The twentieth century has been characterized as an era of cultural revolution with old values and mores giving way to new. It has been a revolution in the common sense, the common style and the common mood of the people.

This century has also been characterized as a time consumed with the question of power. E. C. Goddard, speaking at a teachers' meeting shortly before the turn of this century (in a time when electricity was just being introduced) spoke of the goals of mathematical instruction. He likened teaching to this story:

There is a suggestive sign on homes in this college town. "Rooms to let in the upper story, unfurnished." No doubt our schoolboys' and girls' upper stories ought, when ready to let, to be furnished. And yet I had rather their condition be expressed by that other sign, so often used in modern blocks in these days of machinery—"Room to let, with power"(29, p. 380).

This comment epitomizes the joys and the frustrations of teaching mathematics. Students come with "room to let, with power," Helping them to "furnish the room," to "harness and apply the power" is still the challenge.
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CHAPTER III

TWENTIETH CENTURY TRENDS IN MATHEMATICS EDUCATION
IN THE STATE OF TEXAS

Introduction

A second part of this work concerns trends in mathematics education during this century from another perspective. Dramatic and significant events happened on a national level through committee and commission actions, professional organization recommendations, research projects and governmental directives. Societal forces, scientific and technological discoveries changed the direction of the teaching of mathematics on the secondary level as well. These trends have been examined at length in the previous chapter. Equally as significant and valid to the understanding of the trends in mathematics is the common everyday experience of teacher and student.

Mathematics education may well have been affected by the events in society, decisions in national councils, and discoveries in laboratories; but the education occurred in the local classrooms with teachers and students. The classroom teacher continues to be the single most important factor affecting the achievement of students.
This section examines the trends in mathematics education from the perspective of the teacher and student in the state of Texas. The sources of data are the responses to questionnaires and interviews as well as visits to archives of local school districts. Recorded reflections from teachers in this region published during the national bicentennial year and results from three national surveys published in the fall of 1979 also supplement the sources of data (1, 2).

A considerable lag has occurred historically between the advancement of new theories and their application. That lag has been shown to be geographically determined and the particular region sampled for this study has consistently been behind in educational innovations. Responses were collected from both retired and active teachers of mathematics. The subjective nature of their responses and regional character of their experience will preclude any generalizations.

Analysis of the Data

Tabulated results from the questionnaires and interviews appear in the Appendix in Table II through Table VII. The variable analyzed for this study was the designated time period of earliest experience as a student and as a teacher of secondary mathematics. Table II illustrates the
percentages of the sampling for each of the time periods. Tables III through VI report the most typical responses to the structured questions of the survey for the respective time periods. The typical demographic characteristics (sex, age, ethnicity, and education) for the respondents in each time period are reported in footnotes to each table. For each item, the mode is the statistical average reported. Data gathered from interviews are recorded separately in parentheses for each item with a different mode.

The demographic statistics for the entire questionnaire and interview sampling are included in Table VII in the Appendix. They are reported here as descriptors of the population sample. No further analysis of trends was attempted in this study using these demographic variables.

The questionnaire was structured to solicit responses from perspectives as student and as teacher of mathematics. This procedure allowed the overlapping insight of one generation to another and revealed several notable trends. In the following paragraphs, the trends noted across the time periods and within the time periods are discussed.

In the period from 1901 to 1920, there was a shift from rural to urban experience and enlarging school enrollments and class sizes. The relative stability of the population is revealed in the fact that the experience of
the respondents both as student and as teacher during this period was in the same region. Student populations remained racially segregated. There was a shift away from the use of corporal punishment during this period. One hundred percent of the respondents said their teachers used the textbook and chalkboard exclusively. Ninety-two percent highly respected their teachers. Oral drill was recorded in student experience but short written assignments were more predominant in teaching experience.

In the period from 1921 to 1940, the population's mobility is evident in more varied locations being reported from schools attended and those where the first teaching experience occurred. More of this group had a first course in general mathematics than in algebra as students. Attendance at a junior high school was noted for the first time. Use of other teaching devices such as workbooks and visual aids became more prevalent. Furniture was more mobile and discipline more relaxed. Class sizes tended to be larger during the teaching experience of this period. The category of permissive discipline was noted for the first time. Members of this group (and all subsequent groups) found mathematics exciting rather than difficult.

The category of suburban schools became most typical for the first time in the period from 1941 to 1960.
Classes were dramatically larger. Eight-five percent responded that class sizes were over thirty. The use of posters and other items of decor were noted for the first time. First year salaries showed significant increases during this period. More use of workbooks, overhead projectors and other classroom aids occurred. Teaching assignments included "other" courses, some experimental. There seemed to be a revival of corporal punishment during this period. Fewer activities and games were played and teachers of mathematics were predominantly female for the first time.

The period from 1961 to 1980 revealed the dramatic changes in student population from segregated schools to racially mixed schools. Total school enrollments were larger. More variability occurred among teaching assignments. Early in the period the new curriculum materials of modern mathematics were used but the basics were stressed later in the period. Courses like elementary analysis, calculus, introductory algebra and computer science appeared for the first time. The teaching materials used encompassed the widest range with the category of "other" being more predominant than the textbook category for the first time. These included items such as: outside publications, worksheets, teacher-made materials, learning packets, filmstrips, cassettes, movies,
games, calculators, manipulatives, and computer terminals. Flexible and permissive discipline was more predominant than formal discipline for the first time. The attitudes about teaching and students included responses about frustrations and difficulties with discipline and teaching for the first time reflecting the changing and more diverse student populations. There were twice as many female teachers as males reported in the sample. Starting salaries were $5,000 and over for the first time.

Responses from every time period included homework as a predominant method of assignment. Ninety-six percent of all respondents reported their respect for their own teacher. All groups, except the 1901 to 1920 period, indicated their attitude towards mathematics was one of excitement or pleasure. The 1901 to 1920 group found mathematics difficult. The overall trend in the explosion of curricula materials, enlarging enrollments, diversity in assignments, modes of teaching, student populations and the shift from rural to urban to suburban settings were evident.

Seventy-three percent of the sample responded to the questionnaire. One hundred percent of the interviews was completed. Fifty-two percent of the respondents included comments to the open-ended questions and an additional seven percent responded with lengthy unsolicited letters.
or supplementary details. A sampling of the responses is included below.

Years ago: drill, dull, very hard drill.

We came to class and sat quietly, Hand was held up for permission to speak. We memorized and recited.

I think we need a reversal of the politicians' attitude of penny-pinching so that schools may have a proper environment for the students.

Students are socially promoted, allowed to "get by" and not learn.

Mathematics teaching: more hills than valleys.

There is a lack of pride today in student's work.

Modern math was somewhat of a disaster. The return to traditional math with some of its ideas incorporated would be a far better method.

Students can learn higher levels of math at an earlier age now.

Back to basics!

Computers will definitely be a part of the future.

I don't think we require our students to think and learn to solve problems now as I had to do when I was in the 7th grade.

We should teach basics in elementary school and save the theory for advanced students in high school.

We should dissspell the fear of math.

Additional help is needed for teachers regarding routines in order to free (the) teacher for teaching.

In the curriculum movement of my latter years, the curriculum was up-graded and the students down-graded.
The one change in all education that stands out above the others is the revelation that every child is not the same. It takes all the "tricks" of all the teachers to teach those less well-equipped.

I think that math should be departmentalized from K-12 because in lower grades too many teachers who hate math are presenting it.

James T. Fey reported on three recent national surveys conducted by the National Science Foundation and the National Council of Teachers of Mathematics in an October 1979 issue of The Mathematics Teacher (2, pp. 490-504). Below are a sample of the comments of secondary mathematics teachers from across the nation. They show remarkable similarity to those from the questionnaire of this survey.

We've found that traditional methods (of instruction) work. I don't think kids can handle inquiry...They just don't have the background or sophistication.

Modern mathematics? I dislike it... (The text) shows three ways when one will do. The brass tacks are learning addition and subtraction. That's it.

My kids understand the basic number system much better than I ever did at their age...They know, for instance, what multiplication is, rather than just knowing some tables by heart.

I wish they were almost taught no theory down there. Back to basics. That means delaying teaching of theory.

...they're spoiled, they don't care, they don't try (2, pp. 490-504).
Historical Vignettes of Texas Classrooms

The concluding analysis of the trends in mathematics education during this century in the state of Texas is embodied in the following vignettes. The substance and style of these word pictures is taken from the open-ended questions of the survey as respondents reflected on their own experience. Other data for the situations depicted are from the reflections of Texas retired teachers published in 1976 entitled As We Remember (1). The NCTM yearbook, A History of Mathematics Education, is also a major source (3). The tabulated results of this survey provide the typical settings and other demographic information (See Appendix, Tables II through VII).

1901 to 1920: The Rural Schoolroom

A typical day in the classroom began early for the teacher. As one of three teachers on the staff, he had responsibilities for janitorial work and other administrative chores. The schools were poor, and it had only been a few years ago that the Rural Electric Cooperative had brought that great luxury of electricity to the school. Having finished the daily preparations and his letters of inquiry to the teachers college he hoped to attend during the summer break, he was ready for the day. He still felt a little tired from the weekend's trip to visit his
parents. He was lucky; the women teachers in the elementary school were not allowed that privilege of leaving town. He mused on the school board's latest rulings on the moral conduct of teachers and its double standard, but understood its propriety.

The secondary mathematics class was the first in the morning and he was hopeful that all the students would be present today. The Johnson brothers were needed for the spring planting, especially since the oldest son had been lost in France. When the weather was good, students did not always come to school. When the weather was bad, they could not.

Where were those extra problems he had found for his algebra students from that new textbook his father had given him? Someday, he hoped, the schools would provide more materials. Sixty-two dollars a month was a good salary, but it did not buy many books.

The boys had done a good job of cleaning the blackboard last Friday. He had found someone's lunch pail left in the cloak room, however. He knew the guilty party would have to be reminded of the need for thrift and attentiveness; perhaps a switching would do.

Three Negro boys and girls had just passed by on their way to their school; his students would be arriving
now. The morning routine had begun, and he felt warmed at the thought of the courteous greetings soon to be his.

The class was called to order and homework presented at the chalkboard. The assignment was long but almost all of the students had fared well. Oral drill was next on squares, square roots and cube roots. The students had so much to memorize, but it was good for them. The lecture last Friday was long, he was afraid that many had drifted off into day-dreaming. His mind began to drift too. It would not be long before he had taught all the algebra he knew. The only teacher in the school with a college degree, he still felt somewhat uneasy about his college mathematics training.

The students worked now at their desks. Seven out of twelve were present. It was a good day. Maybe he would have time for a ball game with the boys at noon. The classes were mixed but during the recess time, girls and boys were separated. Only one interruption disturbed the hour. He was called to help in the elementary classes across the hall. He looked forward to his "surprise" for the day. His students would be excited about the homework problems he had prepared. They included some stated problems on farming and ranching.
There were blackboards on every wall in the new classrooms with plenty of room for the students to do board drill. But where would they all sit? Having that many students in a general mathematics class was proving to be a challenge. There were not many more students in the entire grade school he had attended.

The students were eager: they liked their new room, new books, and especially the new radio in the principal's office. Some of the young teachers were even talking about using it to teach, though he could not see how it would be helpful. Everyone was looking a little brighter these days as many people were back to work, most in government sponsored projects. He was pleased that he had been able to last through the lean days and now could bring home $165 each month and not in script!

The students had complained yesterday about the dull drill work at the board. Perhaps he would have to give in to these new activity games he heard others talking about. The general mathematics course outline was new; it seemed to change every year. A lot of problems on banking, purchasing and lending were included, but it did not seem to go into much depth on any concept.

The forty-five minutes went so fast, he did not see how the students could have time to be bored. Most of the
time, students took pride in their work. The challenge did not really seem to be in mathematics any more; the challenge was in the students. He thought of the "testing" a few boys had given him earlier in the year. The times were more permissive and the whipping he had given them caused quite a stir.

He would have a multiplication race at the board to end the class and then give them their homework assignment. That was one thing he would not change!

1941 to 1960: A Suburban Dream

The new equipment had just arrived in the mathematics department office. It was exciting to open the boxes. She had used filmstrips and audio-visuals for some time, but these were to be for a special class of seniors in higher math. The course was so new that it did not have a name yet. The curriculum revision committee had adopted the course, and she had attended last summer's institute for training the gifted college-bound student. She was pleased that the institute would count toward the completion of her master's degree.

New concepts and new approaches to old concepts concerned her. Her own experience in school and her college courses really did not prepare her for this and she worried about measuring up to the task. The students, mostly boys,
were serious, too. Many of them had plans already for engineering and science degrees. She would have to do a lot of planning every day for this class.

The school was large on a beautiful campus, but this class would be small. So many activities claimed her attention and her students—the debate team, student council and the new Christian Athletes Club. Would they have the time to devote to all this new material?

The overhead projector worked fine and without it those circular functions of trigonometry would have been meaningless. The text and materials were very detailed. They almost taught the course without her. She began to think of her other classes in algebra and general mathematics. How could some of these ideas be used? It was not likely that the school would buy materials for them, however. She was not sure that they were even available at that level.

The class passed too quickly. She would have to take the teacher's commentary home tonight; there had been more than one question she could not answer. And why, she asked herself for the hundredth time, had she been persuaded not to enroll in that physics course in college!

Before the next class came in, she made a note to schedule the field trip to the university's new computer center.
1961 to 1980: An Inner City Team

"There must be something we can do about the fact that we are the only school in the city without air conditioning."

"Thirty-seven students in any class is too many -- but a seventh grade junior high, mostly black and chicano -- that is ridiculous,"

"The federation representative was here again soliciting union memberships."

"Look, here's another article on school vandalism, the third one this month."

"Glad I'm not on the textbook committee this year; it looks like there will be more than one militant group at the adoption meeting tonight."

These were just a few of the comments that ran through her mind as she left the teacher's lounge headed for the math department meeting. She had the new learning packet under her arm from the ESEA resource teacher. That meant an extra hour of duplicating time today after school in order to have the materials ready for her class tomorrow. She would urge again in the department meeting for a team-teaching approach. She held little hope for its adoption. Too many teachers told of bad experiences with the method and in an old building there was little room to house and re-group so many students.
The struggle to individualize the instruction of every child in the general mathematics class was sometimes too much for the staff and she saw some signs of frustration and even "teacher burn-out." The packets were helpful at first, but with so many students at so many levels of ability, she could not get to all of them. Classroom discipline, for the first time, had become a problem and consumed much of her time.

Maybe her salary had doubled since she first started teaching, but it was also true that the cost of living had tripled and continued to soar. The work seemed to have tripled too. When those two students were accepted in the language learning clinic, perhaps she could "handle" the remaining thirty-five.

The morning had gone smoothly and those students she had taken across town to the computer-assisted instruction center were excited about their work. It was unfortunate that they would not be in line to go again for another three weeks. Much of the good effects of today would be gone by then.

The afternoon classes were restless, but she managed to lecture briefly to all the class on the meaning of decimals and set the five groups to work on their worksheets. One bright moment came when the student who had been absent for a week showed her his completed fractions
chart, his face beaming. Two groups finished the audio tape set up in the learning center and one group finished the entire packet in class. The two lowest level groups were still working on whole numbers and common fractions. The young man in the third row fell asleep for the second day, and she made a note to see the school nurse about him tomorrow.

There was so much to do when the last bell rang and the hour of duplicating was still before her. The department meeting had gone just as she had expected. One short fellow lagged behind as the class hurried out to the waiting buses. When he hugged her neck, she knew she would be back tomorrow.
CHAPTER BIBLIOGRAPHY

1. As We Remember, Dallas, Texas Retired Teachers Association, 1976.


CHAPTER IV

CONCLUSIONS AND RECOMMENDATIONS

A Gestalt of the Trends

The trends in mathematics education have been summarized and recorded in the gestalt (Table I), on the following page. The complexities of forces and issues are difficult to hold in a single phrase or category. Issues and innovations have recurred and overlapped in each time period. The chart does not begin to hold the dynamics of human society as implementation of some new approach or idea continues to ripple through succeeding eras. There were reactions and reappraisals from one period to the next. The forces of tradition are strong. Just as all human beings model the behaviors of their own experience, teachers model the teaching they experienced.

Educational institutions have rarely led the way, but have rather been responsive to new demands of society. A particular example of this fact is illustrated in the chart in the early periods of this century. Progressivism was the overriding image of the period from 1901 to 1920 in society, but the progressive era in mathematics education did not become the dominant trend until the late twenties
<table>
<thead>
<tr>
<th>Generations</th>
<th>Historical Image</th>
<th>Curriculum and Instruction</th>
<th>Learning Theory</th>
<th>Major Contradiction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1901 to 1920</td>
<td>The Progressives</td>
<td>Drilling the facts</td>
<td>Mental Discipline</td>
<td>Obsolescence of curriculum</td>
</tr>
<tr>
<td>1921 to 1940</td>
<td>Economic Collapse</td>
<td>Focusing on usefulness</td>
<td>Activity Learning</td>
<td>Loss of excellence</td>
</tr>
<tr>
<td>1941 to 1960</td>
<td>Victory without Peace</td>
<td>Formulating the &quot;new math&quot;</td>
<td>Guided Discovery</td>
<td>Exclusivism of teaching</td>
</tr>
<tr>
<td>1961 to 1980</td>
<td>Cultural Revolt</td>
<td>Individualizing the instruction</td>
<td>Cognitive Readiness</td>
<td>Process of implementation</td>
</tr>
<tr>
<td>1981 to 2000*</td>
<td>Maturing Technology</td>
<td>Unifying the curriculum</td>
<td>Selective Technology</td>
<td>Ethics of choices</td>
</tr>
</tbody>
</table>

* projections
and early thirties. The "New Math" revolution was a product of the period from 1941 to 1960, but it did not reach its peak of implementation and controversy until the next twenty-year period. Learning theories were similarly affected as new techniques in the behavioral sciences emerged and old approaches were discarded only to be re-introduced with new jargon and emphasis. The major contradictions offered in the chart are those issues that were of primary concern as the periods ended. As such they are not inclusive of every concern but rather turning points for the future.

Future trends for the final period of this century are also included on the chart as one possible view of the future. Following is an exposition of those future possibilities.

The Period from 1981 to 2000: The Future Prospects

Introduction

As Broudy noted, "Predictions of affairs that involve human resources are liable to a peculiar sort of fallibility" (3, p. 21). Standing in such times of crises as today in international relationships, economic confusion and technological complexities, predictions are even more perilous. A helpful analogy to describe the dilemma is suggested by the Chinese language. The Chinese word for
crisis is written by combining two characters -- one that means danger, the other opportunity. The element of danger is certainly one option for the course of history. The other, opportunity, is more appealing. Without doubt, the costs of a nuclear confrontation would be staggering, perhaps final. The costs for building an appropriate resolution to global conflict over power and resources will be equally staggering. Opting for the course of opportunity, the following historical trends are postulated.

**Historical Image: Maturing Technology**

This section offers a forecast of possible directions in the distribution of use of natural resources, nuclear proliferation, and expanding technology. All of these will require a mature stewardship and corporate responsibility on the peoples of this nation and the world. In the United States, there will be a reduction in the consumption of goods and a conservation of all resources. Development of alternate sources of power will also consume the best minds and technology of the next twenty years. Americans have been the greatest producers of goods and services for themselves and the world, but also the most gluttonous consumers. A moderating force around the globe will narrow the gap between the "haves" and the "have-nots" and that will have profound effects on the consumerism that has marked the present age.
The struggle to resolve the crises of political power and nuclear might will engage the will of all peoples. The block of "Third World" nations will occupy a large share of influence and indeed, act as a balance point for the conflict between East and West.

Expanding technology and the continued explosion of knowledge will test the imagination, individual freedom and sense of morality of this nation. Technology will mature because of the nature of the human response to it -- a response that will call forth an ethical posture that has human dignity and human control at its center. Speaking of this human dimension of technological use, Broudy notes, "if we understand a trend of events, we can by a change of attitude change the trend itself" (3, p. 24). He gives this example:

The computer is unaware of changing human behavior. Concern about the population explosion and such matters as pollution [of natural resources] has already had profound effects. The rate of population growth has slowed. The other solution was in technology itself, for example: the finding of new energy resources (3, p. 24).

All of these trends will affect the nation domestically as well. A conserving society will be concerned about human resources and potentials as well. The economic impact of moving from a consuming society to a conserving society will be borne in the large by the workers of America. Finding a political concensus in international
relationships can affect and be affected by finding similar solutions of equity for the diverse groups and cultures within this nation. A mature information-based society will require new concepts of work, of personal worth, knowledge and responsible citizenship.

Curriculum and Instruction: Unity in Diversity

Dwight Allen, writing in The Future of Education: 1975 - 2000, notes that "there are many present forces which combine to constrain what education might be: lack of funding, racism, and disparate values in society.... but the greatest single obstacle is made up of our beliefs" (1, p. 19). In light of the present realities, these trends are postulated for the next twenty years in the curriculum and instruction of mathematics. First, unification of the curriculum, a proposal which has been present throughout the century, will be given practical form by the end of this period. Some practical implications will follow. Students will study a spiraling curriculum, Math K through Math 12. There will be a converging of the art and the science of mathematics with students engaging in a balance of theoretical conceptualizations and practical applications at each stage. Elective specialization could occur at the eleventh year with the possibility of "apprenticeship" to a collaborating industry, university
or other career institution. Open access to the courses would enable enrollments from across generational lines with persons seeking up-dating in a field, career change opportunities or fulfilling requirements for a deferred college career. Allen reminds educators concerned about credentials that "prerequisites and credentialing have always been scrapped when a military crises arose with no major loss in quality occurring" (1, p. 6). Transdisciplinary curricula could also develop within such a structure (1, p. 7). Departmentalization of mathematics in both secondary and elementary grades would be necessitated.

Second, differentiated staffing will revive and refine the team-teaching approach experimented with in the previous period. Research in the design and implementation of a teaching cadre will have to be done to model the possibilities and limitations of such an approach. Broudy notes that schooling takes place mostly through didactics or programmable interaction, but the other roles of a teacher are equally as needed:

**Heuristics:** interchange between pupils and teachers, in which problems of one kind or another are discussed and structured by teacher and learners. ....process and product is both the outcome and the means.
Philetics is merely a Greek word meaning love. It stands for the type of teacher-pupil relationship currently expressed by the term relating,... The philetic teacher is more a counselor than a teacher, and even something of a therapist in an unobtrusive way (3, p. 41).

A "first" teacher or master teacher will be crucial in such a working model. To deal with the economic limitations, part of the teaching team (perhaps the lecturer) might be mobile and other parts of the team made up of para-professionals or local neighborhood volunteers. Teacher training courses in team-teaching and classroom management would be necessary in colleges.

Third, the use of computers or programmed instruction will increase. The growth will be steady rather than dramatic due to the uncertainties of educational budgets (5, p. 150). Refinements of multi-media and multi-sensory approaches will affect individualized and remedial instruction. Diagnostic procedures will be refined for tailoring the instruction to meet the specific needs of students and subject matter. Data banks of illustrations, demonstration and practical applications will enlarge and keep current resources for problems and exercises.

Learning Theory: Selective Technology

Two directions are postulated for the research in the theories of learning during the next twenty years. First,
research will be more closely related to the schools, school practice, and the classroom. Criticism of the various theories that have been proposed in the past is that they work well within the framework of individuals or controlled laboratories, but that is not where students attend school. "Despite the recent criticisms of educational processes, most critics have assumed the continuation of the school and the classroom. They ask only for changes in operation within this framework" (7, p. 134). Action research will maximize the role of the teacher in research in learning theory and curriculum development.

Second, researchers will direct their attentions to the specific and selective processes of learning both in its physiological structures and in the patterns of cognition. The computer with its present ability to construct simulated intelligence, mostly in the form of puzzle solving, will become more effective in understanding the processes of problem-solving. By the end of the century, educational psychologists should be able to identify specifically the structure of memory and how learning takes place in the human individual and the human environment.

Major Contradictions: Ethical Choices

New directions will bring with them new issues of concern. "The successes and failures of education in the
seventies suggest that the educational enterprise potentially will respond to creative leadership, but that the development and articulation of such leadership will not be a simple task" (8, p. 23). How will that leadership be formed and what presuppositions will direct its choices? Universal education is an accomplished fact of twentieth century education, but with an unabating crisis in the economy, the question of the validity of a universal education for all the citizenry in the public schools must be raised. Who will make the choices? What will be the alternative forms of access to knowledge? The commitment to education as a life-long enterprise with open access must be backed with financial support of much broader base. Will the conscience of the corporate society in industry and management be shifted towards a share of the responsibility for education? Broudy mentions the issues of "genetic and chemical tampering with the learning process" (3, p. 42). The question of who decides and what heroic means are used to change the ability or potential of human beings will be profound. New knowledge has to be absorbed by a culture, but it does not necessarily have to be absorbed uniformly by everyone. An information based society suggests that new definitions of intelligence and education are necessary. All of these issues and questions point to a new ethical framework. As
Robert Havighurst notes, "Our view of the future is clouded by our knowledge -- not by our ignorance. We know too much about change, complexity, and the unexpected to be too confident" (4, p. 83).

A Classroom Vignette of the Future

At the risk then, of too much confidence, the following description is offered of a mathematics classroom of the future. It is an optimistic, but practical, point of view. Data for the vignette are primarily from the predicted trends in education submitted in the previous sections and from the collection of essays by leading educators entitled The Future of Education: 1975 - 2000 (1, 3, 4).

A Big Day In Math Ten-Ten

The math team had decided to meet for breakfast this morning. It seemed the only way to get all the last minute planning done. They did not want to admit their excitement, but it was evident in the fact that everyone on the team was making final adjustments in the room arrangements and fussing over wall decor when the student committee assigned to that task had done a perfectly fine job the day before.

The entire volunteer staff had decided to be at school today, so the "first teacher" had drawn up a new assignment
schedule for consensus. It had started out to be a simple
guest lecture/demonstration but several local school dis-
tricts had heard of the work in this small community and
had asked if they could visit. The district adminis-
tration had decided to turn it into a public relations
opportunity. They did have a lot invested in the school's
solar collector. Some corporation representatives promised
to be present and were considering a cooperative venture.

The visiting professor had been responsible for one
of the first such approaches to a teaching cadre. He was
very pleased with this school's local adaptation of the
model; it had gone further than he had dreamed possible.

The video-taping crew arrived from the regional
service center and they were interviewing some of the
students. The young man who was talking about the Ten-Ten
label for the course was doing a good job of explaining the
ten aspects of the spiraling curriculum students covered in
their tenth year. One volunteer member of the cadre was
having a great time talking with the visiting businessmen.
Forty years as a craftsman, and now engaged in practical
training of young people to take the same pride in their
work, showed on his face. In fact, the teaching cadre
would not have been except for the volunteer help in the
community. It had spread from parents to retired citizens
and now professionals were calling to volunteer some
program time. Teachers' salaries had not taken the dramatic leap that was hoped for ten years ago, but everyone was living a simpler life-style these days.

The tenth year was so important for the young people because they could decide the speciality course they would take the following year. Although access to the junior and senior courses was open, students and parents took the decision time very seriously. Human nature being the same as ever, the young seem to think every decision is irreversible. They would have to think through ways to deal with this in their next evaluation session. Some nagging questions about the subtle pressures or overt value judgments that may have been made by the instruction or curriculum would have to be dealt with as well.

The students' achievement was better than it had been in many years, but there were still wide differences in their abilities. Some great strides had been made with the low achievers in the past ten years. And then, there were always the students who fared well, no matter what or how they were taught. Discipline in the classroom wasn't a problem with all the help. Time/task management was one of the first things the team worked out. Some people said the students were doing better because they saw good models of cooperative planning and shared responsibility and how special talents could be used. The team
had its moments though when one or another student found ways to manipulate "the system" and get "in-between."

The lecture lasted longer than usual, but this was an unusual day and they were on the roof! The solar collector, built by the senior class two years before with the science team and help from local businesses, supplied the power for the whole school. The savings were not great but it made a small difference in the community tax rate. The lecture was right on target -- exploring the role of algebra in the theory and applications of such a system and describing various levels of career opportunities in the field of energy.

The students were enjoying all the attention they were getting. It would be hard to get them back to earth-bound problems, but the team had a plan!

Recommendations

The limitations of this study have precluded any generalizations. The historical literature researched reflected primarily the experience of the dominant culture in our society; the survey conducted was regional in nature; and the analysis of data was restricted to a single variable. In consideration of these items, several recommendations emerge from the completion of this work. The specific recommendations are
1. similar historical surveys be conducted in other regions across the nation (perhaps along the regional lines drawn by the NAEP) to discern any significant geographical differences in the trends in mathematics education,

2. similar historical studies be conducted of the minority cultures in America, particularly the black community with its history of separate educational structures, as a contribution to understanding the total educational forces at work in this society,

3. other variables be isolated for analysis of trends in mathematics education (sex, age, ethnicity, educational level, and other perceptions of teachers) with appropriate refinement of questionnaire and sampling controls in order that data might be tabulated and analyzed by computer, and

4. the findings of this study be submitted to the National Council of Teachers of Mathematics committee on curriculum revision for immediate consideration in its 1980 report.


APPENDIX
January 29, 1980

Dear Colleague:

I am conducting research for the preparation of a master's thesis on the historical trends of secondary mathematics education in this nation during the twentieth century. I will be writing a descriptive vignette of a typical mathematics classroom during each of four twenty year periods.

Will you please complete the enclosed questionnaire recalling your earliest experience as either a student or a teacher of secondary mathematics? You are encouraged to use the spaces provided for comments and to include any reflections as well as significant changes that you have encountered throughout your experience.

A self addressed and stamped envelope is enclosed for your convenience. You need not record your name on the questionnaire. Please mail your completed response by February 12th, if possible.

Thank you for your contribution to the education of our young people and for your cooperation in this endeavor.

Yours truly,

Letty Lynn Maloney
North Texas State University

LLM/sej

Enclosure
QUESTIONNAIRE

Please complete the questionnaire by checking the appropriate blanks that describe your recollection of your earliest experiences as a student and a teacher of secondary mathematics. Your comments are encouraged.

A. Personal Data

<table>
<thead>
<tr>
<th>Sex: Male</th>
<th>Age: 65-over</th>
<th>25-44</th>
<th>Race: Black</th>
<th>Hispanic</th>
<th>Female</th>
<th>45-64</th>
<th>under 25</th>
<th>White</th>
<th>Other</th>
</tr>
</thead>
</table>

Education (highest level attained): High School | College Graduate | Masters Degree | Doctorate

B. Descriptions of your experience

**AS A STUDENT**

|----------------|-----------|-----------|-----------|-----------|

1. Type of School:

<table>
<thead>
<tr>
<th>Middle</th>
<th>Rural</th>
<th>Private</th>
<th>Jr. Hi</th>
<th>Urban</th>
<th>Public</th>
<th>Sr. Hi</th>
<th>Suburban</th>
<th>Parochial</th>
</tr>
</thead>
</table>

2. Location of School

<table>
<thead>
<tr>
<th>South</th>
<th>Southwest</th>
<th>West</th>
<th>Northeast</th>
<th>Other:</th>
</tr>
</thead>
</table>

3. Enrollment of School

<table>
<thead>
<tr>
<th>under 50</th>
<th>50-100</th>
<th>100-300</th>
<th>300-500</th>
<th>500-1000</th>
<th>over 1000</th>
</tr>
</thead>
</table>

4. Class Size:

<table>
<thead>
<tr>
<th>under 10</th>
<th>10-20</th>
<th>20-30</th>
<th>over 30</th>
</tr>
</thead>
</table>

**AS A TEACHER**

|----------------|-----------|-----------|-----------|-----------|

1. Type of School:

<table>
<thead>
<tr>
<th>Middle</th>
<th>Rural</th>
<th>Private</th>
<th>Jr. Hi</th>
<th>Urban</th>
<th>Public</th>
<th>Sr. Hi</th>
<th>Suburban</th>
<th>Parochial</th>
</tr>
</thead>
</table>

2. Location of School

<table>
<thead>
<tr>
<th>South</th>
<th>Southwest</th>
<th>West</th>
<th>Northeast</th>
<th>Other:</th>
</tr>
</thead>
</table>

3. Enrollment of School

<table>
<thead>
<tr>
<th>under 50</th>
<th>50-100</th>
<th>100-300</th>
<th>300-500</th>
<th>500-1000</th>
<th>over 1000</th>
</tr>
</thead>
</table>

4. Class Size:

<table>
<thead>
<tr>
<th>under 10</th>
<th>10-20</th>
<th>20-30</th>
<th>over 30</th>
</tr>
</thead>
</table>
5. Student Population:
   __all white __all black
   __ racially mixed __other:
   ______________________
   ______________________

6. Course of Study:
   (Check one or more)
   __General Math __Algebra
   __Geometry __Trigonometry
   __Remedial Math __Other:
   ______________________
   ______________________

7. Teaching Materials Used:
   (Check one or more)
   __Textbook __Workbook
   __Chalkboard __Overhead projector
   __Other: or visual aids
   ______________________
   ______________________

8. Style of Teaching:
   (Check one or more)
   __Lecture __Demonstration
   __Activities/games
   __Board drill __Other:
   ______________________
   ______________________

9. Room/Equipment:
   (Check one or more)
   __Well lighted __Poorly lighted
   __Fixed arm chairs
   __Movable tables/chairs
   __Posters/bulletin boards
   __Other:__________________
   ______________________

10. Assignments:
    (Check one or more)
    __Oral drill __Short written
    __Long written __Homework
    __Projects __Other:________
    ______________________
    ______________________
11. Discipline in Classroom:  
(Check one or more)

- Formal
- Permissive
- Fair and just
- Corporal punishment administered
- Other:

12. Relationship to Mathematics Study:

- Excited
- Indifferent
- Difficult
- Hated
- Other:

13. Relationship to Teacher:

- Respected
- Not respected
- Caring
- Indifferent
- Other:

14. The teacher was:

- Male
- Female
- under 30
- 30-50
- over 50
- Black
- White
- Other

14. First year salary was:

- under $1,000 per year
- $1,000 - $3,000
- $3,000 - $5,000
- over $5,000

C. Please comment on the following topics in the space provided below or on the back of this sheet:

1. Reflections on a typical day in class:

2. Significant changes you have noted in mathematics education:

Thank you for your cooperation.

Letty Lynn Maloney  
P. O. Box 66, Euless, TX 76039
<table>
<thead>
<tr>
<th>Earliest Experience Described</th>
<th>Questionnaire*</th>
<th>Interviews**</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>As A Student</td>
<td>As A Teacher</td>
</tr>
<tr>
<td></td>
<td>As A Student</td>
<td>As A Teacher</td>
</tr>
<tr>
<td>1901 to 1920</td>
<td>13%</td>
<td>8%</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>12%</td>
</tr>
<tr>
<td>1921 to 1940</td>
<td>18%</td>
<td>17%</td>
</tr>
<tr>
<td></td>
<td>25%</td>
<td>22%</td>
</tr>
<tr>
<td>1941 to 1960</td>
<td>40%</td>
<td>23%</td>
</tr>
<tr>
<td></td>
<td>35%</td>
<td>28%</td>
</tr>
<tr>
<td>1961 to 1980</td>
<td>28%</td>
<td>48%</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>38%</td>
</tr>
</tbody>
</table>

* 146 responses (N = 200)

** 40 responses (N = 40)
# TABLE III

TYPICAL EXPERIENCE OF TEXAS RETIRED AND ACTIVE TEACHERS, SURVEY RESULTS: 1901 to 1920

<table>
<thead>
<tr>
<th>Description of School</th>
<th>As A Student</th>
<th>As A Teacher</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Type of School</strong></td>
<td>rural, public senior high</td>
<td>urban, public junior high</td>
</tr>
<tr>
<td><strong>Location</strong></td>
<td>southwest</td>
<td>southwest</td>
</tr>
<tr>
<td><strong>Enrollment of School</strong></td>
<td>50 - 100</td>
<td>500 - 1000</td>
</tr>
<tr>
<td><strong>Class Size</strong></td>
<td>10 - 20</td>
<td>over 30</td>
</tr>
<tr>
<td><strong>Student Population</strong></td>
<td>all white</td>
<td>all white</td>
</tr>
<tr>
<td><strong>Course of Study</strong></td>
<td>algebra, general math</td>
<td>algebra, general math</td>
</tr>
<tr>
<td><strong>Teaching Materials</strong></td>
<td>textbook, chalkboard</td>
<td>text, chalkboard, workbook</td>
</tr>
<tr>
<td><strong>Teaching Style</strong></td>
<td>drill, lecture demonstration</td>
<td>lecture, drill, demonstration activities</td>
</tr>
<tr>
<td><strong>Room/Equipment</strong></td>
<td>fixed arm desks, good lighting</td>
<td>movable desks, posters</td>
</tr>
<tr>
<td><strong>Assignments</strong></td>
<td>oral drill, homework</td>
<td>short written, homework (oral drill)*</td>
</tr>
</tbody>
</table>
TABLE III
(Continued)

<table>
<thead>
<tr>
<th>Attitudes</th>
<th>As A Student</th>
<th>As A Teacher</th>
</tr>
</thead>
<tbody>
<tr>
<td>** Discipline</td>
<td>formal, corp.</td>
<td>formal, fair</td>
</tr>
<tr>
<td></td>
<td>punishment</td>
<td>and just,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(corporal punishment)</td>
</tr>
<tr>
<td>** Relationship to Math or Teaching</td>
<td>difficult</td>
<td>rewarding,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>successful</td>
</tr>
<tr>
<td>** Relationship to Teacher/Student</td>
<td>respected</td>
<td>comfortable,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>no discipline</td>
</tr>
<tr>
<td></td>
<td></td>
<td>problems</td>
</tr>
<tr>
<td>Misc.</td>
<td>Teacher/Salary</td>
<td>White, male</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30 - 50 yrs.</td>
</tr>
</tbody>
</table>

Characteristics: Female, 65 and over, white, college graduate.

Responses recorded are the mode(s).

* Figures in parentheses are for interview responses where different from the questionnaire responses.

** Questions called for responding to one or more.
<table>
<thead>
<tr>
<th>Descriptions of School</th>
<th>As A Student</th>
<th>As A Teacher</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of School</td>
<td>public, senior high, urban and rural (Junior High)*</td>
<td>public, urban, junior high</td>
</tr>
<tr>
<td>Location</td>
<td>southwest, south</td>
<td>southwest</td>
</tr>
<tr>
<td>Enrollment of School</td>
<td>500 - 1000</td>
<td>over 1000</td>
</tr>
<tr>
<td>Class Size</td>
<td>20 - 30 (over 30)*</td>
<td>over 30</td>
</tr>
<tr>
<td>Student Population</td>
<td>all white</td>
<td>all white, racially mixed</td>
</tr>
<tr>
<td>** Course of Study</td>
<td>general math</td>
<td>general math (algebra)*</td>
</tr>
<tr>
<td>** Teaching Materials</td>
<td>text, chalk</td>
<td>text, chalk, workbook</td>
</tr>
<tr>
<td>** Teaching Style</td>
<td>board drill, lecture</td>
<td>board drill, demonstration, activities</td>
</tr>
<tr>
<td>** Room/Equipment</td>
<td>well lighted, fixed chairs</td>
<td>well lighted, movable desks, posters</td>
</tr>
<tr>
<td>** Assignments</td>
<td>homework, short written, drill</td>
<td>homework, short written, projects</td>
</tr>
</tbody>
</table>
**TABLE IV**

(Continued)

<table>
<thead>
<tr>
<th>Attitudes</th>
<th>As A Student</th>
<th>As A Teacher</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Discipline</strong></td>
<td>formal, fair</td>
<td>fair and just, permissive</td>
</tr>
<tr>
<td><strong>Relationship to Math or Teaching</strong></td>
<td>Excited</td>
<td>Rewarding</td>
</tr>
<tr>
<td><strong>Relationship to Teacher/Student</strong></td>
<td>respected</td>
<td>comfortable</td>
</tr>
<tr>
<td><strong>Misc.</strong></td>
<td><strong>Teacher/Salary</strong></td>
<td><strong>white, male</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>30 - 50 yrs.</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>under $1000/yr.</strong></td>
</tr>
</tbody>
</table>

Characteristics: Male, 45-64, white, master's degree.

Responses recorded are the mode(s).

* Figures in parentheses are for interview responses where different from the questionnaire responses.

** Questions called for responding to one or more.
<table>
<thead>
<tr>
<th>Description of School</th>
<th>As A Student</th>
<th>As A Teacher</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of School</td>
<td>urban, public, senior high, (junior high)*</td>
<td>public, suburban, senior high</td>
</tr>
<tr>
<td>Location</td>
<td>southwest</td>
<td>southwest</td>
</tr>
<tr>
<td>Enrollment of School</td>
<td>300 - 500</td>
<td>over 1000</td>
</tr>
<tr>
<td>Class Size</td>
<td>20 - 30</td>
<td>over 30</td>
</tr>
<tr>
<td>Student Population</td>
<td>all white</td>
<td>racially mixed</td>
</tr>
<tr>
<td>** Course of Study</td>
<td>algebra, geometry, (general math)*</td>
<td>general math, algebra, other</td>
</tr>
<tr>
<td>** Teaching Materials</td>
<td>textbook, chalkboard, workbook,</td>
<td>textbook, workbook, audiovisual, other</td>
</tr>
<tr>
<td>** Teaching Style</td>
<td>lecture, drill, demonstration</td>
<td>lecture, demonstration, activities</td>
</tr>
<tr>
<td>** Room/Equipment</td>
<td>well lighted, fixed chairs, posters</td>
<td>movable desks, posters, other (audiovisual equipment)*</td>
</tr>
<tr>
<td>** Assignments</td>
<td>short written, homework</td>
<td>short written, homework, projects</td>
</tr>
</tbody>
</table>
TABLE V
(Continued)

<table>
<thead>
<tr>
<th>Attitudes</th>
<th>As A Student</th>
<th>As A Teacher</th>
</tr>
</thead>
<tbody>
<tr>
<td>** Discipline</td>
<td>corporal punishment, formal</td>
<td>fair, permissive</td>
</tr>
<tr>
<td>** Relationship to Math or Teaching</td>
<td>excited</td>
<td>rewarding, difficult</td>
</tr>
<tr>
<td>** Relationship to Teacher/Student</td>
<td>respected</td>
<td>comfortable, discipline problems</td>
</tr>
<tr>
<td>Misc.</td>
<td>Teacher/Salary</td>
<td>$3,000-$5,000</td>
</tr>
<tr>
<td></td>
<td>white, female, 30 - 50 yrs.</td>
<td></td>
</tr>
</tbody>
</table>

Characteristics: Female, 25-44, white, master's degree.

Responses recorded are the mode(s).

* Figures in parentheses are for interview responses where different from the questionnaire responses.

** Questions called for responding to one or more.
<table>
<thead>
<tr>
<th>Descriptions of School</th>
<th>As A Student</th>
<th>As A Teacher</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of School</td>
<td>public, suburban, senior high</td>
<td>public, urban, junior high</td>
</tr>
<tr>
<td>Location</td>
<td>southwest</td>
<td>southwest</td>
</tr>
<tr>
<td>Enrollment of School</td>
<td>over 1000</td>
<td>500 - 1000 (over 1000)*</td>
</tr>
<tr>
<td>Class Size</td>
<td>20 - 30 (over 30)*</td>
<td>over 30</td>
</tr>
<tr>
<td>Student Population</td>
<td>all white, racially mixed</td>
<td>racially mixed</td>
</tr>
</tbody>
</table>

** Course of Study

- algebra,
- geometry
- general math
- other: I.A., Cal., E.A.

** Teaching

- textbook,
- chalkboard,
- overhead
- other, textbook, chalkboard, workbooks (packets, computers)*

** Teaching Style

- lecture,
- demonstration,
- board drill
- lecture, demonstration, activities

** Room/Equipment

- well lighted,
- movable,
- posters/bulletin boards
- well lighted, movable, posters/decor


<table>
<thead>
<tr>
<th><strong>Attitudes</strong></th>
<th>As A Student</th>
<th>As A Teacher</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Assignments</strong></td>
<td>homework, short, long written</td>
<td>homework, short written, oral drill, other</td>
</tr>
<tr>
<td><strong>Discipline</strong></td>
<td>formal, fair and just (permissive)*</td>
<td>fair and just, permissive</td>
</tr>
<tr>
<td><strong>Relationship to Math or Teaching</strong></td>
<td>excited</td>
<td>rewarding, difficult</td>
</tr>
<tr>
<td><strong>Relationship to Teacher/Student</strong></td>
<td>respected</td>
<td>comfortable, difficult with discipline</td>
</tr>
<tr>
<td><strong>Misc.</strong></td>
<td>Teacher/Salary</td>
<td>over $5,000/yr.</td>
</tr>
<tr>
<td></td>
<td>female, white, 30 - 50 yrs.</td>
<td></td>
</tr>
</tbody>
</table>

Characteristics: Female, 25-44, white, college graduate.

Responses recorded are the mode(s).

* Figures in parentheses are for interview responses where different from the questionnaire responses.

** Questions called for responding to one or more.
### TABLE VII

**SURVEY RESULTS: DEMOGRAPHIC INFORMATION**

**RETIRED AND ACTIVE TEXAS TEACHERS**

<table>
<thead>
<tr>
<th></th>
<th>Questionnaires*</th>
<th>Interviews**</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sex:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Male</td>
<td>25%</td>
<td>57%</td>
</tr>
<tr>
<td>Female</td>
<td>17%</td>
<td>43%</td>
</tr>
<tr>
<td><strong>Age:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>65 and over</td>
<td>26%</td>
<td>25%</td>
</tr>
<tr>
<td>45 to 64</td>
<td>17%</td>
<td>25%</td>
</tr>
<tr>
<td>25 to 44</td>
<td>44%</td>
<td>25%</td>
</tr>
<tr>
<td>under 25</td>
<td>12%</td>
<td>25%</td>
</tr>
<tr>
<td><strong>Ethnicity:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Black</td>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td>White</td>
<td>87%</td>
<td>77%</td>
</tr>
<tr>
<td>Hispanic</td>
<td>2%</td>
<td>13%</td>
</tr>
<tr>
<td>Other</td>
<td>1%</td>
<td>0%</td>
</tr>
<tr>
<td><strong>Education:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High School</td>
<td>1%</td>
<td>5%</td>
</tr>
<tr>
<td>College Graduate</td>
<td>44%</td>
<td>65%</td>
</tr>
<tr>
<td>Masters Degree</td>
<td>47%</td>
<td>20%</td>
</tr>
<tr>
<td>Doctorate</td>
<td>5%</td>
<td>10%</td>
</tr>
</tbody>
</table>

* 146 responses (N = 200)

** 40 responses (N = 40)
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