DESIGN STUDY OF FLUID ENGINE POWER SYSTEMS

Prepared by
C. H. Baker          R. S. Pauliukonis
T. A. Hunter          A. V. Pradhan

April 12, 1963

Cleveland Pneumatic Tool Company
Cleveland, Ohio
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REPORT ON
THE DESIGN STUDY OF
FLUID ENGINE POWER SYSTEMS

PREPARED BY:
C. H. BAKER  R. S. PAULIUKONIS
T. A. HUNTER  A. V. PRADHAN

April 12, 1963

SUBMITTED BY
THE CLEVELAND PNEUMATIC TOOL COMPANY
A Division of Cleveland Pneumatic Industries, Inc.
Cleveland 5, Ohio

TO
UNITED STATES ATOMIC ENERGY COMMISSION
UNDER CONTRACT AT(30-1)-3022
THE DESIGN STUDY OF FLUID ENGINE POWER SYSTEMS

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Appendix A

Appendix B
LIST OF SYMBOLS

$c_p$  Specific heat at constant pressure
$c_v$  Volumetric specific heat
$H$    Enthalpy
$P$    Pressure
$P_c$  Critical pressure of the fluid
$P_r$  Pressure at reduced condition
$Q$    Heat exchanger duty
$R$    Gas constant
$s$    Entropy
$T$    Temperature
$T_c$  Critical temperature of the fluid
$T_r$  Temperature at reduced condition
$U$    Internal energy
$v$    Specific volume
$W$    Net work from cycle
$W_c$  Work of compression
$W_e$  Work of expansion
$Z$    Compressibility factor
$Z_c$  Compressibility factor at critical pressure and temperature
* Indicator representing ideal gas condition
\( S_c \) Density of fluid critical pressure and temperature
\( S_r \) Density at reduced condition
\( \gamma \) Gas specific heat ratio
\( \eta \) Ideal cycle efficiency
\( \varepsilon_c \) Compressor efficiency
\( \varepsilon_e \) Expander efficiency
\( \varepsilon_m \) Mechanical efficiency
\( \varepsilon_o \) Overall efficiency
THE DESIGN STUDY OF FLUID ENGINE POWER SYSTEMS *

By C. H. Baker, T. A. Hunter, R. S. Pauliukonis and A. V. Pradhan

ABSTRACT

This report presents information generated during a six month feasibility study of an engine which uses a supercritical working fluid as the secondary portion of nuclear powered electric generating systems. Detailed analysis of the PM-2A secondary system 1) showed significant cost, weight, and size reductions to be possible by conversion to the supercritical engine. Similar analysis of the ML-1 secondary system 2) showed a significant efficiency improvement, but this is offset by increased cost, a 10% increase in weight, and essentially the same volume.

Thermodynamic operating principles are given, and details of operation of the supercritical fluid engine are described. Technical results of investigations into the problems of heat transfer at high pressures and background information on supercritical thermodynamics are presented. Preliminary design layouts of two sizes of engines are included by reference and the basic parameters for a wide range of heat exchangers and engines are discussed.

This work was performed under Contract AT(30-1)-3022 for The United States Atomic Energy Commission in accordance with a proposal submitted to the Commission by The Cleveland Pneumatic Tool Company.

1) PM-2A is the Army's pressurized water reactor designed for arctic applications.

2) ML-1 is the Army's mobile gas cooled reactor.

*Work performed under Contract No. AT(30-1)-3022
Chapter 1

SUMMARY

This report presents the record and results of a six month study effort on the part of The Cleveland Pneumatic Tool Company to determine the feasibility of a new energy conversion system when applied to nuclear powered electric generating plants. The work was done under contract AT (30-1)-3022 with the New York Operations Office of the United States Atomic Energy Commission.

An operating temperature range of from 2500 to 1200°F was examined for four types of reactors, including pressurized water, boiling water, nitrogen, and liquid metal cooled. Thermodynamic cycles were calculated for 720 different conditions of engine operation to cover this temperature range. These cycles were then analyzed and the results plotted to show the effects of various operating parameters on engine performance. Heaters and coolers, to work with the engine in the conversion system, were then designed to use the reactors as heat sources and air or water as heat sinks under a variety of conditions. Considerable effort was put forth to establish the values for the heat transfer coefficients to use in the heat exchanger design. A complete and detailed design procedure was worked out, and is included in this report. Detailed comparison of the new fluid engine system with two existing reactor power plants was made, one system being the PM-2A pressurized water system, and the other being the nitrogen cooled ML-1 system. Weight, size, and cost comparisons indicate that the fluid engine system can make substantial weight and size improvements over the PM-2A, but that it is not competitive with the ML-1. Cost comparisons are inconclusive because of lack of adequate available information. Detailed operating analyses were made for the fluid engine, and they indicate that there is very little reflection into the primary system since the response time of the secondary system is much faster than that of the reactor. An effort was undertaken specifically to examine the extent of interaction between the reactor and the fluid engine from the primary system viewpoint.

Because the fluid engine concept is a new one, this report contains a detailed description of the cycle, its operation, the design of the engine and considerable background material on the supercritical thermodynamics involved. Additional information on the transport properties of the selected working fluid, carbon dioxide, and on the method of generalized characteristics is included for reference.
Chapter 2

RESULTS AND CONCLUSIONS

An item-by-item comparison between the existing PM-2A secondary system and the fluid engine system was made by substituting equivalent components and computing the net changes in volume, weight, and cost. Capabilities of both systems were kept identical. No component comparisons were made for the primary system, however, it is believed that significant improvement of primary system specifics is possible by use of the fluid engine concept.

Specific results from the PM-2A system comparison are:

1. Use of the fluid engine would reduce the total system weight by 22.5%, decrease the total system volume by 26.2%, and reduce the total cost by 5.5%. For the particular secondary system components affected by the substitutions, the fluid engine reduces the weight by 69%, decreases the volume by 90%, and saves 20.9% of the cost.

2. Predicted overall thermal efficiency of the fluid engine system is 15.8% compared with 15.6% for the existing system.

3. No additional manpower is required to operate the fluid engine system beyond that already required by the PM-2A.

4. Energy stored in the power conversion system using the fluid engine is reduced by 87% compared to the PM-2A. Total energy storage of the primary and secondary loops is reduced to 58% of the total present energy storage. This reduction in stored energy should permit an appreciable reduction in the size, weight, and cost of the containment system, however, no detail calculations of this effect were made.

5. No additional hazards are introduced into the system by substituting the fluid engine for the existing PM-2A conversion plant.

6. The fluid engine system is appreciably simpler than the existing PM-2A secondary system, eliminates the need for the glycol system and the gear box, and reduces the need for auxiliary equipment.
An item-by-item comparison between the existing ML-1 secondary system and the fluid engine system was made in a manner similar to the PM-2A comparison. Again, no component comparisons were made on the primary system, and overall system capability was kept identical.

Specific results from the ML-1 system comparison are:

1. Use of the fluid engine would increase the system weight by 10.5%, decrease the system volume by 0.9%, and increase the cost by approximately 6.8%.

2. Predicted overall thermal efficiency of the fluid engine system is 19.6% compared to 13.3% for the existing system.

3. Manpower requirements for the fluid engine system are expected to be the same as for the existing system.

4. The overall safety of the ML-1 system is not compromised by a change to the fluid engine concept.

5. The fluid engine system is somewhat more complex than the ML-1 system because the fluid engine uses a binary fluid concept. This requires heat exchangers not needed by the ML-1.

General conclusions from the study are:

1. Existing data on the thermophysical properties of carbon dioxide are subject to doubt in regions near the critical pressure.

2. Little information is available on the performance of reciprocating machinery which operates near the critical region of the working fluid.

3. Computation of more than 700 possible operating cycles showed that the fluid engine concept can be applied to power generation systems over the temperature range from 250°F to 1200°F.

4. Heat exchangers required for the fluid engine system can be fabricated from existing materials using existing techniques, however, further study will probably develop more economical designs than present methods allow.

5. A fluid cycle engine has been designed which can be assembled using only one tool and which requires no field adjustments for proper operation.
6. The fluid engine is suitable for operation in arctic environments using carbon dioxide as the working fluid. For operation in warm or hot regions one of the Freons may be substituted for the carbon dioxide.
Chapter 3

RECOMMENDED DEVELOPMENT PROGRAM

Short range problems in the development of the fluid engine concept lie in three general areas; the properties of the working media, the detail design problems of the heat exchangers, and the evaluation of more sophisticated heat balance cycles. Long range problems are related to the proper methods for matching the unique capabilities of the fluid engine to a nuclear power source in such a manner as to produce the optimum combined system. Both short and long range programs are considered in the following discussion. The schedule and estimated costs of the one year short range program are shown in chart form in Figure 3-1. The overall six year program is shown in Figure 3-2, with estimates for reactor development based on the published figures for the PM-2A system.

The short range program is composed of eight tasks to be performed in one year. Five of these are directed toward improving the knowledge of the physical properties of the working fluid, two relate to heat transfer, and one is intended to assess the possibilities of improving the fluid engine system performance by introducing more complex cycles into the heat balance. The total effort involved in the short range program is 52 man-months of engineering, 17 man-months of technician time, and about $43,000 in equipment expenditure. Of the equipment items, $18,000 is for computer rentals, and $25,000 is for laboratory equipment. Total cost of the suggested short range program is $144,150.

It has become apparent during this study, that the present knowledge of the properties of the chosen working fluid is far from reliable in the supercritical thermodynamic region. The first recommended task is to compute the properties of carbon dioxide in the pressure range from 14.7 psia to 10,000 psia, and in the temperature range from 0°F to 1,200°F. The attack proposed is to utilize the generalized characteristics method to prepare programs for machine computation. Values for specific volumes, enthalpies and entropies would be tabulated. It is expected that this work would require the service of three engineers for a period of six months, plus 240 hours of computer time.

Although a reasonably thorough search of the existing literature has already been made during this feasibility study, it is recommended that the second task be a three man-month literature search for existing
### RECOMMENDED SHORT RANGE DEVELOPMENT PROGRAM

<table>
<thead>
<tr>
<th>Task</th>
<th>Time-months</th>
<th>1</th>
<th>2</th>
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<th>5</th>
<th>6</th>
<th>7</th>
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<th>10</th>
<th>11</th>
<th>12</th>
<th>TOTALS</th>
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<tr>
<td>1. Compute CO₂ thermodynamic tables by generalized characteristics.</td>
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<td>2. Conduct literature search on CO₂ properties.</td>
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<td>3. Correlate computed and reported thermodynamic values for CO₂.</td>
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<td></td>
<td>10,000</td>
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<td>4. Perform Joule-Thompson experiments in regions of poor correlation.</td>
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<td>3</td>
<td>3</td>
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<tr>
<td>5. Tabulate final properties and prepare Mollier chart.</td>
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<tr>
<td>6. Conduct heat transfer tests.</td>
<td>2 2 2 2 2 2 2 2 2 2</td>
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<td>12</td>
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<td>15,000</td>
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<td>7. Conduct heat exchanger design studies.</td>
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<td>8</td>
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<td>8. Perform computer studies on complex heat balance cycles.</td>
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<td>6,000</td>
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<td>Tech.</td>
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## RECOMMENDED LONG RANGE DEVELOPMENT PROGRAM

<table>
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<th>Task</th>
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<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>Cost</th>
</tr>
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<tr>
<td>1. Short range program on CO₂ (See Figure 1).</td>
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<td></td>
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<td>144,150</td>
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<tr>
<td>2. Establish properties and characteristics of other working media, pure substances, and mixtures.</td>
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<td>235,000</td>
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<td>3. Design, construct, test, and evaluate 100 kw fluid engine (By CPT).</td>
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<td>150,000</td>
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<tr>
<td>4. Design, construct, test, and evaluate 2,000 kw fluid engine.</td>
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<td>900,000</td>
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<tr>
<td>5. Conduct design studies on matching of reactors and fluid engines.</td>
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<td>200,000</td>
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<tr>
<td>6. Design, develop, build, and test integrated system.</td>
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<td>250,000</td>
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$1,879,150 + reactor
data about carbon dioxide. This search should include work in the
Russian and European literature in particular, and may be mechanized,
in part, through use of information retrieval services. It is estimated
that this task will require three man-months of engineering effort.

The third task is suggested as the comparison of the computed
data with the existing data to define the areas of discrepancy and un-
certainty which will need to be investigated experimentally. When these
areas are defined, then test requirements, test specifications, and test
apparatus can be prepared. The services of one engineer for three
months and one technician for one month are estimated to be required
for this task. Equipment costs are estimated at $10,000.

With the test program defined, the fourth recommended task is the
performance of Joule-Thompson experiments in the thermodynamic region
of uncertainty, with necessary verification checks at known conditions.
Tabulation, checking against the calculated values, and revisions are in-
cluded here. It is estimated that this effort will require two engineers
and two technicians for a period of three months.

The final task on the basic properties is to tabulate the results of
the computations and experiments and to plot them in the form of a Mollier
diagram. It is estimated that this effort will require one engineer and one
technician for one man-month each.

In conjunction with the research on thermodynamic properties, a
major effort on the heat transfer characteristics of carbon dioxide is also
recommended. This work should cover the temperature range from 60°F
to 1200°F, and pressures up to 2,500 psia. Any further increase in the
pressure is not felt to be necessary, since heat transfer above the critical
point is not so much dependent on the pressure as it is on the temperature.
This activity would require the services of one heat transfer engineer and
one technician for 12 months. In addition, a test facility costing an esti-
mated $15,000 would be needed.

A seventh recommended task is the performance of design studies
on high pressure heat exchangers. In the course of preparing cost esti-
mates for the present study, it became obvious that the heat exchanger
costs were excessive when conventional design and fabrication practices
were followed. It is felt that the costs can be reduced substantially by
application of unconventional techniques which could be developed in four
months by two engineers, or a total of eight man-months of effort.

The final task in the short range program is recommended as the
computation of system performances based on the use of more complex
heat balance cycles than the simple cycle used in the present study. It is
expected that regeneration, re-heating, and intercooling devices can be
applied to the system to improve efficiency, but the amount of improve-
ment and the cost to obtain it cannot be estimated with any degree of accuracy. A study is needed to evaluate the possible cycle improvements, and a four man-month study is suggested, including $6,000 of computer time.

The recommended long range development program includes five tasks which are to be performed within a six year period, in addition to the already described short range program. The objective of the long range program is to provide a power generation system consisting of a reactor and a fluid engine, so well integrated as to exploit the advantages of each portion to the utmost. The effort falls into three categories; research on working fluids; development of fluid engines; and reactor development. Each category has a task which should be started immediately and another follow-on task.

Immediately recommended effort on the working media has already been set forth as the short range program. The follow-on effort is intended to establish the properties and characteristics of pure fluid substances, other than carbon dioxide, when they are used in the critical and supercritical ranges. In addition, it is recommended that mixtures of pure substances be investigated also. This effort would utilize the equipment already assembled for the short range program, but would require two years and would cost approximately $235,000.

Recommended immediate effort on the fluid engine development program involves the design, construction, testing and evaluation of a 100 Kwe version. While listed in Figure 3-2 this item is part of the present program at Cleveland Pneumatic, and will be undertaken by the company with its own funds. The basic design layouts have already been completed. Detailing and development work to determine final configuration, fits and finishes remains to be done. The follow-on effort is directed at the development of a 2,000 Kwe fluid engine, a program taking two years and costing about $900,000.

Immediate recommended effort on reactor development is to conduct design studies on reactors whose characteristics could be matched to the characteristics of the fluid engine. It is anticipated that this effort would lead to a lightweight, relatively low temperature reactor of simple construction, low hazard, and low cost. This effort is expected to take two years and to cost about $200,000. The follow-on effort to this study is the design, development, construction, test, and evaluation of an integrated reactor-engine system. This effort would achieve the objective of the entire program, and would take four years and $250,000 plus the cost of the reactor to complete. Most of this effort would involve the reactor, since a 2,000 Kwe engine would already be proven out by the time the reactor design is completed.
In summary, the entire recommended program to provide a matched reactor and fluid engine system in the 2 Mwe range is estimated to take six years and to cost approximately $879,150 plus the reactor cost.
Chapter 4

BACKGROUND INFORMATION

The idea for an engine operating on a high density fluid cycle was conceived by W. B. Westcott of The Cleveland Pneumatic Tool Company early in 1958. The idea came from some of Westcott's previous work with liquid filled springs, and showed promise as a new and more efficient way of converting heat energy into mechanical work.

For two years, Westcott and Cleveland Pneumatic investigated the scientific principles on which the proposed engine was based, and worked out the thermodynamics involved. The results were then sent to several outside organizations for their review and criticism. None of them were able to find any fundamental reason why the system should not work.

In 1960 a small, one cylinder model of the fluid engine was constructed. It used a silicone oil as the working medium and it proved conclusively that an engine could be built and made to run on a high density fluid cycle. With the basic objective achieved, the next order of business was to improve the performance of the engine. This effort involved still more thermodynamic investigations. They showed that instead of a liquid medium, the engine would work still better if a fluid in the super-critical state were used. A search for suitable fluids showed that there were several in existence, all of them being of the class of materials commonly used for refrigerants. A choice was quickly made to use carbon dioxide as the working fluid since it is cheap, non-toxic, odor free, and generally non-hazardous but still had the required thermodynamic properties.

Conversion of the engine to carbon dioxide made a marked improvement in the performance, and an extensive testing program was begun. Simultaneously, more exacting thermodynamic analyses were made to substantiate the predicted performance estimates. Efforts were also initiated to find suitable applications for the engine system as soon as its basic operating characteristics became known.

It became obvious that one of the most fruitful areas of application would be where there was only a relatively small difference in temperature available for operation. Preliminary calculations indicated that the new engine should work under those conditions, and the test engine verified the predictions.
Another advantage of the fluid engine system was noticed to be its complete flexibility as regards the source of heat. Since it is a closed system, almost any means can be used to generate the temperature differential. It should operate on waste heat, fossil fuel, or any other source with the same characteristics. A third advantage of the fluid engine, exclusive of the heater and cooler, was its very high power density. It appeared that a more compact, and lighter weight engine could be made for use where these advantages were of interest.

Combination of the features of flexibility of heat source, low operating temperature differential, and high power density suggested the application of the system to remotely located power plants operating on nuclear heat sources. A presentation of the general characteristics of the fluid engine system was therefore made to the Army Corps of Engineers. A proposal to investigate the possibilities of applying the fluid engine concepts to the design of the secondary portions of a wide range of nuclear systems was submitted to the Atomic Energy Commission on April 2, 1962. Contract AT(30-1)-3022 was then made between the AEC and Cleveland Pneumatic for a six month feasibility study on the design of fluid engine power systems.
Chapter 5

FLUID ENGINE PRINCIPLES

PART A. GENERAL DESCRIPTION

The engine used in the energy conversion system in this study is a reciprocating configuration, operating on a closed Brayton or Joule cycle which uses a working fluid in the thermodynamic region above its critical temperature and pressure. The engine uses a conventional four stroke cycle, (suction, compression, power, and exhaust), but is arranged in such a manner that each piston delivers a power stroke for each revolution. In addition to the engine, the conversion system requires a source of heat for the heater and heat sink for the cooler in order to provide the difference in temperature which any heat engine must have in order to operate.

Several different possible cylinder arrangements are possible for such an engine, in-line multi-cylinder, radial, pancake, and Vee being the most common forms. In this study, however, the engine layout uses pairs of free pistons which move back and forth in a direction parallel to the axis of rotation of the engine. Such an engine is called a barrel engine, and its use permits a very compact arrangement with simple valving and very high power output per cubic inch of piston displacement. Layouts for two different sizes of barrel type engines have been submitted to the Commission for record purposes, as noted on the Table of Contents. Basic calculations for the heaters and coolers required for several operating cycles are also presented in the appendix.

The proposed new engine incorporates a wealth of existing data on materials, lubrication, valving and controls, and includes the principal advantages offered by the new working fluid at supercritical pressures. These are the short stroke and high power density, which result in small size, lightweight engine. This engine incorporates all of the design improvements which have been obtained from operation of the existing laboratory model. As seen from the enclosed drawing, Figure 5A-1, the proposed engine is a double piston, two stroke, cam operated, barrel-type machine.
ROTATING DRUM
INSULATION

DRIVE SHAFT

CAM & BALLS

STATIONARY REACTION MEMBER (CAMs)

FLOW PASSAGES

HEADER

FROM HEATER
 AT HIGH PRESSURE & TEMPERATURE FOR EXPANSION
TO COOLER
LOW PRESSURE EXHAUST
TO HEATER
HIGH PRESSURE EXHAUST
FROM COOLER
AT LOW PRESSURE FOR COMPRESSION

MAIN BEARING

STATIONARY PINGLE VALVE

OPPOSING PISTONS

GEARS

OIL PUMP

OIL PUMP

FROM OIL PUMP
COOLED FROM HIGH PRESSURE HEATER
TEMPERATURE FOR EXPANSION TO HEATER

FROM AT LOW PRESSURE FOR COMPRESSION TO HIGH PRESSURE SCALING. 5 IN. = 1.0 IN.

FIGURE 5A-1
500 KW
BARREL TYPE FLUID ENGINE
SCALE .5 IN. = 1.0 IN.
In this engine the cam acts as a reaction member. The cylinder assembly revolves together with the integrally mounted flanged shaft for direct coupling with the generator shaft. This revolving cylindrical drum acts as a flywheel. The pistons follow around the cam. Each piston goes through four complete cycles for the PM-2A system during each engine revolution. Consequently, the 1800 RPM shaft speed results in 7200 power strokes per minute, respectively, for each piston. The ball connection between the pistons and the cam is floating in oil, which is provided by a high pressure lubricating pump. The valving arrangement is provided through a multi-passage pintle valve with high pressure fluids channeled in sequence and separated by individual passages so as to prevent leakage. The seals are of high precision type. The critical events of admission, cut-off, release and compression, are controlled by a fixed angle of advance and the optimum port opening, predetermined by tests in the model engine.

The final engine would then be provided with optimum valving and the power control would be simply performed by the by-pass valve alone. This simplifies engine mechanical design and assures greater reliability as the engine will require no complicated actuating mechanism such as eccentrics, pivot cams, flywheels, controlled change-of-throw, etc. Consequently, simplicity in design, in fabrication, and in engine use, is assured.

In operation, the working fluid is introduced to the center of the cylinders through passages in the pintle valve. The high pressure, high temperature fluid expands, pushing the opposing pistons apart on the power stroke while the remote end of each piston admits cold fluid for compression. On the return stroke, this cold fluid is compressed isentropically while the expansion stroke is exhausting.

The internal forces set up during the process of expansion-compression are always in balance, cancelling each other through the cylinder arrangement axially as well as radially. Due to the piston configuration, the force acting on the ball is always undirectional, eliminating all noise normally associated with the free floating balls.

One distinctive feature of the internal engine arrangement is the use of a piston having two different diameters. Both ends of the piston are in contact with the working fluid during the cycle, so that two of the four strokes on the cycle are always taking place simultaneously. This tandem coupling permits a ball and cam arrangement to be used to translate the reciprocating motion into rotary motion without the use of a crankshaft.
By using the working fluid in the supercritical condition, all phase changes may be avoided. Heat input is through a heater rather than a boiler, and heat is rejected by a cooler rather than a condenser. Because the compression stroke is applied to a relatively dense, liquid-like material, the pumping work is relatively low and the pumping efficiency is relatively high. The expansion stroke takes place with the fluid in the gas-like condition. The heating and the cooling equipment is designed to take advantage of the higher than usual heat transfer values inherent in the high density fluid medium used, in this case carbon dioxide, although several other fluids are usable.

Because the engine works on a closed cycle, the mass of fluid pumped on each compression stroke must exactly equal the mass of fluid admitted for each power stroke, otherwise the laws of conservation of mass would be violated. This condition helps to determine the two different diameters for the pistons in the engine. Once the diameters are chosen, the pumping ratio is fixed, but it must be noted that in the design stages, there is complete freedom of choice of operating cycle within rather wide limits. By choosing a cycle which requires the fluid to stay in the liquid-like condition, the power density of the engine can be made very high. By choosing a cycle where the fluid is more like a gas, particularly in the expansion stroke, the thermal efficiency can be made very good. It is believed that this design feature is possessed by no other type of engine.

Heat energy for conversion by the engine can come from almost any source. In this study, the heat is taken from a nuclear reactor, and fed to the engine through the heat exchanger. This implies that the system is a binary fluid system, and it therefore possesses all of the limitations which the use of more than one fluid places on any energy conversion scheme.
PART B. IDEAL OPERATING CYCLE

The thermodynamic cycle on which the engine operates is the Brayton cycle, shown in its ideal form in Fig. 5B-1. Because the engine has a tandem arrangement of power and pumping strokes, the ideal cycle will be treated in segments, each applicable to the compressor and expansion portions. Figures 5B-2 through 5B-5 illustrate the ideal behavior for each stroke.

Fig. 5B-1a
Ideal Brayton Cycles

Fig. 5B-1b

Fig. 5B-2
Ideal Compressor

Fig. 5B-3
Ideal Expander

Fig. 5B-4
Actual Compressor
(with clearance volume and pressure losses)

Fig. 5B-5
Actual Expander
(with clearance volume and pressure losses)
APPLICATION OF CYCLE TO ENGINE (compressor and expander)

Ideal Operation. Numbers at crank-pin positions correspond to points on p-v diagrams below.

Fig. 5B-6

Engine Cycle Operation
SUCTION, COMPRESSION AND DELIVERY

To visualize the operation of the engine through a complete cycle, a unit mass of the working fluid will be traced through the system. First, consider the piston to be at the extreme right end of its stroke, ready to move toward the left. On the right side of the piston, valve A opens, as the piston moves to the left, a unit mass of the working fluid will be sucked into the cylinder, filling the volume swept out behind the moving piston. This is the suction stroke, and it takes place at the relatively constant, low pressure and low temperature prevailing at the outlet of the cooler. The suction stroke ends when the piston has traveled to the extreme left end of the cylinder, at which time valve A closes.

From the left end of the cylinder, the piston next begins to travel toward the right again, compressing the fluid trapped behind the rod side of the piston. This is the compression part of the stroke. After the pressure on the fluid has been raised to the level prevailing in the heater, valve B opens and the warm, high pressure fluid charge moves into the heater, as the piston continues to move to the right. This is the delivery part of the stroke. It must be observed that compression and delivery operations take place in 180° of crank rotation. At the end of the delivery, when the piston is at the extreme right end again, valve B closes and the right side of the piston is ready to suck another charge of cool, low pressure fluid for the next stroke.

Referring to the ideal compressor diagram, 1'' - 1 is the suction stroke, 1-2 is the compression part and 2-2'' is the delivery stroke. 1''-1 is constant pressure suction; in the compression 1-2, pressure is increased from $P_{1''}$ to $P_{2''}$ but the volume is decreased from $V_1$ to $V_2$; 2-2'' is constant pressure delivery.

HEATING

The unit charge under observation is delivered from the compression side of the cylinder into the heater. It is to be noted that the internal volume of the heater is such that many unit charges of fluid are in transit at any one time, making the heater a semi-infinite source of fluid during the cycle, and smoothing out the pulsations developed by the positive displacement pumping strokes. As the warm, high pressure fluid moves through the heater, it takes on heat and expands by virtue of its thermal expansion properties, but it remains at essentially a constant pressure all during its travel through the heater, i.e. referring to the diagrams, the volume of unit mass of fluid changes from $V_2$ to $V_3$, but the pressure remains constant $P_2'' = P_3''$. It finally comes out of the heater to valve C as hot, high pressure, energy laden fluid.
INTAKE, EXPANSION AND EXHAUST

Valve C opens, admitting hot, high pressure fluid into the left end of the cylinder when the piston is at the extreme left end of its travel. This begins the intake part of the power stroke. The valve stays open for only a limited time, admitting one unit mass of the working fluid, after which time it closes. Following closure of valve C, the fluid expands during the rest of the piston travel toward the extreme right end of its stroke.

At the end of the power stroke, with the piston at the right end of the cylinder, valve D opens, and the fluid is forced out into the cooler by the travel of the piston toward the left. This is the exhaust stroke, and it is completed when the piston is again at the extreme left end of its travel.

Referring to the ideal expander diagram, 3'' - 3 is the intake part of expansion stroke, 3-4 is the expansion and 4-4'' is the exhaust stroke. During 3'' - 3 pressure is constant, in the expansion process 3-4, volume increases from \( \nu_3 \) to \( \nu_4 \) but the pressure drops down from \( P_3'' \) to \( P_4'' \) and 4 - 4'' is constant pressure exhaust.

COOLING

From valve C, the warm, low pressure fluid travels through the cooler, where it loses heat and shrinks in volume due to thermal contraction. This process takes place at essentially constant pressure. The cooler like the heater, contains many unit charges of fluid and therefore acts as another semi-infinite fluid source. Referring to the previous quantities, the volume of unit mass of fluid changes from \( \nu_4 \) to \( \nu_1 \) in passing through the cooler, whereas pressure remains constant \( P_4'' = P_1'' \). After the cool, low pressure fluid returns through the cooler to valve A, it is ready to begin its cycle all over again.

COMMENTS

It must be noted that cycle is a closed one. The mass of fluid taken in during the compressor suction stroke and the intake stroke of the expander is the same. The mass of fluid at state 1, when suction is complete is exactly equal to the mass of fluid at state 4, when expansion is complete. The length of stroke for compressor and expander is the same, hence for mass conservation, the available cross-sectional areas on compression and expansion sides of the piston must be in the ratio of \( \nu_1 \) to \( \nu_4 \).
As one side of the piston is used as compressor and the other as expander, there is one complete cycle each of compressor and expander for one revolution of the engine. This means that there is one power stroke per revolution. For the engine shown in Figure 5B-6, the compression and power strokes both take place while the piston is moving from the left to the right end of the cylinder. The suction and exhaust strokes also occur simultaneously, and take place while the piston travels from right to left. For the barrel engine shown in Figure 5A-1, the power and suction strokes occur while the pistons are moving away from each other; the exhaust and compression strokes take place while the pistons move toward each other.

BASIC THERMODYNAMICS

In the following paragraphs a review of the thermodynamics of the Brayton cycle is presented, primarily for understanding and completeness. Nothing not already in many text books is presented. The law of conservation of energy requires that the new work obtained from the engine per cycle be equal to the ideal work obtained in the expander per cycle minus the ideal work required by the compressor per cycle. In the following discussion work output is considered positive. Thus when the work is done on the piston by the fluid it is a gain, because it produces power. When the work is done by the piston on the fluid, it is a loss, because it utilizes the part of the available work from the crankshaft.

Ideal work of the expander is given in pieces as shown by Figure 5B-7,

\[ W_e = \text{work done in intake process (P}_3\nu - 3) \]
\[ + \text{work done in expansion process (} \int P \, d\nu \) \]
\[ - \text{work required for exhaust (P}_4\nu - 4) \]
\[ = \text{area } 3'' - 3 - 4'' = \int \nu \, dP \]

Fig. 5B-7
Expander Work
Ideal work required by the compressor is given by Fig. 5B-8 as

\[ W_e = \text{work done in suction stroke (P}_1''\nu 1) \]
\[ - \text{work required for compression process } \int P d\nu \]
\[ - \text{work required for delivery process (P}_2''\nu 2) \]
\[ = -\text{area 1''-1-2-2''} = -\int^2 \nu\,dP \]

Fig. 5B-8
Compressor Work

Work of compression is negative because work is done by the piston on the fluid and hence represents a loss. The net work from the cycle is the difference between the expansion work and the compression work.

\[
W = \int^3 \nu\,dP - \int^2 \nu\,dP
\]
\[
= \text{area 1-2-3-4 of Fig. 5B-1} \]

because \(P_1 = P_4\) and \(P_2 = P_3\).

It must be noted that the work required for delivery of the fluid from the compressor is automatically taken care of when the work done in the cycle is considered as the area 1-2-3-4.

In the ideal case, both compression and expansion are assumed to take place isentropically (reversible, adiabatic process). Then, as

\[
dH = \nu\,dT + T\,d\nu
\]

where \(H\) = enthalpy, \(S\) = entropy, \(\nu\) = volume, \(P\) = pressure

and \(T\) = temperature.

Since \(dS = 0\) for isentropic or constant entropy process,

\[
dH = \nu\,dP
\]

since the \(T\,d\nu\) term vanishes. The integrals become

\[
\int^2 \nu\,dP = \int^3 dH = H_2 - H_1 \text{ and } \int^3 \nu\,dP = \int^3 dH = H_3 - H_4.
\]
Hence the work required by the compressor is \( H_2 - H_1 \) per unit mass and the work done by the expander is \( H_3 - H_4 \) per unit mass. The net work of the cycle is then the difference between the charges in enthalpies.

\[
W = (H_3 - H_4) - (H_2 - H_1).
\]

Because the Brayton cycle deals with constant pressure heating and cooling it is advantageous to write the work equation in terms of heat added or rejected at \( Q \). The work equation can be written in slightly different way by rearranging the terms to read

\[
W = (H_3 - H_2) - (H_4 - H_1)
\]

Now \( dH = Td\mu + \nu dP \) as before, but also \( Td\mu = dQ \). Substituting, \( dH = dQ + \nu dP \) where \( Q \) is heat added or rejected for constant pressure processes, state 2 to state 3 in the heater and state 4 to state 1 in the cooler, \( dP = 0 \), hence the \( \nu dP \) term vanishes and \( dH = dQ \). Using the integral form, \( \int_4^3 dH = \int_4^3 dQ \). Therefore, \( H_3 - H_2 = Q_3 - 2 \). Similarly, \( H_4 - H_1 = Q_4 - 1 \).

The net work done in the cycle may be written in terms of heat added or rejected as

\[
W = (Q_3 - 2 - Q_4 - 1)
\]

= Heated added in heater - Heat rejected in the cooler.

**ACTUAL OPERATION OF COMPRESSOR AND EXPANDER**

In the actual engine, there are always certain deviations from the ideal cycle conditions just discussed. The following section examines the cycles shown in Fig. 5B-4 and 5B-5 for the effects of pressure losses, clearances, heat transfer and friction etc. on the ideal cycles and their efficiencies.

To begin with, the efficiency of the compressor is defined as the ratio of the ideal isentropic work to the actual work required by the compressor. Efficiency of expander is defined as the ratio of the actual work obtained from the expander to the isentropic work.

Pressure losses are incurred in port openings and by fluid friction. As the compressor must pump to the final required pressure, any loss of pressure in the compressor involves additional pumping work and shows as a decrease in efficiency. Pressure loss in the expander reduces the
available work and hence results in a further decrease in efficiency. By proper design these pressure losses may be kept to a minimum. Clearances in compressors and expanders are there because of unswept volume. The compressor clearance generally does not adversely affect the work required by the compressor, but only decreases its volumetric efficiency. This efficiency is defined as the quotient of the volume of the inlet fluid at the inlet conditions divided by the swept volume. Although the compression work is done on all the mass of fluid contained in the cylinder (both swept and unswept), the unswept mass expands again on the return stroke of the piston, thus doing an equivalent amount of useful work.

Clearance in the expander cylinder may result in loss of work because hot fluid coming in from the heater may lose part of its enthalpy by mixing with the cooler fluid in the clearance space. If the valve timings are so adjusted that, on the return stroke of the expander piston, the fluid in the clearance space is compressed to nearly the pressure of the incoming fluid, some loss of work caused by loss of pressure due to mixing of high pressure incoming fluid and low pressure fluid in the clearance space can be avoided.

Heat transfer takes place because the walls of the cylinder are not the same temperature as the fluid inside the cylinder. In the compressor if the walls are cooler than the fluid, it would decrease the work required for compression, however, if they are hotter than the fluid, it would result in requiring more work for compression. In the expander, as is usually the case, the walls are cooler than the incoming fluid, which results in loss of enthalpy and hence of available work. Of course, heat transfer does depend on many other factors, duration of contact between the fluid and the walls being quite important. Running the engine at higher speeds reduces this time and hence the heat transfer. Jacketing of the expander and cooling of the compressor of course reduce the losses because of heat transfer.

Internal friction results because of turbulence and fluid flow. In the compressor it results in a requirement for more work, but this shows up as reduction of heater duty. In the expander it results in a loss of work and also increases the cooler duty. Internal friction can be considerably reduced by proper design, thereby reducing the turbulence.

There are some other factors which also finally result in the reduction of the net work, such as incomplete compression and expansion. Proper adjustment of valve closings and openings would reduce these to a minimum.
All the effects which are considered above show up as distortion of ideal $P - \nu$ diagrams for compressor and expander, and reduction of their efficiencies. Figures 5B-4 and 5B-5 show the actual $P - \nu$ diagrams as compared to the theoretical. A detailed discussion of the efficiencies assumed in the fluid engine calculations is given in the next chapter.
PART C. THERMODYNAMICS

As an introduction to the thermodynamic evaluation of the cycle performance data, the procedure by which these performance data have been calculated will be described and the possible errors examined and evaluated.

BASIC THEORY

Since the working fluid in the system is far from a perfect gas, the usual ideal gas equations cannot successfully be applied to calculate cycle performance. Therefore, the method of generalized characteristics based on the Principle of Corresponding States, is used in these performance calculations. The method of generalized characteristics may be described briefly as follows:

The equation of state may be written as
\[ PV = Z RT \text{ per mol} \]  
(1)
where \( Z \) is termed the compressibility factor and is itself a function of pressure, temperature and the nature of the fluid. The ideal gas law may then be considered as representing a special case in which the compressibility factor is equal to unity. It is advantageous to relate the conditions of the fluid to its critical values by introducing the concept of a "reduced" condition. The reduced conditions of temperature, pressure, volume and density are defined by the ratios
\[ \tau_r = \frac{T}{T_c}, \quad \rho_r = \frac{P}{P_c}, \quad \gamma_r = \frac{V}{V_c} \]
and \( \rho_f = \frac{\rho}{\rho_e} \), where properties at the critical point are denoted by the subscript \( e \).

Pure substances are said to be in corresponding states when they exist at the same reduced conditions of temperature and pressure. The Principle of Corresponding States requires that all pure fluids manifest the same compressibility factors when measured at the same reduced conditions of pressure and temperature. According to this principle, the actual thermodynamic properties of different pure fluids would manifest the same departure from the properties of these substances in their ideal gaseous states if they are examined at the same reduced conditions of temperature and pressure.

Thus
\[ Z = F(\rho_r, \tau_r) \]  
(2)
when the equations of state are written in reduced quantities, such equations are called generalized equations of state. This equation implies that the critical compressibility factor, \( Z_c \), should be the same for all substances. Actually, the values range from .20 to .30. To account
for this deviation, the critical compressibility $Z_c$ is used as a third parameter. With this modification, the compressibility factor becomes

$$Z = f(\rho, \tau, Z_c)$$  (3)

Several other third parameters have been tried (Ref. 1), such as the reduced dipole moment, bond length, and the normal heat of vaporization, with less favorable results. With $Z_c$ as the third parameter, Lydersen-Greenkorn-Hougen (Ref. 2) assembled the $\rho\nu\tau$ data of 82 different compounds for both gaseous and liquid states, and constructed tables for compressibility factors and reduced densities. These were arranged in four groups according to values of $Z_c$ which were based on 8000 values for various liquids and gases. The average deviation between experimental and tabulated values of $Z$ varies from zero at low pressure to 2.5% in the critical region, and to 2% at high pressures.

In this report, compressibility factors are used directly for calculating pressure, density or temperature of a gas between any two of these values, $Z$ is related to density by the ratio

$$Z = \rho \rho T \ f_c$$  (4)

For liquids, density is not greatly influenced by pressure, hence $Z$ is a measure of the ratio $\rho / \rho_T$ rather than of $f$. For this reason, values of $f_T$ rather than $Z$ are used in calculating liquid densities. The average error in estimating liquid densities is about 1%.

The generalized tables used in the calculations in this report were calculated from existing $Z$ tables and contain derived properties, such as departures from ideal gas behavior of enthalpy, entropy, and internal energy. The enthalpy of a fluid relative to that of its ideal gas when both are at the same temperature was obtained by integrating

$$\left( \frac{dH}{d\rho} \right)_T = \nu - T \left( \frac{d\nu}{dT} \right)_P$$  (5)

between the limits of zero and the given pressure. However, at zero pressure all fluids are in their ideal gaseous states and the enthalpy $H$ becomes independent of pressure. Using the * to indicate ideal gas conditions,

$$H^* = H = \int_0^\rho \left[ T \left( \frac{d\nu}{dT} \right)_P - \nu \right] \ d\rho$$  (6)

which after a few steps reduces to

$$\left( \frac{H^* - H}{T_e} \right)_T = \left[ R_T \int_0^\rho \left( \frac{dZ}{dT} \right)_P \frac{d\rho}{R_T} \right] \frac{dP}{R_T} - T$$  (7)

The entropy values are obtained similarly, by integrating

$$\Delta S = \left( \frac{dS}{dT} \right)_P \ d\rho + \frac{C_P}{T} \ dT$$  (8)

at constant temperature from pressure $P$ to zero

$$\left( S_P - S^*_o \right)_T = -\left[ \int_0^\rho \left( \frac{d\nu}{dT} \right)_P \ d\rho \right]$$  (9)
which reduces to
\[
\left( S_p - S_r \right)_T = - \int_0^\infty \frac{1 - \frac{Z}{P}}{R_T} \frac{dP}{P} + R_T \int_0^\infty \frac{d\theta}{T_r} \frac{dP}{P_r} \quad (10)
\]

In the same way, from the equation
\[
H = U + Z \bar{R} T
\]
the departure of internal energy from ideal-gas behavior is expressed as
\[
\frac{U^* - U}{\bar{R}} = \frac{H^* - H}{\bar{R}} - (1 - Z) \bar{R} T_r
\]
and finally heat capacity departures are given by
\[
C_P - C_P^* = \frac{d(H^* - H)}{dT_r}
\]
The values are calculated from the enthalpy deviation tables given in Ref. 2.

The tables of departures of these various properties from ideal gas behavior have been calculated in Ref. 3 over a wide range of pressures and temperatures, and are within 1% error. These tables have been used for the cycle calculations in this report.

POINT-TO-POINT PROCEDURE

It is proposed to operate the engine on a Brayton cycle as shown in Figure 5C-1. Line 1-2 is isentropic compression, 2-3 is constant pressure heating, 3-4 is isentropic expansion, 4-5 is constant volume exhaust, and 5-1 is constant pressure cooling. In all the simple cycles calculated, points 4 and 5 coincide. That is, expansion is carried to the base pressure.

Figure 5C-1
P-V Diagram for Brayton Cycle

For carbon dioxide, the molecular weight, the critical point data and the heat capacity of the gas at zero pressure are known very accurately and provide good initial points from which to begin the integrations. Then given the initial cycle pressure and temperature, \( P_i \), and \( T_i \), \( V_i \) can be found by either \( \frac{Z}{\bar{R}} \) or \( \int \) tables. (Tables 49 or 48, Ref. 3). With the maximum pressure, \( P_\alpha \), given, and with
\[
S_2 - S_1 = (S_i^* - S_i) - (S_2^* - S_2) + \left( S_2^* - S_i^* \right) = 0
\]
\( T_\alpha \) can be calculated by the trial and error method. Then knowing \( \xi \) and \( T_\alpha \), \( V_\alpha \) can be calculated.
Entropy deviations are obtained from the tables mentioned above (Table 52, Ref. 3) and the relationship

\[ S_2^{\#} - S_1^{\#} = \int_{T_1}^{T_2} c_p^* \, dT \]  

(15)

With the maximum temperature \( T_3 \) given, and \( P_3 = P_2 \), \( \gamma_3 \) can be calculated directly. Knowing the final pressure, \( P_f \), then \( T_f \) can be calculated by the trial and error method and then \( V_f \) calculated. Thus the calculations for \( P \), \( V \), \( T \) data for all points can be determined.

**DATA PRINT-OUT**

In the printed performance data sheets, \( P \) is recorded in psi, \( V \) in cft/lb x 10^3, \( f \) in lb/cft, \( T \) in °R and °F. Listed also are the reduced pressures, volumes and temperatures. Several other items are listed at the bottom of the printed pages. Heat added is calculated from the relationship

\[ H_{add} = \varphi_{3-2} = H_f - H_i = \left( H_f^{\#} - H_i \right) - \left( H_i^{\#} - H_i \right) + \left( H_i^{\#} - H_i^{\#} \right) \]  

(16)

Enthalpy deviations are taken from Table 50, Ref. 3, and the expression

\[ H_1 - H_2 = \int_{T_1}^{T_2} c_p \, dT \]  

(17)

Heat rejected is simply the sum of the heat rejected at constant volume plus the heat rejected at constant pressure, or

\[ H_{rej} = \left( \varphi_{4-5} \right) + \left( \varphi_{5-1} \right) \cdot \left( u_f - u_i \right) + \left( H_f - H_i \right) \]  

(18)

\[ = \left( u_f^{\#} - u_i \right) - \left( u_i^{\#} - u_i \right) + \left( u_i^{\#} - u_i \right) + \left( H_f - H_i \right) \]  

(19)

\[ = \left( u_f^{\#} - u_i \right) - \left( u_i^{\#} - u_i \right) + \int_{T_1}^{T_2} c_v^* \, dT + \left( H_f^{\#} - H_i \right) - \left( H_i^{\#} - H_i \right) + \int_{T_1}^{T_2} c_v^* \, dT \]  

(20)

Internal energy deviations are obtained from Table 53, Ref. 3, and

\[ C_v^* = C_v^* - R \]  

(21)

If points 4 and 5 coincide, then \( \varphi_{4-5} = 0 \). The work done is the difference between the heat added and the heat rejected, and the efficiency is the ratio of the work done to the heat added, and is expressed in percent. Horsepower per in^3 displacement at 1000 rpm is calculated from the equation

\[ HP/\text{in}^3 \text{ displ. } @ 1000 \text{ rpm} = \frac{\text{Work done}}{V_s \times 1728 \times \frac{1}{1} \times \frac{778}{33000}} \]  

(22)

Work required for compression is the difference between \( H_i \) and \( H_f \), while the enthalpy drop in expansion is the difference between \( H_i \) and \( H_f \). The work done in expansion is given by

\[ W_{exp} = H_3 - H_f \times \left( \frac{P_f - P_i}{T_f - T_f} \right) V_f \times \frac{1}{1} \text{ Btu/lb} \]  

and is the difference between \( H_3 \) and \( H_f \) if points 4 and 5 coincide. The
indicated mean effective pressure of the cycle is
work done in cycle \( \times 778 \frac{1}{144} \) psi, while the indicated mean effective
pressure in the compressor is given by \( \frac{(H_2 - H_1)}{V_f} \times 778 \times \frac{1}{144} \) psi
and the indicated mean effective pressure in the expander is
\( \frac{(H_3 - H_4)}{V_f} \times 778 \times \frac{1}{144} \) psi.

ERROR ANALYSIS

Thus, it is seen that for the reported cycle calculations, the basic
data are the molecular weight, critical point, and heat capacity at zero
pressure as functions of the temperature of the fluid. The compressibility,
reduced density, entropy, enthalpy and internal energy deviations were
obtained from Tables in Ref. 3. As all the computations were done with
proven routines on either IBM 7070 or IBM 650, there is slight possibility
of any arithmetic error. All the input data to the computer were rechecked
and verified, and there is negligible possibility of error in instructing the
machine. The other input data of operating pressures and temperatures
were done by permutations and combinations and were generated on IBM
650.

As stated above, the average deviation of experimental values of \( \theta \)
from tabulated values is within 2.5% - perhaps it can be slightly more.
These errors will appear also in the entropy, enthalpy and internal energy
deviation tables. It is claimed by their authors that these tables are with-
in 1% error. Therefore, it can be safely assumed that all the calculations
are within 2.5% error. Of course, there is the possibility of some pring-
ing error in the original tables, and a stray point should indicate this when
the results are plotted.
PART D. FLUID PROPERTIES

It can be noted from the previous thermodynamic analysis of the cycle by the method of generalized characteristics that the only special information required for the cycle calculations is the critical point data and the variation of the specific heat of the ideal gas with changes in temperature of the fluid used in the cycle. Appendix H gives this data for a number of fluids.

The choice of any working fluid depends upon the specific requirements which the engine must satisfy. For instance, the choice of hot and cold sink temperatures is one of the important governing factors. In the previous section, it was shown that if the compression was accomplished in the liquid region and the expansion in the gaseous region, full advantage can then be taken of low work of liquid compression and high work of gaseous expansion. It is important that the critical point of the chosen fluid fall between the maximum and the minimum temperatures of the cycle so that this advantage may be realized.

A number of cycles have been run with a variety of fluids and a general feel for the performance with different fluids has been attained; however, it has become apparent that every specific application has to be studied individually. The exact influence of the variation of specific heat with temperature on the cycle performance is not known as yet.

In the section on "Cycle Selection," a cycle with Freon 12 will be discussed and compared with a CO₂ cycle as an illustrative example to show that efficiency and cycle performance for a given hot and cold sink temperatures can be improved by choosing another suitable fluid.

It is also possible to run the system on two different fluids without any mechanical modifications if the temperatures of the hot and cold sinks are suitably varied and still obtain good performance under various conditions. To illustrate this point, suppose a system was designed for operation in the arctic region with CO₂ as the fluid. The same system could be run in the desert at comparable efficiency by choosing a different working fluid which has higher critical temperature. Freon 12 would be a logical choice for desert use since its properties show that the compression can be accomplished in the liquid region.

The other fluid properties which influence the system are the transport properties, in that, they determine the heat transfer characteristics of the fluid and thus the sizes of the heat exchangers. The transport properties of all the fluids used in this study are included in Appendix A.
PART E. DESIGN FEATURES

Once the operating cycle has been chosen for the engine, the bores and strokes are chosen to match the volume changes required by the cycle. One of the features of the fluid engine system is that it can be sized to match a very wide variety of operating conditions, temperature differences, and loads without doing violence to the basic design. In addition to this, the cycle analysis in the next chapter will show that the engine will operate in "off design" conditions without appreciable degradation of efficiency over a considerable range of temperatures, and temperature differences.

A further degree of design flexibility is allowed by the possibility of choosing any one of a number of operating fluids to suit the design requirements. It is even possible to convert a given engine from one application to another merely by changing the working fluid, other factors remaining unchanged. Generally it has been found desirable to have the critical temperature of the working fluid lie between the maximum and minimum operating temperatures. Fortunately there are materials presently available which cover an extensive range of temperatures and pressures, most of the better ones being compounds commonly used as refrigerants. While the present study has been limited primarily to carbon dioxide for the working fluid, this must not be taken as a limitation.

Because of the high pressures used, the engine has a very high power density as contrasted to modern high compression internal combustion engines. This feature permits very compact arrangements to be used. In addition, the barrel configuration shown in Fig. 5A-1 results in a very efficient use of space and savings in weight.

The high pressures also yield above normal heat transfer rates, thereby permitting the heat exchangers to be made smaller than those the conventional systems require for the same duty.

Like all heat engines, the supercritical fluid engine requires a difference in temperature between the hot side and the cold side to make it operate. However, with the freedom of choice of available working fluids, the fluid engine can use heat from almost any source. It can be used in simple or compounded cycles, as a topping or bottoming cycle in conjunction with other systems, or even as a refrigerator under certain circumstances. Because it operates on a closed system, there is no contamination problem, and because the low pressure side is always above atmospheric pressure, there is no problem of ingesting non-condensible gases into the engine as happens with conventional condensing steam turbine systems.
In the configuration proposed in this study, the engine is completely balanced both statically and dynamically at all times. It operates at the design speed of the generator to which it is coupled, and therefore needs no gear boxes. The valve design is a simple, one piece pintle type and is designed to minimize internal leakages. The loadings on the piston are such that the pressure on each side are in balance during most of each stroke, thus minimizing leakage past the piston and wear on the piston rings.

The use of the double diameter piston provides a continuously compressive force of varying magnitude on the balls which drive the cam surfaces. This condition eliminates any tendency for the balls to float or clatter against their races. The entire engine can be assembled with only one tool, a plain wrench, and can be easily taken apart for inspection or servicing, as required, by one person.
Chapter 6

CYCLE CALCULATION ANALYSIS AND SELECTION

PART A. CYCLE CALCULATION

THERMODYNAMIC REGION OF OPERATION OF CYCLES

Seven hundred twenty cycles with all possible combinations of the following temperature and pressure parameters were run as set forth in the Cleveland Pneumatic Proposal P62-7B. Complete copies of all of the cycles have already been supplied to the Commission, and are not included in this report.

\[
\begin{align*}
T_{\text{max}} \, (^{\circ}\text{F}) & \quad 250, \, 350, \, 450, \, 550, \, 600, \, 700, \, 900, \, 1000, \, 1200 \\
T_{\text{min}} \, (^{\circ}\text{F}) & \quad 40, \, 80, \, 120, \, 150 \\
P_{\text{max}} \, (\text{psia}) & \quad 2000, \, 3500, \, 5000, \, 10000 \\
P_{\text{min}} \, (\text{psia}) & \quad 500, \, 800, \, 1100, \, 1500, \, 1800
\end{align*}
\]

Graph "A," shows the pressure-volume diagram for CO2 and shows the saturated liquid line, the saturated vapor line, the critical point, the critical, (88.9°F), 40°F, 80°F, 120°F, and 150°F isotherms. Above the critical point, although the critical isotherm is supposed to separate the liquid and gas regions, this distinction is purely arbitrary. The fluid is always in a single phase, and there is no change of phase while crossing the critical isotherm.

It is intended that the cycle should take advantage of the low compressibility of liquid-like fluid in the process of compression, (thus resulting in low work of compression) and high compressibility of gas-like fluid in the process of expansion, (resulting in high work of expansion). The diagram shows the steep slope of the isotherms in the liquid region and the gradual slope in the gaseous region and gives an indication of the slopes of the isentropes in these respective regions. To illustrate the point more clearly, two complete cycles are shown on Graph A, operating between the same two pressure limits of 800 psi and 2000 psi, and both having a maximum temperature of 450°F. Cycle 147 has a minimum temperature of 40°F and the compression takes place in the liquid region, the compression isentrope (point 1 to 2) being very steep; cycle 225 has a
GRAPH A PRESSURE-VOLUME DIAGRAM FOR CO₂

DOTTED LINES SHOW 2 CYCLES 12545 °F 12565 WITH
P_MIN = 800 PSI  P_MAX = 2000 PSI
T_MAX = 450 °F
AND T_MIN = 40 °F - 150 °F
minimum temperature of 150°F, the compression isentrope (point 1' to 2') lies completely in the gaseous region and has a gradual slope. The work of compression in the first cycle is only 3.728 BTU/lb, whereas the work of compression in the second cycle is 22.766 BTU/lb. As both the cycles follow the same expansion isentrope, the work of expansion is equal in both cases, being 31.000 BTU/lb.

The maximum pressure of all the cycles run is always above the critical pressure of 1071 psia, whereas the minimum pressure may be either above or below the critical. The expansion process in every cycle is always in the gaseous region but compression takes place in either the liquid or gaseous region, depending upon the location of point 1. For example, cycles with point 1 at 40°F and at 800, 1100, 1500, or 1800 psi, or at 80°F and 1100, 1500, or 1800 psi, have their compression isentropes in the liquid region. All the cycles with an initial temperature, T1 of 120°F and 150°F, have their compression isentropes in the gaseous regions. Fluid may pass through the vapor region only in the cooler during the path from point 5 back to point 1.

As expected, the cycle working with completely incompressible fluid has a very low ideal efficiency, whereas with a perfect gas, the ideal efficiency is determined only the pressure ratio and increases with increase in this ratio. As the cycle is shifted from left to right on the P-v diagram, that is, from liquid to gaseous region, the ideal efficiency slowly increases. Similarly, HP/in³ displacement decreases because the fluid density is reduced in going from left to right on the diagram. These two effects are shown by cycles 147 and 255 on Graph A. Cycle 147 operates in the high density fluid range during compression and has an ideal efficiency of 14.613% and HP/in³ displacement = 1.785. Cycle 255, operating more in the gaseous region, has an ideal efficiency of 17.788% and HP/in³ displacement = .539 respectively. There is an apparent trade-off between power density and ideal efficiency.

The accuracy of the results can be verified to some extent by comparing the value of the work done in the cycle obtained by enthalpy considerations and by calculation of the area of the P-v diagram. For cycle 255 the work done by enthalpy consideration is 8.235 BTU = 6406.8 ft lb, whereas the area of the P-v diagram is 11.6 square inches. With proper scales, this area gives 6681.6 ft lb, or a deviation of 3.5%. For cycle 147, the corresponding figures are 27.272 BTU = 21217.6 ft lb, and 36.7 square inches = 21139.2 ft lb, giving a deviation of only 0.37%. Hence, it may be seen that the method of generalized characteristics gives fairly accurate results, especially in the region of interest.
This portion of the report deals with some of the theoretical aspects of the Brayton thermodynamic cycle as an aid to defining the best operating conditions for the engine. With carbon dioxide as the working fluid, the effects of variation in temperature up to 1200°F and pressures up to 10,000 psi are calculated, plotted, and analysed. Discussion also includes the effects of ideal thermodynamic efficiency, power density, and net work of the various cycles. Analysis of actual rather than ideal cycles is presented in Part C, which follows.

In accordance with the description included in the thermodynamic evaluation of the cycle, this engine is intended to operate on a closed Brayton cycle with CO₂ as the working fluid. The ideal cycle is shown in Fig. 6B-1 and consists of an isentropic compression from point 1 to point 2, heating at constant pressure from point 2 to point 3, isentropic expansion from point 3 to point 4, and cooling at constant pressure from point 4 back to point 1.

The thermodynamic performance of the Brayton cycle, operating with a real fluid such as CO₂, in the region of our interest depends upon the four independent parameters:

- Minimum pressure, $P_1$
- Maximum pressure, $P_2$
- Minimum temperature, $T_1$
- Maximum temperature, $T_3$

In this analysis, study is made of the influences of these four parameters on the ideal efficiency, power density, work of expansion and work of compression of the cycle.

Efficiency and power density measured as horsepower per cubic inch displacement at 1000 rpm are essentially the measure of the performance of any cycle. As the efficiency of the cycle determines the specific fuel consumption, (which in turn influences the economics of the system), the power density affects the design by determining the volume and weight of the engine. The thermodynamic work done in a cycle is the difference between the work obtained in the expansion process and the work required for compression process. It must be noted that the work of expansion or compression does include suction and delivery work. Knowledge of the separate work of expansion and compression is essential for the study of the influence of expansion and compression efficiencies on the design of the engine.
For convenience, some of the terms which will be used in the discussion to follow have been defined below.

- \( T_1 \) = the minimum temperature of the cycle.
- \( T_2 \) = the temperature after isentropic compression.
- \( T_3 \) = the maximum temperature of the cycle.
- \( T_4 \) = the temperature after isentropic expansion.
- \( P_1 \) = the minimum pressure of the cycle.
- \( P_2 \) = the maximum pressure of the cycle.
- \( V_4 \) = the specific volume after isentropic expansion.
- \( C_p \) = the specific heat at constant pressure.
- \( C_v \) = the specific heat at constant volume.
- \( \gamma \) = the specific heat ratio \( \gamma = \frac{C_p}{C_v} = \frac{C_p}{C_p - R} \)
- \( R \) = the specific gas constant.
- \( \eta \) = the thermal efficiency of the ideal cycle, defined as work done divided by heat added.

It will be noted that in some of the cycles, the temperature after compression, \( T_2 \), exceeds the maximum temperature, \( T_3 \), of the cycle. This condition automatically gives negative values for heat added, and the figures for thermal efficiency are meaningless since the cycle denotes a refrigeration process. These refrigeration cycles are omitted from the engine cycle analysis but the data sheets are retained for completeness.

The thermal efficiency effects will be discussed first. To understand the behavior of thermal efficiency when operating with real fluids, it is convenient to assume that the fluid behaves first as a perfect gas, and then to introduce certain deviations from perfect gas behavior. Using a perfect gas with constant specific heats, the thermal efficiency of a Brayton cycle may be derived as follows:
Heat added in the cycle \( H_3 - H_2 = C_p (T_3 - T_2) \) where \( H \) = enthalpy \( \text{(1)} \)

Heat rejected in the cycle \( H_4 - H_1 = C_p (T_4 - T_1) \) \( \text{(2)} \)

Net work done in the cycle = Heat added - Heat rejected
\( = C_p (T_3 - T_2) - C_p (T_4 - T_1) \) \( \text{(3)} \)

Ideal efficiency of the cycle \( \eta = \frac{\text{net work done}}{ \text{heat added}} \)
\( = \frac{C_p (T_3 - T_2) - C_p (T_4 - T_1)}{C_p (T_3 - T_2)} \)
\( = 1 - \frac{T_4 - T_1}{T_3 - T_2} \) \( \text{(4)} \)

For isentropic compression and expansion of an ideal gas, we have,
\( T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{\frac{\gamma - 1}{\gamma}} \) \( \text{(5a)} \)
\( T_3 = T_4 \left( \frac{P_3}{P_4} \right)^{\frac{\gamma - 1}{\gamma}} \) \( \text{(5b)} \)

Since the compression and expansion ratios are the same in this cycle \( \frac{P_2}{P_1} = \frac{P_3}{P_4} \) \( \text{(6)} \)

Inserting these relationships into the equation for the thermal efficiency shows that
\( \eta = 1 - \left( \frac{P_1}{P_2} \right)^{\frac{\gamma - 1}{\gamma}} \) \( \text{(7)} \)

The ideal thermal efficiency of the Brayton cycle for a perfect gas with constant specific heats is therefore a function only of the maximum and minimum pressures of the cycle and is not dependent upon the temperatures. For a given pressure ratio, ideal thermal efficiency will be a straight horizontal line when plotted against the maximum temperature, \( T_3 \), of the cycle. This result is shown on Graph 1 for various pressure ratios.

(Note: Because the number of graphs discussed in this and the following sections is so large, for convenience they are collected in one place as Appendix B.)

As one of the deviations from perfect gas behavior, it is known that the specific heats of any gas are not constant, but increase as the temperature is increased. It follows that \( \gamma \), which is equal to
\( \frac{C_p}{C_v} = \frac{C_p}{C_p - R} = 1 + \frac{R}{C_p - R} \)
decreases as the temperature of the gas is increased. The values of \( \gamma \) for CO\(_2\) are plotted as a function of temperature in Graph 2, and indicate a variation of about 10% between 450\(^{\circ}\)R and 1650\(^{\circ}\)R.
If CO₂ is treated like a gas with temperature dependent specific heats, the ideal efficiency of any cycle with a given pressure ratio will be reduced slightly as the average temperature \( T_1 + T_3 \) of the cycle is increased. This effect is shown plotted in Graph 3 as ideal efficiency versus maximum temperature, \( T_3 \), of the cycle for various pressure ratios, the minimum temperature, \( T_1 \), being constant. In the following analysis of the various cycle ideal efficiencies, this result will be used in the explanation of the observed phenomena.

An ideal engine, operating between a given maximum temperature \( T_3 \) and a given minimum temperature \( T_1 \), will have the maximum possible thermal efficiency if it operates on any thermodynamically reversible cycle such as a Carnot cycle. In this case, the thermal efficiency would be \( \frac{T_3}{T_3 - T_1} \). The thermal efficiency of any Brayton cycle is below the thermal efficiency of the corresponding Carnot cycle by a factor called the relative efficiency. This factor is inherent because of the irreversibilities which are introduced in the constant pressure heating and cooling processes when operating between finite limits. As \( T_3 - T_2 \) and/ or \( T_4 - T_1 \) increases, this inherent irreversibility is likewise increased. Additional irreversibilities are introduced in the compression and expansion processes in an actual engine cycle. These and their effects are discussed in the next section on actual cycle analysis.

If the fluid in the engine were to act like a perfect gas, it would be sufficient to plot the various performance parameters such as thermal efficiency, power density, etc., simply as functions of pressure and temperature ratios. However, in the case of real fluids, especially when working in the critical region where large deviations from ideal behavior are to be expected, a parametric analysis is required. The performance parameters must be plotted as functions of maximum and minimum temperatures independently, and not just as their ratios. The behavior of an engine operating in the sub-critical thermodynamic region may be very different from its behavior if it were operating in the super-critical region, even though the temperature and pressure ratios are the same for both regions.

The next (49) graphs show the results of the parametric analysis of the ideal engine cycle. Operating variables of temperature and pressure are plotted against ideal efficiency and power density. One of the first effects to be examined is the influence on the cycle thermal efficiency of the variation in the maximum temperature, \( T_3 \), and the maximum pressure, \( P_2 \), for a variety of constant values of minimum temperature, \( T_1 \), and minimum pressure, \( P_1 \). The results are shown in Graphs 4, 5, 6, 7, 8 and 9.
On Graph 4, the Carnot efficiency is plotted for comparison purposes, for a given $T_1$ of 40°F and $P_1$ of 500 psi. With these given initial conditions, the temperature, $T_2$, after ideal compression to pressure, $P_2$, of 2000, 3500, 5000, and 10,000 psi are 173.2, 241.2, 288.6 and 367.6°F respectively. These are the limiting temperature values, since any cycles with $T_3$'s below these corresponding $T_2$'s represent refrigeration. If the corresponding $T_3$'s are equal to these $T_2$'s, then the efficiencies of the Brayton cycles are equal to the efficiencies of the corresponding Carnot cycles. As an example, for $T_1 = 40°F$, $P_1 = 500$ psi, and $P_2 = 2000$ psi, $T_2$ ideally will be 173.2°F. If $T_3$ is actually less than 173.2°F, the engine cycle is not possible. If $T_3 = 173.2°F$, the cycle ideal efficiency will theoretically equal the corresponding Carnot efficiency of 21.0%. If $T_3$ is actually greater than 173.2°F, then an engine cycle is possible and the ideal efficiency will be less than 21.0%. From Graph 4, comparison with the Carnot efficiency shows that the Brayton efficiency for real fluids increases initially with temperature as does the Carnot, but after a maximum is reached, the effect of irreversibility becomes more pronounced, the ideal efficiency drops down and the fluid behaves more like a perfect gas with temperature-dependent specific heats.

An obvious discrepancy occurs on Graph 4 at the point on the ideal efficiency curve for $P_2 = 10,000$ psi where $T_3 = 450°F$. This point should be above the corresponding point for $P_2 = 5000$ psi. For cycles operating at the same values of $T_1$, $T_3$, and $P_1$, the irreversibility of the cycle is reduced as $P_2$ is increased. Thus, the efficiency should be greater for higher values of $P_2$, for given $P_1$, $T_1$, and $T_3$. The explanation for this discrepancy is a slight error in the values given in the table of published data. This is indicative of the need for improved information in this area.

There are also some inconsistent points in Graph 7 at $T_3 = 350°F$ and $T_3 = 450°F$ on the curve for $P_2 = 10,000$ psi, and in Graph 9 at $T_3 = 250°F$ on the $P_2 = 10,000$ psi curve. In Graph 7, the value of efficiency at $T_3 = 350°F$ and $P_2 = 10,000$ psi, is obviously wrong, because it is greater than the Carnot efficiency for an engine operating between the given $T_1$ and $T_3$. This discrepancy is caused by the necessity for working with the differences between closely spaced small numbers. Any small errors in the absolute values of these numbers are magnified into large errors during calculations of the efficiency. As an example, for $T_3 = 350°F$ and $P_2 = 10,000$ psi on Graph 7 the heat added is only 2.839 Btu/lb. The heat rejected is 1.791 Btu/lb. leaving the work done as 1.048 Btu/lb. The ideal efficiency is therefore 1.048/2.839 or 37.0% which far exceeds the Carnot value of 24.7%. Note that an error of only
0.01 Btu/lb. on both of the enthalpy values, can cause the ideal efficiency value to vary anywhere from 36.3% to 37.5%, or a 1.2% variation per 0.01 Btu/lb. The discrepancy at $T_3 = 450^\circ F$ on this same curve, and at the point mentioned in Graph 9, is because the efficiency values are lower than those at the corresponding lower pressures. Once again, this has to be attributed to errors in the tables used.

Graphs 4, 5, 6, 7, 8, and 9 shows the effects of pressure and temperature variations on the ideal cycle efficiency. Note that the initial starting points ($P_1$ and $T_1$) are different for each of them. In Graphs 4, 5, 6, 7, and 8, the compression isentrope lies completely in the so-called gaseous regions, whereas for Graph 9, it lies in the liquid region. There is no distinct difference in the behavior of ideal efficiency in these graphs. As expected, the ideal efficiency increase with increase in the maximum pressure for given initial conditions. It also shows a slight initial increase, reaches a maximum, and then slowly decreases as the maximum temperature is increased.

In the next six plots, 10, 11, 12, 13, 14, and 15, the effects of the minimum pressure, $P_1$, on the ideal cycle efficiency is investigated. Ideal efficiency is plotted for constant values of minimum temperature, $T_1$, and maximum pressure, $P_2$, with minimum pressure $P_1$, as the variable parameter. The constant parameters are changed from one graph to another. In general, the behavior of ideal efficiency with maximum temperature, $T_3$, follows the same trends as the previous set of graphs and for the same reasons discussed above.

One new point is brought out in these graphs, however. As the minimum pressure of the cycle, $P_1$, is increased, with $T_1$, $T_3$, and $P_2$ held constant, the ideal efficiency of the cycle drops. This is because, as $P_1$ is increased, $T_3 - T_2$ and $T_4 - T_1$ become greater, thus increasing the irreversibility in heat addition and heat rejection processes, and thus lowering the ideal efficiency of the Brayton cycle. This is also true in the case of a perfect gas with constant or temperature-dependent specific heats, wherein as the pressure ratio is reduced, the ideal efficiency of the Brayton cycle also decreases.

Once again, inconsistent points are observed in Graph 13 at $T_3 = 350^\circ F$ on $P_1 = 1800$ psi and at $T_3 = 700^\circ F$ on $P_1 = 500$ psi curve. The first point has an ideal efficiency greater than Carnot, and the second has the efficiency lower than that at the corresponding temperature on the curve for $P_1 = 800$ psi. Once again these stray points are attributed to small errors in the tables used.
In the next six plots, the effect of the minimum temperature, \( T_1 \), on the ideal cycle efficiency is investigated. On Graphs 16, 17, 18, 19, 20 and 21, the ideal efficiency is plotted against the maximum temperature of the engine cycle, \( T_3 \), with constant values of minimum pressure, \( P_1 \), and maximum pressure, \( P_2 \), with minimum temperature, \( T_1 \), as the behavior of ideal efficiency as a function of \( T_3 \) follows the same general trends as discussed before. The grouping of all of the efficiency curves, as seen especially in Graph 16, shows that the fluid behavior is akin to a perfect gas, that is, ideal efficiency is dependent only on pressure ratio and is independent of temperature. A stray point is noticeable on Graph 19, at \( T_3 = 350^\circ\text{F} \) on \( T_1 =-150^\circ\text{F} \) curve, and can be attributed, as before, to small errors in the tables.

One definite conclusion may be drawn from Graphs 4 thru 21, and that is the insensitivity of the ideal cycle efficiency to a wide range of maximum temperature variation.

On Graphs 22, 23, and 24, ideal efficiency is plotted against the minimum temperature \( T_1 \) of the engine cycle. All three graphs show the insensitivity of the ideal engine cycle efficiency to variations in minimum temperature. There is a slight initial increase in efficiency as the minimum temperature, \( T_1 \), is increased, until the maximum is reached. This is followed by a very slight efficiency drop with further increase of \( T_1 \). In explaining this action, it should be noted that the Carnot efficiency is reduced as \( T_1 \) is increased. But as \( T_1 \) is increased, \( T_4 - T_1 \) is reduced, and the irreversibility caused by the constant pressure rejection of heat is also reduced. Initially, the effect of reduction in irreversibility predominates and the ideal efficiency increases slightly, whereas later the effect of reduction in Carnot efficiency predominates and the ideal efficiency drops off. Once again, it must be noted that these increases and decreases in ideal efficiency are quite small.

On Graphs 25, 26, and 27, ideal efficiency is plotted against the minimum pressure, \( P_1 \), of the engine cycle. It is obvious from all three graphs that as \( P_1 \) increases, efficiency drops. This behavior comes about because of the decrease in the pressure ratio, \( P_2 / P_1 \). A unique behavior is shown on Graph 26. For \( P_1 \) equal to 500 psi, the values of ideal efficiency for all, for values of \( T_1 = 40^\circ\text{F}, 80^\circ\text{F}, 120^\circ\text{F} \) and \( 150^\circ\text{F} \) are almost identical. All of these points are in the gaseous region. For \( P_1 \) of 800 psi, the ideal efficiency values for \( T_1 = 80^\circ\text{F}, 120^\circ\text{F} \), where the fluid is in the gaseous region, are close together, whereas the efficiency point for \( T_1 = 40^\circ\text{F} \), where the fluid is in the liquid state, is separated. It may be concluded that if the initial point \((P_1, T_1)\) is the gaseous state, that is, if the entire cycle is in the gaseous state, then there
is no variation at all of ideal efficiency with respect to $T_1$. However, if the initial point ($P_1, T_1$) is in the liquid region, thereby causing part of the cycle to be in the liquid region, then ideal efficiency does vary slightly with $T_1$.

On Graphs 28, 29 and 30, ideal efficiency is plotted against the maximum pressure, $P_2$, of the engine cycle. As the maximum pressure increases, the ideal efficiency increases in accordance with effect of increased pressure ratio, and this is to be seen from all three graphs. The influence of variation of $P_1$ is to be seen in Graph 28, and is in accordance with previous discussion. In Graph 29, note the close spacing of efficiency curves, again indicating the relative insensitivity of ideal engine efficiency to minimum temperature, $T_1$. It must also be noted here that all the cycles plotted in Graph 29 lie completely in the gaseous region. The influence of maximum temperature, $T_3$, on ideal cycle efficiency, as seen from Graph 30, is nothing different from the previous observations.

Graphs 31, 32, 33, 34, and 35, are envelope plots, showing the highest attainable ideal efficiency versus the maximum temperature, $T_3$, for given values of maximum pressure, $P_2$, with minimum temperature, $T_1$, as the variable parameter. Graphs 31, 32, 33, and 34 are respectively for $P_2$'s of 2000, 3500, 5000 and 10,000 psi and, finally Graph 35 is a plot of the maximum ideal efficiency versus maximum temperature, irrespective of the maximum pressure of the cycle. As expected, Graph 35 corresponds closely with Graph 34, because it includes the highest pressure ratios.

The second part of the investigation relates to the effects of cycle variation on power density. The power density is defined as the net work done divided by $V_4$, the volume after the isentropic expansion. It has the units of horsepower per cubic inch of displacement, and is calculated at an assumed speed of 1000 power strokes per minute. In Graphs 4, 5, 6, 7, 8 and 9, the power density is plotted against the maximum temperature, $T_3$, with maximum pressure, $P_3$, as the variable parameter, and constant values of minimum temperature and minimum pressure.

The power density value tends to zero when the work done in the cycle tends toward zero. This happens when $T_2$ approaches $T_3$. For any given initial conditions, $P_1$ and $T_1$, the temperature $T_2$ always increases as $P_2$ is increased because of the use of the isentropic compression temperature. Hence the power densities for the higher pressures reduce in value very rapidly and approach zero at the higher values. The crossing of the curves over one another is because of this effect and is noticeable in Graphs 4 through 8.
With the other three parameters fixed on any one curve, the effect of increasing $T_3$ is to increase both the area of the $P-\nu$ diagram, i.e., work done in the cycle, as well as $V_4$. If the rate of increase of the area of the $P-\nu$ diagram is greater than the rate of increase of $V_4$, then power density increases directly with $T_3$. This effect may be observed in Graphs 4 through 8, where the entire cycles are in the gaseous region. Power density increases initially with $T_3$ and then tends to be asymptotic or even drops off slightly. Cycles shown in Graph 9 have their compression isentropes lying in the liquid region, and there it seems power density is invariant over a wide range of maximum temperature, $T_3$. Once the initial criss-crossing of curves for different maximum pressures is over, the curves uniformly show higher power density at higher values of $P_2$. For given $P_1$, $T_1$, and $T_3$, the $P-\nu$ diagram becomes taller and slimmer as $P_2$ is increased, but $V_4$, and hence width of the base, decreases. It is observed in Graphs 4, 5, and 8, that at some higher maximum temperatures, the curves tend to cross each other again. This indicates that at final temperatures higher than about $1200^\circ F$, the power density decreases as $P_2$ is increased. Although $V_4$ is reduced, the area of the $P-\nu$ diagram is reduced at an even greater rate, and hence the power density shows a net reduction.

In Graphs 10, 11, 12, 13, 14, and 15, the power density is plotted against the maximum temperature, for given values of minimum temperature and maximum pressure. The minimum pressure, $P_1$, is the variable parameter. As the minimum pressure is increased, keeping the other parameters fixed, the area of the $P-\nu$ diagram is decreased because the total height of the diagram decreases, but at the same time $V_4$ decreases. Depending upon which effect predominates, the power density may increase or decrease. In general, power density increases as the minimum pressure is increased, although there are exceptions.

Graphs 16, 17, 18, 19, and 21 are plots of power density versus maximum temperature for given values of minimum pressure and maximum pressure. The minimum temperature is the variable parameter. As minimum temperature is increased, power density drops off. This is expected because the area of $P-\nu$ diagram (and hence work) is decreased, while $V_4$ is kept constant. Graphs 22, 23, and 24 are plots of power density versus minimum temperature. All graphs show that power density decreases as the minimum temperature is increased for reasons discussed above.

Graphs 25, 26, and 27 represent plots of power density versus minimum pressure. For low initial pressures, the power density rises with $P_1$, and then either levels off or drops off slowly. The point at which this rise stops is at $P_1 = 1100$ psi in all three graphs. It is significant
that the critical pressure of CO\textsubscript{2} is also about 1100 psi. In Graphs 25 and 27, the transition pressure corresponds to the change from liquid compression to gaseous compression in the cycle. In Graph 26, at T\textsubscript{1} of 40\textdegree F, transition from liquid to gaseous compression takes place at 800 psi, and at T\textsubscript{1} of 80\textdegree F transition is at 1100 psi. Both these pressures seem to be points at which the specific curves change their slopes drastically. At T\textsubscript{1} of 120\textdegree and 150\textdegree F, compression is entirely in the gaseous state, hence the curves have smooth slopes. It may be concluded that as P\textsubscript{1} is increased, power density increases rapidly if the compression isentrope of the cycle is in the liquid region, but it remains almost constant or decreases slowly if the compression isentrope lies in the gaseous region. One obvious reason for this is that both isentropes and isotherms are much steeper in the liquid region than in the gaseous region. In Graphs 28, 29, and 30, the power density is plotted against the maximum pressure of the cycle. These graphs confirm the influence of maximum pressure on power density in accordance with the previous discussion.

Graphs 36, 37, 38, and 39 are envelope plots of the highest obtainable power density versus maximum temperature for maximum pressure of cycles of 2,000, 3,500, 5,000 and 10,000 psi, respectively. Minimum temperature is the variable parameter. All of the curves seem to be well behaved, except for some on Graph 39. There are stray points at 350\textdegree F on all the three curves for T\textsubscript{1} of 400, 800, and 120\textdegree F. These stray points are because of small errors in the tables as discussed previously.

To design an actual engine, it is not sufficient to know only the theoretical net work of the cycle. It is also necessary to know the theoretical works of compression and expansion independently. The adiabatic efficiencies of compression and expansion both have an effect on the net work. To make the point clear, consider two cycles, each with a net work of 10 Btu/lbs. In the first cycle the work of compression is assumed at 20 Btu/lb. and expansion work is 30 Btu/lb., whereas in the second cycle compression work is 10 Btu/lb. and expansion work is 20. Now if the adiabatic efficiencies of compression and expansion are both 90\% for both the cycles, then for the first cycle we have a net work of \((30 \times 0.9 - 20) = 4.78\) Btu/lbs., and for the second cycle, we have net work of \((20 \times 0.9 - \frac{10}{9}) = 6.89\) Btu/lbs. This example shows that even though net work and adiabatic efficiencies were the same for both cycles, the cycle with lower ideal works of compression and expansion, is better overall. The effects of various parameters on works of compression and expansion will now be examined.
In Graphs 40 and 41, the ideal work of compression, \( W_c \), and of ideal expansion, \( W_e \), are plotted against the maximum temperature, \( T_3 \), of the cycle, with maximum pressure, \( P_2 \), as the variable parameter. In Graph 40, \( T_1 = 80^\circ F \) and \( P_1 = 800 \text{ psi} \), whereas in Graph 41, \( T_1 = 80^\circ F \) and \( P_1 = 1100 \text{ psi} \). As expected, the ideal work of compression is independent of maximum temperature, whereas ideal work of expansion increases with \( T_3 \). As \( P_2 \) is increased, the ideal work of compression and expansion both increase because of the increased pressure ratio.

Comparing Graphs 40 and 41, it is apparent that the curves for both \( W_c \) and \( W_e \) for given pressures are lower in Graph 41 than the corresponding curves in Graph 40. This difference is because the compression shown on Graph 41 takes place in the liquid region, whereas for Graph 40 it is in the gaseous region. Work of compression is given by \( W_c = \int P \, dv \). In the liquid region, the change of volume, \( dv \), is small, and so is the work of compression. Note also that in Graph 40, \( W_c \) for a pressure ratio of \( 2000/800 = 2.5 \), is 18 Btu/lb., whereas in Graph 41, the corresponding figure for \( 2750/1100 = 2.5 \text{ psi} \), is about 6.5 Btu/lb. This verifies the statement made earlier in the report that it is not adequate to plot values for ideal work just as functions of pressure and temperature ratios, but it is necessary to specify the exact pressure and temperature conditions. Reduction in the work of expansion from Graph 40 to 41 can be explained as due to reduction in pressure ratio.

Graphs 42 and 43 are plots of \( W_c \) and \( W_e \) against maximum temperature. Graph 42 shows that when \( P_1 \) is increased, both \( W_c \) and \( W_e \) drop off. Once again, negligible ideal work of compression in the liquid region is noticeable in this graph. Graph 43 shows that ideal work of expansion is independent of \( T_1 \) since the curves for all four minimum temperatures are coincident.

The ideal works of compression and expansion are plotted against the minimum temperature in Graphs 44, 45, and 46, against the minimum pressure in Graphs 47, 48, and 49, and against the maximum pressure in Graphs 50, 51, and 52. In each one of the graphs in the sets of three, one of the remaining three parameters is varied and the others are held fixed. These graphs substantiate the previous discussion of the influences of minimum temperature, minimum pressure and the maximum pressure on the ideal work of compression and expansion, and are included here for completeness.

In summarizing the work performed in this analysis, it may be concluded that:

1. Ideal efficiency is invariant over a wide range of minimum temperatures, \( T_1 \), of the cycle.
2. Ideal efficiency increases initially with maximum temperature, \( T_3 \), of the cycle, then remains substantially constant, with a gradual decrease at the higher temperature.

3. Ideal efficiency increases with pressure ratio, that is, efficiency increases, either by decreasing \( P_{\text{min}} \) or increasing \( P_{\text{max}} \).

4. Power density decreases as the minimum temperature is increased.

5. Power density increases with the minimum pressure if the compression isentrope lies in the liquid region, but it almost invariant over a wide range of \( P_1 \), if the entire cycle is in the gaseous region.

6. Power density increases initially with maximum temperature, \( T_3 \), and then tends to be constant or even drops off slightly if the cycle is completely in the gaseous region. If the compression isentrope is in the liquid region, then the power density is invariant with respect to \( T_3 \).

7. Power density increases or decreases with the maximum pressure, depending upon the maximum temperature of the cycle.

8. Ideal work of compression is reduced if the compression is accomplished in the liquid region and hence increases as minimum temperature is increased.

9. Ideal work of compression and expansion increases with the pressure ratio.

10. Ideal work of expansion increases with the maximum temperature, \( T_3 \), of the cycle.
PART C. CHOICE OF EFFICIENCIES FOR SYSTEM ANALYSIS

No previous experimental data are available on the efficiencies of compression and expansion of CO$_2$ in the thermodynamic region in which it is proposed to operate the fluid engine. However, one may expect to get about 85% efficiency in compression of any gas and about 95% efficiency in compression of a liquid. In the expansion of gases and liquids, efficiencies depend considerably on the initial heat transfer as soon as hot fluid comes in contact with cold cylinder walls. In hydraulic motors, where there is negligible heat transfer, these efficiencies are as high as 95%. In the case of steam engines, however, where there is considerable initial condensation, these efficiencies tend to be quite low, often about 50%.

Compression and expansion efficiencies have already been defined in terms of actual work and ideal work for each process. The definitions take into account all the internal losses in the compressor and expander respectively. The internal losses include pressure losses in port openings, loss of pressure and enthalpy because of clearance spaces, change of enthalpy because of heat transfer from the fluid to the walls, loss of pressure and change of enthalpy because of fluid flow, and loss of output work because of incomplete expansion.

Besides the internal losses discussed above, there are two predominant external losses. They are the external mechanical friction of the engine and the pressure losses in the heater, cooler, and associated piping.

Mechanical efficiency is defined as the ratio of the work available at the engine shaft to the net indicated work of the cycle. Mechanical losses consist mainly of friction at the piston rings and the bearings. Internal combustion engines have mechanical efficiencies of the order of 90 to 92%, whereas in well designed steam engines this figure is about 94%. The proposed barrel type of engine has considerably less mechanical friction as compared to a conventional engine because of the short stroke design, hence a figure of 95% or even better is expected. A mechanical efficiency of 95% is assumed for this analysis and is considered by Cleveland Pneumatic to be conservative.

The external loss of pressure in the heater and cooler results in some additional pumping work for the compressor. To illustrate this point, if the expansion takes place from 3980 psi to 840 psi, then the compressor would have to pump from, say 800 psi to 4000 psi, allowing for 20 psi pressure drop in the heater and 40 psi in the cooler. On the IBM 7070 cycle analysis program it was not possible to analyze the cycle.
directly by allowing for pressure losses in heat exchangers and efficiencies of compression and expansion because of difficulties in matching the tables. To allow for pressure losses, two simple cycles are run with slightly different pressure ratios and their outputs combined to allow for the additional work of compression.

Another program was used on the IBM 650 which combines these two cycles and also takes into consideration the efficiencies of compression and expansion. For the PM-2A system analysis this combined output of two cycles is used to account for pressure losses in the heat exchangers and for efficiencies of compression and expansion. For the heat exchanger parametric study, it was not considered economical to run two cycles along with each other to allow for pressure losses in heat exchanger because of the bulk of additional work involved. However, efficiencies of compression and expansion are accounted for.

Considerable discussion was held regarding the choice of the compression and expansion efficiencies. It has already been stated that there are no previous experimental data available. Considering that CO₂ acts somewhere between a gas and a liquid in the proposed cycle, a value of 90% for internal efficiency of the compressor and a value of 90% for the expander efficiency was considered reasonable. When compression is done more like a liquid, there is a good possibility of getting better than 90% efficiency, whereas if the incoming fluid is very hot compared to the equilibrium temperature of the engine, then the expander efficiency could be lower than the assumed figure.

To get some idea of actual efficiencies obtained from the lab engine, several tests were run. It must first be appreciated that the lab engine is of a very primitive design. It has an overhanging crank, crude crossheads and 79 feet of single-pass 1/2" diameter tube in the heater and 64 1/2 feet of a single 1/2" tube in the cooler. The engine has 1.5 cubic inches of expansion cylinder volume with about 10% clearance volume.

In the test run, when the engine was running on a suitable cycle, the compressor and the expander inlet conditions were \( P_1 = 970 \text{ psi} \) at \( T_1 = 720°F \), and \( P_3 = 2700 \text{ psi} \) at \( T_3 = 4420°F \) respectively. The engine was running at 516 rpm and developed 0.873 horsepower. That is, it generated a power density of 1.128 horsepower per cubic inch of displacement at 1000 rpm, assuming no clearance volume and \( \frac{1.128}{9} = 1.253 \) horsepower per cubic inch of displacement at 1000 rpm taking the clearance volume into consideration.

Because of the crude mechanical design, it is expected that the lab engine is operating at no better than 85% mechanical efficiency. Even
this figure might be high because of external mechanical friction. The lab engine was then indicating \( \frac{1.253}{85} = 1.474 \) horsepower per cubic inch of displacement at 1000 rpm. Two additional factors which reduce the overall efficiency are the efficiencies of compression and expansion and the pressure losses in the heater and the cooler. On the lab engine there is no provision for measuring either of these, hence they are neglected.

Next a theoretical cycle was run on the computer to match the input and output operating temperatures and pressures. With \( P_1 = 970 \text{ psi} \), \( T_1 = 720 \text{F} \), \( P_3 = 2700 \text{ psi} \), and \( T_3 = 4420 \text{F} \), the output data was:

- Heat added: 151.338 BTU/lb
- Heat rejected: 126.395 BTU/lb
- Work done: 24.943 BTU/lb
- Efficiency, Ideal cycle: 16.482 %
- Power density: 2.120 hp/in\(^3\) displ. at 1000 rpm
- Work of compressor: 7.798 BTU/lb
- Work of expander: 32.741 BTU/lb

As a trial, if we take 82.0% efficiency of compression and expansion, then net work is \( 0.82 \times 32.741 - \frac{7.798}{17.338} = 26.848 - 9.510 = 17.338 \) BTU/lb. The power density is then \( \frac{0.82 \times 17.338}{24.943} \times 2.120 = 1.474 \) hp/in\(^3\) displ. at 1000 rpm. This value exactly equals the power density actually obtained from the lab engine without consideration of compression and expansion efficiencies and the pressure loss in the heat exchanger. That is, the lab engine is attaining at least 82% efficiency in compression and expansion.

A properly designed engine would have much lower internal losses than the lab engine and also lower flow past the piston. In conclusion, it can be said that a figure of 90% efficiency for both compression and for expansion in a properly designed engine is reasonable.
PART D. ACTUAL CYCLE ANALYSIS

After choosing the expected efficiencies as $\eta_c = 90\%$ for the compression, $\eta_e = 90\%$ for the expansion, and $\eta_m = 95\%$ for mechanical, it was necessary to review the ideal cycle analysis, by applying these efficiencies to each of the cycles. Of the 720 ideal cycles originally run, about 350 cycles with the best performance were rerun to give actual performance data. Graphs of overall efficiency, actual work of compression and actual work of expansion have been plotted against the four parameters; maximum pressure, $P_2$ and minimum pressure, $P_1$, just as in the previous case of the ideal cycle analysis. It is not considered necessary to discuss the characteristics of all these curves in detail, as they generally behave in the same manner as the ideal curves already discussed. Only the deviations from the ideal behavior will be discussed and an effort will be made to explain the deviations.

Graphs 53 through 57 are for overall efficiency, $\eta$, versus maximum temperature $T_3$, with maximum pressure, $P_2$ as a variable parameter. Maximum temperature, $T_1$, and minimum pressure, $P_1$, are held fixed. In the first two graphs the compression curve lies in the liquid region, whereas in the last three it is in the gaseous region. General behavior of $\eta$ with respect to $T_3$ and $P_2$ does not differ from the ideal case. The tendency of the higher $P_2$ curves to hit the zero output axis at higher temperatures than shown for the ideal case is quite noticeable in the past three curves because of the higher work of compression in the gaseous region. Overall efficiencies are naturally lower than the ideal ones, as may be seen by reference to Graphs 4, 6, and 9.

Graphs 58 and 59 are for overall efficiency, $\eta$, versus maximum temperature $T_3$ with minimum pressure, $P_1$, as a variable parameter, the two other parameters being held constant. The initial rise and subsequent leveling off is more prominent in these curves than those for the ideal case, as shown on Graph 11. Zero efficiencies starting at comparatively higher temperatures for the higher pressure ratios is also noticeable, again caused by the higher compression work required.

Graphs 60 and 61 are for $\eta$ versus $T_3$ with $T_1$ as the variable parameter, $P_1$ and $P_2$ being held constant. Comparing Graph 60 with 18, it is obvious that at lower maximum temperatures, the minimum temperature influences the efficiency much more in the actual case than in the ideal one. This effect is still more pronounced in Graph 61. The explanation for this behavior is that the ratios of actual net work to ideal net work are lower for cycles with initially low ideal work, as the efficiencies $\eta$ and $\eta_c$ affect $W_c$ and $W_e$ respectively, and not just the net work alone.
In Graphs 62 and 63 $\gamma_0$ is plotted against $T_1$ with $P_1$ as the variable parameter, and $P_2$ and $T_3$ held constant. The same behavior discussed in the last paragraph is noticeable in these graphs. In Graph 62, for $T_3 = 900^\circ F$, $T_1$ seems to have very little effect on $\gamma_0$ except when the pressure ratio increases and the ideal net work diagram slims down. In Graph 63, it appears that $T_1$ seems to affect $\gamma_0$, but even then one could operate between $T_1 = 40^\circ F$ to $120^\circ F$ for $P_2 = 5000$ psi. without much variation in $\gamma_0$ if a sufficiently high $P_1$ is used.

Graphs 64 thru 67 show a plot of $\gamma_0$ versus $T_1$ with $P_2$ as the variable parameter, and $P_1$ and $T_3$ held fixed. Once again at $T_3 = 900^\circ F$, $\gamma_0$ seems fairly independent of $T_1$ except when the pressure ratio is high. At $T_3 = 600^\circ F$, the same behavior is observed, except that $T_1$ influences $\gamma_0$ a bit more.

Graphs 68 and 69 are for $\gamma_0$ against $T_1$ with $T_3$ as the variable parameter, the other two parameters being held fixed. The behavior observed in the last three paragraphs is repeated here. In Graphs 70, 71, and 72, $\gamma_0$ is plotted against $P_1$ with $P_2$ as the variable parameter, $T_1$ and $T_3$ being held fixed. In each of these graphs, low $P_2$ curves behave the same way as the ideal curves. For $P_2 = 10000$ psi in Graphs 70 and 71, and for $P_2 = 5000$ psi in Graph 72, the reason for dropping off of $\gamma_0$ at $P_1 = 500$ psi is not apparent.

Graphs 73 and 74 show a plot of $\gamma_0$ versus $P_1$ with $T_1$ as the variable parameter and with $P_2$ and $T_3$ held fixed. Once again it is obvious that $\gamma_0$ is almost independent of $P_1$ over a large range of $P_1$. On Graph 74 the tendency of the curve for $T_1 = 40^\circ F$ to rise and for the curves for $T_1 = 80^\circ$ and $120^\circ F$ to fall at $P_1 = 500$ psi, may be explained by the increase in work of compression in the latter two cases.

In Graphs 75 and 76, $\gamma_0$ is plotted against $P_1$ with $T_3$ as the variable parameter and with $P_2$ and $T_1$ held fixed. Except for points at $P_1 = 500$ psi, the curves seem to behave quite normally, with a slight drop of $\gamma_0$ as $P_1$ increases. Behavior with respect to $T_3$ has already been discussed.

Graphs 77 through 82 are for $\gamma_0$ versus $P_2$. In Graph 77 $P_1$ is the variable parameter. In 78 thru 80 it is $T_1$, and in 81 and 82 $T_3$ is the variable parameter. The remaining parameters are held constant in each case. The behavior of these curves is essentially the same as that in the ideal case.

In Graphs 83 through 85 actual work of compression ($W_c$) and actual work of expansion ($W_e$) are plotted against $T_3$ with $P_2$, $P_1$, and $T_1$ as the variable parameters respectively, the other two parameters being held
fixed in each case. Similarly in Graphs 86 thru 88 $W_C$ and $W_E$ are plotted versus $T_1$; in Graphs 89 thru 91 the plot is against $P_1$; and in Graphs 92 thru 94 the plot is against $P_2$; the variables are rotated successively. The graphs of actual work of compression and actual work of expansion are identical to graphs of ideal work of compression and ideal work of expansion respectively, except that they are scaled up by factor of $\frac{1}{9}$ in case of work of compression and scaled down by a factor of .9 in case of work of expansion.

In Graph 95, the overall efficiency is plotted against the temperature ratio, whereas in Graphs 96 and 97 it is plotted against pressure ratio. Earlier a remark was made concerning different cycle performance being obtainable for the same pressure and temperature ratios, if the cycles were operating in different thermodynamic regions, this showed the necessity for plotting performance data against dimensional pressures and temperatures rather than on non-dimensional plots. This remark is very well substantiated by these graphs. For example consider curves C and D of Graph 95. For a pressure ratio of $\frac{10000}{800} = 12.5$ and a temperature ratio of 2.0, we have $\eta = 15\%$ for $T_{\text{min}}$ of 800°F and $\eta = 23\%$ for $T_{\text{min}}$ of 400°F. In the first case compression was in the gaseous region, whereas in the second case it was in the liquid region. As the pressure ratio decreases and temperature ratio increases, this deviation starts to narrow down. The criss-crossing of the curves at lower temperature ratios and for $P_{\text{min}}$ of 500 and 800 psi all show that temperature and pressure ratios alone are not the determining factors in fluid engine performance.

Contrary to the belief that $\eta$ always increases with the pressure ratio for a Brayton cycle, from Graphs 96 and 97 it is apparent that at low maximum temperatures for cycles lying completely in the gaseous region, $\eta$ decreases as the pressure ratio is increased. At slightly higher $T_{\text{max}}$, $\eta$ first increases and then decreases with the pressure ratio.

For high $T_{\text{max}}$, the $\eta$ continuously increases with the pressure ratio. For cycles which have their compression in the liquid region, $\eta$ always increases with the pressure ratio. Criss-crossing of curves A, B, C, and D on Graph 95 and not of E and F also shows the same effect.
PART E. SELECTION OF CYCLE

For any specific application, the hot and cold sink temperatures are fixed requirements. The problem then is to fit the energy conversion system in between these two limits in the best possible manner. The basic problem is to determine the best operating temperature limits for the engine cycle, and is done in several steps as follows. First, allow for certain reasonable temperature differences on either side to give the heat exchangers enough driving force to keep them to a reasonable size. This fixes the maximum and minimum temperatures of the engine cycle. Next, the choice of maximum and minimum pressures of the cycle must be made. Series of cycles are run with different initial pressures and different pressure ratios to get the best conditions. Effects of assumed compression and expansion efficiencies and the pressure drops on the theoretical cycles are also studied at this stage. Some actual cycles with good performance, by way of efficiency and power density are then chosen for heat exchanger calculations. Then heat exchangers to go along with these cycles are designed for the given application. If a cycle with suitable efficiency, good power density and reasonable equipment size is obtained, then it can be listed as a feasible cycle. If the heat exchangers tend to be too big, larger temperature differences are allowed at the high and low temperature ends, and the evaluation procedure is repeated. Of course, in every optimization study, it is necessary to know exactly what the system is to be designed for and what weight is given to each design factor.

Efficiency, weight, and reliability are some of the criteria. Of course, all these design criteria cannot be satisfied at the same time and the final configuration is a compromise. The chosen cycle must also have reasonably low zero power efficiencies. That is, the efficiencies of compression and expansion which give no work from the cycle should be fairly low. This assures that the system will run even under adverse conditions.

With the hot and cold sink temperatures given, it can immediately be determined which fluids can be utilized as the working media in the cycle. Preferably the critical temperature of the fluid chosen should lie between the high and low sink temperatures, and the critical pressure should not be too high. If the critical pressure is too high the system must be operated at high pressure, thus resulting in a heavy and costly system. With the critical temperature in between the maximum and minimum temperatures of the cycle, it is possible to accomplish compression in the liquid region and expansion in the gaseous region, thus taking advantage of the possible high actual efficiencies in these processes.

To illustrate this point, assume that the hot water temperature from a pressurized water reactor is given as 290°F and the atmospheric
cooling air temperature is 80°F. Then a possible choice for the maximum and minimum temperatures of the cycle are 265°F and 100°F respectively. If CO₂ with a critical temperature of 88°F is used as the working fluid, both compression and expansion have to be done in the gaseous region and the resulting cycles do not give any substantial work output, thereby giving very low overall efficiencies. If the cycle is now operated with Freon 12, which has a critical temperature of 234°F, the compression can be accomplished in the liquid region and expansion in the gaseous region, thereby giving much better system performance.

With Freon 12, cycle calculations show that under operating conditions of \( P_1 = 132 \) psi, \( P_2 = 676 \) psi, \( T_1 = 100°F \) and \( T_3 = 265°F \), a theoretical efficiency of 13.3% is attained. Assuming 90% efficiency for compression and expansion each, an indicated efficiency of 11.6% results. Because compression is done with very little change of volume, efficiency of compression could be even better than 90%, hence the 11.6% value is considered by Cleveland Pneumatic to be conservative.

If the hot water reservoir is operated at 320°F, then the cycle could be operated between 100°F and 300°F to get 13.76% actual efficiency with \( \gamma_c = \gamma_e = .9 \). These efficiencies are much better than corresponding steam cycle efficiencies which run around 8.5%.

In conclusion, it can be said that with such a wide choice of available fluids and operating pressures, it is possible to obtain cycles for use with the fluid engine which will permit operation over a wide range of ambient temperature conditions, including both arctic and desert locations.
Chapter 7

SYSTEM ANALYSIS AND DESIGN, PM-2A AND ML-1

GENERAL ANALYSIS

This section of the report analyzes PM-2A and ML-1 systems which are modified in accordance with the requirements necessary for operation with CO₂ as the working fluid. The results are used for a comparison with the existing ML-1 and PM-2A system designs.

A literature study on the ML-1 and PM-2A systems indicated that a design employing CO₂ as the working fluid would need to be somewhat different from the existing systems for several reasons. As an illustration, the ML-1 system described in ML-1 Design Report (4), employs nitrogen gas as the primary coolant. This fluid is directly expanded in a turbine and returned to the reactor by means of a compressor via a recuperator and an air cooler. The compressor is coupled directly to the turbine shaft for maintaining system pressures at values necessary to satisfy cycle conditions. A number of blowers provide the required cooling.

Using CO₂, the system arrangement is significantly different in that the nitrogen gas is not employed directly as the working fluid, but is used indirectly for heating CO₂ in a heat exchanger. The system requires a major change to convert it from a single fluid to a binary fluid system. The engine is placed in a secondary loop which needs an extra cooler for the CO₂. Consequently, the new system which employs CO₂ will be different from the existing ML-1 systems in that an additional loop with its hardware is needed.

The PM-2A system, described in ALCO Products Report (5) is not subjected to any major change, because it is already a binary fluid system. There is a considerable similarity in the arrangements possible by substituting CO₂ for the presently used steam as the working fluid. In particular, this is true for the primary system which may be used as is in the new power plant employing CO₂. The secondary system, however, must be changed completely, resulting in a rather greatly simplified PM-2A power plant.
The modified PM-2A and ML-1 power generating systems will both employ their existing nuclear reactors for a continuous supply of heat to the coolant which circulates in the primary loop. The primary coolant then provides thermal energy to the secondary loop, which includes an energy conversion device to change the heat into the useful mechanical work. This work will then be converted into electrical energy for further use, as desired. The electricity generating power plant systems, thus will employ essentially similar components, which are:

(a) Nuclear reactor and associated equipment in the primary loop.
(b) Engine-generator set with the associated heat exchangers-coolers in the secondary loop.
(c) Control center for instrumentation and controls.
(d) Auxiliary subsystem for start-up and shut-down.

Figure 7-1 is a schematic of a typical reactor system which incorporates the primary and the secondary loops in PM-2A and ML-1 power plants operating on the CO₂ cycle. The differences in the components used for PM-2A and ML-1 power plants will be mainly in the equipment size, due to the capacity variation of the two systems, and the necessity for an extra evaporator to generate steam with the PM-2A system.

The Primary Systems

The primary loop of the PM-2A system will be composed of the reactor, primary coolant pump, heat exchanger for CO₂ heating, pressurizer, interconnecting piping and auxiliaries.

The above components in their entirety are typical for the pressurized water reactor system and, as such, are detailed in report APAE No. 49, with the exception of heat exchanger. The heat exchanger that will utilize the primary water coolant as a heat source and the CO₂ as the heat sink will be described separately in further treatment of the respective systems.

The ML-1 system primary loop will also employ existing components with the exception of the turbine-compressor sets which will
FIGURE 7-1 SCHEMATIC OF BASIC COMPONENTS FOR MODIFIED PM-2A AND ML-1 SYSTEMS
have to be replaced by a circulator-compressor. The circulator, however, will have to be designed specifically for use with a CO₂ system employing a reactor cooled by nitrogen at a pressure of approximately 300 psi, and a temperature of 800°F. The allowable pressure drop is 31 psi, of which 23 psi are required for the reactor and 8 psi for the exchanger pressure drops.

Employing CO₂ in the secondary loop, the primary loop of the ML-1 system will have to operate between 800°F and 1200°F, which in turn imposes sealing problems on the circulator. In comparison with the existing ML-1 system, which uses a compressor with inlet conditions of 118°F and 115 psi pressure, the recirculator sealing problem is further aggravated by the inlet pressure rise to approximately 300 psi in a system that employs CO₂.

Under the circumstances, the CO₂ system could not consider the commercially available recirculators which are bulky and heavy. For the present, it is decided to consider the existing package unit of turbine-compressor which is 24" diameter x 49" long and weighs 1600 lbs., and use these values for the circulator in the CO₂ system.

With consideration to the above statements, it appears that the primary loops of the existing PM-2A and ML-1 systems could be employed in their entirety in the systems that employ CO₂ as the working fluid. By reference to the material available on these systems, the existing system weights and volumes were tabulated in Table I.

Secondary Systems

The principal function of the secondary system is to supply heat energy to the prime mover for conversion by an engine which is coupled directly to the electric generator. The primary loop receives its heat from the nuclear reactor which is cooled by pressurized water in a PM-2A system and by nitrogen in an ML-1 system. The secondary loop employs CO₂ as the working fluid. It circulates inside the tubes of the heat exchangers for heating and cooling purposes and operates in a closed Brayton cycle in the fluid engine. Thus, the principal difference between the existing systems and the proposed systems under consideration is in the working fluid and the associated equipment such as the engine, the exchangers, the controls and the secondary supporting equipment.
### TABLE I

EXISTING SYSTEM WEIGHTS AND VOLUMES

<table>
<thead>
<tr>
<th>ITEM</th>
<th>COMPONENT</th>
<th>ML-1</th>
<th>PM-2A</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Weight LB</td>
<td>Volume FT³</td>
</tr>
<tr>
<td>1</td>
<td>Reactor package and/or assy: Reactor core, shielding and reactor auxiliaries.</td>
<td>30000</td>
<td>635</td>
</tr>
<tr>
<td>2</td>
<td>Power conversion system: Turbine-compressor set, recuperators, precoolers, alternator and/or generator, elect. switch gear, etc.</td>
<td>30000</td>
<td>1020</td>
</tr>
<tr>
<td>3</td>
<td>Control cab: Instruments and controls, control console for remote plant start up.</td>
<td>5000</td>
<td>558</td>
</tr>
<tr>
<td>4</td>
<td>Auxiliaries - separately mounted</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Cable reel</td>
<td>6000</td>
<td>380</td>
</tr>
<tr>
<td></td>
<td>Reactor drying equipment</td>
<td>900</td>
<td>180</td>
</tr>
<tr>
<td></td>
<td>Water make up</td>
<td>900</td>
<td>180</td>
</tr>
<tr>
<td></td>
<td>Waste Gas system</td>
<td>2000</td>
<td>160</td>
</tr>
<tr>
<td></td>
<td>Gas supply</td>
<td>2000</td>
<td>94</td>
</tr>
<tr>
<td></td>
<td>Boron compound system</td>
<td>3050</td>
<td>300</td>
</tr>
<tr>
<td></td>
<td>Diesel-generator and accessories. (45 KW unit for ML-1 system and 3-300 KW units for PM-2A system)</td>
<td>1000</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Erection Tools, Etc.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>TOTAL</td>
<td>80850</td>
<td>3517</td>
</tr>
</tbody>
</table>

### Specifics:

- **LB/FT³**
  - ML-1: 23.0
  - PM-2A: 15.75
- **FT³/LB**
  - ML-1: 0.0435
  - PM-2A: 0.0635
- **LB/KWe**
  - ML-1: \(\frac{80850}{400} = 201\)
  - PM-2A: \(\frac{904105}{1560} = 580\)
- **FT³/KWe**
  - ML-1: \(\frac{3517}{400} = 8.7\)
  - PM-2A: \(\frac{57460}{1560} = 36.7\)

*(1) Data from report IDO-28550

(2) Data from shipping list

(3) Maximum allowable skid weight

(4) Estimated
Engine

From the numerous design principles that could be employed in positive displacement machines, the principle of the rotary engine offers undoubted advantages. It permits a wide flexibility in design, simplicity in fabrication, good load balance and compactness. The positive acting rotary engine, designed with multiple pistons and cylinders which are actuated through a set of stationary cams, was considered as the best choice for the PM-2A and ML-1 power plants. Such engines have already been described in Chapter 5. The estimated engine weights and volumes for PM-2A and ML-1 systems are presented in Table II.

Heat Exchangers

Both systems will employ horizontal tubular heat exchangers of tube and shell design with "U" type tubes to compensate for the differential expansion between the shell and tubes, and comprising a two pass unit. The physical exchanger design and fabrication are in accordance with Section VIII of the ASME Unfired Pressure Vessel Code. The thermal design is in accordance with the procedures set forth in the theoretical treatment of the heat transfer capabilities which CO₂ offers at supercritical conditions. The heat exchanger physical design is considered in detail in Chapter 9. The design details of each exchanger are presented on the computer data sheets for each power plant system under consideration in Appendix D. The exchanger function in the secondary system will be described under the respective chapters discussing individual systems in detail.

General Controls

For the systems under consideration, a completely automatic universal type control of reactor and power conversion equipment is recommended. The universal control system should be capable of handling any program ranging from constant pressure of the working fluid to the constant average reactor temperature. The universal controls will automatically control the following:

1. Fast and slow power changes in the power plant and reactor. This will include period control at low power levels.

2. Initial start-up of reactor and power plant.

3. Controlled shut-down of reactor and power plant.
<table>
<thead>
<tr>
<th>Capacity KWe</th>
<th>RPM</th>
<th>Displacement IN(^3)</th>
<th>Weight LB</th>
<th>Volume FT(^3)</th>
<th>Specifics LB/FT(^3)</th>
<th>Specifics LB/KW</th>
<th>Cycle No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>1800</td>
<td>101</td>
<td>1000</td>
<td>7.3</td>
<td>137</td>
<td>2.0</td>
<td>192</td>
</tr>
<tr>
<td>1250</td>
<td>1800</td>
<td>319</td>
<td>3100</td>
<td>21.8</td>
<td>142</td>
<td>2.47</td>
<td>192</td>
</tr>
<tr>
<td>2000</td>
<td>1800</td>
<td>516</td>
<td>4120</td>
<td>27.5</td>
<td>151</td>
<td>2.09</td>
<td>192</td>
</tr>
</tbody>
</table>

**TABLE II - PROPOSED FLUID ENGINES**
4. Scram reactor and power plant shut-down.

5. Restart system after reactor scram.

The system control depends on the coordination of the following.

1. Engine power output controlled by the bypass line.

2. Throttle valve control. Throttle valve will be used for start-up, shut-down, and large excursions in load.

3. Alternator speed control, based on alternator output frequency.

4. CO₂ temperatures and pressures.

5. CO₂ accumulation due to excess pressures, if any.

6. Steam generation in the PM-2A system.

7. Alternator motoring control. During shut-down and start-up, the circulating water pump could be driven by motoring the alternator, or from the standby (auxiliary) diesel generator sets.

8. Reactivity control by changing the position of the reflectors. Neutron flux must be accurately measured in the counter, period and power ranges for determination of reactor power level and period.

9. Reactor safety and scram due to abnormalities in the system. Other controls shown will be either manually operated or of conventional type suitable for radio-active operation. The control valves could be actuated by high pressure hydraulic cylinders.

The operation of the systems thus will need no special attendance and will require minimum manpower (one man operation.) For details of an automatic control system for the nuclear reactor, see also preprint 100-LA-61 by Instrument Society of America, on PM-3A Nuclear Power Plant Instruments and Controls, by Robert L. Schimmel, The Martin Company, Nuclear Division, Baltimore, Maryland.

The secondary supporting equipment, such as the electricity generating and distribution system, the standby equipment, etc., will be covered in the detailed description of each system separately.
Physical Arrangement

Figures 7-2 and 7-3 show equipment arrangement for PM-2A and ML-1 power plants, respectively, for use with CO$_2$ as a working fluid.

The ML-1 system represents a package unit compatible with the requirements set forth for such a portable system. It employs nitrogen as the primary coolant and CO$_2$ as the working fluid flowing in a separate secondary loop.

The PM-2A system employs pressurized water as the reactor coolant, with CO$_2$ as the working fluid flowing in a secondary loop. It allows assembly of the engine relatively close to the CO$_2$ heater in order to shorten fluid transfer lines. This enables construction of modular sections separately for easy transportation and relatively fast erection on location.

Both system arrangements are believed to comply with the present standards for safe operation within the radioactive environments.

System Start-Up

In the overall system start-up, two major events will take place separately and at different time intervals. These are the reactor start-up and the engine start-up. The reactor start-up will take place in accordance with existing procedures as described in the referenced reports. Initially, and until the reactor is completely stabilized, the system will employ auxiliary power sources such as a Diesel engine and battery. Thus, each power plant under consideration will be started from the remotely situated main control center in the following anticipated sequence:

1. Actuate starter switch for Diesel engine to motor the generator and to supply electricity to the system.

2. Charge primary and/or secondary system with working fluids, if and when necessary, and purge the system.

3. Switch-in universal controls for monitoring system start-up and automatic control. This will involve control of the primary system heat-up and reactor stabilization. The engine and the secondary loop will start operating only after the working fluid CO$_2$ in the secondary loop receives enough heat from the primary coolant which circulates through the reactor and heat exchanger at rates predetermined by the reactor heat-up program. When CO$_2$ pressure and
FIGURE 7-2 PM 2A EQUIPMENT ARRANGEMENT
FIGURE 7-3 ML-1 SYSTEM EQUIPMENT ARRANGEMENT
temperature in the secondary loop reach an operating value, the
start-up throttle valve will open and permit fluid flow into the
engine. The start of the engine will be instantaneous. After the
engine obtains the speed of the alternator, a clutch of the engine-
alternator will engage. This will unload the Diesel engine and the
system will become an automatic electricity producing installation.
(For details of the reactor start-up, see Section 9.0 of the Report,
APAE No. 49 and the report IDO 28550).

Due to the requirement for slow build up of reactor pressure in the
lower temperature range in order to stay within the limits imposed by
radiation dosage, the heat-up time of the primary system is rather long.
The manual for operation of the PM-2A system requires maintaining the
maximum permissible heat-up rate at $300^\circ$F/HR, with a proportional pres-
sure buildup. Similar requirements are present for the ML-1 reactor
system. Consequently, the system start-up is purely reactor dependent,
with typical heat-up time of 10 to 12 hours for the primary loop alone.

The secondary loop heat-up period depends on the amount of CO$_2$
present in the exchanger prior to engine start-up, and varies from system
to system, depending on the power output capacity of the specific power
plant. The CO$_2$ preheating also depends on the flow pattern of the primary
coolant which is period dependent and controlled by the reactor heat-up
programmer. Consequently, the exact time involved in starting the sec-
dary loop is extremely difficult to predict theoretically, as it would require
a rather accurate determination of exchanger void - volumes and CO$_2$
charging pressures, as well as the exact heat input during the transients
of the reactor heat-up. It is believed, however, that the systems under con-
sideration offer sufficient flexibility in controls and equipment to be able
to absorb a rather large deviation in the characteristics of secondary loop
starting.

During start-up, two situations may arise. Should the initial CO$_2$
charge in the exchanger voids be too small, the booster will open and
supplement the necessary volume to raise system pressure to the pre-
scribed value. Should the initial CO$_2$ charge in the exchanger result in too
great a system pressure, the pressure relief valves will open, discharging
the excess into the CO$_2$ accumulator. Both provisions are capable of serv-
ing the secondary loop during start-up as well as at steady state operation.
At start-up, with the reactor heat-up times of approximately 12 hours, it
does not seem important to exactly define the relatively short start-up time
of the secondary loop. In fact, the start-up time of the secondary loop can
be considered as an integer of the primary loop.
Starting and acceleration of an unloaded engine is purely dependent on the breakaway torque and the mass of the body to be set into motion. The torque required to accelerate a body from rest to a speed of N-RPM in \( t \) seconds is \( T = \frac{W R^2 N}{308 t} \) LB-FT, where \( W \) = weight of the body \( R \) = radius of gyration

With a disengaged clutch or an unloading device between the engine and the generator, the conditions of starting and coming up to speed against little resistance are extremely favorable. Estimates indicate that the engine so started will come up to speed in a few seconds. The generator load to the running engine can be imposed almost instantaneously when the engine speed matches the generator speed. Additional discussion on system start-up will be found in the description of each power plant system under consideration.

System Shut-Down

The normal system shut-down is associated with the controlled cool-down of the primary loop after the transfer of power plant load to the standby equipment. This takes place after the load has been gradually reduced to a value below the useful range of the power output capacity of the CO\(_2\), which receives heat from the primary loop and acts as a sink for the primary coolant. The associated cool-down procedure is described in the reference reports APAE No. 49 and IDO 28550.

The emergency shut-down is dependent on the type of component failure. If the emergency arises due to the failure of the components in both the primary and the secondary loops, the reactor could be scrammed by dropping the control rods to full "down" position. The determination of the necessity to scram the reactor depends on the interpretation of the failure. The systems employing universal controls would depend on the automatic integrator receiving control signals from system components. The decision for scramming the reactor will also depend on the power plant superintendent in his evaluation of the existing system conditions and the magnitude of the difficulties in operation. For an emergency, however, the secondary loop is provided with pressure relief valves which would automatically render the system safe. For details in the emergency procedures on reactor systems, see Sections 8 and 9 of Report, APAE 49 and pages 61-63 of the Report, IDO 28550. Additional discussion on system shut-down will be found in the description of each power plant system under consideration.
Discussion

It is interesting to note that the choice of the equipment for secondary loops in both systems is reactor dependent and, as such, appears to be subject to the following discussion:

The use of CO₂ in the existing ML-1 reactor system does not appear desirable due to the inferior heat transfer characteristics of the primary coolant, N₂, which requires complex additional heat exchange equipment which, when changed to CO₂, exhibits superior characteristics in efficiencies and system simplicity, including substantial reductions in weights and volumes. Although the use of CO₂ as the working fluid in the secondary loop of the present ML-1 system is not attractive, a change in reactor design to operate with Na or NaK instead of N₂ appears to offer possibilities. These have been investigated, and the results do not appear favorable from a weight standpoint but they may offer advantages in improved overall efficiency.

DETAILED DESCRIPTION OF SYSTEMS

PM-2A System

The PM-2A system is a 1560 net KWe pressurized water power reactor system designed for arctic conditions. The existing system performance characteristics are presented in the Report APAE No. 49, and may be summarized as follows:

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electric power - gross</td>
<td>2000 KW</td>
</tr>
<tr>
<td>Electric power - net</td>
<td>1560 KW</td>
</tr>
<tr>
<td>Auxiliary power</td>
<td>440 KW</td>
</tr>
<tr>
<td>Reactor thermal power</td>
<td>10 MW</td>
</tr>
<tr>
<td>Primary coolant - water</td>
<td></td>
</tr>
<tr>
<td>Flow</td>
<td>4890 GPM</td>
</tr>
<tr>
<td>Max. temperature</td>
<td>517.6°F</td>
</tr>
<tr>
<td>Min. temperature</td>
<td>500°F</td>
</tr>
<tr>
<td>System pressure</td>
<td>1750 psia</td>
</tr>
<tr>
<td>Design pressure</td>
<td>2000 psia</td>
</tr>
<tr>
<td>Design temperature</td>
<td>600°F</td>
</tr>
<tr>
<td>Secondary coolant - water</td>
<td></td>
</tr>
<tr>
<td>Turbine flow</td>
<td>37700 Lb/Hr</td>
</tr>
<tr>
<td>Inlet temperature</td>
<td>306°F</td>
</tr>
<tr>
<td>Steam pressure</td>
<td>480 psia</td>
</tr>
<tr>
<td>Steam temperature</td>
<td>465°F</td>
</tr>
</tbody>
</table>
Estimated thermal efficiency (2000 KW/1000 KW = .20) ... 20%
Overall system efficiency (1560/10000 = 15.6%) ............ 15.6%

Reactor average heat flux (at start) .................. 72400 \( \frac{\text{BTU}}{\text{Hr Ft}^2} \)

Steam generator overall heat transfer coefficient, \( U = 847 \frac{\text{BTU}}{\text{Hr/}^\circ \text{F/Ft}^2} \)

Coolant - air at max. 48\(^\circ\)F, min. -63.5\(^\circ\)F and annual average -10\(^\circ\)F

An air to ethylene glycol cooler is used in the secondary loop for rejection of heat from the system.

The existing system components are described in Report APAE No. 49. Page 3-1, paragraph 3.0 covers basic components of the primary loop, including system operation, and page 4-1, paragraph 4 covers basic components of the secondary system. In essence, the existing PM-2A system employs a pressurized water nuclear reactor for a continuous heat supply in the primary loop and a Rankine cycle turbine operation on steam for power conversion in the secondary loop. The proposed new PM-2A system will employ the existing nuclear reactor, including the accessories for heat generation in the primary loop and an engine operating on a Brayton cycle which employs \( \text{CO}_2 \) as the working fluid at supercritical pressures and temperatures for power conversion in the secondary loop. Consequently, the proposed new working fluid, \( \text{CO}_2 \) will represent the only major change in the PM-2A system. Since \( \text{CO}_2 \) will be flowing in the secondary loop, the PM-2A system will be subjected to changes in the secondary loop alone. This is because of the design of the secondary loop which is intended to be compatible with the capabilities of the present primary loop and the design of the existing reactor. Therefore, the operation of the primary loop will not be affected by the introduction of the new working fluid, \( \text{CO}_2 \), and the reactor control as well as the system response characteristics will be nearly identical to the characteristics described in Report APAE-49. Since the \( \text{CO}_2 \) in the secondary loop exhibits different physical properties from the steam, it is obvious that the system proposed will require completely new instruments and somewhat different controls.

The basic simplification the \( \text{CO}_2 \) system offers may be summarized as follows:

1. Elimination of ethylene glycol cooler system with auxiliaries as the \( \text{CO}_2 \) can be cooled directly by air.

2. Elimination of de-aerating system as the \( \text{CO}_2 \) requires no condensers.
3. Elimination of condensate and boiler feedwater pumps, and high and low pressure feedwater heaters and air ejectors.

4. Elimination of instruments that measure radioactivity in the hot well.

5. Simplification of primary and secondary makeup systems and demineralizing.

6. Elimination of gear reducer as the generator will be directly driven from the engine shaft running at 1800 rpm.

7. Elimination of pumps associated with the equipment above.

8. Simplification of system and system controls.

9. Reduction of the auxiliary power by approximately 100 KW.

A simplified system diagram identifying basic secondary loop components, including system controls necessary for a CO$_2$ power plant is shown in Figure 7-4.

PHYSICAL ARRANGEMENT

The proposed PM-2A plant will be laid out in the form of a cross as shown in Figure 7-5. The reactor would be located sufficiently remote from the main tunnel so as to provide necessary shielding. Arrangement of components is slanted toward safety of personnel, short fluid lines and accessibility to service the engine, generator and reactor.

Safety Considerations

The primary heat exchanger is located within the vapor container housing the reactor. A pressure failure in the heat exchanger will contain working fluid inside the vapor container. A pressure failure in the engine-generator compartment will be isolated by automatic valves A and L in the fluid lines in the vapor containers, as shown in Figure 7-6. Therefore the amount of hot working fluid released into the personnel area will be relatively small.

A pressure failure in the cooler section will be isolated by the above valves, so that only the fluid in the coolers will be discharged into the personnel area. In this event, the Diesel backup power would activate
FIGURE 7-4 SIMPLIFIED SYSTEM DIAGRAM AND CONTROLS
PM 2A SYSTEM
NOTE - FANS ON COOLER DRAW AIR FROM TUNNELS, THUS REMOVING IMPURITIES.

PHYSICAL ARRANGEMENT
FIGURE 7-5
FLUID MANAGEMENT SYSTEM

![Diagram of fluid management system](image)

**Figure 7-6**

<table>
<thead>
<tr>
<th>VALVE IDENT.</th>
<th>NAME</th>
<th>TYPE</th>
<th>NORMAL POSITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>THROTTLE</td>
<td>METERING</td>
<td>OPEN</td>
</tr>
<tr>
<td>B</td>
<td>SHUT OFF</td>
<td>MANUAL</td>
<td>CLOSED</td>
</tr>
<tr>
<td>C</td>
<td>SAFETY</td>
<td>AUTOMATIC</td>
<td>INOPERATIVE</td>
</tr>
<tr>
<td>D</td>
<td>SHUT OFF</td>
<td>ELECTRIC</td>
<td>CLOSED</td>
</tr>
<tr>
<td>E</td>
<td>SHUT OFF</td>
<td>ELECTRIC</td>
<td>CLOSED</td>
</tr>
<tr>
<td>F</td>
<td>MOTORING BYPASS</td>
<td>ELECTRIC</td>
<td>CLOSED</td>
</tr>
<tr>
<td>G</td>
<td>SHUT OFF</td>
<td>ELECTRIC</td>
<td>CLOSED</td>
</tr>
<tr>
<td>H</td>
<td>CHECK</td>
<td>AUTOMATIC</td>
<td>AUTOMATIC</td>
</tr>
<tr>
<td>J</td>
<td>SHUT OFF</td>
<td>ELECTRIC</td>
<td>CLOSED</td>
</tr>
<tr>
<td>K</td>
<td>SHUT OFF</td>
<td>ELECTRIC</td>
<td>CLOSED</td>
</tr>
<tr>
<td>L</td>
<td>CHECK</td>
<td>AUTOMATIC</td>
<td>INOPERATIVE</td>
</tr>
<tr>
<td>M</td>
<td>BYPASS CONTROL</td>
<td>ELECTRIC</td>
<td>CLOSED</td>
</tr>
<tr>
<td>N</td>
<td>MOMENTARY SHUT OFF</td>
<td>ELECTRIC</td>
<td>CLOSED</td>
</tr>
<tr>
<td>O</td>
<td>SHUT OFF</td>
<td>MANUAL</td>
<td>OPEN (At Bottles)</td>
</tr>
</tbody>
</table>

**STORAGE IDENT.**

| I            | HIGH PRESSURE START AND STORE - BOTTLES |
| II           | MEDIUM PRESSURE SYSTEM - BOTTLES        |
| III          | SALVAGE SYSTEM ACCUMULATOR              |
the fans and clear the area through the air exit ducts. Excessive pressure in the high pressure system is relieved through safety valve C and check valve H back into the high pressure bottles. Protection from "shrapnel" from a catastrophic failure in the cooler chamber will be afforded by steel grill work between the engine-generator room and the cooler area.

DESCRIPTION OF COMPONENTS

Reactor

The PM-2A reactor is designed for continuous operation and is provided with radioactive waste disposal facilities, refueling provisions, and a signal and control system to provide control for normal operation and correct response in case of leaks or other abnormalities. No changes in the reactor design, as set forth in Report APAE No. 49, are contemplated in this report. The control of the reactor is to be coordinated with the overall system, as required.

Primary system

The primary system includes the pressurized water, pumps, pressure and temperature sensitive devices, water treatment facilities, pressurizer, and other associated details. The present PM-2A plant includes a steam generator, which will be replaced by the heat exchanger for the fluid engine working fluid.

Since the design philosophy of PM-2A is to include all elements of the primary system in a vapor container for retention of radioactive material in case of failure, the fluid engine heat exchanger will be arranged within the vapor container also.

Heat Exchanger

The fluid engine will take heat from the primary system by means of a heat exchanger enclosed in the vapor container as described above. Vapor container stresses in case of rupture of engine fluid conduits, either in or associated with the heat exchanger, will be recalculated for the engine fluid as they have been considered in the steam cycle of the existing PM-2A plant. If necessary to reduce the mass of engine fluid released into the vapor container in case of failure, quick acting shutoff valves in the engine fluid lines at the point of entrance to and exit from the vapor container, will isolate fluid leakage to the amount contained in the heat exchanger rather than the total mass of working fluid in the system.
Steam Generating System

The auxiliary steam generating system consists of a separate, second coil in the primary heat exchanger shell, an evaporator, a working fluid circulating pump and associated plumbing. The engine system is connected to the steam system only by a charging line fitted with a shut-off valve (B) for charging the steam system with working fluid.

This system may be operated by auxiliary power during engine shutdown. It may also be used to cool the reactor during engine shutdown. By its location, the engine system is not affected by steam demand, nor is primary water circulated in the evaporator.

Fluid Engine

The fluid engine will be designed with multiple short-stroke double diameter pistons. The cylinder block containing the pistons will be cylindrical and carries the piston parallel to the axis of rotation. Piston power is communicated to the stationary housing by means of cam plates and hydrostatic balls. Provision has been made to vary the hydrostatic supporting pressure on the balls in proportion to the loading thereon.

Working Fluid Management System

This system as shown in Figure 7-6 is designed to provide means for:

1. Purging the engine system at initial startup.

2. Adding to or withdrawing fluid from the active working fluid as required by power output and speed of the engine.

3. Recovery of leakage of working fluid by the use of secondary seals containing leaked fluid in lower pressure cavities around valve stems or rotating shafts.

4. Pumping of working fluid back into storage cylinders in case the system must be depressurized for maintenance.

5. Starting the engine after routine shutdown.

The working fluid management system is more or less complex, according to the size of the plant installation, its degree of performance.
the anticipated period of operation, frequency of stoppages and degree of portability. The presence or absence of external power for starting purposes also affects the system. Basically, the system consists of pressure storage containers for the working fluid together with the required valves and fittings.

High Pressure Storage and Start

These containers are kept nominally charged to a pressure not greater than the high system pressure but at a higher level than the low pressure side. It is noted that the opening valves, E and D, admit high pressure to the expansion side of the engine, and cause rotation which in turn causes circulation through the heater and starts the engine operating. As the unloaded engine comes up to speed, hot-side pressure builds up, and the fluid admitted to the system to start the engine is returned to the storage and start bottles.

If additional fluid is to be added, it is now admitted to the lower pressure cool-side by opening valves J and G. It cannot be added through E and D because hot-side pressure now exceeds the storage pressure. If fluid is to be removed, it is allowed to flow through E and D. Safety valve C is provided to relieve excessive pressure into storage.

Salvage System

This system is not needed on small portable units for intermittent use. It is recommended for continuous service in remote areas where a gradual loss of working fluid is not to be tolerated. The system involves secondary backup seals on shafts and valve stems, creating a space in which to confine, at low pressure, any leakage past primary high pressure seals, and to store the leakage in the low pressure storage containers.

When it is desirable to return low pressure fluid back to the working system or back to storage, M is closed, K is opened, and momentary valve N is closed for a period of 1-3 engine piston strokes, drawing low pressure fluid into compression chambers and discharging it into the engine system. Since engine system flows are upset during this salvage period, N opens momentarily and the engine system restabilizes itself. Since this process is expected to occur only periodically at an interval of several hours, it may be desired to be performed manually during periods of light engine load and reduced speed.
Commercial Fluid Addition

This system is provided to make possible the addition of fluid from commercial shipping containers when the cool-side pressure in the engine system is above the pressure of commercial shipping bottles. Valve J is opened, and by the use of the momentary valve, the fluid is pumped into the system in the same manner as salvage fluid is regained. By opening valves D and E, as required, the high pressure start and storage bottles may be recharged.

Purge System

The engine system will be initially purged of air by passing the working fluid through the lines and heat exchangers, and venting to the atmosphere. The final purge will be accomplished by some type of oxygen "getter," such as copper, lithium, etc., in a cartridge in parallel with the working fluid. This will remove final traces of oxygen and prevent internal corrosion and reaction in the lubricant.

Generator and Controls

The 2000 KW electrical system is conventional so far as electrical output is concerned and is similar to the existing system. It is possible that the generating system will be arranged so that the engine may be motored by the generator which is, in turn, powered by the standby Diesels for purposes of starting, purging or checkout after maintenance, without activation of the primary system.

Cooler System

Cooling of the engine fluid is presently planned in a manner like the existing PM-2A system, using blowers and air-to-fluid heat exchangers. No glycol system is needed, however.

Diesel Backup System

No changes are planned for the standby generating capacity. It is anticipated that after initial startup, their only operation over long periods will be scheduled preservation runs to keep them in a state of readiness.
Control System

The basic control for speed governing, starting and stopping, is the throttle valve (A) in the high pressure line to the engine expansion chambers.

The throttle valve control is supplemented by a cooler bypass control (M). It is noted that decreasing the heat rejection of the cool, low pressure side reduces the power output without seriously affecting efficiency. Bypassing the cooler achieves this purpose; the mixture of warm exhaust fluid with cold fluid from the cooler is at a higher temperature at compression. At the same time, fluid travels through the cooler more slowly, and would be cooled to a lower temperature if the cooling air flow remained constant. It is intended, therefore, to reduce the blower output to maintain a constant temperature of the fluid out of the cooler. This will produce a small but useful gain in efficiency at part load by reducing the auxiliary power required to run the blowers. It is intended that the throttle be capable of handling rapid fluctuation in power demand and that the cooler bypass valve will follow throttle action. As the bypass control effect is felt, the throttle valve returns to a more open position and the pressure drop and efficiency loss at the throttle are reduced to a minimum.

Reduction in power, followed by throttle closure, will initiate bypass valve opening after a suitable time delay. Increase in power demand, followed by wide open throttle, will be immediately followed by closure of the bypass valve and increased cooling.

SYSTEM OPERATION PM-2A

Initial Startup

It is assumed that the complete plant has been assembled, the primary system is in readiness, and the fluid engine power conversion installation is complete. The system must then be pressurized and put into operation. Purging, charging, and two methods of starting will be described. It is assumed that sufficient auxiliary power is available to rotate the engine under some pressure but without performing any compression function.
Purging

The removal of air from the system is necessary for proper behavior of the fluid cycle and to remove the possibility of internal corrosion and oil oxidation. Air purging is done with the working fluid. It is necessary to motor the engine for proper purging of passages and cylinders. Also, by motoring the engine, the working fluid is circulated through heat exchanger tubes, and will pick up and carry along air which might otherwise collect in heaters and remain dormant.

Because the displacement of the expansion piston is greater than that of compression, the engine will try to pump fluid into the low pressure side and build up pressure. This will create a back pressure and a deterrent to motoring freely. Therefore, the motoring bypass loop valve (F) is opened and excess fluid merely circulates through the expansion chambers, while the volume displaced by the compression system will circulate through the system.

The system is charged to 400-500 PSI, motored at some convenient speed and, after running for a period of minutes, the fluid is allowed to escape from any convenient valve, carrying with it entrained air. This process is repeated three or four times until fluid samples indicate a minimum quantity of the residual oxygen by chemical test.

Pressure must not be allowed to drop to atmospheric when motoring the engine, or the piston and balls will not be compelled by pressure to follow the cam and might suffer damage.

After the engine is running normally, the "getter" cartridge in parallel with some portion of the fluid flow, is used to scrub out the residual oxygen.

Charging

There are three charge-and-start procedures, as described below.

1. High pressure bottle source - with primary system hot.

When bottled fluid at nearly the maximum cycle pressure is available and the primary system is hot, fluid may be charged into the engine intake through E and D with the throttle A closed, causing the engine at first to run like a simple air motor. Pressure will be built up in the hot-side heat exchanger. When at an operating level the throttle is opened, E and D are closed, and the engine will continue to run. Further additions of fluid may
now be made into the low pressure intake to compression. This charge-and-start procedure does not require motoring.

2. Fluid available at about cold side pressure, but insufficient to cause rotation.

Starting with the primary system cold, fluid is charged into the engine until both hot and cold sides are at cold-side pressure. The throttle valve is closed and the valves to the heat exchanger and steam generator are closed. Primary heat is added until fluid trapped in the heat exchanger reaches operating pressure. The throttle valve and the valve from compression are opened together and the trapped high pressure causes engine rotation.

3. Fluid available at lower than cold side pressure - insufficient pressure to charge engine for operation.

Fluid is charged into the compression intake while the engine is motored by external power. By repeated closure of momentary valve, low pressure is gradually pumped to a working charge pressure. The motoring bypass loop valve is open during this period. When sufficient charge pressure is reached, the starting procedure of the previous section 2 is employed.

Routine Starts and Stops

Routine engine starting and stopping depend on maintaining the high pressure storage and start containers at near to maximum operating pressure. The containers are kept charged by bleeding hot high pressure fluid into them as required.

Fluid additions to the system, while charging the storage and start bottles, are made into the compression intake line from bottles at commercial pressure levels, if cold-side pressure permits. If cold-side pressure is above bottle pressure, the momentary valve technique is used.

After checking the charge in the storage bottles, the engine is stopped by closing the throttle. Primary heat is cut back to avoid excessive pressure in the heat exchanger. Pressure trapped in the heat exchanger loop is available for an immediate start at any time by opening the throttle.
Speed and Power Control

The piston type liquid engine is inherently capable of operating with a virtually flat torque curve throughout its entire speed range. If locomotive engine type application is required, manual control from dead stop, through a low speed "lugging period" and up to any desired speed, is feasible. However, for electric power generation where frequency must remain constant while power requirements fluctuate, automatic governing will be necessary.

Automatic governing will sense RPM and, in turn, cause response in the throttle and bypass control. Reactor output, in turn, must be automatically modulated. Sophistication in automatic control will be dictated by any frequency excursion permitted. For example, torque sensing in the generator drive could enable the control system to "anticipate" a speed change and initiate a control response before a speed sensitive device could act.

Servicing

Servicing, refueling, and other reactor maintenance will be as presently performed and will not be discussed further here. "Engine" service will include filter changes, fluid additions, and maintenance of valves, solenoids, hydraulic components and accessories, as required.

Spare parts will be an important consideration, especially in a remote, inaccessible location. Thought must be given to standardizing control and accessory components as much as possible in order to reduce the inventory of separate items and to minimize special tooling. Electrical servicing will be the same as in the present system.

Operating Manpower

No rigid manpower schedules are offered. In its simplest form, the engine and generating system could require two men constantly in attendance, manually managing the working fluid, overseeing speed control, monitoring gages, keeping records, etc. This is in addition to manpower requirements for managing the reactor. This would constitute three technicians and a supervisor per shift, or twelve men per day, with a backup crew for time off and sickness. From this viewpoint, a crew of sixteen to twenty men would be required.
At the opposite extreme, using sophisticated instrumentation, controls and recorders, the system could operate for considerable periods entirely unattended, as some present day power plants operate. In this approach, a crew of three to four men, on call but not on duty, would be sufficient. Maintenance would be scheduled as a part time operation on a single shift, and the system could function independently through the second and third shifts.

The desired balance between manpower and automation should be discussed and studied for the specific application.

SYSTEM SCHEMATIC

The schematic relation of components in the system is shown in Figure 7-6. All valves have been previously discussed in the text. It is noted that the main fluid lines contain but three valves, one of which is (L), a normally open check valve, while a second valve (A) serves as a throttle.

Other lines, as used in fluid management and bypassing, are smaller than the main lines, and are not so critical insofar as pressure drop is concerned. It is assumed that commercial valves suitable for this service already exist. It is expected that existing valves, made for high working pressure, low pressure-drop steam service, may be used in the main lines.

It may be desirable to operate valves, position indicators, meters, etc. from a battery system rather than from plant output or standby power. This would insure the necessary control function in an emergency, prior to startup of Diesel standby power.

ML-1 SYSTEM

The existing ML-1 system employs relatively new reactor technology which utilizes a water moderated reactor cooled by nitrogen gas, and operated at high temperatures. The system is quite simple in that it comprises of a single loop with an integral turbine-compressor set for generating electricity in a high speed alternator. The system is transportable by several types of aircraft, railroad flat car, standard army trailers, ship or barge. This mobile, closed cycle, gas-turbine power plant is designed to produce 300 to 500 KW in various environments with ambient temperatures of -65 to 100°F.
The existing system performance characteristics are presented in the ML-1 design report, and may be summarized as follows:

Electric power - gross .................... 400 KW
Electric power - net ........................ 330 KW
Reactor thermal power .... 3.0 MW to gas; ... 3.4 MW Total
Primary coolant - nitrogen
    Flow .............................. 95,000 Lb/Hr
    Max. temperature .......... 1200°F
    Min. Temperature .......... 800°F
    Design pressure .......... 345 psia
    Design temperature ..... 400°F
Plant thermal efficiency .......................... 9.7%
Cycle efficiency .............................. 13.3%
Reactor average heat flux ...................... 77,000 BTU
Ambient coolant - air at max. ... 100°F, Min. - 65°F Hr Ft²

System is cooled by air at temperatures stated above.

In essence, the existing ML-1 system employs a nitrogen-cooled heterogeneous nuclear reactor for a continuous supply of heat energy in the primary loop, and a turbine operating on a Brayton cycle for generation of power through a direct expansion of the reactor coolant supplied.

The proposed new ML-1 system will employ the existing nuclear reactor, including the accessories for heat generation in the primary loop and an engine operating on a Brayton cycle which employs CO₂ as a working fluid at supercritical pressures and temperatures in the secondary loop. The use of CO₂ at supercritical conditions in the primary loop is not permissible, as the present reactor design can not tolerate pressures higher than the 300 psi. This eliminates the use of CO₂ as a primary reactor coolant and restricts the possible competition of CO₂ vs nitrogen operating in a simple one loop reactor system.

The introduction of the secondary loop will invariably make the CO₂ - reactor system less attractive from a complexity point of view. Based on the known heat transfer properties of nitrogen, which are poor, one can conclude immediately that the binary system will be non-competitive weight-wise in comparison with systems employing a single loop design. In particular, this is true in using CO₂ at pressures which are 10 or more times larger than the pressure used in the present nitrogen system. However, an attempt will be made to analyze the ML-1 system for use with CO₂ working fluid in the secondary loop because of the higher overall cycle efficiencies attainable.
Under the circumstances, the proposed CO\textsubscript{2} - reactor system will concern itself only with the secondary loop, the only major change the ML-1 system will undergo. The design of the secondary loop will consider the present capabilities of the primary loop including the existing reactor design. Hence, the operation of the primary loop and the reactor control, as well as the system respond characteristics, will be identical to the characteristics described in the report IDO-28550. The system proposed, however, will require completely new instruments and controls. A simplified system diagram identifying basic secondary loop components, including system controls, is shown in Figure 7-7.

Primary Loop

Detailed discussion of the primary loop is believed to be unnecessary here, as report IDO 28550 covers all pertinent details of the primary loop. It should be mentioned, however, that the CO\textsubscript{2} reactor system will require no recuperator, which at present is used in the existing ML-1 system. Instead, it will use a counterflow heat exchanger which will be covered in the description of the secondary loop. As stated in the introductory chapters of this report, the primary loop will require a new, specially designed recirculator-compressor instead of the present turbine-compressor set. Estimates indicate that this recirculator will require a motor of 450 hp capacity. The proposed ML-1 system will require an alternator supplying 794 KW instead of 400 KW presently in use, and an engine of 814 KW power capacity. Finally, the control of the reactor in the new system will have to be coordinated with the overall system control as shown in Figure 7-7.

Secondary Loop

The secondary system of the ML-1 reactor-CO\textsubscript{2} power plant will be very similar to the PM-2A system. The only differences in these two systems is that the secondary loop of the ML-1 system will require no evaporator and that the equipment for the ML-1 system will be smaller because of the smaller system output. The detailed description of the secondary loop components on the ML-1 system is omitted in order to prevent repetition. The CO\textsubscript{2} system engine will be designed to supply 814 KW power of which 394 KW will be used for the recirculator.
FIGURE 7-7 SIMPLIFIED SYSTEM DIAGRAM AND CONTROLS
ML-1 SYSTEM
CHAPTER 8
SYSTEM OPTIMIZATION AND COMPARISON

GENERAL

Since no changes are contemplated in the reactor portions of either PM-2A or ML-1 systems, the optimization of the proposed system involves screening of the computer data which set forth the calculated results on the fundamental equipment to be employed with CO₂ in the secondary loop. The equipment calculated by the computer is as follows:

1. CO₂ heater employing primary fluid in the shell, and the secondary fluid, CO₂, in the tubes, and comprising a heat exchanger consistent with general practice.

2. CO₂ cooler employing secondary fluid in the tubes, externally finned and cooled by air, and comprising a cooler consistent with general practice.

Additional equipment, such as the evaporator, the accumulator and the piping are more or less dependent on the choices of the heater and cooler. These choices, in turn, define the thermodynamic cycle, including the CO₂ specific volumes that are necessary for sizing all other system components. The emphasis here will be placed primarily on the heater and cooler selection.

Final Selection of Heater-Cooler

Each power plant under consideration could operate on anyone of a certain number of chosen cycles which were carefully selected thru the process of elimination. The selection of these cycles was based on the following characteristics:

1. Reasonably good theoretical cycle efficiency
2. Reasonable power density
3. Reasonably low working pressures in order to stay close to the existing practice in the component fabrication and to reduce possible materials problems.
4. Overall gains in system efficiency when compared with the present system.
With reference to the data on cycle analysis, six cycles were chosen for heater-cooler optimization for the PM-2A system and six cycles were chosen for the ML-1 system. These cycles were then used for sizing the heaters and the coolers. The results of the heater-cooler calculations performed by the computer are tabulated in Table III. As seen from Table III, the best cycle for the PM-2A system appears to be Cycle 8. This cycle requires the nearly least amount of primary coolant flow, namely 468.3 lbs/sec. of the reactor water instead of the *540 lbs/sec. presently used. It also yields the highest ideal thermodynamic cycle efficiency, namely, 25.45%. Although the cooler for this cycle is found to be larger than some of the other cycle coolers, the advantages in efficiency and in reduction of the primary coolant flow more than offset the disadvantages cited in the cooler. Cycle 8 will enable a reduction of the reactor power in the proportion of the coolant flow requirements, namely, \(468.3/540 = 0.87\), which in turn will lower the size of the primary coolant loop, increase the life of the reactor and render the overall system more efficient. The power for the primary coolant flow pump will be reduced in the same proportion from the present 47 hp to a new 40.8 hp. Consequently, Cycle 8 could be considered the optimum for the PM-2A system. This, in turn, optimizes the fundamental system components, namely, the heater and the cooler.

For the ML-1 system, cycle 133 was similarly chosen as it offers both the highest efficiency and the other gains associated with it.

It must be noted here that the ML-1 data presented in Table IV is based on 95% efficient processes of adiabatic compression and expansion in the fluid engine, whereas the PM-2A system calculations were based on 90% efficiency. Using 90% adiabatic expansion and compression efficiencies for the ML-1 system, the overall cycle efficiency would assume the following values:

<table>
<thead>
<tr>
<th>Cycle</th>
<th>123</th>
<th>124</th>
<th>132</th>
<th>276</th>
<th>278</th>
</tr>
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<tbody>
<tr>
<td>(\eta)</td>
<td>19.6</td>
<td>12.1</td>
<td>19.3</td>
<td>2.4</td>
<td>14.8</td>
</tr>
</tbody>
</table>

Since the ML-1 system with CO\(_2\) was considered undesirable due to the system complications and an increase in weights over those of the existing system, no correction for efficiency were made on the heater and cooler data shown in Table IV.

* Report APAE 49, Pg. 1-12
<table>
<thead>
<tr>
<th>CYCLE NUMBER</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
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<td>CYCLE PRESSURES, psi</td>
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<td>4050/900</td>
<td>4500/1000</td>
<td>4000/800</td>
<td>4500/900</td>
<td>5000/1000</td>
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<td>CYCLE EFFICIENCY (Ideal)</td>
<td>17.92</td>
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<td>18.9</td>
<td>18.01</td>
<td>18.01</td>
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<tr>
<td>OVERALL EFFICIENCY</td>
<td>23.89</td>
<td>24.18</td>
<td>25.3</td>
<td>25.45</td>
<td>25.3</td>
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<td>Flow lb/sec.</td>
<td>CO₂</td>
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<td>81.9</td>
<td>92.2</td>
<td>75</td>
<td>80.5</td>
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<td>495</td>
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<td>468.3</td>
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<td>Temp. °F</td>
<td>CO₂</td>
<td>Tin</td>
<td>135</td>
<td>148</td>
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<tr>
<td></td>
<td>Tout</td>
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<td>450</td>
<td>450</td>
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<td>Velocities fps</td>
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<td>20</td>
<td>20</td>
<td>20</td>
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<td>Shell Size</td>
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<td></td>
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<td>.375</td>
<td>.375</td>
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<tr>
<td></td>
<td>I. D. In.</td>
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<td>.288</td>
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<td>7173</td>
<td>6817</td>
<td>6929</td>
<td>6798</td>
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<td>Design Parameter</td>
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<td>4500</td>
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<td>Stress</td>
<td>Tube</td>
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<td>17500</td>
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<td>13000</td>
<td>13000</td>
</tr>
<tr>
<td>Temp. °F</td>
<td>Air</td>
<td>In/Out</td>
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<td>35/115</td>
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<td>In/Out</td>
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<td>211/70</td>
<td>213/80</td>
<td>194/60</td>
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<td>CO₂ in tub</td>
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<td>8</td>
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<td>.325</td>
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<td>.332</td>
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<td>.375</td>
<td>.375</td>
<td>.375</td>
<td>.375</td>
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<td>Wide-In.</td>
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<td></td>
<td>L Ft</td>
<td>4x21.7'</td>
<td>3x25'</td>
<td>3x21'</td>
<td>4x23.7'</td>
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<td>20876</td>
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<td>21494</td>
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* Selected Cycles

(1) All calculations are based on 90% expansion & 90% compression efficiency.
### TABLE IV
HEATER-COOLER DATA FOR ML-1 SYSTEM(2)

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<thead>
<tr>
<th>CYCLE NUMBER</th>
<th>133</th>
<th>123</th>
<th>124</th>
<th>132</th>
<th>276</th>
<th>278</th>
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<td>CYCLE PRESSURES, psi</td>
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<td>3500/500</td>
<td>5000/500</td>
<td>5000/800</td>
<td>5000/800</td>
</tr>
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<td>CYCLE EFFICIENCY (Ideal)</td>
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<td>32.34</td>
<td>38.59</td>
<td>31.66</td>
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<td>26.03</td>
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<td>24.69</td>
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</table>

<table>
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<tr>
<th>Flow lb/sec.</th>
<th>CO2</th>
<th>24.81</th>
<th>43.8</th>
<th>23.4</th>
<th>58.55</th>
<th>40.2</th>
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<td></td>
<td>Coolant</td>
<td>24.016</td>
<td>26.4</td>
<td>26.6</td>
<td>24.45</td>
<td>27.84</td>
<td>28.13</td>
</tr>
<tr>
<td>Temp°F CO2</td>
<td>Tin</td>
<td>542</td>
<td>475</td>
<td>474</td>
<td>542</td>
<td>452</td>
<td>452</td>
</tr>
<tr>
<td></td>
<td>Tout</td>
<td>900</td>
<td>700</td>
<td>900</td>
<td>700</td>
<td>700</td>
<td>1000</td>
</tr>
<tr>
<td>Velocities fps</td>
<td>CO2 in tube</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>Coolant in tube</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Shell Size</td>
<td>O. D. In.</td>
<td>15.78</td>
<td>15.83</td>
<td>12.19</td>
<td>17.83</td>
<td>14.25</td>
<td>10.54</td>
</tr>
<tr>
<td></td>
<td>I. D. In.</td>
<td>15.11</td>
<td>15.15</td>
<td>11.66</td>
<td>17.06</td>
<td>13.63</td>
<td>10.08</td>
</tr>
<tr>
<td>Tube Size</td>
<td>O. D. In.</td>
<td>0.375</td>
<td>0.375</td>
<td>0.375</td>
<td>0.375</td>
<td>0.375</td>
<td>0.375</td>
</tr>
<tr>
<td></td>
<td>I. D. In.</td>
<td>0.258</td>
<td>0.290</td>
<td>0.290</td>
<td>0.258</td>
<td>0.258</td>
<td>0.258</td>
</tr>
<tr>
<td></td>
<td>Length Ft</td>
<td>10.3</td>
<td>6.6</td>
<td>15</td>
<td>5.24</td>
<td>8.0</td>
<td>24.2</td>
</tr>
<tr>
<td>Total Heater</td>
<td>Vol. Ft³</td>
<td>15.4</td>
<td>9.9</td>
<td>13.3</td>
<td>10</td>
<td>9.8</td>
<td>16.12</td>
</tr>
<tr>
<td></td>
<td>Wt. Lb</td>
<td>2688</td>
<td>1641</td>
<td>2210</td>
<td>1746</td>
<td>1710</td>
<td>2816</td>
</tr>
<tr>
<td>Design</td>
<td>Shell psi</td>
<td>5000</td>
<td>3500</td>
<td>3500</td>
<td>5000</td>
<td>5000</td>
<td>5000</td>
</tr>
<tr>
<td>Parameter</td>
<td>T&amp;P °F</td>
<td>1200</td>
<td>1200</td>
<td>1200</td>
<td>1200</td>
<td>1200</td>
<td>1200</td>
</tr>
<tr>
<td>Temp. °F Air In/Out</td>
<td>100/220</td>
<td>100/220</td>
<td>100/220</td>
<td>100/220</td>
<td>100/220</td>
<td>100/220</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CO2 In/Out</td>
<td>420/150</td>
<td>329/150</td>
<td>490/150</td>
<td>266/150</td>
<td>343/150</td>
<td>591/150</td>
</tr>
<tr>
<td>Veloc. fps</td>
<td>CO2 in tube</td>
<td>45</td>
<td>30</td>
<td>45</td>
<td>45</td>
<td>45</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>Air</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Tube Size</td>
<td>I. D. In.</td>
<td>0.350</td>
<td>0.350</td>
<td>0.294</td>
<td>0.350</td>
<td>0.331</td>
<td>0.245</td>
</tr>
<tr>
<td></td>
<td>O. D. In.</td>
<td>0.375</td>
<td>0.375</td>
<td>0.375</td>
<td>0.375</td>
<td>0.375</td>
<td>0.375</td>
</tr>
<tr>
<td></td>
<td>L Ft</td>
<td>22.5</td>
<td>17.7</td>
<td>17.2</td>
<td>21</td>
<td>22.7</td>
<td>18.31</td>
</tr>
<tr>
<td>Cooler</td>
<td>Wide-In.</td>
<td>17.5</td>
<td>27.44</td>
<td>20.54</td>
<td>25.14</td>
<td>17.42</td>
<td>17.96</td>
</tr>
<tr>
<td></td>
<td>L Ft</td>
<td>23</td>
<td>18</td>
<td>18</td>
<td>22</td>
<td>23</td>
<td>19</td>
</tr>
<tr>
<td>Design Stress</td>
<td>psi</td>
<td>7350</td>
<td>7350</td>
<td>7350</td>
<td>7350</td>
<td>7350</td>
<td>7350</td>
</tr>
<tr>
<td>Total</td>
<td>Vol. Ft³</td>
<td>42</td>
<td>80</td>
<td>44</td>
<td>80</td>
<td>42</td>
<td>36</td>
</tr>
<tr>
<td>Cooler</td>
<td>Wt. Lb</td>
<td>1762</td>
<td>3274</td>
<td>2100</td>
<td>3274</td>
<td>1860</td>
<td>1901</td>
</tr>
</tbody>
</table>

* Selected Cycles

(2) All calculations are based on 95% expansion & 95% compression efficiency.
Heater - Cooler Specifics

From the numerous calculations in sizing the heaters and coolers, it was established that the exchange surface is dependent on the tube size. It was found that by maintaining the velocity constant, an increase in the tube diameter requires significantly more exchange surface area. The effect is such that the unit length, the overall size and the weight are all decreasing if smaller diameter tubes are used. Also, the pressure drop appeared to follow the same reducing pattern with reducing tube diameter. Consequently, the smallest practical tube diameter was selected from among the sizes considered. As an example, using 1/2" and 1/4" OD tubes, while the velocity was held constant, indicated that the following characteristics would result on the equipment sized for a similar application:

1/2 inch OD at \( V = 15.7 \) fps, required 171 ft. tube while \( \Delta P = 67 \) psi
1/4 inch OD at \( V = 15.7 \) fps, required 73 ft. tube resulting in \( \Delta P = 58 \) psi

The 1/2 inch tube exchanger requires less joints in the tube sheet by approximately 40%. In order to reduce fabrication problems, it was decided to compromise and to select 3/8 inch OD diameter tubes for this project.

Employing 3/8 inch OD tubes, the heater and cooler data summarized in Tables III and IV was further analyzed in order to establish equipment packaging density as well as other specifics. Table V summarizes heater and cooler specifics, identifying the specific weights, the volumes and also the packaging density in terms of exchange surface in \( \text{Ft}^2 \) versus the packaging volume in \( \text{Ft}^3 \). The packaging density, \( P = (\text{Ft}^2/\text{Ft}^3) \), depends on the physical constants, such as the tube OD and the pitch only. In our case, for the fixed tube diameter and the fixed size of fins, the following relationship could be established:

\[
A = \frac{Q}{U} (\text{LMTD}) (\text{Ft}^2) \\
\text{Packaging Density, } P = \frac{A}{V} (\text{Ft}^2/\text{Ft}^3) \\
V = \frac{Q}{U} (\text{LMTD}) P (\text{Ft}^3)
\]

Employing specific weights and packaging density shown in Table V, further sizing of the heaters and coolers utilizing 3/8 inch finned or unfinned tubes will then be simple to calculate. The only additional calculation that the heaters or coolers will require is the establishment of the exchange area, \( A \).
<table>
<thead>
<tr>
<th>Item</th>
<th>HEATER - 3/8 OD x 563&quot; Pitch</th>
<th>COOLER - 3/8 OD x 1.25 &amp; Fins x 0.012th Pitch 11 Fins/inch, Pitch .85&quot; Air Cooled</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cycle</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>$\frac{Q}{HR} \times 10^6$</td>
<td>37.7</td>
<td>37.4</td>
</tr>
<tr>
<td>LMTD</td>
<td>176.7</td>
<td>172.7</td>
</tr>
<tr>
<td>$A (Ft^2) \times 10^3$</td>
<td>.403</td>
<td>.411</td>
</tr>
<tr>
<td>$\frac{U}{HR Ft^\circ F}$</td>
<td>661</td>
<td>658</td>
</tr>
<tr>
<td>W (Lb)</td>
<td>7420</td>
<td>7174</td>
</tr>
<tr>
<td>V (Ft')</td>
<td>44.23</td>
<td>42</td>
</tr>
<tr>
<td>$S = \frac{W (Lb)}{V (Ft')}$</td>
<td>168</td>
<td>171</td>
</tr>
<tr>
<td>$P = \frac{A (Ft^2)}{V (Ft')}$</td>
<td>9.1</td>
<td>9.8</td>
</tr>
</tbody>
</table>

---

<table>
<thead>
<tr>
<th>Item</th>
<th>PM-2A</th>
<th>103</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cycle</td>
<td>278</td>
<td>276</td>
</tr>
<tr>
<td>$\frac{Q}{HR} \times 10^6$</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>LMTD</td>
<td>240</td>
<td>378</td>
</tr>
<tr>
<td>$A (Ft^2)$</td>
<td>2500</td>
<td>1550</td>
</tr>
<tr>
<td>$\frac{U}{HR Ft^\circ F}$</td>
<td>256</td>
<td>267</td>
</tr>
<tr>
<td>W (Lb)</td>
<td>2816</td>
<td>1710</td>
</tr>
<tr>
<td>V (Ft')</td>
<td>16.1</td>
<td>10</td>
</tr>
<tr>
<td>$= \frac{W (Lb)}{V (Ft')}$</td>
<td>175</td>
<td>171</td>
</tr>
<tr>
<td>$P = \frac{A (Ft^2)}{V (Ft')}$</td>
<td>156</td>
<td>155</td>
</tr>
</tbody>
</table>
Other Component Sizing

This pertains to the secondary system components in the CO$_2$ system, namely, the accumulator, the evaporator and the piping for the CO$_2$ loop. All of these components were calculated employing fluid conditions defined in cycle 8 for the PM-2A system, and the results are summarized in Table VI. Table VI identifies weights and volumes of all PM-2A system components removed from the existing system, and indicates new components placed instead. Table VII indicates weights and volumes of all ML-1 major components removed from the existing system and indicates new components placed instead.

SYSTEM SUMMARY

As seen from Table VI the PM-2A system using CO$_2$ will require considerably fewer system components and, as such, will be a more reliable and less expensive system. The data shown in Table VI indicates the improvement that CO$_2$ offers for power generation in the PM-2A system. Although the overall thermal efficiency of cycle 8 is only 19%. The overall plant efficiency of the PM-2A system using CO$_2$ was estimated to be 15.75% as follows:

Required gross electric power output 2000 kw
Existing net electric power output 1560 kw
Power required for auxiliaries 440 kw
Power reduction using CO$_2$ system 90 kw
Auxiliary power using CO$_2$ system 350 kw
CO$_2$ system net power output 1650 kw
CO$_2$ system gross power input (2000 kw/.19 eff.) 10550 kw
CO$_2$ system overall plant efficiency 15.75%
(1650 kw/10550 kw = 15.75%)
Existing system overall plant efficiency 15.60%
(1560 kw/10000 kw = 15.60%)

Table VII exhibits characteristics for the ML-1 system. The ML-1 system employing CO$_2$ in the secondary loop cannot be considered competitive with the existing system in weight, simplicity and, consequently, reliability, as seen from the complexity in components shown in Table VII. Due to the disadvantages a binary ML-1 system would present, the CO$_2$ working fluid for this system is not recommended, unless the emphasis is shifted to the operational gains which the CO$_2$ cycle offers by means of efficiency increase. Under the circumstances, any further discussion of the ML-1 system is omitted.
TABLE VI - PM-2A SUMMARY
REPLACEMENT OF MAJOR COMPONENTS

<table>
<thead>
<tr>
<th>EQUIPMENT REMOVED FROM EXISTING SYSTEM</th>
<th>NEW EQUIPMENT PLACED IN CO₂ SYSTEM</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Component</strong></td>
<td><strong>Component</strong></td>
</tr>
<tr>
<td><strong>Lb. Ft³ Dollars Hp</strong></td>
<td><strong>Lb. Ft³ Dollars Hp</strong></td>
</tr>
<tr>
<td>1. Steam Generator 17600 620 100,000 -</td>
<td>CO₂ Exchanger 6798 40 34,000 -</td>
</tr>
<tr>
<td>2. Turbine-Generator 3280 225,940 -</td>
<td>Engine-Generator 11420 178 260,000 -</td>
</tr>
<tr>
<td>3. Condenser Package 28250 1850 37,180 -</td>
<td>CO₂ Supply 11000 332 6,670 -</td>
</tr>
<tr>
<td>4. Glycol Air Coolers 75600 5950 300,000 -</td>
<td>CO₂ Coolers 21500 503 195,000 -</td>
</tr>
<tr>
<td>5. Evaporator 2280 80 20,000 -</td>
<td>Evaporator 1000 21 8,800 -</td>
</tr>
<tr>
<td>6. Feed Water Package 28900 2220 36,280 -</td>
<td>Feed Water Package 8000 62 10,000 -</td>
</tr>
<tr>
<td>7. Glycol 55245 1673 8,850 -</td>
<td>Glycol 2130 64 532 -</td>
</tr>
<tr>
<td>8. Expansion Tanks-Glycol 9220 787 15,000 -</td>
<td>Exp. Tank-Glycol 354 30 580 -</td>
</tr>
<tr>
<td>9. Primary Coolant Pump 7160 206 - 47</td>
<td>Primary Coolant Pump 7160 206 - 41</td>
</tr>
<tr>
<td>10. Glycol Pumps Main - - - - 50 - - - -</td>
<td>- - - - - - - - - -</td>
</tr>
<tr>
<td>11. Condensate Pump - - - - 25 - - - -</td>
<td>- - - - - - - - - -</td>
</tr>
<tr>
<td>12. Boiler Feed Pump - - - - 40 - - - -</td>
<td>- - - - - - - - - -</td>
</tr>
<tr>
<td>13. Transmission Lines - - - - - - - -</td>
<td>CO₂ Transmission Lines 9770 49 19,540 -</td>
</tr>
<tr>
<td>14. - - - - - - - - - - - - - - - -</td>
<td>CO₂ Accumulator 373 51 3,730 -</td>
</tr>
<tr>
<td><strong>Subtotal</strong> 293255 1666 $743250 162</td>
<td><strong>Subtotal</strong> 82205 1536 $538,852 -</td>
</tr>
<tr>
<td><strong>Less</strong> 90425 1689 592,737 41</td>
<td><strong>Total</strong> 90425 1689 $592,737 41</td>
</tr>
<tr>
<td><strong>Difference or Net Gain</strong> 202830 14977 150,513 121</td>
<td></td>
</tr>
</tbody>
</table>

(1) $85/Ft² cost from Chem. Engrg. Progress Pg. 103, Vol. 57, No. 12, Dec. 1961
(2) $2.5/Ft² estimate based on cost (Fin area) shown in Chem. Engrg. Progress Pg. 104, Vol. 57, No. 12, Dec. 1961
(3) Includes 1 full scale engine and generator and a reduced size prototype engine system to check out design.
(4) Estimated by prorating weights between existing and CO₂ systems.
(5) Prorated on duty basis $1.1 \times 10^6 \cdot 0.385
(6) Glycol cost on this quantity $.16/Lb. ($ .25/Lb for less) obtained from Union Carbide Chem. Div., Cleveland
(7) Estimated.
**TABLE VII - ML-1 SUMMARY**

**REPLACEMENT TABLE FOR WEIGHTS & VOLUMES OF MAJOR COMPONENTS**

<table>
<thead>
<tr>
<th>Component</th>
<th>Wt. (Lb)</th>
<th>Vol. (Ft³)</th>
<th>HP Req'd</th>
<th>Component</th>
<th>Wt. (Lb)</th>
<th>Vol. (Ft³)</th>
<th>HP Req'd</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Turbo-Compressor Set</td>
<td>1600</td>
<td>12.9</td>
<td></td>
<td>Compressor</td>
<td>1600</td>
<td>12.9</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Drive Motor (1)</td>
<td>3500</td>
<td>60.0</td>
<td>450</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Gear Box (2)</td>
<td>450</td>
<td>4.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Engine</td>
<td>1476</td>
<td>21.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Recuperator</td>
<td>3800</td>
<td>79.0</td>
<td></td>
<td>CO₂ Exchanger</td>
<td>2688</td>
<td>15.4</td>
<td></td>
</tr>
<tr>
<td>3. N₂ Coolers</td>
<td>4630</td>
<td>276</td>
<td></td>
<td>CO₂ Coolers</td>
<td>1762</td>
<td>42.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>CO₂ Supply</td>
<td>3000</td>
<td>90</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>CO₂ Accumulator</td>
<td>150</td>
<td>21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Alternator-starting motor (3000 rpm)</td>
<td>3400</td>
<td>22.2</td>
<td></td>
<td>Alternator-starting motor (1800 rpm)</td>
<td>5350</td>
<td>56.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Sub-total</td>
<td>19976</td>
<td>323.6</td>
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<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10% Misc.</td>
<td>1997</td>
<td>32.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>13430</td>
<td>390.1</td>
<td></td>
<td></td>
<td>21973</td>
<td>356.0</td>
<td>450</td>
</tr>
</tbody>
</table>

(1) Drive Motor 1800 rpm Wound Rotor design for speed change, 60 cycle, 3 phase line.

(2) Compressor speed is 12, 350 rpm requiring step up gear box.
PM-2A System Flow Diagram

Employing cycle 8.1, the system flow conditions are illustrated in Fig. 8-1.

System Specifics

Employing Table I, which identifies the existing system weights, volumes and specifics, modifications were made to incorporate new values specified in Tables VI and VII that identify the CO₂ system requirements, and the results are presented in Table VIII.

Table VIII indicates total weights and volumes removed from the existing system and makes corrections accordingly, resulting in new system specifics. Again, improvements in both the overall efficiency and the system simplicity are noted in PM-2A system while employing CO₂ instead of the existing working fluid, whereas ML-1 system shows significant loss in specifics.
FIGURE 8-1 PM-2A SYSTEM FLOW DIAGRAM
TABLE VIII - SYSTEM SPECIFICS; ML-1, PM-2A

<table>
<thead>
<tr>
<th>SYSTEM</th>
<th>PM-2A SYSTEM</th>
<th>ML-1 SYSTEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>ITEM</td>
<td>W(LB)</td>
<td>V(Ft$^3$)</td>
</tr>
<tr>
<td>Existing System</td>
<td>904,105</td>
<td>57,460</td>
</tr>
<tr>
<td>Eliminated</td>
<td>293,255</td>
<td>16,666</td>
</tr>
<tr>
<td>Remainder</td>
<td>610,850</td>
<td>40,794</td>
</tr>
<tr>
<td>New Addition</td>
<td>90,425</td>
<td>1,689</td>
</tr>
<tr>
<td>CO$_2$ System Total</td>
<td>701,275</td>
<td>42,483</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SPECIFICS</th>
<th>Existing</th>
<th>New</th>
<th></th>
<th>Existing</th>
<th>New</th>
</tr>
</thead>
<tbody>
<tr>
<td>LB/$\text{KW}_e$</td>
<td>580</td>
<td>450</td>
<td>201</td>
<td>223</td>
<td></td>
</tr>
<tr>
<td>$\text{Ft}^3$/KWe</td>
<td>36.7</td>
<td>27.2</td>
<td>8.7</td>
<td>8.7</td>
<td></td>
</tr>
<tr>
<td>LB/$\text{Ft}^3$</td>
<td>15.75</td>
<td>16.6</td>
<td>23</td>
<td>25.7</td>
<td></td>
</tr>
<tr>
<td>$\text{Ft}^3$/LB</td>
<td>.0635</td>
<td>.0602</td>
<td>.0435</td>
<td>.039</td>
<td></td>
</tr>
<tr>
<td>KWe/LB</td>
<td>.00172</td>
<td>.0022</td>
<td>.00495</td>
<td>.00446</td>
<td></td>
</tr>
<tr>
<td>KWe/$\text{Ft}^3$</td>
<td>.0271</td>
<td>.0368</td>
<td>.1135</td>
<td>.1145</td>
<td></td>
</tr>
<tr>
<td>$$/\text{KWe}$</td>
<td>1800</td>
<td>1695</td>
<td>7380</td>
<td>- - -</td>
<td></td>
</tr>
<tr>
<td>$$/\text{LB}$</td>
<td>3.1</td>
<td>3.77</td>
<td>36.5</td>
<td>- - -</td>
<td></td>
</tr>
<tr>
<td>$$/\text{Ft}^3$</td>
<td>48.7</td>
<td>62.3</td>
<td>840</td>
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<td>KWe$/ $</td>
<td>.00056</td>
<td>.00059</td>
<td>.000136</td>
<td>- - -</td>
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<tr>
<td>System</td>
<td>15.6%</td>
<td>15.75%</td>
<td>9.7</td>
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</table>

NOTE: Net Electric Power outputs are as follows:

PM-2A Net Output - 1560 KWe
ML-1 Net Output - 400 KWe
Chapter 9

HEAT EXCHANGER THERMODYNAMICS AND DESIGN

In the following pages are given the form of print-out data sheets from IBM 650 computer and the step by step procedure for calculations of various types of heat exchangers used in this analysis.

The various types are:

1. Shell and tube type counterflow heater. Heating medium - pressurized or supercritical water - Set I A.
2. Circular finned tube type crossflow cooler. Cooling medium - air. Set II A.
3. Shell and tube type counterflow cooler. Cooling medium - water. Sets IIIA, IVA.
5. Shell and tube type counterflow heater. Heating medium - liquid metal. Set VI A.

The print-out data sheets include the input data to the computer and the output data calculated by the computer following the given procedure. There are two data sheets for each heat exchanger. All the quantities on the data sheets are self explanatory. Input data items 10 and 11, the maximum allowable stresses, are obtained from the ASME, Unfired Pressure Vessels Code. Some of the input data, particularly the physical parameters of the design such as velocities of the fluids, size and spacing of tubes and fins, etc. which are more frequently changed to obtain a suitable physical design are separated and recorded on the second sheet along with the output data affected by these quantities.

In the calculation procedures, the circled number indicates the input data quantity corresponding to that number, and the circled step number indicates the output data from that output step. e.g.

\[ \text{3} \] means input No. 3 from print-out sheet

\[ \text{Step 12} \] means output of Step 12

The step by step procedure of calculating output is believed to be quite self explanatory. Some further explanation of the relationship used is obtained in Appendix E.
**INPUT DATA**

<table>
<thead>
<tr>
<th>No.</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Theoretical Cycle Eff.</td>
<td>( \eta_{th} ) Percent</td>
</tr>
<tr>
<td>2</td>
<td>Volumetric Efficiency</td>
<td>( \eta_v ) Percent</td>
</tr>
<tr>
<td>3</td>
<td>Mechanical Efficiency</td>
<td>( \eta_m ) Percent</td>
</tr>
<tr>
<td>4</td>
<td>Brake Horsepower</td>
<td>BHP BHP</td>
</tr>
<tr>
<td>5</td>
<td>Work Done/LB of Fluid</td>
<td>W.D. BTU/LB</td>
</tr>
<tr>
<td>6</td>
<td>Heat Add/LB of Fluid</td>
<td>H.A. BTU/LB</td>
</tr>
<tr>
<td>7</td>
<td>Heat Lost/LB of Water</td>
<td>H.L. BTU/LB</td>
</tr>
<tr>
<td>8</td>
<td>Delta Temp TA Entr</td>
<td>( \Delta T_a ) F</td>
</tr>
<tr>
<td>9</td>
<td>Delta Temp TB Exit</td>
<td>( \Delta T_b ) F</td>
</tr>
<tr>
<td>10</td>
<td>Max Allow Stress Tube</td>
<td>ft PSI</td>
</tr>
<tr>
<td>11</td>
<td>Max Allow Stress Shell</td>
<td>fs PSI</td>
</tr>
<tr>
<td>12</td>
<td>Max Pressure of Fluid</td>
<td>psi PSI</td>
</tr>
<tr>
<td>13</td>
<td>Max Pressure of Water</td>
<td>psi PSI</td>
</tr>
<tr>
<td>14</td>
<td>Density of Fluid @ BTP</td>
<td>( \rho_i ) LB/FT³</td>
</tr>
</tbody>
</table>

**FLUID ENTRANCE TEMP.** DEG F  **WATER ENTRANCE TEMP.** DEG F  **FLUID EXIT TEMP.** DEG F  **WATER EXIT TEMP.** DEG F  **FLUID BULK MEAN TEMP.** DEG F  **WATER BULK MEAN TEMP.** DEG F  **FLUID WALL MEAN TEMP.** DEG F  **WATER WALL MEAN TEMP.** DEG F

**OUTPUT DATA**

<table>
<thead>
<tr>
<th>No.</th>
<th>Description</th>
<th>Units</th>
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<tbody>
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<td>1</td>
<td>Overall Efficiency</td>
<td>( \eta_o ) Percent</td>
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<tr>
<td>2</td>
<td>Indicated Horsepower</td>
<td>IHP IHP</td>
</tr>
<tr>
<td>3</td>
<td>Fluid in the System</td>
<td>( W_i ) LB/SEC</td>
</tr>
<tr>
<td>4</td>
<td>Heat Added in Heater</td>
<td>( Q ) BTU/SEC</td>
</tr>
<tr>
<td>5</td>
<td>Ditto X 10³</td>
<td>( G ) BTU/HR</td>
</tr>
<tr>
<td>6</td>
<td>Quan. of Water Req'd</td>
<td>( W_o ) LB/SEC</td>
</tr>
<tr>
<td>7</td>
<td>Log Mean Temp Diff.</td>
<td>LMTD F</td>
</tr>
</tbody>
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### Input Data

<table>
<thead>
<tr>
<th>HEATER NO.</th>
<th>BASED ON CYCLE NO.</th>
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<th>SUBTRIAL NO.</th>
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#### External Dia of Tubes
- **29**: External Dia of Tubes
- **do**: Inches

#### Velocity of Fluid
- **30**: Velocity of Fluid
- **vi**: Ft/Sec

#### Velocity of Water
- **31**: Velocity of Water
- **vo**: Ft/Sec

### Output Data

<table>
<thead>
<tr>
<th>8</th>
<th>Internal Dia of Tubes</th>
<th>di</th>
<th>Inches</th>
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<tr>
<td>9</td>
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<td>X Area All Tubes</td>
<td>Xc</td>
<td>SQ.INS.</td>
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<td>Xo</td>
<td>SQ.INS.</td>
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<td>Total X-Area</td>
<td>X</td>
<td>SQ.INS.</td>
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<td>Internal Dia of Drum</td>
<td>Di</td>
<td>Inches</td>
</tr>
<tr>
<td>14</td>
<td>External Dia of Drum</td>
<td>Do</td>
<td>Inches</td>
</tr>
<tr>
<td>15</td>
<td>Trvse Pitch of Tubes</td>
<td>Sr</td>
<td>Inches</td>
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<td>Lgtndnl Pitch of Tubes</td>
<td>Sl</td>
<td>Inches</td>
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<td>Min Id of Drum By Geo</td>
<td>Dig</td>
<td>Inches</td>
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<td></td>
</tr>
<tr>
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<td>Nusselt No for Fluid @ BTP</td>
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<tr>
<td>20</td>
<td>Heat Trans Coeff Fluid</td>
<td>Ai</td>
<td>BTU/HRFT2F</td>
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<td>Equiv Dia of Tube Bank</td>
<td>de</td>
<td>Inches</td>
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<td>Prandtl No for Water @ BTP</td>
<td>Prwb</td>
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<tr>
<td>23</td>
<td>Heat Trans Coeff for Water</td>
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<td>BTU/HRFT2F</td>
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<td>25</td>
<td>Length of Each Tube</td>
<td>L</td>
<td>FEET</td>
</tr>
<tr>
<td>26</td>
<td>Heat Trans Coeff for Water</td>
<td>Nu</td>
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<tr>
<td>27</td>
<td>Fluid Pressure Loss</td>
<td>Pi</td>
<td>PSI</td>
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<td>28</td>
<td>Water Pressure Loss</td>
<td>Pd</td>
<td>PSI</td>
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<td>Total Length of Tubing</td>
<td>NL</td>
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<td>Total Surface Area</td>
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<td>SQ.FT.</td>
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<td>Fluid Pressure Loss</td>
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<td>HP</td>
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<td>We</td>
<td>LBS.</td>
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<td>HP to Drive Waterpump</td>
<td>Ws</td>
<td>LBS.</td>
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<td>Weight of Tubes</td>
<td>W</td>
<td>LBS.</td>
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<td>35</td>
<td>Weight of Shell</td>
<td>Wx</td>
<td>LBS.</td>
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<td>36</td>
<td>Wt of End Plate Plus Aux.</td>
<td>W</td>
<td>LBS.</td>
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<td>37</td>
<td>Total Weight of Heater</td>
<td>V</td>
<td>CUBIC FT.</td>
</tr>
<tr>
<td>38</td>
<td>Total Volume of Heater</td>
<td>Vd</td>
<td>CUBIC FT.</td>
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SHELL AND TUBE TYPE HEAT EXCHANGER DESIGN CALCULATIONS-
HEATER
HEATING MEDIUM - PRESSURIZED WATER (OUTSIDE TUBES)

PROCEDURE FOR CALCULATING OUTPUT DATA

Step 1. Overall Efficiency
\[ \eta_0 = \frac{\eta_h}{100} \times \frac{\eta_t}{100} \times \frac{\eta_m}{100} \times 100 \]
\[ = \frac{0 \times 2 \times 3}{10,000} \text{ Percentage} \]

Step 2. Indicated Horsepower
\[ IH_P = \eta_h \times \frac{\eta_m}{\eta_0} \]
\[ = \frac{\text{Step 1}}{4} \times \frac{\text{0}}{1} \text{ HP} \]

Step 3. Fluid in the System
\[ W_r = \frac{IH_P \times 42.418}{W.D. \times 60} \]
\[ = \frac{.707 \times \text{Step 2}}{5} \text{ M lb/sec} \]

Step 4. Heat added in the Heater
\[ Q_r = W_r \times H_A \]
\[ = \text{Step 3} \times \text{Step 6} \text{ Btu/sec} \]

Step 5. Heat added in the Heater
\[ Q = 3600 \text{ Q} \]
\[ = 3600 \times \text{Step 4} \text{ Btu/hr} \]

Step 6. Quantity of Water Required
\[ W_0 = \frac{Q}{HL} \]
\[ = \frac{\text{Step 4}}{1} \text{ M lb/sec} \]
Step 7. Log Mean Temperature Difference

\[ LMTD = \frac{\Delta t_A - \Delta t_B}{\ln \Delta t_A / \Delta t_B} = \frac{(\Delta t_A - \Delta t_B)}{\ln \left(\frac{\Delta t_A}{\Delta t_B}\right)} \text{ deg F} \]

Step 8. Internal Dia. of Tubes

\[ d_i = d_o \left[\frac{(ft - pi)/(ft + pi)}{ft}\right]^{1/2} \]

By Lames Thick Cylinder formula

\[ = \frac{29}{2} \left[\frac{(10 - 12)}{(10 + 12)}\right]^{1/2} \text{ inches} \]

Step 9. Number of Tubes

\[ N = \frac{W_t}{6.0 \times \frac{\text{Step 3}}{\text{Step 2}}} \text{ tubes} \]

Step 10. Cross-Sectional Area for All Tubes

\[ x_t = \frac{\pi}{4} d_o^2 N \]

\[ = \frac{\pi}{4} \left(\frac{29}{2}\right) \text{ Step 9} \text{ sq. inches} \]

Step 11. Cross-Sectional Area for Water

\[ x_o = \frac{W_o \times 144}{80 \times V_0} \]

\[ = \frac{144 \times \text{Step 6}}{15} \text{ sq. inches} \]

Step 12. Total Cross-Sectional Area

\[ x = x_t + x_o \]

\[ = \text{Step 10} + \text{Step 11} \text{ sq. inches} \]

Step 13. Internal Diameter of Drum

\[ D_i = \left(\frac{4 \times x}{\pi}\right)^{1/2} \]

\[ = \left(\frac{4}{\pi} \text{Step 12}\right)^{1/2} \text{ inches} \]

Step 14. External Diameter of Drum

By Lames Thick Cylinder Formula

\[ D_o = D_i \left[\frac{fs + po}{fs - po}\right]^{1/2} \]

\[ = \text{Step 13} \left[\frac{10 + 13}{10 - 13}\right]^{1/2} \text{ inches} \]
Step 15. Transverse Pitch of Tubes
\[ S_T = 1.5 \, d_0 \]
\[ = 1.5 \left( \frac{29}{2} \right) \text{ inches} \]

Step 16. Longitudinal Pitch of Tubes
\[ S_L = \frac{\sqrt{3}}{2} \, S_T \]
\[ = \frac{\sqrt{3}}{2} \left( \text{Step 15} \right) \text{ inches} \]

Step 17. Min. Internal Dia. of Drum by Geometry
\[ D_{\text{drum}} = \left( \frac{2 \sqrt{3}}{\pi} \, N \right)^{\frac{1}{2}} \, S_T \]
Ref. 7
\[ = \left( 1.5 \, \text{Step 9} \right)^{\frac{1}{2}} \, \text{Step 15} \text{ inches} \]
assuming efficiency \( \phi = 0.735 \)

Step 18. Reynolds No. for Fluid at Bulk Cond.
\[ \text{Re}_i = \left( \frac{d_i \cdot \rho_i \cdot V_i}{12 \, \mu_{ib}} \right) \]
\[ = \left( \frac{\text{Step 8} \, 14 \, 30}{12 \, 16} \right) \]

Step 19. Prandtl No. for Fluid at Bulk Cond.
\[ \text{Pr}_i = \left( \frac{3600 \, \mu_{ib} \, \mu_{ib}}{\kappa_{ib}} \right) \]
\[ = 3600 \left( \frac{20}{16} \right) \]

\[ \text{Nu}_{i}^' = \text{Nu}_{i}^' \left( \frac{\mu_{ib}}{\mu_{iw}} \right)^{0.11} \left( \frac{\kappa_{ib}}{\kappa_{iw}} \right)^{-0.33} \left( \frac{\text{Pr}_i}{\text{Pr}_{ib}} \right)^{0.35} \]
where
\[ \text{Nu}_{i}^' = \frac{8 \left( 1.82 \, \log \left( \text{Step 13} \right) - 1.64 \right) \left[ \text{Step 18} \, \text{Step 19} \right]}{\left( 1.82 \, \log \left( \text{Step 18} \right) - 1.64 \right)^{2/3} + 1.07} \]
and
Ref. 8
\[ \left( \frac{\mu_{ib}}{\mu_{iw}} \right) = \frac{14}{17}, \quad \left( \frac{\kappa_{ib}}{\kappa_{iw}} \right) = \frac{23}{24}, \quad \left( \frac{\text{Pr}_i}{\text{Pr}_{ib}} \right) = \frac{21}{20} \]

Step 21. Heat Transfer Coeff. for Fluid
\[ h_i = 12 \, \frac{\kappa_{ib}}{d_i} \, \text{Nu}_{i}^' \]
\[ = 12 \, \frac{23}{\text{Step 8}} \, \text{Step 20} \quad \text{BTU/hr ft}^2 \text{°F} \]
Step 22. Equivalent Dia. for Tube Bank
\[
de = \frac{4 \times 0}{\pi d_{o1}} = \frac{4 \text{ Step 11}}{\pi (29 \text{ Step 9})} \text{ inches}
\]

Step 23. Reynolds No. for Water at Bulk Conditions
\[
Re_{ob} = \frac{d_{o1} \rho_0 V_o}{12 \mu_0} = \frac{\text{ Step 22}}{15 \frac{31}{12} \frac{18}{12}}
\]

Step 24. Prandtl No. for Water at Bulk Conditions
\[
Pr_{ob} = 3600 \frac{c_{ob} \mu_0}{K_{ob}} = 3600 \frac{18}{25}
\]

Step 25. Nusselt No. for Water at Bulk Conditions
\[
Nu_0 = 0.023 (Re_{ob})^{0.8} (Pr_{ob})^{1/3} \left( \frac{\mu_0}{\mu_{ow}} \right)^{0.14}
\]
Ref. 9 & 10
\[
= 0.023 \left( \text{ Step 23} \right)^{0.8} \left( \text{ Step 24} \right)^{1/3} \left( \frac{18}{19} \right)^{0.14}
\]

Step 26. Heat Transfer Coeff. for Water
\[
\delta_0 = \frac{12 \mu_0}{d_{o1} N_{u0}} = \frac{12 \text{ Step 25}}{15 \frac{31}{12} \text{ Btu/hr ft}^2 \text{ °F}}
\]

Step 27. Overall Heat Transfer Coeff. Based on Outside Dia. of Tube
\[
U = \frac{1}{\frac{1}{h_0} + \frac{d_{o1}}{24} \frac{ln(d_{o1}/d_i)}{K_t}} = \frac{1}{\frac{1}{\text{ Step 26}} + \frac{29}{24} \frac{ln(69)}{26} + \frac{29}{\text{ Step 21} \text{ Step 27}}}
\]

Step 28. Length of Each Tube
\[
L = 1.25 \times \frac{12}{11} \frac{\theta}{d_{o1} N (LMTD) U} = 1.25 \times \frac{\text{ Step 5}}{29 \text{ Step 9 Step 7 Step 27}} \text{ ft}
\]

(Increased 25% to take care of inconsistencies in property evaluation & data)
Step 29. Total Length of Tubing

\[ NL = \text{Step 28} \] ft

Step 30. Total Surface Area

\[ F_s = \pi \frac{d_0}{12} NL \]

\[ = \frac{\pi}{12} \text{Step 29} \text{ sq ft} \]

Step 31. Fluid Pressure Loss in Flow

Ref. 9 assuming friction factor \( f = 0.005 \)

2nd term for entrance and exit losses

\[ \rho_f = \left( \frac{48 \times 0.005 \times L}{d_t} + 0.75 \right) \frac{V_i^2}{64.4 \times 144} \]

Step 32. Water Pressure Loss in Flow

Ref. 9 assuming \( f = 0.004 \)

\[ P_0 = \left( 0.5 + 0.048 \frac{\pi d_0 L N + \pi d_i L}{12 \times 144} \right) \frac{V_0^2}{64.4 \times 144} \]

Step 33. Horsepower to Drive 70% Efficient Water Pump

\[ WHP = \frac{W_o}{550} \left( \frac{P_o}{29} + \frac{V_o^2}{29} \right) \times \frac{1}{.7} \]

\[ = \frac{\text{Step 6}}{550 \times .7} \left( \frac{\text{Step 32} \times 144}{\text{Step 15}} + \frac{\text{Step 31}^2}{\text{Step 14}} \right) \text{ HP} \]

Step 34. Weight of Tubes

\[ W_t = \frac{\pi}{4} \left( d_0^2 - d_i^2 \right) \times L \times N \times 12 \times P_t \]

\[ = 3 \pi \left\{ \text{Step 27} - \text{Step 8} \right\} \text{ Step 29 \ Step 27} \text{ lb} \]

Step 35. Weight of Shell

\[ W_s = \frac{\pi}{4} \left( D_0^2 - D_i^2 \right) \times L \times 12 \times P_s \]

\[ = 3 \pi \left\{ \text{Step 28} - \text{Step 13} \right\} \text{ Step 28 \ Step 28} \text{ lb} \]
Step 36. Weight of End Plates Plus Auxiliaries

\[ W_{ex} = 30\% \text{ of } (W_t + W_s) \text{ assumed} \]

\[ W_{ex} = 0.3 \times \left( \text{Step 34} + \text{Step 35} \right) \text{ lbs.} \]

Step 37. Total Weight of Heater

\[ W = W_t + W_s + W_{ex} \]

\[ W = \text{Step 34} + \text{Step 35} + \text{Step 36} \text{ lbs.} \]

Step 38. Total Volume of Heater

\[ V = \frac{\pi}{4} \times \frac{D_0^2}{144} \times L \]

\[ V = \frac{\pi}{4} \times \left( \text{Step 14} \right)^2 \times \left( \text{Step 28} \right) / 144 \text{ ft.} \]
CIRCULAR FINNED TUBE TYPE CROSSFLOW HEAT EXCHANGER DESIGN CALCULATIONS - COOLER
COOLING MEDIUM - AIR OUTSIDE TUBES
FLUID -
TUBING MATERIAL -
FIN MATERIAL -
SHELL MATERIAL -
COOLER NO. -
BASED ON CYCLE NO. -
TRIAL

INPUT DATA

1. THEORETICAL CYCLE EFF. \[ \eta_{th} \] PERCENT
2. VOLUMETRIC EFFICIENCY - \[ \eta_{v} \] PERCENT
3. MECHANICAL EFFICIENCY \[ \eta_{m} \] PERCENT
4. BRAKE HORSEPOWER \[ BHP \]
5. WORK DONE/LB OF FLUID \[ W.D. \] BTU/LB
6. HEAT LOST/LB OF FLUID \[ H.L. \] BTU/LB
7. HEAT GAINED/LB OF AIR \[ H.G. \] BTU/LB
8. DELTA TEMP TA ENTRANCE \[ \Delta T_a \] DEG F
9. DELTA TEMP TB EXIT \[ \Delta T_b \] DEG F
10. MAX. ALLOW STRESS TUBES \[ f_t \] PSI
11. MAX. ALLOW STRESS SHELL \[ f_s \] PSI
12. MAX. PRESSURE OF FLUID \[ P_i \] PSI
13. MAX. PRESSURE OF AIR \[ P_o \] PSI
14. DENSITY OF FLUID @ BTP \[ \rho_i \] LB/FT^3
15. DENSITY OF AIR @ BTP \[ \rho_o \] LB/FT^3
16. VISC OF FLUID @BTP X10^3 \[ \eta_{bf} \] LB/SECFT

FLUID ENTRANCE TEMP \[ \theta_{in} \] DEG F
FLUID EXIT TEMP \[ \theta_{out} \] DEG F
FLUID BULK MEAN TEMP \[ \theta_{fb} \] DEG F
FLUID WALL MEAN TEMP \[ \theta_{fw} \] DEG F
AIR ENTRANCE TEMPERATURE \[ \theta_{a} \] DEG F
AIR EXIT TEMPERATURE \[ \theta_{e} \] DEG F
AIR BULK MEAN TEMPERATURE \[ \theta_{ba} \] DEG F
AIR WALL MEAN TEMPERATURE \[ \theta_{wa} \] DEG F

OUTPUT DATA

1. OVERALL EFFICIENCY \[ \eta_o \] PERCENT
2. INDICATED HORSEPOWER \[ IHP \]
3. FLUID IN THE SYSTEM \[ W_i \] LB/SEC
4. HEAT REJECTED IN COOLER \[ q_i \] BTU/SEC
5. HEAT REJECTED IN COOLER X10^3 \[ \bar{q}_i \] BTU/HR
6. QUANTITY OF AIR REQUIRED \[ \bar{q}_a \] LB/SEC
7. LOG MEAN TEMP DIFFERENCE \[ \Delta T_{lm} \] DEG F
### DATA CONTINUED

<table>
<thead>
<tr>
<th>Input Data</th>
<th>External Dia. of Tubes</th>
<th>Velocity of Fluid</th>
<th>Velocity of Air</th>
<th>Thickness of Fins</th>
<th>Spacing of Fins</th>
<th>Transverse Pitch of Tubes</th>
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</thead>
<tbody>
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<td>( V_i ) FT/SEC</td>
<td>( V_o ) FT/SEC</td>
<td>( t_f ) INCHES</td>
<td>( c_f ) INCHES</td>
<td>( s_T ) INCHES</td>
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</table>

| Output Data      | Internal Dia. of Tubes | Number of Tubes | Internal Dia. of Drum by Geo. | External Dia. of Drum | Reynolds No. for Fluid @ BTP | Prandtl No. for Fluid @ BTP | Nusselt No. for Fluid @ BTP | Heat Transfer Coeff for Fluid | Reynolds No. for Air | Prandtl No. for Air @ FTP | Nusselt No. for Air @ FTP | Heat Transfer Coeff. for Air | Effectiveness of Fin | Overall Heat Transfer Coeff. |
|------------------|------------------------|-----------------|-----------------------------|-----------------------|-----------------------------|-----------------------------|-------------------------------|------------------------|--------------------------|--------------------------|---------------------------|--------------------------|--------------------------|
| No.              | \( d_i \) INCHES       | \( N \) TUBES   | \( d_i \) INCHES            | \( D_o \) INCHES      | \( K_i \)                     | \( F_i \)                     | \( N_u \)                     | \( \eta \)                 | \( E_i \)                 | \( F_i \)                 | \( \eta \)                 | \( \eta \)                 | \( \eta \)                 |
| 8                |                        |                 |                             |                       |                             |                             |                               |                        |                         |                          |                          |                          |                          |
| 9                |                        |                 |                             |                       |                             |                             |                               |                        |                         |                          |                          |                          |                          |
| 10               |                        |                 |                             |                       |                             |                             |                               |                        |                         |                          |                          |                          |                          |
| 11               |                        |                 |                             |                       |                             |                             |                               |                        |                         |                          |                          |                          |                          |
| 12               |                        |                 |                             |                       |                             |                             |                               |                        |                         |                          |                          |                          |                          |
| 13               |                        |                 |                             |                       |                             |                             |                               |                        |                         |                          |                          |                          |                          |
| 14               |                        |                 |                             |                       |                             |                             |                               |                        |                         |                          |                          |                          |                          |
| 15               |                        |                 |                             |                       |                             |                             |                               |                        |                         |                          |                          |                          |                          |
| 16               |                        |                 |                             |                       |                             |                             |                               |                        |                         |                          |                          |                          |                          |
| 17               |                        |                 |                             |                       |                             |                             |                               |                        |                         |                          |                          |                          |                          |
| 18               |                        |                 |                             |                       |                             |                             |                               |                        |                         |                          |                          |                          |                          |
| 19               |                        |                 |                             |                       |                             |                             |                               |                        |                         |                          |                          |                          |                          |
| 20               |                        |                 |                             |                       |                             |                             |                               |                        |                         |                          |                          |                          |                          |
| 21               |                        |                 |                             |                       |                             |                             |                               |                        |                         |                          |                          |                          |                          |
IIB

CIRCULAR FINNED TUBE TYPE CROSSFLOW HEAT EXCHANGER
DESIGN CALCULATIONS - COOLER
COOLING MEDIUM - AIR OUTSIDE TUBES

PROCEDURE FOR CALCULATING OUTPUT DATA

Step 1. Overall Efficiency
\[ \eta_0 = \frac{\eta_h}{100} \times \frac{\eta_v}{100} \times \frac{\eta_m}{100} \times 100 \]
\[ = \frac{1 \times 2 \times 3}{10,000} \text{ Percent} \]

Step 2. Indicated Horsepower
\[ \text{IHP} = \text{BHP} \times \frac{\eta_m}{\eta_0} \]
\[ = \frac{4 \times (1 \text{ Step 1})}{\text{IHP}} \]

Step 3. Fluid in the System
\[ \text{Wi} = \frac{\text{IHP} \times 42.418}{\text{WD} \times 60} \]
\[ = \frac{707 \times (\text{Step 2})}{\text{60}/\text{sec}} \]

Step 4. Heat rejected in the Cooler
\[ q = \text{Wi} \times \text{H.G} \]
\[ = \frac{(\text{Step 3}) \times (6)}{\text{BTU} / \text{sec}} \]

Step 5. Heat rejected in the Cooler
\[ \delta = 3600 \times \frac{q}{\text{H.G}} \]
\[ = 3600 \times (\text{Step 4}) \text{ BTU} / \text{hr}. \]

Step 6. Quantity of Air Required
\[ \text{WO} = \frac{q}{\text{H.G}} \]
\[ = \frac{(\text{Step 4})}{\text{(7) 60}/\text{sec}} \]
Step 7. Log Mean Temperature Difference

\[ \text{LMTD} = F_r \frac{\Delta t_a - \Delta t_b}{\ln \left( \frac{\Delta t_a}{\Delta t_b} \right)} \]

Assuming correction factor for cross flow as .9

\[ F_t = 0.9 \times \frac{8 - 9}{\ln (8/9)} \]

Step 8. Internal Dia. of Tubes

By Lame's Thick Cylinder Formula

\[ d_i = d_o \left[ \frac{ft - pi}{ft + pi} \right]^{\frac{1}{2}} \]

\[ = 30 \left[ \frac{10 - 12}{10 + 12} \right]^{\frac{1}{2}} \text{ inches} \]

Step 9. Number of Tubes

\[ N = \frac{W_i}{\pi \cdot V_i \cdot d_i^2 / 4 \times 144} \]

\[ = \frac{576}{\pi} \times \left( \frac{\text{Step 3}}{10} \right)^2 \text{ tubes.} \]

Step 10. Min. Internal Dia. of Drum by Geometry

Ref. 7

Assuming efficiency \( \Phi = 0.735 \)

\[ D_{ib} = \left( \frac{2 \sqrt{3}}{\pi \cdot \Phi \cdot N} \right)^\frac{1}{2} \times S_r \]

\[ = \left( 1.5 \times \text{Step 9} \right)^\frac{1}{2} \times 30 \text{ inches} \]

Step 11. External Dia. of Drum

\[ D_o = D_{ib} + 0.25 \]

\[ = \text{Step 10} + 0.25 \text{ inches} \]

Step 12. Reynolds No. for Fluid at Bulk Condition

\[ Re_i = \frac{d_i \cdot \dot{V}_i}{12 \cdot \mu_b} \]

\[ = \left( \frac{\text{Step 8}}{12} \right) \left( \frac{14}{10} \right) \]

Step 13. Prandtl No. for Fluid at Bulk Cond.

\[ \Pr_i = \frac{3600}{\mu_b \cdot M_b} \]

\[ = \frac{3600}{19 \times 16} \]
Step 14. Nusselt No. for Fluid at Bulk Condition

\[ Nu_i = Nu_i' \left( \frac{d_{in}}{d_{out}} \right)^{0.11} \left( \frac{L_{ib}}{k_{ib}} \right)^{-0.33} \left( \frac{G_i}{\phi_{ib}} \right)^{0.35} \]

Ref. 8

\[
= \left( \frac{1}{10} \right)^{0.11} \left( \frac{22}{22} \right)^{-0.33} \left( \frac{20}{19} \right)^{0.35} \left[ \frac{8 (1.82 \log \text{Step} 12 - 1.64)^2}{12.7 \sqrt{1.82 \log \text{Step} 12 - 1.64} (\text{Step} 13)^{3/2} - 1} + 1.07 \right]
\]

Step 15. Heat Transfer Coeff. for Fluid

\[ h_i = 12 \ \frac{k_{ib}}{d_i} \ \text{Nu}_i \]

\[ = 12 \ \frac{22}{30} \ \text{Step 14} \quad \text{BTU/h ft}^2 \ ^\circ F \]

Step 16. Reynolds No. for Air

\[ Re_0 = \frac{d_0 \rho_0 V_0}{12 \mu_0} \]

\[ = \frac{30 \ 15 \ 32}{12 \ 18} \]

Step 17. Prandtl No. for Air at Film Condition

\[ Pr_{ow} = \frac{3600 \ C_{pow} \mu_0}{k_{ow}} \]

\[ = \frac{3600 \ 21 \ 18}{24} \]

Step 18. Nusselt No. for Air at Bulk Condition

\[ Nu_0 = 0.33 \ (Re_0)^{0.6} (Pr_{ow})^{1/3} \]

\[ = 0.33 \ \text{Step 16}^{0.6} \ \text{Step 17}^{1/3} \]

Step 19. Heat Transfer Coeff. for Air

\[ h_0 = \frac{12 \ k_{ow}}{d_0} \ Nu_0 \]

\[ = \frac{12 \ 24}{30} \ \text{Step 18} \quad \text{BTU/h ft}^2 \ ^\circ F \]

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Step 20. Effectiveness of Circular Rectangular Fin  \[ E_f = E_c \times E_s \]
Assume correction for curvature  
\[ E_c = 0.9 \]
Ref. 11
\[ E_s = \text{effectiveness of straight rectangular fin} \]
where  
\[ E_s = \left( \frac{w + \delta}{2} \right)^{3/2} \sqrt{\frac{h_0}{k g t_f}} \left( \frac{d_f - d_o}{2} + \frac{t_f}{2} \right) \]
\[ = \left( \frac{d_f - d_o}{2} + \frac{t_f}{2} \right) \left( \frac{h_0}{k g t_f} \left( \frac{d_f - d_o + t_f}{2} \right) \right) \]
\[ = \left( \frac{33}{12^2 (30)} \right) \left( \frac{\text{Step 19}}{34} \right) \left( \frac{\text{Step 19}}{12} \right) \left( \frac{\text{Step 19}}{12} \right) \]

Step 21. Overall Heat Transfer Coeff. Based on Outside Dia. of Tube
\[ U = \frac{1}{k_0 \left( 1 + \frac{f_t}{4} \frac{d_f - d_o}{d_o} \cdot \frac{1}{T_f} \right)} + \frac{d_0 \ln(d_0/d_1)}{24 \ k c} + \frac{1}{h t} \frac{d_0}{d_1} \]
\[ = \left[ \frac{30}{\text{Step 15}} \frac{\text{Step 8}}{\text{Step 9}} \right] + \left[ \frac{\text{Step 20}}{4} \frac{33}{30} \frac{1}{30} \frac{33}{34} \right] + \frac{30}{24} \ln(30/34) \]

Step 22. Length of Each tube
(increased 25% to take care of inconsistencies in properties)
\[ L = 1.25 \times \frac{12 \ \pi}{L} \times \frac{\theta}{d_0 \ \text{LMTD}} \]
\[ = 1.25 \times \frac{12 \ \pi}{L} \times \frac{\theta}{d_0 \ \text{LMTD}} \]

Step 23. Total Length of Tubing
\[ N \ L = \text{Step 9} \ \text{Step 22} \]

Step 24. Surface Area of Bare Tubing
\[ A_t = \frac{\pi d_0}{12} \ N \ L \]
\[ = \frac{\pi}{12} \ \text{Step 23} \]
Step 25. Surface Area of Finned Surface

\[ A_f = \frac{\pi}{4} \cdot \frac{d_1^2 - d_0^2}{14} \cdot \frac{NL \times 12}{S_f} \]
\[ = \frac{\pi}{48} \left( \frac{33^2 - 30^2}{35} \right) \times \frac{\text{Step 22} \times 3}{35} \text{sq. ft.} \]

Step 26. Total Surface Area

\[ A_T = A_f + A_r \]
\[ = \text{Step 24} + \text{Step 25} \]

Step 27. Fluid Pressure Loss in Flow

\[ P_f = \left( \frac{48 \times 0.25 \times L}{d_i \times 64.4 + .75} \right) \frac{V_i^2}{144} \times \frac{\rho_i}{144} \]
\[ = \left( \frac{\text{Step 22}}{\text{Step 8}} + .75 \right) \frac{30^2}{64.4} \times \frac{14}{144} \text{ psf} \]

Step 28. Air Pressure Loss in Flow

Ref. 12 Assuming \[ P_0 = \frac{4G_{\text{max}} f_{iB} N_1}{29c^2} \]
\[ N_1 = \frac{\sqrt{N}}{2} \]
\[ = \frac{4 \cdot P_0^2 \cdot V^2}{2.9c^2} - \frac{2}{k_c e^2} \times \frac{\sqrt{N}}{144} \times \frac{1}{144} \]
\[ = .43 \times 10^{-3} \times \left( \frac{\text{Step 9}}{\text{Step 10}} \right)^{0.2} \text{ psf} \]

Step 29. Horsepower to Drive Air Blower (AHP) Approx.

Assuming \[ \gamma = 0.7 \]
\[ A_{\text{HP}} = \frac{W_0}{550} \times \left( \frac{P_0 \times 144}{80} + \frac{V_0^2}{29} \right) \times \frac{1}{0.7} \]

Step 30. Weight of Tubing with Fins

\[ W_t = \frac{\pi}{4} \left( d_0^2 - d_1^2 \right) \times LN \times 12 \times P_t \]
\[ + \frac{\pi}{4} \left( d_1^2 - d_0^2 \right) \times \frac{LN \times 12 \times t_f \times P_f}{S_f} \]
\[ = 3 \pi \left( \frac{\text{Step 22} \times (30^2 - \text{Step 8}^2)}{64.4} \right) \frac{12}{144} + (33^2 - 30^2) \]

Step 31. Weight of Shell

\[ W_s = \frac{\pi}{4} \left( D_0^2 - D_0^2 \right) \times LN \times 12 \times \rho_s \]
\[ = 3 \pi \left( \frac{\text{Step 11}^2 - \text{Step 10}^2}{\text{Step 22} \times 19} \right) \text{ lb} \]
Step 32. Weight of End Plates and Auxiliaries

\[ W_{ex} = 30\% \ of \ (W_t + W_s) \]
\[ = 0.3 \left( \text{Step 30} + \text{Step 31} \right) \text{ lbs} \]

Step 33. Total Weight of the Air Blast Cooler

\[ W = W_t + W_s + W_e \]
\[ = \text{Step 30} + \text{Step 31} + \text{Step 32} \text{ lbs} \]

Step 34. Total Volume of the Cooler

\[ V = \frac{\pi}{4} \cdot \frac{d_0^2}{144} \times L \]
\[ = \frac{\pi}{576} \cdot \text{Step 11}^2 \cdot \text{Step 22} \text{ cft} \]
### Shell and Tube Type Counterflow Heat Exchanger Design Calculations - Cooler

**Cooling Medium -** Water Outside Tubing

**Tubing Material -** Shell Material

**Cooler No.** Based on Cycle No.

#### Input Data

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<th>No.</th>
<th>Description</th>
<th>Units</th>
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<td>Theoretical Cycle Eff.</td>
<td>Percent (%)</td>
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<tr>
<td>2</td>
<td>Volumetric Efficiency</td>
<td>Percent (%)</td>
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<tr>
<td>3</td>
<td>Mechanical Efficiency</td>
<td>Percent (%)</td>
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<tr>
<td>4</td>
<td>Brake Horsepower</td>
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<td>Work Done/LB of Fluid</td>
<td>BTU/LB</td>
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<td>6</td>
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<td>Density of Fluid @ BTPT</td>
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### Fluid -

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III B

SHELL AND TUBE TYPE HEAT EXCHANGER DESIGN CALCULATIONS - COOLER

COOLER MEDIUM - WATER OUTSIDE TUBES

PROCEDURE FOR CALCULATING OUTPUT DATA -
- SAME AS FOR HEATER WITH PRESSURIZED OR SUPER-CRITICAL WATER AS HEATING MEDIUM.
### Input Data

1. **Theoretical Cycle Eff.** \( \eta_{th} \) \( \text{Percent} \)
2. **Volumetric Efficiency** \( \eta_v \) \( \text{Percent} \)
3. **Mechanical Efficiency** \( \eta_m \) \( \text{Percent} \)
4. **Brake Horsepower** \( BHP \)
5. **Work Done/LB of Fluid** \( W.D \) \( \text{BTU/LB} \)
6. **Heat Gained/LB of Fluid** \( H.G \) \( \text{BTU/LB} \)
7. **Heat Lost/LB of \( N_2 \)** \( H.L \) \( \text{BTU/LB} \)
8. **Delta Temp TA Entrance** \( \Delta T_a \) \( \text{Deg F} \)
9. **Delta Temp TB Exit** \( \Delta T_b \) \( \text{Deg F} \)
10. **Max. Allow Stress Tubes** \( a_t \) \( \text{PSI} \)
11. **Max. Allow Stress Shell** \( a_s \) \( \text{PSI} \)
12. **Max. Pressure of Fluid** \( \rho_f \) \( \text{PSI} \)
13. **Max. Pressure of \( N_2 \)** \( \rho_o \) \( \text{PSI} \)
14. **Density of Fluid @ BTP** \( \rho_i \) \( \text{LB/FT}^3 \)
15. **Density of \( N_2 \) @ BTP** \( \rho_o \) \( \text{LB/FT}^3 \)
16. **Visc of Fluid @ BTP X10P-3** \( \eta_{vis} \) \( \text{LB/SECFT} \)
17. **Viscosity of Fluid @ WTP X10P-3** \( \mu_{vis} \) \( \text{LB/SECFT} \)
18. **Viscosity of \( N_2 \) @ WTP X10P-3** \( \mu_{vis} \) \( \text{LB/SECFT} \)
19. **Specific Heat of Fluid @ BTP** \( c_p \) \( \text{BTU/LB F} \)
20. **Specific Heat of Fluid @ WTP** \( c_p \) \( \text{BTU/LB F} \)
21. **Specific Heat of \( N_2 \) @ WTP** \( c_v \) \( \text{BTU/LB F} \)
22. **Thermal Cond of Fluid @ BTP** \( k_f \) \( \text{BTU/HRFT F} \)
23. **Thermal Cond of Fluid @ WTP** \( k_f \) \( \text{BTU/HRFT F} \)
24. **Thermal Cond of Tube Mat.** \( k_t \) \( \text{BTU/HRFT F} \)
25. **Density of Tube Mat.** \( \rho_t \) \( \text{LB/IN}^3 \)
26. **Density of Fin Mat.** \( \rho_f \) \( \text{LB/IN}^3 \)
27. **Density of Shell Mat.** \( \rho_s \) \( \text{LB/IN}^3 \)
28. **Fluid Entrance Temp** \( T_{in} \) \( \text{Deg F} \)
29. **Fluid Exit Temp** \( T_{out} \) \( \text{Deg F} \)
30. **Fluid Bulk Mean Temp** \( T_{b} \) \( \text{Deg F} \)
31. **Fluid Wall Mean Temp** \( T_{w} \) \( \text{Deg F} \)

### Output Data

1. **Overall Efficiency** \( \eta_o \) \( \text{Percent} \)
2. **Indicated Horsepower** \( I_{hp} \) \( \text{IHP} \)
3. **Fluid in the System** \( W_i \) \( \text{LB/SEC} \)
4. **Heat Added in Heater** \( q \) \( \text{BTU/SEC} \)
5. **Heat Added in Heater X10P-3** \( \delta \) \( \text{BTU/HR} \)
6. **Quantity of \( N_2 \) Required** \( \rho_o \) \( \text{LB/SEC} \)
7. **Log Mean Temp Difference** \( L_{MID} \) \( \text{Deg F} \)
### Input Data Continued

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<th>Based on Cycle No.</th>
<th>Trial</th>
<th>Subtrial</th>
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#### Input Data

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<th>Unit</th>
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<td>Velocity of $N_2$</td>
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#### Output Data

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<tbody>
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<td>Internal Dia. of Tubes</td>
<td>di</td>
<td>Inches</td>
<td>8</td>
</tr>
<tr>
<td>Number of Tubes</td>
<td>N</td>
<td>TUBES</td>
<td>9</td>
</tr>
<tr>
<td>Internal Dia. of Drum by Geo.</td>
<td>Dip</td>
<td>Inches</td>
<td>10</td>
</tr>
<tr>
<td>External Dia. of Drum</td>
<td>D0</td>
<td>Inches</td>
<td>11</td>
</tr>
<tr>
<td>Reynolds No. for Fluid @ BTP</td>
<td>Re</td>
<td></td>
<td>12</td>
</tr>
<tr>
<td>Prandtl No. for Fluid @ BTP</td>
<td>Prv</td>
<td></td>
<td>13</td>
</tr>
<tr>
<td>Nusselt No. for Fluid @ BTP</td>
<td>Nu</td>
<td></td>
<td>14</td>
</tr>
<tr>
<td>Heat Transfer Coeff. for Fluid</td>
<td>hi</td>
<td>BTU/HRFT2F</td>
<td>15</td>
</tr>
<tr>
<td>Reynolds No. for $N_2$ @ BTP</td>
<td>Reo</td>
<td></td>
<td>16</td>
</tr>
<tr>
<td>Prandtl No. for $N_2$ @ FTP</td>
<td>Prn</td>
<td></td>
<td>17</td>
</tr>
<tr>
<td>Nusselt No. for $N_2$ @ BTP</td>
<td>Nuo</td>
<td></td>
<td>18</td>
</tr>
<tr>
<td>Heat Transfer Coeff. for $N_2$</td>
<td>ho</td>
<td>BTU/HRFT2F</td>
<td>19</td>
</tr>
<tr>
<td>Effectiveness of Pin</td>
<td>Ef</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>Overall Heat Transfer Coeff.</td>
<td>U</td>
<td>BTU/HRFT2F</td>
<td>21</td>
</tr>
</tbody>
</table>
Step 11  External Diameter of Drum

By Lame's Thick Cylinder Formula

\[ D_o = D_i \left( \frac{f_s + k_o}{f_s - k_o} \right)^{1/2} \]

\[ = 10 \left[ \frac{11 + 13}{11 - 13} \right]^{1/2} \text{ inches} \]
CIRCULAR PINNED TUBE TYPE CROSSFLOW HEAT EXCHANGER DESIGN CALCULATIONS - HEATER

HEATING MEDIUM - NITROGEN GAS OUTSIDE TUBES

PROCEDURE FOR CALCULATING OUTPUT DATA -

SAME AS FOR COOLER WITH AIR AS COOLING MEDIUM EXCEPT

STEP (11) EXTERNAL DIAMETER OF DRUM

By Zenz's Tank Cylinder formula

\[ D_0 = D_1 \cdot \sqrt{\frac{\frac{1}{L} + \frac{1}{D}}{\frac{1}{L} - \frac{1}{D}}} \]

\[ = 10 \cdot \sqrt{\frac{10 + 12}{10 - 12}} \] inches
### INPUT DATA

<table>
<thead>
<tr>
<th></th>
<th>THEORETICAL CYCLE EFF. ( \eta_{th} )</th>
<th>PERCENT</th>
<th>VOLUMETRIC EFFICIENCY ( \eta_{v} )</th>
<th>PERCENT</th>
<th>MECHANICAL EFFICIENCY ( \eta_{m} )</th>
<th>PERCENT</th>
<th>BRAKE HORSEPOWER ( BHP )</th>
<th>HEATING MEDIUM</th>
<th>LIQUID METAL OUTSIDE TUBES</th>
<th>TUBING MATERIAL</th>
<th>SHELL MATERIAL</th>
<th>HEATER NO.</th>
<th>BASED ON CYCLE NO.</th>
<th>FLUID -</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \eta_{th} )</td>
<td>PERCENT</td>
<td>15</td>
<td>DENSITY OF LIQ. METAL @ BTP</td>
<td>( \rho_{0} )</td>
<td>LB/FT³</td>
<td>( \eta_{v} )</td>
<td>PERCENT</td>
<td>16</td>
<td>VISC OF FLUID @ BTP ( X \times 10^3 )</td>
<td>( \mu )</td>
<td>LB/SECFT</td>
<td>( \eta_{m} )</td>
<td>PERCENT</td>
</tr>
</tbody>
</table>

**FLUID**

|   | FLUID ENTRANCE TEMP. | DEG F | LIQ. METAL ENTRANCE TEMP. | DEG F | FLUID EXIT TEMP. | DEG F | LIQ. METAL EXIT TEMP. | DEG F | FLUID BULK MEAN TEMP. | DEG F | LIQ. METAL MEAN TEMP. | DEG F | FLUID WALL MEAN TEMP. | DEG F | LIQ. METAL WALL MEAN TEMP. | DEG F |

### OUTPUT DATA

<table>
<thead>
<tr>
<th></th>
<th>OVERALL EFFICIENCY</th>
<th>PERCENT</th>
<th>INDICATED HORSEPOWER</th>
<th>IHP</th>
<th>FLUID IN THE SYSTEM</th>
<th>LB/SEC</th>
<th>HEAT ADDED IN WATER</th>
<th>BTU/SEC</th>
<th>DITTO X 10³</th>
<th>BTU/HR</th>
<th>QUAN. OF LIQ. METAL</th>
<th>LB/SEC</th>
<th>LOG MEAN TEMP DIFFR</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \eta_{o} )</td>
<td>PERCENT</td>
<td>2</td>
<td>( \delta_{f} )</td>
<td>IHP</td>
<td>3</td>
<td>( W_{f} )</td>
<td>LB/SEC</td>
<td>4</td>
<td>( \gamma_{f} )</td>
<td>BTU/SEC</td>
<td>5</td>
<td>( \delta )</td>
<td>BTU/HR</td>
</tr>
</tbody>
</table>

---

*VI A*

**DATE**

**SHELL AND TUBE TYPE COUNTERFLOW HEAT EXCHANGER DESIGN CALCULATIONS - HEATER**

**HEATING MEDIUM - LIQUID METAL OUTSIDE TUBES**

**TUBING MATERIAL - SHELL MATERIAL -**

**HEATER NO. BASED ON CYCLE NO.**

**FLUID -**

**TRIAL A**
<table>
<thead>
<tr>
<th>Input Data</th>
<th>Output Data</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Heater No.</strong></td>
<td><strong>Based On Cycle No.</strong></td>
</tr>
</tbody>
</table>

**Input Data**
- Heater No.
- Based On Cycle No.
- Trial A
- Subtrial No.

**Input Data**
- External Dia of Tubes
- Velocity of Fluid
- Velocity of Liq. Metal

**Output Data**

<table>
<thead>
<tr>
<th>Column</th>
<th>Entry</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>Internal Dia of Tubes</td>
</tr>
<tr>
<td>9</td>
<td>Number of Tubes</td>
</tr>
<tr>
<td>10</td>
<td>X Area All Tubes</td>
</tr>
<tr>
<td>11</td>
<td>X Area For Liq. Metal</td>
</tr>
<tr>
<td>12</td>
<td>Total X-Area</td>
</tr>
<tr>
<td>13</td>
<td>Internal Dia of Drum</td>
</tr>
<tr>
<td>14</td>
<td>External Dia of Drum</td>
</tr>
<tr>
<td>15</td>
<td>Trvse Pitch of Tubes</td>
</tr>
<tr>
<td>16</td>
<td>LGTUDNL Pitch of Tubes</td>
</tr>
<tr>
<td>17</td>
<td>MIN ID of Drum By Geo</td>
</tr>
<tr>
<td>18</td>
<td>REY No For Fluid @ BTP</td>
</tr>
<tr>
<td>19</td>
<td>FRLD No For Fluid @ BTP</td>
</tr>
<tr>
<td>20</td>
<td>HUST No For Fluid @ BTP</td>
</tr>
<tr>
<td>21</td>
<td>Heat Trans Coeff Fluid</td>
</tr>
<tr>
<td>22</td>
<td>EQUIV Dia of Tube Bank</td>
</tr>
<tr>
<td>23</td>
<td>REY No For Liq. Metal @ BTP</td>
</tr>
</tbody>
</table>

**Output Data**
- Prandtl No For Liq. Metal @ BTP
- Nusselt No For Liq. Metal @ BTP
- Heat Trans Coeff For Liq. Metal
- Overall Heat Trans Coeff
- Length of Each Tube
- Total Length of Tubing
- Total Surface Area
- Fluid Pressure Loss
- PSI
- Total Pressure Loss
- HP To Drive Liq. Metal Pump
- Weight of Tubes
- Weight of Shell
- Weight of End Plate Plus Aux.
- Total Weight of Heater
- Total Volume of Heater
- CUBIC FT
VI B

SHELL AND TUBE TYPE HEAT EXCHANGER DESIGN CALCULATIONS - HEATER
HEATING MEDIUM - LIQUID METAL OUTSIDE TUBES

PROCEDURE FOR CALCULATING OUTPUT DATA -
SAME AS FOR HEATER WITH PRESSURIZED OR SUPERCRITICAL WATER AS HEATING MEDIUM EXCEPT

Step 24 Instead of Prandtl No. Calculate Peclet No. given by

\[
P_{eb} = \frac{\rho_{ob} \beta_{ob} \nu_{o} d_{o}}{K_{ob}}
\]

\[
= \frac{22}{15} \times \frac{(31/29)}{25} \times \frac{3000}{12}
\]

Step 25 Nusselt No. for Liquid Metal

\[
N_{wo} = 7 + 0.025 \left( P_{eb} \right)^{0.8}
\]

\[
= 7 + 0.025 \left( \text{Step 24} \right)^{0.8}
\]
HEAT EXCHANGER DESIGN CONSIDERATIONS

The physical considerations in the design of heat exchangers are size and weight of the unit, pressure drop of fluid through the heat exchanger, and horsepower required to drive the other medium through the system. Calculations with different sizes of tubing revealed that the smallest practicable tubing was the most suitable considering weight, length and pressure drop of fluid. Though the number of tubes required was the largest, the total length of tubing was the least. Of course, the heat exchanger diameter increased as the size of tubing was decreased; the horsepower to drive the medium through the heat exchanger decreased if it was a counterflow heat exchanger and increased if it was of a cross-flow type.

Increasing the velocity of the fluid and the transfer medium increased the heat transfer coefficients, thus decreasing the surface area. Increase in the fluid velocity usually results in a smaller number of longer tubes, also increasing the pressure drop. Increasing the medium velocity naturally increased the horsepower required to drive the medium through the heat exchangers. Medium velocity in case of counterflow designs had to be such that sufficient cross-section area was available to fit all the tubes with the given transverse and longitudinal pitch.

To obtain the least weight and length of heat exchangers for easy transportation, the smallest practicable size of tubing was 3/8" and was used throughout. For heat exchangers for PM-2A and ML-1 systems, fluid and transfer medium velocities were varied over a wide range to obtain reasonable pressure drops, horsepower requirements for the medium pump, and lengths of the tubing. It must be noted that decreasing the fluid velocity results in a large number of shorter tubes.

Because of the bulk of work involved in the parametric study, it was not possible to vary all the parameters. Hence all the physical parameters such as size and spacing of tubes and fins, and the velocities of fluid and medium were held fixed, only power levels and conditions and temperature differences on the mediums were varied. No consideration was given to refining the pressure drops of fluid and the pumping horsepower requirements.

The weights of the heat exchangers were approximated by calculating the weight of the tubes and the shell and adding 30% of the sum for the weight of end plates and auxiliaries to give the weight of the heat exchangers. No attempt was made to determine exact weight, since the detail work required was considered to be unjustified for a study of feasibility.
Chapter 10

PARAMETRIC ANALYSIS

As described in considerable detail in the previous chapters, the fluid engine system consists of two principal classes of components plus the necessary auxiliaries. The two principal components are the engine itself and the heat exchangers which are connected to it. This portion of the report will enumerate several of the factors which affect the design of these major components, and will discuss the effects of variations in these factors.

The two dominant factors in the determination of the size and weight of engine are the operating cycle and the basic mechanical configuration. In this report only one configuration has been considered, the rotating barrel type, because of its compactness, low internal friction, inherent dynamic and static balance, and simplicity of valving. It is believed that the barrel arrangement provides the best values of weight and volume for any size of fluid engine design.

All other engine design parameters are strictly cycle dependent. As discussed in Chapter 6, there is a wide latitude permissible in the choice of operating pressures, pressure ratios, and temperatures, but for a particular case, the operating temperatures are the first limitation which must be recognized. These temperatures are determined by the nature of the heat source and heat sink. As for all other types of heat engines the larger the difference in temperature available to the cycle, the greater the overall engine efficiency will be, and the smaller will be the specific weights and volumes of the engine.

With the temperatures established, the next factor to be determined is the pressure ratio in which the engine is to operate if the best efficiency is to be obtained. Pressure ratio values are determined from curves such as those shown in Graphs 95-97 in Appendix B, and are usually not a matter of choice if efficiency is important. The only remaining cycle factor to be determined is the maximum operating pressure and this is a matter of choice depending upon the trade-off between high power density, which produces a small engine, and the high pressures required, which call for heavy cylinder walls. Once the cycle is chosen, the engine must be designed with sufficient strength and cylinder wall thickness to withstand the operating pressures and temperatures, and be
arranged internally to give the desired speed at the output shaft. As with all positive displacement engines, the fluid engine becomes lighter if the speed is increased. Some of the factors affecting engine specific weight and volume are summarized in Table IX.

**TABLE IX. ENGINE PARAMETER VARIATIONS**

<table>
<thead>
<tr>
<th>FOR AN INCREASE IN</th>
<th>SPECIFIC WEIGHT</th>
<th>SPECIFIC VOLUME</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power</td>
<td>Decreases</td>
<td>Decreases</td>
</tr>
<tr>
<td>Operating Pressure</td>
<td>Increases</td>
<td>Decreases</td>
</tr>
<tr>
<td>Shaft Speed</td>
<td>Decreases</td>
<td>Decreases</td>
</tr>
<tr>
<td>Temperature Differential</td>
<td>Decreases</td>
<td>Decreases</td>
</tr>
</tbody>
</table>

Several different engine designs have been carried out in sizes from 50 HP to 15,000 KW, and the results collected in Table X to confirm some of the general trends given in Table IX.

Several fundamental factors determine the design of the heat exchangers. Among these are the total heat transfer duty, the nature and condition of the transfer media, the physical geometry involved, and the materials selected for construction. Under the scope of the contract it was not possible to study the effect of variation of all these parameters independently. In fact, some of these parameters are not independent variables, but are influenced by other quantities. However, comments on each parameter will be made, discussing the quantities that influence them, how they affect the heat exchanger design and the values chosen in the present calculation.

Heat exchanger duty is determined by the flow rate of the fluid and the heat added or rejected per pound of fluid. The flow rate is determined by the power level, the overall efficiency of the system, and the chosen cycle. Heat added or rejected per pound of fluid is also directly determined by the cycle. The overall efficiency is the product of the cycle efficiency and the engine mechanical and generator efficiencies. In all the calculations the engine mechanical efficiency is assumed to be fixed at 95% and the generator efficiency is assumed a constant 93%.
<table>
<thead>
<tr>
<th>HP OR KW RATING</th>
<th>50 HP</th>
<th>200 HP</th>
<th>300 HP</th>
<th>400 HP</th>
<th>750 HP</th>
<th>2000 KW</th>
<th>2000 KW</th>
<th>3000 KW</th>
<th>10,000 KW</th>
<th>15,000 KW</th>
</tr>
</thead>
<tbody>
<tr>
<td>HP/IN³ Displacement at 1000 S/M</td>
<td>2.64</td>
<td>2.64</td>
<td>2.64</td>
<td>2.64</td>
<td>.643</td>
<td>1.234</td>
<td>.870</td>
<td>1.022</td>
<td>.937</td>
<td></td>
</tr>
<tr>
<td>Max. Pressure PSI</td>
<td>6000</td>
<td>6000</td>
<td>6000</td>
<td>6000</td>
<td>3500</td>
<td>3500</td>
<td>4500</td>
<td>3000</td>
<td>3000</td>
<td></td>
</tr>
<tr>
<td>Strokes/Min.</td>
<td>5400</td>
<td>5400</td>
<td>6000</td>
<td>7200</td>
<td>1800</td>
<td>6000</td>
<td>7200</td>
<td>5400</td>
<td>4500</td>
<td></td>
</tr>
<tr>
<td>Lobes on Cam</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td>4</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>RPM Output Shaft</td>
<td>1800</td>
<td>1800</td>
<td>1200</td>
<td>1800</td>
<td>300</td>
<td>1200</td>
<td>1800</td>
<td>1800</td>
<td>900</td>
<td></td>
</tr>
<tr>
<td>IN³ Displacement</td>
<td>4.56</td>
<td>19.38</td>
<td>26.58</td>
<td>29.45</td>
<td>2048</td>
<td>712.7</td>
<td>514.5</td>
<td>1113.6</td>
<td>3819</td>
<td></td>
</tr>
<tr>
<td>No. of Cylinders</td>
<td>22</td>
<td>26</td>
<td>30</td>
<td>30</td>
<td>60</td>
<td>60</td>
<td>60</td>
<td>60</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>Bore (INS)</td>
<td>.593</td>
<td>1.125</td>
<td>1.062</td>
<td>1.00</td>
<td>4.170</td>
<td>2.75</td>
<td>2.50</td>
<td>3.437</td>
<td>4.812</td>
<td></td>
</tr>
<tr>
<td>Stroke (INS)</td>
<td>.750</td>
<td>.750</td>
<td>1.000</td>
<td>1.25</td>
<td>2.50</td>
<td>2.00</td>
<td>1.75</td>
<td>2.00</td>
<td>3.50</td>
<td></td>
</tr>
<tr>
<td>Est. Engine Weight LBS</td>
<td>100</td>
<td>350</td>
<td>450</td>
<td>600</td>
<td>11,000</td>
<td>4,985</td>
<td>4,120</td>
<td>6,000</td>
<td>17,000</td>
<td></td>
</tr>
<tr>
<td>LB/HP</td>
<td>2</td>
<td>1.75</td>
<td>1.5</td>
<td>1.5</td>
<td>1.466</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LB/KW</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2.49</td>
<td>2.60</td>
<td>2.00</td>
<td>1.70</td>
<td>1.46</td>
<td></td>
</tr>
</tbody>
</table>
Thus the overall efficiency is automatically determined when the cycle and its particular efficiency are chosen. The cycle efficiency in turn depends on the efficiency of expansion and compression, which are assumed 90% each, and the theoretical performance. So the duty finally depends only upon the power level and the chosen cycle.

The number of tubes and the quantities of fluid and medium flows are directly proportional to the power level and, hence, a heat exchanger calculated for a given cycle at one power level can be very easily scaled to other power levels. All the heat exchangers discussed are designed for 1250 KWe net.

The nature and condition of the heat transfer media examined in this report are shown in Table XI. They cover the requirements for the heater and coolers as set forth in the statement of work, except that cooling water is used only at 100°. Of course, the higher pressure media require stronger and, therefore, heavier heat exchangers. The gaseous media require the use of finned tubes in order to efficiently utilize the space and weight involved.

The total flow rates of the media are determined by the cycle conditions, but the individual velocities are chosen to provide a high heat transfer rate by developing turbulent flow conditions. This results in shorter, lighter heat exchangers but tends to increase the pumping work. Generally, the highest practicable media velocities are specified.

In this study of shell and tube heat exchangers, the physical geometry involved rapidly settled onto the smallest practical tube size as being the best from both the weight and size criteria. All tubes are therefore 3/8" O.D. Where fins are used, the ML-1 practice is continued by calling out 1.25" O.D., a thickness of 0.012" and a spacing of 11 fins per inch.

Construction materials for the heaters were limited to stainless steel for its strength-weight characteristics, while carbon steel was used for the lower pressure coolers. Aluminum has been called out for the air blast coolers as being most desirable from the weight standpoint. In all cases, the stresses have been determined in accordance with the A.S.M.E. Unfired Pressure Vessels Code.

About 300 heat exchangers were designed on the computer to cover the range of conditions given in Table XI. Sixty-one of the best of these have been selected and tabulated in Appendix D. Because several of the governing parameters are interdependent, it was not considered meaningful to make elaborate cross-plots of the exchanger characteristics.
### TABLE XI. HEAT TRANSFER MEDIA CONDITIONS

<table>
<thead>
<tr>
<th>HEATING MEDIUM</th>
<th>Temp, °F</th>
<th>ΔT, °F</th>
<th>Pressure, PSI</th>
<th>Velocity, Ft/Sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressurized Water</td>
<td>300, 450, 600</td>
<td>20, 40</td>
<td>Saturation+ 100°F</td>
<td>7</td>
</tr>
<tr>
<td>Nitrogen Gas</td>
<td>1000, 1200, 1400</td>
<td>400</td>
<td>300, 500</td>
<td>45</td>
</tr>
<tr>
<td>Supercritical Water</td>
<td>800, 1000</td>
<td>20, 100</td>
<td>3500</td>
<td>7</td>
</tr>
<tr>
<td>NaK</td>
<td>600, 1000, 1400</td>
<td>200</td>
<td>150</td>
<td>30</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>COOLING MEDIUM</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>100</td>
<td>Suitable</td>
<td>Atmos.</td>
<td>7</td>
</tr>
<tr>
<td>Air</td>
<td>40, 80</td>
<td>Suitable</td>
<td>Atmos.</td>
<td>45</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>WORKING MEDIUM</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>CO2</td>
<td></td>
<td></td>
<td>Determined by operating cycle</td>
<td>30</td>
</tr>
</tbody>
</table>
REFERENCES


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## APPENDIX A

### TRANSPORT PROPERTIES-VARIATION WITH TEMPERATURE-SOURCES

<table>
<thead>
<tr>
<th></th>
<th>ATMOSPHERIC PRESSURE</th>
<th>HIGHER PRESSURES</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Water</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Density</td>
<td>Ref. 3 &amp; 4</td>
<td>Ref. 3</td>
</tr>
<tr>
<td>Specific Heat</td>
<td>Ref. 3 &amp; 4</td>
<td>Ref. 3</td>
</tr>
<tr>
<td>Thermal Conductivity</td>
<td>Ref. 3</td>
<td>Ref. 3</td>
</tr>
<tr>
<td>Viscosity</td>
<td>Ref. 3</td>
<td>Ref. 3</td>
</tr>
<tr>
<td><strong>Air</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Density</td>
<td>Ref. 3</td>
<td></td>
</tr>
<tr>
<td>Specific Heat</td>
<td>Ref. 5</td>
<td>Ref. 5</td>
</tr>
<tr>
<td>Thermal Conductivity</td>
<td>Ref. 1</td>
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<tr>
<td>Viscosity</td>
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<td><strong>CO₂</strong></td>
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<td>Specific Heat</td>
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<td>Ref. 1</td>
<td>Ref. 2</td>
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<td>Viscosity</td>
<td>Ref. 1</td>
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<td>Density</td>
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<td>Specific Heat</td>
<td>Ref. 1</td>
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<td>Thermal Conductivity</td>
<td>Ref. 1</td>
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<td>Viscosity</td>
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<tr>
<td>Viscosity</td>
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DENSITY OF LIQUID WATER VS TEMPERATURE
(WITH PRESSURE AS THE VARIABLE PARAMETER)
SPECIFIC HEAT OF LIQUID WATER VS TEMPERATURE
(WITH PRESSURE AS THE VARIABLE PARAMETER)
THERMAL CONDUCTIVITY OF LIQUID WATER VS TEMPERATURE
(WITH PRESSURE AS THE VARIABLE PARAMETER)

THERMAL CONDUCTIVITY, $K$, [BTU/(hr)(ft)(°F/ft)]

Temperature, $T$, (°F)

Pressures:
- $P = 14.7$ psi
- 1000 psi
- 2000 psi
- 3000 psi
- 4000 psi
VISCOSITY OF LIQUID WATER VS TEMPERATURE (WITH PRESSURE AS THE VARIABLE PARAMETER)
DENSITY OF AIR VS TEMPERATURE AT ATMOSPHERIC PRESSURE

The graph shows the relationship between density, \( \rho \) (in lbs/ft\(^3\)), and temperature, \( T \) (in °F). The density decreases as the temperature increases, following a downward trend on the graph.
SPECIFIC HEAT OF AIR VS TEMPERATURE AT ATMOSPHERIC PRESSURE

TEMPERATURE, $T$, ($^\circ$F)

SPECIFIC HEAT, $C_p$, (BTU/ib$^\circ$F)
THERMAL CONDUCTIVITY OF AIR VS TEMPERATURE AT ATMOSPHERIC PRESSURE

THERMAL CONDUCTIVITY, $K$, [BTU/(hr*ft) ($^\circ$F/ft)]

TEMPERATURE, $T$, ($^\circ$F)
VISCOSITY OF AIR VS TEMPERATURE AT ATMOSPHERIC PRESSURE

![Graph showing the relationship between viscosity and temperature. The y-axis represents the viscosity in lb/(ft·sec), and the x-axis represents temperature in °F. The graph shows an increasing trend as temperature increases.]

Temperature, $T$, (°F)

Viscosity, $\mu \times 10^{-3}$ (lb/(ft·sec))

-100 0 100 200 300 400 500 600 700 800 900 1000
DENSITY OF CO\textsubscript{2} VS TEMPERATURE (WITH PRESSURE AS THE VARIABLE PARAMETER)
SPECIFIC HEAT OF \( CO_2 \) VS TEMPERATURE (WITH PRESSURE AS THE VARIABLE PARAMETER)
THERMAL CONDUCTIVITY OF CO\textsubscript{2} VS TEMPERATURE (WITH PRESSURE AS THE VARIABLE PARAMETER)
VISCOSITY OF CO₂ VS TEMPERATURE (WITH PRESSURE AS THE VARIABLE PARAMETER)
DENSITY OF $N_2$ VS TEMPERATURE (WITH PRESSURE AS THE VARIABLE PARAMETER)
SPECIFIC HEAT OF N$_2$ VS TEMPERATURE AT PRESSURES FROM 14.7 TO 500 PSI

Specific Heat, $C_p$, (BTU/(lb)(°F))

Temperature, T (°F)

$P = 14.7, 300, 500$ psi
THERMAL CONDUCTIVITY OF $N_2$ VS TEMPERATURE
(WITH PRESSURE AS THE VARIABLE PARAMETER)
VISCOSITY OF N₂ VS TEMPERATURE AT PRESSURES FROM 14.7 TO 500PSI
Density of Na, k, Na\textsubscript{k}(56,44), and Na\textsubscript{k}(22,78) vs Temperature

Na\textsubscript{k}(56,44) = 56\text{wt} - \% \text{Na}, 44\text{wt} - \% \text{k}

Na\textsubscript{k}(22,78) = 22\text{wt} - \% \text{Na}, 78\text{wt} - \% \text{k}
SPECIFIC HEAT OF Na, k, Nak(56,44), AND Nak(22,78) VS TEMPERATURE

Nak(56,44) = 56wt-% Na, 44wt-% k
Nak(22,78) = 22wt-% Na, 78wt-% k

SPECIFIC HEAT, CP, (BTU/lb°F)

TEMPERATURE, T (°F)
THERMAL CONDUCTIVITY OF Nak (56,44), AND Nak (22,78) VS TEMPERATURE

Nak (56,44) = 56 wt-% Na, 44 wt-% k
Nak (22,78) = 22 wt-% Na, 78 wt-% k
VISCOSITY OF Na, K, Nak (56,44), AND Nak (22,78) VS TEMPERATURE

Nak (56,44) = 56 wt-% Na, 44 wt-% K
Nak (22,78) = 22 wt-% Na, 78 wt-% K
REFERENCES


7. "Perfect Gas Law in Thermodynamics PV = mRT."

IDEAL CYCLE EFFICIENCY VS MAXIMUM TEMPERATURE (IDEAL GAS)
(CONSTANT $\gamma$)

MINIMUM PRESSURE $P_1 = 500$ PSI
MINIMUM TEMPERATURE $T_1 = 40^\circ$ F

P.R. = PRESSURE RATIO

$P_{max} = 10000$ psi (P.R. = 20)
$5000$ psi (P.R. = 10)
$3500$ psi (P.R. = 7)
$2000$ psi (P.R. = 4)

MAXIMUM TEMPERATURE (DEGREES FAHRENHEIT) $T_3$
$\gamma$ VS TEMPERATURE FOR CO$_2$

$\gamma = \frac{C_p}{C_v}$

GRAPH N° 2

TEMPERATURE, $T$, ($^\circ$F)

$\gamma$

0 100 200 300 400 500 600 700 800 900 1000 1100

1.1 1.2 1.3 1.4
IDEAL CYCLE EFFICIENCY VS MAXIMUM TEMPERATURE (CO$_2$) (VARIABLE $\gamma$)

MINIMUM PRESSURE $P_1 = 500$ PSI
MINIMUM TEMPERATURE $T_1 = 40^\circ$

GRAPH N° 3

EFFICIENCY (PERCENT)

P$_{\text{max}} = 10000$ psi (P.R. = 20)
5000 psi (P.R. = 10)
3500 psi (P.R. = 7)
2000 psi (P.R. = 4)

MAXIMUM TEMPERATURE (DEGREES FAHRENHEIT) $T_3$
Ideal cycle efficiency and power density vs maximum temperature

Minimum temperature $T_1 = 40^\circ F$
Minimum pressure $P_1 = 500$ PSI
--- HP/IN$^3$ displacement @ 1000 RPM
--- Efficiency

The variable parameter is maximum pressure, $P_2$

Carnot efficiency

Graph No. 4

Efficiency (Percent)

HP/IN$^3$ displacement @ 1000 RPM

Maximum temperature (degrees Fahrenheit), $T_3$
IDEAL CYCLE EFFICIENCY AND POWER DENSITY VS MAXIMUM TEMPERATURE

MINIMUM TEMPERATURE $T_1 = 80^\circ\text{F}$

MINIMUM PRESSURE $P_1 = 800\text{ PSI}$

---

$\text{HP/IN}^3$ DISPLACEMENT

EFFICIENCY

THE VARIABLE PARAMETER IS MAXIMUM PRESSURE, $P_2$

$P_{\text{max}} = 10000\text{ psi}$

$5000\text{ psi}$

$3500\text{ psi}$

$10000\text{ psi}$

$5000\text{ psi}$

$3500\text{ psi}$

$2000\text{ psi}$

$200\text{ psi}$

MAXIMUM TEMPERATURE (DEGREES FAHRENHEIT) $T_3$
IDEAL CYCLE EFFICIENCY AND POWER DENSITY VS MAXIMUM TEMPERATURE

MINIMUM TEMPERATURE $T_1 = 120^\circ F$
MINIMUM PRESSURE $P_1 = 1100$ PSI

--- HP/IN$^3$ DISPLACEMENT
--- EFFICIENCY

THE VARIABLE PARAMETER IS MAXIMUM PRESSURE, $P_2$

THE VARIABLE PARAMETER IS MAXIMUM PRESSURE, $P_2$

MAXIMUM TEMPERATURE (DEGREES FAHRENHEIT), $T_3$
IDEAL CYCLE EFFICIENCY AND POWER DENSITY VS MAXIMUM TEMPERATURE

MINIMUM TEMPERATURE $T_1 = 150^\circ F$
MINIMUM PRESSURE $P_1 = 1800$ PSI

--- HP/IN$^3$ DISPLACEMENT
--- EFFICIENCY

THE VARIABLE PARAMETER IS MAXIMUM PRESSURE, $P_2$

P$\max = 10,000$ psi
5,000 psi

MAXIMUM TEMPERATURE (DEGREES FAHRENHEIT), $T_3$

EFFICIENCY (PERCENT)

HP/IN$^3$ DISPLACEMENT @ 1000 RPM

GRAPH No. 7
IDEAL CYCLE EFFICIENCY AND POWER DENSITY VS MAXIMUM TEMPERATURE

MINIMUM TEMPERATURE $T_1 = 120^\circ F$
MINIMUM PRESSURE $P_1 = 800$ PSI

--- HP/IN$^3$ DISPLACEMENT
--- EFFICIENCY

THE VARIABLE PARAMETER IS MAXIMUM PRESSURE, $P_2$

GRAPH N° 8

The graph shows the variation of efficiency and power density with maximum temperature for different values of maximum pressure ($P_\text{max}$) from 2,000 psi to 10,000 psi. The efficiency decreases as the maximum temperature increases for each constant maximum pressure value.
IDEAL CYCLE EFFICIENCY AND POWER DENSITY VS MAXIMUM TEMPERATURE

MINIMUM TEMPERATURE $T_1 = 80^\circ F$
MINIMUM PRESSURE $P_1 = 1100$ PSI

---

GRAPH NO. 9

THE VARIABLE PARAMETER IS MAXIMUM PRESSURE, $P_2$

$P_{\text{max}} = 10,000$ psi

5,000 psi

10,000 psi

3,500 psi

5,000 psi

3,500 psi

2,000 psi

1 HP/IN$^3$ DISPLACEMENT

EFFICIENCY

EFFICIENCY (PERCENT)

MAXIMUM TEMPERATURE (DEGREES FAHRENHEIT), $T_3$

HP/IN$^3$ DISPLACEMENT @ 1000 RPM
IDEAL CYCLE EFFICIENCY AND POWER DENSITY VS MAXIMUM TEMPERATURE

MINIMUM TEMPERATURE $T_1 = 40^\circ F$
MAXIMUM PRESSURE $P_2 = 2000$ PSI

--- HP/IN$^3$ DISPLACEMENT
--- EFFICIENCY

THE VARIABLE PARAMETER IS MINIMUM PRESSURE, $P_1$

GRAPH NO. 10

--- HP/IN$^3$ DISPLACEMENT @ 1000 RPM

MAXIMUM TEMPERATURE (DEGREES FAHRENHEIT), $T_3$
IDEAL CYCLE EFFICIENCY AND POWER DENSITY VS MAXIMUM TEMPERATURE

MINIMUM TEMPERATURE $T_1 = 80^\circ F$
MAXIMUM PRESSURE $P_2 = 3500$ PSI

GRAPH No II

--- HP/IN$^3$ DISPLACEMENT
--- EFFICIENCY

THE VARIABLE PARAMETER IS MINIMUM PRESSURE, $P_1$

Efficiency (%)

HP/IN$^3$ DISPLACEMENT @ 1000 RPM

MAXIMUM TEMPERATURE (DEGREES FAHRENHEIT), $T_3$

Efficiency (%)

50 40 30 20 10 0

5 4 3 2 1

500 psi
800 psi
1100 psi
Pmin = 500 psi
IDEAL CYCLE EFFICIENCY AND POWER DENSITY VS MAXIMUM TEMPERATURE

MINIMUM TEMPERATURE $T_1 = 120^\circ F$
MAXIMUM PRESSURE $P_2 = 5000$ PSI

---

GRAPH Nº 12

THE VARIABLE PARAMETER IS MINIMUM PRESSURE, $P_1$

THE VARIABLE PARAMETER IS MINIMUM PRESSURE, $P_1$

---

HP/IN$^3$ DISPLACEMENT

EFFICIENCY

MAXIMUM TEMPERATURE (DEGREES FAHRENHEIT), $T_3$

MAXIMUM TEMPERATURE (DEGREES FAHRENHEIT), $T_3$

---

$P_{\text{min}} = 500$ psi

$800$ psi

$1100$ psi

$1800$ psi

$500$ psi

$1100$ psi

$1800$ psi

$800$ psi

$500$ psi
IDEAL CYCLE EFFICIENCY AND POWER DENSITY VS MAXIMUM TEMPERATURE

MINIMUM TEMPERATURE $T_1 = 150^\circ F$

MAXIMUM PRESSURE $P_2 = 10,000$ PSI

--- HP/IN$^3$ DISPLACEMENT
--- EFFICIENCY

THE VARIABLE PARAMETER IS MINIMUM PRESSURE, $P_{min}$

GRAPH № 13

THE VARIABLE PARAMETER IS MINIMUM PRESSURE, $P_{min}$
IDEAL CYCLE EFFICIENCY AND POWER DENSITY VS MAXIMUM TEMPERATURE

MINIMUM TEMPERATURE $T_1 = 80^\circ F$
MAXIMUM PRESSURE $P_2 = 5000$PSI

---

HP/IN$^3$ DISPLACEMENT
EFFICIENCY

THE VARIABLE PARAMETER IS MINIMUM PRESSURE, $P_1$

$P_{\text{min}} = 500$psi

MAXIMUM TEMPERATURE (DEGREES FAHRENHEIT), $T_3$

EFFICIENCY (PERCENT)

GRAPH N° 14

HP/IN$^3$ DISPLACEMENT @ 1000RPM
IDEAL CYCLE EFFICIENCY AND POWER DENSITY VS MAXIMUM TEMPERATURE

MINIMUM TEMPERATURE $T_1 = 120^\circ F$
MAXIMUM PRESSURE $P_2 = 3500$ PSI

--- HP/IN$^3$ DISPLACEMENT
--- EFFICIENCY

THE VARIABLE PARAMETER IS MINIMUM PRESSURE, $P_1$

MAXIMUM TEMPERATURE (DEGREES FAHRENHEIT), $T_3$
IDEAL CYCLE EFFICIENCY AND POWER DENSITY VS MAXIMUM TEMPERATURE

MINIMUM PRESSURE \( P_1 = 500 \text{ PSI} \)
MAXIMUM PRESSURE \( P_2 = 2000 \text{ PSI} \)

--- HP/IN\(^3\) DISPLACEMENT
--- EFFICIENCY

THE VARIABLE PARAMETER IS MINIMUM TEMPERATURE, \( T_1 \)

--- DISPLACEMENT @ 1000 RPM

MAXIMUM TEMPERATURE (DEGREES FAHRENHEIT), \( T_3 \)
IDEAL CYCLE EFFICIENCY AND POWER DENSITY VS MAXIMUM TEMPERATURE

MINIMUM PRESSURE $P_1 = 800$ PSI
MAXIMUM PRESSURE $P_2 = 5000$ PSI

--- HP/IN$^3$ DISPLACEMENT
--- EFFICIENCY

THE VARIABLE PARAMETER IS MINIMUM TEMPERATURE, $T_{min}$

GRAPH N° 17

MAXIMUM TEMPERATURE (DEGREES FAHRENHEIT), $T_3$
IDEAL CYCLE EFFICIENCY AND POWER DENSITY VS MAXIMUM TEMPERATURE

MINIMUM PRESSURE $P_1 = 1100$ PSI
MAXIMUM PRESSURE $P_2 = 5000$ PSI

--- HP/IN³ DISPLACEMENT
--- EFFICIENCY

THE VARIABLE PARAMETER IS MINIMUM TEMPERATURE, $T_{min}$

$T_{min} = 40^\circ F$
$80^\circ F$
$120^\circ F$
$150^\circ F$

EFFICIENCY (PERCENT)

MAXIMUM TEMPERATURE (DEGREES FAHRENHEIT), $T_3$

HP/IN³ DISPLACEMENT @ 1000RPM
EFFICIENCY AND POWER DENSITY VS MAXIMUM TEMPERATURE
(IDEAL CYCLE)

THE VARIABLE PARAMETER IS MINIMUM TEMPERATURE, $T_1$
MINIMUM PRESSURE $P_1 = 1800$ PSI
MAXIMUM PRESSURE $P_2 = 1000$ PSI

GRAPH Nº 19

$T_{min} = 40^\circ F$

$80^\circ F$

$120^\circ F$

$150^\circ F$

$40^\circ F$

$50^\circ F$

$70^\circ F$

$90^\circ F$

$110^\circ F$

$130^\circ F$

$150^\circ F$

$170^\circ F$

$190^\circ F$

$200^\circ F$

$220^\circ F$

$240^\circ F$

$260^\circ F$

$280^\circ F$

$300^\circ F$

$320^\circ F$

$340^\circ F$

$360^\circ F$

$380^\circ F$

$400^\circ F$

$420^\circ F$

$440^\circ F$

$460^\circ F$

$480^\circ F$

$500^\circ F$

$520^\circ F$

$540^\circ F$

$560^\circ F$

$580^\circ F$

$600^\circ F$

$620^\circ F$

$640^\circ F$

$660^\circ F$

$680^\circ F$

$700^\circ F$

$720^\circ F$

$740^\circ F$

$760^\circ F$

$780^\circ F$

$800^\circ F$

$820^\circ F$

$840^\circ F$

$860^\circ F$

$880^\circ F$

$900^\circ F$

$920^\circ F$

$940^\circ F$

$960^\circ F$

$980^\circ F$

$1000^\circ F$

$1020^\circ F$

$1040^\circ F$

$1060^\circ F$

$1080^\circ F$

$1100^\circ F$

$1120^\circ F$

$1140^\circ F$

$1160^\circ F$

$1180^\circ F$

$1200^\circ F$

EFFICIENCY (PERCENT)

HP/IN$^3$ DISPLACEMENT @ 1000 RPM

MAXIMUM TEMPERATURE (DEGREES FAHRENHEIT), $T_3$
IDEAL CYCLE EFFICIENCY AND POWER DENSITY VS MAXIMUM TEMPERATURE

MINIMUM PRESSURE $P_1 = 800$ PSI
MAXIMUM PRESSURE $P_2 = 3500$ PSI

---

HP/IN$^3$ DISPLACEMENT

EFFICIENCY

THE VARIABLE PARAMETER IS MINIMUM TEMPERATURE, $T_1$

GRAPH No 20

MAXIMUM TEMPERATURE (DEGREES FAHRENHEIT), $T_3$

EFFICIENCY (PERCENT)

HP/IN$^3$ DISPLACEMENT @ 1000 RPM
IDEAL CYCLE EFFICIENCY AND POWER DENSITY VS MAXIMUM TEMPERATURE

MINIMUM PRESSURE \( P = 1100 \text{ PSI} \)
MAXIMUM PRESSURE \( P = 3500 \text{ PSI} \)

\[ \text{GRAPH N° 21} \]

--- HP/IN\(^3\) DISPLACEMENT
--- EFFICIENCY

THE VARIABLE PARAMETER IS MINIMUM TEMPERATURE, \( T_1 \)

\[
\begin{align*}
\text{EFFICIENCY (PERCENT)} & \quad \text{HP/IN}^3 \text{ DISPLACEMENT @ 1000RPM} \\
0 & \quad 0 \\
10 & \quad 1 \\
20 & \quad 2 \\
30 & \quad 3 \\
40 & \quad 4 \\
50 & \quad 5 \\
\end{align*}
\]

\[
\begin{align*}
\text{MAXIMUM TEMPERATURE (DEGREES FAHRENHEIT), } T_3 & \\
300 & \quad 300 \\
400 & \quad 400 \\
500 & \quad 500 \\
600 & \quad 600 \\
700 & \quad 700 \\
800 & \quad 800 \\
900 & \quad 900 \\
1000 & \quad 1000 \\
1100 & \quad 1100 \\
1200 & \quad 1200 \\
\end{align*}
\]
IDEAL CYCLE EFFICIENCY AND POWER DENSITY VS MINIMUM TEMPERATURE

THE VARIABLE PARAMETER IS MINIMUM PRESSURE, $P_1$

MAXIMUM TEMPERATURE $T_3 = 400^\circ F$

MAXIMUM PRESSURE $P_2 = 10,000$ PSI

--- HP/IN$^3$ DISPLACEMENT

--- EFFICIENCY

GRAPH No. 22
IDEAL CYCLE EFFICIENCY AND POWER DENSITY VS MINIMUM TEMPERATURE

THE VARIABLE PARAMETER IS MAXIMUM PRESSURE, $P_2$

MAXIMUM TEMPERATURE $T_3 = 1200^\circ F$
MINIMUM PRESSURE $P_1 = 500^\circ F$

---

HP/IN$^3$ DISPLACEMENT

EFFICIENCY

GRAPH Nº 23

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<thead>
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<th>EFFICIENCY (PERCENT)</th>
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<td>4</td>
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<td>5</td>
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</table>

MINIMUM TEMPERATURE (DEGREES FAHRENHEIT), $T_1$

$P_{max} = 10,000$ psi

$5000$ psi

$3500$ psi

$10,000$ psi

$5000$ psi

$2000$ psi

$3500$ psi
IDEAL CYCLE EFFICIENCY AND POWER DENSITY VS MINIMUM TEMPERATURE

THE VARIABLE PARAMETER IS MAXIMUM TEMPERATURE, \( T_3 \)

MINIMUM PRESSURE \( P_1 = 800 \text{PSI} \)
MAXIMUM PRESSURE \( P_2 = 5000 \text{PSI} \)

--- HP/IN\(^3\) DISPLACEMENT
--- EFFICIENCY

GRAPH Nº 24

MINIMUM TEMPERATURE \( T_1 \) (DEGREES FAHRENHEIT)
IDEAL CYCLE EFFICIENCY AND POWER DENSITY VS MINIMUM PRESSURE

THE VARIABLE PARAMETER IS MAXIMUM PRESSURE, \( P_2 \)

--- HP/IN\(^3\) DISPLACEMENT

--- EFFICIENCY

MAXIMUM TEMPERATURE \( T_3 = 550^\circ \)

MINIMUM TEMPERATURE \( T_1 = 80^\circ \)

\( P_{\text{max}} = 10000 \text{ psi} \)

GRAPH № 25
IDEAL CYCLE EFFICIENCY AND POWER DENSITY VS MINIMUM PRESSURE

THE VARIABLE PARAMETER IS MINIMUM TEMPERATURE, $T_1$

--- HP/IN$^3$ DISPLACEMENT
--- EFFICIENCY

MAXIMUM PRESSURE $P_2 = 3500$ PSI
MAXIMUM TEMPERATURE $T_3 = 550^\circ F$

GRAPH N° 26
IDEAL CYCLE EFFICIENCY AND POWER DENSITY VS MINIMUM PRESSURE

THE VARIABLE PARAMETER IS MAXIMUM TEMPERATURE, $T_3$

- - - HP/IN$^3$ DISPLACEMENT
- - - EFFICIENCY

MAXIMUM PRESSURE $P_2 = 5000$ PSI
MINIMUM TEMPERATURE $T_1 = 80^\circ F$

GRAPH NO. 27
IDEAL CYCLE EFFICIENCY AND POWER DENSITY VS MAXIMUM PRESSURE

MAXIMUM TEMPERATURE $T_3 = 900^\circ F$

MINIMUM TEMPERATURE $T_1 = 80^\circ F$

--- HP/IN$^3$ DISPLACEMENT

--- EFFICIENCY

THE VARIABLE PARAMETER IS

MINIMUM PRESSURE, $P_1$

GRAPH N° 28

Pmin = 1800 psi

1100 psi

5000 psi

800 psi

1100 psi

800 psi

1800 psi

500 psi

1000 2000 3000 4000 5000 6000 7000 8000 9000 10000

MAXIMUM PRESSURE, $P_2$ (psi)

0 10 20 30 40 50

EFFICIENCY (PERCENT)

0 1000 2000 3000 4000 5000 6000 7000 8000 9000 10000

HP/IN$^3$ DISPLACEMENT @ 1000 RPM
IDEAL CYCLE  EFFICIENCY AND POWER DENSITY VS MAXIMUM PRESSURE

MAXIMUM TEMPERATURE $T_3 = 1200^\circ F$
MINIMUM PRESSURE $P_1 = 3500$ PSI

--- HP/IN$^3$ DISPLACEMENT
--- EFFICIENCY

THE VARIABLE PARAMETER IS MINIMUM TEMPERATURE, $T_1$

![Graph showing efficiency and power density vs maximum pressure]
IDEAL CYCLE EFFICIENCY AND POWER DENSITY VS MAXIMUM PRESSURE

MINIMUM TEMPERATURE $T_i = 80^\circ F$
MINIMUM PRESSURE $P_i = 800$PSI

--- HP/IN$^3$ DISPLACEMENT
--- EFFICIENCY

THE VARIABLE PARAMETER IS MAXIMUM TEMPERATURE, $T_3$

GRAPH NO. 30

MAXIMUM PRESSURE (PSI), $P_2$
EFFECTIVENESS (PERCENT)

--- $T_{\text{max}} = 1200^\circ F$
--- $550^\circ F$
--- $900^\circ F$
--- $350^\circ F$

HP/IN$^3$ DISPLACEMENT @ 1000RPM

--- $1200^\circ F$
--- $900^\circ F$
--- $550^\circ F$
--- $350^\circ F$
MAXIMUM EFFICIENCY VS MAXIMUM TEMPERATURE
(IDEAL CYCLE)

MAXIMUM PRESSURE \( P_2 = 2000 \text{ PSI} \)

THE VARIABLE PARAMETER IS MINIMUM TEMPERATURE, \( T_1 \)

\[ T_{\text{min}} = 40^\circ \text{F} \]

\[ 80^\circ \text{F} \]

\[ 120^\circ \text{F} \]

\[ 150^\circ \text{F} \]
MAXIMUM EFFICIENCY VS MAXIMUM TEMPERATURE
(IDEAL CYCLE)

MAXIMUM PRESSURE \( P_2 = 3500 \text{ PSI} \)

THE VARIABLE PARAMETER IS MINIMUM TEMPERATURE, \( T_1 \)

\[ T_{\text{min}} = 40^\circ\text{F} \]

\[ 80^\circ\text{F} \]

\[ 120^\circ\text{F} \]

\[ 150^\circ\text{F} \]

MAXIMUM TEMPERATURE (DEGREES FAHRENHEIT), \( T_3 \)
MAXIMUM EFFICIENCY VS MAXIMUM TEMPERATURE
(IDEAL CYCLE)

MAXIMUM PRESSURE $p_2 = 5000$ PSI

THE VARIABLE PARAMETER IS MINIMUM TEMPERATURE, $T_{min}$
MAXIMUM EFFICIENCY VS MAXIMUM TEMPERATURE
(IDEAL CYCLE)

MAXIMUM PRESSURE $P_2 = 10000$ PSI

THE VARIABLE PARAMETER IS MINIMUM TEMPERATURE, $T_{min}$

MAXIMUM TEMPERATURE $T_3$ (DEGREES FAHRENHEIT), $T_{min} = 120^\circ F$
MAXIMUM EFFICIENCY VS MAXIMUM TEMPERATURE (IDEAL CYCLE)

THE VARIABLE PARAMETER IS MINIMUM TEMPERATURE, $T_1$

Graph No. 35

The variable parameter is minimum temperature, $T_1$. The graph shows the relationship between maximum efficiency, percentage ($\eta$), and maximum temperature (degrees Fahrenheit), $T_3$. The graph includes curves for different minimum temperatures: $T_{min} = 120^\circ F$, $80^\circ F$, $40^\circ F$, and $150^\circ F$. The x-axis represents maximum temperature in degrees Fahrenheit, ranging from 300 to 1200, while the y-axis represents maximum efficiency in percent, ranging from 0 to 50.
MAXIMUM POWER DENSITY VS MAXIMUM TEMPERATURE
(IDEAL CYCLE)

MAXIMUM PRESSURE $P_2 = 2000$ PSI

MINIMUM TEMPERATURE IS THE VARIABLE PARAMETER

GRAPH NO. 36

MAXIMUM HP/IN$^3$ DISPLACEMENT @ 1000 RPM

MAXIMUM TEMPERATURE ($^\circ$ FAHRENHEIT) $T_3$

$T_{\text{min}} = 40^\circ F$

$80^\circ F$

$120^\circ F$

$150^\circ F$
MAXIMUM POWER DENSITY VS MAXIMUM TEMPERATURE
(IDEAL CYCLE)

MAXIMUM HP/IN³ DISPLACEMENT

MAXIMUM PRESSURE = 3,500 PSI

THE VARIABLE PARAMETER IS MINIMUM TEMPERATURE, $T_1$

MAXIMUM TEMPERATURE, $T_3$
MAXIMUM POWER DENSITY VS MAXIMUM TEMPERATURE
(IDEAL CYCLE)

GRAPH N° 38
MAXIMUM PRESSURE = 5,000 PSI
THE VARIABLE PARAMETER IS MINIMUM TEMPERATURE, \( T_{min} \)

MAXIMUM HP / IN \(^3 \) DISPLACEMENT @ 1000 RPM

\( T_{min} = 40^\circ F \)
\( 80^\circ F \)
\( 120^\circ F \)
\( 150^\circ F \)

MAXIMUM TEMPERATURE, \( T_3 \)

PRESSURE = 5,000 PSI

THE VARIABLE PARAMETER IS MINIMUM TEMPERATURE, \( T_{min} \)
MAXIMUM POWER DENSITY VS MAXIMUM TEMPERATURE

(IDEAL CYCLE)

MAXIMUM PRESSURE = 10,000 PSI

THE VARIABLE PARAMETER IS MINIMUM TEMPERATURE, T_1

GRAPH N° 39

T_{min} = 40°F

80°F

120°F

150°F
WORK OF COMPRESSION AND WORK OF EXPANSION VS MAXIMUM TEMPERATURE (IDEAL CYCLE)

MINIMUM TEMPERATURE \( T_i = 80^\circ \)
MINIMUM PRESSURE \( P_i = 800 \text{PSI} \)

--- WORK OF EXPANSION
--- WORK OF COMPRESSION

THE VARIABLE PARAMETER IS MAXIMUM PRESSURE, \( P_2 \)

--- GRAPH N° 40 ---
WORK OF COMPRESSION AND WORK OF EXPANSION VS MAXIMUM TEMPERATURE
(IDEAL CYCLE)

MINIMUM TEMPERATURE $T_1 = 80^\circ F$
MINIMUM PRESSURE $P_1 = 1100$ PSI

--- WORK OF EXPANSION
--- WORK OF COMPRESSION

THE VARIABLE PARAMETER IS MAXIMUM PRESSURE, $P_2$

The variable parameter is maximum pressure, $P_2$. The graph shows the work of compression and work of expansion as a function of maximum temperature, $T_3$. The minimum temperature is $80^\circ F$ and the minimum pressure is 1100 PSI. The graph includes lines for different maximum pressures: 10,000 psi, 5,000 psi, 3500 psi, and 2000 psi.
WORK OF COMPRESSION AND WORK OF EXPANSION VS MAXIMUM TEMPERATURE (IDEAL CYCLE)

MINIMUM TEMPERATURE \( T_1 = 40^\circ\text{F} \)
MAXIMUM PRESSURE \( P_2 = 2000\text{psi} \)

Graph No. 42

The variable parameter is minimum pressure, \( p_1 \)
WORK OF COMPRESSION AND WORK OF EXPANSION VS MAXIMUM TEMPERATURE (IDEAL CYCLE)

MINIMUM PRESSURE $P_1 = 1100$ PSI
MAXIMUM PRESSURE $P_2 = 5000$ PSI

--- WORK OF EXPANSION
--- WORK OF COMPRESSION

THE VARIABLE PARAMETER IS MINIMUM TEMPERATURE, $T_1$

$T_{min} = 40^\circ F, 80^\circ F, 120^\circ F, 150^\circ F$

GRAPH NO 43

WORK OF COMPRESSION (BTU/LB)

WORK OF EXPANSION (BTU/LB)

MAXIMUM TEMPERATURE (DEGREES FAHRENHEIT), $T_3$
WORK OF COMPRESSION AND WORK OF EXPANSION VS MINIMUM TEMPERATURE (IDEAL CYCLE)

THE VARIABLE PARAMETER IS MAXIMUM TEMPERATURE, $T_3$

MINIMUM PRESSURE $P_1 = 800$ PSI

MAXIMUM PRESSURE $P_2 = 5000$ PSI

--- WORK OF EXPANSION

--- WORK OF COMPRESSION

--- WORK OF COMPRESSION

--- WORK OF EXPANSION

GRAPH NO. 44
WORK OF COMPRESSION AND WORK OF EXPANSION VS MINIMUM TEMPERATURE (IDEAL CYCLE)
THE VARIABLE PARAMETER IS MAXIMUM PRESSURE

MINIMUM PRESSURE \( P_1 = 500 \text{ PSI} \)
MAXIMUM TEMPERATURE \( T_2 = 1200^\circ \text{F} \)

--- WORK OF EXPANSION
--- WORK OF COMPRESSION

GRAPH Nº 45
WORK OF COMPRESSION AND WORK OF EXPANSION VS MINIMUM TEMPERATURE (IDEAL CYCLE)

THE VARIABLE PARAMETER IS MINIMUM PRESSURE, $P_1$

MAXIMUM TEMPERATURE $T_3 = 900^\circ F$
MAXIMUM PRESSURE $P_2 = 10000$ PSI

--- WORK OF EXPANSION
--- WORK OF COMPRESSION

GRAPH NO. 46
WORK OF COMPRESSION AND WORK OF EXPANSION VS MINIMUM PRESSURE
(IDEAL CYCLE)

THE VARIABLE PARAMETER IS MAXIMUM PRESSURE, $P_2$

MAXIMUM TEMPERATURE $T_3 = 550°F$
MINIMUM TEMPERATURE $T_1 = 80°F$

--- WORK OF EXPANSION
--- WORK OF COMPRESSION

GRAPH № 47
WORK OF COMPRESSION AND WORK OF EXPANSION VS MINIMUM PRESSURE
(IDEAL CYCLE)

THE VARIABLE PARAMETER IS MINIMUM TEMPERATURE, $T_1$

MAXIMUM PRESSURE $P_2 = 3500$ PSI
MAXIMUM TEMPERATURE $T_3 = 550^\circ F$

--- WORK OF EXPANSION
--- WORK OF COMPRESSION

GRAPH No 48

WORK OF COMPRESSION (BTU/LB) vs MINIMUM PRESSURE (PSI), $P_1$
WORK OF COMPRESSION AND WORK OF EXPANSION VS MINIMUM PRESSURE

(IDEAL CYCLE)

THE VARIABLE PARAMETER IS MAXIMUM TEMPERATURE, $T_3$

MAXIMUM PRESSURE $P_2 = 5000$ PSI

MINIMUM TEMPERATURE $T_1 = 80^\circ F$

--- WORK OF EXPANSION

--- WORK OF COMPRESSION

GRAPH Nº 49
WORK OF COMPRESSION AND WORK OF EXPANSION VS MAXIMUM PRESSURE

(IDEAL CYCLE)

MAXIMUM TEMPERATURE $T = 1200^\circ F$
MINIMUM PRESSURE $P_1 = 500\text{PSI}$.  

--- WORK OF EXPANSION
--- WORK OF COMPRESSION

THE VARIABLE PARAMETER IS MINIMUM TEMPERATURE, $T_1$

Graph No. 50

$T_{\text{min}} = 40^\circ F, 80^\circ F, 120^\circ F, 150^\circ F$

**Graph**

- Y-axis: BTU/LB
- X-axis: MAXIMUM PRESSURE (PSI) $P_2$
- HP/IN$^3$ DISPLACEMENT @ 1000 RPM

---

$40^\circ F$
$80^\circ F$
$120^\circ F$
$150^\circ F$
WORK OF COMPRESSION AND WORK OF EXPANSION VS MAXIMUM PRESSURE
(IDEAL CYCLE)

MINIMUM TEMPERATURE $T_1 = 80^\circ F$
MINIMUM PRESSURE $P_1 = 800$ PSI

GRAPH N° 51

THE VARIABLE PARAMETER IS MAXIMUM TEMPERATURE, $T_3$

![Graph showing work of compression and expansion vs maximum pressure](image-url)
WORK OF COMPRESSION AND WORK OF EXPANSION VS MAXIMUM PRESSURE (IDEAL CYCLE)

MAXIMUM TEMPERATURE $T_3 = 900^\circ F$
MINIMUM TEMPERATURE $T_1 = 80^\circ F$

GRAPH No. 52

THE VARIABLE PARAMETER IS MINIMUM PRESSURE, $P_i$

WORK OF EXPANSION
WORK OF COMPRESSION

THE VARIABLE PARAMETER IS MINIMUM PRESSURE, $P_i$
OVERALL EFFICIENCY VS MAXIMUM TEMPERATURE

MINIMUM TEMPERATURE $T_1 = 40^\circ F$
MINIMUM PRESSURE $P_1 = 1100$ PSI

THE VARIABLE PARAMETER IS MAXIMUM PRESSURE, $P_2$

GRAPH NO. 53

EFFICIENCY (PERCENT)

MAXIMUM TEMPERATURE, $T_3$ (DEGREES FAHRENHEIT)
OVERALL EFFICIENCY VS MAXIMUM TEMPERATURE

MINIMUM TEMPERATURE $T_1 = 80^\circ F$

MINIMUM PRESSURE $P_1 = 1100$ PSI

THE VARIABLE PARAMETER IS MAXIMUM PRESSURE, $P_2$

Graph No. 54
OVERALL EFFICIENCY VS MAXIMUM TEMPERATURE

MINIMUM TEMPERATURE $T_1 = 40^\circ F$

MINIMUM PRESSURE $P_1 = 500$PSI

GRAPH NO. 55

THE VARIABLE PARAMETER IS MAXIMUM PRESSURE, $P_2$

EACH CURVE IS FOR A DIFFERENT MAXIMUM PRESSURE:

- 10,000 PSI
- 5,000 PSI
- 3,500 PSI
- 2,000 PSI

EFFICIENCY (PERCENT)

MAXIMUM TEMPERATURE, $T_3$ (DEGREES FAHRENHEIT)
OVERALL EFFICIENCY VS MAXIMUM TEMPERATURE

MINIMUM TEMPERATURE $T_1 = 80^\circ F$
MINIMUM PRESSURE $P_1 = 500$ PSI

THE VARIABLE PARAMETER IS MAXIMUM PRESSURE, $P_2$

GRAPH NO. 56
OVERALL EFFICIENCY VS MAXIMUM TEMPERATURE

MINIMUM TEMPERATURE $T_1 = 120^\circ F$

MINIMUM PRESSURE $P_1 = 1100$ PSI

THE VARIABLE PARAMETER IS MAXIMUM PRESSURE, $P_2$

GRAPH NO. 57

EFFICIENCY (PERCENT)

MAXIMUM TEMPERATURE, $T_3$ (DEGREES FAHRENHEIT)

- 10000 PSI
- 5000
- 3500
OVERALL EFFICIENCY VS MAXIMUM TEMPERATURE

MINIMUM TEMPERATURE $T_1 = 80^\circ F$
MAXIMUM PRESSURE $P_2 = 3500$ PSI

THE VARIABLE PARAMETER IS MINIMUM PRESSURE, $P_1$. 

![Diagram showing overall efficiency vs maximum temperature, with various curves for different minimum pressures.](image-url)
OVERALL EFFICIENCY VS MAXIMUM TEMPERATURE

MINIMUM TEMPERATURE \( T_1 = 40^\circ F \)

MAXIMUM PRESSURE \( P_2 = 5000 \text{PSI} \)

THE VARIABLE PARAMETER IS MINIMUM PRESSURE, \( P_1 \)

GRAPH NO. 59

EFFICIENCY (PERCENT)

MAXIMUM TEMPERATURE, \( T_3 \) (DEGREES FAHRENHEIT)

500 PSI
800
1100
OVERALL EFFICIENCY VS MAXIMUM TEMPERATURE

MAXIMUM PRESSURE $P_2 = 5000$ PSI
MINIMUM PRESSURE $P_1 = 1100$ PSI

GRAPH Nº 60

THE VARIABLE PARAMETER IS MINIMUM TEMPERATURE, $T_1$
OVERALL EFFICIENCY VS MAXIMUM TEMPERATURE

MAXIMUM PRESSURE \( P_2 = 5000 \text{ PSI} \)
MINIMUM PRESSURE \( P_1 = 500 \text{ PSI} \)

THE VARIABLE PARAMETER IS MINIMUM TEMPERATURE, \( T_1 \)

EFFICIENCY (PERCENT)

MAXIMUM TEMPERATURE, \( T_3 \) (DEGREES FAHRENHEIT)
OVERALL EFFICIENCY VS MINIMUM TEMPERATURE

MAXIMUM TEMPERATURE \( T_3 = 900^\circ F \)

MAXIMUM PRESSURE \( P_2 = 3500 \text{ PSI} \)

THE VARIABLE PARAMETER IS MINIMUM PRESSURE, \( P_1 \)

Graph No. 62

Efficiency (Percent) vs Minimum Temperature, \( T_1 \) (Degrees Fahrenheit)

- 500 PSI
- 800
- 1100
OVERALL EFFICIENCY VS MINIMUM TEMPERATURE

MAXIMUM TEMPERATURE $T_3 = 600^\circ\text{F}$

MAXIMUM PRESSURE $P_2 = 5000 \text{ PSI}$

THE VARIABLE PARAMETER IS MINIMUM PRESSURE, $P_1$
OVERALL EFFICIENCY VS MINIMUM TEMPERATURE

MAXIMUM TEMPERATURE $T_3 = 900^\circ F$

MINIMUM PRESSURE $P_1 = 1100$

THE VARIABLE PARAMETER IS MAXIMUM PRESSURE, $P_2$

Graph N° 64

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<th>PRESSURE (PSI)</th>
<th>EFFICIENCY (PERCENT)</th>
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<tbody>
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MINIMUM TEMPERATURE $T_1$ (DEGREES FAHRENHEIT)
OVERALL EFFICIENCY VS MINIMUM TEMPERATURE

MAXIMUM TEMPERATURE $T_3 = 900^\circ F$

MINIMUM PRESSURE $P_1 = 500 \text{ PSI}$

THE VARIABLE PARAMETER IS MAXIMUM PRESSURE, $P_2$
OVERALL EFFICIENCY VS MINIMUM TEMPERATURE

MAXIMUM TEMPERATURE $T_3 = 600^\circ F$

MINIMUM PRESSURE $P_1 = 1100$ PSI

THE VARIABLE PARAMETER IS MAXIMUM PRESSURE, $P_2$

MINIMUM TEMPERATURE, $T_1$ (DEGREES FAHRENHEIT)
OVERALL EFFICIENCY VS MINIMUM TEMPERATURE

MAXIMUM TEMPERATURE $T_3 = 600^\circ F$

MINIMUM PRESSURE $P_1 = 800$ PSI

THE VARIABLE PARAMETER IS MAXIMUM PRESSURE, $P_2$
OVERALL EFFICIENCY VS MINIMUM TEMPERATURE

MAXIMUM PRESSURE $P_2 = 5000$ PSI
MINIMUM PRESSURE $P_1 = 1100$ PSI

THE VARIABLE PARAMETER IS MAXIMUM TEMPERATURE $T_3$

GRAPH NO. 68

The diagram shows the relationship between overall efficiency and minimum temperature $T_1$ (degrees Fahrenheit). The variable parameter is maximum temperature $T_3$. The graph includes points for different maximum temperatures: 900°F, 700°F, 600°F, and 550°F. The efficiency is plotted on the y-axis, and the minimum temperature is plotted on the x-axis.
OVERALL EFFICIENCY VS MINIMUM TEMPERATURE

MINIMUM PRESSURE $P_1 = 500$ PSI

MAXIMUM PRESSURE $P_2 = 5000$ PSI

THE VARIABLE PARAMETER IS MAXIMUM TEMPERATURE, $T_3$

![Graph showing efficiency vs minimum temperature](image)

MINIMUM TEMPERATURE, $T_1$ (DEGREES FAHRENHEIT)

EFFICIENCY (PERCENT)

- $900^\circ F$
- $600^\circ F$
- $550^\circ F$
- $1200^\circ F$
- $1000^\circ F$
- $900^\circ F$
- $600^\circ F$
- $700^\circ F$
OVERALL EFFICIENCY VS MINIMUM PRESSURE

MAXIMUM TEMPERATURE $T_3 = 600^\circ F$

MINIMUM TEMPERATURE $T_1 = 40^\circ F$

THE VARIABLE PARAMETER IS MAXIMUM PRESSURE, $P_2$

![Graph showing overall efficiency vs minimum pressure with points at 5000 and 3500 PSI and a line for 10,000 PSI.](image)
OVERALL EFFICIENCY VS MINIMUM PRESSURE

MAXIMUM TEMPERATURE $T_3 = 900^\circ F$

MINIMUM TEMPERATURE $T_1 = 80^\circ F$

THE VARIABLE PARAMETER IS MAXIMUM PRESSURE $P_2$

GRAPH N° 71

Efficiency (Percent)

MINIMUM PRESSURE $P_1$ (PSI)
OVERALL EFFICIENCY VS MINIMUM PRESSURE

MAXIMUM TEMPERATURE $T_3 = 700^\circ F$

MINIMUM TEMPERATURE $T_1 = 120^\circ F$

THE VARIABLE PARAMETER IS MAXIMUM PRESSURE, $P_2$

![Graph showing efficiency vs minimum pressure](image-url)
OVERALL EFFICIENCY VS MINIMUM PRESSURE

MAXIMUM PRESSURE $P_3 = 5000$ PSI

MAXIMUM TEMPERATURE $T_3 = 900^\circ F$

THE VARIABLE PARAMETER IS MINIMUM TEMPERATURE, $T_1$
OVERALL EFFICIENCY VS MINIMUM PRESSURE

MAXIMUM PRESSURE $P_2 = 3500$ PSI

MAXIMUM TEMPERATURE $T_3 = 600^\circ$F

THE VARIABLE PARAMETER IS MINIMUM TEMPERATURE, $T_1$
OVERALL EFFICIENCY VS MINIMUM PRESSURE

MAXIMUM PRESSURE $P_2 = 5000$ PSI

MINIMUM TEMPERATURE $T_1 = 40^\circ F$

THE VARIABLE PARAMETER IS MAXIMUM TEMPERATURE, $T_3$
OVERALL EFFICIENCY VS MINIMUM PRESSURE

MAXIMUM PRESSURE $P_2 = 3500$ PSI

MINIMUM TEMPERATURE $T_1 = 80^\circ F$

THE VARIABLE PARAMETER IS MAXIMUM TEMPERATURE, $T_3$
OVERALL EFFICIENCY VS MAXIMUM PRESSURE

MAXIMUM TEMPERATURE $T_3 = 900^\circ F$

MINIMUM TEMPERATURE $T_1 = 80^\circ F$

THE VARIABLE PARAMETER IS MINIMUM PRESSURE, $P_1$. 

Graph Number 77
OVERALL EFFICIENCY VS MAXIMUM PRESSURE

MAXIMUM TEMPERATURE $T_3 = 900^\circ\text{F}$

MINIMUM PRESSURE $P_1 = 500\text{PSI}$

THE VARIABLE PARAMETER IS MINIMUM TEMPERATURE, $T_1$
OVERALL EFFICIENCY VS MAXIMUM PRESSURE

MAXIMUM TEMPERATURE $T_3 = 600^\circ F$

MINIMUM PRESSURE $P_1 = 500$ PSI

THE VARIABLE PARAMETER IS MINIMUM TEMPERATURE, $T_1$
OVERALL EFFICIENCY VS MAXIMUM PRESSURE

MAXIMUM TEMPERATURE $T_3 = 600^\circ F$

MINIMUM PRESSURE $P_1 = 1100$ PSI

THE VARIABLE PARAMETER IS MINIMUM TEMPERATURE, $T_1$

MAXIMUM PRESSURE, $P_2$ (PSI)
OVERALL EFFICIENCY VS MAXIMUM PRESSURE

MINIMUM TEMPERATURE, $T_1 = 80^\circ F$

MINIMUM PRESSURE $P_1 = 500$ PSI

THE VARIABLE PARAMETER IS MAXIMUM TEMPERATURE, $T_3$
OVERALL EFFICIENCY VS MAXIMUM PRESSURE

MINIMUM TEMPERATURE $T_i = 80^\circ F$

MINIMUM PRESSURE $P_i = 1100$ PSI

THE VARIABLE PARAMETER IS MAXIMUM TEMPERATURE, $T_3$
ACTUAL WORK OF COMPRESSION AND WORK OF EXPANSION VS MAXIMUM TEMPERATURE

THE VARIABLE PARAMETER IS MAXIMUM PRESSURE, $P_2$

MINIMUM TEMPERATURE $T_1 = 80^\circ F$
MINIMUM PRESSURE $P_1 = 1100$ PSI

GRAPH N° 83
ACTUAL WORK OF COMPRESSION AND WORK OF EXPANSION VS MAXIMUM TEMPERATURE

THE VARIABLE PARAMETER IS MINIMUM PRESSURE, $P_1$

MINIMUM TEMPERATURE $T_1 = 80^\circ F$
MAXIMUM PRESSURE $P_2 = 5000$ PSI

WORK OF EXPANSION
WORK OF COMPRESSION

GRAPH NO. 84
ACTUAL WORK OF COMPRESSION AND WORK OF EXPANSION VS MINIMUM TEMPERATURE

THE VARIABLE PARAMETER IS MAXIMUM TEMPERATURE, $T_3$

MAXIMUM PRESSURE $P_2 = 5000$ PSI
MINIMUM PRESSURE $P_1 = 1100$ PSI

WORK OF EXPANSION
WORK OF COMPRESSION

GRAPH № 86

![Graph showing work of expansion and compression vs minimum temperature](image-url)
ACTUAL WORK OF COMPRESSION AND WORK OF EXPANSION VS MAXIMUM TEMPERATURE

THE VARIABLE PARAMETER IS MINIMUM TEMPERATURE, $T_1$

MINIMUM PRESSURE $P_1 = 1100$ PSI
MAXIMUM PRESSURE $P_2 = 5000$ PSI

--- WORK OF EXPANSION
-- WORK OF COMPRESSION

GRAPH NO. 85

$T_{min} = 40^\circ F$
$80^\circ$
$120^\circ$
$150^\circ$

WORK OF EXPANSION (BTU/LB)

WORK OF COMPRESSION (BTU/LB)

MAXIMUM TEMPERATURE, $T_3$ (DEGREES FAHRENHEIT)
ACTUAL WORK OF COMPRESSION AND WORK OF EXPANSION VS MINIMUM TEMPERATURE

THE VARIABLE PARAMETER IS MAXIMUM PRESSURE

MAXIMUM TEMPERATURE $T_3 = 600^\circ F$
MINIMUM PRESSURE $p_1 = 1100\text{PSI}$

--- WORK OF EXPANSION
--- WORK OF COMPRESSION

GRAPH NO. 87

WORK OF EXPANSION (BTU/LB)
WORK OF COMPRESSION (BTU/LB)

MINIMUM TEMPERATURE, $T_1$ (DEGREES FAHRENHEIT)
ACTUAL WORK OF COMPRESSION AND WORK OF EXPANSION VS MINIMUM TEMPERATURE

THE VARIABLE PARAMETER IS MINIMUM PRESSURE, $P_1$

MAXIMUM TEMPERATURE $T_3 = 600^\circ\text{F}$
MAXIMUM PRESSURE $P_2 = 5000\text{PSI}$

- WORK OF EXPANSION
- WORK OF COMPRESSION

GRAPH NO. 88
ACTUAL WORK OF COMPRESSION AND WORK OF EXPANSION VS MINIMUM PRESSURE

THE VARIABLE PARAMETER IS MAXIMUM PRESSURE, $p_2$

MAXIMUM TEMPERATURE $T_3 = 600^\circ F$

MINIMUM TEMPERATURE $T_1 = 80^\circ F$

GRAPH N° 89
ACTUAL WORK OF COMPRESSION AND WORK OF EXPANSION VS MINIMUM PRESSURE

THE VARIABLE PARAMETER IS MINIMUM TEMPERATURE, $T_1$

MAXIMUM TEMPERATURE $T_3 = 600^\circ F$
MAXIMUM PRESSURE $P_2 = 5,000$ PSI

- - - WORK OF EXPANSION
- - - WORK OF COMPRESSION

GRAPH NO 90

MINIMUM PRESSURE, $P_1$ (PSI)

WORK OF EXPANSION (BTU/LB)

WORK OF COMPRESSION (BTU/LB)
ACTUAL WORK OF COMPRESSION AND WORK OF EXPANSION VS MINIMUM PRESSURE

THE VARIABLE PARAMETER IS MAXIMUM TEMPERATURE $T_3$

MINIMUM TEMPERATURE $T_1 = 80^\circ F$
MAXIMUM PRESSURE $P_2 = 5000$ PSI

WORK OF EXPANSION
WORK OF COMPRESSION

GRAPH Nº 91
ACTUAL WORK OF COMPRESSION AND WORK OF EXPANSION VS MAXIMUM PRESSURE

THE VARIABLE PARAMETER IS MINIMUM TEMPERATURE, $T_1$

MAXIMUM TEMPERATURE $T_3 = 600^\circ\text{F}$
MINIMUM PRESSURE $P_1 = 1100$ PSI

---

GRAPH N° 92

---

MAXIMUM PRESSURE, $P_2$ (PSI)

---

WORK OF EXPANSION

WORK OF COMPRESSION

---

WORK OF EXPANSION (BTU/LB)

WORK OF COMPRESSION (BTU/LB)

---

3000 4 5 6 7 8 9 10000

---

$40^\circ\text{F}$

$80^\circ\text{F}$

$120^\circ\text{F}$

$150^\circ\text{F}$
ACTUAL WORK OF COMPRESSION AND WORK OF EXPANSION VS MAXIMUM PRESSURE

THE VARIABLE PARAMETER IS MAXIMUM TEMPERATURE, $T_3$

MINIMUM TEMPERATURE $T_1 = 80^\circ F$
MINIMUM PRESSURE $P_1 = 1100$ PSI

- - - - - - WORK OF EXPANSION
- - - - - - WORK OF COMPRESSION

GRAPH NO. 93
ACTUAL WORK OF COMPRESSION AND WORK OF EXPANSION VS MAXIMUM PRESSURE

THE VARIABLE PARAMETER IS MINIMUM PRESSURE, $P_1$

MAXIMUM TEMPERATURE $T_3 = 600^\circ F$

MINIMUM TEMPERATURE $T_1 = 80^\circ F$

WORK OF EXPANSION

WORK OF COMPRESSION

GRAPH NO. 94

MAXIMUM PRESSURE, $P_2$ (PSI)
GRAPH No. 97 PRESSURE RATIO VS OVERALL EFFICIENCY

- **Tmax = 1200°F**
  - 1000°F
  - 900°F
  - 700°F

- **Pmin = 500 psi**
  - Tmin = 150°F (gas)

- **Tmax = 900°F**
  - 700°F
  - 600°F
  - 550°F

- **Pmin = 800 psi**
  - Tmin = 150°F (gas)

- **Tmax = 1200°F**
  - 1000°F
  - 900°F

- **Pmin = 500 psi**
  - Tmin = 120°F (gas)

- **Tmax = 1200°F**
  - 1000°F
  - 900°F

- **Pmin = 800 psi**
  - Tmin = 120°F (gas)

- **Tmax = 1200°F**
  - 1000°F
  - 900°F

- **Pmin = 1100 psi**
  - Tmin = 120°F (gas)
## APPENDIX D

**Heat Exchanger - Parametric Study**  
Water Coolers (Counter Flow - Shell & Tube Type)

Outside Dia. of Tubes = .375"  
Transverse Pitch = .563"  
Longitudinal Pitch = .487"

Velocity of Fluid = 301/sec.  
Assumed \( \gamma_{comp} = \gamma \, \text{exp} = .9 \quad \gamma_{mech} = .95 \)

### Tube Data

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### Tube Data

- **Pressure Loss**
- **Tray Loss**
- **Total Loss**
- **Water Data**
- **Oxy Data**

**Tube Type**

- **Length**
- **Number**
- **Surface Area**
- **Total**

**Weight C. ft.**

**Volume C. ft.**

**Volume C. ft./lb.**

**k**

**p**

### Notes

- Prepared by:
- Checked by:

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**Heat Exchanger - Parametric Study**

**Outside Dia. of Tubes** = .375"

**Velocity of Fluid** = 30'/Sec.

**Pressurized Water Heaters (Counter Flow - Shell & Tube Type)**

**Transverse Pitch** = .563"  
**Longitudinal Pitch** = .487"  
**Assumed** $\gamma = \beta = .9$  
**$\gamma_{m} = .95$**

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**PREPARED BY: ___________________________**

**CHECKED BY: ___________________________**

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268
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Heat Exchanger - Parametric Study.

Air Blast Cooler (Crossflow - Circular Fin Type)

Outside Dia. of Tubes = .375"  Outside Dia. of Fins = 1.25"  Thickness of Fins = .091"

Transverse Pitch of Tubes = .850"  Assumed \( \gamma_{comp} = \gamma_{exp} = 0.9 \)  \( \gamma_{mech} = .95 \)

Velocity of Fluid = 30 Ft/Sec.  Velocity of Air = 45 Ft/Sec.
Heat Exchanger - Parametric Study.

Outside Dia. of Tubes = 0.375

Velocity of Fluid = 30'/Sec.

Transverse Pitch = 0.563

Longitudinal Pitch = 0.487

Assumed $\gamma_{\text{comp}} = \gamma_{\text{exp}} = 0.9$

Mech = 0.95

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APPENDIX E

RELATIONSHIPS USED FOR HEAT TRANSFER CALCULATIONS

Shell and tube type counterflow heat exchangers are used to give minimum surface area.

1. Lame's formula for thick cylinders to obtain thickness of tubing.

\[ \frac{pr}{r^2} = a \quad \frac{fr}{r^2} = b \]

where \( p \) = pressure, \( f \) = stress, \( a \) and \( b \) are constants, and if subscripts \( o \) and \( i \) are outside and inside conditions when \( p_o = 0 \), the above equations reduce to

\[ f_i = p_i \left( r_o^2 + r_i^2 \right) / \left( r_o^2 - r_i^2 \right) \quad \text{or} \]

\[ r_o = r_i \left[ \left( f_i + p_i \right) / \left( f_i - p_i \right) \right]^{1/2} \]

Ref. 1

2. Minimum internal diameter of drum by geometry

\[ \text{Dig} = \left( \frac{2 \sqrt{3}}{\pi \phi} \right) N \]

where \( N \) = number of tubes
\( \phi \) = packing efficiency
\( \phi = \frac{\text{area of circumscribing hexagons}}{\text{area of container}} = \frac{2 \sqrt{3} N}{\pi \text{Dig/St}} \]

Ref. 2

3. Heat transmitted per unit time

\[ Q = U \Delta tm \]

where \( U \) = overall heat transfer coefficient
\( A \) = surface area
\( \Delta tm \) = log mean temperature difference

Ref. 3 or 4
4. Log mean temperature difference

$$LMTD = \Delta tm = \frac{\Delta ta - \Delta tb}{\log \frac{\Delta ta}{\Delta tb}}$$

where \(\Delta ta\) and \(\Delta tb\) are temperature differences between external and internal fluids at ends a and b

Ref. 3 or 1

5. Overall heat transfer coefficient is given by

$$\frac{1}{UA} = \frac{1}{(hA)_i} + \frac{\log (r_o / r_i)}{K \pi L} + \frac{1}{(hA)_o}$$

where \(U\) = overall heat transfer coefficient
\(A\) = surface area
\(K\) = thermal conductivity of the material
\(r_o\) and \(r_i\) are outer and inner radii of tubes
\(h_o\) and \(h_i\) are heat transfer coefficients outside and inside tubes

Ref. 3 or 1

6. Heat transfer coefficient for CO\(_2\) flowing inside the tubes is given by

$$Nu_i = Nu_i \left(\frac{\mu_{ib}}{\mu_{iw}}\right)^{0.11} \left(\frac{K_{ib}}{K_{iw}}\right)^{-0.33} \left(\frac{C_{pi}}{C_{pib}}\right)^{0.35}$$

where \(Nu_i\) = Nusselt number = \(\frac{h_i d_i}{K_{ib}}\)

\(\mu_{ib}\) = viscosity of CO\(_2\) at bulk conditions
\(\mu_{iw}\) = viscosity of CO\(_2\) at wall conditions
\(K_{ib}\) = thermal conductivity of CO\(_2\) at bulk conditions
\(K_{iw}\) = thermal conductivity of CO\(_2\) at wall conditions
\(C_{pi}\) = specific heat of CO\(_2\) at mean conditions between bulk and wall
\(C_{pib}\) = specific heat of CO\(_2\) at bulk conditions

$$Nu_i = \frac{\dot{J}}{8} \frac{Re_{ib} Pr_{ib}}{12.7 \sqrt{\frac{\dot{J}}{8}} (Pr_{ib}^{2/3} - 1) + 1.07}$$

where \(\dot{J} = \frac{1}{(1.82 \log Re_{ib} - 1.64)^2}\)

\(Re_{ib}\) = Reynolds number for CO\(_2\) based on bulk conditions
\(Pr_{ib}\) = Prandtl number for CO\(_2\) based on bulk conditions

Ref. 5

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7. Heat transfer coefficient for pressurized or supercritical water for turbulent flow outside tube banks with water flowing parallel to the axis of the tubes, is given by

\[
\frac{h_0}{C_{P_0} \rho_{ob}} \left( \frac{C_{P_0} \mu_{ob}}{\mu_{ow}} \right)^{2/3} \left( \frac{\mu_{ob}}{\mu_{ow}} \right)^{0.14} = \frac{0.023}{(\frac{d_e}{C_{t_{ob}}})^{0.2}}
\]

\[0.7 < Pr_{ob} < 120, \quad 10,000 < Re_{ob} < 120,000\]

where \(C_{P_0}\) = specific heat of water at bulk conditions
\(C_{t_{ob}}\) = mass flow velocity of water
\(\rho_{ob}\) = density of water at bulk conditions
\(V_0\) = velocity of water
\(Pr_{ob}\) = Prandtl no. for water at bulk conditions

\[= C_{P_0} \mu_{ob} \frac{K_{ob}}{\mu_{ob}}\]

\(K_{ob}\) = thermal conductivity of water at bulk conditions
\(\mu_{ob}\) = viscosity of water at bulk conditions
\(\mu_{ow}\) = viscosity of water at wall conditions
\(Re_{ob}\) = Reynolds no. for water at bulk conditions based on equivalent dia \(d_e\)
\[= \frac{d_e C_{t_{ob}}}{\mu_{ob}}\]

\[d_e = \text{crossflow area for water} / \text{wetted perimeter} = \frac{X_0}{\pi d_0 N}\]

\(X_0\) = x-area for water flow
\(d_0\) = outside dia. of tubes
\(N\) = no. of tubes Ref. 3 & 7

8. Heat transfer coefficient for Liquid Sodium is given by

\[Nu_{ob} = 7 + 0.025 (Pe)^{0.8}\]

\[\begin{cases} Pe > 100 \\ L/d_0 > 60 \end{cases}\]

uniform heat flux

where \(Nu_{ob}\) = Nusselt's number for liquid sodium at bulk conditions

\[= \frac{h_0 d_0}{K_{ob}}\]

\(K_{ob}\) = thermal conductivity of liquid sodium at bulk conditions
\(Pe\) = Peclet number for liquid sodium at bulk conditions
\[= \frac{C_{P_0} \int_{ob} V_0}{d_0} \frac{D_0}{K_{ob}}\]
where \( C_{p_{ob}} \) = specific heat of liquid sodium at bulk conditions
\( \rho_{ob} \) = density of liquid sodium at bulk conditions
\( V_o \) = velocity of liquid sodium
\( D_o \) = outside diameter of tubing

9. Heat transfer coefficient for hot gas or air flowing across a bank of tubes is given by

\[
\frac{h_o D_o}{K_{of}} = 0.33 \left( \frac{Re_{of}}{Pr_{of}} \right)^{0.6} \left( \frac{Pr_{of}}{Pr_{of}} \right)^{1/3}
\]

where all quantities are evaluated at film conditions

Ref. 3
Eq. (10-11a)

10. Pressure drop of CO\(_2\) flowing through the tubes

a. Pressure drop in tubes:

\[
p_1 - p_2 = \frac{G_i^2 (v_2 - v_1)}{\alpha g} + \frac{f_m G_i^2 v_m L}{2g r_{hi}}
\]

\( \alpha = 1/2 \) for laminar flow, = 1 for turbulent flow
\( G_i \) = mass velocity, \( v \) = specific volume
\( f_m \) = friction factor, from figure 6-11, Ref. 3
\( r_{hi} \) = hydraulic radium = \( \frac{X \text{ sectional area}}{\text{wetted perimeter}} = \frac{S}{b} \)

Ref. 3
Eq. 6-9b

b. Pressure drop due to sudden contraction

\[
p_o - p_1 = \frac{1}{v_{oi}} \left( \frac{V_1^2 - V_o^2}{2g} + \frac{K_c V_1^2}{2g} \right)
\]

\( K_c \) from Figure 6-13, Ref. 3

Ref. 3
Eq. 6-10

\( V \) = velocity
\( p \) = pressure

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(a) + (b) + (c) = Total Pressure Loss

Take (b) + (c) = \(0.75 \frac{V_i^2}{2g} \times \frac{1}{v_m}\)

and in (a), take \(f'_m = 1.1 f_m\) to account for the effects of compressibility

\[
\text{Total pressure loss} = \left(\frac{4f'_m L V_i^2}{d_i 2g d_i} + 0.75 \frac{V_i^2}{2g}\right) \times \int_{ib} \text{m} \text{L} \text{d} \text{m} \text{d} \text{m}
\]

11. Pressure drop of water flowing outside the tubes

Total head loss \(H = \text{frictional head loss} + \text{head loss due to sudden contraction}\)

Take head loss due to sudden contraction and expansion = \(.5 \frac{V^2}{2g}\)

and to account for change of density at entrance and exit, taking \(f' = 1.1 f\)

We have \(H = \left(0.5 + \frac{f' L}{r_{ho}}\right) \frac{V_o^2}{2g} \text{ ft.}\)

where \(r_{ho} = \text{hydraulic radius, based on flow conditions outside tubes}\)

\(V_o = \text{velocity of water outside tubes}\)

12. Horsepower to drive water pump

\[
\text{HP} = W_o \left(1.5 + \frac{f' L}{r_{ho}}\right) \frac{V_o^2}{2g} \times \frac{1}{\eta} \times \frac{1}{550} \times \pi \frac{X}{d_o L N + \pi D_i L}
\]

where \(X = \text{cross-sectional area for water}\)

\(D_i = \text{inside diameter of shell}\)

\(L_i = \text{length of each tube}\)

\(N = \text{number of tubes}\)

\(d_o = \text{outside diameter of tube}\)

\(\eta = \text{pump efficiency}\)
13. Effectiveness of straight rectangular fins

\[ e_s = \frac{1}{\sqrt{2}} \tanh \sqrt{2} \gamma \]

where \( \gamma = \frac{w_c^{3/2} \sqrt{h / KA}}{A} \)

\( w_c \) = height of the fin + \( \delta \)

\( A = 2 \delta w_c \)

\( 2 \delta \) = thickness of the fin

\( h \) = heat transfer coefficient

\( k \) = thermal conductivity of fin material

Ref. 6

14. Effectiveness of circular fin of rectangular profile

\[ e_c = \frac{\sqrt{2}}{\gamma} \left[ \frac{I_1(R_a^2 \gamma) K_1(R_b^2 \gamma) + I_1(R_b^2 \gamma) K_1(R_a^2 \gamma)}{(1 + r_{2c}/r_1)(I_1(R_a^2 \gamma) K_0(R_b^2 \gamma) + I_0(R_b^2 \gamma) K_1(R_a^2 \gamma))} \right] \]

where \( \gamma \) is defined as above

\( w_c = r_{2c} - r_1 \)

\( r_{2c} = r_2 + \delta \)

\( R_a = \sqrt{2} / (1 - r_1 / r_{2c}) \)

\( R_b = (r_1 / r_{2c}) R_a \)

and \( I_0 \) and \( I_1 \), and \( K_0 \) and \( K_1 \) are zero and first order and first order and first and second kind modified Bessel functions.

\( r_1 \) and \( r_2 \) are base and tip radii of the fin

Ref. 6
REFERENCES


### APPENDIX F

**CRITICAL CONSTANTS OF FLUIDS**

<table>
<thead>
<tr>
<th>INORGANIC FLUIDS</th>
<th>FORMULA</th>
<th>$T_c , ^\circ K$</th>
<th>$p_c , \text{atm}$</th>
<th>$V_c , \text{litres g-mole}$</th>
<th>$Z_c$</th>
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</thead>
<tbody>
<tr>
<td>1. Ammonia</td>
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<th>$p_c , \text{atm}$</th>
<th>$V_c , \text{litres g-mole}$</th>
<th>$Z_c$</th>
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<td>p_c (atm)</td>
<td>V_c (litres g-mole)</td>
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ADDITIONS

| 1. Xenon               | Xe             | 289.81  | 58       | .119                | .290 |

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MOLAL HEAT CAPACITIES OF GASES AT ZERO PRESSURE

\[ C_p^\circ = a + bT + cT^2 + dT^3 \] (T = °K)

\[ \text{gm-cal/gm-mole °K or BTU/lb mole °R} \]

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<tr>
<th></th>
<th>Temp Range</th>
<th>Error</th>
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<td>°K</td>
<td>Max %</td>
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<td>12.432</td>
<td>273-1500</td>
</tr>
</tbody>
</table>