STUDIES OF REACTOR CONTAINMENT

Summary Report No. 4

August 1, 1960 to January 31, 1961

Edited by

T. A. Zaker

AEC Research and Development Report
Contract No. AT(11-1)-528

25 years of research
STUDIES OF REACTOR CONTAINMENT

Summary Report No. 4
August 1, 1960 to January 31, 1961

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T. A. Zaker

United States Atomic Energy Commission
Contract No. AT(11-1)-528
ARF No. 4132

February, 1961
This technical summary report on "Studies of Reactor Containment" covers the period August 1, 1960 to January 31, 1961. It is prepared by Armour Research Foundation in response to Article V of Contract No. AT(ll-1)-528 and its Modification No. 6, as elaborated in correspondence with personnel of the U.S. Atomic Energy Commission's Chicago Operations Office.

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Respectfully submitted,

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STUDIES OF REACTOR CONTAINMENT

I. INTRODUCTION

This is the fourth technical summary report issued by Armour Research Foundation on a program of "Studies of Reactor Containment" conducted for the United States Atomic Energy Commission under Contract No. AT(11-1)-528. Work under this contract is generally directed toward determination of the properties and effectiveness of containment structures when subjected to shock or other loads of a transient nature. This work includes examination of the containment problem from the generation of transient loads at the source to the ultimate effects upon the containment.

The present report covers the period August 1, 1960 through January 31, 1961.

Activities on the program during the present report period have been subdivided into five problem areas or tasks, each in the charge of a task engineer. The entire effort is coordinated by a project engineer. The titles and numbers of the five tasks active during the report period are as follows:

1. Design Procedures for Blast Shields (Task No. 9)
2. Shock Crushing of Reactor Shield Materials (Task No. 12)
3. Explosive Decompression of Water Resulting from Pressure Vessel Rupture (Task No. 13)
4. Explosives Test Evaluation of Blast Shields and Blast Shield Materials for Nuclear Reactors (Task No. 14)
5. Design Criteria for External Containment Structures (Task No. 17)
The tasks have as their general objectives the theoretical and experimental investigation of the loads to which external containment structures for nuclear reactors are subjected in the event of a violent incident at the reactor core, the evaluation of methods of reducing that loading, and the study of the response of and design criteria for external containment structures as a result of such loading. For a discussion of background relevant to the program, project organization, and reporting procedures, reference is made to an earlier summary report. The purpose of the present report is to describe activities during the report period and to indicate significant results in each problem area.

II. TECHNICAL PROGRESS

As indicated earlier, activities during the report period were divided into five problem areas or tasks. The following paragraphs summarize technical progress on each of the tasks.

A. Task No. 9 - Design Procedures for Blast Shields (T. A. Zaker)

The essential aim of work in this problem area is to develop systematic techniques for analyzing the effectiveness of crushable barriers or shields as protective devices against the effects of a sudden release of energy nearby. This effort will ultimately provide the hazards analyst with rapid methods for calculation of the transmission of compressive disturbances through complex configurations of material surrounding an energy source. The methods must necessarily be based on a realistic model of the behavior of the material and they must be

relatively simple to apply in order to reduce the analysis to a tractable form. In addition, prescribed loadings at boundaries need not be restricted to suddenly applied constant-pressure pulses.

During the report period effort has been concerned with application of analytical methods developed in earlier work to the case of one-dimensional systems composed of crushable material subjected to prescribed loadings at an exposed surface. Pressure or stress in the material considered is strongly dependent on density, weakly dependent on temperature. With negligible body forces during fully confined dynamic compression (uniaxial strain), the axial normal stress is the only stress component which enters the problem. The conservation laws for mass and momentum, together with appropriate pressure-volume relations for compression, unloading, and recompression in uniaxial strain, are sufficient for the solution of the problem. Energy losses and the exchange of mechanically available energy between potential and kinetic forms can be evaluated at any given instant during the motion by consideration of the state of the overall system.

For completeness we first recapitulate the basic assumptions and some results of the analysis given in the preceding report of this series. The analytical technique utilizes piecewise linear idealizations of actual pressure-volume relations for typical crushable materials. The piecewise linear relations for balsa wood are shown in Fig. A-1. Such piecewise linear relations exhibit initial linear elasticity (line AB); large volume changes when the crushing pressure or yield point is exceeded (line BCD);

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Fig. A-1 PIECEWISE LINEAR PRESSURE-VOLUME RELATIONS
final compaction at a definite value of relative volume (line DE); and pronounced hysteresis upon unloading (lines FG, HJ, DK). The utility of the linearized relations lies in the simplification introduced into the numerical work at no sacrifice to realistic modeling of material properties. No restrictions other than the applicable physical constraints are placed on the magnitude of particle displacements.

Up to the present time attention has been restricted to cases of constant-pressure loading at one boundary for specified finite time durations. For such cases solutions have been constructed based on the assumption of regions of uniform stress, density, and velocity state in the plane of independent variables, distance x and time t. These uniform states are assumed to be separated by steep-fronted elastic waves, plastic waves or shocks, unloading waves, and well-defined composition discontinuities or interfaces. Residual density discontinuities develop in such problems owing to the interaction of unloading waves with plastic waves, and as a result of the properties of hysteresis exhibited in the piecewise linear pressure-volume relations.

With the assumption of regions of uniform state separated by sharply defined discontinuities, analysis of problems of this type reduces to the repeated application of purely algebraic relations for mass and momentum conservation across the jumps in state properties, and the simultaneous satisfaction of the applicable linear pressure-volume relations. This technique is readily extended to the case of laminar shields (layered materials), in which discontinuities in material properties exist initially in the problem. The refraction of wave fronts at such interfaces requires no additional methods of analysis; indeed, it has been pointed out that the formation of residual density discontinuities in an initially...
uniform crushable material is an essential feature of the behavior of such a medium.

The relations which must be satisfied across a discontinuity separating states denoted by the subscripts 1 and 2 are

\[(u_2 - u_1)^2 = (P_2 - P_1)(v_1 - v_2), \quad (A-1)\]

\[(c_{21} - u_1)^2 = v_1 \frac{P_2 - P_1}{v_1 - v_2}, \quad (A-2)\]

where \(c_{21}\) is the velocity of propagation of the discontinuity into state 1 relative to a frame of reference fixed in space, and

- \(P\) = normal stress or pressure,
- \(v\) = specific volume,
- \(u\) = particle velocity.

Equation (A-1) represents a relation between pressure jump and particle velocity change, since \(P\) and \(v\) are connected by the piecewise linear pressure-volume relations of Fig. A-1. Equation (A-2) then provides the relation for calculation of the wave velocity. For a compressive wave \((P_2 > P_1)\) propagating into state 1, the positive or negative root of Eq. (A-1) or of Eq. (A-2) applies according as the wave propagates in the positive or negative direction relative to the material particles. The reverse is true of a rarefaction wave \((P_2 < P_1)\) propagating into state 1.

The acoustic impedance \(\rho c\) is constant on each linear segment of the pressure-volume relations, where \(\rho (=1/v)\) is the density and \(c\) is the local sound speed relative to the material particles, defined by \(c^2 = dP/d\rho\). For changes of state which take place between points on the same linear segment of the pressure-volume relations, as in unloading
waves and weak compression waves, the jump equations (A-1) and (A-2) simplify to

\[ u_2 - u_1 = \pm \frac{P_2 - P_1}{\rho c} \tag{A-3} \]

\[ c_{21} - c_1 = \pm \frac{\rho c}{\rho_1} \tag{A-4} \]

where the double signs refer to waves propagating in the positive and negative directions with respect to the material particles.

The utility of the piecewise linear pressure-volume relations lies in part in the fact that the acoustic impedance \( \rho c \) is constant on each linear segment in the pressure-volume state plane. The system of differential equations governing one-dimensional unsteady motion of a compressible medium is of hyperbolic type and possesses so-called characteristic lines along which are propagated certain integral invariants of the system\(^2\). In consequence of the property of constant acoustic impedance, the characteristic lines in the present problem are straight and parallel to each other throughout any region of the physical (x-t) plane in which all state points lie on one linear segment of the pressure-volume relations. As a result, it is possible to show that solutions obtained on the basis of assumed uniform states separated by steep-fronted waves and ideal contact interfaces are in fact exact solutions for piecewise constant boundary values of normal stress and velocity.

The intersection of two wave fronts in x-t space, or of a wave front and a composition discontinuity, in general gives rise to leftward and rightward moving waves of such magnitude and type as to satisfy the

conditions of continuity of stress and particle velocity in the zone between the resulting waves after the interaction. The pressure-volume relations of Fig. A-1 must also be satisfied simultaneously. After a typical wave-wave interaction, for example, leftward and rightward moving waves are hypothesized and the appropriate one of Eqs. (A-1) or (A-3) is applied across each assumed wave, together with the two conditions of equality of stresses and particle velocities behind the two opposite-facing waves. The resulting simple system of simultaneous algebraic equations is solved for the stress and particle velocity in the zone between the waves. It then remains to verify that the final states behind the waves satisfy the piecewise linear pressure-volume relations; if this is not the case, the procedure is repeated with another hypothesis for the wave system after interaction.

The results of analysis by these methods of a semi-infinite column of initially uniform material having the properties exhibited in Fig. A-1 are shown in Fig. A-2 for an assumed loading at the boundary of 122 bars for a duration of three milliseconds. Numbered regions in the plane of independent variables refer to pressure-volume states in the inset figure. The progressive weakening of the plastic wave which trails an elastic precursor rightward into the material is due to successive reflections of high-speed unloading and recompression waves in the region to the left of the plastic wave. On the basis of such a so-called free-field problem, solutions may be constructed for finite depths of the same material backed by a rigid surface at given initial locations relative to the loaded surface, and subjected to the same loading as the free-field case. The stress-time history at the rigid surface is then deduced, and provides criteria for assessing the effectiveness of the crushable shield.
Fig. A-2 FREE-FIELD PROBLEM
It can be seen in Fig. A-2 that residual density discontinuities which travel at the local particle velocity are generated by the intersection of successive unloading waves with the plastic wave. Furthermore, unloading and recompression waves which traverse the field behind the plastic wave are refracted at each of the residual density discontinuities thus formed, thereby complicating the problem rapidly.

It is evident from inspection of the state plane inset in Fig. A-2 that, in certain regions of the field behind the plastic wave, residual density discontinuities are relatively small. That is, the unloading paths followed by adjacent material particles which have been compressed nearly to the compacted density by prior passage of the plastic wave lie relatively close together. Since it is the decay of the plastic wave itself which is of primary interest in these problems, this suggests that a simplification of the computations can be effected by neglecting some of the interactions of density discontinuities with recompression waves and with later unloading waves. Such a procedure was used to extend the solution of the basic free-field problem of Fig. A-2 in an approximate manner up to time $t = 18$ milliseconds. The procedure consists of calculating in the usual way the residual density discontinuities formed by the overtaking of the plastic wave by unloading waves, and neglecting the effects of each such discontinuity after passage of the next succeeding unloading wave originating from the surface. This is accomplished in cases where the density discontinuity is small by using an average density over the region to either side of the discontinuity, and replacing the corresponding adjacent unloading and recompression lines in the pressure-volume relations by a single average path. In addition, the interaction of very weak signals with other waves and with interfaces may
also be neglected. The approximate extension of the basic free-field problem is shown in Fig. A-3, where numbered regions in the x-t plane again refer to pressure-volume states in the inset figure. Average (replacement) unloading paths are indicated by broken lines in the pressure-volume relations. It may be seen that, at t = 14 milliseconds, the plastic wave stress (state 16") has decayed to approximately 28 bars, a value quite close to the crushing strength of the material.

The approximate procedure outlined above unfortunately calls for a great deal of judgment on the part of the analyst. It is the objective of the present work to reduce such computations to explicit systematic procedures which can be carried out rapidly by the analyst. One possibility in this connection is that of adapting the present computational procedures to digital machine calculation.

During the report period, a generalized computational procedure was formulated for digital machine calculation of the elastic waves, plastic waves or shocks, unloading waves, and interfaces generated within a crushable solid under impact loading. The variety of types and increasing number of waves in the crushing solid are to be continually followed by the computer memory units. This computational procedure is to be adaptable to initially non-uniform (layered) materials characterized by piecewise linear pressure-volume relations, with piecewise constant surface pressure-time loadings.

The Remington Rand 1105 computer at Armour Research Foundation is one of the larger digital computers with a memory capacity of 40,960 separate words. The memory is composed of a magnetic drum with a 32,768-word storage, and two rapid-access magnetic cores with a 8,192-word storage. Each word in the drum and in the core contains 10 digits.
Fig. A-3 EXTENDED FREE-FIELD PROBLEM
The 10-digit words may be used to describe integers (fixed point numbers) up to $2^{35} - 1$, or floating point numbers ranging from $10^{-38}$ to $10^{38}$ in absolute value.

Of the 40,960 words available, 32,768 are on a 4-in. revolving magnetic drum. The time required to read a number from the drum varies according to the location of the number on the drum; this access time is usually the slowest operation of the computer, since times of the order of milliseconds are required for the mechanical revolution of the drum. Therefore, the revolving drum is not used for all of the storage; instead, non-mechanical storage units are used for the remaining 8,192 words. These 8,192 words are placed on two 4,096-word magnetic cores, from which any given word may be read in approximately 6 microseconds.

Since approximately 3,000 words are to be required for recording constants used throughout the execution of the program, for addresses of variables of the wave interactions, and for temporary storage, it is possible to use one of the 4,096-word, rapid-access magnetic cores for this purpose. The other magnetic core may be used to store the program which directs the computation. The use of only two magnetic core units permits quick access and rapid execution of each step.

Figure A-4 shows a typical wave diagram for a finite-depth shield consisting of two layers of crushable material, each characterized by a set of piecewise linear pressure-volume relations. As may be noted in Fig. A-4, some of the constant-state regions vanish as time increases. When a region vanishes, a wave-wave or wave-contact surface interaction occurs. Since linear equations describe the trajectories of wave fronts and contact surfaces, the representation of each front may be described in the form
Fig. A-4  TYPICAL PRESSURE-VOLUME RELATIONS FOR A CRUSHABLE MATERIAL AND WAVE DIAGRAM FOR TWO-LAYER FINITE-DEPTH SHIELD
\[ S = a \, t + b, \]  
where  
\begin{align*} 
S &= \text{distance transversed by the wave (ft)}, \\
 t &= \text{time (ms)}, \\
a &= \text{slope of the wave line (ft/ms)}, \\
b &= \text{distance-axis intercept of the wave line (ft)}. 
\end{align*}

Solving pairs of equations of the above form simultaneously, it is possible to obtain wave-wave and wave-contact surface interaction points in the \( x-t \) space. In the solution of such equations, the computer (1) solves for all possible points of interaction for each region present, (2) eliminates all points of interaction which have already occurred, and (3) determines the next possible interaction.

One section of the program is devoted entirely to decision making, i.e., what resulting wave pattern exists after a wave-wave or wave-contact surface interaction occurs. For example, at point A of Fig. A-4, region 1 vanishes. The remaining regions surrounding this point are then regions 2 and 1'. The computer then compares (1) the pressure in region 2 to the pressure in region 1', and (2) the density in region 2 to the density in region 1', and (3) the particle velocity in region 2 to the particle velocity in region 1'. As a result of these comparisons, one of 110 different combinations of conditions in the adjacent states 2 and 1' exists. Two of these 110 are the special cases of a wave-free surface and a wave-rigid wall interaction. Approximately 70 per cent of these combinations may be characterized by four different major basic calculation procedures to follow. The remaining 30 per cent are either miscellaneous or fall into smaller groups of procedures which occur very infrequently. After comparison of the two remaining regions (here regions 2 and 1'), the decision section of the program refers.
the computer to the proper section of the program containing the numerical procedure to be used to determine the pressures, the particle velocities, and the volume ratios for the newly formed regions.

The input surface loading may be approximated by as much as a four-step function, and a variety of wave shapes can easily be handled. Two typical shapes of such a wave are shown below along with the approximation to the wave for computer use:

a. **Peaked Wave**

b. **Finite Rise Time Wave**

In further work on this task, effort will be concentrated on program refinement. Upon completion of program refinement, the completed program will be transferred from punched paper tape to magnetic tape for use by the computer. A sample calculation performed manually will then be introduced into the computer and necessary program revisions will...
be made. Various combinations of initial data will then be read into the computer, varying (1) the number of shield layers, (2) the densities of the materials, (3) the pressure-volume relations for each layer, (4) the depth of each layer, and (5) the input surface loading. The solution of such a series of one-dimensional wave motion problems in crushable shields will lead to an assessment of the influence of shields of various types on the stress-time history at the rigid backup surface.

B. Task No. 12 - Shock Crushing of Reactor Shield Materials (P. Lieberman)

The objective of the work of this task has been to establish the loci of stress-specific volume states attained by shock compression (the Hugoniot curves) of various crushable materials, and to obtain unloading and recompression curves in the stress-specific volume plane traced by the material in the crushed state. These data are useful for the solution of impact problems such as are encountered in the design of a blast shield for a nuclear reactor in the event of a violent excursion in the core. In addition to supplying data which would be the basis for a blast shield design, experimentation can also verify the effectiveness of proposed blast shield designs by tests in the laboratory. In previous operation, an apparatus has been constructed for the attainment of these objectives, and preliminary experiments have been completed on 6-in. and 42-in. lengths of balsa wood.

Previously published reports have described these earlier efforts.

This section of the present report summarizes work on this task during the period August 1960 through January 1961.

During the report period several tests were performed on 42-in. lengths of balsa wood, and the data were analyzed to determine elastic, plastic, and unloading wave velocities and particle velocities, and stress amplitudes in the impacted solid. This information is used to determine the dynamic equation of state of the crushable material. The analysis of a typical test suffices to demonstrate how equation-of-state data are derived, and how the equation of state is used to predict attenuation of a stress wave in the material.

Conditions in a typical test (Run No. 197) are displayed on the wave diagram of Fig. B-1. Piezoelectric crystal pressure gages were located at stations 5, 6, KP-1, KP-2, and KE as indicated along the x-axis in the diagram. Since the pressure taps were mounted in the sidewall and end plate of the tube, the sensors recorded the pressure or stress histories (Fig. B-2) at points fixed in laboratory coordinates. In the construction of the wave diagram from the experimental data, the wave pattern in the driven gas was determined from the pressure records taken upstream of the gas-solid interface and from shock tube gasdynamic relations; that in the solid was determined indirectly from the observed motion of the gas-solid interface and from the arrival time of waves at the end of the specimen (station KE).

With reference to the wave diagram sketched below, the results obtained for the experiment cited are
Fig. B-1 DIAGRAM OF WAVE MOTION IN SHOCK TUBE AND CRUSHABLE MATERIAL, LABORATORY COORDINATES
Fig. B-2 DATA RECORDS FOR RUN NUMBER 197
State Properties

\[ P_5 = \sigma_{1,3} = 213 \text{ psia} \]
\[ \sigma_{1,2} = 54.4 \text{ psia} \]

Wave Velocities

\[ W_{1,3} = 412 \text{ ft/sec} \]
\[ W_{1,1} = 1850 \text{ ft/sec} \]

Particle Velocities

\[ U_{1,3} = 204 \text{ ft/sec} \]
\[ U_{1,2} = 0 \]

Assuming a bi-linear dynamic stress-volume relation for the material in the range of stress encountered here, one uses the values given above in the jump equations across each wave of the system shown to obtain the acoustic impedances, the so-called Hugoniot elastic limit, and the state point corresponding to the crushed material immediately behind the plastic wave (state 1.3). The latter represents a point on the Hugoniot curve of the material. The initial specific gravity of the wood in this case was 0.156. Initially we have \[ P_1 = \sigma_{1,0} = 24.5 \text{ psia} \]. The results for longitudinal stress and specific volume ratio at the Hugoniot elastic limit and at the crushed state point are respectively.
\[ \sigma_{1.1} = 47.5 \text{ psia}, \quad \frac{v_{1.1}}{v_{1.0}} = 0.475 \]

\[ \sigma_{1.3} = 213 \text{ psia}, \quad \frac{v_{1.3}}{v_{1.0}} = 0.513 \]

Furthermore, the pressure record taken at station 6 in the driven gas showed the arrival of a strong expansion wave at 3.25 milliseconds from the instant of impingement, and a strong unloading wave appeared at the end of the specimen (station KE) at 12 milliseconds. This corresponds to an average unloading wave velocity in the wood of approximately 530 ft/sec relative to laboratory coordinates, if account is taken of the current strain distribution in the material.

The multi-linear stress-volume relations postulated on the basis of the above data are shown in Fig. B-3. These relations differ appreciably from static longitudinal stress-density relations in uniaxial strain reported earlier. Utilizing the tentative stress-volume relations so obtained and shown in Fig. B-3, one can calculate the detail of wave motion in the solid by techniques discussed in Section II-A above and in earlier reports of this series. The graphical analysis employed is given by the constructions shown in Figs. B-4 and B-5. This leads to a prediction of the detail of the stress history at the end of the specimen (station KE), and the results may be compared with the observed values.

Figure B-4 represents the piecewise linear stress-velocity relations derivable from the tentative stress-volume relations given in Fig. B-3. The lines shown are so-called characteristic lines which represent changes of stress and velocity state across the waves shown in the physical plane, Fig. B-5. In the latter diagram, the pressure-time history at the gas-solid
Fig. B-3 TENTATIVE PIECEWISE LINEAR DYNAMIC STRESS-VOLUME RELATIONS FOR BALSA WOOD
Fig. B-4 STRESS-VELOCITY RELATIONS FOR BALSA WOOD
Fig. B-5 DIAGRAM OF WAVE MOTION IN BALSA WOOD SPECIMEN, PARTICLE COORDINATES

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interface, obtained from the gage record at station 6, has been replaced by an equivalent piecewise constant pressure loading. The resulting system of uniform states and discontinuity lines is shown in Fig. B-5 with the initial particle position coordinate and the time regarded as the independent variables. In the figure, numbered regions correspond to the numbered stress-volume states shown in Fig. B-3 and to the stress-velocity states shown in Fig. B-4. The graphical method of analysis indicated in Fig. B-4, and in the so-called Lagrangian frame of reference of Fig. B-5, has been utilized in connection with the work of Task No. 9 above and will be presented in detail in a later report of this series. The results of analysis of wave motion in the solid so obtained on the basis of the tentative stress-volume relations of Fig. B-3 are transformed to laboratory coordinates and are shown as the detailed wave pattern in the solid in Fig. B-1.

The calculated stress-time history at the end of the solid (states 10, 11, 17 of Figs. B-4 and B-5) is seen to correspond well with the observed stress history at station KE (Fig. B-2) insofar as the arrival times of compression and unloading waves are concerned. However, the agreement of the calculated and observed stress amplitudes in states 11, 17, and 18 is poor. The agreement between the calculated and observed stress histories can be improved by a new hypothesis regarding the slope of unloading lines in Fig. B-3. The wave motion in the specimen can be recalculated on the basis of the new assumption for the stress-volume relations in dynamic uniaxial compression and unloading. Such an iterative procedure for the determination of stress-volume relations so as to bring into agreement the calculated and observed stress histories at the end of
the specimen is lengthy, but can be utilized with effectiveness if the results of several experiments are analyzed simultaneously and cross comparisons are made.

The measurement of transient stress in the interior of the specimen represents a more direct approach to obtaining equation-of-state data for crushable materials than is the indirect determination of the wave motion in the solid from measurements taken in the adjacent gas and at the end surface of the specimen.

On the one hand, it is possible to utilize standard piezoelectric crystal gages mounted in the sidewall of the shock tube adjacent to the crushable solid specimen to observe lateral pressure exerted on the wall during passage of stress waves through the material; this of course does not represent a measurement of transient longitudinal stress, but provides a direct determination of wave arrival times in laboratory coordinates. In the present experiments, such gages (stations KP-1 and KP-2 in Fig. B-1) were employed (1) to observe wave arrival times, (2) to give a rough indication of the relative strength of waves in the specimen, and (3) to detect passage of the gas-solid interface and hence provide a measure of its velocity. Records obtained from these gages are shown in Fig. B-6.

Another gage of this type was mounted centrally in the blind flange at the end of the specimen (station KE) face-on to the direction of propagation to provide a direct measurement of the stress-time history at the backup surface. Typical records taken at this station have been indicated in Fig. B-2. The gages mounted in the sidewall and end plate of the shock tube were recessed slightly to protect the sensing elements from abrasion, and
Fig. B-6 DATA RECORDS FOR SENSORS MOUNTED AT THE SIDE SURFACE OF A TEST SPECIMEN

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the small recess was filled with a thin film of grease in tight contact with the surface of the solid. The grease film was sufficiently thin so as to minimize the effects of possible mismatch in acoustic impedance between the test material, grease film, and sensing element.

On the other hand, it is desirable to explore the possibility of inserting sensing elements within the interior of crushable solid test specimens to measure transient longitudinal stress conditions directly. Such sensors must be designed so as to minimize interference with the field in which the measurements are made. Accordingly, a stress sensing device was developed for such measurements in crushable solid materials.

The stress sensors consisted of thin wafers of high-polymer plastic in which were embedded micron-size metallic particles. The effect of an increase in applied compressive stress is to decrease the electrical resistance of such a composite. Changes in voltage impressed across the wafer through a battery circuit were fed into an oscilloscope through associated circuitry, and the output signal was calibrated in terms of the applied compressive stress. Construction of special electrical leads to the wafer was necessitated owing to the large particle displacements encountered in the motion of 42-in. test sample lengths of crushable material, in which the sensor may displace as much as two feet under conditions of shock impingement.

**Mechanical Aspects**

Assuming that the sensor is subjected to static and dynamic calibration prior to use in experiments, it is desirable that the sensor material behave elastically in the anticipated experimental stress range. Therefore, it is necessary to consider the extent of interference with stress wave motion caused by the presence of a stress sensor of elastic material inserted in
solids whose physical properties are such that undisturbed stress waves are transmitted with (1) no change in stress profile, (2) "shock-up" changes in profile, and (3) dispersive changes in profile.

A wafer of elastic material suspended in a test specimen of a different elastic material will not influence the transmitted wave motion if the acoustic impedances $p_c$ of the two materials are equal. Unfortunately, it is not always possible to match the impedance of such materials. Therefore it is necessary to design the sensor so as to compensate for the impedance mismatch between the sensor and the test specimen.

Although it is possible that some sensor materials may have lower impedance values than the test specimen material, and the following analysis would have to be extended to that case, the high impedance values are more typical. Figure B-7 shows the stress-time histories upstream of the wafer, in the wafer, and downstream of the wafer. When the input wave is flat-topped, the impedance mismatch gives rise to three principal effects: (1) the stress history in the test specimen upstream of the sensor is modified; (2) the sensor will respond in a damped oscillatory fashion about the input stress level, and the wave form appears sawtoothed because the output consists of the change in voltage across the entire wafer thickness; and (3) the stress transmitted past the wafer approaches the input stress level monotonically with time. For a general input stress wave form, the analysis is more complicated than that indicated in Fig. B-7, where a flat-topped input wave form is depicted.

The sensor output oscillations are rapidly damped if the wafer is thin. For the same reason, for a given wafer thickness and impedance mismatch, high acoustic velocities are desirable for sensor materials (Fig. B-8). For
When \( \frac{\sigma - \sigma_0}{\sigma_0} \times 100 \leq \eta \%), then the subsequent oscillations are neglected.

Fig. B-7 STRESS HISTORIES IN THE MATERIAL AND IN THE SENSOR FOR A GIVEN FLAT-TOPPED INPUT STRESS PULSE.
Fig. B-8 EFFECT OF ACOUSTIC VELOCITY OF THE SENSOR MATERIAL ON THE STRESS PULSE IN THE SPECIMEN
example, if the impedance mismatch and desired damping time are given, it is possible to obtain the required sensor thickness from Fig. B-9.

Suppose that the ratio of sensor and material impedances is 10, and a 5 per cent deviation from the input stress level is permitted after one microsecond; then Fig. B-9, gives $n = 7$. Thus

$$\frac{nL}{c} = 1.0 \times 10^{-6} \text{ sec}$$

or

$$L = \frac{(1.0 \times 10^{-6} \text{ sec})(2100 \text{ ft/sec})}{0.0003 \text{ ft}} = 0.0036 \text{ in.}$$

Therefore a sensor having an acoustic velocity of 2100 ft/sec should be 0.0036 in. thick. Note that, if the wafer thickness is halved, the deviation after one microsecond is reduced to 0.5 per cent. In experiments performed on wafer-type sensors of this kind, erratic electrical properties were encountered in sensors of very small thickness. The results of experiments on somewhat thicker wafers are shown in Fig. B-10. Figure B-11 shows a general view of the strong shock tube used in shock impingement experiments on crushable solids.

The single flat-topped, steep-fronted input wave, after transmission through the wafer, becomes a series of flat-topped, steep-fronted wavelets in the downstream specimen. These wavelets propagate without dispersion through the remainder of the elastic specimen. A sensor of given acoustic impedance and of the proper thickness will therefore yield an acceptable level of accuracy in output and a tolerable perturbation of the stress wave profile in the test specimen. This conclusion is also applicable to cases in which an elastic wafer is inserted in "shocking-up" or dispersive test specimen materials (Fig. B-12).
Fig. B-9 CHART FOR DETERMINATION OF SENSOR THICKNESS
Fig. 3-10 COMPARISON OF STRESS SENSOR RECORDS WITH PRESSURE RECORDS TAKEN IN ADJACENT GAS
a. "SHOCKING UP" TEST SAMPLE MATERIAL

b. ELASTIC TEST SAMPLE MATERIAL

c. DISPERSIVE TEST MATERIAL

Fig. B-12 WAVE TRANSMISSION THROUGH AN ELASTIC WAFER EMBEDDED IN VARIOUS TYPES OF TEST MATERIALS
The wave forms in a "shocking-up" material are least affected by the insertion of a sensor. A shock incident upon the wafer reflects with factors in general exceeding the elastic stress reflection factor of two, so that a higher stress is transmitted through the wafer to the opposite face, where transmission to the downstream material will bring the stress level at the downstream side to the correct magnitude in a shorter time. In addition, the reduced dispersion in the downstream material will be further improved by the "shocking-up" process in the downstream material, so that the initial input profile will redevelop in a short distance. The stress wave form in a dispersive test specimen material, on the other hand, is severely modified by the presence of a sensor. All the factors which favor the "shocking-up" test specimen materials for sensor insertion operate in reverse in dispersive materials.

The insertion of a sensor in a test specimen is, fortunately, not wholly a one-dimensional effect. When the wafer is smaller in diameter than the test specimen, a major portion of the shock may diffract around the wafer and reinforce the wave transmitted through the sensor. Thus it is desirable to make the wafer as thin and as small in radius as mechanically and electrically feasible.

**Electrical Aspects**

In order to select possible sensor materials, it is necessary to consider the mechanism of resistance change. In this way sensor materials can be selected on the basis of electrical properties and the previously discussed mechanical properties with a minimum of preliminary testing.

The sensor material must be such that it undergoes a change in some electrical property with an increase or decrease of strain. Also, the rate
of electrical property change should be capable of following the rate of change of stress. Fortunately, several such materials are available. Since these materials have resistivities of the order of 10 ohm-cm, conducting impurities can be added which act as donors to lower the electrical resistivity.

Many of the materials have an equation of state of the form

\[ P v^{k_1} = k_2, \]  

where \( P \) is pressure, \( v \) is specific volume, and \( k_1 \) and \( k_2 \) are constants. Since increased pressure places more conducting material within a given volume, it may be expected that change in specific volume causes an inverse effect on the resistance. Also, if the added impurities are conductors, resistance may be changed by improved contact between conducting particles. Therefore, the form of the pressure-resistance relation should be

\[ P (k_3 R)^{-k_1} = k_2, \]

where \( R \) is the resistance of the element and \( k_3 \) is another constant. The results of static compression tests shown in Fig. B-13 indicate this trend.

The logarithmic stress-resistance curve indicates that sensitivity at high pressure ranges in the test specimen is relatively poor and pressure fluctuations in this range would be difficult to detect on a data record. This distortion may be remedied somewhat by electrically subtracting the lower portion of the trace and amplifying the remainder.

Under high transient pressure (greater than 30 kbars) some plastics generate an electric current. Tests of non-polar teflon and polyethylene by other investigators show no current generation; weakly polar polystyrene
Fig. B-13 STRESS-RESISTANCE CURVES FOR A SENSOR ELEMENT
gives a large current signal. On this basis it has been suggested that
dipole orientation in the shocked material is the predominant mechanism
for current generation. It has also been shown that epoxy and nylon
exhibit this same property. Under high transient pressure, crystalline
substances also exhibit decreases in electrical resistivity.

Other mechanisms may also be postulated for explaining the change
of resistance with applied stress in materials. However, it can be stated
that it is possible to obtain logarithmic stress-resistance curves which are
fairly linear, and which can be shifted by the addition of impurities or by
inducing residual stresses in fabrication. Experiments have indicated that
there is also a slight change in capacitance. This effect is understandable
because of the high percentage of metallic particles, suspended in the di­
electric matrix of plastic, which act as miniature capacitors. Unfortunately,
creep effects cause static tests to show changes in electrical properties
without change in applied stress; this leads to difficulties in correlating
static test results with those of short-duration dynamic tests.

C. Task No. 13 - Explosive Decompression of Water Resulting From
Pressure Vessel Rupture (E. Terner)

Sudden venting of coolant pipes in power reactor systems may impose
dangerous loading conditions on the surrounding containment structures. In
order to assess such loadings, it is necessary to study the manner in which
the coolant (frequently hot and highly pressurized water) flashes into steam
upon explosive decompression. A water-driven shock tube which has been
constructed for the purpose of studying such phenomena has been described
in the preceding summary report. The tube is essentially a 2-in. diameter stainless steel pipe, mounted vertically and consisting of two parts, the upper or driven section which contains air or other gases, and the lower or driver section which contains the heated pressurized water. The two sections are connected by a pair of steel flanges, between which is mounted a frangible diaphragm separating the two chambers initially. The design of a suitable diaphragm presents considerable difficulty in liquid-driven shock tubes.

The preceding report of this series described the solution of the diaphragm problem and dealt also with several exploratory experiments with heated liquid driver. During the present report period the equipment was relocated to more suitable laboratory space. Provisions were made to accommodate additional instrumentation; fast-response thermocouples were provided in the driver and driven sections, and quartz glass windows were installed in both chambers to permit possible visual observation or optical density measurement. The entire assembly has been housed in a light steel frame enclosure with a waterproof fabric cover. A photograph of the test facility is shown in Fig. C-1. Modifications were made to improve the flanges and sealing arrangement at the diaphragm station.

The technique of making scored tempered glass diaphragms which initially separate the heated, pressurized liquid from the low-pressure chamber gas was further improved. High-speed motion picture photography was employed in a series of 20 tests with the driven chamber removed to observe the rupture characteristics of the glass diaphragms. Consistent brittle rupture was observed as illustrated in Fig. C-2.
Fig. C-1 PHOTOGRAPH OF LIQUID-DRIVEN SHOCK TUBE FACILITY
A series of decompression experiments was carried out in which the pressure rise in the driven chamber following diaphragm rupture was observed. In these tests, the procedure is briefly as follows: The lower or driver section of the shock tube is filled with water and heated slowly under pressure which is steadily increased by means of a hand pump to prevent boiling at any time during the heating process. In this series of experiments, temperatures up to 280°F and pressures up to 800 psi were attained initially in the driver chamber. Upon sudden rupture of the diaphragm, the heated liquid will decompress, vaporize, and discharge in the form of steam into the upper or driven section. There the change of pressure is recorded as a function of time by a piezoelectric quartz crystal gage mounted in the sidewall of the tube approximately one foot from the diaphragm station; the output of the gage is displayed on an oscilloscope and the trace is photographed.

The relevant records are shown in Fig. Q-3; these were selected as typical examples from a number of experiments conducted with heated pressurized water. The two traces displayed in each line represent the same phenomenon, the left-hand trace having been recorded at a much slower oscilloscope beam sweep speed than the right-hand trace. Thus the latter trace in each instance is essentially an enlargement and detail of the initial portion of the left-hand trace.

The uppermost trace A was obtained when water, pressurized to 750 psi and heated to about 460°F, was allowed to discharge or vaporize into the driven chamber, which was loaded with argon at an initial pressure of 400 psi. The second and third traces, B and C respectively, were recorded in experiments in which the water was heated to nearly 300 °F and allowed to...
Fig. C-3 PRESSURE-TIME HISTORY IN DRIVEN CHAMBER
discharge, at initial pressures of about 400 and 500 psi respectively, into the driven chamber containing air and held initially at atmosphere pressure. In test C, the blind flange at the top of the driven chamber was removed, having been in place for tests A and B. The interesting details of wave reflection from the open end of the tube can be observed on trace C.

The right-hand traces, recorded at fast sweep rates, exhibit rather slow buildup of pressure to what appears to be an equilibrium value. The left-hand traces, recorded at slower sweeps, show that the time required to reach equilibrium under the present experimental conditions was of the order of tens of milliseconds. In trace A the saturation pressure level, once it is reached, persists for the duration of the record. In trace B a pressure decrease is observed once a maximum is attained which corresponds to the saturation pressure at the initial temperature of the liquid. This decrease is possibly attributable to energy losses to the walls of the driven chamber due to heat conduction effects. In trace C the pressure decrease is of course much more pronounced, since in addition to energy losses by heat conduction, the vapor exhausts to the atmosphere through the open end of the driven chamber.

It can be seen quite clearly from the right-hand traces that wave motion in the driven gas and in the liquid-vapor mixture following diaphragm breakage appears as a slight modulation of the pressure buildup due to vaporization. Although superimposed pressure signals of relatively low amplitude can be clearly discerned, the phenomena of wave propagation and vaporization appear to be largely decoupled and proceed at entirely different time rates.

In summary, it has been shown that the sudden decompression of pressurized nuclear reactor coolant can be successfully simulated in a water-driven shock tube. No well-defined shocks have been observed in the
driven chamber; it was found that vaporization proceeds at a rather slow pace, requiring times of the order of $10^{-2}$ seconds to reach equilibrium in the driven chamber under the present initial test conditions. These results are not surprising if it is considered that a diffusion process rather than a wave propagation phenomenon underlies the process of vaporization.

D. Task No. 14 - Explosives Test Evaluation of Blast Shields and Blast Shield Materials for Nuclear Reactors (H.S. Napadensky)

The general objective of this task consists of the evaluation of properties of crushable shields both of uniform and of layered material under transient loading conditions. For this purpose, an experimental technique utilizing an explosive-driven metal plate has been developed in previous work for observation of the dynamic response of compressible solid materials.

During the major portion of the report period this task was inactive. Experimentation was resumed in the latter part of the period. The experiments which were conducted were explosive-driven plate impact tests on laminates consisting of alternate layers of metal and of crushable porous materials. The simple laminates were made up of two 1/2-in. thicknesses of insulating wallboard between which was placed an aluminum or steel plate ranging in thickness from 0.075 in. to 1/2 in.

In each experiment, the laminar specimen is placed between a driver plate of aluminum ranging in thickness from 1 in. to 5 in. and a massive steel anvil which represents a rigid backup surface. A charge of low-density explosive propels the driving plate which compresses the specimen between the driving plate and the anvil. A schematic diagram of the experimental arrangement as given in earlier reports of this series is reproduced in Fig. D-1.
Fig. D-1 EXPLOSIVE-DRIVEN PLATE EXPERIMENTAL ARRANGEMENT
The events following impact are recorded on the moving film of a streak camera, in which a reference grid stenciled on the lateral surface of the specimen is focused on a thin slit immediately in front of the film. A typical film record of a test on a laminate is shown in Fig. D-2. In the experiment shown, the driver plate, test specimen, and anvil form a cylindrical assembly 3-3/4 in. in diameter. A 50-gm charge of granular tetryl explosive drives a 3 in. long, 3-1/4 lb. aluminum cylinder against a simple laminate consisting of a 1/2 in. thick steel plate sandwiched between two 1/2-in. thicknesses of wallboard.

By varying the quantity of low-density explosive and the mass of the driver plate, a wide range of impact velocities may be attained. Initial driver plate velocities in these experiments ranged from about 50 ft/sec to about 500 ft/sec. Impact pressures up to approximately 1000 bars were attained in these experiments.

Particle velocities, particle displacements, and wave speeds are readily derived from a record such as that shown in Fig. D-2. During the next report period the photographic records obtained from these plate-impact experiments will be analyzed. A preliminary inspection of these records indicate that there is a significant difference in energy absorption for laminates of the type being studied here over that of initially uniform (non-layered) crushable specimens subjected to identical initial impact velocities.

E. Task No. 17 - Design Criteria for External Containment Structures

(M. A. Salmon)

Previous work on this task has included the development of analytical methods for determining the strength of cylindrical shells of finite length under internal pressure, which are applicable in cases where the deformations are large. The strength of a thin-walled cylinder at large deflections is
Fig. D-2 STREAK CAMERA RECORD OF EXPLOSIVE-DRIVEN PLATE IMPACT EXPERIMENT ON SIMPLE WALLBOARD-STEEL LAMINATE
primarily determined by its strength as an ideally thin membrane. The
effects of longitudinal bending moments are small. Two methods for analyzing
the strength of cylindrical shells acting as ideal membranes have been
developed.

The first of these\textsuperscript{1} assumes the shell to be composed of a rigid-linear
strain hardening material which obeys the maximum shear stress yield
criterion, the Tresca yield condition. Exact solutions for a range of length-
diameter ratios have been obtained for three values of the hardening constant.
These solutions were obtained by numerical integration on the Foundation's
UNIVAC 1105.

In an effort to reduce the amount of calculation required to obtain
solutions for particular values of the shell dimensions and material constants,
a simple approximate analysis has been developed by N. A. Weil\textsuperscript{2}. In this
method the shell material is assumed to be a rigid-strain hardening material
which obeys the maximum octahedral yield stress criterion (Mises yield
condition). A parabolic strain hardening law is assumed in the original
development of the method.

This section of the present report presents a comparison of the results
of the two methods. In addition, a comparison of these theories with the data
from a test of a ductile cylinder is made. In order to make these comparisons,
it was necessary to adapt the approximate method to linear hardening materials.

This modification is given in the following paragraphs.

\textsuperscript{1} T. A. Zaker, ed. Studies of Reactor Containment. Summary Report No. 3,
AT(11-1)-528, Armour Research Foundation (August 1960), Section II-E

\textsuperscript{2} N. A. Weil. An Approximate Solution for the Bursting of Thin-Walled
Cylinders, Report No. ARF 4132-17, U.S. Atomic Energy Commission
Contract No. AT(11-1)-528, Armour Research Foundation (to be published)
The purpose of comparing the two methods was to determine whether it is possible to use the simpler approximate method in practical calculations. If the two are in substantial agreement, the approximate method is preferable. It would of course be more desirable to choose between the methods on the basis of their degree of agreement with test results. There is, however, insufficient data to make this possible.

**Approximate Solution for a Linear-Hardening Mises Material**

The approximate solution is based on the assumption that the generators of the cylinder deform into circular arcs. It is further assumed that the ends of the cylinder do not move during the deformation process, so that the length of the cylinder in the axial direction is constant. A sketch of the assumed profile is shown in Fig. E-1. These assumptions fix the radius of the circular profile for given values of the initial length of the cylinder and its maximum radius \( r_1 \) in the deformed state as

\[
R = \frac{j_0^2 + w_1}{2w_1},
\]

where

\[
\begin{align*}
r_o & \text{ radius of a meridian,} \\
2r_o j_o & \text{ length of the cylinder,} \\
r_0 w_1 & = r_1 - r_o \text{ radial displacement at the center of the cylinder.}
\end{align*}
\]

With the geometry fixed, the ratio of the meridional stress \( \sigma_t \) to the circumferential stress \( \sigma_c \) at the center of the shell is determined from equilibrium considerations to be

\[
\beta = \frac{\sigma_t}{\sigma_c} = \frac{1}{2r_1/Rr_o}.
\]

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ARF 4132-16

Summary No. 4
Fig. E-1  ASSUMED PROFILE OF DEFORMED CYLINDER
or, using Eq. (E-1),

$$\beta = \frac{f_o^2 + w_1^2}{2[f_o^2 - w_1]}.$$  \hspace{1cm} (E-3)

The value of $\beta$ increases from 0.5 at the start of deformation and reaches a value of unity when the radial displacement equals

$$w_1 = \sqrt{1 + \frac{f_o^2}{2}} - 1,$$

or

$$p_1 = 1 + w_1 = \sqrt{1 + \frac{f_o^2}{2}},$$  \hspace{1cm} (E-4)

where

$$p_1 = \frac{r_1}{r_0}.$$

At this point the meridional and circumferential radii of curvature are equal and the surface of the shell is spherical. The present analysis is restricted to cases for which $\beta \leq 1$; this covers most cases of practical interest. For greater values of radial displacement, it would be reasonable to abandon the assumption that the ends of the cylinder do not move in the axial direction and to assume instead that the shell continues to deform as a sphere.

The relationship between the internal pressure and the maximum radial displacement is found by requiring that the stresses at the center of the shell satisfy the equilibrium condition

$$\sigma_t = \frac{p}{2t} r_1,$$  \hspace{1cm} (E-5)

in which

- $p$ = internal pressure,
- $t$ = thickness of the shell,

and by requiring also that the stresses satisfy the Mises yield criterion. For
In the present case of plane stress, this criterion may be written

\[ (\sigma_c^2 - \sigma_t \sigma_c + \sigma_t^2) \frac{1}{2} = \sigma, \]  

where \( \sigma \), the current yield stress in simple tension, depends on the state of hardening. The current yield stress is assumed to be a linear function of the effective strain \( \varepsilon \). That is,

\[ \sigma = \sigma_0 + c \varepsilon, \]  

where

- \( \sigma_0 \) = initial yield stress in tension,
- \( c \) = slope of the true stress-strain curve in uniaxial stress,

and where the effective strain is defined as

\[ \varepsilon = \sqrt{\frac{2}{3} \left( \varepsilon_c^2 + \varepsilon_t^2 + \varepsilon_n^2 \right)^{\frac{1}{2}}}. \]

In this equation, \( \varepsilon_t \), \( \varepsilon_c \), and \( \varepsilon_n \) are the meridional, circumferential, and thickness strains respectively. Since for incompressibility

\[ \varepsilon_c + \varepsilon_t + \varepsilon_n = 0, \]

the effective strain may be written

\[ \varepsilon = \frac{2}{\sqrt{3}} \left( \varepsilon_c^2 + \varepsilon_t^2 + \varepsilon_n^2 \right)^{\frac{1}{2}}. \]

The circumferential strain at the center of the shell is

\[ \varepsilon_c = \log \frac{r_1}{r_0} = \log p_1, \]

but an additional assumption is required to specify the meridional strain.

The flow rule corresponding to the Mises yield condition specifies the ratio of the strain rates to be

\[ \frac{\dot{\varepsilon}_t}{\dot{\varepsilon}_c} = \frac{2\sigma_t}{2\sigma_c - \sigma_t}. \]
If it is assumed that during deformation the ratio of the stresses is constant, Eq. (E-11) can be integrated to give

$$\epsilon_t = \frac{2 \sigma_t - \sigma_c}{2 \sigma_c - \sigma_t} \epsilon_c = \frac{2\beta - 1}{2 - \beta} \epsilon_c. \quad (E-12)$$

Since $\beta$ is known from Eq. (E-3), Eq. (E-12) determines the meridional strain and consequently fixes the current state of hardening. Using Eqs. (E-12) and (E-10) in (E-9), we obtain

$$\epsilon = \frac{2}{2 \beta} (1 - \beta + \beta^2) \frac{1}{2} \log \rho_1. \quad (E-13)$$

Equation (E-6) may be solved for the meridional stress to give

$$\sigma_t = \beta (1 - \beta + \beta^2) \frac{1}{2} \bar{\sigma}. \quad (E-14)$$

where

$$\bar{\sigma} = \frac{\sigma}{\sigma_0}.$$

Using Eqs. (E-13) and (E-7) in (E-14), we obtain

$$\sigma_t = \beta (1 - \beta + \beta^2) \frac{1}{2} \left[1 + \frac{2}{2 \beta} (1 - \beta + \beta^2) \frac{1}{2} H \log \rho_1 \right] \quad (E-15)$$

as the expression for the meridional stress, in which $H = \frac{\sigma}{\sigma_0}$ is the dimensionless hardening coefficient. Since the circumferential and meridional strains are known, the thickness can be found from the condition for incompressibility given by Eq. (E-8) as follows:

$$\epsilon_n = \log \frac{t}{t_0} = (\epsilon_c + \epsilon_t) = \frac{1 + \beta}{2 - \beta} \epsilon_c.$$

or since $\epsilon_c = \log \rho_1$,

$$\eta = \frac{t}{t_0} = \rho_1^{-\left(1+\beta\right)/\left(2-\beta\right)}. \quad (E-16)$$
Equation (E-5) can be solved for the pressure to give
\[ p = \frac{2t \sigma_1}{r_1}, \]
or in dimensionless form
\[ \alpha = \frac{\eta_0}{\rho_1} = \eta_0^{-\gamma/2(2-\beta)}, \quad \text{(E-17)} \]
where
\[ \alpha = \frac{p r_0}{2\sigma_0^2}. \quad \text{(E-18)} \]

Substituting Eqs. (E-15) and (E-16) into Eq. (E-17), we have
\[ \alpha = \rho_1 \frac{3(2-\beta)}{\beta^2} \left[ \frac{1}{1 + \frac{2}{Z(2-\beta^2)} \frac{1}{2} \log \rho_1} \right] \quad \text{(E-19)} \]
as the expression for the pressure ratio as a function of \( \beta \) and \( \rho_1 \), where \( \beta \) is given by Eq. (E-3). The determination of the pressure-radius function for a cylinder of given length is simply a matter of evaluating Eq. (E-19) for a given values of \( \rho_1 \) and \( \rho_0 \), while the determination of the exact solution for a Tresca material requires a numerical integration for each point on the pressure-radius curve.

Comparison with the Exact Solution for a Tresca Material

The Tresca yield condition states that flow occurs where the maximum shear stress becomes equal to half the yield stress in simple tension. For states of plane stress this yield condition is represented geometrically by a hexagon in the \( \sigma_c \), \( \sigma_t \) stress plane as shown in Fig. E-2. The Mises yield condition, Eq. (E-6), is also shown in this figure. The Tresca yield condition predicts that the circumferential stress in an infinite cylinder under internal pressure at yield will be equal to the yield stress in tension. On the other hand, the Mises yield condition predicts that in this case the circumferential...
Fig. E-2 COMPARISON OF MISES AND TRESCA YIELD CONDITIONS FOR PLANE STRESS
stress at yielding will be
\[ \sigma_c = \frac{2}{\sqrt{3}} \sigma . \]

The two theories differ by 15 per cent in this case. Since the Mises theory is
in better agreement with test results than is the Tresca theory, it is desirable
to modify the latter so as to minimize this discrepancy. This can be done by
using as the yield condition a hexagon circumscribed about the Mises ellipse as
shown in Fig. E-2. This is equivalent to assuming an effective tensile yield
stress
\[ \sigma^* = \frac{2}{\sqrt{3}} \sigma . \quad (E-20) \]

For the case of the infinite cylinder, the stress ratio \( \beta \) is equal to 0.5.
In this case Eq. (E-19) becomes
\[ \sigma = \frac{2}{\sqrt{3}} \left[ 1 + \frac{2}{\sqrt{3}} H \log \rho_1 \right] / 2 \rho_1^2 \quad (E-21) \]

In the case of a linear hardening Tresca material, the circumferential
stress is
\[ \bar{\sigma}_t = 1 + H \log \rho_1 = 2 \bar{\sigma}_t , \]
while the thickness ratio is
\[ \eta = 1/\rho_1 . \]
Therefore the pressure ratio
\[ \sigma = \frac{\eta \bar{\sigma}_t}{\rho_1} \]
becomes
\[ \sigma = \left( 1 + H \log \rho_1 \right) / 2 \rho_1^2 . \quad (E-22) \]
The values of the pressure given by Eqs. (E-21) and (E-22) can be made
equal for all values of \( \rho_1 \) if for the Tresca material we take an effective
initial yield stress
\[ \sigma^* = \frac{2}{\sqrt{3}} \sigma_0. \]  
(E-23)

and an effective hardening coefficient
\[ H^* = \frac{2}{\sqrt{3}} H. \]  
(E-24)

Pressure-radius curves for the two methods are given in Figs. E-3 and E-4 for three values of the hardening coefficient $H (0, \sqrt{3}, 4.48)$. The agreement between the two methods is fairly good for the largest value of $H$ used, while for the rigid-perfectly plastic material ($H=0$) there is a substantial difference in the predictions of the two methods. Experimental evidence is required to assess the relative merits of the analyses.

A comparison of the theoretical predictions with the results of a single test is made in the following paragraphs.

Comparison of Theory with Experiment

The results of a test of a cylinder with rigid end closures\(^3\) conducted at the Naval Ordnance Laboratory are used for this comparison. A cylinder 10 in. long with a 5-in. inside diameter was tested under internal pressure to the point of failure. The test data are tabulated in Fig. E-5. The material used was Type 304 stainless steel, a very ductile steel. The stress-strain curve in simple tension is shown in Fig. E-6. It is almost perfectly linear up to a strain of 0.6. The values of $\sigma_0$ and $H$ obtained from this curve are 25,500 psi and 6.32 respectively.

Fig. E-3 COMPARISON OF THE APPROXIMATE SOLUTION FOR A MISES MATERIAL WITH THE EXACT SOLUTION FOR AN EQUIVALENT TRESCA MATERIAL, H=4.48
Fig. E-4 COMPARISON OF THE APPROXIMATE SOLUTION FOR A MISES MATERIAL WITH THE EXACT SOLUTION FOR AN EQUIVALENT TRESCA MATERIAL, $H = 0.3$
<table>
<thead>
<tr>
<th>Internal Pressure $p$ (psi)</th>
<th>Maximum Outside Radius $r_o$ (in.)</th>
<th>Maximum Inside Radius $r_i$ (in.)</th>
<th>Average Strain at Inside Wall $\frac{\varepsilon}{\varepsilon_c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.75</td>
<td>2.50</td>
<td>1</td>
</tr>
<tr>
<td>3000</td>
<td>2.7936</td>
<td>2.5479</td>
<td>0.911</td>
</tr>
<tr>
<td>4000</td>
<td>2.8862</td>
<td>2.6491</td>
<td>0.913</td>
</tr>
<tr>
<td>4500</td>
<td>2.9902</td>
<td>2.7620</td>
<td>0.916</td>
</tr>
<tr>
<td>5000</td>
<td>3.1587</td>
<td>2.9436</td>
<td>0.921</td>
</tr>
<tr>
<td>5400</td>
<td>3.3326</td>
<td>3.1295</td>
<td>0.925</td>
</tr>
<tr>
<td>5800</td>
<td>3.5141</td>
<td>3.3221</td>
<td>0.928</td>
</tr>
<tr>
<td>6200</td>
<td>3.6998</td>
<td>3.5180</td>
<td>0.932</td>
</tr>
<tr>
<td>6400</td>
<td>3.8032</td>
<td>3.6265</td>
<td>0.933</td>
</tr>
<tr>
<td>6700</td>
<td>3.9320</td>
<td>3.7614</td>
<td>0.935</td>
</tr>
</tbody>
</table>

Values computed on the assumption of zero tangential strain

Fig. E-5 DATA FOR NOL TEST OF CYLINDER WITH RIGID END CAPS UNDER INTERNAL PRESSURE; LENGTH 10 in., INSIDE DIAMETER 5 in.
Fig. E-6  TRUE STRESS-STRAIN CURVE FOR TYPE 304 STAINLESS STEEL USED IN NOL TEST

$\sigma_0 = 25,500$ psi
$\sigma = 161,200$ psi
$H = \frac{\sigma}{\sigma_0} = 6.32$
The theory is based on the assumption that the cylinder is an ideally thin membrane. However, the inside radius-thickness ratio for the test specimen (10:1) is so small that the effect of wall thickness cannot be neglected in interpreting the test results. The following procedure is used to account for the effect of the finite wall thickness of the test specimen.

The radius of the equivalent ideally thin-walled cylinder is assumed to be equal to the inside radius of the actual cylinder and a reduced value of the hardening coefficient \( H \) is used. It is possible to determine errors involved in this procedure for the case of the infinite cylinder.

In the case of the ideally thin cylinder, the pressure is given by

\[
P_{\text{I}} = \frac{\sigma_0}{t},
\]

or since, for incompressibility,

\[
t = \frac{r_0 t_0}{t},
\]

the pressure is

\[
P_{\text{I}} = \frac{\sigma r_0 t_0}{r^2},
\]

in which the circumferential stress

\[
\sigma = \sigma_0 (1 + H \epsilon_c).
\]

The pressure in an infinite cylinder of finite wall thickness is given by

\[
P_{\text{II}} = \frac{\sigma_0 t}{r_1^2},
\]

where \( r_1 \) is the inside radius of the cylinder and \( \sigma_0 \) is the average circumferential stress through the thickness. In this case the condition of incompressibility requires that
in which $r_{10}$ and $t_o$ are the initial inside radius and thickness respectively.
The pressure is then

$$p_a = \frac{\sigma_a}{r_{10}^2} \frac{1 + t_o/2}{1 + t/2}$$

The ratio of the pressure in the idealized cylinder of radius $r_i$ to the actual pressure is

$$\frac{p_i}{p_a} = \frac{\sigma}{\sigma_{av}} \frac{1 + t/2}{1 + t_o/2}$$

For the range of displacements considered, the maximum value of this difference is less in absolute value than 0.03.

The first factor in Eq. (E-30) can be written

$$\frac{\sigma}{\sigma_{av}} = \frac{1 + H\overline{e}_c}{1 + H\overline{e}_c}$$

where $\overline{e}_c$ is the average circumferential strain through the thickness and $e_c$ is the strain at the inner surface. In analyzing the NOL test result, a value of $H = H$ was selected in such a way that

$$H e_c = H \overline{e}_c$$

for the range of displacements considered.

If we denote the current radius of a particle by $x$ and its initial radius by $y$, incompressibility requires that

$$y^2 = x^2 - (r_i^2 - r_{10}^2).$$
The circumferential strain is then
\[ \varepsilon_c = \log \frac{x}{y} = \frac{1}{2} \log \frac{\frac{x^2}{y}}{x^2 - a^2} \]  \hspace{1cm} (E-33)

where
\[ a^2 = r_i^2 - r_{10}^2 \]  \hspace{1cm} (E-34)

The average strain through the thickness is
\[ \bar{\varepsilon}_c = \frac{1}{t} \int_{r_i}^{r_e} \varepsilon_c \, dx, \]  \hspace{1cm} (E-35)

where \( r_e \), the outside radius, is given by
\[ r_e^2 = r_i^2 + 2r_{10} (2r_{10} + t_0), \]  \hspace{1cm} (E-36)

and the thickness \( t = r_e - r_i \).

Integration of Eq. (E-35) gives
\[ \bar{\varepsilon}_c = \frac{1}{t} \left( r_e \log \frac{r_e}{r_{10}} - r_i \log \frac{r_i}{r_{10}} + \frac{a}{2} \log \left[ \frac{r_e - a}{r_i - a} \left( \frac{r_i + a}{r_e + a} \right) \right] \right) \]  \hspace{1cm} (E-37)

Values of \( \bar{\varepsilon}_c / \varepsilon_c \) were computed for the NOL test data and are tabulated in Fig. E-5. The values of the inside radius \( r_i \) computed from the measured values of the outside radius with the assumption of zero axial strain are also tabulated. The ratios of the average strain to the strain at the inner surface range from 0.911 to 0.935. A value of \( H = 0.925 \) was used in the theoretical calculations in an attempt to account for the variation of strain through the thickness.

A comparison of the results given by the two analytical methods with the NOL data is given in Fig. E-7. The values of \( \sigma_0 \) and \( \bar{H} \) used to compute the pressures by the approximate method for a Mises material were
\[ \sigma_0 = 25,500 \text{ psi}, \]
\[ \bar{H} = 0.925 \times 6.32 = 5.85. \]
Fig. E.7  COMPARISON OF THEORETICAL RESULTS WITH NOL TEST DATA
while for the second analytical method these values were multiplied by the factor $\sqrt{\frac{2}{3}}$. The analytical results agree fairly well with the test data.

The maximum error for the approximate analytical method is 6 per cent while for the other method it is 12 per cent. It is not possible to draw general conclusions on the basis of a single test, but these results are encouraging.

To summarize the results of the comparison of the analyses with each other and with the test data, it appears that the agreement is in general good enough to permit the use of the simple approximate method for design purposes.