ON THE POSSIBILITY OF EXTRACTING ENERGY
FROM GRAVITATIONAL SYSTEMS
BY NAVIGATING SPACE VEHICLES

By

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Alamos Scientific Laboratory or the final opinion of the authors on the subject.
It is intended to outline in this brief report a number of problems of the following type: We assume an astronomical system composed of two or more stellar bodies and a space vehicle which, as an additional body of infinitely small mass compared to the celestial objects, forms part of a many-(e.g., 3)-body system. We assume that the "rocket" not only describes the trajectory under the action of the gravitational forces, but also that it has still a reserve energy available for steering by suitably emitted impulses. This energy in the discussion below will be assumed to be roughly of the order of the kinetic energy which the rocket already possesses. The problem, broadly speaking, involves the possibility of using this reserve energy in such a way as to acquire, by suitable near collisions with one or the other of the celestial bodies, much more kinetic energy than it possesses — more by an order of magnitude than the available reserve energy would allow it to acquire by itself.

As examples of the situation we have in mind: Assume a rocket cruising between the sun and Jupiter, i.e., in an orbit approximately that of Mars, with an energy in reserve which would allow the kinetic energy of the vehicle to increase by a factor like 2. The question is
whether, by planning suitable approaches to Jupiter and then closer approaches to the sun, it could acquire, say, 10 times more energy. Another example would be a space vehicle moving in a double star system "half way in between." Then the question is whether, by using additional impulses of its own, it could acquire again a kinetic energy much greater than what it already possesses.

As a purely mathematical problem we could consider the case of two mass points each of mass 1 forming a Keplerian system, and a rocket of mass vanishingly small compared to 1 in an orbit which forms a curve between the two mass points. Suppose that the reserve power of the rocket is such that it could double its kinetic energy. Question: Can one, in this idealized condition, obtain a velocity arbitrarily large (i.e., close to light velocity)?

That this possibility exists seems extremely probable from the theorems on ergodic transformations. It has been shown that arbitrarily near to any given transformation, like the one given by the Hamiltonian describing the n-body system above, there exist transformations which are metrically transitive, that is, to say, in particular, Liouville flows such that the trajectory of the system will penetrate arbitrarily near any point on the phase space. The theorem has been proved for bounded phase spaces. This does not make our theorem inapplicable to the problem. We could put in cut-offs in the distance of approach and

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assume at a finite but very great distance from the gravitational bodies another cut-off. The theorem would imply that arbitrarily near the given dynamical motion there exists one which will make the rocket approach as close to the cut-off sphere surrounding any one of the given mass points as we please, which would in particular imply obtaining arbitrarily high velocities. The theorem asserts the existence of such motions arbitrarily near given ones. The question whether these can be obtained by changes effected through emitting additional impulses inside the rocket is not essentially answered, but in view of the prevalence of ergodic motions near a given one this seems extremely likely. Such an ergodic trajectory would, of course, in particular provide arbitrarily high velocities. Nothing is said, however, about the times necessary for effecting this. They might be of super-astronomical lengths. It is clear, on the other hand, on general thermodynamical grounds that "in general" the equipartition of energy may take place. This implies that the body with the small mass of the rocket will acquire very high velocities. This is well known even in systems of a moderate number of particles. The energy distribution is Maxwellian, again tending to provide the small masses with high velocity. The problem is whether, by steering the rocket, one can to some modest extent acquire the properties of a Maxwell demon, i.e., plan the changes in the trajectory in such a way as to shorten by many orders of magnitude the time necessary for acquisition of very high velocities.

As is well known, the perturbations of Jupiter on the motion of
some comets provide them occasionally with velocities of escape from the solar system. It has been noticed also that one can use the attraction of the moon to provide a rocket with additional kinetic energy, enabling it to escape from the earth's gravitational field even if it did not have enough energy to do that to begin with.

Our problem is whether one can do it repeatedly to obtain essentially arbitrary kinetic energies by repeated and suitably timed approaches to the two or more celestial bodies.

The question is that of finding general recipes for a 3-(or more)-body problem to achieve that aim as quickly as possible. It is proposed to calculate some very schematic, simple, but perhaps instructive cases for a "strategy" of steering the rocket.

1. The first case involves a problem in one dimension. Suppose two masses oscillate at the end of a segment with given amplitude, say harmonically; a point of vanishingly small mass is rocketed in between and, possessing some initial kinetic energy, collides elastically with the two oscillating end points. If these should be in phase, the calculation will show the increase of kinetic energy of the small mass. If the phases of the two oscillators should be randomly independent, the question arises how to plan the emission of additional small impulses by the middle point so as to make it increase its kinetic energy most efficiently. Obviously, one should plan to collide head on with the two oscillators as much as possible. In other words, through additional impulses,

collisions that lead to a gain in energy for the "free" point should be maximized as far as possible. The ones that involve colliding by overtaking the receding end point should be diminished in their effect. Without a strategy of changes in the velocity of the "rocket" the gain of energy towards an eventual equipartition would be a very slow process—the rates at which this approach to equipartition takes place are unknown in statistical mechanics but certainly the gains in a random process increase with the square root of time or slower. With an operating intelligence perhaps this approach to near-equilibrium could be made vastly more rapid.

2. This problem will involve two mass points describing quite elongated Keplerian ellipses around their center of mass. The rocket moves initially in a roughly circular orbit in between the two masses around their common center. Of course, the actual trajectory in this 3-body problem is very complicated. The question is again to plan a strategy of changing, by small amounts, the energy of the small object so as to approach one or the other of the large bodies to gain kinetic energy. If this is to take place, the approach to either of the two bodies must be increasingly closer. This involves great elongation and an increase in apastron of the rocket. Again the plan is to make near collisions head on. Presumably, the planned changes, that is to say the emitted impulses from the rocket, will be most efficient when the body is at maximum distance from the center of gravity of the two celestial points. It is there that a small increase in velocity will enable one to make
changes in the time of the next approach.

The above discussion is, of course, intended for a purely theoretical, mathematical question. Even so, during the next few decades large objects may be constructed with a cruising velocity of 20 kilometers a second, and there will be still some additional energy left for changes in this velocity. It is obvious that the process of increasing the kinetic energy of the rocket by such extraction of gravitational energy from celestial motions is, at best, very slow. The computations required to plan changes in the trajectory might be of prohibitive length and complication. This little note is meant merely as an introduction to exploratory analyses and calculations undertaken with Kenneth W. Ford and C. J. Everett of IASL.