LAMS-618 Instrumentation

Photostat Price $ 3.30
Microfilm Price $ 2.40

Available from the
Office of Technical Services
Department of Commerce
Washington 25, D. C.

11 November 1947

FRAUNHOFER DIFFRACTION PATTERN PRODUCED BY A SLIT OF VARYING WIDTH AND ITS APPLICATION TO HIGH SPEED CAMERAS

Work Done By:

B. Brixner
F. E. Geiger

Report Written By:

F. E. Geiger

Abstract

A theoretical and experimental investigation is made of the diffraction pattern produced by a slit, whose aperture varies uniformly from a constant value A to zero. The results of this investigation are applied to a proposed high speed camera. It is shown that diffraction effects are very serious and cannot be neglected. It seems, unless the suggested design of this high speed camera is changed, the camera will be of little use for accurate measurements, and photographs will show too much blur to give details.

Fraunhofer Diffraction Pattern Produced by a Slit of Varying Width and its Application to High Speed Cameras

Dr. J. E. Mack has suggested a high speed camera* capable of taking pictures at the rate of about 10^6 per second. One of the limitations of this camera is the diffraction effects produced by narrow slits placed in front of the camera lenses. It is a simple matter to calculate the Fraunhofer diffraction produced by a slit of narrow aperture. But in these calculations the effect produced by the shutter has been neglected. The shutter is a rotating disk, having slits of the same width as the stationary slits in front of the camera lenses. Obviously, the shutter moving in front of the camera slit decreases the aperture of the latter, producing diffraction which spreads the image to a considerably greater degree, as one might suspect from the simple single slit, constant width calculations.

It is the purpose of this report to investigate, both theoretically and experimentally, the Fraunhofer diffraction due to a slit whose width varies uniformly from some value A to zero.

OFFICIAL USE ONLY

*Official communication of J. E. Mack to B. Brixner

Theoretical Investigation. In carrying out the solution of this problem, a few simplifying assumptions are made. It is assumed that the luminous object is a narrow line source at infinity, and that the action of the shutter is equivalent to the motion of the jaws of a bilateral slit. The width of the slit is decreased uniformly to zero. The latter assumption is justified, because the diffraction pattern is independent, within limits, of the position of the diffracting aperture in its own plane*. The slit width of the shutter equals that of the slit in front of the camera lens.

Fig. 1 is a schematic drawing of the camera slit, and the shutter.

If the luminous object is a line source, and the diffracting aperture is long and narrow, the intensity distribution in the focal plane of the camera is given by,

$$I = I_o \left(\frac{\sin kaA}{kaA}\right)^2 \qquad (1)$$

where, $k = \pi/\lambda$

A, is the width of the slit in front of the camera lens

$a = (x + x_0)/f$, x_o being the coordinate of the geometrical image,

x, the coordinate of a point in the image plane. (See Fig. 2.)

f, is the focal length of the camera lens.

λ, the wavelength of light

I, the intensity at point $Q(x)$, see Fig. 2

I_o, the intensity at $P(X_o)$

---

*Actually, the moving shutter exposes portions of the camera slit which are slightly off the camera axis.

Since P, the geometrical image, of the object O, is the optical axis, $x_o = 0$, and (1) becomes,

$$I = I_0 \left(\frac{\sin kxA/f}{kxA/f}\right)^2 \qquad (2)$$

In this equation A and I_0 are variables. I_o is directly proportional to the aperture, but $A = ct$, where c is the velocity of the shutter. Therefore, $I_o = bt$. We are only interested in relative values of intensities, hence, b may be set equal to one.

The energy the film or screen receives at some point x in the time interval dt will be,

$$Idt = t. \left(\frac{\sin kxct/f}{kxct/f}\right)^2 . dt \qquad (3)$$

The total energy,

$$\int_0^T Idt = \int_0^T t. \left(\frac{\sin kxct/f}{kxct/f}\right)^2 dt \; ; \; T = \frac{A}{C}$$

Changing variables,

$$E = \int_0^T Idt = \int_0^{mT} 1/m^2 . \frac{\sin^2 u}{u} \, du \qquad (4)$$

where $m = kxc/f \cdot \pi \;\; xc/\lambda f$

and $u = mt$

$$E = 1/2m^2 \int_0^{2mT} \frac{1 - \cos 2u}{2u} \, d(2u) \qquad (5)$$

$$E = 1/2m^2 . (\log_e \gamma . Y - Ci(Y))^* \qquad (6)$$

---

*Tables of Functions, Jahnke & Emde, 1943 Dover Publication, pp. 3 & 6.

324843

where,

$$Y = 2mT = 2\pi\, xA/f\lambda$$

$$Ci(Y) = (\log_e \gamma + \log_e Y - \int_0^Y \frac{1 - \cos Y}{Y}\, dY)$$

$$\log_e \gamma = 0.5772$$

Or,

$$E = 2A^2/c^2 \,(\, 0.5772 + \log_e Y - Ci(Y))/Y^2 \qquad (7)$$

This equation then gives the total amount of energy, (ergs/cm^2), which a point Q(x) on the film receives in time T.

Now since,

$$E = Ixt = \frac{2A^2}{C^2} \,(\, 0.5772 + \log_e Y - Ci(Y)\,)/Y^2$$

we may set t = 1, and write

$$I = \frac{2A^2}{C^2} \,(\, 0.5772 + \log_e Y - Ci(Y)\,)/Y^2 \qquad (8)$$

or,

$$I/I_o = \left[0.5772 + \log_e Y - Ci(Y)\right] /0.25Y^2$$

where I_o, the intensity at X = 0, equals 0.25 $\frac{2A^2}{C^2}$ * .

$\left[0.5772 \log_e Y - Ci(Y)\right] /Y^2$ has been tabulated for values of Y from 0.2 to 130. See Table I.

Graph 1 shows a plot of Eq. (8) as a function of Y. This graph may be used very easily to compute the intensity at any point x, if λ, A and f are known. For comparison the single slit, constant width, diffraction pattern has been drawn in. This shows that for Y = 2π, the single slit intensity (A = constant) is zero, while for the varying slit, the intensity is still 25% of the maximum.

---

* $\lim_{Y\to 0} \left[0.5772 \log_e Y - Ci(Y)\right] /Y^2 = 0.25$

324844

Experimental Verification. The experimental arrangement is essentially the same as that shown in Fig. 2. The light source is a narrow slit, 0.035 mm wide. The slit is illuminated by the light of a sodium vapor lamp. Parallel light, emerging from an eight inch collimating lens, passes through a second slit in front of the camera lens. The initial aperture of this bilateral slit is 0.070 cm. The camera lens has a focal length of 200 cm. Both the collimating and camera lenses are achromats. Since it proved quite impossible to reduce the width of the shutter uniformly (exposure time about three hours), it was reduced in steps of 0.050 mm each.

The film is exposed for 10 minutes for each new slit setting. The photograph of the diffraction pattern was densitometered on a Leeds & Northrup densitometer. A gelatin stepwedge was used to determine the H & D curve of the film, this film, in turn, was densitometered on the same instrument as the photograph of the diffraction pattern, and the trace of the photograph of the gelatin stepwedge, densities of the slit image as a function of distance were obtained. It was then an easy matter to convert these densities into relative intensities by means of the previously obtained H & D curve. The results of the experimental investigation are plotted on Graph 2. The theoretically computed intensity distribution for this special case has also been drawn. The agreement between the two curves is very good down to intensities of 40%. Below 40% the observed curve shows intensities considerably less than the predicted values. This discrepancy cannot be attributed to a failure of the theory at least not definitely so, since the experimental data is not sufficient to show where the difficulty lies.

Conclusion. Both theoretical and experimental investigations show that the proposed Megacycle camera will produce a considerable spread of the image due to diffraction effects. This spread will overshadow all other optical aberrations, so that they may be neglected in this discussion. Let us assume, that the focal length of the camera is 100 cm, the slit width 0.070 cm, and the object, a luminous line, is 10,000 yards away, then the total spread of the image will be 1.44 mm. (The intensity distribution for this special case is shown on Graph 3.) This figure has been arrived at by assuming that at 35% of the maximum intensity of the image, this image will appear to have a reasonably sharp edge. This is illustrated in Fig. 3. Since the width of the geometrical image may be neglected in this case, this image spread corresponds to a distance of 14.4 meters in space.

These results may be applied to an object of finite width, if we make the assumption that the "fuzzyness" of the image is about the same as the spread for a very narrow line source.* Thus, for example, for an object 30 meters wide, the image will appear larger by almost 50%. See Fig. 4.

It appears then that the pictures obtained with the megacycle camera will not give the desired definition to show, say, details in the shockfront of an explosive. Furthermore, it seems that it will be very difficult to get accurate space time relationships (say, the diameter of the shockfront as a function of time) from pictures of this type.

Acknowledgements. Dr. R. T. Landshoff was helpful in evaluating integral (5) in terms of tabulated functions.

---

*Calculations are in progress to determine the effect of an object of finite width.

TABLE I

Y	$[0.5772 \ \log_e Y - Ci(Y)]/Y^2$	Y	$[0.5772 \ \log_e Y - Ci(Y)]/Y^2$
0.2	0.2500	11.0	0.025328
0.5	0.2470	12.0	0.021611
1.0	0.23982	13.0	0.018434
1.5	0.22768	14.0	0.016056
2.0	0.21184	15.0	0.014395
2.2	0.21466	20.0	0.008821
2.4	0.19712	25.0	0.006085
2.6	0.18926	30.0	0.004457
2.8	0.18117	35.0	0.003383
3.0	0.17291	40.0	0.002654
3.2	0.16456	45.0	0.002156
3.4	0.15619	50.0	0.001798
3.6	0.14785	55.0	0.001522
3.8	0.13961	60.0	0.001299
4.0	0.13153	65.0	0.001125
4.2	0.12365	70.0	0.000983
4.4	0.11604	75.0	0.000880
4.6	0.108709	80.0	0.000777
4.8	0.101712	85.0	0.000695
5.0	0.095066	90.0	0.000626
6.0	0.067695	95.0	0.000568
7.0	0.049910	100.0	0.000519
8.0	0.039596	110.0	0.000436
9.0	0.033569	120.0	0.000372
10.0	0.029253	130.0	0.000323

324847

[redacted]

LAMS-618

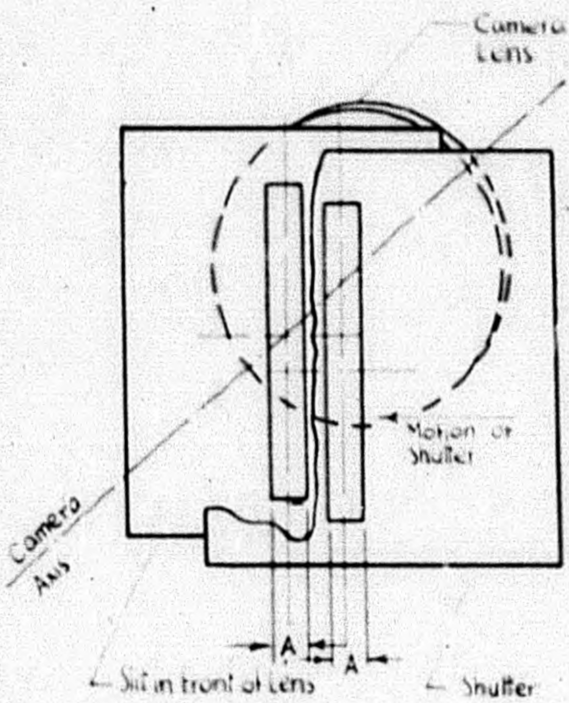


Fig 1

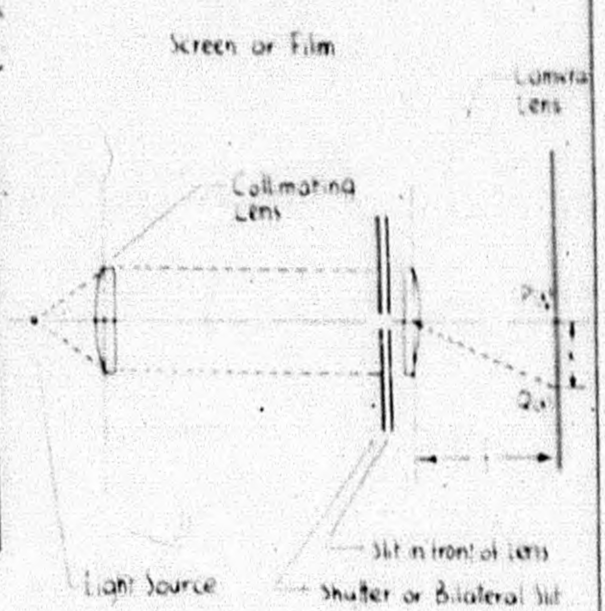


Fig 2

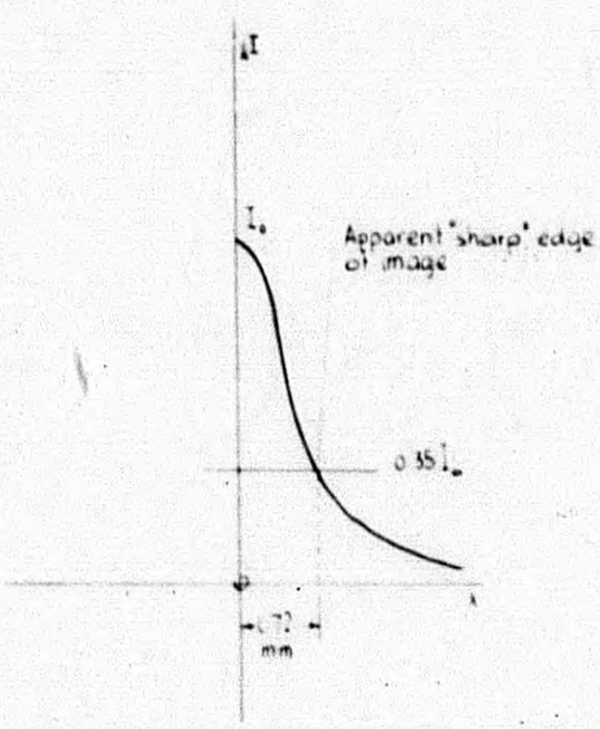


Fig 3

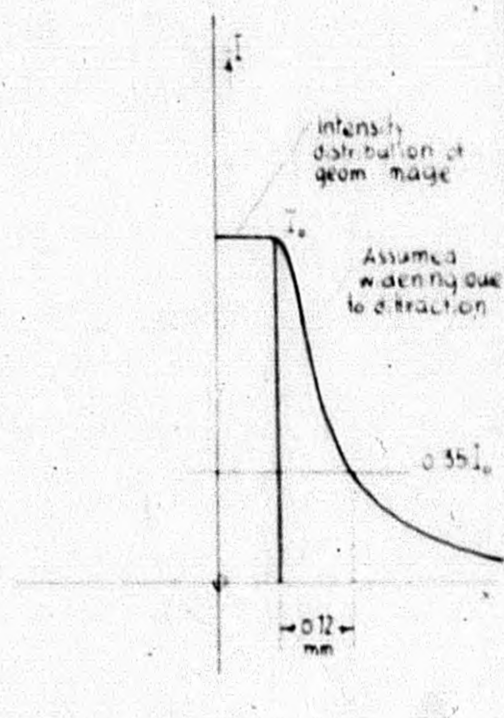
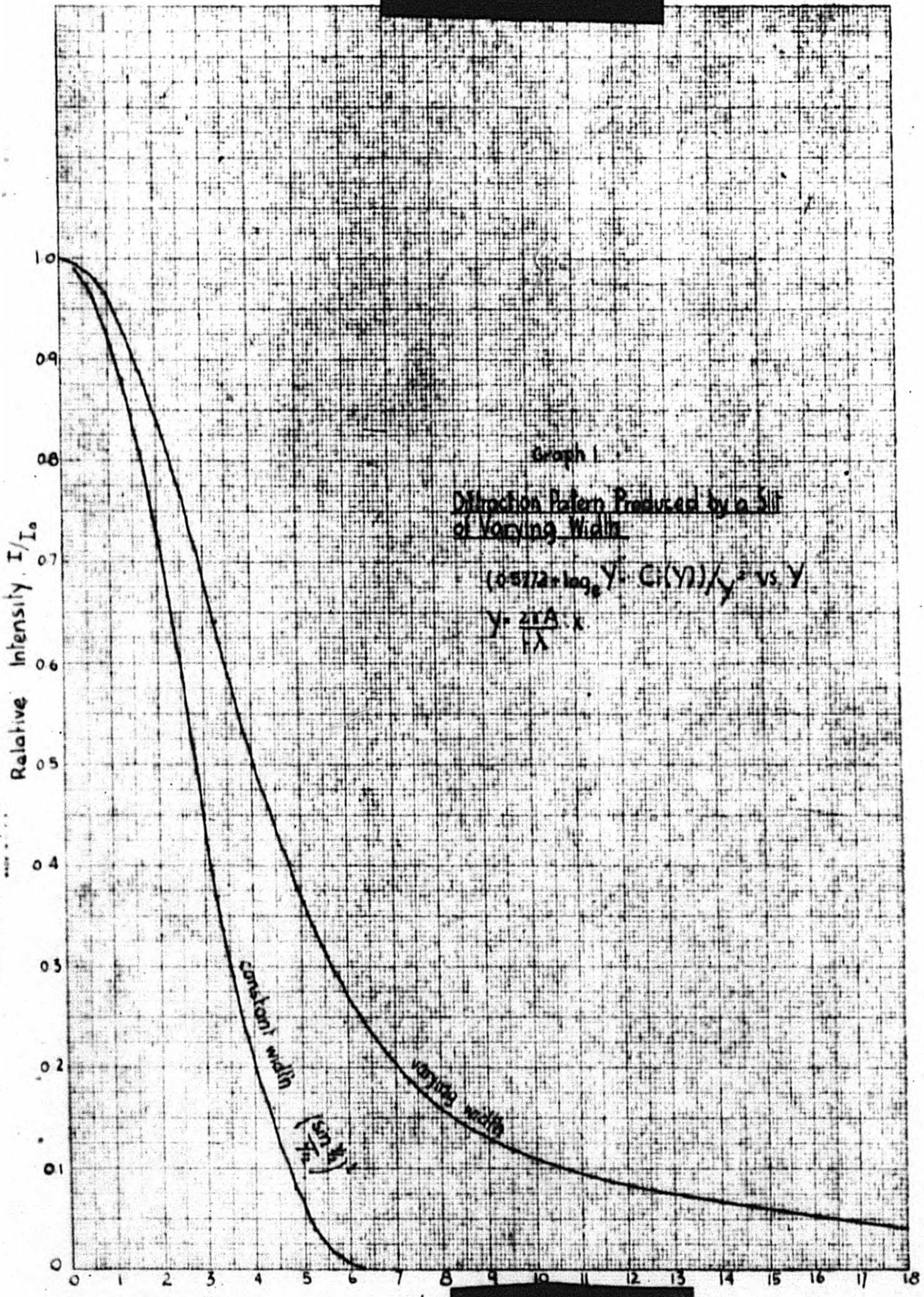


Fig 4

[redacted]

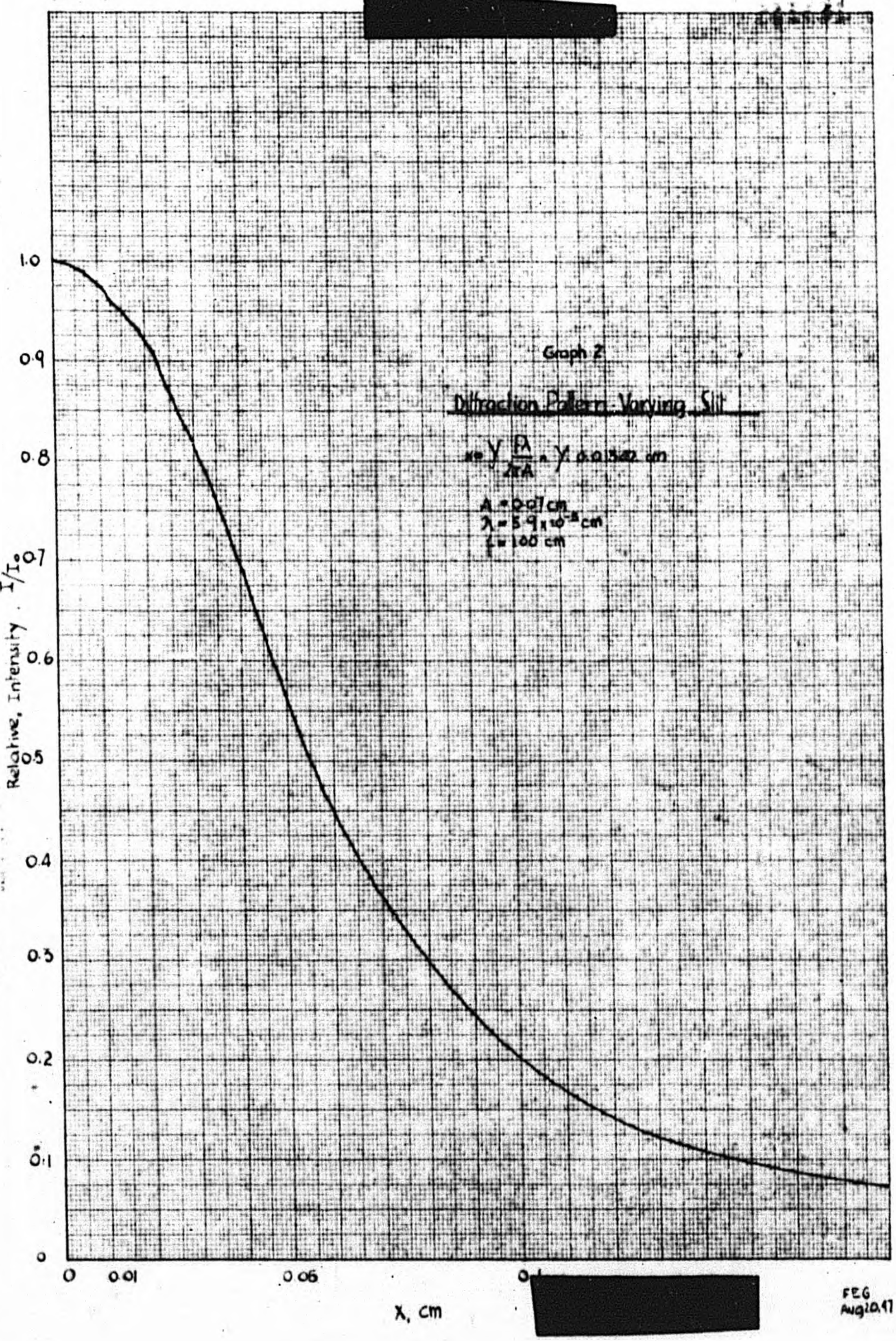
[redacted]

335430

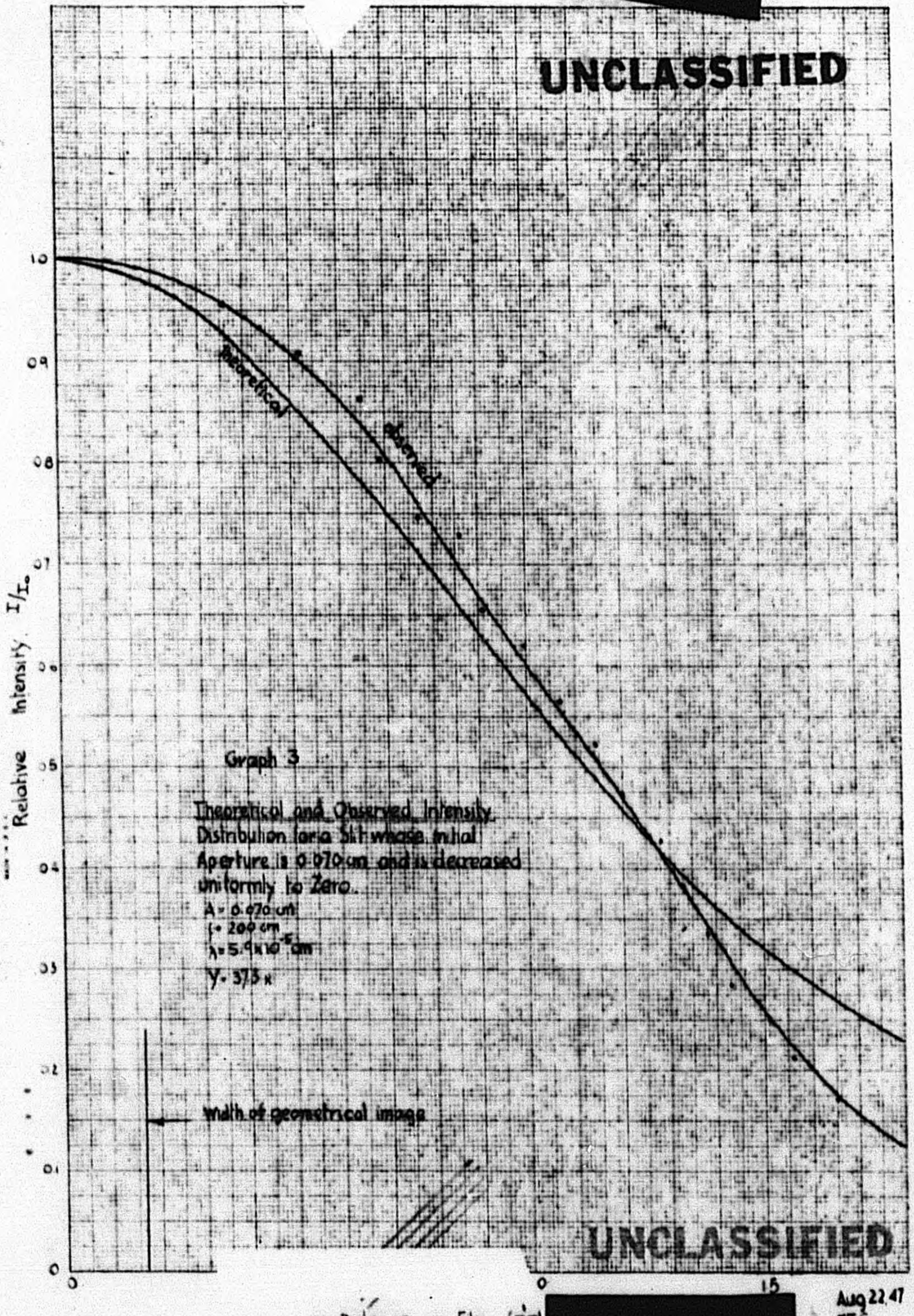


[redacted]

[redacted]



[redacted]


UNCLASSIFIED
Relative Intensity I/I_0
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Theoretical
Observed
Graph 3
Theoretical and Observed Intensity Distribution for a Slit whose Initial Aperture is 0.070 cm and is decreased uniformly to Zero.
A = 0.070 cm
f = 200 cm
$\lambda = 5.9 \times 10^{-5}$ cm
Y = 373 x
Width of geometrical image
UNCLASSIFIED
0
0
15
x, Distance on Film, (mm) [redacted]
Aug 22,47
FEG