FORECASTING QUARTERLY SALES TAX REVENUES: A COMPARATIVE STUDY

THESIS

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The purpose of this study is to determine which of three forecasting methods provides the most accurate short-term forecasts, in terms of absolute and mean absolute percentage error, for a unique set of data. The study applies three forecasting techniques--the Box-Jenkins or ARIMA method, cycle regression analysis, and multiple regression analysis--to quarterly sales tax revenue data.

The final results show that, with varying success, each model identifies the direction of change in the future, but does not closely identify the period to period fluctuations. Indeed, each model overestimated revenues for every period forecasted. Cycle regression analysis, with a mean absolute percentage error of 7.21, is the most accurate model. Multiple regression analysis has the smallest absolute percentage error of 3.13.

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OVERVIEW AND INTRODUCTION TO THE STUDY

Overview

The purpose of this study is to determine which of three forecasting methods provides the most accurate short-term forecasts, in terms of absolute percentage error, for a unique set of data. The study applies three time series forecasting techniques--the Box-Jenkins method, cycle regression analysis, and multiple regression analysis--to quarterly sales tax revenue data. It introduces these three methods and outlines the final models that were fitted to the data.

Introduction

Time series analysis is an approach which has gained increased use in forecasting since the late 1960s. "The basic premise underlying time series methods is that the best predictors of the future values of a data series are the past values of the series itself" (2, p. 104). Time series analysis can be divided into two categories--time domain and frequency domain. In the former case, the analysis is based on measuring and identifying the components of a series. These components include trend, cycle, seasonality, and random shocks. Examples of time domain techniques range from very simple methods such as trend lines and exponential smoothing to more sophisticated techniques such as multiple regression and the Box-Jenkins, or ARIMA, method. The acronymn stands for autoregressive integrated moving average and represents the family of models made popular by Box and Jenkins' book. These terms are defined in Chapter III. (The reader will note these two terms Box-Jenkins and ARIMA, are used interchangeably in this study).

The second category, frequency domain techniques, decomposes a time series into sine waves with varying frequencies; therefore, the component parts are related to frequency rather than time. Examples of this analysis include the periodogram method, Fourier analysis, spectral analysis, and cycle regression analysis (3).

These examples are only a sample of the numerous forecasting techniques available to the analyst today. With so many techniques available, the trend in forecasting is to look at these separate techniques not in terms of winning and losing, but in terms of how each differs from the other, and under what circumstances a certain technique is most appropriate (1). In the same manner, this study does not attempt to determine the best of these three forecasting methods in the universal sense but seeks only to identify the one that performs most accurately on one time series data set.

Description of Data

The time series data employed in this study are a set of 45 observations of quarterly sales tax revenues for the City of Denton, Texas from the final quarter of 1974 to the final quarter of 1985. The data are listed in Appendix A.

Limitations of the Study

This study evaluated only three forecasting techniques. Clearly, there are numerous other methods that would have been applicable to the quarterly time series data. The data set was not large and was, therefore, a limitation; furthermore, two methods only using 26 of the original 45 observations is a serious consideration.

The results of this study pertain only to the one data set employed, and are not generalizable.

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CHAPTER II

REVIEW OF THE LITERATURE

Introduction

In the first section of this chapter, two major forecasting studies are presented with emphasis on three separate issues: (1) general findings, (2) common recommendations, and (3) comments on the specific methods employed in this thesis. In the second section, relevant articles dealing with ARIMA models, multiple regression analysis, and cycle regression analysis are reviewed. The third section discusses the practice of combining forecasting techniques.

Background

Two of the most comprehensive forecasting studies to date are Mahmoud's "Accuracy in Forecasting: A Survey" and "Accuracy in Forecasting: An Empirical Investigation" by Makridakis, *et al*. The former study is a summary of empirical investigations pertaining to forecasting. Special emphasis is given to assessing the accuracy of different forecasting techniques (4). After reviewing approximately 120 sources, the author provides three general conclusions. First, he found that

> simple forecasting methods perform reasonably well in comparison to the sophisticated forecasting methods. Secondly, most of the studies indicate that quantitative methods are more accurate than qualitative methods. Thirdly, forecasting accuracy can be improved by combining techniques (4, p. 155).

The Makridakis study analyzes and reports the results of a forecasting competition in which expert participants analyzed and forecasted many real life time series. (It is usually referred to as the M-Competition.) The study is concerned mainly with the post-sample forecasting accuracy of the time series methods. Specifically, twenty-four extrapolation methods were tested on 1001 time series for 6-18 time horizons. (The Box-Jenkins, Lewandowski, and Parzen methodologies used a sample of 111 time series) (5).

The Makridakis study has been described as "the most exhaustive undertaken to date in terms of number and variety of extrapolation methods considered" (7, p. 273). Results of the analysis show that (1) Lewandowski's method is best for yearly and quarterly data, and both Holt and Holt-Winter's exponential smoothing methods are equivalent; (2) the Bayesian forecasting method and the Box-Jenkins method are about the same as single exponential smoothing; (3) on the macro level, the simple methods are superior to the sophisticated methods; (4) on the micro level, the sophisticated methods are the best; and (5) for seasonal data, exponential smoothing, deseasonalized regression, Bayesian forecasting and Parzen methods are almost identical (4).

One very clear finding, evident in both studies, is that there does not exist one forecasting technique that is "best" or "right" for every situation. In the commentary on the M-Competition, one author listed that conclusion as one of the three major contributions of the study (7). The authors of the M-Competition urge the forecasting user to discriminate in his/her choice of methods according to the type of data (yearly, quarterly, monthly), the type of series (macro, micro, etc.) and the time horizon (5).

The two studies' recommendations are based on this idea of no "best" technique. One should try "rather to understand how various forecasting approaches and methods differ from each other and how information can be provided so that forecasting users can be able to make rational choices for their situation" (5, p. 112). Mahmoud echoes this conclusion by stating, "What is needed is an understanding of when and under what circumstances one method is to be preferred over the other" (4, p. 154). The forecasting trend of the future, as recommended by Armstrong and Lusk, is that "specific hypotheses should be formulated and tested [as opposed to exploratory research] to determine which methods will be most effective in given situations" (2, p. 261).

Of the three methods being evaluated in this paper--the Box-Jenkins method, cycle regression analysis, and multiple regression analysis--the Box-Jenkins method received the most attention in both the Mahmoud and Makridakis studies. Simple linear regression trend fitting is evaluated in both studies and will be presented as the method most similar to multiple regression analysis. Cycle regression analysis *per se* is not evaluated in either study.

As mentioned in Chapter I, the Box-Jenkins technique is considered to be a sophisticated forecasting method and, as such, its results are quite often compared to those of the simple forecasting techniques in terms of accuracy. Mahmoud concludes that simple forecasting methods perform more accurately than, or at least as accurately as, sophisticated methods. He cites three leading articles (one of which is the M-Competition) that confirm this conclusion. In his own study of fourteen time series, the Box-Jenkins technique outperformed simple methods in only one series (4).

Following are brief summaries of the findings of studies reviewed by Mahmoud that employed the Box-Jenkins technique:

1. The Box-Jenkins method and other quantitative methods provided better forecasting than qualitative ones in the majority of the studies cited.

2. Six studies reported that the Box-Jenkins method was superior to regression analysis. One study indicated both methods performed about the same.

3. The Box-Jenkins analysis was found to be 20 percent more accurate than those using heuristics.

4. The Box-Jenkins method was found to be more robust than econometric models in three studies; two studies showed that the Box-Jenkins method did at least as well as large econometric models.

5. Comparing Box-Jenkins and exponential smoothing methods, one study showed Box-Jenkins superior, one study showed exponential smoothing superior, and one study showed them performing equally.

6. In a comprehensive study of 111 time series, (one by Makridakis that preceeded the M-Competition), results showed Box-Jenkins models were outperformed by the simple naive, moving average, and exponential smoothing methods (4).

The M-Competition evoked the following reactions and observations regarding the Box-Jenkins method:

1. The participants stated that the Box-Jenkins methodology was the most time-comsuming (5).

2. Newbold questions how an "ARIMA model would be seriously outperformed by an exponential smoothing procedure which is really based on a specific ARIMA model, which might have been chosen. . . ignoring the presence of outliers can lead to the choice of the 'wrong' ARIMA structure" (9, p. 278).

3. Pack, a Box-Jenkins forecaster, defends time and human interference in the ARIMA methodology. He believes there were serious questions regarding the validity of the experimental design of the M-Competition. These include a single forecast time origin, averaging over different time series, averaging over lead times, and focusing on a sequence of highly positively correlated lead times (10).

4. The Box-Jenkins analyst of the M-Competition argues that, if the geometric mean squared one-step-ahead error is applied to the quarterly and monthly series of the M-Competition, results show, for short term forecasts, "The more complex techniques, in particular Box-Jenkins, are a little better; whereas for longer term forecasts it does not really matter which method is used" (1, p. 287).

Makridakis, for whom the M-Competition is named, points out that "sophisticated methods, such as Box-Jenkins . . . do their best on forecasts approximately three to four periods ahead" (5, p. 297). He says, however, they were designed to minimize the mean squared error on one-period ahead forecasts. He theorizes that this discrepancy can be attributed to the sophisticated methods "correctly identifying the overall trend, but [not following] period-to-period fluctuations because of the high amount of randomness involved" (5, p. 297).

Regression Methods

Since the relatively new cycle regression algorithm was not incorporated in either Mahmoud's survey or the

M-Competition, findings concerning the regression method must be addressed. In Mahmoud's survey, regression analysis was compared to moving average and exponential smoothing in four different studies from 1965-1972. Results showed that regression analysis was the most accurate technique for longer-term forecasts; however, it was consistently outperformed by the other two techniques in short-term forecasts. In 1981, a study by Dalrymple and King comparing these three techniques indicated that the regression method was the most accurate used in the study (4). Linear regression analysis, however, did not fare as well in the M-Competition. In fact, it was considered one of the worst overall forecasting tools (5). However, deseasonalized regression analysis used on seasonal data had approximately the same overall mean absolute percentage error as exponential smoothing, the Bayesian method, and the Parzen method (4). A summary statistic of the M-Competition estimated the percentage of time that the Box-Jenkins method is better than the regression method. For the first four forecasting horizon periods, those percentages are 64.0, 68.5, 72.1, and 65.8 respectively (5). (It should be noted that this is simple linear regression analysis, not multiple regression analysis.)

Other Studies and Methods

This section briefly covers additional literature on the Box-Jenkins method and regression models. It also introduces the development of cycle regression analysis.

The Box-Jenkins technique has become quite popular since the publication of their book, Time Series Analysis, Forecasting and Control, in 1970. Throughout the 1970s, this technique was compared to other forecasting methods. Some researchers believe the popularity of the Box-Jenkins method was based on the fact that several studies found it to be at least as accurate as the complex econometric models of the U. S. economy (5). One such study, for example, concluded "the Box-Jenkins results were significantly better in all cases and, except for GNP, they provide better forecasts by a factor of almost two to one" (8, p. 153). Numerous other studies reached similar conclusions; however, some researchers concluded that it was the inability of econometric models to accommodate structural change in the economy that truly gave the ARIMA models the edge (6). In at least one study, the outcome was reversed between these two methods, and the ARIMA model was the worst performing of the two compared (6). Other studies comparing Box-Jenkins with methods such as exponential smoothing, moving averages, stepwise autoregression, etc. are equally as contradictory; yet, until Makridakis & Hibron (1979), Makridakis et al (1982), and Mahmoud (1982), the Box-Jenkins method was generally believed to be at or near the top of the forecasting techniques. As seen earlier in this chapter, these more recent studies have reversed this thought; however, these particular studies are not completely accepted. There continues to be questions regarding the ability of the particular analyst in a specific study to fit precisely the proper Box-Jenkins model, so a fair comparison can be made.

The majority of the literature in the last decade dealing with regression forecasting techniques has centered around econometric models. These structural models are usually quite complex and use numerous equations containing mainly macroeconomic variables (3). Multiple regression analysis then is a simplified version of these larger models.

In 1972, Cooper compared regression forecasts and autoregressive forecasts of thirty-three dependent variables from seven macroeconomic models. Results showed the autoregressive models outperformed the regression models in eighteen of thirty-three cases (3). Similar conclusions were reached by Cooper and Nelson when they compared the Federal Reserve Bank of St. Louis' regression method and an autoregressive model on forecasts of six variables. One year later, Behravesh found forecasts obtained from regressive equations to be superior to those generated by an autoregressive scheme (3). Another study, titled "The Forecasting Record of the 1970's," compared econometric models only to other econometric models. As the M-Competition and Mahmoud's survey showed, no one forecasting technique was the most accurate for the seventeen variables reviewed. Each technique had strengths and weaknesses (3).

As noted with the somewhat contradictory studies of Box-Jenkins methodology, not all regression models perform equally well. In fact, just as the Box-Jenkins analyst of the 1980s attributes the poor performance of the ARIMA method to the misspecification of the model, the econometricians of the 1970s felt that their methods were also being applied inappropriately and therefore were not actually outperformed by the ARIMA models (5; 10).

Finally, a study incorporating both an ARIMA model and regression analysis must be considered. These two methods were applied to monthly sales tax revenues, and forecasts were made for five months. Multiple regression analysis was utilized with a trend variable and dummy variables for seasonality. With few exceptions, this is the regression model used in this thesis. Final results showed that the ARIMA model and the regression method had average absolute percentage errors of 3.2 percent and 3.6 percent, respectively. The author concluded that "both techniques capture the directional changes. . . but neither method anticipated the sharp drop in receipts" in the final forecasting period (3, p. 387). The study pointed out that the higher cost of constructing and maintaining the ARIMA model could make the regression model the more attractive of the two methods, even though the ARIMA model slightly outperformed it (3).

Cycle Regression Analysis

The final method used in this study is the frequency domain technique called cycle regression analysis. It was first introduced by Simmons and Williams in 1982. The family of frequency domain techniques includes Fourier analysis, spectral analysis, the periodogram, and the Cosinor test. Until recently, these techniques have been used extensively and successfully in biological and physical sciences as well as in engineering. Now they are being adapted to the business world (15).

The other methods have limitations which cycle regression attempts to overcome. For example, unlike the periodogram, cycle regression analysis is not restricted to equally spaced observations. The Cosinor test and the periodogram method require a priori knowledge of the sinusoidal periods. The cycle regression method does not require such knowledge; therefore, it is more objective than the other two methods. The Fourier method also requires a priori knowledge of "either the number of trigonometric terms to be included or the fundamental frequency" (12, p. 46). Again in cycle regression analysis, these are done objectively by the heuristic steps of the method (13).

In their 1982 introductory article, Simmons and Williams found cycle regression analysis superior to the periodogram method in estimating amplitudes, angular frequencies, and phases. The algorithm was also successful in forecasting Texas Instrument common stock prices in the short-term future (15). A second study, by the same authors, compared cycle regression analysis with stepwise multiple regression analysis for three separate dependent variables: political regime, civil conflict, and energy consumption. In every case, cycle regression analysis had a higher percentage variation explained than did multiple regression analysis (16).

Although the cycle regression algorithm used in this study is objective and automatic (computer generated with no human interference), this is not true of the algorithm used in the two studies previously discussed. The one subjective step in the earlier algorithm involved analyzing the autocorrelations of the residuals to find an initial estimate for the cycle period. Specifically, the analyst was required to determine significant valleys and subsequent peaks in the autocorrelations. Designation of significance was left to the analyst's judgment (14). Since 1983, three methods have been presented to eliminate this subjective step. Two of the methods will be discussed here; the second method is the one employed in this study.

Simmons et al first developed an objective method that employed a t-like statistic to determine when "the autocorrelations have dropped to a significant negative value, and increased to a significant positive value" (14, p. 99-100). The method was based upon a procedure involving sample autocorrelations from the Box-Jenkins methodology. Validation tests of this procedure show that, in almost all cases, "estimates obtained by the t-value method were closer to the true periods than those obtained by the original procedure" (14, p. 100). Their second method incorporates spectral analysis in estimating the cycle period. Validation tests on the technique currently are being conducted; preliminary findings show that the spectral analysis step enhances the overall analytical capabilities of cycle regression methodology (13).

Combining Forecasts

In Mahmoud's survey of forecasting techniques, an entire section is dedicated to reviewing the literature on combining techniques. Without exception, every study he reviewed reported some type of improvement when a composite forecast was used (4). Although there are a number of methods of combining techniques, this study will limit itself to taking a simple average of forecasts. The M-Competition chose only to look at two methods of combining forecasts--the simple and the weighted average methods. The author called the simple average "Combining A," and it consisted of an average of the following six methods: exponential smoothing, adaptive response rate exponential smoothing, Holt's exponential smoothing, Brown's linear exponential smoothing (all of which were deasonalized), Holt-Winters linear and seasonal exponential smoothing, and the Automatic Carbone-Longini. "Combining A" was a weighted average based on the sample covariance matrix of percentage errors of the same six methods (5). Both "Combining A" and "Combining B" performed better than virtually all of the individual methods, including the six methods used in the combination, as well as the other methods in the M-Competition; futhermore, the simple average technique performed better than the weighted average method (5).

A later study by Makridakis and Winkler, however, showed differential weighting outperforming a simple average method (4). The most recent study by Makridakis and Winkler evaluated only simple average combinations. Fourteen forecasting methods and 111 time series were used. The following three conclusions were reached: (1) The specific methods included in a combination had little influence on the accuracy of the forecasts; (2) although accuracy increased with each additional method used, a saturation point was reached after four or five methods; (3) as the number of methods used in a particular combination increased, the variability of the accuracy of the different combinations decreased (4).

In conclusion, Reeves and Lawrence point out that interactive computer forecasting packages allow analysts to produce multiple forecasts with respect to a single objective. Generally, "The single forecast which comes closest to satisfying the chosen objective is then selected, and the remaining forecasts are discarded" (11, p. 271). They believe this process may not make the best use of all the available information because discarded information may contain information not available in the selected forecast. This is possible because the discarded information may be based on "different assumptions, different variables, or different relationships among variables" (11, p. 271).

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CHAPTER III

METHODOLOGY

Introduction

This chapter outlines the time series methodology employed in this study. Specifically, it discusses (1) the research type, (2) the research models and (3) the application of the models.

Research Type

In this paper, time series analysis is tested for its forecasting capabilities. Time series analysis can be divided into two categories, time domain and frequency domain, as defined in Chapter I. Of the three types of time series analysis tested, two--Box-Jenkins and multiple regression--are time domain; the third--cycle regression--is a frequency domain technique. The two time domain techniques can also be subcategorized as follows: Box-Jenkins is an extrapolation method which predicts future values based on relevant statistical features of past data (1). Multiple regression is considered a causal model.

Research Models

Three distinct models were used in this study. In statistical forecasting, a model is defined as an "algebraic statement telling how one thing is statistically related to one or more other things" (5, p. 5). The first model tested in this study is the Box-Jenkins or ARIMA model. Basically, Box-Jenkins is an algebraic statement showing how "observations on the same variable are statistically related to past observations on the same variable" (6, p. 5). It is important to note that an ARIMA model is really a family of models from which the researcher chooses the most appropriate specification.

Box-Jenkins analysis is based on the idea that the time-sequenced observations in a data series may be statistically dependent. An ARIMA model can include both an autoregressive term (AR) and a moving average (MA) term. An AR term is expressed as a function of past values of itself at varying time lags. The MA term is a function of previous error values. In addition, differencing enters the period to period change in the series. (It can be expressed algebraically as: $X_t = X_t - X_{t-1}$.) The I in ARIMA stands for integrated, which is the process that converts the differenced series back to the original forms (5). Algebraically, the AR and MA terms are expressed:

AR (1): $z_t = C + \emptyset_1 z_t - 1 + a_t$

MA (1): $z_t = C - \emptyset_1 a_t - 1 + a_t$

where z_t is the variable whose time structure is described by the process; C is a constant term related to the mean of the process; $\emptyset_1 z_{t-1}$ is the AR term and is a fixed coefficient, $\emptyset_1$, multiplied by a past z term; $\emptyset_1 a_{t-1}$ is the MA term and $\emptyset_1$ is a fixed coefficient multiplied by a past random shock, a_{t-1}; and a_t is a current random shock (6).

The ARIMA model is further described as an ARIMA (p,d,q)(P,D,Q)s where p is the number of lagged autoregressive terms, d is the order of differencing used to make the series stationary, and q is the number of lagged errors, or shocks, in the moving average component. The capital letters (P, D, Q) represent the seasonal component in the model. The s is the periodicity of the process. The actual development of the model involves three stages (5):

1. Identification. One of the keys to this stage is transforming the time series so it is stationary. Transformation options include logarithms, raising to powers, and centering. An additional transformation for eliminating trend or drift is differencing. Plots of the estimated autocorrelation functions (acf) and partial autocorrelation functions (pacf) are examined to give the researcher a feel for any patterns in the data. An autocorrelation function is a graphic representation of autocorrelation coefficients. These coefficients "measure the

direction and strength of the statistical relationship between ordered pairs of observations on two variables" (6, p. 35). The partial autocorrelation function is similar to the acf, but instead of representing the relationships only between ordered pairs, the pacf takes into account the effects of intervening values (6). The acf and pacf are then used as guides for choosing an appropriate ARIMA (p,d,q)(P,Q,D)s model.

2. Estimation. Estimates of the coefficients Ø and θ are chosen during this stage. An iterative nonlinear estimation process is used. Several computer programs perform this estimation.

3. Diagnostic Checking. This stage primarily checks that the random shocks (a_t) are independent. Since one cannot actually observe shocks, the residuals ($\hat{a}_t$) are observed. More precisely, a residual acf is constructed, and t-values and the summary Box-Pierce chi-square statistic are calculated (6). The Box-Pierce statistic is used to "determine whether several autocorrelation coefficients are significantly different than zero" (4, p. 269). It is based on the chi-squared distribution of the autocorrelation coefficients. If the autocorrelation coefficients are not significantly different from zero, the data generating those autocorrelations are considered random (4). If these diagnostic checks do not prove satisfactory, the iterative process, common in ARIMA modeling, is begun again.

The second method used in this study is a cycle regression algorithm involving spectral analysis. The algorithm estimates the sinusoidal or harmonic components in times series data. This is done by employing Marquardt's compromise to estimate amplitude, period, and phase simultaneously (8). Marquardt's compromise is a nonliner regression procedure that accurately and quickly converges to least squares estimates (6).

Spectral analysis is used in this algorithm to determine the initial period of the harmonic. Technically, the method is used to estimate the angular frequency of the dominant harmonic which is in turn used to estimate the period. The spectral analysis method estimates a spectrum, or the spectral density function, of the time series being analyzed. Its primary objective is to determine the "most significant components in terms of their contribution to the total variance in the data" (7, p. 23). A spectrum decomposes this total variance into that explained by components of various frequencies. In this way, spectrums can be used to detect sinusoidal components in the data (7).

The following steps outline the general algorithm for cycle regression. They include a "starting procedure that fits trend and one harmonic, steps for adding additional harmonics, and a stopping procedure" (8, p. 31).

Step 1: The starting procedure is to estimate the trend and first harmonic. Linear regression is used to obtain the initial estimates of the intercept and slope parameters. Autocorrelations are computed from residuals, and the spectral density function is computed from these autocorrelations (8). The spectral density function is estimated at different frequencies with the frequency corresponding to the highest peak, (i.e., highest spectral density function value) singled out. This frequency is multiplied by 2π to obtain the initial estimate of the angular frequency of the dominant harmonic (7; 8).

The initial estimate is used by cycle regression analysis to determine the estimates of amplitude, phase, and period of the harmonic (7). It is at this point that Marquardt's Compromise is used to move from the initial estimates described above to the final estimate for the trend and the first harmonic. The appropriate equation is as follows:

$$X_t = B_o + B_1 t \quad B_2 \text{ Sin } B_3 \ (t + B_4) + e_t$$

where:

$B_o + B_1 t$ represents the linear trend;

the parameters B_2, B_3 and B_4 determine the amplitude, period, and phase respectively of the first harmonic;

e_t is a random variable (7).

The stopping procedure involves a partial F-test to establish significance of the harmonic. If it is not found significant, Step 1 is repeated. If it is found to be significant, Step 2 is followed.

Step 2: Parameters for k harmonics when k = 1 are estimated in this step. Estimates obtained for trend in Step 1 are used again. For the initial estimate of B_{3i} (angular frequency), the estimate from B_3 is used for all harmonics until the kth harmonic where the spectral density function procedure is used to obtain the initial estimate. For the initial estimate of B_{3+1} (phase), the estimate of B_4 in Step 1 is used for all harmonics except the kth harmonic. The model uses 0.1 for that harmonic. For the initial estimate of B_{3i-1} (amplitude), the amplitudes found in Step 1 are summed and divided by k. The final estimates for trend and k harmonics are found through Marquardt's compromise using the above estimates in the following equation:

$$X_t = B_0 + B_1 t + \sum_{i=1}^{K} B_{3i-1} \operatorname{Sin} B_{3i} (t + B_{3i+1}) + e_t$$

The stopping procedure again applies a partial F-test to determine the significance of the kth harmonic. If significance is established, k is increased by 1, and Step 2 is repeated. If it is not significant, the model obtained from the equation above is used (8; 7).

The third method employed in this study is multiple regression analysis. It is merely an extension of simple linear regression, and attempts to explain changes in the dependent variable (Y) in terms of changes in two or more independent variables (X_1 X_2 . . . X_k). In this study, as in many regression studies, dummy variables are used to account for seasonal variation in the data. In general terms, the multiple regression model is written:

$$Y_t = a + b_1 X_{1t} + bX_{2t} + . . . b_k + X_{kt} + e_t$$

where Y represents the dependent variable whose values are explained by the model. Parameters a and b are estimated by the least squares principle; a is considered the intercept and b is described as a partial slope since the regression equation no longer fits a simple straight line, and e denotes the error term (3). The error term is a very important factor when applying multiple regression analysis to time series data. As in the ARIMA model, the researcher must be aware of the possibility of autocorrelated error terms. Examining the autocorrelations of the residuals visually as well as calculating the Durbin-Watson statistic are two means of testing for autocorrelation. The Durbin-Watson statistic tests for the most important type of autocorrelation--first-order linear correlation. The statistic is defined as the "ratio of the sum of squares of successive differences of residuals to the sum of the squared residuals" (2, p. 161). A value of approximately 2 indicates the absence of first order autocorrelation (2).

Application

The time series data analyzed in this study are quarterly sales tax revenues for the City of Denton, Texas from the final quarter of 1974 to the final quarter of 1985. This constitutes 45 observations; however, the ARIMA model eventually fit the last 26 observations only. The pattern of the data appeared to change around the 20th observation; consequently, after numerous attempts at fitting an ARIMA model to the entire set proved unsuccessful, the decision was made to model the most recent 26 observations. The same decision was made with regard to the cycle regression model. The multiple regression model, however, used the original data in its entirety.

The final ARIMA model, using (p, d, q) (P, D, Q)s notation, is (0,1,0) (1,1,1)4. To achieve a more stationary time series, a natural logarithmic transformation was performed. The nonseasonal component fits one autoregressive term and one moving average term, both at lag four. Seasonal differencing completes the final model.

The ARIMA model was chosen using two statistical packages--IDA and Minitab. Minitab was only used in calculating initial estimates for the parameters needed in the model. IDA was used primarily in a trial and error fashion with the acf and pacf plots, histograms, and the Box-Pierce statistic serving as the major sources of feedback.

The cycle regression method was applied through a computer program obtained from Dr. LeRoy Simmons of the Business Computer Information Systems Department of North

Texas State University. All parameters needed for the final forecasting model were computed automatically by Dr. Simmons' program. A short program that can be run on an IBM personal computer was written to solve the final forecasting equation. The final cycle regression model fit three significant harmonics to the tax revenue data.

In the multiple regression model, the quarterly tax revenues served as the dependent variable, and six independent variables were assigned to the equation. X_1 was a dummy variable that basically divided the data set into two sections--before and after the 20th observation; X_2 was time (1 . . . 45) which accounted for trend; X_3 was time squared which accounted for the nonlinearity of the time series; X_4 through X_6 were dummy variables representing the seasonal factor of the quarterly data. The statistical package of SPSSx was used for this regression analysis.

Chapter IV presents the final equations used in these three models. The point forecasts, generated from these equations, are discussed, and interpretations and conclusions are given.

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CHAPTER IV

FINDINGS AND CONCLUSIONS

Introduction

In the first section of Chapter IV, final equations, diagnostic statistics, forecasts, and interpretations are presented separately for each model. The second section compares the forecasts of the three models in terms of absolute percentage error and mean absolute percentage error; two summary tables are provided. The third section discusses overall conclusions regarding individual models as well as the study itself.

The Box-Jenkins Method

Numerous variations from the ARIMA family models were fitted to the tax data, both in its original state and after a natural logrithmic transformation had been performed. The most satisfactory model was found using the transformed data. As discussed in Chapter II, only the final 26 observations from the original 45 quarters were used in the analysis.

The multiplicative model fitted to the transformed time series is (0,1,0)(1,1,1)4. It is expressed:

$$\phi(B^4)\nabla^d\nabla_4^D\tilde{z}_t = \theta_Q(B^4)a_t$$

where

$\phi(B^4)$ is the seasonal autoregression term at lag 4,

$\nabla^d\nabla_4^D$ is nonseasonal differencing at length 1 multiplied by seasonal differencing at length 4

z_t is the variable whose time structure is described by the process

$\theta_Q(B^4)$ is the seasonal moving average term at lag 4

a_t represents random shocks

(Note: B represents backshift notation where $B^4z_t = z_{t-4}$) (8). The following parameter estimates are used in the equation:

seasonal autoregressive coefficient (ϕ) = -.9
seasonal moving average coefficient (θ) = .65

Both coefficients are significant at the .05 level with t-values of 86.46 and 6.05, respectively. The Box-Pierce statistic is 17.96, which is below the critical value of 30.1 for 19 degrees of freedom (8). This statistic, as discussed in Chapter III, is a test for correlation among the random shocks of a time series and is based on the residual autocorrelations.

Forecasts of sales tax revenues for two time horizons (the first two quarters of 1986) were calculated from this model. Table I shows the forecasts, the actual revenues, and the absolute percentage errors.

Table I

BOX-JENKINS METHOD

Time Horizon	Actual	Forecast	Absolute Percentage Error
1	$1,270,681	$1,420,057	11.76
2	$1,119,925	$1,295,177	15.65

The Box-Jenkins model projects revenues of $1,420,057 for the first quarter of 1985. This is approximately $150,000 over the actual revenues generated for that quarter. The second quarter projection is approximately $175,250 over the actual amount generated. The absolute percentage errors (APE) of 11.76 and 15.63 were the largest of the three models used in the study. These two errors produce a mean absolute percentage error (MAPE) for the combined time horizons of 13.71. The M-Competition reported the MAPE for the Box-Jenkins method, applied to quarterly data for the time horizons 1 and 2, as 7.6 and 8.2, respectively (4).

There are several possible explanations for these large errors. The most obvious is rather simple and straightforward--the model was not specified correctly. The Box-Jenkins methodology is complex, and most analysts believe time and experience are required to truly fit the proper models consistently. In fact, C.W.G. Granger noted that "it has been said to be so difficult that it should never be tried for the first time" (6, p. 112). In addition, the amount of subjectivity in fitting the model allows for a wide range of possible solutions; consequently, the risk of not specifying the "right" process increases. In the M-Competition commentary, Geurts states "that it is probable for two experienced Box-Jenkins forecasters to examine identical autocorrelations, partial autocorrelations. . . and then to specify different models" (2, p. 268). Another possibility is that the overall trend is being detected, since the direction of both forecasts are correct, but the process is not following the "period-to-period fluctuations because of the high amount of randomness" (5, p. 297).

Cycle Regression Analysis

The most satisfactory cycle regression model, in terms of forecasting, used only the final 26 observations of the time series. This abbreviated set of data limited the search for significant harmonics. Instead of proceeding until an insignificant one was identified, the algorithm stopped after three significant harmonics were identified. This is due to a limit imposed by the program. In an attempt to prevent overfitting of the data, half the number of observations is the maximum number of parameters that can be used in this program; therefore, with 26 observations, the program will fit a maximum of 13 parameters. Since each sine wave has three parameters, (amplitude, period, and phase) and trend accounts for two more parameters (constant and slope), a fourth harmonic would have fit a total of 14 parameters--one more than the maximum allowed by the program.

As outlined in Chapter II, the general equation for cycle regression analysis is

$$x_t = B_0 + B_{1t} + B_2 \text{Sin } B_3(t + B_4) + e_t$$

where

B_0 and B_{1t} are the intercept and slope parameters, respectively

B_2, B_3, B_4 are the parameters for amplitude, period, and phase respectively

e_t is a random variable (9).

The following equation includes the final parameters generated by the cycle regression algorithm:

$$\begin{aligned} Y = 380069.4 + 30325.12t &+ 67998.81\sin(.2923369(t+2.024837)) \\ &+ 79239.75\sin(3.128417(t+.1025962)) \\ &- 58777.58\sin(1.546734(t+.6198372)) \end{aligned}$$

$t = 27, 28$

Table II presents the forecasts, the actual revenues, and the percentage errors for the cycle regression method.

TABLE II

CYCLE REGRESSION ANALYSIS

Time Horizon	Actual	Forecast	Absolute Percentage Error
1	$1,270,681	$1,359,366	6.98
2	$1,119,925	$1,203,288	7.44

The forecast generated from cycle regression analysis overestimates revenues for two time horizons by $88,685 and $83,363, respectively. This method provided the smallest MAPE (7.21) of the three methods used in the study. Since this method requires no human intervention, subjectivity is not an issue. There are, however, certain advantages and disadvantages that can be addressed. The method's objectiveness allows for extremely quick and easy forecasts. The only time consuming step is solving the final equation, and this step was shortened considerably by a basic program run on a personal computer. One disadvantage of the method is its relatively newness to the family of frequency domain techniques. It has not been tested to the extent of the other two methods. In addition, the program for cycle regression analysis cannot, at present, be purchased in a large statistical package as can the other two methods.

Multiple Regression

All 45 observations of the time series data are included in the multiple regression model. Tax revenue serves as the dependent variable, and it is regressed on six independent variables. Of the six independent variables used, four are dummy coded. (Descriptions of the variables are provided at the end of Chapter III.) The following equation represents a multiple regression model with six independent variables.

$$Y_t = a + b_1X_{1t} + b_{2t}X_2 . \; . \; . + B_6X_{6t} + e_t$$

To forecast from this equation for two time horizons, t is given the values of 46 and 47, and the coefficients for a, b, and X_1 through X_6 are inserted:

$$Y = 242581.38 + 9387.24 + 1772.17(46) + 432.96(46^2) + 60770.90$$

$$Y = 242581.38 + 9387.24 + 1772.17(47) + 432.96(47^2) - 36787.63$$

(Note: In quarter 46, X_4 and X_6 have values of zero; in quarter 47, X_4 and X_5 have values of zero)

The value of 2.03 for the Durbin-Watson statistic is significant at the .05 level. To accept the null hypothesis, i.e., to conclude that first order autocorrelation is not present, the following equation must be satisfied:

$$d_u < d < 4 - d_u$$

where d_u, for n = 45 and k = 7, = 1.24

$1.24 < 2.03 < 2.76$ (7;3).

Table III provides the summary values of actual revenues, forecasts, and percentage errors for the multiple regression model.

TABLE III

MULTIPLE REGRESSION ANALYSIS

Time Horizon	Actual	Forecast	Absolute Percentage Error
1	$1,270,681	$1,310,403	3.13
2	$1,119,925	$1,254,882	12.05

Multiple regression analysis provides the single best forecast estimate in this comparative study. The forecast overestimated the revenues for the first time horizon by only $39,722. This is less than half of the error associated with the next best model--cycle regression analysis. This estimate, of course, provides the smallest APE (3.13) of the study. The model, however, did not prove consistent in the second time horizon; revenues are overestimated by almost $315,000 in that case, and the APE quadrupled to 12.05. The MAPE of 7.59 drops multiple regression analysis to second in the overall rankings.

As the only causal model used in the study, multiple regression analysis requires human intervention to determine the number and make-up of the independent variables. X_1 in this study was chosen to divide the data into two time periods--before and after the building of a large retail mall in the City of Denton. It is believed this event could have contributed to a pattern change seen in the data around the 20th observation. This independent variable, however, is not significant. In fact, in addition to the constant that proved to be significant, there are only two other independent variables that are significant--X_5 which represents the first quarter of the year and accounts for seasonality and X_3 which represents time squared and accounts for nonlinearity in the data.

Even though the Durbin-Watson statistic shows this model to be free of first order autocorrelation, a plot of the residuals still contains a wave-like characteristic that is indicative of autocorrelation. Differencing the series is one method of reducing autocorrelation; however, first order differencing would only address first order autocorrelation, which has already been ruled out. The differencing, therefore, would need to be performed at varying lengths to try to reduce or eliminate the auto-correlation that is not of the first order. A model with additional dummy coded variables that further explains the cyclical component of this data set might produce improved results. Finally, a model that includes independent variables that reflect the changing nature of the local economy might also generate improved forecasts.

Summary Tables

Table IV is a composite of the three previous tables presented in this chapter.

TABLE IV

SUMMARY

Methodology	Actual	Forecast	Absolute Percentage Error
<u>Time Horizon 1</u>			
Box-Jenkins	$1,270,681	$1,420,057	11.76
Cycle regression		$1,359,366	6.98
Multiple regression		$1,310,403	3.13
<u>Time horizon 2</u>			
Box-Jenkins	$1,119,925	$1,295,177	15.65
Cycle regression		$1,203,288	7.44
Multiple regression		$1,254,882	12.05

Table V presents the mean absolute percentage error (MAPE) for each method for two combined time horizons.

TABLE V

MAPE--TWO TIME HORIZONS

Methodology	MAPE
Box-Jenkins	13.71
Cycle regression	7.21
Multiple regression	7.59

Conclusions

Armstrong and Rusk, in their opening commentary of the M-Competition, suggest that "determining which methods will be most effective in given situations," as opposed to exploratory research, should be the pursuit of further research (1, p. 261). This comparative study attempts to follow their suggestion. The purpose of the study is to find the most accurate of the three forecasting techniques, in terms of APE and MAPE, for one set of time series data.

The three methods employed in this study are dissimilar in many ways--two are time domain techniques and one is a frequency domain technique. The Box-Jenkins method is a very complex and subjective one; cycle regression analysis, on the other hand, is a quick, automatic, and objective method; and multiple regression analysis is a causal model.

Yet each of these methods is a time series technique and, as such, must address the possibility of autocorrelation among the residuals. The various diagnostic checks were reported throughout the study, but a warning still needs to be included regarding autocorrelation. If it is present, the variance of parameter estimates of a model are affected and the precision, or significance, of those estimates can be greatly overestimated. This does not, however, affect the "unbiased" status of the estimates (7).

In Chapter II, the benefits of combining forecasts were discussed; however, this procedure does not provide superior forecasts here because each model in this study overestimates future revenues.

In summary, the multiple regression analysis had the smallest APE for any method for either time horizon--3.13. Over both time horizons, however, cycle regression analysis was the most accurate. But, when considering which method should be used as the best predictor of quarterly sales tax revenues, a case by case analysis is still recommended. In this way, the analyst can decide how factors, such as the sophistication of the methodology, time required in analysis, subjectivity, experience, and human intervention, count in selecting a final forecasting model.

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APPENDIX A

QUARTERLY SALES TAX REVENUES
CITY OF DENTON

Year	Sales Tax Revenues
Quarter 4, 1974	238051
1975	237199
	281928
	243754
	293110
1976	281460
	250067
	335475
	281156
1977	363058
	263610
	290138
	366233
1978	399293
	370928
	399173
	460436
1979	403093
	425124
	471492
	516990
1980	552794
	517176
	629642
	585387
1981	709389
	584962
	608122
	658992
1982	739050
	627057
	744828
	704844

QUARTERLY SALES TAX REVENUES CITY OF DENTON--Continued

Year	Sales Tax Revenues
1983	837146
	755165
	808769
	900216
1984	1062377
	926680
	1012962
	1074098
1985	1304289
	1047116
	1162843
	1241014

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