THE DEVELOPMENT AND VALIDATION OF A COMPUTER-AIDED INSTRUCTIONAL PROGRAM IN MATHEMATICS FOR BUSINESS AND ECONOMICS MAJORS

DISSERTATION

Presented to the Graduate Council of the North Texas State University in Partial Fulfillment of the Requirements

For the Degree of

DOCTOR OF PHILOSOPHY

By

Kenneth B. McCool, M. S.

Denton, Texas

August, 1973

The problem with which this study is concerned is that of comparing the results of teaching community college students enrolled in a transferable mathematics sequence for business and economics majors by a computer-aided instructional program and by the traditional lecture method. In order to effectively resolve this problem, an A Programming Language System 360 (APL/360)-aided instructional program was developed and an experimental study was conducted.

The APL/360-aided instructional program consisted of three sets of materials: a manuscript on APL/360, a list of APL programs defining operators relevant to a computer-aided study of calculus, and a collection of problems based on these programs and calculus concepts. The subjects for the experiment were forty-four students enrolled in three sections of Mathematics 112 at Mountain View College of the Dallas County Community College District. The control group, students taught by the traditional lecture method, consisted of twenty-one students. The experimental group, students taught by the APL/360-aided instructional program, consisted of twenty-three students. The same instructor taught all students.
The essential difference in the two teaching methods was the use of the computer as a teaching-learning aid in the computer-aided instructional program. The computer was a course supplement to classroom instruction and aided students in obtaining insight into the nature of mathematical concepts as well as serving as a computational aid.

Composite scores on the California Short-Form Test of Mental Maturity, 1963 Revision (Level 5) were used to establish three levels of ability. Since three levels of ability and two teaching methods were used, the experiment conformed to the model for a three-by-two factorial experiment, and the data were analyzed by an unweighted-means-factorial analysis of covariance. Randomization of students involved in the experiment was not feasible.

All findings are based on calculated F-ratios. The level of significance is reported rather than specifying a particular level for accepting the hypotheses. For student achievement, as measured by the Cooperative Mathematics Tests for Calculus, and student attitude toward mathematics, as measured by the Purdue Scale to Measure Attitude toward Any School Subject, it was determined that there is no significant difference between the two teaching methods at any level of significance.

Since far fewer problems to be done by pencil and paper methods were usually assigned to the experimental group than
to the control group, it is concluded that the inclusion of materials on computers in mathematics courses for business and economics majors can help students understand mathematical concepts. On the basis of unsolicited comments received from the experimental and control groups, it is concluded that students taught by a computer-aided instructional program will be more outspoken in their attitude toward the course and that their comments will be generally favorable.

As a result of using the APL/360-aided instructional program, not only did students learn calculus concepts as well as by the traditional lecture method, but they also learned a programming language and used the computer interactively. The extent to which students developed independent learning ability as a result of using the computer was not measured. Since most business schools require knowledge of a programming language, it is recommended that a computer-aided approach to the teaching of mathematics courses for business and economics majors be continued.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>LIST OF TABLES</th>
<th>...............</th>
<th>v</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chapter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I. STATEMENT OF THE PROBLEM</td>
<td>...............</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Statement of the Problem</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Purposes of the Study</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Hypotheses</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Background and Significance of the Study</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Definition of Terms</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Limitations</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Basic Assumptions</td>
<td></td>
</tr>
<tr>
<td>II. SURVEY OF RELATED RESEARCH LITERATURE</td>
<td>...............</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>Overview of Changes in Mathematics</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Curricula and Methodology</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Computer-Managed Instruction and Computer-Assisted Instruction</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Using Computers Interactively</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Computers and Calculus</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Educational Uses of APL</td>
<td></td>
</tr>
<tr>
<td></td>
<td>The Computer and the Two-Year College</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Future Uses of Computers with Mathematics</td>
<td></td>
</tr>
<tr>
<td>III. EXPERIMENTAL DESIGN AND EXPERIMENTAL PROCEDURES</td>
<td>...............</td>
<td>39</td>
</tr>
<tr>
<td></td>
<td>The Setting of the Study</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Pilot Study</td>
<td></td>
</tr>
<tr>
<td></td>
<td>The Experimental Design</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Instruments</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Procedures for Collecting Data</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Procedures for Treating Data</td>
<td></td>
</tr>
<tr>
<td>Chapter</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>---------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>IV. THE RESULTS OF THE EXPERIMENTAL STUDY</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>The Findings for Student Achievement in Calculus</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The Findings for Student Attitude toward Mathematics</td>
<td></td>
<td></td>
</tr>
<tr>
<td>V. SUMMARY, FINDINGS AND CONCLUSIONS, AND RECOMMENDATIONS</td>
<td>67</td>
<td></td>
</tr>
<tr>
<td>Summary</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Findings and Conclusions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recommendations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>APPENDICES</td>
<td>76</td>
<td></td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>158</td>
<td></td>
</tr>
</tbody>
</table>
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I.</td>
<td>Analysis of Covariance for Student Achievement in Calculus</td>
<td>63</td>
</tr>
<tr>
<td>II.</td>
<td>Analysis of Covariance for Student Attitude toward Mathematics</td>
<td>65</td>
</tr>
</tbody>
</table>
CHAPTER I

STATEMENT OF THE PROBLEM

The past few years have seen the evolution of the computer. The computer is completely impersonal, and past experience would indicate that all of mankind in general and all educational disciplines in particular may reap the rewards of its constructive use.

While computers have been evolving, a trend has been taking place with students who enroll in mathematics courses. Unfortunately, this trend has been one toward dichotomizing these students. One group enjoys mathematics because of their appreciation for the beauty of the intricate, logical structure of the various mathematical systems. The other group enrolls in mathematics courses because of necessity, usually because of degree requirements. This trend might well be interpreted as one form of a "call for relevance" which echoes throughout society as a whole and education in particular. There is no need to justify a place for mathematics in society, for such a need is obvious. How much mathematics and how it is taught seem to be the questions.

The impact of the computer and of mathematics, primarily calculus, on the business world has been extensive. Computers and calculus are not separated in the business
world. Thus, it seems natural to integrate the computer with calculus in mathematics courses for students majoring in business and economics. Add to all this the development and potential of A Programming Language (APL), a formal, mathematically-oriented programming language, and the stage is set for a natural combination. Such a combination and its effects on business and economics students merit study.

Statement of the Problem

The problem of this study was to compare the results of teaching community college students enrolled in a transferable mathematics sequence for business and economics majors by a computer-aided instructional program and by the traditional lecture method.

Purposes of the Study

The purposes of this study were

(1) to develop an A Programming Language System 360 (APL/360)-aided instructional program for community college students enrolled in a transferable mathematics sequence for business and economics majors,

(2) to determine the effect of the above program by comparing student achievement when taught under this program and student achievement when taught by the traditional lecture method, and

(3) to determine the effect of the computer-aided program by comparing student attitude toward mathematics
when the computer-aided program and the traditional lecture method were employed as teaching techniques.

Hypotheses

To carry out the purposes of this study, the following hypotheses were formulated:

1. For student achievement, as measured by the Cooperative Mathematics Tests for Calculus, there will not be significant interaction between the teaching techniques used (APL/360-aided instructional program and the traditional lecture method) and levels (high, middle, and low) of student ability.

2. Business and economics students taught with the APL/360-aided instructional program (hereafter referred to as the experimental group) will score significantly higher on the Cooperative Mathematics Tests for Calculus than students taught by the traditional lecture method (hereafter referred to as the control group).

3. There will be a significant difference in achievement between the three levels of student ability, both teaching techniques being used, as measured by the Cooperative Mathematics Tests for Calculus.

4. For student attitude toward mathematics, as measured by the Purdue Scale to Measure Attitude toward Any School Subject, there will not be significant interaction between the teaching techniques and levels of student ability.
5. The experimental group will score significantly higher on the *Purdue Scale to Measure Attitude toward Any School Subject* for mathematics than the control group.

6. There will be no significant difference in attitude toward mathematics between the three levels of student ability, both teaching techniques being used, as measured by the *Purdue Scale to Measure Attitude toward Any School Subject*.

**Background and Significance of the Study**

The number of computers in operation in the United States is expected to grow from 35,000 in 1967 to 100,000 in 1975.¹ Hence, one of the major tasks of the 1970's will be continued effort on the part of educators to determine the role of the computer in the curriculum. Many studies have been conducted concerning the use of the computer, with a multitude of different types of subjects in a wide variety of areas. One example of such a study is *Computer Assisted Instruction for Vocational Rehabilitation of the Mentally Retarded*.²

---


Literature on computers quickly becomes dated for several reasons. One reason is that computer technology itself changes so rapidly. Also, "industry experience in using computing machinery and the experience of educators in teaching students how to use it continually uncover new aspects of both technique and pedagogy." Therefore, it is clear that "educators face demanding responsibilities of designing, maintaining, and evaluating curricular efforts."4

The two-year college will be a vast experimental ground for innovation during the immediate future. In regard to methods, it has been claimed that "questions of methods are more important in the two-year college than in the four-year college, or even in the secondary school."5 The computer fits well into this experimental framework, for non-data-processing faculty, if they are qualified, "should participate in computer education in their respective areas."6 "The computer is a tool that can fit into existing curricula and help convert routine courses into exciting experimental subjects."7

4Ibid.
5G. S. Young, "The Opportunities and Problems of the Two-Year College," The American Mathematical Monthly, LXXIX (April, 1972), 388.
6Brightman, op. cit., p. 9.
7Ibid., p. 11.
One of the most fruitful areas of integration of the computer with mathematics has been with calculus, but primarily only for science and mathematics majors. Many mathematicians are of the conviction that, since calculus is usually the first mathematics course which introduces students to limit concepts, calculus is where mathematics really begins. Many claims have been advanced for the effectiveness of the computer in aiding the student in learning calculus concepts, but until 1969 no studies were noted that verified such generous claims. Some studies have been conducted in this area, but far too many of the investigators appear to have been too liberal in generalizing results.

There were only a few programming languages in 1957; now there are approximately 200. Each language has its own advantages and disadvantages. Major disadvantages resulted in new languages to alleviate the various shortcomings. Minor disadvantages have caused many refinements in the original language, formally known as subsets and extensions. For example, Formula Translator (FORTRAN) has been extended to FORTRAN IV, and FORTRAN IV has also been extended. To claim that students learn calculus concepts better under computer-assisted, computer-aided, computer-extended, or computer-oriented programs is too general a statement. There are a multiplicity of computer languages, many grade levels of students, and many disciplines which lean heavily on
calculus. It is recognized that "there is at present no single CAI (Computer Assisted Instruction) system which is best for all occasions."³

One programming language which may well have a definite effect on mathematical education is A Programming Language.⁹ APL is best classified as a problem-solving language and is based on standard mathematical symbols and logical functions. APL/360 is International Business Machines' implementation (adaptation to a computer) of APL. APL/360 is a remote, time-sharing interactive system that reduces the time and effort in solving a wide variety of problems. APL/360's two major elements, or components, are the APL language and a remote terminal which permits many users the opportunity to conduct desired programs or computations at the same time. This computer system has been installed in the Dallas County Community College District, with numerous terminals available at each of its four campuses. It is anticipated that this system will be extensively integrated into the District, both in curriculum and operations.

It has been noted that the business world makes the most extensive use of computers. A great many courses in

³Max Jerman, "The Use of Computers to Individualize Instruction," The Mathematics Teacher, LXV (May, 1972), 467.

the business field will require extensive computer use by students during the immediate future. Mathematics will be failing to meet the needs of these students if they cannot acquire the necessary computer proficiency in their mathematics courses. Further, "revisions of mathematics courses to meet this threat are almost nonexistent."10 "Every student pursuing a business related program of study should have some exposure to the computer."11 As a result of the integration of the computer with calculus, it is anticipated that community college students enrolled in a transferable mathematics sequence for business and economics majors will be more enthusiastic about what is often considered an "I have to take it" mathematics sequence. Since this sequence involves a study and application of calculus concepts to their fields of study; and, since the computer has proved in a few limited studies to be effective when integrated with calculus; and, since APL is based on standard mathematical symbols, the results of this study should have strong implications for other school systems.

Definition of Terms

Computer-aided instructional program is defined as a teaching method in which students use a computer as a course

11 Brightman, op. cit., p. 9.
supplement to classroom instruction in constructing computer programs and performing calculations, through a knowledge of an appropriate programming language and a knowledge of necessary computer operations. "Computer-aided" is to be distinguished from "computer-assisted," for in the latter the student need not have a knowledge of a programming language and need only be capable of giving responses to material presented in a "programmed learning" format by the computer.

Traditional lecture method, as used in this study, is a teaching technique in which the teacher presents course material in organized units in a classroom environment, with no visual or instructional aids other than chalkboards, a textbook and/or supplementary course material to the text, and requires students to complete pencil and paper homework assignments related to this material. Although there are sufficient minor variations to justify more than one traditional lecture method, this study will consider traditional lecture method as singular and will be referred to as the traditional lecture method.

Mathematics 111-112 is a mathematics sequence at Mountain View College of the Dallas County Community College District consisting of two, three-semester-hour courses designed for students who will be majoring in business and economics. Mathematics 111 is commonly referred to as
precalculus and hence consists of those topics which are considered necessary to prepare students for a study of calculus. Mathematics 112 is a continuation of Mathematics 111, and a study of calculus and appropriate applications.

Limitations

This study was limited to students enrolled in Mathematics 112 at Mountain View College during the spring semester of the 1972-1973 school year. The reason for limiting the study to students enrolled in Mathematics 112 is because it is in this course that calculus is studied, and a study of calculus is usually considered to be the major content area of the Mathematics 111-112 sequence. Also, calculus has been shown by other studies to be fruitful for integration with the computer. With regard to the study being conducted at Mountain View College, there is no reason to suppose that students at other two-year institutions would differ significantly from those included in this study.

Basic Assumptions

1. It is assumed that the sections of Mathematics 112 used in this study were representative of sections of similar courses at other two-year institutions.

2. It is assumed that any interaction outside the mathematics classroom related to the learning of calculus
was balanced between the experimental and control groups and hence did not invalidate the study.

3. It is assumed that students involved in this study responded honestly and carefully to the instruments used to obtain data.
CHAPTER II

SURVEY OF RELATED RESEARCH LITERATURE

The purpose of this chapter is to present a summary of research literature related to this study. The development of an APL/360-aided instructional program in mathematics for business and economics majors represents a change both in mathematics curricula and methodology. Hence, this chapter begins with a brief overview of changes in mathematics curricula and methodology. Curriculum and method mean, essentially, what is taught and how it is taught, respectively. The changes discussed are limited to 1957 and thereafter. No attempt is made to identify and/or describe all changes in mathematics curricula and methodology, for this period is far too rich in effort, creativity, and implementation of innovations in mathematics education.

The first educational uses of the computer were primarily in the areas of CAI and computer-managed instruction (CMI). Thus, a description and/or discussion of the advantages and disadvantages of these uses of the computer are presented. As the potential of the educational use of the computer continued to be recognized, an interactive, algorithmic approach to the integration of the computer with mathematics
was begun; therefore, a description of this approach to
the study of calculus is presented.

APL is but one of many programming languages. However,
APL is mathematically oriented, and hence it has consider-
able potential in bringing about a more natural and
effective integration of the computer with mathematics.
Following a discussion of some of the educational uses of
APL, a presentation of the impact, need and uses of the
computer in the two-year college is given. Finally, a
discussion of the use of the computer in future mathematics
programs is included.

Overview of Changes in Mathematics
Curricula and Methodology

In the late 1950's there was a so-called revolution in
mathematics.¹ The Rockefeller Report on Education pointed
out that the USSR was not the "cause" of this revolution,
but rather the sudden movement into a new technological era.
The USSR served primarily as a "rude stimulus" to this
awareness. In meeting the challenge of the technological
revolution, mathematics curricula and methodology came
under close scrutiny.²

¹The Revolution in School Mathematics (Washington, 1961),
pp. 1-5.

²Ibid., p. 11.
In 1959, the Board of Directors of the National Council of Teachers of Mathematics (NCTM) authorized the establishment of the Committee on the Analysis of Experimental Mathematics Programs. Most of the innovations in mathematics were occurring in the secondary schools. For example, the Boston College Mathematics Institute began informally in 1953 and became a formal organization in 1957. One of its goals was the preparation of modern mathematics courses; also, these new courses were designed to increase and develop the professional competence of the teacher. The first course text developed by the Institute was *Sets, Operations, and Patterns: A Course in Basic Mathematics*; this text was originally intended for the ninth grade. The Committee, in its analysis, was quick to point out the wide range of topics that were included in a ninth grade level book. The structure of mathematics was developed rigorously and in the language of sets, as well as such other topics as inequalities and factoring. This represented a new approach to ninth grade mathematics and certainly challenged the thinking habits of students. In the area of methodology, there was a compromise between teacher-presentation and student-discovery methods. Thus, this single text indicated changes to come in mathematics curricula and methodology.³

The Committee also analyzed the School Mathematics Study Group (SMSG) and the University of Illinois Committee on School Mathematics (UICSM), both of which were concerned with the improvement of the teaching of mathematics in the schools. Among other curriculum-revision projects reviewed by the Committee were the following: Greater Cleveland Mathematics Program, The Syracuse University-Webster College Madison Project, University of Maryland Mathematics Project, The Ontario Mathematics Commission, and the Developmental Project in Secondary Mathematics at Southern Illinois University. The Commission on Mathematics of the College Entrance Examination Board also made some innovative proposals in the area of mathematics curricula and methodology.

Thus, traditionally in mathematics education, educators have been attacking the problem of teaching better mathematics and the problem of teaching mathematics better. And although there has been improvement in teaching better mathematics, teaching mathematics better has in no wise been effectively controlled.

In the late 50's numerous efforts were made to teach by means of movies or television. Next came teaching machines and programmed learning with the promise that these would make it possible for all students to learn, although perhaps at different rates. Team teaching once commanded, and to some extent still does command, considerable attention. The discovery method of teaching has been held

4Ibid., p. 8.
out as the answer to our problem of teaching mathematics and other subjects better. More recently, individually prescribed instruction has been proposed, as has been computer assisted instruction. We have been offered modular scheduling and flexible scheduling. Quite recently our attention has been called to mathematics laboratories.

However, the actual state of affairs is that for each of these panaceas either there is very little empirical evidence of any kind, or else there is a great deal of empirical evidence which demonstrates that the new way of teaching is no better, though often no worse than our old-fashioned ways.5

The above statements reflect great concern about the future of the teaching of mathematics. Two laws have been stated concerning mathematics education. One of these laws is that "the validity of an idea about mathematics education and the plausibility of that idea are uncorrelated." The other law is that "mathematics education is much more complicated than you expected even though you expected it to be more complicated than you expected." The suggestion has been made that new efforts should be made in mathematics education, keeping the above two laws in mind.6

Computer-Managed Instruction and Computer-Assisted Instruction

The idea of using computers in education seemed very simple at first glance. However, the use of the computer in

5A Conference on Responsibilities for School Mathematics in the 70's (Stanford, 1971), p. 27.
6Ibid., pp. 29-30.
education has become increasingly widespread and complex. The computer is no longer viewed strictly as a tool for the scientist, but rather an educational aid in many disciplines, whether this use be made through CAI or otherwise. No longer is it sufficient to state merely that the computer is being used in education, but rather how it is being used.

An increasing number of colleges and universities are offering advanced degrees in computer science. Thus, the study of the computer has itself become a discipline. But the influence of the computer in education cannot be limited only to a discipline in itself. 7

No matter how peripheral or how central the student's concern with computers will someday be, unless he is exposed early to the rapidly changing computer world, he will not be in a psychological position to appreciate them properly or to understand their role in his own occupation and cultural environment. 8

One area in which the computer has had a pronounced effect is in computer-managed instruction. 9

In computer-managed instruction the computer is used to aid the teacher in guiding and administering the instructional process. The computer activates hardware and learning materials that are extraneous to the computer system itself.


The student is usually not on line with the computer system and the processing of data need not be in real time. As an example of computer-managed instruction (CMI), we will assume that the computer is to assist in managing a course in geography. The computer has been programmed to process information concerning Joey Jones's progress in geography, and based upon its data input it concludes that it is appropriate for Joey to hear a tape about Eskimo culture and to view a set of slides coordinated with the tape. The computer would activate appropriate hardware to play the tape, and it would monitor the selection of the slides to be shown on a viewer in Joey's instructional carrel. The computer may also print out a memorandum to Joey's guidance counselor advising him to speak to Joey about a problem that has been detected from an analysis of Joey's updated instructional and personal file.

The distinction between computer-aided instruction and computer-assisted instruction was made in Chapter I of this study. In the November 15, 1967 issue of Educational Technology, CAI was described as having evolved from a vague dream by computer specialists, engineers and technologists, to a specific, demonstratable and potentially effective instructional aid. Since CAI was in the developmental stage, it defied precise definition. But even in its simplest form, a computer-connected typewriter could provide drill materials to students in many areas, such as arithmetic and spelling.

---

The various content areas could be presented in such a way that the student received continuous feedback on his past performance. The material could be provided in printed, audio, visual and graphic modes. The student could respond by written, typed, response selection, audio, or light pen modes. Feedback techniques would permit student control of the learning process. "Major problems appear to be in cost, program development, curriculum decision-making and potential dehumanization of the educational process."11

Many advantages have been cited concerning the usefulness of CAI with regard to the student:

1. CAI can provide individualization of instruction--there is a one-to-one correspondence between the instructor (via the computer program) and the student.
2. CAI can give remedial help to the slow or average student.
3. CAI can present enriched material to the advanced student.
4. CAI can replace problem sessions by presenting a large number of exercises of varying difficulty and by scoring a student's work as he performs.
5. CAI can solve the problem of "pacing" by letting each student work at his own speed.
6. CAI can reinforce a student by informing him of his progress (and, through branching, letting him proceed at his own level of ability) as he learns.12


The following advantages have been cited concerning the usefulness of CAI with regard to the instructor:

1. CAI can offer a means of presenting "background" material, i.e., that knowledge (such as definitions, basic techniques) which an instructor would like his students to possess before coming to a lecture.
2. CAI can raise the level of material an instructor can present in a given time.
3. CAI can continuously compare students' performances according to criteria established by the instructor, and inform him of students' strengths and weaknesses.
4. CAI can serve as a continuous testing device.
5. CAI can be changed at any time, since the course material stored in the computer's memory bank can be easily modified or replaced. Thus, when the costs of updating printed materials, preparing new curriculums, and revising other media are taken into account, CAI offers flexibility in changing course content.13

Using Computers Interactively

The student does not really learn much about computers through CAI. The student need not know a programming language, and the only computer operation which he must know is usually merely how to respond (usually by a type-writer at a computer terminal) to the various instructional units presented to him by the computer. Only through a more interactive relationship, student with computer, can he take full advantage of the computer in an educational setting.

13Ibid., p. 6.
The most common subject taught around computers—besides data processing itself—is mathematics. Experiments have shown that teaching students to program a computer and then requiring them to solve problems with it is effective in improving their problem-solving abilities. The success of this technique has been attributed to the need for precise, detailed, step-by-step procedures in computer usage.\textsuperscript{14}

The Committee on the Undergraduate Program in Mathematics (CUPM) has recognized the potential of using the computer interactively:

The prevalence of the high-speed automatic computer affects the teaching of mathematics in a very general way. Many mathematically trained students will work closely with computers, and even those who do not should be taught to appreciate the type of algorithmic approach that enables a problem to be handled by a machine . . . .

If there is a computer on campus, or if one is otherwise accessible, it is likely that elementary programming instruction will be available to students early in their academic careers. This, in turn, makes it possible to take advantage of the computer throughout the mathematics program, and material should be presented to make use of this opportunity.\textsuperscript{15}

A project designed to utilize computer programming in the teaching of mathematics was developed by a computer science-curriculum group of Colorado schools. One end-product of the project was the text \textit{A Second Course in Algebra and Trigonometry with Computer Programming}. This


was not a computer course, but computer concepts were integrated directly with the mathematics throughout the text. "No attempt is made to develop exceptionally proficient programmers, but rather to introduce computer concepts and techniques that will strengthen the understanding of mathematics." An example of the desired procedure would be to use the computer to find an approximation to the square root of any positive number by Newton's method with a tolerance of 0.0001. A computer has to be told exactly what to do, and a flow chart (schematic description) of the steps involved in the solution of a problem is of first-order importance in understanding various mathematical concepts.

At a meeting of the Conference on Computer Oriented Mathematics and the Secondary School, the general opinion was that most of the experimental teaching of computer programming had taken place in connection with mathematical applications. The assertion was made that students who had programmed mathematical problems for solution on the digital computer had acquired surprisingly high insight into the mathematical concepts involved.


17 Ibid., p. 136.
As an example, if a student is asked to write a program for the solution of quadratic equations, data can be supplied with, for example, a leading coefficient of zero, which quickly convinces the student that he must consider the usually over-looked condition that the quadratic formula does not work if the leading coefficient is zero. To generalize, by using the computer the student is forced to acquire insight into the general algorithm, the conditions under which it applies, and the general class of problems to which a given procedure can be applied as well as to those special cases to which the general solution is not applicable. This is clearly what we have always been trying to teach mathematics students, and contact with the computer makes it easier to accomplish the desired ends.18

Thus, using computers interactively should consist of students learning a programming language and using the language to assist in solving mathematical problems. As a result, students should gain greater insight into the nature of the mathematical concepts involved and greater understanding and appreciation of the role of the computer.

Computers and Calculus

It has been stated that the right way to do an algorithm is to program it so that it can be used with a computer. In essence, the student must explain the algorithm to the computer in a programming language which the computer can translate into machine instructions.19 If the student is


successful in such an effort, he will have achieved understanding of the first order. The availability of the computer presents a tremendous opportunity for students of all levels to learn and apply mathematical concepts.

The computer can be integrated effectively with many branches of mathematics.

Various reasons have been advanced for introducing computers into college calculus, including:
1. Freshman calculus, because of the nature of the subject and its placement in the undergraduate curriculum, is a good college course for the introduction of computers into the student's thinking and working habits.
2. Overall student interest and comprehension is increased as learning is turned into a more active, less passive experience, and more emphasis is given to the "constructive" and algorithmic aspects of the calculus. The computer gives the student the opportunity for experimentation.
3. Problems become more real, more challenging, more interesting, and less tedious when programmed for a computer.
4. Students who have become acquainted with calculus or computers in high school mathematics curricula will have greater interest in a college calculus course if it uses computing.

---


The above statements are appropriately called "reasons," for no supportative data (experimental research) was listed or even suggested as having been initiated. The potential of the integration of calculus with computers has resulted in a CUPM publication entitled *Calculus and the Computer Revolution*.

Several colleges have been listed by the CUPM as using the computer in calculus courses. One such school is the University of New Mexico.

The University of New Mexico has a computer laboratory in conjunction with the last three semesters of the calculus sequence. There are approximately 500 students involved and each student is scheduled for one hour per week in the lab. Each student uses one of 16 IBM 2260 Display Units (scopes) which are hooked up on a time sharing basis with an IBM 360, Model 44. At the beginning of each semester instruction is given using locally produced programming manuals and scope operations manuals. The language used is a minimal subset of FORTRAN IV. After the basic instruction period, a problem is assigned every two or three weeks to be done by each student. The problems are chosen to fit closely with the material the student is studying in his textbook, which is *Modern University Calculus* by Bell, Blum, Lewis, and Rosenblatt.

The philosophy of the program is that the computer is mainly a tool to help the student learn mathematics. If a student is able to explain to a very literal-minded machine how a problem should be done, then it is felt that he has a good understanding of it. Secondly, homework problems are assigned to be done on the machine which are

---

useful and illuminating, but which would be
too tedious to do by hand. Lastly, the
experience in programming a computer is seen
as very valuable to many students, though
this is not a primary goal.\(^\text{24}\)

However, each description of school programs integrating
calculus with computers was void of any experimental
research data as to the effectiveness of these changes in
mathematics curricula and methodology.

In 1968, the Center for Research in College Instruction
of Science and Mathematics (CRICISAM) published the text
CRICISAM Computer-Oriented Calculus. A questionnaire was
sent out to the institutions known to be using this text in
1971. Approximately 74 per cent of the students using the
text were classified as "average" in ability. The following
question was asked of the instructors using the text:
"Compared with your experience with teaching a traditional
calculus course, how are things going"?\(^\text{25}\) Forty per cent
of the instructors reported that they had a better experi-
ence with the CRICISAM approach than the traditional
approach; 54 per cent claimed that they could identify no
apparent difference between the two approaches, and 6 per
cent of the instructors felt that they had better success
with the traditional approach. Each instructor was asked

\(^{24}\)Committee on the Undergraduate Program in Mathematics

\(^{25}\)The Center for Research in College Instruction of
Science and Mathematics, \textit{Calculus: A Computer Oriented
Presentation}, Newsletter No. 2, edited by Robert and Carol
about the attitudes of his fellow instructors, in other departments as well as mathematics, toward the CRICISAM approach to the study of calculus. Sixty-five per cent of the instructors were favorable toward computer calculus, 33 per cent were neutral, and 2 per cent were against computer calculus. No formal research data was either reported or noted as being conducted as to the comparative effectiveness of the traditional approach and the CRICISAM approach to the study of calculus.\(^{26}\)

In 1971, several units on calculus were published as a result of the Computing and Mathematics Curriculum Project (CMCP). The unit on derivatives was designed as follows:

The computer will be used to find an approximate value of the derivative at the point \(X = A\) in the domain of the function under consideration. In this way, one can often obtain the information necessary to predict the value of the derivative at the point \(X = A\). Since the derivative of a function at a point requires evaluating a certain limit, we recall from the unit on limits that the computer can only provide the information to "guess" at the limit. The computer is not capable of assuring us that this "guess" is correct. By "guessing" at the derivative of a function at many points it is possible that a conjecture can be made about the derivative of the function at any point in the domain of the function. This type of experimentation is done in the unit, and the conjectures made are either disproved by counterexamples or are stated as theorems and proved.

Concepts of which the student should gain a better understanding from this unit include:

\(^{26}\)Ibid.
(1) What is meant by the derivative of a function?
(2) Does the derivative of a function exist at all points where the function is defined?
(3) Once a "guess" is made for the derivative of a function how can the guess be proved or disproved?
(4) What is the connection between continuous functions and differentiable functions?
(5) What are some of the properties that derivatives of functions have in common? 27

Another publication integrating computers with calculus was published by the General Electric Company, entitled Computer Applications to Calculus. 28

Thus, it is clear that work has been done in the area of integrating computers with calculus. However, the literature indicates that this integration has had three limitations: (1) there are not a significant number of studies to validate the claims made for the enhancement of instruction, (2) the level of calculus has been for science and mathematics majors, and (3) the programming languages used in this integration have been almost exclusively FORTRAN or Beginner's All-Purpose Symbolic Instruction Code (BASIC).

Educational Uses of APL

"The APL language was first defined by K. E. Iverson in A Programming Language (Wiley, 1962) and has since been


28 Lee M. Ellwood, Computer Applications to Calculus (Dallas, 1969).
developed in collaboration with A. D. Falkoff." Jean
Sammet recognizes APL as a programming language, stating
that it has been known in public form since 1962 and that
it has undergone subsequent development over the past few
years. APL is well suited "not only to the advanced
scientific or technical user, but also to the occasional
user and to the user with little or no previous experience
with computers." "Since it [APL] is similar in many
respects to algebraic notation and, in addition, contains
many useful functions not expressible concisely with con-
ventional symbols, it has proved to be very efficient for
describing algorithms (problem-solving procedures)." In
fact, a text in algebra, entitled Elementary Algebra, has
been published using APL in an algorithmic approach. A
textbook in algebra, Elementary Functions, has also been published;
this text is algorithmic in its treatment and utilizes APL
as the programming language.

By communicating with the computer via APL, the student and the computer are working together as a team to discover and demonstrate mathematical concepts with a minimum of error. As was pointed out in Chapter I, there are approximately 200 programming languages. There is no reason to assume that the list will not continue to grow. One of the more popular programming languages has been BASIC. It has been chosen over other programming languages for such reasons as "the small number of commands needed, its easy application in solving problems, and its practicality in our evolving educational setting." 

John Clark maintains that there are several factors which give APL an edge over BASIC or any other language. He considers the design difference to be the most important factor. "APL is a notation for the concepts of man which have been implemented on a computer. BASIC is a series of computer programming techniques which have been implemented on man." 

With regards to instruction, APL is easier to learn than BASIC, although APL has at least ten times as many


operators. Many of the operators can be understood before the first program is prepared. The student may analyze each step of his program before formally adding it to his program. "It [APL] is a true interactive language."38

Students can begin programming within only a few hours of instruction concerning the procedure for editing functions and operating the computer terminals. After that, the learning process is individualized, that is, the student may proceed at his own pace; most students will not need a lot of assistance from the instructor. "The instructor and student can spend their classroom time working with non-trivial problems rather than coding techniques. Actual learning rather than coding tricks can be brought into the classroom."39

BASIC, on the other hand, requires a program for the simplest operation. Much time is spent in BASIC classes in the iterative solution routines required by the language rather than the discussion of the problem. The student tends to concentrate on the coding of the problem, e.g., flow charting, data structure, looping techniques, rather than the problem and how it relates to his universe. For many programmers and instructors, the concept of using the computer to assist the individual student has taken second place to teaching the student coding tricks. To them, BASIC is familiar while APL is strange. For the student with no background in either languages, both are strange. However, the language of APL is

38Ibid., p. 2.
39Ibid.
heuristic and the student becomes a self-generator rather than a follower who waits for the next rule to be handed down by the instructor.\textsuperscript{40}

When working with mathematical algorithms, the number of lines necessary to code the algorithm in APL should be equal to, or less than, the number of lines required in English. But this is not true for any of the first generation languages like BASIC or FORTRAN. Most applications of mathematical algorithms can be written in one line of APL. However, there are many one-line-APL programs which would be practically impossible to write in one line of BASIC code. One of the most important advantages of APL is that the techniques learned carry over to other languages. Input and output can both be accomplished on one line of APL code, certainly an advantage over other languages. And, although this study is concerned with computer-aided instruction rather than CAI, it is true that APL is such a universal language that any instructor may combine CAI segments in a program of study that will best serve the individual needs of his class. "Using a system of sub-functions it is possible to create segments which range from simple drill and practice through simulation and problem solving."\textsuperscript{41}

In summary, John Clark has made the following statements concerning APL:

\textsuperscript{40}Ibid., p. 3.

\textsuperscript{41}Ibid., p. 5.
Once the initial shock of those [APL] strange symbols is overcome, the natural flow of APL allows the non-dedicated programmer to work with problems which are non-trivial and are of interest to him. Instead of becoming bogged down with counters, loops, flow charts, etc., the instructor and student can concentrate on the problem. The first generation languages, FORTRAN and BASIC, were quite valuable in getting many people coding computers, but they did enforce much concentration on coding techniques as on solving the problem. APL has eliminated this programming trivia and allowed the individual to work with the computer rather than drive it through a series of detail instruction.42

The Computer and the Two-Year College

"The importance of the computer in today's community college cannot be overemphasized."43

In education, it is becoming apparent that the use of the computer as a "tool for instruction" is an important and desirable adjunct to its use as a "subject of instruction." This usage of the computer in the community college environment is virtually untapped and has almost unlimited growth potential.44

Students in community colleges should be required to develop "computer literacy," regardless of their field of specialization.45 The use of the computer would serve to accomplish the following objectives:

42 Ibid., p. 6.


45 Brightman, op. cit., p. 6.
1. Develop student appreciation of the computer's role in society both now and in the future.
2. Motivate students and teachers to more individual, challenging instruction.
3. Enrich existing programs through use of the computer.
4. Develop an early recognition of, and a deeper insight into, concepts of mathematics, logic, and science.
5. Encourage faculty and students to apply computer concepts creatively to other areas of the instructional program.

It has been recognized that the computer can be most effective in teaching such courses as mathematics, physics, chemistry, and engineering. "The student, via a programming language, can command the power of the computer to aid him, rather than being concerned with the computer as a subject area of its own." The computer has potential in many non-scientific disciplines, such as the social sciences, business, medical services, electronics, and the library-technician field.

The past few years have seen substantial educational experimentation occurring in junior colleges. But in order for experimentation to continue, materials must be developed. "Community college instructors in all disciplines

---

46 Ibid., p. 11.
47 Hill and Sedral, op. cit.
48 Ibid.
49 G. S. Young, "The Opportunities and Problems of the Two-Year College," The American Mathematical Monthly, LXXIX (April, 1972), 388.
can be of help in developing materials (programs) for the computer."

A course in mathematics, physics, business, etc., that is to feature computer usage will require a more imaginative set of instructional materials and problems than may have been used previously. Typically, the student can be expected to accomplish more sophisticated types of problem solving and will be more likely to ask complex questions when the solutions are feasible. Publishers of instructional materials have been slow to catch up with this trend; therefore, instructors often find themselves in the position of writing their own texts.

All students who will use the computer for problem solving must have a short course which teaches the language that they will use to describe their problems for the computer as well as some introduction to computer concepts. The latter is important to enable students to use the computer most effectively and to keep to a minimum the frustrations created by mistakes that are unique to computing. This training can be provided by formal courses over a quarter or semester, short cram courses, or self-taught methods with tutors available for questions.

Every business curriculum should include the fundamentals of a programming language. If a business curriculum does not require a knowledge of a programming language, then those students who had the opportunity to take a business mathematics sequence which had the calculus portion integrated with a programming language, such as APL,

---

50 Hill and Sedral, op. cit., p. 28.
51 Ibid., p. 27.
52 Ibid., p. 8.
would still learn the fundamentals of a programming language. And, since the techniques learned in using APL carry over to other programming languages, those students who would be required to take a business-related programming language, such as COBOL, would find themselves amply prepared.

Future Uses of Computers with Mathematics

One of the major tasks of the seventies will be determining the role of the computer in the school program. It is already clear that much more will happen than a senior course in programming for the college-bound. Such concepts as flow diagrams have proven themselves valuable in the actual exposition of mathematical ideas, and will certainly work their way down further and further in the grades. But I believe that more than that will occur. I wish I were technically competent to map out the changes. But I cannot even guess as to whether, for example, sixth graders will have direct access to computers by 1980... All I know for sure is that we must give most serious attention to the implications of the computer for the school.53

Future undergraduate mathematics programs will be revolutionized by the introduction of computers.54

---


To my mind, the use of computers is analogous to the use of logarithm tables, tables of integrals, carefully drawn graphs of the trigonometric functions, or carefully drawn figures of the conic sections. Far from muddying the limpid waters of clear mathematical thinking, they make them more transparent by filtering out most of the messy drudgery which would otherwise accompany the working out of specific illustrations. Moreover, they give a much more adequate idea of the range to which the ideas expressed are applicable, than could be given by a purely deductive general discussion unaccompanied by carefully worked-out examples.55

It has been maintained that unless mathematics courses undergo revisions to embody the use of computers, that most of the students who would ordinarily take these courses will instead be taking computer courses in 1984. Also, by 1984, a great many courses in economics will require extensive computer use by students. If these students cannot acquire the necessary computer proficiency in their mathematics courses, then they will be forced to take appropriate computer courses which will fill this void.56

What they [students] will not be taught in the computer course is the habit of taking some thought before performing a computer calculation in order to see if some mathematical insight might permit the use of a


more efficient or reliable algorithm, or might even give the answer without resorting to a machine calculation.\textsuperscript{57}

Interest and relevance of calculus courses should be greatly enhanced by the judicious use of computers.\textsuperscript{58}

\textsuperscript{57}Ibid., 640.

\textsuperscript{58}Birkhoff, \textit{op. cit.}, 649.
CHAPTER III

EXPERIMENTAL DESIGN AND EXPERIMENTAL PROCEDURES

Despite accomplishments in the area of integrating computers with calculus, the following limitations are clear from the preceding chapter: (1) claims for enhancement of instruction have not been sufficiently validated, (2) the level of calculus has been primarily only for science and mathematics majors, and (3) the only programming languages being used on a large scale have been FORTRAN (or a version thereof) and BASIC. The purpose of this study was to develop and validate an APL/360-aided instructional program in mathematics for business and economics majors, calculus being the mathematical topic. The purpose of this chapter is to describe the study that was conducted.

The Setting of the Study

The purpose of this section is to describe the school at which the experiment was conducted, the subjects used in the experiment, and the course of study in which these subjects were enrolled. The experiment was conducted during the spring semester of the 1972-1973 school year at Mountain View College of the Dallas County Community College District.
Dallas county voters created the district in May, 1965. The district is now composed of four colleges: Mountain View, El Centro, Eastfield, and Richland. In addition to these four colleges, sites have been purchased for three future colleges: Brookhaven, Cedar Valley, and North Lake. The colleges of the district are comprehensive, open-door institutions. A variety of programs are offered, including technical and professional, college transfer, and community service courses. Each college offers associate degree and certificate programs.

Mountain View College first enrolled students in August, 1970, and is a fully accredited two-year college. The enrollment at Mountain View College during the spring term of the 1972-1973 school year was approximately 4,000. Of all students, approximately 70 per cent were male students, 60 per cent were day students, and 50 per cent were full-time students. The evening college offers the same broad spectrum of educational programs that is available to day students. The science and mathematics division had twenty-six full-time instructors during the fall semester of 1972, seven of which were mathematics instructors.

The subjects for the study were students enrolled in all three sections of Mathematics 112 taught at Mountain View College during the spring semester of the 1972-1973 school year. Mathematics 111-112 is a required sequence
in most Texas institutions for business and economics majors. The general content of this sequence was described in Chapter I.

Pilot Study

The instructor for both the pilot study and the experiment was a full-time staff member of the Science and Mathematics Division at Mountain View College. The instructor had had several years of experience in teaching mathematics for business and economics majors, at two different universities. The instructor held a Master's degree in mathematics and had taken graduate courses in computer science. The textbook for the pilot study and the experiment was *Foundations of Mathematics*.¹

A manuscript on APL/360 was developed by the instructor during the summer of 1972. (See Appendix A.) The purpose of the manuscript was to give the students a "short course" on APL/360. A comprehensive survey of the APL language was made, and all parts of the language which were considered relevant to a study of calculus by undergraduate business and economics majors were included in the manuscript. APL programs defining operators of significance in a computer-aided study of calculus and special problem sets based on

these programs and calculus concepts in the text were also developed. (See Appendices B and C.)

Copies were made of the manuscript and problem sets and used in teaching the only day section of Mathematics 112 during the fall semester of the 1972-1973 school year. Experience was also obtained in administering instruments to be used in the experiment. As a result of the pilot study, corrections and/or revisions were made on all developed materials.

The Experimental Design

In this section the nature of the experimental design is presented and the teaching methods are described. The experiment conducted was designed to conform to the model for a three-by-two factorial experiment. The two independent variables were level of ability and method of teaching. Since validation was sought in terms of student achievement and attitude, the dependent variables were achievement and attitude. Hence, the three-by-two factorial design was used twice, first for the dependent variable, achievement, and then for the dependent variable, attitude.

Traditional Lecture Method

All course material was presented in organized units in a classroom environment. Chapter thirteen in the text
was comprised of topics from differential calculus. Some extensions and/or additions were made to the content of chapter thirteen. The definition of limit of a function was extended to include the cases where the domain and/or range values increased or decreased without bound. Such an extension was made in order that an intuitive approach to the definite integral could be presented in chapter fourteen, without covering the section on limits of sequences. For completeness, the definition and test for inflection points were added to the section on maximum and minimum values of a function, as well as the terms "concave up," "concave down," "increasing," and "decreasing." Also, the formula for differentiating the natural logarithm function was included in the section on differentiation formulas in chapter thirteen.

Chapter fourteen in the text was comprised of topics from integral calculus. Several changes were made to chapter fourteen. The only additions made were the anologue indefinite integrals for definite integrals and the change-of-variable method for evaluating integrals. These additions were considered desirable since students might need an introduction to these topics for future courses. Sections 14.2, 14.3, and 14.4 were omitted since these sections were highly theoretical in the approach to the concept of definite integral. Since the anologue concept of derivative had been developed rigorously, students were expected to understand
the concept of area under a curve intuitively through a study of the definition of the definite integral.

No visual or instructional aids were used other than chalkboards and the text used in the pilot study and/or the above mentioned supplementary topics to the text. Students were required to complete pencil and paper homework assignments related to this material.

Computer-Aided Instructional Program

The same course content on calculus as described above was included in the computer-aided instructional program. Prior to the study of calculus, copies of the manuscript on APL and problem sets based on the manuscript were distributed to students. Eight computer terminals were available for general student use between the hours of eight a.m. and ten p.m., Monday through Friday. Use of computer terminals was on a "first-come, first-serve" basis. However, no great difficulty was encountered by the students included in the experiment in gaining access to the computer terminals either as a result of other students using the terminals or computer and/or computer-terminal malfunctions. The terminals were located in one spacious room beneath the business division on a lower floor. Students were not restricted by a time limit in using the terminals.

After an initial class period for orientation on the computer terminals, such as signing on and signing off,
traditional class lectures were presented on the manuscript on APL/360. The problem sets on APL/360 were solved primarily on the terminals; a few problems were solved by pencil and paper methods, where appropriate. This phase of the experiment lasted approximately three weeks, concluding with a teacher-made test over the manuscript and problem sets. During the study of APL/360 by the experimental group, the control group studied topics on permutations, combinations, and probability in chapters five and six of the text; this study was also concluded with a teacher-made test.

During the study of calculus, the problem sets which were developed for a computer-aided study of calculus were distributed to students. The problem sets required students to write APL programs and/or use APL programs in order to accomplish the following objectives:

1. help students gain insight into calculus concepts intuitively,

2. obtain numerical approximations for functions, derivatives, and definite integrals,

3. obtain graphs for functions, derivatives, and antiderivatives, and

4. determine derivatives and antiderivatives for polynomial functions.
Intuitive insight into such concepts as limit of a function, continuity, derivative, and definite integral was sought.

The operator \( F \) was designed to evaluate any algebraic expression. The operator \( \text{SLOSEC} \) gave a numerical approximation to the slope of the tangent line to a function at a given point for an appropriately small value of \( \Delta x \); that is, \( \text{SLOSEC} \) gave a numerical approximation to the derivative. \( \text{SLOPT} \) was designed more as an intuitive aid than for numerical approximation since, for a given value of \( \Delta x \), \( \text{SLOPT} \) would generate a new \( \Delta x \) each time the slope of a secant line was calculated. The program would automatically stop when the absolute value of \( \Delta x \) became less than 0.001. \( \text{POINTS} \) determined the points, as a vector, which divided an arbitrary closed interval into an arbitrary number of equal intervals. \( \text{SLOSEC} \) and \( \text{POINTS} \) were used together (in a single \( \text{APL} \) statement) to assist the student by numerical methods in determining critical numbers and using the first and second derivative tests for maximum and minimum values.

The operators \( \text{RTXSUM}, \text{LTXSUM}, \text{RECTIN}, \) and \( \text{RECTEX} \) gave numerical approximations to the definite integral. That is, these programs approximated the area under the graph of a function over a closed interval for any number of rectangles by using the right end-point of rectangles, the left end-point of rectangles, inscribed rectangles, and escribed rectangles, respectively. \( \text{PLOT} \) and \( \text{POINTS} \) were used
together to obtain graphs of a function, its derivative, and one of its antiderivatives. These graphs were then interpreted by students in terms of critical numbers, the first and second derivative tests, and the fundamental theorem of integral calculus. The operators DER and INT determined the derivative and an antiderivative for any polynomial function. This demonstrated that differentiation and integration formulas could be programmed and that exact results could be obtained in lieu of numerical approximations. The value of numerical approximations was in the independence of formulas and sole reliance on the definition of the concept involved, such as slope and area.

Not all of calculus was amenable to computer methods. The emphasis on homework was on both computer and pencil and paper methods for solving calculus problems. The number of problems assigned to the experimental group to be accomplished by pencil and paper were generally far fewer than those assigned to the control group since the experimental group was also assigned problems to be accomplished by computer methods. Every effort was made to assign the same types of problems to the experimental and control groups when paper and pencil methods were required. However, the experimental group was not assigned as many problems of each type, usually only one. No effort was made to keep homework assignments exactly the same length, time-wise.
In using the computer terminals, it was determined from the pilot study that considerable time was expended by students in making errors at the terminals, errors which could not be corrected as rapidly as on paper by using an eraser. Also, many students forgot portions of the APL language and had to refer back to their copy of the manuscript on APL/360. This variable was "allowed for" in the instructor's assignments. Records on the time devoted to paper and pencil methods and computer methods, respectively, were maintained by the instructor. (See Appendix D.)

Each class lecture on calculus was divided into two parts. The first part of class was devoted to a discussion of the homework assigned the previous day and the assignment for the next day; the second part was devoted to a presentation of those topics necessary to accomplish the assigned homework. The topics presented in the second part of class were on calculus, not on the computer. The computer-problem sets were considered self-explanatory and gave the student the opportunity to develop independent learning ability by negotiating the material without assistance in class. Some difficulty was encountered in always devoting the same time for both parts of class for both experimental and control groups. However, every effort was made to keep this time in class as equally divided as possible.

The instructor for the experiment maintained liberal office hours. In helping students outside of class, it was
the instructor's philosophy to assist the student in understanding the concepts involved, and not working the assigned homework for the student. The experimental group required more out-of-class time than the control group, primarily because of students encountering some minor difficulties with terminals or forgetting some APL/360 commands and procedures.

Thus, the essential difference between the traditional lecture method and the computer-aided instructional program was the use of the computer as a course supplement to classroom instruction in the latter method. All data collected and analyzed were for the purpose of determining the effectiveness of the computer as a teaching-learning aid.

Instruments

**California Short-Form Test of Mental Maturity, 1963 Revision (Level 5)**

The California Short-Form Test of Mental Maturity has been used by educators, counselors, psychologists, and employers in a wide variety of testing situations. It is appropriate for a mental age range common to grades twelve through college and adults. The Short-Form is a major revision of the 1957 Edition and consists of seven test units, each a different mental exercise, and takes thirty-nine minutes of actual testing time. Test units one through four, Opposites, Similarities, Analogies, and Numerical
Values, compose the Non-Language Section. A minimum of verbal material is presented and the recognition or logical analysis of abstract relationships reflect the measurement of an examinee's mental capacities. The Language Section is composed of tests five through seven, Number Problems, Verbal Comprehension, and Delayed Recall, which "sample the ability to comprehend verbal and numerical concepts of various types, and test the extent and accuracy of recall." ²

The reported coefficient of reliability for the Short-Form was .91.³ This coefficient was computed on raw-score data based on the normative population characteristics using Kuder-Richardson Formula Twenty-One.⁴ Content validity was claimed on the basis of coefficients of correlations with other standardized tests of mental ability. A correlation coefficient of .96 was obtained between the Short-Form and the School and College Ability Tests.⁵

Cooperative Mathematics Tests for Calculus

The Cooperative Mathematics Tests for Calculus, Form A and Form B, consist of five-choice items. Each form


⁴Ibid., p. 15. ⁵Ibid., p. 26.
consists of two parts, thirty questions to each part, and each takes forty minutes to administer.

Achievement is assessed in terms of students' comprehension of the basic concepts, techniques, and unifying principles in each content area. Where possible, many of the newer trends and emphases in mathematics are represented in the tests, but content has been selected carefully to ensure the appropriateness of the tests for most students. Ability to apply understanding of mathematical ideas to new situations and to reason with insight are emphasized. Factual recall and computation are minimized.\(^6\)

The Cooperative Mathematics Tests are purported to be useful in "appraising average levels of achievement for evaluating instructional programs."\(^7\)

It has been noted that "content validity is best insured by entrusting test construction to persons well-qualified to judge the relationship of test content to teaching objectives."\(^8\)

The Cooperative Mathematics Tests represent the combined judgment of many teachers, scholars, and mathematics specialists concerning the important elements of mathematics that are included in the nation's school mathematics programs, whether the approach be "modern" or more traditional in nature.\(^9\)


\(^{7}\) Ibid., p. 8.

\(^{8}\) Ibid., p. 62.

\(^{9}\) Ibid., p. 15.
The reported coefficients of reliabilities for Forms A and B of the calculus examination were .87 and .84, respectively. These coefficients were obtained by using the Kuder-Richardson Formula Twenty.10

Equivalence of Forms A and B was claimed for essentially two reasons. First, parallel content was claimed on the basis of the similarity of tables listing item-content classifications. Secondly, a table depicting representative converted scores, which differed by less than 3 per cent of the smaller score for each pair of converted scores, was purported to indicate that the two forms were very similar in difficulty.11

Purdue Scale to Measure Attitude toward Any School Subject

The Purdue Master Attitude Scales consist of nine separate scales. For example, three of these scales can be used to measure attitude toward "any vocation," "any school subject," and "any high school." "The scaling procedure for each of these scales is the psycho-physical principle that equally often observed differences are equal, often referred to as the Thurstone attitude scaling technique."12

---

10 Ibid., p. 62.
11 Ibid., p. 67.
only scale used in this study was the Purdue Scale to Measure Attitude toward Any School Subject; both Form A and Form B of this instrument were used.

The claim was made that the Purdue Master Attitude Scales have one unique advantage, namely, "that a single scale can validly measure attitude toward any one of a large number of specific attitudes with a known, adequate degree of reliability." ¹³

Beyond their face validity, these scales have demonstrated validity both against Thurstone's specific scales with which they show typically almost perfect correlations and in differentiating among attitudes known to differ among groups.¹⁴

The original scales usually contained in excess of forty items per form. The present scales have all been restricted to seventeen items per form as a result of an experimental study which demonstrated that the reliability of the instrument was not appreciably lowered by reducing the number of items.¹⁵ No time limit was specified for completion of the instrument. Scale values are available for Form A and Form B for each of the seventeen items. The median scale value of the items checked is the attitude

¹³Ibid.
¹⁴Ibid., p. 2.
¹⁵Ibid., p. 5.
score. This method of scoring has been validated by Sigerfoos.\textsuperscript{16} The reliabilities of the original scales for various population samples was purported to range from .71 to .92.\textsuperscript{17}

**Procedures for Collecting Data**

Due to schedule conflicts and other factors, random assignment of students to the three sections of Mathematics 112 was not possible. One section met from 11:00 a.m. to 11:50 a.m. on Monday, Wednesday, and Friday during each week of the semester. Another section met from 12:15 p.m. to 1:30 p.m. on Tuesdays and Thursdays. The other section met from 5:30 p.m. to 6:45 p.m. on Mondays and Wednesdays. The section meeting three times a week was the experimental group. The sections meeting twice a week were collectively the control group.

In the design of the experiment and in the analysis of data, three levels of ability were considered. In order to determine the level of ability of each student, the student’s composite score on the California Short-Form Test of Mental Maturity, 1963 Revision (Level 5) was utilized. The levels of ability were determined on the basis of composite scores for all students enrolled in Mathematics 112. Two classes

\textsuperscript{16}Ibid.

\textsuperscript{17}Ibid.
were tested on January 31, 1973. The other class was tested on February 1, 1973. The test was administered by a member of the counseling staff of Mountain View College who had had substantial experience in administering this test. On the basis of these scores it was determined that students with a composite score of ninety-five or above ranked above the sixty-sixth percentile, while students with a score of seventy-nine or below ranked below the thirty-third percentile. As a result, in the experiment the first level (high) consisted of all students with a composite score of ninety-five or above. The second level (middle) consisted of students with a composite score higher than seventy-nine but lower than ninety-five, and the third level (low) consisted of students with a composite score of seventy-nine or lower.

As criteria for determining the relative effectiveness of the two teaching methods involved in the experiment, two dependent variables were considered. One dependent variable was student achievement. Student achievement was measured by the standardized test Cooperative Mathematics Tests for Calculus, Form A and Form B. Each form was administered to the control and experimental groups, either as a pre-test or as a post-test, in order to control for some initial differences between the experimental and control groups since randomization was not possible.
It was randomly determined, by flip of a coin, to administer Form B as the pre-test and Form A as the post-test. Part I of Form B was administered to the experimental group on February 2, 1973 and Part II on February 5, 1973. Part I was administered to the two sections composing the control group on February 5 and 6 and Part II on February 7 and 8, respectively. Both pre-tests and post-tests were administered by the instructor conducting the experiment. The experiment lasted approximately fourteen weeks, one week of which was a "spring break." Part I of Form A was administered to the experimental group on May 2, 1973 and Part II on May 4, 1973. Part I was administered to the two sections composing the control group on April 30 and May 1 and Part II on May 2 and 4, respectively. Raw scores, total number correct on parts I and II, were obtained on each administration of each form.

The second dependent variable was student attitude. Student attitude was measured by the Purdue Scale to Measure Attitude toward Any School Subject, Form A and Form B, for attitude toward mathematics. Each form was administered in a similar manner to the achievement test, that is, as a pre-test and post-test.

It was randomly determined, by flip of a coin, to administer Form B as the pre-test and Form A as the post-test. Form B was administered on the same days as Part I
of Form B of the achievement test was administered. Form A was administered on the same days as Part I of Form A of the achievement test was administered. In each case, the attitude instrument was administered before the achievement test. Ten minutes were allowed for students to complete the attitude test.

All students were told that scores on the Short-Form and attitude tests would be kept anonymous. Students were also told that scores on the calculus pre-test would not count in student evaluation. However, students were informed that scores on the calculus post-test would be used in student evaluation. Teacher-made tests were also given over calculus; two such tests were prepared and administered. The teacher-made tests on calculus for the experimental group were equivalent forms over calculus only, not over, nor using, computer methods. Teacher-made test scores obtained by students were not included in the analysis of data for the experiment. The raw data collected and analyzed are listed in Appendix E.

Procedures for Treating Data

For purposes of statistical analysis, the raw scores on each of the tests, achievement and attitude, pre-test and post-test, were divided into six categories. Data were analyzed only for those students who completed the experiment and for whom there was usable data. Fifty-six students
were officially enrolled when the experiment began. Two students in the experimental group were course repeaters. Forty-six students were still enrolled when the experiment was concluded, including the two course repeaters. Data collected from the course repeaters were not included in the analysis of data. Thus, the analysis of data was made for the data collected from forty-four students. The control group was composed of twenty-one students, eleven in the day section and ten in the evening section. Six, nine, and six students composed the high, middle, and low levels of student ability, respectively, for the control group; two, five, and four were day students, and four, four, and two were evening students. Eight, eight, and seven students composed the high, middle, and low levels of student ability, respectively, for the experimental group. Appendix F contains further descriptive information for the forty-four students involved in the study.

The analysis of data used was an unweighted-means-factorial analysis of covariance. The computational formulas are those given by Winer. These formulas do not depend on an equal number of entries in each category. All calculations

were accomplished by using verified APL programs at one of the computer terminals used in the experiment. The level of significance was reported rather than specifying a particular level for accepting the hypotheses.
CHAPTER IV

THE RESULTS OF THE EXPERIMENTAL STUDY

An experimental study was conducted in which the relative effectiveness of an APL/360-aided instructional program and the traditional lecture method were compared. The comparison was made in terms of student achievement in calculus and student attitude toward mathematics. The subjects for the study were students enrolled in three sections of Mathematics 112 at Mountain View College during the spring semester of the 1972-1973 school year. A total of forty-four subjects participated in the study. Two criterion tests were used, one to measure student achievement in calculus, and the other to measure student attitude toward mathematics. An equivalent form of each test was administered as pre-test and post-test, respectively, in order to control for some initial differences between the experimental and control groups. It was not feasible to assign students randomly to the experimental and control groups.

Three levels of ability were considered. The raw data were divided into six categories, determined by the two teaching methods and the three levels of ability. The
experimental and control groups were composed of twenty-three and twenty-one students, respectively. The smallest of the six categories consisted of six students. The largest category consisted of nine students. The data were analyzed using an unweighted-means analysis of covariance. The raw data used in the statistical analysis can be found in Appendix E.

The statistics which were relevant were the F-ratio for the two groups determined by the two teaching methods and the three groups determined by the three levels of ability, the F-ratio for the two groups determined by the two teaching methods, and the F-ratio for the three groups determined by the three levels of ability. An F-ratio of one or less would indicate that the combination of factors for which the ratio was calculated did not significantly affect the scores achieved by the subjects on the criterion test. In order for the F-ratio to indicate that a given combination of factors had a significant effect, the F-ratio must be larger than one. The exact size which the ratio must be in order to indicate a significant effect is dependent on the criteria for significance used and the degrees of freedom for the factors. The level of significance is reported instead of specifying a particular level for accepting the hypotheses.

For each criterion, a "highly" significant F-ratio (a significant F-ratio at a low level of significance; that is,
an F-ratio unlikely to have occurred by chance) for the interaction effect would indicate that the two teaching methods were dependent on ability and that the effects of the two teaching methods may be different at each level of ability. An insignificant F-ratio for the interaction effect would indicate that the two teaching methods have similar effects on a criterion variable at each level of ability. A highly significant F-ratio for the teaching method factor would indicate that one of the teaching methods is more effective with respect to the criterion variable. An insignificant F-ratio for the teaching methods would indicate that the teaching methods are equally effective with respect to the criterion variable. A highly significant F-ratio for the ability factor would indicate that the students in the three levels of ability differ with respect to the criterion variable, both teaching methods being used.

The Findings for Student Achievement in Calculus

The criterion scores for student achievement in calculus were the raw scores achieved by the subjects on the Cooperative Mathematics Tests for Calculus. This test consists of two, forty-minute, thirty-item, multiple-choice parts. The test is designed to measure achievement with regard to comprehension and application of basic calculus concepts.
From Table I it can be seen that the F for the interaction effect of the ability factor and the factor determined by the teaching methods is less than one. Thus, the following hypothesis was accepted:

1. For student achievement, as measured by the Cooperative Mathematics Tests for Calculus, there will not be significant interaction between the teaching techniques used (APL/360-aided instructional program and the traditional lecture method) and levels (high, middle, and low) of student ability.

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Squares</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
<td>.561</td>
<td>1</td>
<td>.561</td>
<td>.041</td>
<td></td>
</tr>
<tr>
<td>Ability</td>
<td>333.608</td>
<td>2</td>
<td>166.804</td>
<td>12.325</td>
<td>.0002</td>
</tr>
<tr>
<td>Interaction</td>
<td>17.450</td>
<td>2</td>
<td>8.725</td>
<td>.645</td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>500.762</td>
<td>37</td>
<td>13.534</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>852.381</td>
<td>42</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The F for the factor determined by the teaching methods is also less than one. On this basis, the following hypothesis was rejected:

2. Business and economics students taught with the APL/360-aided instructional program will score significantly higher on the Cooperative Mathematics Tests for Calculus.
than those students who were taught by the traditional lecture method.

The $F$ for the effect of the ability factor is 12.325. The probability of obtaining an $F$ as large or larger than 12.325 is .0002; that is, such an $F$ occurring by chance is highly unlikely. Hence, the following hypothesis was accepted:

3. There will be a significant difference in achievement between the three levels of student ability, both teaching techniques being used, as measured by the Cooperative Mathematics Tests for Calculus.

The Findings for Student Attitude toward Mathematics

The criterion scores for student attitude toward mathematics were the raw scores achieved by the subjects on the Purdue Scale to Measure Attitude toward Any School Subject. This instrument consists of seventeen items, the median scale value of the items checked being the attitude score.

From Table II it can be seen that the $F$ for the interaction effect of the ability factor and the factor determined by the teaching methods is 1.092. The probability of obtaining an $F$ as large or larger than 1.092 is .3470; that is, an $F$ of 1.092 occurring by chance is quite likely (better than one chance out of three). Hence, the following hypothesis was accepted:
4. For student attitude toward mathematics, as measured by the Purdue Scale to Measure Attitude toward Any School Subject, there will not be significant interaction between the teaching techniques and levels of student ability.

TABLE II

ANALYSIS OF COVARIANCE FOR STUDENT ATTITUDE TOWARD MATHEMATICS (N = 44)

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Squares</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
<td>.786</td>
<td>1</td>
<td>.786</td>
<td>.721</td>
<td></td>
</tr>
<tr>
<td>Ability</td>
<td>3.823</td>
<td>2</td>
<td>1.911</td>
<td>1.751</td>
<td>p = .1861</td>
</tr>
<tr>
<td>Interaction</td>
<td>2.383</td>
<td>2</td>
<td>1.192</td>
<td>1.092</td>
<td>p = .3470</td>
</tr>
<tr>
<td>Error</td>
<td>40.382</td>
<td>37</td>
<td>1.091</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>47.374</td>
<td>42</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The F for the factor determined by the teaching methods is less than one. Therefore, the following hypothesis was rejected:

5. The experimental group will score significantly higher on the Purdue Scale to Measure Attitude toward Any School Subject for mathematics than the control group.

The F for the ability factor is 1.751. The probability of obtaining an F as large or larger than 1.751 is .1861, or about one chance out of five; that is, such an F occurring by chance is quite likely. Thus the following hypothesis was accepted:
6. There will be no significant difference in attitude toward mathematics between the three levels of student ability, both teaching techniques being used, as measured by the Purdue Scale to Measure Attitude toward Any School Subject.

In summary, the primary findings were that there was no significant difference between the APL/360-aided instructional program and the traditional lecture method when the criteria were student achievement in calculus and student attitude toward mathematics.
CHAPTER V

SUMMARY, FINDINGS AND CONCLUSIONS,
AND RECOMMENDATIONS

Summary

One purpose of this study was to develop an APL/360-aided instructional program for community college students enrolled in a transferable mathematics sequence for business and economics majors. Three sets of materials were developed: a manuscript on APL/360, a list of APL programs defining operators relevant to a computer-aided study of calculus, and a collection of problems based on these programs and calculus concepts. A pilot study was conducted during the fall semester of 1972 using these materials. Corrections and/or revisions were then made on all developed materials.

The other purposes of this study were to determine the relative effectiveness of the APL/360-aided instructional program and the traditional lecture method in terms of student achievement and student attitude toward mathematics, respectively. To accomplish these purposes, an experimental study was conducted during the spring semester of the 1972-1973 school year. The subjects for the experiment were forty-four students enrolled in three sections of Mathematics 112 at Mountain View College of the Dallas County
Community College District. Mathematics 111-112 is a required sequence for business and economics majors in most Texas colleges and universities. The control group, students taught by the traditional lecture method, consisted of eleven day students and ten evening students. The experimental group, students taught by the APL/360-aided instructional program, consisted of twenty-three day students. All three sections were taught by the same instructor.

Composite scores on the California Short-Form Test of Mental Maturity, 1963 Revision (Level 5) were used to establish three levels of ability. The first level (high) consisted of all students with a composite score of ninety-five or above. The second level (middle) consisted of students with a composite score higher than seventy-nine but lower than ninety-five. The third level (low) consisted of students with a composite score of seventy-nine or lower. Since three levels of ability and two teaching methods were used, all data collected and analyzed were divided into six categories. Hence, the experiment conducted conformed to the model for a three-by-two factorial experiment. The smallest number of students in the six categories was six; the largest category consisted of nine students.

The essential difference in the two teaching methods was the use of the computer as a teaching-learning aid in the computer-aided instructional program. All material
taught in class was presented in organized units for both the control and experimental groups. The computer was a course supplement to classroom instruction and aided students in obtaining insight into the nature of mathematical concepts as well as serving as a computational aid.

The Cooperative Mathematics Tests for Calculus, Form A and Form B, and the Purdue Scale to Measure Attitude toward Any School Subject, Form A and Form B, were administered to obtain the data necessary to determine the relative effectiveness of the two teaching methods in terms of student achievement and student attitude toward mathematics, respectively. Form B of each of the above instruments was administered as a pre-test and Form A as a post-test. Raw scores were obtained and analyzed by an unweighted-means-factorial analysis of covariance. An analysis of covariance was used since randomization of students involved in the experiment was not feasible. All findings were based on calculated F-ratios. The level of significance was reported rather than specifying a particular level for accepting the hypotheses.

Findings and Conclusions

For each of the two criterion measures (student achievement in calculus and attitude toward mathematics), the F-ratio for the interaction effects of the two teaching methods and the three levels of ability was calculated.
The F-ratio for the criterion student achievement was less than one and hence did not indicate significant interaction for any level of significance. The F-ratio for the criterion student attitude toward mathematics was significant at the .3470 level of significance; hence, such an F-ratio was quite likely to have occurred by chance. Thus, of the six hypotheses formulated at the beginning of the study, the following were accepted:

For student achievement, as measured by the **Cooperative Mathematics Tests for Calculus**, there will not be significant interaction between the teaching techniques used (APL/360-aided instructional program and the traditional lecture method) and levels (high, middle, and low) of student ability.

For student attitude toward mathematics, as measured by the **Purdue Scale to Measure Attitude toward Any School Subject**, there will not be significant interaction between the teaching techniques and levels of student ability.

For each of the two criterion measures, the F-ratio for the effects of the two teaching methods was calculated. Both F-ratios were less than one; hence, there was no indication of a significant difference between the two teaching methods at any level of significance. Thus, the following hypotheses were rejected:

Business and economics students taught with the **APL/360-aided instructional program** will score significantly higher...
on the Cooperative Mathematics Tests for Calculus than
students taught by the traditional lecture method.

The experimental group will score significantly higher
on the Purdue Scale to Measure Attitude toward Any School
Subject for mathematics than the control group.

For each of the two criterion measures, the F-ratio
for the effects of the three levels of student ability,
both teaching methods being used, was calculated. The F-
ratio for the criterion student achievement was significant
at the .0002 level of significance. The F-ratio for the
criterion student attitude toward mathematics was significant
at the .1861 level of significance; that is, such an F-ratio
occurring by chance was fairly likely. Therefore, the follow-
ing hypotheses were accepted:

There will be a significant difference in achievement
between the three levels of student ability, both teaching
techniques being used, as measured by the Cooperative Mathe-
ematics Tests for Calculus.

There will be no significant difference in attitude
toward mathematics between the three levels of student
ability, both teaching techniques being used, as measured
by the Purdue Scale to Measure Attitude toward Any School
Subject.

Hypotheses were formulated and findings made for the
interaction effects of the two teaching methods and the
three levels of student ability and for the effects of the three levels of student ability. However, primary attention should be focused on the hypotheses and findings for the relative effects of the two teaching methods. On the basis of the testing program and the statistical analysis, it is concluded that, in a transferable mathematics sequence for business and economics majors, the APL/360-aided instructional program and the traditional lecture method are equally effective when the criterion measures are student achievement in calculus and student attitude toward mathematics.

As was pointed out in Chapter III, the number of problems assigned to the experimental group to be accomplished by pencil and paper methods were generally far fewer than those assigned to the control group, usually only one of each type. Thus, it is concluded that the inclusion of materials on computers in mathematics courses for business and economics majors can help students understand mathematical concepts.

The instructor who taught the three sections of Mathematics 112 involved in the study received many unsolicited comments from the experimental group concerning the use of the computer in the course. All such comments were favorable. Some students reported that they enjoyed "playing" with the computer. Some students reported that they felt that the computer problems aided them in learning mathematical concepts. Also, some students reported that they felt the
course was more relevant as a result of combining the computer with mathematics. No unfavorable comments concerning the use of the computer were received. No unsolicited comments concerning the course were received by the instructor from the control group. Thus, it is concluded that business and economics students taught by a computer-aided instructional program will be more outspoken in their attitude toward the course and that their comments will be generally favorable.

Recommendations

Since it has been concluded that the two teaching methods involved in this study are equally effective, a mathematics department could use either technique. However, this conclusion was based on only two criteria, student achievement in calculus and student attitude toward mathematics. As a result of using the APL/360-aided instructional program, not only did students learn calculus concepts as well as by the traditional lecture method, but they also learned a programming language and used the computer interactively. The extent to which students developed independent learning ability as a result of using the computer was not measured. The problem sets developed by the instructor conducting the study were considered to be quite challenging.

Thus, as a result of this study, any recommendation for the use of the computer as a course aid must be on the basis
of factors other than learning calculus concepts better or enhancing student attitude toward mathematics. Since most business schools require knowledge of a programming language in junior or higher level courses, it is recommended that a computer-aided approach to the teaching of mathematics courses for business and economics majors be continued. Since some business schools do not require knowledge of a programming language and the use of the computer, the use of a computer-aided approach could well be the only opportunity for many business and economics students to develop "computer literacy," which was noted in Chapter II as being an important educational objective. In order for mathematics departments to offer a mathematics sequence for business and economics majors to be taught from a computer-aided approach, mathematics teachers must have sufficient training in computer science, appropriate textbooks need to be written and used, and there must be a proper balance between theory and applications.

The results of this study do not represent final answers to major questions concerning the use of a computer-aided instructional program in teaching mathematics courses for business and economics majors. In particular, research is needed which will help answer the following questions:

1. Does APL offer more advantages than other programming languages in computer-aided instructional programs in mathematics for business and economics majors?
2. Do business and economics students who have been taught by a computer-aided instructional program in mathematics achieve more in later computer courses for business and economics majors than those students not taught by such programs?

3. In which content areas can a computer-aided instructional program be effectively used in a mathematics sequence for business and economics majors?

4. Should students learn a programming language before taking a mathematics sequence taught by a computer-aided instructional program?

5. Which cognitive skills and attitudes are most effectively enhanced through computer-aided instruction?
APPENDIX A

APL/360 MANUSCRIPT
INTRODUCTION

Just as there are many varieties of most machines (automobiles, for example), there are many varieties of computers. Some computers are faster in carrying out calculations than others. Also, some computers require the use of instruction and data cards whereas many computers can be used without a need for any cards at all. One attribute common to all computers is their capability for carrying out the tasks for which they were designed at a rate far exceeding that of man.

In order to communicate with the computer, the concept of programming languages must be considered, for the computer will do no more or no less than what it is instructed to do. A suitable definition for a programming language is any language that is used to prepare computer programs (a computer program is a set of instructions which will direct the computer in carrying out some desired activity).
MANY PROGRAMMING LANGUAGES HAVE BEEN DEVELOPED OVER THE PAST FEW YEARS. THERE WERE ONLY A FEW SUCH LANGUAGES IN 1957; NOW THERE ARE APPROXIMATELY 200. ONE MAY THINK OF PROGRAMMING LANGUAGES AS HE DOES THE VARIOUS LANGUAGES OF MEN. GERMAN, FRENCH, ENGLISH, OR ANY OTHER LANGUAGE HAS ITS OWN GRAMMAR. ANALOGOUS TO GRAMMAR IS SYNTAX FOR PROGRAMMING LANGUAGES. THE SYNTAX OF A PROGRAMMING LANGUAGE DESCRIBES HOW IT IS STRUCTURED.

WHY DO WE HAVE SO MANY PROGRAMMING LANGUAGES? WELL, FIRST, SINCE THERE ARE MANY PROGRAMMERS (INDIVIDUALS WHO PREPARE COMPUTER PROGRAMS), THERE ARISÉS A DESIRE ON THE PART OF MANY OF THESE PROGRAMMERS TO SLIGHTLY MODIFY SOME PROGRAMMING LANGUAGES IN ORDER TO CARRY OUT ACTIVITIES RELATED TO THE PRIMARY PURPOSES FOR WHICH THESE LANGUAGES WERE DEVELOPED. SUCH MODIFIED LANGUAGES, FORMALLY CALLED SUBSETS OR EXTENSIONS, QUALIFIES THE CHANGED LANGUAGE AS A DIFFERENT (NOT COMPLETELY, OF COURSE) PROGRAMMING LANGUAGE. FOR EXAMPLE, THERE ARE MANY DIFFERENT PROGRAMMING LANGUAGES UNDER THE GENERAL TITLE FORMULA TRANSLATOR (FORTRAN), SUCH AS FORTRAN II AND FORTRAN IV. BY WAY OF ANALOGY, CONSIDER THE FACT THAT MOST AMERICANS REALLY SPEAK DIFFERENT TYPES OF ENGLISH. THAT IS, THOSE WHO WISH TO SPEAK USING CONTRACTIONS SUCH AS CAN'T, WON'T OR DON'T, MIGHT BE SAID TO BE USING ENGLISH VI, FOR EXAMPLE. THIS IS A RELATIVELY
MINOR CHANGE BUT A CHANGE NONETHELESS.

A MORE FAR REACHING REASON FOR THE MULTITUDE OF PROGRAMMING LANGUAGES IS THE NEED FOR LANGUAGES WHICH WILL BE OF THE MOST USE IN A WIDE VARIETY OF AREAS. FOR EXAMPLE, DYNAMICS ANALYZER (DYANA) IS EXCELLENT FOR DESCRIBING VIBRATIONAL AND OTHER DYNAMIC SYSTEMS WHILE COMMON BUSINESS ORIENTED LANGUAGE (COBOL) IS A WIDELY USED LANGUAGE IN BUSINESS.

A PROGRAMMING LANGUAGE (APL) WAS ORIGINALLY DEFINED BY DR. KENNETH E. IVerson AND THIS LANGUAGE IS CONSIDERED BY MANY AUTHORITIES TO BE CAPABLE OF REPLACING MANY OF THE PROGRAMMING LANGUAGES CURRENTLY BEING USED. APL IS BEST CLASSIFIED AS A PROBLEM SOLVING LANGUAGE. A PROGRAMMING LANGUAGE SYSTEM 360 (APL/360) IS INTERNATIONAL BUSINESS MACHINES' IMPLEMENTATION (ADAPTATION TO A COMPUTER) OF APL. APL/360 CONSISTS OF TWO ELEMENTS, OR COMPONENTS: THE APL LANGUAGE, AND A REMOTE, TIME SHARING INTERACTIVE TERMINAL. THE APL LANGUAGE IS THE MEANS BY WHICH THE USER COMMUNICATES WITH THE COMPUTER AND THE TERMINAL IS WHERE INTERACTION BETWEEN THE USER AND THE COMPUTER VIA THE APL LANGUAGE TAKES PLACE.

A KNOWLEDGE OF THE INTERNAL STRUCTURE OF COMPUTERS IS NOT NECESSARY. THAT SUCH A SYSTEM
IS HIGHLY COMPLEX IS EASILY TAKEN FOR GRANTED. THE ONLY COMPUTER KNOWLEDGE REQUIRED WILL BE THE USE OF A FEW OF THE MANY TERMINALS OF APL/360. THESE TERMINALS ARE SAID TO BE REMOTE BECAUSE OF THEIR SEPARATION, DISTANCE WISE, FROM THE CENTRAL COMPUTER. THEY ARE NOT COMPLETELY PHYSICALLY SEPARATED, HOWEVER, FOR A TELEPHONE COUPLER OR OTHER TYPE OF CONNECTION IS USED TO TRANSFER INFORMATION BACK AND FORTH BETWEEN THE CENTRAL COMPUTER AND THE TERMINALS. THUS, EACH TERMINAL MIGHT BE CONSIDERED AS AN ARM OF THE CENTRAL COMPUTER. THE USE OF ONE OF THESE TERMINALS WILL BE DEMONSTRATED IN CLASS. DURING THE SEMESTER, MANY TERMINALS WILL BE AVAILABLE FOR STUDENT USE.

EACH OF THE TERMINALS HAS AN ELECTRIC TYPEWRITER. WHEN AN ENTRY IS MADE AT THE TERMINAL VIA THE TYPEWRITER, THIS INFORMATION IS RELAYED TO THE CENTRAL COMPUTER. WHEN THE COMPUTER IS DIRECTED TO CARRY OUT SOME COMPUTATION, THE COMPUTATION WILL BE ACCOMPLISHED AND THEN THE COMPUTER WILL TAKE CONTROL OF THE TYPEWRITER AND TYPE THE RESULTS. THESE TERMINALS ARE SAID TO BE ON A TIME SHARING BASIS BECAUSE THE CENTRAL COMPUTER SERVES MANY USERS AT THE SAME TIME. NORMALLY, COMPUTATION IS CARRIED OUT SO QUICKLY THAT THE USER IS UNAWARE OF ANYONE ELSE USING THE COMPUTER AT OTHER TERMINALS.

A PRESENTATION OF THOSE FUNDAMENTALS OF APL/360 REQUIRED FOR THIS COURSE NOW FOLLOWS.
PRIMITIVE OPERATORS

OPERATORS ARE OF TWO TYPES: PRIMITIVE AND DEFINED. DEFINED OPERATORS WILL BE CONSIDERED LATER. PRIMITIVE OPERATORS ARE AVAILABLE ON THE ELECTRIC TYPEWRITER BECAUSE OF THE FREQUENCY OF THEIR USE. ONE SUCH OPERATOR IS THE USUAL MULTIPLICATION FOR THE REAL NUMBERS, THE FAMILIAR \( \times \). FOR EXAMPLE, IF WE WISH TO MULTIPLY 3 AND 1.2 THEN THE FOLLOWING INTERACTION WOULD OCCUR AT THE TERMINAL:

\[3 \times 1.2\]

3.6

THE USER OF THE TERMINAL WOULD TYPE \(3 \times 1.2\) AND THEN HIT THE CARRIER-RETURN KEY. THIS WOULD INDICATE TO THE COMPUTER THAT THE TRANSMISSION IS COMPLETE AND THEN THE COMPUTER WOULD MAKE THE CALCULATION AND ENTER THE RESULT AT THE LEFT MARGIN OF THE NEXT LINE. AFTER PRINTING THE RESULT, THE COMPUTER WOULD SKIP A LINE AND INDENT SIX SPACES TO AVOID THE NEXT REQUEST. THE TERMINAL DIALOGUE WILL REFER TO THE INSTRUCTIONS TYPED BY THE USER FOLLOWED BY PRESSING THE RETURN KEY, FOLLOWED BY THE COMPUTER RESPONSE TO THE INSTRUCTIONS.
WE ALSO HAVE THE FAMILIAR SYMBOLS FOR ADDITION, SUBTRACTION, AND DIVISION, NAMELY, +, -, AND *.

AS EXAMPLES, CONSIDER THE FOLLOWING DIALOGUE:

8 \times 9

72

6 + 4

10

2 - 6

-4

5 \div 3

1.666666667

SEVERAL OBSERVATIONS SHOULD BE MADE CONCERNING THE ABOVE CALCULATIONS. FIRST, A SPACE MAY OR MAY NOT APPEAR BETWEEN AN OPERATOR AND ITS ARGUMENTS (IN APL, THE TERM ARGUMENT(S) IS USED INSTEAD OF INDEPENDENT VARIABLE(S), FOR THE TERM VARIABLE IS USED IN A SPECIAL WAY, TO BE SEEN LATER). IT IS PERMISSIBLE TO TYPE $8 \times 9$, $8 \times 9$, $8 \times 9$, OR $8 \times 9$, FOR ALL WILL BE COMPUTED TO BE 72 ($8 \times 9$ IS USUALLY PREFERRED SINCE SPACING TAKES TIME). IN ORDER TO DISTINGUISH BETWEEN THE
OPERATOR SUBTRACTION AND THE ADDITIVE INVERSE OF A NUMBER, A HIGH BAR WILL BE USED TO DESIGNATE THE ADDITIVE INVERSE, SUCH AS \(-5\). THE COMPUTER WILL PRINT AT MOST TEN SIGNIFICANT FIGURES IN YIELDING RESULTS. THE COMPUTER ACTUALLY CARRIES OUT ALL CALCULATIONS TO SIXTEEN PLACES BUT-rounds off to ten places in the output.

MANY OTHER PRIMITIVE OPERATORS ARE AVAILABLE WHICH WILL BE OF USE IN THIS COURSE. THE EXPONENTIAL OPERATOR IS \(*\). THUS, IF WE DESIRE FOUR CUBED AND THE POSITIVE SQUARE ROOT OF 16, WE WOULD HAVE THE FOLLOWING DIALOGUE:

\[4*3\]

64

16* .5

4

IF NO LEFT ARGUMENT IS USED, THEN \(e\) WILL BE RAISED TO THE EXPONENT TO THE RIGHT OF \(*\) AND EVALUATED. FOR EXAMPLE,

\[\ast 1\]

2.718281828
\[ 20.08553692 \]

The expression used to obtain the natural logarithm is \( \ln \), the * being overstruck by o, where \( n \) is any positive real number. For example:

\[
\begin{align*}
1 & \quad \ln \quad 0 \\
10 & \quad \ln \quad 0
\end{align*}
\]

\[ 2.302585093 \]

The familiar symbols \( <, \leq, =, \geq, > \), and \( \neq \) are used in a special way. For example:

\[
\begin{align*}
3 & \leq 4 \\
1 \\
4 & < 3 \\
0 \\
4 & = 3 \\
0
\end{align*}
\]
IF THE RELATIONSHIP BETWEEN THE TWO ARGUMENTS IS TRUE, A 1 RESULTS, OR IF FALSE, A 0 RESULTS.
IT IS TRUE THAT 3 IS LESS THAN 4, BUT IT IS FALSE THAT 4 IS LESS THAN 3 AND THAT 4 IS EQUAL TO 3.

IT WILL BE VERY CONVENIENT TO HAVE SIN, COS, AND TAN ACCESSIBLE AS PRIMITIVE OPERATORS.
A 1, 2, OR 3 WILL INDICATE WHETHER WE WISH THE SIN, COS, OR TAN, RESPECTIVELY, OF AN ANGLE IN RADIANS.
THE SYMBOL FOR THE TRIGONOMETRIC OPERATOR IS O. FOR EXAMPLE,

101
0.8414709848

201
0.5403023059

301
1.557407725

THAT IS, SIN 1 = 0.8414709848, COS 1 = 0.5403023059, AND TAN 1 = 1.557407725. TABLE ONE LISTS MANY, BUT BY NO MEANS ALL, OF THE PRIMITIVE OPERATORS.

<table>
<thead>
<tr>
<th>OPERATOR</th>
<th>NAME</th>
<th>EXAMPLE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Operation</td>
<td>Example</td>
<td>Result</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Addition</td>
<td>(1 + 2.5)</td>
<td>3.5</td>
</tr>
<tr>
<td>Subtraction</td>
<td>(0.3 - 1)</td>
<td>-0.7</td>
</tr>
<tr>
<td>Multiplication</td>
<td>(4 \times 1.2)</td>
<td>4.8</td>
</tr>
<tr>
<td>Division</td>
<td>(8 \div 2)</td>
<td>4</td>
</tr>
<tr>
<td>Exponential</td>
<td>(5^{-2})</td>
<td>0.04</td>
</tr>
<tr>
<td>Absolute Value</td>
<td>(</td>
<td>-2</td>
</tr>
<tr>
<td>Natural Logarithm</td>
<td>(\log(3))</td>
<td>1.09861229</td>
</tr>
<tr>
<td>Maximum</td>
<td>(-210)</td>
<td>0</td>
</tr>
<tr>
<td>Minimum</td>
<td>(-210)</td>
<td>-2</td>
</tr>
<tr>
<td>Less Than</td>
<td>(-3.1 &lt; 5.8)</td>
<td>1</td>
</tr>
<tr>
<td>Less Than or Equal</td>
<td>(2 \leq 2)</td>
<td>1</td>
</tr>
<tr>
<td>Equal</td>
<td>(2.9 = 3)</td>
<td>0</td>
</tr>
<tr>
<td>Greater Than or Equal</td>
<td>(6 \geq 8)</td>
<td>0</td>
</tr>
</tbody>
</table>
>      GREATER THAN       1>0       1
≠       NOT EQUAL       3≠3       0
°      TRIGONOMETRIC
 1  SIN       102  0.9092974268
 2  COS       202 -0.4161468365
 3  TAN       302 -2.185039863

NOTE THAT THE ABOVE OPERATORS ARE USED WITH SCALAR (REAL NUMBER) ARGUMENTS AND
GIVE A SCALAR RESULT. THESE OPERATORS CAN BE EXTENDED TO VECTORS, AS WILL BE SEEN LATER.
ALSO, IT SHOULD BE POINTED OUT THAT MANY OF THE ABOVE OPERATORS CAN BE USED WITH EITHER ONE OR
TWO ARGUMENTS, SUCH AS *; HOWEVER, THE ABOVE TABLE WILL BE ADEQUATE FOR THE PURPOSES OF MATH
112.

ONE OF THE CONVENIENT FEATURES OF APL/360 IS THE INTERRUPT (INTER) KEY. A CALCULATION
IN PROGRESS MAY BE TERMINATED BY HITTING THIS KEY. THE CALCULATIONS OF TABLE ONE ARE CARRIED
OUT SO QUICKLY THAT INTERRUPTION OF THE CALCULATIONS WOULD NOT SAVE ANY TIME. BUT WHEN WE
CONSIDER VECTORS AND START OBTAINING MULTIPLE ANSWERS, A HALT TO THE CALCULATIONS IN PROGRESS
MAY BE DESIRED AND THE INTERRUPT KEY PROVIDES THIS LUXURY.

APL, UNLIKE MANY OTHER PROGRAMMING LANGUAGES, DOES NOT HAVE AN ORDER OF PRECEDENCE FOR OPERATORS. IN APL, THE ORDER OF EXECUTION DEPENDS ON ONLY TWO THINGS: PARENTHESES, AND THE ORDER IN WHICH THE OPERATORS APPEAR IN THE INSTRUCTION. FOR EXAMPLE, $2 \times 5 + 10 \div 5$ IS EXECUTED FROM RIGHTMOST TO LEFTMOST AND SIMPLIFIES TO 14. BUT $2 \times (5 + 10) \div 5$ EVALUATES TO 6, AS THE PARENTHEtical EXPRESSION $(5 + 10)$ IS THE FIRST EXPRESSION TO BE SIMPLIFIED. CARE MUST BE EXERCISED IN THE FORMULATION OF SUCH COMPOUND EXPRESSIONS AS THE ABOVE AND PARENTHESES SHOULD BE USED, IF NECESSARY, TO INDICATE PRECISELY THE ORDER IN WHICH CALCULATIONS ARE TO BE PERFORMED.
STATEMENTS

THERE ARE TWO MAIN TYPES OF APL STATEMENTS. THE SPECIFICATION STATEMENT IS DENOTED BY ← AND THE BRANCH STATEMENT IS DENOTED BY →. A TYPICAL SPECIFICATION STATEMENT IS OF THE FORM:

INTEREST ← .06

THIS SPECIFICATION WOULD CAUSE THE VALUE .06 TO BE ASSOCIATED WITH THE NAME INTEREST. SINCE INTEREST IS A NAME TO WHICH WE ARE FREE TO ASSIGN ANY VALUE WE CHOOSE, EVEN THOUGH .06 WAS CHOSEN HERE, IT AND OTHER NAMES WHICH ARE USED IN AN ANALOGOUS MANNER ARE USUALLY CALLED VARIABLES. A VARIABLE MUST ALWAYS HAVE BOTH A NAME AND A VALUE. A NAME WITHOUT A VALUE IS MEANINGLESS IN APL. NO VALUE CAN BE STORED WITHOUT A NAME. THE NAME OF A VARIABLE MUST ALWAYS BEGIN WITH A LETTER OF THE ALPHABET (Δ IS CONSIDERED A LETTER OF THE ALPHABET); AFTER THAT, THE NAME MAY CONTAIN ANY COMBINATION OF LETTERS OR NUMERALS, AND MAY BE AS LONG AS DESIRED (USUALLY SHORT NAMES WILL BE DESIRED BECAUSE OF THE TIME SAVED IN TYPING). NAMES CANNOT CONTAIN A SPACE, ANY FORM OF PUNCTUATION, OR ANY OF THE SYMBOLS USED FOR FUNCTIONS (OTHER THAN Δ).
THE BRANCH STATEMENT TELLS THE COMPUTER WHICH LINE TO GO TO NEXT IN A PROGRAM. THE POWER OF THIS STATEMENT WILL BECOME CLEARER WHEN WE DISCUSS PROGRAMS. BUT FOR NOW IT IS SUFFICIENT TO SAY THAT →3 INSTRUCTS THE COMPUTER TO GO TO LINE 3 IN A PROGRAM; →0 INSTRUCTS THE COMPUTER TO STOP WORK AND EXIT FROM THE PROGRAM. SUPPOSE THAT THE USER DEFINES THE FOLLOWING INSTRUCTION:

→3×x<k

FIRST, THIS CALLS FOR A TEST TO SEE WHETHER OR NOT X IS LESS THAN K. IF X IS LESS THAN K, THEN THE EXPRESSION x<k HAS THE VALUE 1. IF X IS NOT LESS THAN K, THEN x<k HAS THE VALUE 0. HENCE, 3×x<k IS EITHER GOING TO COMPUTE TO BE 3 OR 0 AND →3×x<k WILL INSTRUCT THE COMPUTER TO GO TO EITHER LINE 3 OR LINE 0 (THAT IS, EXIT FROM THE PROGRAM).
ERRORS AND CORRECTIONS

Even the best of secretaries make typing errors. Hence, it is only reasonable to assume that the user of the computer will make typing errors at his terminal. Such errors are easily rectified. For example, suppose that the user types 3 x 23 when he really wanted 3 x 53. Then in order to correct this, the user should backspace the typeball over to where the error begins. Then the interrupt key should be hit. This results in an inverted caret (\^) being typed under the error, as follows:

\[
\begin{align*}
3 \times 23 \\
\text{\^} \\
53
\end{align*}
\]

After typing the \^, the computer moves the paper up a line to await the correction. Everything beginning with and to the right of the \^ must be retyped. The corrected calculation of 3 times 53 is as follows:

\[
\begin{align*}
3 \times 23 \\
\text{\^} \\
53
\end{align*}
\]
THERE ARE MANY TYPES OF ERRORS OTHER THAN ORDINARY TYPING ERRORS. THE FOLLOWING TABLE LISTS SOME OF THE MOST COMMON ERRORS WITH THE PROBABLE CAUSE. FOLLOWING THE TABLE WILL BE EXAMPLES OF THESE ERRORS.

<table>
<thead>
<tr>
<th>ERROR</th>
<th>CAUSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>CHARACTER</td>
<td>NON-PERMISSIBLE OVERSTRIKE</td>
</tr>
<tr>
<td>DEFN</td>
<td>INCORRECT USE OF 7 OR []</td>
</tr>
<tr>
<td>DOMAIN</td>
<td>ATTEMPTED EXECUTION OF A OPERATOR FOR ARGUMENTS NOT IN ITS DOMAIN</td>
</tr>
<tr>
<td>RESEND</td>
<td>TRANSMISSION FAILURE</td>
</tr>
<tr>
<td>SYNTAX</td>
<td>INVALID STRUCTURE OF INSTRUCTIONS</td>
</tr>
<tr>
<td>VALUE</td>
<td>ATTEMPTED USE OF A NAME WHICH HAS NOT PREVIOUSLY BEEN ASSIGNED A VALUE</td>
</tr>
</tbody>
</table>

TABLE TWO

EXAMPLES OF THE ABOVE ERRORS AND CORRECTIVE ACTION WILL NOW BE GIVEN. SUPPOSE THAT ONE WISHES TO MULTIPLY 29 AND 32 BUT TYPES 29×42 AND ATTEMPTS TO CORRECT THIS BY OVERSTRIKING THE 4 WITH A 3. THEN THE FOLLOWING DIALOGUE WOULD OCCUR:

---

92
BENEATH CHARACTER ERROR THE STATEMENT IS RETYPED BY THE COMPUTER UNTIL THE POINT OF ERROR IS
DENOTED BY A CARET ON THE NEXT LINE. CORRECTIVE ACTION CONSISTS OF RETYPING THE ENTIRE
INSTRUCTION, VOID OF ANY OVERSTRIKE. THE CAUSE OF THIS ERROR WAS SAID TO BE A NON-PERMISSIBLE
OVERSTRIKE BECAUSE IN ORDER TO COMPLETE SOME SYMBOLS AN OVERSTRIKE IS REQUIRED (RECALL 
). AN EXAMPLE OF A DEFINITION ERROR WILL BE GIVEN IN THE SECTION ON DEFINED OPERATORS. WE
NOW LOOK AT AN EXAMPLE OF A DOMAIN ERROR.

SUPPOSE THE USER ATTEMPTS TO DIVIDE 3 BY 0. THEN THE FOLLOWING DIALOGUE WOULD OCCUR:

3÷0

DOMAIN ERROR

3÷0

THE CARET APPEARS BENEATH THE DIVISION SYMBOL, INDICATING THAT FOR THIS OPERATOR THAT ITS
RIGHT ARGUMENT IS NOT IN ITS DOMAIN. CORRECTIVE ACTION WOULD CONSIST OF RE-EXECUTING THE
OPERATOR ONLY FOR VALUES IN ITS DOMAIN.

SUPPOSE THAT THE FOLLOWING DIALOGUE TAKES PLACE:

6+3×68
RESEND

ALL WE KNOW AT THIS POINT IS THAT A TRANSMISSION ERROR (NOT THE USER'S FAULT) HAS OCCURRED.
THE EXPRESSION 6+3×68 SHOULD BE RETYPED. THIS TYPE OF ERROR IS RARE.

AN EXAMPLE OF A SYNTAX ERROR WOULD BE THE FOLLOWING:

9AKL+3.2
SYNTAX ERROR

9 AKL+3.2
∧

HERE, THERE IS A SPACE BETWEEN THE NUMBER 9 AND AKL+3.2, THE CARET LYING BENEATH THE A; THIS
INDICATES THAT 9 IS NOT PERMISSIBLE AS THE FIRST ENTRY IN A NAME AND THAT IF IT IS OMITTED,
THEN AKL IS AN ACCEPTABLE NAME. ANOTHER COMMON SYNTAX ERROR IS UNMATCHED PARENCETHESES;
ALWAYS CHECK EXPRESSIONS TO MAKE SURE THERE ARE THE SAME NUMBER OF LEFT AND RIGHT PARENTHESES.
CONSIDER THE FOLLOWING DIALOGUE:

3+KL*XY-2.1

VALUE ERROR

3+KL*XY

∧

IN THIS CASE IT SHOULD BE CLEAR WHAT IS WRONG. KL IS A NAME WHICH HAS EVIDENTLY BEEN PREVIOUSLY ASSIGNED A VALUE, BUT NOT SO FOR THE NAME XY. CORRECTIVE ACTION CONSISTS OF ASSIGNING XY A VALUE BEFORE ATTEMPTING TO USE IT IN ANY CALCULATION.

THERE ARE OTHER TYPES OF ERRORS, BUT THE ONES ABOVE WOULD ORDINARILY BE THE MOST COMMON. IN THE EVENT THAT SOME OTHER TYPE OF ERROR IS INDICATED BY THE COMPUTER OR THAT TROUBLE IS ENCOUNTERED IN UNDERSTANDING OR CORRECTING ANY OF THE ABOVE ERRORS, THE INSTRUCTOR OR SOME OTHER INDIVIDUAL KNOWLEDGEABLE ABOUT THE COMPUTER SHOULD BE CONSULTED FOR ASSISTANCE.
DEFINED OPERATORS

It is convenient to have primitive operators. However, it is possible to have too many
primitive operators since, for example, we would not wish to have to sort through pages of
primitive operators in order to find an operator which is suitable for each of our needs (and
it would be difficult to have many more primitive operators on the keyboard than there are
now). It would be simpler merely to define the needed operators and use them accordingly.

In order to further introduce the idea of defined operators, we will now take a look at
the Pythagorean theorem. For any right triangle we know that \( c^2 = (a^2) + (b^2) \), where \( c \)
represents the hypotenuse and \( a \) and \( b \) are the legs of the triangle. Thus, if \( a \) is 3 and
\( b \) is 4, then \( c^2 \) will be 25 and hence \( c \) will be 5; that is, the length of the hypotenuse is
5 units. One way of accomplishing this calculation is the following:

\[
\begin{align*}
A &= 3 \\
B &= 4 \\
C &= ((a^2) + (b^2))^{.5} \\
&= ((3^2) + (4^2))^{.5} \\
&= 5
\end{align*}
\]
Typing the letter C and hitting the return key instructs the computer to display the value assigned to C.

A better way to go about this is to devise a method for assigning values to A and B and requesting the computation of C all at the same time. That is, we would like to have something analogous to the form 3×4, so that if we had something like a hypotenuse B, we could have the following dialogue:

3  hypotenuse  4

Of course, this assumes that we have previously defined the appropriate operator hypotenuse. A program for defining such an operator would look something like the following:

\[ z \leftarrow a \text{ hypotenuse } b \]

\[ z \leftarrow (a^2 + b^2)^{.5} \]

The mechanics of how to define hypotenuse will be illustrated below. Once we have it, we may use it for any values of A and B, similar to the use of \( \times \). For example,
6 HYPOTENUSE 8

12 HYPOTENUSE 5

13 WE SHALL NOW OUTLINE THE PROCEDURE FOR DEFINING OPERATORS. THE DEL SYMBOL (nę) IS TYPED TO INDICATE TO THE COMPUTER THAT THE USER WISHES TO DEFINE AN OPERATOR. ONCE ń IS TYPED, THE COMPUTER IS IN DEFINITION MODE. EVERYTHING TYPED AFTER THE ń WILL BE STORED BY THE COMPUTER. A RETURN TO EXECUTION MODE IS INITIATED BY TYPING ANOTHER ń. THE FIRST LINE OF OUR PROGRAM IS CALLED THE HEADER. IN THE ABOVE PROGRAM, ń2*A HYPOTENUSE B IS THE HEADER. AFTER THE HEADER IS TYPED AND THE CARRIER-RETURN KEY IS HIT, THE COMPUTER WILL TYPE A 1 IN SQUARE BRACKETS AND THEN SKIP OVER TO THE SIXTH SPACE TO AVOID AN INSTRUCTION OR EXPRESSION TO BE INCLUDED AS LINE 1 OF THE PROGRAM. IN THE ABOVE PROGRAM, THE FIRST LINE CONSISTS OF Z*(A*2)+B*2)*.5. NO OTHER LINES ARE NEEDED TO COMPLETE THIS PROGRAM, SO THE .5 IS FOLLOWED BY A ń, WHICH INDICATES TO THE COMPUTER THAT THE PROGRAM (DEFINITION OF AN OPERATOR) IS COMPLETE AND THAT THE USER WISHES TO RETURN TO EXECUTION MODE. THE NAME OF THIS OPERATOR IS HYPOTENUSE AND IT HAS TWO ARGUMENTS, A AND B. AS HAS BEEN SHOWN ABOVE, FOR VALUES OF A AND B THIS
OPERATOR WILL YIELD THE LENGTH OF THE HYPOTENUSE OF A RIGHT TRIANGLE WHOSE OTHER SIDES ARE A AND B.

THUS, HYPOTENUSE ACTS LIKE THE PRIMITIVE OPERATOR \( \times \). AND JUST AS \( \times \) CAN BE USED IN COMPOUND EXPRESSIONS SUCH AS \( 3+4 \times 6+2 \), SO TOO CAN HYPOTENUSE. FOR EXAMPLE,

\[
3+(6 \text{ HYPOTENUSE } 8)+2
\]

WHY ARE PARENTHESES PLACED ABOUT 6 HYPOTENUSE 8? WHY WERE PARENTHESES NOT PLACED ABOUT 4\times6? THERE SHOULD BE A QUESTION IN THE MIND OF THE READER CONCERNING THE USE OF 2 IN THE DEFINITION OF THE OPERATOR HYPOTENUSE. THE NEXT SECTION WILL ANSWER THIS QUESTION.
TYPES OF VARIABLES

THE VARIABLE Z IN THE DEFINITION OF THE OPERATOR HYPOTENUSE IS CALLED A DUMMY VARIABLE. THE ARGUMENTS A AND B ARE ALSO DUMMY VARIABLES WITH REGARD TO THIS OPERATOR. A AND B MERELY SUPPLY THE INFORMATION REQUIRED BEFORE HYPOTENUSE CAN BE EXECUTED. Z IS USED ONLY TO INDICATE THAT THE COMPUTED VALUE OF HYPOTENUSE FOR VALUES OF A AND B WILL BE STORED AS Z (RECALL THAT A VALUE CANNOT BE STORED WITHOUT A NAME). OTHER LETTERS, SUCH AS K, L AND M COULD JUST AS EASILY HAVE BEEN USED. IT IS NOT THE DUMMY VARIABLE THAT IS IMPORTANT, BUT RATHER THE VALUE ASSIGNED TO IT WITH REGARD TO THE OPERATOR.

SUPPOSE THAT THE USER HAD EARLIER MADE THE ASSIGNMENTS A ← 6.2 AND Z ← 12.1. WE COULD HAVE THE FOLLOWING DIALOGUE:

3 HYPOTENUSE

5

A

6.2

B
VALUE ERROR

B
A
Z

-12.1

Why do we not have values of 3.4 and 5 for A, B and Z, respectively? Why do A and Z have values of 6.2 and -12.1, respectively, and why does calling for B result in a value error?

The answer to the first question is that A, B and Z are dummy variables with regard to the operator hypotenuse and hence will not carry with them the values they may have taken on during the execution of the operator hypotenuse. And because of this, A, B and Z must therefore have the values which they were last assigned. Since A and Z last had the values of 6.2 and -12.1 before the execution of hypotenuse, they still retain these values. The value error for B has occurred since evidently B had not been previously assigned a value.

We will now add to the confusion by stating that A and Z are also global variables. Since A and Z had previously been assigned values of 6.2 and -12.1, respectively, and since...
A AND Z ARE DUMMY VARIABLES WITH REGARD TO HYPOTENUSE. THEY HAVE RETAINED THESE VALUES.

SINCE THEY WILL RETAIN THESE VALUES UNTIL THEY ARE RESPECIFIED, SUCH AS

\[ A \leftarrow A + 3 \]

\[ A \]

9.2

THEY ARE CALLED GLOBAL VARIABLES. B QUALIFIED ONLY AS A DUMMY VARIABLE SINCE IT WAS A NAME WITHOUT A VALUE, BEYOND THE OPERATOR HYPOTENUSE. IN THE PRECEDING, A AND Z WERE GLOBAL VARIABLES WITH REGARD TO THE OPERATOR HYPOTENUSE.

SUPPOSE THAT K HAD BEEN PREVIOUSLY ASSIGNED THE VALUE 3 AND THAT OUR OPERATOR HYPOTENUSE HAD APPEARED AS FOLLOWS:

\[ \forall z + A \ \text{HYPOTENUSE} \ B \]

[1] \[ k \leftarrow A + B \]

[2] \[ z + ((A \times 2) + B \times 2)^{0.5} \]

NOW WE EXECUTE HYPOTENUSE AS FOLLOWS:

3 \ HYPOTENUSE \ 4
Our results are the same as before. But what will happen if we call for the value of k? Will k be 3 or 7? The answer is 7. Is k a global or dummy variable? The answer is that k is a global variable. Confused? Well, in unconfusing the issue, first note that k cannot be a dummy variable because it does not appear in the header (\( \text{VZ} \rightarrow A \ \text{HYPOTENUSE} \ \text{B} \)). But since k now has the value 7, can it be a global variable, or is it still another type of variable? Well, k is a global variable in APL. What happened to k was that it was not a dummy variable and it was respecified during the execution of the operator hypotenuse.

How can we keep k from losing its original value of 3? Well, (here we go again) by making k a local variable with regard to the operator hypotenuse. In order to do this we would have to place a semicolon after B and type k. The program would now look like the following:

\[
\text{VZ} \rightarrow A \ \text{HYPOTENUSE} \ \text{B}; \text{K}
\]

[1] \( K \rightarrow A+B \)

[2] \( Z \rightarrow ((A*2)+B*2)*.5V \)

Now assuming that k had previously been assigned the value 3 we could have the following
DIALOGUE:

3 HYPOTENUSE 4

K

EVEN THOUGH K TOOK ON THE VALUE 7 DURING THE COURSE OF THE PROGRAM, IT WAS LOCALIZED AND HENCE RETAINED ITS FORMER VALUE. THIS IS AN IMPORTANT FEATURE OF THIS COMPUTER SYSTEM, FOR MANY SERIOUS ERRORS HAVE BEEN MADE BY USERS WHO USE A NAME THINKING IT HAD ONE VALUE WHEN IT ACTUALLY HAD ANOTHER. FOR SUPPOSE THAT K HAD BEEN USED IN A LATER COMPOUND EXPRESSION, SUCH AS T+K*2+(X*9) AND K HAD NOT BEEN LOCALIZED TO HYPOTENUSE. THEN T COULD HAVE QUITE A DIFFERENT RESULT, DEPENDING ON WHETHER K WAS 3 OR 7, THAT IS, WHETHER OR NOT K HAD BEEN LOCALIZED OR NOT TO HYPOTENUSE.  THUS, A GOOD RULE OF THUMB WOULD BE TO LOCALIZE ANY AND ALL VARIABLES IN ORDER TO KEEP THEM FROM ACCIDENTALLY BEING RESPECIFIED. THE DANGER OF SUCH AN OCCURRENCE FOR THE NEEDS OF THIS COURSE ARE MINIMAL. BUT SUCH AN EVENT CAN OCCUR IF NOT SAEGUARDED AGAINST. ANY NUMBER OF LOCAL VARIABLES CAN BE USED BY MERELY SEPARATING THEM BY SEMICOLONS, SUCH AS
\text{VZ+A HYPOotenuse B;K;L;M;N;O;P.}

\text{NOW IS A GOOD TIME TO INTRODUCE THE CONCEPT OF A DEFINED OPERATOR WITH ONE ARGUMENT. FOR WE CAN DO THIS AND AT THE SAME TIME FURTHER ILLUSTRATE THE USE OF A DUMMY VARIABLE.}

\text{SUPPOSE THAT WE HAVE PREVIOUSLY DEFINED AN OPERATOR SQUAREFOOT AS FOLLOWS:}

\text{VZ+SQUAREFOOT X}

\text{[1] Z=X^{0.5}}

\text{IN THIS CASE WE HAVE AN OPERATOR BY THE NAME OF SQUAREFOOT WITH ONE ARGUMENT (THE ARGUMENT MUST ALWAYS LIE TO THE RIGHT OF THE OPERATOR NAME), X, AND TWO DUMMY VARIABLES, Z AND X. WE NOW USE SQUAREFOOT IN STILL ANOTHER VERSION OF HYPOtenuse, NAMELY,}

\text{VZ+A HYPOtenuse B}

\text{[1] Z=SQUAREFOOT ((A^{2}+B^{2})^{3}}

\text{DO NOT BE CONFUSED BY THE USE OF Z IN BOTH OF THESE OPERATORS, FOR IT IS MERELY A DUMMY VARIABLE. R AND S COULD JUST AS EASILY HAVE BEEN USED IN PLACE OF Z IN THE DEFINITION OF SQUAREFOOT AND HYPOtenuse, RESPECTIVELY. THE IMPORTANT THING IS USING SQUAREFOOT PROPERLY, AND IT IS BEING USED PROPERLY SINCE FOR VALUES OF A AND B IN EXECUTING HYPOtenuse, SQUAREFOOT}
WILL HAVE A SINGLE ARGUMENT X, NAMELY, THE QUANTITY \((A^2 + B^2)\). HENCE, WE COULD HAVE THE
FOLLOWING DIALOGUE AND OBTAIN THE SAME RESULT AS BEFORE:

3 HYPOTENUSE 4

5

IT SHOULD BE NOTED THAT WE CANNOT HAVE SEVERAL VERSIONS OF HYPOTENUSE AT THE SAME TIME
UNDER THE SAME NAME:

\[ VZ+A \text{ HYPOTENUSE } B \]

[\[ (A^2 + B^2)^{0.5} \]

\[ VZ+A \text{ HYPOTENUSE } B \]

[1] \[ K=A+B \]

[2] \[ z=((A^2+B^2)^{0.5} \]

\[ VZ+A \text{ HYPOTENUSE } B;K \]

[1] \[ K=A+B \]

[2] \[ z=((A^2+B^2)^{0.5} \]

ASSUMING THAT WE HAD ALREADY STORED OUR ORIGINAL OPERATOR HYPOTENUSE, AN ATTEMPT TO USE THE
NAME HYPOTENUSE FOR THE SECOND OF THE ABOVE THREE PROGRAMS WOULD RESULT IN THE FOLLOWING DIALOGUE:

VZ+A HYPOTENUSE B
DEFN ERROR
VZ+A HYPOTENUSE B

^ 

THE ABOVE WOULD INDICATE THAT HYPOTENUSE HAS ALREADY BEEN USED AS AN OPERATOR NAME. IF FOR SOME REASON ALL THREE OF THE PROGRAMS WERE DESIRED, THEY WOULD HAVE TO HAVE DIFFERENT NAMES, SUCH AS YP, HYPOT AND HYPOTENUSE, RESPECTIVELY.
EDITING

SUPPOSE THAT THE USER WISHES TO LOOK AT THE OPERATOR HYPOTENUSE WHICH HE HAD DEFINED EARLIER IN HIS WORK SESSION AT HIS TERMINAL. THEN TO DISPLAY THE PROGRAM WHICH DEFINED THIS OPERATOR HE WOULD ONLY HAVE TO TYPE `HYPOTENUSE[]\n` AND HIT THE CARRIER RETURN KEY. THE FOLLOWING DIALOGUE WOULD OCCURR:

```
@HYPOTENUSE[]\n@ 2+A HYPOTENUSE B
[1]  Z=((A*2)+B*2)*0.5
```

THE COMPUTER LEAVES A SPACE BETWEEN `\n` AND `Z` AND TYPES A `\n` AT THE END ON THE NEXT LINE; THE USER NEED NOT DO THIS.

SUPPOSE THE USER WISHES TO ADD ANOTHER LINE TO THE ABOVE PROGRAM. THEN THE PROCEDURE WOULD BE TO TYPE `HYPOTENUSE` AND HIT THE CARRIER RETURN KEY. THE FOLLOWING DIALOGUE WOULD OCCURR:

```
@HYPOTENUSE
THE USER WOULD THEN TYPE IN THE DESIRED INFORMATION FOR LINE 2 FOLLOWED BY ANOTHER V, UNLESS HE WISHES TO ADD STILL ANOTHER LINE. NOTICE THAT THE ABOVE PROCEDURE DOES NOT RESULT IN A DEFINITION ERROR. THIS IS BECAUSE SINCE WE ARE USING ONLY THE OPERATOR NAME INSTEAD OF THE COMPLETE HEADER, THE COMPUTER KNOWS THAT THE DESIRES OF THE USER IS TO ADD ANOTHER LINE TO THE PROGRAM WHICH DEFINES THE OPERATOR HYPOTENUSE.

THE READER MAY BEGIN TO WONDER AT THIS POINT WHETHER OR NOT THE EDITING OF OPERATOR DEFINITIONS IS WORTHWHILE, FOR EVEN THE EDITING PROCEDURE TAKES TIME. WELL, FIRST OF ALL, RECALL THAT WE CANNOT DEFINE ANOTHER OPERATOR BY THE NAME OF HYPOTENUSE; SUCH A TRY WOULD GIVE A DEFINITION ERROR AS WAS SEEN EARLIER. THIS IS AN ADVANTAGE RATHER THAN A DISADVANTAGE, FOR THE COMPUTER HELPS US KEEP FROM INADVERTANTLY DESTROYING SOME OPERATOR WE MIGHT LIKE TO HAVE FOR LATER USE. ONE ALTERNATIVE WOULD BE TO USE A SIMILAR NAME, SUCH AS HYP. BUT A BETTER WAY IS TO HAVE EDITING FEATURES. IF ONE HAD A PROGRAM CONSISTING OF FIFTEEN LINES AND HE WISHED TO ADD A SIXTEENTH, THEN A LOT OF TIME WOULD BE REQUIRED TO CHANGE THE NAME OF THE OPERATOR AND RETYPE THE FIRST FIFTEEN LINES. IN SUMMARY, EDITING FEATURES MAY INCREASE THE NOTATION AND
PROCEDURES, BUT ULTIMATELY THE TIME REQUIRED IN ACTUAL USE OF THE TERMINAL IS CONSIDERABLY
REDUCED.

SUPPOSE THAT INSTEAD OF ADDING A SECOND LINE TO THE PROGRAM DEFINING HYPOTENUSE WE WISHED
TO HAVE A NEW FIRST LINE. THEN WE START OFF AS ABOVE, BY TYPING VHYPOTENUSE. THE COMPUTER
RESPONDS AS ABOVE BY TYPING [2]. BUT SINCE THE USER DESIRES A NEW LINE PRIOR TO LINE 1, THE
USER TYPES [.9] (ANY NUMBER WILL DO AS LONG AS IT IS BETWEEN 0 AND 1, IN THIS CASE). IF THE
USER WISHES TO CHANGE A GLOBAL VARIABLE K WITH A VALUE OF 3 TO A VALUE OF A+B, HE WOULD TYPE
K=A+B. IF NO OTHER CHANGES WERE DESIRED, THEN A DEL WOULD BE TYPED AFTER B. THE COMPLETE
DIALOGUE WOULD APPEAR AS FOLLOWS:

VHYPOTENUSE
[2] [.9]
[0.9] K=A+B

WHAT HAPPENS WHEN WE ASK THE COMPUTER TO DISPLAY OUR PROGRAM DEFINING HYPOTENUSE? WELL, THE
FOLLOWING WOULD OCCUR:

VHYPOTENUSE[[]]V
\[ z+a \text{ HYPOTENUSE } b \]

\[ k+a+b \]

\[ z+((a*2)+b*2)*0.5 \]

\[ \text{\textdagger} \]

Since lines with decimal numbers are often undesirable, the computer has automatically renumbered the lines of the program in integers. This procedure will work for inserting a line between any two lines of a program.

Suppose that the user wishes to change \( k \) to \( m \) in the above displayed program. Then the following dialogue will result in the desired change:

\[ \text{\textdagger} \text{HYPOTENUSE} \]

\[ 3 \quad [1] \]

\[ 1 \quad m+a+b \text{\textdagger} \]

\[ 1 \quad m+a+b \]

\[ \text{\textdagger} \text{HYPOTENUSE}[\!\!\!]\text{\textdagger} \]

\[ \text{\textdagger} z+a \text{ HYPOTENUSE } b \]

\[ 1 \quad m+a+b \]
[2]  \[ z = \sqrt{(A^2 + B^2)} \]

\[ \checkmark \]

Thus, in general, this procedure can be used to replace a line with another line. Suppose

Now that the user wishes to go back to the way he originally had hypotenuse defined. Well, in order to accomplish this, he must eliminate, not replace, line 1. The procedure is to begin by typing \texttt{hypotenuse[1]}. The computer types [1], and then to delete line 1 the user hits the interrupt key, followed by the return key. The computer responds with an inverted caret. The user then types another \checkmark and the deletion of line 1 is complete. The dialogue would have appeared as follows:

\texttt{hypotenuse[1]}

[1]

\[ \checkmark \]

[2]  \[ \checkmark \]

A request for a display of the program defining hypotenuse reveals the original program:
\[ \text{HYPOTENUSE} \]  
\( \sqrt{Z} = A \cdot \text{HYPOTENUSE} \cdot B \)  
\[ [1] \quad Z = (A^2 + B^2)^{0.5} \]  

Note that in requesting a display of the above program, two DELS have been used, one at the beginning and one at the end of the display request. Hence, after the program has been displayed, the computer is in execution mode, not definition mode. If the user types an even number of DELS, the computer will be in execution mode; otherwise, the computer will be in definition mode.
VECTORS

A VECTOR IS A ONE-DIMENSIONAL ARRAY. FOR EXAMPLE, IF WE TYPE X=1 2 3, THEN X IS A VECTOR WHOSE ELEMENTS ARE LISTED FROM LEFT TO RIGHT AS 1, 2 AND 3, RESPECTIVELY. AT LEAST ONE SPACE MUST APPEAR BETWEEN ELEMENTS OF A VECTOR. NOTE THAT A ONE-ELEMENT VECTOR IS A SCALAR. A TWO-DIMENSIONAL ARRAY IS Called A MATRIX. FOR EXAMPLE, 1 2 3 IS A TWO BY THREE MATRIX (TWO ROWS AND THREE COLUMNS). WE WILL NOT BE CONCERNED WITH MATRICES OR HIGHER DIMENSIONAL ARRAYS, BUT IT IS WORTHY OF NOTE THAT MOST OF OUR OPERATORS ON VECTORS HAVE NATURAL EXTENSIONS TO HIGHER DIMENSIONAL ARRAYS. IF WE TYPE Y=2 1 3, THEN X AND Y ARE NOT THE SAME BECAUSE OF A CHANGE IN POSITION OF THE ELEMENTS. ALSO, Z=2 1 3 IS NOT THE SAME VECTOR AS X SINCE ONE OF THE ELEMENTS IS DIFFERENT. FOR TWO VECTORS TO BE IDENTICAL, THEY MUST HAVE THE SAME NUMBER OF ELEMENTS (THREE, IN THE ABOVE EXAMPLES) AND EACH POSITION MUST BE FILLED BY THE SAME ELEMENT. pX WILL GIVE THE NUMBER OF ELEMENTS FOR ANY VECTOR. FOR EXAMPLE,

pX
TO IDENTIFY ELEMENTS OF A VECTOR WE PROCEED AS FOLLOWS:

\[ y[1] \]

\[ y[2] \]

\[ y[3] \]

CONSIDER THE FOLLOWING COMPUTATIONS:

\[ 1 \ 3 \ 5 \times 2 \ 4 \ 6 \]

\[ 2 \ 12 \ 30 \]
\[ 1 - 2 \ 0 \ 1 - 3 \ 4 \ 9 \]
\[ 2 \ 2 \ 9 \]
\[ 3 \ 1 \ 6 \ 9 \ 12 \]
\[ 1 \ 3 \ 3 \ 3 \]
\[ 2 \ 4 \ 6 \ 8 \div 2 \]
\[ 1 \ 2 \ 3 \ 4 \]
\[ 1 \ 3 \ 5 \ 0 \ 4 \ 8 \]
\[ 1 \ 4 \ 8 \]
\[ 1 \ 4 \ 6 \ 8 \times 5 \]
\[ 1 \ 1 \ 0 \ 0 \]
\[ \otimes 1 \ 5 \ 10 \ 12 \]

0 1.098612289 2.302585093 2.48490655

The above results suggest that when using the operators of Table One that each such operator applies to a pair of vectors element by element or to a vector and a scalar, the scalar being used with each element of the vector. The exponential operator also applies to a single
VECTOR TO OBTAIN POWERS OF $\epsilon$ (ACTUALLY, MOST OF THE OPERATORS OF TABLE ONE WILL APPLY TO A SINGLE VECTOR, BUT SUCH USE OF THESE OPERATORS WILL NOT BE NEEDED). THE NATURAL LOGARITHM OPERATOR IS THE ONLY OPERATOR WHICH IS TO BE USED ONLY WITH A SINGLE VECTOR.

ONE PARTICULAR OBSERVATION SHOULD HAVE BEEN MADE BY THE READER CONCERNING THE OPERATORS OF TABLE ONE. AS LONG AS THE PRIMITIVE OPERATORS OF TABLE ONE WERE BEING USED WITH A SCALAR OR SCALARS, A SCALAR RESULT WAS OBTAINED. NOTE THAT WHEN USING THESE OPERATORS WITH VECTORS WITH MULTIPLE ELEMENTS A VECTOR RESULT IS OBTAINED. Thus, WE SAY THAT THE PRIMITIVE OPERATORS HAVE BEEN EXTENDED TO VECTORS.

BUT OTHER TYPES OF OPERATORS ARE CONVENIENT WHEN WORKING WITH VECTORS. WE HAVE ALREADY MET TWO OF THESE OPERATORS IN THE BEGINNING OF THIS SECTION, NAMELY, $\varphi$ AND $[ ]$. $\varphi$ GAVE US THE NUMBER OF ELEMENTS COMPRISING A VECTOR. $[ ]$ ENABLES US TO IDENTIFY THE ELEMENTS IN THE POSITION DESIGNATED BY THE NUMBER APPEARING BETWEEN THE BRACKETS. FOR THE VECTOR $0 \ 3 \ -2 \ 4 \ 1$ CONSIDER THE FOLLOWING:

$2 \varphi C$

$4 \ 1$


\[-1+C\]

\[3 -2 4\]

\(\phi C\)

\[1 4 -2 3\]

\(+/C\)

6

6, C

6 3 -2 4 1

2+C resulted in the first two elements beginning on the left being dropped from the vector C. 
\(-1+C\) resulted in the rightmost element being dropped from vector C. \(\phi C\) completely reversed 
the positions of the elements of C. +/C caused all of the elements of C to be added together 
and is called the sum reduction of a vector. 6,C caused the number 6 to be concatenated to the 
vector C, resulting in a new vector in which 6 occupies the first position. One other 
operator of use to us will be \(x_{+13}\). If we assign \(x_{+13}\) then \(x\) will be a vector whose entries 
are 1, 2 and 3. That is,
Also, \( \mathbf{13} \) will the vector \( \mathbf{1} \mathbf{2} \mathbf{3} \). Table three lists the above operators with further examples of their use.

<table>
<thead>
<tr>
<th>OPERATOR</th>
<th>NAME</th>
<th>EXAMPLE</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho )</td>
<td>RHO</td>
<td>( \rho \mathbf{1 9 6 2} )</td>
<td>4</td>
</tr>
<tr>
<td>[ ]</td>
<td>INDEX</td>
<td>( A[3] ), where ( A=0 \mathbf{4 2 5} )</td>
<td>2</td>
</tr>
<tr>
<td>( + )</td>
<td>DROP</td>
<td>( 3+\mathbf{A} )</td>
<td>5</td>
</tr>
<tr>
<td>( \phi )</td>
<td>REVERSE</td>
<td>( \phi \mathbf{1 6 -3} )</td>
<td>( -3 6 1 )</td>
</tr>
<tr>
<td>( +/ )</td>
<td>SUM REDUCTION</td>
<td>( +/-\mathbf{2 0 3} )</td>
<td>1</td>
</tr>
<tr>
<td>( \cdot )</td>
<td>CATENATION</td>
<td>( -3,2,1 )</td>
<td>( -3 2 1 )</td>
</tr>
<tr>
<td>( \mathbf{1} )</td>
<td>IOTA</td>
<td>( \mathbf{14} )</td>
<td>( \mathbf{1234} )</td>
</tr>
</tbody>
</table>

**Table Three**

The usefulness of the operators of Table Three will become most apparent when we study
INTEGRATION. A MORE IMMEDIATE USE OF VECTORS IS IN THE REPEATED EXECUTION OF THE DEFINED
OPERATOR HYPOTENUSE IN ITS ORIGINAL FORM:

\[ \sqrt{z^2 + a^2} \text{ HYPOGENUSE B} \]

\[ z = \sqrt{(a^2 + b^2)} \]

[1] \[ z = ((a^2) + b^2) ^ {1/2} \]

WE WILL NOW EVALUATE HYPOTENUSE FOR SEVERAL DIFFERENT PAIRS OF LENGTHS
OF THE LEGS OF A RIGHT TRIANGLE.

3 6 8 9 6 11 4 12 18

3.60551275 6.08276253 8.94427191 15 18.9736596

THUS, WE OBTAIN THE SOLUTION TO SEVERAL PROBLEMS AT ONCE. IF WE WISH TO HOLD ONE LEG CONSTANT
AT A VALUE OF 5 AND COMPUTE THE LENGTH OF THE HYPOTENUSE WHEN THE OTHER LEG TAKES ON EVERY
INTEGER FROM 1 TO 10, WE CAN ACCOMPLISH THIS AS FOLLOWS:

5 10

WORKSPACES

When a user signs on to the computer, a block of internal storage (quite often referred to as memory) is set aside for his use. It is in this memory area of the computer that the users calculations will be executed. This memory area is called a workspace. As we shall see later, there are many types of workspaces, but the one workspace in which the user is currently working is called an active workspace. The size (how much information it will hold) is set at the central computer, but for the purposes of this course we should not have to be concerned about filling up our workspace.

Several commands can be initiated by the user of a terminal concerning his active workspace. Suppose that the user has made the following assignments during his use of the terminal at a particular terminal session: \(a=3\), \(b=-6\) and \(k=32\). Then we can request a listing of these variables as follows (they will be printed out in alphabetical order):

)VARS

A B K
THE COMPUTER HAS RESPONDED BY TYPING A B K AS THE VARIABLES WE HAVE USED SO FAR. SUPPOSE THE
USER HAS USED THE NAMES BAT, MATH, F, AND DER AS OPERATOR NAMES. THEN HE CAN REQUEST A LISTING
OF THESE OPERATOR NAMES AS FOLLOWS (THEY WILL ALSO BE PRINTED OUT IN ALPHABETICAL ORDER):

)FNS
BAT DER F MATH
THE COMPUTER HAS RESPONDED BY TYPING THE OPERATOR NAMES WHICH THE USER HAS USED IN HIS ACTIVE
WORKSPACE.

IF WE WISH TO ERASE EVERYTHING IN OUR ACTIVE WORKSPACE, THEN THE FOLLOWING DIALOGUE WILL
ACCOMPLISH THIS:

)CLEAR
CLEAR WS
THE COMPUTER HAS RESPONDED CLEAR WS AND WE NOW HAVE A COMPLETELY NEW ACTIVE WORKSPACE, WITH NO
OLD VARIABLE OR OPERATOR NAMES. IT SHOULD ALSO BE POINTED OUT THAT EACH TIME WE SIGN ON TO
THE COMPUTER WE RECEIVE A NEW ACTIVE WORKSPACE AND WE LOSE EVERYTHING IN OUR ACTIVE WORKSPACE
WHEN WE SIGN OFF UNLESS WE SAVE IT.
IF THE USER WISHES TO SAVE A COPY OF HIS ACTIVE WORKSPACE FOR FUTURE USE, HE MUST FIRST GIVE THE WORKSPACE A NAME, SAY DERIV. THE DIALOGUE WITH THE COMPUTER WOULD BE AS FOLLOWS:

)SAVE DERIV

9.32.23 10/13/72

A COPY OF THE ACTIVE WORKSPACE HAS BEEN STORED UNDER THE NAME DERIV. THE COMPUTER HAS GIVEN THE TIME AND DATE OF STORAGE.

A SET OF SAVED WORKSPACES BY THE USER CONSTITUTES HIS PRIVATE LIBRARY. TO SEE WHAT WORKSPACES ARE IN HIS LIBRARY THE FOLLOWING DIALOGUE WOULD BE NECESSARY:

)LIB

INTEG

DERIV

THE COMMAND )LIB DIRECTS THE COMPUTER TO LIST THE NAMES OF STORED WORKSPACES BY THE USER. SINCE INTEG HAS APPEARED IN THE LIST THIS INDICATES THAT THE USER HAD STORED A WORKSPACE BY THE NAME OF INTEG AT AN EARLIER DATE.
SUPPOSE THAT A FEW DAYS LATER A USER WISHES TO WORK WITH SOME OF THE OPERATORS HE HAD
STORED UNDER DERIV. THEN THE FOLLOWING DIALOGUE WOULD OCCURR:

)LOAD DERIV

SAVED 9.32.23 10/13/72

THE COMPUTER HAS RESPONDED BY INDICATING THE TIME AND DATE THE WORKSPACE WAS SAVED. THE ACT
OF LOADING A STORED WORKSPACE AUTOMATICALLY CLEARS THE ACTIVE WORKSPACE FROM WHENCE THE LOAD
COMMAND WAS GIVEN. SUPPOSE THAT THE VARIABLES AND OPERATORS DEFINED IN THE WORKSPACE DERIV
WERE A, K, L AND N AND POWER, EXPONENT AND TRIG, RESPECTIVELY. THEN THE FOLLOWING DIALOGUE
COULD OCCURR:

)VARS
A K L N

)FNS

EXPONENT POLY TRIG

THE COMPUTER, AS REQUESTED, HAS LISTED THE VARIABLE AND OPERATOR NAMES DEFINED IN THE
WORKSPACE DERIV. THESE VARIABLES AND OPERATORS CAN THEN BE USED JUST AS ON THE DAY THEY WERE
STORED. WHEN THE USER IS THROUGH WORKING WITH DERIV HE NEED ONLY GIVE THE COMMAND \texttt{CLEAR} AND THE COMPUTER ASSIGNS HIM A NEW ACTIVE WORKSPACE. ACTUALLY, WHEN THE USER LOADED DERIV, HE ONLY LOADED A COPY OF DERIV SO THAT WHEN HE GIVES THE COMMAND \texttt{CLEAR} THIS COPY IS DESTROYED BUT DERIV STILL REMAINS IN HIS LIBRARY FOR FUTURE ACCESS.

SUPPOSE THAT THE FOLLOWING DIALOGUE OCCURRS:

\texttt{)SAVE FCT}

NOT SAVED, WS QUOTA USED UP

THE COMPUTER HAS TOLD THE USER THAT HE CANNOT STORE ANOTHER WORKSPACE. BUT SUPPOSE THE USER NO LONGER NEEDS INTEG. THEN HE COULD REPLACE INTEG BY FCT AS FOLLOWS:

\texttt{)DROP INTEG}

15.16.32 08/06/72

\texttt{)SAVE FCT}

15.16.56 08/06/72

THE COMPUTER HAS REPLACED INTEG BY FCT, WITH TIMES AND DATE AS SHOWN. NOW THE FOLLOWING DIALOGUE COULD OCCURR:
FINALLY, SUPPOSE THE USER WISHES TO ADD THE OPERATOR LOP TO THE WORKSPACE DERIV. THEN
THE FOLLOWING DIALOGUE WOULD BE NECESSARY:

)LOAD DERIV

9.32.23 10/13/72

V2+LOP T

[1] Z+(T*2)-1V

)SAVE

15.18.03 08/06/72 DERIV

IT WAS NOT NECESSARY TO ENTER DERIV AFTER SAVE SINCE WE ARE ALREADY WORKING IN THIS WORKSPACE
AND THE COMPUTER CONFIRMS THIS BY TYPING DERIV AFTER THE DATE OF STORAGE. BUT JUST TO PURSUE
THE MATTER FURTHER WE WILL CHECK AS FOLLOWS:

)LOAD DERIV
THE WORKSPACES MENTIONED ABOVE WERE STORED IN PRIVATE LIBRARIES. THERE IS ONE OTHER TYPE OF LIBRARY THAT WILL BE OF INTEREST TO US, THE PUBLIC LIBRARY. THERE ARE SEVERAL PUBLIC LIBRARIES, ALL CONSISTING OF SAVED WORKSPACES. WE WILL BE CONCERNED ONLY WITH ONE OF THESE WORKSPACES, IN LIBRARY 1. TO FIND OUT WHAT WORKSPACES ARE IN LIBRARY 1, THE DIALOGUE WOULD BE AS FOLLOWS:

)LIB 1

FORMAT
ADVANCEDEX
TEXT
WSFNS
PLOTFORMAT
NEWS
CLASS

THE SAVED WORKSPACE WE WISH TO UTILIZE IS PLOTFORMAT, SO WE LOAD PLOTFORMAT AS FOLLOWS:

)LOAD 1 PLOTFORMAT

SAVED 15.00.56 07/24/72

NOTICE THAT A 1 IS INCLUDED IN THIS LOADING PROCEDURE, FOR WE ARE ASKING THE COMPUTER TO LOAD
A WORKSPACE FROM PUBLIC LIBRARY NUMBER 1, NOT OUR PRIVATE LIBRARY.

WHAT OPERATORS ARE INCLUDED IN THE WORKSPACE PLOTFORMAT? WELL, TO FIND OUT WE PROCEED AS
FOLLOW:

)PNS

AND DESCRIBE DFT EFT PLOT VS

IT IS THE PLOT OPERATOR WE WISH TO USE, FOR THIS PROGRAM WILL PLOT THE GRAPH OF ONE OR MORE
EXPRESSIONS.

SUPPOSE WE WISH THE GRAPH OF \( y = (2x(x^2)) - 1 \) FOR \( x = -3.6, -2.4, -1.2, 0, 1.2, 2.4, 3.6 \). THEN WE
OBTAIN THIS AS FOLLOWS:

40 PLOT ((2*(x*(x^2)))-1) VS x=-3.6 -2.4 -1.2 0 1.2 2.4 3.6
THE 40 APPEARING TO THE LEFT OF PLOT HAS TO DO WITH THE SPACING AVAILABLE FOR THE AXIES. WE
WILL ALWAYS USE 40. THE ACTUAL SCALESIZE IS DETERMINED BY THE COMPUTER AND WE WILL HAVE LITTLE
CONTROL OVER IT. THE \(((2\times(x^2))^{-1})\) VS \(x+3.6\) -2.4 -1.2 0 1.2 2.4 3.6 TELLS THE COMPUTER TO
PLOT \((2\times(x^2))^{-1}\) FOR EACH VALUE OF \(x\) LISTED. THE PLOTTING SYMBOL USED IN THIS CASE WAS *. 
THE SYMBOL USED FOR PLOTTING WILL BE DETERMINED BY THE COMPUTER.

THERE WILL BE TIMES WHEN PLOTTING THAT PART OF A GRAPH MAY LOOK LIKE *** WHEN ACTUALLY
THE GRAPH SHOULD BE **. THIS IS DUE TO THE SCALESIZE USED BY THE COMPUTER, WHICH AS

* 
MENTIONED BEFORE, WE HAVE LITTLE CONTROL OVER. THIS SHOULD BE NO GREAT HANDICAP FOR OUR
PURPOSES, HOWEVER.
PRIMITIVE OPERATORS

1. PERFORM THE FOLLOWING CALCULATIONS ON THE COMPUTER:
   
   (A) \( (3 \times (4+6)) + (2+8) \)
   
   (B) \( ((3 \times 4)+6) + (2+8) \)
   
   (C) \( ((3 \times 4)+(6+2))+8 \)
   
   (D) \( ((3 \times (4+6))+2)+8 \)

2. EXPRESS THE FOLLOWING IN APL FORM:

   (A) \( 6x^3 - 5x^2 - 2x + 5 \)

   (B) \( 6x \times 6 - 5 \)

   (C) \( \sqrt[6]{2x} - 6 + \sqrt{6x LN 8} \)
DEFINED OPERATORS

1. WRITE A PROGRAM TO DEFINE AN OPERATOR WHICH WILL CUBE ANY REAL NUMBER X. USE YOUR PROGRAM TO CUBE THE FOLLOWING NUMBERS: -10, -5, -1.0, 1.0, 5.0, AND 10.

2. I=PRT IS A FAMILIAR FORMULA IN BUSINESS, WHERE I REPRESENTS THE SIMPLE INTEREST, IN DOLLARS AND CENTS, FOR A GIVEN PRINCIPLE P, IN DOLLARS AND CENTS, FOR A PERIOD OF TIME T, IN YEARS, AT AN ANNUAL INTEREST RATE R, IN DECIMAL FORM. FOR EXAMPLE, IF P=$10.00, T=6 MONTHS, AND R=6%, THEN I=(10.00)(.06)(.5)=$.30.

WRITE A PROGRAM WHICH WILL YIELD THE INTEREST RECEIVED ON AN INVESTMENT OF $1.00 (HINT: USE A PROGRAM HEADER OF THE FORM R INI T). USE YOUR PROGRAM TO DETERMINE THE INTEREST RECEIVED ON AN INVESTMENT OF $1.00 WHERE R=8% AND T IS 3 MONTHS, 9 MONTHS 1 YEAR, 3 YEARS, AND 10 YEARS, RESPECTIVELY. FIX T AT 9 MONTHS AND DETERMINE THE INTEREST RECEIVED ON AN INVESTMENT OF $1.00 WHERE R IS 3%, 5%, 7%, AND 9%, RESPECTIVELY.
TYPES OF VARIABLES

WRITE A PROGRAM WHICH WILL USE THE OPERATOR INT1 TO YIELD THE INTEREST RECEIVED ON
AN INVESTMENT OF P DOLLARS (HINT: USE A PROGRAM HEADER OF THE FORM INTP P). USE
INTP TO OBTAIN THE INTEREST RECEIVED ON AN INVESTMENT OF $5,000.00 FOR 12
YEARS AT AN INTEREST RATE OF 3%.
EDITING

1. DEFINE INT1 AND INTP AS EARLIER. REQUEST A DISPLAY OF INT1 AND INTP. EDIT INTP SO THAT INTP LOOKS AS FOLLOWS:

   VZ+INTP P
   [1] K+P*R INT1 T
   [2] Z+P+K

EXECUTE INTP FOR R=6%, T=6 MONTHS, AND P=$2000.00.

2. EDIT INTP SO THAT INTP LOOKS AS FOLLOWS:

   VZ+INTP P
   [1] K+P*R INT1 T
   [3] Z+P+K

REQUEST A DISPLAY OF INTP. EXECUTE INTP FOR THE ABOVE VALUES OF R, T AND P.

WHAT DOES LINE [2] DO FOR US?

3. EDIT INTP SO THAT INTP LOOKS AS FOLLOWS:

   VZ+INTP P
   [1] Z+P*1+R INT1 TV

REQUEST A DISPLAY OF INTP. EXECUTE INTP FOR THE ABOVE VALUES OF R, T AND P.
VECTORS

USE INT1 TO OBTAIN THE INTEREST RECEIVED ON AN INVESTMENT OF $1.00 AT AN INTEREST RATE OF 6% FOR T = .5 .75 1 2 5 10. USE INT1 TO OBTAIN THE INTEREST RECEIVED ON AN INVESTMENT OF $1.00 FOR 5 YEARS FOR R = .02 .04 .08 .10 .15 .20. USE INT1 TO OBTAIN THE INTEREST RECEIVED ON AN INVESTMENT OF $1.00 UNDER THE FOLLOWING CONDITIONS: R = .02 .04 .08 .10 .15 .20 AND T = .5 .75 1 2 5 10.
WORKSPACES

1. MAKE THE FOLLOWING ASSIGNMENTS: A<3, B<6 AND K<32. REQUEST A LISTING OF THESE VARIABLE NAMES. DEFINE INT1 AS FOLLOWS:

\[ z2 + r \times int1 \]

[1] \[ z + r \times t4 \]

REQUEST A LISTING OF OPERATOR NAMES. CLEAR YOUR WORKSPACE. ASK FOR A LISTING OF VARIABLE AND OPERATOR NAMES, RESPECTIVELY. RE-DEFINE INT1 AND ASSIGN A THE VALUE 10. SAVE THIS WORKSPACE UNDER THE NAME DOG.

2. REQUEST A LISTING OF YOUR STORED WORKSPACES. LOAD DOG. REQUEST A LISTING OF VARIABLE AND OPERATOR NAMES, RESPECTIVELY. CLEAR YOUR WORKSPACE. DEFINE F AS FOLLOWS:

\[ z2 + f \times x \]

[1] \[ z + (x \times 3) + (-3 \times x \times 2) + 5 \times v \]

TRY TO SAVE THIS WORKSPACE UNDER THE NAME CAT.
3. LOAD DOG. DEFINE P AS ABOVE. GIVE THE COMMAND \textit{SAVE}. AGAIN LOAD DOG. ASK FOR A LISTING OF OPERATOR NAMES. CLEAR YOUR ACTIVE WORKSPACE. COPY A OUT OF DOG. ASK FOR A DISPLAY OF A. ERASE A. NOW LOAD DOG. ASK FOR A LISTING OF VARIABLE NAMES. ERASE A. GIVE THE COMMAND \textit{SAVE}. ASK FOR A LISTING OF VARIABLE NAMES. CLEAR YOUR WORKSPACE.

4. LOAD PLOTFORMAT INTO YOUR ACTIVE WORKSPACE. REQUEST A LISTING OF OPERATOR NAMES. ASK FOR A DISPLAY OF THE OPERATOR PLOT. COPY F OUT OF DOG. OBTAIN GRAPHS OF THE FUNCTIONS DETERMINED BY THE FOLLOWING RULES OF CORRESPONDENCE FOR THE INDICATED VALUES OF X:

\begin{enumerate}
\item[(A)] \( y = f(x, x \leftarrow -2.5 \ -2 \ -1.5 \ -1 \ -0.5 \ 0 \ 1.5 \ 2 \ 2.5 \ 3 \ 3.5 \ 4 \ 4.5) \)
\item[(B)] \( y = 2x - 1, \ x \leftarrow -3.6 \ -2.4 \ -1.2 \ 0 \ 1.2 \ 2.4 \ 3.6 \)
\item[(C)] \( y = e^x, \ x \leftarrow -1 \ 0 \ 1 \ 2 \ 3 \ 4 \)
\item[(D)] \( y = \ln x, \ x \leftarrow -0.5 \ 1 \ 3 \ 9 \ 27 \)
\end{enumerate}

CLEAR YOUR WORKSPACE. DROP THE WORKSPACE DOG.
APL PROGRAMS

139

\[ VZ + F X \]

\[ Z \leftarrow (A N Y ~ A L G E B R A I C ~ E X P R E S S I O N ~ I N ~ X) \]

\[ VZ + \Delta X S L O S E C X \]

\[ \Delta X \]

\[ Z \leftarrow \left( (F (X + \Delta X) - F X) * \Delta X \right) \]

\[ VZ + \Delta X S L O P T X \]

\[ \Delta X \]

\[ Z \leftarrow \left( (F (X + \Delta X) - F X) * \Delta X \right) \]

\[ Z \]

\[ \Delta X = \Delta X + 2 \]

\[ \rightarrow (X + |\Delta X|) * X + 0.01 V \]

\[ VZ + D E R ~ P O L Y \]

\[ Z \leftarrow (-1 + POLY) * \phi - 1 + p POLY V \]

\[ VZ + N P O I N T S ~ I N T A B ; K \]


\[ K = \Delta X * ; N \]

\[ Z \leftarrow I N T A B[1] * 0, K V \]

\[ VZ + N R T X S U M ~ I N T A B ; K ; L \]

\[ K = F N P O I N T S ~ I N T A B \]

\[ L = 1 + K \]

\[ Z = \Delta X * + / L V \]

\[ VZ + N L T X S U M ~ I N T A B ; K ; L \]

\[ K = F N P O I N T S ~ I N T A B \]

\[ L = -1 + K \]

\[ Z = \Delta X * + / L V \]

\[ VZ + N R E C T I N ~ I N T A B ; K ; L \]

\[ K = F N P O I N T S ~ I N T A B \]

\[ L = (-1 + K) * 1 + K \]

\[ Z = \Delta X * + / L V \]

\[ VZ + N R E C T E X ~ I N T A B ; K ; L \]

\[ K = F N P O I N T S ~ I N T A B \]

\[ L = (-1 + K) * 1 + K \]

\[ Z = \Delta X * + / L V \]

\[ VZ + I N T ~ P O L Y \]

\[ Z \leftarrow (POLY * \phi * p POLY) * 2 V \]
APPENDIX C

COMPUTER—CALCULUS PROBLEMS
13.1

1. USE SLOSEC TO SOLVE PROBLEM 1, PAGE 342, WHERE \( y = 3x^2 \). USE SLOSEC FOR \( \Delta x \) VALUES OF -.1, -.01 AND -.001. RE-DO PROBLEM 1 FOR \( y = 3x^2 \) USING A \( \Delta x \) OF .5 FOR \( x = -4 \) -3 -1 0 1 3 4. USE THE OPERATOR PLOT TO GRAPH THE FUNCTION DETERMINED BY \( y = 3x^2 \) FOR THESE VALUES OF \( x \). DRAW THE SECANT LINES FOR THE POINTS (-1,3), (0,0) AND (1,3), WHERE \( \Delta x \) IS .5.

2. RE-DO PROBLEM 1 FOR \( y = 3x^2 \) USING \( \Delta x = .5 \), 25, .125, .0625, .03125, .015625, .0078125.

SUPPOSE WE DEFINE THE OPERATOR SLOPT AS FOLLOWS:

\[ \forall z + \Delta x \text{ SLOPT } x \]

[1] \( \Delta x \)

[2] \( z + ((F \ x + \Delta x) - F \ x) \times \Delta x \)

[3] \( z \)

[4] \( \Delta x + \Delta x \times 2 \)

[5] \( \rightarrow (x + \Delta x) \geq x + .001 \)

TRY TO FIGURE OUT WHAT SLOPT WILL DO FOR US AND HOW.
13.2

1. Write a program to compute \( y = (x^2 - 9) / (x - 3) \). Use your program to find \( y \) for the following values of \( x \): 5, 4, 3.5, 3.1, 2.9, 2.5, 2 and 1. What is \( \lim_{x \to 3} (x^2 - 9) / (x - 3) \)? Use as many values of \( x \) as necessary to find \( \lim_{x \to \infty} (x^2 - 9) / (x - 3) \) and \( \lim_{x \to -\infty} (x^2 - 9) / (x - 3) \).

2. Use the operator plot to graph the function determined by \( y = (x^2 - 9) / (x - 3) \) for the above listed values of \( x \). Does your graph verify the above limits?
1. Write a program to compute \((x^4 - 16) / (x^2 + 4)\). Is the function determined by the rule of correspondence \(f(x) = (x^4 - 16) / (x^2 + 4)\), \(x \neq -4\), and \(f(-4) = 10\) continuous at \(x = -4\)? Use your program to help determine whether it is or not.

2. Suppose \(f(x) = x^2\). Consider the following programs:

\[
\text{vz+} f(x) \\
\text{[1]} \quad z + x \times 2v \\
\text{vz+ax slosec x} \\
\text{[1]} \quad \Delta x \\
\text{[2]} \quad z + ((f(x + \Delta x) - f(x)) \times \Delta x)
\]

Try fixing \(x\), say \(x=2\), and let \(\Delta x\) take on different values, say \(\Delta x \approx 1.5, 1.01\). Try \(\Delta x \approx 1.01, 0.5, 0.1, 0.01\). What do your results mean (use plot to graph the function determined by \(f(x) = x^2\) and study the graph in conjunction with the operator slosec to see what is happening)? If \(\Delta x\) is set as \(0.0001\) and \(x=0, 1, 2, 3, 4, 5\), what is the significance of the results for \(\Delta x\) slosec \(x\)?
13.5

**USE SLOPE WITH ΔX+4, X+2, AND THEN ΔX−2, X+2, TO DETERMINE \( \lim_{{Δx \to 0}} \frac{F(x+Δx) - F(x)}{Δx} \) WHERE**

(A) \( F(x) = 2x^2 - 5x + 2 \),

(B) \( F(x) = x^3 \).
1. Use SLOSEC with \( \Delta x = 0.001 \) to approximate \( f'(2) \) in Problem 2, Page 356.

2. Use SLOSEC with \( \Delta x = 0.001 \) to approximate \( g'(-3) \) for Problem 1(b) and \( f'(4) \) for Problem 5(b), Page 356; then calculate these derivatives using the formulas for differentiation. Now use SLOSEC with \( \Delta x = 0.001 \) to approximate \( g'(-3) \) and \( f''(4) \); then calculate these derivatives using the formulas for differentiation. Compare your results.

3. Consider \( g(x) = 81x^4 - 17x^2 + 26x - 54 \). We can identify \( g(x) \) by the coefficients of the powers of \( x \). That is, let \( \text{POLY} = 81 \ 0 \ -17 \ 26 \ -54 \); thus, POLY is a vector of coefficients. The following program will differentiate \( g(x) \), the result being the vector of coefficients 324 0 -34 26, where \( g'(x) = 324x^3 - 34x + 26 \):

   \[ \text{VZ} = \text{DER POLY} \]

   \[[1] \quad z \leftarrow (\text{POLY}) \times (1 + \text{POLY}) \]

   Go through this program, working from right to left as the computer would do, and write down the results of each step (\( \text{POLY} \), then \( 1 + \text{POLY} \), etc.) to see that for \( \text{POLY} = 81 \ 0 \ -17 \ 26 \ -54 \) we do indeed obtain 324 0 -34 26 for \( \text{DER POLY} \).

4. Use \( \text{DER POLY} \) to differentiate \( y = 3x^7 - 4x^3 + 2x - 1 \).
1. USE SLOSEC WITH ΔX=.001 TO APPROXIMATE H'(0) IN PROBLEM 3 AND H'(3) IN PROBLEM 11, PAGE 359. COMPARE YOUR RESULTS WITH THE RESULTS OBTAINED BY USING THE FORMULAS FOR DIFFERENTIATION.

2. CONSIDER THE CLOSED INTERVAL [A,B]. FOR THE MOMENT LET US ASSUME A=0 AND B=1 AND THAT WE WANT TO DIVIDE [0,1] INTO 10 EQUAL INTERVALS. THEN 1/10=.1 AND THE POINTS 0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1 WOULD BE THE POINTS WHICH WOULD ACCOMPLISH THIS SUBDIVISION. WRITE A PROGRAM WHICH WILL YIELD THE SUBDIVISION POINTS AS A VECTOR (SUCH AS THE 0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1 ABOVE) WHICH DIVIDE THE INTERVAL [A,B] INTO N EQUAL PARTS. [HINT: USE A PROGRAM HEADER OF THE FORM N POINTS INTAB, WHERE INTAB IS THE TWO ELEMENT VECTOR A B, N IS THE NUMBER OF EQUAL PARTS, AND POINTS IS THE NAME OF THE OPERATOR WHICH WILL GIVE THE POINTS, AS A VECTOR, WHICH ACCOMPLISH THE DIVISION OF [A,B] INTO N EQUAL PARTS].
1. For Problem 5, Page 365, use SLOSEC, with \( \Delta x = .001 \), and points to approximate \( f'(x) \) at the points which divide the interval \([-1, 3]\) into 40 equal parts. Reviewing the definition of critical number and Theorem 13.20, what do the above results tell you about \( f(x) \)? Differentiate \( f(x) \); use SLOSEC, with \( \Delta x = .001 \) and points to approximate \( f''(x) \) at the points which divide the interval \([-1, 3]\) into 40 equal parts.

2. Use PLOT to graph the functions determined by \( f(x) \), \( f'(x) \) and \( f''(x) \), respectively, for Problem 12, Page 365, over the interval \([-1, 5]\) using \( n = 12 \). First, does the graph of the function determined by \( f(x) \) agree with the graph you drew by hand? Secondly, do the graphs of the functions determined by \( f'(x) \) and \( f''(x) \) agree with the definition of critical number, Theorem 13.20, and Theorem 13.21?
1. ONE WAY OF WRITING THE POINTS OPERATOR IS THE FOLLOWING:

\[ \forall z \in \mathbb{N} \text{ POINTS INTAB; } k \]

[1] \( \Delta x \leftarrow (\text{INTAB}[2] - \text{INTAB}[1]) \times n \)

[2] \( k \leftarrow \Delta x \times n \)

[3] \( z \leftarrow \text{INTAB}[1] + 0, k \)

CONSIDER THE FOLLOWING TWO PROGRAMS:

\[ \forall z \in \mathbb{N} \text{ RTXSUM INTAB; } k; l \]

[1] \( k \leftarrow F N \text{ POINTS INTAB} \)

[2] \( l \leftarrow 1 + k \)

[3] \( z \leftarrow \Delta x \times l + l \)

\[ \forall z \in \mathbb{N} \text{ LTXSUM INTAB; } k; l \]

[1] \( k \leftarrow F N \text{ POINTS INTAB} \)

[2] \( l \leftarrow -1 + k \)

[3] \( z \leftarrow \Delta x \times l + l \)
Use \texttt{rtxsum} with \( n=4 \) to approximate the area under the curve \( y=x^3 \) over the interval \([0,1]\) (see Figure 14.1(a) and Figure 14.2 in the text). Compare your result with the result obtained in the text for Figure 14.2. Use \texttt{ltxsum} with \( n=4 \) to approximate the area under the curve \( y=x^3 \) over the interval \([0,1]\) (see Figure 14.1(a) and Figure 14.1(b)). Do you obtain the same approximation as obtained in the text?
2. DRAW A GRAPH OF A FUNCTION OVER [0, 1] SUCH THAT RTXSUM AND LTXSUM WILL NOT USE ONLY RECTANGLES ALL OF WHICH ARE ABOVE OR ALL OF WHICH ARE BELOW, BUT SOME RECTANGLES WHICH ARE ABOVE AND SOME BELOW THE CURVE. NOW WRITE A PROGRAM WHICH WILL USE ONLY RECTANGLES BELOW THIS OR ANY OTHER CURVE TO APPROXIMATE THE AREA UNDER THE CURVE. (HINT: USE A HEADER OF THE FORM N RECTIN INTAB). WRITE A PROGRAM WHICH WILL USE ONLY RECTANGLES ABOVE THIS OR ANY OTHER CURVE TO APPROXIMATE THE AREA UNDER THE CURVE.

3. USE RECTIN AND RECTEX TO FIND THE APPROXIMATE AREA BOUNDED BY THE CURVE AND THE X AXIS IN THE STATED DOMAIN OF X, USING 2, 4, 8, 20, 50, AND 100 RECTANGLES FOR PROBLEMS 2, 4 AND 6 IN THE TEXT AND ALSO FOR \( Y = (X-2)^3 \), \( X \in [0, 3] \), \( Y = e^X \), \( X \in [0, 2] \), AND \( Y = \ln X \), \( X \in [1, 3] \).

4. USE PLOT TO OBTAIN GRAPHS OF THE FUNCTIONS DETERMINED BY THE ABOVE RULES OF CORRESPONDENCE (EXCEPT FOR \( Y = 1 + X^2 \)) FOR INCREMENTS OF .5 IN THE RESPECTIVE DOMAINS. DOES THE AREA YOU CALCULATE CORRESPOND FAIRLY CLOSELY TO THE AREA UNDER THE CURVE THAT YOU WOULD ESTIMATE VISUALLY?
14.5

Consider the polynomial \( g(x) = 31x^4 - 17x^2 + 26x - 54 \) from problem set 13.6. Here, as before, we let \( \text{POLY} \rightarrow [81, 0, -17, 26, -54] \). That is, \( \text{POLY} \) is a vector whose elements are the coefficients of powers of \( x \) in \( g(x) \). The following program will yield one antiderivative of \( g(x) \) in terms of coefficients of powers of \( x \); that is, \( \int g(x) \, dx = 16.2x^5 - 5.66x^3 + 13x^2 - 54x + 2 \), the program indicating the coefficients by printing the vector \( 16.2 \, 0 \, -5.666666667 \, 13 \, -54 \, 2 \):

\[
\text{VZ} <- \text{INT POLY}
\]

\[1\]

\[
\text{Z} \rightarrow (\text{POLY} \times \text{POLY}), 2 \text{V}
\]

Go through \( \text{INT} \), working from right to left as the computer would do and write down the results of each step. Use \( \text{INT} \) to determine an antiderivative for \( f(x) = x^5 - 2x^3 + 3x - 1 \).

Use \( \text{DER} \) to differentiate \( f(x) \).

2. Use \( \text{PLOT} \) to graph the functions determined by \( f(x) \), \( f'(x) \), and \( \text{INT} \) over the interval \([-1, 2]\) using \( h=6 \). From your graphs, what are \( f'(0) \) and \( f'(2) \)? Also determine \( \int_{-1}^{2} f(x) \, dx \) (use theorem 14.7 with the appropriate graph).
3. Use SLOSEC to approximate $F'(0)$ and $F'(2)$ with $\Delta x = 0.001$. Use RECTIN with $n = 10$ to approximate $\int_0^2 F(x) \, dx$.

4. Approximate the integrals in problems 3, 8, and 11 in the text and $\int_3^4 x \, dx$ using RECTIN for $n = 10$, 15, 20, and 25, respectively. Compare results of problems 3, 8, and 11 by formula and by approximation.
APPENDIX D

TIME DEVOTED TO HOMEWORK BY THE
CONTROL AND EXPERIMENTAL GROUPS

The time devoted to paper and pencil methods by the control group and the time devoted to paper and pencil methods and computer methods by the experimental group were recorded by the instructor who conducted the experiment. The control group averaged 2.18 hours for fifteen homework assignments; the day section averaged 2.12 hours and the evening section 2.25 hours. The experimental group averaged 1.41 hours for fifteen homework assignments by paper and pencil methods, 1.63 hours for nine assignments by computer methods, and 2.44 hours for a combination of pencil and paper and computer methods for fifteen assignments.
APPENDIX E

THE RAW DATA

In order to facilitate the handling of the raw data, each student was assigned a student number. Numbers from one to thirty-one were used to identify students in the experimental group. Numbers fifty-two to ninety-one were used to identify students in the control group, with numbers sixty-four and below designating day students and the higher numbers designating evening students. The raw data are listed only for those students for whom the data were analyzed.

In the column labeled "Ability Score," the composite raw scores on the California Short-Form Test of Mental Maturity, 1963 Revision (Level 5) are listed. The next two columns contain the data obtained by administering the Purdue Scale to Measure Attitude toward Any School Subject for mathematics, Form B being used as the pre-test and Form A as the post-test. The last two columns contain the data obtained by administering the Cooperative Mathematics Tests for Calculus; Form B was used as the pre-test and Form A as the post-test.
<table>
<thead>
<tr>
<th>Student Number</th>
<th>Ability Score</th>
<th>Attitude Pre-test</th>
<th>Attitude Post-test</th>
<th>Calculus Pre-test</th>
<th>Calculus Post-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>86</td>
<td>7.3</td>
<td>7.45</td>
<td>12</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>90</td>
<td>7.5</td>
<td>8.1</td>
<td>10</td>
<td>24</td>
</tr>
<tr>
<td>3</td>
<td>84</td>
<td>6.5</td>
<td>8.1</td>
<td>9</td>
<td>17</td>
</tr>
<tr>
<td>4</td>
<td>98</td>
<td>7.7</td>
<td>8.1</td>
<td>4</td>
<td>21</td>
</tr>
<tr>
<td>5</td>
<td>76</td>
<td>8.1</td>
<td>8.9</td>
<td>7</td>
<td>17</td>
</tr>
<tr>
<td>7</td>
<td>73</td>
<td>7.1</td>
<td>8.1</td>
<td>12</td>
<td>17</td>
</tr>
<tr>
<td>8</td>
<td>88</td>
<td>6.5</td>
<td>6.85</td>
<td>11</td>
<td>14</td>
</tr>
<tr>
<td>9</td>
<td>88</td>
<td>2.6</td>
<td>5.75</td>
<td>3</td>
<td>19</td>
</tr>
<tr>
<td>11</td>
<td>84</td>
<td>6.2</td>
<td>7.7</td>
<td>8</td>
<td>22</td>
</tr>
<tr>
<td>12</td>
<td>99</td>
<td>7.1</td>
<td>8.5</td>
<td>16</td>
<td>23</td>
</tr>
<tr>
<td>15</td>
<td>62</td>
<td>8.7</td>
<td>8.1</td>
<td>1</td>
<td>13</td>
</tr>
<tr>
<td>16</td>
<td>96</td>
<td>5.5</td>
<td>4.3</td>
<td>12</td>
<td>21</td>
</tr>
<tr>
<td>18</td>
<td>96</td>
<td>8.5</td>
<td>8.5</td>
<td>5</td>
<td>32</td>
</tr>
<tr>
<td>19</td>
<td>88</td>
<td>8.5</td>
<td>4.3</td>
<td>15</td>
<td>18</td>
</tr>
<tr>
<td>20</td>
<td>58</td>
<td>8.5</td>
<td>8.5</td>
<td>14</td>
<td>12</td>
</tr>
<tr>
<td>21</td>
<td>79</td>
<td>5.1</td>
<td>6.85</td>
<td>15</td>
<td>16</td>
</tr>
<tr>
<td>22</td>
<td>95</td>
<td>5.5</td>
<td>7.7</td>
<td>14</td>
<td>19</td>
</tr>
<tr>
<td>23</td>
<td>99</td>
<td>7.5</td>
<td>8.5</td>
<td>11</td>
<td>21</td>
</tr>
<tr>
<td>24</td>
<td>66</td>
<td>6.0</td>
<td>7.7</td>
<td>11</td>
<td>15</td>
</tr>
<tr>
<td>26</td>
<td>84</td>
<td>6.8</td>
<td>8.1</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>27</td>
<td>96</td>
<td>5.9</td>
<td>7.7</td>
<td>11</td>
<td>25</td>
</tr>
<tr>
<td>30</td>
<td>79</td>
<td>8.1</td>
<td>8.1</td>
<td>10</td>
<td>24</td>
</tr>
<tr>
<td>31</td>
<td>98</td>
<td>8.1</td>
<td>8.5</td>
<td>12</td>
<td>24</td>
</tr>
<tr>
<td>52</td>
<td>92</td>
<td>8.5</td>
<td>8.3</td>
<td>16</td>
<td>27</td>
</tr>
<tr>
<td>53</td>
<td>92</td>
<td>8.8</td>
<td>8.3</td>
<td>9</td>
<td>17</td>
</tr>
<tr>
<td>54</td>
<td>82</td>
<td>7.9</td>
<td>6.0</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>56</td>
<td>79</td>
<td>8.5</td>
<td>8.1</td>
<td>11</td>
<td>17</td>
</tr>
<tr>
<td>57</td>
<td>97</td>
<td>7.5</td>
<td>8.5</td>
<td>17</td>
<td>23</td>
</tr>
<tr>
<td>58</td>
<td>84</td>
<td>7.1</td>
<td>8.5</td>
<td>12</td>
<td>21</td>
</tr>
<tr>
<td>59</td>
<td>99</td>
<td>6.5</td>
<td>8.1</td>
<td>9</td>
<td>22</td>
</tr>
<tr>
<td>60</td>
<td>90</td>
<td>6.0</td>
<td>8.1</td>
<td>13</td>
<td>16</td>
</tr>
<tr>
<td>62</td>
<td>76</td>
<td>8.3</td>
<td>8.5</td>
<td>7</td>
<td>22</td>
</tr>
<tr>
<td>63</td>
<td>73</td>
<td>2.8</td>
<td>7.45</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>64</td>
<td>54</td>
<td>7.7</td>
<td>8.9</td>
<td>5</td>
<td>14</td>
</tr>
<tr>
<td>Student Number</td>
<td>Ability Score</td>
<td>Attitude Pre-test</td>
<td>Attitude Post-test</td>
<td>Calculus Pre-test</td>
<td>Calculus Post-test</td>
</tr>
<tr>
<td>----------------</td>
<td>---------------</td>
<td>-------------------</td>
<td>--------------------</td>
<td>-------------------</td>
<td>-------------------</td>
</tr>
<tr>
<td>80</td>
<td>86</td>
<td>4.8</td>
<td>8.9</td>
<td>9</td>
<td>17</td>
</tr>
<tr>
<td>81</td>
<td>92</td>
<td>8.8</td>
<td>7.7</td>
<td>13</td>
<td>24</td>
</tr>
<tr>
<td>82</td>
<td>88</td>
<td>8.3</td>
<td>8.1</td>
<td>7</td>
<td>20</td>
</tr>
<tr>
<td>83</td>
<td>58</td>
<td>8.1</td>
<td>8.7</td>
<td>6</td>
<td>16</td>
</tr>
<tr>
<td>84</td>
<td>99</td>
<td>6.5</td>
<td>8.1</td>
<td>14</td>
<td>21</td>
</tr>
<tr>
<td>86</td>
<td>54</td>
<td>6.0</td>
<td>8.5</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>87</td>
<td>99</td>
<td>8.7</td>
<td>7.7</td>
<td>13</td>
<td>28</td>
</tr>
<tr>
<td>88</td>
<td>98</td>
<td>6.2</td>
<td>6.0</td>
<td>11</td>
<td>21</td>
</tr>
<tr>
<td>90</td>
<td>92</td>
<td>8.8</td>
<td>8.5</td>
<td>13</td>
<td>22</td>
</tr>
<tr>
<td>91</td>
<td>96</td>
<td>6.5</td>
<td>5.75</td>
<td>8</td>
<td>20</td>
</tr>
</tbody>
</table>
APPENDIX F

DESCRIPTIVE INFORMATION FOR STUDENTS WHO PARTICIPATED IN THE EXPERIMENTAL STUDY

<table>
<thead>
<tr>
<th>Student Group</th>
<th>Average Age</th>
<th>Sex</th>
<th>Enrollment</th>
<th>Classification</th>
<th>Employment</th>
<th>Used APL/360 Before</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>M</td>
<td>F</td>
<td>Full-time</td>
<td>Part-time</td>
<td>Yes</td>
</tr>
<tr>
<td>Control (day)</td>
<td>21.5</td>
<td>11</td>
<td>0</td>
<td>11</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>Control (evening)</td>
<td>26.9</td>
<td>9</td>
<td>1</td>
<td>4</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>Control (combined)</td>
<td>24.0</td>
<td>20</td>
<td>1</td>
<td>15</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>Experimental</td>
<td>23.5</td>
<td>16</td>
<td>7</td>
<td>19</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>Control and Experimental</td>
<td>23.8</td>
<td>36</td>
<td>9</td>
<td>34</td>
<td>10</td>
<td>17</td>
</tr>
</tbody>
</table>
BIBLIOGRAPHY

Books


Hamming, R. W., Calculus and the Computer Revolution, Berkeley, California, Mathematical Association of America, 1966.


Articles


Reports


Committee on the Undergraduate Program in Mathematics, *A Transfer Curriculum in Mathematics for Two Year Colleges*, Berkeley, Calif., Committee on the Undergraduate Program in Mathematics, 1969.
Committee on the Undergraduate Program in Mathematics Panel
on Computing, Calculus with Computers, Newsletter No. 4,
edited by George Pedrick and Gerald Leibowitz, Berkeley,
Calif., Committee on the Undergraduate Program in

Mathematical Association of America, A General Curriculum in
Mathematics for Colleges, Berkeley, Calif., Committee
on the Undergraduate Program in Mathematics, 1965.

Mathematical Association of America, Recommendations for an
Undergraduate Program in Computational Mathematics,
Berkeley, Calif., Committee on the Undergraduate
Program in Mathematics, 1971.

National Council of Teachers of Mathematics, Report of the
Conference on Computer Oriented Mathematics and the
Secondary School, Washington, National Council of

National Council of Teachers of Mathematics, The Revolution
in School Mathematics, A Report of Regional Orienta-
tion Conferences in Mathematics, Washington, National

National School Public Relations Association, Computers:
New Era for Education?, Washington, D. C., National

School Mathematics Study Group, A Conference on Future
Responsibilities for School Mathematics, Stanford,

A Conference on Mathematics
Education for Below Average Achievers, Stanford, Calif.,
School Mathematics Study Group, 1964.

A Conference on Responsi-
bilities for School Mathematics in the 70's, Stanford,

Technical Report on the California Test of Mental Maturity
Series, 1963 Revision, Monterey, Calif., McGraw-Hill,
Publications of Learned Organizations


Unpublished Materials


