A COMPARISON OF AN INDUCTIVE AND A DEDUCTIVE PROCEDURE
OF TEACHING IN A COLLEGE MATHEMATICS COURSE
FOR PROSPECTIVE ELEMENTARY TEACHERS

DISSERTATION

Presented to the Graduate Council of the
North Texas State University in Partial
Fulfillment of the Requirements

For the Degree of

DOCTOR OF EDUCATION

By

James K. Morris, M. T.
Denton, Texas
December, 1973

To obtain information regarding the effects of two divergent thought processes used in a college mathematics course for prospective elementary school teachers, this study compared the effectiveness of an adaptation of the traditional, deductive teaching method with that of an inductive method reflecting the recommendations of the Committee on the Undergraduate Program in Mathematics. In the spring semester of 1973, two sections of Mathematics for Elementary Teachers I, at Cameron College, Lawton, Oklahoma, served as experimental groups to test the two adaptations. The course followed the Committee on the Undergraduate Program in Mathematics recommendations for a first course in mathematics for prospective elementary teachers.

Each section was divided into two groups, to which students were randomly assigned in such a way as to balance the groups by ability as determined by composite scores on the American College Testing Program Examination. One group from each section was randomly selected to receive the
inductive procedure, the other to be taught by the deductive procedure. Of the eight tests used to compare the two procedures, seven were the subtests of *A Test of Arithmetic Understanding*: "Knowledge," "Comprehension," "Application," "Analysis," "Computation," "Number and Number Systems," and "Figures and Graphs." The eighth comparison of methods was based upon composite scores from three teacher-made tests.

On the "Application" test a significant F-ratio for the effects of the two teaching procedures was obtained at the .005 level. For each of the other seven tests none of the F-ratios was large enough to indicate a significant difference between the two teaching procedures. None of the eight tests showed a significant interaction between procedure and ability level. Therefore, it was concluded that the two procedures are equally effective in the college mathematics course for prospective elementary school teachers. The only exception was in the student's application of his knowledge and comprehension to problem situations. The deductive procedure was superior in this area, contrary to the direction of this study's hypothesis.
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CHAPTER I

PURPOSE OF THE STUDY

Introduction

The Committee on the Undergraduate Program in Mathematics panel on the training of teachers of Mathematics (1971) has developed recommendations for the preparation of teachers of elementary school mathematics (grades K through 6). The committee has designed two sequences of four three-semester hour courses as alternate ways of organizing the material which it believes a "really first-rate teacher of elementary school mathematics should know." These two sequences of four courses each were designed to achieve the following objectives of mathematical training:

1. Understanding of the concepts, structure, and style of mathematics
2. Facility with mathematical applications
3. Ability to solve mathematical problems
4. Development of computational skills.

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1 Committee on the Undergraduate Program in Mathematics will be abbreviated as CUPM for the remainder of this study.

2 Committee on the Undergraduate Program in Mathematics, Recommendations on Course Content for the Training of Teachers of Mathematics, Mathematics Association of America (August, 1971), p. 23.
Among these objectives, the objective of understanding is emphasized because the teacher's "understanding affects his views and attitudes; and in the classroom, the views and attitudes of the teacher are crucial." Along with this emphasis on understanding is the need for these four courses to prepare teachers "to have the flexibility of outlook necessary to adjust to the curriculum changes which will surely take place in the course of their professional careers."

In order to achieve these objectives and to prepare the prospective teacher to be flexible, the CUPM panel urges experimentation and diversity in providing appropriate material for an integrated course.

Cameron College, Lawton, Oklahoma, has responded to the recommendations of the CUPM panel and is currently offering course number 1 of the second recommended CUPM sequence, Number Systems and Their Origins, under the title Mathematics for Elementary School Teachers I. The study described here was designed to investigate one approach, in a localized setting, to the challenge of the CUPM panel for experimentation.

The study was designed to compare an inductive procedure to a deductive procedure in teaching Mathematics for Elementary Teachers. These two teaching procedures were

3Ibid., p. 7. 4Ibid., p. 8. 5Ibid., pp. 34-37.
designed to correlate, as closely as possible, with an inductive approach and a deductive approach, respectively, while arranging for each experimental group to achieve the same objectives in approximately the same amount of time. A more detailed description of these procedures is given in the section on definitions of terms.

Purpose of the Study

The purpose of this study was to compare the effectiveness of two procedures of instruction in a mathematics course, Mathematics for Elementary Teachers I: a) a deductive procedure designed to represent a typical, traditional approach to mathematics instruction, and b) an inductive procedure designed to reflect the recommendations of CUPM.

Hypotheses

This study was designed to test the following hypotheses:

1. Students taught by an inductive procedure will score significantly higher on each of the following five subtests of A Test of Arithmetic Understanding\(^6\) than students taught by a deductive procedure: a) Number and Number Systems, b) Figures and Graphs, c) Comprehension, d) Application, and e) Analysis.

2. Students taught by a deductive procedure will score significantly higher on each of the following subtests of A Test of Arithmetic Understanding than students taught by an inductive procedure: a) Computation, b) Knowledge.

3. Students taught by an inductive procedure will score significantly higher than those taught by a deductive procedure on teacher-made tests containing test items designed to be solved by inductive thought processes as well as test items measuring recall.

4. The relative effects of an inductive procedure of teaching and a deductive procedure of teaching on arithmetic understanding, as measured by each of the seven subtests (Computation, Numbers and Number Systems, Figures and Graphs, Knowledge, Comprehension, Application, and Analysis) of A Test of Arithmetic Understanding, will not be dependent upon student ability.

5. The relative effects of an inductive procedure of teaching and a deductive procedure of teaching on arithmetic understanding, as measured by the student's grade on teacher-made tests, will not be dependent upon student ability.

Definition of Terms

1. Inductive procedure of teaching. The inductive procedure of teaching used in this study is an adaptation
of an inductive teaching-learning thought process. However, a truly inductive teaching-learning process would have required a great deal of time to achieve a designated pattern of objectives. Since Mathematics for Elementary Teachers I was limited by time and was the first course in a two-course sequence, the inductive teaching-learning thought process was modified by allowing the instructor to provide the students with a degree of prescriptive guidance. This guidance emphasized discovery and understanding on the student's part by the use of specific examples designed to lead the student to discover the desired concepts. These examples became increasingly more specific until the student discovered the desired concept. Before or during this guidance period occasional impromptu discoveries were made by students. This guidance did not preclude the possibility of discussing them, but did lessen the chance for impromptu discoveries.

The inductive procedure used in this study can be described as follows: a) Students were given, as a first step, a set of exercises pertaining to a concept to be learned. These exercises consisted of material gleaned

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from state-adopted mathematics textbooks (grades K through 6) and were designed so that the student would be led to discover the concept to be learned. b) A time lapse of four days or longer was provided for individual contemplation of these exercises. c) After the student completed the set of exercises, the instructor asked carefully selected questions to determine if discovery of the concept had occurred. These questions were selected so that if discovery had occurred students would answer readily, but if discovery had not occurred students would answer with difficulty, if at all. During this questioning period, the students were urged to make enquiries of their own and to discuss any discoveries they may have made. d) If student response indicated that discovery had not taken place by a majority of the students, the instructor would ask the students questions designed to lead them as a group to discovery, or the instructor could assign a new set of exercises to lead the students to discovery. Since, on this step and step c, the instructor was asking questions and giving more specific examples, students were giving their individual responses (discoveries, intuitions, etc.), and a general discussion about the problems involved was being conducted, the instructor was not stating the desired concept for the students but was aiding the students to see weaknesses in their false conclusions. Through this process the students would discover the desired concept. e) When a majority of
the students had discovered the concept, the students as a group would develop a formal statement of the concept and the concept would be named. f) As a final step, the students would be given a set of exercises which gave them a chance to use what they had discovered. If a student could not solve a problem in this followup set of exercises, he was referred to an example that was similar or he was asked leading questions. When possible, this final set of exercises was used as a basis for discovering the next concept. (As an example of how this method of teaching was conducted, see Appendix B.)

2. Deductive procedure of teaching. The deductive procedure of teaching used in this study was designed to reflect a deductive, a priori, general-to-particular approach to conceptualization. Like the inductive procedure, the deductive procedure stressed understanding. However, understanding for the deductive procedure was the result of arguments based on previous definitions, premises, and theorems which were followed by illustrative examples. Also, through the deductive procedure, students studied the same content material in approximately the same order, and with written lessons of approximately the same length, as the students taught by the inductive procedure.

8Ibid.
The deductive procedure can be described as follows:
a) Students were given, as a first step, a statement of the concept to be learned and a name for the concept. b) Following the statement of the concept, the instructor discussed the concept including deductive arguments based on previous definitions, premises, and theorems. c) Examples illustrating the concept were given. d) Exercises were then assigned so that the student could use the concept to solve specific problems. e) If a student could not solve these practice problems, he was referred to a specific formula or principle that could be used to solve the problem.

3. Mathematics for Elementary Teachers I. The course Mathematics for Elementary Teachers I, Mathematics 3353, is a required three-hour course for all prospective elementary school teachers. The course is taught at Cameron College, Lawton, Oklahoma, and is designed to follow CUPM's recommended course I, Number Systems and Their Origins. (For a course outline see Appendix A.)

4. Ability. Ability in this study was measured by the student's composite score on the American College Testing Program Examination.

5. Arithmetic understanding. Arithmetic understanding was measured by the student's score on A Test of Arithmetic Understanding.
6. Grade on teacher-made tests. The student's grade on teacher-made tests was the student's total score received on three teacher-made tests. Each of these tests was constructed by the instructor and was designed to measure only the student's understanding of concepts discussed during each unit of instruction. To discover the reliability and validity of these teacher-made tests, a Kuder-Richardson formula 20 coefficient was computed for each test, and an item-analysis coefficient consisting of a point-biserial correlation coefficient between the score on each item and the score on its respective test was computed for each item's content validity.

Background and Significance

As a premise for the development of this study, the "view of the teacher as one continually searching for answers to the question: 'What procedures, under what conditions, and for what kinds of pupils, are effective in achieving various types of learning?'"\(^9\) was adopted.

In searching for an answer to the question of what procedures are effective in achieving learning based on the CUPM panel's objectives for course number 1, several approaches were considered. However, since one of the CUPM objectives involved problem solving and "problem solving . . . and discovery learning are synonymous when the

\(^9\) Emmer, op. cit.
solution in problem solving is discovered," and since the inductive teaching-learning thought process is a discovery learning process, the inductive teaching-learning process was the most logical choice for achieving the CUPM panel's objectives. Studies supporting this choice are given in Chapter II, Survey of Related Research Literature.

According to P. S. Jones, the inductive teaching-learning thought process, which is one of the modern heuristic teaching methods, had its beginnings with the Socratic method. Although Jones points out that the Socratic method is not the same as the modern heuristic methods, he states that "there are significant common elements in the methodologies. . . . " Others, such as R. M. Gagné, feel that these heuristic methods have their "origin partly in the writings of members of the Gestalt school of Psychology."
Whatever the origin, most advocates (e.g., Johnson, Bruner, and Jones) of discovery teaching would agree with the Cambridge Conference when they argue that anyone can do and create mathematics, but that discovery teaching is slow and that not all lessons for all students at all times should be taught in this manner. They also concede that some of the goals are not easily tested behaviorally. However, the conference concludes that since so many experienced teachers think that discovery teaching contributes significantly toward these goals, it should not be discarded for lack of proof of its value in terms of meeting behavioral objectives. Rather, teaching by discovery should be encouraged while further studies of its nature and effectiveness are conducted.

As an example of discovery-teaching advocates' agreement with the Cambridge Conference, Mauritz Johnson quoted William C. Bagley as writing that in the process of creating mathematics "the pupil is not to be told but led to see. . . . Whatever the pupil gains, whatever thought connections he works out, must be gained with the consciousness that he, the pupil, is the active agent--that he is in a


16Cambridge Conference on School Mathematics, Goals for School Mathematics (Boston, 1963), pp. 9, 17, 28, and 80.
sense at least, the discoverer." Also, P. S. Jones pointed out that even though discovery learning is slow, the teacher should not hide from the student the teacher's own wanderings, false starts, and originally roundabout procedures for solving problems.

According to the Cambridge Conference, some of the goals achieved by the discovery method are increased interest, induced research, understanding, confidence, appreciation, experience in generalizing, testing conjectures and discarding or modifying false ones, and formulating rules or theorems. B. Kersh supported these goals when he stated that...

However, P. S. Jones states that the contention that "gains in understanding, retention and applicability more than

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17 Johnson, op. cit., p. 120.
20 Cambridge Conference, op. cit.
offset any loss due to a slower pace . . . needs objective support . . . obtained by controlled experimentation based on a viable definition of discovery teaching." Also, reviews by Cummings and Hermann point out that despite the number of research studies comparing discovery methods of teaching with other methods of teaching, many questions remain unanswered.

Although the inductive teaching-learning thought process was the most logical choice for achieving the CUPM panel's objectives, it became apparent that the inductive teaching-learning thought process was not entirely applicable as a method of instruction at Cameron College. For example, the course Mathematics for Elementary Teachers I is the first course in a sequence of two courses, and the instructor is obligated to achieve a designated pattern of objectives. Since the inductive teaching-learning thought process requires a considerable amount of time to discover each concept to be learned, the process was modified to the inductive procedure used in this study. Thus, this study was designed to compare the two procedures of teaching, as

22Jones, op. cit., p. 504.

described in the section on definitions of terms, in this instructional situation.

With these limitations in mind, this study was designed to supply more information on a local level which, when combined with information from other localized studies, could be used as a basis for answering the question of whether or not the inductive teaching-learning thought process enhances learning. In the same vein, this study was designed to shed light on the question of whether the inductive or the deductive approach was superior for use in teaching students in each of the ability categories. Also, this study was designed to be of significance in relation to determining which teaching method might be the best approach to teaching CUPM's recommended course Natural Numbers and Their Origins.

Limitations

The study was limited to those students who completed the course Mathematics for Elementary Teachers I at Cameron College during the 1973 spring semester.

Assumptions

It was assumed that extraneous conditions affecting the results of the instruction being studied were evenly balanced between the groups and thus would not significantly affect the results of the study.
CHAPTER II

SURVEY OF RELATED RESEARCH LITERATURE

Introduction

The purpose of this chapter is to provide a review of studies which were designed to compare heuristic teaching methods to other teaching methods. Since there are several different heuristic teaching methods and each has been used in many studies, this chapter is designed to present a representative sample of these comparative studies and is not intended to be comprehensive. To provide a logical way for presenting these studies, the studies are categorized on the basis of their basic significant results. Five categories were used for categorizing the studies on the basis of results indicating that students taught by an inductive method were significantly 1) more able to transfer learning, 2) more able to retain learning, 3) more interested in what was learned, 4) more able to adjust imperfect learning and obtain and discover more correct reasons for facts, and 5) better learners, in general, due to active participation than students taught by other teaching approaches. A sixth category was included to cover more recent studies and a recent critical review of studies involving heuristic teaching methods.
Also presented are studies in which the opposite results were obtained. It should also be noted that many studies obtained non-significant results. G. Hermann's critical review and the section on current studies are intended to emphasize the comparison of the number of studies in which the results significantly favored the discovery approach, significantly favored the deductive approach, or were non-significant.

Since the inductive procedure used in this study is considered a heuristic procedure,¹ the results of these studies were used as reasons for selecting the inductive teaching-learning thought process as the most logical teaching process for achieving the CUPM panel's objectives. To a large extent the studies discussed under categories one through five were used to form this conclusion. On the other hand, the studies discussed under category six were basically used to design the instructional procedures and the experimental format used in this study.

Research Relating to Transfer of Learning

There were several reasons for selecting the inductive teaching-learning thought process as the most logical

teaching process for achieving the CUPM panel's objectives. First, Joseph Scandura and B. R. Worthen supported the concept that students taught by an inductive approach were significantly better able to transfer learning than students taught using a deductive approach. In particular, Joseph Scandura found that students who succeeded in discovering rules themselves were better able to discover new rules of the same general type than were students given rules first and then an opportunity to discover new rules. His experiment involved three programs. The first, the introductory program, introduced the idea of a number series and the terminology used throughout the experiment. The second, the rule (R) program, introduced, along with examples and practice, rules for summing each of three different arithmetic series. And the third, the discovery (D) program, required the subjects to discover respective formulas for the same three series. Hints of increasing specificity were provided to aid the subjects. Each program was given in booklet form, with the introductory program containing twelve frames, the R-program containing ten frames, and the D-program containing seven frames.

These two programs were given to 135 ninth-, eleventh-, and twelfth-grade mathematics students, who were assigned

randomly to three different treatment groups. The first group was given the R-program only; the second was given the R-program and then the D-program; and the third was given the D-program and then the R-program. These three treatments were administered over three consecutive class meetings, each meeting lasting forty-eight minutes. Each student's performance was measured by the percentage of errors on the learning frames in programs R and D. These performance scores were then analyzed by a $x^2$.

In B. R. Worthen's study a comparison was made between a guided discovery method of teaching and an exposition method of teaching. These two methods were used to teach a six-week mathematics unit involving sixth-grade students. Sixteen classes were involved in which eight were taught using a discovery approach and eight using an exposition approach. Eight teachers were used, with each teacher teaching one discovery class and one exposition class. Quasi-textual instructional programs were used to present the following mathematical concepts: 1) notation, addition, and multiplication of integers, 2) the distributive principle, and 3) exponential notation and multiplication and division of numbers expressed in exponential notation.

Four subtests, each designed as a unit test, were given during the instructional period. These four subtests

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were used to test the relative effectiveness of the two teaching methods. Also, a comprehensive test was given five weeks after completion of instruction and again seven weeks after completion of instruction. The comprehensive test was designed to test transfer of learning. Two attitude scales were also given.

Although Worthen's study seemed to support the superiority of the discovery method, the basic significant finding was that the discovery group was superior in ability to transfer heuristics. Other aspects tested in his study were retention of learning, concept transfer, and change in attitude toward mathematics. The findings for these aspects were non-significant.

Research relating to transfer of learning was also conducted by Haslerud and Meyers. They performed an experiment involving the ability of college students to decipher twenty codes. In the experiment one hundred students were given material which was designed to be learned inductively and material which was designed to be learned by the rule-example format. This material was given to the students in the form of a testing program. The testing program consisted of problems for which the students were asked to decipher twenty codes. Half of these problems

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were stated inductively and half deductively. A test consisting of twenty codes was constructed so that the two types of problems were alternated. This allowed the students to work an equal number of each type. This test was given one week prior to a second test. The second test was constructed so that the students had to identify only the correct solution to the twenty codes. The experimental group, consisting of seventy-six students, were present for the first administration. The remaining twenty-four, called the control group, were included on the second administration.

In this situation it was found that independently derived principles are more transferable than those which are given to the student. This conclusion was based on the fact that the experimental group solved forty-six per cent more of the inductively stated codes on the second administration than they solved on the first administration. The increase for the rule-example material was ten per cent. The difference in these two increases was very significant.

Other studies seem to support the concept that students taught by the discovery method are better able to transfer learning. In discussing the different instructional strategies of the discovery method, Bert Kersh summarized this supportive evidence when he stated that "... when a student learns by discovery he (1) understands what he learned,
and is better able to remember and transfer it, . . . "5

It should be noted, however, that S. Krebs6 conducted an experiment, similar to that of Haslerud and Meyers, but using ninth-grade students, in which the opposite results were obtained.

S. Krebs' experiment involved having ninety students decipher codes presented to them through deductive and inductive test items. A sequence of four test administrations was conducted, with the first administration consisting of a learning test, and the last three administrations consisting of test items based on similar principles as the first test. Thirty-two of the ninety students took the initial test and were considered the experimental group. The remaining fifty-eight students, called the control group, were included on the last three administrations. The last three tests were given in a time sequence consisting of zero days, six days, and forty-three days after the conclusion of the experiment. On the basis of analysis of the test data, the hypothesis that independently derived principles are more transferable than those which are given to the student was rejected.

5Bert Y. Kersh, op. cit., p. 414.

Research Related to Retention of Learning

A second reason for selecting the inductive teaching-learning thought process was that studies have shown that students taught by an inductive approach have significantly better retention of learning than do students taught by a deductive approach. To exemplify these studies an experiment conducted by Max Sobel using ninth-grade algebra students and an experiment by George Katona are cited. In his card trick and match stick experiments, George Katona concluded that the discovery group's retention and performance of the learned tasks seemed to endure longer than the memorization group's. In the card trick experiment two groups of students were given a learning task which consisted of learning to perform card tricks. The "memorization" group was required to memorize the tricks. The "understanding" group was given an explanation of the tricks. Following their instructional period the two groups were given similar card tricks to perform. On this new set of tricks the "understanding" group performed better than the "memorization" group. This "understanding" group also out-performed the "memorization" group when they were asked, one or more weeks later, to perform the initial learned card tricks.

Following the same design, G. Katona repeated the experiment using as a learning task the transformation of a set of squares into different arrangements. These arrangements consisted of figures containing more or fewer...
squares. To make it easier to transform the arrangement of the squares, the sides of the squares were composed of match sticks. By changing the position of one or more of the sticks, the number of squares in the figure could be altered. Again the "understanding" group out-performed the "memorization" group.7

Max Sobel8 compared an inductive approach to a deductive approach for teaching the first four weeks of an algebra course. In the inductive approach, the students were guided by the teacher to discover and verbalize concepts from their experience with applications. In the deductive approach, the concepts were defined and discussed by the teacher who then supplied the students with practice exercises. The students taught by these two approaches were statistically divided into two levels of ability, high and average. To test the significance of the study, two tests designed to test performance and retention were constructed. Each test contained a section dealing with concepts and a section dealing with fundamental skills. His findings showed that high ability students had superior test performance and retention as late as eleven weeks after the four-week unit. He also determined that there

7George Katona, Organizing and Memorizing (New York, 1940), pp. 1-31.

was no significant difference between the test performance and retention of the average ability students taught by the inductive approach and the average ability students taught by the deductive approach.

However, Charles Reimer's findings were non-significant for effects of the methods of teaching and interaction effects of ability levels with method of teaching. Although Charles Reimer's study was conducted using students enrolled in a basic college mathematics class, the contrast between his findings and Max Sobel's seem to place some doubt on the significance of interaction between student ability and these two approaches to teaching. Charles Reimer's study will be discussed in more detail later in this chapter.

Research Relating to Interest in Learning

A third reason for selecting the inductive teaching-learning thought process was that research has shown that learning by discovery increases the student's interest in what is learned. For example, Bert Kersh concluded, after he required high school boys to learn a mathematical formula,

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that learning by discovery does produce a greater interest in what is learned than learning by rote. This conclusion was based on his analysis of a questionnaire given to three groups of students. The analysis showed that students in the inductive group used rules to solve problems significantly more often after the learning unit than did the other groups. Retention and transfer of learning were also tested in his experiment, with no significant difference found between the discovery group, the rote learning group, and the programmed-materials-learning group. The discovery group was required to make inferences from their performance on specific algebraic manipulations. These inferences were to be obtained without help, but the teacher provided guidance to aid the students to discover an explanation of the rules. The rote learning group was given the rules only and told to memorize them.

The findings of the study were based on the test performance of ninety geometry students who were taught a set of rules for summing series. To determine this performance, tests of recall and transfer were given in a sequence of administrations. The first administration was given to ten students from each group three days after completion of the unit. Two weeks after completion of the unit a different set of ten students from each group was given the test. Then, six weeks after completion of the unit, the remaining ten students in each group were given the test. Transfer
was measured by the number of students who used the appropriate rule to solve a problem and retention was measured by the number of students who stated the rule correctly.

The motivational superiority of the discovery approach, as found by Kersh, was also pointed out by Kuslan, who concluded that the discovery approach gives children increased confidence in their ability to think and the strength to be independent. According to Kuslan, this increased confidence and independence are reasons for students having increased interest.  

Research Relating to Adjustment to Learning

A conclusion by Swenson, Anderson, and Stacey is cited as a fourth reason for selecting the inductive teaching-learning thought process. They concluded that

3. The imperfect learning that occurs during a process of self-discovery is less detrimental to learning than that which occurs during a process in which the responses are identified for the learner.

4. The learner obtains as many or more facts, and discovers more correct reasons for them, by a process of self-discovery than by a process of authoritative identification.

These conclusions were formed on the basis of their experiment on the learning of word relationships by one


13Ibid., p. 100.
hundred sixth-grade students. The students were randomly assigned to five instructional groups. Each of the groups was given fifty sets of words. Each set of words consisted of five words from which the students were to learn which word was not related to the other four words. In each set four of the words were related by some principle. To teach the sets of words, five instructional procedures were used. These instructional procedures were 1) no information was given as to why one of the words did not belong or that there was a reason for a correct choice, 2) information was given to the effect that there was a reason for a correct choice but the reason was not given, 3) the correct choices were given on the first trial but no reasons were given for the choice, 4) the correct choices were given on the first trial and it was explained that a reason existed for the choice, but the reason was not given, and 5) the correct choices were given on the first trial and the reasons for the choices were given. The number of incorrect responses to the sets of words on each of five tries were used as criterion scores. For additional information the students were asked for their reasons for each response on the fifth trial. Also, a second test similar to the one used in the five trials was used as a pretest and a posttest.

Research Relating to Students as Better Learners

As a fifth reason for selecting the inductive teaching-learning thought process, it has been observed that students
taught by this process are better learners. In particular, Alan Gastner, Swenson, Anderson, and Stacey observed that active participation by the learner makes a better learner,\textsuperscript{14} and Max Wertheimer found that the discovery method is superior as a process for discovering the essential nature of the problematic situation.\textsuperscript{15} Also, as found by Lewis Smith, discovery lessons may lead to unexpected and equally important discoveries as well as to those that were intended.\textsuperscript{16}

It should be noted that D. P. Ausubel argues that discovery methods are more relevant to children under twelve years of age than to older school pupils, since the latter can learn by meaningful reception learning. He concludes his discussion of the rationale for discovery learning as follows: "The crucial points at issue, however, are not whether learning by discovery enhances learning, retention, and transferability; but (a) whether it does so sufficiently, for learners who are capable of learning principles meaningfully without it, . . . ; and (b) whether, . . . , the discovery method is a feasible technique for transmitting the substantive content of an intellectual or scientific

\textsuperscript{14}Ibid., p. 100; Alan Gastner, \textit{Children Teach Children}, (New York, 1971).


\textsuperscript{16}Lewis Smith, "A Discovery Lesson in Elementary Mathematics," \textit{Arithmetic Teacher}, XVIII (February, 1971), 73-76.
discipline to cognitively mature students who have mastered its rudiments and basic vocabulary."\textsuperscript{17}

Since the students in Mathematics for Elementary School Teachers I are college students with reasonably mature, cognitive abilities and have mastered the basic language and operations in arithmetic, it would seem that they are similar to the students Ausubel is discussing. This would imply that a deductive approach to conceptualization might be significantly more satisfactory for these students. Thus the inductive teaching-learning thought process and the deductive approach to conceptualization are logical choices for comparative teaching processes.

**Recent Research and a Critical Review**

Several recent studies comparing the inductive and deductive teaching processes were reviewed before designing the instructional procedures used in this study. A critical review of studies prior to 1969 and seven studies conducted since 1969 were selected as representative of these comparative studies. G. Hermann\textsuperscript{18} made a critical review of


learning by discovery\textsuperscript{19} in 1969 and summarized the results as follows:

a) The discovery method implies better transfer of learning than the deductive method.

b) The deductive method implies better retention of learning.

c) The discovery method is more effective when learning tasks involve material such as that taught in schools.

d) The discovery method is more effective when background knowledge is limited.

e) Discovery method is more effective for low ability groups than for high ability groups.

f) A large number of studies obtained non-significant findings.

g) In the discovery method, a reasonable degree of guidance is better than little guidance.

In forming this summary Hermann computed a ratio comparing the number of studies supporting the inductive method as significantly superior to the number of studies supporting the deductive method as significantly superior. However, this ratio was computed only for parts (a) and (b) above. For part (a), transfer of learning, the ratio was ten to one in favor of the inductive method, and for part (b), retention of learning, the ratio was four to two in favor of the inductive method.

Also, Hermann summarized the number of studies having findings supporting the inductive method as significantly

\textsuperscript{19}George Hermann points out that wherever he uses the word "discovery" he means "inductive."
superior, the deductive method as significantly superior, or having no significant findings. This summary was separated on the basis of three different age levels: college, high school, and elementary. On the college level three studies found the inductive method superior to the deductive method; one study found the deductive method superior to the inductive method; and twelve studies found non-significant results. On the high school level, eight studies found the inductive method superior to the deductive method; one study found the deductive method superior to the inductive method; and fourteen studies found non-significant results. On the elementary school level, one study found the inductive method superior to the deductive method; three studies found the deductive method superior to the inductive method; and three found non-significant results.

In concluding his review, Hermann points out that it would be easier to compare these studies and future studies if some consideration were given to the verbal content of the material and the organization of the material used by both the deductive and inductive methods. Also, some care should be taken to show the reader the amount of activity the inductive group has in the learning process. Hermann also feels that the material should be made as meaningful as possible and that the time spent in learning for the inductive and deductive groups should be made as equal as possible.
Studies by M. R. Rizzuto, M. L. Koran, R. L. Sparks and E. McCallon, V. Richardson, R. Wong, C. Reimer, and Lois Lackner were selected as representative of the comparative studies conducted between 1969 and 1972. The results of these studies are intended to illustrate the findings obtained by G. Hermann.

Malcolm F. Rizzuto compared inductive and deductive methods of teaching concepts of language structure to a group of eighth-grade students. The group of one hundred sixty-five pupils of the Fox Lane Middle School, Bedford, New York, was separated into two groups. The students were randomly assigned to six teachers and six classrooms. Guidance with open-ended questions was given to the inductive group, and rules followed by examples were given to the deductive group. The two groups were given twenty forty-five minute lessons over a period of five weeks. One of these lessons was video taped for each teacher during the first two weeks of the study, and one lesson was video taped for each teacher during the final two weeks of the study. At the conclusion of the five-week period, a recognition and transfer criterion test was given. The test consisted of thirty-five multiple choice items designed to test recognition and thirty multiple choice items designed to test transfer of learning. To investigate the relative effectiveness of the inductive and deductive methods of instruction and to determine the interaction
among ability, sex, and method, a $2 \times 3 \times 2$ analysis-of-variance was used. This analysis led to one significant result at the .05 level of confidence: females in the inductive group performed better than the females in the deductive group.\textsuperscript{20}

Mary Lou Koran conducted a study using programmed material for necessary arithmetic operations. In her study four groups of students were each given a different set of programmed material. These four sets of programmed material were

1. Inductive high-alternation (Example statements and rule statements alternated in the sequencing of the material, with examples preceding a corresponding rule)

2. Deductive high-alternation (Example statements and rule statements alternated in the sequencing of the material, with rule statements preceding corresponding examples)

3. Inductive low-alternation (All rule statements listed together and all example statements listed together, with examples preceding the corresponding rules)

4. Deductive low-alternation (All rule statements listed together and all example statements listed together, with rules preceding the corresponding examples)

She found that "Necessary Arithmetic Operations produced significant .05 level interaction with both program time

and criterion test time. Differences between inductive high-alternation treatments appeared to contribute heavily to interaction effects. Test scores were negatively related to these time variables in the inductive high-alternation treatment, and not significantly related to time in the deductive low-alternation treatment.\(^{21}\)

She also showed that, in general, the low-ability group did better using the deductive approach,\(^{22}\) which contradicts earlier studies.

The purpose of a study by V. Richardson was to determine the effectiveness of having students apply the inquiry method to their chemistry laboratory experiments. Since freshman, sophomore, junior, and senior college students were enrolled in the laboratory sections used in the study and since the enrollment procedure was assumed to place the students randomly in the laboratory sections, the group's ACT composite scores were tested to determine the difference in the academic superiority of the experimental group and the control group. The results of a t-test showed no significant difference in the two group's academic superiority. In order to determine the effectiveness of using the inquiry method, several experiments were written so that the experimental group would have to use the inquiry method, and experiments covering similar principles were written so that


\(^{22}\)Ibid., pp. 301-307.
the control group would have to use the conventional laboratory procedures. To obtain data to determine this effectiveness, eight short unit exams were given during the semester and one final exam was given at the end of the term. The examination of this data showed that the students using the inquiry method of learning, as opposed to the students using the conventional laboratory procedure, performed significantly better on the measures.  

Ruth Wong taught six units of geometric content to small classes of prospective elementary school teachers enrolled at the University of Hawaii. These units were 1) congruence, 2) geometric inequalities, 3) parallelism in planes and space, 4) polygons and polyhedrons, 5) area and volume, and 6) transformations in the plane. Two groups of students were formed. The inductive group was given informal exploratory activities which served as a basis for drawing conclusions. These activities were followed by a discussion of the conclusions drawn and expositions of major ideas involved, including proofs when appropriate. The second group used a conventional rule-example method.

A limited testing program was designed for two groups composed of eighteen and twenty-four students. The testing program consisted of a multiple choice geometry test and a

geometry attitude scale. The results showed that the inductive group had significant gains in knowledge of geometric content, more positive feelings toward geometry, and a greater ability to utilize geometric knowledge in tasks related to teaching elementary pupils.24

As an example of studies in which the findings were statistically non-significant, a study by E. McCallon and Sparks, a study by C. Reimer, and a study by Lois Lackner are cited. Although the findings of McCallon and Sparks were statistically non-significant, the results indicated that the use of the discovery approach produced changes in a favorable direction. In their study the discovery approach was said to coincide with categories of indirect verbal behavior as categorized by the Flanders system of interaction analysis. According to Flanders these basic categories are the categories in which the teacher accepts the feelings of students, praises or encourages the students, and accepts or uses the ideas of the students. This indirect verbal behavior was observed as pre-service elementary school teachers and elementary pupils interacted in a micro-teaching laboratory. The subjects consisted of one class of twenty-six university juniors and seniors who had been accepted into an elementary education program.

The class they were taking was randomly selected from a total of four required science method classes meeting during the semester. Also observed in the micro-teaching laboratory were the categories which Flanders supports as measuring direct verbal behavior. These basic categories are the categories in which the teacher gives directions and criticizes or justifies authority.

From these observations a comparison was formed between indirect verbal behavior and direct verbal behavior. This comparison is called an indirect/direct ratio. An indirect/direct ratio was computed for each pre-service teacher as he taught an initial ten-minute micro-lesson and, eight weeks later, when each pre-service teacher re-taught the initial micro-lesson. During the eight-week period each pre-service teacher taught three micro-lessons, critiqued each lesson with the researchers, and re-taught the lessons to different pupils. Also, all lessons were video taped.  

Charles Reimer's study involved two groups of students taught using an inductive approach and two groups using a deductive approach. The students involved were 104 students enrolled in a college freshman mathematics course for non-mathematics and non-science majors. These students were

randomly assigned to four sections taught by two instructors. Each instructor taught one section using a deductive approach and one section using an inductive approach. Special instructional materials were prepared for both approaches to insure that the material would be given in the appropriate manner.

Comparisons between the two approaches were made on the basis of students' mathematics achievement scores, critical thinking scores, and grades on teacher-made tests. For a measure of mathematics achievement, the Cooperative Mathematics Test, Structure of the Number System, and the Cooperative Mathematics Test, Algebra I were used. The Watson-Glaser Critical Thinking Appraisal was used to measure critical thinking, and the total scores on four teacher-made unit tests were used as a single measure for the teacher-made test. It was hypothesized that the inductive group would perform significantly better in each of these categories. However, no significant findings were obtained. 26

L. M. Lackner presented the derivative concept in terms of limits by an inductive and a deductive method. The purpose of the experiment was to determine the effectiveness of the two approaches in beginning calculus. To remove the teacher variable, she prepared programmed

26Reimer, op. cit.
test material for the two methods. Each set of materials consisted of three hundred eighty-one frames. The deductive material followed the format of rule-example; the inductive material followed the format of example-statement of a problem situation-conclusion. The deductive programmed material was given to beginning calculus classes in two high schools located in Urbana, Illinois. The inductive programmed material was given to beginning calculus classes in two high schools located in Midlothian, Illinois.

As a criterion test, all four classes were given a twenty-eight-item teacher-made test during two forty-minute sessions. No significant difference in the performances of the two groups was found. However, Lackner did conclude that the degree to which the student mastered the unit on the limit concept appeared to have a powerful effect on the student's mastery of the unit on the derivative. This conclusion was also independent of the method of instruction.  

Summary

The purpose of this chapter was to provide a review of studies which were designed to compare heuristic teaching methods to other teaching methods. Representative samples

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of these comparative studies were presented in five categories determined by their basic significant results. These five categories were studies which found that students taught by an inductive method were significantly 1) more able to transfer learning, 2) more able to retain learning, 3) more interested in what was learned, 4) more able to adjust imperfect learning and obtain and discover more correct reasons for facts, and 5) better learners, in general, due to active participation than students taught by other teaching approaches. Also, a sixth category was presented in which a critical review, by G. Hermann, of studies prior to 1969 and more recent studies were reviewed.

Under the first category, three studies significantly supported the conclusion that students taught by an inductive method were better able to transfer learning than students taught by other teaching approaches, and one study resulted in a conclusion opposed to the first three. These four studies were taken from the elementary, junior high school, high school, and college levels.

For the second category, three studies supported the conclusion that students taught by an inductive method were better able to retain learning than students taught by other teaching approaches. Two of these three studies were done by George Katona, the third by Max Sobel. Also, one study
was cited in which no significant difference was found between the two teaching procedures.

A study in which students were required to learn the word contained in a list of five words which was not related to the other four words was cited as representative of studies supporting category four. Most studies reviewed cited some evidence supportive to category four.

Most studies which found the inductive method a superior teaching method also found some evidence in support of category five. Four sources were cited as representative of this category.

All of these studies seem to be representative of the findings of G. Hermann in his critical review of studies using heuristic methods. Hermann's review, along with seven other studies, were given to represent recent research using heuristic teaching methods. Four of these studies supported the inductive method as superior and three found no significant difference between their experimental groups.

If all of these studies were summarized into a ratio, the inductive approach would prove superior almost two to one. However, as Hermann stated, it is difficult to compare studies due to the many different heuristic teaching procedures and the many different experimental designs.
Also, Phillip Jones, in a review of heuristic teaching methods, has found that many questions remain unanswered. Thus, in attempting to interpret results of experiments involving heuristic teaching methods, it is important to analyze the characteristics of each experiment carefully.

28 Phillip Jones, *op. cit.*
CHAPTER III

EXPERIMENTAL DESIGN AND EXPERIMENTAL PROCEDURES

Introduction

The present study was undertaken to provide further evidence, in a localized setting, concerning the effectiveness of using an inductive procedure of teaching to teach a mathematics course for prospective elementary school teachers. The purpose of this chapter is to describe the study. The setting of the study, the experimental design used in the study, the teaching procedures compared, the testing program conducted to determine the effectiveness of the teaching procedure, and the procedures used in analyzing the test data are discussed in this chapter.

The Setting of the Study

In this section a description of the students participating in the study, the course in which they were enrolled, and the school offering the course will be provided. The study was conducted at Cameron College during the Spring semester of 1973. Cameron College is located in Lawton, Oklahoma, which is a city of approximately 80,000 people. Lawton is bordered by Fort Sill, an army post having a population of approximately 25,000.
Most of the students attending Cameron College are drawn from families living in the immediate Lawton-Fort Sill area (within forty miles of Lawton). Most of these students enroll as business or education majors. The students participating in this study were elementary education majors enrolled in two sections of Mathematics for Elementary Teachers I. Mathematics for Elementary Teachers I is a required three-hour course for all prospective elementary school teachers and is a prerequisite to the mathematics methods course. The course is designed to follow CUPM's recommended course I, Number Systems and Their Origins. Mathematics for Elementary Teachers I is the first half of a two-course sequence in mathematics for prospective elementary school teachers and is a prerequisite for twelve hours of elective mathematics courses for prospective elementary school teachers. Thus the course is designed to achieve a designated pattern of objectives. (For a course outline of Mathematics for Elementary Teachers I, see Appendix A.) The students enrolling in this course are generally sophomores, juniors, and seniors. Most of them have a weak background in mathematics and express a dislike for, or a fear of, mathematics.

The Experimental Design

In this section the design of the study is presented in the same order as it was executed. Briefly, the study
was designed to conform to a two by three treatment by levels design.¹ The two treatments were the two teaching methods, and the three levels were the three levels of ability (high, average, and low). Two dependent variables were selected. The first dependent variable was arithmetic understanding, and the second was the student's grade on teacher-made tests. Arithmetic understanding was measured by the student's performance on seven subtests of *A Test of Arithmetic Understanding*. The student's grade on teacher-made tests was the student's total grade on three teacher-made unit tests.

Before the course began, the instructor's name was associated with two sections of Mathematics for Elementary Teachers I, Mathematics 3353, to be scheduled during two comparable time periods, one at 9:30 and one at 10:30. The assumption was made that these two sections would be composed of students selected at random to enter either the 9:30 or the 10:30 section. After the enrollment was completed, each student's *American College Testing Program Examination*² score was obtained from school records. These ACT scores were used to separate the students in each section into three levels of ability. To determine these levels of

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¹E. F. Lindquist, *Design and Analysis of Experiments in Psychology and Education* (Boston, 1953).

²Oscar Buros, editor, *The Sixth Mental Measurements Yearbook* (Highland Park, 1965), pp. 1-13. The *American College Testing Program* will be abbreviated as ACT.
ability, the scores for all students enrolled in Mathematics 3353 were used to compute the 33 1/3 percentile and the 66 2/3 percentile scores. When computed the 33 1/3 percentile score was 14.25 and the 66 2/3 percentile score was 17.5. Since ACT scores are based on whole units, the low-ability group had scores equal to 14 or below, the average-ability group had scores equal to 15, 16, or 17, and the high-ability group had scores equal to 18 or above. It is worth noting that the average of these scores is significantly below the regional average.

Once the students enrolled in the 9:30 section were classified as belonging to one of these levels of ability, they were randomly assigned to one of two groups. This was done in such a way that the students classified as having high ability were randomly assigned to each of the two groups; the students classified as having average ability were randomly assigned to each of the two groups; and the students classified as having low ability were randomly assigned to each of the two groups. In this way the two groups were matched according to ability. Similarly, the 10:30 section was randomly split into two groups matched according to ability. Thus, four groups were formed and were matched according to three levels of ability (high, average, and low). These four groups were numbered 1, 2, 3 and 4, and randomly assigned to receive one of the two treatments (a deductive procedure or an inductive procedure).
Table I shows the distribution of the students according to level of ability, time of class, and instructional procedure used.

After the students were assigned to one of the four groups, their names were listed on class rolls as if two distinct classes were meeting during each time period. This necessitated that the instructor meet with each group for one and one-half hours per week during their regularly scheduled time. The one and one-half hour per week figure is actually an average, since the groups alternated attending class as follows: one group per section would receive instruction on a regularly scheduled class day, would not receive instruction on the next regularly scheduled class day, and then would receive instruction on the third regularly scheduled class day. During the time they were not in class, the second group formed from that section would receive instruction. Each group attended a mathematics laboratory to conduct independent research while their companion group (the other group meeting during the time they would normally attend the three-hour course) was receiving instruction in the scheduled classroom. A schedule showing the students in each group and the dates they were to attend class was posted to help keep attendance orderly. The students not receiving instruction were allowed to attend the laboratory for any length of time, selected at their discretion, between the two sessions in which they
### TABLE I

DISTRIBUTION OF STUDENTS IN EACH GROUP BASED ON ABILITY

<table>
<thead>
<tr>
<th>ACT Composite Score</th>
<th>9:30 Inductive Procedure Class</th>
<th>9:30 Deductive Procedure Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>18 and above</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>15, 16, or 17</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>14 and below</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ACT Composite Score</th>
<th>10:30 Inductive Procedure Class</th>
<th>10:30 Deductive Procedure Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>18 and above</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>15, 16, or 17</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>14 and below</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>
were to receive instruction. This independent research consisted, basically, of time to contemplate materials designed to lead the inductive group to discover concepts or time to use materials designed to provide the deductive group with practice in using concepts.

An F-ratio was computed to determine if there was any significant difference in terms of those abilities measured by the student's ACT scores between the inductive group and the deductive group at the beginning of the experiment. This F-ratio was computed using a treatment by levels design (two teaching procedures and three levels of ability) to compare two sets of scores. These scores were obtained by combining the composite ACT scores for the two inductive groups into one set of scores and combining the composite ACT scores for the two deductive groups into one set of scores. From Table II it can be seen that the F-ratio for the two groups determined by the two teaching procedures did not reach significance and the F-ratio for interaction did not reach significance. This indicates that the means of the two groups were not significantly different and that the two groups were similar at each level of ability. Thus the two groups were found not to be significantly different at the beginning of the experiment.

In summary, the study was designed to conform to a two by three, treatment by levels design, in which the two treatments were the procedures of teaching and the three
levels were the three levels of ability. The effects of the two treatments were compared on the basis of their effect on student arithmetic understanding and student grades on three teacher-made tests. There were fifty-four students enrolled in two sections of Mathematics for Elementary School Teachers I who comprised the subjects of the study. These students were enrolled at Cameron College during the spring semester of 1973. After enrolling in the course, the students were randomly separated into four groups, with two of the groups receiving instruction through an inductive procedure and two of the groups receiving instruction through a deductive procedure. These four groups were matched according to student ability as measured by the students' composite ACT scores. In order to statistically analyze the data collected in this study, all data for the two deductive groups were
combined into one group of data and all data for the two inductive groups were combined into one group of data.

The Teaching Methods

In this section a description of the two teaching procedures used in this study is given. This description shows in which ways the two teaching procedures were similar and in which ways they differed. The two teaching procedures differed in five ways and were similar in five ways. They were similar in that they both stressed understanding on the student's part; they both were intended to teach the same concepts, in approximately the same order; they both required the same number of class meetings, involving the same number of class hours, to achieve the objectives of the course; they both used the same number of homework assignments, which were of approximately the same length and difficulty; and they both made use of the same laboratory arrangement. While the two teaching procedures are similar in these five ways, it should be pointed out that the approach of the two procedures to some of these categories of similarity was quite different. For example, in the inductive procedure attempts were made to help students understand what they learned by requiring the students to learn through examples and through active participation in the investigation of these examples. On the other hand,
in the deductive procedure attempts were made to help the student understand what they learned by justifying each concept through deductive arguments based on previous definitions, premises, and theorems, and by illustrating the concept through examples after the concept had been presented.

The two teaching procedures were different in that in the inductive procedure the concepts to be learned were introduced by example while in the deductive procedure the examples were given to the student after the instructor had stated the concept to be learned; in the inductive procedure the student used the examples to discover the concept to be learned and, occasionally, other just as important facts were discovered, but in the deductive procedure the students were given a formal statement of the concept to be learned; in the inductive procedure the students were given prescriptive guidance to aid discovery and to increase understanding of the concepts while in the deductive procedure the instructor gave deductive arguments to increase understanding of the formally stated concept; in the inductive procedure written material was provided which included exercises containing material gleaned from state adopted mathematics textbooks (grades K through 6) and designed so that the student would be led to discover the concept to be learned, but in the deductive procedure the written material contained only practice problems; and
in the inductive procedure, if a student could not work a homework problem, he was referred to an example that was similar or he was asked leading questions, while in the deductive procedure, if a student could not solve a homework problem, he was referred to a specific formula or principle that could be used to solve the problem. A more detailed description of these two teaching procedures is presented next.

The Inductive Procedure

The inductive procedure used in this study is an adaptation of an inductive teaching-learning thought process. However, a truly inductive teaching-learning process would require a great deal of time to achieve a designated pattern of objectives. This requirement was one of the factors, according to G. Hermann, which made the results of many studies involving the inductive teaching-learning thought process questionable, since more instructional time was devoted to the students who learned

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by the inductive process than to the students who learned by some other approach to teaching. Thus he concluded that future studies should attempt to equate the instructional time used for each approach to instruction. Due to G. Hermann's conclusion, and since Mathematics for Elementary Teachers I is limited by time and is the first course in a two-course sequence, the inductive teaching-learning thought process was compromised by allowing the instructor to provide the students with a degree of prescriptive guidance. This guidance emphasized discovery and understanding on the student's part by giving specific examples designed to lead the student to discover the desired concepts. These examples became increasingly more specific until the student discovered the desired concept. When impromptu discoveries were made by students before or during this guidance period, the guidance did not preclude the possibility of discussing them, but this guidance did lessen the chance for impromptu discoveries. According to G. Hermann and A. TerKeurst and J. Martin, this guidance was not only effective in equating the instructional time but was found to produce a desirable effect on the learning process. Thus, by providing specific forms of guidance for the students taught by an inductive procedure, it was possible to equate the instructional times

for the two procedures. The instructional sequence used in the inductive procedure can be described by the sequence of steps below.

Students were given, as a first step, a set of exercises pertaining to the concept to be learned. These exercises consisted of material gleaned from state adopted mathematics textbooks (grades K through 6). The material was selected so that the exercises exemplified the concept to be learned, and the example exercises became more closely related to the concept to be learned as the exercises proceeded. Through this process the exercises were selected so that the student would be led to discover the concept to be learned.

As a second step, the students were provided a time lapse of four days or longer for individual contemplation of these exercises. During this time the student visited an available mathematics laboratory, staffed by a full-time, highly qualified and successful teacher. The student was required to attend the mathematics lab, but the selection of a time and the length of each attendance was left to the student's discretion.

After the students completed the set of exercises, the students as a group embarked on the third step of the sequence. This step began with the instructor asking carefully selected questions designed to determine if
discovery of the concept had occurred. These questions were selected so that if discovery had occurred the students would answer readily, but if discovery had not occurred, the students would answer with difficulty, or not at all. If it was determined that discovery had occurred, the instruction involving the discovered concept proceeded to step five of the instructional sequence. If discovery had not occurred, the instruction for the desired concept proceeded to step four of the instructional sequence. It should be noted that occasionally students made unexpected impromptu discoveries during this questioning period. During such occasions the students were urged to make inquiries of their own and to discuss any discoveries they had made. Each such impromptu discovery was considered as important, if not more important, than the discovery of the desired concept.

The fourth step began when student response indicated that discovery had not taken place by a majority of the students. In this step the instructor did one or both of two things. First, the instructor asked the students questions designed to lead the students as a group to discovery. These questions involved either new example situations or queries using whatever insight the students had been able to verbalize. These example situations or queries became more closely related to the concept as this questioning continued. If this first procedure was totally ineffective, the instructor selected a second procedure. This procedure was
to assign a new set of exercises designed to lead the student to discovery.

Since, on the third and fourth steps, the instructor was asking questions and giving more specific examples, the students were giving their individual responses (discoveries, intuitions, etc.), and a general discussion about the problems involved was being conducted, the instructor was not stating the desired concept for the students, but was aiding the students to see weaknesses in their false conclusions. Thus this discussion was individualized as well as being a group process, since the students were forced to decide upon the most appropriate or strongest conclusion. In this way the students discovered the desired concept along with other important facts.

When a majority of the students had discovered the desired concept, step five was begun. This step consisted of the development of a formal statement of the discovered concept. This was done by the students as a group. If the development proved especially difficult, the instructor rephrased the students' statements in a more formal or precise form and named the concept.

As a final step, the students were given a set of exercises which gave them a chance to use what they had discovered. If a student could not solve a problem in this followup set of exercises, he was referred to an example that was similar or he was asked leading questions. When
possible this final set of exercises was used to introduce examples designed to lead the student to discover the next desired concept.

The Deductive Procedure

The deductive procedure of teaching used in this study was designed to reflect a deductive, a priori, general-to-particular approach to conceptualization. Like the inductive procedure the deductive procedure stressed understanding. However, understanding for the deductive procedure was the result of arguments based on previous definitions, premises, and theorems which were followed by illustrative examples. Also, through the deductive procedure students studied the same content material in approximately the same order and with written lessons of approximately the same length as the students taught by the inductive procedure. The instructional sequence used in the deductive procedure can be described by the sequence of steps below.

Students were given, as a first step, a statement of the concept to be learned and a name for the concept. This statement was presented to the students by the instructor or by means of instructional materials. All instructional material was designed to help the student understand the concept to be learned and to convince the student that the concept was valid. Following the statement of the concept,

\[6^\text{Emmer, op. cit.}\]
the instructor conducted a discussion of the concept. This discussion was highly teacher-centered. Both the discussion and the instructional material were structured so that student understanding was the result of arguments based on previous definitions, premises, and theorems. These arguments consisted of some simple deductive proofs and examples of applications illustrating the stated concept.

In the third step the students were provided with opportunities to use the concept presented to solve specific problems. These were usually in the form of exercise assignments to be completed by the student before the next class session. During the time the student was not involved in a class conducted by the instructor, he was required to visit the mathematics laboratory as described in the section on the inductive procedure. The only difference was that, in the deductive procedure, the student was working with a different set of exercises.

In contrast to the inductive procedure, all of the steps of the instructional sequence for the deductive procedure were usually presented in the same class period. All the instruction related to a given concept was given as a complete unit. However, interrelationships among the concepts were recognized and in most cases presented to the students.

In relation to these exercise assignments, a fourth step was conducted. This step involved what took
place if a student could not solve a practice problem. In this situation the student was referred to a specific formula or concept that could be used to solve the problem.

Before the beginning of the course, a complete course outline of topics to be presented, including course objectives, was written. This outline was the same for both teaching procedures and is supplied in Appendix A. As each lesson was given, a recording of the classroom verbal activity was made. These recordings are available on request. So that the reader may compare the inductive teaching procedure with the deductive teaching procedure, a log for each of four concepts as taught by the inductive teaching procedure and a log for each of the same four concepts as taught by the deductive procedure are supplied in Appendix B. These logs include a brief description of the types of verbal and/or written activity used to introduce these four specific concepts and a transcript of the recorded class activity used in discussing each of the four specific concepts. Since the same qualified teacher was available in the mathematics laboratory to all students, it was assumed that the effect of the mathematics laboratory was evenly balanced between the two groups and thus did not significantly affect the study.

To supply evidence of the validity and reliability of the two teaching procedures, an indirect to direct ratio,
I/D, was obtained using Flander's interaction analysis on the verbal content of eight fifty-minute lectures chosen at random. Four of these lectures were from the deductive group and four from the inductive group. According to Flanders, a higher I/D ratio implies extensive use of indirect teacher influence which characterizes the teaching activity of the inductive procedure, and a low I/D ratio implies extensive use of direct teacher influence which characterizes the teaching activity of the deductive procedure.

These I/D ratios were then statistically compared using a t-test for two related samples. From Table III it can be seen that the t-value was significant at the 0.0005 level. This significance indicates that the two teaching procedures were conducted in significantly different manners. And, according to Flanders, the inductive procedure made significantly greater use of indirect teaching activity.

**TABLE III**

<table>
<thead>
<tr>
<th>Mean of Differences Between Pairs of Scores</th>
<th>Standard Error of Difference Between Means of Two Samples</th>
<th>t</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.413</td>
<td>0.02397</td>
<td>17.23</td>
<td>0.0005</td>
</tr>
</tbody>
</table>

The Testing Program

In order to provide a means for comparing the effectiveness of the two teaching procedures, a testing program was conducted in which seven subtests of *A Test of Arithmetic Understanding* and three teacher-made tests were administered. Each of the subtests was chosen because it was designed to measure some form of arithmetic understanding emphasized by objectives of the course Mathematics for Elementary Teachers I. Together, the seven subtests measured the student's knowledge for almost all course objectives stated in the course outline. The teacher-made tests were included since they were constructed by the instructor and could possibly provide a more sensitive measurement of the student's knowledge of content areas and cognitive ability to use that knowledge for the stated course objectives. Also included in the testing program were the students' composite scores received on the American College Testing Program Examination. The scores from this examination were used to measure student ability. This measure of student ability was used to establish the three levels of ability which allowed the instructor to match the four groups and determine whether the relative effects of the two teaching procedures were dependent upon student ability. In the following sections a detailed description of the tests used in the testing program is provided.
The American College Testing Program Examination

The American College Testing Program Examination is a three-hour test battery designed to test knowledge of facts as well as ability to use knowledge in the solution of complex problems. The examination was developed in order to provide a predictor for the success of college-bound high school seniors and junior college students who intend to transfer to a four-year college. During each year over 300,000 students complete the test and the results are sent to over 700 colleges.

The test consists of four subtests. The items in each subtest are multiple-choice items. Test I is an eighty-item, fifty-minute test of English usage. Test II is a forty-item, fifty-minute test of mathematics usage. Test III is a fifty-two-item, forty-minute social studies reading test, and Test IV is a fifty-two-item, forty-minute natural science reading test. Each edition of the examination is published as a single thirty-two-page booklet containing the four subtests.

In developing new forms of the test, specifications for test items are developed. Then writers are employed to select and write test items which meet these specifications. "Tryout" units are then administered to large representative samples of students. Then item analysis is conducted and

8 Oscar Krisen Buros, op. cit.
the results on the new units are compared with the scores the students in the sample have achieved on the Iowa Tests of Educational Development. On the basis of this program of analysis, national percentile norms are developed. In addition, local norms and other data are provided for colleges that participate in the program.

The odd-even reliability coefficients of the four subtests are .90, .89, .86, and .83 for English, mathematics, social studies, and natural science respectively. The reliability coefficient of the composite score is .95.

A Test of Arithmetic Understanding

A Test of Arithmetic Understanding is a test battery designed to test a fifth-grade student's understanding of arithmetic. The test measures arithmetic understanding on the basis of two general categories—student understanding of content areas and student understanding of cognitive objectives.

Seven subtests were used to measure the students' understanding related to these two categories. Three subtests were used for content areas and four subtests for cognitive objectives. "Computation," "Number and Number Systems," and "Figures and Graphs" were the titles given to the three subtests used to measure understanding of content.

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areas. "Knowledge," "Comprehension," "Application," and "Analysis" were the titles given to the subtests used to measure understanding of cognitive objectives.

A description of these cognitive objectives was given by Von Brock as follows:

Knowledge--Knowledge of specifics, of ways and means of dealing with specifics, and of universals and abstractions in the field of arithmetic.

Comprehension--The ability to translate, interpret, and extrapolate arithmetic concepts.

Application--The ability to apply knowledge and comprehension to problem situations.

Analysis--The ability to analyze elements, relationships, and organizational principles in arithmetic.\(^\text{10}\)

Most of the items were multiple choice, but a few were questions requiring the students to compute a direct solution or construct a desired graph. The number of items on each subtest were: Knowledge--19, Comprehension--13, Application--11, Analysis--12, Computation--24, Number and Number Systems--21, and Figures and Graphs--10.

The reliability of A Test of Arithmetic Understanding as indicated by a split half correlation and corrected by the Spearman-Brown formula was .86, and the standard error of measurement for each subtest was .65, .43, .43, .41, .71, .59, and .43, respectively. These figures were obtained in a study by Von Brock of fifth-grade classes in five different schools. Although the test was designed

\(^{10}\text{Ibid.}, \text{ p. 538.}\)
for fifth-grade students, the test items are appropriate for measurement of the content of the course used in this study. To verify this contention, the test was administered to one section of Mathematics for Elementary Teachers I at the end of the fall semester of 1972. On a 120-point scale the scores ranged from 80 to 114, and 70 per cent of the scores were distributed between 84 and 107. These results seemed to show that the test provides a satisfactory distribution of scores and was of appropriate difficulty.

Teacher-Made Test

In addition to the seven subtests, a series of three teacher-made tests was administered during the course of the semester. These tests were designed to measure certain response probabilities regarding the specific course content covered during the time period between the administration of the previous test and the given test. None of these tests was comprehensive in nature. None of these tests was designed to measure content learned for the entire semester. The raw scores on these tests were totaled and this total score was used as the criterion score for the teacher-made test.

To support statistically the validity and reliability of the teacher-made tests, a Kuder-Richardson Formula 20 coefficient was computed for each test, and an item-analysis coefficient consisting of a point-biserial correlation
coefficient between the score on each item and the score on its respective test was computed for each item's content validity. The Kuder-Richardson coefficient for Test I, II, and III were .98, 1.06, and .98, respectively. A .20 coefficient was selected as the minimum acceptable item-analysis coefficient for any test item. All items whose item-analysis coefficients were below this level were discarded before the scores were totaled. Copies of the three tests are supplied in Appendix C. Each item's item-analysis coefficient is also given in Appendix C.

Procedures for Analysis of the Data

For purposes of statistical analysis, the scores on each of eight criterion tests taken by fifty-four students were divided according to two teaching procedures and three levels of ability. The two teaching procedures were the inductive procedure and the deductive procedure. The three ability levels were determined by each student's composite score on the ACT examination. Since these students were enrolled in two different sections of Mathematics 3353, each section was divided into two groups. One group in each section was taught by the inductive procedure and one group from each section was taught by the deductive procedure.

In order to analyze statistically the data collected from these groups, the scores for the students in the two
inductive groups were combined into one set of inductive group scores. Similarly, the students' scores in the two deductive groups were combined into one set. To analyze this data, a treatment by levels design was used for the two treatments and the three levels of ability. This design followed the following steps:

1. Raw scores (on each of the eight criterion tests) for students will be listed in a 3 x 2 table for the 3 levels of ability and the 2 methods of teaching.
2. The sum of raw scores for each group and each level will be computed.
3. The sum of squares will be computed.
4. The correction term, \( C = (\text{Total sum})^2 / \text{Total N} \), will be computed.
5. The sum of squares total, \( SS_t = SS - C \), will be computed.
6. The effects of levels, \( SS_{\text{levels}} \), for the three ability levels, 1, 2, and 3, will be computed as follows:
   \[
   SS_{\text{levels}} = (S_{t,1})^2 / N_1 + (S_{t,2})^2 / N_2 + (S_{t,3})^2 / N_3 - C
   \]
   where \( S_{t,1} \) = total sum for level one over the two methods of teaching, \( S_{t,2} \) = total sum for level two over the two methods of teaching, and \( S_{t,3} \) = total sum for level three over the two methods of teaching.
7. The treatment effects for the two methods of teaching, \( a \) and \( b \), will be computed as follows:
8. The interactive effects for the treatments x levels will be computed as follows:

\[ SS_{\text{treat x levels}} = (S_{1,a})^2/N_{1,a} + (S_{1,b})^2/N_{1,b} + (S_{2,a})^2/N_{2,a} + (S_{2,b})^2/N_{2,b} + (S_{3,a})^2/N_{3,a} + (S_{3,b})^2/N_{3,b} - C - SS_{\text{levels}} - SS_{\text{treatments}} \]

where

- \( S_{1,a} \) = sum of raw scores for ability one and method a,
- \( S_{1,b} \) = sum of raw scores for ability one and method b,
- \( S_{2,a} \) = sum of raw scores for ability two and method a,
- \( S_{2,b} \) = sum of raw scores for ability two and method b,
- \( S_{3,a} \) = sum of raw scores for ability three and method a,
- \( S_{3,b} \) = sum of raw scores for ability three and method b,
- \( N_{1,a} \) = N for ability one and method a,
- \( N_{1,b} \) = N for ability one and method b,
- \( N_{2,a} \) = N for ability two and method a,
- \( N_{2,b} \) = N for ability two and method b,
- \( N_{3,a} \) = N for ability three and method a,
- \( N_{3,b} \) = N for ability three and method b.

9. The error term will be computed as follows:

\[ SS_{\text{error}} = SS_t - SS_{\text{levels}} - SS_{\text{treatments}} - SS_{\text{treatments x levels}} \]
10. The degrees of freedom will be calculated as follows:
\[
df \text{ for } SS_t = N_t - 1, \; df \text{ for } SS_{\text{levels}} = \text{number of levels} - 1, \; df \text{ for } SS_{\text{treatments}} = \text{number of treatments} - 1, \; df \text{ for } SS_{\text{treat x levels}} = (df \text{ for } SS_{\text{levels}}) \times (df \text{ for } SS_{\text{treat}}), \text{ and } df \text{ for } SS_{\text{error}} = df \text{ for } SS_t - df \text{ for } SS_{\text{levels}} - df \text{ for } SS_{\text{treat}} - df \text{ for } SS_{\text{treat x levels}}.
\]

11. The mean squares will be computed as follows:
\[
MS_{\text{levels}} = \frac{SS_{\text{levels}}}{(df \text{ for } SS_{\text{levels}})}, \; MS_{\text{treat}} = \frac{SS_{\text{treat}}}{(df \text{ for } SS_{\text{treat}})}, \; MS_{\text{treat x levels}} = \frac{SS_{\text{treat x levels}}}{(df \text{ for } SS_{\text{treat x levels}})}, \; MS_{\text{error}} = \frac{SS_{\text{error}}}{(df \text{ for } error)}.
\]

12. The F ratios will be computed as follows:
\[
MS_{\text{levels}}/ MS_{\text{error}}, \; MS_{\text{treatments}}/ MS_{\text{error}}, \; \text{ and } MS_{\text{treat x levels}}/ MS_{\text{error}}.
\]

13. The F ratios for treatments will be used to compute the significance of the measures of differences for each part of hypotheses 1, 2, and 3.

14. The F ratios for levels x treatments will be used to compute the significance of the measures of relative effects for each part of hypotheses 4 and 5.

15. The .05 level of significance will be arbitrarily chosen as the level of significance for all statistical tests.
16. On the basis of part 16, 17, and 18, conclusions will be stated.

Summary

A study was conducted in which two teaching procedures were compared. The teaching procedures were an inductive procedure and a deductive procedure. Four groups of students were used in the study with each group matched according to three ability levels. Two of the groups were taught by one procedure and the other two by the second procedure. Fifty-four students enrolled in Mathematics for Elementary School Teachers I at Cameron College, Lawton, Oklahoma, were the subjects of the study.

Seven subtests of A Test of Arithmetic Understanding and the total score on three teacher-made tests were used as criterion tests. The seven subtests were administered during the last week of the 1973 spring semester. The three teacher-made tests were administered during the semester as unit tests.

The scores from these eight criterion tests were analyzed using a two-by-three, treatment-by-levels design. This constituted dividing the scores on each criterion test into six categories according to the two teaching procedures and the three levels of ability. The .05 level of significance was arbitrarily chosen as the level of significance for all statistical tests.
CHAPTER IV
THE RESULTS OF THE EXPERIMENTAL STUDY

Introduction

A study was conducted to compare the effectiveness of two procedures of instruction in a mathematics course, Mathematics for Elementary Teachers I: a) a deductive procedure designed to represent a typical, traditional approach to mathematics instruction, and b) an inductive procedure designed to reflect the recommendations of CUPM. The comparison of the two procedures was made by using two groups of students taught by a deductive procedure and two groups of students taught by an inductive procedure.

In order to determine the effectiveness of the inductive procedure and the deductive procedure, the scores of students on eight criterion tests were analyzed. The scores for the two groups taught by the deductive procedure were combined into one set of scores for analysis. And the scores for the two groups taught by the inductive procedure were combined into one set of scores for analysis.
To treat the data statistically, a treatment-by-levels design\(^1\) was used in which a two by three (two procedures and three levels of ability) matrix was employed. With this design, hypotheses were tested to determine if 1) there was a significant difference between the effectiveness of the treatments, 2) the effectiveness of the treatments depends upon the level at which they were used (e.g., the deductive procedure is a better procedure to use with low-ability students than the inductive procedure), and 3) the effects of the procedures on arithmetic understanding was dependent upon ability. This third class of hypotheses have been well established and are of no particular interest in this study. Therefore, the first two classes of hypotheses are of primary interest. However, in using a treatment by levels design, F-ratios for each of the three classes appear in the tabled data. Thus, each table of data will have three main categories: level of ability, teaching procedure used, and interaction between the procedures and the levels of ability. A significant F-ratio on the ability factor would indicate that the students in the three levels of ability performed differently on the eight criterion tests.

A significant difference between the two teaching procedures can be indicated in two ways. First, if the F-ratio for the interaction effect is insignificant, then a significant F-ratio for the teaching procedure factor would indicate that one teaching procedure is more effective than the other, and an examination of the means would indicate the procedure that was more effective. Second, if the F-ratio for the interaction effect is significant, then there would be an indication that a significant difference may exist between the relative effectiveness of the two teaching procedures. On the other hand, an insignificant F-ratio on the teaching procedure factor would indicate that both teaching procedures were equally effective.

A significant F-ratio for interaction would also show that the effects of the two teaching procedures are dependent on ability. An insignificant F-ratio for interaction would indicate that the two teaching procedures have similar effects on arithmetic understanding at each level of ability. In all cases the .05 level of significance was used to determine the significance of the effects of the various factors. The raw data used in the statistical analysis are given in Appendix D.

The Findings for the Subtest: Knowledge

"Knowledge" is a subtest of A Test of Arithmetic Understanding and consists of nineteen items. Most of these
items are multiple choice, but a few are questions requiring the students to compute a direct solution. The test is designed to measure "knowledge of specifics, of ways and means of dealing with specifics, and of universals and abstractions in the field of arithmetic."²

From Table IV it can be seen that the F for the interaction effect of the ability factor and the factor determined by teaching procedures is not large enough to indicate a significant effect at any acceptable level of significance. On this basis, the following hypothesis was accepted: the relative effects of an inductive procedure of teaching and a deductive procedure of teaching on arithmetic understanding, as measured by the subtest "Knowledge" of A Test of Arithmetic Understanding, will not be dependent upon student ability.

TABLE IV

TREATMENT BY LEVELS TABLE FOR SUBTEST: KNOWLEDGE

(N=54)

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>225.88</td>
<td>53</td>
<td>--</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>Ability</td>
<td>102.37</td>
<td>2</td>
<td>51.19</td>
<td>21.24</td>
<td>p&lt;0.001</td>
</tr>
<tr>
<td>Procedure</td>
<td>2.29</td>
<td>1</td>
<td>2.29</td>
<td>0.95</td>
<td>n.s.</td>
</tr>
<tr>
<td>Interaction</td>
<td>5.49</td>
<td>2</td>
<td>2.74</td>
<td>1.13</td>
<td>n.s.</td>
</tr>
<tr>
<td>Error</td>
<td>115.73</td>
<td>48</td>
<td>2.41</td>
<td>----</td>
<td>----</td>
</tr>
</tbody>
</table>

From Table IV it can be seen that the F for the factor determined by the teaching procedures is not large enough to indicate a significant effect at any acceptable level of significance. This indicates that the inductive procedure for teaching and the deductive procedure for teaching are equally effective when the criterion is that of arithmetic understanding as measured by the subtest "Knowledge." On this basis the following hypothesis was rejected: students taught by a deductive procedure will score significantly higher on the subtest "Knowledge" of A Test of Arithmetic Understanding than students taught by an inductive procedure.

The Findings for the Subtest: Comprehension

"Comprehension" is a subtest of A Test of Arithmetic Understanding and consists of thirteen items. All of these items are multiple choice. The test is designed to measure "the ability to translate, interpret, and extrapolate arithmetic concepts."³

From Table V it can be seen that the F for the interaction effects of the ability factor and the factor determined by teaching procedures is not large enough to indicate a significant effect at any acceptable level of significance. On this basis, the following hypothesis was accepted: the relative effects of an inductive procedure of teaching and

³Ibid.
a deductive procedure of teaching on arithmetic understanding, as measured by the subtest "Comprehension" of A Test of Arithmetic Understanding, will not be dependent upon student ability.

TABLE V

TREATMENT BY LEVELS TABLE FOR SUBTEST: COMPREHENSION

(N=54)

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>201.21</td>
<td>53</td>
<td>----</td>
<td>-------</td>
<td>----------</td>
</tr>
<tr>
<td>Ability</td>
<td>76.93</td>
<td>2</td>
<td>38.46</td>
<td>8.39</td>
<td>p &lt; 0.001</td>
</tr>
<tr>
<td>Procedure</td>
<td>3.13</td>
<td>1</td>
<td>3.13</td>
<td>0.68</td>
<td>n.s.</td>
</tr>
<tr>
<td>Interaction</td>
<td>1.15</td>
<td>2</td>
<td>0.57</td>
<td>0.12</td>
<td>n.s.</td>
</tr>
<tr>
<td>Error</td>
<td>220.00</td>
<td>48</td>
<td>4.58</td>
<td>-------</td>
<td>----------</td>
</tr>
</tbody>
</table>

From Table V it can be seen that the F for the factor determined by the teaching procedures is not large enough to indicate a significant effect at any acceptable level of significance. This indicates that the inductive procedure for teaching and the deductive procedure for teaching are equally effective when the criterion is that of arithmetic understanding as measured by the subtest "Comprehension." On this basis the following hypothesis was rejected: Students taught by an inductive procedure will score significantly higher on the subtest "Comprehension" of A Test of
Arithmetic Understanding than students taught by a deductive procedure.

The Findings for the Subtest: Application

"Application" is a subtest of *A Test of Arithmetic Understanding* and consists of eleven items. Most of these items are multiple choice, but a few are questions requiring the students to compute a direct solution. The test is designed to measure "the ability to apply knowledge and comprehension to problem situations." 4

From Table VI it can be seen that the F for the interaction effects of the ability factor and the factor determined by teaching procedure is not large enough to indicate a significant effect at any acceptable level of significance. On this basis, the following hypothesis was accepted: the relative effects of an inductive procedure of teaching and a deductive procedure of teaching on arithmetic understanding, as measured by the subtest "Application" of *A Test of Arithmetic Understanding*, will not be dependent upon student ability.

4Ibid.
From Table VI it can be seen that the F for the factor determined by the teaching procedures is significant at the 0.005 level. Since the mean for the deductive group was calculated to be 9.29 and the mean for the inductive group was calculated to be 8.25, these data imply that the deductive group performed significantly better on the subtest "Application" than the inductive group. On this basis the following hypothesis was reversed: students taught by an inductive procedure will score significantly higher on the subtest "Application" of A Test of Arithmetic Understanding than students taught by a deductive procedure.

The Findings for the Subtest: Analysis

"Analysis" is a subtest of A Test of Arithmetic Understanding and consists of twelve items. Most of these items
are multiple choice, but a few are questions requiring the students to compute a direct solution. The test is designed to measure "the ability to analyze elements, relationships, and organizational principles in arithmetic."^5

From Table VII it can be seen that the F for the interaction effect of the ability factor and the factor determined by teaching procedures is not large enough to indicate a significant effect at any acceptable level of significance. On this basis, the following hypothesis was accepted: the relative effects of an inductive procedure of teaching and a deductive procedure of teaching on arithmetic understanding, as measured by the subtest "Analysis" of A Test of Arithmetic Understanding, will not be dependent upon student ability.

### TABLE VII

TREATMENT BY LEVELS TABLE FOR SUBTEST: ANALYSIS

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>p</th>
</tr>
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<tbody>
<tr>
<td>Total</td>
<td>99.04</td>
<td>53</td>
<td>------</td>
<td>-------</td>
<td>-------</td>
</tr>
<tr>
<td>Ability</td>
<td>23.80</td>
<td>2</td>
<td>11.90</td>
<td>7.93</td>
<td>p &lt; 0.001</td>
</tr>
<tr>
<td>Procedure</td>
<td>0.07</td>
<td>1</td>
<td>0.07</td>
<td>0.04</td>
<td>n.s.</td>
</tr>
<tr>
<td>Interaction</td>
<td>2.72</td>
<td>2</td>
<td>1.36</td>
<td>0.90</td>
<td>n.s.</td>
</tr>
<tr>
<td>Error</td>
<td>72.45</td>
<td>48</td>
<td>1.50</td>
<td>-------</td>
<td>-------</td>
</tr>
</tbody>
</table>

^5Ibid.
From Table VII it can be seen that the F for the factor determined by the teaching procedures is not large enough to indicate a significant effect at any acceptable level of significance. This indicates that the inductive procedure for teaching and the deductive procedure for teaching are equally effective when the criterion is that of arithmetic understanding as measured by the subtest "Analysis" of A Test of Arithmetic Understanding. On this basis the following hypothesis was rejected: Students taught by an inductive procedure will score significantly higher on the subtest "Analysis" of A Test of Arithmetic Understanding than students taught by a deductive procedure.

The Findings for the Subtest: Computation

"Computation" is a subtest of A Test of Arithmetic Understanding and consists of twenty-four items. Most of these items are multiple choice, but a few are questions requiring the students to compute a direct solution. The test consists of items which are used to measure the abilities characterized by the four cognitive subtests and which are measuring concepts directly related to computation.

From Table VIII it can be seen that the F for the interaction effects of the ability factor and the factor determined by teaching procedures is not large enough to indicate a significant effect at any level above the .20 level of significance. On this basis, the following hypothesis was
accepted: the relative effects of an inductive procedure of teaching and a deductive procedure of teaching on arithmetic understanding, as measured by the subtest "Computation" of A Test of Arithmetic Understanding, will not be dependent upon student ability.

### TABLE VIII

TREATMENT BY LEVELS TABLE FOR SUBTEST: COMPUTATION

(N=54)

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
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<tr>
<td>Total</td>
<td>500.84</td>
<td>53</td>
<td>----</td>
<td>----</td>
<td>-------</td>
</tr>
<tr>
<td>Ability</td>
<td>216.44</td>
<td>2</td>
<td>108.22</td>
<td>19.96</td>
<td>p &lt; 0.001</td>
</tr>
<tr>
<td>Procedure</td>
<td>9.81</td>
<td>1</td>
<td>9.81</td>
<td>1.81</td>
<td>p &lt; 0.20</td>
</tr>
<tr>
<td>Interaction</td>
<td>14.36</td>
<td>2</td>
<td>7.18</td>
<td>1.32</td>
<td>n.s.</td>
</tr>
<tr>
<td>Error</td>
<td>260.23</td>
<td>48</td>
<td>5.42</td>
<td>----</td>
<td>-------</td>
</tr>
</tbody>
</table>

From Table VIII it can be seen that the F for the factor determined by the teaching procedures is not large enough to indicate a significant effect at any acceptable level of significance above the .20 level. This indicates that the inductive procedure for teaching and the deductive procedure for teaching are equally effective when the criterion is that of arithmetic understanding as measured by the subtest "Computation." On this basis the following hypothesis was rejected: students taught by a deductive procedure will score significantly higher on the subtest "Computation" of A Test.
of Arithmetic Understanding than students taught by an inductive procedure.

The Findings for the Subtest: Number and Number Systems

"Number and Number Systems" is a subtest of A Test of Arithmetic Understanding and consists of twenty-one items. These items are all multiple choice items. The test consists of items which are used to measure the abilities characterized by the four cognitive subtests and which are measuring concepts directly related to number and number systems.

From Table IX it can be seen that the F for the interaction effects of the ability factor and the factor determined by teaching procedures is not large enough to indicate a significant effect at any acceptable level of significance. On this basis, the following hypothesis was accepted: The relative effect of an inductive procedure of teaching and a deductive procedure of teaching on arithmetic understanding, as measured by the subtest "Number and Number Systems" of A Test of Arithmetic Understanding, will not be dependent upon student ability.
TABLE IX
TREATMENT BY LEVELS TABLE FOR SUBTEST:
NUMBER AND NUMBER SYSTEMS
(N=54)

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>340.82</td>
<td>53</td>
<td>----</td>
<td>-----</td>
<td>---------</td>
</tr>
<tr>
<td>Ability</td>
<td>54.37</td>
<td>2</td>
<td>27.18</td>
<td>4.74</td>
<td>p &lt; 0.025</td>
</tr>
<tr>
<td>Procedure</td>
<td>10.67</td>
<td>1</td>
<td>10.67</td>
<td>1.86</td>
<td>n.s.</td>
</tr>
<tr>
<td>Interaction</td>
<td>0.81</td>
<td>2</td>
<td>0.40</td>
<td>0.06</td>
<td>n.s.</td>
</tr>
<tr>
<td>Error</td>
<td>275.34</td>
<td>48</td>
<td>5.73</td>
<td>-----</td>
<td>---------</td>
</tr>
</tbody>
</table>

From Table IX it can be seen that the F for the factor determined by the teaching procedures is not large enough to indicate a significant effect at any acceptable level of significance. This indicated that the inductive procedure for teaching and the deductive procedure for teaching are equally effective when the criterion is that of arithmetic understanding as measured by the subtest "Number and Number Systems." On this basis the following hypothesis was rejected: Students taught by an inductive procedure will score significantly higher on the subtest "Number and Number Systems" of A Test of Arithmetic Understanding than students taught by a deductive procedure.

The Findings for the Subtest:
Figures and Graphs

"Figures and Graphs" is a subtest of A Test of Arithmetic Understanding and consists of ten items. Most of these
items are multiple choice, but one item requires the students to construct a desired graph. The test consists of items which are used to measure the abilities characterized by the three cognitive subtests "Knowledge," "Comprehension," and "Analysis," and which are measuring concepts directly related to figures and graphs.

From Table X it can be seen that the F for the interaction effects of the ability factor and the factor determined by teaching procedures is not large enough to indicate a significant effect at any acceptable level of significance. On this basis, the following hypothesis was accepted: The relative effect of an inductive procedure of teaching and a deductive procedure of teaching on arithmetic understanding, as measured by the subtest "Figures and Graphs" of A Test of Arithmetic Understanding, will not be dependent upon student ability.

TABLE X
TREATMENT BY LEVELS TABLE FOR SUBTEST:
FIGURES AND GRAPHS
(N=54)

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>p</th>
</tr>
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<tr>
<td>Total</td>
<td>141.34</td>
<td>53</td>
<td>-----</td>
<td>-----</td>
<td>----------</td>
</tr>
<tr>
<td>Ability</td>
<td>38.99</td>
<td>2</td>
<td>19.49</td>
<td>9.46</td>
<td>p &lt; 0.001</td>
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<tr>
<td>Procedure</td>
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<td>1</td>
<td>2.67</td>
<td>1.29</td>
<td>n.s.</td>
</tr>
<tr>
<td>Interaction</td>
<td>0.79</td>
<td>2</td>
<td>0.38</td>
<td>0.14</td>
<td>n.s.</td>
</tr>
<tr>
<td>Error</td>
<td>98.89</td>
<td>48</td>
<td>2.06</td>
<td>-----</td>
<td>----------</td>
</tr>
</tbody>
</table>
From Table X it can be seen that the F for the factor determined by the teaching procedures is not large enough to indicate a significant effect at any acceptable level of significance. This indicates that the inductive procedure for teaching and the deductive procedure for teaching are equally effective when the criterion is that of arithmetic understanding as measured by the subtest "Figures and Graphs." On this basis, the following hypothesis was rejected: students taught by an inductive procedure will score significantly higher on the subtest "Figures and Graphs" of A Test of Arithmetic Understanding than students taught by a deductive procedure.

The Findings for the Teacher-Made Test

The teacher-made test was composed of three unit tests. The tests included true-false, multiple choice, and short essay items, as well as items requiring the students to compute a direct solution. The tests were designed to measure the unit objectives for their individual units of instruction.

From Table XI it can be seen that the F for the interaction effects of the ability factor and the factor determined by teaching procedures is not large enough to indicate a significant effect at any acceptable level of significance. On this basis, the following hypothesis was accepted: The relative effects of an inductive procedure of teaching
and a deductive procedure of teaching on arithmetic understanding, as measured by the student's grade on teacher-made tests, will not be dependent upon student ability.

**TABLE XI**

**TREATMENT BY LEVELS TABLE FOR TEACHER-MADE TEST**

(N=54)

<table>
<thead>
<tr>
<th>Source</th>
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<th>Mean Square</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
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<td>53</td>
<td>------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>Ability</td>
<td>50001.60</td>
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<td>25000.80</td>
<td>20.44</td>
<td>p &lt; 0.001</td>
</tr>
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<td>Procedure</td>
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<td>1</td>
<td>400.17</td>
<td>0.32</td>
<td>n.s.</td>
</tr>
<tr>
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<td>0.21</td>
<td>n.s.</td>
</tr>
<tr>
<td>Error</td>
<td>58682.67</td>
<td>48</td>
<td>1222.55</td>
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<td></td>
</tr>
</tbody>
</table>

From Table XI it can be seen that the F for the factor determined by the teaching procedure is not large enough to indicate a significant effect at any acceptable level of significance. This indicates that the inductive procedure for teaching and the deductive procedure for teaching are equally effective when the criterion is that of arithmetic understanding as measured by the teacher-made test. On this basis the following hypothesis was rejected: students taught by an inductive procedure will score a significantly higher grade on teacher-made tests than students taught by a deductive procedure.
Summary

Eight criterion tests were used to determine the effectiveness of two procedures of instruction in a mathematics course Mathematics for Elementary Teachers I: a) a deductive procedure designed to represent a typical, traditional approach to mathematics instruction, and b) an inductive procedure designed to reflect the recommendations of CUPM. The scores on each test were analyzed using a two-by-three, treatment-by-levels design. The F-ratio was calculated for the interaction effect of the two procedures of teaching and the three levels of ability. The F-ratio for the effects of the two teaching procedures was also calculated.

On each of the eight criterion tests, the F-ratio for the interaction of the ability factor and the teaching procedure factor was not large enough to indicate a significant interaction of these factors. This implies that the comparative effectiveness of the inductive procedure of teaching and the deductive procedure of teaching was similar at each level of ability. As a result, the following hypotheses were accepted:

1. The relative effects of an inductive procedure of teaching and a deductive procedure of teaching on arithmetic understanding, as measured by each of the seven subtests ("Computation," "Numbers and Number Systems," "Figures and Graphs," "Knowledge," "Comprehension," "Application," and
"Analysis") of A Test of Arithmetic Understanding, will not be dependent upon student ability.

2. The relative effects of an inductive procedure of teaching and a deductive procedure of teaching on arithmetic understanding, as measured by the student's grade on teacher-made tests, will not be dependent upon student ability.

On one of the criterion tests, the F-ratio for the effects of the two teaching procedures was large enough to indicate a significant difference between the two teaching procedures. As a result the following hypothesis was accepted: Students taught by an inductive procedure will score significantly higher on the subtest "Application" of A Test of Arithmetic Understanding than students taught by a deductive procedure.

On the other seven criterion tests the F-ratio for the effects of the two teaching procedures was not large enough to indicate a significant difference between the two teaching procedures. As a result the following hypotheses were rejected:

1. Students taught by an inductive procedure will score significantly higher on each of the following four subtests of A Test of Arithmetic Understanding than students taught by a deductive procedure: a) "Number and Number Systems," b) "Figures and Graphs," c) "Comprehension," and d) "Analysis."
2. Students taught by a deductive procedure will score significantly higher on each of the following subtests of *A Test of Arithmetic Understanding* than students taught by an inductive procedure: a) "Computation" and b) "Knowledge."

3. Students taught by an inductive procedure will score a significantly higher grade on teacher-made tests than students taught by a deductive procedure.

In summary, only one of the eight factors considered gave a significant difference between the inductive procedure of teaching and the deductive procedure of teaching when the criterion was student arithmetic understanding. That factor was the factor entitled "Application" on *A Test of Arithmetic Understanding*. No other measures showed significant difference.
CHAPTER V

SUMMARY, FINDINGS AND CONCLUSIONS,
AND IMPLICATIONS

Summary

A study was conducted to obtain information regarding the effects of the application of an adaptation of an inductive thought process in a college mathematics course for prospective elementary school teachers.

The study was designed to compare the effects of two teaching procedures, one inductive and the other deductive, on student arithmetic understanding. The students involved in the study were fifty-four students enrolled in two sections of Mathematics for Elementary Teachers I at Cameron College, Lawton, Oklahoma. The experiment was conducted during the spring semester of 1973. Mathematics for Elementary Teachers I is a required three-hour course for all prospective elementary school teachers and is designed to follow CUPM's recommended course I, Number Systems and Their Origins. The two sections of Mathematics for Elementary Teachers I were taught at 9:30 and 10:30 respectively. Each section was separated into two groups. All of the four resulting groups were taught by the same instructor. This
made it necessary to have each group attend class, conducted by the instructor, for one and one-half hours per week and attend a mathematics laboratory, taught by a full-time, qualified teacher, for the remaining one and one-half hours per week. The groups alternated their class attendance so that the instructor could work with each group separately. The students were allowed to attend the laboratory on an individual basis. One group from each section was taught by the inductive procedure, and one group from each section was taught by the deductive procedure. Thus, two groups were taught by the inductive procedure and two by the deductive procedure.

Composite scores on the American College Testing Program Examination were used to establish three ability-level groups of approximately equal size. The first group consisted of students with composite scores of eighteen or higher. The second group consisted of students with composite scores of fifteen, sixteen or seventeen, and the third group consisted of students with composite scores of fourteen or below. The four class groups were matched by assuring approximately equal distribution of students in the three ability-level groups among the classes. It was assumed that the students were randomly assigned to the two sections of Mathematics for Elementary Teachers I by the enrollment process. To match the four groups, the students
in each section were categorized according to ability. The students in each of these categories were randomly assigned to one of the two teaching procedure groups. Thus, each of the two teaching procedure groups had the same number of students from each ability category or level. The teaching procedure to be used by each group was determined by a random process.

The purpose of this study was to compare the effectiveness of two teaching procedures of instruction in a mathematics course Mathematics for Elementary Teachers I: a) a deductive procedure designed to represent a typical, traditional approach to mathematics instruction and b) an inductive procedure designed to reflect the recommendations of CUPM. The inductive procedure of teaching is a procedure in which students learn through examples. In this procedure formal statements of concepts to be learned are not given to the students until the students have had experience with examples illustrating the concept to be learned and until the students are convinced that the concept to be learned is valid. The deductive procedure of teaching is a procedure in which students learn through the acceptance of authoritative statements of concepts to be learned. In this procedure a formal statement of the concept to be learned is given as an introductory statement, and this statement is followed by illustrative examples.
In order to determine the effectiveness of the two teaching procedures, scores on seven subtests of *A Test of Arithmetic Understanding* and the total score on three teacher-made unit tests were used. The seven subtests were entitled: "Knowledge," "Comprehension," "Application," "Analysis," "Computation," "Number and Number Systems," and "Figures and Graphs." These seven subtests were administered during the last week of the semester, and the three teacher-made tests were administered during the semester. The raw scores on each of the seven subtests and the total of the student's three teacher-made test scores were used as eight criterion scores. These eight criterion scores were analyzed by using a two-by-three, treatment-by-levels, design. The two treatments were the teaching procedures, and the three levels were the ability levels.

**Findings and Conclusions**

For each of the eight criterion scores the F-ratio for the interaction effects of the two teaching procedures and the three levels of ability was computed. Of the eight F-ratios computed, none was large enough to indicate a significant interaction between the teaching procedure factor and the ability factor. As a result, the following hypotheses were accepted:

1. The relative effects of an inductive procedure of teaching and a deductive procedure of teaching on arithmetic
understanding, as measured by each of the seven subtests ("Computation," "Numbers and Number Systems," "Figures and Graphs," "Knowledge," "Comprehension," "Application," and "Analysis") of A Test of Arithmetic Understanding, will not be dependent upon student ability.

2. The relative effects of an inductive procedure of teaching and a deductive procedure of teaching on arithmetic understanding, as measured by the student's grade on teacher-made tests, will not be dependent upon student ability.

On one of the seven subtests of A Test of Arithmetic Understanding, a significant F-ratio for the effects of the two teaching procedures was obtained at the .005 level. This subtest was the subtest "Application." Since the mean for the deductive group was higher than the mean for the inductive group, the following hypothesis was reversed: Students taught by an inductive procedure will score significantly higher on the subtest "Application" of A Test of Arithmetic Understanding than students taught by a deductive procedure.

For each of the other seven criterion scores, the F-ratio for the effects of the two teaching procedures was computed. Of the seven F-ratios, none was large enough to indicate a significant difference between the two teaching procedures at the .05 level. As a result, the following hypotheses were rejected:
1. Students taught by an inductive procedure will score significantly higher on each of the following four subtests of *A Test of Arithmetic Understanding* than students taught by a deductive procedure: a) "Number and Number Systems," b) "Figures and Graphs," c) "Comprehension," and d) "Analysis."

2. Students taught by a deductive procedure will score significantly higher on each of the following subtests of *A Test of Arithmetic Understanding* than students taught by an inductive procedure: a) "Computation" and b) "Knowledge."

3. Students taught by an inductive procedure will score a significantly higher grade on teacher-made tests than students taught by a deductive procedure.

On the basis of the testing program and the statistical analysis, it is concluded that, in a college mathematics course for prospective elementary school teachers, the inductive procedure of teaching and the deductive procedure of teaching are equally effective when the criterion is arithmetic understanding as measured by the six subtests, "Knowledge," "Comprehension," "Analysis," "Computation," "Number and Number Systems," and "Figures and Graphs," of *A Test of Arithmetic Understanding* and as measured by three teacher-made tests. However, the deductive procedure was found to be significantly better than the inductive procedure when used to teach a college mathematics course for
prospective elementary school teachers when the criterion was arithmetic understanding as measured by the subtest "Application" of A Test of Arithmetic Understanding. Since the subtest "Application" was designed to measure the student's ability to apply knowledge and comprehension to problem situations, this study shows that the deductive procedure is significantly more effective than the inductive procedure when used to teach this ability. Also, the mean for the deductive group on each criterion score was superior to the mean for the inductive group in all cases except when the criterion was the teacher-made tests.

Implications

The results of this study imply that in a college mathematics course designed for prospective elementary school teachers, a deductive procedure of teaching is superior to an inductive procedure. This implication might have been expected if the educational backgrounds of the students had been examined. If such an examination had been conducted, it might have been found that these students had been exposed to a deductive procedure of teaching during the majority of their educational experiences. If this were the case, the students would be able to perform better in a similar setting. However, the majority of studies found the heuristic methods superior when compared to the more traditional teaching methods.
With this in mind, it would seem that further research is needed to determine what effect the student's background has on the two teaching procedures. Also, since most of these comparative studies have used subjects whose age is younger than eighteen, it would seem that similar students who are now college-age would benefit in a similar way from the teaching procedures. Since the subjects of this study did not benefit similarly and since the results of most studies using college-age subjects obtained non-significant results, more research would seem to be needed to determine what effect the age of the subjects has on heuristic teaching methods.

If the teaching procedures used in a study are designed for a localized setting, factors inherent to the locale may affect the results. For example, the philosophy of the teacher or the degree of prescriptive guidance can affect the results. If the teacher has a teaching philosophy which favors one of the teaching methods, he may elicit more interest in the subject from students taught by that teaching method, or, if the local teaching situation requires liberal use of prescriptive guidance, the pure inductive thought process can be altered to the extent that the advantages it offers are decreased. In particular, the process may no longer foster a confidence in the learner that he has the ability to operate independently, learn for himself, and
cope with the problems he faces through the use of his own intellectual abilities. Also, the amount of prescriptive guidance may affect the way the learner behaves when he encounters an unprecedented problem or the way he behaves when he forgets how to solve a problem. Thus, it would seem that more localized studies should be conducted to provide information to be used as a basis for answering the question of whether or not the heuristic methods of teaching enhance learning.
APPENDICES
APPENDIX A

A COURSE OUTLINE FOR MATHEMATICS
FOR ELEMENTARY TEACHERS, I

(Based on CUPM recommendations on course content)¹

Unit 1: Sets and Functions

Unit one is designed as a "review, at an intuitive level, of the basic concepts associated with sets and functions in order to establish the language and notation that will be used throughout the course. Set concepts covered are: membership, inclusion, and equality for sets; various ways of describing sets (rosters, set-builder notation, Venn diagrams); special subsets that often lead to misunderstanding (empty set, singletons, the universal set); common operations on sets (intersection, union, complementation, cartesian product); illustration of the above concepts in various ways from real objects and from geometry....

The major function concepts to be covered include an intuitive rule-of-assignment definition; various ways of specifying this rule (arrow diagram, table graph, set of ordered pairs, formula); notions of domain and range; input-machine-output analogy; one-one properties; one-one

¹CUPM, op. cit., pp. 34-37.
correspondences between finite sets and between infinite sets; brief look at composition and inverses with a view toward later ties to rational arithmetic.²

Lesson 1: The meaning of set.

Objectives: 1.10³ Students will be able to recall the meanings of: set, element of a set, and well defined set, subset, proper subset, universal set, and empty set.

2.20 Students will be able to perform activities as prescribed by written directions using the terms: set, element of a set and well defined set, subset, proper subset, universal set, and the empty set.

Lesson 2: Ways of describing sets.

Objectives: 1.10 Students will be able to recall the meanings of: roster notation, set builder notation, and Venn Diagram.

2.20 Students will be able to perform activities as prescribed by written directions using the terms given in 1.10.

Lesson 3: Operations on sets.

Objectives: 1.10 Students will be able to recall

²Ibid., p. 35.

the meanings of: intersection, union, binary
operation, and grid.

2.20 Students will be able to perform
activities as prescribed by written directions
using the terms in 1.10.

Lesson 4: Comparison of sets.
Objectives: 1.10 Students will be able to recall
the meanings of: one-one correspondence, equiva-

tent sets, equal sets, negation notation, and
arrow diagram.

1.30 Students will be able to recall the
fundamental properties: reflexive, symmetric, and
transitive properties.

2.20 Students will be able to perform
activities as prescribed by written directions
using the terms in 1.10.

Lesson 5: Concept of number.
Objectives: 1.10 Students will be able to recall
the meanings of: number notation for sets, number,
numeral, order of numbers, ordinal numbers, cardin-

al numbers, finite sets, and infinite sets.

1.30 Students will be able to recall the
fundamental properties: trichotomy property.

2.20 Students will be able to perform activi-
ties as prescribed by written directions using the
terms in 1.10.
Note: The notion of function, domain, range, and input-machine-output analogy will be introduced in this unit only as intuitive concepts involved in assigned exercises.

Unit 2: Systems of Numeration.

Unit two is designed to contrast the Hindu-Arabic numeration system with historical and artificial numeration systems "in order to emphasize the roles of base and place value." 4

Lesson 1: Inventing a numeration system.

Objectives: 1.10 Students will be able to recall the meanings of: numeration system, and algorithm.

1.20 The students will be able to list the essential elements needed to form a numeration system.

2.20 Students will be able to perform activities as prescribed by written directions using the terms in 1.10.

5.30 The students will be able to invent a positional value numeration system.

Lesson 2: Historical numeration systems (Egyptian, Babylonian, and Roman).

Objectives: 1.10 Students will be able to recall

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4CUPM, op. cit., p. 35.
the meanings of: place value, base, exponent, and power.

2.20 Students will be able to perform activities as prescribed by written directions using the terms in 1.10.

4.10 The students will be able to recognize the points of similarity between any of the numeration systems (Egyptian, Babylonian, and Roman) and the Hindu-Arabic numeration system.

Lesson 3: Artificial numeration systems (base 2, 5, and 12).

Objectives: 3.00 The students will be able to count using symbols in each of the numeration systems (Egyptian, Babylonian, Roman, base 2, 5, and 12); the students will be able to convert numerals written in each of the numeration systems given above to numerals written in base 10, and vs.; the students will be able to extend these principles and methods to similar numeration systems with a base the student has not used before.

4.10 The students will be able to recognize the points of similarity between any of the numeration systems (base 2, 5, 12) and the Hindu-Arabic numeration system.
Note: Each of these numeration systems are compared to the Hindu-Arabic system as often as possible.

Unit 3: The Natural Number System and Whole Number System.

Unit three is designed to extend the concept of counting in Unit one, to the concepts of Natural number and Whole number. Also through the natural desire to specify the size of finite sets and the desire to overcome the inadequacies of historical numeration systems, the Natural Number System and Whole Number System will be developed.

Lesson 1: Definition for Addition.

Objectives: 1.10 Students will be able to recall the meanings of: natural number, whole number, sum, disjoint, ordered pair, first component, and second component.

1.30 Students will be able to recall the fundamental properties: addition operation.

2.20 Students will be able to perform activities as prescribed by written directions using the terms in 1.10.

Lesson 2: Basic addition facts.

Objectives: 1.10 Students will be able to recall the meanings of: grid, basic facts for addition of whole and natural numbers, symmetry, and function notation.

2.20 Students will be able to perform activities as prescribed by written directions
using the terms in 1.10.

2.20 The students will be able to explain how to find the sum, given as a function of an ordered pair of numbers, using an addition grid.

Lesson 3: Properties for addition.

Objectives: 1.30 Students will be able to recall the fundamental properties: closure, commutative, associative, renaming, and the identity element properties for addition.

2.20 The students will be able to use an addition grid to check the closure and commutative properties for addition.

6.10 The students will be able to show by counterexample why a set is not closed for addition.

Lesson 4: The subtraction process.

Objectives: 1.10 Students will be able to recall the meanings of: difference.

1.30 Students will be able to recall the fundamental properties: subtraction operation.

2.20 Students will be able to perform activities as prescribed by written directions using the terms in 1.10.

Lesson 5: The multiplication process (as repeated addition and Cartesian products).
Objectives: 1.10 Students will be able to recall the meanings of: basic facts for multiplication, Cartesian product, product, factor, perfect square.

1.30 Students will be able to recall the fundamental properties: multiplication operation.

2.20 Students will be able to perform activities as prescribed by written directions using the terms in 1.10.

2.20 The students will be able to explain how to use a multiplication grid to find the product, given as a function of an ordered pair of numbers.

Lesson 6: Properties for multiplication.

Objectives: 1.30 Students will be able to recall the fundamental properties: closure, commutative, associative, renaming, identity element, and distributive properties for multiplication.

2.20 The students will be able to use a multiplication grid to check the closure and commutative properties for multiplication.

5.30 The students will be able to illustrate how the distributive property may be used to find ax(a+1) when axa is known.

6.10 The students will be able to show by counterexample why a set is not closed for
multiplication.

Lesson 7: The division process.

Objectives: 1.10 Students will be able to recall the meanings of: quotient, dividend, and divisor.

1.30 Students will be able to recall the fundamental properties: division operation.

2.20 Students will be able to perform activities as prescribed by written directions using the terms in 1.10.

Lesson 8: Order of whole numbers and the number line.

Objectives: 1.10 Students will be able to recall the meanings of: number line.

2.20 Students will be able to perform activities as prescribed by written directions using the terms in 1.10.

Lesson 9: Solution sets of sentences.

Objectives: 1.10 Students will be able to recall the meanings of: solution set, solution frame, mathematical sentence, variable, and graph.

2.10 The students will be able to translate a given verbal sentence into a mathematical sentence and vs.

2.20 Students will be able to perform activities as prescribed by written directions using the terms in 1.10.
3.00 The students will be able to illustrate, by using a number line graph, the solution set of a given open sentence.

Unit 4: Arithmetic of Whole Numbers.

Unit four is designed to introduce the concepts about whole numbers which will be incorporated in the rational number system. Also, the algorithms of whole number arithmetic are justified (as in the elementary classroom) by reference to manipulating and grouping finite sets.

Lesson 1: Factors, multiples, prime numbers, composite numbers, and prime factorization.
Objectives: 1.10 Students will be able to recall the meanings of: multiple, divisible, composite number, prime number, odd number, even number, prime factor, and factor tree.

1.30 Students will be able to recall the fundamental properties: sieve of Eratosthenes.

2.20 Students will be able to perform activities as prescribed by written directions using the terms in 1.10.

Lesson 2: Greatest common factor and least common multiple.
Objectives: 1.10 Students will be able to recall the meanings of: divisibility tests, common factors, greatest common factor, relatively prime, common multiple, least common multiple.
2.20 Students will be able to perform activities as prescribed by written directions using the terms in 1.10.

Lesson 3: The algorithms for the operations on whole numbers.

Objectives: 1.10 Students will be able to recall the meanings of: partial product, remainder, and partial quotient.

1.30 Students will be able to recall the fundamental properties: algorithms for binary operations.

2.20 Students will be able to perform activities as prescribed by written directions using the terms in 1.10.

3.00 The students will be able to illustrate step by step how to use the different forms of each of the addition, subtraction, multiplication, and division algorithms to add, subtract, multiply, and divide, respectively.

4.10 Given an arithmetic problem worked out in steps, the students will be able to list the properties which justify the validity of each step.

Lesson 4: Arithmetic in different bases.

Objectives: 4.20 The students will be able to explain the points of similarity between an
addition problem performed in each of the numeration systems (base 2, 5, 12, and 10).

Unit 5: System of integers.

Unit five is designed to extend the operations of addition, subtraction, and multiplication of whole numbers into a new set, based on the need for a more appropriate solution set for the subtraction operation.

Lesson 1: The set of Integers and the addition operation.
Objectives: 1.10 Students will be able to recall the meanings of: integer, negative integer, non-negative integer, positive integer, and additive inverse.

1.30 Students will be able to recall the fundamental properties: opposite of additive inverse.

2.20 Students will be able to perform activities as prescribed by written directions using the terms in 1.10.

Lesson 2: Order of integers, the number line, and absolute value.
Objectives: 1.10 Students will be able to recall the meanings of: distance, and absolute value.

2.20 Students will be able to perform activities as prescribed by written directions using the terms in 1.10.
Lesson 3: Subtraction and multiplication in the set of integers.

Objectives: 4.20 The students will be able to contrast the natural, whole, and integral number systems.

6.10 The students will be able to show by counterexample why a set is not closed for a given operation.

Lesson 4: Solution sets of sentences.

Objectives: 1.30 Students will be able to recall the fundamental properties: cancelation property of binary operations for "equals" relation, and uniqueness property of binary operations for "equals" relation and "less than" relation.

Unit 6: System of Rationals.

Unit six is designed to introduce the system of rational numbers by a comparison of two finite sets in a "ratio" situation. Following this intuitive approach to fractions, a more detailed discussion of the set of rationals' algebraic structure, as compared to the set of integers, will be made.

Lesson 1: The set of Rationals and the multiplication operation.

Objectives: 1.10 Students will be able to recall the meanings of: rational, multiplicative inverse.
1.30 Students will be able to recall the fundamental properties: multiplication of fractions.

2.20 Students will be able to perform activities as prescribed by written directions using the terms: rational, and multiplicative inverse.

6.10 The students will be able to repeat a classroom proof of one of the basic generalizations of the property of multiplication of fractions.

Lesson 2: Equivalent fractions.

Objectives: 1.10 Students will be able to recall the meanings of: fraction, numerator, denominator, equivalent, and simplest form.

2.20 Students will be able to perform activities as prescribed by written directions using the terms: fraction, numerator, denominator, equivalent, and simplest form.

Lesson 3: Addition and Subtraction in the set of rationals.

Objectives: 1.10 Students will be able to recall the meanings of: common denominator, and least common denominator.

2.20 Students will be able to perform activities as prescribed by written directions
using the terms: common denominator, and least common denominator.

Lesson 4: Order of rationals.

Objectives: 1.30 Students will be able to recall the fundamental properties: denseness.

Lesson 5: Division of rationals and decimal notation.

Objectives: 1.10 Students will be able to recall the meanings of: division, decimal notation, mixed form, integral part, and fractional part.

2.20 Students will be able to perform activities as prescribed by written directions using the terms: division, decimal notation, mixed form, integral part, and fractional part.
APPENDIX B

LOGS FOR THE TWO TEACHING PROCEDURES
LOGS FOR THE INDUCTIVE PROCEDURE

Concept I

Concept: To teach the meaning of place value in the Babylonian numeration system.

One class meeting before this concept was to be discussed, a preliminary exercise sheet was given to the students for their independent consideration. The following problem was a problem in this exercise set designed to lead the students to discover the concept individually:

Babylonians used cuneiform symbols consisting of \( \text{ } \) for one and \( \text{ } \) for ten. Numbers like 4; 12; 59; 60; 70; and 72; and 119 would be symbolized as \( \text{ } \); \( \text{ } \); \( \text{ } \); \( \text{ } \) respectively.

a) How do you think a Babylonian would symbolize the following numbers? 42; 80; 82; 104; 120; and 180.

b) Would the Babylonian who wrote \( \text{ } \) be naming the same number as the Babylonian who wrote \( \text{ } \) ? Why or why not?

c) Count from 108 to 130 using Babylonian symbols.

Before the concept was discussed, the students were involved in discussions of powers and bases in relation to base ten and a discussion of the Egyptian numeration system.
The following is the class discussion of the concept. T represents teacher talking and S represents student talking.

T) Let's look at a Babylonian number or symbol. I gave you several examples of Babylonian numerals. Could you write a Babylonian symbol or numeral for 42? (Pause.) T) I don't think you would have a great deal of trouble with symbols for numbers like 10, 11, or 12. But what about 42? Do you have a name for the symbol. (Pause.) T) This symbol that's on its side, 𒐈, we call ten, and the symbol, 𒐂, we call? S) One. T) OK. How would you name 42? S) Four, oh, four of the tens. T) Four tens? S) And two ones. T) Do you all agree with that? S) Yes. T) How would I symbolize a number like 52? S's) Five tens and two ones. T) Do you all agree? S's) Yes. T) How would I symbolize a number like 57? S) Five tens and seven ones. T) How would I symbolize a number like 59? S) Add two more ones to the last number. T) How would I symbolize 60? S) With just a one. S) That one was given. S) That's a problem. T) OK. That one was given and looked like a one. You were also given an example for 70, 72, and 80. What did 72 look like? S) A one and a ten followed by two ones. T) What did 80 look like? S) One followed by two tens. T) Do the rest of you agree with that? (Pause.) T) What's the symbol 𒐂 representing? S)
One times 60. S) Sixty. T) This symbol, $<\text{ },$ is representing? S) Ten. T) And this symbol, $\checkmark<\checkmark,$ is representing what? S) Sixty, ten and ten. T) So the actual answer is 60 plus what? S) Ten. T) Plus? S) Ten. T) To give what? S) Eighty. T) That's how we are forming these numbers. Did we do something similar to that in the Egyptian system? S) Yes, we added the symbols to get the number. T) How did we know that four scrolls, two heel bones, and three strokes equalled 423? S's) Added their values. T) And, we do the same thing here. But this system does something a little different than the Egyptian system and it's all tied in with the concept we have been talking about. Let's go a little bit further with this.

How would you write a number like 112 in Babylonian symbols? S) Start out with a sixty, then four tens. No! Five tens and two units. T) Do you all agree? S's) Yes. T) How would I write 119? S"s) One sixty, five tens and nine ones. T) Would you still have five tens? S's) Yes. T) So it might look like $\checkmark<\checkmark<< \checkmark \checkmark \checkmark \checkmark$? S's) Yes. S) How did you come up with that one thing for sixty? T) That symbol was the one used by the Babylonians. Does that bother you? S) Well, it could be ten. S) It looks like they would have used it for something else. T) Does this symbol, $\checkmark,$ look like the symbol for ten? S) No, it looks like one, but it's a sixty. S) It's supposed to look like the symbol for one. S) OK. Then how do you get that same
symbol for one and a sixty? T) That's a good question. But that's what they used. S) Is it larger than the one symbol? S) They had place value. T) If it were written in this form $\overline{V<}$, could you tell if it was supposed to be a one or a sixty? S) Yes. T) And for these examples, $\overline{V<VV}$ and $\overline{V<}$, you can tell; but if all you saw was $\overline{V}$, you might think it was one. S) Yes. T) This was a problem in their system and the only way they could tell was to write a statement explaining the situation in which the numeral was used so that the reader could determine its value. For example, $\overline{V}$ might be the temperature in April in Oklahoma. Would you think that was a one or a sixty? S) Sixty. T) In $\overline{V<}$ it was easy to see that $\overline{V}$ represents sixty. Why? S) Because it comes before a larger valued symbol. T) How would I write 120 in their system? S's) Use two sixties. T) Does that make sense? S's) Yes. T) Actually if I put one more unit symbol on this $\overline{V<><><><><><><><><}$ symbol, I would have had ten unit symbols. How would they have written ten ones? S) With a ten symbol. T) This would give us $\overline{V<><><><><><><><><}$ with how many tens? S) Six. T) And six tens were usually written how? S) With a one. T) So, 120 was written as $\overline{VV}$.

T) OK. In counting in our system, how large was the set before we were forced to use a symbol whose value was different because it was in a different place? S) Ten.
T) In the Babylonian system, how large was the set before we changed to a different valued symbol? S's) Sixty. T) This characteristic of symbols having different values in different places is called place value.

Concept II

Concept: To teach the meaning of the subtraction principle used in the Roman numeration system.

One class meeting before this concept was to be discussed, a preliminary exercise sheet was given to the students for their independent consideration. The following problem was a problem in this exercise set designed to lead the students to discover the concept individually:

a) Answer the questions given on pages nine and ten of Silver Burdett's Grade five mathematics text. (See duplication 1 and 2.)

b) Also, answer the questions at the bottom of each duplicated page.

c) How did the Romans use subtraction in writing numerals?

The following is the class discussion of the concept:

T) Can you count using Roman numerals? S's) Yes. T) If I ask you to write a number like sixteen in Roman numerals, you could do that? S's) Yes. XVI. T) If I ask you to write a number like fourteen, what would you write? S's) XIV. T) Do we need to review any of those
things about Roman Numerals? (Pause.) S) Hey, which way do you go? Do you go VIII or IX? Which is correct? T) In their system—before I answer that, maybe we can all answer that for ourselves. In their system, what were some of the characteristics of the system? (Pause.) T) They have the symbols: I, V, X, and so on. Do they have some characteristics similar to the Egyptian system or Babylonian system? S's) Yes. T) Like what? S) Like they could write a smaller valued symbol before another symbol and subtract. T) Is that a characteristic similar to the Egyptian system? (Pause.) Or the Babylonian system? (Pause.) If they write an I before an X, this means that that I is supposed to be what? S's) Subtracted off. T) Then you are saying that IX is really the symbol for nine? S) Right. Where IX is 10 - 1. T) Did they ever write something like an IIX? (Pause.) If they did that, they would be saying what subtraction problem? S's) Ten, minus one, minus one. T) Was it possible for them to write IIX? S) Yes. But I don't think they did. T) Actually they did not; but they could have. In their system, they only subtracted two symbols. The only time an I could preceed a symbol was when I preceeded what? S's) A V or an X. T) Where IV stands for what? S's) Four. T) And IX stands for? S's) Nine. T) So, the characteristic or generalization you can make about this is that what? S's) A smaller symbol could be written before a larger symbol to mean
subtraction of the smaller symbol from the larger. T) What could we call this characteristic? S) Subtractive, since the addition characteristic was called additive. T) So, eight would be written as VIII. How would you write a number like 49? S's) XLIX. T) OK. So X can be written before what? S's) D or M. T) Do we need to make our generalization more specific? S's) Yes. Add that I comes before V or X, X comes before C or L, and C before D or M.

Concept III

Concept: To extend the concept of counting in base five to counting in base twelve.

Prior to the discussion of this concept the base five numeration system had been discussed in detail.

The following is the class discussion of the concept:

T) We can do the same thing (counting in base five) in base twelve or any other base. If I give you something like $24_{12}$, can any of you tell me what the next number is? S) What do you mean? T) What is the next number in the counting sequence? (Pause.) S) $25_{12}$. T) Would $25_{12}$ be the answer if I added one onto $24_{12}$? S's) Yes. T) What is the next number in the sequence? S's) $26_{12}$. T) The next? S's) $27_{12}$, $28_{12}$, $29_{12}$, (pause.) T) What is the next number? (Pause.) S) I'm not sure. T) What do you think it should be? S) 2 ten$_{12}$. T) And the next? S) 2 eleven$_{12}$. T) And the next? S) 30$_{12}$. T) Does this
follow the same pattern as base five? S) Yes, but I can't write \(210_{12}\). (Pause.) That looks like 156 instead of 34.

T) If that's true, we have a problem in base twelve. Can you find a way around this problem? (Pause.) Could we invent a symbol for 10 or 11? (Pause.) Actually, the symbols T and E are used to represent ten and eleven. So 2 ten\(_{12}\) becomes what? S) 2T\(_{12}\). T) And 2 eleven\(_{12}\) becomes what? S's) 2E\(_{12}\). T) Any questions? S) Well, this is the same thing as base five. Whenever you add that 12 on, you still add, change it to one, you still carry, it's still the same principle, isn't it? T) Yes. It will be the same for any system I give you regardless of the base. Do we need to count any further than this? S) Would there be any problem of picking up the counting if I had something like 2T\(_{12}\)? Would that present a problem? T) If I had 2T\(_{12}\)? S) Yes, would that present a problem? T) Well, the next number would still be 2E\(_{12}\). S) Oh! I see. OK. T) Suppose I start a little further down the list, with 44\(_5\) or EE\(_{12}\), can you give me the next number? S's) 100\(_5\) and 100\(_{12}\). S) 50\(_5\). T) If you think it would be 50\(_5\), then remember about the digits. How large can the digits be in base ten? S) Nine. T) In base five, how large can the digits be? S) Four. T) In base twelve? S) Eleven. T) In base eight? S) Seven. T) So in base five, we cannot write 50\(_5\). What should we write? S) 100\(_5\). T) So 44\(_5\) goes to 100\(_5\) and EE\(_{12}\) goes to what? S's) 100\(_{12}\). T) The
only way we can tell the difference between these numerals is what? S) The small number five or twelve. T) OK. In one case, the one represents one set of twelve-twelves and in the other the one represents what? S) One set of five-fives. T) Now, we can go further and find the next number in the sequence $444_5$ and $EEE_{12}$. What are the numbers? S) One thousand. T) They look like 1000 but they are $1000_5$ and $1000_{12}$. S) Yes. T) Any questions? S's) No.

Concept IV

Concept: To define addition of Natural numbers.

One class meeting before this concept was to be discussed, a preliminary exercise sheet was given to the students for their independent consideration. The following problem was a problem in this exercise set designed to lead the students to discover the concept individually:

The set of numerals $\{1, 2, 3, \ldots\}$ we have been using to answer the question of "how many elements a set contains," is called the set of Natural numbers; $N$: $N=\{1, 2, 3, \ldots\}$. If we let the numeral 0 symbolize $N(\emptyset)$, and union $N$ with $\{0\}$ we have the set of Whole numbers; $W$: $W=\{0, 1, 2, 3, \ldots\}$.

Duplication one from Mathematics We Need, Book One is an example of how the authors of Ginn Modern Mathematics series introduce the concept of addition of natural and whole numbers.
a) Using picture story 3, make up a word story which describes what is happening in story 3.

b) What is \( N(\{\text{dogs laying down}\}) \)?

c) What is \( N(\{\text{dogs playing with the ball in the left}\text{ hand set}\}) \)?

d) What is \( N(\{\text{dogs in the right hand set}\}) \)?

e) If you were a first grade teacher, how would you describe the concept of addition of two whole numbers using these illustrations? (Use set terminology in your description.)

The following is the class discussion of the concept:

T) I asked you a few questions concerning the set of natural numbers and whole numbers in terms of the word addition. All of us, I am fairly certain, could add three and four and come up with an answer, that was correct 90 per cent of the time. What does this concept mean to a six-year-old? How are you going to explain that \( 3 + 4 = 7 \) to a six-year-old or a seven-year-old, which is where that concept comes into being? (Pause.) In order to do that, you are going to have to decide what addition means to you. You are going to have to define addition for yourself. (Pause.) I will just ask you if you can come up with a description or definition for addition from some of those examples you were given. (Pause.) S) You could say addition is a process of grouping of two sets into one, or combining. T) Would you repeat that, please? S) Addition
2 dogs and 1 dog
3 dogs

Dogs and dogs
Dogs

Dogs and dog
Dogs

Dogs and dogs
Dogs
is a process of combining two or more sets into one. T) Would you agree with that for a description of what we mean by addition? In other words, could we use that description to explain why $3 + 4 = 7$? Could you? If you can, then it is a pretty good definition. (Pause.) T) Try it. That's the only way to tell. S) Put three pennies in one set and four pennies in another set and combine them to form a set with seven pennies. T) So it would look like this:

$$A = \{\emptyset, \emptyset, \emptyset\} \quad \text{and} \quad B = \{\emptyset, \emptyset, \emptyset, \emptyset\}$$

to get

$$C = \{\emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset\}.$$ Would this be a good description for what took place? S) That wouldn't be. That's not true. T) Why not? S) That definition wouldn't make that description a true statement, because if you try to combine two or more sets, and you have two sets up there, into one—all right, two sets. If you combine sets A and B. That would be two sets. Could we combine those two sets to get set C? S) You could combine the objects of the sets. T) OK. How do you combine sets; maybe that's what your question is? What does the word "combine" mean in that description? S) Put them together into one. T) If we had a group of three pennies and a group of four pennies and pushed them together, would that be what you mean? S) I think that's what he meant. T) Would that work for this? S) Not really. T) Maybe we need to explain more what the word "combine" means. That is a pretty general word, but it is probably the one you would use to
explain addition to a six-year-old. The word that would actually be used in most elementary text books would be the word "join." S) You wouldn't need the word elements of a set? That's what's bothering me. T) Oh! You mean you want to add the phrase elements of a set in the description. S) Yes. You are really combining the elements of the sets and not the sets. T) OK. You want to change it to: Addition is a process of combining the elements of two or more sets into one set. That certainly won't hurt anything.

What do you mean by the word "process"? S's) A way. T) OK. This description is certainly a good description that makes this true. Could you take this description and use set terminology to replace the word "combine" or "join."

S) Unioned the two sets. T) So we could replace the word "combining" with a phrase using "union." How could we write it using "union"? S) The union of two sets will equal the sum of two sets. T) When you form a sum, what do you form a sum of? Two sets or two numbers? S) Change two sets to two numbers. T) Does that make sense to say that the union of two sets will equal a number or would it make more sense to say that the number of elements in the union will equal a number? The three is actually equal to what? S) \(N(A)\). T) And the four is equal to? S) \(N(B)\). T) And seven? S) \(N\) of the union. T) So we can't say the sum of the two sets, but we can say the union of two sets. So how could I get the number idea in the statement? S) You could say the
number of elements in the union of two sets will equal the
sum of the number of elements in the two sets. T) Yes,
because we want this to express the concept of number and
sum. If I gave you two sets could you show me that the sum
is what it is supposed to be? S) Yes. T) OK. Show me
that $2 + 3 = 5$, using sets. S) Write a set with two ob-
jects, a set with three objects and form their union. The
union will have Five objects. T) OK. Suppose I gave you
$A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$. What is $N(A)$? And
$N(B)$? S) Three and three. T) What is $A \cup B$? S) $\{1,$
$2, 3, 4\}$. T) What is $N(A \cup B)$? S) Oh! A and B have to
be different. T) In what way? S) They have to have
different elements. T) What is the name for two sets
which have no elements in common. S) Disjoint. T) OK.
Suppose I gave you set $A = \{1, 2, 3, \ldots\}$, what is
$N(A)$? S) The sets have to be finite. T) Good. So we
need to add to our definition that the two sets have to be
finite and disjoint. This formal definition should help you
to select examples appropriate for teaching addition. This
definition is actually for addition in the set of whole
numbers. If you select for set $A$, the empty set, is there
a symbol in the set of $N$ for $N(A)$? S) No. T) Thus, for
a definition of addition in the set of natural numbers, $A$
and $B$ would have to be what? S) Non-empty. T) Are there
any questions? S' s) No.
Concept I

Concept: To teach the meaning of place value in the Babylonian numeration system.

Before the concept was discussed, the students were involved in discussions of powers and bases in relation to base ten and a discussion of the Egyptian numeration system.

The following is the class discussion of the concept:

T) The next system that we want to look at is the Babylonian System. It is more complicated than the Egyptian system and is an improvement over the Egyptian system, because it involves the property of our system called place value. And it was invented about the same time as the Egyptian system.

Rather than to have to invent new symbols every time they wanted to talk about a larger number, they used only two symbols in their system. One was a wedge-shaped symbol shaped like this, \( \overline{\|} \), with a value of one, and the other was a wedge-shaped symbol shaped like this, \( \overline{\|} \), with a value of ten. In order to print these, they used a wedge-shaped stylus to press the imprint into soft clay. And for that
reason, they called it cuneiform writing, meaning wedge-shaped writing. If they wanted to count or symbolize the number of elements in a set, they would put the elements in one-to-one correspondence with the symbols: \( V \) for the first element, \( VV \) for the second element, \( VVV \) for the third element, and so on up to \( VVVVVVVVV \) for nine, \( V \) for ten. So we would have one, two, three, and so on up to nine, and then ten. In their system rather than starting with one-zero and then one-one like we do in our system, they wrote \( <V \) for eleven, for twelve, \( <<V \), thirteen, \( <<<V \), and so on until they got up to nineteen, \( <VVVVVVVVVV \). Then the next number would be what? S) Two tens. T) Yes, \( <VV \), twenty. They repeated that process through thirty, forty, and fifty, until they got to fifty-nine or \( <VVVVVVVVVV \). Once they got to fifty-nine, the next number they wrote looked like this, \( V \). Which looked like what? S) One. T) Actually, it represented sixty. This was a problem in their system. That symbol looks exactly like the symbol they started out with. So, in their system, in order to tell you that that represented sixty, they had to write a statement explaining the use of the symbol. For example, that might represent the temperature in April in Lawton, Oklahoma. You probably would not think that it would be one degree. Most likely, it would be sixty degrees. So they needed a statement. But more important than that, they have thrown in something
we call place value. Instead of grouping things in sets of
tens like it looked like they started out doing, they
grouped things in sets of sixty. Every time they got to a
set of sixty, they would go back to their smaller symbol,
one, and represent the one set of sixty as, \( \text{V} \). For seventy
they would write it like this, \( \text{V} < \), where the \( \text{V} \) repre-
sents sixty and the \( < \) represents ten. Which gives me
sixty plus ten to make seventy. This looks a little dif-
ferent than the symbol they had for eleven because the one
is in a different position. So for a symbol like \( \text{V} < \),
it is not hard to tell that the \( \text{V} \) represents a sixty. Any
time the smaller valued symbol is to the left of a ten, it
had a higher value. For that reason, we say that their
system had place value, because the same symbol in different
places had different value, just like our system. In our
system, if you write 121, the one on the left has a value
one hundred times greater than the one on the right. We do
repeat our symbols, also. If I were to write a number that
had place value, like \( \text{VV}<\text{VV} < \), could you tell me what
number I was expressing in Babylonian notation? S's) 142.
T) This \( \text{V} \) would be representing sixty, the next \( \text{V} \),
sixty, \( < \), ten, \( < \) ten, \( \text{V} \) one, and \( \text{V} \), one. So we have
120 plus 20, plus two, or 142. So this system has place
value where there are two places (\( \text{VV} \)) (\( <\text{VV} \)). What about
this one: \( <\text{VV} < \text{V} \)? (Pause.) S's) 741. T) This system
is much better than the Egyptian system for this reason.
Now their system was based on groups of sixty, and not groups of ten. Thus, their places increased in value for units to groups of sixty, and so on. Our system goes for groups of units to groups of ten, and so on. So their system was very similar to ours.

How many places would this number have: \( \text{\texttt{V\texttt{I}}} \text{\texttt{I}} \)?
S) Two. T) What is the value of the second place? S) 133 sixties. T) In our system, in the number 123, the one has what value? S) One hundred. T) Or one set of ten-tens. The value of the first two one symbols in \( \text{\texttt{V\texttt{I}}} \text{\texttt{I}} \) is two sets of sixty-sixties. So this number has three places, just like 123 in our system. They are \( \text{\texttt{V\texttt{I}}} \text{\texttt{I}} \). What number is represented by \( \text{\texttt{V\texttt{I}}} \text{\texttt{I}} \text{\texttt{V}} \)? (Pause.) S's) 4,871. T) And the value of the on the left is what? S's) 3,600. T) Any questions? S's) No.

Concept II

Concept: To teach the meaning of the subtraction principle used in the Roman numeration system.

The following is the class discussion of the concept:

T) Are most of you familiar with the symbols used in the Roman System? Or have you forgotten them? S) I have forgotten some of the higher ones. T) All right, these--the basic symbols that they used were I for one, V for five, X for ten, L for fifty, C for one hundred, D for five hundred
and M for one thousand. These are basically the only seven symbols they used. They did have a symbol where if they put a bar over any one of these seven symbols, the value of the symbol would be multiplied by one thousand. So an X with a bar over it means the same thing as ten times one thousand or, in this case, ten thousand. Or a C would be the same thing as 100 x 1000. The bar only means multiply the symbol value by one thousand. If they put two bars over it, that meant multiply the symbol value by one thousand and then multiply that product by one thousand. Or, multiply by (1000)^2. If they wanted to symbolize a number like three, they would use an additive property just like the Egyptians used, by repeating these symbols, using enough of the symbols to add up to three. E.g., 3, III. To symbolize a number like eight, they would use enough symbols like 5 + 3 to give 8. So eight would be what? S's) VIII. T) If I wanted to write a number like 257, could you symbolize that in the Roman system? (Pause.) S's) It would be CCLVII. T) Any questions? (Pause.) All you had to do was find the correct arrangement of symbols to add up to 257. Or the least number of symbols you could use to add up to 257. Now, if they had a number like 900, you could write that with a D and four C's. S's) Right. T) Actually, they must have felt like that was too much to write down or chip out of stone, because they did invent a system where for special numbers like nine or four they
could rewrite those using a subtraction process. Where this 900 can be thought of as 1000 - 100. And the symbol for 1000 in this system is M and for 100 is C. So, I could have written CM where when I write a smaller valued symbol just before a larger valued symbol, it means to subtract its value off. So 900 is an example of a number which can be written with fewer symbols using that process. But it can only be like subtracting a one from a five or a one from a ten. I can only subtract that one from the next two higher valued numbers. I can only subtract the X from the next two higher valued numbers. And I can only subtract the C from the next two higher valued numbers. I can't subtract an X from a D or an M to get 490 or 990. I can't do that particular thing. Only the valued symbol can be subtracted from the next two larger symbols. They actually could have subtracted a V from an X, but it wouldn't make much sense to do that, because what is 10 - 5? S) Five. T) So this didn't decrease the number of symbols. So they took these symbols in order: I from V or X, X from L or C, and C from D or M. That's the only way they could use this subtraction operation. So if you wanted to write four, there would be two ways to write it, and one way would use less symbols. What would be the way to write four using the least symbols? S's) IV. T) How could I write the number nine? S's) IX. T) Or the number 40? S's) XL. T) And the number 90? S's) XC. T) And I could go up to 400 and 900. Any
questions on that? S's) No. T) Then you can write in
Roman numerals, say the year 1973? What would that look
like? S's) MCMLXXIII. T) Do you agree with that? S's)
Yes. T) How could you check to see if that was the correct
symbol? (Pause.) T) One thing we can do is say what is
the value of this symbol, M. S) 1,000. T) And write that
number down and add it to whatever the value of each of the
rest of them are. What is the value of this symbol, C?
S's) 100. T) But can I just write it down as 100? S's)
No, because the next symbol is 1,000. T) OK. Because the
CM is one symbol and I would think of that as 1000 - 100,
or 900. L is what? S's) 50. T) XX is what? S's)
Twenty. T) And III is what? S's) Three. T) So we have
1000 + 900 + 50 + 20 + 3, which is 1973. Can you do some
of these? S) Try 49. Would you have to have four X's
then? T) Would you have to use four X's to write 49? S's)
Could you use XLIX? T) Do you agree with that? S) Yes.
T) Could I write it like XXXXVIII and be correct? S's)
Yes. T) Yes, but the Romans probably would have used the
least number of symbols such as XLIX. There are probably
other ways it could be written and still be correct. All
you have to do is be sure the symbol values add up to the
numeral value. S) Could you have IL? T) No. It would
have been nice if they did that, but they didn't. T) Any
questions on this? S's) No.
Concept III

Concept: To extend the concept of counting in base five to counting in base twelve.

Prior to the discussion of this concept the base five numeration system had been discussed in detail.

The following is the class discussion of the concept:

T) If I gave you any base, could you count in that base? S) I don't know. T) Suppose I asked you to count in base twelve, could you write out the numerals used in counting? (Pause.) T) OK. In base twelve we have the numerals 0, 1, 2, 3, . . ., 9, just like we have in base ten. But, like in base five and base ten, we have to have the same number of symbols as the number of elements in our grouping set. So base twelve has to have twelve symbols. For ten and eleven, we use the letters T and E. So our symbols are 0, 1, 2, 3, and so on up to 9, T, and E. This is similar to base five where we have the symbols 0, 1, 2, 3, and 4. So in counting we start out with 0, 1, 2, 3, and so on up to 9. What numeral would follow 9 in this counting sequence? S) T. T) Then what? S) E. T) OK. What would be a numeral for a set of twelve? S) 10₁₂. T) Yes, one set of twelve and no units. What would be a numeral for a set of thirteen? S) 11₁₂. T) OK. One set of twelve with one unit left over. What about fourteen? S) 12₁₂. T) And fifteen? S) 13₁₂. T) And so on up to 19₁₂.
What would the next numeral be? S) 1T₁₂. T) And the
T) OK. If you select the right numbers, you can even
spell words. What is the next number in the sequence after
questions? (Pause.) OK. Can you count up to any number I
give you? For example, up to a set with 144 objects? S's)
Yes. T) What would the last numeral in that sequence
look like? S) 1₀₀₁₂. T) OK. Any questions on any of
this? S's) No.

Concept IV

Concept: To define addition of Natural numbers.

The following is the class discussion of the concept:

T) In working with Natural numbers we have discussed
how to count, how a child might learn to count, and so on,
using examples of sets where we had sets of toys or sets of
objects a child might want to count. Now the next step in
the elementary school program involves how do you go about
adding two Natural numbers. This is the next logical proce-
dure after counting. What I am going to do is give you a
definition of addition and then we will see why that defini-
tion is true. This definition will be quite a bit different
from the way you learned addition.

Addition will be defined this way in most teachers'
editions of elementary textbooks. This definition will not
be the one you state for the child, but you will teach all of the concepts in the definition whether they are stated or not. Addition of two Natural numbers, a and b, will be defined as the sum, c, where c is the number of elements in the union of two disjoint, finite, and non-empty sets, A and B, and where N(A)=a, and N(B)=b. What this really says is that N(A) + N(B) = N(A ∪ B), where A and B are finite, disjoint, non-empty sets. This sounds like an awfully complicated way of defining a simple topic like addition. Quite obviously, you are not going to give this definition to a first grader. In the first place, he couldn't read the terms. All the concepts you need to know to teach addition are in this statement. And that's the reason why we state it this way in this course. What does all of this mean? Do you understand all of this? (Pause.)

Now, I think we will look at a few examples first, and then explain where some of this comes from. If you are, let's just say we are a child being introduced to addition, and are given a set of toys or balloons, say when you went to a shoe store. One store gives you a red and a yellow balloon, and another store gives you a toy whistle. As a child, you have all of these objects in a shopping bag or the back seat of a car. One of the common questions you might ask would be, "How many gifts do I have?" To answer the question, you would have to add the number of elements in the two sets to get the number three. How do you do
that? How does a child answer this question? What is the number of elements in the first set? S) Two. T) What is the number of elements in the second set? S's) One. T) Label the sets A and B, and we have \( N(A) = 2 \), and \( N(B) = 1 \). Now, when you are counting these gifts, what does the child actually do to the two sets? S's) He puts them together into one set. T) He joins the two sets, which we call the union operation on two sets. What is \( A \cup B \)? S's) Red balloon, yellow balloon, whistle. T) What is \( N(A \cup B) \)? S's) Three. T) So \( N(A \cup B) = N(A) + N(B) \). That's all that is involved in addition, and we said all of that in the definition. Plus, we described what kind of sets we could use for examples. Why do the sets have to be finite? (Pause.) Suppose I gave you the sets, \( A = \{1, 2, 3, \ldots \} \) and \( B = \{2, 3, 4\} \), could you find \( N(A) \)? S) You can't find that since \( A \) is infinite. S) You can't add those. T) So, when you start out teaching addition, you will always use finite sets. These usually have cardinality of one, two, three, or four. So that the sums are never larger than five.

There are some other words in the definition that we haven't discussed, such as disjoint or non-empty. Why does a set have to be disjoint? S's) If the sets had one element in common, the union wouldn't have enough elements. T) OK. If we had \( A = \{1, 2, 3\} \), and \( B = \{2, 3, 4\} \), then \( A \cup B = \{1, 2, 3, 4\} \). Is the sum of \( N(A) \) and \( N(B) \) equal to
N(A ∪ B)? S's) No. T) So the sets have to be disjoint. Any questions? S's) No. T) Why did the sets have to be non-empty? S) Because you don't have zero in the naturals. T) OK. This definition is for natural numbers and this set has no zero element. Thus N(∅) has no symbol in N. Suppose we used the set of Whole numbers, would we have to specify that the sets be non-empty? S's) No. T) So we actually have two definitions for addition, one for the natural numbers and one for the Whole numbers. Are there any questions? (Pause.) T) OK. If I ask you to convince me that the sum of two numbers, such as three and four is seven. Could you do it? (Pause.) S's) Take a set \{1, 2, 3\} and a set \{4, 5, 6, 7\} which are disjoint and finite and non-empty. Then--a--. (Pause.) T) What would I do with these sets? S) Combine the two sets. T) How? S) Join them. T) OK. How does that convince you that 3 + 4 = 7? What set is the join? S's) The set \{1, 2, 3, 4, 5, 6, 7\}. T) OK. How would you use that set to show 3 + 4 = 7? S's) Count the elements in the set to get seven. T) OK. So N(A) + N(B) = N(A ∪ B) where N(A) = 3, N(B) = 4, and N(A ∪ B) = 7. Now you, as a teacher, will have to make sure your example sets are all of what type? S's) Disjoint, finite, and non-empty. T) OK. These are the things you, as a teacher, will have to be aware of. Any questions? (S's) No.
APPENDIX C

TABLE OF ITEM-ANALYSIS COEFFICIENTS,
RELIABILITY COEFFICIENTS,
AND TESTS
Test I

ITEM-ANALYSIS COEFFICIENT AND RELIABILITY COEFFICIENT

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I. Circle the correct answer.

T F 1. A set is a well defined collection of objects.
T F 2. B is an element of \{Bottles\}.
T F 3. If D\subseteq E, then D and E are disjoint sets.
T F 4. If A=\{box, cup, can\} and B=\{draw, box, dog\},
    then A\cup B= \{box\}.
T F 5. If A=\{1, 2, 3, 4, \ldots, 60\}, then \#(A) = 60.
T F 6. The basis for grouping Roman symbols is 5 and 10.

II. Circle the correct answer.

7. If A=\{1, 2, 3, 4\} then which of the following is
    a list of all proper subsets of A which contain
    only three elements.
    (a) \{1,2,3\}, \{1,2,4\}  (c) \{2,3,2\}, \{1,2,1\}, \{1,3,1\}
    (b) \{2,3,4\}, \{1,3,4\}  (d) a combination of (a)
        and (b)
    (e) a combination of
         (a), (b), and (c)  (f) none of these

8. If A=\{4,6,8\} and B=\{116,120,138\}, which of the
    following would be a good choice for a universal
    set.
    (a) \{odd counting numbers\}  (c) \{grades on a 100
        point test\}
    (b) \{fractions\}  (d) \{2,4,6, \ldots, 90\}
    (e) \{116,120,128\}
9. If \( A = \{4, 6, 8\} \), which of the following would be the correct set builder notation for set \( A \).

(a) \( \{x \in N: x > 2 \text{ and } x < 10, N = \text{even natural numbers} \} \)

(b) \( \{x \in J: x > 2 \text{ and } x < 10, J = \text{whole numbers} \} \)

(c) \( \{x \in A: x < 22 \text{ and } x > 10, A = \{1, 2, 3, \ldots, 10\} \} \)

(d) \( \{x: x \text{ is an even number}\} \)

(e) none of these.

10. Which of the following numeration systems has the characteristic of place value.

(a) Egyptian  (b) Babylonian  (c) Roman

(d) none of these

11. Which of the following is the correct decimal notation for the Babylonian notation: \(  \) \( \) \( \) \( \)

(a) 471632  (b) 472340  (c) 220340

(d) none of these

12. Which of the following is the correct expression in Roman numerals for the decimal number: 3,724.

(a) MMMCCLXXIV  (c) MMMDCXXIV

(b) IIDCXXIV  (d) IIDCCXXIV
III. Solve:

13. Use the following Roman counting board to add the numbers: CCLXXIV + MCMXXXVIII

\[ \text{I} \]
\[ \text{V} \]
\[ \text{X} \]
\[ \text{L} \]
\[ \text{C} \]
\[ \text{D} \]
\[ \text{M} \]

14. Explain how to count the elements in the set \{○, ★, ?\}

15. In a class of 25 students, 10 have passed their daily spelling test, 15 have passed their daily math test, and 8 have passed neither test. If only those students who have passed both tests can play a new game called fun-go, use a Venn diagram to find how many students may play fun-go.
## Test II

**ITEM-ANALYSIS COEFFICIENT AND RELIABILITY COEFFICIENT**

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1. Write the numeral $3041_{10} 1702_8$ in expanded notation.

2. Given the following set, show the number of objects in the set using a base two numeral.

3. Convert 152 to a base five numeral.

4. Convert $10TE_{12}$ to a base 10 numeral

5. Convert $1101101_2$ to a base 5 numeral.

6. Count from four to ten using base three numerals.

7. Given the numeral $233_4$, write the numeral which follows $233_4$ in a counting sequence.

8. The sum of 15 and 3 is 20. What base was used in writing these numerals.

9. Show that $1 \times 6 = 6$ using the definition of multiplication.

10. Describe where all sums of the form $(b,b+1)$ are found in a table of basic addition facts for W. Also, replace b and b + 1

11. If a table of basic multiplication facts has symmetry, what property does the set of factors have?

12. Find the Cartesian product of A and B, if A= a,b,c and B= a,b.

13. Justify each step in the following computations by giving the property used.

   (a) $3 + (7 + 6) =$

   $3 + (6 + 7) = (1)$
(3 + 6) + 7 =  (2)
(6 + 3) + 7 =  (3)
6 + (3 + 7) =  (4)

(b) 4 \times (6+1) + 0 =
4 \times 6 + 4 \times 1 + 0 =  (1)
6 \times 4 + 1 \times 4 + 0 =  (2)
24 + 1 \times 4 + 0 =  (3)
24 + (1 \times 4 + 0) =  (4)
24 + (4 + 0) =  (5)
24 + 4 =  (6)
**Test III**

**ITEM-ANALYSIS COEFFICIENT AND RELIABILITY COEFFICIENT**

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1. Find the set of factors of 370. Show your work.

2. Find the set of multiples of 8.

3. Find the complete prime factored form of 84. Show your work.

4. Find the greatest common factor of 84 and 90.

5. Find the least common multiple of 84 and 90.

6. Find the sum of 46 and 78. Show your work, and do not use the carry method.

7. Justify that the sum of 46 and 38 is 84 using properties.

8. Find the difference of 48 - 19. Show your work, and do not use the borrow method.

9. Find the product of 48 x 24. Show your work, and do not use multiplication involving carries.

10. Justify that the product of 24 x 3 is 72. Use properties.

11. Find the quotient and remainder for 894 - 14. Show your work. Use the "guessing" method.

12. (Bonus 5 points.) Find the sum of 432₁₅. Show your work and use only base five numerals. Do not convert to base ten.
APPENDIX D

TABLES OF RAW SCORES
### EIGHT CRITERION TESTS RAW SCORES FOR INDUCTIVE GROUP

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BIBLIOGRAPHY

Books


Kibler, Robert J., Behavioral Objectives and Instruction, Boston, Allyn and Bacon, 1970.


Articles


Johnson, Mauritz, "Who Discovered Discovery?", Phi Delta Kappan, XLVIII (November, 1966), 120.


Sparks, Rebecca, and Earl McCallon, "Fostering Indirect Teaching Behavior in an Elementary Science Methods Course," School Science and Mathematics, LXXI (May, 1971), 381-383.


Publications of Learned Organizations

Committee on the Undergraduate Program in Mathematics, Recommendations on Course Content for the Training of Teachers of Mathematics, Mathematics Association of America, August, 1971.


Public Documents


Unpublished Materials
