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Turbulent Wind Effects on Tension Leg Platform Surge

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Turbulent Wind Effects on Tension Leg Platform Surge

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ABSTRACT

A procedure is presented for estimating surge response to turbulent wind in the presence of current and waves. The procedure accounts for the nonlinearity of the hydrodynamic forces with respect to surge and for the coupling of aerodynamic and hydrodynamic effects. It is shown that current wind spectra do not model correctly the wind speed fluctuations at very low frequencies and an alternative model of the wind spectrum, consistent with fundamental principles, is presented. The equation of surge motion under turbulent wind in the presence of current and waves is solved for typical tension leg platforms, and it is shown that under extreme wave conditions the damping provided by the hydrodynamic forces precludes the occurrence of significant wind-induced resonant amplification effects even if the drag coefficient in the Morison equation is very small (e.g., $C_d = 0.1$). It is verified that for the platforms being investigated the use of a one-minute wind speed to represent the effect of the mean wind and of the turbulent wind fluctuations is acceptable for the purpose of estimating peak surge response.

Key Words: compliant platforms; ocean engineering; offshore platforms; structural engineering; tension leg platforms; turbulence; waves; wind loads.
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>iii</td>
</tr>
<tr>
<td>ACKNOWLEDGMENTS</td>
<td>iv</td>
</tr>
<tr>
<td>LIST OF SYMBOLS</td>
<td>vi</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>x</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>xi</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2. WIND LOADS</td>
<td>3</td>
</tr>
<tr>
<td>2.1 Basic Expressions</td>
<td>3</td>
</tr>
<tr>
<td>2.2 Mean Wind Speeds</td>
<td>3</td>
</tr>
<tr>
<td>2.3 Fluctuating Wind Speeds: Critique of NBC Spectrum</td>
<td>4</td>
</tr>
<tr>
<td>2.4 Fluctuating Wind Speeds: Proposed Spectrum</td>
<td>6</td>
</tr>
<tr>
<td>2.5 Fluctuating Wind Speeds: Spatial Coherence</td>
<td>6</td>
</tr>
<tr>
<td>2.6 Mean Wind Loads</td>
<td>10</td>
</tr>
<tr>
<td>2.7 Fluctuating Wind Loads</td>
<td>10</td>
</tr>
<tr>
<td>3. Hydrodynamic Loads</td>
<td>15</td>
</tr>
<tr>
<td>4. Restoring Force</td>
<td>17</td>
</tr>
<tr>
<td>5. Numerical Estimates</td>
<td>19</td>
</tr>
<tr>
<td>5.1 Equation of Surge Motion</td>
<td>19</td>
</tr>
<tr>
<td>5.2 Assumed Characteristics of Environment and of Platforms</td>
<td>19</td>
</tr>
<tr>
<td>5.3 Linear vs. Nonlinear Restoring Force</td>
<td>21</td>
</tr>
<tr>
<td>5.4 Nominal Damping Ratio of Pseudo-Linear System Equivalent to Equation 38</td>
<td>24</td>
</tr>
<tr>
<td>5.5 Peak Surge Response Under Turbulent Winds</td>
<td>27</td>
</tr>
<tr>
<td>5.6 Surge Response Estimated Under Various Simplifying Assumptions</td>
<td>27</td>
</tr>
<tr>
<td>5.7 Sensitivity of Surge Response to Changes in Values of Wind Environment and Aerodynamic Parameters</td>
<td>30</td>
</tr>
<tr>
<td>6. SUMMARY AND CONCLUSIONS</td>
<td>33</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>35</td>
</tr>
<tr>
<td>APPENDIX - Expression for the Spectrum of the Longitudinal Velocity Fluctuations</td>
<td>39</td>
</tr>
</tbody>
</table>
### LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>surge added mass</td>
</tr>
<tr>
<td>(A_a)</td>
<td>projection of above-water part of platform on plane normal to wind speed</td>
</tr>
<tr>
<td>(A_{p_{ij}})</td>
<td>elemental area of submerged structure projected on a plane normal to direction of current and wave motion</td>
</tr>
<tr>
<td>b</td>
<td>width of main deck</td>
</tr>
<tr>
<td>B</td>
<td>surge wave-radiation damping coefficient</td>
</tr>
<tr>
<td>(C_a)</td>
<td>overall aerodynamic drag coefficient defined by Eq. 17</td>
</tr>
<tr>
<td>(C_{c_{ij}},C_{m_{ij}})</td>
<td>drag inertia coefficient in Morison's equation, respectively</td>
</tr>
<tr>
<td>(C_{D_{sea}})</td>
<td>sea drag coefficient (Eq. 5)</td>
</tr>
<tr>
<td>(C_{d_{ij}},C_{m_{ij}})</td>
<td>drag, inertia coefficient in Morison's equation corresponding to volume (V_{ij}), respectively</td>
</tr>
<tr>
<td>(C_{NL})</td>
<td>downdraw coefficient (Eq. 37)</td>
</tr>
<tr>
<td>Coh</td>
<td>square root of coherence function (Eq. 15)</td>
</tr>
<tr>
<td>(C_p)</td>
<td>pressure coefficient (Eq. 1)</td>
</tr>
<tr>
<td>(C_y,C_z)</td>
<td>exponential decay coefficients (Eq. 16)</td>
</tr>
<tr>
<td>D, (D_c)</td>
<td>diameter of immersed body or component, diameter of buoyant columns, respectively</td>
</tr>
<tr>
<td>E</td>
<td>function defined by Eq. 29</td>
</tr>
<tr>
<td>(f, f_m, f_s)</td>
<td>nondimensional frequency, nondimensional frequency at which product (nS_u) is maximum, nondimensional frequency above which Eq. A1c is valid</td>
</tr>
<tr>
<td>(F_h, F_e, F_{equ.r}, F'_e, F'_r, F_v)</td>
<td>total hydrodynamic load, total wave-induced exciting force, equivalent force defined by Eq. 23, total wind load, fluctuating part of wind load acting on body at rest, total hydrodynamic viscous force</td>
</tr>
<tr>
<td>g</td>
<td>acceleration of gravity</td>
</tr>
<tr>
<td>H</td>
<td>wave height</td>
</tr>
<tr>
<td>J</td>
<td>reduction factor defined by Eq. 28</td>
</tr>
</tbody>
</table>
LIST OF SYMBOLS (Continued)

k = von Kármán's constant
k_w = wave number given by Eq. 34
K = Keulegan-Carpenter number
\( \xi_n \) = nominal length of tethers
L = wave length
\( L_u \) = integral scale of longitudinal wind velocity fluctuations (in direction of mean wind speed)
M = total mass of platform
M_i = point in space
Max(x_{max}) = sample maximum value of x_{max}
n, n_m = frequency
p = wind pressure
R = restoring force
Re = Reynolds number
\( R_u \) = autocovariance function of longitudinal velocity fluctuations
s(x_{max}) = sample standard deviation of x_{max}
\( S_{F_{equ,r}}, S_{F_{u,r}} \) = spectrum of \( F_{equ,r}' \), of \( F_{u,r}' \), respectively
\( S_u, S_{u_1 u_2}, S_{u_1 u_2}^C \) = spectrum, cross-spectrum, cospectrum of longitudinal velocity fluctuations, respectively
\( S_{ueq} \) = spectrum of equivalent wind speed fluctuations, defined by Eq. 21
t = time
T = total tension in tethers at x = 0
\( T_f \) = period of harmonic relative fluid-body motion
\( T_n \) = nominal natural period of surge motion
\( T_w \) = wave period
LIST OF SYMBOLS (Continued)

\( u, u', \bar{u}_{eq}, \bar{u}'_{eq} \) = longitudinal wind velocity, longitudinal velocity fluctuation, equivalent wind speed fluctuation, component of equivalent wind speed fluctuation

\( u_\ast \) = friction velocity

\( \bar{v}_i \) = current velocity

\( v_{ij} \) = horizontal wave-induced velocity at coordinates \( z_i, X_j \)

\( \bar{V} \) = amplitude of relative fluid-body velocity

\( \mathcal{V}_{ij} \) = elemental submerged volume

\( x \) = platform surge displacement

\( x_{\text{max}} \) = peak surge displacement

\( x_{\text{umax}} \) = amplitude of contribution to surge of a harmonic force (Eq. 14)

\( X \) = horizontal coordinate

\( y, y_i \) = horizontal coordinate, horizontal coordinate of point \( M_i \), respectively

\( Y_i \) = arbitrary coordinate

\( z \) = vertical coordinate above mean water level

\( z_{ij} \) = vertical distance from free surface to center of volume \( \mathcal{V}_{ij} \)

\( z_0 \) = roughness length

\( \beta \) = coefficient defining mean square value of turbulent fluctuation in terms of friction velocity (Eq. 14)

\( \Delta \) = denotes increment of a quantity

\( \zeta \) = nominal damping ratio

\( \nu \) = dynamic viscosity

\( \rho_a, \rho_w \) = air, water density, respectively

\( \tau \) = time lag

\( \phi \) = arbitrary function

\( \phi_j \) = phase angle
LIST OF SYMBOLS (Continued)

- = denotes differentiation with respect to time

- = denotes mean value

' = denotes fluctuating part
<table>
<thead>
<tr>
<th>Figure</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1</td>
<td>Spectra of Longitudinal Velocity Fluctuations</td>
<td>7</td>
</tr>
<tr>
<td>Figure 2</td>
<td>Spectra of Longitudinal Velocity Fluctuations</td>
<td>8</td>
</tr>
<tr>
<td>Figure 3</td>
<td>Spectra of Longitudinal Velocity Fluctuations</td>
<td>9</td>
</tr>
<tr>
<td>Figure 4</td>
<td>Integration Domain and Transformation of Variables</td>
<td>12</td>
</tr>
<tr>
<td>Figure 5</td>
<td>Geometry of Tension Leg Platform</td>
<td>20</td>
</tr>
<tr>
<td>Figure 6</td>
<td>Surge Response of Tension Leg Platform with Nonlinear Restoring Force</td>
<td>22</td>
</tr>
<tr>
<td>Figure 7</td>
<td>Surge Response of Tension Leg Platform with Linear Restoring Force</td>
<td>23</td>
</tr>
<tr>
<td>Figure 8</td>
<td>Surge Response of Tension Leg Platform to Harmonic Wind Load in the Presence of Current and Waves, $C_m = 1.8$, $C_d = 0.8$</td>
<td>26</td>
</tr>
<tr>
<td>Figure 9</td>
<td>Surge Response of Tension Leg Platform to Turbulent Wind Load in the Presence of Current and Waves</td>
<td>28</td>
</tr>
</tbody>
</table>
LIST OF TABLES

Table 1. Assumed Current Speeds at Various Depths $z_1$ ........................... 19

Table 2. Estimated Nominal Damping Ratio, $\zeta$, and Mean and Peak Surge Response $\bar{x}$ and $x_{pk}$, in Meters .................................................. 25

Table 3. Statistics of Peak Surge Response, $x_{max}$, Under Turbulent Winds (in meters) ................................................................. 27

Table 4. Estimates of Surge Response Corresponding to Various Deterministic Loading Assumptions (in meters) ......................... 29

Table 5. Dependence of $J^{1/2}(n)$ Upon Exponential Decay Coefficient, $C_y$ .... 31

Table 6. Dependence of $u(z)$ Upon $z_0$ ($u(35 \text{ m}) = 45 \text{ m/s}$) ............... 31
1. INTRODUCTION

Wind effects on tension leg platform (TLP) surge response can be divided into two categories: the wind-induced steady drift, due in practice solely to the mean wind speed, and oscillatory motions induced by the turbulent wind speed fluctuations.

If the requisite aerodynamic information is available the estimation of the wind-induced steady drift is straightforward. However, the estimation of the wind-induced oscillation raises a number of problems that merit careful investigation. It has been stated [24] that "determining the response to wind is possibly of greater importance to the design of compliant structures than the wave and current aspects. Wind has a spectrum which has its peak near the structure's natural frequency in surge. In many instances, the movements derived from varying wind will be greater than those for the wave drift."

Investigations into wind-induced oscillatory compliant platform motions have been reported recently in references 6 and 28. However, these investigations do not take into account nonlinearities due to the hydrodynamic viscous forces, and assume instead that the response to wind is described by a system with proportional damping, the damping ratio being of the order of 5 percent. It appears that no research has been reported that takes into account explicitly the turbulent nature of wind as reflected by spectral and cross-spectral information, while considering simultaneously the nonlinear effects of current and waves. Rather, estimates of the effect of wind speed fluctuations in the presence of current and waves have been advanced that assume a constant (1-minute wind) or harmonic loading in lieu of the actual turbulent wind load [11, 17, 19]. The extent to which these simplified representations are acceptable has not yet been established in the literature. This was noted in a recent evaluation of the Hutton TLP response to environmental loads, which states: "Wind gusts are typically broad-banded and may contain energy which could excite surge motions at the natural period. These would be controlled by surge damping. Theoretical and experimental research is required to clarify the importance of this matter" [11]. The investigation presented here was undertaken in response to this need.

In this paper the surge response is estimated by solving the equation of surge motion in the time domain. The forces represented in this equation consist of: the forces of inertia, the external forces (hydrodynamic and aerodynamic), and the internal forces (restoring, and damping due to internal friction within the structure).

In the following sections the models used in this investigation for the external and internal forces are described, and the equation of surge motion is solved for TLP's with specified characteristics in specified wave, current, and wind environments. To gain insights into the behavior of the platform viewed as a hydrodynamically damped system, the equation of surge motion is also solved in the idealized case where the fluctuating wind load is represented by a harmonic function. Numerical results are presented, and their sensitivity to changes in the values of various parameters is assessed. Current simplified wind loading models are evaluated in light of these results.
2. WIND LOADS

2.1 BASIC EXPRESSIONS

Like the hydrodynamic loads, the wind loads consist of a component due to the presence of viscosity and the consequent flow separation, and an inertial component associated with the relative fluid-body accelerations. However, it can be verified that the inertial component is about two orders of magnitude smaller than the component due to flow separation, and can therefore be neglected in practical applications.

To estimate the wind-induced drag force it is assumed that the elemental drag force per unit of area projected on a plane, \( P \), normal to the mean wind speed can be written as:

\[
p(y,z,t) = \frac{1}{2} \rho_a C_{p}(y,z)[u(y,z,t) - \dot{x}(t)]^2
\]

where \( \rho_a \) = air density, \( C_{p}(y,z) \) = pressure coefficient at elevation \( z \) and horizontal coordinate \( y \) in the plane \( P \), \( t \) = time, \( x \) = surge displacement, the dot denotes differentiation with respect to time, and \( u(y,z,t) \) = wind speed upwind of the structure in the direction of the mean wind. It is assumed that the directions of the mean wind and of the surge motion coincide. The speed, \( u(y,z,t) \), can be expressed as:

\[
u(y,z,t) = \overline{u(z)} + u'(y,z,t)
\]

where the bar and the prime indicate mean value and fluctuating part, respectively. The total wind-induced drag force is

\[
F_u(t) = \int_A p(y,z,t) \, dy \, dz
\]

where \( A \) = projection of above-water part of the platform on a plane normal to the mean wind speed.

It is seen that a prerequisite for the modeling of the elemental and total drag forces is the modeling of the wind speed, \( u(y,z,t) \).

2.2 MEAN WIND SPEEDS

The mean wind speed can be modeled by the equation:

\[
u(z) = u(z_{\text{ref}}) \frac{\ln \frac{z}{z_0}}{\ln \frac{z_{\text{ref}}}{z_0}}
\]

where \( z_{\text{ref}} \) = reference elevation (usually in meteorological practice \( z_{\text{ref}} = 10 \) m), and \( z_0 \) = roughness length.
Information on the roughness length, $z_0$, over the ocean is commonly provided by specifying the value of the sea drag coefficient, defined as

$$C_{D_{sea}} = \left[ \frac{k}{\eta n(10/z_0)} \right]^2 \quad (5)$$

where $z_0$ is expressed in meters and $k = \text{von Kármán constant} (k = 0.4)$. Measurements have shown that the sea drag coefficient increases with wind speed. This dependence is not completely understood; however, on the basis of a large number of measurements, the following empirical relations were proposed for the range $4 < \bar{u}(10) < 21 \text{ m/s}$:

$$C_{D_{sea}} = 5.1 \times 10^{-4} \left[ \bar{u}(10) \right]^{0.46} \quad (6)$$

or

$$C_{D_{sea}} = 10^{-4} \left[ 7.5 + 0.67 \bar{u}(10) \right] \quad (7)$$

where $\bar{u}(10)$ is expressed in m/s [25]. For values of about $\bar{u}(10) > 20 \text{ m/s}$ the variation of $C_{D_{sea}}$ with wind speed appears to be insufficiently documented. According to reference 29, for such values $C_{D_{sea}}$ is constant. On the other hand, results of limited studies summarized in reference 5 would suggest that for about $\bar{u}(10) > 20 \text{ m/s}$ $C_{D_{sea}}$ can be expressed as an average of the values given by equations 6 and 7.

It is shown subsequently that uncertainties with respect to the actual value of $C_{D_{sea}}$ are of little consequence in the estimation of the total surge response; errors in the estimation of $C_{D_{sea}}$ of the order of 50 percent result in differences in the calculated total surge of less than 5 percent. Note that, even if the actual value of $z_0$ were known, equation 4 should not be regarded as "exact," particularly near the mean water surface. Indeed, it is argued in reference 9 that owing to the presence of waves "wind profiles are distorted to lower wind speeds compared to flow above a rigid surface." The use of equation 4 in structural calculations is therefore likely to be conservative.

2.3 **FLUCTUATING WIND SPEEDS: CRITIQUE OF NBC SPECTRUM**

For design purposes, it is necessary to describe the longitudinal wind speed fluctuations, $u'$, in equation 2 in terms of their spectra and cospectra [23, p. 446].

The expression for the spectrum of the longitudinal velocity fluctuations used in the National Building Code of Canada (NBC) [12] and the American National Standard ANSI A58.1 − 1972 [1] is

$$S_{n}(n) = \left[ \frac{1,200/\bar{u}(10)}{[1+1200n/\bar{u}(10)]^{4/3}} \right]^{2} \quad (8)$$

where $n = \text{frequency}$, $\bar{u}(10)$ is expressed in m/s, and the friction velocity, $u_*$ is defined as
\[ u_* = \frac{k u(10)}{\frac{10}{z_o}} \]  

Equation 8 is represented in figure 1 for \( u_* = 1.76 \text{ m/s} \) and \( C_{D_{\text{sea}}} = 0.002 \) (\( z_o = 0.0013 \text{ m} \)). It is seen that according to equation 8 the ordinates of the spectrum at frequencies of interest in deep water platform design (i.e., \( n = 0.01 \text{ Hz} \)) are quite small. However, this is actually not the case. As pointed out by Lumley and Panofsky [10], the assumption inherent in equation 8 to the effect that the spectrum vanishes at \( n = 0 \) is incorrect. This can be shown by considering the relationship between the spectrum and the autocovariance function, \( R_u(\tau) \):

\[ S_u(n) = 4 \int_0^\infty R_u(\tau) \cos 2\pi n \tau d\tau \]  

For \( n = 0 \):

\[ S_u(0) = 4 \int_0^\infty R_u(\tau) d\tau \]  

The function \( R_u(\tau) \) decreases with increasing \( \tau \) from \( R_u(0) = u'^2 \) to \( R_u(\tau) = 0 \) for large values of \( \tau \). Defining the integral length scale of the longitudinal turbulence as

\[ L_u = \frac{\overline{u^2}}{\overline{u'^2}} \int_0^\infty R_u(\tau) d\tau \]  

it follows immediately from equations 11 and 12 that

\[ S_u(0) = 4 \frac{u'^2}{u} L_u \]  

Information on \( L_u \) and its variation in the atmosphere cannot be obtained from wind tunnel measurements, owing to the difficulty of modeling the mesometeorological features that presumably control the lower frequency portions of the spectra [13]. However, values of \( L_u \) obtained from numerous full-scale measurements are listed in the literature, and in particular in reference 2. These show that \( L_u \) increases with height above ground, and that it also increases if the surface roughness decreases. From these measurements it may be inferred roughly that at elevations over water of about 20 to 60 m, which are of primary interest in platform design, \( 100 \text{ m} < L_u < 240 \text{ m} \). Even larger variations (\( 60 \text{ m} < L_u < 450 \text{ m} \)) were reported by Shiotani [22] (see also reference 23, p. 55). The effect of such variations upon the magnitude of surge response is examined subsequently.
2.4 FLUCTUATING WIND SPEEDS: PROPOSED SPECTRUM

No expression for the wind spectrum compatible with equation 13 has been developed so far in the literature primarily because the shape of the spectrum in the very low frequency range has little effect on the design of land-based structures or fixed offshore platforms [23]. To develop such an expression the following conditions, in addition to equation 13, must be satisfied (for details, see reference 23, pp. 53-55):

1. In the inertial subrange \( f > f_8 \), where the nondimensional frequency \( f = nz/u(z) \), and \( f_8 = 0.2 \), Kolmogorov's first and second hypothesis apply, and the energy production is approximately balanced by the energy dissipation.

2. The product \( nS_u(n) \) is a maximum (i.e., the derivative of the function \( nS_u(n) \) vanishes) at some nondimensional frequency, \( f_m < f_8 \). Measurements at elevations of interest in platform design suggest that \( f_m = 0.05 \) to 0.09.

3. The area under the spectral curve is

\[
\frac{u^1}{u^2} = \beta u^2
\]

(14)

where \( u^* \) is defined by equation 9 and \( \beta = 6.0 \).

An expression for the spectrum derived from these conditions and Eq. 13 is given in the appendix (equation A1). This expression is plotted in figure 1 for \( k = 0.4 \), \( z_0 = 0.001266 \) m, \( z = 35 \) m, \( u(35) = 45 \) m/s \( (u = 1.76 \) m/s), \( \beta = 6.0 \), \( f_8 = 0.2 \), \( f_m = 0.07 \), and \( L_u = 180 \) m. For the same values of the parameters \( k \) through \( f_8 \), the expression for the spectrum is also plotted in figure 2 for \( f_m = 0.07 \) and \( L_u = 100 \) m, \( 180 \) m, and \( 240 \) m, and in figure 3 for \( L_u = 180 \) m and \( f_m = 0.05, 0.07, \) and 0.09. It is seen from figure 3 that the influence of the nondimensional frequency \( f_m \) on the shape of the spectrum is weak.

2.5 FLUCTUATING WIND SPEEDS: SPATIAL COHERENCE

It is a characteristic of turbulent flows that the velocity fluctuations at any two points separated by some distance, \( d \), do not exhibit perfect mutual coherence. Owing to imperfect coherence such fluctuations do not attain their maxima or minima at the same time, and the resultant wind-induced fluctuating loads are lower than would be the case if perfect spatial coherence were assumed. Mathematically, the imperfect coherence is reflected in the expression for the cross-spectrum of the velocity fluctuations, \( S_{u_1 u_2}(n) \), whose absolute value can be written as:

\[
|S_{u_1 u_2}(n)| = S_{u_1}^{1/2}(n) S_{u_2}^{1/2}(n) \text{Coh} (M_1, M_2, n)
\]

(15)

where \( S_{u_1}(n) \) and \( S_{u_2}(n) \) = spectra of the velocity fluctuations, \( u' \) and \( u' \), at points \( M_1 \) and \( M_2 \), respectively, and \( \text{Coh} (M_1, M_2, n) \) is referred to as the...
Figure 1. Spectra of longitudinal velocity fluctuations
Figure 2. Spectra of longitudinal velocity fluctuations
Figure 3. Spectra of longitudinal velocity fluctuations
square root of the coherence function. Coh \((M_1, M_2, n)\) is unity for \(M_1 = M_2\) and decreases with increasing distance between \(M_1\) and \(M_2\). For any given \(M_1\) and \(M_2\) the coherence decreases with increasing \(n\). An expression similar to equation 15 can be written for the cross-spectrum (i.e., the real part of the cross-spectrum) of \(u'\) and \(u'_1 S_{u_1 u_2}^C(n)\). Measurements have shown that if \(M_1\) and \(M_2\) are contained in a plane normal to the mean velocity, it is possible to write [3]:

\[
S_{u_1 u_2}^C(n) = S_{u_1}^{1/2}(n) S_{u_2}^{1/2}(n) \exp \left[ \frac{-\left\{ C_y (y_1 - y_2)^2 + C_z (z_1 - z_2)^2 \right\}^{1/2}}{\frac{1}{2} \left\{ u(z_1) + u(z_2) \right\}} \right]
\]

where \(C_y\), \(C_z\) are empirical factors referred to as exponential decay coefficients. Commonly accepted values are \(C_y = 10\), \(C_z = 16\) [28]. However, actual values as obtained from experiments vary widely [8, 22, 23]. The effect of this variation on the calculated surge response is examined subsequently.

Equation 16 can be modified to include the effect of longitudinal separation (in the direction of the mean wind) as well. However, it follows from information presented in reference 7 that this effect is negligible as far as fluctuating aerodynamic loads on offshore structures are concerned.

2.6 MEAN WIND LOADS

It can be verified that the mean square values of \(u'\) and \(x\) and the mean value of the product \(u'x\) are small compared to the square of \(u\). It then follows from equations 1, 2, and 3 that the mean drag load can be written as

\[
\bar{F}_u(t) = \frac{1}{2} \rho_a C_a A_a u(z_a)^2
\]

where the overall aerodynamic drag coefficient is

\[
C_a = \frac{1}{A_a u(z_a)^2} \int_{A_a} C_p(y,z) \overline{u(z)^2} \ dy \ dz
\]

and \(z_a\) = elevation of aerodynamic center of above-water part of the platform.

2.7 FLUCTUATING WIND LOADS

From equation 1, 2, 3, 17, and 18 it follows that the fluctuating part of the wind drag load that would act on the platform at rest (i.e., with \(x = 0\)) is

\[
F'_{u,x}(t) = \rho_a \int_{A_a} C_p(y,z) \overline{u(z)^2} u'(z,t) \ dy \ dz
\]
where the subscript \( r \) refers to the fact that the platform is at rest. The Fourier transform of the autocovariance function of \( F_{u,r}(t) \) yields:

\[
S_{F_{u,r}}(n) = \rho_w^2 \int \int \frac{C_p(y_1, z_1) C_p(y_2, z_2) \overline{u(z_1)} \overline{u(z_2)}}{A_a A_a} \, dy_1 dy_2 dz_1 dz_2
\]

\[
C_{S_{u_{1,2}}}(y_1, y_2, z_1, z_2) \, dy_1 dy_2 dz_1 dz_2
\]

(20)

where \( C_{S_{u_{1,2}}} \) is given by equation 16. The spectrum \( S_{F_{u,r}}(n) \) can be estimated numerically by assuming \( C_p(y_i, z_i) = C_{a_i} \) for \( i = 1, 2 \) and using equations 4, 16, and 11 (see appendix). An equivalent wind fluctuation spectrum can then be defined as

\[
S_{u,eq}(n) = \frac{S_{F_{u,r}}(n)}{[\rho_w C_{A a} \overline{u(z_a)}]^2}
\]

(21)

From \( S_{u,eq}(n) \) it is possible to generate by Monte Carlo simulation realizations of the process \( u_{eq}(t) \):

\[
u_{eq}(t) = \sum_{j} u_{eqj} \cos(2\pi n_j t + \phi_j)
\]

(22)

In equation 22 the phase angle, \( \phi_j \), is generated by random sampling from a uniform distribution in the interval \( 0 < \phi_j < 2\pi \).

Let the spectrum of the force, \( F_{equ,r}'(t) \), defined as

\[
F_{equ,r}'(t) = \rho_w C_{A a} \overline{u(z_a)} \, u_{eq}'(t)
\]

be denoted by \( S_{F_{equ,r}}(n) \). Clearly,

\[
S_{F_{equ,r}}(n) = S_{F_{u,r}}(n)
\]

(24)

Thus, \( u_{eq}'(t) \) can be viewed as an equivalent wind speed fluctuation which is perfectly coherent over the area \( A_a \) and whose effect upon the structure at rest is the same as that of the actual fluctuating wind field.

The total wind load acting on the platform can thus be expressed as

\[
F_u(t) = \frac{1}{2} \rho_w C_a A_a \left[ \overline{u(z_a)} + u_{eq}'(t) - \dot{x}(t) \right]^2
\]

(25)
Numerical calculations have shown that if the difference between the elevation of the helideck (or the top of the crew quarters) and the underside of the lower deck in a typical drilling and production platform is less than about two-thirds to three quarters of the width of the main deck, the term $C_z^2 (z_1-z_2)^2$ of equation 16 can be neglected when evaluating the integral of equation 20. This is a consequence of the fact that $C_z$ is smaller than $C_y$ by a factor of about 1.5. The approximation inherent in the neglect of $C_z^2 (z_1-z_2)^2$ is slightly conservative from a structural engineering point of view (though insignificantly so). Noting, then, that for any arbitrary function, $\phi$,

$$
\int_0^1 \int_0^1 \phi(|Y_1-Y_2|) \, dY_1 \, dY_2 = \int_0^1 \phi(t) (1-t) \, dt
$$

(figure 4), and assuming $C_p(y_1, z_1) = C_a$, $u(z_1) = u(z_a)$, and $S_{u_1}(n) = S_u(z_a, n)$, ($i=1, 2$), it follows after some algebra from equations 20, 16, and 21 that

$$
S_{u_{eq}}(n) = S_u(z_a, n) J(n)
$$

(27)

**Figure 4. Integration domain and transformation of variables**
where $S_u(z_a,n) = \text{spectrum of longitudinal velocity fluctuations at elevation } z_a$
given by equations A1, and $J(n) = \text{reduction factor accounting for the imperfect}
coherence among the fluctuating wind pressures at different points of the
platform, given by the expression

$$J(n) = -2 \frac{E}{E} \left\{ -\exp(-E) + (1-1)\left[ \exp(-E)-1 \right] \right\}$$  \hspace{1cm} (28)$$

$$E = C_y b \frac{n}{u(z_a)}$$  \hspace{1cm} (29)$$

In equation 29 $b = \text{width of main deck}$. Equation 27 can be used in lieu of
equations 20 and 21 for the Monte Carlo simulation of the equivalent velocity
fluctuations $u_{eq}(t)$ (see equation 22) needed in the expression for the
total wind load acting on the platform, $F_u(t)$.  

$$\text{...}$$
3. HYDRODYNAMIC LOADS

The total hydrodynamic load, \( F_h \), is written in the form:

\[
F_h = F_v + F_e - Ax - Bx
\]  
(30)

where \( F_v \) = total hydrodynamic viscous force, \( F_e \) = total wave-induced exciting force, \( A \) = surge added mass, and \( B \) = surge wave-radiation damping coefficient. Following reference 17, it is assumed for convenience that the wave motion is monochromatic, hence the absence of second-order drift forces in equation 30 [16]. It is assumed in addition that \( B = 0 \) since the radiation damping at low frequencies is negligible (17, 26).

The total wave-induced exciting force and the surge added mass can be estimated numerically on the basis of potential theory. Alternatively, these two terms may be assumed to be given by the inertia component of the Morison equation, i.e.,

\[
A = \rho_w \sum_{ij} \psi_{ij}(C_{m_{ij}} - 1) \]  
(31)

\[
F_e = \rho_w \sum_{ij} \psi_{ij} C_{m_{ij}} \left[ \frac{\partial v_{ij}}{\partial t} + [\bar{v}_i + v_{ij} - x] \frac{\partial v_{ij}}{\partial x} \right] \]  
(32)

[18, p. 31], where \( \rho_w \) = water density, \( \psi_{ij} \) = elemental volume of submerged structure, \( C_{m_{ij}} \) = surge inertia coefficient corresponding to \( \psi_{ij} \), \( X \) = horizontal distance from some arbitrary origin to center of \( \psi_{ij} \) along direction parallel to surge motion, \( v_i \) and \( v_{ij} \) = current velocity and horizontal particle velocity due to wave motion, respectively, at center of \( \psi_{ij} \). Equations 31 and 32 may be employed if for the component being considered the ratio of diameter to wave length, \( D/L \leq 0.2 \) [18, p.283]. Since for \( T_w = 15 \) sec, \( L = g T_w^2 / 2\pi \approx 350 \) m [18, p. 283], where \( T_w \) = wave period, and \( g \) = acceleration of gravity, it follows that for members of typical TLP structures, for which \( D < 20 \) m or so, the use of equations 31 and 32 is acceptable if three-dimensional flow effects are not taken into account. The wave motion is assumed to be described by deep water linear theory, so that

\[
v_{ij} = \frac{\pi H}{T_w} e^{\frac{-kz_i}{T_w}} \cos(kw x_j - \frac{2\pi t}{T_w}) \]  
(33)

where \( H \) = wave height, \( kw \) = wave number given by

\[
k_w = \frac{1}{g} \left( \frac{2\pi}{T_w} \right)^2 \]  
(34)

[18, p. 157]. It is recalled that the directions of the mean wind, current, and wave propagation are assumed to coincide.
The total hydrodynamic viscous load is assumed to be given by the viscous component of Morison's equation

\[ F_v = 0.5 \rho w \sum_{ij} C_{d_{ij}} A_{p_{ij}} \left[ \bar{v}_1 + v_{ij} - x \right] \left[ \bar{v}_1 + v_{ij} - x \right] \]  \hspace{1cm} (35)

where \( A_{p_{ij}} \) = area of elemental volume \( V_{ij} \) projected on a plane normal to the direction of the current, and \( C_{d_{ij}} \) = drag coefficient corresponding to \( A_{p_{ij}} \).

If the relative motion of the body with respect to the fluid is harmonic, the drag and inertia coefficients in Morison's equation can be determined on the basis of experimental results as functions of local oscillatory Reynolds number, \( Re = \frac{2\pi D^2}{\nu T_f} \), Keulegan-Carpenter number, \( K = \frac{\bar{V}}{T_f} \), and relative body surface roughness, where \( D = \) diameter of body, \( \nu = \) kinematic viscosity, \( \bar{V} \) and \( T_f \) = amplitude and period of relative fluid-body velocity. However, actual relative fluid-body motions are not harmonic. This introduces uncertainties in the determination of the drag and inertia coefficients even if experimental information for harmonic relative motions were available in terms of \( Re \) and \( K \). Unfortunately, such information is not available for the small \( K \) numbers (of the order of 2) and large Reynolds numbers (of the order of \( 10^6 \)) of interest in TLP design. For this reason calculations will be carried out for various sets of values \( C_d \), \( C_m \), and the sensitivity of the results to changes in these values will be investigated.
4. RESTORING FORCE

The restoring force, \( R \), at any instant, \( t \), is equal to the horizontal projection of the resultant of the tether tensions at that instant, i.e.,

\[
R(t) = -(T + \Delta T) \frac{x}{\ell_n + \Delta \ell_n}
\]

(36)

where \( T \) = initial pretensioning force, \( \Delta T \) = incremental tension due to surge motion, \( \ell_n \) = nominal length of tethers at \( x = 0 \) \( \Delta \ell_n \) = incremental length, and

\[
\frac{T + \Delta T}{\ell_n + \Delta \ell_n} = \frac{T}{\ell_n} + C_{NL} \left[ 1 - \sqrt{1-(x/\ell)^2} \right]
\]

(37)

where \( C_{NL} \) = downdraw coefficient, equal to the weight of water displaced as the draft is increased by a unit length [17].
5. NUMERICAL ESTIMATES

5.1 EQUATION OF SURGE MOTION

Numerical estimates of surge motion are obtained by solving the equation of surge motion for structures with specified characteristics in specified wind, current, and wave environments. The equation of surge motion can be written as:

\[
(M + A)\ddot{x} + \left[\frac{T}{\xi_n} + C_{NL} \left(1 - \sqrt{1-(x/L)^2}\right)\right] x =
\]

\[
0.5 \rho_w \sum_{ij} C_{d_{ij}} A_{p_{ij}} \left| \overline{v_i} + v_{ij} - \dot{x} \right| [v_i + v_{ij} - \dot{x}]
\]

viscous hydrodynamic force, \( F_v(t) \) [equation 35]

\[
+ \rho_w \sum_{ij} V_{ij} C_{m_{ij}} \left( \frac{\partial v_{ij}}{\partial t} + [\overline{\dot{v}_i} + v_{ij} - \dot{x}] \frac{\partial v_i}{\partial x} \right)
\]

wave-induced exciting force, \( F_a(t) \) [equation 32]

\[
+ 0.5 \rho_a C_a A_a \left[ \overline{\left( u(z_a) - \sum u_{eq,k} \cos(2\pi n_k t + \phi_k) - x(t) \right)^2} \right]
\]

wind force, \( F_u(t) \) [equations 25 and 22]

The equation of motion is solved by using the PORT Ordinary Differential Equations Solution Subroutine [4]. For details on computational aspects of differential equation solutions, see references 20 and 21.

5.2 ASSUMED CHARACTERISTICS OF ENVIRONMENT AND OF PLATFORMS

The assumed characteristics of the environment and of the platforms are listed below.

(a) The current speed, \( v_i \), varies linearly between the values given in table 1.

<table>
<thead>
<tr>
<th>( z_i ) (meters)</th>
<th>( \overline{v_i} ) (meters/second)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.40</td>
</tr>
<tr>
<td>35</td>
<td>0.90</td>
</tr>
<tr>
<td>300</td>
<td>0.30</td>
</tr>
<tr>
<td>550</td>
<td>0.15</td>
</tr>
</tbody>
</table>

| 19 |
(b) The horizontal velocities, $v_{14}$, associated with the wave motion, are given by equation 33 in which $H = 25$ m and $T_w = 15$ sec.

(c) The mean wind speed is specified as $\bar{u}(z_a) = 45$ m/s, unless otherwise stated.

(d) The equivalent wind speed fluctuation components, $u_{eq}$, are obtained by Monte Carlo simulation from the spectrum $S_{u,eq}(n)$, given by equation 27, unless otherwise stated. In equation 27, the spectrum of the longitudinal velocity fluctuations, $S_u(n)$, is given by equation A1 and A2, where the friction velocity, $u_*$, is obtained from equations 9 and 5, and where, unless otherwise stated, it is assumed $C_{D,sea} = 0.002$, $L_u = 180$ m, $\beta = 6.0$ (equation 14), $f_m = 0.07$, and $f_s = 0.2$. The experimental decay coefficient in the function $J(n)$ of equation 27 (see also equations 28 and 29) is $C_v = 16$. However, values of $C_{D,sea}$, $L_u$, $\beta$, $f_m$, $f_s$, and $C_v$ different from those just listed are considered in subsequent sensitivity studies.

(e) Geometric characteristics of the platforms are shown in figure 5. Unless otherwise stated, the nominal length of the tethers is assumed to be $L_n = 590$ m, corresponding to about 600 m depth of water. The case $L_n = 150$ m is also considered where noted.

Figure 5. Geometry of tension leg platform
(f) **Mechanical characteristics** of the platforms are specified as follows: mass of platform, \( M = 34.3 \times 10^6 \) kg, total tension in legs, \( T = 1.56 \times 10^5 \) kN. From figure 5 it follows that \( C_{NL} = 4(\pi D_c^2/4) \rho_w = 1.03 \times 10^4 \) kN/m, where \( D_c \) = diameter of buoyant columns. Note that the values of \( M \) and \( T \) and the geometric characteristics of the platform shown in figure 5 are similar to those assumed in reference 17. This choice was aimed at facilitating comparisons with results obtained therein. Damping due to internal friction within the structure is assumed to be negligible.

(g) **Hydrodynamic characteristics** of the structure consist of the Morison equation coefficients. Unless otherwise stated, these are specified as \( C_m = 1.8, C_d = 0.6 \). (Note that \( C_m = 1.8 \) yields the surge added mass, \( A_s \), obtained from calculations based on potential theory in reference 17.) In addition, the sets (a) \( C_m = 1.9, C_d = 0.1 \), (b) \( C_m = 1.9, C_d = 0.2 \), (c) \( C_m = 1.8, C_d = 0.8 \), and (d) \( C_m = 1.3, C_d = 1.2 \) (assumed to correspond to the case of submerged surfaces roughened by biofouling) are considered in sensitivity studies. Note that low \( C_d \) values have been suggested for small \( K \) numbers in low Reynolds number flow [26]. Corresponding values in the high Reynolds number regime do not appear to be available in the literature.

The nominal natural period of the surge motion, in seconds, is given by

\[
T_n = 2\pi \left( \frac{M + A_s}{T} \right)^{1/2} \kappa_n
\]

(39)

where \( \kappa_n \) = nominal length of tethers. For the platforms with \( \kappa_n = 590 \) m and \( \kappa_n = 150 \) m, if \( C_m = 1.8, T_n = 103 \) sec (as in reference 17) and \( T_n = 52 \) sec, respectively.

(h) **Aerodynamic characteristics** of the structure consist of the product \( 0.5 \rho_a C_a A_a \) (equations 17 and 19), and the elevation of the aerodynamic center, \( z_a \). The values specified for these parameters are \( 0.5 \rho_a C_a A_a = 2700 \) kg/m, as in reference 17, and \( z_a = 50 \) m. In addition, the value \( z_a = 35 \) m is used to investigate the influence of \( z_a \) upon the calculated surge.

5.3 **LINEAR VS. NONLINEAR RESTORING FORCE**

To assess the effect of neglecting the nonlinearity of the restoring force, solutions of equation 38 were obtained in which, all other parameters being the same, the cases \( C_{NL} = 1.03 \times 10^4 \) kN/m and \( C_{NL} = 0 \) were considered. In both cases it was assumed that \( u_{eq}(t) \) (see equation 22) is given by the harmonic function

\[
u_{eq}(t) = 5.9 \cos \frac{2\pi}{T_n} t
\]

(40)

in meters per second, where \( T_n = 103 \) sec. It is seen from figures 6 and 7 that if the restoring force is assumed to be linear i.e., if it is assumed \( C_{NL} = 0 \), the surge response is amplified with respect to the nonlinear case by about 7
Figure 6. Surge response of tension leg platform with nonlinear restoring force
Figure 7. Surge response of tension leg platform with linear restoring force
percent. This is comparable with results reported in reference 17. Once the approximate magnitude of these differences was established, all estimates of the surge response reported in reference 17 were based on the assumption that the restoring force is linear. To facilitate comparisons with the results of reference 17, the same assumption is used henceforth in the present work.

5.4 NOMINAL DAMPING RATIO OF PSEUDO-LINEAR SYSTEM EQUIVALENT TO EQUATION 38

It was indicated previously that the estimation of the effect of turbulent wind on surge was attempted in the literature on the basis of the assumption that the equation of surge motion represents a linear system with a viscous damping term characterized by a nominal damping ratio, \( \zeta \). The effect of this term is postulated to be equivalent to the damping effect of the hydrodynamic viscous force. This approach was used, for instance, in references 6 and 28, where calculations are presented based on the assumption that \( \zeta \) is of the order of 5 percent.

Such an approach is acceptable if the order of magnitude of the nominal damping ratio is consistent with the hydrodynamic behavior of the system. Calculations are now presented that illustrate how such nominal damping ratios can be estimated.

It is assumed that \( u'_{eq}(t) \) (see equation 22) is given by the harmonic function

\[
u'_{eq}(t) = u'_{eq1} \cos \frac{2\pi}{T_n} t
\]

where \( T_n = \) natural period in surge, and that the system under consideration is linear with mass \( M + A \), natural period \( T_n \), and damping ratio \( \zeta \). The amplitude of the contribution to the surge response of a harmonic force \( \rho_a C_a A_a u(z_a) u_{eq1} \cos \frac{2\pi}{T_n} t \) is denoted by \( x_{umax} \), and is given by the relation:

\[
x_{umax} = \frac{\rho_a C_a A_a u(z_a) u'_{eq1}}{(M+A)(\frac{2\pi}{T_n})^2 \left[1-(nT_n)^2\right]^{1/2}}
\]

The nominal damping ratio, \( \zeta \), is estimated from equation 42 by equating \( x_{umax} \) and the contribution to the surge response of the force \( \rho_a C_a A_a u(z_a) u'_{eq1} \cos \frac{2\pi}{T_n} t \) as obtained by solving equation 38. By substituting \( 1/T_n \) for \( n \) in equation 42, it follows that

\[
\zeta = \frac{1}{2} \frac{\rho_a C_a A_a u(z_a) u'_{eq1}}{x_{umax} (M+A) \left(\frac{2\pi}{T_n}\right)^2}
\]

Calculated values of \( \zeta \) are shown in table 2 for the platform described previously with nominal tether lengths \( l_n = 590 \) m and 150 m. These values are
based on the assumption $u(z_a) = 40 \text{ m/s}, u'_{eq1} = 4 \text{ m/s}$. For example, in the case $\lambda_n = 590 \text{ m}, C_m = 1.8, C_d = 0.8$, equation 38 yields $x_{\text{umax}} = 2.5 \text{ m}$ (i.e., half the double amplitude of the low frequency oscillation in figure 8). Since $M+A = 71.1 \times 10^6 \text{ kg}, T_n = 103 \text{ sec}, \text{ and } 0.5 \rho \alpha A _{a} A_a = 2,700 \text{ kg/m}$, the estimated nominal damping ratio in this case is $\zeta = 0.65$. Results are given in table 2 for each of the five sets of values $C_m, C_d$ listed previously. Also shown in table 2 are calculated values of the steady and peak surge response in meters. It is seen that as the damping coefficient, $C_d$, increases, both the steady and the peak response increase, even though the contribution to the response due to wind speed fluctuations decreases as a result of the stronger hydrodynamic damping.

It is seen in table 2 that the estimated damping ratios are considerably larger than assumed, e.g., in references 6 and 28. Large values of $\zeta$ indicate that the damping inherent in the viscous hydrodynamic forces is sufficiently strong to preclude the occurrence of significant resonant amplification effects under fluctuating wind loads. Note that results similar to those of table 2 are obtained from the surge response calculations reported in reference 17.

Table 2. Estimated Nominal Damping Ratio, $\zeta$, and Mean and Peak Surge Response $\bar{x}$ and $x_{pk}$, in Meters

<table>
<thead>
<tr>
<th>$C_m$</th>
<th>$C_d$</th>
<th>$\lambda_n = 590 \text{ meters}$</th>
<th>$\lambda_n = 150 \text{ meters}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\bar{x}$ meters</td>
<td>$x_{pk}$ meters</td>
</tr>
<tr>
<td>-------</td>
<td>-------</td>
<td>------------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>1.9</td>
<td>0.1</td>
<td>0.25</td>
<td>25.</td>
</tr>
<tr>
<td>1.9</td>
<td>0.2</td>
<td>0.30</td>
<td>28.5</td>
</tr>
<tr>
<td>1.8</td>
<td>0.6</td>
<td>0.55</td>
<td>34.</td>
</tr>
<tr>
<td>1.8</td>
<td>0.8</td>
<td>0.65</td>
<td>37.</td>
</tr>
<tr>
<td>1.3</td>
<td>1.2</td>
<td>0.80</td>
<td>46.</td>
</tr>
</tbody>
</table>

\text{a N.C. = not calculated}

The results of table 2 suggest that, unless biofouling effects are significant, it is reasonable to assume for the purpose of estimating peak surge response that $C_d = 0.6, C_m = 1.8$. This set of values is used subsequently in this paper for the estimation of surge response under turbulent wind loads.
Figure 8. Surge response of tension leg platform to harmonic wind load in the presence of current and waves, $C_m = 1.8$, $C_d = 0.8$. 

$\text{SURGE [meters]}$ 

$t$ (seconds)
5.5 PEAK SURGE RESPONSE UNDER TURBULENT WINDS

The equation of surge motion (equation 38), with the environmental, geometric, mechanical, hydrodynamic, and aerodynamic parameters specified previously, was solved for fifty different realizations of the random process \( u'_{eq}(t) \) (equation 22) in each of the following cases: \( \ell_n = 590 \) m and \( \ell_n = 150 \) m. The sample mean, \( \bar{x}_{\text{max}} \), the sample standard deviation, \( s(x_{\text{max}}) \), and the sample maximum \( \text{Max}(x_{\text{max}}) \), of the peak surge response, \( x_{\text{max}} \), are listed in line (1) of table 3 (corresponding to \( L_u = 180 \) m). A typical solution of equation 38 is shown in figure 9 for the platform with \( \ell_n = 590 \) m (\( T_n = 103 \) sec), and \( L_u = 180 \) m.

Table 3. Statistics of Peak Surge Response, \( x_{\text{max}} \), Under Turbulent Winds (in meters)

<table>
<thead>
<tr>
<th>( \ell_n = 590 ) m (( T_n = 103 ) sec)</th>
<th>( \ell_n = 150 ) m (( T_n = 52 ) sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{x}_{\text{max}} )</td>
<td>( s(x_{\text{max}}) )</td>
</tr>
<tr>
<td>(1) ( L_u = 180 ) m</td>
<td>48.7</td>
</tr>
<tr>
<td>(2) ( L_u = 100 ) m</td>
<td>48.1</td>
</tr>
<tr>
<td>(3) ( L_u = 240 ) m</td>
<td>48.9</td>
</tr>
</tbody>
</table>

5.6 SURGE RESPONSE ESTIMATED UNDER VARIOUS SIMPLIFYING ASSUMPTIONS

It is of interest to estimate the surge response that would occur if the platform were subjected to forces induced by:

- **Case A:** Mean wind alone
- **Case B:** Current and waves alone
- **Case C:** Current, waves, and mean wind alone (no wind speed fluctuations)
- **Case D:** Current, waves, mean wind, and harmonic wind speed fluctuations, \( u'_{eq}(t) = 0.1 u(z_a) \cos (2\pi t/T_n) \)
- **Case E:** Current, waves, mean wind, and harmonic wind speed fluctuations, \( u'_{eq}(t) = 1.41 \sqrt{\bar{u}} \cos 2\pi t/T_n \) [recall that \( \sqrt{\bar{u}} \) = r.m.s. of turbulent wind speed fluctuations — see equation 14].
- **Case F:** Current, waves, and steady (1-minute) wind speed [No wind speed fluctuations]. (Note that 1-minute speed = 1.24 \( u(z_a) \), see reference 23, p. 62.)
Figure 9. Surge response of tension leg platform to turbulent wind load in the presence of current and waves.
The results of the calculations are shown in table 4, which lists for each of these cases the peak surge, as well as the steady surge, the amplitude of the wave-induced fluctuating surge, and the amplitude of the wind-induced fluctuating surge.

The results of tables 3 and 4 show that:

- The contribution of the mean wind to the peak surge response is about 40 percent and 25 percent for the platforms with \( L_n = 590 \) m, and \( L_n = 150 \) m, respectively (case A, table 4, versus Max(\( m_{\text{max}} \)), line (1), table 3).

- The contribution of the wind speed fluctuations to the peak surge response is about 12 percent for both values of \( L_n \) (case C, table 4, versus Max(\( x_{\text{max}} \)), line (1), table 3).

- The contribution of current and waves to the peak surge response is about 50 percent and 60 percent for the platforms with \( L_n = 590 \) m and \( L_n = 150 \) m, respectively (case B, table 4, versus Max(\( x_{\text{max}} \)), table 3).

- Representing the turbulent wind speed fluctuations by a harmonic fluctuation with amplitude equal to one-tenth of the mean wind speed, as done in reference 17, (case D), results in an underestimation of peak surge response by 5 percent to 10 percent. Calculations were carried out which showed that, in order for harmonic and turbulent wind speed fluctuations to be equivalent in terms of surge response, the amplitude of the harmonic wind speed fluctuations should be about twice the r.m.s. of the turbulent wind fluctuations.

Table 4. Estimates of Surge Response Corresponding to Various Deterministic Loading Assumptions (in meters)

<table>
<thead>
<tr>
<th>Case</th>
<th>( L_n = 590 ) m</th>
<th>( L_n = 150 ) m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Wave-Induced Fluct.</td>
<td>Wind-Induced Fluct.</td>
</tr>
<tr>
<td>A</td>
<td>20.5</td>
<td>-</td>
</tr>
<tr>
<td>B</td>
<td>18.0</td>
<td>6.5</td>
</tr>
<tr>
<td>C</td>
<td>39.0</td>
<td>6.0</td>
</tr>
<tr>
<td>D</td>
<td>39.0</td>
<td>5.0</td>
</tr>
<tr>
<td>E</td>
<td>39.0</td>
<td>5.0</td>
</tr>
<tr>
<td>F</td>
<td>50.0</td>
<td>6.0</td>
</tr>
</tbody>
</table>
Accounting for the contribution of the wind speed fluctuations to the peak surge response by assuming $u_{dq}(t) = 0$ and replacing $u(z_a)$ in equation 25 by the one-minute wind speed, as done in references 11 and 18 (case F), results in an overestimation of the total peak surge by about 10 percent for the platform with $L_n = 590$ m, and 1 percent for the platform with $L_n = 150$ m.

5.7 SENSITIVITY OF SURGE RESPONSE TO CHANGES IN VALUES OF WIND ENVIRONMENT AND AERODYNAMIC PARAMETERS

This section examines the effect upon the surge response of changes in the values of the wind environment parameters $L_u$, $f_m$, $f_g$, $C_z$ and $C_y$, and $C_{Dsea}$ (or $z_0$ - see equation 5), and of the aerodynamic parameter $z_a$.

The effect of the magnitude of the integral turbulence scale, $L_u$, is investigated by estimating the surge response under the same assumptions that led to the results of line (1) of table 3, except that in lieu of $L_u = 180$ m the values $L_u = 100$ m and $L_u = 240$ m are used. Again, for each of the cases $L_n = 590$ and $L_n = 150$ m, equation 38 was solved for fifty different realizations of the random process $u_{dq}(t)$. The results are given in lines (2) and (3) of table 3. It is seen that the statistics of the peak response are not affected significantly by $L_u$; the sample maxima, Max$(X_{max})$, differ in both cases by 3.5 percent or less when the integral scale of turbulence increases by a factor of 2.4.

An inspection of figure 2 and 3 shows that the influence upon the spectral shape of the nondimensional frequency, $f_m$, at which $nS_u(n)$ is maximum (see equation A1), is considerably weaker than the influence of $L_u$. Hence, it can be expected that the influence of $f_m$ on the surge response is minimal. Solutions of equation 38 in which the values $f_m = 0.04$ and $f_m = 0.10$, in lieu of $f_m = 0.07$, were used, all other parameters being unchanged, showed that this is indeed the case.

The nondimensional frequency $f_g$ (see equation A1) affects the higher frequency portion of the spectrum, which has no perceptible influence on the surge response. Hence, even large variations of $f_g$ (between 0.1 and 1, say) are inconsequential as far as their effect on surge is concerned.

It is well known that the average size of the longitudinal turbulent eddies in the atmosphere is considerably less in the transverse than in the longitudinal direction [22, 23]. For this reason $C_z < C_y$, the ratio $C_z/C_y$ being about 0.6 or less. It was pointed out previously that for typical platform shapes this results in the effect of $C_z$ being negligible with respect to the effect of $C_y$ in equation 16 and 20, so that equation 27 through 29 may be used in lieu of equation 21. If the value $C_y = 24$ in lieu of $C_y = 16$ is used, the values of $J^{1/2}(n)$ in and near the range of frequencies where the energy of the wind speed fluctuations is large ($0 < n < 0.5$ or so - see figure 2) decrease as shown in table 5.
Table 5. Dependence of $J^{1/2}(n)$ Upon Exponential Decay Coefficient, $C_y$

<table>
<thead>
<tr>
<th>$n$</th>
<th>0.01</th>
<th>0.02</th>
<th>0.05</th>
<th>0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J^{1/2}$ ($C_y = 16$, $n$)</td>
<td>0.95</td>
<td>0.91</td>
<td>0.79</td>
<td>0.68</td>
</tr>
<tr>
<td>$J^{1/2}$ ($C_y = 24$, $n$)</td>
<td>0.93</td>
<td>0.87</td>
<td>0.73</td>
<td>0.59</td>
</tr>
</tbody>
</table>

It is seen that the influence of large changes in the value of $C_y$ upon $J^{1/2}(n)$ in this frequency range is on the average of a few percent. It follows that the influence of such changes on the peak surge response is negligible for practical purposes.

The choice of value $C_{DSEA}$ affects both the mean wind profile and the spectrum of the longitudinal velocity fluctuations. To $C_{DSEA} = 0.002$ and 0.003, for example, there correspond values $z_0 = 0.0013$ m and 0.0067 m, respectively (equation 5). Based on equation 4, the respective calculated mean speeds at $z = 20$ m and $z = 50$ m above mean water level, given that $u(35 \text{ m}) = 45$ m/s, are listed in Table 6.

Table 6. Dependence of $\overline{u(z)}$ Upon $z_0$ ($u(35 \text{ m}) = 45$ m/s)

<table>
<thead>
<tr>
<th>Elevation, $z$</th>
<th>20 m</th>
<th>50 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\overline{u(z)}$ for $z_0 = 0.0013$ m</td>
<td>42.4 m/s</td>
<td>46.5 m/s</td>
</tr>
<tr>
<td>$\overline{u(z)}$ for $z_0 = 0.0067$ m</td>
<td>42.0 m/s</td>
<td>46.8 m/s</td>
</tr>
</tbody>
</table>

The influence of $C_{DSEA}$ is more significant in the case of the spectra, $S_u(n)$, which are proportional to $u^2$ (equations A1). To differences between values of $C_{DSEA}$ of the order of 50 percent there correspond differences in the magnitude of $S_u^{1/2}(n)$ of about 20 percent. Owing to nonlinearity effects, the differences between the corresponding contributions of the wind speed fluctuations to the surge response are somewhat less. Since these contributions were previously estimated to be of about 12 percent of the peak surge response, differences between $C_{DSEA}$ values of the order of 50 percent result in differences between the corresponding values of the peak surge response of less than 3 percent.

To summarize, although the uncertainties with respect to the parameters defining the wind environmental are large, the effects of these uncertainties upon the peak response translate in all cases into differences of the order of a few percent.
The aerodynamic parameter $z_a$ used in the calculations presented so far was specified as $z_a = 50 \text{ m}$, as indicated previously. Lower values, e.g. $z_a = 35 \text{ m}$, may be encountered in typical practical situations. Calculations based on the assumption $z_a = 35 \text{ m}$, all other parameters being unchanged (including the speed $u(z_a) = 45 \text{ m/s}$), have shown that the peak surge response is smaller by less than 2 percent than the corresponding response based on the value $z_a = 50 \text{ m}$. 
6. SUMMARY AND CONCLUSIONS

This paper presents a procedure for estimating surge response to the action of turbulent winds in the presence of current and waves. The procedure accounts for the nonlinearity of the hydrodynamic forces and the coupling of aerodynamic and hydrodynamic effects.

It is demonstrated that, unlike wave spectra, wind spectra peak at zero frequency, where they are proportional to the integral scale of turbulence. An expression for the wind spectrum consistent with this requirement is derived, and a critique is presented of current expressions in which this requirement is violated. It is shown that such expressions may lead to the gross underestimation of wind speed fluctuation components with frequencies comparable to the natural frequencies of compliant platforms.

A simple expression is derived that accounts for the imperfect coherence of fluctuating wind pressures acting at different points on the surface of the platform.

For illustrative purposes the procedure for estimating surge response is applied to typical tension leg platforms (TLP's) for water depths of about 600 m and 150 m. Solutions of the equation of surge motion are obtained, which suggest that for these platforms the hydrodynamic damping is sufficiently large to preclude the occurrence of significant resonant amplification effects due to wind loads, even if the assumed drag coefficients in Morison's equation are as low as $C_d = 0.1$. Equivalent linear damping ratios with respect to wind effects were estimated to be of the order of 25 percent to 55 percent corresponding to $C_d = 0.1$ and $C_d = 0.6$, respectively, in 600 m deep waters, and 10 percent to 20 percent for $C_d = 0.1$ and $C_d = 0.6$, respectively, in 150 m deep waters. The calculations show that as the viscous damping due to hydrodynamic effects increases, the peak total surge response also increases, even though the contribution of the fluctuating wind effects to the total surge is reduced. The use of a Morison equation drag coefficient $C_d = 0.6$ appears to be reasonable for the purpose of estimating peak total surge.

Parameters defining the wind environment are reviewed, and it is shown that uncertainties with respect to the actual values of these parameters have little effect on the estimated peak surge response. However, uncertainties with respect to the sea surface drag coefficient (or equivalently, to the roughness length parameter for the sea surface) may cause differences of the order of 20 percent in the estimated contribution of the wind load fluctuations to the surge response.

For the platforms considered in this work, and under the assumption that the drag coefficient in Morison's equation is $C_d = 0.6$, a fictitious harmonic wind speed fluctuation and the actual turbulent wind speed fluctuations are equivalent from the standpoint of their contribution to the total surge if the amplitude of the harmonic fluctuations is equal to about twice the r.m.s. of the turbulent fluctuations. Finally, it is concluded that, under the same assumption, the use of a one-minute wind speed to represent the effect of the mean wind and of the turbulent wind fluctuations is acceptable for the purpose of estimating peak surge response.
REFERENCES


APPENDIX: EXPRESSION FOR THE SPECTRUM OF THE LONGITUDINAL VELOCITY FLUCTUATIONS

\[
\frac{n S_u(z,n)}{u^2} = \begin{cases} 
  a_1 f + b_1 f^2 + d_1 f^3 & f \leq f_m \\
  c_2 + a_2 f + b_2 f^2 & f_m < f < f_s \\
  0.26 f^{-2/3} & f \geq f_s 
\end{cases}
\]  

(Al)

where \( u^* \) is given by equations 9 and 4, \( f = nz/u(z) \), and

\[
a_1 = \frac{4 L_u \beta}{z} 
\]

(A2a)

\[
\beta_1 = 0.26 f_s^{-2/3} 
\]

(A2b)

\[
b_2 = \frac{\frac{1}{3} a_1 f_m + \left( \frac{7}{3} + \ln \frac{f_s}{f_m} \right) \beta_1 - \beta}{\frac{5}{6} (f_m - f_s)^2 + \frac{1}{2} (f_m^2 - f_s^2) + 2 f_m (f_s - f_m) + f_s (f_s - 2 f_m) \ln \frac{f_s}{f_m}} 
\]

(A2c)

\[
a_2 = -2 b_2 f_m 
\]

(A2d)

\[
d_1 = \frac{2}{f_m^3} \left[ \frac{a_1 f_m}{2} - \beta_1 + b_2 (f_m - f_s)^2 \right] 
\]

(A2e)

\[
b_1 = \frac{-a_1}{2 f_m} - 1.5 f_m d_1 
\]

(A2f)

\[
c_2 = \beta_1 - a_2 f_s - b_2 f_s^2 
\]

(A2g)
Turbulent Wind Effects on Tension Leg Platform Surge

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A procedure is presented for estimating surge response to turbulent wind in the presence of current and waves. The procedure accounts for the nonlinearity of the hydrodynamic forces with respect to surge and for the coupling of aerodynamic and hydrodynamic effects. It is shown that current wind spectra do not model correctly the wind speed fluctuations at very low frequencies and an alternative model of the wind spectrum, consistent with fundamental principles, is presented. The equation of surge motion under turbulent wind in the presence of current and waves is solved for typical tension leg platforms, and it is shown that under extreme wave conditions the damping provided by the hydrodynamic forces precludes the occurrence of significant wind-induced resonant amplification effects even if the drag coefficient in the Morison equation is very small (e.g., $C_d = 0.1$). It is verified that for the platforms being investigated the use of a one-minute wind speed to represent the effect of the mean wind and of the turbulent wind fluctuations is acceptable for the purpose of estimating peak surge response.

Compliant platforms; ocean engineering; offshore platforms; structural engineering; tension leg platforms; turbulence; waves; wind loads.

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