

NBS BUILDING SCIENCE SERIES 139

# **Application of Method of Characteristics to Model the Transport of Discrete Solids in Partially-Filled Pipe Flow**

U.S. DEPARTMENT OF COMMERCE • NATIONAL BUREAU OF STANDARDS

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## Application of Method of Characteristics to Model the Transport of Discrete Solids in Partially-Filled Pipe Flow

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## PREFACE

This report is one of a group documenting National Bureau of Standards (NBS) research and analysis efforts in developing water conservation test methods, analysis, economics, and strategies for implementation and acceptance. This work is sponsored by the Department of Housing and Urban Development/Office of Policy Development and Research, Division of Energy Building Technology and Standards, under HUD Interagency Agreement H-48-78.

The report was prepared by Dr. J. A. Swaffield, Senior Lecturer, Drainage Research Group, Department of Building Technology, Brunel University, Uxbridge, UK., during a study leave period as a guest research worker at NBS/Stevens Institute of Technology.

Experimental results included in this report are drawn from the published work of the Drainage Research Group at Brunel University.

## ABSTRACT

The flow depth and velocity changes across a moving solid in partially-filled pipe flow are predicted by means of the application of the method of characteristics to solve the unsteady flow equations.

Simplified force models are presented which, when used in conjunction with empirical relationships linking leakage flow past the solid to upstream specific energy, are sufficient to provide the required moving solid boundary conditions that allow solid velocity prediction.

A wide range of simulated transport conditions are presented that confirm the applicability of this technique as a basis for the future evaluation of more complex body force models.

The predicted solid velocity during drain transport is shown to be compatible with laboratory observations of the influence of solid dimensions and position in inflow profile on transport characteristics.

**Key words:** computer based model; drainage; solid transport; unsteady flow

## CONTENTS

	<u>Page</u>
<b>Notation .....</b>	<b>ix</b>
<b>1. INTRODUCTION .....</b>	<b>1</b>
<b>2. DEVELOPMENT OF THE NUMERICAL SOLUTION .....</b>	<b>3</b>
<b>2.1 Summary of the Method of Characteristics Solution of the Equations Defining Unsteady Flow in Partially Filled Pipe Flow .....</b>	<b>3</b>
<b>2.2 Partially Filled Pipe Flow Regimes .....</b>	<b>5</b>
<b>2.3 Application of the Method of Characteristics Solution to Drainage Flow .....</b>	<b>9</b>
<b>2.3.1 Initial Steady Conditions Along the Pipe Length .....</b>	<b>10</b>
<b>2.3.2 Internal or Nodal Points .....</b>	<b>11</b>
<b>2.3.3 Entry Boundary Conditions, Supercritical Flow .....</b>	<b>11</b>
<b>2.3.4 Entry Boundary Conditions, Subcritical Flow .....</b>	<b>12</b>
<b>2.3.5 Exit Boundary Conditions, Supercritical Flow .....</b>	<b>12</b>
<b>2.3.6 Exit Boundary Conditions, Subcritical Flow .....</b>	<b>13</b>
<b>2.4 Application of Method of Characteristics to Waste Solid Boundary, Stationary Condition .....</b>	<b>13</b>
<b>2.5 Application of Method of Characteristics to Waste Solid Boundary, Moving Condition .....</b>	<b>14</b>
<b>2.5.1 Model of Forces Acting on the Solid .....</b>	<b>14</b>
<b>2.5.2 Equations Governing Flow Past the Moving Solid .....</b>	<b>16</b>
<b>2.6 Simulation Cases Covered by the Proposed Model .....</b>	<b>17</b>
<b>3. CALCULATION TECHNIQUES AND PRESENTATION OF RESULTS .....</b>	<b>19</b>
<b>3.1 Determination of Normal and Critical Depths .....</b>	<b>19</b>
<b>3.2 Numerical Integration for Surface Profiles .....</b>	<b>20</b>
<b>3.3 Choice of Time Step .....</b>	<b>20</b>
<b>3.4 Presentation of Results .....</b>	<b>21</b>
<b>3.5 Choice of Calculation Constants .....</b>	<b>21</b>

CONTENTS (Continued)

	<u>Page</u>
4. DISCUSSION OF UNSTEADY FLOW SIMULATION RESULTS .....	23
4.1 Influence of Body Force Model on Predicted Solid Velocity .....	24
4.2 Influence of Solid Dimensions and Position in the Inflow Profile on Predicted Solid Velocities .....	25
4.3 Comparison of Solid Motion from Both Initial Deposition and Insertion at Water Velocity Models .....	25
4.4 Comparison of Predicted Solid Velocities to Empirical Relationships .....	26
4.5 Limitations to the Simulation .....	27
5. CONCLUSIONS AND FURTHER WORK .....	29
6. REFERENCES .....	31
Figures .....	33
APPENDIX 1 Program TRANSCD .....	55
APPENDIX 2 Derivation of Solid Boundary Equations Used in Subroutine SOLID .....	85
APPENDIX 3 Typical Output Program TRANSCD .....	89

## LIST OF FIGURES

		<u>Page</u>
<b><u>Figure</u></b>		
1	Application of momentum equation to unsteady flow in a general open channel .....	33
2	Modification to specified time interval grid for subcritical and supercritical flow regimes .....	34
3	Solution at pipe entry, exit and internal nodes for subcritical and supercritical flow regimes .....	35
4	Measurement of the flow specific energy change across a solid tethered in the waste pipe under steady flow conditions .....	36
5	Solution of $C^+$ characteristic with body equivalent $B^+$ characteristic .....	37
6	Equation of motion applied to a rectangular section solid .....	38
7	Calculation of buoyancy forces .....	39
8	Summary of alternative force models .....	40
9	Schematic representation of solid motion in the xt plane .....	41
10	Modifications to xt grid adjacent to the solid position for subcritical flow .....	42
11	Modifications to xt grid adjacent to the solid position for supercritical flow .....	43
12	Propagation of steep fronted wave upstream from the solid position .....	44
13	Comparison of maternity pad transport in UPVC and cast iron 100 mm diameter pipe .....	45
14	Influence of force model on the predicted solid velocity .....	46
15	Influence of solid position in the inflow profile on predicted solid velocity .....	47
16	Influence of solid size on predicted velocity .....	48

LIST OF FIGURES (Continued)

<u>Figure</u>		<u>Page</u>
17	Initially stationary solid, predicted water and solid velocities illustrating relative flow over the body .....	49
18	Initially stationary solid, depth upstream of the solid during motion .....	50
19	Solid injected into flow, predicted water and solid velocities illustrating relative flow over the body .....	51
20	Solid injected into flow, predicted depth differential across solid during motion along simulated drain .....	52
21	Predicted solid velocity plotted vs $\sqrt{L/G}$ , pipe slopes 1/40 and 1/100 .....	53
22	Predicted solid velocity plotted vs $\sqrt{L/G}$ , pipe slopes 1/40 to 1/300 .....	54

### Notation

A	Pipe flow cross sectional area, $\text{m}^2$
$s^+$	Solid characteristic in $xt$ plane
$c^+, c^-$	Notation referring to the positive and negative characteristics
c	Wave speed, $\text{m/sec}$
$F_r$	Froude N° = $V/\sqrt{gh}$
F	Forces on solid, $\text{gm}$ or $\text{kg}$
G	Pipe slope or Pipe Gradient
g	Acceleration due to gravity, $\text{m/sec}^2$
h	Flow depth, $\text{m}$
$\ell$	Solid length, $\text{m}$
L	Distances in flow direction
m	Hydraulic mean depth, $\text{m}$
N, n	N° of pipe length sections employed
P	Wetted channel perimeter, $\text{m}$
Q	Flow rate, $\text{l/s}$
SE	Specific energy = $h + V^2/2g$ , $\text{m}$
$SE_0$	Minimum specific energy required to initiate flow past solid, $\text{m}$
$s_o$	Pipe slope
S	Slope of energy grade line, defined by Manning's Equation
T	Surface width of flow within partially filled channel, $\text{m}$
t	Time, sec (also t = thickness dimension solid in forces acting on body, $\text{m}$ ).
V	Local mean velocity, $\text{m/sec}$
$v_B$	Solid velocity, $\text{m/sec}$

## NOTATIONS (Continued)

w Solid width, m

X1-4 Functions of h, V, c and S calculated at each base point at each time step

x Distance, positive in initial flow direction, m

$\alpha$   $S_0 = \sin \alpha$  - Angle of solid relative to pipe

$\Delta t$  Time step, sec

$\Delta x$  Pipe section length, m

$\theta$   $\Delta t / \Delta x$ , sec/m

$\rho$  Fluid density gmsec<sup>2</sup>/m<sup>4</sup>

$\tau_0$  Wall to fluid shear stress

### Suffixes

A,B,C Calculated points in an x-t grid at time t

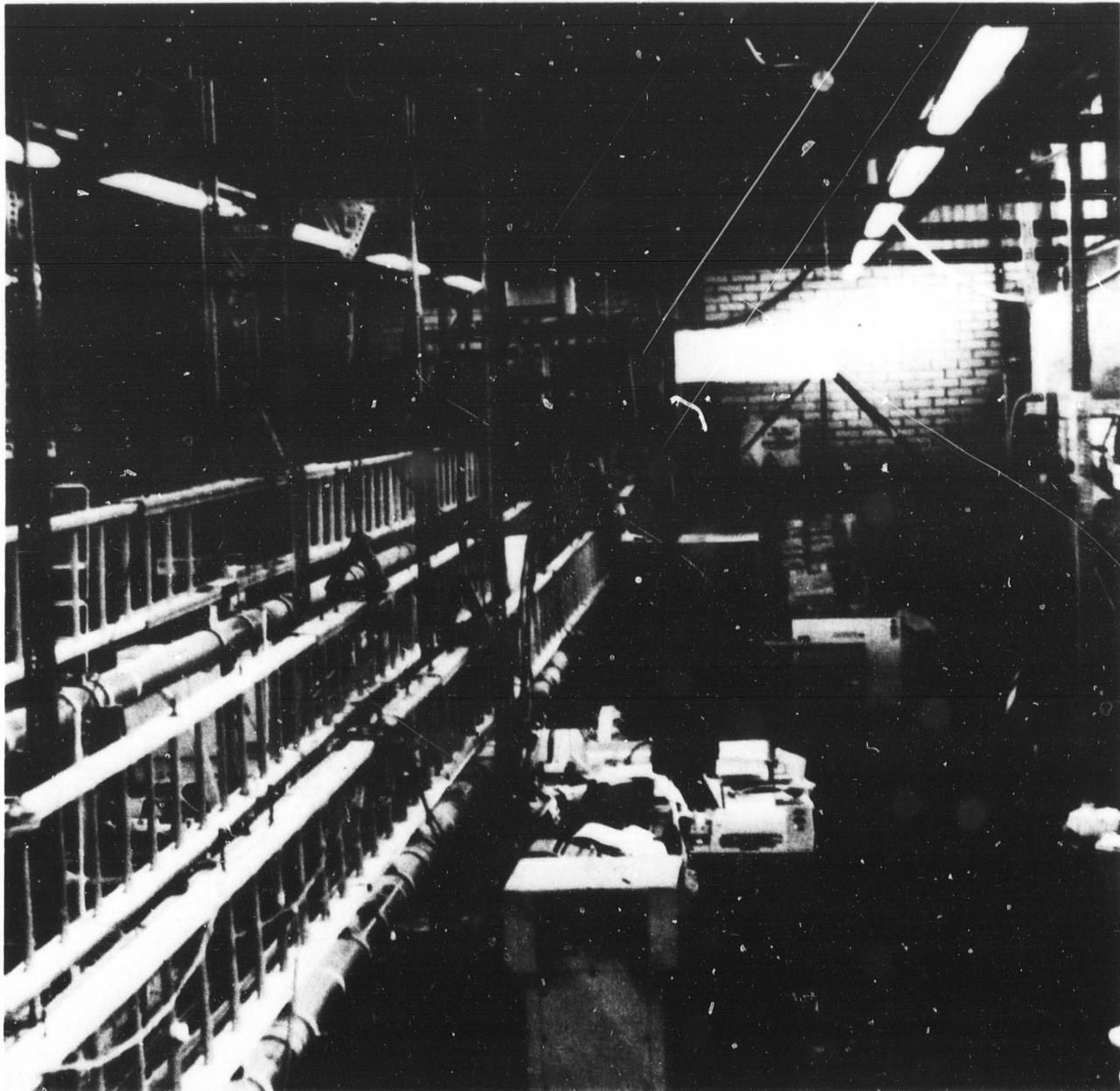
abs Absolute velocity

P Calculated points in an x-t grid at time t +  $\Delta t$

R,S Interpolated points in an x-t grid at time t

re Relative velocity

U,D Upstream and downstream conditions relative to the solid boundary



## 1. INTRODUCTION

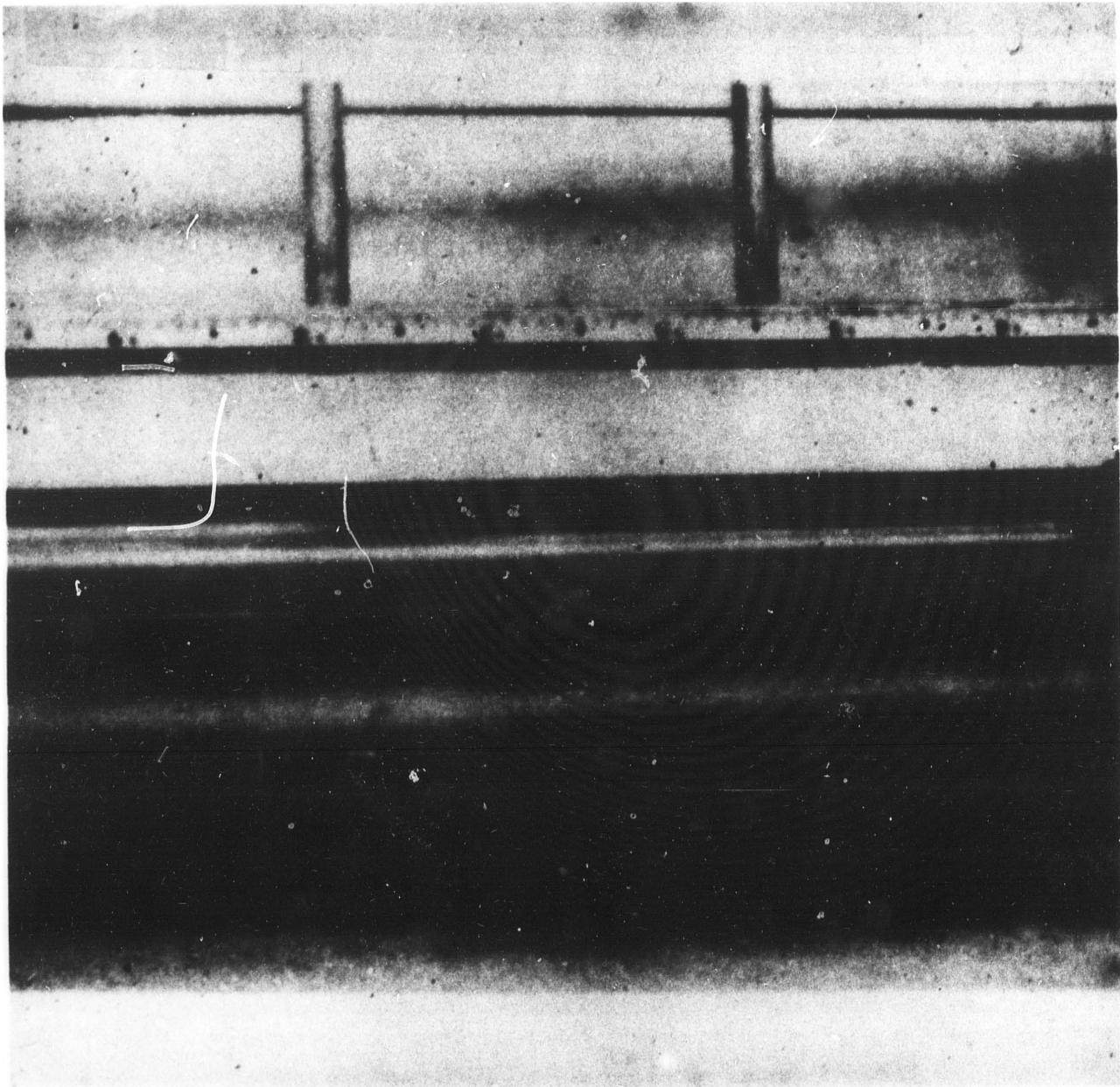
The prediction of solid transport in partially filled drainage pipe flow requires a knowledge of the forces acting on the solid and their relative importance. The treatment of the force and momentum equations for the body and surrounding flow is analytically complex; however, a numerical model capable of simulating the unsteady flow equations could be used to evaluate models of the body forces for comparison to laboratory observations.

Previous reports [1,2] have developed the application of the method of characteristics solution to the unsteady partially filled pipe flow equations. The introduction of a moving solid into the simulation requires the knowledge of leakage flow past the body as a function of upstream conditions, with a suitable model for the forces acting on the body. The passage of the simulated solid through the pipe may then be treated as a moving boundary condition by a similar technique to that applicable to the tracing of a shock wave or steep fronted wave motion.

The necessary computational techniques to allow inclusion of the solid boundary conditions are presented in this report with computer simulations involving three force models. The predicted solid motion characteristics are also compared to laboratory observed solid motion in drains which are set at a wide range of gradients.

The development of the method of characteristics solution is also summarized, and there is a statement of the necessary boundary conditions at the drain inlet and exit in both subcritical and supercritical flow regimes.

It is stressed that the objective of the study was the development of the necessary computational techniques to demonstrate the feasibility of the method coupling the hydraulic and solid motion. The development illustrates the applications of the basic modeling approach and the simultaneous solution of the governing equations of hydraulic interaction with that of the motion of the solid. A basis has been established for future comparison with development of other body force models representative of the test conditions for a wide range of body configurations for which observed laboratory solid transport results are obtained.



## 2. DEVELOPMENT OF THE NUMERICAL SOLUTION

### 2.1 SUMMARY OF THE METHOD OF CHARACTERISTICS SOLUTION OF THE EQUATIONS DEFINING UNSTEADY FLOW IN PARTIALLY FILLED PIPE FLOW

Referring to figure 1, it has been shown [1] that the equations of motion and continuity may be expressed as

$$g \frac{\partial h}{\partial x} + g(s - s_0) + v \frac{\partial v}{\partial x} + \frac{\partial v}{\partial t} = 0 \quad (1)$$

$$VT \frac{\partial h}{\partial x} + T \frac{\partial h}{\partial t} + A \frac{\partial V}{\partial x} = 0. \quad (2)$$

These equations may be combined to form a total derivative expression,

$$\frac{dV}{dt} + \frac{g}{c} \frac{dh}{dt} + g(S - S_o) = 0, \quad (3)$$

subject to

$$\frac{dx}{dt} = V + c, \quad (4)$$

the wave speed being defined by

$$c = \sqrt{\frac{gA}{T}} \quad (5)$$

where  $V$  = local average flow velocity

$c$  = local wave speed

$h$  = local flow depth

$S_o$  = pipe slope

$S$  = slope of the local energy grade line =  $\frac{n^2 V^2}{m^{4/3}}$ ,  $n$  the Manning

coefficient and  $m$  the hydraulic mean depth

$T$  = water surface width

$A$  = water depth cross sectional area.

Referring to figure 2, if the variables  $V$  and  $h$  are known at  $R$  and  $S$  then four equations may be written in terms of the unknowns at point  $P$ , by means of a first order approximation,

$$V_p - V_R + \frac{g}{c_R} (h_p - h_R) + g(S_R - S_o) \Delta t = 0 \quad (6)$$

$$x_p - x_R = (V_R + c_R) \Delta t \quad (7)$$

$$V_p - V_S - \frac{g}{c_S} (h_p - h_S) + g(S_R - S_o) \Delta t = 0 \quad (8)$$

$$x_p - x_S = (V_S - c_S) \Delta t. \quad (9)$$

It is stressed that these equations are paired and that equations 6 and 8 only apply as long as equation 7 and 9 are satisfied. This introduces the characteristics lines  $C^+$  and  $C^-$ , shown in figure 3.

It is also necessary that the time step  $\Delta t$  is sufficiently small for  $R$  and  $S$  (points in the  $x-t$  plane) to fall within  $\pm \Delta x$  of point  $P$  as shown in figure 2.

From figure 3 it will be seen, provided boundary equations governing the conditions at the extremities of the system are known, that an orderly solution may proceed yielding flow depth and velocity at each pipe section at each time increment.

The basis for the system boundary conditions is set out below.

## 2.2 PARTIALLY FILLED PIPE FLOW REGIMES

Two flow regimes may be identified for open channels or partially filled pipes:

$$(1) \text{ Subcritical flow, Froude } N^o = \frac{V}{\sqrt{gh}} < 1$$

Here the local wave speed exceeds the flow average velocity, thus waves may be propagated both upstream and downstream in the flow, i.e.,  $c > V$ .

$$(2) \text{ Supercritical flow, Froude } N^o > 1$$

Here the local wave speed is less than the average flow velocity at that section and hence waves cannot be propagated upstream, i.e.,  $V > c$ .

The flow regime applicable to any partially filled pipe flow may be determined by a comparison of the flow normal and critical depths.

Under steady uniform flow conditions the force balance equation for an element of flow is normally expressed by the Chezy equation

$$V = C \sqrt{m S_o} \quad (10)$$

$m$  = hydraulic mean depth  $A/P$ ,  $m$  (where  $A$  is the pipeflow cross section area)

$S_o$  =  $\sin \theta$  -duct slope

$V$  = mean velocity  $m/s$

$C$  = Chezy constant.

$P$  = pipe wetted perimeter.

The value of loss coefficient  $C$  was found by Manning to be dependent on hydraulic mean depth and duct surface roughness  $n$ . The Manning formula is the simplest of the open channel equations:

$$V = \frac{1}{n} m^{2/3} S_o^{1/2}$$

$$Q = \frac{1}{n} A m^{2/3} S_o^{1/2} \quad (11)$$

where  $Q$  is the flow rate  $m^3/s$   
 $A$  is the flow cross sectional area,  $m^2$

The value of the Manning coefficient,  $n$ , varies with pipe or channel material; values in the range 0.009 to 0.020 apply to for materials commonly found in building drainage systems.

Equation 10 effectively determines the flow depth under steady, uniform conditions with only one value of  $h$  yielding the values of  $A$  and  $m$  necessary to satisfy the equation. As this depth is by definition constant downstream,  $dh/dx = 0$ , it must also be the terminal depth corresponding to the flow terminal velocity at that channel slope.

This depth,  $h_n$ , is commonly referred to as the normal depth.

The specific energy of the flow may be defined as

$$E = h + \frac{v^2}{2g} \quad (12)$$

where  $h$  = local flow depth, m

$v$  = local average flow velocity, m/s

From equation 12 it may be seen that the flow specific energy has a minimum value below which the given flow conditions cannot exist. In a general, non-rectangular channel this value may be determined:

$$\begin{aligned} E &= h + \frac{Q^2}{2gA^2} \\ \frac{dE}{dh} &= 0 = 1 - \frac{Q^2}{gA^3} \frac{dA}{dh}. \end{aligned} \quad (13)$$

From figure 3

$$dA = T dh \quad (14)$$

where  $T$  is the surface width at any depth,  $h$ .

From equations (13) and (14) the minimum value of  $E$  will occur at a depth value,  $h_c$ , that satisfies the expression

$$1 - Q^2 T / g A^3 = 0 \quad (15)$$

where  $T$  and  $A$  are both  $f(h)$

This value of  $h$  is referred to as the flow critical depth  $h_c$ .

If the normal flow depth  $h_n$  exceeds  $h_c$  then the terminal flow would be termed subcritical, or tranquil flow. If  $h_n$  is less than  $h_c$  then the flow is termed rapid or supercritical.

It should be stressed that  $h_c$  is independent of pipe slope and pipe surface roughness, while the normal depth is dependent on both. Thus the same volume flow rate in any particular pipe may be rapid or tranquil depending on pipe slope, and similarly, the same flow rate in a series of constant diameter pipes will be tranquil or rapid depending on roughness.

Pipes or channels in which rapid flow is normal are termed steep; pipes or channels in which tranquil flow is normal are termed of mild slope.

Figure 2 illustrates the importance of these two flow regimes on the solution of equations 6 to 9.

If  $c > V$ , then the conditions at P are determined by the intersection of the  $C^+$  and  $C^-$  drawn from P into the AC and BC sections.

If  $c < V$ , then conditions in the downstream section BC cannot affect point P. The slope of the  $C^-$  characteristic, PS, becomes positive, and both R and S lie in the AC section as shown.

Both subcritical and supercritical flow are encountered in the drainage applications considered and the equations derived below cover both conditions.

Referring to figure 2 for subcritical flow:

$$\frac{V_C - V_R}{V_C - V_A} = \frac{x_C - x_R}{x_C - x_A} = (V_R + c_R) \frac{\Delta t}{\Delta x}$$

$$\frac{c_C - c_R}{c_C - c_A} = \frac{x_C - x_R}{x_C - x_A} = (V_R + c_R) \frac{\Delta t}{\Delta x}$$

and  $\frac{h_C - h_R}{h_C - h_A} = (V_R + c_R) \frac{\Delta t}{\Delta x}$

as  $x_P = x_C$  and  $x_P - x_R = (V_R + c_R)\Delta t$ .

The solution yields

$$V_R = \frac{V_C + \theta (-V_C c_A + c_C V_A)}{1 + \theta (V_C - V_A + c_C - c_A)} \quad (16)$$

$$c_R = \frac{c_C (1 - V_R \theta) + c_A V_R \theta}{1 + c_C \theta - c_A \theta} \quad (17)$$

$$h_R = h_C - (h_C - h_A)(\theta (V_R + c_R)) \quad (18)$$

where  $\theta = \Delta t / \Delta x$ .

Similarly,

$$v_s = \frac{v_c - \theta(v_c c_B - c_C v_B)}{1 - \theta(v_c - v_B - c_C + c_B)} \quad (19)$$

$$c_s = \frac{c_c + v_s \theta(c_C - c_B)}{1 + \theta(c_C - c_B)} \quad (20)$$

$$h_s = h_C + \theta(v_s - c_s)(h_C - h_B) \quad (21)$$

For the supercritical flow regime, the equations determining R in figure 2 remain unchanged. The interpolation equations for S' in figure 2 may be determined by an identical technique to that shown above,

$$v_c - v_{s'} = (v_c - v_A) \theta(v_{s'} - c_{s'})$$

$$c_c - c_{s'} = (c_c - c_A) \theta(v_{s'} - c_{s'})$$

$$\text{and } h_c - h_{s'} = (h_c - h_A) \theta(v_{s'} - c_{s'}).$$

The solution yields

$$v_{s'} = \frac{v_c(1 - c_A \theta) - v_A c_C \theta}{1 + \theta(v_c - v_A + c_A - c_C)} \quad (22)$$

$$c_{s'} = \frac{c_c + v_{s'} \theta(c_A - c_C)}{1 + c_A \theta - c_C \theta}. \quad (23)$$

From equations 7 and 9 it will be seen that

$$\frac{dt}{dx} = \frac{1}{v + c} \quad (24)$$

hence, if  $(v + c)$  becomes large, then  $\Delta t$  becomes small for a constant  $\Delta x$ . This implies that the progress of the numerical solution could become prohibitively slow for supercritical flow conditions. Fox [3] suggests a check within any program that will terminate the solution if  $\Delta t$  falls below a specified value; however, this comment by Fox applies to the existing applications of the method of characteristics, which have been limited to large civil engineering open channel or river flooding flow problems. It is likely that the reduction in  $\Delta t$  will not be a significant problem with the relative values of  $v$  and  $c$  encountered in drainage sized pipe applications due to their dependence on pipe geometry and flow rate.

The determination of conditions at P at time  $t + \Delta t$  requires the following steps (i-iv), for either subcritical or supercritical flow conditions:

- (i) All conditions known at time  $t$  for nodal points A, B, C etc. (figure 2).

- (ii) Values of  $V$ ,  $h$  and  $c$  at interpolation points  $R$ ,  $S$ , or  $S'$  calculated from equations 16 - 23.
- (iii) Using these values of  $V$ ,  $h$ , and  $c$ , the conditions at  $P$ , i.e. velocity  $V$  and depth  $h$ , at time  $t + \Delta t$ , are calculated by means of equations 6 and 8.
- (iv) The value of wave speed  $c$  at  $P$  at time  $t + \Delta t$  is calculated from equation 5. The value of flow surface width and cross sectional area are calculated from flow depth,  $h$ , and the channel shape relationships.
- (v) The sequence is repeated at each time step.

### 2.3 APPLICATION OF THE METHOD OF CHARACTERISTICS SOLUTION TO DRAINAGE FLOW

For convenience, the application of the solution developed above to drainage pipe flow may be considered under two headings; namely, boundary conditions and characteristic equation solution at intermediate pipe sections. Both of these headings must be further subdivided depending on whether the flow is termed subcritical or supercritical.

The equations 6 to 9 may be restated as

$$V_p = X_2 - X_1 h_p \quad C^+ \quad (25)$$

$$x_p - x_R = (V_R + c_R)\Delta t$$

$$V_p = X_4 + X_3 h_p \quad C^- \quad (26)$$

$$x_p - x_S = (V_S - c_S)\Delta t$$

where  $X_1 = g/c_R$

$$X_3 = g/c_S$$

$$X_2 = V_R + g \frac{h_R}{c_R} - g(s_R - s_o)\Delta t$$

$$X_4 = V_S - g \frac{h_S}{c_S} - g(s_S - s_o)\Delta t.$$

It will be noted that these equations apply in either subcritical or supercritical flow, the interpolated values of the conditions at  $S$  or  $S'$  being sufficient to define the flow regime, figure 2.

### 2.3.1 Initial Steady Conditions Along the Pipe Length

As will be seen from figure 3, the initial conditions along the whole pipe length at time  $t = 0$  must be known in order for the solution to proceed. It is therefore necessary to calculate the steady state flow velocity and depth throughout the pipe length initially with the initial wave speed. This process may be carried out by the following steps:

- (i) Determine the steady flow normal and critical depths as set out previously. This determines whether the flow is subcritical or supercritical.
- (ii) For supercritical flow, the normal flow depth may be assumed to apply throughout the pipe length. As the velocity exceeds the wave speed, there is no effect propagated upstream from the pipe discharge point. This implies that the flow leaves the pipe at normal depth and, for a known flow rate and pipe dimension, allows the local velocity and wave speed to be calculated along the whole pipe length.
- (iii) For subcritical flow, the initial water surface profile is more complicated as the effect of the pipe discharge is propagated upstream. In subcritical flow, it has been found that the depth of flow is at its critical value at the discharge point, with the water depth then rising upstream until the normal steady flow depth is achieved. Calculation of this depth profile is presented in [2] and summarized below in terms of the gradually varied flow profile prediction technique.

Gradually varied flow is steady non-uniform flow of a special type. The flow parameters are assumed to change slowly, if at all, in the flow direction. The basic assumption in the treatment of this type of flow is that the local head loss at any section is given by the Manning expression (11), for the identical local flow depth and rate under assumed steady, uniform flow conditions.

Depth profile predictions by numerical integration are based on this assumption,

$$\frac{d}{dL} \left\{ \frac{V^2}{2g} + (z_o - S_o L) + h \right\} = - \left\{ \frac{nQ}{A_m^{2/3}} \right\}^2 \quad (27)$$

where  $(z_o - S_o L)$  is the elevation at distance  $L$  along the channel, measured in the downstream direction;  $S_o$  is  $\sin \theta$ , channel bed slope,

$$\text{hence } - \frac{V}{g} \frac{dV}{dL} + S_o - \frac{dh}{dL} = \left( \frac{nQ}{A_m^{2/3}} \right)^2 \quad (28)$$

and as,  $Q = VA$

$$\frac{dV}{dL} A + V \frac{dA}{dL} = 0$$

and as  $\frac{dA}{dh} = T$  from equation 5, it follows that

$$\frac{dV}{dL} = \frac{V}{A} \frac{dA}{dL} = - \frac{VT}{A} \frac{dh}{dL} = - \frac{QT}{A^2} \frac{dh}{dL},$$

and substituting in equation 28 yields

$$\frac{Q^2 T}{g A^3} \frac{dh}{dL} + S_o - \frac{dh}{dL} = \left\{ \frac{n Q}{A m^{2/3}} \right\}^2 \quad (29)$$

$$dL = \left\{ \frac{1 - Q^2 T / g A^3}{S_o - (n Q / A m^{2/3})^2} \right\} dh$$

$$L = \int_{h_o}^{h_1} \frac{1 - Q^2 T / g A^3}{S_o - (n Q / A m^{2/3})^2} dh \quad (30)$$

where  $L$  is the distance between two known depths  $h_o, h_1$ .

The initial depth at each section  $\Delta x$  apart along the pipe may then be calculated from the profile produced by the integration of equation 30. Flow velocity is then calculated based on a constant flow rate through the pipe, and similarly, wave speed is determined based on flow depth and channel geometry.

Once the initial flow depth, velocity and wave speed have been determined the unsteady flow calculation procedure may begin.

### 2.3.2 Internal or Nodal Points

Simultaneous solution of equations 25 and 26 at all points  $\Delta x$  apart between  $x = \Delta x$  and  $x = (L - \Delta x)$  will yield the required values of flow depth and velocity at the end of each time step. Wave speed may then also be determined from equation 5. This process applies to either sub or supercritical flow conditions as the particular regime is represented in the interpolations required to fix points R, S or S', figure 2.

### 2.3.3 Entry Boundary Conditions, Supercritical Flow

In this case the inflow profile alone determines the flow depth at pipe entry as downstream conditions, that would have been represented by the C' characteristic in subcritical flow, cannot effect the flow conditions at the upstream boundary, as by definition the flow velocity exceeds the wave speed.

Hence the boundary condition is obtained from the flow profile  $Q = f(t)$  solved with the normal depth expression

$$Q = \frac{1}{n} A m^{2/3} s_o^{1/2}$$

where  $A$  and  $m$  are both  $f(h)$ . This equation may be rewritten in the form

$$1 - \frac{(nQ)^2}{A^2 m^{4/3} s_o} = 0 \quad (31)$$

and this boundary expression may be solved at each time step with a known  $Q$  by the bisection technique, this technique is described later.

#### 2.3.4 Entry Boundary Conditions, Subcritical Flow

For subcritical flow the downstream conditions do contribute to the entry flow depth and velocity. In this case the inflow profile  $Q = f(t)$  is solved with the  $C^-$  characteristic

$$\begin{aligned} Q &= f(t) = v_1 A_1 \\ v_1 &= x_4 + x_3 h_1 \\ Q(t) &= A_1 (x_4 + x_3 h_1) \end{aligned} \quad (32)$$

where suffix 1 refers to the entry boundary location and  $A = f(h)$  dependent on the pipe cross sectional geometry.

In the form

$$Q(t) - f(h_1)(x_4 + x_3 h_1) = 0 \quad (33)$$

this boundary equation may be solved by the bisection technique.

#### 2.3.5 Exit Boundary Conditions, Supercritical Flow

As the flow velocity exceeds the local wave speed the exit boundary condition may be determined in the same manner as the upstream nodal points; namely, by the simultaneous solution of the  $C^+$  and  $C^-$  characteristics. With reference to figure 2, both the R and S' points lie upstream of the pipe exit.

The exit condition may be included in the nodal point calculations for the supercritical flow case, no separate exit subroutine being necessary in the program, as equations 25 and 26 are sufficient.

### 2.3.6 Exit Boundary Conditions, Subcritical Flow

At pipe exit in the subcritical flow regime, the flow depth approaches the critical depth value, given by zero value of equation 15:

$$\frac{Q^2}{g A_{\text{crit}}^3} T_{\text{crit}} = 1,$$

where  $A$  and  $T$  are  $f(h)$ .

This condition may be solved with the  $C^+$  characteristic

$$V_{N+1} = X_2 - X_1 h_{N+1}$$

where  $N = N^\circ$  of pipe length sections,  $\Delta x$ .

The boundary condition becomes

$$\left[ (X_2 - X_1 h_{N+1}) A_{N+1} \right]^2 \frac{T_{N+1}}{g A_{N+1}^3} - 1 = 0. \quad (34)$$

The solution may again be achieved by use of the bisection method with the use of the area to depth relationship for the channel.

### 2.4 APPLICATION OF METHOD OF CHARACTERISTICS TO WASTE SOLID BOUNDARY, STATIONARY CONDITION

Considering a stationary solid deposited at some point along the waste pipe, the water depth and velocity upstream of the solid may be predicted if a suitable boundary equation may be written linking flow past the solid to upstream conditions.

Figure 4 illustrates the relationship between flow past a stationary solid and the specific energy upstream. These results were compiled during a Brunel University Drainage Research Group study of solid transport in drainage systems.

The flow past the solid (experimentally determined) may be expressed as

$$Q = K \left( h + \frac{V^2}{2g} - SE_o \right)^2 \quad (35)$$

where  $SE = h + V^2/2g$ , flow specific energy and  $SE_o$  is the flow specific energy required for flow initiation past the solid.

Equation 35 may then be solved with the  $C^+$  characteristic, of figure 5,

$$V_I = X_2 - X_1 h_I$$

where  $Q = V_I A_I$

so that

$$A_I(X_2 - X_1 h_I) = K \left| h_I + \frac{1}{2g} (X_2 - X_1 h_I)^2 - SE_o \right|^2.$$

This expression results in a quartic in terms of depth upstream of the solid,  $h_I$  (see appendix 2). This quartic must be solved by an iterative technique as the flow area,  $A_{N+1}$ , is a function of  $h_I$ . The Newton Raphson method may be employed to carry out the necessary iterative solution. Once the value of  $h_I$  has been determined, the value of  $V_I$  and  $c_I$  may be determined.

As mentioned, the  $SE_o$  term is the flow specific energy required to initiate flow past the stationary solid. If the value of flow specific energy at time  $t$  is less than that of  $SE_o$ , then the value of flow velocity at the solid at time  $t + \Delta t$  is set equal to zero. The flow depth then comes directly from equation 25 as:

$$h_I = X_2/X_1. \quad (36)$$

This implies that the flow depth upstream of the solid must rise to  $SE_o$  prior to the initiation of flow past the solid. This solution is set out in detail in appendix 2, and it has been shown to be capable of simulating depth increase upstream of a stationary solid [1].

It should be noted that the analysis above applies to both subcritical and supercritical flow regimes, as only the  $C^+$  characteristic is involved.

## 2.5 APPLICATION OF METHOD OF CHARACTERISTICS TO WASTE SOLID BOUJNDARY, MOVING CONDITION

Prior to application of the moving boundary conditions representing the solid motion, it is necessary to determine the solid motion initiation time. This may be accomplished by monitoring the net force acting on the solid.

### 2.5.1 Model of Forces Acting on the Solid

Figure 6 illustrates the forces assumed to act on the rectangular section solid, namely, hydrostatic pressure forces,  $F_{P1}$ ,  $F_{P2}$ , acting on the trailing and leading edge projected areas, body weight force, frictional force due to wall to body contact,  $F_S$ , and a bouyancy force,  $F_B$ , dependent on the solid position in the flow.

The net force acting on the solid may be expressed as, figure 6,

$$F_{P1} - F_{P2} + mg \sin\theta - F_S = F_{BODY} \quad (37)$$

where  $F_S = f(mg \cos\theta - F_B)$

and  $f$  is the wall to solid friction factor, its value being reduced from the static friction coefficient to the sliding friction value if  $F_{BODY}$  becomes positive.

If  $F_{BODY} > 0$ , then solid motion is initiated and the solid velocity at the end of a computational time step  $\Delta t$  is given by:

$$F_{BODY} = \frac{m dV}{dt} = \frac{m \Delta V}{\Delta t} \quad (38)$$

where  $\Delta V = V_{t+\Delta t} - V_t$  and  $m$  = mass of body. (39)

(note values of  $V_t = 0$  for the first time step)

$$\text{Hence, } \Delta V = V_{t+\Delta t} = V_t + \frac{\Delta t}{m} F_{BODY}. \quad (40)$$

The distance traveled by the solid in the time step may be approximated by

$$X_B = X_o + \frac{1}{2}(V_t + \Delta t + V_{t+\Delta t})\Delta t \quad (41)$$

where  $X_o$  is the solid position at time  $t$  and  $X_B$  the final position at the end of the time step.

Equations 37 to 41 apply to subsequent time steps with the mentioned modifications to the value of wall to solid friction coefficient in equation 37. The net force on the body may become negative in subsequent time steps; however, this represents solid deceleration and no modification to the equations is necessary until the predicted value of  $V_t + \Delta t$  (equation 40) becomes either zero or negative (the condition for the body coming to rest).

Figure 7 illustrates the forces assumed to act on the solid. The calculation of the buoyancy force,  $F_B$ , (equation 37) requires the solution of the body force system. Values of  $F_1$ ,  $F_2$  and  $F_D$  may be approximated by the hydrostatic equation; however, the force  $F_u$  is not readily estimated due to the curvature of the solid to pipe boundaries.

Laboratory observations have shown, that the model solids, maternity pads [4], due to their flexibility, tend to take up the shape shown in figure 7. Taking moments about the leading edge point A, as shown in figure 7, allows  $F_u$  to be estimated.

The buoyancy force  $F_B$  may then be determined as

$$F_B = (F_u - F_D) \cos \alpha \quad (42)$$

where  $\alpha$  is the slope of the solid surface relative to the pipe wall.

Figure 8 illustrates three force models investigated in this study:

Model 1 - buoyancy forces neglected, the value of  $F_B = 0$

Model 2 - downthrust force  $F_D = 0$ . The justification for this model comes from test observations. In the boundary equations for the solid leading edge, i.e. downstream, the leakage flow is assumed to take up its normal depth at once. Laboratory observations indicate a downstream transition length so that the flow depth immediately ahead of the solid is less than that predicted.

Model 3 - all forces shown in figure 7 included.

### 2.5.2 Equations Governing Flow Past the Moving Solid

It may be assumed that the relationship between specific energy and flow past the solid may be employed in the moving solid case, provided that the absolute fluid velocity employed in equation 35 is replaced by a relative water to solid velocity. The value of  $SE_0$ , the specific energy term to initiate flow past the body, remains unchanged.

Equation 35 is therefore rewritten as

$$Q = K(h + \frac{(V - V_B)^2}{2g} - SE_0)^2 \quad (43)$$

where  $Q$  is the leakage past the moving solid, because

$$Q = V_{re}A = (V_{abs} - V_B)A = (X_2 - X_1 h - V_B)A \quad (44)$$

where  $A$  is the flow area upstream of the solid and  $V_{abs}$  is the fluid velocity expressed by the  $C^+$  characteristic, figure 5.

Appendix 2 presents the full solution to this boundary condition in a general form, applicable to both the initially stationary and moving solid.

The prediction of solid velocity allows the solid path to be drawn in the  $x-t$  plane as shown in figure 9. The  $B^+$  lines drawn in the plane are the equivalent to the fluid characteristics; the gradient of  $B^+$  is hence given by

$$\frac{dx}{dt} = V_B. \quad (45)$$

Figures 10 and 11 illustrate the necessary techniques to allow the solution to proceed with a moving solid. A slightly different solution is required, depending on whether the flow is subcritical or supercritical.

Figure 10 presents the subcritical case. Assume that the solid was at point C at time  $t$  and is predicted to move to  $P'$  by time  $t + \Delta t$ . In order to calculate

the conditions at  $t + \Delta t$ , it is necessary to set up the interpolated base values at R and S; therefore, for conditions immediately upstream and downstream of the solid, a new  $\Delta x$  grid A'P' to P'C' must be set up.

Using the position P' predicted from solid velocity, the points A' and C' are determined. The conditions at R and S are then calculated by interpolation (equations 16-21).

Point R may be used to yield the C<sup>+</sup> characteristic RP' that may be solved with the solid leakage equation 43 to yield depth and flow velocity upstream of the solid.

The conditions on the downstream face of the solid are calculated by a similar technique applied to C'R'. The C<sup>-</sup> characteristic is solved with the flow rate at the solid by the technique utilized at pipe entry in subcritical flow (equations 32-33).

Points W, X and Z, figure 10, may be dealt with by the nodal point equations, 25 and 26, as the necessary interpolations are not affected by the presence of the solid at P'.

Conditions at P and Y, however, cannot be readily calculated due to the B<sup>+</sup> characteristic. Conditions at these points, however are required as base conditions for the next time step. As the  $\Delta x$  and  $\Delta t$  values are small, it is reasonable to determine conditions at P and Y by interpolation between X and P' and P' and Z respectively.

Figure 11 illustrates the solution for the supercritical case. It remains necessary to approximate conditions at P and Y and the conditions downstream of the solid are determined by the use of equation 31 (the normal depth calculation).

## 2.6 SIMULATION CASES COVERED BY THE PROPOSED MODEL

Two types of solid motion must be covered by a model of the type presented. The motion of the solid subsequent to injection into the flow with a downstream velocity must be dealt with, as this represents the introduction of solids with finite velocities in the drain from water closet discharge. Similarly, the motion of a deposited solid in response to a flow must be considered, as this covers the partial system clearance that results in any long drainage pipe exhibiting a series of depositions along its length, each of which is moved on slightly by each incoming flow.

Both cases may be dealt with by the proposed model. The injection case is covered by assuming that, at some instant of time and position, the solid appears in the flow, moving at the local flow velocity that was calculated at that section by the characteristics solution run up to this time with no solid present.

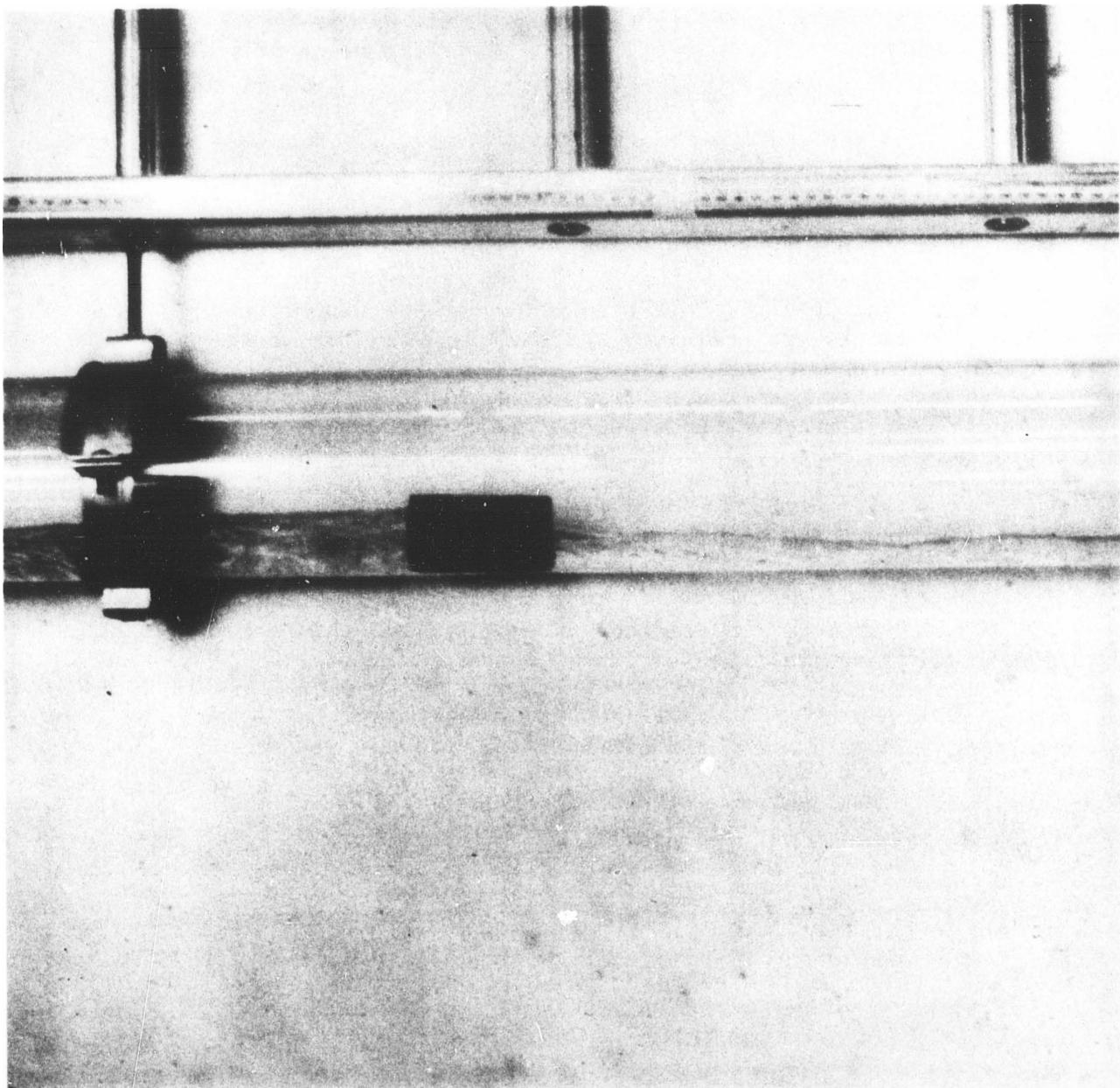
The deposited solid case, at any downstream location, is simply dealt with by the technique described above and more fully in [1]. In this case, the solid is assumed to be present from the first computing time step.

Similarly, the model must be capable of dealing with both subcritical and supercritical flow regimes. The techniques for these cases have been set out; however, care must be taken in the supercritical case.

Figure 12 illustrates the problem encountered in supercritical flow. In the case of a stationary, or even moving solid, the depth increase upstream of the solid may be sufficient to produce a subcritical zone in an otherwise supercritical flow condition. In order for the upstream supercritical flow to become subcritical behind the solid, a depth change wave, or series of depth change waves must be propagated upstream. If the depth change upstream of the solid is rapid, as would happen with a rapidly increasing inflow profile, the depth change waves propagating upstream at the wave speed  $\sqrt{g A/T}$  combine to form a steep fronted wave that moves at a velocity greater than  $\sqrt{g A/T}$ , and hence the solution breaks down.

If the inflow profile is not sufficiently rapid, then the transition from supercritical to subcritical flow may be accommodated by introducing a flow condition check into the calculations at the stage represented by figure 2. If the velocity at a section at time  $t$  is less than the calculated wave speed at that section in an initially supercritical flow condition, then the flow locally is assumed to have become subcritical and the interpolated values, (figure 2) switch from  $R, S'$  to  $R$  and  $S$ .

The speed of propagation of a steep-fronted wave may be calculated and included in the model in a manner similar to that described for the solid motion. Further studies will include this facility. The test cases presented in this report were not affected by this effect, due to the inflow profile shape and the initial position of the solid in the pipe.



### 3. CALCULATION TECHNIQUES AND PRESENTATION OF RESULTS

#### 3.1 DETERMINATION OF NORMAL AND CRITICAL DEPTHS

The bisection method was used to solve the equation defining both critical flow depth

$$X = 1 - Q^2 T / g A^3$$

and normal flow depth

$$Y = S_o - (n Q/A_m^{2/3})^2$$

that feature as boundary conditions.

It may be assumed that both X and Y have zero values for some value of depth h in the range  $0 < h < D$  for the pipe case.

This initial interval,  $0 < h < D$  is bisected and  $h = D/2$  used to evaluate X, Y. If the resulting values are positive, the root is less than the midpoint; then, the upper limit is reset equal to the h value just used and the remaining interval 0 to  $D/2$  bisected. The process is repeated, with the upper limit replaced until the value of X and Y become zero, i.e., the solution desired for the critical and normal flow depths. If the X or Y value had been negative, then the root would be greater than the trial h value; in this case the lower limit would be initially reset to the trial h value,  $D/2 < h < D$ , and the interval bisected to a new value of  $3/4D$ . The process is repeated until a root is obtained.

Due to the need to include the area depth relationship, this solution must be undertaken by an iterative process. The time taken depends on the complexity of the area-depth function.

### 3.2 NUMERICAL INTEGRATION FOR SURFACE PROFILES

The integration of the position vs depth profile

$$L = \int_{h_1}^{h_2} \frac{1 - QT^2/gA^3}{S_o - (nQAm^{2/3})^2} dh$$

is achieved by means of Simpson's Rule. Let the integral  $X = \int_{h_0}^{h_1} F(h) dh$ ; then, if the interval  $h_1 - h_0$  is divided into 2 equal increments, the value of X is given by

$$X = \frac{1}{3} dh [F(h_0) + 4F(h_0 + dh) + F(h_0 + 2 dh)].$$

As the integration moves on, the length traversed may be accumulated as the added interval X with the prior L,  $L = L + X$ , at the completion of each integration.

### 3.3 CHOICE OF TIME STEP

Referring to figure 4, it will be seen that the time step chosen must be such that the points R and S fall within  $\pm \Delta x$  of point P. In order for this to occur for all sections along the pipe, it follows that

$$\Delta t = \frac{\Delta x}{(V + c)_{\max}}. \quad (35)$$

This expression yields the smallest possible time step, as the maximum values of flow velocity and wave speed at any pipe section have been used.

In order to ensure that the computation proceeds as quickly as possible, the computer program presented calculates a new time step magnitude at each time increment so that the time step increases when V and c decrease, but decreases to maintain a stable solution when V and c are increasing in response to an inflow surge.

### 3.4 PRESENTATION OF RESULTS

The objectives of the numerical method for computation of the transport mechanisms of solids impartially filled pipe flow were to: (a) identify the potential application of coupling the method of characteristics solution for the hydraulic phenomena with the solution of the equation of motion for the solid based upon modeling the liquid/solid interface forces; and (b) highlight any limitations inherent to the technique. The simulated pipe flow/solids motion numerical data were developed from use of the Fortran computer program TRANSCC, run on the NBS CBT Perkin Elmer 732 computer.

The following test cases were investigated and are reported here

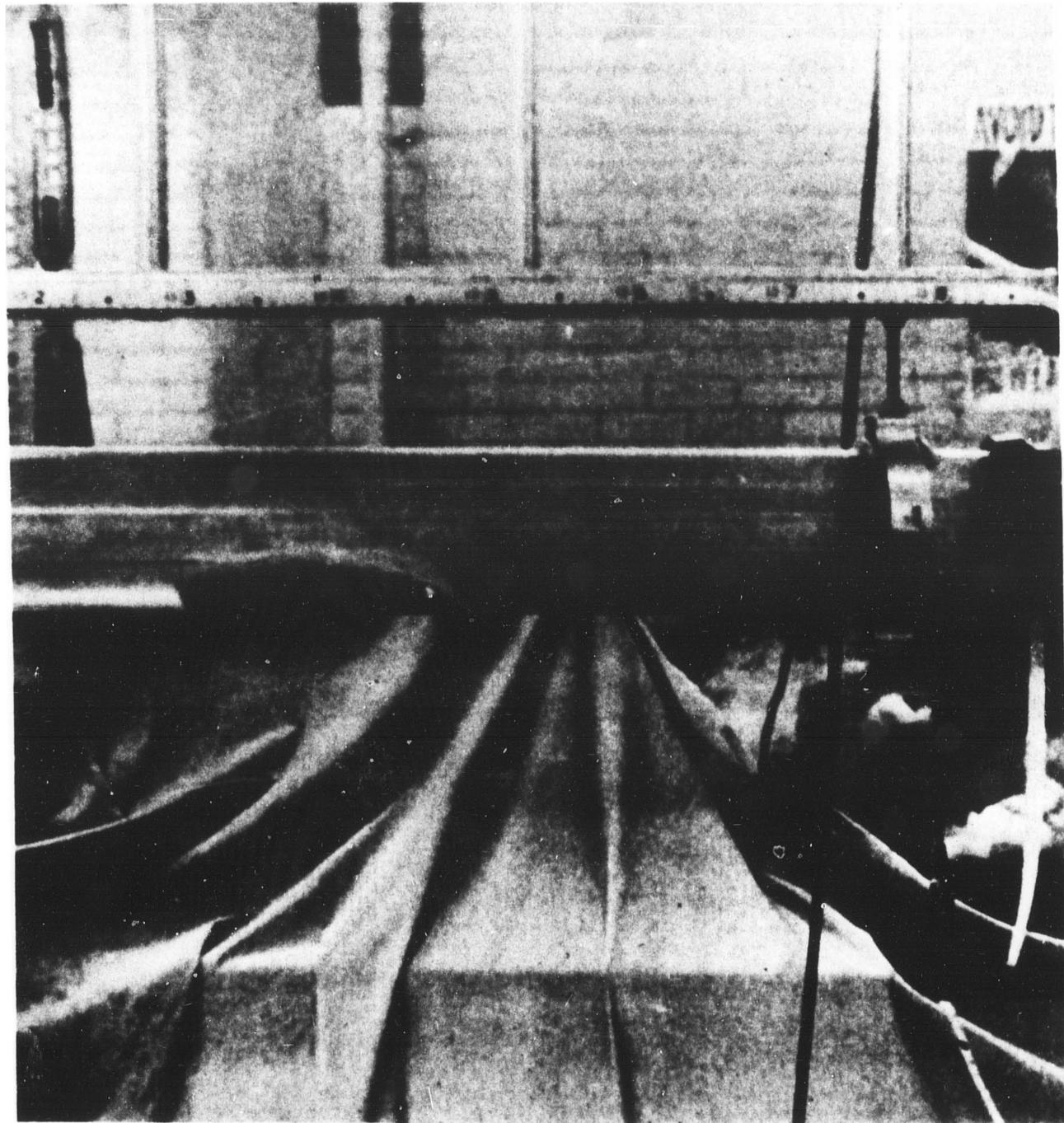
- (1) Solid types -- two cases represented by figure 4, 270 mm x 20 mm x 70 mm and 270 mm x 20 mm x 35 mm with saturated weights of 250 g and 125 g
- (2) Pipe gradients -- 1/40 to 1/300
- (3) Inflow profile -- constant profile employed with peak flow of 4.2 l/s, overall duration 9 seconds
- (4) Pipe roughness coefficient -- constant at 0.015
- (5) Solid friction factors -- 0.10 sliding and 0.15 static
- (6) Solid position in flush, 3 s and 4 s from flow initiation in the moving solid insert case, at 2.7 m from entry
- (7) Solid position -- 4.2 m from entry for deposited solid
- (8) Force models -- three models as illustrated in Figure 8.

### 3.5 CHOICE OF CALCULATION CONSTANTS

In order to undertake the calculation procedure described, it is necessary to have values of leakage constants and initial specific energy for the solid. In the simulations presented, the values of these constants are drawn from the

results illustrated in figure 4, referring to transport tests at Brunel using large deformable solids.

The surface to pipe wall friction factors are also required. No values are at present available for the deformable solids tested; however, similar tests conducted in the Plumbing Research Laboratory at the National Bureau of Standards utilizing impermeable cylindrical solids suggest values for friction factors in the range 0.6-0.8 for the sliding and static cases. It is likely that the deformable solids will have lower values, due to the presence of the material saturating water that will tend to provide a degree of lubrication. All the simulations presented were carried out with values of 0.1 sliding friction factor and 0.15 static friction coefficient.



#### 4. DISCUSSION OF UNSTEADY FLOW SIMULATION RESULTS

As outlined in the introduction, the objective of the study was the development of a technique, based on the method of characteristics, that would allow the transport of discrete solids in partially filled pipe flow to be modeled mathematically.

It was recognized that insufficient data on the frictional characteristics of model solids used in laboratory test programs at Brunel and NBS were available to attempt to predict actual solid transport velocity profiles. However, the

models of the forces acting on the solid were based on current research at Brunel and NBS, and the values of friction factors assumed were, as already discussed, based on the best available information.

It is more important in this effort to demonstrate the ability of the analysis techniques to yield the solid velocity and the associated flow depth and velocities during the transport along the pipe. Subsequently, modifications based upon improved force models, friction factors derived from tests and other empirical adjustments, can be introduced to fit predicted data to experimental results.

From the examples computed for the conditions of 3.4, it is worth noting the form of the relationships of solid velocity to pipe length and gradients. Figure 13 represents typical empirical results for the single maternity pad solid tested at Brunel. This solid has dimensions 270 mm x 20 mm x 70 mm, a saturated weight of 250 g, and a specific energy to leakage flow relationship in a 100 mm pipe as shown in figure 4. This solid, with a half width version, was used as a basis for the computer simulations presented.

It will be seen from figure 13 that the velocity of the solid over the major length of the drain is governed by the  $\sqrt{L/G}$  term; i.e., the square root of distance travelled divided by pipe gradient. This effect will be studied by plotting the computed velocity results against this term. Wherever possible, simulations are presented at pipe slopes of 1/40 and 1/100 in order to represent both supercritical and subcritical flow regimes. In addition, these gradients represent the common spread of slopes employed in drainage system design.

It should also be noted that the water depth and velocity results presented are immediately upstream and downstream of the solid, and thus refer to a location that moves down the pipe at the solid velocity.

#### 4.1 INFLUENCE OF BODY FORCE MODEL ON PREDICTED SOLID VELOCITY

Figure 14 illustrates the predicted solid velocity for the three force models presented in figure 8. It was found from the values of force predicted by these models that, for the range of model sizes simulated, the downthrust on the leading surface of the solid materially increased the surface frictional force. This is represented in figure 14 by the observation that at both 1/40 and 1/100 gradients, the model 3 results yielded the lowest solid velocity. As expected, therefore, the exclusion of the downthrust force,  $F_D$ , (figure 8) yielded the highest solid velocity. The omission of  $F_D$  is reasonable on the basis of laboratory observations. The water depth tends to require a transition length downstream of the solid to achieve the normal depth appropriate to the flow past the solid. Hence, the water depth predicted by the analysis, the effective normal depth, will be an overestimate of the depth of this location.

Figure 14 is also based on the assumption that the solid appears in the flow at a particular  $x,t$  coordinate traveling at local water speed. This is a

reasonable assumption that leads inevitably to a rapid deceleration of the solid accompanied by a buildup of water depth immediately upstream of the solid. The adjustment to a new water-driven velocity is rapid, and any non-realistic effects due to the insertion model are likely to decay rapidly.

All force models were used in the collection of the data presented; each figure carries a note as to the applicable model.

#### 4.2 INFLUENCE OF SOLID DIMENSIONS AND POSITION IN THE INFLOW PROFILE ON PREDICTED SOLID VELOCITIES

Laboratory experiments and data analysis presented in [4] have shown that the volume available behind a solid in the flush significantly determines its transport properties.

Referring to figure 15, it will be seen that introducing the solid at 2.7 m from entry at 4 seconds into the flush, as opposed to 3 seconds, does support these observations. Although the predicted differences are small, they are consistent.

Similarly, laboratory tests have shown that base area, defined in the case of a rectangular model solid (figure 6), as thickness,  $t$ , times width,  $w$ , is a major factor in determining transport performance. Small base area solids (e.g., tampons or sheets of toilet tissue) travel badly when compared to larger solids. This is due to the increased flow past the solid that reduces the depth buildup behind the solid, and from equation 37, leads to smaller body forces. Figure 16 clearly demonstrates this effect for both the subcritical flow at 1/100 and the supercritical flow at 1/40.

#### 4.3 COMPARISON OF SOLID MOTION FROM BOTH INITIAL DEPOSITION AND INSERTION AT WATER VELOCITY MODELS

Figures 17 and 18, and 19 and 20 compare (for slopes of 1/40 and 1/100) the flow, solid velocity, and depth changes for a solid accelerated from rest to that for a solid assumed to enter the flow at local water speed.

A number of general observations may be made from these results that find confirmation in previous observations of laboratory tests:

- (1) At both pipe gradients, the maximum depth upstream of the solid occurs in the acceleration from rest case.
- (2) Perhaps more surprisingly, the maximum depth occurs in both test cases in the steeper pipe. This is a direct result of the application of equation 36.

In the supercritical flow at slope 1/40, the velocity term is greater than at 1/100, and consequently, the flow depth is less. Thus the "destruction" of the flow momentum by the partial blockage formed by the solid results

in a greater depth change in the steeper pipe. Similarly, this enhanced depth change leads to a higher force acting on the solid and results in the earlier motion of the solid in the 1/40 pipe case. Figure 17 indicates solid motion at 1.4 s for 1/40 and 4.2 s at 1/100 pipe gradients.

- (3) Both the depth upstream of the solid as it moves along the pipe and the associated solid and water velocities display oscillations. This is entirely attributable to the choice of time step, and the fact that the simulation equations 40, 41 are only first approximations; no return iterative technique has been introduced. Subsequent time steps may then underestimate the solid velocity. These oscillations are reflected in the water depth upstream of the solid. This link between upstream depth and solid velocity is clearly demonstrated in figure 18 by the sharp dip motion at  $t = 1.4$  seconds. All simulations involved a grid length of 15/50 m, time step dependent on wavespeed.

#### 4.4 COMPARISON OF PREDICTED SOLID VELOCITIES TO EMPIRICAL RELATIONSHIPS

Figure 13 indicated that solid velocity is dependent on the experimentally derived group  $\sqrt{L/G}$ ,

$$V_B = C_1 - C_2 \sqrt{L/G},$$

where  $C_1$  and  $C_2$  are experimentally measured.

Figures 21 and 22 present simulated solid velocity results plotted against the  $\sqrt{L/G}$  term for two force models and pipe slopes from 1/40 to 1/300.

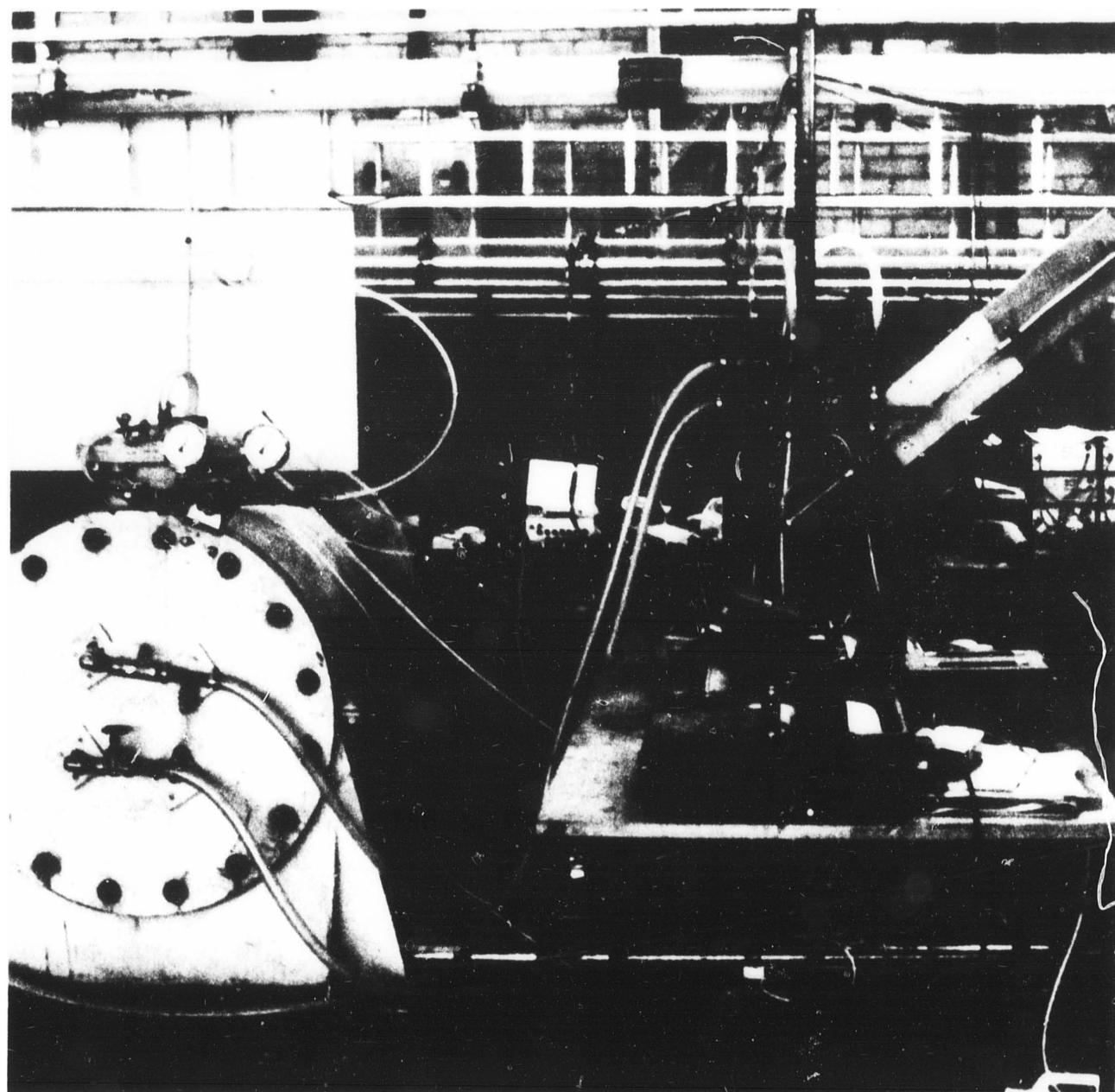
The results indicate that the solid velocity in each case is linearly dependent on  $\sqrt{L}$  over the major portion of the pipe length. The dependence on  $G^{-1/2}$  is not clearly defined by the results; however, it is clear that a pipe gradient term is present and would have an index greater than 0.5. Alternatively, and more probably, the deviation is due to factors not yet included in the force models, for example, the surface to water shear force arising from the water flow over the solid has not been included, as the surface shear stress is unknown. However, the results are encouraging in that the general form of the predicted solid velocity curves approximates the observed laboratory results. This also indicates that the force models employed were reasonable, as well as the values of surface to wall friction coefficients assumed.

The forces acting on the solid have not been presented in graphical form, as they were generally found to remain roughly constant during any simulation, although exhibiting the oscillations mentioned previously. Typical values for the zero downthrust model, figure 14, at 1/40 were -0.2 N during the initial deceleration, falling to -0.01 N during the subsequent motion. At 1/100 the comparable values were in the range -0.2 N initially, falling to around -0.02 N.

#### **4.5 LIMITATIONS TO THE SIMULATION**

As fully discussed in [2], the method of characteristics solution requires an initial flow in the pipe that continues beyond the termination of any inflow profile. This effectively means that a simulated solid will achieve a terminal velocity in the pipe, or alternatively, will be deposited and moved on continuously as the residual flow acts to increase upstream depth. This effect is not readily apparent from the results presented due to both the relatively short pipe length, 15 m, and curtailed run duration of 30 seconds. For the purpose of investigating the force model to be used, this limitation is not major and, indeed, could be duplicated in any parallel experimental work.





## 5. CONCLUSIONS AND FURTHER WORK

The objective of this study was the evaluation of the coupling of the method of characteristics with the equation of motion of a solid based upon an assumed force model to provide a numerical analysis base for modelling the transport of discrete solids in partially filled pipe flow.

The results presented show that the method is applicable, and that the motion of the solid may be satisfactorily tracked through an x-t grid representing the

pipe by introducing a solid characteristic whose slope in the plane is governed by the forces acting on the solid.

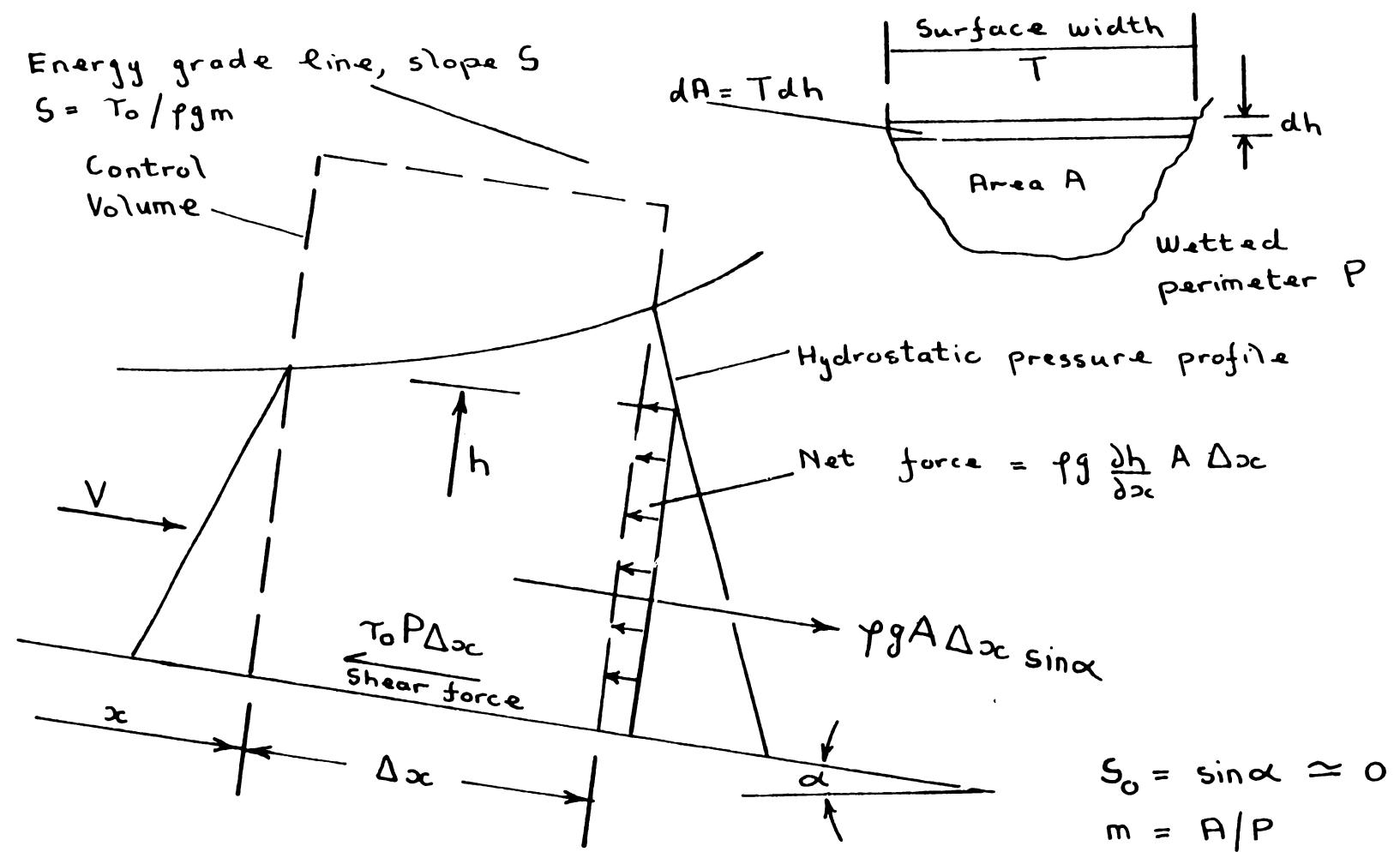
A wide range of simulations in both subcritical and supercritical flow regimes yielded solid velocity results that were compatible with laboratory observations.

Further work is required to establish the true values of the solid sliding friction coefficient, with further study of the force models to be used as the moving solid boundary condition. Similarly, the specific energy vs. relative flow rate over the solid requires further investigation to extend the range of solid geometry available at present.

## **6. REFERENCES**

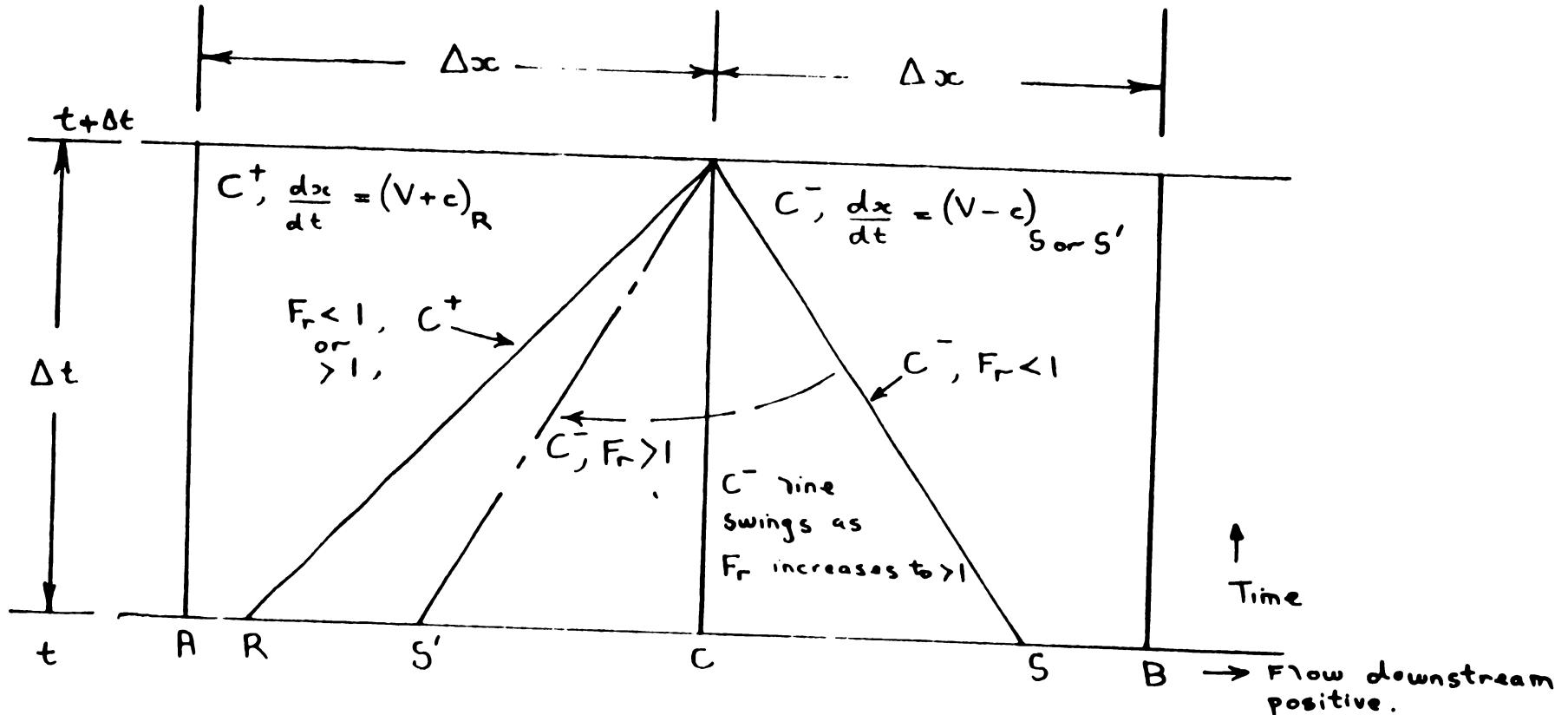
- [1] Swaffield, J. A. "An Initial Study of the Application of the Method of Characteristis to Unsteady Flow Analysis in Partially Filled Pipe Flow." NBS Report August 1980.
- [2] Swaffield, J. A. "Application of the Method of Characteristis to Predict Attenuation in Unsteady Partially Filled Pipe Flow." NBS Report October 1980.
- [3] Fox, J. A. "Hydraulic Analysis of Unsteady Flow in Pipe Networks." Macmillan Press, London 1977.
- [4] Swaffield, J. A. "Dependence of Model Waste Solid Transport Characterstics in Drainage Systems on Solid Geometry, Mass and Pipe System Parameters." NBS Report August 1980.





Note 2nd order terms neglected

Figure 1. Application of momentum equation to unsteady flow in a general open channel



$F_r = \text{Froude N}^{\circ}$        $< 1$  subcritical flow, S lies in downstream section CB  
 $> 1$  supercritical flow, S' lies in upstream section AC

Figure 2. Modification to specified time interval grid for subcritical and supercritical flow regimes

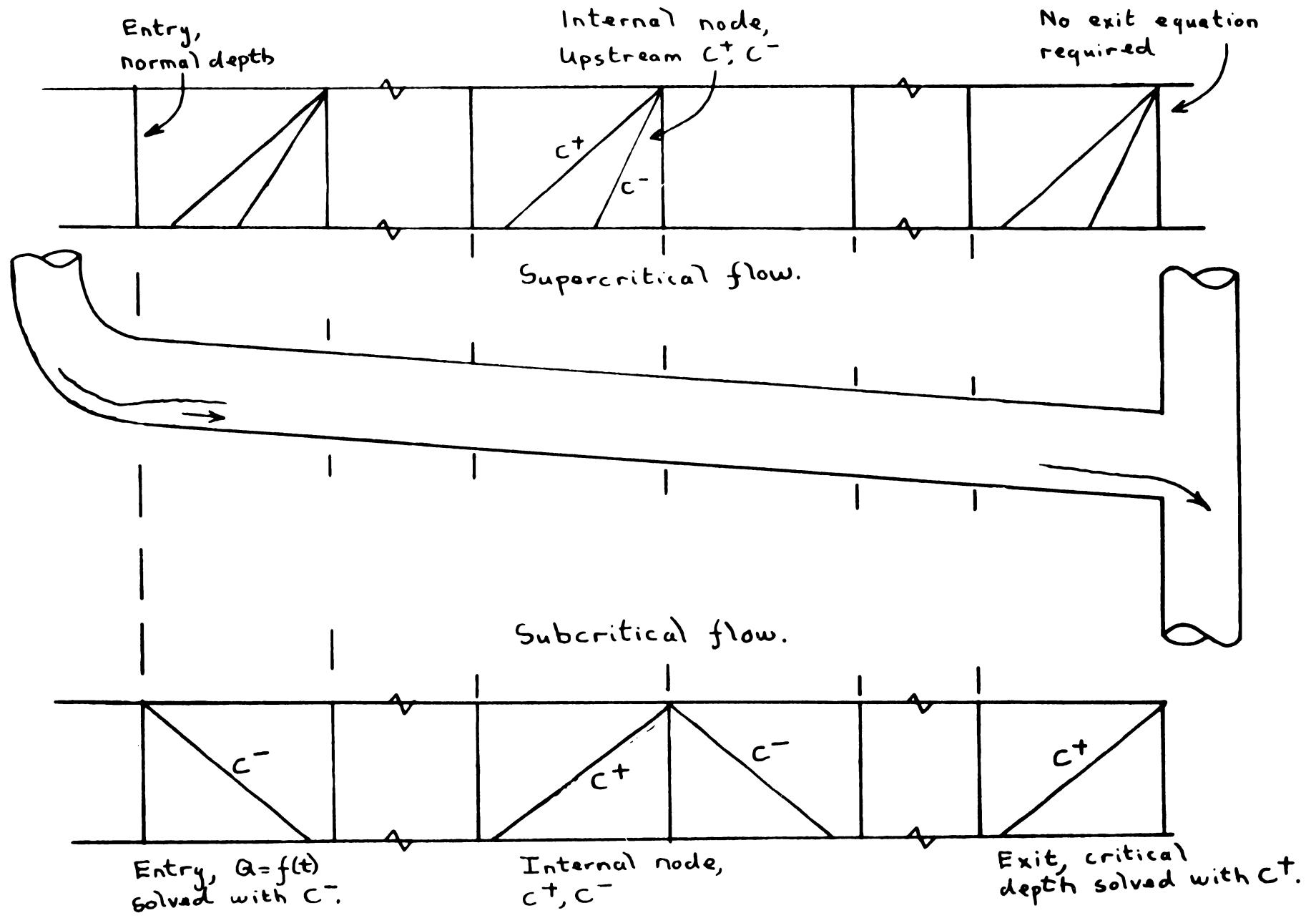


Figure 3. Solution at pipe entry, exit and internal nodes for subcritical and supercritical flow regimes

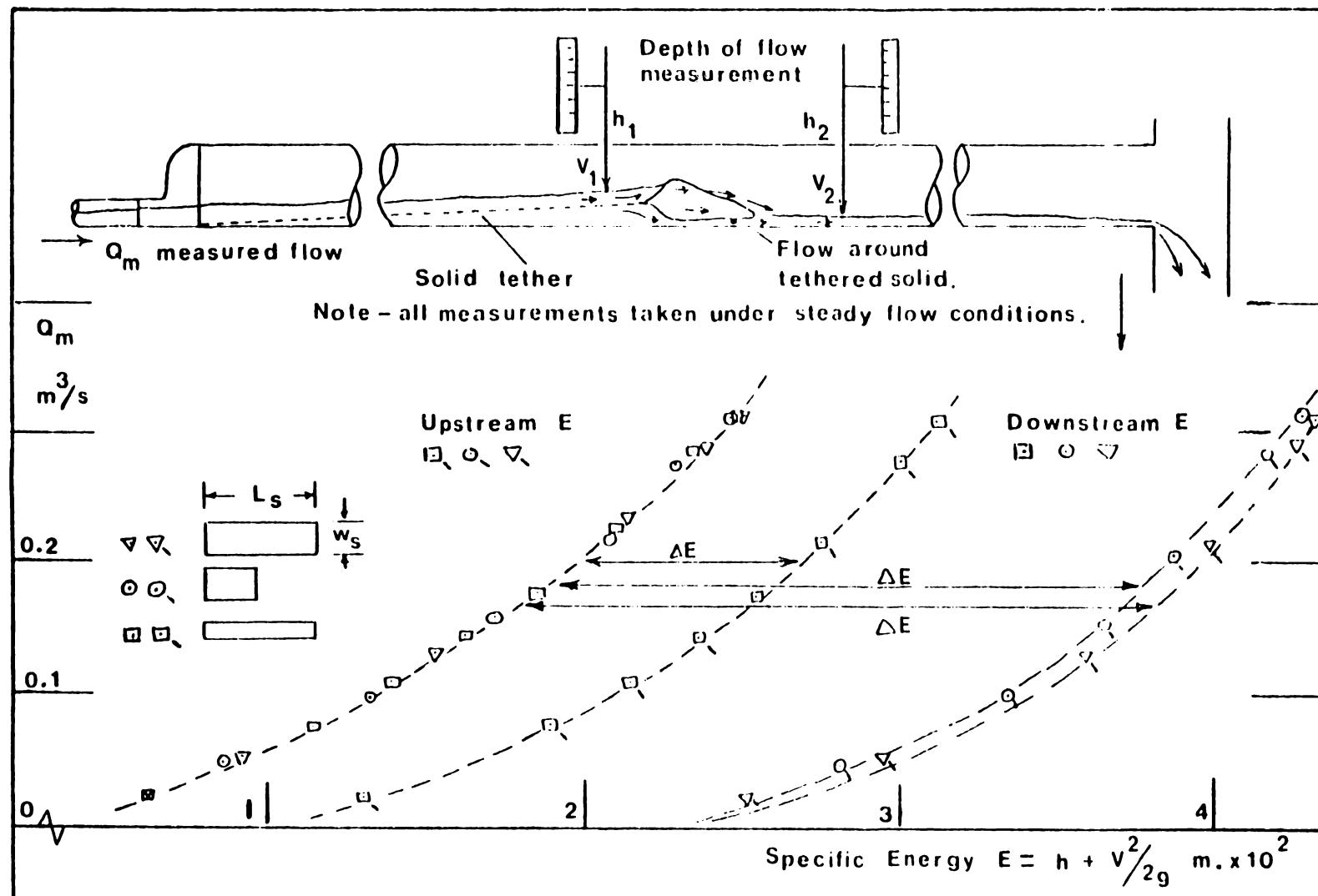


Figure 4. Measurement of the flow specific energy change across a solid tethered in the waste pipe under steady flow conditions

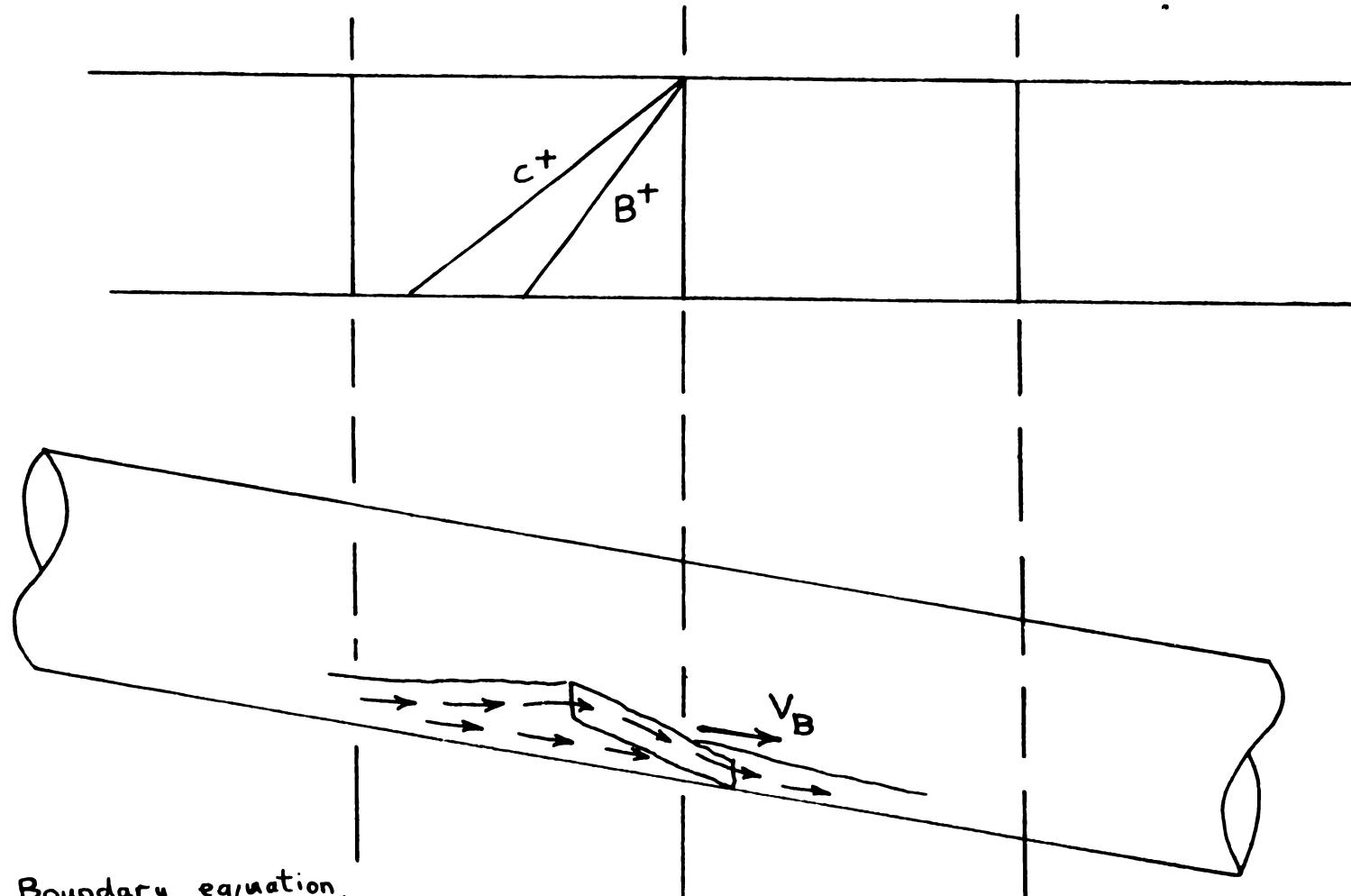


Figure 5. Solution of  $C^+$  characteristic with body equivalent  $B^+$  characteristic

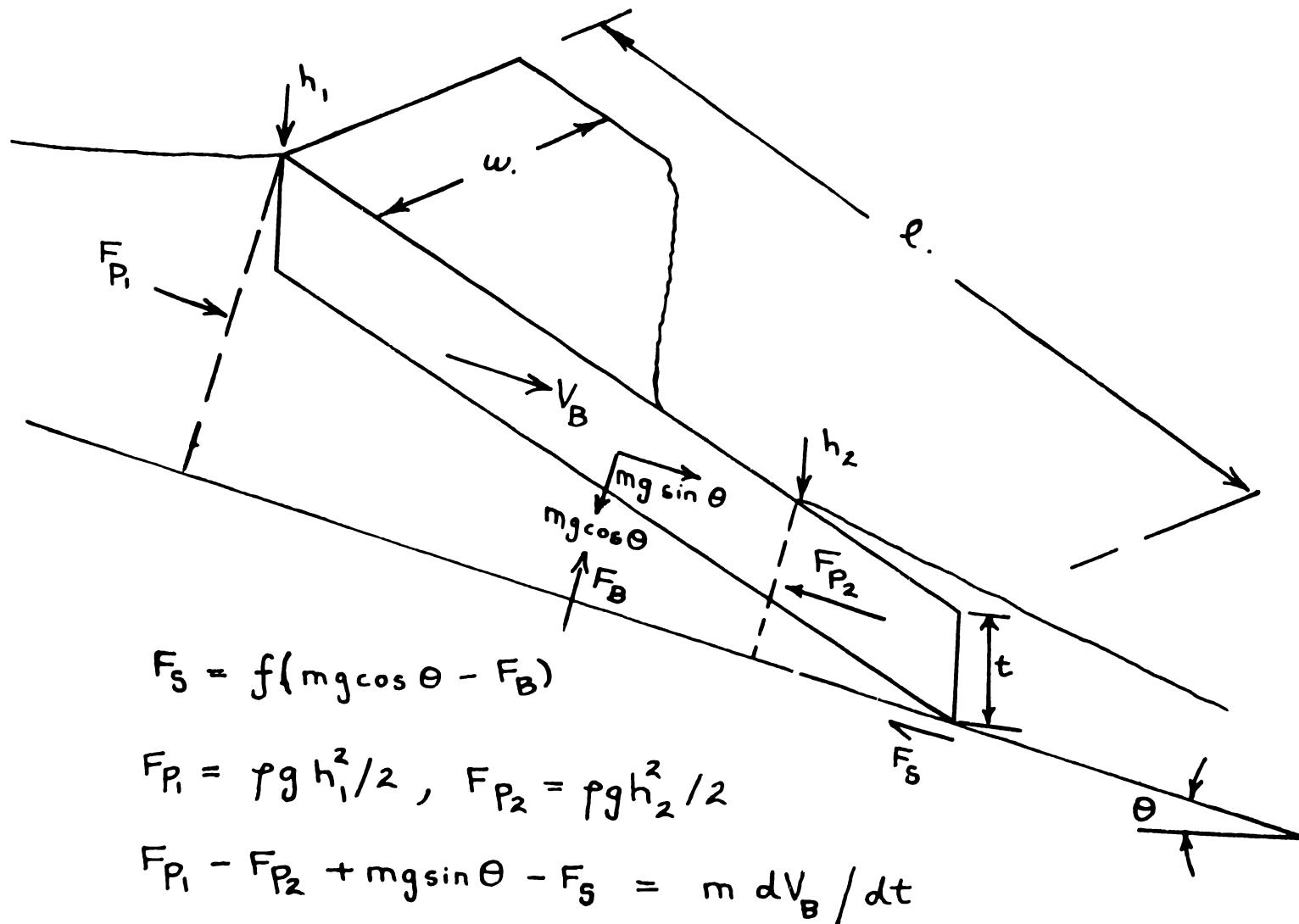
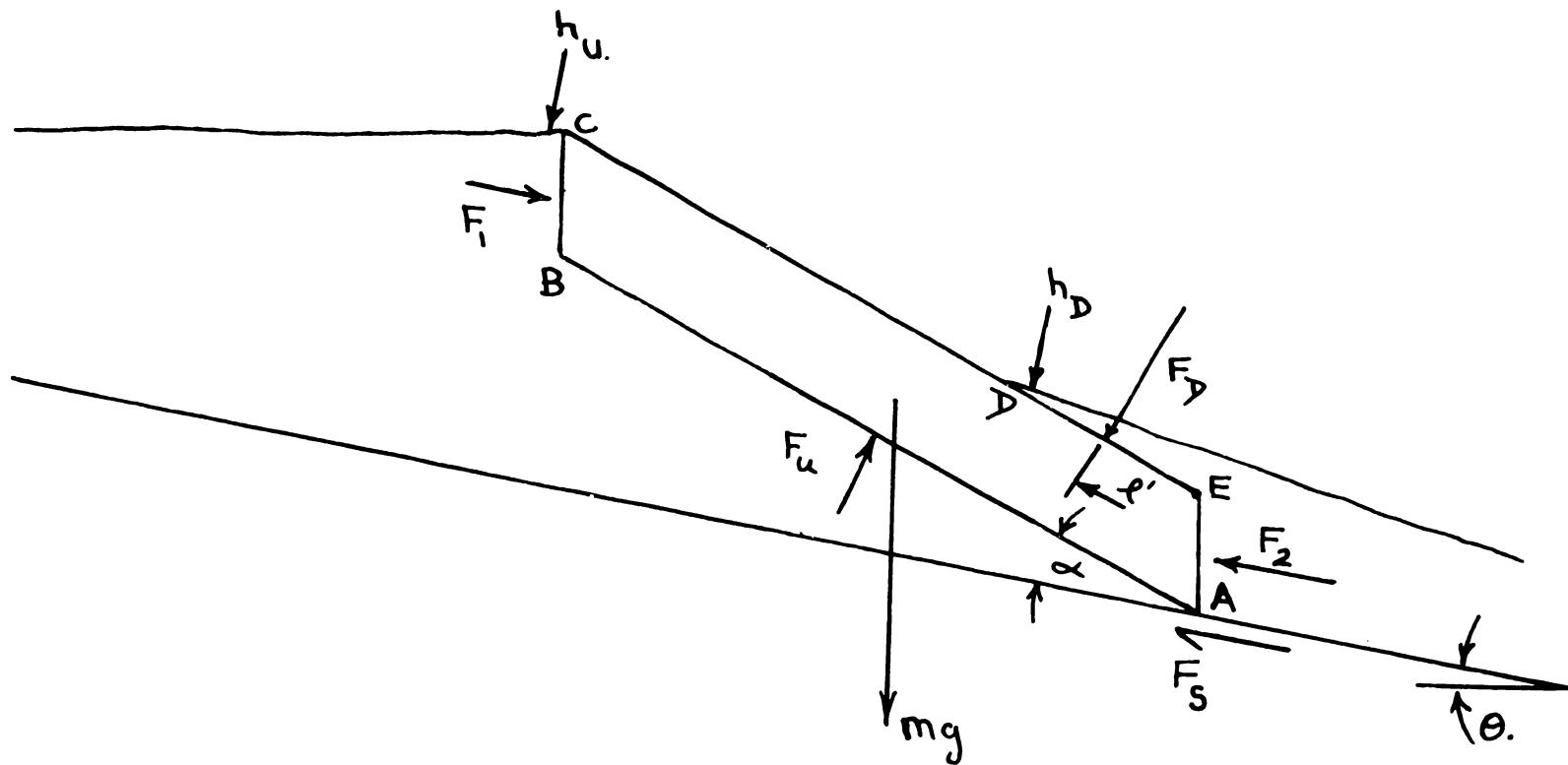


Figure 6. Equation of motion applied to a rectangular section solid



$$F_1(h_u - t/2) + F_u \cdot \ell/2 - mg(\cos\theta)\ell/2 - F_D \cdot \ell' - F_2 t/2 = 0$$

(Equation for small  $\alpha$ )

Figure 7. Calculation of buoyancy forces

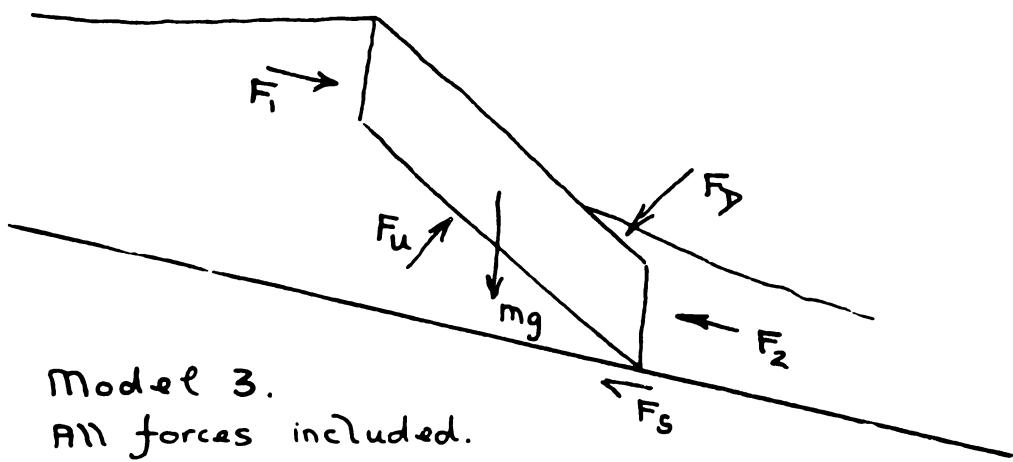
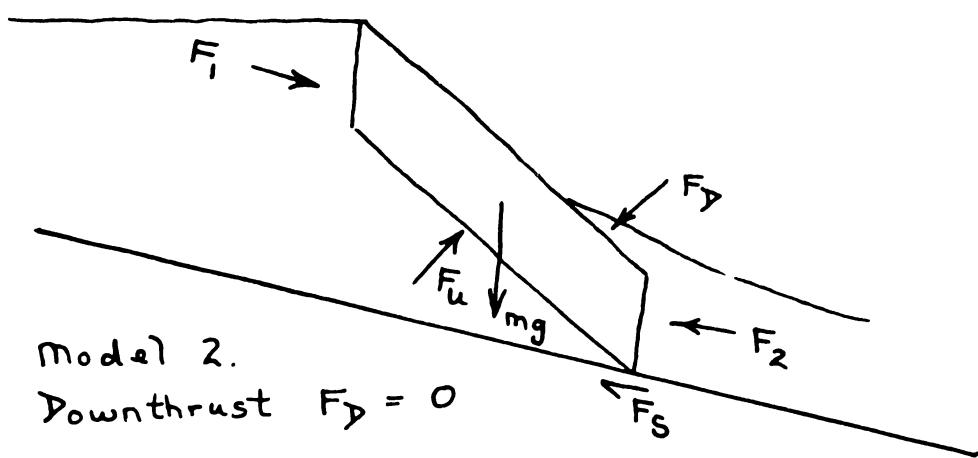
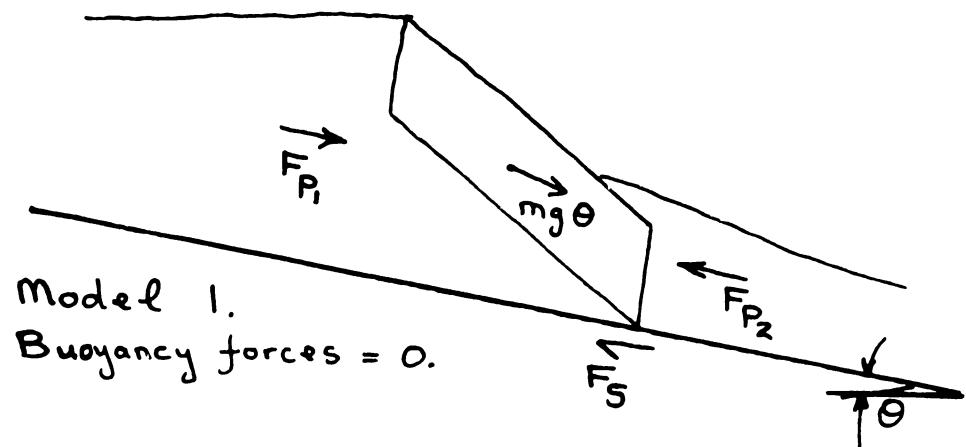


Figure 8. Summary of alternative force models

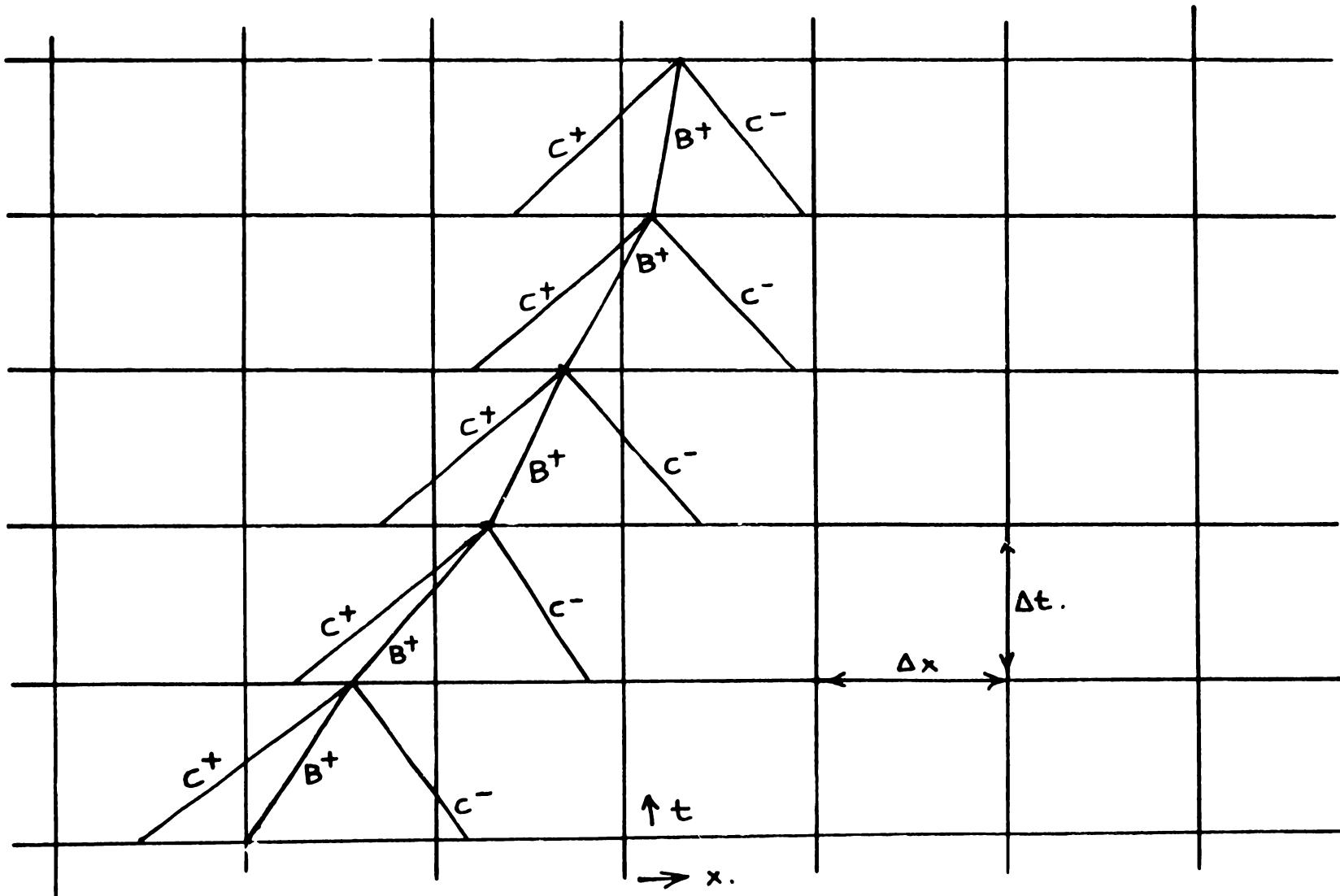
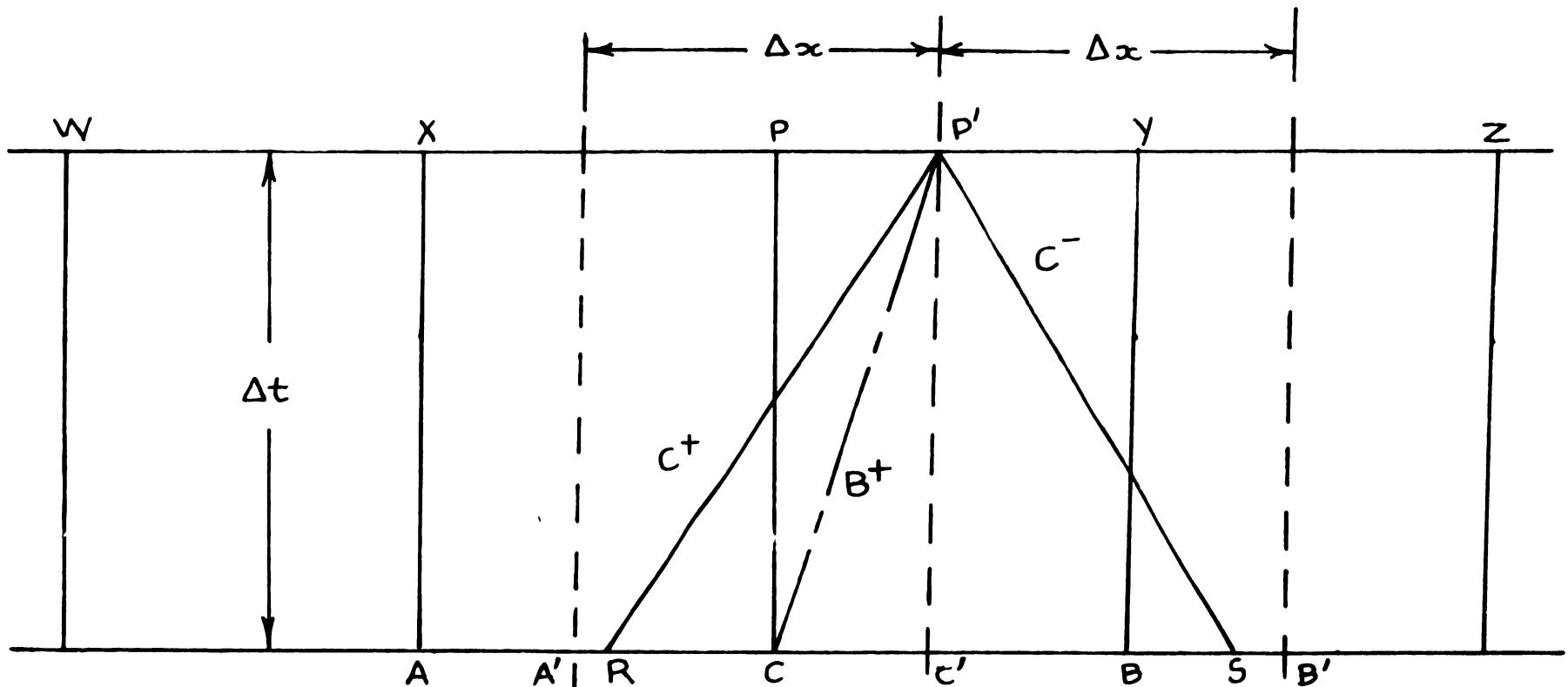
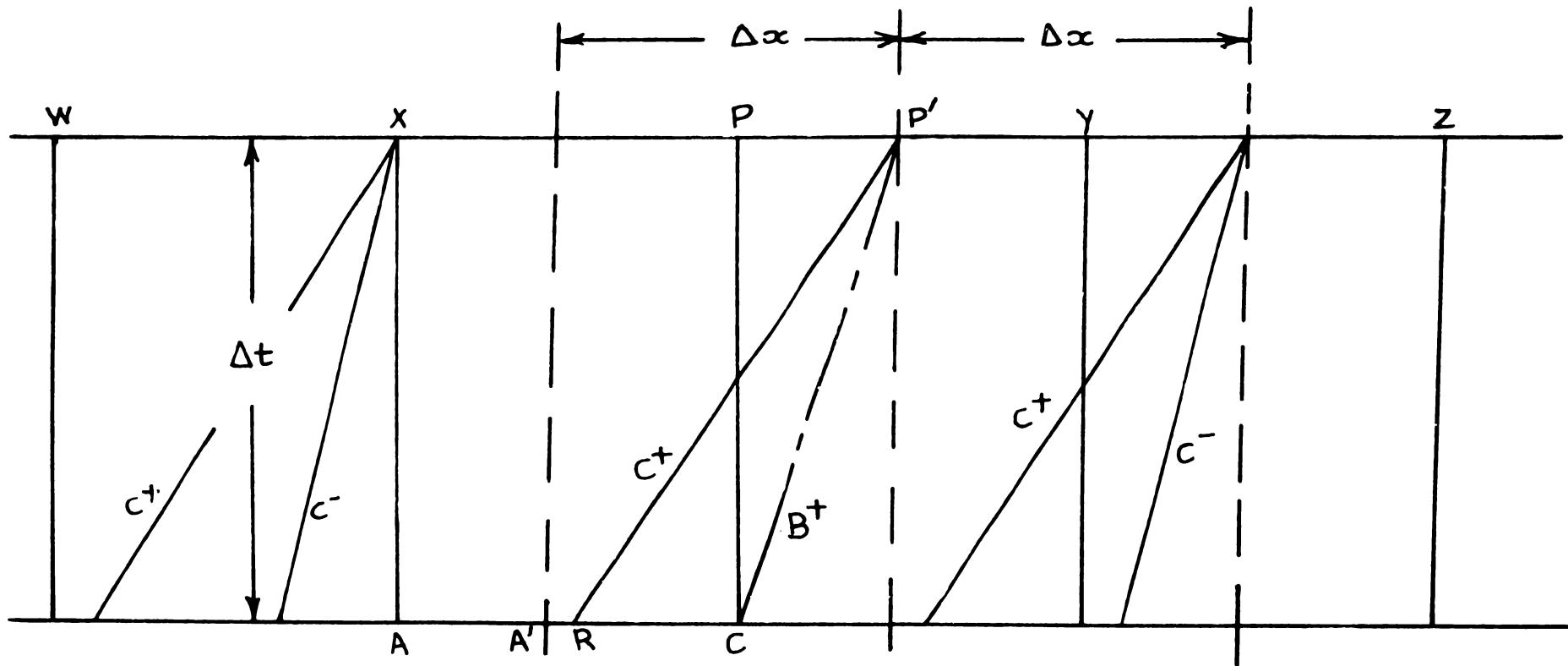


Figure 9. Schematic representation of solid motion in the  $xt$  plane



$$c^+ \frac{dx}{dt} = (V + c)_R, \quad c^- \frac{dx}{dt} = (V - c)_S \\ B^+ \frac{dx}{dt} = V_{BODY}.$$

Figure 10. Modifications to xt grid adjacent to the solid position for subcritical flow



$$c^+ \frac{dx}{dt} = (V + c)_R, \quad c^- \frac{dx}{dt} = (V - c)_{S'}$$

$$B^+ \frac{dx}{dt} = V_{\text{Body}}$$

Figure 11. Modifications to xt grid adjacent to the solid position for supercritical flow

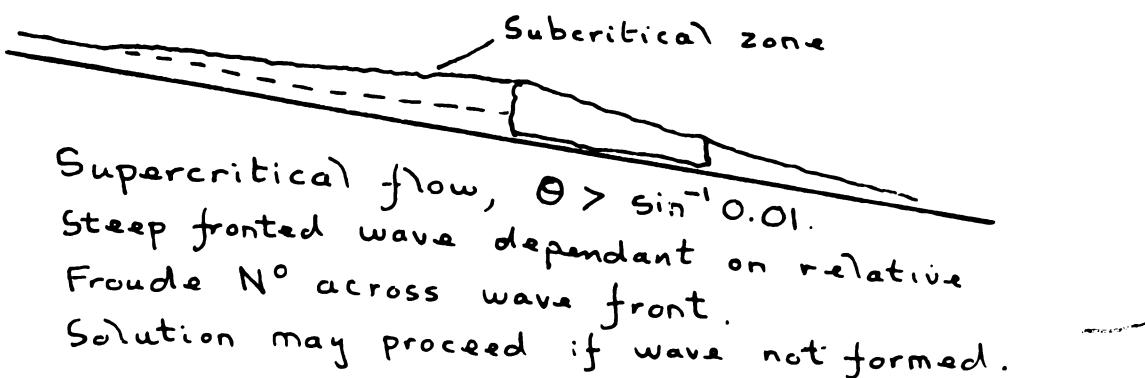
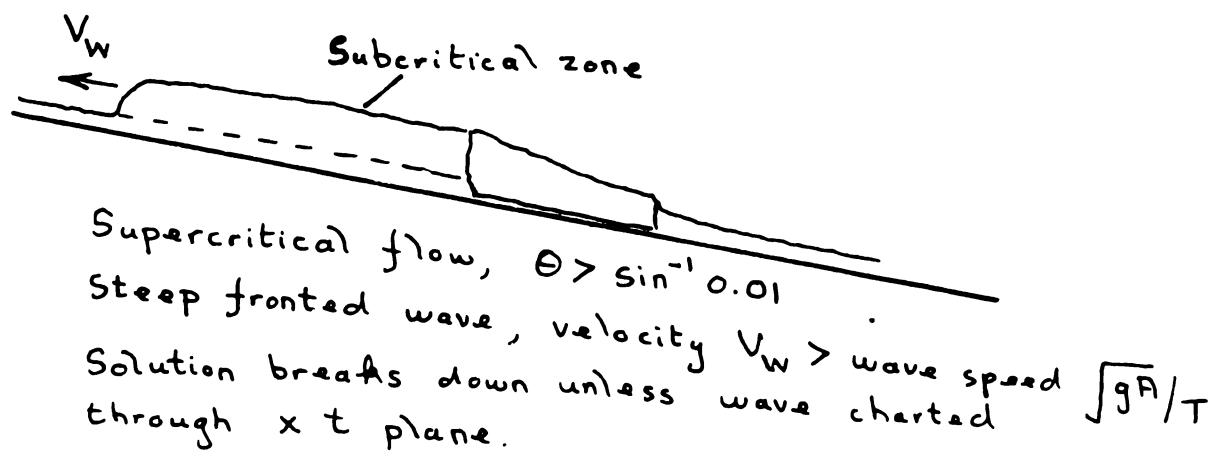
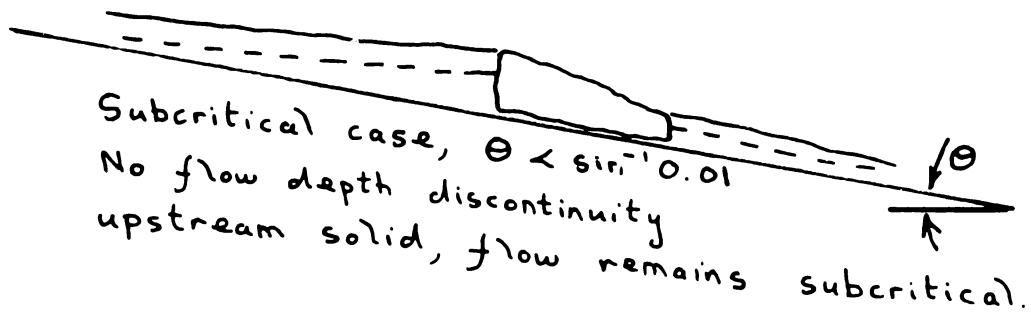


Figure 12. Propagation of steep fronted wave upstream from the solid position

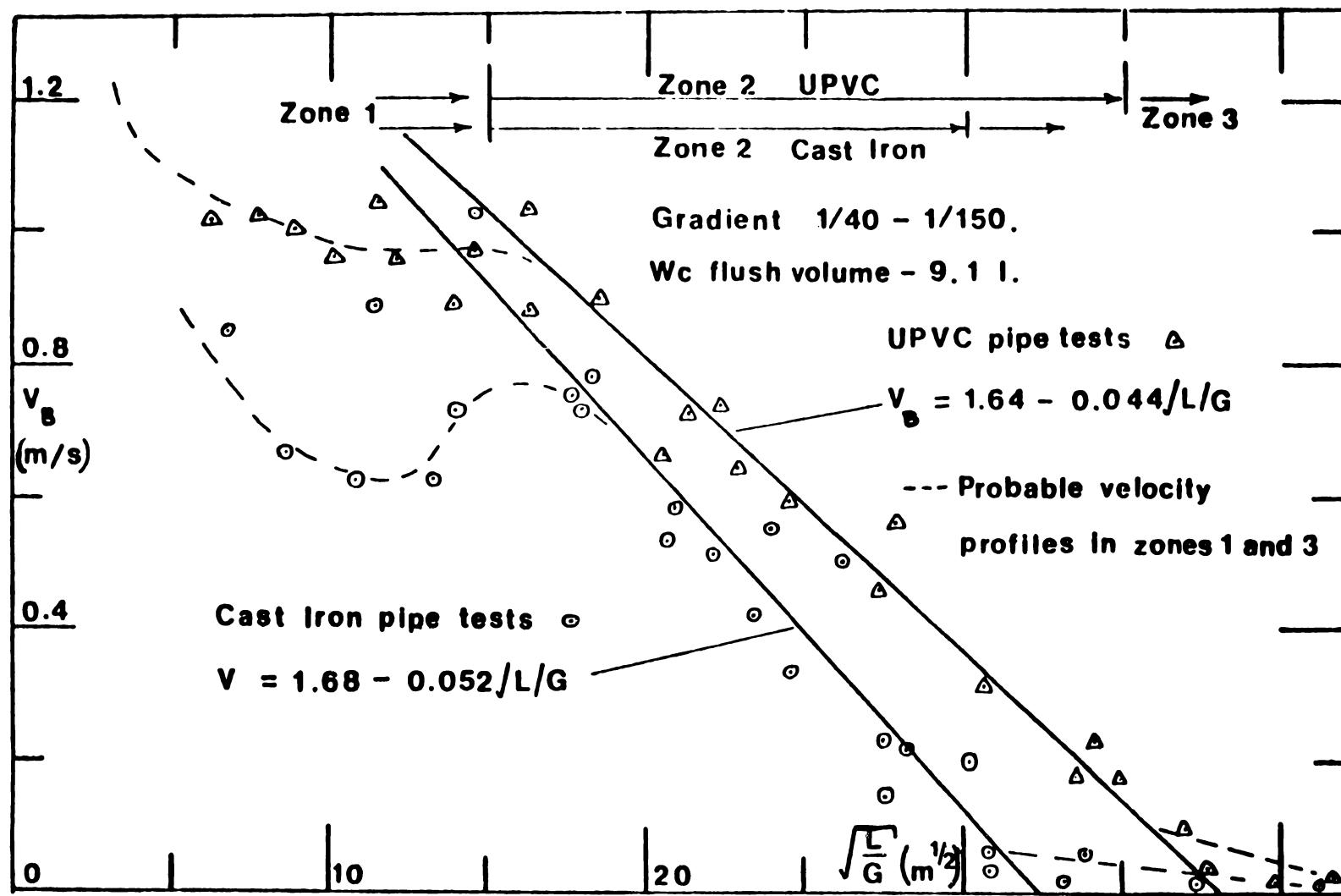


Figure 13. Comparison of maternity pad transport in UPVC or cast iron 100 mm diameter pipe

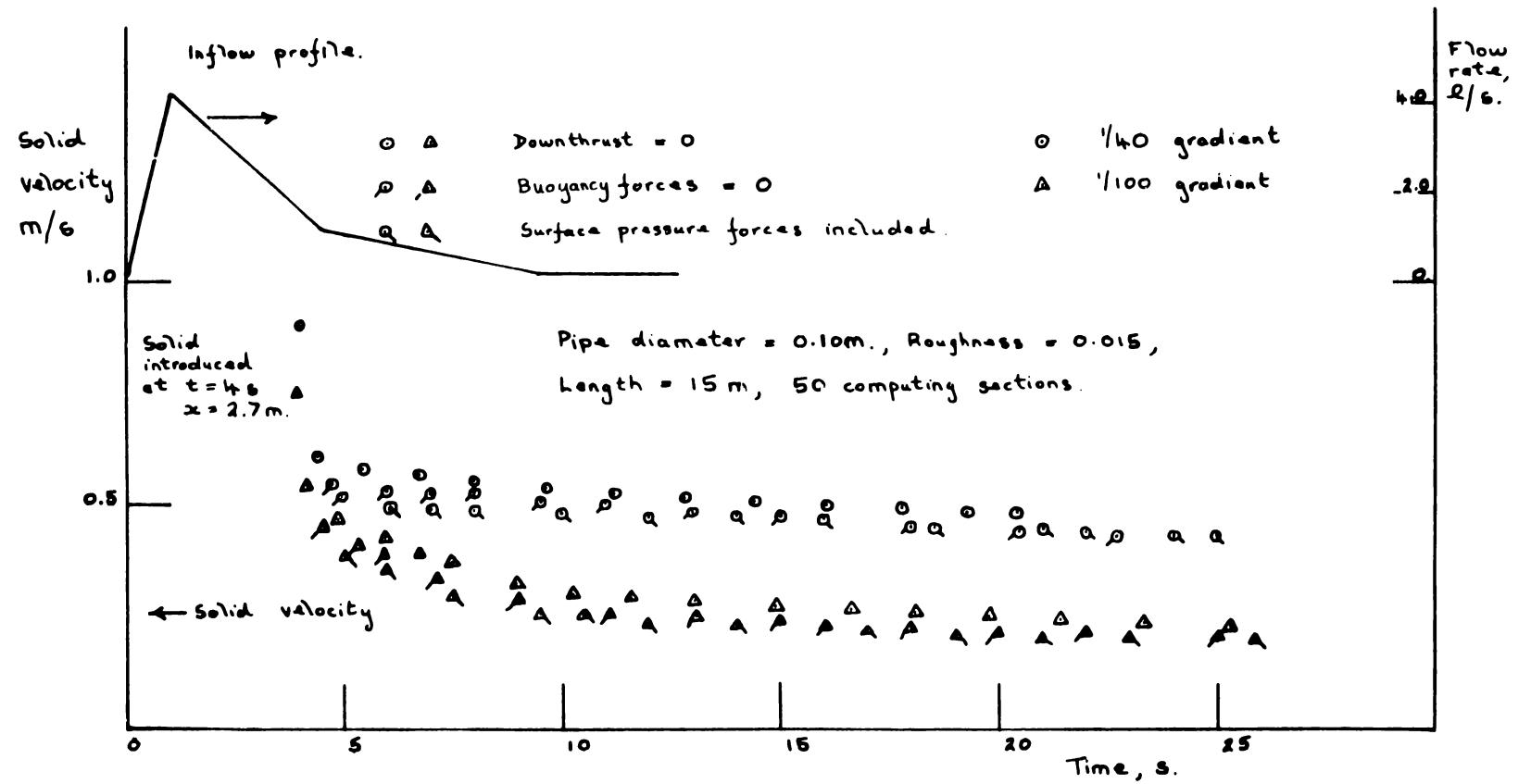


Figure 14. Influence of force model on the predicted solid velocity

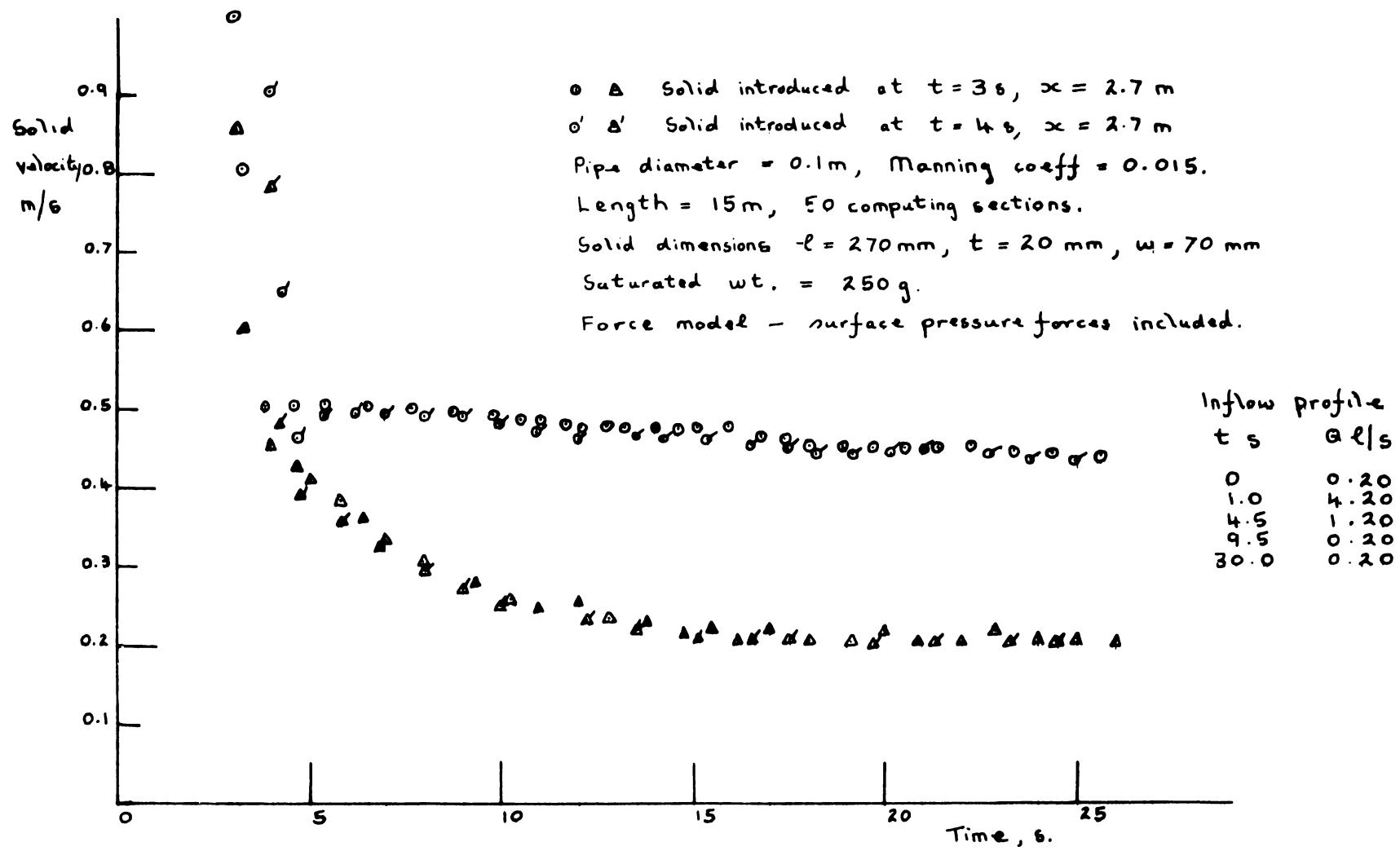


Figure 15. Influence of solid position in the inflow profile on predicted solid velocity

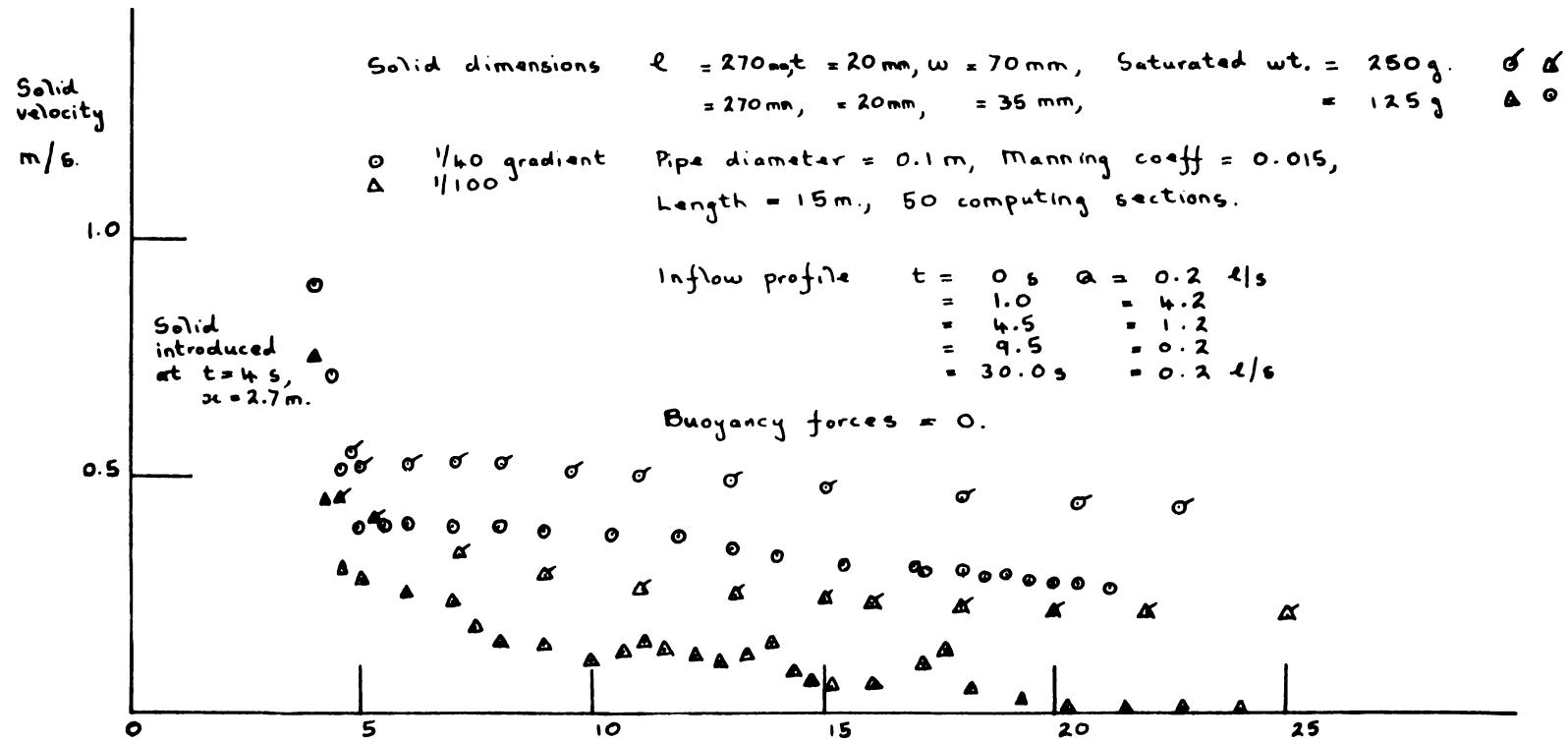


Figure 16. Influence of solid size on predicted velocity

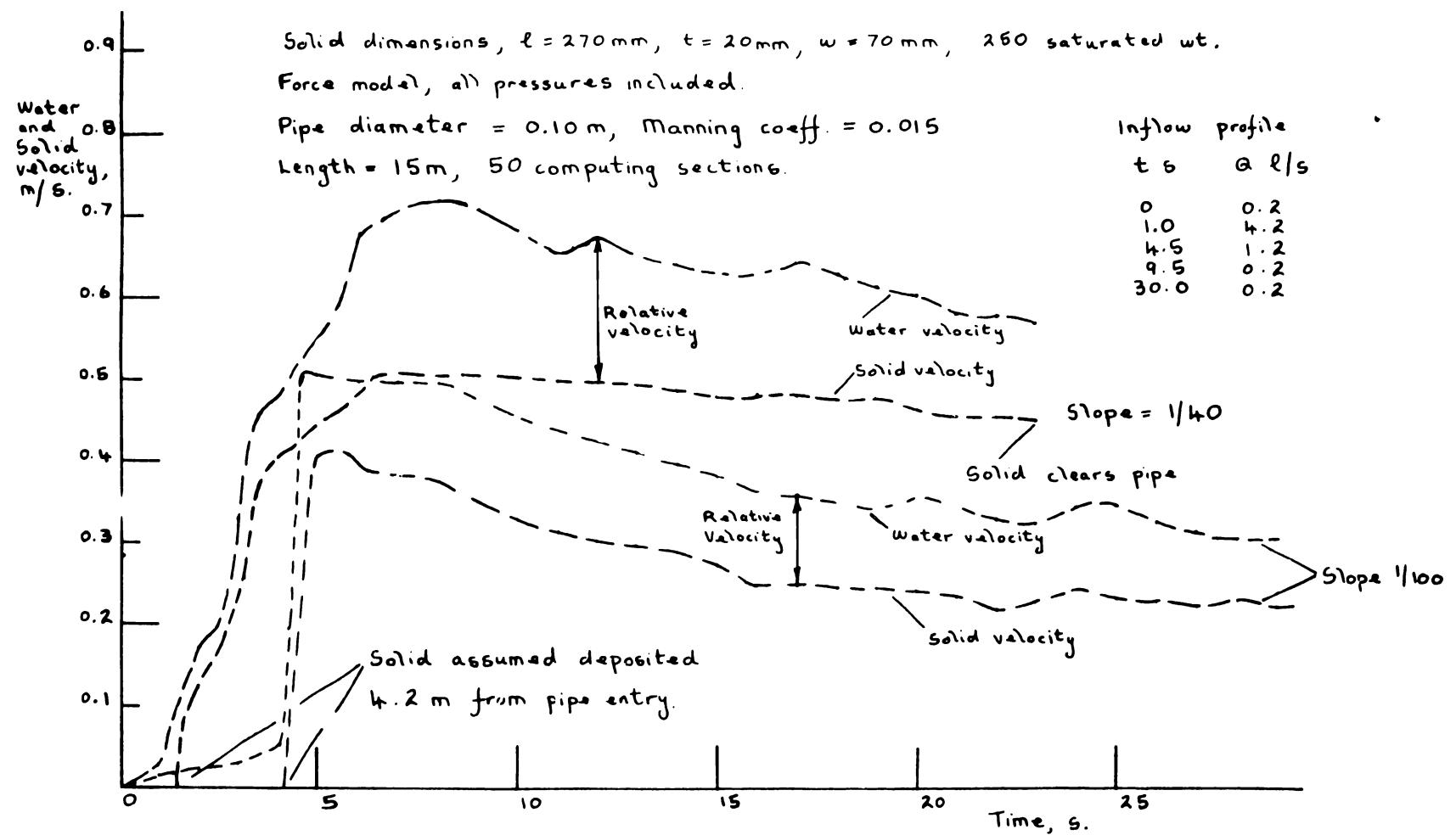


Figure 17. Initially stationary solid, predicted water and solid velocities illustrating relative flow over the body

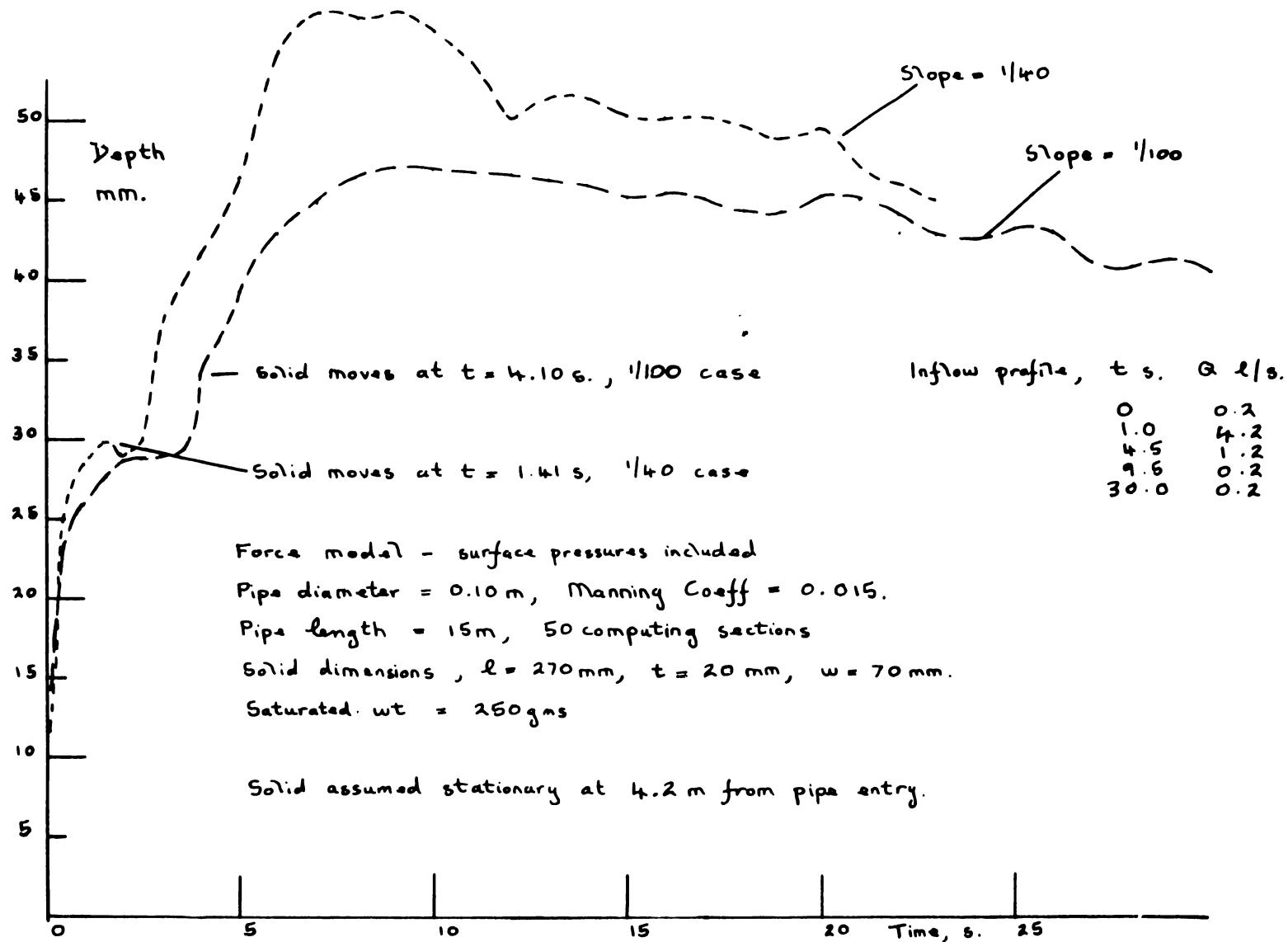


Figure 18. Initially stationary solid, depth upstream of the solid during motion

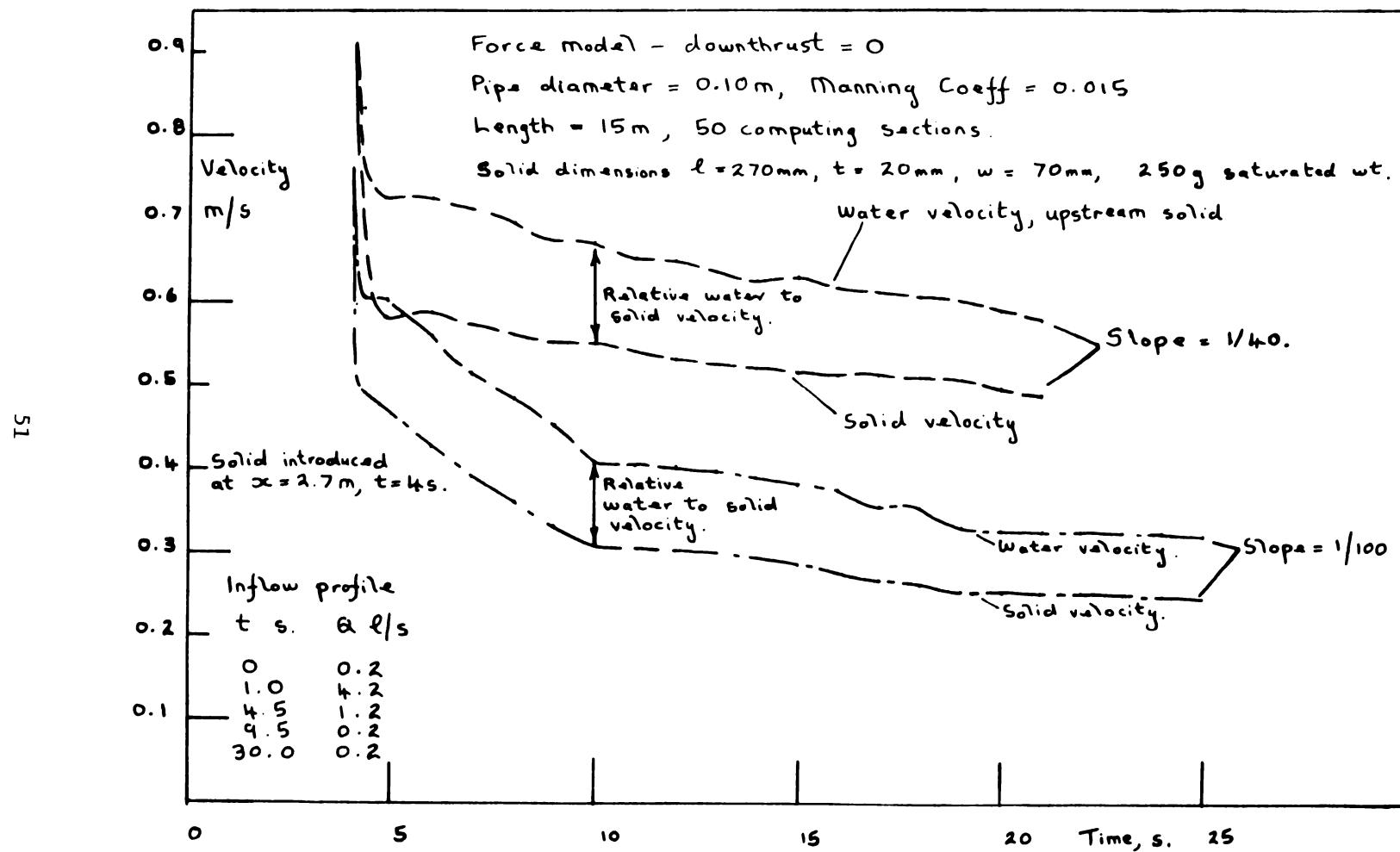


Figure 19. Solid injected into flow, predicted water and solid velocities illustrating relative flow over the body

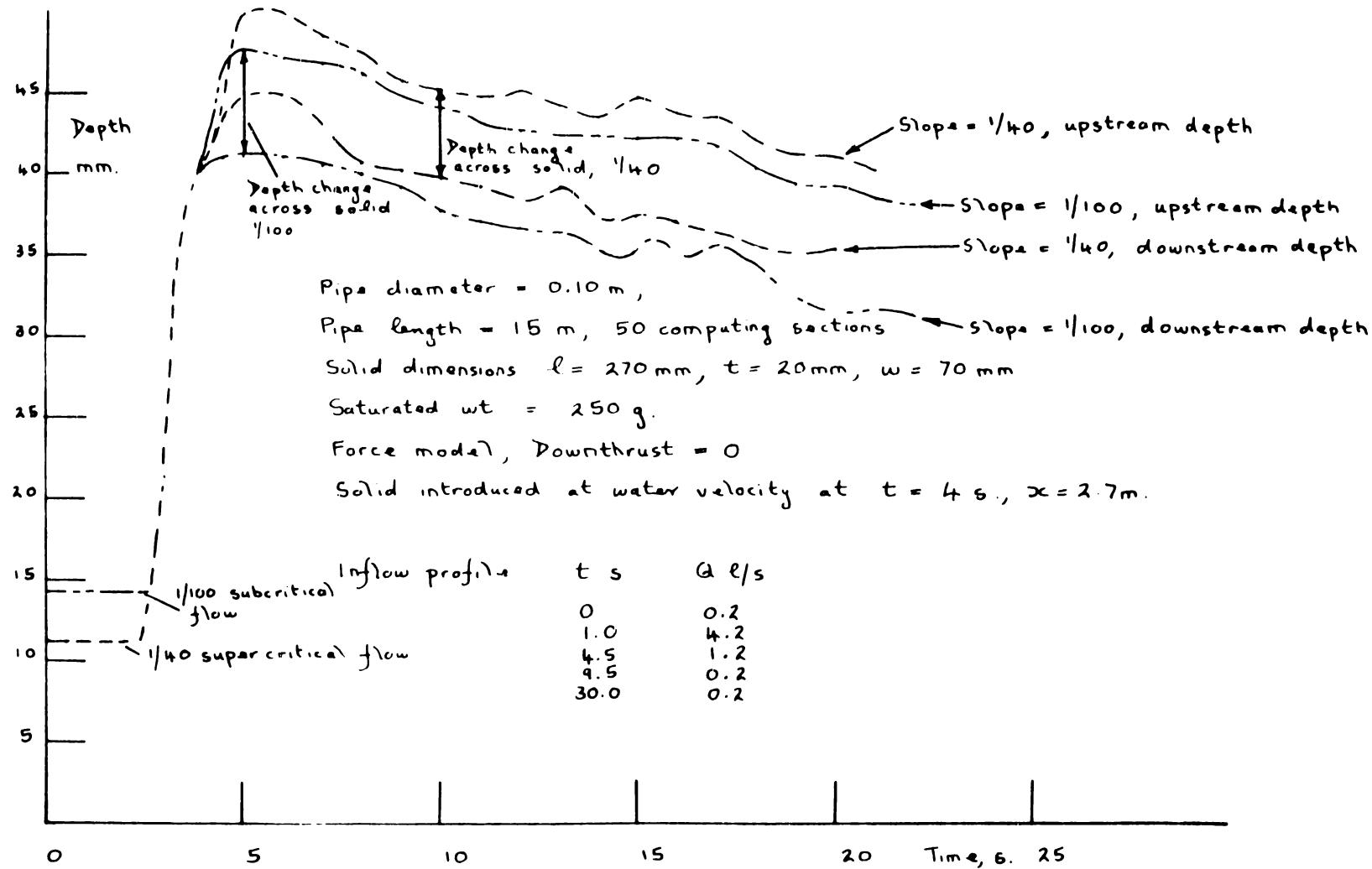


Figure 20. Solid injected into flow, predicted depth differential across solid during motion along simulated drain

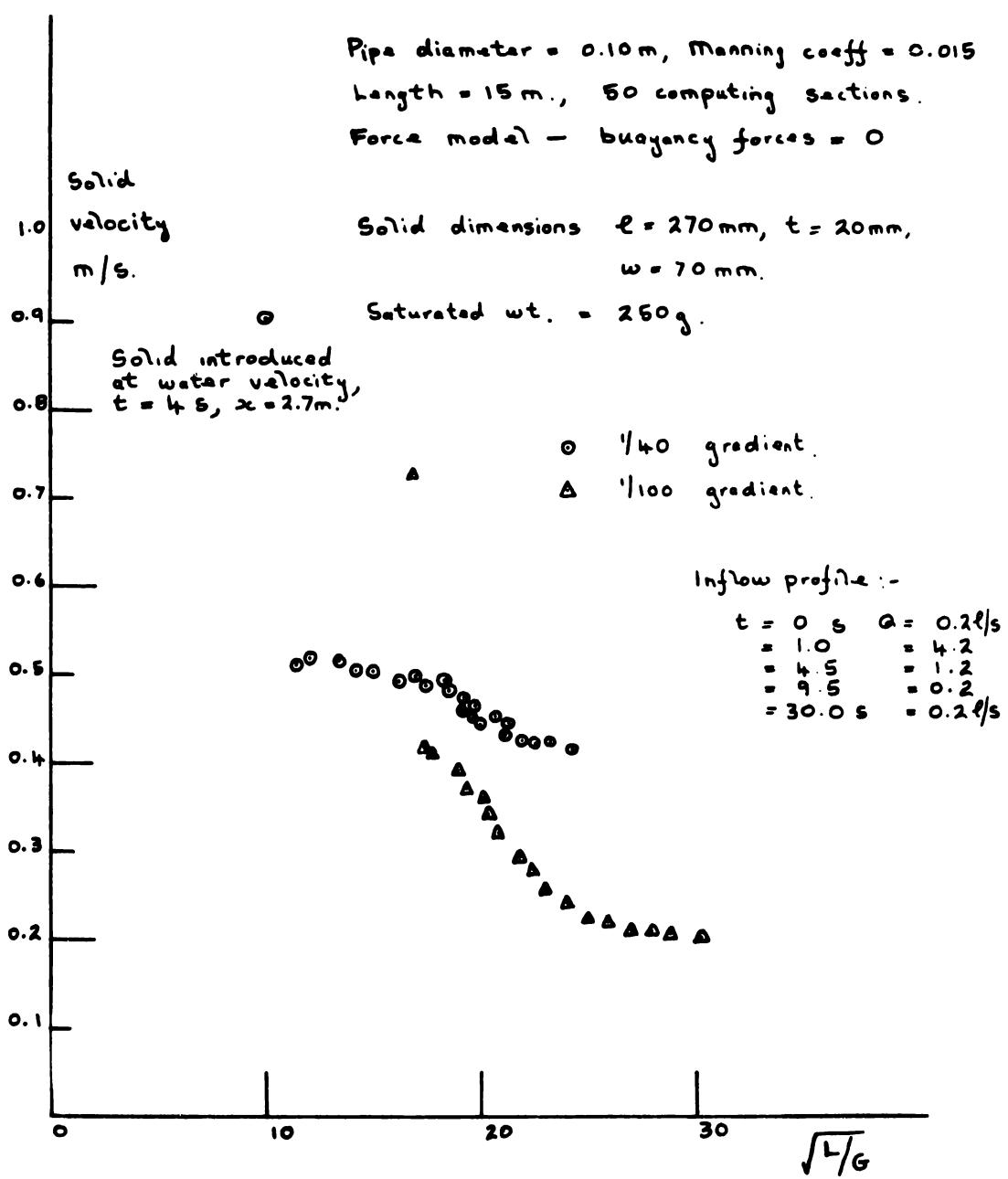


Figure 21. Predicted solid velocity plotted vs  $\sqrt{L/G}$ , pipe slopes 1/40 and 1/100

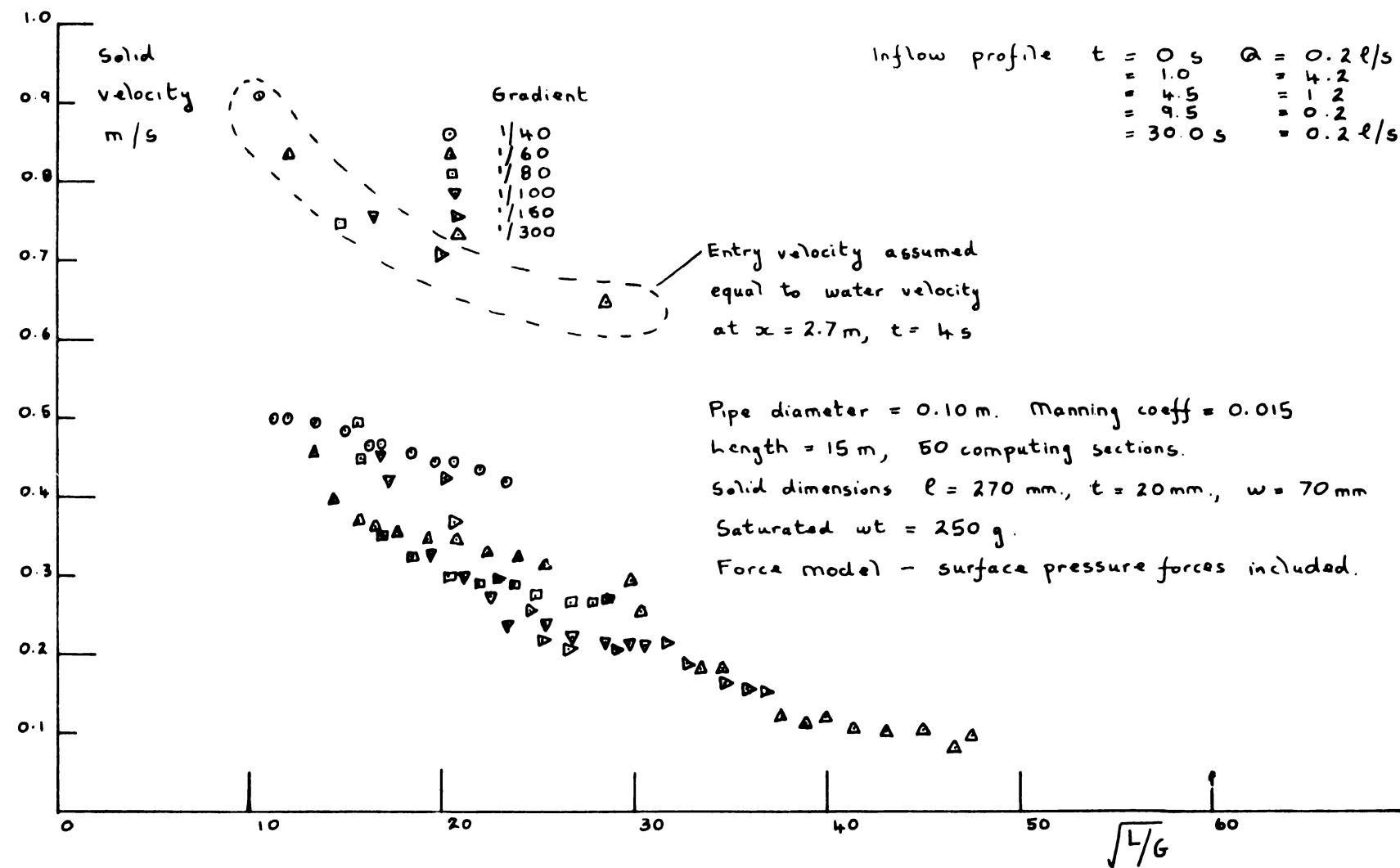


Figure 22. Predicted solid velocity plotted vs  $\sqrt{L/G}$ , pipe slopes 1/40 to 1/300

**APPENDIX I**  
**Program TRANS CD**

### Program TRANSCD

This appendix presents a complete print out of this program, written in Fortran, a flow chart, and sample input data. The program was run on the NBS Center for Building Technology Perkin Elmer 732 computer.

The program accepts data in SI units with the exception of the inflow profile, which is read in as litres/second and corrected to  $m^3/s$  within the program.

The program is written in terms of a series of subroutines that deal with specific aspects of the numerical solution presented in the report. In order to aid the understanding of the program, each subroutine is discussed individually, with the calculation methods outlined in each case. The titles used refer to the program printout included in this appendix.

Note that the notation employed in this appendix in describing the various subroutines is compatible with that used in the main report and not the program notation, although in most cases the variation is small.

The program as written applies only to simple straight pipes with choice of constant gradient, diameter and roughness; however, these subroutines can obviously be utilized in the modelling of more complex pipe networks in future programs.

#### Subroutine TIMINC

This subroutine determines the maximum values of wave speed and velocity at any section along the pipe at each time step. This ensures that the next time increment

$$\Delta t = \frac{\Delta x}{(V + c)}$$

is always sufficiently small to ensure a stable solution. This effectively removes the need for a time step factor, but this is retained in the program and normally set to unity.

#### Subroutine LOSS

This subroutine calculates the equivalent steady flow friction loss over each pipe section by means of the Manning equation, defined as

$$S = V |V| n^2/m^{4/3}.$$

Note the use of  $V$  times absolute value of  $V$  to ensure that frictional forces always oppose the local flow direction.

#### **Subroutine DEPTH**

This subroutine is only called at the initial time zero to calculate the normal and critical depths based on the initial assumed flow. The values of  $h_c$  and  $h_n$  are then used to guide the solution whenever a subcritical or supercritical flow calculation choice arises.

#### **Subroutine INFLOW**

This subroutine calculates the inflow to the pipe at any time during the simulation, based on linear interpolation between successive pairs of Q, T coordinates.

#### **Subroutine SHAPE**

This subroutine calculates flow surface width, T, area A, and wetted perimeter depth.

It is also used, at the initial time zero, to calculate the terms necessary to provide the subcritical water surface profile.

It should be noted that this subroutine could be rewritten for any other pipe cross sectional shape and that the program in general is not restricted to circular cross section pipes. SHAPE is called from many of the other subroutines.

#### **Subroutine WAVSPD**

In this subroutine, the local wavespeed is simply based on

$$c = \sqrt{\frac{gA}{T}}$$

and utilizes SHAPE.

#### **Subroutine PROFIL**

This subroutine is only fully employed if the initial flow is shown to be subcritical as a result of the normal and critical depths calculated via DEPTH.

The water surface profile is calculated from an assumed critical depth at pipe exit by means of the techniques described in the report. The calculation utilizes SHAPE. If the flow is supercritical, then PROFIL sets all depths equal to the normal flow depth.

#### Subroutine INTER

This subroutine carries out the necessary interpolations to fix conditions at points R and S in subcritical flow and R' and S' in supercritical flow (figure 4).

#### Subroutine ENTRY

This subroutine deals with the flow depth, velocity and wave speed at the upstream boundary. The inflow at any time is calculated from INFLOW and this flow rate is then employed, in the subcritical case, in conjunction with the appropriate C<sup>+</sup> characteristic to yield the required parameter values referred to in figure 5.

In the supercrititcal flow case, identified in terms of the initial flow normal to critical flow depth comparison, the inflow curve is solved in conjunction with the normal depth relationship as the downstream conditions, represented by the C<sup>-</sup> characteristic previously employed, can no longer effect conditions at pipe entry.

#### Subroutine NODAL

This subroutine solves the C<sup>+</sup> and C<sup>-</sup> characteristics at all internal pipe sections between  $x = \Delta x$  and  $x = L - \Delta x$  (figure 4) for both subcritical and supercritical flow.

In addition for supercritical flow, it also calculates the flow exit condition, which is based solely on upstream conditions.

#### Subroutine EXIT

This subroutine is only utilized for the subcritical flow case. The exit boundary conditions are determined by solution of the appropriate C<sup>+</sup> characteristic (figure 4) with the critical depth expression

$$\frac{Q^2}{g A^3} T - 1 = 0$$

and  $Q = VA = A(X_2 - X_1 h)$ .

As this is effectively a loss coefficient concentrated at pipe exit, the boundary equation must be solved in the form

$$\frac{Q |Q|}{g A^3} T - 1 = 0$$

$$\text{or } (X_2 - X_1 h) |(X_2 - X_1 h)| \frac{T}{g A^3} = 1$$

where  $| |$  indicate the absolute value of the term enclosed.

The solution is performed by means of the bisection technique described in the main text.

#### Subroutine ASSIGN

This subroutine reassigns the values calculated at the end of a time step, points P in figure 4 to allow the calculation procedures to move on one time increment. This is necessary to avoid program storage space problems.

#### Subroutine INSERT

This subroutine ensures that the solid upstream water velocity, depth, and wave speed are equated to those calculated at the insertion time and pipe section preset in the input data. The solid velocity is set equal to the fluid velocity at this stage. INSERT is only called once; at subsequent time steps, subroutine SOLID calculates the conditions at the solid.

#### Subroutine SOLID

This subroutine calculates the water velocity and depth upstream of the solid by means of the specific energy vs. relative flow rate boundary equation. The flow conditions downstream of the solid are determined in the same manner as ENTRY, dependent on flow regime.

#### Subroutine FORCE

This subroutine calculates the force acting on the solid. Initially it determines the inception of motion by monitoring the force acting on the solid. Various force models may be investigated by modifying this routine.

#### FLOW CHART PROGRAM TRANSCD

Set up initial conditions.

Time = 0

Read pipe data - Line 1 data table

Read calculation data - Line 2 data table

Read inflow profile - Lines 3-8 in example data

Read solid position - Line 9 in example data

Read solid dimensions - Line 10 in example data

Read specific energy to flow over solid coefficients - Line 11 in example data

Read force model indicators and friction coefficients - Line 12 in example data

Adjust inflow rate from l/s to m<sup>3</sup>/s solid dimensions from mm to m solid saturated weight from g to kg.

Calculation of initial steady conditions at time zero.

CALL DEPTH - Calculation of normal and critical depths at initial flow.

$h_c < h_n$

$h_n < h_c$

Subcritical flow

CALL PROFIL - calculate  
water surface profile and  
interpolate for section  
depths.

Supercritical flow

CALL PROFIL - set all depths  
to normal value.

CALL ASSIGN - set up base arrays H, V, C

CALL TIMINC - identify maximum V, C to yield stable time step.

$\Delta\infty = PL/N$

$\Delta t = \Delta\infty / (V + c)_{max}$

Output - Initial conditions and pipe description

- Solid dimensions and model indicators
- Depth, velocity, wavespeed at each increment if  $N < 10$ ,  
at 1/10 pipe length points if  $N > 10$  and a multiple of 10

A - Update time and commence unsteady flow simulation.

CALL TIMINC - Identify new time step based on  $(V, C)_{max}$

Check Time vs. TMAX

Time < TMAX

Time > TMAX

Go to B

Go to G

B - Unsteady flow calculations.

CALL INFLOW - Calculate pipe inflow rate from data profile.

CALL INTER - Interpolate to obtain base conditions HR, HS or HS' etc.  
Contains choices based on flow regime and presence of the  
solid between any two computing points.

CALL LOSS - Calculation of equivalent steady flow loss terms, SR, SS.

CALL ENTRY - Solves upstream boundary condition for either sub or super-  
critical flow.

CALL NODAL - Solves  $C^+C^-$  characteristics based on output of INTER and LOSS.  
Omits the two nodes bracketting a moving solid and the intial  
location node of a stationary solid. In supercritical flow,  
NODAL also provides exit conditions.

For subcritical flow only, CALL EXIT - exit depth based on critical depth at pipe discharges.

Check solid model being simulated:

Injected solid, time < injection time, Go to C

Injected solid, time > injection time, on the first occasion, Go to D

Initially stationary solid, Go to E

Injected solid, time > injection time, following first occasion, Go to E

D - CALL INSERT - assigns water velocity at solid entry station and time to be solid velocity.

Go to C

E - CALL SOLID - calculate conditions on upstream and downstream solid faces.  
Downstream velocity given by same technique as ENTRY dependnet on flow regime.

Go to C

C - CALL ASSIGN - sets up base arrays for next time step.  
Output of simulation results.

Velocity, depth, flow and wavespread at solid and at 2 points each 1/10 pipe length upstream and downstream of the moving solid.

Solid velocity and body forces.

Solid position.

Check presence of solid, no solid Go to B, solid present Go to F.

F - CALL FORCE - calculation of forces acting on solid over next time step.  
Calculation of solid position and velocity at end of next time step.

Check solid still in pipe at end of next time step:

Solid position > pipe length, set solid indicators to no solid and continue calculation to TMAX, Go to B.

G - Program terminates.

SAMPLE INPUT DATA, PROGRAM TRANSCD.

Line 1.

Pipe diameter, Manning coeff., slope, length.

Format 4F10.4.

VVVV - .1000 VVVV 0.0150 VVVV 0.0100 VVVV 15.0000

Line 2.

Nº calculation steps, Run time, Time step factor. (Note, Nº calculation sections < 10 or multiple of 10 to match output format, time step factor, normally set = 1.)

Format I3, 2F10.4

V 50 VVV 30.0000 VVVV 1.000

Line 3.

Nº pairs of coordinates on inflow-time curve.

Format I3

VV 5

Line 4 to 8 (note actual number depends on Line 3).

Inflwo QIN or time TIN

Format 2F10.4

VVVV 0.2000 VVVV 0.0000

VVVV 4.2000 VVVV 1.0000

VVVV 1.2000 VVVV 4.5000

VVVV 0.2000 VVVV 9.5000

VVVV 0.2000 VVV 50.0000

Line 9.

Solid initial position in pipe, must be at a section not an intermediate location. Applies to both initially stationary and injected models.

Format I3

V 10

Line 10.

Solid dimensions, length, width, thickness, saturated weight.

Format 4F10.4

vv 270.0000 vvv 70.0000 vvvv 20.0000 vv 250.0000.

Line 11.

Solid minimum specific energy to allow bypass flow, Specific energy vs. Q coefficient.

Format 2F10.4

vvvv 0.0200 vvvv 0.6000

Line 12.

Buoyancy force indicator, 0. force omitted, 1. force included. Downthrust force indicator, 0. force omitted, 1. force included. Solid surface shear stress, set 0. Sliding friction factor, static friction coefficient.

Format 5F10.4

vvvv 1.0000 vvvv 0.0000 vvvv 0.0000 vvvv 0.1000 vvvv 0.1500

Line 13.

Solid insertion time, zero for stationary solid. Solid insertion model indicator, zero for stationary solid, > 0 for moving solid.

Format F10.4 I3

vvvv 4.0000 vv 4

Units - SI units used in data field except for inflow QIN in ls, converted to m<sup>3</sup>/s in program. Outut in S.I., except flow rate in ls. Solid dimensions and saturated weight read in mm and g, converted to m and kg in program.

	RRRRRRRR	AAAAAA	NNNN	NN	SSSSSS	CCCCC	UUUUUU
	RRRRRRRR	AAAAAA	NNNN	NN	SSSSSS	CCCCC	UUUUUU
	RR RR	AA AA	NNNN	NN	SS SS	CC CC	UU UU
	RR RR	AA AA	NNNNN	NN	SS SSSS	CC CC	UU UU
	RRRRRRRR	AAAAAA	NN NNN	NN	SSSSSS	CC	UU
	RRRRRRR	AAAAAA	NN NNN NN	NN	SSSSSS	CC	UU
	RR RR	AA AA	NN NNNNN	NN	SS	CC CC	UU UU
	RR RR	AA AA	NN NNNNN	NN	SS SS	CC CC	UU UU
	RR RR	AA AA	NN NNN	NN	SSSSSS	CCCCCCC	UUUUUUU
	RR RR	AA AA	NN NNN	NN	SSSSSS	CCCCC	UUUUUU

00	00	00	00	00	00	00
AAAAAA	CCCC	000000000	000000000	000000000	3333333	222222
AAAAAAA	CCCC	000000000	000000000	000000000	3333333	2222222
AA AA	CC CC	-----	00 000	00 000	333	222 22
AA AA	CC CC	-----	00 0000	00 0000	3333	222
AAAAAA	CC	00000000	00000000	00000000	33333	222
AAAAAA	CC	00000000	00000000	00000000	33	222
AA AA	CC CC	-----	0000 00	0000 00	33	222
AA AA	CC CC	-----	000 00	000 00	33	222
AA AA	CCCC	000000000	000000000	000000000	3333333	2222222
AA AA	CCCC	000000000	000000000	000000000	3333333	2222222

00	00	00	00	00	00	00	
				666666	666666	0000000000	222222
				66666666	66666666	0000000000	22222222
				66 66	66 66	00 000	222 22
				66	66	00 0000	222
				66666666	66666666	00 00000	222
				66666666	66666666	00000 00	222
				66 66	66 66	0000 00	222
				66 66	66 66	000 00	222
				66666666	66666666	0000000000	22222222
				666666	666666	0000000000	22222222

00	00	00	00	00	00	00	
	0000000000			222222	222222	4444444	0000000000
	0000000000			22222222	22222222	44444444	0000000000
	00 000			222	222	44 44	00 000
	00 0000			222	222	44 44	00 0000
	00 00000			222	222	4444444	00000000
	00 000000			222	222	4444444	00000000
	00 0000000			222	222	4444444	00000000
	00 00000000			222222	222222	44444444	00000000
	00 00000000			22222222	22222222	44444444	00000000
	00 00000000			222222222	222222222	444444444	00000000

```

$BATCH
C   TRANSC ARE A SERIES OF PROGRAMS DESIGNED TO APPLY THE METHOD OF
C   CHARACTERISTICS SOLUTION TO THE EQUATIONS DEFINING
C   UNSTEADY FLOW IN PARTIALLY FILLED CHANNELS.
C
C
DIMENSION QP(61),QIN(30),TIN(30),VP(61),MP(61),CP(61)
DIMENSION Y(61),M(61)
DIMENSION VR(61),MR(61),CR(61),VS(61),HS(61),CS(61),SS(61)
DIMENSION SR(61),XR(61),XS(61),XM(61),CE(61)
COMMON/CM1/LT0,UT,DX,D,S0,PL,SE0,XK,2H
COMMON/CM2/M,TMAX
COMMON/CM3/NPTS,QIN,TIN
COMMON/CM4/V,M,C,VR,MR,CR,XR,SP,VS,MS,CS,XS,SS,XN
COMMON/CM5/MC,MN
COMMON/CM6/OP,VP,MP,CP
COMMON/CM7/EL
COMMON/CM8/INSOL,NSOL,1,NSOL
COMMON/CM9/SGLVEL,HDS,VDS,CDS,MPLS,VPOS,CPDS
COMMON/CM10/XPOS,YSON,VPSOL,VSC1,OPSUL,HUS,VJS,CUS,HPUS,Vpus,CPUS
COMMON/CM11/TL,TH,TT,SW
COMMON/CM12/XFAC2,XFAC,TAU,FMEV,FSTAT
C
C
C
C
C
C   SET INITIAL CONDITIONS
TIME=0.0
VPSOL=0.0
C   READ PIPE DESCRIPTION DATA
READ(4,100)R,RP,S0,PL
C   D-DIA. RP-MANNING COEFF. S0-PIPE SLOPE. PL-PIPE LENGTH
100 FORMAT(4F10.4)
C   READ CALCULATION DATA
READ(4,103)N,TMAX,TFAC
IF(N.LE.10) N2=1
IF(N.GT.10) N2=N/10
C   N2 INCREMENT BETWEEN OUTPUT NODES. ALLOWS MORE THAN 10
C   CALCULATION NODES TO BE USED.
C   N-NUMBER OF PIPE SECTIONS CONSIDERED, THX-MAX-INCREMENT OF CALC.
C   TFAC-TIME STEP FACTOR, 1-10.
103 FORMAT(13,2F10.4)
C   READ INFLOW DATA PROFILE. INFLOW PROFILE USED IS BASED ON
C   A LINEAR INTERPOLATION BETWEEN THESE DATA POINTS. NOTE DATA
C   READ IN IN L/S BUT USED IN PROGRAM IN M3/S.
READ(4,102)FTS
102 FORMAT(13)
DO 50 I=1,NPTS
READ(4,104)QIN(I),TIN(I)
104 FORMAT(2F10.4)
QIN(I)=QIN(I)/1000.0
50 CONTINUE
C   READ SOLID DESCRIPTIVE DATA. NSOL-INITIAL POSITION,
C   TL-LENGTH, TB-WIDTH, TT-THICKNESS, SH-SATURATED HT.,
C   SEO-SPECIFIC ENERGY REQUIRED TO INITIATE FLOW PAST
C   SOLID, XK-FLOW TO SPECIFIC ENERGY COEFF. AT SP
C   VALUES ABOVE SEO
C   TAU-SOLID SURFACE COEFF OF SHEAR, FMUV-SLIDING SOLID
C   TO PIPE WALL FRICTION COEFF, STAT-STATIC SOLID TO
C   PIPE WALL FRICTION COEFF.

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C      SOLTIM-TIME POSITION OF SOLID IN INFLOW PROFILE,NOTE
C      ZERO VALUE INDICATES STATIONARY IN PIPE AT DESIGNATED
C      SECTION NSOL.INSOL-INDICATOR OF SOLID VELOCITY,GT.0
C      INDICATES THAT FLOW VELOCITY AT SECTION MSOL IS ASSUMED
C      AS CALCULATION START POINT.
C      READ(4,102) NSOL
C      IF (NSOL.EQ.0)GOTO 729
C      READ(4,100) TL,TW,TT,SW
C      READ(4,104) SEO,XK
C      READ(4,109)XFAC,XFAC2,TAU,FMOV,FSTAT
C      109   FORMAT(F10.4)
C      READ(4,105)SOLTIM,INSOL
C      105   FORMAT(F10.4,13)
C      729   CONTINUE
C      MSOL1=NSOL
C      IF(INSOL.GT.0) NSOL=0
C
C
C
C
C      CALCULATION OF INITIAL CONDITIONS ALONG THE PIPE
C      BASED ON GRADUALLY VARIED FLOW EQUATIONS. THE
C      TERMINAL CONDITIONS IN THE PIPE ARE BASED ON
C      CRITICAL DEPTH AT PIPE EXIT.
C      CALL DEPTH(TIME)
C      CALL PROFIL(GIN(1),MN,MC,XN)
C      CALL ASSIGN(N)
C      IF(TIME.LE.SOLTIM.AN).INSCL.GT.0) VSJL=V(NSOL)
C      CALCULATION OF TIME STEP AND LENGTH SECTION.
C      CALL TIMINC(N,VP,CP,VN,CN)
C      DX=PL/FLOAT(N)
C      DT=DX/(TFAC*(CN+VN))
C      DTO=DT
C
C      CALCULATION OF SOLID INITIAL POSITION
C      IF(NSOL1.GT.0) XSQL=DX*FLCAT(NSCL1-1)
C      IF(NSOL1.GT.0) XPOS=DX*FLCAT(NSCL1-1)
C
C
C
C      OUTPUT TEST DESCRIPTION PLUS INITIAL CONDITIONS.
C      WRITE(3,202)D,RM,SD,PL
C      202   FORMAT(1H1,/10X,'TEST PIPE CONFIGURATION:-',
C      1 /10X,'DIAMETER = ',F10.4,' M.',/
C      1 /10X,'MANNING COEFFICIENT: ',F10.9,/,
C      1 /10X,'PIPE SLOPE = ',F10.4,' PIPE LENGTH = ',
C      1 F10.4,' M.',//)
C      IF(HC.GT.HN) WRITE(3,200)HN,HC
C      IF(HC.LE.HN) WRITE(3,201)HN,HC
C      200   FORMAT(10X,'FLOW SUPERCRITICAL, ','NORMAL DEPTH = ',F10.4,
C      1 ' M.', ' AND CRITICAL DEPTH = ',F10.4,' M.',//)
C      201   FORMAT(10X,'FLOW SURCRITICAL, ','NORMAL DEPTH = ',
C      1 F10.4,' M. AND CRITICAL DEPTH = ',F10.4,' M.',//)
C      QIN(1)=QIN(1)*1000.0
C      WRITE(3,203)QIN(1),HN,CN
C      QIN(1)=QIN(1)/1000.0
C      203   FORMAT(10X,'INITIAL FLOW RATE = ',F10.4,' L/S.',',
C      1 ' INITIAL DEPTH = ',F10.4,' M.',',
C      2 ' INITIAL WAVE SPEED = ',F10.4,' M/S.',/,)
C      N1=M+1

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      WRITE(3,204)UT,EX,NL,TFAC
204  FORMAT(10X,'CALCULATION TIME STEP = ',F10.4,' S.',/
1   'LENGTH INCREMENT = ',F10.4,' M.',' NUMBER OF NODES = ',I3,
2   ' TFAC = ',F5.2,/)

      IF(NSOL.LT.0) GOTO 730
      WRITE(3,220)
220  FORMAT(10X,'SOLID DESCRIPTION DATA.',/)
      WRITE(3,221)TL,TW,TT,SW,NSCL1,XSOL
221  FORMAT(10X,'LENGTH = ',F7.2,' MM.',' WIDTH = ',F7.2,
1   ' MM.',' THICKNESS = ',F7.2,' MM.',' SATURATED WT. = ',
2   F7.2,' GM.',//)
3   10X,'SOLID POSITION, SECTION NUMBER = ',I3,
4   ' DISTANCE FROM ENTRY = ',F7.3,' M.',/)
      WRITE(3,222)FSTAT,FMOV,TAU
222  FORMAT(10X,'STATIC FRICTION FACTOR = ',F6.3,
1   ' SLIDING FRICTION FACTOR = ',F6.3,
2   ',10X,'SOLID SURFACE SHEAR COEFF. = ',F7.4,/)

      WRITE(3,271)XFAC,XFAC2
271  FORMAT(10X,'POLYANCY FACTOR XFACT = ',F5.2,/)

1   ,10X,'DOWNTHRUST FACTOR XFACT2 = ',F5.2,/)

      IF(SOLTIM.GT.0.0) WRITE(3,223)SOLTIM
223  FORMAT(10X,'SOLID POSITION IN INFLOW PROFILE = ',
1   F6.2,' SECONDS FROM FLOW INCEPTION.',//)

      TL=TL/1000.0
      TW=TW/1000.0
      TT=TT/1000.0
      SW=SW/1000.0

730  CONTINUE
C     OUTPUT TEST SIMULATION RESULTS.
      IF(NSOL.GT.0) GOTO 733
      WRITE(3,205)(XN(I),I=1,N1,N2)
205  FORMAT(10X,' POSITION FRST ',/,18X,' ENTRY N.',EX,11F7.3,/)

      WRITE(3,206)(TIME,(HP(I),I=1,N1,N2)
206  FORMAT(2X,'TIME = ',F6.2,' S.',' DEPTH',7X,'M=',11F7.4)

      WRITE(3,207)(VP(I),I=1,N1,N2)
      WRITE(3,307)(CP(I),I=1,N1,N2)
307  FORMAT(18X,' FLOW RATE L/S=',11F7.4)
      WRITE(3,208)(CF(I),I=1,N1,N2)
208  FORMAT(18X,' VELOCITY M/S=',11F7.4)
      FORMAT(18X,' WAVE SPD. M/S=',11F7.4,/)

C     GOTO 734
733  CONTINUE
      N3=NSOL-2*N2
      N4=NSOL-1/2
      N5=NSOL+N2
      N6=NSOL+2*N2
      IF(N3.LE.0) N3=NSOL-2
      IF(N4.LE.0) N4=NSOL-1
      WRITE(3,305) VSOL
305  FORMAT(10X,'SOLID POSITION',/54X,'SOLID VELOCITY',/,
1   54X,' = ',F6.3,' M/S.')
      WRITE(3,205)XN(1),XN(N3),YN(N4),XSOL,XSOL,XN(N5),XN(N6),AN(N1)
      WRITE(3,206)TIME,HP(1),HP(N3),HP(N4),HPUS,HPDS,HP(N5),HP(N6),
1   HP(N1)
      WRITE(3,207)VP(1),VP(N3),VP(N4),VPUS,VPDS,VP(N5),VP(N6),VP(N1)
      WRITE(3,307)CF(1),CP(N3),CP(N4),CPUS,CPDS,CP(N5),CP(N6),CP(N1)
      WRITE(3,208)CP(1),CP(N3),CP(N4),CPUS,CPDS,CP(N5),CP(N6),LP(N1)

734  CONTINUE
C
C     UPDATE TIME AND CHECK CALCULATION LENGTH.
600  CONTINUE

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```

501    CONTINUE
      CALL TIMINCIN, VP, CP, VN, CM
      DT=DX/ETFAC*(CM+VN)
      DTO=DT
      TIME=TIME+DT

      IF(TIME.GT.TMAX) GOTO 500

C
C
C
C
C   SET UP CALCULATIONS FOR THIS TIME STEP.
      DTRAT=DT/DTO
      VSOL=VPSUL
      IF(DTRAT.LT.0.06) GOTO 500
      CALL INFLOW(TIME,Q)
      J1=1
      CALL INTER(N)
      J1=2
      CALL LOSS(VR,VS,MR,MS,M,SR,SS,RF)
      J1=3
      CALL ENTRY(TIME)
      J1=4
      CALL NODAL(N)
      IF(TIME.GE.SOLTIM.AN).AND.NSOL.GT.0) CALL SOLID(TIME)
      IF(TIME.GT.SOLTIM.AND.INSLL.GT.0) CALL INSERT(XN,TIME)
      IF(TIME.GE.SOLTIM.AND.NSDL.GT.0) XPOS=XSL
      J1=5
      IF(HN.GT.HC)CALL EXIT(TIME)
      IF(DT.LT.DTO) GOTO 500
      DT=DTO
      J1=6
      CALL ASSIGN(N)
      IF(TIME.GE.SOLTIM.4N).AND.NSOL.GT.0) CALL FORCE(TIME,XN,FSOL)
      IF(NSOL.GT.0) GOTO 731
      J1=7
787    CONTINUE
      WRITE(3,206) TIME,(HP(I),I=1,N1,N2)
      WRITE(3,207)(VP(I),I=1,N1,N2)
      WRITE(3,307)(CF(I),I=1,N1,N2)
      WRITE(3,208)(CF(I),I=1,N1,N2)
      GOTO 501
731    CONTINUE
      N3=NSOL-2*N2
      N4=NSOL-N2
      N5=NSOL+N2
      N6=NSOL+2*N2
      IF(N6.GE.N1) N5=NSOL+1
      IF(N6.GE.N1) N6=NSOL+2
      IF(NSOL.GE.N-1) NSOL=0
      IF(N3.LE.0) N4=NSOL-1
      IF(N3.LE.0) N3=NSOL-2
      IF(NSOL.EQ.0) WRITE(3,224)
224    FORMAT(//30X,'SOLID WITHIN < SECTIONS OF PIPE DISCHARGE.',/,/
1 30X,'SOLID ASSUMED TO CLEAR PIPE, CALCULATION TERMINATED, SOLUTION'
2 ,/30X,'CONTINUES WITH SOLID FREE PIPE FLOW.',///)
      IF(NSOL.EQ.0) WRITE(3,205) XN(1),XN(N3),XN(N4),XPOS,YPOS,XN(N5),
1 XN(N6),XN(N1)
      IF(NSOL.EQ.0) GOTO 787
      WRITE(3,308) SOLVEL
308    FORMAT(/54X,'SOLID VELOCITY',/54X,' = ',F6.3,' M/S.')
      'IF(VSUL.GT.0.0) GOTO 577

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      GOTO 578
577  CONTINUE
      WRITE(3,205)XM(1),XM(N3),XM(N4),XPOS,XPOS,XM(N5),XM(N6),XM(N1)
578  CONTINUE
      WRITE(3,206)TIME,MP(1),MP(N3),MP(N4),MPUS,MPDS,MP(N5),MP(N6),
1   MP(N1)
      WRITE(3,207)VP(1),VP(N3),VP(N4),VPUS,VPDS,VP(N5),VP(N6),VP(N1)
1   WRITE(3,307)OP(1),OP(N3),OP(N4),OPSOL,OPSL,OP(N5),OP(N6),OP(N1)
      WRITE(3,208)CP(1),CP(N3),CP(N4),CPUS,CPDS,CP(N5),CP(N6),CP(N1)
      GOTO 501
500  CONTINUE
      END
C
C
C
C
C
C
      SUBROUTINE TIMINC(M,VP,CP,VN,CN)
COMMON/CM8/NSCL,NSOL1,NSOL
COMMON/CM9/SOLVEL,MDS,VDS,CDS,MPDS,VPDS,CPDS
COMMON/CM10/XPOS,XSOL,VPSOL,VSCL,OPSL,MUS,VUS,CUS,MPUS,VPUS,CPUS
C THIS SUBROUTINE IDENTIFIES THE HIGHEST WAVE SPEED
C AND THE HIGHEST AVERAGE FLOW VELOCITY IN
C THE SIMULATED FLOW IN ORDER TO ENSURE THAT THE TIME STEP
C CHOSEN IS THE SMALLEST, HENCE ENSURING STABILITY.
      DIMENSION VF(61), CP(61)
      CN=0.0
      DO 1 I=1,M+1
      IF(I.EQ.NSCL)GOTO 1
      IF(CP(I).GE.CN) CN=CP(I)
1   CONTINUE
      VN=0.0
      DO 2 I=1,M+1
      IF(I.EQ.NSOL) GOTO 2
      IF(VP(I).GE.VN) VN=VP(I)
2   CONTINUE
      IF(CPUS.GT.CN) CN=CPUS
      IF(CPDS.GT.CN) CN=CPDS
      IF(VPUS.GT.VN) VN=VPUS
      IF(VPDS.GT.VN) VN=VPDS
      RETURN
      END
C
C
C
C
C
C
      SUBROUTINE LOSS(VR,VS,MR,HS,N,SR,SS,CM)
DIMENSION VF(61),VS(61),MR(61),RS(61),SR(61),SS(61)
COMMON/CM5/MC,MM
C THIS SUBROUTINE CALCULATES THE EQUIVALENT STEADY STATE
C LOSS BASED ON THE MANNING COEFFICIENTS.
C
      DO 1 I=2,M+1
      CALL SHAPF(TIME,MR(I),A,T,PER)
      SR(I)=((VF(I)*ABS(VR(I))*T**2)/(A*PER))**1.333
1   CONTINUE
      N2=M
      NY=1

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M2=MC+DH*FLOAT(I+1)
M3=MC+DH*FLOAT(I)
CALL SHAPE(TIME,M1,A,T,PER)
DL1=DL
CALL SHAPE(TIME,M2,A,T,PER)
DL2=DL
CALL SHAPE(TIME,M3,A,T,PER)
DL3=DL
DXP=DH*(DL1+DL2+4.0*DL3)/3.0
SL=SL-DXP
M1=M2
IF(SL.GE.PL) GOTO 81
X(IS)=PL-SL
IF(M1.GE.HN) GOTO 83
DEP(IS)=M1
80 CONTINUE
81 X(IS)=0.0
NIS=IS
IF(M1.GE.HN) GOTO 83
DEP(IS)=M1
GOTO 84
83 DEP(IS)=HN
GOTO 84
84 IF(X(IS).GT.0.0) GOTO 85
GOTO 86
85 X(IS)=0.0
DEP(IS)=HN
NIS=IS
86 CONTINUE
C
C
C INTERPOLATION REQUIRED TO DETERMINE DEPTH AT EACH NODE.
900 CONTINUE
DX=PL/FLOAT(P)
XN(1)=0.0
DO 87 I=2,N+1
XN(I)=XN(I-1)+DX
87 CONTINUE
IF(HN.LE.MC) GOTO 901
N2=NIS+1
DO 94 J=1,NIS
N2=N2-1
X1(J)=X(N2)
DEP1(J)=DEP(N2)
WRITE(3,93)X1(J),DEP1(J)
93 FORMAT(10X,2F10.5)
94 CONTINUE
DO 95 J=1,NIS
X(J)=X1(J)
DEP(J)=DEP1(J)
95 CONTINUE
NP(1)=DEP(1)
NP(N+1)=MC
DO 98 I=2,N
DO 99 K=1,NIS-1
IF(XN(I).GT.X(K).AND.XN(I).LT.X(K+1)) GOTO 90
98 CONTINUE
90 NP(I)=DEP(K)+(DEP(K+1)-DEP(K))*(XN(I)-X(K))/(X(K+1)-X(K))
99 CONTINUE
C SET UP PASE CONDITIONS AT TIME ZERO.
901 CONTINUE
END

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8      V(I)=VCU
      M(I)=HCU
      C(I)=CCU
      V(I-1)=VA
      M(I-1)=MA
      C(I-1)=CA
CONTINUE
9      IF(VSOL.LE.0.0.AND.I.EQ.NSOL) V(I)=VUS
      IF(VSOL.LE.0.0.AND.I.EQ.NSOL) C(I)=CUS
      IF(VSOL.LE.C.0.AND.I.EQ.NSCL) M(I)=HJS
      IF(VSOL.LE.0.0.AND.I.EQ.NSOL+1) V(I-1)=VDS
      IF(VSOL.LE.0.0.AND.I.EQ.NSOL+1) C(I-1)=CDS
      IF(VSOL.LE.0.0.AND.I.EQ.NSCL+1) M(I-1)=HDS
      VR(I)=(V(I)+THETA*(C(I)*V(I-1)-V(I)*C(I-1)))
1      / (1.0+THETA*(V(I)-V(I-1)*C(I)-C(I-1)))
      CR(I)=(C(I)*(1.0-VR(I)*THETA)+(C(I-1)*VR(I)*THETA))
1      / (1.0+C(I)*THETA-C(I-1)*THETA)
      MR(I)=M(I)-(H(I)-H(I-1))*THETA*(VR(I)*CR(I))
1      CONTINUE
      IF(MN.LE.HC) GOTO 4
      DO 2 I=1,N
      IF(I.EQ.NSOL.AND.VSOL.GT.0.0) GOTO 10
      GOTO 11
10     V(I)=VCD
      M(I)=HCD
      C(I)=CCD
      V(I+1)=VB
      M(I+1)=HB
      C(I+1)=CB
11     CONTINUE
      IF(VSOL.LE.C.0.AND.I.EQ.NSCL) V(I)=VDS
      IF(VSOL.LE.C.0.AND.I.EQ.NSCL) C(I)=CUS
      IF(VSOL.LE.C.0.AND.I.EQ.NSCL) M(I)=HJS
      IF(VSOL.LE.C.0.AND.I.EQ.NSCL-1) V(I+1)=VUS
      IF(VSOL.LE.0.0.AND.I.EQ.NSCL-1) C(I+1)=CUS
      IF(VSOL.LE.C.0.AND.I.EQ.NSCL-1) M(I+1)=HJS
      VS(I)=(V(I)-THETA*(V(I)*C(I+1)-C(I)*V(I+1)))
1      / (1.0-THETA*(V(I)-V(I+1)*C(I)+C(I+1)))
      CS(I)=(C(I)+VS(I)*THETA*(C(I)-C(I+1)))
1      / (1.0+THETA*(C(I)-C(I+1)))
      HS(I)=H(I)+THETA*(VS(I)-CS(I))*(H(I)-H(I+1))
2      CONTINUE
      GOTO 3
3      DO 5 I=2,N+1
      IF(VSOL.GT.0.0.AND.I.EQ.NSCL+1) GOTO 5
      IF(NSOL.GT.0.0.AND.I.EQ.NSCL+1) GOTO 20
      GOTO 80
20     V(I-1)=VDS
      M(I-1)=HDS
      C(I-1)=CDS
80     CONTINUE
      VS(I)=(V(I)*(1.0+THETA*C(I-1))-V(I-1)*THETA*C(I))/
1      / (1.0+THETA*(V(I)-V(I-1)*C(I-1)-C(I)))
      CS(I)=(C(I)+VS(I)*(C(I)-C(I-1)))/(1.0+THETA*(C(I-1)-C(I)))
      HS(I)=H(I)-(H(I)-H(I-1))*THETA*(VS(I)-CS(I))
5      CONTINUE
3      CONTINUE
      IF(NSOL.EQ.0) GOTO 13
      IF(MN.GE.HC) GOTO 14
      DO 17 I=1,NSLL-1
      IF(V(I).GT.C(I)) GOTO 17
      VS(I)=(V(I)-THETA*(V(I)*C(I+1)-C(I)*V(I+1)))
1      / (1.0-THETA*(V(I)-V(I+1)*C(I)+C(I+1)))

```



```

      MN=MNN
      GOTO 9
14      MN=MNN
11      CONTINUE
      IF(TIME.EQ.0.0) GOTO 500
C
C
C      CALCULATION OF BOUNDARY DEPTH.
600      CONTINUE
      IF(MN.LE.HC) GOTO 700
      UP=D
      DN=0.0
      HB=(UP+DN)/2.0
15      CONTINUE
      CALL SHAPE(TIME,HB,AREA,T,PER)
      X3=C/CS(1)
      X4=VS(1)-G*(SS(1)-SO)*DT-X3*HS(1)
      MFLW=0-(X4*X3*HB)*AREA
      IF(MFLW)16,17,18
16      UP=HB
      GOTO 19
18      DN=HB
19      HBB=(UP+DN)/2.0
      IF(ABS((HFF-HB)/HB).LE.0.001) GOTO 2J
      HB=HBB
      GOTO 15
20      HB=HBB
17      CONTINUE
      HP(1)=HB
      VP(1)=X4*X3*HP(1)
      CALL SHAPE(TIME,HB,AREA,T,PER)
      QP(1)=AREA*VP(1)*1000.0
      CP(1)=SO*TS(C*AREA/T)
500      CONTINUE
      GOTO 800
700      CONTINUE
C      CALCULATION OF NORMAL DEPTH.
      UP=D
      DN=0.0
      HB=(UP+DN)/2.0
79      CONTINUE
      CALL SHAPE(TIME,HB,AREA,T,PER)
      MNORM=1.0-(C**2)*CDN/((AREA**3.333)/(PER**1.333))
      IF(MNORM)80, 1,82
80      DN=HB
      GOTO 83
82      UP=HB
83      HBB=(UP+DN)/2.0
      IF(ABS((HFF-HB)/HB).LE.0.001) GOTO 84
      HB=HBB
      GOTO 79
84      HB=HBB
81      CONTINUE
      HP(1)=HB
      CALL SHAPE(TIME,HB,AREA,T,PER)
      VP(1)=0/AREA
      GF(i)=0*1000.0
      CP(1)=SO*TS*AREA/T
800      CONTINUE
      RETURN
      END
C

```

```

C C C C C
C
SUBROUTINE MODAL(M)
C
DIMENSION OP(61),QIN(30),TIN(30),VP(61),HP(61),CP(61)
DIMENSION V(61),M(61)
DIMENSION VR(61),HR(61),CR(61),VS(61),HS(61),CS(61),SS(61)
DIMENSION SP(61),XR(61),XS(61),XN(61),C(61)
COMMON/CM1/DT0,DT,DX,D,S0,PL,SEO,XK,RM
COMMON/CM3/NPTS,QIN,TIN
COMMON/CM4/V,H,C,VR,HR,CR,XR,SR,VS,HS,CS,XS,SS,XN
COMMON/CM5/MC,MN
COMMON/CM6/QP,VP,HP,CP
COMMON/CM8/INSOL,NSOL1,NSOL
COMMON/CM10/XPOS,XSOL,VPSOL,VSOL,OPSOL,HUS,VUS,CUS,HPUS,Vpus,CPus
THIS SUBROUTINE CALCULATES THE FLOW VELOCITY AND DEPTH
AND WAVE SPEED AT EACH OF THE NODES BETWEEN THE UPSTREAM
AND DOWNSTREAM BOUNDARIES BY SOLUTION OF THE TWO WAVE
EQUATIONS.
G=9.81
NZ=M
IF(MN.LE.MC) NZ=M+1
DO 1 I=2,NZ
IF(I.EQ.NSOL1) GOTO 1
IF(VSOL.GT.0.0.AND.I.EQ.NSOL+1) GOTO 1
X1=G/CR(1)
X3=G/CS(1)
X2=X1+HR(1)+VR(1)-G*(SR(1)-S0)*DT
X4=VS(1)-X3*HS(1)-G*(SS(1))-S0)*DT
HP(I)=(X2-X4)/(X1*X3)
VP(I)=X4+X3*HP(I)
CALL WAVSPD(HP(I),CP(1))
CALL SHAPE(TIME,HP(I),X1,X3,X4)
QP(I)=X1*VP(I)*1000.0
1 CONTINUE
RETURN
END
C C C C C

SUBROUTINE EXIT(TIME)
DIMENSION OP(61),VP(61),HP(61),CP(61)
DIMENSION V(61),M(61)
DIMENSION VR(61),HR(61),CR(61),VS(61),HS(61),CS(61),SS(61)
DIMENSION SP(61),XR(61),XS(61),XN(61),C(61)
COMMON/CM1/DT0,DT,DX,D,S0,PL,SEO,XK,RM
COMMON/CM2/N,TMAX
COMMON/CM5/MC,MN
COMMON/CM4/V,H,C,VR,HR,CR,XR,SR,VS,HS,CS,XS,SS,XN
COMMON/CM6/QP,VP,HP,CP
C
C
THIS SUBROUTINE CALCULATES THE FLOW DEPTH AT PIPE DISCHARGE
BASED ON THE ASSUMPTION OF CRITICAL DEPTH AT SUCH A BOUNDARY.

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G=9.81

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X1=G/CR(M+1)
X2=X1*HR(M+1)+VR(M+1)-G*(SR(M+1)-SO)*DT
UP=D
DN=0.0
HB=(UP+DN)/2.0
CALL SHAPE(TIME,HB,A,T,PER)
HEXIT=1.0-(ABS(X2-X1*HB)*(X2-X1*HB))*T/(G+A)
IF(HEXIT>1.2) GOTO 4
DN=HB
GOTO 4
UP=HB
GOTO 4
HBB=(UP+DN)/2.0
IF(ABS((HBB-HB)/HBB).LT.0.00001) GOTO 2
HB=HBB
GOTO 5
CONTINUE
VP(M+1)=X2-X1*HB
NP(M+1)=HB
CALL SHAPE(TIME,HB,A,T,PER)
CALL WAVSPD(HB,CP(M+1))
QP(M+1)=1000.0*A*VP(M+1)
RETURN
END

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#### SUBROUTINE SOLID(TIME)

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DIMENSION CP(6),JIN(30),TIN(30),VP(61),NP(61),CP(61)
DIMENSION V(61),H(61)
DIMENSION VF(61),HR(61),CR(61),VS(61),HS(61),CS(61),SS(61)
DIMENSION FS(2),SR(61),XR(61),XS(61),XN(61),C(61)
COMMON/CM1/TO,DT,DY,D,SO,FL,SEG,YR,CM
COMMON/CM2/R,TMAX
COMMON/CM3/NPTS,GIM,TIN
COMMON/CM4/V,F,C,VRF,HR,CR,XL,SH,VS,HS,CS,XS,SS,XN
COMMON/CM5/MC,MN
COMMON/CM6/CF,VP,HP,CP
COMMON/CM8/INSCL,NSOL1,NSCL
COMMON/CM9/SCLVEL,HDS,VDS,CDS,MPUS,VPOS,CPUS
COMMON/CM10/XPOS,XSOL,VPSOL,VSEL,OPSQL,MUS,VUS,CUS,MPUS,VPOS,CPUS
COMMON/CM11/TL,TH,TT,SH
C THIS SUBROUTINE CALCULATES THE FLOW DEPTH, LEAKAGE RATE AND
C WAVE SPEED AT THE DOWNSTREAM BOUNDARY FORMED BY AN INITIALLY
C STATIONARY SOLID.
C G=9.81
C CON=RM**2/SC
C OPSOL=OPSOL/1000.0
IZ=0
SEV=SEO
J=1
M0=MUS
60 CONTINUE
J=1
8000 CALL SHAPE(TIME,M0,A,T,PER)
X1=G/CR(NSCL)
X2=X1*HR(NSCL)+VP(NSCL)-G*(SR(NSCL)-SO)*DT
SE=IIUS*(VUS*2)/(2.0*G)
IF(SE.LT.SEO) GOTO 6

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 5  CONTINUE
 U=(X2-VPSOL)*A1/XK
 B=X1*A1/XK
 W=(X1**2)/(2.0**G)
 V=(X2**2)/(2.0**G)-SE0-VPSOL*X2/G+(VPSOL**2)/(2.0**G)
 Z=X1*VPSOL/G+1.0-X2*X2/G
 Z4=W*Z
 Z3=2.0*W*Z
 Z2=Z**2+2.0*W*Y
 Z1=2.0*Y**2+B
 ZC=Y*ARC(Y)-U
 F=Z4*H0**4+Z3*H0**3+Z2*H0**2+Z1*H0+Z0
 FS(J)=F
 IF(J.EQ.2) GOTO 7000
 J=2
 H0=H0+0.0001*HO
 GOTO 8000
 7000 DF=(FS(2)-FS(1))/(40/10000.0)
 DH=FS(1)/DF
 H1=H0-DH
 IF(ABS((H1-H0)/H0).LE.0.005) GOTO 70
 IF(H1.GT.D)H1=1.1*HO
 H0=H1
 IZ=IZ+1
 GOTO 60
 70 CALL SHAPE(TIME,H1,A2,T,PER)
 IF(ABS((A2-A1)/A1).LE.0.005) GOTO 80
 A1=(A1+A2)/2.0
 H0=H1
 GOTO 65
 80 CONTINUE
 HPUS=H1
 VPUS=X2-X1*H1
 QPSOL=VPUS*A2
 IF(VPUS.LT.VPSOL) GOTO 37
 GOTO 5
 37 CONTINUE
 VPUS=VPSOL
 HPUS=(X2-VPUS)/X1
 CALL SHAPE(TIME,HPUS,A2,T,PER)
 QPSOL=VPUS*A2
 GOTO 5
 6 VPUS=VPSOL
 HPUS=(X2-VPUS)/X1
 CALL SHAPE(TIME,HPUS,A2,T,PER)
 QPSOL=VPUS*A2
 GOTO 5
 5 CALL WAVSPD(HPUS,CBUS)
 SE0=SEV
 DT=DTO
 GOTO 806
 406 CONTINUE
 IF(HN.LE.HC) GOTO 8061
 GOTO 8062
 8061 CONTINUE
 Q=QPSOL
 IF(Q.LT.0.0) Q=0.05/(10.0*10.0**4)
 C CALCULATION OF NORMAL DEPTH.
 UP=D
 DN=0.0
 HB=(UP+DN)/2.0
 CON1HUL
 CALL SHAPE(TIME,HB,AREA,T,PER)

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HNDRM=1.0-(G*0.2)*COM/(C*AREA*3.333)/(PER*1.333)
IF(HNDRM)90,91,92
90  DB78893
92  UP=HB
93  HBB=(UP+DN)/2.0
    IF(ABS((HEB-HB)/HB).LE.0.001) GOTO 94
    HB=HBB
    GOTO 79
94  HB=HBB
91  CONTINUE
HPDS=HB
CALL SHAPE(TIME,HB,AREA,T,PER)
VPDS=0/AREA
OPDS=0*1000.0
OPSOL=OPDS
CPDS=SORT(G*AREA/T)
GOTO 934
8062 X3=G/CS(NSOL)
X4=VS(NSOL)-G*(SS(NS)L1-S0)*DT-X3*HS(NSOL)
IF(OPSOL.EC.C.C) GOTO 932
UP=D
DN=0.0
HB=(UP+DN)/2.0
15  CONTINUE
CALL SHAPE(TIME,HB,AREA,T,PER)
HFLUN=OPSOL-(X4*X3*18)*AREA
IF(HFLUN)16,17,18
16  UP=HB
GOTO 19
18  DN=HB
19  HBB=(UP+DN)/2.0
    IF(ABS((HBF-HB)/HB).LE.0.001) GOTO 20
    HB=HBB
    GOTO 15
20  HB=HBB
17  CONTINUE
HPDS=HB
VPDS=X4*X3*HPDS
CALL SHAPE(TIME,HB,AREA,T,PER)
OPSOL=AREA*VPDS*100.0
CPDS=SORT(G*AREA/T)
GOTO 933
932  CONTINUE
HPDS=-X4/X3
VPDS=0.0
CALL MAVSPD(1,PDS,CPDS)
933  CONTINUE
IF(VPSOL.GT.0.0) GOTO 934 .
GOTO 935
934  CONTINUE
DXX=DX/(XSOL-XN(NSOL-1))
VP(NSOL)=VP(NSOL-1)+JXX*(VPUS-VP(NSOL-1))
HP(NSOL)=HP(NSOL-1)+JXX*(HPUS-HP(NSOL-1))
CP(NSOL)=CP(NSOL-1)+JXX*(CPUS-CP(NSOL-1))
DXX=(XN(NSOL+1)-XSOL)/(XN(NSOL+2)-XSOL)
VP(NSOL+1)=VPDS+DXX*(VP(NSOL+2)-VPDS)
HP(NSOL+1)=HPDS+DXX*(HP(NSOL+2)-HPDS)
CP(NSOL+1)=CPDS+DXX*(CP(NSOL+2)-CPDS)
935  CONTINUE
RETURN
END
C
C

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SUBROUTINE INSERT(XN,TIMED)
DIMENSION XN(61),QP(61),VP(61),HP(61),CP(61)
COMMON/CM6/QP,VP,IP,CP
COMMON/CM6/INSOL,NSOL1,NSOL
COMMON/CM4/SOLVEL,HDS,VDS,CDS,MPDS,VPDS,CPDS
COMMON/CM10/XPOS,XSOL,VPSOL,VSOL,OPSOL,MUS,VUS,CUS,MPUS,VPUS,CPUS
C
INSOL=0
NSOL=NSOL1
VPUS=VP(NSOL)
MPUS=HP(NSOL)
CPUS=CP(NSOL)
VPDS=VP(NSOL)
MPDS=HP(NSOL)
CPDS=CP(NSOL)
OPSOL=OP(NSOL)
VSOL=VPUS
VPSOL=VSOL
DX=XN(NSOL)-XN(NSOL-1)
DXX=DX/(XSOL-XN(NSOL-1))
VP(NSOL)=VP(NSOL-1)+DXX*(VPUS-VP(NSOL-1))
HP(NSOL)=HP(NSOL-1)+DXX*(MPUS-MP(NSOL-1))
CP(NSOL)=CP(NSOL-1)+DXX*(CPUS-CP(NSOL-1))
DX=(XN(NSOL+1)-XSOL)/(XN(NSOL+2)-XSOL)
VP(NSOL+1)=VPDS+DXX*(VP(NSOL+2)-VPDS)
HP(NSOL+1)=HDS+DXX*(HP(NSOL+2)-MPDS)
CP(NSOL+1)=CPDS+DXX*(CP(NSOL+2)-CPDS)
RETURN
END
C
C
SUBROUTINE FORCE(TIME,XN,FSOL)
C
C THIS SUBROUTINE CALCULATES THE FORCE ACTING ON THE SOLID
C AND DETERMINES ITS VELOCITY AND POSITION AT THE END OF THE
C NEXT TIME STEP.
C
DIMENSION YN(61)
COMMON/CM1/DTC,DT,DX,D,S0,PL,SEC,XK,RM
COMMON/CM2/R,THMAX
COMMON/CM8/INSOL,NSOL1,NSOL
COMMON/CM9/SCLVEL,HDS,VDS,CDS,MPUS,VPDS,CPDS
COMMON/CM10/XPOS,XSOL,VPSOL,VSOL,OPSOL,MUS,VUS,CUS,MPUS,VPUS,CPUS
COMMON/CM11/TL,TW,TT,SH
COMMON/CM12/XFAC2,XFAC,TAU,FMOV,FSTAT
C
G=9.81
RHO=1000.0
FRICT=FSTAT
FM1=RHO*G*(HUS-TT/2.0)*TH*TT*TT/2.0
FM2=RHO*G*(TT/2.0)*TH*TT*0.5*(HDS-TT+HDS)
FM3=SH*G*TL*G*0.5*COS(A SIN((HUS-TT)/TL))
TX=(HDS-TT)/SIN(A SIN((HUS-TT)/TL))
FM4=XFAC2*FM1*G*0.5*(HDS-TT)*TH*G*0.5*TX**2
IF(HDS.LE.TT) FM4=0.0
FP=(0.5/TL)*(FM2+FM3+FM4-FM1)
FP2=(FP+FM4/10.5*TX)*COS(A SIN((HUS-TT)/TL))
IF(VSOL.GT.0.0)FRICT=FM0V
HSH=TT+0.5*(HPUS-TT)
CALL SHAPE(TIME,HSH,ASH,TSH,PER)
FLOOR=ASH-TT*TH

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...  
VPUTS=VP(I)  
VPDS=VP(I)  
CPUS=CP(I)  
CPDS=CP(I)  
GOTO 7  
1    CONTINUE  
SOLVE L=VSOL  
VSOL=VPSOL  
IF(VPSOL.GE.0.0) GOTO 3  
GOTO 4  
3    VSOL=VPSOL  
HUS=VPUTS  
VUS=VPUTS  
CUS=CPUS  
HDS=HPDS  
VDS=VPDS  
CDS=CPDS  
4    CONTINUE  
RETURN  
END  
$BEND
```



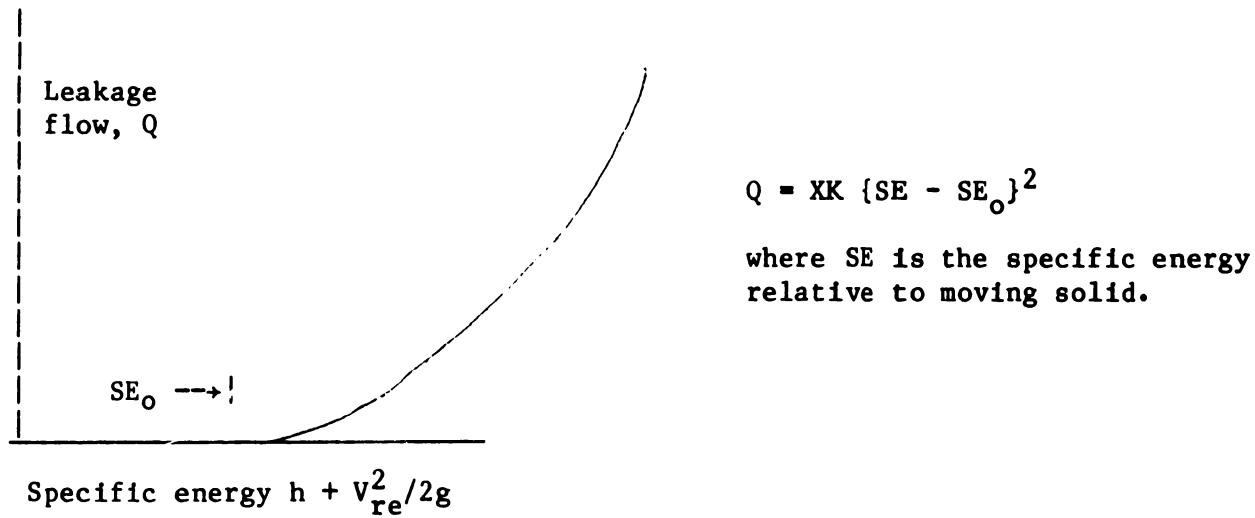
## **Appendix 2**

**Derivation of solid boundary equations  
used in Subroutine SOLID**

Derivation of solid boundary equations employed in Subroutine SOLID.

Available equations at moving solid boundary.

1) Leakage flow past solid.

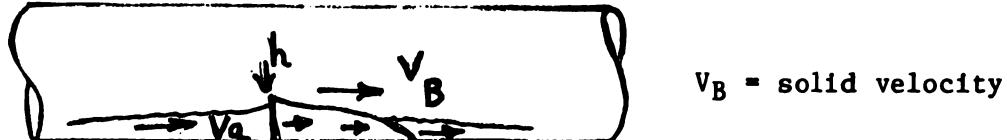


$$Q = XK \{SE - SE_0\}^2$$

where  $SE$  is the specific energy relative to moving solid.

$XK, SE_0$  determined by measurement.

2) Flow over solid in terms of flow area



$v_B$  = solid velocity

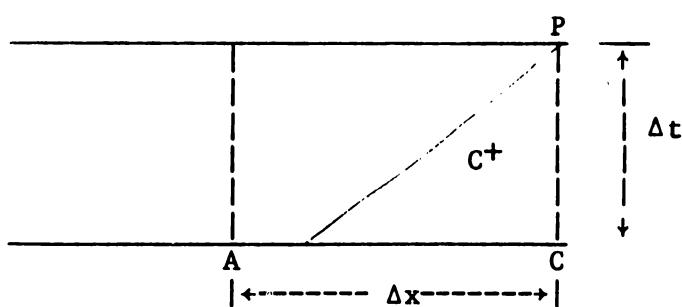
$$\text{Relative velocity } v_{re} = v_a - v_B$$

$$\text{Flow area } A = f(h)$$

$$\text{Total flow} = v_a A = v_a f(h)$$

$$\text{Flow past solid} = v_{re} A = v_{re} f(h).$$

3) Characteristic equation,  $C^+$  in both sub and supercritical flow conditions.



$C^+$  links R at  $t$  to P at  $t + \Delta t$

$C^+$  equation  $v_p = x_2 - z_1 h_p$

$$x_1 = g/cR$$

$$x_2 = v_R + gh_R/cR - g(s_R - s_o)\Delta t$$

Note characteristics equations in terms of absolute velocity.

The flow Q over the solid is given by the empirical relation,

$$Q = XK \left( h + \frac{v_{re}^2}{2g} - SE_o \right)^2$$

where  $v_{re}$  is the flow velocity relative to the moving solid.

$$Q = v_{re}A = (v_{abs} - v_B)A = (x_2 - x_1 h - v_B)A$$

$$\text{Also, } Q = XK \{ SE - SE_o \}^2$$

$$= XK \left\{ h + \frac{v_{abs}^2 - v_B^2}{2g} - SE_o \right\}^2$$

Hence

$$(x_2 - x_1 h - v_B)A = XK \left\{ h + \frac{(x_2 - x_1 h - v_B)^2}{2g} - SE_o \right\}^2$$

Collecting terms

$$U = (x_2 - v_B)A/XK; B = XI A/XK;$$

$$W = (x_1^2)/2g; Y = x_2^2/2g - SE_o - v_B x_2/g + v_B^2/2g;$$

$$Z = XI \cdot v_B/g + 1.0 \cdot X_2 X_1/g$$

$$U - Bh = \{Zh + Y + Wh^2\}^2$$

$$= Z^2 h^2 + Y^2 + W^2 h^4 + 2YZh^2 + 2ZYh + 2ZHh^3$$

$$W^2 h^4 + 2ZHh^3 + (Z^2 + 2Yh)h^2 + (2ZY + B)h + (Y \cdot 1Y1 - U) = 0$$

Note:  $Y.ABS |Y|$  retains sign Y.

This quartic is solved by the Newton Raphson method as B and U involve f(h) through the flow area term.



**APPENDIX 3**  
**Typical Output Program TRANSCD**

TEST PIPE CONFIGURATION:-  
 DIAMETER = 0.1000 M. ROLLING COEFFICIENTS = 0.0150  
 PIPE SLOPE = 0.0101 PIPE LENGTH = 15.0000 M.  
  
 FLOW SUPERCRITICAL, NORMAL DEPTH = 0.0144 M. AND CRITICAL DEPTH = 0.3118 M.  
  
 INITIAL FLOW RATE = 0.2000 L/S. INITIAL DEPTH = 0.0144 M. INITIAL WAVE SPEED = 0.3118 M/S.  
 CALCULATION TIME STEP = 0.4961 S.LENGTH INCREMENT = 0.3000 M. NUMBER OF NODES = 51 XFAC = 1.00  
 SOLID DESCRIPTION DATA.  
 LENGTH = 270.00 MM. WIDTH = 20.00 MM. THICKNESS = 2.00 MM. SATURATED WT. = 250.00 GM.  
 SOLID POSITION, SECTION NUMBER = 10 DISTANCE FROM ENTRY = 2.700 M.  
  
 STATIC FRICTION FACTOR = 0.150 SLIDING FRICTION FACTOR = 0.100  
 SOLID SURFACE SWELL COEFF. = 0.  
  
 BOUNCING FACTOR XFAC = 1.00  
 DOWNTHRUST FACTOR XFAC2 = 0.  
  
 SOLID POSITION IN INFLOW PROFILE = 4.00 SECONDS FROM FLOW INCEPTION.

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	POSITION FROM ENTRY M.	0.	1.500	3.000	4.500	6.000	7.500	9.000	10.500	12.000	13.500	15.000
TIME = 0. S. DEPTH	M=	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0130
VELOCITY M/S=	0.2075	0.2075	0.2076	0.2077	0.2078	0.2080	0.2082	0.2083	0.2084	0.2085	0.2085	0.3054
FLOW RATE L/S=	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000
WAVE SPD. M/S=	0.3118	0.3118	0.3117	0.3117	0.3116	0.3116	0.3115	0.3115	0.3114	0.3114	0.3114	0.3052
TIME = 0.49 S. DEPTH	M=	0.0341	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0130
VELOCITY M/S=	0.9077	0.2075	0.2076	0.2077	0.2078	0.2078	0.2079	0.2079	0.2080	0.2080	0.2080	0.3051
FLOW RATE L/S=	0.1466	0.1999	0.1999	0.1999	0.1999	0.1999	0.1999	0.1999	0.1999	0.1999	0.1999	0.1999
WAVE SPD. M/S=	0.4947	0.3118	0.3117	0.3117	0.3116	0.3116	0.3115	0.3115	0.3114	0.3114	0.3114	0.3051
TIME = 0.70 S. DEPTH	M=	0.0395	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0130
VELOCITY M/S=	1.0396	0.2075	0.2075	0.2076	0.2076	0.2077	0.2077	0.2078	0.2078	0.2078	0.2078	0.3046
FLOW RATE L/S=	0.0021	0.1999	0.1999	0.1999	0.1999	0.1999	0.1999	0.1999	0.1999	0.1999	0.1999	0.1999
WAVE SPD. M/S=	0.9374	0.3118	0.3117	0.3117	0.3116	0.3116	0.3115	0.3115	0.3114	0.3114	0.3114	0.3046
TIME = 0.89 S. DEPTH	M=	0.0451	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0137
VELOCITY M/S=	1.0930	0.2075	0.2075	0.2075	0.2076	0.2076	0.2077	0.2077	0.2077	0.2077	0.2077	0.3042
FLOW RATE L/S=	0.0026	0.1999	0.1999	0.1999	0.1999	0.1999	0.1999	0.1999	0.1999	0.1999	0.1999	0.1976
WAVE SPD. M/S=	0.9821	0.3118	0.3117	0.3117	0.3116	0.3116	0.3115	0.3115	0.3114	0.3114	0.3114	0.3042
TIME = 1.07 S. DEPTH	M=	0.0491	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0137
VELOCITY M/S=	1.0941	0.2075	0.2075	0.2075	0.2075	0.2075	0.2075	0.2075	0.2076	0.2076	0.2076	0.3040
FLOW RATE L/S=	0.0029	0.1999	0.1999	0.1999	0.1999	0.1999	0.1999	0.1999	0.1999	0.1999	0.1999	0.1970
WAVE SPD. M/S=	0.9147	0.3118	0.3117	0.3117	0.3116	0.3116	0.3115	0.3115	0.3114	0.3114	0.3114	0.3040
TIME = 1.25 S. DEPTH	M=	0.0491	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0137
VELOCITY M/S=	1.0420	0.2075	0.2075	0.2075	0.2075	0.2075	0.2075	0.2075	0.2075	0.2075	0.2075	0.3038
FLOW RATE L/S=	0.0033	0.1999	0.1999	0.1999	0.1999	0.1999	0.1999	0.1999	0.1999	0.1999	0.1999	0.1966
WAVE SPD. M/S=	0.6134	0.3118	0.3117	0.3117	0.3116	0.3116	0.3115	0.3115	0.3114	0.3114	0.3114	0.3038

TIME = 1.42 S. DEPTH = M= 0.0495 0.0194 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0137  
 VELOCITY M/S= 0.6914 0.3179 0.2874 0.2875 0.2875 0.2875 0.2875 0.2875 0.2875 0.2875 0.2875 0.2875 0.3037  
 FLOW RATE L/S= 3.6794 0.2430 0.2149 0.1946 0.1947 0.1946 0.1945 0.1945 0.1945 0.1945 0.1945 0.1945 0.1963  
 WAVE SPD. M/S= 0.6184 0.1225 0.1117 0.1117 0.3116 0.3116 0.3115 0.3115 0.3115 0.3115 0.3115 0.3114 0.3037  
  
 TIME = 1.60 S. DEPTH = M= 0.0496 0.0202 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0137  
 VELOCITY M/S= 0.6429 0.3179 0.2874 0.2874 0.2874 0.2874 0.2874 0.2874 0.2874 0.2874 0.2874 0.2874 0.3036  
 FLOW RATE L/S= 3.6799 0.2432 0.1948 0.1947 0.1947 0.1946 0.1945 0.1945 0.1945 0.1945 0.1945 0.1945 0.1961  
 WAVE SPD. M/S= 0.6187 0.1220 0.1117 0.1117 0.3116 0.3116 0.3115 0.3115 0.3115 0.3115 0.3115 0.3114 0.3036  
  
 TIME = 1.79 S. DEPTH = M= 0.0500 0.0245 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0137  
 VELOCITY M/S= 0.6791 0.3179 0.2874 0.2874 0.2874 0.2874 0.2874 0.2874 0.2874 0.2874 0.2874 0.2874 0.3036  
 FLOW RATE L/S= 3.6185 1.7721 0.1948 0.1947 0.1947 0.1946 0.1945 0.1945 0.1945 0.1945 0.1945 0.1945 0.1959  
 WAVE SPD. M/S= 0.6213 0.6180 0.1117 0.1117 0.3115 0.3116 0.3115 0.3115 0.3115 0.3115 0.3115 0.3114 0.3036  
  
 TIME = 1.98 S. DEPTH = M= 0.0500 0.0246 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0137  
 VELOCITY M/S= 0.6543 0.3159 0.2874 0.2874 0.2874 0.2874 0.2874 0.2874 0.2874 0.2874 0.2874 0.2874 0.3035  
 FLOW RATE L/S= 3.3598 1.0475 0.1948 0.1947 0.1946 0.1946 0.1945 0.1945 0.1945 0.1945 0.1945 0.1945 0.1958  
 WAVE SPD. M/S= 0.6281 0.6274 0.1117 0.1117 0.3116 0.3116 0.3115 0.3115 0.3115 0.3115 0.3115 0.3114 0.3035  
  
 TIME = 2.17 S. DEPTH = M= 0.0500 0.0245 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0137  
 VELOCITY M/S= 0.6145 0.3179 0.2874 0.2874 0.2874 0.2874 0.2874 0.2874 0.2874 0.2874 0.2874 0.2874 0.3035  
 FLOW RATE L/S= 3.2034 3.0682 0.1948 0.1947 0.1946 0.1945 0.1945 0.1945 0.1945 0.1945 0.1945 0.1945 0.1957  
 WAVE SPD. M/S= 0.6281 0.5649 0.3117 0.3117 0.3116 0.3116 0.3115 0.3115 0.3115 0.3115 0.3115 0.3114 0.3035  
  
 TIME = 2.36 S. DEPTH = M= 0.0499 0.0243 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0137  
 VELOCITY M/S= 0.7762 0.4000 0.2876 0.2876 0.2876 0.2876 0.2876 0.2876 0.2876 0.2876 0.2876 0.2876 0.3035  
 FLOW RATE L/S= 3.0367 3.0245 0.2001 0.1947 0.1946 0.1945 0.1945 0.1945 0.1945 0.1945 0.1945 0.1945 0.1957  
 WAVE SPD. M/S= 0.6195 0.7260 0.3116 0.3117 0.3116 0.3116 0.3115 0.3115 0.3115 0.3115 0.3115 0.3114 0.3035  
  
 TIME = 2.56 S. DEPTH = M= 0.0495 0.0240 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0137  
 VELOCITY M/S= 0.7396 0.3853 0.2897 0.2874 0.2873 0.2873 0.2873 0.2873 0.2873 0.2873 0.2873 0.2872 0.3035  
 FLOW RATE L/S= 2.8648 2.9669 0.2024 0.1947 0.1946 0.1945 0.1945 0.1945 0.1945 0.1945 0.1945 0.1945 0.1956  
 WAVE SPD. M/S= 0.6164 0.5814 0.3126 0.3117 0.3116 0.3116 0.3115 0.3115 0.3115 0.3115 0.3115 0.3114 0.3035  
  
 TIME = 2.75 S. DEPTH = M= 0.0490 0.0245 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0137  
 VELOCITY M/S= 0.7064 0.3831 0.3120 0.2874 0.2873 0.2873 0.2873 0.2873 0.2873 0.2873 0.2872 0.2872 0.3035  
 FLOW RATE L/S= 2.7015 2.8459 0.2194 0.1947 0.1946 0.1945 0.1945 0.1945 0.1945 0.1945 0.1945 0.1945 0.1956  
 WAVE SPD. M/S= 0.6126 0.5851 0.3205 0.3117 0.3116 0.3116 0.3115 0.3115 0.3115 0.3115 0.3115 0.3114 0.3035  
  
 TIME = 2.95 S. DEPTH = M= 0.0482 0.0240 0.0226 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0137  
 VELOCITY M/S= 0.6734 0.3834 0.5357 0.2873 0.2873 0.2873 0.2873 0.2873 0.2873 0.2872 0.2872 0.2872 0.3035  
 FLOW RATE L/S= 2.5243 2.5147 0.7108 0.1947 0.1946 0.1945 0.1945 0.1945 0.1945 0.1945 0.1945 0.1945 0.1959  
 WAVE SPD. M/S= 0.6046 0.5873 0.3966 0.3117 0.3116 0.3116 0.3115 0.3115 0.3115 0.3115 0.3115 0.3114 0.3035  
  
 TIME = 3.16 S. DEPTH = M= 0.0473 0.0249 0.0141 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0137  
 VELOCITY M/S= 0.6430 0.7759 0.7905 0.2873 0.2873 0.2873 0.2873 0.2873 0.2872 0.2872 0.2872 0.2871 0.3035  
 FLOW RATE L/S= 2.7946 2.7771 1.8847 0.1947 0.1946 0.1945 0.1945 0.1945 0.1945 0.1945 0.1945 0.1945 0.1959  
 WAVE SPD. M/S= 0.5992 0.5882 0.4147 0.3117 0.3116 0.3116 0.3115 0.3115 0.3115 0.3115 0.3115 0.3114 0.3035  
  
 TIME = 3.36 S. DEPTH = M= 0.0464 0.0258 0.0170 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0137  
 VELOCITY M/S= 0.6126 0.7445 0.4321 0.2873 0.2873 0.2873 0.2873 0.2873 0.2872 0.2872 0.2872 0.2871 0.3035  
 FLOW RATE L/S= 2.1744 2.0284 2.0210 0.1947 0.1946 0.1945 0.1945 0.1945 0.1945 0.1945 0.1945 0.1945 0.1959  
 WAVE SPD. M/S= 0.5909 0.5876 0.5182 0.3117 0.3116 0.3116 0.3115 0.3115 0.3115 0.3115 0.3115 0.3114 0.3035  
  
 TIME = 3.57 S. DEPTH = M= 0.0449 0.0255 0.0184 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0137  
 VELOCITY M/S= 0.5834 0.7218 0.4147 0.2873 0.2873 0.2873 0.2873 0.2873 0.2872 0.2872 0.2871 0.3034  
 FLOW RATE L/S= 1.4934 2.5200 2.6761 0.1947 0.1946 0.1945 0.1945 0.1945 0.1945 0.1945 0.1945 0.1945 0.1955  
 WAVE SPD. M/S= 0.6815 0.4457 0.5970 0.3117 0.3116 0.3116 0.3115 0.3115 0.3115 0.3115 0.3115 0.3114 0.3034  
  
 TIME = 3.79 S. DEPTH = M= 0.0434 0.0252 0.0195 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0144 0.0137  
 VELOCITY M/S= 0.6443 0.6994 0.7719 0.2873 0.2873 0.2873 0.2873 0.2873 0.2872 0.2872 0.2871 0.3034  
 FLOW RATE L/S= 1.8111 2.6113 2.2616 0.2001 0.1946 0.1945 0.1945 0.1945 0.1945 0.1945 0.1945 0.1945 0.1955

WAVE SPD. M/S= 0.5997 0.61829 0.60277 0.6168 1.1117 0.6116 0.6115 0.6115 0.6115 0.6115 0.6115 0.6115

FSOL= -0.1843 FP2= 0.6194 FSUR= 0.1843

SOLID VELOCITY  
= 0.758 M/S.

POSITION FROM  
ENTRY 4. 0. 2.100 2.900 2.700 2.700 4.200 5.700 15.000  
TIME = 4.00 S. DEPTH = 0.0417 0.0440 0.0477 0.0416 0.0413 0.0183 0.0199 0.0177  
VELOCITY M/S= 0.5250 0.7167 0.7567 0.7562 0.9069 0.2671 0.1034  
FLOW RATE L/S= 1.0267 2.1942 2.1599 2.1598 0.1796 0.1976 0.1955  
WAVE SPD. M/S= 0.5997 0.6137 0.5632 0.5528 0.5556 0.3530 0.3117 0.3034  
FSOL= -0.1207 FP2= 0.6079 FSUR= 0.1843

SOLID VELOCITY  
= 0.599 M/S.

POSITION FROM  
ENTRY 4. 0. 2.100 2.900 2.997 2.897 4.200 5.700 15.000  
TIME = 4.22 S. DEPTH = 0.0398 0.0439 0.0424 0.0436 0.0417 0.0280 0.0199 0.0137  
VELOCITY M/S= 0.4950 0.6494 0.7242 0.7055 0.7471 0.6650 0.2671 0.3034  
FLOW RATE L/S= 1.0411 2.3078 2.0138 2.0120 2.0137 1.1485 0.1976 0.1955  
WAVE SPD. M/S= 0.5992 0.6171 0.5610 0.5706 0.5556 0.4435 0.3117 0.3034  
FSOL= -0.0012 FP2= 0.6051 FSUR= 0.1847

SOLID VELOCITY  
= 0.496 M/S.

POSITION FROM  
ENTRY 4. 0. 0. 1.500 2.900 2.906 2.906 4.500 6.000 15.000  
TIME = 4.44 S. DEPTH = 0.0376 0.0376 0.0434 0.0477 0.0418 0.0243 0.0199 0.0137  
VELOCITY M/S= 0.4919 0.6464 0.6286 0.6168 0.7176 0.5758 0.2871 0.3034  
FLOW RATE L/S= 1.0259 1.2459 2.0539 2.2233 2.2923 0.6474 0.1976 0.1956  
WAVE SPD. M/S= 0.5927 0.6127 0.5682 0.6049 0.5569 0.4104 0.3117 0.3034  
FSOL= -0.0165 FP2= 0.6054 FSUR= 0.1847

SOLID VELOCITY  
= 0.449 M/S.

POSITION FROM  
ENTRY 4. 0. 0. 1.500 3.074 3.074 4.500 6.000 15.000  
TIME = 4.66 S. DEPTH = 0.0365 0.0365 0.0426 0.0479 0.0420 0.0111 0.0199 0.0137  
VELOCITY M/S= 0.4910 0.6460 0.6595 0.6156 0.7117 0.7149 0.2871 0.3034  
FLOW RATE L/S= 1.0165 1.1665 1.9312 2.2613 2.2633 1.4912 0.1976 0.1956  
WAVE SPD. M/S= 0.5136 0.5136 0.5623 0.6006 0.5573 0.4701 0.3117 0.3034  
FSOL= -0.0061 FP2= 0.6052 FSUR= 0.1847

SOLID VELOCITY  
= 0.470 M/S.

POSITION FROM  
ENTRY 4. 0. 0. 1.500 3.179 3.179 4.500 6.000 15.000  
TIME = 4.89 S. DEPTH = 0.0357 0.0357 0.0417 0.0477 0.0417 0.0115 0.0199 0.0117  
VELOCITY M/S= 0.4901 0.6451 0.6553 0.6131 0.6073 0.7149 0.7125 0.2871 0.3034  
FLOW RATE L/S= 1.0125 1.1705 1.9367 2.2305 2.2302 1.6023 0.1976 0.1956  
WAVE SPD. M/S= 0.5173 0.5173 0.5552 0.6027 0.5571 1.4999 0.3117 0.3034  
FSOL= -0.0028 FP2= 0.6052 FSUR= 0.1847

SOLID VELOCITY  
= 0.465 m/s.

POSITION FROM  
ENTRY N.  
TIME = 5.11 S. DEPTH = 0.0349 0.0121 0.0415 0.0578 0.0421 0.0121 0.0144 0.0137  
VELOCITY M/S = 0.4411 0.0269 0.5154 0.5903 0.7095 0.7117 0.2673 0.3034  
FLOW RATE L/S = 1.0700 1.1916 1.1056 2.2192 2.2192 1.5557 0.1976 1.1490  
WAVE SPD. M/S = 0.5019 0.9778 0.5541 0.6017 0.5597 0.4785 0.3117 0.3034  
FSOL = -0.0177 FP2 = 0.6054 FSUR = 0.1847

SOLID VELOCITY  
= 0.458 m/s.

POSITION FROM  
ENTRY N.  
TIME = 5.34 S. DEPTH = 0.0342 0.0116 0.0406 0.0474 0.0420 0.0138 0.0144 0.0137  
VELOCITY M/S = 0.4349 0.0152 0.5636 0.5886 0.6903 0.7104 0.2673 0.3034  
FLOW RATE L/S = 1.0304 1.0490 1.6885 2.1288 2.1588 1.6560 0.1997 1.1456  
WAVE SPD. M/S = 0.4496 0.9793 0.5670 0.6003 0.5575 0.4917 0.3117 0.3034  
FSOL = -0.0093 FP2 = 0.6092 FSUR = 0.1847

SOLID VELOCITY  
= 0.442 m/s.

POSITION FROM  
ENTRY N.  
TIME = 5.57 S. DEPTH = 0.0334 0.0112 0.0396 0.0477 0.0418 0.0151 0.0144 0.0137  
VELOCITY M/S = 0.4291 0.5035 0.5917 0.5741 0.6807 0.7013 0.2673 0.3034  
FLOW RATE L/S = 0.4771 1.0537 1.5705 2.1190 2.1190 1.7230 0.2603 1.1956  
WAVE SPD. M/S = 0.4891 0.9707 0.5392 0.6022 0.5569 0.5025 0.3118 0.3034  
FSOL = -0.0049 FP2 = 0.6091 FSUR = 0.1847

SOLID VELOCITY  
= 0.437 m/s.

POSITION FROM  
ENTRY N.  
TIME = 5.80 S. DEPTH = 0.0326 0.0109 0.0397 0.0479 0.0421 0.0149 0.0144 0.0137  
VELOCITY M/S = 0.4219 0.4496 0.5646 0.5710 0.6762 0.6927 0.2673 0.3034  
FLOW RATE L/S = 0.4345 1.0592 1.5818 2.1244 2.1244 1.6309 0.1979 1.1956  
WAVE SPD. M/S = 0.4826 0.6938 0.5196 0.6094 0.5587 0.4918 0.3117 0.3034  
FSOL = -0.0195 FP2 = 0.6056 FSUR = 0.1847

SOLID VELOCITY  
= 0.432 m/s.

POSITION FROM  
ENTRY N.  
TIME = 6.03 S. DEPTH = 0.0319 0.0105 0.0407 0.0477 0.0413 0.0151 0.0145 0.0137  
VELOCITY M/S = 0.4113 0.4199 0.5616 0.5616 0.6572 0.6658 0.2673 0.3034  
FLOW RATE L/S = 0.4793 1.0155 1.5476 2.0566 2.0566 1.6262 0.2603 1.1956  
WAVE SPD. M/S = 0.4700 0.6196 0.5118 0.5995 0.5563 0.5040 0.3126 0.3034  
FSOL = -0.0033 FP2 = 0.6091 FSUR = 0.1847

\* 0.414 m/s.

POSITION FROM  
ENTRY N.  
TIME = 6.26 s. DEPTH = 0.0711 0.0129 0.0378 0.0674 0.0915 0.0364 0.0120 0.0117  
VELOCITY M/S= 0.4023 0.4109 0.5097 0.5951 0.6692 0.6781 0.3070 0.1735  
FLOW RATE L/S= 0.8495 0.8716 1.1703 1.7989 1.7959 1.7510 0.2271 0.1956  
WAVE SPD. M/S= 0.4700 0.4951 0.5241 0.6054 0.5517 0.5231 0.3137 0.3039  
FSOL= -0.0010 FP2= 0.6042 FSUR= 0.1847

SOLID VELOCITY  
\* 0.411 m/s.

POSITION FROM  
ENTRY N.  
TIME = 6.30 s. DEPTH = 0.0303 0.0137 0.0359 0.0477 0.0418 0.0355 0.0197 0.0137  
VELOCITY M/S= 0.3900 0.4279 0.5460 0.5490 0.6479 0.6717 0.2969 0.3035  
FLOW RATE L/S= 0.7997 0.8792 1.1135 2.0120 2.0120 1.6754 0.2127 0.1956  
WAVE SPD. M/S= 0.4631 0.4913 0.5040 0.6028 0.5660 0.5057 0.3171 0.3039  
FSOL= -0.0205 FP2= 0.6050 FSUR= 0.1847

SOLID VELOCITY  
\* 0.410 m/s.

POSITION FROM  
ENTRY N.  
TIME = 6.73 s. DEPTH = 0.0295 0.0331 0.0351 0.0467 0.0412 0.0364 0.0161 0.0137  
VELOCITY M/S= 0.3895 0.4204 0.5302 0.5355 0.6792 0.6639 0.3397 0.3035  
FLOW RATE L/S= 0.7947 0.8559 1.3051 1.9223 1.7223 1.7168 0.2779 0.1957  
WAVE SPD. M/S= 0.4415 0.4667 0.5029 0.5945 0.5517 0.5134 0.3336 0.3035  
FSOL= -0.0030 FP2= 0.6055 FSUR= 0.1847

SOLID VELOCITY  
\* 0.390 m/s.

POSITION FROM  
ENTRY N.  
TIME = 6.97 s. DEPTH = 0.0287 0.0329 0.0345 0.0469 0.0408 0.0372 0.0208 0.0137  
VELOCITY M/S= 0.3747 0.4127 0.5169 0.5173 0.6146 0.6559 0.4545 0.3035  
FLOW RATE L/S= 0.7071 0.7153 1.2918 1.8560 1.8660 1.7952 0.5132 0.1957  
WAVE SPD. M/S= 0.4444 0.4619 0.4979 0.5959 0.5447 0.5198 0.3746 0.3035  
FSOL= -0.0013 FP2= 0.6054 FSUR= 0.1847

SOLID VELOCITY  
\* 0.387 m/s.

POSITION FROM  
ENTRY N.  
TIME = 7.21 s. DEPTH = 0.0278 0.0329 0.0344 0.0471 0.0411 0.0364 0.0110 0.0137  
VELOCITY M/S= 0.3687 0.4106 0.4951 0.5156 0.5141 0.6512 0.3960 0.1035  
FLOW RATE L/S= 0.6773 0.6917 1.2796 1.8779 1.8779 1.6916 0.3775 0.1957  
WAVE SPD. M/S= 0.4416 0.4590 0.5292 0.5762 0.5526 0.5114 0.3679 0.1035  
FSOL= -0.0211 FP2= 0.6054 FSUR= 0.1847

SOLID VELOCITY  
\* 0.385 m/s.

POSITION FROM

ENTRY 4. 0. 1.200 2.200 4.250 4.250 5.200 7.200 15.000  
 TIME = 2.45 S. DEPTH M= 0.0749 0.0121 0.0177 1.0460 0.1911 0.0171 0.0241 0.0117  
 VELOCITY M/S= 0.1724 0.4111 0.6904 0.5969 0.5973 0.6527 0.5177 0.3035  
 FLOW RATE L/S= 0.7191 0.1119 1.0151 1.7197 1.7197 1.7117 0.6993 0.1737  
 WAVE SPD. M/S= 0.4161 0.8101 0.5719 0.5111 1.0509 0.1134 0.3062 0.1119  
 FSOL= -0.0050 FP2= 0.6054 FSUR= 0.1847

SOLID VELOCITY  
 = 0.360 M/S.

POSITION FROM  
 ENTRY 4. 0. 1.200 2.200 4.349 4.349 5.200 7.200 15.000  
 TIME = 2.69 S. DEPTH M= 0.0740 0.0117 0.0171 0.0460 0.0400 0.0176 0.0234 0.0117  
 VELOCITY M/S= 0.1749 0.4014 0.6906 0.4870 0.5877 0.6133 0.5966 0.3035  
 FLOW RATE L/S= 0.7178 0.1039 1.0150 1.7224 1.7224 1.7110 1.0027 0.1957  
 WAVE SPD. M/S= 0.4174 0.8151 0.5142 0.5896 0.5417 0.5230 0.4328 0.3035  
 FSOL= -0.0003 FP2= 0.6056 FSUR= 0.1847

SOLID VELOCITY  
 = 0.360 M/S.

POSITION FROM  
 ENTRY 4. 0. 1.500 3.000 4.437 4.437 6.000 7.500 15.000  
 TIME = 2.94 S. DEPTH M= 0.0251 0.0120 0.0380 0.0460 0.0400 0.0169 0.0247 0.0117  
 VELOCITY M/S= 0.3319 0.6114 0.6274 0.4933 0.5457 0.6303 0.5575 0.3035  
 FLOW RATE L/S= 0.5118 0.8933 1.0165 1.7190 1.7190 1.6613 0.8527 0.1950  
 WAVE SPD. M/S= 0.4127 0.4727 0.5427 0.5911 0.5421 0.5176 0.4139 0.3035  
 FSOL= -0.0202 FP2= 0.6064 FSUR= 0.1846

SOLID VELOCITY  
 = 0.360 M/S.

POSITION FROM  
 ENTRY 4. 0. 1.500 3.000 4.524 4.524 6.000 7.500 15.000  
 TIME = 3.18 S. DEPTH M= 0.0241 0.0114 0.0274 0.0451 0.0394 0.0174 0.0278 0.0117  
 VELOCITY M/S= 0.3180 0.6142 0.6216 0.4754 0.5665 0.6210 0.6039 0.3035  
 FLOW RATE L/S= 0.4739 0.1146 1.0297 1.6361 1.6361 1.6635 1.0777 0.1958  
 WAVE SPD. M/S= 0.4090 0.4726 0.5212 0.5821 0.5374 0.5211 0.4420 0.3035  
 FSOL= -0.0049 FP2= 0.6062 FSUR= 0.1847

SOLID VELOCITY  
 = 0.360 M/S.

POSITION FROM  
 ENTRY 4. 0. 1.500 3.000 4.610 4.610 6.000 7.500 15.000  
 TIME = 3.43 S. DEPTH M= 0.0231 0.0109 0.0169 0.0451 0.0369 0.0177 0.0277 0.0117  
 VELOCITY M/S= 0.3025 0.1961 0.4169 0.4561 0.5560 0.6110 0.6168 0.3035  
 FLOW RATE L/S= 0.4149 0.1151 1.0261 1.5607 1.5602 1.6539 1.2029 0.1758  
 WAVE SPD. M/S= 0.3490 0.5625 0.5169 0.5811 0.5224 0.5215 0.4531 0.3035  
 FSOL= -0.0011 FP2= 0.6062 FSUR= 0.1846

SOLID VELOCITY  
 = 0.360 M/S.

POSITION FROM  
 ENTRY 4. 0. 1.500 3.000 4.694 4.694 6.000 7.500 15.000  
 TIME = 3.68 S. DEPTH M= 0.0220 0.0107 0.0161 0.0451 0.0349 0.0179 0.0311 0.0117  
 VELOCITY M/S= 0.2849 0.1841 0.4119 0.4516 0.5525 0.6087 0.6167 0.3035

FLOW RATE L/S= 0.3697 0.7748 1.09603 1.55505 1.55504 1.6169 1.7975 0.1959  
 WAVE SPD. M/S= 0.3193 0.4622 0.5126 0.5820 0.5116 0.5250 0.4711 0.3035  
 FSOL= 0.0062 FP2= 0.6074 FSUR= 0.1847

SOLID VELOCITY  
 = 0.335 M/S.

POSITION FROM  
 ENTRY N. 0. 1.000 3.300 4.777 4.777 6.300 7.800 15.000  
 TIME = 0.92 S. DEPTH M= 0.0208 0.3308 0.0371 0.0456 0.0391 0.0173 0.0301 0.0137  
 VELOCITY M/S= 0.2648 0.1964 0.4099 0.4542 0.5576 0.5493 0.6110 0.3035  
 FLOW RATE L/S= 0.3194 0.8058 1.0883 1.5460 1.5460 1.6025 1.2473 0.1959  
 WAVE SPD. M/S= 0.3784 0.4652 0.4192 0.5864 0.5347 0.5207 0.4616 0.3035  
 FSOL= -0.0197 FP2= 0.6062 FSUR= 0.1846

SOLID VELOCITY  
 = 0.341 M/S.

POSITION FROM  
 ENTRY N. 0. 1.000 3.300 4.861 4.861 6.300 7.800 15.000  
 TIME = 0.97 S. DEPTH M= 0.0196 0.0249 0.0366 0.0443 0.0385 0.0374 0.0319 0.0137  
 VELOCITY M/S= 0.2436 0.1487 0.4055 0.4521 0.5443 0.5887 0.6104 0.3035  
 FLOW RATE L/S= 0.2442 0.7679 1.0558 1.5177 1.5177 1.5901 1.2839 0.1959  
 WAVE SPD. M/S= 0.3463 0.4600 0.5144 0.5762 0.5335 0.5216 0.4621 0.3035  
 FSOL= -0.0002 FP2= 0.6074 FSUR= 0.1846

SOLID VELOCITY  
 = 0.321 M/S.

POSITION FROM  
 ENTRY N. 0. 1.000 3.300 4.944 4.944 6.300 7.800 15.000  
 TIME = 0.93 S. DEPTH M= 0.0182 0.0293 0.0361 0.0446 0.0381 0.0374 0.0329 0.0137  
 VELOCITY M/S= 0.2189 0.1407 0.4010 0.4517 0.5342 0.5780 0.6076 0.3036  
 FLOW RATE L/S= 0.2186 0.7748 1.0239 1.4971 1.4971 1.5921 1.2936 0.1959  
 WAVE SPD. M/S= 0.3529 0.4595 0.5106 0.5779 0.5266 0.5217 0.4808 0.3036  
 FSOL= 0.0004 FP2= 0.6074 FSUR= 0.1846

SOLID VELOCITY  
 = 0.321 M/S.

POSITION FROM  
 ENTRY N. 0. 2.100 3.600 5.025 5.025 6.600 8.100 15.000  
 TIME = 0.68 S. DEPTH M= 0.0172 0.0295 0.0369 0.0447 0.0392 0.0370 0.0316 0.0137  
 VELOCITY M/S= 0.2141 0.1397 0.4096 0.4513 0.5354 0.5775 0.6024 0.3036  
 FLOW RATE L/S= 0.2003 0.7598 1.0526 1.4750 1.4750 1.5263 1.2822 0.1959  
 WAVE SPD. M/S= 0.3479 0.4562 0.5174 0.5707 0.5273 0.5182 0.4739 0.3036  
 FSOL= -0.0146 FP2= 0.6068 FSUR= 0.1846

SOLID VELOCITY  
 = 0.321 M/S.

POSITION FROM  
 ENTRY N. 0. 2.100 3.600 5.106 5.106 6.600 8.100 15.000  
 TIME = 0.93 S. DEPTH M= 0.0174 0.0297 0.0364 0.0448 0.0393 0.0372 0.0325 0.0137  
 VELOCITY M/S= 0.2143 0.1397 0.4052 0.4297 0.5283 0.5665 0.5929 0.3036  
 FLOW RATE L/S= 0.2144 0.7729 1.0209 1.4519 1.4519 1.5026 1.2927 0.1959  
 WAVE SPD. M/S= 0.3444 0.4519 0.5131 0.5719 0.5233 0.5177 0.4616 0.3036  
 FSOL= -0.0027 FP2= 0.6074 FSUR= 0.1846

SOLID VELOCITY  
= 0.307 m/s.

POSITION FROM  
ENTRY N.  
TIME = 10.19 S. DEPTH = 0.0172 0.0181 0.0152 0.0431 0.0372 0.0169 0.0332 0.0117  
VELOCITY M/S= 0.2222 0.1763 0.3707 0.4143 0.2167 0.2579 0.5917 0.1038  
FLOW RATE L/S= 0.1442 0.0854 0.9555 1.1713 1.1713 1.6685 1.3549 0.1960  
WAVE SPD. M/S= 0.3418 0.0456 0.5087 0.5717 0.5172 0.5175 0.4675 0.3036  
FSOL= 0.0000 FP2= 0.6042 FSUR= 0.1846

SOLID VELOCITY  
= 0.309 m/s.

POSITION FROM  
ENTRY N.  
TIME = 10.45 S. DEPTH = 0.0170 0.0276 0.0353 0.0437 0.0711 0.0370 0.0338 0.0137  
VELOCITY M/S= 0.2267 0.1676 0.1861 0.4114 0.5126 0.5508 0.5875 0.3036  
FLOW RATE L/S= 0.2004 0.0491 0.9564 1.3584 1.3584 1.4532 1.3710 0.1960  
WAVE SPD. M/S= 0.3348 0.0401 0.5043 0.5715 0.5187 0.5178 0.4923 0.3036  
FSOL= 0.0000 FP2= 0.6075 FSUR= 0.1846

SOLID VELOCITY  
= 0.304 m/s.

POSITION FROM  
ENTRY N.  
TIME = 10.70 S. DEPTH = 0.0166 0.0282 0.0363 0.0441 0.0373 0.0366 0.0332 0.0137  
VELOCITY M/S= 0.2306 0.1575 0.3032 0.4139 0.5173 0.5510 0.5847 0.3036  
FLOW RATE L/S= 0.2005 0.0470 0.9862 1.1830 1.1830 1.4357 1.3339 0.1960  
WAVE SPD. M/S= 0.3374 0.0451 0.5124 0.5797 0.5207 0.5191 0.4670 0.3036  
FSOL= -0.0131 FP2= 0.6070 FSUR= 0.1846

SOLID VELOCITY  
= 0.309 m/s.

POSITION FROM  
ENTRY N.  
TIME = 10.96 S. DEPTH = 0.0166 0.0276 0.0357 0.0432 0.0370 0.0366 0.0337 0.0137  
VELOCITY M/S= 0.2240 0.1628 0.3740 0.4130 0.5091 0.5417 0.5741 0.3036  
FLOW RATE L/S= 0.2010 0.0414 0.7547 1.1944 1.1944 1.4157 1.3430 0.1960  
WAVE SPD. M/S= 0.3352 0.0403 0.5078 0.5773 0.5182 0.5149 0.4911 0.3036  
FSOL= -0.0001 FP2= 0.6048 FSUR= 0.1846

SOLID VELOCITY  
= 0.295 m/s.

POSITION FROM  
ENTRY N.  
TIME = 11.22 S. DEPTH = 0.0164 0.0271 0.0351 0.0413 0.0360 0.0360 0.0359 0.0137  
VELOCITY M/S= 0.2277 0.1511 0.3747 0.4001 0.5117 0.5518 0.5739 0.3036  
FLOW RATE L/S= 0.2014 0.0559 0.7729 1.1065 1.1065 1.4157 1.3432 0.1960  
WAVE SPD. M/S= 0.3349 0.0413 0.5110 0.5661 0.5184 0.5149 0.4922 0.3036  
FSOL= 0.0007 FP2= 0.6048 FSUR= 0.1846

SOLID VEL CITY

POSITION FROM  
 ENTRY N.  
 TIME = 11.49 S. DEPTH = 0.0100 0.0105 0.0105 0.0100 0.0100 0.0100 0.0100  
 VELOCITY M/S = 0.2410 0.1911 0.1711 0.1700 0.1700 0.1700 0.1700  
 FLOW RATE L/S = 0.1906 0.1708 0.1611 0.1610 0.1610 0.1615 0.1616  
 WAVE SPD. M/S = 0.1111 0.0910 0.0810 0.0800 0.0800 0.0800 0.0800  
 FSOL = 0.0067 FP2 = 0.0000 FSUM = 0.0000

SOLID VELOCITY  
= 0.2496 M/S.

POSITION FROM  
 ENTRY N.  
 TIME = 11.75 S. DEPTH = 0.0101 0.0108 0.0105 0.0108 0.0108 0.0108 0.0108  
 VELOCITY M/S = 0.2445 0.2150 0.1895 0.1914 0.1916 0.1916 0.1916  
 FLOW RATE L/S = 0.2001 0.1901 0.1738 0.1733 0.1733 0.1734 0.1734  
 WAVE SPD. M/S = 0.3705 0.3421 0.3063 0.3116 0.3116 0.3119 0.3122  
 FSOL = -0.0144 FP2 = 0.0072 FSUM = 0.0045

SOLID VELOCITY  
= 0.3014 M/S.

POSITION FROM  
 ENTRY N.  
 TIME = 12.01 S. DEPTH = 0.0100 0.0102 0.0149 0.0165 0.0165 0.0165 0.0165  
 VELOCITY M/S = 0.2470 0.1278 0.1652 0.1602 0.1602 0.1602 0.1602  
 FLOW RATE L/S = 0.2000 0.1657 0.1475 0.1771 0.1771 0.1760 0.1761  
 WAVE SPD. M/S = 0.3289 0.4162 0.5014 0.5017 0.5017 0.5106 0.4961  
 FSOL = 0.0020 FP2 = 0.0070 FSUM = 0.0046

SOLID VELOCITY  
= 0.2498 M/S.

POSITION FROM  
 ENTRY N.  
 TIME = 12.28 S. DEPTH = 0.0156 0.0265 0.0363 0.0429 0.0361 0.0360 0.0361  
 VELOCITY M/S = 0.2103 0.1207 0.1609 0.1901 0.1902 0.1902 0.1902  
 FLOW RATE L/S = 0.1945 0.1144 0.0811 0.2594 0.2554 0.2330 0.3069 0.1761  
 WAVE SPD. M/S = 0.3279 0.3104 0.4965 0.5691 0.5691 0.5104 0.4961 0.1937  
 FSOL = -0.0010 FP2 = 0.0070 FSUM = 0.0045

SOLID VELOCITY  
= 0.2490 M/S.

POSITION FROM  
 ENTRY N.  
 TIME = 12.54 S. DEPTH = 0.0157 0.0275 0.0337 0.0469 0.0361 0.0360 0.0365  
 VELOCITY M/S = 0.2153 0.1240 0.1664 0.1921 0.1941 0.1941 0.1941  
 FLOW RATE L/S = 0.1992 0.1153 0.1208 0.2591 0.2629 0.2055 0.3037 0.1761  
 WAVE SPD. M/S = 0.3270 0.3149 0.4919 0.5697 0.5697 0.5097 0.4977 0.1937  
 FSOL = 0.0101 FP2 = 0.0070 FSUM = 0.0046

SOLID VELOCITY  
= 0.2490 M/S.

POSITION FROM

ENTRY NO. 0. 10.000 4.500 5.963 5.963 7.500 9.000 15.000  
 TIME = 12.81 S. DEPTH NO. 0.0154 0.0157 0.0159 0.0165 0.0169 0.0150 0.0151 0.0137  
 VELOCITY M/S. 0.2612 0.1100 0.3921 0.3920 0.3921 0.3157 0.3577 0.3037  
 FLOW RATE L/S. 0.2715 0.0287 0.4630 0.4226 0.4223 0.2797 0.2660 0.1962  
 WAVE SPD. M/S. 0.3156 0.4121 0.5002 0.5676 0.5113 0.5081 0.4767 0.3037  
 FSOL= -0.0175 FP2= 0.6074 FSUR= 0.1845

SOLID VELOCITY  
 = 0.100 M/S.

POSITION FROM  
 ENTRY NO. 0. 10.000 4.500 5.942 5.942 7.500 9.000 15.000  
 TIME = 13.08 S. DEPTH NO. 0.0155 0.0161 0.0142 0.0123 0.0161 0.0157 0.0153 0.0137  
 VELOCITY M/S. 0.2586 0.1142 0.3517 0.3901 0.3913 0.3107 0.3577 0.3037  
 FLOW RATE L/S. 0.2745 0.0213 0.4836 0.2577 0.2577 0.2864 0.2609 0.1962  
 WAVE SPD. M/S. 0.3237 0.4271 0.4952 0.5579 0.5115 0.5078 0.4962 0.3037  
 FSOL= 0.0032 FP2= 0.6071 FSUR= 0.1845

SOLID VELOCITY  
 = 0.281 M/S.

POSITION FROM  
 ENTRY NO. 0. 10.000 4.500 6.119 6.119 7.500 9.000 15.000  
 TIME = 13.35 S. DEPTH NO. 0.0154 0.0155 0.0135 0.0126 0.0156 0.0157 0.0144 0.0137  
 VELOCITY M/S. 0.2614 0.1090 0.3969 0.3912 0.3913 0.3080 0.3520 0.3037  
 FLOW RATE L/S. 0.2010 0.04887 0.8020 1.2120 1.2120 1.2797 1.2730 0.1962  
 WAVE SPD. M/S. 0.3226 0.4218 0.4901 0.5622 0.5066 0.5078 0.4971 0.3037  
 FSOL= 0.0000 FP2= 0.6071 FSUR= 0.1845

SOLID VELOCITY  
 = 0.284 M/S.

POSITION FROM  
 ENTRY NO. 0. 10.000 4.500 6.146 6.146 7.500 9.000 15.000  
 TIME = 13.62 S. DEPTH NO. 0.0153 0.0150 0.0124 0.0125 0.0156 0.0156 0.0146 0.0137  
 VELOCITY M/S. 0.2635 0.1065 0.3924 0.3942 0.3962 0.3037 0.3566 0.3037  
 FLOW RATE L/S. 0.1998 0.04669 0.7714 1.2200 1.2200 1.2644 1.2639 0.1962  
 WAVE SPD. M/S. 0.3215 0.4168 0.4899 0.5617 0.5067 0.5070 0.4977 0.3037  
 FSOL= 0.0099 FP2= 0.6078 FSUR= 0.1846

SOLID VELOCITY  
 = 0.284 M/S.

POSITION FROM  
 ENTRY NO. 0. 10.000 4.500 6.222 6.222 7.500 9.000 15.000  
 TIME = 13.89 S. DEPTH NO. 0.0152 0.0159 0.0140 0.0132 0.0161 0.0155 0.0142 0.0137  
 VELOCITY M/S. 0.2681 0.1079 0.3920 0.3945 0.3953 0.3048 0.3566 0.3037  
 FLOW RATE L/S. 0.2003 0.04790 0.8006 1.2615 1.2615 1.2599 1.2534 0.1962  
 WAVE SPD. M/S. 0.3202 0.4249 0.4941 0.5671 0.5132 0.5058 0.4982 0.3037  
 FSOL= -0.0175 FP2= 0.6075 FSUR= 0.1845

SOLID VELOCITY  
 = 0.285 M/S.

POSITION FROM  
 ENTRY NO. 0. 10.000 4.500 6.151 6.151 7.500 9.000 15.000  
 TIME = 14.16 S. DEPTH NO. 0.0151 0.0251 0.0145 0.0149 0.0156 0.0155 0.0141 0.0137  
 VELOCITY M/S. 0.2684 0.1057 0.3379 0.3799 0.3863 0.3018 0.3223 0.3037

FLOW RATE L/S= 0.2105 0.4788 0.7767 1.2206 1.2206 1.2510 1.2439 0.1961  
 WAVE SPD. M/S= 0.3194 0.4701 0.6689 0.5569 0.5067 0.5056 0.4961 0.3037  
 FSOL= 0.0022 FP2= 0.6072 FSUR= 0.1845

SOLID VELOCITY  
 = 0.276 M/S.

POSITION FROM  
 ENTRY N. 0. 3.100 4.800 6.628 6.428 7.800 9.330 15.000  
 TIME = 14.43 S. DEPTH M= 0.0191 0.0248 0.0328 0.0422 0.0351 0.0355 0.0346 0.0137  
 VELOCITY M/S= 0.2700 0.3125 0.3138 0.3736 0.4778 0.4799 0.5112 0.3037  
 FLOW RATE L/S= 0.2004 0.4603 0.7474 1.1765 1.1765 1.2466 1.2304 0.1963  
 WAVE SPD. M/S= 0.3191 0.4156 0.6637 0.5591 0.5030 0.5056 0.4960 0.3037  
 FSOL= 0.0019 FP2= 0.6072 FSUR= 0.1845

SOLID VELOCITY  
 = 0.278 M/S.

POSITION FROM  
 ENTRY N. 0. 3.100 4.800 6.504 6.504 7.800 9.330 15.000  
 TIME = 14.70 S. DEPTH M= 0.0190 0.0244 0.0321 0.0422 0.0352 0.0353 0.0349 0.0137  
 VELOCITY M/S= 0.2714 0.3000 0.3248 0.3762 0.4779 0.4953 0.5146 0.3037  
 FLOW RATE L/S= 0.2001 0.4440 0.7190 1.1839 1.1839 1.2305 1.2303 0.1963  
 WAVE SPD. M/S= 0.3183 0.4113 0.6785 0.5572 0.5034 0.5066 0.4974 0.3037  
 FSOL= 0.0019 FP2= 0.6069 FSUR= 0.1846

SOLID VELOCITY  
 = 0.280 M/S.

POSITION FROM  
 ENTRY N. 0. 3.600 5.100 6.590 6.590 8.100 9.600 15.000  
 TIME = 14.98 S. DEPTH M= 0.0189 0.0253 0.0333 0.0424 0.0352 0.0352 0.0342 0.0137  
 VELOCITY M/S= 0.2726 0.3138 0.3302 0.3823 0.4931 0.4969 0.5148 0.3037  
 FLOW RATE L/S= 0.1995 0.4731 0.7349 1.2319 1.2319 1.2279 1.2216 0.1963  
 WAVE SPD. M/S= 0.3175 0.4193 0.6879 0.5651 0.5073 0.5037 0.4953 0.3037  
 FSOL= -0.0195 FP2= 0.6072 FSUR= 0.1845

SOLID VELOCITY  
 = 0.292 M/S.

POSITION FROM  
 ENTRY N. 0. 3.600 5.100 6.659 6.659 8.100 9.600 15.000  
 TIME = 15.25 S. DEPTH M= 0.0189 0.0248 0.0327 0.0415 0.0352 0.0352 0.0343 0.0137  
 VELOCITY M/S= 0.2744 0.3116 0.3265 0.3959 0.4952 0.4942 0.5120 0.3038  
 FLOW RATE L/S= 0.1994 0.4575 0.7276 1.1854 1.1854 1.2182 1.2211 0.1964  
 WAVE SPD. M/S= 0.3172 0.4151 0.6476 0.5536 0.5036 0.5032 0.4962 0.3038  
 FSOL= 0.0023 FP2= 0.6073 FSUR= 0.1845

SOLID VELOCITY  
 = 0.271 M/S.

POSITION FROM  
 ENTRY N. 0. 3.600 5.100 6.736 6.736 8.100 9.600 15.000  
 TIME = 15.53 S. DEPTH M= 0.0188 0.0241 0.0320 0.0418 0.0346 0.0352 0.0344 0.0137  
 VELOCITY M/S= 0.2745 0.3107 0.3131 0.3906 0.4792 0.4917 0.5017 0.3038  
 FLOW RATE L/S= 0.1994 0.4517 0.7013 1.1776 1.1776 1.2145 1.2117 0.1965  
 WAVE SPD. M/S= 0.3174 0.4111 0.6577 0.5591 0.5030 0.5032 0.4967 0.3038  
 FSOL= 0.0019 FP2= 0.6073 FSUR= 0.1845

SOLID VELOCITY  
= 0.274 m/s.

POSITION FROM  
ENTRY N. 0. 3.600 5.100 6.811 6.811 8.100 9.630 15.000  
TIME = 15.01 S. DEPTH N= 0.0140 0.0239 0.0314 0.0418 0.0517 0.0610 0.0714 0.0817  
VELOCITY M/S= 0.2760 0.2703 0.1199 0.1700 0.4741 0.4890 0.5063 0.3060  
FLOW RATE L/S= 0.1946 0.9102 0.6765 1.1496 1.1496 1.1990 1.2117 0.1269  
WAVE SPD. M/S= 0.3124 0.6072 0.4727 0.5553 0.4911 0.5021 0.4972 0.3040  
FSOL= 0.0127 FP2= 0.6004 FSUK= 0.1846

SOLID VELOCITY  
= 0.276 m/s.

POSITION FROM  
ENTRY N. 0. 3.400 5.400 6.888 6.888 8.400 9.600 15.000  
TIME = 16.08 S. DEPTH N= 0.0142 0.0248 0.0326 0.0427 0.0533 0.0632 0.0732 0.0837  
VELOCITY M/S= 0.2760 0.2017 0.1201 0.3769 0.4963 0.4901 0.5064 0.3045  
FLOW RATE L/S= 0.2103 0.6570 0.7119 1.2066 1.2106 1.1974 1.2027 0.1932  
WAVE SPD. M/S= 0.3156 0.6152 0.6024 0.5612 0.5096 0.5014 0.4955 0.3045  
FSOL= -0.0212 FP2= 0.6078 FSUK= 0.1845

SOLID VELOCITY  
= 0.290 m/s.

POSITION FROM  
ENTRY N. 0. 3.400 5.400 6.966 6.966 8.400 9.600 15.000  
TIME = 16.36 S. DEPTH N= 0.0142 0.0246 0.0320 0.0411 0.0519 0.0619 0.0719 0.0819  
VELOCITY M/S= 0.2760 0.2000 0.1172 0.3411 0.4759 0.4876 0.5063 0.3060  
FLOW RATE L/S= 0.1442 0.6591 0.6803 1.1562 1.1562 1.1884 1.2018 0.2020  
WAVE SPD. M/S= 0.3151 0.6113 0.4776 0.5513 0.5002 0.5002 0.4962 0.3060  
FSOL= 0.0024 FP2= 0.6074 FSUK= 0.1845

SOLID VELOCITY  
= 0.267 m/s.

POSITION FROM  
ENTRY N. 0. 3.400 5.400 7.043 7.043 8.400 9.600 15.000  
TIME = 16.64 S. DEPTH N= 0.0142 0.0240 0.0314 0.0414 0.0513 0.0617 0.0711 0.0812  
VELOCITY M/S= 0.2760 0.2007 0.1147 0.3400 0.4662 0.4865 0.5015 0.3101  
FLOW RATE L/S= 0.2041 0.6319 0.6658 1.1040 1.1040 1.1467 1.1706 0.2121  
WAVE SPD. M/S= 0.3146 0.6076 0.5728 0.5534 0.4956 0.5017 0.4965 0.3101  
FSOL= 0.0030 FP2= 0.6074 FSUK= 0.1845

SOLID VELOCITY  
= 0.269 m/s.

POSITION FROM  
ENTRY N. 0. 3.400 5.400 7.118 7.118 8.400 9.600 15.000  
TIME = 16.92 S. DEPTH N= 0.0140 0.0236 0.0309 0.0415 0.0514 0.0617 0.0711 0.0811  
VELOCITY M/S= 0.2760 0.2007 0.1124 0.3403 0.4576 0.4925 0.5036 0.3129  
FLOW RATE L/S= 0.2041 0.6305 0.6658 1.1210 1.1210 1.1677 1.1912 0.2126  
WAVE SPD. M/S= 0.3145 0.6040 0.5681 0.5562 0.4966 0.5016 0.4966 0.3129  
FSOL= 0.0149 FP2= 0.6076 FSUK= 0.1846

SOLID VELOCITY

\* 0.271 M/S.

POSITION FROM  
ENTRY N. 0. 4.200 5.700 7.175 7.175 4.200 10.200 15.030  
TIME = 17.20 S. DEPTH = 0.0146 0.0244 0.1321 0.0472 0.2120 0.0346 0.0392 0.0157  
VELOCITY M/S= 0.2714 0.2709 0.1722 0.1723 0.0887 0.4617 0.4993 0.1917  
FLOW RATE L/S= 0.2713 0.4956 0.6793 1.1327 1.1327 1.1672 1.1655 0.2711  
WAVE SPD. M/S= 0.3143 0.3111 0.5781 0.5626 0.3910 0.4989 0.4953 0.1117  
FSOL= -0.0240 FP2= 0.6075 FSUR= 0.1845

SOLID VELOCITY  
\* 0.289 M/S.

POSITION FROM  
ENTRY N. 0. 4.200 5.700 7.273 7.273 4.200 10.200 15.030  
TIME = 17.40 S. DEPTH = 0.0146 0.0240 0.0315 0.0407 0.0346 0.0395 0.0392 0.0190  
VELOCITY M/S= 0.2722 0.2703 0.3103 0.1700 0.4729 0.4814 0.4975 0.1606  
FLOW RATE L/S= 0.2702 0.4931 0.6592 1.1324 1.1329 1.1587 1.1626 0.3752  
WAVE SPD. M/S= 0.3146 0.3082 0.6735 0.5473 0.4979 0.4983 0.4937 0.3606  
FSOL= 0.0036 FP2= 0.6075 FSUR= 0.1845

SOLID VELOCITY  
\* 0.262 M/S.

POSITION FROM  
ENTRY N. 0. 4.200 5.700 7.350 7.350 4.200 10.200 15.030  
TIME = 17.76 S. DEPTH = 0.0146 0.0236 0.0310 0.0412 0.0339 0.0365 0.0392 0.0215  
VELOCITY M/S= 0.2727 0.2703 0.3096 0.1542 0.4617 0.4808 0.4999 0.3899  
FLOW RATE L/S= 0.2701 0.4730 0.6490 1.0811 1.0811 1.1567 1.1767 0.4775  
WAVE SPD. M/S= 0.3137 0.3107 0.6690 0.5517 0.4977 0.4982 0.4953 0.3899  
FSOL= 0.0031 FP2= 0.6075 FSUR= 0.1845

SOLID VELOCITY  
\* 0.267 M/S.

POSITION FROM  
ENTRY N. 0. 4.500 6.000 7.429 7.429 9.000 10.500 15.030  
TIME = 18.04 S. DEPTH = 0.0145 0.0245 0.0322 0.0413 0.0340 0.0345 0.0391 0.0215  
VELOCITY M/S= 0.2736 0.3000 0.3082 0.1706 0.4659 0.4817 0.4959 0.3939  
FLOW RATE L/S= 0.1946 0.4477 0.6742 1.0975 1.0975 1.1561 1.1711 0.5630  
WAVE SPD. M/S= 0.3134 0.4126 0.6792 0.5520 0.4977 0.4996 0.4939  
FSOL= -0.0151 FP2= 0.6070 FSUR= 0.1844

SOLID VELOCITY  
\* 0.270 M/S.

POSITION FROM  
ENTRY N. 0. 4.500 6.000 7.500 7.500 9.000 10.500 15.030  
TIME = 18.33 S. DEPTH = 0.0145 0.0241 0.0317 0.0402 0.0335 0.0343 0.0391 0.0250  
VELOCITY M/S= 0.2733 0.2702 0.3067 0.1758 0.4567 0.4795 0.4931 0.4174  
FLOW RATE L/S= 0.1945 0.4166 0.6726 1.0513 1.0513 1.1411 1.1629 0.6627  
WAVE SPD. M/S= 0.3134 0.4090 0.6747 0.5453 0.4977 0.4992 0.4956 0.4174  
FSOL= -0.0011 FP2= 0.6074 FSUR= 0.1845

SOLID VELOCITY  
\* 0.251 M/S.

POSITION FROM

ENTRY N. 0. 4.500 6.000 7.679 7.679 4.000 10.500 15.000  
 TIME = 18.61 S. DEPTH N= 0.0145 0.0233 0.0312 0.0401 0.0491 0.0583 0.0681 0.0781  
 VELOCITY M/S= 0.2842 0.2479 0.3043 0.3169 0.4439 0.4769 0.4919 0.4773  
 FLOW RATE L/S= 0.2004 0.6255 0.6178 0.7701 1.1730 1.1631 0.6951 0.6971  
 WAVE SPD. M/S= 0.3112 0.6054 0.4703 0.5412 0.4842 0.4958 0.4989 0.6271  
 FSOL= 0.0001 FP2= 0.6074 FSUR= 0.1845

SOLID VELOCITY  
 = 0.252 M/S.

POSITION FROM  
 ENTRY N. 0. 4.500 6.000 7.646 7.646 4.000 10.500 15.000  
 TIME = 18.90 S. DEPTH N= 0.0145 0.0233 0.0306 0.0401 0.0491 0.0583 0.0681 0.0781  
 VELOCITY M/S= 0.2842 0.2479 0.3043 0.3169 0.4439 0.4769 0.4919 0.4773  
 FLOW RATE L/S= 0.2004 0.6150 0.6210 0.4905 0.4905 1.1330 1.1559 0.7959  
 WAVE SPD. M/S= 0.3129 0.6014 0.4666 0.5424 0.4810 0.4958 0.4986 0.6350  
 FSOL= 0.0001 FP2= 0.6077 FSUR= 0.1845

SOLID VELOCITY  
 = 0.252 M/S.

POSITION FROM  
 ENTRY N. 0. 4.500 6.000 7.718 7.718 4.000 10.500 15.000  
 TIME = 19.18 S. DEPTH N= 0.0145 0.0230 0.0301 0.0405 0.0491 0.0581 0.0681 0.0781  
 VELOCITY M/S= 0.2849 0.2473 0.3034 0.3192 0.4436 0.4727 0.4865 0.4912  
 FLOW RATE L/S= 0.2002 0.6049 0.6050 1.0115 1.0115 1.1162 1.1451 0.7891  
 WAVE SPD. M/S= 0.3129 0.3095 0.4517 0.4956 0.4946 0.4949 0.4999 0.6912  
 FSOL= 0.0139 FP2= 0.6074 FSUR= 0.1845

SOLID VELOCITY  
 = 0.254 M/S.

POSITION FROM  
 ENTRY N. 0. 4.800 6.300 7.791 7.791 9.300 10.500 15.000  
 TIME = 19.47 S. DEPTH N= 0.0145 0.0236 0.0313 0.0413 0.0336 0.0341 0.0390 0.0283  
 VELOCITY M/S= 0.2848 0.2483 0.3041 0.3181 0.4649 0.4738 0.4872 0.4964  
 FLOW RATE L/S= 0.1946 0.6281 0.6340 1.0754 1.0754 1.1180 1.1458 0.9171  
 WAVE SPD. M/S= 0.3126 0.4064 0.5714 0.5522 0.4903 0.4942 0.4935 0.4964  
 FSOL= -0.0192 FP2= 0.6084 FSUR= 0.1844

SOLID VELOCITY  
 = 0.274 M/S.

POSITION FROM  
 ENTRY N. 0. 4.800 6.300 7.866 7.866 9.300 10.500 15.000  
 TIME = 19.75 S. DEPTH N= 0.0145 0.0231 0.0308 0.0397 0.0331 0.0338 0.0350 0.0288  
 VELOCITY M/S= 0.2846 0.2479 0.3032 0.3176 0.4567 0.4698 0.4879 0.4957  
 FLOW RATE L/S= 0.2000 0.6179 0.6215 1.0176 1.0174 1.0936 1.1511 0.7454  
 WAVE SPD. M/S= 0.3126 0.4060 0.5671 0.5599 0.4866 0.4927 0.4917 0.4957  
 FSOL= 0.0032 FP2= 0.6176 FSUR= 0.1845

SOLID VELOCITY  
 = 0.274 M/S.

POSITION FROM  
 ENTRY N. 0. 4.800 6.300 7.930 7.930 9.300 10.500 15.000  
 TIME = 20.03 S. DEPTH N= 0.0145 0.0231 0.0303 0.0369 0.0331 0.0338 0.0350 0.0288  
 VELOCITY M/S= 0.2846 0.2473 0.3032 0.3172 0.4567 0.4698 0.4879 0.4957

FLOW RATE L/S= 0.6003 0.6079 0.6084 0.69972 0.69972 1.07111 1.1161 0.9710  
 WAVE SPD. M/S= 0.3124 0.3129 0.30629 0.55412 0.55412 0.41927 0.59113 0.45597  
 FSOL= 0.0025 FP2= 0.6079 FSUM= 0.1845

SOLID VELOCITY  
 = 0.257 M/S.

POSITION FROM  
 ENTRY 4. 0. 5.100 6.600 8.004 8.004 9.600 11.100 15.000  
 TIME = 20.31 S. DEPTH M= 0.0144 0.0227 0.0298 0.0403 0.0318 0.0332 0.0339 0.0277  
 VELOCITY M/S= 0.2855 0.2768 0.3026 0.3423 0.3513 0.3584 0.3616 0.4579  
 FLOW RATE L/S= 0.1494 0.1784 0.5938 1.0176 1.0176 1.0459 1.1277 0.9461  
 WAVE SPD. M/S= 0.3123 0.3164 0.4587 0.5942 0.4842 0.4874 0.4930 0.4579  
 FSOL= 0.0162 FP2= 0.6079 FSUM= 0.1845

SOLID VFLJCITY  
 = 0.259 M/S.

POSITION FROM  
 ENTRY 4. 0. 5.100 6.600 8.004 8.004 9.600 11.100 15.000  
 TIME = 20.59 S. DEPTH M= 0.0144 0.0236 0.0310 0.0413 0.0334 0.0333 0.0338 0.0300  
 VELOCITY M/S= 0.2859 0.2976 0.3019 0.3516 0.3663 0.3603 0.3621 0.4609  
 FLOW RATE L/S= 0.1494 0.1710 0.6262 1.0725 1.0725 1.0917 1.1276 0.9848  
 WAVE SPD. M/S= 0.3123 0.4043 0.4640 0.5514 0.4891 0.4877 0.4923 0.4609  
 FSOL= -0.0229 FP2= 0.6063 FSUM= 0.1844

SOLID VFLJCITY  
 = 0.278 M/S.

POSITION FROM  
 ENTRY 4. 0. 5.100 6.600 8.159 8.159 9.600 11.100 15.000  
 TIME = 20.87 S. DEPTH M= 0.0144 0.0237 0.0305 0.0345 0.0310 0.0331 0.0338 0.0304  
 VELOCITY M/S= 0.2864 0.2771 0.3009 0.3591 0.3578 0.4582 0.4613 0.4635  
 FLOW RATE L/S= 0.2001 0.4113 0.6113 1.0341 1.0341 1.0405 1.1253 0.9136  
 WAVE SPD. M/S= 0.3123 0.4010 0.4651 0.5378 0.4856 0.4865 0.4924 0.4635  
 FSOL= 0.0024 FP2= 0.6079 FSUM= 0.1845

SOLID VELOCITY  
 = 0.252 M/S.

POSITION FROM  
 ENTRY 4. 0. 5.100 6.600 8.233 8.233 9.600 11.100 15.000  
 TIME = 21.15 S. DEPTH M= 0.0144 0.0229 0.0301 0.0349 0.0324 0.0331 0.0338 0.0306  
 VELOCITY M/S= 0.2845 0.2966 0.3000 0.3594 0.3660 0.4591 0.4711 0.4698  
 FLOW RATE L/S= 0.2003 0.4017 0.5967 0.9847 0.9847 1.0471 1.1105 0.9501  
 WAVE SPD. M/S= 0.3123 0.4177 0.4612 0.5915 0.4836 0.4864 0.4920 0.4639  
 FSOL= 0.0018 FP2= 0.6079 FSUM= 0.1845

SOLID VELOCITY  
 = 0.255 M/S.

POSITION FROM  
 ENTRY 4. 0. 5.100 6.600 8.304 8.304 9.600 11.100 15.000  
 TIME = 21.43 S. DEPTH M= 0.0144 0.0225 0.0296 0.0300 0.0325 0.0330 0.0317 0.0109  
 VELOCITY M/S= 0.2845 0.2771 0.3293 0.3611 0.4512 0.4524 0.4704 0.4630  
 FLOW RATE L/S= 0.2003 0.4025 0.5923 1.0005 1.0005 1.0349 1.1071 0.9679  
 WAVE SPD. M/S= 0.3123 0.4145 0.4574 0.5921 0.4881 0.4855 0.4911 0.4640  
 FSOL= 0.0152 FP2= 0.6079 FSUM= 0.1845

SOLID VELOCITY  
= 0.257 4/5.

POSITION FROM  
ENTRY N. 0. 5.400 6.400 8.176 8.376 9.900 11.400 15.000  
TIME = 21.71 S. DEPTH N= 0.0144 0.0234 0.0310 0.0419 0.0511 0.0610 0.0716 0.0811  
VELOCITY M/S= 0.2864 0.2967 0.2764 0.1977 0.5612 0.5587 0.4771 0.4678  
FLOW RATE L/S= 0.1996 0.6147 0.6110 1.0510 1.7506 1.6177 1.1017 0.7738  
WAVE SPD. M/S= 0.3121 0.5129 0.6106 0.5915 0.4902 0.4857 0.4917 0.4630  
FSOL= -0.0221 FP2= 0.0164 FSUR= 0.1844

SOLID VELOCITY  
= 0.274 4/5.

POSITION FROM  
ENTRY N. 0. 5.400 6.400 8.451 8.451 9.900 11.400 15.000  
TIME = 21.99 S. DEPTH N= 0.0144 0.0230 0.0305 0.0392 0.0376 0.0430 0.0515 0.0613  
VELOCITY M/S= 0.2869 0.2963 0.2961 0.1545 0.5547 0.5587 0.4715 0.4719  
FLOW RATE L/S= 0.1996 0.6053 0.5999 1.0125 1.0123 1.0348 1.0412 0.9910  
WAVE SPD. M/S= 0.3128 0.5192 0.6146 0.5157 0.4926 0.4852 0.4874 0.4719  
FSOL= 0.0019 FP2= 0.0060 FSUR= 0.1845

SOLID VELOCITY  
= 0.250 M/S.

POSITION FROM  
ENTRY N. 0. 5.400 6.400 8.523 8.573 9.900 11.400 15.000  
TIME = 22.27 S. DEPTH N= 0.0144 0.0227 0.0300 0.0396 0.0321 0.0330 0.0313 0.0315  
VELOCITY M/S= 0.2866 0.2959 0.2958 0.1121 0.4437 0.4597 0.4674 0.4729  
FLOW RATE L/S= 0.1994 0.1960 0.5862 0.4616 0.7616 1.0376 1.0235 1.0015  
WAVE SPD. M/S= 0.3128 0.1959 0.6106 0.5390 0.4779 0.4853 0.4879 0.4729  
FSOL= 0.0026 FP2= 0.0079 FSUR= 0.1845

SOLID VELOCITY  
= 0.252 4/5.

POSITION FROM  
ENTRY N. 0. 5.400 6.400 8.594 8.594 9.900 11.400 15.000  
TIME = 22.55 S. DEPTH N= 0.0144 0.0223 0.0295 0.0397 0.0321 0.0329 0.0311 0.0316  
VELOCITY M/S= 0.2868 0.2959 0.2955 0.1346 0.4450 0.4576 0.4663 0.4742  
FLOW RATE L/S= 0.2000 0.1920 0.5729 0.4708 0.7208 1.0268 1.0271 1.0139  
WAVE SPD. M/S= 0.3128 0.1922 0.6106 0.5375 0.4732 0.4841 0.4867 0.4742  
FSOL= 0.0016 FP2= 0.0176 FSUR= 0.1845

SOLID VELOCITY  
= 0.255 4/5.

POSITION FROM  
ENTRY N. 0. 5.700 7.200 9.666 8.666 10.200 11.700 15.000  
TIME = 22.83 S. DEPTH N= 0.0144 0.0212 0.0198 0.0405 0.0376 0.0328 0.0311 0.0316  
VELOCITY M/S= 0.2719 0.2761 0.2702 0.1421 0.4553 0.4594 0.4671 0.4723  
FLOW RATE L/S= 0.2011 0.4048 0.6097 1.0204 1.1204 1.0101 1.0617 1.0171  
WAVE SPD. M/S= 0.3121 0.6016 0.6070 0.5596 0.4876 0.4843 0.4865 0.4753  
FSOL= -0.0116 FP2= 0.0184 FSUR= 0.1844

SOLID VELOCITY

\* 0.269 m/s.

POSITION FROM

ENTRY N. 0. 5.700 7.200 8.700 8.712 10.200 11.700 15.000  
TIME = 23.81 S. DEPTH M= 0.0144 0.0228 0.0303 0.0310 0.0312 0.0314 0.0330 0.0339  
VELOCITY M/S= 0.2874 0.2757 0.2750 0.1463 0.4912 0.4974 0.4630 0.4763  
FLOW RATE L/S= 0.2102 0.1996 0.5973 0.4730 1.0212 1.0212 1.0223 1.0265  
WAVE SPD. M/S= 0.3121 0.3973 0.6610 0.5318 0.4793 0.4835 0.4839 0.4763  
FSOL= 0.0004 FP2= 0.6061 FSUR= 0.1844

SOLID VELOCITY  
\* 0.247 m/s.

POSITION FROM

ENTRY N. 0. 5.700 7.200 8.812 8.912 10.200 11.700 15.000  
TIME = 23.90 S. DEPTH M= 0.0144 0.0229 0.0298 0.0342 0.0316 0.0327 0.0330 0.0320  
VELOCITY M/S= 0.2872 0.2953 0.2948 0.1275 0.4749 0.4576 0.4699 0.4771  
FLOW RATE L/S= 0.2003 0.3409 0.5784 0.4351 0.7151 1.0222 1.0531 1.0323  
WAVE SPD. M/S= 0.3121 0.3981 0.6589 0.5357 0.4761 0.4833 0.4835 0.4771  
FSOL= 0.0014 FP2= 0.6071 FSUR= 0.1844

SOLID VELOCITY  
\* 0.248 m/s.

POSITION FROM

ENTRY N. 0. 5.700 7.200 8.882 8.882 10.200 11.700 15.000  
TIME = 23.98 S. DEPTH M= 0.0144 0.0222 0.0293 0.0393 0.0317 0.0326 0.0329 0.0321  
VELOCITY M/S= 0.2872 0.2949 0.2946 0.1263 0.4913 0.4549 0.4661 0.4778  
FLOW RATE L/S= 0.2003 0.3416 0.5658 0.4244 0.7424 1.0092 1.0462 1.0380  
WAVE SPD. M/S= 0.3121 0.3909 0.6549 0.5354 0.4765 0.4819 0.4851 0.4778  
FSOL= 0.0011 FP2= 0.6078 FSUR= 0.1845

SOLID VELOCITY  
\* 0.249 m/s.

POSITION FROM

ENTRY N. 0. 6.000 7.500 8.953 8.953 10.500 12.000 15.000  
TIME = 23.97 S. DEPTH M= 0.0144 0.0230 0.0306 0.0400 0.0321 0.0326 0.0329 0.0321  
VELOCITY M/S= 0.2873 0.2952 0.2942 0.1339 0.4506 0.4560 0.4669 0.4784  
FLOW RATE L/S= 0.2004 0.3429 0.5682 0.4767 0.7387 1.0328 1.0471 1.0426  
WAVE SPD. M/S= 0.3121 0.3949 0.6657 0.5417 0.4778 0.4821 0.4869 0.4759  
FSOL= -0.0019 FP2= 0.6075 FSUR= 0.1844

SOLID VELOCITY  
\* 0.262 m/s.

POSITION FROM

ENTRY N. 0. 6.000 7.500 9.026 9.026 10.500 12.000 15.000  
TIME = 24.75 S. DEPTH M= 0.0144 0.0227 0.0301 0.0406 0.0317 0.0324 0.0329 0.0322  
VELOCITY M/S= 0.2874 0.2950 0.2940 0.1303 0.4915 0.4539 0.4697 0.4719  
FLOW RATE L/S= 0.2004 0.3417 0.5648 0.4941 0.7471 1.0022 1.0438 1.0432  
WAVE SPD. M/S= 0.3121 0.3956 0.6617 0.5107 0.4762 0.4810 0.4867 0.4739  
FSOL= 0.0012 FP2= 0.6082 FSUR= 0.1844

SOLID VELOCITY  
\* 0.261 m/s.

POSITION FROM

TIME = 24.94 S. DEPTH = 0.0144 0.0274 0.0300 0.0339 0.0371 0.0402 0.0432 0.0462  
 ENTRY NO. 0. 6.000 7.000 8.000 9.000 10.000 11.000 12.000 13.000  
 VELOCITY M/S = 0.2974 0.3141 0.3218 0.3293 0.3369 0.3443 0.3517 0.3591  
 FLOW RATE L/S = 0.0144 0.0167 0.0191 0.0204 0.0216 0.0226 0.0237 0.0248  
 WAVE SPD. M/S = 0.0121 0.0159 0.0192 0.0213 0.0232 0.0251 0.0271 0.0291  
 FSOL = 0.0001 FP2 = 0.0004 FSUH = 0.1849

SOLID VELOCITY  
= 0.244 M/S.

POSITION FROM  
 ENTRY NO. 0. 6.000 7.000 8.000 9.000 10.000 11.000 12.000 13.000  
 TIME = 24.93 S. DEPTH = 0.0144 0.0274 0.0300 0.0339 0.0371 0.0402 0.0432 0.0462  
 VELOCITY M/S = 0.2975 0.3141 0.3218 0.3293 0.3369 0.3443 0.3517 0.3591  
 FLOW RATE L/S = 0.0145 0.0167 0.0191 0.0204 0.0216 0.0226 0.0237 0.0248  
 WAVE SPD. M/S = 0.0121 0.0159 0.0192 0.0213 0.0232 0.0251 0.0271 0.0291  
 FSOL = 0.0000 FP2 = 0.0004 FSUH = 0.1849

SOLID VELOCITY  
= 0.245 M/S.

POSITION FROM  
 ENTRY NO. 0. 6.100 7.000 8.000 9.000 10.000 11.000 12.000 13.000  
 TIME = 25.12 S. DEPTH = 0.0144 0.0274 0.0304 0.0335 0.0365 0.0422 0.0432 0.0462  
 VELOCITY M/S = 0.2975 0.3141 0.3218 0.3293 0.3369 0.3443 0.3517 0.3591  
 FLOW RATE L/S = 0.0145 0.0167 0.0191 0.0204 0.0216 0.0226 0.0237 0.0248  
 WAVE SPD. M/S = 0.0121 0.0159 0.0192 0.0213 0.0232 0.0251 0.0271 0.0291  
 FSOL = -0.0150 FP2 = 0.0005 FSUH = 0.1849

SOLID VELOCITY  
= 0.256 M/S.

POSITION FROM  
 ENTRY NO. 0. 6.300 7.000 8.000 9.000 10.000 11.000 12.000 13.000  
 TIME = 25.41 S. DEPTH = 0.0144 0.0274 0.0304 0.0335 0.0365 0.0422 0.0432 0.0462  
 VELOCITY M/S = 0.2975 0.3141 0.3218 0.3293 0.3369 0.3443 0.3517 0.3591  
 FLOW RATE L/S = 0.0145 0.0167 0.0191 0.0204 0.0216 0.0226 0.0237 0.0248  
 WAVE SPD. M/S = 0.0118 0.0150 0.0182 0.0213 0.0243 0.0273 0.0303 0.0333  
 FSOL = 0.0013 FP2 = 0.0003 FSUH = 0.1849

SOLID VELOCITY  
= 0.234 M/S.

POSITION FROM  
 ENTRY NO. 0. 6.300 7.000 8.000 9.000 10.000 11.000 12.000 13.000  
 TIME = 25.70 S. DEPTH = 0.0144 0.0272 0.0294 0.0365 0.0370 0.0420 0.0432 0.0462  
 VELOCITY M/S = 0.2974 0.3128 0.3205 0.3280 0.3354 0.3428 0.3491 0.3564  
 FLOW RATE L/S = 0.0144 0.0167 0.0191 0.0204 0.0216 0.0226 0.0237 0.0248  
 WAVE SPD. M/S = 0.0118 0.0150 0.0182 0.0213 0.0243 0.0273 0.0303 0.0333  
 FSOL = 0.0002 FP2 = 0.0003 FSUH = 0.1849

SOLID VELOCITY  
= 0.240 M/S.

POSITION FROM  
 ENTRY NO. 0. 6.300 7.000 8.000 9.000 10.000 11.000 12.000 13.000  
 TIME = 25.99 S. DEPTH = 0.0144 0.0271 0.0294 0.0365 0.0370 0.0420 0.0432 0.0462  
 VELOCITY M/S = 0.2974 0.3128 0.3205 0.3280 0.3354 0.3428 0.3491 0.3564

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<p>The flow depth and velocity changes across a moving solid in partially-filled pipe flow are predicted by means of the application of the method of characteristics to solve the unsteady flow equations.</p> <p>Simplified force models are presented which, when used in conjunction with empirical relationships linking leakage flow past the solid to upstream specific energy, are sufficient to provide the required moving solid boundary conditions that allow solid velocity prediction.</p> <p>A wide range of simulated transport conditions are presented that confirm the applicability of this technique as a basis for the future evaluation of more complex body force models.</p> <p>The predicted solid velocity during drain transport is shown to be compatible with laboratory observations of the influence of solid dimensions and position in inflow profile on transport characteristics.</p>						
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