A COMPARISON OF THE EFFECTIVENESS OF AN ABSTRACT AND A CONCRETE APPROACH IN TEACHING SELECTED ALGEBRAIC CONCEPTS TO NINTH AND TENTH GRADE STUDENTS

DISSERTATION

Presented to the Graduate Council of the University of North Texas in Partial Fulfillment of the Requirements For the Degree of DOCTOR OF PHILOSOPHY

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One purpose of this study was to determine whether any differences in immediate achievement or retention existed between students using manipulatives and students not using manipulatives. Also addressed in this study is whether or not the use of manipulatives is more beneficial for girls than boys and whether the use of manipulatives is more beneficial for low-ability students than for high-ability students.

Students selected for this study were from a large suburban school district in Texas. The students were from eight intact classes, four of which were designated as the experimental group and the other four as the control group. The sample consisted of one hundred eighty-seven students.

All students were tested with a test developed by the researcher. This same test was administered as a pretest, posttest, and retention test. The following supplemental data were also gathered on the students: mathematics scores from the California Test of Basic Skills and scores from the mathematics section of the Texas Educational Assessment of Minimum Skills test.

Analysis of the data revealed no statistical difference in the mean scores of students instructed with or without manipulatives when the test was administered immediately after instruction. Nor was there any statistical difference in the mean scores when the test was administered two months after instruction. There was no statistical difference in the mean gain scores from the
pretest to the posttest between boys and girls or between high- and low-
achieving students. Nor was there any statistical difference between the mean
gain scores from the pretest to the retention test between boys and girls or
between high- and low-achieving students.

It is recommended that further studies be conducted to investigate
achievement and retention of students using manipulatives at the secondary
level. It is also recommended that variables other than achievement be studied
to determine the effects of manipulatives on secondary students.
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CHAPTER I

INTRODUCTION

The Underachieving Curriculum (1987), which was a report of the second international mathematics study comparing the mathematical ability of American students to those in other industrialized nations, did not paint a favorable picture of mathematics instruction in the United States. Test scores earned by the American students in this study were among the lowest of students in the industrialized nations. Algebra was one of the weakest areas for American students.

Results from the Second Mathematics Assessment of the National Assessment of Educational Progress (1981) showed that there was no significant difference in the results that were obtained in the first national assessment with regards to algebraic concepts. From the students in this study, only about 40% of the seventeen-year-olds who had completed Algebra I could solve a linear equation. About 25% of these same students were able to solve systems of equations and quadratic equations that could be solved by factoring. When it came to solving quadratic equations using the quadratic formula and applying algebraic skills to solving verbal problems, only 20% of these students were successful.

Both of these studies would seem to indicate that algebra instruction in the recent past has not been universally successful. Driscoll (1986) also points out in an article in a publication of the National Council of Teachers of Mathematics that many of the students taking Algebra II have mastered only
mechanical skills and cannot adapt them to new situations. This inability to adapt mechanical skills to new situations would suggest that even those students who had been successful in Algebra I did not have a good understanding of the concepts underlying the mechanical skills.

All of this would suggest that it may be time for a new approach to the teaching of the fundamental concepts in Algebra I. Learning theorists have provided some insight that may be helpful at this point. Both Jean Piaget (1952) and Richard Skemp (1982) believe that individuals pass through stages as they mature and develop. Piaget believes that maturation and development occur in four stages and that manipulative materials play a significant role at all four stages because abstract ideas are based on experiences. Skemp believes that learning occurs by means of a two-level system. Manipulation of objects at the first level provides the learner with physical activities that form the basis for learning. At the second level, mental activities build on the physical activities from the first level so that further learning can take place.

Jerome Bruner (1960), who also believes that learning takes place by progressing from the concrete to the abstract, shed some light on this subject when he presented a paper to the National Council of Teachers of Mathematics. Bruner, a great proponent of discovery learning, feels that this method should be used in developing mathematical concepts. He says that materials such as Cuisenaire rods, which are manipulatives familiar to all elementary teachers, give students a chance to break away from the banal manipulation of numbers to the possibility of discovery. Bruner also notes that most of our present teaching is much like that of a speaker giving a speech: listeners sometimes fall asleep, but it is rare for a speaker to do so. Speakers do not fall asleep
because they are active. A wise expositor knows that to hold the attention of his audience, he must share some of his role with them.

The use of manipulative materials would seem to be a logical choice for a different approach to teaching algebraic concepts. One of the recommendations of The National Council of Teachers of Mathematics in its Agenda for Action (1980) was that teachers should use diverse instructional strategies, materials, and resources, such as manipulative materials, where suited, to illustrate or develop a concept or skill.

Manipulative materials are readily available at the elementary level; several mathematics programs have been developed to take advantage of this approach. The Math Solution by Marilyn Burns (1984) and Mathematics Their Way by Mary Barratta-Lorton (1976) are two of the best known such programs. The state of Texas has also produced some staff development modules for training teachers to use manipulatives on a wide range of mathematical topics at the elementary level. Also, research on the use of manipulatives at the elementary level is abundant and encouraging. Materials and programs at the secondary level, however, are not so readily available. Texas is only now in the process of producing staff development modules on the use of manipulatives for teaching mathematical topics at the secondary level. Since there has been so little done in this area at the secondary level, research on the effectiveness of using manipulative materials at this level is virtually nonexistent.
Statement of the Problem

The problem for this study is to determine whether the use of manipulatives will provide a better understanding of selected algebraic concepts for ninth and tenth grade Algebra I students than a nonmanipulative method. Also addressed in this study is whether or not the use of manipulatives is more beneficial for girls than boys and whether the use of manipulatives is more beneficial for low-ability students than for high-ability students.

Hypotheses

1. The mean score on an achievement test over selected algebraic concepts for a group of Algebra I students taught using manipulatives will be significantly higher than will the mean score of a control group of Algebra I students studying those same concepts without the use of manipulatives when the test is given immediately following instruction.

   a. Students using manipulatives will score significantly higher on the recall questions from the achievement test given immediately following instruction than students not using manipulatives.

   b. Students using manipulatives will score significantly higher on the application questions from the achievement test given immediately following instruction than students not using manipulatives.

2. The mean score on an achievement test over selected algebraic concepts for a group of Algebra I students taught using manipulatives will be significantly higher than will the mean score of a control group of Algebra I students covering those same concepts without the use of manipulatives when the test is administered two months after instruction.
a. Students using manipulatives will score significantly higher on the recall questions from the achievement test given two months after instruction than the students not using manipulatives.

b. Students using manipulatives will score significantly higher on the application questions from the achievement test given two months after instruction than the students not using manipulatives.

3. The mean score on an achievement test over selected algebraic concepts for a group of female Algebra I students taught using manipulatives will be significantly higher than the mean score of male Algebra I students covering those same objectives using manipulatives when the test is given immediately following instruction.

a. Female students using manipulatives will score significantly higher on the recall questions from the achievement test given immediately following instruction than the male students using manipulatives.

b. Female students using manipulatives will score significantly higher on the application questions from the achievement test given immediately following instruction than male students using manipulatives.

4. The mean score on an achievement test over selected algebraic concepts for a group of female Algebra I students taught using manipulatives will be significantly higher than the mean score of male Algebra I students covering those same objectives using manipulatives when the test is given two months after instruction.

a. Female students using manipulatives will score significantly higher on the recall questions from the achievement test given two months after instruction than male students using manipulatives.
b. Female students using manipulatives will score significantly higher on the application questions from the achievement test given two months after instruction than the male students using manipulatives.

5. The mean gain score on an achievement test over selected algebraic concepts for the fifteen Algebra I students with the lowest mathematics TEAMS scores taught using manipulatives will be significantly higher than the mean gain score of the fifteen Algebra I students with the highest mathematics TEAMS scores covering those same objectives using manipulatives when computed from pretest to posttest.

a. Fifteen students with the lowest mathematics TEAMS scores will have a significantly higher gain on the recall questions from the achievement test given immediately following instruction than the fifteen students with the highest mathematics TEAMS scores.

b. Fifteen students with the lowest mathematics TEAMS scores will have a significantly higher gain on the application questions from the achievement test given immediately following instruction than the fifteen students with the highest mathematics TEAMS scores.

6. The mean gain score on an achievement test over selected algebraic concepts for the fifteen Algebra I students with the lowest mathematics TEAMS scores taught using manipulatives will be significantly higher than the mean gain score of the fifteen Algebra I students with the highest mathematics TEAMS scores covering those same objectives using manipulatives when computed from pretest to retention test.

a. Fifteen students with the lowest mathematics TEAMS scores will have a significantly higher gain on the recall questions from the achievement test
given two months after instruction than the fifteen students with the highest mathematics TEAMS scores.

b. Fifteen students with the lowest mathematics TEAMS scores will have a significantly higher gain on the application questions from the achievement test given two months after instruction than the fifteen students with the highest mathematics TEAMS scores.

Limitations

This study will provide information about the effectiveness of manipulative materials for teaching Algebra I concepts. Broad generalizations about the use of manipulatives for teaching Algebra I concepts will not be possible because of the particular population used. Only ninth and tenth grade students will be involved in the study even though Algebra I is a course of study offered to a wide range of students from middle school to college. Another limitation is the fact that only a specific topic from algebra rather than the entire range of topics covered by the course will be used in the study.

Another limitation may arise because the same teachers will be teaching the experimental groups and the control groups, so it may be difficult for them not to use the manipulative ideas for the control group classes, especially if the manipulatives seem to provide a promising approach. The study may also be limited because the students are in the same school, so students from the experimental group and the control group may work with each other on their Algebra I outside of class. The students from the experimental group may share information learned in class with students from the control group.
Definition of Terms

For the purposes of this study, the following definitions were selected:

**Algebra I** -- a course covering operations on rational numbers and polynomials, linear equations and inequalities in one or two variables, systems of linear equations and quadratic equations.

**Manipulative** -- any object that a student or learner can physically handle.

**TEAMs** -- Texas Educational Assessment of Minimum Skills, an examination of minimum skills in mathematics and English administered to all students attending Texas schools in one of the odd numbered grades.

**CTBS** -- California Test of Basic Skills
CHAPTER II

REVIEW OF LITERATURE

Introduction

This review of literature is divided into eight sections: a brief history of manipulatives, a general review of selected articles on manipulatives, research on the use of manipulatives at the elementary level, at the secondary level, with college students, with college students who are prospective elementary teachers, sex differences with regards to mathematics, and mathematical performance of students at different ability levels. Research included in this review was collected only from the ten year period 1977 to 1987. The only research older than ten years that is included in this review is that which was mentioned in other research falling within the ten year period already mentioned and which seemed to have a direct bearing on this study.

A Brief History

It would be very difficult to pinpoint the beginning of the use of manipulatives in mathematics instruction. The use of manipulatives has played a subordinate role in the broader quest to introduce meaning into mathematical concepts. Although the use of sticks, stones, and other concrete objects by early civilizations could be considered among the earliest attempts to introduce meaning into mathematical concepts, this brief history will begin with specific references to manipulatives.

Brubacher (1966) considers Comenius (1592-1670) to be the first modern educator and feels Comenius' most outstanding achievement was his
writing of the first textbook that used pictures as an aid to explanations. Comenius felt that instruction was of the utmost importance and that everything should be taught through the senses (Trask, 1972). Comenius also felt that more than one sense at a time should be used in teaching because the several senses would help to reinforce each other.

Another contributor to the use of manipulatives for instruction was Johann Heinrich Pestalozzi, who in the late seventeen hundreds and early eighteen hundreds (National Council of Teachers of Mathematics, 1970). Pestalozzi was greatly influenced by other writers, including Locke and such eighteenth century authors as Hecker, Froebel, and Rousseau; however, it was Pestalozzi who really made an impact on instruction. He believed that the basis of all knowledge is "sense impressions" and the obtaining of sense perceptions through the use of any available concrete objects was a necessity.

The writings of Pestalozzi greatly influenced Warren Colburn, who wrote and introduced the book First Lessons in 1821 (NCTM, 1970). Because of this book, a number of instructional changes, such as arithmetic instruction at an earlier age made possible by the use of concrete and manipulative materials, took place in the United States. Colburn also advocated postponing the use of drill and practice until understanding was established. As always, however, the ideas put forth by Colburn were not universally accepted, and some of these concepts are still being pursued today.

John Dewey had a great influence on American education in general, but he also played an important part in the development of mathematics. Dewey made his contributions to the field of education in the late nineteenth and early twentieth centuries in the area of methodology (NCTM, 1970). Dewey felt that educators should rely on a child's natural tendency for activity. He felt that they
should use this natural activity to help clarify understanding and stimulate interest. Dewey felt that a child's activity should be directed to the desired learning. Manipulatives could be a central focus for the child's activities and could easily be used to give direction to those activities.

Another influential figure in the twentieth century was William Brownell (Pa, 1986). Brownell was a proponent of the structural school of thought which says that meaning is established by understanding the structure of a subject. Although Brownell stressed the importance of teaching the structure of arithmetic, he also argued that the use of concrete materials had a definite place in the structural school. In an article written in 1956 (reprinted in Arithmetic Teacher, 1987), Brownell stated that perhaps the most common educational error is the acceptance of memorized responses in place of insistence on understanding. He also emphasized that less practice would be necessary to promote and fix learning if the degree of understanding were greater. After conducting a research project comparing programs employing conventional, Cuisenaire, and Dienes approaches, Brownell (1968) suggested that conventional approaches needed to be overhauled to accommodate the experiences with discrete objects stressed in the other programs.

Van Engen, another twentieth century mathematics educator, was a principal contributor to the operational school of thought (Pa, 1986). This school of thought stresses the study of operations as the predominant concern of arithmetic. Van Engen professed that physical or sensory-motor activity is the source of meaning for subject matter. He stresses the fact that the foundations for abstract ideas arise from reactions to concrete objects (NCTM, 1973).

Another twentieth century mathematics educator who has had a profound impact on mathematics instruction is Zoltan Dienes. Dienes developed a set of
manipulatives called Dienes Blocks that have been widely used in the United States and abroad. According to Dienes (1961), learning takes place as we move from abstractions to generalizations. He says that one of the reasons so few people really understand mathematics is that they cannot internalize a sufficient number of abstractions from our present form of teaching. Dienes (1968) says that if children are given enough variety of mathematical materials to play with, they will be likely to develop the abstractions they need through the course of their play with these materials. Dienes (NCTM, 1973) feels that from his research, organisms want to manipulate and explore their environment in order to make predictions. He feels that mathematics learning would follow this same path if children were provided the time for mathematical play. He feels that this mathematical play could be generated by simply providing a large variety of constructed mathematical materials. Dienes (1963) says the most effective and permanent learning takes place as a result of concrete experiences.

About the same time that Dienes was advocating the use of the Dienes' blocks, Gattegno and Cuisenaire (1960) were making contributions with the Cuisenaire rods. Cuisenaire rods are manipulatives that are widely utilized today, and new uses for the rods are continually being developed.

The use of manipulative materials is still an issue in mathematics education which commands a great deal of attention. However, the issues at present seem to be how to get them into widespread use and when they should be used rather than whether or not they should be used. E. G. Begle (1979), one of the leaders of the "New Math" movement of the late fifties and early sixties, says that we know manipulatives can play an important role in mathematics instruction, but the question now is when and how to use them
most effectively.

**Articles on the Use of Manipulatives**

An article by Eleanor Duckworth (1964) discusses the ideas of Piaget and their implications for mathematics instruction. Duckworth points out Piaget's observation that adults as well as children learn better by doing things than by being told about them. Duckworth quotes Piaget as saying that all learning that is not the result of the learner's own activity will be deformed. Duckworth includes another quotation in which Piaget indicates that he specifically means acting on material things when he refers to activity by the learner. Elizabeth Fennema (1972) remarks that the idea of using concrete materials prior to introducing symbolic models is not new or unique. She cites Piaget's theory of cognitive development as an early work supporting the use of concrete materials. She notes that such practices are gaining increasing importance. She goes on to say that children are able to use symbols effectively only after they have experienced ideas actively through the manipulation of concrete models.

The *Arithmetic Teacher* (1986) devotes an entire issue to manipulatives and reiterates in the introduction the call of the National Council of Teachers of Mathematics for the use of manipulatives in its *Agenda for Action* (1986). Leonard Kennedy (1986) states that research supports the use of manipulatives at all school levels. He also states that many classroom teachers are skeptical about the use of manipulatives, but they see the value of these materials once they have used them with their classes. Kennedy also emphasizes the support of the use of manipulatives by learning theorists such as Piaget and Skemp. James Moser (1986) says that the proper use of manipulatives may eliminate
the need for later remediation of students. He also states that all students should use manipulative materials to develop concepts and establish a good concrete base for learning. However, some students may not need to use the manipulatives as long as others because some move quickly from the concrete to the abstract level. Michael Hynes (1986) agrees with Moser that there seems to be little disagreement among mathematics educators that experiencing ideas on the concrete level is important to learning mathematics. He further says that it is the teacher's responsibility to use concrete models to foster understanding of the abstract.

James Heddens (1986) points out the importance of bridging the gap between the concrete and the abstract. Ian Beattie (1986) emphasizes the importance of understanding algorithms in this age of calculators and computers; then he goes on to say that manipulative materials can show why algorithms work and clarify interpretations of each of the mathematical operations. Douglas Clements and Michael Battista (1986) suggest that manipulatives can play a vital part in the study of geometry. They say that both experience and research suggest that manipulatives not only prepare students for the formal study of geometry but also help students to develop knowledge and intuition about their spatial environment. They also point out that because geometry and geometric measurement are based on the study of the physical world, the use of manipulatives is imperative.

Carol Thornton and Barbara Wilmot (1986) emphasize the use of manipulative materials for two special groups of students, those with learning difficulties and those who are gifted. Both of these groups can benefit from the use of manipulatives according to Thornton and Wilmot.
Elizabeth Herbert (1985), a classroom teacher, describes her use of manipulatives at the middle school level. According to Herbert, manipulatives can improve student attitudes about mathematics. She feels that manipulative materials should be used on a regular basis in the mathematics classroom along with a variety of other tools and methods. Most teachers recognize and support the claim that manipulatives are very helpful in developing concepts at the primary level, but according to David Williams (1986), manipulatives can also be very useful at the pre-algebra and algebra level. According to Mark Driscoll (1984), all levels of students, not just those at the primary level, need experience with concrete materials.

Marilyn Suydam (1984) summarizes responses to a questionnaire and finds that most teachers believe that manipulatives should be used for mathematics instructions. Her summary of research done on manipulatives led to the following conclusions: lessons using manipulative materials have a higher probability of producing mathematics achievement, and the use of manipulatives enhances achievement across a variety of topics at all grade levels and at every ability level. She does, however, caution that the needs of the individual child must be kept in mind because not all children need to use manipulatives for the same length of time. In another article, Suydam (1986) points out that when manipulative materials are used to introduce content, problem-solving scores can improve. She also states that the use of manipulative materials for reinforcement can improve retention. In addition, she asserts that a major learning difficulty of students arises because so few teachers use manipulative materials.

Maureen Herman (1983) cautions teachers about using manipulatives with high school students having difficulty. Although she emphasizes the need
for an approach that is more concrete for these students, she says that teachers
must be sensitive to the students' feelings that they are being treated like
elementary school students; they may refuse to use the manipulatives.
Friedman (1978) says that on the basis of research, any instructional strategy
after the first grade that gives preeminence to the use of manipulatives is
unwarranted. However, he does state that the manipulative material strategy
should be included in an instructional repertoire. Kieren (1969) says that
research on manipulative activity has suffered from the bias of having the effort
focused on the development of manipulatives rather than on the application of
them. He suggests that further research might be more beneficial if it
concentrated on such questions as these: Who needs to do the actual
manipulation at different grade levels? Is a multimodel approach more
beneficial than a single model approach? What role can the computer play in
supplying manipulative-type activities? One last word of caution is expressed
by Robert Jackson (1979). Although he recognizes the value of manipulatives
for the instruction of mathematics, he says that manipulatives are not a
panacea. They demand thoughtful selection and skillful use.

Research on the use of Manipulatives at the Elementary Level

There is a large amount of research on the use of manipulatives at the
elementary level. In their extensive review of the research in this area, Suydam
and Higgins (1977) conclude that mathematics topics taught with manipulatives
have a high probability of producing greater achievement than the same topics
taught without the use of manipulatives. Twenty-three studies on a variety of
mathematical topics comparing only use to non-use of manipulatives are
considered in their review. Suydam and Higgins report that of the twenty-three
studies, eleven favor the use of manipulatives, ten find no significant difference between the achievement of students using manipulatives and those not using manipulatives, and two favor the lessons taught without manipulatives. In their conclusion, the reviewers support the use of manipulatives, stating that if there is really no difference in achievement between students using manipulatives and those not using manipulatives, then the number of studies favoring the use of manipulatives should be equal to the number of studies favoring non-use, and the number of each of these should be less than those showing no significant difference, approximating a normal distribution. However, far more studies support the use of manipulatives than those that support non-use. With this in mind, the researchers conclude that in a simple comparison of lessons taught with manipulatives and those taught without manipulatives, the non-manipulative lessons taught at the elementary level cannot be expected to produce superior results. The reviewers also suggest that a possible reason for so many of the studies showing no significant difference is that the way manipulative materials are used in lessons may be of critical importance. The reviewers also find that the use of manipulative materials is equally effective for all ability levels. High-ability students profit from the use of manipulatives just as much as the low-ability students do.

A similar conclusion is reached by Parham (1983) in a meta-analysis of the research on the use of manipulative materials, which treat a variety of mathematical topics, at the elementary level. In her analysis of the results of sixty-four studies comparing use to non-use of manipulative materials, she concludes that the use of manipulative materials has a positive effect on student achievement. Parham finds that students who used manipulatives scored at approximately the 85th percentile, whereas students who did not use
manipulatives scored at the 50th percentile. From a step-wise multiple regression analysis, Parham also concludes that the grade level of the students and the type of study done were the two variables that had the greatest effect on the results of the studies considered in the meta-analysis. She finds that as the grade level increases, the effect of the manipulatives decreases. The types of manipulatives compared were concrete, pictorial, and symbolic. She also finds that the concrete manipulative was the most effective.

Canny (1983) investigated the relationship of manipulative materials to achievement in fourth-grade mathematics. She considered achievement in the areas of computation, concept development, and problem solving. Canny had three experimental groups of students using manipulatives in different ways and a control group which had no lessons involving manipulatives. The first experimental group used manipulatives only during the introduction of a concept, the second group used the manipulatives after a concept had been introduced using the traditional lecture method, and the third group used the manipulatives for introduction and reinforcement of a concept. Canny found that there is no significant difference in the scores of students in the three groups on SRA tests measuring computation and concept development. However, the group that used manipulatives for introduction purposes scored significantly higher on the problem-solving test. On a test designed by the researcher, there was a significant difference in the scores from the three groups on both achievement and retention. Both the group that used manipulatives for introduction and the group that used them for reinforcement scored significantly higher on these tests than the other group.

A study by Usnick (1985) does not support the use of manipulatives. Second-grade students were used in this study to determine whether or not the
use of manipulatives would improve learning of the addition algorithm with three digit numbers including regrouping. In a comparison of instructional methods, the direct instruction approach was favored over the approach using manipulative materials. However, Usnick points out that a limiting factor of this study was the lack of expertise in the use of manipulative materials on the part of the teachers as well as the students.

Another 1985 study found the use of manipulatives to be very beneficial (Beattys, 1985). This study compared three different approaches to learning the concept of area in the upper elementary grades. Results statistically favor the manipulative groups on the post and retention tests. More specifically, 60% of the students from the manipulative groups could solve area measurement problems that they were unable to solve before the treatment, whereas only 18% to 30% of the textbook approach groups were able to do so.

Toney (1968) conducted a study with fourth grade students which compared the effectiveness of manipulatives used two different ways. In her study, students in the experimental group used manipulatives, whereas the control group only saw a teacher demonstration manipulatives. She finds that the students using the manipulatives make greater gains in achievement than those that only see a teacher demonstrate manipulatives. She also found that boys in the manipulative group made greater gains in achievement and understanding than boys in the demonstration group, whereas girls in the manipulative group made greater gains than girls in the control group on understanding but not on achievement. She also found that girls in the manipulative group made greater gains on understanding than boys in that group.
A study by Bright, Harvey, and Wheeler (1981) investigated the use of manipulative materials for playing mathematical games that involve the ordering of common fractions. The fifth- and seventh-grade students in the study were given three different treatments. Some of the students were allowed to use manipulative materials while playing the games, some were given pictorial representations of the manipulatives during the game, and the third group was given neither the manipulatives nor the pictures. They found that there was no significant difference in the scores on a posttest among the three groups.

A study by Zirkle (1981) considered the use of manipulative materials for teaching area measurement to sixth- and seventh-grade students. Zirkle investigated whether the students actually handling the manipulatives or the teachers handling the manipulatives with the students only watching would have the greater effect on achievement and retention. He found that it did not seem to matter whether the students actually handled the manipulatives or whether they only watched the teacher manipulate the materials. He also found that scores on achievement and retention show that pictorial aids were as effective as manipulatives. However, he does say that less able students seem to learn best from the pictorial aids.

Another publication reviewed many of the studies of the use of manipulative materials at the elementary level (Perry and Grossnickle, 1987). Perry and Grossnickle state that one of the universally accepted principles among leading authorities in the field of elementary mathematics teaching is that pupils should have the opportunity to manipulate materials to help them discover a concept or procedure. They also found that research predominantly favors the use of manipulatives. Perry and Grossnickle also present results
from a survey of elementary teachers on the frequency of use of manipulative materials in their classrooms. They report a wide variation in the frequency of use; however, all students received at least some time using manipulative materials during their class instruction in mathematics.

Research on the Use of Manipulatives at the Secondary Level

There is not an abundance of research on the use of manipulatives for teaching mathematical concepts at the secondary level. The results of the existing studies are mixed and not as positive as those for the elementary level. With regards to algebra in particular, Werth (1985) points out that there have been a great many changes in the content and teaching of basic algebra between 1950 and 1985. He states that there is now a great diversification of content as well as teaching of basic algebra.

Three studies at the seventh-grade level have mixed results. Purser (1973) finds that seventh-grade students of all ability levels who receive instruction with the aid of manipulative materials score significantly higher on achievement and retention tests than do seventh-grade students taught without the manipulatives. In a study of some concepts of fractions involving seventh grade inner-city students, Rich (1972) used a single group of seventh grade, inner-city students with repeated trials for gathering data. He finds that manipulatives do not negatively affect the achievement of inner-city students, and that the use of manipulative materials benefits the below-average student more than the average or above-average student in achievement. However, in another study with seventh-grade students, McMillian (1972) found that the use of manipulative materials did not significantly affect learning, transfer, or retention of the concept of place value in a place value system of numeration,
as measured by achievement tests.

A study on the use of manipulative materials in an eighth-grade pre-algebra class was conducted by Altizer (1977). She finds the use of manipulative materials favorable for teaching multiplication and factoring of polynomials. Results of the posttest given at the conclusion of the unit did not favor the group using manipulatives over the group not using manipulatives. However, scores for the group using the manipulatives on the retention test were significantly higher than those for the group not using manipulatives. The study is somewhat limited because negative numbers were not used in the problems or test items in the study. It was felt that the students in the study were not familiar enough with negatives for them to be comfortable using them in the topics being studied.

Two studies of high school students do not find that the use of manipulative materials significantly improves achievement. Collins (1984) found no significant difference in achievement scores of basic mathematics students using manipulatives when compared to basic mathematics students using formal instruction with no manipulatives. Corwin (1977) found that there was no significant difference in achievement test scores between groups of geometry students who used manipulative materials and those who did not.

Dobelstein (1978) investigated the use of supplementary materials and activities in a first-year algebra course. Dobelstein looked at the effect of the materials on the attitude and achievement of the students. He found that the supplementary materials had no effect on the attitude of first-year algebra students. He also found that the materials did not affect the achievement of the students.
Spiegel (1985) conducted a study on the use of the computer to present selected basic algebra topics. Subjects in the study were secondary school students enrolled in first-year algebra, and the topics were multiplication of binomials and the factoring of polynomials that are the products of two binomials. Material was presented on the computer to one group using a tile model and to the other group using a conventional model. Both models were presented on the computer; therefore, the students using the tile model did not physically handle any of the tiles. She found that the students using the tile model did not score significantly higher on achievement, retention, or transfer tests than the students using the conventional model. She also found that there is no statistically significant interaction of ability levels and test scores. She found that the students had positive attitudes toward both models presented on the computer.

Stockdale (1980) conducted a study comparing the use of two different types of manipulatives for teaching multiplication of binomials and factoring of second-degree polynomials. The subjects for this study were secondary students enrolled in Algebra I. The manipulatives used were the Rasmussen mathtiles (a commercial product) and color-coded rectangles that were designed by Stockdale. The subjects in the study were divided into three groups. One group received instruction using the Rasmussen mathtiles, one group received instruction using the Stockdale model, and the third group received instruction without the use of any manipulatives. Stockdale found that neither of the manipulative groups performed significantly better on the achievement or retention tests than the group that was instructed without manipulatives. He found that students instructed with the mathtiles performed significantly better on one-variable items than did students using the Stockdale
model. However, he found that students instructed in either model did equally well on items involving two variables and items involving negatives.

Research on the Use of Manipulative Materials With College Students

Results of research in this area are again mixed but generally more favorable than the research at the secondary level. The first three studies reviewed deal with remedial college students while the remainder deal with students in required general college mathematics courses.

According to Maynard (1983) and Chew (1984), the use of manipulatives with remedial college students does not produce a significant overall difference from students not using manipulatives. Although the students in the Maynard (1983) study using the manipulatives score significantly higher on teacher-made unit tests than the students not using manipulatives, the overall effect of the manipulatives, taking into consideration posttests and covariates, is not significant. Chew (1984) also finds no significant difference between achievement scores of remedial students using manipulatives and those not using manipulatives. Chew does point out, however, that there is a lower dropout rate in classes that use manipulatives.

The findings of C. Smith (1975) are contrary to those of the two studies discussed above. Smith's research involves students enrolled in an introductory mathematics course who were classified as at risk according to a placement test or because of a deficiency in their high school mathematics. He finds a significant difference in mathematics achievement in favor of the at risk students using manipulative materials compared to those not using manipulatives.
G. Smith (1974) studied the effects of manipulative materials on the attitudes and achievement of college students in a required college mathematics class. Smith found that the use of manipulatives not only improves achievement but also encourages a more positive attitude toward the required mathematics course. J. Smith (1970) finds similar results among a comparable group of college students studying numeration systems with bases other than ten. He found that the students using manipulative materials significantly improved in skills performance and retention; he concludes that students who are beginners in a discipline may learn better when they progress from the concrete to the abstract by means of models. Archer (1972) also found that students using manipulative materials scored significantly higher on achievement and retention tests than students who did not use manipulatives. She concludes that college students who use manipulative materials may be able to achieve, retain, and transfer better than students who do not use manipulatives.

Drapac (1980) developed the manipulative materials necessary for presenting elementary algebra concepts and considered the effect of these materials on achievement, attitude toward mathematics, and math anxiety. She found that the use of manipulatives significantly improved achievement, significantly improved attitudes toward mathematics, created more confidence in the students' ability to learn mathematics, and produced significantly lower levels of mathematics anxiety.

Buckley (1977) conducted a study on the effect of concrete referents in the form of physical science applications on achievement and attitude toward algebra. The subjects were community college students enrolled in a developmental program in which remedial instruction was given for both
algebra and science. Buckley found that the concrete referents did not affect the attitudes of these students; however, the achievement scores of the students receiving the concrete referents were significantly higher than those of the students who did not receive the referents.

Research on Manipulatives With College Students Who Are Prospective Elementary Teachers

Results of research on the use of manipulatives with prospective elementary teachers are less favorable than any of the research findings considered here. Most of the research in this area is negative, but this conclusion may result because concepts taught to these students using manipulatives are concepts already well established, and thus the effects of a new approach on achievement would be minimal at best. The only positive aspects of the research in this area focus on the attitudes of the prospective teachers toward teaching with the aid of manipulatives.

Skipper (1972) finds that prospective elementary teachers using manipulatives perform no better than those taught with the lecture method. Another study by Hendrickson (1969) has similar results. He finds that there is no significant difference in mathematics achievement between prospective elementary teachers using manipulatives and those who do not use the manipulatives. Hall (1974) concurs with these findings. He concludes that approaches using manipulatives and approaches not using manipulatives are equally effective in the learning of mathematical ideas for prospective elementary teachers. A study by Barnett and Eastman (1978) also agrees with these findings and concludes that prospective elementary teachers do not actually have to handle manipulative aids in order to learn how to use them for
demonstration purposes. Their research, however, shows that the group actually handling the manipulative materials may better understand the concept involved. After further study on this topic, Eastman and Barnett (1979) report that groups using manipulatives do no better than groups not using manipulatives on paper and pencil tasks or tasks in which they are asked to demonstrate the manipulative materials. They conclude that watching manipulatives used or demonstrated is just as effective as handling the manipulatives. They also state that the attitudes toward mathematics of the manipulative and non-manipulative groups is not significantly different.

A study by Tyderle (1983) concludes that the use of manipulative materials does not appear to increase the knowledge of mathematical concepts or improve attitudes toward mathematics. However, the use of manipulatives increased the inclination of prospective elementary teachers to write lesson plans incorporating the use of manipulative materials and increased the prospective teachers' ability to demonstrate mathematical concepts using manipulative materials. Warkentin (1975) reports that although the achievement test results in his study favored the non-manipulative group, the manipulative group did not finish all of the material covered on the test, whereas the non-manipulative group did. He points out, however, that the use of manipulatives had a positive effect on the attitude of the prospective elementary teachers toward mathematics.

Research on Gender and Mathematical Performance

Fennema (1985) finds that although there has been a great deal of research done on the differences in mathematics attributable to gender, there has not been much agreement either on the extent of such differences or on the
relative importance of the different factors contributing to them. She finds that there are few consistent gender-related differences at the elementary level; however, there is a great deal more evidence to suggest that by the beginning of secondary school, boys frequently perform better than girls. She also says that the differences seem to increase as the difficulty level of mathematical examinations increases and are especially apparent when above-average performance is considered. She points out that male superiority seems to be especially evident in such tasks as true problem solving.

She found that although some of the gender differences could be attributed to biological factors, the overwhelming amount of evidence seems to suggest that the differences are more likely attributable to social and cultural stereotypes. For example, the first international study of student performance in mathematics reported that boys perform better than girls in mathematics in each of the countries studied, even though girls as a whole in some countries perform better than boys on the whole in other countries. She also points out that notions about gender-appropriate behavior and achievement continue to be perpetuated by the news media. She also says that beliefs about appropriate behavior for the sexes are reflected in the expectations of parents, peers, school, and society. She asserts that parents more often encourage their sons to pursue mathematical studies than their daughters, and that this parental influence affects their children's mathematical performance.

Fennema also finds that boys prefer pastimes that involve mastery of objects, whereas girls choose activities that reinforce interpersonal relationships, both of which conform to adult expectations. She says that the different areas in which boys and girls choose to excel contribute to early career choices and that these choices can eventually lead to decisions to drop out of
mathematics and not take the more advanced courses. She finds that boys in grades six through twelve have greater confidence in their ability to do mathematics and that there is a high correlation between confidence and performance. She also stresses that for girls to be highly successful in an area that society considers to be more appropriate for men, can result in anxiety, which in turn can affect performance.

In another study Fennema (1979) reports that when boys and girls study the same amount of mathematics, the differences in learning are minimal. She does, however, point out that fewer females elect to study mathematics and that some of the factors that contribute to this situation are their belief that mathematics is not useful to them and their lack of confidence in their ability to do mathematics. She finds that some other important factors which contribute to the differences exhibited between males and females in mathematics is the differentiated treatment that teachers give to males and females and the stereotyping of mathematics as a male activity. She goes on to say that there is nothing inherent that keeps females from learning mathematics at the same level as males. A meta-analysis done by Freeman (1985) found that there was no strong support for overall gender differences in mathematics.

Badger (1981) reviewed of research on gender differences in mathematics and concludes that girls' declining achievement in mathematics as they get older can largely be accounted for in social terms. He says that societal attitudes are probably the determining factor as to whether or not girls succeed in mathematics. He points out that until adolescence there does not seem to be any difference in mathematical achievement between boys and girls; however, with the onset of adolescence there is a marked decline in the performance of girls in relation to boys. He also finds that many girls fail to
continue their mathematical studies. He concludes that there does not seem to be one single factor to explain the gender differences in mathematics but several factors. He says that differences in spatial ability seem to favor boys, but then he asserts that practice can improve this skill and that girls do not choose to participate in activities that would improve their spatial ability. He also points out that the influence of self confidence as well as stereotyping of mathematics as a male domain work against females. Another important factor is the increased attention that teachers give male students in mathematics classes and the fact that teachers can have a major influence on girls' persistence in mathematics.

Luchins (1981) reviewed the literature about gender differences in mathematics and found that some of the accepted reasons may not be well founded. She says that differences in various aptitudes and abilities, such as males excelling in spatial ability and females excelling in verbal ability, are often cited as contributing to gender differences in mathematics. She feels that these stereotypes have been drummed into generations of teachers, counselors, and students even though the scientific evidence does not exist. She says, however, that some recent studies are not finding such clear-cut gender differences in spatial ability. She also refutes the idea that spatial ability is hereditary because it is not an all-or-nothing trait but one that lies on a continuum. She also says that there is some evidence that spatial ability can be affected by training and culture. She also rejects the idea that spatial ability in males is superior because of a lack of the hormone estrogen. Luchins also refutes the idea that boys have better spatial ability and thus are better at mathematics than girls because of the dominance of the right side of the male brain. She says that mathematical abilities are attributed to the left side of the
brain and therefore girls should be better at mathematics. She does feel that the differences in spatial ability between the sexes may be attributable to the training that boys in our society receive in tasks involving these abilities. She also points out that although spatial ability of students has been studied for over half a century, little is known about the spatial ability of mathematicians.

Burton (1978) found that the scores of children given an intelligence test with the items balanced so as not to favor either gender did not differ with respect to general intelligence. However, she finds that there are differences on subtests, with males excelling in mathematical areas, especially in the area of visual-spatial tasks. She says that there may be some innate biological differences that could account for the differences in mathematical ability between boys and girls; however, they do not seem to account for all the differences. She feels that cultural pressures can explain some of the differences. She also feels that sex-role identification as it relates to appropriate behavior and preferences may account for some of the differences in mathematical ability between boys and girls.

A review of literature by Kirschner (1982) concludes that teachers treat boys and girls differently in mathematics classes, with high-achieving boys receiving the most attention. She finds that this preferential treatment reinforces the stereotyping of mathematics as a male domain. She finds that girls who regard mathematics as a subject for both boys and girls tended to take more math. Signorella and Jamison (1986) support this idea and say that a girl with a masculine self-concept has a significantly better chance of performing well on mental rotations or spatial perception tasks. They also find that a masculine self-concept could be an important factor in explaining sex differences in cognition. Kirschner (1982) goes on to say that in grades six through twelve
boys were significantly more confident in their mathematical ability than girls. She also points out the positive effect that parents can have on students' attitudes toward mathematics and that fathers' expectations influence the mathematics achievement of girls.

Individual studies have also contributed some interesting results. In a study of eighth-grade students in Ontario, Hanna (1986) found that boys scored significantly higher on the geometry and measurement sections of the test which she says could possibly be attributed to informal training that boys may receive in these areas. She also reported that girls had a much higher omission rate on the test on all topics. She also says that instruction at this level had about the same influence on girls as boys. Fennema and Tartre (1985) found that in a problem-solving situation emphasizing the use of spatial visualization, students with high spatial visualization skills solved no more problems than students with low spatial visualization skills. They question the idea that spatial visualization skills are highly important in the learning of mathematics. Pesch (1985) studied the differences in performance on Piagetian tasks by eighth-grade students based on gender. He found that males did better on these tasks than did females at this level and suggested that females might benefit from receiving a different type of instruction. A study by Pepin, Beaulieu, Matte, and Leroux (1985) focused on spatial ability in a computer-game format. They found that boys performed significantly better than girls on the game involving spatial ability. They suggest, however, that girls can match boys' performance in spatial ability by simple training. Meyers (1986) found that the affective variables, confidence, usefulness, sex-role congruency, teacher influence, and patterns of causal attributions are often cited as reasons for gender-related differences in mathematics achievement and participation.
She says that the selected variables predicted achievement and participation when used with prior achievement. Moreover, the affective variables were better predictors for females than for males, and confidence was the most predictive variable for both males and females. Koehler (1986) found that in the Algebra I classes she studied, males received more of all types of interactions; however, there was no apparent relation between this preferential treatment and the algebra achievement of the males and females. She also found that the class that most effectively enhanced achievement in algebra for females was one in which the teacher fostered the development of autonomous learning behaviors.

In a study with tenth-grade students, Tartre (1985) found that females with a high spatial orientation skill scored as well as or better than the males in solving mathematics problems. She also observed that females with low spatial orientation skills scored lower than any other group on mathematics achievement and on many tasks involved in solving mathematics problems. A study by Brown (1980) of ninth-grade algebra students found that females performed better than males, but that gender did not contribute significantly to attitude toward mathematics. He also found that boys did somewhat better when enrolled in individualized classes than in traditional classes, whereas girls performed equally as well in either type of class.

Research on Students With Different Ability Levels

A review of literature by Suydam and Higgins (1977) suggests that many educators believe low achievers need to use manipulative materials more than high achievers; however, they point out that research does not support this idea. They assert that research done in this area has had very mixed results, but
overall the research shows no significant difference in gains between high- and low-achieving students when manipulatives are used. Suydam (1983) states that high and low achieving students receive different treatment from their teachers. She says low achieving students are called on less frequently, given less praise, given fewer clues, given less attention, and given less time to respond. She says that low achieving students are being given a self-image that will probably result in poorer achievement. Suydam (1984) says that the nature and cause of a student’s low achievement must be determined in order for a teacher to help because that information may indicate the best instructional method for that student. She goes on to say that diagnosis is imperative because there are very few methods that work for every student in every instance. Based on her review of research, she observes that certain things are applicable for remediation with many students, one of which is to encourage active involvement of the student with a variety of manipulative materials at all age levels.

A review of literature by Alesandrini (1982) points out that the imagery-eliciting strategies of pictures, concreteness, and imagery instructions generally help to bring about meaningful learning for a variety of learners under a variety of conditions. She suggests that a strategy which employs the use of pictures may be most beneficial for younger and more disadvantaged children.

Purser (1973) studied seventh-grade mathematics students and compared the effectiveness of manipulative materials for students of different ability levels. He found that high, medium, and low ability students who use manipulatives scored significantly higher on both a posttest and a retention test than those students with the same abilities who did not use manipulatives.

Brace (1976) studied the difference between using only one model to represent
a mathematical concept as compared to using several models for the same concepts. She found that students taught using several models scored significantly higher on a posttest than students using only a single model. She also found that the high ability students had significantly higher mean scores than the low ability students. Simpson (1974) investigated the effect of a laboratory approach to instruction for slow learners in mathematics. He found that among the seventh-graders in the study there was a difference in achievement between the students given the laboratory approach and those students taught with a teacher-centered approach. However, the differences were sometimes in favor of the laboratory approach and sometimes in favor of the teacher-centered approach. He concluded that neither approach is always favorable for higher achievement, nor does one approach seem to foster a better attitude toward mathematics than the other.

Calvano (1986) found that significant differences existed between the learning styles of high- and low-achieving students at the middle school level. She found that high-achieving students prefer responsibility, persistence, intake, and warmth during educational activities. Low-achieving students prefer tactile learning experiences, teacher motivation, the presence of authority figures and mobility while learning. She also points out that learning style preferences differ by grade level and gender. A study of Algebra I students by Johnston (1983) compares the effects of specificity of objectives on high and low reasoning ability students. He found that there was no significant effect on achievement between the students with high reasoning ability or low reasoning ability when the objectives for the course were very specific or when the objectives were very general.
Two studies also considered mathematical achievement as it relates to Piagetian levels. Bruce (1986) found that gifted students had significantly higher achievement on the Social Science Piagetian Inventory test than non-gifted students in the fifth and seventh grades. She also found that gender did not affect the achievement scores on the test. Johnson (1978) studied college algebra students to test the relationship between the Piagetian level of the students with their success in the class. He found that there was no statistically significant relationship between the Piagetian level of the students in college algebra and their success in the class.

Summary

The studies cited in this section give mixed results. Research supports the use of manipulatives in mathematics instruction at the elementary level, where the majority of the research has been done. Research at the secondary level is not as favorable as that at the elementary level. Lack of favorable data could be due to several different factors, some of which were pointed out in the discussions of the studies. Secondary teachers are not accustomed to using manipulatives and are not comfortable with them. Students at the secondary level do not readily embrace the use of manipulatives, viewing them as somewhat childish or too elementary for them. Manipulatives may be very difficult for secondary students to use if they have never used them prior to secondary school.

The research on the use of manipulatives for algebra is mixed, and because there is a limited amount of data available, it is difficult to generalize. Research on the effectiveness of manipulatives for girls versus boys and high ability versus low ability students is also mixed. Therefore, research on the use
of manipulatives for elementary algebra concepts with secondary school students seems warranted. The comparison of boys and girls as well as high and low ability students as it relates to the use of manipulatives for instruction of basic algebra concepts also seems warranted.
CHAPTER III

METHODS

Population

The population for the study consisted of all the students from eight intact Algebra I classes. These classes were chosen at random from all the Algebra I classes taught by the four teachers involved in the study. The teachers were volunteers who agreed to teach one class with manipulatives and one class without manipulatives for the same set of objectives. Each of the teachers was teaching three classes of Algebra I at the time of the study. Two of each teacher's classes were chosen at random to participate in the study, one as an experimental (manipulative) group and one as a control (nonmanipulative) group. The students involved in this study were either 9th or 10th graders.

The experimental group originally consisted of 101 students, but 10 of the students were dropped from the study because they left school before the study was completed. The final number of students in the experimental group was 91. Of these 91 students, 53 were boys and 38 were girls. The experimental group contained 58 students who were 9th graders and 33 students who were 10th graders. Prior to their enrollment in the Algebra I course, were as follows, 42 students had taken eighth-grade mathematics, 12 had taken honors eighth-grade mathematics, 27 had taken Pre-Algebra, 6 had taken Algebra I, and 4 who had recently transferred into the district had no records available. The mathematics TEAMS scores for this group ranged from 715 to 999. The mean mathematics TEAMS score for the experimental group
was 853.2, with a median of 854 and a mode of 854. The mean percentile score from the total mathematics score of the CTBS that the students in the experimental group took in the eighth grade was 82.6. The CTBS percentile scores for this group ranged from 43 to 98, with a median of 87 and a mode of 88.

The control group originally consisted of 109 students, but 13 students were removed from the study because they left school before the study was completed. The final number of students in the control group was 96. Of these 96 students, 52 were boys and 44 were girls. The control group consisted of 59 students who were 9th graders and 37 students who were 10th graders. Prior to their enrollment in the Algebra I course, 46 subjects had taken eighth-grade mathematics, 11 had taken honors eighth-grade mathematics, 26 had taken Pre-Algebra, 9 had taken Algebra I, 1 had taken Fundamentals of Mathematics, 1 had taken Geometry, and 2 who had recently transferred into the district had no records available. The mathematics TEAMS scores for the control group ranged from 691 to 999. The mean mathematics TEAMS score for this group was 850.8, with a median of 849 and a mode of 902. The mean percentile score from the total mathematics score of the CTBS that the students in the control group took in the eighth grade was 80.9. The CTBS percentile scores for this group ranged from 46 to 98, with a median of 86 and a mode of 94.

Demographic Information

The school district selected for this study is located in north Texas and has approximately 28,106 students. The city covers a 66 square mile area, but the district itself covers a 114 square mile area, taking in two adjacent cities and branching into three surrounding cities. The suburban district is classified as
middle to upper-middle class. The ethnic composition is 89.5% Anglo, 3.1% Hispanic, 4.0% Black, and 3.4% other nationalities. The district is composed of 23 elementary schools, 8 middle schools, 4 high schools (grades 9 and 10), and 2 senior high schools (grades 11 and 12). About 60% of the teachers in the district hold master's degrees, and 82.8% of the students in the district are college bound.

The four teachers in the study were female. All were experienced teachers who had taught Algebra I. Only one teacher had previously used manipulatives, but she had not used them in Algebra I. She was the only one who had received instruction in the use of manipulatives prior to the study, but again this instruction was not in the use of algebra tiles. At the time of the study, three of the teachers had been teaching Algebra I for eight years, and one had been teaching Algebra I for 11 years. All of the teachers had master's degrees, two of them in mathematics and the other two in education.

Instrumentation

The test used for this study was developed by the researcher. No Algebra I test was available that covered only the objectives being used in this study. The test was an achievement test in a multiple-choice format and was based on five objectives with eight items for each objective. Four of the eight questions on each objective tested higher order thinking skills at the application level and the remaining four questions tested these skills at the simple recall level. The simple recall questions were questions exactly like those the students had worked in class; only the numbers were different. The students had never seen the problems which tested higher order thinking skills, but these could be worked by transferring the knowledge acquired and applying it to this
new type of problem.

The test was checked for reliability using the Kuder-Richardson Formula 20. Before the test was administered, an overall reliability of .80 was chosen as an acceptable level of reliability. The test was administered to 111 Algebra II students who attended a school not involved in the study. Results of the calculations gave an overall reliability coefficient of .88.

Content validity for the test was established by a committee composed of three mathematics supervisors from school districts comparable to the district in which the study was conducted and four mathematics specialists from the Texas Education Agency. The committee reviewed the test items and approved each item as appropriate for the objective that it was testing as well as being either a simple recall item or a higher order thinking skill item.

The same test was used for the pretest, the posttest, and the retention test. A copy of the test is included in Appendix A.

Rationale for the Selection of the Manipulative Materials

Three criteria were considered in the selection of the manipulatives. The first was whether or not the manipulative was an appropriate model for the mathematical concepts being presented. Algebra tiles are well accepted and widely used for teaching the particular objectives that had been selected for the study. The tiles present a clear and accurate concrete picture of the mathematical concepts covered by the objectives in the study.

The second consideration was the cost of the manipulatives. Because all students would need to have their own sets with which to work, the tiles needed to be relatively inexpensive. Poster board was ideal for their construction. The sets of overhead tiles proved to be more expensive.
However, because only four sets were needed, the cost was not prohibitive.

The third consideration was the ease with which the manipulatives could be handled and transported. Students had to be able to carry the manipulatives easily because they would be expected to have them in class each day as well as at home for homework assignments. The poster board tiles were easy to carry around because they fit easily into a reclosable plastic bag, which in turn fit nicely into a notebook. However, the tiles were not so small that handling them was a problem.

Research Design and Treatment

The study involved a pretest, a posttest, and a retention test with an experimental and a control group. All of the students in the study were taught the unit from the regular school district curriculum covering the following objectives:

1. The student will be able to square a binomial.
2. The student will be able to multiply two binomials whose product is the difference of two squares.
3. The student will be able to multiply any two binomials.
4. The student will be able to factor the difference of two squares.
5. The student will be able to factor any second degree polynomial whose factors are two binomials.

The unit was taught as the first unit of the fourth six weeks grading period and took approximately three weeks to complete. Students in the experimental group were told that they would be allowed to keep the manipulatives and that they were permitted to use them whenever they wished. Students in the control group were not given any instructions different from those ordinarily given at the
beginning of a new unit. The pretest was administered to all the students in the experimental group and the control group the day before the teachers began discussing the unit.

Students in the experimental group were taught the designated concepts by using the manipulative materials. Two to three days after the pretest, students in the manipulative group were shown how to use the manipulatives to represent variables. A copy of this introductory lesson is included in Appendix B. The manipulatives used after the introductory lesson were a combination of those suggested by Laycock and Smart (1981), Laycock and Schadler (1987), and Howden (1985). Actual teaching suggestions for using the manipulatives to present the designated concepts were also taken from these three sources. Each student in the experimental group received a set of manipulatives to keep and use throughout the study. A diagram of what each set contained is included in Appendix C.

Students in the experimental group were assigned fewer homework problems than those in the control group because they were expected to use the tiles to work the problems and then draw diagrams of the solutions. This is an important step in bridging the concrete and the symbolic representations, but it does require more time. The assignment of fewer homework problems to the experimental group was an attempt to equalize the time required for students in both groups to complete homework assignments. The control group covered the same topics as the experimental group without the use of manipulatives. When the unit was completed, the students from the experimental group and the control group were given the posttest. Two months after the unit was completed the students were again given the test as a retention test.
Each teacher in the study taught one experimental class and one control class. Each was given a set of overhead algebra tiles, which had pieces similar to the pieces in the experimental students' manipulative kits, for demonstration purposes on the overhead projector. In addition to the tiles, each teacher also had a copy of the manual, written by Hilde Howden (1985), that accompanied the algebra tiles. The teachers were trained in the use of the manipulatives by the researcher during a three-week period immediately prior to the study. Training was done in three two-hour sessions in which the teachers actually used the manipulatives. The first session consisted of activities for the teachers to use as an introduction to manipulatives for their students. A copy of this lesson is included in Appendix B. The second session with the teachers covered the use of the manipulatives for multiplying binomials and factoring into binomials. The two-color method and the cover-up method of using the tiles were presented at this session. The teachers agreed that both methods were valuable and should be presented to the students rather than selecting only one method to use throughout the study. Session three was a planning session in which the four teachers and the researcher actually planned lessons and established a timetable. The teachers were also instructed at this time that they were not to use the manipulatives or any ideas from the manipulative sessions in the classes containing the control group.

The classes involved in the study were observed by the researcher throughout the course of the study. Only the experimental classes were observed; no observations were made while examinations were being administered. Observations were made for entire class periods.
Treatment of Data

Data were collected three times during the study. A pretest was administered to all students the day before the teachers began the unit. A posttest was administered to the students as soon as they completed the unit. A retention test was given to all students approximately two months after completion of the unit.

Hypotheses one through four were tested using analysis of covariance (ANCOVA). Table 1 shows the covariates, dependent variables, and independent variables for each of the hypotheses. Hypotheses three and four were tested using the manipulative group only.

Hypotheses five and six were tested using analysis of variance (ANOVA). The dependent variable in each case was a gain score. Hypothesis five tested the difference in gain from the pretest to the posttest for high and low achieving students. High achieving students were defined as the 15 students in the manipulative group with the highest mathematics TEAMS scores on the ninth grade TEAMS test. Low-achieving students were defined as the 15 students in the manipulative group with the lowest mathematics TEAMS scores on the ninth-grade TEAMS test. Hypothesis six tested the difference in gain from the pretest to the retention test for the high and low achieving students.

For each hypothesis, two tables were prepared. The first table presented is a table of means and standard deviations. The second table presented is an ANOVA or ANCOVA table indicating the significance of the difference between means.
Table 1

Variables in Hypotheses One Through Four

<table>
<thead>
<tr>
<th>Hypothesis Variable</th>
<th>Covariate</th>
<th>Dependent Variable</th>
<th>Independent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-a</td>
<td>pretest-recall</td>
<td>posttest-recall</td>
<td>treatment</td>
</tr>
<tr>
<td>1-b</td>
<td>pretest-application</td>
<td>posttest-application</td>
<td>treatment</td>
</tr>
<tr>
<td>2-a</td>
<td>pretest-recall</td>
<td>retention test-recall</td>
<td>treatment</td>
</tr>
<tr>
<td>2-b</td>
<td>pretest-application</td>
<td>retention test-application</td>
<td>treatment</td>
</tr>
<tr>
<td>3-a</td>
<td>pretest-recall</td>
<td>posttest-recall</td>
<td>sex</td>
</tr>
<tr>
<td>3-b</td>
<td>pretest-application</td>
<td>posttest-application</td>
<td>sex</td>
</tr>
<tr>
<td>4-a</td>
<td>pretest-recall</td>
<td>retention test-recall</td>
<td>sex</td>
</tr>
<tr>
<td>4-b</td>
<td>pretest-application</td>
<td>retention test-application</td>
<td>sex</td>
</tr>
</tbody>
</table>
Hypothesis 1

Hypothesis one states that the mean score on an achievement test over selected algebraic concepts for a group of Algebra I students taught using manipulatives will be significantly higher than will the mean score of a control group of Algebra I students studying those same concepts without the use of manipulatives when the test is given immediately following instruction.

Hypothesis 1a states that students using the manipulatives will score significantly higher on the recall questions from the achievement test given immediately following instruction than the students not using manipulatives. This hypothesis was tested using ANCOVA. The pretest score on the recall items was the covariate, and the posttest score on the recall items was the dependent variable. The means and standard deviations of the scores on the recall items are presented in Table 2. The observed means for the recall items on the posttest, controlling for the pretest, are presented as the adjusted means.

The results of the ANCOVA are presented in Table 3. ANCOVA tests the significance of the difference between the adjusted means. As Table 3 shows, there was no significant difference between the adjusted mean posttest scores on the recall items for the control and manipulative groups.
Table 2
Pretest and Posttest Recall Items by Treatment Groups

<table>
<thead>
<tr>
<th></th>
<th>Pretest Recall Items</th>
<th></th>
<th>Posttest Recall Items</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Standard Mean</td>
<td>Standard Deviation</td>
<td>Observable Mean</td>
<td>Adjusted Mean</td>
</tr>
<tr>
<td>Control</td>
<td>6.31</td>
<td>3.04</td>
<td>17.61</td>
<td>17.61</td>
</tr>
<tr>
<td>Manipulative</td>
<td>6.33</td>
<td>2.87</td>
<td>17.34</td>
<td>17.34</td>
</tr>
<tr>
<td>Total</td>
<td>6.32</td>
<td>2.95</td>
<td>17.48</td>
<td>17.48</td>
</tr>
</tbody>
</table>

Table 3
Effect of Treatment on Posttest Recall Items, Controlling for Pretest Knowledge

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>DF</th>
<th>Mean Square</th>
<th>F</th>
<th>Significance of F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariates</td>
<td>44.329</td>
<td>1</td>
<td>44.329</td>
<td>7.139</td>
<td>.008</td>
</tr>
<tr>
<td>Treatment (between)</td>
<td>3.160</td>
<td>1</td>
<td>3.160</td>
<td>.509</td>
<td>.477</td>
</tr>
<tr>
<td>Residual (within)</td>
<td>1049.418</td>
<td>169</td>
<td>6.210</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1096.907</td>
<td>171</td>
<td>6.415</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Hypothesis 1b states that students using the manipulatives will score significantly higher on the application questions from the achievement test given immediately following instruction than the students not using manipulatives. This hypothesis was tested using ANCOVA. The pretest score on the application items is the covariate and the posttest score on the application items is the dependent variable. The means and standard deviations of the scores on the application items are presented in Table 4. The observed means for the application items on the posttest, controlling for the pretest, are presented as the adjusted means.

Table 4
Pretest and Posttest Application Items by Treatment Groups

<table>
<thead>
<tr>
<th></th>
<th>Pretest Application Items</th>
<th>Posttest Application Items</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Standard Mean</td>
<td>Standard Deviation</td>
</tr>
<tr>
<td>Control</td>
<td>5.75</td>
<td>2.32</td>
</tr>
<tr>
<td>Manipulative</td>
<td>5.20</td>
<td>2.36</td>
</tr>
<tr>
<td>Total</td>
<td>5.49</td>
<td>2.35</td>
</tr>
</tbody>
</table>
The results of the ANCOVA are presented in Table 5. As Table 5 shows, there was no significant difference between the adjusted mean posttest scores on the application items for the control and manipulative groups.

Table 5

Effect of Treatment on Posttest Application Items, Controlling for Pretest Knowledge

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>DE</th>
<th>Mean Square</th>
<th>F</th>
<th>Significance of F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariates</td>
<td>134.207</td>
<td>1</td>
<td>134.207</td>
<td>17.235</td>
<td>.000</td>
</tr>
<tr>
<td>Treatment (between)</td>
<td>6.238</td>
<td>1</td>
<td>6.238</td>
<td>.801</td>
<td>.372</td>
</tr>
<tr>
<td>Residual (within)</td>
<td>1315.992</td>
<td>169</td>
<td>7.787</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1456.436</td>
<td>171</td>
<td>8.517</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Hypothesis 2

Hypothesis 2 states that the mean score on an achievement test over selected algebraic concepts for a group of Algebra I students taught using manipulatives will be significantly higher than will the mean score of a control group of Algebra I students studying those same concepts without the use of manipulatives when the test is administered two months after instruction.

Hypothesis 2a states that students using the manipulatives will score significantly higher on the recall questions from the achievement test given two months after instruction than the students not using the manipulatives.
This hypothesis was tested using ANCOVA. The pretest score on the recall items is the covariate, and the retention test score on the recall items is the dependent variable. The means and standard deviations of the scores on the recall items are presented in Table 6. The observed means for the recall items on the retention test, controlling for the pretest, are presented as the adjusted means.

Table 6
Pretest and Retention Test Recall Items by Treatment Groups

<table>
<thead>
<tr>
<th></th>
<th>Pretest Recall Items</th>
<th>Retention Test Recall Items</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Standard Mean</td>
<td>Observable Mean</td>
</tr>
<tr>
<td></td>
<td>Deviation</td>
<td></td>
</tr>
<tr>
<td>Control</td>
<td>6.31</td>
<td>16.07</td>
</tr>
<tr>
<td>Manipulative</td>
<td>6.33</td>
<td>16.02</td>
</tr>
<tr>
<td>Total</td>
<td>6.32</td>
<td>16.05</td>
</tr>
</tbody>
</table>

The results of the ANCOVA are presented in Table 7. As Table 7 shows, there was no significant difference between the adjusted mean retention test scores on the recall items for the control and manipulative groups.
Table 7

**Effect of Treatment on Retention Test Recall Items, Controlling for Pretest Knowledge**

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>DF</th>
<th>Mean Square</th>
<th>F</th>
<th>Significance of F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariates</td>
<td>117.155</td>
<td>1</td>
<td>117.155</td>
<td>7.152</td>
<td>.008</td>
</tr>
<tr>
<td>Treatment (between)</td>
<td>.092</td>
<td>1</td>
<td>.092</td>
<td>.006</td>
<td>.940</td>
</tr>
<tr>
<td>Residual (within)</td>
<td>2768.381</td>
<td>169</td>
<td>16.381</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>2885.628</td>
<td>171</td>
<td>16.875</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Hypothesis 2b states that students using the manipulatives will score significantly higher on the application questions from the achievement test given two months after instruction than the students not using the manipulatives. This hypothesis was tested using ANCOVA. The pretest score on the application items is the covariate, and the retention test score on the application items is the dependent variable. The means and standard deviations of the scores on the application items are presented in Table 8. The observed means for the application items on the retention test, controlling for the pretest, are presented as the adjusted means.

The results of the ANCOVA are presented in Table 9. As Table 9 shows, there was no significant difference between the adjusted mean retention test scores on the application items for the control and manipulative groups.
Table 8
Pretest and Retention Test Application Items by Treatment Groups

<table>
<thead>
<tr>
<th></th>
<th>Pretest Application Items</th>
<th>Retention Test Application Items</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Standard Mean</td>
<td>Standard Deviation</td>
</tr>
<tr>
<td>Control</td>
<td>5.75</td>
<td>2.32</td>
</tr>
<tr>
<td>Manipulative</td>
<td>5.20</td>
<td>2.36</td>
</tr>
<tr>
<td>Total</td>
<td>5.49</td>
<td>2.35</td>
</tr>
</tbody>
</table>

Table 9
Effect of Treatment on Retention Test Application Items, Controlling for Pretest Knowledge

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>DF</th>
<th>Mean Square</th>
<th>F</th>
<th>Significance of F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariates</td>
<td>48.713</td>
<td>1</td>
<td>48.713</td>
<td>3.705</td>
<td>.056</td>
</tr>
<tr>
<td>Treatment (between)</td>
<td>.897</td>
<td>1</td>
<td>.897</td>
<td>.068</td>
<td>.794</td>
</tr>
<tr>
<td>Residual (within)</td>
<td>2221.832</td>
<td>169</td>
<td>13.147</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>2271.442</td>
<td>171</td>
<td>13.283</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Hypothesis 3

Hypothesis 3 states that the mean score on an achievement test over selected algebraic concepts for a group of female Algebra I students taught using manipulatives will be significantly higher than the mean score of male Algebra I student studying those same objectives using manipulatives when the test is given immediately following instruction.

Hypothesis 3a states that female students using the manipulatives will score significantly higher on the recall questions from the achievement test given immediately following instruction than the male students using the manipulatives. This hypothesis was tested using ANCOVA. The pretest score on the recall items is the covariate, and the posttest score on the recall items is the dependent variable. The means and standard deviations of the scores on the recall items are presented in Table 10. The observed means for the recall items on the posttest, controlling for the pretest, are presented as the adjusted means.

The results of the ANCOVA are presented in Table 11. As Table 11 shows, there was no significant difference between the adjusted mean posttest scores on the recall items for boys and girls.

Hypothesis 3b states that female students using the manipulatives will score significantly higher on the application questions from the achievement test given immediately following instruction than the male students using the manipulatives. This hypothesis was tested using ANCOVA. The pretest score on the application items is the covariate, and the posttest score on the application items is the dependent variable. The means and standard deviations of the scores on the application items are presented in Table 12.
Table 10

Scores on Pretest and Posttest Recall Items by Gender

<table>
<thead>
<tr>
<th></th>
<th>Pretest Recall Items</th>
<th>Posttest Recall Items</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Standard Mean</td>
<td>Standard Deviation</td>
</tr>
<tr>
<td>Boys</td>
<td>6.21</td>
<td>3.14</td>
</tr>
<tr>
<td>Girls</td>
<td>6.47</td>
<td>2.50</td>
</tr>
<tr>
<td>Total</td>
<td>6.33</td>
<td>2.87</td>
</tr>
</tbody>
</table>

Table 11

Effect of Gender on Scores for Posttest Recall Items, Controlling for Pretest Knowledge

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>DF</th>
<th>Mean Square</th>
<th>F</th>
<th>Significance of F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariates</td>
<td>17.911</td>
<td>1</td>
<td>17.911</td>
<td>2.244</td>
<td>.138</td>
</tr>
<tr>
<td>Gender (between)</td>
<td>.000</td>
<td>1</td>
<td>.000</td>
<td>.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Residual (within)</td>
<td>638.643</td>
<td>80</td>
<td>7.983</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>656.554</td>
<td>82</td>
<td>8.007</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The observed means for the application items on the posttest, controlling for the pretest, are presented as the adjusted means.

Table 12
*Pretest and Posttest Application Items by Gender*

<table>
<thead>
<tr>
<th></th>
<th>Pretest Application Items</th>
<th>Posttest Application Items</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Standard Mean</td>
<td>Deviation</td>
</tr>
<tr>
<td>Boys</td>
<td>5.26</td>
<td>2.46</td>
</tr>
<tr>
<td>Girls</td>
<td>5.14</td>
<td>2.26</td>
</tr>
<tr>
<td>Total</td>
<td>5.20</td>
<td>2.36</td>
</tr>
</tbody>
</table>

The results of the ANCOVA are presented in Table 13. As Table 13 shows, there was no significant difference between the adjusted mean posttest scores on the application items for boys and girls.

Hypothesis 4

Hypothesis 4 states that the mean score on an achievement test over selected algebraic concepts for a group of female Algebra I students taught using manipulatives will be significantly higher than the mean score of male Algebra I student studying those same objectives using manipulatives when the test is given two months after instruction.
Table 13

**Effect of Gender on Posttest Application Items, Controlling for Pretest Knowledge**

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>DF</th>
<th>Mean Square</th>
<th>F</th>
<th>Significance of F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariates</td>
<td>51.214</td>
<td>1</td>
<td>51.214</td>
<td>7.016</td>
<td>.010</td>
</tr>
<tr>
<td>Gender (between)</td>
<td>1.084</td>
<td>1</td>
<td>1.084</td>
<td>.149</td>
<td>.701</td>
</tr>
<tr>
<td>Residual (within)</td>
<td>583.991</td>
<td>80</td>
<td>7.300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>636.289</td>
<td>82</td>
<td>7.760</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Hypothesis 4a states that female students using the manipulatives will score significantly higher on the recall questions from the achievement test given two months after instruction than the male students using the manipulatives. This hypothesis was tested using ANCOVA. The pretest score on the recall items is the covariate, and the retention test score on the recall items is the dependent variable. The means and standard deviations of the scores on the recall items are presented in Table 14. The observed means for the recall items on the retention test, controlling for the pretest, are presented as the adjusted means.
Table 14
Pretest and Retention Test Recall Items by Gender

<table>
<thead>
<tr>
<th></th>
<th>Pretest Recall Items</th>
<th>Retention Test Recall Items</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Standard Mean</td>
<td>Standard Deviation</td>
</tr>
<tr>
<td>Boys</td>
<td>6.21</td>
<td>3.14</td>
</tr>
<tr>
<td>Girls</td>
<td>6.47</td>
<td>2.50</td>
</tr>
<tr>
<td>Total</td>
<td>6.33</td>
<td>2.87</td>
</tr>
</tbody>
</table>

The results of the ANCOVA are presented in Table 15. As Table 15 shows, there was no significant difference between the adjusted mean retention test scores on the recall items for boys and girls.

Hypothesis 4b states that female students using the manipulatives will score significantly higher on the application questions from the achievement test given two months after instruction than the male students using the manipulatives. This hypothesis was tested using ANCOVA. The pretest score on the application items is the covariate, and the retention test score on the application items is the dependent variable. The means and standard deviations of the scores on the application items are presented in Table 16. The observed means for the application items on the retention test, controlling for the pretest, are presented as the adjusted means.
Table 15
Effect of Gender on Retention Test Recall Items, Controlling for Pretest Knowledge

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>DF</th>
<th>Mean Square</th>
<th>F</th>
<th>Significance of F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariates</td>
<td>76.663</td>
<td>1</td>
<td>76.663</td>
<td>5.615</td>
<td>.020</td>
</tr>
<tr>
<td>Gender (between)</td>
<td>1.063</td>
<td>1</td>
<td>1.063</td>
<td>.078</td>
<td>.781</td>
</tr>
<tr>
<td>Residual (within)</td>
<td>1092.225</td>
<td>80</td>
<td>13.653</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1169.952</td>
<td>82</td>
<td>14.268</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 16
Pretest and Retention Test Application Items by Gender

<table>
<thead>
<tr>
<th></th>
<th>Pretest Application Items</th>
<th>Retention Test Application Items</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Standard Mean</td>
<td>Deviation</td>
</tr>
<tr>
<td>Boys</td>
<td>5.26</td>
<td>2.46</td>
</tr>
<tr>
<td>Girls</td>
<td>5.14</td>
<td>2.26</td>
</tr>
<tr>
<td>Total</td>
<td>5.20</td>
<td>2.36</td>
</tr>
</tbody>
</table>
The results of the ANCOVA are presented in Table 17. As Table 17 shows, there was no significant difference between the adjusted mean retention test scores on the application items for boys and girls.

Table 17
Effect of Gender on Retention Test Application Items, Controlling for Pretest Knowledge

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>DF</th>
<th>Mean Square</th>
<th>F</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariates</td>
<td>30.223</td>
<td>1</td>
<td>30.223</td>
<td>2.658</td>
<td>.107</td>
</tr>
<tr>
<td>Gender (between)</td>
<td>12.164</td>
<td>1</td>
<td>12.164</td>
<td>1.070</td>
<td>.304</td>
</tr>
<tr>
<td>Residual (within)</td>
<td>909.565</td>
<td>80</td>
<td>11.370</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>951.952</td>
<td>82</td>
<td>11.609</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Hypothesis 5

Hypothesis 5 states that the mean gain score on an achievement test over selected algebraic concepts for the 15 Algebra I students with the lowest mathematics TEAMS scores taught using manipulatives will be significantly higher than the mean gain score of the 15 Algebra I students with the highest mathematics TEAMS scores studying those same objectives using manipulatives when computed from pretest to posttest.

Hypothesis 5a states that the 15 students with the lowest mathematics TEAMS scores will have a significantly higher gain on the recall questions from the achievement test given immediately following instruction than the 15
students with the highest mathematics TEAMS scores. This hypothesis was tested using ANOVA. The achievement level of the students in the manipulative group, as determined by the mathematics section of the TEAMS test, was used as the independent variable, and the gain score on the recall items was the dependent variable. The mean of the gain scores and standard deviations of the gain scores on the recall items are presented in Table 18. The gain scores for the recall items were for the gain in scores from the pretest to the posttest.

The results of the ANOVA are presented in Table 19. ANOVA tests the significance of the difference between the mean gains. As Table 19 shows, there was no significant difference between the mean gain scores on the recall items from the pretest to the posttest for the low achieving and high achieving groups.

Table 18
Gain on Recall Items From Pretest to Posttest by Achievement Level

<table>
<thead>
<tr>
<th>Achievement Level</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low TEAMS</td>
<td>10.07</td>
<td>3.15</td>
</tr>
<tr>
<td>High TEAMS</td>
<td>11.93</td>
<td>3.84</td>
</tr>
<tr>
<td>Total</td>
<td>11.00</td>
<td>3.58</td>
</tr>
</tbody>
</table>
Table 19
Effect of Achievement Level on Gain Scores From Pretest to Posttest for Recall Items

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>DF</th>
<th>Mean Square</th>
<th>F</th>
<th>Significance of F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Achievement Level</td>
<td>26.133</td>
<td>1</td>
<td>26.133</td>
<td>2.116</td>
<td>.157</td>
</tr>
<tr>
<td>Residual (within)</td>
<td>345.867</td>
<td>28</td>
<td>12.352</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>372.000</td>
<td>29</td>
<td>12.828</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Hypothesis 5b states that the 15 students with the lowest mathematics TEAMS scores will have a significantly higher gain on the application questions from the achievement test given immediately following instruction than the 15 students with the highest mathematics TEAMS scores. This hypothesis was tested using ANOVA. The achievement level of the students in the manipulative group, as determined by the mathematics section of the TEAMS test, was used as the independent variable, and the gain score on the application items was the dependent variable. The mean of the gain scores and standard deviations of the gain scores on the application items are presented in Table 20. The gain scores for the application items were for the gain in scores from the pretest to the posttest.
Table 20

Gain on Application Items From Pretest to Posttest by Achievement Level

<table>
<thead>
<tr>
<th>Achievement Level</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low TEAMS</td>
<td>6.47</td>
<td>3.07</td>
</tr>
<tr>
<td>High TEAMS</td>
<td>6.67</td>
<td>2.77</td>
</tr>
<tr>
<td>Total</td>
<td>6.57</td>
<td>2.87</td>
</tr>
</tbody>
</table>

The results of the ANOVA are presented in Table 21. As Table 21 shows, there was no significant difference between the mean gain scores on the application items from the pretest to the posttest for the low achieving and high achieving groups.

Table 21

Effect of Achievement Level on Gain Scores From Pretest to Posttest for Application Items

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>DF</th>
<th>Mean Square</th>
<th>F</th>
<th>Significance of F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Achievement Level</td>
<td>.300</td>
<td>1</td>
<td>.300</td>
<td>.035</td>
<td>.853</td>
</tr>
<tr>
<td>Residual (within)</td>
<td>239.067</td>
<td>28</td>
<td>8.538</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>239.367</td>
<td>29</td>
<td>8.254</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Hypothesis 6

Hypothesis 6 states that the mean gain score on an achievement test over selected algebraic concepts for the fifteen Algebra I students with the lowest mathematics TEAMS scores taught using manipulatives will be significantly higher than the mean gain score of the fifteen Algebra I students with the highest mathematics TEAMS scores studying those same objectives using manipulatives when computed from pretest to retention test.

Hypothesis 6a states that the 15 students with the lowest mathematics TEAMS scores will have a significantly higher gain on the recall questions from the achievement test given two months after instruction than the 15 students with the highest mathematics TEAMS scores. This hypothesis was tested using ANOVA. The achievement level of the students in the manipulative group, as determined by the mathematics section of the TEAMS test, was used as the independent variable, and the gain score on the recall items was the dependent variable. The mean of the gain scores and standard deviations of the gain scores on the recall items are presented in Table 22. The gain scores for the recall items were for the gain in scores from the pretest to the retention test.

Table 22

<table>
<thead>
<tr>
<th>Achievement Level</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low TEAMS</td>
<td>8.00</td>
<td>4.60</td>
</tr>
<tr>
<td>High TEAMS</td>
<td>10.60</td>
<td>4.07</td>
</tr>
<tr>
<td>Total</td>
<td>9.30</td>
<td>4.47</td>
</tr>
</tbody>
</table>
The results of the ANOVA are presented in Table 23. As Table 23 shows, there was no significant difference between the mean gain scores on the recall items from the pretest to the retention test for the low achieving and high achieving groups.

Hypothesis 6b states that the 15 students with the lowest mathematics TEAMS scores will have a significantly higher gain on the application questions from the achievement test given two months after instruction than the 15 students with the highest mathematics TEAMS scores. This hypothesis was tested using ANOVA. The achievement level of the students in the manipulative group, as determined by the mathematics section of the TEAMS test, was used as the independent variable, and the gain score on the application items was the dependent variable. The mean of the gain scores and standard deviations of the gain scores on the application items are presented in Table 24. The gain scores for the application items were for the
gain in scores from the pretest to the retention test.

The results of the ANOVA are presented in Table 25. As Table 25 shows, there was no significant difference between the mean gain scores on the application items from the pretest to the retention test for the low achieving and high achieving groups.

Table 24
Gain on Application Items From Pretest to Retention Test by Achievement Level

<table>
<thead>
<tr>
<th>Achievement Level</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low TEAMS</td>
<td>4.40</td>
<td>3.74</td>
</tr>
<tr>
<td>High TEAMS</td>
<td>4.87</td>
<td>3.89</td>
</tr>
<tr>
<td>Total</td>
<td>4.63</td>
<td>3.76</td>
</tr>
</tbody>
</table>

Table 25
Effect of Achievement Level on Gain Scores From Pretest to Retention Test for Application Items

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>DF</th>
<th>Mean Square</th>
<th>Significance of E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Achievement Level</td>
<td>1.633</td>
<td>1</td>
<td>1.633</td>
<td>.112</td>
</tr>
<tr>
<td>Residual (within)</td>
<td>407.333</td>
<td>28</td>
<td>14.548</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>408.967</td>
<td>29</td>
<td>14.102</td>
<td></td>
</tr>
</tbody>
</table>
CHAPTER V

SUMMARY OF FINDINGS, CONCLUSIONS, AND RECOMMENDATIONS

This study investigated the use of manipulative materials for teaching selected topics in Algebra I compared to the teaching of those same topics without the use of the manipulatives. The study used achievement levels of these two groups as a basis of comparison. The study also investigated the differences in achievement between boys and girls within the group using manipulatives. Differences in achievement between high achieving and low achieving students within the manipulative group were also investigated.

Summary of Findings

Algebra I students enrolled in 9th and 10th grades at two different high schools in a middle to upper-middle class school district in north Texas were studied. The students were from eight randomly selected classes taught by four teachers who volunteered to participate in the study. Each teacher taught one class as a control group and one class as an experimental group. There were 96 students in the control group and 91 students in the experimental group who completed the study.

Data were collected three times during the study using the same test each time. The test was developed by the researcher to cover the objectives designated for the study. Initial data were collected with a pretest which was administered to all students in the control and experimental groups the day before teachers began discussing the unit. Posttest data were collected by
administering the posttest to all students in the control group and the experimental group the day after presentation of the unit was completed. The data for retention were collected by administering the retention test to all students in the control group and the experimental group two months after the unit was completed. In each instance, the tests were machine scored, and the raw scores were statistically analyzed by computer using analysis of covariance (ANCOVA) and analysis of variance (ANOVA) to examine each hypothesis in the research study.

Supplemental data were collected on all of the students in order to compare the control and experimental groups. The eighth grade test scores from the CTBS (California Test of Basic Skills), administered annually to all students in grades one through eight in this district, were collected and analyzed to find the range, mean, median, and mode for each group. The scores from the mathematics section of the ninth grade TEAMS (Texas Educational Assessment of Minimum Skills), administered annually to all students in odd numbered grades in the state of Texas, were also collected and analyzed to find the range, mean, median, and mode for each group. The TEAMS scores were also used to determine the high and low achieving students in this research study.

Findings

The major findings resulting from the analysis of the statistical data presented in this study were the following:

1. No significant statistical difference was found between the mean scores of the students who were taught selected algebraic topics using manipulatives and the students who were taught those same topics without
using manipulatives on the recall items of an achievement test given immediately following instruction.

2. No significant statistical difference was found between the mean scores of the students who were taught selected algebraic topics using manipulatives and the students who were taught those same topics without using manipulatives on the application items of an achievement test given immediately following instruction.

3. No significant statistical difference was found between the mean scores of the students who were taught selected algebraic topics using manipulatives and the students who were taught those same topics without using manipulatives on the recall items of an achievement test given two months after instruction.

4. No significant statistical difference was found between the mean scores of the students who were taught selected algebraic topics using manipulatives and the students who were taught those same topics without using manipulatives on the application items of an achievement test given two months after instruction.

5. No significant statistical difference was found between the mean scores on the recall items of an achievement test given immediately following instruction for male and female students who were taught selected algebraic topics using manipulatives.

6. No significant statistical difference was found between the mean scores on the application items of an achievement test given immediately following instruction for male and female students who were taught selected algebraic topics using manipulatives.
7. No significant statistical difference was found between the mean scores on the recall items of an achievement test given two months after instruction for male and female students who were taught selected algebraic topics using manipulatives.

8. No significant statistical difference was found between the mean scores on the application items of an achievement test given two months after instruction for male and female students who were taught selected algebraic topics using manipulatives.

9. No significant statistical difference was found between the mean gain scores on the recall items of an achievement test given immediately following instruction for the high achieving and low achieving students who were taught selected algebraic topics using manipulatives.

10. No significant statistical difference was found between the mean gain scores on the application items of an achievement test given immediately following instruction for the high achieving and low achieving students who were taught selected algebraic topics using manipulatives.

11. No significant statistical difference was found between the mean gain scores on the recall items of an achievement test given two months after instruction for the high achieving and low achieving students who were taught selected algebraic topics using manipulatives.

12. No significant statistical difference was found between the mean gain scores on the application items of an achievement test given two months after instruction for the high achieving and low achieving students who were taught selected algebraic topics using manipulatives.
Conclusions

Based on the findings of this research study, the following conclusions seem justified:

1. As measured by the instrument in this study, achievement on recall items over selected algebraic topics for 9th- and 10th-grade Algebra I students was not increased by the use of manipulative materials.

2. As measured by the instrument in this study, achievement of 9th- and 10th-grade Algebra I students on application items over selected algebraic topics was not increased by the use of manipulative materials.

3. As measured by the instrument in this study, retention of recall items from selected algebraic topics for 9th- and 10th-grade Algebra I students was not increased by the use of manipulative materials.

4. As measured by the instrument in this study, retention of application items over selected algebraic topics for 9th- and 10th-grade Algebra I students was not increased by the use of manipulative materials.

5. Girls did not appear to benefit from the use of manipulatives more than boys on either the recall or the application items from the selected algebraic topics on the instrument used in this study.

6. Manipulatives did not appear to benefit girls more than boys in the retention of the selected algebraic topics as measured by either the recall or the application items of the instrument used in this study.

7. Low-achieving students did not appear to benefit from the use of manipulatives more than high-achieving students on either the recall or the application items from the selected algebraic topics on the instrument used in this study.
8. The use of manipulatives did not appear to benefit low achieving students more than high achieving students in the retention of the selected algebraic topics as measured in either the recall or the application items of the instrument used in this study.

Recommendations

Based on the findings and conclusions of this study, the following recommendations are made:

1. Achievement should not be the only factor used to assess whether manipulatives are advantageous for the learning of algebraic concepts. Attitude of students and teachers toward mathematics should also be assessed.

2. A study should be conducted that has only students taking Algebra I for the first time to avoid influences on first-time students by those students who are repeating the course.

3. A study should be conducted that covers the entire Algebra I course.

4. A longitudinal study should be conducted to assess whether or not students benefit from the use of manipulatives if they are used from elementary school through Algebra I.

5. A longitudinal study should be conducted to determine whether students who use manipulatives in Algebra I achieve at a higher level in Algebra II.

6. A study should be conducted to determine whether or not there is a point in the educational progression of a student at which manipulatives cease to be effective.

7. A study should be conducted to determine the effect of manipulatives on teachers who use them. Such a study might include the effect the
manipulatives have on the attitude of the teacher toward mathematics as well as the effect they have on the presentation of mathematical concepts.

8. More studies should be done in other populations to determine whether or not manipulatives are beneficial for teaching algebraic concepts to different populations.

Although the study did not produce any significant statistical differences between the test scores of secondary students using manipulatives and secondary students not using manipulatives, more research needs to be done. Such research could further investigate the role played by manipulatives in the achievement of secondary students. Research should focus not only on test scores but also on students' attitudes toward mathematics. Researchers also need to explore at what point in the lesson, for what topics, and for which students the use of manipulatives might be more appropriate than conventional teaching strategies.
APPENDIX A

PRETEST, POSTTEST, AND RETENTION TEST

74
Algebra I Test

1. Multiply \((4y + 7)^2\).
   a. \(16y^2 + 56y + 49\)
   b. \(16y^2 + 49\)
   c. \(8y^2 + 14\)
   d. \(16y^2 + 28y + 14\)

2. Multiply \((5a - b)^2\)
   a. \(25a^2 + b^2\)
   b. \(25a^2 - b^2\)
   c. \(25a^2 + 10ab + b^2\)
   d. \(25a^2 - 10ab + b^2\)

3. Multiply \((3x - y)^2\)
   a. \(9x^2 - y^2\)
   b. \(9x^2 - 3xy + y^2\)
   c. \(9x^2 + y^2\)
   d. \(9x^2 - 6xy + y^2\)

4. Multiply \((x - y)^2\).
   a. \(x^2 - 2xy + y^2\)
   b. \(x^2 + 2xy + y^2\)
   c. \(x^2 - y^2\)
   d. \(x^2 - 2xy - y^2\)
5. What is the area of a square whose side is $2a + 3b$?
   a. $4a^2 + 12ab + 9b^2$
   b. $2a^2 + 12ab + 3b^2$
   c. $4a^2 + 9b^2$
   d. $4a^2 + 6ab + 9b^2$

6. Multiply $(-3 + 5a)^2$.
   a. $-9 - 30a + 25a^2$
   b. $9 - 15a + 25a^2$
   c. $9 - 30a + 25a^2$
   d. $9 + 25a^2$

7. Multiply $(ab - 5)^2$.
   a. $a^2b^2 - 25$
   b. $a^2b^2 - 10ab - 25$
   c. $a^2b^2 - 10ab + 25$
   d. $a^2b^2 - 5ab + 25$

8. Multiply $(4 - 3x^2)^2$.
   a. $16 - 24x^2 + 9x^4$
   b. $16 - 12x^2 + 9x^4$
   c. $16 - 24x^2 + 3x^4$
   d. $16 + 9x^4$
9. Multiply \((s - t)(s + t)\).  
   a. \(s^2 - 2st - t^2\)  
   b. \(s^2 + t^2\)  
   c. \(s^2 - t^2\)  
   d. \(s^2 - 2st + t^2\)

10. Multiply \((x^2 - 5)(x^2 + 5)\)  
    a. \(x^4 + 25\)  
    b. \(x^4 - 10x^2 - 25\)  
    c. \(x^4 - 10x^2 + 25\)  
    d. \(x^4 - 25\)

11. Multiply \((2k + 1)(2k - 1)\).  
    a. \(4k^2 - 1\)  
    b. \(4k^2 - 4k + 1\)  
    c. \(4k^2 + 1\)  
    d. \(4k^2 - 4k - 1\)

12. Multiply \((x - 3)(x + 3)\).  
    a. \(x^2 - 6x - 9\)  
    b. \(x^2 - 9\)  
    c. \(x^2 + 9\)  
    d. \(x^2 - 6x + 9\)
13. Multiply \((-5 + 3b)(5 + 3b)\)
   a. \(9b^2 - 25\)
   b. \(25 - 30b + 9b^2\)
   c. \(-25 - 30b + 9b^2\)
   d. \(9b^2 + 25\)

14. Multiply \(-(a - z)(z + a)\). 
   a. \(z^2 + a^2\)
   b. \(z^2 - a^2\)
   c. \(z^2 - 2az + a^2\)
   d. \(z^2 - 2az - a^2\)

15. Multiply \((8 - n^4)(8 + n^4)\).
   a. \(64 + n^8\)
   b. \(64 - 16n^4 - n^8\)
   c. \(64 - 16n^4 + n^8\)
   d. \(64 - n^8\)

16. Multiply \((xn + ym)(xn - ym)\).
   a. \(xn+2 - yn+2\)
   b. \(x^{2n} - 2xynym - y^{2m}\)
   c. \(x^{2n} - y^{2m}\)
   d. \(x^{2n} + y^{2m}\)
17. Multiply \((a - 4)(a - 5)\).
   a. \(a^2 - 9a - 20\)
   b. \(a^2 - 9a + 20\)
   c. \(a^2 - a + 20\)
   d. \(a^2 + 9a + 20\)

18. Multiply \((c + 3)(c + 2)\).
   a. \(c^2 + c + 6\)
   b. \(c^2 + 5c + 6\)
   c. \(c^2 - 5c + 6\)
   d. \(c^2 + 5c + 6\)

19. Multiply \((3h + 2k)(h - 3k)\).
   a. \(3h^2 - k - 6k^2\)
   b. \(3h^2 - 11hk - 6k^2\)
   c. \(3h^2 - 7hk - 6k^2\)
   d. \(3h^2 - 7hk + 6k^2\)

20. Multiply \((6a - 5)(a + 2)\)
   a. \(6a^2 + 7a - 10\)
   b. \(6a^2 + 7a + 10\)
   c. \(6a^2 - 17a - 10\)
   d. \(6a^2 + 17a - 10\)
21. Multiply \((3a^4 - 5b^2)(a^4 - 2b^2)\)
   a. \(3a^{16} - 11a^4b^2 + 10b^4\)
   b. \(3a^8 - a^4b^2 + 10b^4\)
   c. \(3a^8 - 11a^4b^2 + 10b^4\)
   d. \(3a^2 + a^4b^2 + 10b^4\)

22. Multiply \((2yk - 3)(5yk + 6)\).
   a. \(10y^2k - 3yk - 18\)
   b. \(10yk + 1 - 3yk - 18\)
   c. \(10y^2k + 27yk - 18\)
   d. \(10y^2k + 3yk - 18\)

23. Multiply \((x^4 + 7x^2)(x^2 - 3)\).
   a. \(x^6 + 10x^4 - 21x^2\)
   b. \(x^6 + 4x^4 - 21x^2\)
   c. \(x^6 + 4x^2 - 21x^2\)
   d. \(x^8 + 4x^4 - 21x^2\)

24. Multiply \((x^2 + y^2)(x^2 + 3y^2)\).
   a. \(x^4 - 4x^2y^2 + 3y^4\)
   b. \(x^2 + 4x^2y^2 + 3y^2\)
   c. \(x^4 + 2x^2y^2 + 3y^4\)
   d. \(x^4 + 4x^2y^2 + 3y^4\)
25. Factor completely $36x^2 + 25$.
   a. $(6x + 5)(6x + 5)$
   b. $(6x - 5)(6x + 5)$
   c. $(4x + 5)(9x + 5)$
   d. not factorable

26. Factor completely $r^2 - 9s^2$
   a. $(r - 3s)(r - 3s)$
   b. $(r - 9s)(r - s)$
   c. $(r - 3s)(r + 3s)$
   d. not factorable

27. Factor completely $z^2 - 16$.
   a. $(z - 2)(z - 8)$
   b. $(z - 4)(z + 4)$
   c. $(z - 4)(z - 4)$
   d. not factorable

28. Factor completely $9a^2 - b^2c^2$.
   a. $(3a - bc)(3a + bc)$
   b. $(3a - bc)(3a - bc)$
   c. $(9a - bc)(a - bc)$
   d. not factorable
29. Factor completely \( 4u^4n - v^2n \)
   a. \((4u^2n - vn)(u^2n + v^2n)\)
   b. \((2u^2n - vn)(2u^2n + v^2n)\)
   c. \((2u^4 - v^2)(2un + vn)\)
   d. not factorable

30. Factor completely \(-9 + 16x^4\).
   a. \((-3 + 2x^2)(3 + 8x^4)\)
   b. \((3 + 4x^2)(3 + 4x^4)\)
   c. \((4x^2 + 3)(4x^4 - 3)\)
   d. not factorable

31. Factor completely \(s^4n - t^2\).
   a. \((s^2n - t)(s^2n - t)\)
   b. \((s^2n + t)(s^2n - t)\)
   c. \((sn^2 + 1)(sn^2 - 1)\)
   d. not factorable

32. Factor completely \(y^2 - (y - 1)^2\).
   a. \((2y - 1)\)
   b. \((y + 1)(y - 1)\)
   c. \((2y - 1)(2y + 1)\)
   d. not factorable
33. Factor completely $2n^2 - 11n + 5$.
   a. $(2n + 1)(n + 5)$
   b. $(2n - 1)(n - 5)$
   c. $(2n + 1)(2n - 5)$
   d. not factorable

34. Factor completely $7k^2 + 19k - 6$.
   a. $(7k - 6)(k + 1)$
   b. $(7k + 3)(k - 2)$
   c. $(7k - 2)(k + 3)$
   d. not factorable

35. Factor completely $12a^2 + ab - 6b^2$.
   a. $(4a - 3b)(3a + 2b)$
   b. $(4a + 3b)(3a - 2b)$
   c. $(2a + 2b)(6a - 3b)$
   d. not factorable

36. Factor completely $x^2 + 5xy + 4y^2$.
   a. $(x + 4y)(x + y)$
   b. $(x - 4y)(x - y)$
   c. $(x + 2y)(x + 2y)$
   d. not factorable
37. Factor completely \(-21b^2 + 4b + 12\).
   a. \((-3b + 2)(7b + 6)\)
   b. \((6 + 7b)(2 - 3b)\)
   c. \((6 - 7b)(2 + 3b)\)
   d. not factorable

38. Factor completely \(49 - 7k + k^2\).
   a. \((7 - k)(7 + k)\)
   b. \((7 - k)(7 - k)\)
   c. \((7 + k)(7 + k)\)
   d. not factorable

39. Factor completely \(3(a + b)^2 - 17(a + b) - 6\).
   a. \((3a + 3b + 1)(a + b - 6)\)
   b. \((3a + 3b - 1)(a + b + 6)\)
   c. \((3a + 3b + 2)(a + b - 3)\)
   d. not factorable

40. Factor completely \(3x^{4n} - 10x^{2n} + 3\).
   a. \((3x^{2n} - 3)(x^{2n} - 1)\)
   b. \((3x^{2n} + 1)(x^{2n} + 3)\)
   c. \((3x^{2n} - 1)(x^{2n} - 3)\)
   d. not factorable
APPENDIX B

LESSON PLAN FOR INTRODUCTORY LESSON
INTRODUCTORY LESSON

Part I. Need: 1 packet per group of 4 students containing 6-7 small squares (one color), 4-5 large squares (a different color) and 3 extra 8 1/2 x 11 sheets of a 3rd color of construction paper.

Scissors

Part Ia Participants cut 12 rectangles whose width is the same as the small square and length equal to the length of the large squares from the 3rd color of paper.

Exploration After pieces are cut have participants try making as many different rectangles as they can using:

(a) 2 small squares (1)
(b) 2 large squares (1)
(c) 2 rectangles (2)
(d) 1 square, 1 rectangle (1)

continue as needed-
(e) with 3 rectangles?
(f) with 4 rectangles?

Part Ib Problem:
How many different rectangles can be formed using at least one of every shape (1 small square, 1 large square, 1 rectangle) and no
more than 12 pieces all together. Different rectangles are defined as rectangles of different dimensions, not different designs.
Find a way to record your creations so that you don't duplicate rectangles.
Were any of your rectangles made with the same exact pieces?
Is it possible to make different rectangles using the same piece?
Recreate your favorite rectangle. View the rectangles made by other groups.

Question:
Are there shapes that one group has made that no one else can make?

Try to make the rectangle of the group next to you. Now, try to make a group's rectangle that you think can't be duplicated.

Can everyone make this rectangle?

(Teacher puts rectangle on overhead using algebra tiles or colored acetate squares and rectangles.)

Part II. Make this one.
Conclusions:
Yes, everyone can duplicate someone else's rectangles with their material.

Question: Can we exchange the places of the small and large squares? That is, wherever you have a large square put a small one and wherever there are small squares put large ones. What happens?

Does this always work with any rectangle?
Create a rectangle (up to 12 pieces, same rules as before).
Try to exchange large and small squares.

Part III
(Algebra Tiles) Class:
Using the following \(3a^2+7ab+2b^2=(3a+ b)(a+2b)\), make a rectangle:
3 small squares, 2 large squares, 7 rectangles. (Teacher makes on overhead)
Rearrange small and large squares so that they fall into 4 quadrants.
To test pieces to see if they can make a rectangle, arrange large squares into quadrant II (upper left) and small squares into quadrant IV (lower right). Try to fit the rectangles in (squares may be rearranged as long as at least one of the large squares and one of the small squares is touching the vertical axis). If this can't be done, the pieces cannot make a rectangle.

**Task/Question:** (Worksheet "Possible/Impossible") Which group of pieces can be made into rectangles? Record your arrangements if possible and write "impossible" if not for #'s 1-10. After the groups of 4 students have completed their tasks, ask for feedback (check answers). Any discoveries?

**Part IV**

What is the area of this rectangle? \(2b^2+7ab+3a^2\)

What is the area of each piece?

Large square has dimensions \(b \cdot b\).

\[\text{Area} = b^2\]

Small square has area \(a \cdot a\).

\[\text{Area} = a^2\]

Rectangles have dimensions \(a \cdot b\) or \(ab\).

Area of large figure = \(2b^2+7ab+3a^2\)
(Have student find how each term is represented in the model.)

What are the dimensions of the rectangle? \((b+3a)(2b+a)\)

Connect the concrete representation to FOIL.

\[(b+3a) \times (2b+a)\]

- **first**: \(2b^2\) (2 large squares)
- **outside**: \(ab\) (rectangle above 3 small squares)
- **inside**: \(6ab\) (6 rectangles below 2 large squares)
- **last**: \(3a^2\) (3 small squares)

**Part V**

Find the areas and dimensions of the rectangles in #'s 1, 3, 4, 6, 7-10 of the worksheet.

Let's compare with each other?

Does anyone see any patterns or discoveries?

Were any of the rectangles on the worksheet squares?

Can you make a square using at least one of every piece?

What would the dimension be?

Ex. \(a^2+2ab+b^2\) \((a+b)(a+b)\)

\[
\begin{array}{c|c}
  a & b \\
  \hline \\
  a & b \\
\end{array}
\]

Relate to "perfect squares".

Can you make other perfect squares?

(end of 2 hr session)
Part VI  **Extension:**

Could we let the length of a side of a small square equal 1? The length of 2 small squares would equal 2. A rectangle of length $x$ would then have the dimension $x \cdot 1$ (where $x =$ large square's length or $1x$)

Example: $x^2+3x+2 = \text{Area}$

$(x+2)(x+1) = \text{dimensions}$

Note terms represented by model:

$x^2+1x+2x+2$

$x^2+3x+2$

**POSSIBLE/IMPOSSIBLE**

A RECTANGLE CONSISTING OF

<table>
<thead>
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<th>RECTANGLES</th>
<th>SMALL SQUARES</th>
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<tr>
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<td>1</td>
<td>5</td>
</tr>
<tr>
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<td>2</td>
<td>5</td>
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<tr>
<td>5</td>
<td>3</td>
<td>6</td>
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</tbody>
</table>
**COMPLETE THE FOLLOWING TO MAKE RECTANGLES**

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<th>RECTANGLES</th>
<th>SMALL SQUARES</th>
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<td>7)</td>
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<td>8)</td>
<td>1</td>
<td></td>
<td>4</td>
</tr>
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<td>9)</td>
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</tr>
<tr>
<td>10)</td>
<td>2</td>
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</table>
MANIPULATIVE KIT

Each student manipulative kit contained the following items made out of poster board which were placed in a resealable plastic bag. The poster board was white on one side, which was used to represent positive values, and grey on the other side, which was used to represent negative values.

10 squares like the following.

![Square](image)

20 rectangles like the following.

![Rectangle](image)

40 squares like the following.

![Square](image)
REFERENCES


