A WEAK-FORM EFFICIENT MARKETS TEST OF THE DALLAS-FORT WORTH
OFFICE PROPERTIES REAL ESTATE MARKET

DISSERTATION

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By

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Few areas of research in the finance literature have received greater attention than the efficient market hypothesis. Much of the research has been directed toward the securities market while very little research has been done in the real estate markets. The existing research on real estate market efficiency has been either descriptive or illustrative with very little empirical testing being performed. The major reason for the lack of empirical testing has been the inability to develop an adequate data base. The results of the empirical work that has been done do not support the widely held belief that real estate markets are inefficient.

This study, using the autoregressive-integrative-moving average (ARIMA) time series analysis technique, tests the weak-form efficiency of the Dallas-Fort Worth office properties real estate market. According to the weak-form efficient market hypothesis, all price information should be capitalized into current real estate prices and not provide the basis for earning abnormal returns in trading.
Price data formed from office building sales dating from January, 1979 to January, 1985 are used to test the market. The data was gathered from the files of several professional appraisal firms located in the Dallas-Fort Worth area. The transaction information includes (1) transaction price; (2) location of the property; (3) net rentable area; (4) gross income multiplier (GIM); (5) net income multiplier (NIM); and (6) net operating income.

The results of the study indicate a lack of significant autocorrelation. This suggests that the Dallas-Fort Worth office properties real estate market is weak-form efficient. As further evidence of weak-form market efficiency, ARIMA models are estimated to predict future sales prices but they are unable to outperform a simple mean series forecast. The results indicate that a change in traditional real estate theory concerning market efficiency may be warranted.
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CHAPTER I

INTRODUCTION

General Statement of the Problem

Few areas of research in the finance literature have received greater attention than the efficient market hypothesis (EMH). Much of the research has been directed toward the securities market. Little, however, has been done in the real estate literature to examine the efficiency of the real estate markets. Most of the existing research on real estate market efficiency has been either descriptive, discussing the application of the Capital Asset Pricing Model (CAPM) to real estate, or illustrative, using hypothetical real estate data without empirical testing. The major reason for the lack of empirical testing has been the inability to develop an adequate data source. Empirical research by Gau (1984), Guntermann (1984) and Dale-Johnson, Findlay, Schwartz, and Kapplin (1985) has provided support for real estate market efficiency. These results do not support the widely held belief that the real estate markets are inefficient. Jaffee and Sirmans (1984), among others, have expressed the need for further research.
Fama (1970) describes an efficient market as one in which prices adjust rapidly to new information and all available information, including risk, is fully reflected in current prices. He presented the efficient market theory in terms of a fair game model and divided the hypothesis into three categories referred to as weak-form, semistrong-form, and strong-form market efficiency. Each category of the efficient market hypothesis deals with a different set of information.

The weak-form EMH states that current prices reflect the information implied by the historical sequences of prices. An investor cannot improve his ability to select investments by knowing the history of successive prices and the results of analyzing them in all possible ways.

The semistrong-form EMH asserts that current prices fully reflect all public knowledge about the underlying investment and that efforts to acquire and analyze this knowledge cannot be expected to produce superior results. The strong-form EMH contends that prices fully reflect all information both public and private. Therefore, no group of investors should be able to consistently derive above-average profits.
Objectives of the Study

The objective of this study is to test the efficiency of a real estate market as the theory is defined in the financial literature. The weak-form version of the efficient markets tests will be applied to a time series of office building sales prices in the Dallas-Fort Worth metropolitan area. This test will examine whether past price information in the real estate office building market can be utilized to predict the future prices of these real estate assets. According to the EMH, all price information should be capitalized into current real estate prices and not provide the basis for earning abnormal returns in trading.

Scope and Limitations of the Study

This study uses data about actual office building sales transactions from January 31, 1979 to January 31, 1985 in the Dallas-Fort Worth office properties market. The data were obtained from the files of several professional appraisal firms located in the Dallas-Fort Worth area. In addition to the location of each property, transaction data were collected on (1) sales price (SP); (2) gross building area (GBA); (3) net rentable area (NRA); (4) effective gross annual income (EGI); and (5) annual net operating income
(NOI). To insure reliability, the data were cross-verified by examining several appraisal data banks.

This study is subject to the following limitations:

1. All of the data in this study were gathered from the Dallas-Fort Worth office building market. Thus, generalizations of the findings to other populations or settings may not be appropriate.

2. All of the data were gathered over a specific six-year time period. Thus, generalizations based on the transactions during this time period to other time periods may not be appropriate.

3. Several sales transactions did not contain complete information. As a result, some entire transactions were omitted from analysis. Furthermore, the high frequency of incomplete information on gross building area resulted in the inability to use that variable in this study.

Assumption

It was assumed that all data taken from the appraisal files were verified by the appraiser and were reliable.

Hypothesis

Although theory has consistently promoted that real estate markets are inefficient, recent empirical studies have not supported this position. Therefore, based on
recent empirical work, it is expected that the Dallas-Fort Worth office properties market is efficient. The null hypothesis is that the Dallas-Fort Worth office properties market is weak-form efficient. In other words, real estate investors cannot earn abnormal returns by utilizing commonly available price information.

Organizational Plan

Chapter II contains the review of the related literature. It is divided into two sections. First, it reviews efficient market studies in the financial literature. Second, the efficient market studies in the real estate literature are reviewed. Chapter III describes the methodology, including a discussion of the sample selection and empirical testing procedures. The results are presented in Chapter IV. Chapter V provides the summary and conclusions.
CHAPTER I BIBLIOGRAPHY


CHAPTER II

REVIEW OF RELATED LITERATURE

Introduction

This chapter reviews the literature relative to the efficiency of the financial and real estate markets. The chapter is divided into two sections. The first section reviews efficient capital markets theory. The financial literature involving weak-, semistrong-, and strong-form capital market efficiency tests is examined.

The second section reviews the real estate market efficiency literature. Real estate market efficiency from the descriptive or illustrative viewpoint is examined, followed by reviews of the semistrong-form and weak-form efficiency literature. This includes the works of Miles and Rice (1978), Sirmans and Webb (1978), Wofford and Moses (1978), Roulac (1978), Hoag (1980), Draper and Findlay (1982), Gau (1985), and Dale-Johnson, Findlay, Schwartz, and Kapplin (1985).

Efficient Capital Markets Theory

Many important implications for security valuation and portfolio management may be derived from the existence of efficient capital markets. Much has been written on the efficiency of the capital markets and there are many forms
that the empirical evidence can take. Several excellent reviews already exist on the efficiency of capital markets. This section will draw heavily on the work of Fama (1970) and Reilly (1985).

According to Reilly (1985), an initial and important assumption of an efficient market is that there are many profit-maximizing participants operating independently of each other. The second assumption of efficient markets is that new information regarding securities occurs randomly and independent of other announcements. A third assumption is that investors adjust security prices rapidly to new information. The price adjustment process requires many investors who follow the stock to adjust the price to reflect the new information.

If markets are efficient, security prices that exist at any point in time should be an unbiased reflection of all currently available information. At any point in time, the price of a security is an unbiased estimate of the intrinsic value of the security given all of the information that is available. An efficient market is, therefore, one in which security prices adjust rapidly to new information and all available information, including risk, is fully reflected by current stock prices. The returns implicit in the price reflect the risk involved.
Several authors have defined efficient markets. Fama (1970) stated that market efficiency requires that in setting the prices of securities at any time, the market correctly uses all available information. Further, Jensen (1978) stated that a market is efficient with respect to information if it is impossible to make profits by trading on the basis of that information. Samuelson (1965) and Mandelbrot (1966) say that independence of successive price changes is consistent with efficient markets.

**Alternative Efficient Market Hypotheses**

Fama (1970) was instrumental in helping organize the studies of the efficient market theory. He presented efficient market theory in terms of a fair game model. Prior to his work, an extensive amount of empirical analysis was done using the random walk hypothesis. Random walk has been of interest since the early 1900's. The earliest recorded test of the random walk model was the study of commodity prices by Louis Bachelier (1900). Bachelier's work had important meaning for investors, but it took a long time to develop. Working (1934), Cowles and Jones (1937) and Kendall (1953) worked in this area also. However, modern work on this subject began in 1959, when two important papers were published by Roberts and Osborne.
Roberts (1959) indicated that a series of random numbers had the same appearance as a time series of stock prices. He encouraged further investigation into the apparent similarity of shape between the two series.

Osborne (1959) provoked more controversy. He claimed ignorance of the stock market. By profession, he was a physicist. He observed that stock prices apparently conformed to certain laws governing the motion of physical objects (Brownian motion). He found that the variance of stock prices increased over time exponentially. Through logarithmic transformation, they then became independent of each other.

Differing from the random walk hypothesis which deals with price movements over time, the fair game model deals with price at a specified time. The fair game model assumes that the price of a security fully reflects all available information at that time.

There are three widely discussed theories of price behavior over time: (1) the fair-game model; (2) the martingale or submartingale; and (3) the random walk. The fair game model considers returns as opposed to the entire probability distribution. Using the notation of Fama (1970),

\[ \epsilon_{j,t+1} = \frac{\left( p_{j,t+1} - p_{jt} \right)}{p_{jt}} - \frac{\left[ E(p_{j,t+1|M_t}) - p_{jt} \right]}{p_{jt}} \]  

(1)
where $P_{j,t+1}$ is the actual price of security $j$ next period,

$$E(P_{j,t+1}/\mathcal{H}_t) = \text{the predicted end-of-period price of security } j \text{ given the current information structure, } \mathcal{H}_t,$$

$\varepsilon_{j,t+1}$ is the difference between actual and predicted returns,

written in return form

$$r_{j,t+1} = (P_{j,t+1} - P_{jt})/P_{jt}. \quad (2)$$

Formula (1) may be rewritten as

$$\varepsilon_{j,t+1} = r_{j,t+1} - E(r_{j,t+1}/\mathcal{H}_t), \quad (3)$$

and

$$E(\varepsilon_{j,t+1}) = E[r_{j,t+1} - E(r_{j,t+1}/\mathcal{H}_t)] = 0. \quad (4)$$

A fair game model, will on average, across a large number of samples, have actual returns equal to the expected returns. A fair game implies that expectations will be realized.

A submartingale is a fair game where the price next period is expected to be greater than the price this period shown as

$$E(P_{j,t+1}/\mathcal{H}_t) > P_{jt}. \quad (5)$$

This implies that expected returns are positive and may be written as

$$[E(P_{j,t+1}/\mathcal{H}_t) - P_{jt}]/P_{jt} = E(r_{j,t+1}/\mathcal{H}_t) > 0. \quad (6)$$
A martingale is also a fair game where the price next period is expected to be the same as the price this period. This can be expressed as

\[ E(P_{j,t+1}/\mathcal{N}_t) = P_{jt}, \]  

or in return form as

\[ \frac{E(P_{j,t+1}/\mathcal{N}_t) - P_{jt}}{P_{jt}} = E(r_{j,t+1}/\mathcal{N}_t) = 0. \]  

A random walk states that there is no difference between the distribution of returns conditional on a given information set and an unconditional distribution of returns. Again, according to Fama (1970), a random walk in prices can be expressed as

\[ f_m(P_{lt'},\ldots,P_{nt}/\mathcal{N}_{t-1}) = f(P_{lt},\ldots,P_{nt}/\mathcal{N}_{t-1}), \]  

where \( f_m(P_{lt'},\ldots,P_{nt}/\mathcal{N}_{t-1}) \) is the joint distribution of security prices given the set of information that the market uses to determine prices at \( t-1 \),

\[ f(P_{lt},\ldots,P_{nt}/\mathcal{N}_{t-1}) \] is the joint distribution of prices which would exist if all relevant information available at time \( t-1 \) were used.

If the distribution of prices in time period \( t \), which was predicted from the previous time period, \( t-1 \), and based on market information, is not different from the prices predicted by using all relevant information from the previous time period, then there must be no difference.
between the information the market uses and the set of all relevant information. An efficient market instantaneously reflects all relevant information. In return form the random walk can be written as

$$f(r_1,t+1,\ldots,r_n,t+1) = f(r_1,t+1,\ldots,r_n,t+1/\mathcal{N}_t).$$  \(10\)

Random walk assumptions are stronger than those of fair game models or martingales because they require all of the parameters of a distribution to be the same with (or without) an information structure. Successive drawings over time must appear to be taken from the same distribution and be independent. If returns follow a random walk, the mean of the underlying distribution does not change over time and a fair game exists.

The majority of the empirical evidence shows that security returns do not follow a random walk. The fair-game model, with its less stringent requirements, is a more likely model of security returns. Serial covariance can exist with a fair game model that cannot exist with a random walk.

Fama (1965) showed that security returns are not strictly random walks but could be fair games, particularly submartingales. Research by Alexander (1961) and Fama and Blume (1966) also supports the fair game, submartingale model, by using filter rule tests.
Fama (1970) divided the efficient market hypothesis into three categories known as weak-form efficiency, semistrong-form efficiency, and strong-form market efficiency. Each category of EMH deals with a different set of information.

The weak-form EMH asserts that current prices fully reflect the information implied by the historical sequences of prices. Investors cannot improve their ability to select stocks by knowing the history of successive prices or by analyzing them in any possible way.

The semistrong-form EMH assumes that current prices fully reflect public knowledge about the underlying companies and that efforts to acquire and analyze this knowledge cannot be expected to produce superior results. The semistrong-form hypothesis includes the weak-form hypothesis --since all public information includes all market information and all nonmarket information (e.g., earnings, stock splits, dividends, and economic or political news).

The strong-form EMH asserts that prices fully reflect all information--both public and private. Therefore, no group of investors should be able to consistently derive above-average profits. The strong-form hypothesis includes both the weak and semistrong-forms. Under the strong-form EMH, individuals who supposedly have privileged information cannot use it to secure superior earnings.
Weak-Form Hypothesis

The EMH has been typically divided into two categories of tests for weak-form efficiency: (1) statistical tests of independence between stock price changes, and (2) testing specific trading rules that try to produce investment decisions on the basis of past market information (as opposed to a buy-and-hold strategy).

Statistical tests of independence.—In an efficient capital market, stock price changes should be independent and random. Two statistical techniques have been used to test this. Autocorrelation tests correlate price changes over time to see if they are independent. Several authors, including Kendall (1953), Alexander (1961), Moore (1962), Fama (1965), and Fama and MacBeth (1973), analyzed serial correlations between stock price changes for different intervals of one day, four days, nine days, and sixteen days. The results consistently indicated insignificant correlation in stock price changes over time.

Moore (1962) studied serial correlation between successive price changes of individual stocks. He examined weekly changes of twenty-nine randomly selected stocks for 1951-1958 and found an average serial correlation coefficient of -.06. This value is extremely low and indicates
that data on weekly changes are not helpful in predicting future changes.

Fama (1965) studied daily proportionate price changes of the thirty industrial stocks in the Dow Jones Average over a period of approximately five years. The serial correlation coefficients for the daily changes were small, with the average being 0.03. Fama also investigated lagged price-change dependence. Again, the coefficients did not differ from zero.

The other typically used statistical technique is a runs test. Given a series of price changes, each price change is designated a plus (+) if it is an increase or a minus (-) if it is a decrease. A set of pluses and minuses will result. A sequence of two or three consecutive positive or consecutive negative price changes is one run. The run ends when the price change is to a different sign. The expected number of runs for a random series is 1/3 (2n-1), where n is the number of observations. If the actual number of runs is significantly less than expected, there is positive correlation in successive signs. Further, if the number of runs is significantly greater than expected, there is negative correlation. Runs tests supported the idea that price changes over time are independent.

Fama, in his 1965 study, used the runs test. The daily changes in the prices of each of the thirty Dow Jones stocks
were classified as zero, positive, or negative. The actual number of runs closely conformed to the number expected, though there was a trend toward runs in daily changes. There was minimal departure from randomness and strong evidence supporting the random walk hypothesis.

The same statistical tests for independence were repeated on the Over-the-Counter (OTC) market by Hagerman and Richmond (1973). Their results also supported the efficient market hypothesis.

Neiderhoffer and Osborn (1966) examined price changes in terms of individual transactions on the New York Stock Exchange (NYSE) and found economically significant serial correlation. A study by Carey (1971) supports these results. However, neither study demonstrated that above average returns could be earned by examining the dependence in price movements.

**Trading rules tests.**—To answer the accusations by technical analyst that previous empirical tests were too rigid, several tests were conducted involving the examination of trading rules. Analysts apparently did not believe a mechanical number of positive or negative price changes signals a move to a new equilibrium. However, they did feel that a consistency in trend over time might signal a move to a new equilibrium. This would include both positive and
negative price changes. Although technical analysts believed their trading rules were too sophisticated and complicated to be simulated, researchers made an attempt to investigate them. The EMH asserts that any trading rule used by investors, based solely on past market information and stock prices moving in trends could not derive above average profits.

The trading rule studies were used to simulate conditions under which a specific technical system was used to make decisions based on public information. The results of the investment decisions, including commission costs, were compared with a simple buy-and-hold strategy. There are an infinite number of potential trading rules—the tests are generally of only the simpler trading rules and only for the better known and more heavily traded stocks. One would expect more heavily traded stocks to be priced more efficiently.

Filter rules have been the most often tested trading rule. A filter can be set for a given stock. When the price changes exceed the specified filter, the stock is sold. Various studies have used filters ranging from one-half percent to fifty percent. Alexander (1961 and 1964) found that by using small filters, one can derive above average profits. However, Fama (1965) discussed several major problems with Alexander's work. According to
Fama, Alexander failed to realize that dividends were a cost rather than a benefit when stocks were sold short. Also, he failed to take transactions costs into account and assumed the stocks could be bought and sold at the precise price when the signal to buy or sell was given. The use of small filters resulted in numerous trades and substantial commissions. When commissions are considered, the trading profits turn to losses. Fama and Blume (1966) found large profits with small filters. Again, however, commissions turned profits into losses.

George Pinches (1970) reviewed several studies using past market data other than stock prices. Trading rules using odd lot figures, advance-decline ratios, and short sales showed some small profits but generally a buy-and-hold strategy was superior when commissions were considered. The majority of the evidence generated by simulating trading rules supports weak-form market efficiency.

The Semistrong-Form Efficient Market Hypothesis

The semistrong-form EMH implies that security prices adjust rapidly to the release of all new public information. Studies of the semistrong-form generally examine price movements around the time of an announcement, or event, in an attempt to see when the expected price adjustment took place. In an efficient market prices adjust
quickly. Semistrong-form studies have also examined potential profits from acting after the information becomes public.

**Adjustment for market effects.**—Analysts have used both tests of the semistrong-form hypothesis, generally adjusting individual stock price movements (security returns) for aggregate price movements (market returns). Prior to 1970, authors often assumed that stocks would experience returns (or percentage price changes) equal to those of the market. Reilly (1985) reports that the adjustment process involved the estimation of abnormal returns by subtracting the market return from the security's return as follows:

\[
AR_{it} = R_{it} - R_{mt}
\]

where \(AR_{it}\) = the abnormal rate of return on security \(i\) during period \(t\),

\(R_{it}\) = the rate of return on security \(i\) during period \(t\),

\(R_{mt}\) = the rate of return on a market index during period \(t\).

Fama, Fisher, Jensen, and Roll (1969) used another adjustment technique to examine the effect of stock splits on stock prices. They regressed stock returns on market returns for a period prior to and subsequent to an event as follows:
\[ R_{it} = a_i + \beta_i R_{mt} + \epsilon, \]  

(12)

where \( R_{it} \) = the rate of return on security \( i \) during period \( t \),

\( a_i \) = the intercept or constant for security \( i \) in the regression,

\( \beta_i \) = the regression slope coefficient for security \( i \) equal to \( \text{cov}_{im}/\sigma_m^2 \),

\( R_{mt} \) = the rate of return on a market index during period \( t \),

\( \epsilon \) = a random error term that sums to zero.

Since deviations from the regression line are random, they would be expected to sum to zero over long time periods. Using the appropriate parameters, the expected return for a stock can be derived for a specified market rate of return. The abnormal return \( (AR_{it}) \), would be equal to the security return minus the expected return as follows:

\[ AR_{it} = R_{it} - E(R_{it}), \]  

(13)

where \( AR_{it} \) = the abnormal rates of return on security \( i \) during period \( t \),

\( R_{it} \) = the rate of return on security \( i \) during period \( t \).
E(R_{it}) = the expected rate of return on security i during period t.

Over long time periods, abnormal returns should cancel out and sum to zero. If there is significant new information that is positive or negative, it should be reflected in the abnormal returns during the adjustment period. The impact of an announcement (or event), and its timing, are examined by looking at abnormal returns surrounding the announcement (the event window). The typical procedure is to examine the abnormal returns during individual periods surrounding an event and to derive a series of cumulative abnormal returns to determine the total impact of the event.

Cumulative average returns can be calculated by taking the difference between estimated and actual returns and averaging them across all companies for a stated time period.

\[
AR_t = \frac{1}{N} \sum_{i=1}^{N} AR_{it},
\]

where \( N \) = the number of companies,

\( AR_{it} = \) the abnormal rate of return on security i during period t,

\( AR_t = \) the average abnormal return in a given time period t.
The cumulative average return is the sum of the average returns over all time periods from the start of the data up to and including the current time period, \( T \):

\[
\text{CAR} = \sum_{t=1}^{T} \text{AR}_t, \tag{15}
\]

where \( T \) = the number of time periods being summed
\( (t = 1, 2, \ldots, M) \),

\( M \) = the total number of time periods in the sample,
\( \text{AR}_t \) = the average abnormal return in a given time period \( t \),

\( \text{CAR} \) = the cumulative average return.

According to Copeland and Weston (1983), the Fama, Fisher, Jensen, and Roll (FFJR) model, also known as the market model, has a few inherent problems. The model is not theoretically based. Further, it assumes that the slope and intercept terms are constant over the time during which the model is fit to the data.

There are two other empirical models that are used to estimate expected returns. The excess-returns model is based on capital asset pricing theory; it assumes an intercept equal to the risk-free rate (or the rate of return on the minimum variance, zero-beta portfolio) which changes over time. The excess-returns model is written as follows:

\[
[R_{jt} - R_{ft}] = [R_{mt} - R_{ft}] \beta_j + \epsilon_{jt}, \tag{16}
\]
where \( R_{jt} \) = the return on security \( j \) at time period \( t \),
\[ R_{ft} = \text{the risk-free rate of return during time period} \ t, \]
\[ R_{mt} = \text{the rate of return on a market index during period} \ t, \]
\[ \beta_j = \text{the estimated systematic risk of the} \ j \text{th security}, \]
\[ \epsilon_{jt} = \text{a random error term for the} \ j \text{th security at time period} \ t. \]

This (and the market) model assumes systematic risk remains constant over the estimation interval. Further, the CAPM-based model is subject to Roll's 1977 critique. Specifically, in CAPM-based tests the CAPM and capital market efficiency are joint and inseparable hypotheses. If capital markets are inefficient, then the assumptions of the CAPM are invalid and a different model is required.

The Empirical Market Line (EML) model is written as:
\[ R_{jt} = \hat{\gamma}_0 t + \hat{\gamma}_1 t \beta_{jt} + \epsilon_{jt}, \quad (17) \]
where \( R_{jt} \) = the return on security \( j \) at time period \( t \),
\[ \hat{\gamma}_0 t = \text{the intercept for time period} \ t, \text{a best linear estimate taken from cross-sectional data}, \]
\[ \hat{\gamma}_1 t = \text{the slope of the regression for time period} \ t, \]
and a best linear estimate taken from cross-sectional data,

\[ \beta_{jt} = \text{the quantity of risk for security } j \text{ at time period } t, \]
\[ \varepsilon_{jt} = \text{the random error term for security } j \text{ at time period } t. \]

The EML does not assume parameters to be constant over time. However, it still suffers many of the same criticisms as the CAPM-based model.

Studies of the semistrong-form efficient market hypothesis can be organized in terms of specific events. The following discussion uses such a classification.

**Stock split studies.**—Some believe that the prices of stocks that split increase in value because the shares are priced lower, which increases demand for them. Efficient market supporters would not expect a change in value.

The FFJR (1969) study examined long-run effects of stock splits on returns to stockholders. It provided important evidence on the semistrong EMH because it examined how rapidly stock prices adjusted to this important economic event. The authors used a new market-adjustment technique which has been utilized extensively in later studies.

FFJR hypothesized that splits, which are usually accompanied by dividend increases, were interpreted by the
market as a predictor of a dividend change. Stock splits alone do not cause higher rates of return. A dividend change can convey information about corporate management's confidence concerning future earnings. If the market is efficient, the only price effects of a split would be those associated with the information implied by a possible dividend change.

In addition, in an efficient market, investors would adjust or the forthcoming stock split prior to the announcement. Any relevant information that caused the split has already been discounted. It is felt that the stock price increase that causes a company to split its stock may be attributed to increased earnings or other successes, and this information is known and adjusted for prior to the split announcement.

FFJR examined all stock splits, of twenty-five percent or more, on the NYSE, from January 1927 through December 1959 to find whether the related stock's prices went up or down more than could have been expected. They abstracted from the influences of general market conditions during a period surrounding the time of the split. The market model was run on 622 securities to estimate the monthly rates of return for individual stocks and the rates of return for all stocks listed on the NYSE. The estimators were based on the 420 months during the 1926-1960 time period, excluding the
fifteen months before and the fifteen months after the month of the split. The months around the split were excluded because of the unusual effects they might have on the market model.

Next, the authors estimated the deviation of the actual return from the estimated market model return for each security in each of the twenty-nine months prior to the split and thirty months after the split. The deviations (or residuals) measure the abnormal price changes in those months. For each split, month zero is defined as the month of the split, -1 as the month prior to the split, and month +1 as the month after the split. Average residuals were accumulated across months to measure the average abnormal return over that period relative to a stock split to determine when the effects took place. The authors accumulated the residuals for individual stocks and for all stocks in the sample over time.

The total sample was divided into two groups: (1) stocks that split and also experienced an increase in their dividend rate, and (2) stocks that split but did not increase their dividend rate. This permitted the authors to examine the differential effect of dividend increases. Both groups of stocks had positive abnormal price changes prior to the split. However, stocks that split but did not increase their dividend had price declines following the
split. Within twelve months, the no-dividend increase stocks lost all of their accumulated abnormal gains.

In contrast, the stocks that split and also increased their dividend had no change in their abnormal return pattern after the split as indicated by a flattened abnormal return pattern. This was interpreted to mean that the full impact of the price change took place prior to the stock split. After the split, stocks with dividend increases were able to maintain their positive abnormal price increases, while stocks that did not increase their dividends and confirm expectations, returned to their status prior to the announcement. These results provided evidence to suggest that stock splits alone do not cause higher rates of return. Further, they support the semistrong-form of the EMH because they indicated that the price adjustment occurred prior to the split. It was argued that investors could not have gained by acquiring the stock after the split announcement.

A subsequent study by Hausman, West, and Largay (1971) examined profit opportunities and supported the FFJR results. A study by Charest (1978) found large positive residuals during the period surrounding the announcement. FFJR's conclusions were supported by Reilly and Drzycimski (1981) who extended the earlier work by analyzing the stock price reaction to the public announcement of the split.
Over fifteen days prior to and including the announcement date, stockholders earned positive abnormal returns. This indicated that some of the positive performance prior to the split observed by FFJR was associated with the announcement of the split. After the announcement there was no evidence of abnormal returns.

**Studies of new issues.**—During the 1960's, a large number of closely held corporations decided to go public by selling some of their common stock. It is difficult to determine the appropriate price for a stock that did not trade publicly. Due to the uncertainty of the price and risk involved in underwriting such issues, Reilly and Hatfield (1969) hypothesized that underwriters would tend to underprice the new issues. They felt that investors who acquired the new issues at the offering price would tend to earn abnormal returns. There was also a question as to how fast the market would adjust to the underpricing. To test market efficiency, returns for first investors (at the public price) for various period were examined.

Jaffee (1975), Weinstein (1978), and Block and Stanley (1980). Most of the studies indicated that on average, new issues yield abnormally positive short-run returns assuming purchase at the offering price. Most of the researchers attributed these excess returns to underpricing by the underwriters. The results tend to support the semistrong-form EMH because the market seemed to adjust almost immediately to the underpricing. The returns from acquiring new issues either yielded returns consistent with their risk or they were below expectations. A article by Miller and Reilly (1984) shows that prices adjust by the day after the offering—further evidence of rapid price adjustment.

**Exchange listing.**—The decision to list on a national exchange such as the NYSE, is an economic event that may have an impact on the value of a firm and its stock. Listing may increase the liquidity of the stock and add to the prestige of the firm. Furst (1970) examined the effect of listing on price by regressing price on variables such as dividends, growth, retention rate, earnings stability, leverage, corporate size, and a dummy variable for listing. The results indicated that listing did not have a significant impact on price.

Van Horne (1970) looked at listing price effects and the ability to profit from these effects. He compared price
movements for stocks listed on the American Stock Exchange (ASE) and the NYSE and similar movements of the Standard and Poors (S&P) 500 Index. The results showed positive abnormal price changes for the period between two and four months before listing. After taking into account transaction costs and estimation biases, the change in average price after listing (adjusted for industry price movements) was not significant. Van Horne concluded, after examining several studies, that market participants cannot profit from buying a stock upon the announcement to list and selling it at the time of listing. Further, he did not feel that listing was a thing of value.

Goulet (1974) examined listing related price changes and effects on shares outstanding, sales of stock, and the number of shareholders. Generally, listing caused an increase in shares outstanding and stockholders, but a decrease in stock price after the listing. It was not clear whether the investor could profit from a price change.

Several studies provide contrary evidence on exchange listing. Ying, Lewellen, Schlarbaum, and Lease (YLSL), in 1977, analyzed 248 firms listed on the NYSE and the ASE during 1966-1968. They examined the potential for extraordinary profits surrounding listing and the need for risk adjustment. YLSL used a technique developed by Fama and MacBeth (1973) to examine abnormal returns. Initially, they
examined abnormal returns surrounding the actual listing date and found positive abnormal returns during 24 months prior to listing. They discovered negative abnormal returns during most months after the listing.

The public information related to the listing comes with the announcement to apply for listing. YLSL examined the abnormal returns during the months following the announcement to apply. Results showed a significant positive abnormal return of 7.54 percent during the application month and a 5.00 percent abnormal return, excluding commissions, during the month following the announcement. This is counter to semistrong market efficiency because it implies that an investor can earn an abnormal risk-adjusted return based upon public information. Even with commissions, the abnormal return remains. There were consistently large negative abnormal returns after listing, but not enough to profit from short selling.

McConnell and Sanger (1981) used weekly rather than monthly data and considered several different models for deriving expected returns to re-examine the YLSL results. They also used different market indexes and different OTC stock prices. Even with all of the new analysis, the results support YLSL. They found positive abnormal returns for the five weeks following the public announcement to apply for listing. There was little impact from the
approval of the application. After the actual listing there was negative abnormal returns.

Several studies including Fabozzi (1981), Fabozzi and Hershkoff (1979), Phillips and Zecker (1982), Reints and Vanderberg (1975), and Baker and Spitzfaden (1982) examined the impact of listing on the risk of the stocks involved. The results indicated no impact on risk from listing. There appeared to be no significant change in systematic risk or cost of equity capital caused by listing.

The results for new listings and the EMH are mixed. However, there is more evidence of inefficiency.

**Block trades.**—Sale of blocks of stocks may have two effects: stock prices may change to reflect new information supposedly carried with the block, and (2) if buyers must incur extra costs when they acquire the block, there may be an initial decline in price due to price pressure (or liquidity premium). Using the market model and methodology similar to FFJR, Scholes (1972) and Kraus and Stoll (1972) found evidence of permanent price decline indicated by price drops between the closing prices the day before the block trade and the day of the block trade. Kraus and Stoll also found that temporary intra-day price pressure effects exist.
Dann, Mayers, and Raab (1977) scrutinized the results of the previous studies. They collected and analyzed continuous transaction data during the day of a block trade for a sample of 298 blocks between July 1968 and December 1969. Results of their analysis showed that one would have to react in less than five minutes to earn a positive return. Since prices change so rapidly, it would be hard for an individual trading on publicly available information to earn abnormal returns. Further, they found that fifteen minutes after the block trade, prices completely adjusted to unbiased estimates of closing prices.

For those who can transact at the block price, Dann, Mayers, and Raab found that even after adjusting for risk, transaction costs and taxes, it is possible to earn abnormal returns. This is evidence of strong-form market inefficiency.

It seems, therefore, that the empirical research involving price changes around block trades supports semistrong-form market efficiency. Although there is evidence of both a permanent effect and temporary price pressure effect, the market appears to very rapidly reflect all publicly available information. Evidence that abnormal returns are earned by participants in block trades at the block price indicates that there may be strong-form inefficiency.
World events.—Reilly and Drzycimski (1973) examined the adjustment of stock prices to seven unexpected significant world events. Stock prices were analyzed prior to the announcement, at the market opening after the announcement, and during the two days following the announcement. The major adjustment in stock prices seemed to take place during the time interval between the close before the announcement and before the market opened after the announcement. There were some large stock price changes after the opening stock price that followed the announcement but the direction of the changes was inconsistent. The results, therefore, indicate that investors can not earn abnormal returns from investment in the stocks at the opening following the announcement. This finding supports semistrong-form market efficiency.

Accounting information.—Numerous studies have analyzed the effects of announcements of accounting changes on stock prices. These studies contend that capital markets are relatively efficient. If an accounting change affects the economic value of a firm, there should be rapid change in stock price.

Ball and Brown (1968) examined the differential stock price movement for companies that had good earnings reports, and stock price movements of companies with poor earnings
reports. The FFJR technique was used to derive abnormal price performance of stocks of 261 firms between 1946 and 1965. The abnormal price movements were related to the abnormal earnings changes for the companies. The following regression was used to predict next year's change in earnings, $\Delta I_{j,t+1}$, 

$$
\Delta I_{j,t+1} = \hat{a} + \hat{b}_j \Delta m_{t+1},
$$

where $\hat{a}, \hat{b}$ = coefficients estimated from time-series models, 

$\Delta m_{t+1}$ = the actual change in market average earnings per share (eps) during the (t+1)th time period.

Abnormal earnings changes were derived by examining the historical relationship between firm earnings and aggregate earnings. The model for the change in earnings was 

$$
\Delta I_{jt} = \hat{a} + \hat{b}_j \Delta m_t + \epsilon_{jt},
$$

where $\Delta I_{jt}$ = the change in eps for the jth firm, 

$\Delta m_t$ = the change in the average eps for all firms (other than firm j) in the market.

If the firm did better than expected, based on historical data, it had a good earnings year. Ball and Brown divided the sample into companies with good and bad earnings reports, and examined abnormal stock price returns for the two samples during the year prior to the earnings reports.
An abnormal performance index (API) was calculated representing the value of one dollar invested in a portfolio twelve months before an annual report and held for \( T \) months. The API is computed as follows:

\[
API = \frac{1}{N} \sum_{j=1}^{N} \sum_{t=1}^{T} (1 + \epsilon_{jt}),
\]

(20)

where \( N \) = the number of companies in a portfolio,

\( T = 1, 2, \ldots, 12, \)

\( \epsilon_{jt} = \) abnormal performance measured by deviations from the market model.

Their results indicated that companies with abnormally good earnings reports also experienced positive normal stock price performance. However, most of the stock price adjustment took place prior to the end of the year and release of the annual report. This indicates that stock prices adjust prior to the new earnings information and that new information may have been obtained from prior quarterly reports.

Two studies examined the effect on stock prices of changes in inventory valuation methods from FIFO to LIFO or vice versa during periods of inflation. Sunder (1973 and 1975) examined 110 firms that changed to LIFO and twenty-two firms that changed to FIFO during the twenty-one year period of 1946-1966. He looked at the pattern of cumulative
average residuals from the market model. The results, assuming that risk did not change during the 24-month period, indicated that the cumulative average residuals for the firms switching to LIFO rose by 5.3 percent during the 12 months prior to the announcement of the accounting change. This supports the belief that shareholders actually value cash flow, not eps. Although the switch to LIFO lowered eps, there is no evidence that the switch lowered value.

Reilly, Smith, and Hurt (1975) tested the same hypothesis as Sunder for a sample of thirty-two firms that changed from LIFO to FIFO during the period 1972-1974. The average of the abnormal stock price changes for each of the six months prior to the announcements was always positive. The averages of the abnormal price changes during the announcement month and two subsequent months were positive. The following months exhibited small negative values. It was concluded that the results supported the efficient market hypothesis because positive price movements are consistent with increases in economic value.

Kaplan and Roll (1972) examined investor reactions to two accounting changes that have the effect of increasing the reported eps but have no effect on cash flows. They examined: (1) the switch in 1964 to the flow-through method of reporting investment credit, and (2) the switch back from
reporting accelerated depreciation to reporting straight-line depreciation. Both changes affected only financial statements and had no effect on taxes, cash, or any real economic asset or liability.

The FFJR market model technique was used to examine abnormal stock price movements for the sixty weeks surrounding the announcements. The abnormal price movements were generally negative except during the few weeks surrounding the announcements. The authors believed that the firms making the accounting changes may have been performing poorly. There may be some temporary benefit from the accounting change and the resulting higher reported earnings. However, the average negative price changes resumed afterwards and continued to the end of the test period. Such practices appear to be unsuccessful in permanently affecting stock prices.

Hong, Kaplan, and Mandelker (1978) tested the effect of pooling versus purchasing accounting on stock prices of acquiring firms. They used monthly data between 1954 and 1964 and compared 122 firms that used pooling and thirty-seven that used purchase. A simple log form time series market model was used. The acquired firm had to be at least three percent of the net asset value of the acquiring firm. Mergers were not included in the sample if another merger took place within eighteen months. If the acquiring firm
was not NYSE listed or the merger terms not based on an exchange of shares, they were excluded from the sample.

Cumulative average residual (CAR) patterns revealed no evidence of abnormal performance for the sample when the CARs were centered around the month of actual merger. Results of the study for acquiring firms which had to write off goodwill against their after-tax earnings because they used the purchase technique showed no evidence of negative abnormal returns. There is weak evidence that shareholders of acquiring firms experienced positive abnormal returns when the purchase technique was used. This supports the theory that investors value cash flows and not eps.

Archibald (1972) studied market reaction to changing the depreciation accounting method from accelerated to straight-line depreciation for financial statement purposes. He used a sample of sixty-five firms, all of which experienced an increase in their reported profits because of the change. The results indicated that the majority of abnormal returns before the accounting change were negative, while the abnormal price changes during the twenty-four months after the accounting change were mixed. Price changes during the initial five months after the announcement were negative. These results were interpreted as supporting the EMH.
Several studies have provided evidence inconsistent with the EMH. A discussion of the studies follows.

**Earnings reports.**—Studies reporting the usefulness of quarterly earnings reports do not support the semistrong EMH. Many of these studies were conducted by Latane and other associates including: Latane, Tuttle, and Jones (1969); Latane, Joy, and Jones (1970), Jones and Litzenberger (1970); and Latane, Jones, and Rieke (1974). All of these studies suggested that the market was not completely semistrong efficient. However, there were problems in each of the studies. Most of the problems were apparently eliminated in research completed in 1977 by Joy, Litzenberger, and McEnally (JLM).

JLM selected firms that experienced unanticipated changes in quarterly earnings and also considered different degrees of good and bad earnings. JLM established three categories of earnings based on deviation from expectations: (1) any deviation based on expectations; (2) a deviation of plus or minus twenty percent; and (3) the deviation must be forty percent. Using an FFJR technique to analyze abnormal price changes for the period from thirteen weeks prior to the announcement to twenty-six weeks following the announcement, they examined the abnormal
performance (API) during the period before and after the announcement.

Category one (any deviation) results indicate that the abnormal price movements for the good earnings companies were about 1-2 percent during the period. Transactions costs of 2-3 percent would eliminate trading profits.

The 20 and 40 percent deviation requirements generated much stronger results. For the twenty percent above expectations category, the post-announcement gain was about four percent. The forty percent above expectations category experienced a 5-6 percent gain for the sample. These abnormal returns would be adequate without information and distribution costs. The price adjustment for unfavorable earnings performance appeared to be faster, and there was no abnormal returns for any category.

Analysis of the cumulative API indicated that the post announcement change was statistically significant for the models using twenty and forty percent deviations. These results indicate that favorable information contained in quarterly earnings reports are not instantaneously reflected in stock prices. The authors also examined whether there is a significant relationship between the size of the unexpected earnings performance and the post-announcement stock price change. The results support the idea that the
price change is influenced by the size of the favorable earnings change.

Two review articles have discussed a number of studies in this area. Joy and Jones (1979) noted problems with several of the earlier studies and cited more recent studies where they felt the problems had been solved. Based on the articles they reviewed, Joy and Jones felt that market inefficiencies exists with respect to earnings reports.

Ball (1978) examined anomalous evidence regarding market efficiency. He reviewed twenty studies of price reaction to earnings announcements and found that the post-announcement risk-adjusted abnormal returns are consistently positive, which is inconsistent with market efficiency. Contrasting with Joy and Jones, he argued that the abnormal returns occur because of problems with the two parameter asset pricing model used to derive expected returns, not because of market inefficiencies.

A study by Watts (1978) found significant abnormal returns even after making the adjustments suggested by Ball. He found returns were due to market inefficiencies rather than problems with the two parameter market model.

In conclusion, the evidence from studies on quarterly earnings announcements generally are inconsistent with semistrong market efficiency.
P/E ratios.—A study by Basu (1977) tested the relationship between the price-earnings (P/E) ratios for stocks and the returns on the stocks. Market observers have traditionally contended that stocks with low P/E ratios will tend to outperform stocks with high P/E ratios. There seems to be no well-developed reasoning behind this hypothesis except for a belief that the growth companies enjoy high P/E ratios consistent with that growth. The market may over estimate growth potential and over value the stocks of growth companies. Basu empirically examined whether the investment performance of common stocks is related to their P/E ratios. If there is a definite relationship between stock prices and P/E ratios, the market would be inefficient.

Basu ranked stocks on the basis of their historical P/E ratios to determine the risk and return for portfolios containing high P/E ratio stocks compared to portfolios with low P/E ratio stocks. The stocks were divided into five P/E classes, and returns and alternate measures of risk were computed for each class. The average annual rates of return were substantially different: 9 percent for high P/E ratio stock and 16 percent for the low P/E ratio group. An unexpected result was that the low P/E ratio group also had lower risk. Composite performance measures that consider return and risk indicated that the low P/E ratio
stocks experienced superior results relative to the market, while high P/E ratio stocks experienced inferior results. In the analysis that followed, he attempted to avoid any potential bias in the performance measures and also accounted for taxes, transaction costs, and search costs. Taxes and transaction costs had a small impact. However, Basu concluded that publicly available P/E ratios possess valuable information and should be considered by investors. These results are strictly inconsistent with market efficiency.

**Small firm effect.**—Banz (1981) and Reinganum (1981) did analysis similar to Basu but they examined the impact of size on the risk-adjusted rates of return. They measured size in terms of the total market value of the firm. Banz ranked all the stocks on the NYSE while Reinganum did the same for both the NYSE and the ASE on the basis of the market value of the stocks. The ranked sample was divided into ten portfolios with equal weighting of stocks in the portfolios. Risk was measured in terms of the one-period CAPM. Next, they derived risk-adjusted abnormal returns for the ten portfolios for extended periods of 10-15 years and found that the small firms consistently experienced significantly larger risk-adjusted returns than the larger firms.
They felt that Basu's P/E results were really a small firm effect.

The Basu, Banz and Reinganum studies are really dual test of the efficient market hypothesis and the capital asset pricing model which is used to derive expected rates of return. The abnormal returns derived from these studies may be caused by market inefficiency or a misspecified market model. Reinganum (1981) felt that the simple one-period CAPM is an adequate model for the real world capital markets.

Roll (1981) responded to the Basu, Banz, and Reinganum articles by arguing that the results occurred because the riskiness of the small firms was improperly measured. Since small firms are traded less frequently, risk measures obtained from short interval data may understate the risk. Infrequent trading may cause an increase in serial correlation of prices over time and a decrease in the variance of returns over time. Either of these results will lower the stock's beta. This same phenomena was noted early by Dimson (1979). He suggested adding lagged and small leading market returns to the market model and running the coefficients to arrive at the beta for infrequently traded stocks.

Reinganum responded to these suggestions by studying a number of portfolios ranked by size and then computing betas
for each market value portfolio by the standard ordinary least squares (OLS) model and the Dimson aggregate coefficients model. There was a substantial difference in the beta estimates using the two methods. The results closely supported the contention by Dimson and Roll regarding the underestimation of risk for small firms. To test whether the new betas could explain the large differences in rates of return for the different portfolios, Reinganum compared the returns to the betas and firm size. The firm size coefficients are generally significant after controlling the size effects. Differences in Dimson betas seem to explain only a small portion of the differences in average portfolio returns. The author concluded that the small firm effect is still a significant economic and empirical anomaly.

Stoll and Whaley (1983) contend that the differential impact of transaction cost on small and large firms has not been given adequate attention. They demonstrated that the total market value of common stock equity varies inversely with risk-adjusted returns. Also, there is a strong positive correlation between average price per share and market value. Transactions costs including both dealer's bid-ask spread and the broker's commission must be considered when examining the small firm effect. Based on sample stocks, it was shown that proportional bid-ask spread varies from 2.93 percent for small value stocks to 0.69
percent for large value stocks. The total commission to buy and sell stock was 3.84 percent for small firms and 2.02 percent for large firms. This indicates a total difference in transactions cost of 4.06 percent between large and small firms. This difference in transactions cost can have a significant impact on returns from frequent trading. With daily transactions, the original small firm effects are reversed. The difference in transactions cost for small, lower-priced stocks appears important and subsequent studies on differential performance must consider such costs.

Reinganum (1983) considered the holding period assumption when he examined the strategy of buying and holding securities for longer periods of time. These yielded results similar to the results from daily trading. He developed ten portfolios including all securities traded on the NYSE and the ASE at the end of each year based on year-end market capitalizations. His initial analysis indicated that the rates of return for the various portfolios differ systematically depending on market capitalizations. With small capitalization there was greater return from 1963-1980. Consistent with the Stoll and Whaley study, average market price was positively correlated with firm size. Risk was measured using the Dimson aggregate coefficients estimator.
Two holding period strategies were used: (1) a one-year holding period, which assumed rebalancing every year, and (2) a buy and hold strategy, which assumed an investor held the original portfolios derived at the beginning of 1963 through 1980. With the small firms, one dollar in 1963 grew to be over forty-six dollars without commission, while one dollar with the large firms grew to about four dollars. The results with the passive strategy still indicated the superiority of the small firm investment. There was no explicit consideration of transactions cost and it was felt that this would not significantly change the results. The distribution of individual security returns was analyzed. The results showed that the distributions were generally non-normal and the small firm distributions were substantially different from the large firm distributions in terms of both skewness and kurtosis. The small firm distribution was very skewed, indicating an abnormal proportion of large positive returns. Reinganum felt that small firms outperformed large firms even after considering risk and transactions costs assuming annual rebalancing—which was strongly recommended.

Arbel and Strebel (1983) studied the impact of trading activity (in terms of attention) as an influence on the returns of stocks. They measured attention as the number of analysts who regularly followed a stock and divided the
stocks into three groups: (1) highly followed; (2) moderately followed; and (3) neglected. They measured risk with the single factor CAPM but also considered total risk. They found and isolated the small firm effect and contended that the neglected-firm effect also persists. They felt that the neglected firm effect is due to the lack of information on these firms and limited institutional interest. Since the neglected firm effect exists across size classes, institutions can invest in medium-sized neglected firms and derive maximum benefits recognizing the informative problems and the need for diversification. A potential problem with the study was the risk measurement—the study did not consider the infrequent trading problems and possible effects on betas.

Peavy and Goodman (1983) examined the P/E ratios question adjusting for firm size, industry, and infrequent trading effects. They tried to deal with the size problem by considering only firms with a market value of $100 million and to control the industry effect by examining only firms within certain industries. They dealt with the infrequent trading problem by using quarterly intervals and by including only stocks which had an average monthly trading volume exceeding 25,000 shares. They found risk-adjusted returns for stocks in the lowest P/E ratio quintile. There
is, however, some question as to the adequacy of the risk measure and size adjustment.

James and Edmister (1983) examined the relationship between returns, market volume, and trading activity. They found a significant inverse relationship between firm size and risk-adjusted rates of return, even when an aggregate coefficients technique is used for infrequent trading. There was a strong positive correlation between size and trading activity. Firm size was used as a proxy for trading activity. Trading activity was a relevant factor. The excess returns could then be justified as a liquidity premium.

Results regarding trading activity do not support the EMH. There is considerable evidence of a size effect. The size anomaly is not explained by differential trading activity. There have been several attempts to explain this anomaly by looking at superior risk measurements, transactions costs, analysts' attention, and trading activity. To date, no study has been able to explain the small-firm effect. Some combination of risk measurement and the differential transaction costs may eventually explain the difference in returns on small firms.

Tax selling.--Branch (1977) proposed a trading rule for those interested in taking advantage of tax selling. He noticed that investors tend to sell for tax purposes toward
the end of the year to establish losses on stocks whose prices have declined. After the new year, people tend to reacquire stocks or reinvest the proceeds. It would be expected, therefore, that stock prices would be depressed in November and December and would be higher in January. In an efficient market this should not happen. Arbitrageurs or speculators would eliminate any seasonal patterns by purchasing in December and selling in January.

Dyl (1977) also found evidence of tax selling. He found an unusually high volume of trading in December for stocks that had experienced large losses during the previous year; and low volume trading for high yield stocks. Dyl reports significant abnormal returns during January for stocks that had lost money during the previous year.

Roll (1983) examined the January anomaly along with the small firm effect. He found significant price patterns for the last day of December and the first four days of January. Roll discussed several potential causes for this pattern. Stocks with negative returns over the entire previous year were reported to have higher returns around the first of the year. Roll questioned whether the entire year-end return is caused by tax selling or size. He found that smallness had an effect beyond that attributable to volatility and tax selling.
Roll felt that the tax selling pattern was not eliminated by arbitrage because of transactions costs. Using a simple trading rule he generated high returns from buying on the second-to-last day of the year and selling on the fourth day of the new year. Applying the commissions estimated by Stoll and Whaley, there were still excess returns. As an extreme estimate of the spread, Roll assumed that an investor bought at the high price for the second-to-last day and sold at a low price on the fourth day of the new year. Under these assumptions and adding commissions, there was no profit on the NYSE and little excess profit on the ASE. Roll concluded by discussing some problems with estimating an appropriate long-run risk measure that would explain both the seasonal phenomenon and the remaining small firm effect.

Summary.--The evidence of semistrong market efficiency is mixed. Studies dealing with specific events generally support the efficient market hypothesis--because prices react rapidly to new information and investors cannot derive abnormal returns by acting after the event. There are anomalies, however. Evidence on quarterly earnings does not support the EMH. Small firms seem to generate superior abnormal risk-adjusted returns compared to large firms. There seems to be a January anomaly due to tax selling.
This is practically neutralized if one considers transaction cost.

The Strong-Form Efficient Market Hypothesis

The strong-form EMH argues that stock prices fully reflect all information--no investor possesses information that would allow them to consistently generate above average risk-adjusted returns. Stock prices adjust rapidly to new information and no group has monopoly access to any information. Tests have examined the returns to specific groups of investors to see if they earn abnormal returns. If a specific group can earn abnormal returns, the market is not efficient. Three major groups of investors have been examined: (1) corporate insider trading; (2) stock exchange specialists; and (3) professional money managers with emphasis on mutual funds.

Corporate insiders are required to report their transactions to the Securities and Exchange Commission (SEC) if they involve the company they work for. Approximately six weeks after the reporting period, insider trading information is made public by the SEC. Studies by Lorie and Niederhoffer (1968), Jaffee (1974), and Finnerty (1976) generally indicate that corporate insiders consistently earn significantly above average profits. It appears that stock markets are inefficient in the strong-form.
EMH requires more than an efficient trading market. It requires a perfect information generating process and a perfect information processing market.

There is evidence that public investors who consistently trade with insiders based upon announced insider transactions would also earn returns in excess of a simple buy-and-hold policy. This is further evidence against the strong-form EMH.

Stock exchange specialists have monopoly access to information about unfilled limit orders. An SEC study (1963) found that typically the specialist sells above his last purchase on 83 percent of all his sales and buys below his last sale on 81 percent of all his purchases.

Neiderhoffer and Osborne (1966) analyzed individual transaction data on the NYSE and indicated that specialists apparently use their access to information about unfilled limit orders to generate excess profits. The Institutional Investor Study by the SEC (1971) indicates that returns earned by specialists were above what would be expected. The study showed that the average return on capital exceeded 100 percent.

Reilly and Drzycimski (1975) indicated that specialists, following major unexpected world announcements, and acting as he is directed, would have consistently made profits on trades following such announcements.
Professional money managers do not necessarily have monopolistic access to information. If any normal investor should be able to derive above average profits without inside information it should be this group. Professional money managers are usually well trained and they also do extensive management interviews. Most studies have been done on mutual funds but more recently, bank trust departments, insurance companies, and investment advisers have also been analyzed.

Several studies, including Sharpe (1966), Jensen (1968), and Treynor (1965), examine the performance of mutual funds over extended periods. The results indicate that a majority of the funds were not able to match the performance of a buy-and-hold strategy. Excluding commission costs, only slightly more than half of a large sample of mutual funds did better than the overall market risk-adjusted rate of return. When commission costs were included approximately two-thirds generally did not match the market. Funds were not consistent in their performance.

Klemkosky (1977) examined the consistency in the risk-adjusted performance of a sample of 158 mutual funds at two- and four-year intervals during the eight year period 1968-1975. He concluded that investors should be cautious when using past relative risk-adjusted performance to predict future relative performance.
The performance of mutual fund managers supports the strong-form EMH. Reilly (1985) indicates that the performance for other institutional investors is, generally, consistent with those for mutual funds.

Tests of the strong-form EMH provide mixed results. However, the bulk of the evidence does not support the strong-form EMH. Corporate insiders and stock exchange specialists appear to make high returns. Professional money managers do not make high returns.

**Real Estate Market Efficiency**

Few areas of research in the finance literature have attracted greater attention than the efficient market hypothesis. However, most of the research has been directed toward the securities market and little has been done to examine the efficiency of the real estate markets. Fama (1970) describes an efficient market as one in which prices adjust rapidly to new information and all available information, including risk, is fully reflected in current prices. As Jaffe and Sirmans (1984) indicated, there is some doubt about the degree of efficiency of real estate markets. Researchers have had to face the problems of inadequate data bases, understanding a general market behavior, and incomplete theory about the real estate market operations.
Fisher (1983) and Roulac (1980) indicate that real estate markets have traditionally been viewed as less efficient than other markets. Several reasons for this include: (1) the lack of a national real estate exchange; (2) the uniqueness of each parcel of real estate; (3) difficulty of obtaining data; (4) the large amount of equity investment required in real estate; (5) the impact of financing that is usually required with real estate; (6) valuation problems; (7) the use of professional management; and (8) potential legal problems. However, recent studies by Gau (1984, 1985), Guntermann and Smith (Forthcoming), Dale-Johnson, Findlay, Schwartz, and Kapplin (1985), Locke (1986), and Guntermann and Norrbin (1986) have provided empirical evidence that real estate markets are efficient.

Early research into real estate market efficiency was primarily descriptive or illustrative. Several researchers discussed the relationship between real estate returns and returns in the money and capital markets. In many cases regression analysis and the the Capital Asset Pricing Model (CAPM) were used to estimate yields in the real estate markets.

**Descriptive and Illustrative Literature**

Miles and Rice (1978), Miller (1978), Gau and Kohlhepp (1978), Sirmans and Webb (1978), and Wofford and Moses
(1978) investigated the relationship between real estate investment yields and yields in the money and capital markets. They implicitly assumed efficiency of the money and capital markets and discussed how returns in those markets can be used to forecast the real estate markets. Using capital markets' methodology, Miles and Rice hypothesized that current real estate applications of capital market theory produce unbelievably positive risk-adjusted results for the investor who includes real estate in his individual portfolio. Their results supported their hypothesis. Miles and Rice also found that there was a very low correlation between real property returns and the stock market index and proceeded to develop a new market index for real estate.

Miller found a correlation between real estate investment yields and rates attainable in the money and capital markets. He concluded, using a regression equation, that real estate yields originate in the money and capital markets and it is possible to estimate real estate yields using this information. Gau and Kohlhepp developed a technique of estimating an equity yield rate for real estate called the Capital Market Approach (CMA). The CMA was a version of the CAPM which relied on the S&P stock market index as a proxy for a diversified real estate portfolio.
Sirmans and Webb examined real estate yields versus yields on common stocks, long-term government bonds, long-term corporate bonds, and U.S. Treasury bills for the period 1951 to 1976. They found that real estate returns were related to returns in the money and capital markets.

Hoag (1980) tried to overcome the problem of the lack of a real estate market portfolio index and data base by constructing an index of value and return for non-owner occupied industrial property. A valuation model was developed for the purpose of estimating a quarterly asset value based on a set of macroeconomic, regional, and property-specific variables. His results indicated that equity investment in industrial real estate over a short period had risk and return comparable to the stock market. However, the valuation approach of developing a time series and market index may be subject to considerable estimation error.

Draper and Findlay (1982) examined the CAPM applied to real estate investment analysis and appraisal. They discussed how the CAPM is derived, its theoretical problems, and its empirical validation. They compared the real estate and securities markets and discussed alternatives to the CAPM. Further, it was pointed out that the real estate market must be efficient for the CAPM to be an effective measure of real estate return. However, up to this point,
no one had really demonstrated that the market was efficient. Further, the traditional belief was that real estate markets are inefficient as Roulac (1980) had claimed.

The next logical step in the research was to empirically test the efficiency of the real estate markets. With data for research becoming more readily available, several studies were published dealing with semi-strong form and weak-form real estate market efficiency.

**Semistrong-Form Efficiency in Real Estate**

Semistrong-form studies of the efficiency of real estate markets have been conducted by Davies (1977), Gau (1985), and Dale-Johnson, Findlay, Schwartz and Kapplin (1985). Davies examined the effects of restrictive land use policies on land prices in a Canadian city. Gau also examined a Canadian real estate market. For the period of January 1971 through December 1980, Gau used 1,533 apartment transactions in the Vancouver, British Columbia market. Monthly series of continuously-compounded returns were derived for a unit of homogeneous apartment investment. Investment units were scaled in terms of sales price per square foot, sales price per dollar of gross income, and sales price per suite.

Gau used a three-factor arbitrage pricing theory (APT) model and cluster analysis to form risk classes and estimate
risk-adjusted abnormal returns. He believed that using APT would allow him to get around the market index requirement for the CAPM model and would include other possible factors in the pricing of real assets beyond the market portfolio. The abnormal returns from two types of public information were examined: (1) major changes in government tax shelter and rent control policies; and (2) unanticipated changes in interest rates. In both cases Gau found an absence of significant abnormal returns and no evidence to suggest that investors could use information concerning government policy changes on a risk-adjusted basis. Thus, Gau's results support the semi-strong form of the EMH for real estate markets.

Dale-Johnson, Findlay, Schwartz, and Kapplin examined real estate market efficiency and the impact of creative financing on housing prices. The generally accepted cash equivalence adjustment was compared to an approach known as the financed fee valuation adjustment (FFVA), which they argue is theoretically superior. According to the authors, any test of a creative financing valuation model is, implicitly, a test of the following joint hypotheses: (1) the valuation model is correct; (2) the markets for housing and mortgages are efficient.

It was hypothesized that if markets are efficient in the pricing of creative financing, after adjusting the
observed selling price of a creatively financed house by the appropriate theoretical price adjustment, the remaining price would be the price one would have observed if a cash sale or market financed sale had occurred. This hypothesis was tested by using regression analysis and the results supported real estate market efficiency.

**Weak-Form Efficiency in Real Estate**

Only recently have empirical tests of weak-form real estate market efficiency been conducted. Several studies have been identified involving Markusen (1979), Adams, Milgram, Green and Mansfield (1968), Gau (1984), Locke (1986), Guntermann and Smith (1987), and Guntermann and Norrbin (1986). The results indicate support for the weak-form EMH for real estate markets.

Markusen constructed a model that examined the inter-temporal price and transaction patterns of real assets. He analyzed how the asset price path is affected by changes in real income, the real rate of interest, and the decision maker's time horizon by using the model to estimate elasticities. Markusen concluded that asset purchasers will increase their demands at current prices after a fall in current and expected real interest rates or a rise in current or expected future real income. However, potential suppliers will supply less under these circumstances. An
increase or decrease in quantities transacted depends upon
the nature of intertemporal demand.

Adams et al. used 1,111 land transactions in North-
east Philadelphia over the period 1945 to 1962, together
with selected land characteristics, to study the process by
which vacant land on the edge of a city goes into urban
use. In particular, they examined factors that influenced
the rate of development and rise in prices during the
process. In the case of residential and commercial land the
long run trend of prices was close to a normal return
based on the belief that Northeast Philadelphia development
trends were anticipated early and capitalized into real
estate values. However, industrial land showed substantial
shifts in anticipations because the growth rate of prices
was very high.

Gau performed a rigorous study of weak-form real estate
market efficiency. He developed price series from apartment
and commercial transactions in Vancouver, British Columbia
during the period of January 1971 through December 1980.
Data were collected on 1,533 apartment and 1,084 commercial
transactions. Three scaled prices were calculated for each
apartment transaction and one scaled price was calculated
for each retail transaction.

Gau hypothesized that if significant autocorrelations
were found across the lagged values of time series, the
potential for utilizing such price information to have successfully predicted real estate sales prices could be evaluated through the estimation of a forecasting model. In general there was little evidence from the serial correlation results that past prices and returns contained much information about future real estate values. However, several price series did indicate some serial correlation and the possibility of constructing a forecasting model to estimate future sales prices. After further examination, the forecasting models did not perform significantly better than a mean value forecast. Prediction errors were large for all forecasting models.

The results suggested that the serial correlation present in apartment price and return measures is too small to allow for consistently earning abnormal returns based on past price information.

Locke examined the United Kingdom and Australian Stock Exchange and selected the British FT Property Index, the Australian Statex Accumulation Property Index and thirteen Australian listed property trusts (companies that invest in property) as financial securities in the property sector. To represent real property prices the British Jones Lang Wooten (1981) indices, the Australian Richard Ellis and Associates (1984) indices and Valuer-General's (Vic) indices were used. Returns were calculated on a monthly basis and
runs tests and autocorrelations tests were conducted on each of the securities to ascertain whether the returns move in accordance with the requirements necessary for weak-form efficiency. Locke also performed a normal distribution test using a Kolmogorov-Smirnov two tail test.

The results of Locke's study indicated that the financial securities were weak-form efficient. However, the real property results varied between British and Australian data. The Australian real property market appeared to be efficient while the British real property market was inefficient. Possible explanations for the varying results are that British data are largely based on valuations and British leases are considerably longer.

Guntermann and Smith gathered FHA and Census of Housing data on land and housing prices for fifty-seven metropolitan areas to perform a weak-form EMH test on single-family housing markets. Annual land and property prices per square foot were used to construct annual holding period returns by metropolitan areas. The returns were rank-ordered and cross-correlation coefficients were computed between holding periods for various years.

Efforts to develop a trading rule to take advantage of possible inefficiencies were unsuccessful. Therefore, they concluded that residential real estate markets are weak-form efficient.
Finally, Guntermann and Norrbin conducted an intra-market test of the EMH for real estate markets. They used housing price data for twenty census tracts within the Lubbock, Texas market over the decade of the 1970's. Rate of appreciation in property values in one year was compared with subsequent years by using a correlation analysis. The results indicated a weak negative relationship for one and two year lags of returns and no relationship for lags of three years or more. A trading strategy could not be developed to earn abnormal returns above transactions costs.

Guntermann and Norrbin used another test for market efficiency called the dynamic multiple-indicator multiple-cause (DYMIMIC) model. This technique is supposed to explain house price simultaneously in terms of expected appreciation and traditional house characteristics. Using this technique, the results were consistent with an efficient housing market.

Summary

Contrary to traditional beliefs, the majority of the empirical research on the efficiency of real estate markets overwhelmingly supports weak and semistrong-form market efficiency. However, further research is needed. Much of the previous research has been limited by the quality and
quantity of data available for analysis. This has made it very difficult to make generalizations.

This study involves a rigorous test of weak-form real estate market efficiency using a broad data base. Given the quality and uniqueness of the data base, this research should make a significant contribution to the literature.
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CHAPTER III

METHODOLOGY

Introduction

If a real estate market is weak-form efficient, information contained in past prices is fully incorporated in current prices and therefore cannot be used in attempting to earn an abnormal return. If weak-form real estate market efficiency does not exist, information in past prices is not fully incorporated into the market pricing process and the information may be used to earn abnormal returns.

To test for weak-form real estate market efficiency the following research methodology is used:

1. Obtain six years of office building transaction data dating from January 31, 1979 to January 31, 1985 from the Dallas-Fort Worth office properties market on the following variables: (a) sales price (SP); (b) gross building area (GBA); (c) net rentable area (NRA); (d) effective gross annual income (EGI); and (e) annual net operating income (NOI).

2. Calculate scaled prices and returns for each transaction as follows: (a) sales price per square foot of net rentable area (SPSQFNRA) = sales price /square feet of

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net rentable area; (b) gross income multiplier (GIM) = sales price/gross annual income; and (c) net income multiplier (NIM) = sales price/net operating income. In addition the following return series are estimated for each transaction:

\[ \text{SPSQFNRA RETURN (R1)} = \frac{(\text{SPSQFNRA}_t - \text{SPSQFNRA}_{t-1})}{\text{SPSQFNRA}_{t-1}} \] (21)

\[ \text{GIM RETURN (R2)} = \frac{(\text{GIM}_t - \text{GIM}_{t-1})}{\text{GIM}_{t-1}} \] (22)

\[ \text{NIM RETURN (R3)} = \frac{(\text{NIM}_t - \text{NIM}_{t-1})}{\text{NIM}_{t-1}} \] (23)

The GIM RETURN series becomes a return for a given dollar of gross income. The NIM RETURN series becomes a return for a given dollar of net income. The natural log of the SPSQFNRA, GIM, AND NIM variables are taken to generate a linear data set from a possibly nonlinear data set and to establish a stationary covariance.

3. Identify downtown and suburban submarkets based on locational characteristics of the property.

4. Choose the submarket with the highest transaction frequency and select one transaction per month from the time series.

5. Estimate autocorrelation and partial autocorrelation functions of the monthly price series and rates of return.

6. Test the significance of the autocorrelation functions.
7. Derive forecasting equations using the Box-Jenkins ARIMA Model and check the significance of the estimated moving average and autoregressive parameters.

8. Test the accuracy of forecasting models by re-estimating the ARIMA models over the first two-thirds of the data and using the parameter estimates to forecast the remaining observation values. The accuracy of the forecasts is compared with naive no-change and mean-value forecasts using the mean absolute error (MAE), root mean square error (RMSE), and Theil U statistic criteria.

Hypotheses

The hypothesis is that real estate markets are weak-form inefficient, and prices do not follow a random walk. A random walk means that there is no difference between the distribution of returns conditional on a given information set and the unconditional distribution of returns. Formally, the null hypothesis in prices can be stated as

\[ H_0: f_m(P_{1t}, \ldots, P_{nt}/\mathcal{N}_{t-1}) - f(P_{1t}, \ldots, P_{nt}/\mathcal{N}_{t-1}) = 0 \]

\[ H_a: f_m(P_{1t}, \ldots, P_{nt}/\mathcal{N}_{t-1}) - f(P_{1t}, \ldots, P_{nt}/\mathcal{N}_{t-1}) = 0 \]

where \( f_m(P_{1t}, \ldots, P_{nt}/\mathcal{N}_{t-1}) = \) the joint distribution of real estate prices given the set of information that the market uses to determine prices at \( t-1 \),

\( f(P_{1t}, \ldots, P_{nt}/\mathcal{N}_{t-1}) = \) the joint distribution of
which would exist if all relevant information at time t-1 were used.

If the distribution of prices in time period t, which was predicted from the previous time period, t-1, and based on market information, is not different from the prices predicted by using all relevant information from the previous time period, then there must be no difference between the information the market uses and the set of all relevant information. In return form the null hypothesis can be expressed as

\[ H_0^2: f(r_1,t+1, \ldots, r_n,t+1) - f(r_1,t+1, \ldots, r_n,t+1/ \mathcal{N}_t) = 0 \]

\[ H_a^2: f(r_1,t+1, \ldots, r_n,t+1) - f(r_1,t+1, \ldots, r_n,t+1/ \mathcal{N}_t) = 0 \]

In an economic sense, if the time series can be modeled by a moving average process of order one, an autoregressive process of order one, or an autoregressive process of order two, the null hypothesis cannot be rejected. Intuitively, none of these ARIMA models would provide enough information to overcome the transaction time-period lag for real estate and would thus be consistent with weak-form market efficiency.

**Sample Selection**

The sample time series were selected from several Dallas-Fort Worth appraisers' files. The Dallas-Fort Worth real estate market is one of the largest and most active
real estate markets in the world. Comparable transaction data were collected for a six year time period from January 31, 1979 to January 31, 1985. Transaction information was divided into downtown and suburban submarkets based on location. The submarket with the most transactions and at least one transaction per month was chosen for testing. All transaction data were cross-verified with other appraisal company files.

**Empirical Testing**

Given the above data, scaled prices for each transaction will be calculated including: (1) sales price per square foot of net rentable area; (2) sales price per dollar of gross income (gross income multiplier); and (3) sales price to annual net operating income (net income multiplier).

Geographic submarkets are identified in the Dallas-Fort Worth office properties market based on property location. The market is divided into downtown and suburban markets. One transaction per month is selected for the time series.

An autoregressive-integrated-moving-average (ARIMA) model is fit to the time series data. Box and Jenkins (1970) have been credited with constructing a four step iterative approach for developing linear time series models. The following discussion draws heavily from a
presentation by Vandaele (1983) on Box Jenkins models. The four steps for developing the models are:

1. identification of the preliminary specifications of the model;
2. estimation of the parameters of the model;
3. diagnostic checking of model adequacy; and
4. forecasting future realizations (observations).

In the identification stage, a particular model from the general class of ARIMA models is selected. The analysis of time series requires that the series be stationary. A time series is stationary if the mean and the variance are constant over time (and both are finite) and if the autocorrelation between values of the process at two time periods depends only on the distance between those two time points and not on the time period itself. If the process is stationary, one can estimate the mean, variance, and autocorrelations from a time series.

If the variance of the time series is not constant over time, data transformation may create a constant variance. The data can be transformed by using logarithmic, square root, or power transformation. Logarithmic and square root transformations appear to be the most frequently applied.

Many economic time series are characterized by movements along a trend line. A trend can be represented by any systematic change in the level of a time series. Box and
Jenkins (1970) advocate the use of differencing to remove seasonal and nonseasonal trends. Differencing involves subtracting the values of observations from one another in some time-dependent order. The revised time series (obtained by differencing) is analyzed to forecast more effectively.

The general class of ARIMA models consists of many different types of time series models. A time series is said to be governed by a first-order autoregressive process [AR(1)] if the current value of the time series, $Z_t$, can be expressed as a linear function of the previous value of the series and a random shock, $a_t$. Denoting the previous value of the series $Z_{t-1}$, this process can be written as

$$Z_t = \phi_1 Z_{t-1} + a_t,$$  \hspace{1cm} (24)

where $\phi_1$ is the autoregressive parameter which describes the effect of a unit change in $Z_{t-1}$ on $Z_t$, and which must be estimated. The random shocks, $a_t$ (also known as white noise), are assumed to be independent of $Z_{t-1}$, and normally and independently distributed with mean zero and constant variance $\sigma^2 a$. The process described in equation (24) is called an autoregressive process of order 1, AR(1). The order of the process corresponds to the number of parameters to be estimated. This model can also be viewed as a linear regression model in which the dependent variable, $Z_t$, is explained by and regressed on its previous values $Z_{t-1}$. 
There are certain properties which can be used to distinguish an AR process from any other process. A useful parameter is the autocorrelation parameter. The autocorrelation at lag $k$, $\rho_k$ is defined as the ratio of the autocovariance at lag $k$ to the autocovariance at lag zero ($\text{Var}(Z_t)$) and is therefore a scaled autocovariance. The autocovariance of $Z_t$ at lag 1, denoted $\lambda_1$, is a covariance between $Z_t$ and $Z_{t-1}$, and is defined as

$$\lambda_1 = \text{Cov}(Z_t, Z_{t-1}) = E(Z_t, Z_{t-1}).$$

(25)

Note that because of the stationarity assumptions, the autocovariance solely depends on the lag between $Z_t$ and $Z_{t-1}$. We can also define the variance of $Z_t$ as the covariance between $Z_t$ and $Z_{t-1}$, and denote it as $\lambda_0$. For the variance to be both nonnegative and finite, the absolute value of $\phi_1$ must be less than one.

For the autoregressive process of order 1, the autocorrelations are obtained as

$$\rho_k = \lambda_k / \lambda_0 = \phi_1^k, \ k > 0.$$  

(26)

Another feature of the autoregressive process is its "long memory." Current observations are influenced by shocks that occurred in the distant past. In a stationary process, the effect of shock will gradually dissipate. A memory coefficient at lag one is defined as the coefficient of the $a_{t-1}$ error. The memory coefficient at lag one
indicates the effect on current observation $Z_t$ of a shock that occurred one period ago. The plot of the memory coefficients as a function of the lag $k$, for $k \geq 0$, is called the memory function of the process.

Equation (24) can be expanded to include more lagged variables. If events two periods ago also had an effect on what is happening today, a second-order autoregressive model [AR(2)] is used

$$Z_t = \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + a_t,$$

where $\phi_1$ and $\phi_2$ are autoregressive parameters to be estimated. Autoregressive models rarely require an order higher than two.

A time series may be said to be governed by a first-order moving average process [MA(1)] if the current value of the series, $Z_t$, is a linear function of the current and previous errors or shocks, $a_t$ and $a_{t-1}$. The first-order moving average model may be expressed as

$$Z_t = a_t - \Theta_1 a_{t-1},$$

where $\Theta_1$ is the moving average parameter. The random shocks in a moving average process are assumed to be normally and independently distributed with mean zero and constant variance $\sigma^2 a$. The intercept in the case of an MA(1) process is the mean of the series.
With a moving average process, the number of parameters to be estimated can be drastically reduced. The mean, variance, covariance, and autocorrelation of an MA model are constant over time. The absolute value of $\Theta_1$ must be less than one for an invertibility condition to be met. An invertibility condition imposed on the parameter of a moving average process corresponds to the stationarity condition imposed on the autoregressive parameters.

Invertibility is most easily illustrated with a simple example. Consider an MA(1) process with a zero mean

$$Z_t = a_t - \Theta_1 a_{t-1},$$

(this cannot be estimated as is, since the disturbance $a_{t-1}$ cannot be directly observed. However, equation (29) must also hold for time period $t-1$

$$Z_{t-1} = a_{t-1} - \Theta_1 a_{t-2},$$

from equation (30)

$$a_{t-1} = Z_{t-1} + \Theta_1 a_{t-2},$$

$$a_{t-2} = Z_{t-2} + \Theta_1 a_{t-3},$$

substituting

$$Z_t = a_t - \Theta_1 Z_{t-1} - \Theta_1 a - \Theta_2 a_{t-2},$$

and repeat substitutions $- \Theta^n a_{t-n}$. Clearly, with repeated substitution, an MA(1) is equivalent to an infinite order AR process. This may be arbitrarily truncated. The term
- $\Theta^n n a_t - n$ goes to zero, as $n$ goes to infinity, provided that $0 < \Theta < 1$. This latter condition is known as the invertibility condition for an MA(1) process.

Only the first autocorrelation for an MA(1) process is nonzero. The memory function of an MA(1) process is a plot of the coefficients of the error term and only lasts for one period.

The MA(1) model can easily be extended to include additional lagged residual terms to a qth order MA process or MA(q). For example, an MA(2) process can be expressed as

$$Z_t = a_t - \Theta_1 a_{t-1} - \Theta_2 a_{t-2},$$

(34)

The memory function for an MA(2) process shows that the effect of the shock will last only for two periods.

Time series models can also be expressed as (mixed) autoregressive, moving average models and can be denoted as ARMA (p,q). The p refers to the number of autoregressive parameters and the q refers to the number of moving average parameters. An ARMA (1,1) process is expressed as

$$Z_t = \phi_1 Z_{t-1} + a_t - \Theta_1 a_{t-1},$$

(35)

where the $Z_t$s represent deviations from the mean of the series. ARMA (1,1) model stationarity requires the absolute values of $\phi_1$ and $\Theta_1$ to each be less than one. The autocorrelations of a stationary ARMA (1,1) process "die out" gradually. The rate of decline is determined by the AR
parameter $\phi_1$. However, the first autocorrelation is not equal to $\phi_1$, but is influenced by the AR or MA parameters. It is customary to denote the values of the $\Theta_i$ parameters as negative and $\phi_i$ parameters as positive.

AR, MA, and ARMA models describe a wide variety of stationary time series. Many time series can be made stationary by simple transformations. Nonstationary (and seasonal) time series can be depicted with a more general class of models.

A random walk may be written as a special case of the AR(1) model with $\phi_1$ equal to one (instead of less than one). In this case the AR(1) model is written as

$$z_t = z_{t-1} + a_t. \quad (36)$$

Changes are brought about in the random walk by a white noise disturbance. Stock prices have been hypothesized to follow a random walk with successive price changes being independent (see Fama (1970)).

A random walk model has a persistent memory and each random shock has a value of one for all lags. Both of the characteristics are typical of a nonstationary time series. Nonstationary series may be transformed into stationary series by differencing. If the first differences of a series are stationary, the difference between consecutive values of $z_t$ can be defined as
\[ w_t = z_t - z_{t-1}, \]  

(37)

and \( z_t \) and \( z_{t-1} \) can be replaced in the mixed autoregressive moving average model, the ARMA (1,1) model, with \( w_t \) and \( w_{t-1} \) to obtain

\[ w_t = \phi_1 w_{t-1} + a_t - \Theta_1 a_{t-1}. \]  

(38)

Equation (38) is called an autoregressive integrated moving average (ARIMA) model and may be denoted as ARIMA \((p,d,q)\). The letters \((p,d,q)\) refer to the order of the autoregressive process, the degree of differencing required to induce stationarity, and the order of the moving average process, respectively. In the ARIMA model, the word integrated actually means summed since it can be shown that the realization (observation), \( z_t \), can be written as an infinite sum of past and present differences.

Quarterly and monthly time series frequently follow seasonal patterns. This can be exploited in order to improve the efficiency of the forecasts. The ARIMA model can be extended to analyze these seasonal variations.

A time series is governed by a first-order seasonal autoregressive process if the current value of the series, \( z_t \), can be expressed as a linear function of the value of the series attained one season ago, \( z_{t-s} \), and a random shock, \( a_t \). This model can be noted as an SAR(1) model and written as
\[ Z_t - \Phi_1 Z_{t-s} = a_t, \quad (39) \]

where \( \Phi_1 \) is the seasonal autoregressive parameter and \( s \) equals one. The model may be extended to include more seasonal autoregressive parameters of order \( P \). For example, a seasonal autoregressive model of order two can be written as

\[ \Phi(B^s) W_t = a_t, \quad (40) \]

where

\[ \Phi(B^s) = 1 - \Phi_1 B^s - \Phi_2 B^{2s}, \quad (41) \]

\[ W_t = \nabla^D_s \nabla^d Z_t, \quad (42) \]

and \( \nabla^D_s \) and \( \nabla^d \) are respectively the seasonal and consecutive difference operators used to induce stationarity in the series \( Z_t \).

The autocorrelation function of a seasonal autoregressive model is similar in general characteristics to the regular autoregressive model except that the values of the autocorrelations appear at multiples of the span.

A stationary time series is said to be governed by a first-order seasonal moving average process if the current value of the series \( Z_t \) can be represented by a current shock, \( a_t \), and a shock occurring exactly \( s \) observations earlier, \( a_{t-s} \), where \( s \) equals the span of the seasonal model. This model may be written as
\[ z_t = a_t - \omega_1 a_{t-s} \] (43)

where \( \omega_1 \) is the seasonal moving average parameter. The model may be expanded to a seasonal moving average model of order \( Q \). In general, a seasonal moving average model of order two can be expressed as

\[ W_t = \omega (B^S)a_t, \] (44)

where

\[ \omega (B^S) = 1 - \omega_1 B^S - \omega_2 B^{2S}, \] (45)

\[ W_t = \nabla^D_s \nabla^d z_t, \] (46)

and \( \nabla^D_s \) and \( \nabla^d \), as in the case of the seasonal autoregressive process, are the difference operators representing the degree of seasonal and consecutive differencing, respectively, needed to make the series \( Z_t \) stationary.

The autocorrelation function of a seasonal moving average model behaves similarly to autocorrelation functions of the regular moving average model except that the values of the autocorrelations appear at lags which are multiples of the span. Higher-order seasonal moving average models will have the same number of nonzero autocorrelations as the order of the process, and the autocorrelations will also occur at multiples of the span. The seasonal autoregressive and seasonal moving average processes can be combined into a
single class of models. A series with a mixed seasonal process can be expressed as

$$\Phi(B^S)w_t = \Theta(B^S)a_t,$$  \hspace{1cm} (47)

where

$$\Phi(B^S) = 1 - \Phi_1 B^S - \Phi_2 B^{2S} - \ldots - \Phi_p B^{pS},$$  \hspace{1cm} (48)

$$\Theta(B^S) = 1 - \Theta_1 B^S - \Theta_2 B^{2S} - \ldots - \Theta_q B^{qS},$$  \hspace{1cm} (49)

$$w_t = D^d Z_t,$$  \hspace{1cm} (50)

$$B = \text{back shift operator}$$

$$B^2 z_t = z_{t-1}, \ B^2 z_t = z_{t-2}, \ldots$$

The $D^d$ or $D^d$ are respectively the seasonal and consecutive difference operators used to induce stationarity in the series $Z_t$. The general mixed seasonal models can be denoted as ARIMA $(P,D,Q)s$ where

$P = \text{order of the seasonal autoregressive process},$

$D = \text{number of seasonal differences},$

$Q = \text{order of the seasonal moving average process},$ and

$s = \text{the span of the seasonality}.$

All models discussed to this point can be combined into a single general class of time series models which, for many time series, when properly analyzed, can yield excellent fits, and generate accurate forecasts. This broad class is called multiplicative ARIMA models and can be expressed as
\( \phi(B) \) \( \Pi(B^s)w_t = \phi(B) \theta(B^s)a_t, \) 

(51)

where

\( \phi(B) = 1 - \phi_1B - \phi_2B^2 - \ldots - \phi_pB^p, \)

(52)

\( \Pi(B^s) = 1 - \Pi_1B^s - \Pi_2B^{2s} - \ldots - \Pi_pB^{ps}, \)

(53)

\( \theta(B) = 1 - \theta_1B - \theta_2B^2 - \ldots - \theta_qB^q, \)

(54)

\( \Theta(B^s) = 1 - \Theta_1B^s - \Theta_2B^{2s} - \ldots - \Theta_qB^{qs}, \)

(55)

\[ w_t = \nabla^D_s \nabla^dZ_t . \]

We can summarize equation (51) as ARIMA \((p,d,q) \times (P,D,Q)s.\)

The autocorrelation functions (acfs) are used to determine the appropriate model. An AR(1) process exhibits a stationary series that has an acf that decays to zero in an exponential fashion or oscillating in sign. The autocorrelations of an AR(1) process with positive \( \phi_1 \) decay exponentially to zero, while for \( \phi_1 \) negative their decay oscillates in sign.

Autocorrelations cannot always solely be depended on to distinguish between certain forms of AR(1) or AR(2) processes. Partial autocorrelations (pacfs) are sometimes needed in conjunction with autocorrelations to identify appropriate models. Pacfs can be explained in the context of specific ARIMA models. In constructing autoregressive models we may want to include an additional lagged \( Z_t \) in the
model to represent the data more adequately. Suppose that after fitting an AR(k-1) model we want to see if the data should not be represented by an AR(k) model. We therefore include an additional lagged variable in the model, namely $Z_{t-k}$. If the value of the coefficient $|\phi_k|$ is "large" we should include the $Z_{t-k}$; otherwise we can omit the variable $Z_{t-k}$ and assume that the AR(k-1) representation of the process is adequate. This coefficient $\phi_k$ measures the "excess" correlation not accounted for by the AR model of order (k-1); that is, it measures the "partial" effect the terms $Z_{t-1}, Z_{t-2}, \ldots, Z_{t-(k-1)}$. The highest-order autoregressive coefficient in the model, in this case $\phi_k$, is defined as the partial autocorrelation at lag $k$, and is denoted by $\phi_{kk}$. The plot of $\phi_{kk}$ for different values of $k$, against $k$, is called the partial autocorrelation function, or pacf.

For an MA(q) process, the acf has a cutoff after lag q. This contrasts sharply with a gradual decline in values of the autocorrelations. The first autocorrelation will be of opposite algebraic sign as the value of the moving average parameter, $\Theta_1$. In examining an MA(2) process, different values for $\Theta_1$ and $\Theta_2$ produce different patterns in the acfs, but the cutoff point is always equal to the order of the process. The highest-order autocorrelation
always has the opposite sign of the highest-order MA parameter.

When a mixed model is examined, ARMA(1,1), the acf decays from the starting autocorrelation value. The sign of the first autocorrelation depends on the difference $(\phi_1 - \theta_1)$. It is difficult to distinguish some ARMA (1,1) acfs from AR(1) acfs without using partial autocorrelation functions.

Rewriting an ARMA (p,q) model in terms of an autoregressive process, one would obtain a pure AR process of infinite order, if q ≠ 0. If an ARMA (1,1) is stationary and invertible, the pacf will gradually die out. This pattern is generally true of all mixed processes. The acf of an MA process behaves like the pacf of an AR process.

The pacf of seasonal ARIMA models behave in the same way as the pacf of a nonseasonal model except that the values of the partial autocorrelations appear at multiples of the span. The general characteristics of the acfs and pacfs of multiplicative seasonal models are harder to understand.

In practice, the autocorrelation of the underlying stochastic process, the population autocorrelations, are not known. Sample autocorrelations and partial autocorrelations are only estimates and are subject to sampling error. An estimate of the population autocorrelation, $\rho_k$, can be
calculated by using the formula for the sample autocorrelation, \( r_k \),

\[
r_k = \frac{C_k}{C_0},
\]

where \( C_k \) is defined as

\[
C_k = (1/n) \sum_{t=1}^{n-k} z_t z_{t+k}, \quad k \geq 0,
\]

and is the estimate of the autocovariance \( \lambda_k \). Again, \( z_t \) represents deviations from the mean of the stationary data and \( n \) is the number of observations available after suitable differencing has been made. To use this identification method, it is necessary to know when \( \rho_k \) is effectively zero. Therefore, one must use the standard error of the sample autocorrelations. For lags greater than some value \( q \) beyond which the theoretical autocorrelation functions are said to have died out, Bartlett (1946, 1966) showed that an approximate estimate of the autocorrelation variance is given by

\[
\text{Var}(r_k) \approx \frac{1}{n} [1 + 2 (\rho_1^2 + \rho_2^2 + \ldots + \rho_q^2)], \quad k > q
\]

where \( n \) is defined as the number of pairs of observations.

In practice, the population autocorrelations \( \rho_k \) of a particular ARIMA model are replaced with the estimated autocorrelations \( r_k \) to obtain an estimate of the approximate variance of \( r_k \) as
\[ \text{Var}(r_k) = \frac{1}{n} (1 + 2 \sum_{i=1}^{q} r_i^2 ), \text{ k}>q. \]  

(59)

The square root of (59) is called the large sample error, SE(r_k).

A series of autocorrelations may be examined as a group for evidence of model inadequacies. Box and Pierce (1970) showed that for a purely random process, (all k=0), the statistic

\[ Q(K) = n(n + 2) \sum_{k=1}^{K} \frac{1}{n-k} r_k^2 \]

(60)

is distributed approximately as a \( \chi^2 \) (chi-square) distribution with K degrees of freedom, and K is the number of autocorrelations used in the summation. The test is often referred to as the Portmanteau test. If the computed Q statistic is less than the table value of the \( \chi^2 \) statistic with K degrees of freedom, given a prespecified significance level, the group of autocorrelations used to calculate the test can be assumed to be not different from zero. This indicates that the data generating autocorrelations are random. If the Q statistic is larger than the \( \chi^2 \) value an existence of some pattern is indicated.

To evaluate when population autocorrelations can be considered to be zero, the standard error of \( \hat{\phi}_{kk} \) needs to be evaluated. Quenouille (1949) showed that the variance of
the estimate of the partial autocorrelations, \( \hat{\phi}_{kk} \), is approximately equal to

\[
\text{Var}(\hat{\phi}_{kk}) \approx \frac{1}{n}, \quad k>p, \tag{61}
\]

where \( n \) equals the number of observations after suitable differencing, and \( p \) represents the first \( p \) partial autocorrelations that are assumed to be nonzero. Sample size is important to this calculation. Equation (61) provides a method, after observing \( p \) nonzero partial autocorrelations, to evaluate if all other \( \hat{\phi}_{kk} \)s are different from zero.

The next step after identifying a particular ARIMA model from the general class of multiplicative models as illustrated by equation (51) is to estimate the vectors of parameters

\[
\varphi = (\varphi_1, \varphi_2, \ldots, \varphi_p)', \quad \phi = (\phi_1, \phi_2, \ldots, \phi_p)', \quad \theta = (\theta_1, \theta_2, \ldots, \theta_q)', \quad \gamma = (\gamma_1, \gamma_2, \ldots, \gamma_q)', \quad \text{and} \quad \psi = (\psi_1, \psi_2, \ldots, \psi_q)'.
\]

There are two methods available for estimating these parameters: (1) the least squares method and (2) maximum likelihood. Vandaele (1983) recommends that the least squares method be used because of its computational efficiency.

Once the model has been identified and its parameters estimated, it is necessary to verify whether the model can be improved. Four groups of tests or diagnostic checks can
evaluate the model adequacy. The four groups of diagnostic checks are stationarity analysis, residual analysis, fitting extra parameters, and omitting parameters. The cumulative results of the various checks establish model adequacy.

An evaluation of the properties of the errors of an ARIMA process (known as residual analysis) indicate model adequacy. If a model adequately depicts the ARIMA process governing the series, the errors of the model should be white noise.

If the residuals are white noise, their acf should have no spikes and the sample autocorrelations should all be small. Based on equation (58), the estimate of the approximate large-sample standard error for individual autocorrelations under the assumption that the errors follow a white noise process is $1/\sqrt{n}$. Thus, residual autocorrelations, $r_k$, which lie outside the range, say, $\pm 1.96/\sqrt{n}$, are significantly different from zero for 95 percent confidence limits. However, values of $r_k$ at low lags which are inside the approximate confidence limits may still be significantly different from zero, and may need further analysis.

A second approach for analyzing the residual autocorrelations is to rely on the Ljung-Box $Q$ statistic, also known as the Portmanteau test on the residual. If the fitted model is appropriate, $Q$ is approximately distributed
as a $\chi^2$ variable with $K-p-q-P-Q$ degrees of freedom. The symbol $K$ in $Q(K)$ refers to the numbers of parameters in the ARIMA model. The symbols $p$, $q$, $P$, and $Q$ are the numbers of parameters in the ARIMA model.

A third approach for determining if the errors are white noise is to evaluate the autocorrelations of the first differenced residuals. If the errors form a white noise process, then the first difference should follow an MA(1) process, with the moving average parameter $\theta_1$ equal to one.

A check of model adequacy is to evaluate the current model for redundant parameters. Redundant parameters are identified by the estimate of the large-sample standard error of the coefficient estimates (SE) and the estimate of the large-sample correlations between these coefficient estimates.

The SE can be used to evaluate the statistical significance of a single coefficient. As a rule of thumb, a coefficient is said to be significantly different from zero, if the absolute value of the point estimate is at least twice the value of the standard error. An insignificant parameter indicates that the model is overspecified. If the insignificant parameter estimate is of the highest order, remove it. If the insignificant estimate is not of the highest order, then large-sample correlation between
estimates is examined to determine which to delete. If the insignificant estimate is highly correlated with the highest-order estimate, evaluate the model without the highest-order parameter estimate. If there is no such correlation, the model is reestimated suppressing the insignificant parameter estimate. To assess whether a tentatively identified model contains the appropriate number of parameters an additional parameter is included to see if the original model is improved.

After identifying an adequate model, it can be used to generate forecasts for future periods. Since a variable to be forecasted, $Z_{n+h}$, is a random variable where the current period is $n$ and the number of time periods ahead to be forecasted is $h$, it can only be fully described in terms of its forecast distribution. The forecast distribution is conditional on past and present data as well as on the specification of the ARIMA model. The forecast distribution of $Z_{n+h}$ can be denoted $f_{n,h}(Z)$. There is no other forecast which will produce errors whose squares have smaller expected values than the mean of the forecast distribution.

The mean of the forecast distribution $E(Z_{n+h})$, can be estimated as follows. Let $Z_t$ be a stationary and invertible ARMA($p,q$) process. For the time period $t = n + h$, this process can be expressed as
\[ Z_{n+h} = \phi_1 Z_{n+h-1} + \cdots + \phi_p Z_{n+h-p} + a_{n+h} - \theta_1 a_{n+h-1} - \cdots - \theta_q a_{n+h-q}. \] (62)

The expected value of \( Z_{n+h} \) in (62), calculated using information up to period \( n \), is obtained as follows:

1. replace the current and past errors \( a_{n+j}, j \leq 0 \), with actual residuals;
2. replace each future error \( a_{n+j}, 0 < j < h \), with its expectation, which, since \( a_{n+j} \) is white noise, is just 0;
3. replace current and past observations \( Z_{n+j}, j \leq 0 \), with the actual observed values;
4. replace each future value of \( Z_{n+j}, 0 < j < h \), with the appropriate forecast \( Z_n(j) \); therefore we should first forecast \( Z_{n+1}, Z_{n+2}, \ldots, Z_{n+h-1} \) in order to forecast \( Z_{n+h} \).

In addition, all parameters in the model must be replaced with estimates. For a first-order autoregressive model with mean \( E Z_t = \mu \),

\[ Z_t - \mu = \phi_1 (Z_{t-1} - \mu) + a_t \]

or

\[ Z_t = (1 - \phi_1) + \phi_1 Z_{t-1} + a_t. \] (63)

The mean of the \( h \) periods ahead forecast distribution is given by

\[ Z_n(h) = (1 - \phi_1) \mu + Z_n(h-1), \ h > 1. \] (64)
It is important to note that all future error terms have mean zero and therefore $E(a_{n+h}) = 0$, $h > 0$.

The forecast for an MA(1) process can be generated as follows. For the period $n+1$ an MA(1) process can be expressed as

$$Z_{n+1} = \mu + a_{n+1} - \theta a_n.$$  \hfill (65)

$Z_t$ represents the actual data. At time $n$, the residual $a_{n+1}$ has not been observed and is therefore replaced by its expected value of zero. The value for the residual $a_n$ can be calculated as

$$a_n = Z_n - \mu - \theta a_{n-1}.$$  \hfill (66)

The one period ahead forecast for an MA(1) model is

$$Z_n(1) = \mu.$$  \hfill (67)

For two periods ahead the forecast is simply

$$Z_n(2) = \mu.$$  \hfill (68)

The memory function of an MA(q) model shows that the effect of a shock in the model only lasts for $q$ future periods. Therefore, only the first $q$ values of the forecast profile of an MA(q) process will be determined by past disturbances. All other future values will be equal to the mean of the process.

With an ARMA(1,1) process with mean $\mu$, only the one period ahead forecast is directly influenced by past residuals. The difference from the AR(1) model is that the
current error $a_n$ is used to determine the one step ahead forecast $Z_n(1)$.

In addition to a point forecast, it helps to quantify the uncertainty in the point forecast. This can be done using estimates of forecast error to set confidence limits. To calculate standard errors of the forecast errors, express the ARIMA process in error-shock form. By successive substitution for $Z_{t-1}, Z_{t-2}, \ldots$, the model is written in terms of current and past errors only

$$Z_t = a_t + \psi_1 a_{t-1} + \psi_2 a_{t-2} + \ldots$$  \hspace{1cm} (69)

The values of the parameters $(\psi_1, \psi_2, \ldots)$ depend upon the ARIMA model used and are called the error learning coefficients. For an AR(1) process $\psi_k$ is equal to $\phi^k$. The forecast $Z_n(h)$ can also be expressed using equation (69) in terms of current and past errors

$$Z_n(h) = \psi_h a_n + \psi_{h+1} a_{n-1} + \ldots$$  \hspace{1cm} (70)

As a result the $h$ step ahead forecast error

$$a_n(h) = Z_{n+h} - Z_n(h),$$  \hspace{1cm} (71)

can be written as

$$a_n(h) = a_{n+h} + \psi_1 a_{n+h-1} + \ldots + \psi_{h-1} a_{n+1}.$$  \hspace{1cm} (72)

Since the errors $a_t$ are independent, $a_n(h)$ is an MA($h$-1) process regardless of the form of the ARIMA process being analyzed. The one step ahead errors form a white noise
series. The forecast errors \( a_n(h) \) have mean zero and variance equal to

\[
\text{Var}[a_n(h)] = \text{E}[a_n^2(h)] = \sigma^2 a \sum_{j=0}^{h-1} \psi_j^2, \text{ with } \psi_0 = 1. \tag{73}
\]

As the forecast horizon is lengthened, the error variances are monotonically nondecreasing

\[
\text{Var}[a_n(h)] - \text{Var}[a_n(h-1)] = \sigma^2 a \psi_{h-1}^2 \geq 0. \tag{74}
\]

Based on equation (74), one can never know more as one looks further into the future.

If the error terms \( a_t \) are assumed to be normally distributed then the whole forecast distribution \( f_{n,h}(z) \) can be characterized. The forecast distribution of \( Z_{n+h} f_{n,h}(z) \) will be distributed as a normal random variable with mean \( Z_n(h) \) and variance \( \text{Var}[a_n(h)] \). This error distribution permits probability statements about the future. The 95 percent large-sample confidence interval for \( Z_{n+h} \) is

\[
Z_{n+h} \pm 1.96 \text{SE}[a_n(h)].
\]

In the calculation of the confidence limits, the error learning coefficients are replaced with their estimates and \( \sigma^2 a \) with its estimates. A confidence interval for the mean of \( Z_{n+1} \), based on current and past values of \( Z_t \) may also be obtained.

The ARIMA representatives of an MA(1) model

\[
Z_t = \mu + a_t - \theta a_{t-1}, \tag{75}
\]
is already in the error shock form. Therefore, we have

\[ \psi_1 = -\Theta_1 \text{ and } \psi_j = 0, \ j > 1. \]

The variance of the h step ahead forecast errors are therefore given by

\[ \text{Var}[a_n(1)] = \sigma^2 a, \]

\[ \text{Var}[a_n(h)] = \sigma^2 a (1 + \Theta_1^2), \ h > 1. \] (76)

The variance of forecast errors for two or more steps ahead equals the variance of MA(1) data and this variance remains constant beyond the two steps ahead forecasts.

The fact that the forecast error variance approaches a constant and is equal to the variance of the model as h becomes large is a result that extends to all seasonal and nonseasonal stationary models. For a stationary model this variance increases without limit, and indicates that little is known about distant future values of nonstationary series.

Summary

Using the ARIMA models and six years of real estate office property transaction data, the hypothesis that the Dallas-Fort Worth real estate office properties market is weak-form efficient is tested. The real estate market is divided into submarkets and the submarket with the most observations is tested. Scaled prices and returns are calculated and ARIMA forecasting models are fitted to the
data. The accuracy of the forecasting models is tested by reestimating over the first two-thirds of the data points in each series and then forecasting the remaining observation values. The results are compared with a naive no-change and mean-value forecasts. If the time series can be modeled by a moving average process of order one, an autoregressive process of order one, or an autoregressive process of order two, market efficiency cannot be rejected in an economic sense.
CHAPTER III BIBLIOGRAPHY


CHAPTER IV

RESULTS

Introduction

This chapter presents the results of the investigation of the weak-form efficiency of the Dallas-Fort Worth office properties real estate market. If a real estate market is weak-form efficient, information contained in past prices is fully incorporated in current prices and therefore cannot be used in attempting to earn an abnormal return.

Scaled prices for each transaction were estimated as sales price per square foot of net rentable area, gross income multiplier, net income multiplier, and return series for each of these measures. Autocorrelation functions are estimated over the monthly time series to test for weak-form efficiency. If significant autocorrelations are identified over the time series, the potential for forecasting future sales prices is examined by using the Box-Jenkins autoregressive-integrative-moving average (ARIMA) forecasting model. The four steps used for developing ARIMA models are: (1) identification of the preliminary specifications of the model; (2) estimation of
the model parameters; (3) diagnostic checking of model adequacy; and (4) forecasting future observations.

To see whether the estimated models can be used to accurately forecast future prices, the models are re-estimated over the first two-thirds of the data in each series and the remaining one-third of the data is then forecasted. The results of the forecast are compared with the predictive accuracy of a naive no-change and a mean value forecast.

Serial autocorrelation tests of price and return series indicate a lack of significant autocorrelation. ARIMA models are used to predict future sales prices but are unable to outperform a simple mean series forecast. These results suggest that this real estate office properties market is weak-form efficient.

Identification

Figure 1 shows a plot of the salesprice per square foot of net rentable area (SPSQFNRA) data over time. The data plots and descriptive statistics (means and standard deviations) over the complete data set and three sub-intervals, presented in Table I, indicate that there is an upward trend in the data. The ACF and PACF patterns are shown in Figure 2. The ACF, which represents the correlation of the
Fig. 1—Plot of sales price per square foot of net rentable area over time.
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SPSPFNNA = Sales Price per Square Foot of Net Rentable Area.
GIM = Gross Income Multiplier
NIM = Net Income Multiplier
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**PARTIAL AUTOCORRELATION FUNCTION FOR VARIABLE SPSQFNRA**

**PARTIAL AUTOCORRELATIONS**

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Fig. 2—Autocorrelations and partial autocorrelations for sales price per square foot of net rentable area.
time series with its lags, dies out slowly. Such persistently large ACF values and a lag 1 PACF value, which almost equals +1, support the conclusion of nonstationary data. The trend in the data can be removed by taking the first difference (Δ) of the time series. This will cause the ACF to die out rapidly (usually after four or five lags).

Figure 3 shows the time series data plot for the gross income multiplier (GIM) which is sales price divided by annual gross income. The plots of the ACF and PACF in Figure 4 do not seem to indicate any trend. There are no significant autocorrelations over the 24 lag periods. This indicates that trend has been removed. However, the descriptive statistics over the full time series and the three time intervals indicate that a slight trend may exist. Therefore, first differencing of the time series will also be examined.

The means and standard deviations over the time series also indicate that the variance is increasing over time for SPSQFNRA and, also slightly so for the GIM. This problem is corrected as attempts are made to model the data. The return series calculations, R1 for the series SPSQFNRA and R2 for the GIM series, incorporate both a first differencing of the series data and a data transformation by dividing by the previous lag. If the systematic portion of the stochastic component has been identified, the ACF and
Fig. 3—Plot of gross income multiplier data over time.
**Fig. 4**—Autocorrelations and partial autocorrelations for the gross income multiplier.
PACF should not have any significant spikes. In other words, the autocorrelations should not be significantly different from zero (at the 5% level of statistical significance). However, chance may cause one or two significant lags; therefore, another test should be used to see if the data is white noise (consists entirely of random shocks). According to Vandaele (1983), the Q-statistic, a chi-square goodness of fit test for the ACF, can be used to test a number of lags simultaneously. Twelve and twenty-four lags are examined so that any seasonality in the data will not be ignored. If the estimated ACF is a white noise process, the Q-statistic will not be significant.

The ACF and PACF illustrated in Figure 5 for the R1 time series indicate that the series is white noise. However, the Q-statistics for twelve lags (23.43) indicates that some serial correlation may exist. The presence of this serial correlation indicates that it may be possible to construct a forecasting model to predict future values in the market. The Q-statistic for twenty-four lags (30.08) supports the white noise interpretation.

Figure 6 for the R2 (GIM returns) series indicates that this series is white noise. However, the Q-statistics for twelve (28.46) and twenty-four (35.16) lags do not support this inference. Again, the Q-statistics indicate that some serial correlation may exist and that a model can be
**AUTOCORRELATION FUNCTION FOR VARIABLE R1**

**AUTOCORRELATIONS**

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Fig. 5—Autocorrelations and partial autocorrelations of the rate of return based on sales price per square foot of net rentable area over time.
**AUTOCORRELATION FUNCTION FOR VARIABLE R2**

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Fig. 6—Autocorrelations and partial autocorrelations of the rate of return based on the gross income multiplier over time.
constructed. Caution should be exercised because this may be an indication of over differencing.

The SPSQFNRA and GIM time series data may also be adjusted for trend and expanding variance by simply taking the first difference and natural log of each series. Figures 7 and 8 show the ACFs and PACFs for the first differences of the natural logs of SPSQFNRA (ΔLSPSQFNRA) and GIM (ΔLGIM) respectively. The descriptive statistics, ACFs and PACFs indicate that both series are apparently white noise. The Q-statistics for the ΔLSPSQFNRA time series are 7.88 for the twelve month lag and 14.14 for the twenty-four month lag. The ΔLGIM time series had a Q(12) of 28.87 and a Q(24) of 35. The conclusion that the series are white noise is supported for the ΔLSPSQFNRA series. However, the Q-statistics indicate that the LGIM time series can be modeled.

Figure 9, shows the time series data for the net income multiplier (NIM) which is sales price divided by annual net operating income. The NIM is also known as the inverse of the overall rate of return. From a close look at the time series data plot and an examination of the descriptive statistics in Table I, there does not appear to be any trend or expanding variance in the data. Therefore, any attempt to difference the data would cause over differencing and any other data transformations are unnecessary. The ACF
### Autocorrelation Function for Variable SPSQFNRA

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Fig. 7—Autocorrelations and partial autocorrelations of the first difference of the natural log of the sales price per square foot of net rentable area data.
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Fig. 8—Autocorrelations and partial autocorrelations of the first difference of the natural log of the gross income multiplier over time.
NET INCOME MULTIPLIERS OVER TIME

Fig. 9—Plot of net income multiplier data over time
and PACF in Figure 10, as well as the Q-statistics for twelve (20.16) and twenty-four (30.17) lags, support the white noise inference.

Table II provides a summary of the identification phase of Box-Jenkins analysis for the time series. Based on identification phase analysis, models will be attempted for the SPSQFNRA RETURN, Δ LOG SPSQFNRA, GIM, GIM RETURN, Δ LOG GIM, and NIM time series data. However, it appears that GIM RETURN, Δ LOG GIM, and NIM are the only models that are not white noise. Further, based on ACF and PACF analysis, the GIM RETURN and Δ LOG GIM series should model as an MA process with one significant parameter (MA1). The NIM series should model as an ARIMA process.

When real estate transactions take place, there is usually a one- to two-month time period from the sales contract agreement to actual transfer of ownership (closing). Therefore, forecasting models with two or more significant parameters would be required to predict future prices or returns. It is doubtful, therefore, that any of the tentatively identified time series above can carry much information for future sales prices in real estate markets.

Model Estimation

After tentative forecasting models are identified, the model parameters are estimated using the Box-Jenkins ARIMA
### Partial Autocorrelation Function for Variable NIM

**Partial Autocorrelations**

*Two Standard Error Limits*

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Fig. 10—Autocorrelations and partial autocorrelations for the net income multiplier.
<table>
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<tr>
<th>MEASURE</th>
<th>TREND</th>
<th>SEASONAL TREND</th>
<th>VARIANCE</th>
<th>RATE OF RETURN</th>
<th>Δ NATURAL LOG</th>
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<tr>
<td>SIM</td>
<td>Data plot indicates no trend. However, descriptive statistics indicate that a slight trend exists. Differencing is attempted. Try modeling.</td>
<td>No visual evidence.</td>
<td>Descriptive statistics indicate a slight increasing variance. Transformation is attempted.</td>
<td>Visually white noise due to combination of trend removal and transformation. However, significant Q-statistics exist. This may be due to over differencing. However, try modeling.</td>
<td>Visually white noise due to combination of trend removal and transformation. However, significant Q-statistics exist. This may be due to over differencing. Try modeling.</td>
</tr>
</tbody>
</table>

SPSBFRNRA = Sales Price per Square Foot of Net Rentable Area.
SIM = Gross Income Multiplier
NIM = Net Income Multiplier
technique. Table III shows the results of the estimation process. In all six models the estimated MA and AR parameters are significant at the 5% level. To assure that the models indentified describe the systematic component of the error and leave only uncorrelated error accounted for, the residuals must behave like white noise. For every model the residual autocorrelations individually are not significantly different from zero. Further, the Q-statistics on the residuals as a group for each model are not statistically significant. Therefore, the diagnostics indicate that the hypothesis that each model provides an efficient representation of the behavior of the time series cannot be rejected.

All of the forecasted time series, with the exception of the SPSQFNRA RETURN series, modeled either as first order moving average processes (MA1) or as first order mixed processes (ARIMA). An MA1 process only predicts one period ahead with anything beyond that being forecasted as the mean of the series. With an AR1 process the forecasts beyond one period also tend toward the series mean. The models that can predict only one month in advance are of little help to the real estate investor. Price information in the real estate markets generally are available with one-to two-month lags because of the extended closing time for real estate.
### TABLE III
FORECASTING MODELS

<table>
<thead>
<tr>
<th>MEASURE</th>
<th>CONSTANT</th>
<th>Autoregressive Parameter</th>
<th>Moving Average Parameters</th>
<th>$\sigma^2(\hat{\mu})$</th>
<th>$\hat{\mu}(24)$</th>
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<tr>
<td></td>
<td></td>
<td>$\phi_1$</td>
<td>$\Theta_1$</td>
<td>$\Theta_2$</td>
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<tr>
<td>SPSFNRA RETURN</td>
<td>.018167</td>
<td>(.007147)</td>
<td>.253129</td>
<td>(.1155410)</td>
<td>.1623</td>
</tr>
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<td></td>
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<td></td>
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<tr>
<td>△LOG SPSFNRA</td>
<td>.001449</td>
<td>(.000420)</td>
<td>.998440</td>
<td>(.076787)</td>
<td>13.99</td>
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<tr>
<td>SPM</td>
<td>1.00160</td>
<td>(.000381)</td>
<td>.998044</td>
<td>(.049419)</td>
<td>21.55</td>
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<td></td>
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<tr>
<td>SPM RETURN</td>
<td>.010938</td>
<td>(.004018)</td>
<td>.726560</td>
<td>(.082134)</td>
<td>23.94</td>
</tr>
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</tr>
<tr>
<td>△LOG SPM</td>
<td>.001562</td>
<td>(.005272)</td>
<td>.998844</td>
<td>(.047026)</td>
<td>23.94</td>
</tr>
</tbody>
</table>

SPSFNRA = Sales Price per Square Foot of Net Rentable Area.
SPM = Sales Price Multiplier
SFM = Sales Multiplier

*d.f.* = 22
Forecasts

The SPSQFNRA RETURN model had an additional parameter (MA2) that will allow it to forecast two months ahead. This particular model may be useful in forecasting future real estate returns. To see if this model or any of the other models can be used to accurately forecast future real estate returns or prices, the predicative accuracy of these models is compared to naive no-change and mean-value forecasts. Each model is reestimated over the first two-thirds of its data and used to forecast the remaining observations. The prediction results are shown in Table IV.

The mean absolute error (MAE), root mean square error (MSE), and Theil U statistics are three popular measures used to identify the accuracy of forecasts. The lower the calculated statistical value, the better the predictive capability of the model. The MSE is similar to the MAE with the exception that it severly penalizes the large residuals and is therefore preferable when one cannot afford a single severe residual. The advantage in using the Theil U statistic is that it penalizes systematic linear bias.

The prediction results indicate that, in general, the forecasting models were more accurate than a no-change forecast. However, in most cases the forecasting models could not outperform a simple mean series forecast. These results suggest that the serial correlation present in office
<table>
<thead>
<tr>
<th>MEASURE</th>
<th>FORECAST</th>
<th>MEAN ABSOLUTE ERROR</th>
<th>ROOT MEAN SQUARE ERROR</th>
<th>THEIL U</th>
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<tbody>
<tr>
<td>SPSSFNRA RETURN</td>
<td>NO CHANGE</td>
<td>.191</td>
<td>.046</td>
<td>2.044</td>
</tr>
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<td></td>
<td>SERIES MEAN</td>
<td>.087</td>
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<td>MODEL</td>
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<td>.015</td>
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<tr>
<td>Δ LOG SPSSFNRA</td>
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<td>.120</td>
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<td>GIM</td>
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<td>.702</td>
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<tr>
<td>GIM RETURN</td>
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<td>.046</td>
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<tr>
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SPSSFNRA = Sales Price per Square Foot of Net Rentable Area.
SPSSFNRA RETURN = (SPSSFNRA<sub>t</sub> - SPSSFNRA<sub>t-1</sub>) / SPSSFNRA<sub>t-1</sub>
GIM = Gross Income Multiplier
NIM = Net Income Multiplier
property prices in the Dallas-Fort Worth market is not enough to provide an investor with the ability to consistently earn abnormal returns by using past price information.
CHAPTER IV BIBLIOGRAPHY


CHAPTER V

SUMMARY

The theory of efficient markets has been tested extensively in the finance literature. Financial markets appear to be both weak- and semistrong-form efficient. However, strong-form efficiency has not been supported in the literature.

Only recently has the efficiency of real estate markets been empirically tested. Traditionally, real estate markets were assumed to be inefficient (Gau, 1985). However, empirical studies by Gau (1984, 1985), Guntermann (1985), and Dale-Johnson, et. al. (1985) support the notion of real estate market efficiency. This suggests that there may be a shift in the real estate market paradigm. It also suggests that decision rules and trading strategies may be developed that do not require alteration with each transaction.

Several limitations can be found in the empirical examinations of real estate market efficiency. The quantity and quality of the data analyzed in previous studies were severely limited. This study analyzed weak-form efficiency of a real estate market and property type with a larger and more specific data set. The Box Jenkins ARIMA models were
used to model serial autocorrelation between monthly office property sales prices and returns.

The results of this study indicate that the Dallas-Fort Worth office properties market is weak-form efficient. A model could not be developed to predict the future sales prices or returns with two months or more lead and that could outperform a naive no-change and series mean forecast. The results support a previous empirical examination of weak-form real estate market efficiency conducted by Gau in 1984. It appears, therefore, that real estate markets may be efficient in the weak-form and the traditional paradigm of real estate market inefficiency may not hold.

Limitations

This study has several limitations. First, the data was gathered from a specific real estate market place and property type. Generalization of the findings to other markets and property types may not be appropriate. Second, the properties located in the market studied were assumed to be homogeneous. Any office property sale within the market over the studied time period was assumed to be the same as all other properties in that market. Third, the data was gathered over a specific time period. Generalizations to other time periods may not be appropriate.
Implications for Future Research

Further research is needed in several areas. Additional weak-form market efficiency studies should be conducted in other real estate markets around the world and on other property types before generalizations are justified. The other types of market efficiency, semistrong-form and strong-form, need to be tested. The development of new and better data sets along with a national real estate market index might allow modern portfolio theory to be applied to real estate.

Conclusions

The general conclusion of this research is that real estate markets, contrary to traditional theory, appear to be weak-form efficient. Therefore, traditional beliefs that real estate markets are inefficient should be reconsidered. It appears that investors cannot earn abnormal returns in this real estate market by relying totally on publicly available past price information.
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