DECISION CRITERIA FOR GIFTS UNDER
THE 1976 TAX REFORM ACT

DISSERTATION

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The 1976 Tax Reform Act made many changes in the taxation of estate and gift transfers. Previously gifts and estates were taxed separately and the gift tax rate was 75 percent of the estate tax rate; and there was a $30,000 exemption for gifts and a $60,000 exemption for estate transfers. Under the new law the exemptions were repealed and replaced with a unified credit against the tax; and the tax on estate and gift transfers was combined into one increasing rate schedule. Under the prior law, deathbed gifts were advantageous because the gift tax paid on the transfer was excluded from the taxable estate but was allowed as a credit against the estate tax since gifts within three years of the date of death were included in the gross estate unless the estate could demonstrate that the gifts were not made in contemplation of death. Under the new law, gift taxes paid on transfers which occur within three years of the date of death are included in the taxable estate.

Previously gifts were almost always an advantageous estate planning device. Under the new law, the advantage was substantially reduced. The objective of this dissertation is to determine where gifts are advantageous under the new law.

Chapter I states the problem and objectives and outlines the methodology to be followed. Chapter II develops a model which yields as a product the donee's present value of: (1) property net of taxes if an
inter vivos transfer is made and (2) property net of taxes if the transfer is made by death. Only the following cases were considered transfer to spouses and children of the following types of assets: cash, marketable securities, and raw land. The model incorporated for each possible future year (1) the gift tax, (2) the estate tax, (3) the donor's donee's, and estate's marginal income tax, (4) the donor's basis, (5) market value of the assets, (6) income from asset, (7) the donee's after-tax imputed interest rate, (8) the donor's after-tax rate of return on cash, and (9) the donor's date of death. Initial models were expanded to include a probability density function for the date of death, and a probability density function for all other variables except the date of death. The model's equations were incorporated into a computer program. The model only considered the transfer of $1 of assets for marketable securities.

Chapter III presented some generalizations which can be drawn from the model and some examples of decision-making from the model.

Chapter IV presented conclusions and recommendations for future research. At the start of this research, it was assumed incorrectly that gifts would not be advantageous in most cases. Gifts will be advantageous in most cases and disadvantageous when special basic rules for property acquired before January 1, 1977, are applicable or the donor's marginal income tax rate and estate tax rate are low compared to the donee's marginal income tax rate. The optimal gift year will usually be three years prior to death or an immediate gift. Deathbed gifts can be advantageous since the donee may receive a higher basis for determining capital gains or losses when property is transferred by gift rather than through the
estate. Deathbed sales may now be advantageous since the income tax liability will reduce the gross estate. Although it is not possible to give a precise estimate without detailed computations, many gift transfers will give a donee a present value which is 40 percent to 130 percent greater than the present value for an estate transfer.
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CHAPTER I

INTRODUCTION

Statement of the Problem

An individual in our society, unable to defeat death, nonetheless tries to defeat estate taxes through the use of inter vivos gifts. The 1976 Tax Reform Act substantially reduced the benefits of inter vivos gifts versus transfers at death. However, the act does not result in the complete removal of the advantages and, as a result, the decision criterion becomes much more complex and subject to risk.

Section 2001 of the Internal Revenue Code of 1954 adopts a unified rate schedule for estate and gift taxes.\(^1\) Under Section 2502, prior law, the gift tax rate was 75 percent of the estate tax rate. The prior law allowed an exemption of $30,000 towards the gift tax under Section 2521 and $60,000 towards the estate tax under Section 2052. These have been replaced by a unified credit under Section 2010 which is first offset against gift taxes, with the excess being applied against the estate tax. The $3,000 per year per recipient exclusion for the gift tax under Section 2503(b) is retained. The marital deduction under Section 2056 for estate taxes is changed from one half the taxable estate to one half the taxable estate or $250,000, whichever is larger. Under Section 2035 of the prior law, gifts given within three years of death were presumed to be

\(^1\)Throughout this dissertation, the term "Section" will be used in lieu of Internal Revenue Code of 1954 Section.
in contemplation of death—unless the estate could prove otherwise—and were included in the gross estate. Section 2035 of the new law requires all gifts within three years of death (except those falling under the $3,000 exclusion) to be included in the gross estate. In addition, gift taxes paid on gifts made during the three years prior to the decedent's death must also be included in the gross estate. The prior law had permitted the donor, just before imminent death, to make gifts, pay the gift tax, and then use the gift tax paid on those gifts given in contemplation of death as credits against the estate tax without having to include the gift tax paid in the gross estate.

The new law under Section 2032A contains favorable provisions for the estate tax valuation of real estate used in farming and, in certain restricted cases, to real estate used in other closely held businesses. In general, where the special valuation rules apply, the estate tax value is based on "current use" as opposed to "best use," a valuation procedure intended to exclude the speculative value of such realty from the gross estate. Furthermore, under Section 6166, closely held businesses and farms included in the decedent's estate often result in the privilege of paying the estate tax over a ten-year period. Finally, under Section 6166A, in a few instances—where the estate includes farms or other closely held businesses—the executor is entitled to defer all taxes for five years and pay the tax over the next ten-year period, with a 4 percent interest rate applicable to a portion of the tax. For these reasons, farms and interests in closely held businesses are not usually suitable gift property.

Under Section 1014 of the prior law, inherited property had a basis equal to the estate tax valuation for determining gain or loss under the
income tax law. For properties acquired by the decedent after 1976, the Section 1014, as amended, provides that heirs will generally take the same basis as that of the decedent except for proration of death taxes to appreciated properties. Where the decedent acquired properties prior to 1977, heirs will receive a "fresh start" basis loosely tied to the fair market value of such property on December 31, 1976, with the exception of marketable securities. The rules for determining the fair basis under the "fresh start" rules are extremely complicated and presently very uncertain, especially for properties that are subject to depreciation. Because of the present uncertainties, it is very difficult to determine the advisability of making gifts of depreciable property acquired by the prospective donor before 1977.

Under Section 1015 of the prior law, the donee's basis for determining gain was the donor's basis plus any gift tax paid, not to exceed the property's fair market value at the date of gift. Under Section 1015 as amended, the donee's basis is the donor's basis plus a fraction of the gift taxes paid on the transfer. For properties acquired by the donor after December 31, 1976, the fraction is the fair market value minus the donor's basis divided by the fair market value. The higher the basis, the lower the potential capital gain.

As can be seen from the preceding brief discussion of the estate and gift tax provisions of the 1976 Tax Reform Act, the decision of whether or not to make an inter vivos gift has been considerably complicated. Previously, one had a $30,000 specific exemption, lower rates, and the benefit of splitting the estate between two increasing rate schedules. Offsetting these advantages was the fact that basis was
increased to the estate valuation if held until death. Of course, one of the most important advantages of inter vivos gifts remains unchanged—the benefit of reduced income taxes.

Variables which influence the decision with respect to an inter vivos gift are: (1) the gift tax, (2) the estate tax, (3) the donor's taxable income until the date of death, (4) the donee's taxable income until the date of death of the donor, (5) the expected date of sale of the gift, (6) the tax basis of the property if transferred by gift, (7) the tax basis of the property if transferred by death, (8) the expected market value of the property in the future, (9) mortgage payments in the case of raw land, (10) the expected taxable income produced by the property, (11) the actual date of death, and (12) the effective interest rate which prevails for the taxpayer(s) over the time span under consideration. The problem then is to evaluate these factors to determine if and when an inter vivos transfer is advantageous.

Objectives and Limitations

The objective of this dissertation is to design a model which yields as a product the donee's present value of: (1) property net of taxes if an inter vivos transfer is made and (2) property net of taxes if the transfer is made by death. The present values will be expressed as a probability density function which incorporates the twelve variables, some of which will also be expressed as probability density functions.

In order to limit the scope of this dissertation, only the following cases were considered: transfer to spouses and children of the following types of assets—cash, marketable securities, and raw land. The effect
of delayed payment of estate taxes will be examined in the case of a closely held corporation.

Any model uses certain parameters and makes certain assumptions. Although this research has considered the choice of parameter estimation methods, no significant research effort has been directed in this area. For example, the objective of this research was not to make a forecast about future interest rates, rates of inflation, market values of various kinds of property, nor the correlation between these variables. The models were constructed to accept as input estimates of future values, such as fair market values, dividends, and marginal tax rates. Present values were based on the given estimates. The models did not generate as an output optimal gift size. The models examined an incremental transfer of $1 of assets. The models, once developed and expressed in a computer program, permit one to ascertain the probability that a gift is advantageous and the most appropriate time to make the gift.

The models permit the comparison of gift situations to measure the effect of the 1976 Tax Reform Act and compare it with the avowed congressional intent to substantially eliminate the value of inter vivos gifts. This analysis was restricted to a limited number of examples, and no attempt was made to determine aggregate dollar amounts for the economy as a whole. However, the findings were used to evaluate federal tax policies relating to wealth transfers.

Method of Procedure

The methodology used in the dissertation consists of developing a model for each of the cases enumerated in the objectives. Once models had been developed for the cases under consideration, a computer program was written to correspond to the developed models. Using this program, the parameters were varied to show the effect of changes in the parameters on the output functions. The effect of simplifying assumptions in the general model was examined. The results of this analysis and appropriate conclusions is presented. A portion of the analysis consists of comparing the results under the new and old laws.

The dissertation consists of four chapters. Chapter I defines the objectives and expected contributions of the dissertation. Chapter II develops a model for each combination of donee-type of property listed in the objectives. Chapter III consists of three main parts: (1) a summary of general decision rules presented in Chapter II, along with some simple decision rules which are obtainable under simplifying assumptions, (2) some rather common estate tax situations are analyzed and the pertinent computer output are presented, and (3) the House Committee Report on the Tax Reform Act of 1976, pertaining to estate and gift taxes, is examined to see if the avowed congressional intent was met, and some illustrative examples are given to demonstrate an apparent failure of congressional intent. Chapter IV summarizes the conclusion which can be drawn from the analysis done in Chapter III and suggest possible extensions of this study.
CHAPTER II

DERIVATION OF PRESENT VALUE
COMPARISON BASIS

Introduction

Chapter Objectives

The primary objective of this chapter is to define a set of present value equations in the most general form, which will permit a comparison of the present value to the donee of an inter vivos gift and an estate transfer. Only transfers of cash, marketable securities, and raw land to children and spouses will be considered. In addition, the effect of delayed payment of estate taxes in the case of stock in a closely held corporation will be considered. In the case of cash and marketable securities, only the transfer of a small incremental amount will be considered. In the case of raw land, the assumption will be made that cash is available to pay gift and estate taxes and that the entire property will be transferred.

The secondary objective of this chapter will be to show the effect of variability in the model's parameters. This analysis will center in the variability of the date of death, but will also consider the effect of alternative future variable projections. Due to the fact that many of the variables are highly correlated, each projection will be assigned a probability and the expectation of present value will be based on given projections and their assigned probability.
Selection of Imputed Interest Rates

In order to compare cash flows occurring at different points in time, an imputed after-tax imputed interest rate must be chosen. This imputed interest rate will be the donee's imputed interest rate. The possibility that the donee's marginal utility for cash may vary through time and for different gross amounts will be ignored. The donee will be assumed to desire to maximize the present value of his after-tax cash flows.

A good candidate for the donee's imputed interest rate would be the yield on a high quality municipal bond selling at par and with a maturity equal to the donor's expected date of death. In the event the donee has a low marginal income tax rate, a good candidate for the imputed interest rate would be the after-income-tax yield on a high grade bond selling at par and with a maturity equal to the donor's expected date of death.

In specifying the models, the after-tax imputed interest rate will be assumed to be constant for the time period under consideration. This assumption is not necessary, nor is it necessarily valid; however, actual computations would be much more tedious if this assumption were not made. If one felt that this assumption were not acceptable, \[ \prod_{k=1}^{n} (1+i_k) \] could be substituted for \((1+i)^n\) in the equations developed in the model.

Simplifying Assumptions

The model assumes that gift taxes are paid at the date of gift; estate taxes are paid at the date of death; and income or loss occurs on the last day of the year, with income taxes being paid or refunded at that time.
The model assumes that the market value of the stock or raw land will never decrease. The model assumes that the parent's basis is always less than the fair market value of the stock or land.

Except for transfers of raw land, the marginal tax rate on capital gains will be assumed to be one half the marginal rate on ordinary income, and that the marginal tax rate will always be positive.

**Definitions**

A glossary of symbols is included in Appendix A of this dissertation. For development of the first model—transfer of stock to child when cash is available to pay taxes—a definition of the following variables is required.

\[ V = \text{Number of shares of stock which have a fair market value of $1 at time, } t = 0. \]

\[ V_0 = \text{Fair market value of } V \text{ at time } t = 0 \]

\[ V_d = \text{Fair market value of } V \text{ at the date of death.} \]

\[ V_g = \text{Fair market value of } V \text{ at the date of gift.} \]

\[ V_s = \text{Proceeds after sales commissions, if any, from the sale of } V \text{ in year } s. \]

\[ \alpha = \text{Current marginal gift tax rate.} \]

\[ \beta = \text{Highest marginal estate rate that will be applicable to the estate.} \]

\[ i = \text{After-income-tax imputed interest rate for making present value decisions for the donee.} \]

\[ j_1 = \text{Parent's after-tax rate of return on cash in year } 1. \]

\[ I_k = \text{Dividends in year } k \text{ paid on the fraction of one share of stock represented by } V_0. \]
\[ b_p = \text{Parent's current basis for determining capital gain or loss.} \]

\[ t_{p_k} = \text{Parent's marginal income tax rate in year } k. \]

\[ t_{c_k} = \text{Child's marginal income tax rate in year } k. \]

\[ t_{e_d} = \text{Estate's marginal income tax rate in year } d. \]

**Notation**

Although they represent a single concept, many of the equations developed in this dissertation are lengthy and cumbersome. In order to facilitate comprehension and eliminate lengthy redundant equations, the following notation will be used throughout this dissertation.

Let:

\[ \sum(x,y,c) = \sum_{k=x}^{y} \frac{I_k(1-t_{c_k})}{(1+i)^k} \]

\[ \sum(x,y,p) = \sum_{k=x}^{y} \sum_{l=k}^{y} (1-t_{p_k}) \frac{l=y}{(1+j_1)} \]

\[ \pi(x,y) = \pi(1+j_1) \]

\[ S(x-y) = \text{equation } Sx \text{ minus equation } Sy \]

\[ S(x+y) = \text{equation } Sx \text{ plus equation } Sy \]

**Basis of Comparison**

In the case of all transfers, the model will assume the following situation: The donor has property and, in some cases, cash as well as property, which he is willing at some time, \( t = 0 \), to transfer to a donee,
or the donee can accept the property at any time prior to the donor's death as a gift, or acquire it at death through the estate. The donor or donee will keep the property and cash, including income or loss net of the effect of income taxes, in a separate account. Thus, the basis of comparison will be: Which method, and in the case of gifts, which point in time, will give the donee the greatest present value—-an inter vivos gift or an estate transfer.

Gift of Stock to Child

Cash Available to Pay Taxes

Child will never sell stock.—A taxpayer's current and future estate's assets include stock and cash. Should the parent transfer the stock to the child through an estate or through an inter vivos gift? Consider the case where the child will never sell the stock nor spend any of the cash equivalents. If the parent wants to give \( V_0 = $1 \) of current fair market value of stock and, in addition, is prepared to allocate \( $M \) to the child and \( $M \) is adequate to pay all gift and estate taxes, should the parent give \( V_0 + $M \) to the child, net of gift and estate taxes, or should the stock and \( $M \) plus the after-tax interest accrued on \( $M \) and any dividends, \( I_k \), be passed through the estate?

The question is: If the parent has stock = \( V_0 = $1 \) and cash = \( $M \) and is prepared to segregate these assets at \( t=0 \) and pass them to his child either by gift or through the estate net of all income, estate, and gift taxes, which will give the child the greatest present value, (1) the gift or (2) the estate transfer? In order to make this comparison, the
assumption will be made that in the case of the gift, the child will also segregate the stock and dividends net of income taxes.

If the parent passes the property through the estate, the child would receive (in current dollars)

\[ S_1.1 \]

\[ \frac{V_d}{(1+i)^d} + \frac{Mn(1,d)}{(1+i)^d} + \frac{\Sigma(1,d,p)}{(1+i)^d}, \]

less \( \beta \) times this amount, leaving a net of

\[ S_2. \]

\[ (1-\beta) \left[ \frac{V_d}{(1+i)^d} + \frac{Mn(1,d)}{(1+i)^d} + \frac{\Sigma(1,d,p)}{(1+i)^d} \right]. \]

If the parent makes an inter vivos transfer at \( t = g \), he will on that date have the original stock, \( V_0 \), plus cash of

\[ S_3. \]

\[ Mn(1,g) + \Sigma(1,g,p). \]

From this amount, the parent must pay gift taxes equal to \( \alpha \) times the amount transferred. In the event \( \alpha = \beta \), the gift tax would be

\[ S_4. \]

\[ \alpha V_g + \frac{\alpha}{1+\alpha} \left[ S(3) - \alpha V_g \right]. \]

The child will receive a present value of

\[ S_5 \]

\[ \frac{V_d}{(1+i)^d} + \frac{S(3-4)}{(1+i)^g}, \]

\[ ^1 \text{The prefix } S \text{ indicates that the equation pertains to securities transactions. The prefix } C \text{ is used for Cash transactions. The prefix } L \text{ is used for raw land transactions. The prefix } P \text{ is used for transactions involving probability density functions. The prefix } D \text{ is used for equations which develop decision criteria.} \]
plus dividends net of income taxes

S6.

\[ \Xi(g+1,d,c), \]

for a total of \( S(5+6) \). Substituting for \( S3 \) and \( S4 \) yields

S7.

\[
\frac{V_d}{(1+i)^d} + \Sigma(g+1,d,c) + \frac{M\pi(1,g) + \Sigma(1,g,p) - \alpha V_g}{(1+\alpha)(1+i)^g}.
\]

In the event \( \alpha < \beta \), the parent will have to retain enough cash to pay the estate taxes at \( t=d \), which will be equal to \( \beta \) times the amount of the gift plus \( \beta \) times the amount retained to pay the estate tax less the gift tax paid. The estate tax will be

S8.

\[
(\beta - \alpha) (V_g + C_g) + \Sigma(g+1,d) \left[ S(3) - \alpha (V_g + C_g) - C_g \right],
\]

where \( C_g = \text{cash given at } t = g \). The child will receive

S9.

\[
\frac{V_d}{(1+i)^d} + \Sigma(g+1,d,c) + \frac{C_g}{(1+i)^g}.
\]

\( C_g \) must be chosen such that the cash available at \( t = g \), \( S3 \), less \( C_g \), less gift taxes plus accrued interest to \( t = d \) will be equal to \( S8 \).

S10.

\[
S(3) - (1 + \alpha) C_g - \alpha V_g = \frac{S(8)}{\pi(g+1,d)}.
\]

Solving for \( C_g \) yields

S11.

\[
C_g = \frac{M\pi(1,g) + \Sigma(1,g,p) - V_g \left[ \alpha + \frac{\beta - \alpha}{(1-\beta)\pi(g+1,d)} \right]}{1 + \alpha + \frac{\beta - \alpha}{(1-\beta)\pi(g+1,d)}}.
\]
In the event \( d > g + 3 \), the gift taxes paid at \( t = g \) will be included in the gross estate. The estate tax will be

\[ S(8) + \beta \left[ \alpha (V_g + C_{g+3>d}) \right], \]

where \( C_{g+3>d} = \text{cash given if } g + 3 > d \). Simplifying gives

\[ S(13) - (1 + \alpha)C_{g+3>d} - \alpha V_g = \frac{S(13)}{\pi(g+1,d)}. \]

Substituting for \( S_{13} \) and simplifying gives

\[ C_{g+3>d} = \frac{M(1,g) + \Sigma(1,g,p) - V g \left[ \alpha + \frac{\beta - \alpha + \alpha \beta}{(1 - \beta) \pi(g+1,d)} \right]}{1 + \alpha + \frac{\beta - \alpha + \alpha \beta}{(1 - \beta) \pi(g+1,d)}}. \]

Note that even if \( \alpha = \beta \), an extra estate tax must be paid on the gift taxes included in the gross estate. The child will receive

\[ \frac{V_d}{(1+i)^d} + \Sigma(g+1,d,c) + \frac{C_{g+3>d}}{(1+i)^g}. \]

If cash is available to pay taxes and the child will never sell the stock, the optimal decision can be found by evaluating \( S_2, S_9 \) for all
possible values of \( g \), and \( S16 \) for all possible values of \( g \) and choosing the largest present value available.

Child will sell stock at some \( t > d \).--If cash is available to pay taxes but the child will sell the stock at some \( t > d \), there exist four possible courses of action: (1) the parent can pass the stock through the estate, (2) the parent can make an inter vivos gift of the stock, (3) the parent can sell the stock at some \( t = s < d \) and give the proceeds plus other cash net of all taxes to the child, or (4) the stock can be passed through the estate but rather than give the stock to the child, the estate can sell the stock and transfer cash to the child net of income taxes.

If the stock is transferred through the estate and the child sells the stock, the child's total present value will be

\[
S17. \quad \frac{V_s - V_{d}c_s (V_s - b_d)}{(1+i)^s} + \frac{M_{1}(1,d) + \Sigma(1,d,p)}{(1+i)^d} + \Sigma(d+1,s,c) + \frac{V_d - \Sigma(l,d,p)}{(1+i)^d} + \Sigma(d+1,s,c) = b_d + (V_d - b_e) \cdot V_d^* \text{ or } V_d^*, \text{ whichever is less.}
\]

The term \( b_d \) is defined as follows:

\[
S18. \quad b_d = b_e + (V_d - b_e) \cdot V_d^* \text{ or } V_d^*, \text{ whichever is less.}
\]

where \( V_d^* \) equals average estate tax for the entire estate, and \( b_e = b_p \) for assets acquired by the parent after December 31, 1976, and is equal to the December 31, 1976 fair market value for other assets.
If the estate sells the stock at \( t = d \), the child will receive

\[
S_{19}. \quad V_d - \frac{1}{2} t_c (V_d - b_d) (1+i)^d + M \pi(1,d) + \frac{E(1,d,p)}{(1+i)^d} + \frac{\beta V_d + Mm(1,d) + E(1,d,p)}{(1+i)^d}.
\]

If the stock is transferred by an inter vivos gift and \( d > g + 3 \), the child will receive

\[
S_{20}. \quad V_s - \frac{1}{2} t_c (V_s - b_g) (1+i)^s + E(g+1,s,c) + \frac{C_g}{(1+i)^g}.
\]

The term \( b_g \) is defined as follows:

\[
S_{21}. \quad b_g = b_p + \alpha V_g \left[ \frac{V_g - b_p}{V_g} \right].
\]

If the stock is transferred by an inter vivos gift and \( d < g + 3 \), the child will receive

\[
S_{22}. \quad V_s - \frac{1}{2} t_c (V_s - b_g) (1+i)^s + E(g+1,s,c) + \frac{C_{g+3>d}}{(1+i)^g}.
\]

If the parent sells the stock and makes an inter vivos gift of cash to the child, the parent will have cash at \( t = s \) of

\[
S_{23}. \quad V_s - \frac{1}{2} t_p (V_s - b_p) + M\pi(1,s) + E(1,s,p).
\]
If the parent makes an inter vivos gift of cash, $C_s$, a gift tax of $\alpha C_s$ must be paid and if $s+3<d$, the estate tax will be

$$S_{24} = (\beta - \alpha)C_s + \beta \pi(s+1,d) \left[ S(23) - (1 + \alpha)C_s \right].$$

$C_s$ must be chosen such that $S_{23}$ minus $C_s$ minus the gift tax plus accrued interest will be equal to $S_{24}$:

$$S_{25} = S(23) - (1 + \alpha)C_s = \frac{S(24)}{\pi(s+1,d)}.$$

Solving for $C_s$ gives

$$S_{26} = \frac{S(23)}{1 + \alpha + \frac{\beta - \alpha}{(1 - \beta)\pi(s+1,d)}}.$$

The child will receive

$$S_{27} = \frac{C_s}{(1+i)^s}.$$

If $s + 3 > d$ and the parent makes an inter vivos gift of cash, $C_{s+3>d}$, a gift tax of $\alpha C_{s+3>d}$ must be paid and the estate tax will be

$$S_{28} = (\beta - \alpha)C_{s+3>d} + \beta \pi(s+1,d) \left[ S(23) - (1 + \alpha)C_{s+3>d} \right] + \beta \left[ \alpha C_{s+3>d} \right].$$

$C_{s+3>d}$ must be chosen such that $S_{23}$ minus $C_{s+3>d}$ minus the gift tax plus accrued interest will be equal to $S_{28}$:
S29.

\[ S(23) - (1 + \alpha)C_{s+3>\delta} = \frac{S(28)}{\pi(s+1,d)}. \]

Solving for \( C_{s+3>\delta} \) gives

S30.

\[ C_{s+3>\delta} = \frac{S(23)}{1 + \alpha + \frac{\beta - \alpha + \alpha\pi}{(1 - \beta)\pi(s+1,d)}}. \]

The child will receive

S31.

\[ \frac{C_{s+3>\delta}}{(1+i)^{s+d}}. \]

If cash is available to pay taxes and the child plans to sell the stock at some \( t > \delta \), the optimal decision can be found by evaluating S17, S19, S20, and S22 for all possible values of \( g \), S27 and S31 for all possible values of \( s \), and choosing the largest present value available.

The possibility exists that the sale of stock after \( t = \delta \) but before \( t = s \) might be the optimal decision. The assumption has been made that the value of the stock will never decrease; but, if the stock and dividends net of taxes appreciate at a rate less than \( i \), or if the child's marginal income tax rate is low for some year between \( t = \delta \) and \( t = s \), a sale between those dates might be the optimal decision.

**Stock Must be Sold to Pay Taxes**

Child will never sell stock.--If the parent plans to transfer only stock to the child and the child plans to never sell the stock, the parent must sell enough stock and possibly pay capital gains taxes in
order to generate the cash necessary to pay the gift taxes and estate taxes. In the case of a gift, it will be assumed that the parent will sell enough stock at \( t = g \) to generate enough cash such that the cash plus accrued interest will be equal to the future extra estate taxes. The parent will sell some fraction of \( V_0, \lambda \), such that enough cash will be generated to pay capital gains taxes, gift taxes, and additional estate taxes which will be paid at \( t = d \).

If \( g + 3 < d \), \( \lambda \) will be chosen such that

\[
\Sigma(1, g, p) + \lambda V_s - \lambda \cdot \frac{1}{2} t P_g (V_s - b_p) - \alpha(1 - \lambda) V_g
\]

is equal to zero. Solving for \( \lambda \) gives

\[
\lambda = \frac{V_g \left[ \alpha + \frac{(\beta - \alpha)}{\pi(g+1,d)(1 - \beta)} \right] - \Sigma(1, g, p)}{V_s - \frac{1}{2} t P_g (V_s - b_p) + V_g \left[ \alpha + \frac{(\beta - \alpha)}{\pi(g+1,d)(1 - \beta)} \right]}
\]

The child will receive

\[
\frac{(1 - \lambda) V_d}{(1+i)^d} + (1 - \lambda) \Sigma(g+1,d,c).
\]
In the event \( g+3 > d \), the estate will include the gift taxes and some fraction of \( V_d \), \( \lambda_{g+3 > d} \), must be sold such that enough cash is generated to pay all taxes. \( \lambda_{g+3 > d} \) will be chosen such that

\[
S(32) = \frac{\beta \alpha (1 - \lambda_{g+3 > d}) V_g}{\pi(g+1,d)} = 0.
\]

Solving for \( \lambda_{g+3 > d} \) gives

\[
\lambda_{g+3 > d} = \frac{V_g \left[ \alpha + \frac{\beta - \alpha + \alpha \beta}{\pi(g+1,d)(1 - \beta)} \right] - \Xi(1,g,p)}{V_S - \frac{t}{p_s} (V_S - b_{p}) + V_g \left[ \alpha + \frac{\beta - \alpha + \alpha \beta}{\pi(g+1,d)(1 - \beta)} \right]}.
\]

The child will receive

\[
\frac{(1 - \lambda_{g+3 > d}) V_d}{(1+i)^d} + (1 - \lambda_{g+3 > d}) \Xi(g+1,d,c).
\]

In the event that at \( t = g \) the parent has enough cash to pay all present and future taxes, \( S9 \) or \( S16 \) will be used to determine the child's present value.

Instead of the parent selling stock at \( t = g \), the parent could borrow enough on the stock to pay gift taxes and future estate taxes and transfer the stock to the child subject to the loan. Upon receipt of the gift, the child would immediately sell enough stock to pay off the loan. At \( t = g \), the parent will have stock and cash in the amount of
Let \( L \) represent the borrowed funds. If \( g + 3 > d \), the extra estate tax will be

\[
\sum(1,g,p) - \alpha(V_g - L).
\]

At \( t = g \), the parent will need cash to pay the extra estate tax in the amount of

\[
\frac{S(39)}{\tau(g+1,d)}.
\]

The gift tax will be

\[
\alpha(V_g - L).
\]

\( L \) must be chosen such that

\[
L + \sum(1,g,p) - \alpha(V_g - L) - S(40) = 0.
\]

Solving for \( L \) gives

\[
L = \frac{-\sum(1,g,p) + V_g \left[ \alpha + \frac{\beta - \alpha}{(1 - \beta)\tau(g+1,d)} \right]}{1 + \alpha + \frac{\beta - \alpha}{(1 - \beta)\tau(g+1,d)}}.
\]
If \( g + 3 > d \), then at \( t = g \) the extra estate taxes will be
\[
S(39) + \alpha (V_g - L)
\]
\[
\frac{\beta}{\pi(g+1,d)}
\]

In this case, \( L \) must be chosen such that
\[
L + \Sigma(1, g, p) - \alpha (V_g - L) - S(44) = 0.
\]

Solving for \( L \) gives
\[
L = \frac{-\Sigma(1, g, p) + V_g \left[ \alpha + \frac{\beta - \alpha + \alpha \beta}{(1 - \beta)\pi(g+1,d)} \right]}{1 + \alpha + \frac{(\beta - \alpha + \alpha \beta)}{(1 - \beta)\pi(g+1,d)}}.
\]

The child will have to sell some fraction of the stock, \( \lambda \), such that the proceeds after the child pays capital gains taxes will be equal to \( L \).

The child's basis will be
\[
b_c = b_p + \alpha (V_g - L - b_p).
\]

\( \lambda \) must be chosen such that
\[
L = \lambda \left[ V_s - \frac{1}{c_g} (V_s - b_c) \right].
\]

Solving for \( \lambda \) gives
\[ \lambda = \frac{L}{V_s - \frac{1}{2}t_c (V_s - b_c)} \]

The child will receive

\[ (1 - \lambda)V_d \]
\[ \frac{(1 - \lambda)V_d}{(1+i)^d} + (1 - \lambda)\Sigma(g+1,d,c). \]

If the property, stock plus dividends net of taxes, plus accrued interest is transferred to the child through the estate and enough cash is available to pay the taxes, S2 will be applicable. If S2 is not applicable, then some fraction of \( V_d \) must be sold to pay the estate taxes. Either the estate can sell some fraction of the stock or the estate can borrow enough cash to pay the estate tax and the child can sell the stock. Thus, if the estate's marginal income tax rate, \( t_{ed} \), is less than \( t_{cd} \), the estate should sell the stock; and if \( t_{cd} < t_{ed} \), the child should sell the stock.

If the estate sells the stock, it will sell a fraction, \( \mu \), of \( V_d \) such that the proceeds plus the cash held by the estate less capital gains taxes less estate taxes are equal to zero:

\[ \mu V_d + \Sigma(1,d,p) - \frac{1}{2}t_{ed} (V_d - b_e) - \frac{1}{2} \Sigma(1,d,p) + V_d = 0. \]

Solving for \( \mu \) gives

\[ \mu = \frac{(\beta - 1)\Sigma(1,d,p) + \beta V_d}{V_d - \frac{1}{2}t_{ed} (V_d - b_d)}. \]
In the event \( t_{cd} < t_{ed} \), Sections 52 and 51 are unchanged except one should substitute \( t_{cd} \) for \( t_{ed} \). The child will receive

\[ S53. \]
\[
\frac{(1-\mu)V_d}{(1+i)^d}.
\]

Instead of selling stock to pay gift taxes, the parent can sell enough stock to pay future estate taxes, give the rest of the stock to the child, and let the child sell enough stock to pay the gift taxes. At \( t = g \), the parent will have cash equal to

\[ S54. \]
\[ \Sigma(1,g,p). \]

If the parent sells a fraction, \( \lambda \), of the stock, the total cash will be

\[ S55. \]
\[ \Sigma(1,g,p) + \lambda \left[ V_s - \frac{1}{2} p_s (V_s - b_p) \right]. \]

The child will pay gift taxes equal to

\[ S56. \]
\[ \alpha(1-\lambda)V_g. \]

If \( g + 3 < d \), the parent will pay future estate taxes equal to

\[ S57. \]
\[ (\beta-\alpha)(1-\lambda)V_g + \frac{p}{\Sigma(1,g,p) + \lambda \left[ V_s - \frac{1}{2} p_s (V_s - b_p) \right]} \pi(g+1,d). \]

\( \lambda \) must be chosen such that

\[ S58. \]
\[ S(55)\pi(g+1,d) = S(57). \]
Solving for $\lambda$ gives

\[ \lambda = \frac{-E(1,g,p) + \frac{(\beta - \alpha)V_g}{(1-\beta)\pi(g+1,d)}}{V_s - \frac{1}{2}t_{p_s}(V_s - b_p) + \frac{(\beta - \alpha)V_g}{(1-\beta)\pi(g+1,d)}}. \]

The child must sell a fraction of $V_g$, $\mu$, such that

\[ \mu \left[ V_s - \frac{1}{2}t_{c_s} (V_s - b_g) \right] = S(56). \]

Solving for $\mu$ gives

\[ \mu = \frac{\alpha(1-\lambda)V_g}{V_s - \frac{1}{2}t_{c_s} (V_s - b_g)}. \]

The child will receive

\[ \frac{(1-\lambda-\mu)V_d}{(1+i)^d} + (1-\lambda-\mu)E(g+1,d,c). \]

In the event $g+3 > d$, $\lambda$ will be $\lambda_{g+3>d}$, which will be equal to

\[ \lambda = \frac{-E(1,g,p) + \frac{(\beta - \alpha + \alpha\beta)V_g}{(1-\beta)\pi(g+1,d)}}{V_s - \frac{1}{2}t_{p_s}(V_s - b_p) + \frac{(\beta - \alpha + \alpha\beta)V_g}{(1-\beta)\pi(g+1,d)}}. \]
If the child plans to keep the stock and at \( t=0 \) the parent has no cash, \( M=0 \), the optimal decision can be found by evaluating \( S_{34}, S_{37}, S_{50}, \) and \( S_{62} \) for all possible values of \( g \) and \( S_{53} \) and picking the highest present value obtained. In some cases, although \( M=0 \), enough cash will be accumulated from dividends to pay gift and estate taxes without selling part of the stock. Thus, for some \( t=k \), instead of evaluating \( S_{34}, S_{37}, S_{50}, \) and \( S_{62} \), it may be necessary to evaluate \( S_{2}, S_{9}, \) and \( S_{16} \).

If the child plans to sell the stock at some \( t>d \). If the child plans to sell the stock at some \( t=s>d \), the stock is received through an inter vivos gift, \( g+3<d \), and stock must be sold to pay taxes, the child will receive

\[
S_{64} = \frac{(1-\lambda) V^s_s}{(1+i)^s} + \frac{(1-\lambda)^{\frac{s}{2}} c_s (V^s_s - b^g)}{(1+i)^s} .
\]

In the event \( g+3>d \), the child will receive

\[
S_{65} = \frac{(1-\lambda) V^s_{g+3>d}}{(1+i)^{s}} + \frac{(1-\lambda)^{\frac{g+3+d}{i}} c_s (V^s_s - b^g)}{(1+i)^s} .
\]

If the child receives the stock subject to a loan, \( \lambda \) is determined by \( S_{49} \) and the child will receive

\[
S_{66} = \frac{(1-\lambda) V^s_s}{(1+i)^s} + \frac{(1-\lambda)^{\frac{s}{2}} c_s (V^s_s - b^g)}{(1+i)^s} .
\]

If the child receives the stock through an estate transfer, the present value to the child will be
An optimal decision can be found by evaluating $S27$, $S31$, $S64$, $S65$, and $S66$ for all possible values of $g$ and $S67$ and then choosing the highest present value available. In making this decision it will be possible for $S19$, $S20$, or $S22$ to be applicable for some $t=k$, and $S27$, $S31$, $S64$, $S65$, $S66$, and $S67$ to be applicable to other transfer years. Thus, it is necessary to determine for each $t=k$ which equation is applicable.

**Gift of Stock to Spouse**

Under Section 2523 of the Internal Revenue Code, the first $100,000$ of gifts to a spouse are not subject to gift taxes, the second $100,000$ of gifts are taxed at the normal rate, and subsequent gifts are reduced by a 50 percent marital deduction before computing the gift tax. Under Section 2056 of the Internal Revenue Code, an estate gets a marital deduction of $250,000$ or 50 percent of the gross estate if that much property is transferred to the spouse. Thus, above certain limits in the case of transfers to a spouse, $\alpha$ and $\beta$ will be equal to one-half the usual rate.

Assuming that a joint return is filed, there will be no potential savings due to income taxes on dividends. Thus, equations $S1$ through $S67$ will be applicable to transfers to spouses if appropriate values for $\alpha$ and $\beta$ are used.

**Gift of Cash to Child**

If the parent has $\$1$ which will be given to the child through a gift or through the estate, which transfer would be most advantageous to
the child? If the $1 = C$ is passed through the estate, the child will receive

\[ C_1 = \frac{(1 - \beta) C \tau(1, d)}{(1 + i)^d}. \]

If an inter vivos gift is made and \( g + 3 < d \), the extra estate taxes will be

\[ C_2 = \beta(C_g + C_e) - \alpha C_g, \]

where \( C_e \) is the cash retained to pay the estate taxes on the gift \( C_g \). At \( t = g \), \( C \) is equal to

\[ C_3 = C_g + \alpha C_g + \frac{C_e}{\tau(g + 1, d)}. \]

Solving \( C_2 \) for \( C_e \) gives

\[ C_4 = \frac{(\beta - \alpha)}{(1 - \beta) C_g}. \]

Substituting \( C_4 \) in \( C_3 \) gives

\[ C_5 = C_g \left[ 1 + \alpha + \frac{\beta - \alpha}{(1 - \beta) \pi(g + 1, d)} \right]. \]

The child will receive

\[ C_6 = \frac{C_g}{(1 + i)^g} = \frac{C_5(1, g)}{\left[ 1 + \alpha + \frac{(\beta - \alpha)}{(1 - \beta) \pi(g + 1, d)} \right] (1 + i)^g}. \]
In the event $g + 3 > d$, the extra tax paid by the estate will be $C_e$.

\[ C_e = \beta(C_{g+3>d} + C_e + \alpha C_{g+3>d}) - \alpha C_{g+3>d}. \]

Solving for $C_e$ gives

\[ C_e = C_{g+3>d} \left[ \frac{\beta + \alpha \beta - \alpha}{1 - \beta} \right]. \]

Substituting $C_e$ in $C_3$ gives

\[ C_\pi(1,g) = C_{g+3>d} + \alpha C_{g+3>d} + \frac{C_{g+3>d}}{\pi(g+1,d)} \left[ \frac{\beta + \alpha \beta - \beta}{1 - \beta} \right]. \]

Solving for $C_{g+3>d}$ gives

\[ C_{g+3>d} = \frac{C_\pi(1,g)}{1 + \alpha + \left[ \frac{\beta + \alpha \beta - \beta}{1 - \beta} \right]} \frac{1}{\pi(g+1,d)}. \]

The child will receive $C_{11}$.

\[ \frac{C_{g+3>d}}{(1+i)^g} = \frac{C_\pi(1,g)}{1 + \alpha + \left[ \frac{\beta + \alpha \beta - \alpha}{1 - \beta} \right]} \frac{1}{\pi(g+1,d)} (1+i)^g. \]

If more than $1$ is to be transferred to the child, the amount the child will receive can be found by summing $C_1$, $C_6$, and $C_{11}$ for each dollar transferred. This process would require keeping track of the total gifts
and the size of the estate since $\alpha$ and $\beta$ increase in a discrete step function as total gifts and estate size increase.

Let:

$C_g(C) = \text{Total amount that the child will receive from a gift of } C \text{ dollars}$

$G = \text{Gifts since December 31, 1976}$

$E = \text{Total size in dollars of the taxable estate assuming that } C \text{ does not exist. In the case of raw land to be considered later, the land, mortgage, and cash to be transferred would be assumed not to exist.}$

$T(x,y) = \text{A function which represents the extra gift or estate tax due when a prior amount, } x, \text{ is increased to } x+y \text{ (see Appendix B).}$

In the case of an estate transfer, $C_1$, the child will receive

$C_{12}.$

$$\frac{C_g(C)}{(1+i)^d} = \frac{1}{(1+i)^d} \left[ C_{m}(1,d) - T[E,C_{m}(1,d)] \right].$$

In the case of an inter vivos transfer, $C_6$ or $C_{11}$--the amount the child receives--is a function of $\alpha$ and $\beta$, which are a function of $G$ and $E$ respectively.

Let

$$\alpha_n = \alpha(G, \sum_{k=1}^{n} C_{g_k}).$$

where $\alpha(g,x)$ represents the gift tax rate applicable to a transfer at $G+x$, and $C_{g_k}$ is the cash transferred by $C_6$ or $C_{11}$ when $C = \$1$. 
Let
\[ \beta_n = \beta \left[ E, \sum_{k=1}^{n} q_k \sum_{k=1}^{n} \pi(k+1,d) \right]. \]

where \( \beta(E,x) \) represents the estate tax rate applicable to a transfer at \( E+x \).

In the case of a gift where \( g+3 < d \), C6, the child will receive

\[ C_{13}. \]

\[ \frac{C_g(C)}{(1+i)^g} = \frac{1}{(1+i)^g} \sum_{k=1}^{k=C} \frac{\pi(1,g)}{1 + \alpha_k + \frac{(\beta_k - \alpha_k)}{(1 - \beta_k)\pi(g+1,d)}}. \]

In the case of a gift, when \( g+1 > d \), C11, the child will receive

\[ C_{14}. \]

\[ \frac{C_{g+3>d}(C)}{(1+i)^g} = \frac{1}{(1+i)^g} \sum_{k=1}^{k=C} \frac{\pi(1,g)}{1 + \alpha_k + \frac{\beta_k + \alpha_k \beta_k - \alpha_k}{(1 - \beta_k)\pi(g+1,d)}}. \]

Evaluating C13 or C14 requires the use of a computer if the evaluation is to be accomplished before \( t = d \).

Gift of Cash to Spouse

Equations C1 through C14 can be made applicable for a transfer to a spouse by defining \( \alpha(G,x) \) and \( \beta(E,x) \) to take into consideration the special treatment given to spouses and the marital deduction for estate transfers. This can be accomplished by modifying Appendix B.
Gift of Raw Land to Child

Introduction

If the parent is planning to transfer raw land to his child, has adequate cash to pay gift and estate taxes, and the property produces a positive taxable cash flow, the decision as to whether an inter vivos gift or transfer via the estate is most advantageous to the child can be made by using the same equations which were applicable to marketable securities. However, caution must be exercised since the equations developed for the marketable securities assumed a $1 transaction. In the case of raw land, the quantity sold can affect market prices and, in some cases, only the entire property could be sold in an economical fashion. These difficulties can be overcome by using an average value for $\alpha$, $\beta$, and $t_x$, or by letting $N_x$ be a negative number in the following equations.

In the case of raw land, the number of economically feasible alternatives is much more limited than in the case of marketable securities. Unlike marketable securities, a small fraction of raw land cannot be readily sold; hence, only cases where the entire property is sold will be considered.

The assumption will be made that the parent has adequate cash to pay gift and estate taxes, and that either the parent has enough cash to make a gift to the child adequate to cover future negative cash flows or the child has such a cash flow. The $3,000 per year gift exclusion will be assumed to be used elsewhere. The objective of the following analysis will be to produce a set of equations similar to those developed for marketable securities. There will be two general cases to be considered:
(1) the child will sell the land at some \( t > d \), and (2) the child will never sell the land.

**Child Will Never Sell Land**

If the parent has raw land, which has a fair market value of \( V_0 \) at \( t_0 \), cash in the amount of \( D \), and the property will have a negative cash flow\(^2\) in year \( x \) of \( N_x \) (if one is dealing with a positive cash flow \( N_x \) is a negative number), and the parent segregates the land, cash, and income tax savings on the negative cash flow and transfers them to the child through the estate, the child will receive land with a present value of \( L1 \).

\[
\frac{V_d - M_d}{(1+i)^d},
\]

where \( M_x \) is the amount of a mortgage in year \( x \) and \( P_x \) is the amount of a principal payment on a mortgage in year \( x \). The estate taxes on the land net of the mortgage will be \( L2 \).

\[
T(E, V_d - M_d).
\]

The estate will have cash equal to

\[
\begin{align*}
L3. & \quad C_0 = D & \text{at } t = 0; \\
L4. & \quad C_1 = D(1+j_1) - \left(1 - \frac{E}{P_1}\right)N_1 - P_1 & \text{at } t = 1; \\
L5. & \quad C_2 = C_1(1+j_2) - \left(1 - \frac{E}{P_2}\right)N_2 - P_2 & \text{at } t = 2; \\
\end{align*}
\]

and

\[
\begin{align*}
L6. & \quad C_k = C_{k-1}(1+j_k) - \left(1 - \frac{E}{P_k}\right)N_k - P_k & \text{at } t = k.
\end{align*}
\]

\(^2\)For example, if taxes exceed rental income there would be a negative cash flow.
At \( t = d \), the estate will have cash equal to \( C_d \). Unless \( N_k \) and \( P_k \) are zero for all \( k \), a computer program will be needed to evaluate this function efficiently. The child will receive

\[
\frac{V_d - M_d + C_d - T(E, V_d - M_d + C_d)}{(1+i)^d}.
\]

If the parent makes the transfer by an inter vivos gift and \( g+3 < d \), the child will receive land subject to a mortgage with a present value of \( L^7 \).

\[
\frac{V_d}{(1+i)^d} - \frac{M_d}{(1+i)^d}.
\]

At \( t = g \), the parent will have cash equal to \( C_g \) as determined by \( L^6 \). Part of this cash, \( C_t \), can be given to the child at \( t = g \), and part, \( C_e \), must be retained by the parent to pay extra estate taxes. At \( t = d \), \( C_e \) will have accrued interest and will total \( L^9 \).

\[
C_d = C_e \pi(g+1,d).
\]

\( C_e \) must be chosen such that \( L^{10} \).

\[
C_e = C_g - T(G, V_g - M_g + C_t) - C_t
\]

and

\[
L^{11}.
\]

\[
C_e \pi(g+1,d) = T \left[ E, V_g - M_g + C_t + C_e \pi(g+1,d) \right] - T(G, V_g - M_g + C_t).
\]

Unless \( \alpha \) and \( \beta \) are constant over \( G, G + V_g - M_g + C_t \) and \( E, E + V_g - M_g + C_e \pi(g+1,d) \), an exact solution to this set of equations cannot be immediately
found. The solution can be found by making a guess at the value of $C_e$ and increasing or decreasing $C_e$ until L10 and L11 hold. A low guess at $C_e$, which would constitute a starting point, would be

$$L_{12}. \quad C_e = \frac{T(E, V_g - M_g) - T(G, V_g - M_g)}{\pi(g+1,d)(1+\beta)}$$

where $\beta = \beta(E,0)$. Again, use of a computer program will be expedient.

The child will receive

$$L_{13}. \quad \frac{V_d}{(1+i)^d} + \frac{C_d - M_d}{(1+i)^d},$$

where $C_d$ is defined by L6, $D = C_t$, $k = d = g$, and $\pi_{px}$ is replaced with $\pi_{cx}$.

If the parent makes the transfer by an inter vivos gift and $g+3 > d$, the gift taxes paid on the transfer will be included in the estate. As a result, $C_t$ will be smaller. L11 will become

$$L_{14}. \quad C_e \pi(g+1,d) = T\left[E, V_g - M_g + C_t + C_e \pi(g+1,d)\right] + T(G, V_g - M_g + C_e) - T(G, V_g - M_g + C_t).$$

The solution to L10, L11, and L14 can be found by guessing $C_e = L_{12}$ and increasing $C_e$ until the equations hold. Again, utilization of a computer program is indicated.

The optimal decision can be found by evaluating L7 and L13 for all values of $g$ and selecting the largest present value available.
Child Will Sell Land at Some \( t > d \)

If the child receives the land subject to a mortgage through the estate, he will have cash in the amount of

\[
C_0 = C_d - T(E, V_d - M_d + C_d) \quad \text{at } t = d.
\]

From \( t = d \) to \( t = s \), the child will have to make principal payments of \( P_k \) and cash expenses of \( N_k \) less the tax savings of \( T_c \cdot N_k \). The child will have

\[
C_{d+1} = C_o (1+i) - P_k - (1-\bar{E}_{c_k})N_k \quad \text{at } t, k = d+1;
\]

\[
C_{d+2} = C_1 (1+i) - P_k - (1-\bar{E}_{c_k})N_k \quad \text{at } t, k = d+2;
\]

and

\[
C_k = C_{k-1}(1+i) - (1-\bar{E}_{c_k})N_k - P_k \quad \text{at } s, k = s.
\]

At \( t = s \), the child will have cash equal to \( C_s \) as determined by L18. The child will have a present value of

\[
C_s \left( \frac{1}{1+i} \right)^s,
\]

plus proceeds from the sale of the land with a present value of

\[
\frac{V_s - M_s - T_c (V_s - b_e_d)}{(1+i)^s}.
\]

The term \( T_{c_s} \) is the appropriate long-term capital gains tax rate for the child in year \( s \). The term \( T_{p_g} \) is the appropriate long-term capital gains
tax rate for the parent in year \( g \). The alternative capital gains tax, the minimum tax, the maximum tax, and the income averaging provisions of the tax law could all have an effect on the appropriate tax rate. The child will receive \( L(19 + 20) \):

\[
L21. \quad \frac{C_s + V_s - M_s - T_c (V_s - b_d)}{(1+i)^s}.
\]

If the child receives the land subject to a mortgage plus cash at \( t = g \), at \( t = 2 \) the child will have cash equal to \( C_s \) as determined by \( L18 \), except \( C_0 \) will be equal to \( C_t \) as determined by \( L11 \) or \( L14 \), and \( k \) will range from \( g+1 \) to \( s \). The child will receive

\[
L22. \quad \frac{C_s + V_s - M_s - T_c (V_s - b_g)}{(1+i)^s}.
\]

If the parent sells the land at \( t = g \), the after-tax proceeds plus other cash from \( D \), less expenses net of taxes, less mortgage payment will be

\[
L23. \quad C = V_s - T_p (V_s - b_p) - M_g + C_g,
\]

where \( C_g \) is determined by \( L6 \).

If \( g + 3 < d \), \( C13 \) is applicable and the child will receive

\[
L24. \quad \frac{C_d(C)}{(1+i)^g}.
\]
If $g + 3 > d$, C14 is applicable and the child will receive L25.

$$\frac{C_{g+3} d(C)}{(1+i)^g}.$$ 

The optimal decision can be found by evaluating L21, L22, L24, and L25 for all values of $g$ and selecting the largest present value available.

**Gift of Raw Land to Spouse**

Equations L1 through L25 can be used to determine the optimal decision in making a transfer to the spouse by redefining $T(x,y)$ to take into consideration the special treatment given to gifts to spouses and the marital deduction for estate transfers. $T(x,y)$ for gifts must be adjusted for prior gifts to the spouse in the case of gifts. $T(x,y)$ must be adjusted for prior gifts, the current gift of raw land, and other transfers to the spouse, in the case of estate transfers. In some cases, the estate tax will be zero but a gift tax would have to be paid on the transfer. This will be accomplished by modifying Appendix B.

**Delayed Payment of Estate Taxes**

Section 6166 and Section 6166A permit an estate to pay the estate taxes out over a period of years when, in general, immediate payment of the estate taxes would require sale of a closely held stock or perhaps partial liquidation of the corporation. Determining the present value of this to the child is difficult unless the effect of potential capital gains taxes and loss of earnings is ignored. If the assumption is made that the debt can be paid without extra tax liability from dividends, stock redemptions, or stock sales to outsiders, the benefit to the child
is the lower interest rates which will be charged by the government as compared to the cost of a commercial loan.

Let:

\[
I_g^k = \text{After-tax interest charged by the government in year } k.
\]

\[
I_c^k = \text{After-tax interest charged by a commercial lender in year } k.
\]

The present value to the child will be:

\[
\frac{1}{(1+i)^d} \sum_{k=d+1}^{k=n} \frac{I_c^k - I_g^k}{(1+i)^k}.
\]

In many cases this will be insignificant.

Effect of Probabilistic Variables

Date of Death

The model developed in this chapter has assumed that the date of death is known to be a specific date. This assumption is patently false. The actual date of death will occur sometime after \( t = 0 \) and before some date, which is normally assumed to be one hundred years of age in actuarial tables. For each year \( k \), there is a non-zero probability, \( p_k \), that the parent will die in that year. The word probability by definition requires that

\[
\sum_{k} p_k = 1.
\]

If the parent has cash and stock and decides to pass this property to the child through the estate, the child will receive \( S_2 \). The present value represented by \( S_2 \) is a function of the date of death, since \( V_d \).
(1+i)^d, \pi(1,d), and \Sigma(1,d,p) are a function of the date of death. Also, \beta is a function of the date of death. \beta is the highest marginal estate tax rate that the estate will pay, and \beta will possibly increase or decrease for changes in the size of the total estate. Let S2(k) represent the value of S2 if k is the date of death. The child's expectation in current dollars will be

\[ P_2. \]

\[ \sum_{k} p_k S2(k). \]

If a decision is made to make a gift in year five, S2, S9, or S16 will be applicable depending on when the parent dies. If the parent dies in the first five years, S2 will be applicable; if the parent dies in year six, seven, or eight, S16 will be applicable; if the parent dies in year nine or ten, S9 will be applicable. Thus, the date of death not only influences the child's expectation, but how the child's expectation will be computed is a function of the date of death. In this case, the child's expectation in current dollars will be

\[ P_3. \]

\[ \sum_{k=5} \Sigma_{k=6} p_k S2(k) + \sum_{k=6} p_k S16(k) + \sum_{k=9} p_k S9(k). \]

Let A(k) be the child's present value if an estate transfer is made; let B(k) be the child's present value if an inter vivos gift is made and g + 3 < d; and let C(k) be the child's present value if an inter vivos gift is made and g + 3 > d. If a decision is made to make an inter vivos gift on the last day of year g, and n is the highest possible date of death, the child's expectation will be
If a decision is made to make an estate transfer, the child expectation will be

\[ p_5. \]

\[ \sum_{k=1}^{g+1} p_k A(k) + \sum_{k=g+4}^{k=n} p_k B(g) + \sum_{k=g+1}^{k=g+3} p_k C(g). \]

It is also possible to determine the probability that the child will receive at least a specified present value for a decision. This can be accomplished by finding the sum of all \( p_k \) where \( A(k), B(k), \) or \( C(k) \)--whichever is applicable to the decision--is greater than the specified value.

**Forecasting Market Values and Tax Rates**

The model developed in this chapter has assumed that all the variables pertaining to future conditions are known at \( t = 0 \). Again, this is a patently false assumption. This problem is more complicated than the date of death problem since there are many variables and they are interconnected. For example, \( V_d \) will determine in part \( B \) and \( \bar{B} \). Assuming a company has a stable dividend policy, \( V_d \) implies a range of possible values of \( I_d \). \( I_d \) in turn implies \( t_{p_d} \). The choice of \( i \) is a function of \( V_d \). Stated inversely, \( V_d \) may be a function of \( i \). \( j_k \) is a function of \( i \) and \( t_{p_k} \).
Thus, in making projections, it is necessary to consider the whole picture and not just individual variables. In applying probability to the variable, the only tenable solution given the current state of knowledge is to predict the variables together but make several projections and assign probabilities to these projections. Let \( m_1 \) be the probability that projection 1 will be accurate. If the date of death is known and a decision is made to pass cash and stock through the estate, the child's expectation in current dollars will be

\[ P_6. \]

\[ \sum_{l} m_{1} S_{2}(l). \]

If \( S_{2}(l) \) is known for all values of \( l \), then the probability that the child will receive more than some specific value can be determined by summing \( m_{1} \) for all \( l \) where \( S_{2}(l) \) is greater than the specific value. In a similar fashion, the expectation of gift decisions and the probability that the expectation under a gift will exceed a given value can be determined.

**Date of Death Combined with Other Variables**

If \( p_k \) and \( m_1 \) are assigned values, one can consider their joint probability density function. This model will assume that \( p_k \) and \( m_1 \) are independent. As used, the mathematical term—indepedent—means that the date of death has no effect on future values and that future values have no effect on the date of death. Thus, if \( m_5 \) assumes a worldwide depression or World War III, \( p_k \) will be assumed to be the same even if the parent under consideration is subject to atomic attack or an incipient coronary.
If the parent has cash and stock and decides to pass the property to the child through the estate, the child's expectation in current dollars will be

\[ P_7 = \sum_{k} p_k m_1 S(k, l). \]

If a decision is made to make an inter vivos gift in year \( g \), and \( n \) is the highest possible date of death, the child's expectation is

\[ P_8 = \sum_{k=g}^{k=g+3} \left[ \sum_{k=1}^{k=n} p_k A(k, l) + \sum_{k=g+4}^{k=g+1} p_k B(g, l) + \sum_{k=g+2}^{k=g+1} p_k C(g, l) \right]. \]

The evaluation of \( P_8 \) will require an immense amount of computation. Initially, \( A(k, l) \), \( B(k, l) \), and \( C(k, l) \) must be evaluated for each possible combination of \( (k, l) \). Then, \( P_8 \) can be determined as the product of three matrices.

In making a decision, it will be necessary to evaluate \( P_8 \) for all possible dates of gift. The optimal decision will be the date of gift that has the highest expectation as determined by \( P_8 \), or the expectation of an estate transfer as determined by \( P_7 \), whichever is greater.

Given a matrix of the joint probability density function and a corresponding matrix containing the expectations determined by \( A(k, l) \), \( B(k, l) \), or \( C(k, l) \), the probability that a decision will yield a present value above a given value can be determined by summing \( p_k m_1 \) for all values of \( k, l \) where the expectation is greater than the given value.
CHAPTER III

DECISION CRITERIA FOR GIFTS

Introduction

Chapter II developed a set of equations for cash, marketable securities, and raw land which, when evaluated, give the donee's present value for alternative decisions. The possible decisions are an estate transfer or an inter vivos gift; of course, there are many different possible dates for a gift. In addition, in the case of an estate, whether the estate or the donee can sell stock to obtain cash, and in the case of a gift, there are several alternatives for paying the gift taxes and obtaining cash to pay future incremental estate taxes. This cash can be generated from the donor selling stock, the donor borrowing on the stock and making a gift subject to a loan, or selling stock to pay future incremental estate taxes and having the donee sell enough stock to pay the gift taxes. Other alternatives exist but models in Chapter II were limited to these methods. This chapter will develop some of the implications which can be drawn from the models developed in Chapter II.

This chapter will consist of three main parts: (1) a summary of general decision rules presented in Chapter II, along with some simple decision rules which are obtainable under simplifying assumptions, (2) some rather common estate tax situations will be analyzed and the pertinent computer output will be presented, and (3) the House Committee Report on the Tax Reform Act of 1976, pertaining to estate and gift taxes,
will be examined to see if the avowed congressional intent was met, and some illustrative examples will be given to demonstrate an apparent failure of congressional intent.

Before presenting a detailed analysis of the models developed in Chapter II, an overall or general view of what factors make a gift advantageous or disadvantageous in comparison to an estate transfer will be useful. An inter vivos gift will tend to be advantageous when any of the following conditions are true: (1) the donor's marginal income tax rate is greater than the donee's marginal income tax rate; (2) the date of gift occurs more than three years before the date of death, since the gift tax paid will not be included in the gross estate but will be a credit against the estate; (3) the gift's value at death is substantially greater than its value at the gift date since the gift tax is based on the value on the gift date and the estate tax is based on the value at death; (4) the stock will be sold and the marginal gift tax rate is greater than the average estate tax rate since a gift will give a larger basis for determining capital gains, because the donee's basis for determining gain or loss if the gift property is sold is increased by a fraction of the marginal gift taxes.

An estate transfer will tend to be advantageous when any of the following conditions are true: (1) the value at death is close in value to the value on the gift date; (2) the stock will be sold and the marginal gift tax rate is less than the average estate tax rate, so the estate transfer gives a higher basis for determining capital gain; (3) the estate will have a low income tax rate compared to the parent or donee; (4) the estate consists primarily of closely held stock and the extended payment provisions of Sections 6166 and 6166A are applicable; (5) the
donee's marginal income tax rate is higher than the donor's marginal income tax rate; and (6) the donor is likely to die within three years after the date of gift which would result in the gift taxes being added into the gross estate and estate taxes being paid on the gift taxes.

A tax practitioner is faced with the problem of evaluating these variables and making a recommendation to his client. The models developed in Chapter II give the practitioner the probability that a gift in a specific year will be more advantageous than an estate transfer. An illustrative final output on which a decision may be made would appear as follows:

TABLE I
AN ILLUSTRATION OF COMPARATIVE PRESENT VALUES: GIFT VERSUS ESTATE

<table>
<thead>
<tr>
<th>Gift when t is equal to:</th>
<th>Expected Present Value of (Gift/Estate):</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.40</td>
</tr>
<tr>
<td>1</td>
<td>1.45</td>
</tr>
<tr>
<td>2</td>
<td>1.50</td>
</tr>
<tr>
<td>3</td>
<td>1.55</td>
</tr>
<tr>
<td>4</td>
<td>1.60</td>
</tr>
<tr>
<td>5</td>
<td>1.15</td>
</tr>
<tr>
<td>6</td>
<td>1.10</td>
</tr>
<tr>
<td>7</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table I shows the expected present value of a gift in a given year divided by the expected present value of an estate transfer. As used, the word "expected" means average value taking into consideration different possible dates of death and different possible future economic
conditions as they affect the model's parameters. The highest number, 1.60, is for a gift in year 4, and this would be the optimal decision.

Table II shows the probability that the present expected value of the gift will exceed some multiple of the estate transfer present value. Inspection of this table shows, for example, in the second entry that the gift present value will exceed 110 percent of the estate present value 90 percent of the time. In this example, Table II confirms the decision

<table>
<thead>
<tr>
<th>Expected Present Value of Gift at t = 4 (Gift/Estate)</th>
<th>Probability of Occurrence</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 1</td>
<td>1.00</td>
</tr>
<tr>
<td>&gt; 1.1</td>
<td>0.90</td>
</tr>
<tr>
<td>&gt; 1.2</td>
<td>0.85</td>
</tr>
<tr>
<td>&gt; 1.3</td>
<td>0.80</td>
</tr>
<tr>
<td>&gt; 1.4</td>
<td>0.76</td>
</tr>
<tr>
<td>&gt; 1.5</td>
<td>0.72</td>
</tr>
<tr>
<td>&gt; 1.6</td>
<td>0.65</td>
</tr>
<tr>
<td>&gt; 1.7</td>
<td>0.21</td>
</tr>
</tbody>
</table>

to make a gift in year 4. However, if Table II had different values—for example, of 20 percent of the time the estate transfer would be a better decision—then a practitioner would have to consider how the client felt about this type of risk. Fortunately, as will be shown, disadvantageous conditions are relatively rare.

This example of the final output computations was presented at this point in the analysis to demonstrate that, irrespective of the complexities of Chapter II, the problem of parameter estimation and performing
thousands of computations, the final result can be expressed in a simple
and readily understood format. If the practitioner can give a valid
estimate of input parameters, the model will give a simple, concise, and
readily understood answer.

The practitioner faces three major problems in utilizing the model:
(1) making a prediction of the input variables, (2) performing the compu-
tations required to generate the above information, and (3) explaining
the analysis to the client and convincing him to make the optimum decision.

The models developed are of limited value unless the input variables
bear a reasonably close relation to future events. The introduction of
probability density functions into the model permits a practitioner to
develop a feel for decision variability based on input parameter vari-
bility. Therefore, most analyses should include the probabilistic models.

The computations required by the model can readily be accomplished
with computer programs. A large portion of the time spent in the devel-
opment of this study consisted of writing computer programs which evaluate
the equations developed in Chapter II and permit computation of present
value for a specific example. Examples of this output will be presented
in the examples shown in this chapter. If a practitioner does not have
access to a computer, the practitioner has a good estimate of the input
parameters, and the marginal income tax rates for the donor and donee are
stable over the time period under consideration, one can obtain a good
guess at the optimal decision by evaluating the appropriate equations for
the date of death, three and four years before the date of death. If
four years before death is the largest present value, then the earliest
two gift dates should be evaluated. While this is not demonstrated in
the dissertation, processing numerous examples gives a sound basis for this decision rule. This rule must be used with extreme care. If dividend rates, tax rates, or market values fluctuate significantly, another year might give the best result. For example, the donor might be in a 70 percent income-tax bracket but will retire in two years when the donor will be in a 35 percent income-tax bracket.

The problem of explaining the results to the client is in all likelihood insoluble in most instances. The client will, in most cases, have to rely upon the practitioner's advice without really understanding the nature of the decision. This will present the practitioner with a serious ethical problem when, for example, a gift in year two is optimal if the client dies in year five, but a gift in year one is optimal if the client dies in year four. The decision is subject to risk and, like a stockbroker, the practitioner will have to evaluate the client's attitude towards risk and make a recommendation based on a subjective evaluation of the client's attitude towards risk.

Utilization of Model

Cash

By evaluating equations C1, C6, and C11, one can determine for a $1 gift which decision is best—a gift or an estate transfer—and, in the case of a gift, the optimum gift year. For a gift exceeding $1, the optimum decision can be found by evaluating equations C12, C13, and C14.

The following analysis will demonstrate mathematically that, if the donee's after-tax rate of return on cash is greater than or equal to the donor's after-tax rate of return on cash, a gift will never result
in the donee receiving a lower present value than the donee would have received under an estate transfer. That is, a gift should be made in every situation where the above conditions exist. This does not, however, determine the optimal gift year. A reader uninterested in the details of the mathematical proof of the preceding assertion can ignore the following without loss of content.

Under what circumstances will a gift of cash be advantageous? If an estate transfer is made, the child will receive C1

\[ q = \frac{(1-g)C_\pi(1,d)}{(1+i)^d} \]

If \( g + 3 < d \) and a gift is made, the child will receive C6

\[ r = \frac{C_\pi(1,g)}{1 + \alpha + \frac{\beta - \alpha}{(1-\beta)C_\pi(g+1,d)}}(1+i)^g \]

If \( \pi(1,x) = (1+i)^x \), then

\[ q = (1-\beta)C \]

and

\[ r = \frac{C}{1 + \alpha + \frac{\beta - \alpha}{(1-\beta)(1+i)^{d-g}}} \]
A gift will be advantageous if \( r > q \)

\[
\frac{C}{1 + \alpha + \frac{\beta - \alpha}{(1 - \beta)(1+i)^{d-g}}} > (1-\beta)C.
\]

Rearranging gives

\[
0 > (\beta - \alpha) \left[ \frac{1 - (1+i)^{d-g}}{(1+i)^{d-g}} \right] - \alpha \beta.
\]

\((\beta - \alpha)\) will always be \(\geq 0\), \((1+i)^{d-g}\) will always be \(\geq 1\), \(1 - (1+i)^{d-g}\) will always be \(\leq 0\), so

\[
(\beta - \alpha) \left[ \frac{1 - (1+i)^{d-g}}{(1+i)^{d-g}} \right] \leq 0
\]

and

\[
-\alpha \beta \text{ will always be } \leq 0.
\]

Therefore,

\[
(\beta - \alpha) \left[ \frac{1 - (1+i)^{d-g}}{(1+i)^{d-g}} \right] - \alpha \beta
\]

will always be \(\leq 0\). This means that a gift will always be advantageous if (1) \(g + 3 < d\), (2) the donor's after-tax rate of return is equal to the donee's after-tax rate of return, and (3) \(i > 0\).
If \( g + 3 > d \) and a gift is made, the donee will receive C11

\[ s = \frac{C\pi(1,g)}{1 + \alpha + \frac{(\beta - \alpha + \alpha \beta)}{(1 - \beta)\pi(g+1,d)}}\]

A gift will be advantageous if \( s > q \). If \( \pi(1,x) = (1+i)^x \), then

\[ C > (1-\beta)C. \]

Rearranging gives

\[ 0 > (\beta - \alpha + \alpha \beta)\left[\frac{1 - (1+i)^{d-g}}{(1+i)^{d-g}}\right]. \]

\( \beta - \alpha \) and \( \alpha \beta \) will always be \( > 0 \); \( 1 - (1+i)^{d-g} \) will always be \( \leq 0 \); so, D12 will hold for all cases except \( i = 0 \). If \( \pi(1,x) = (1+i)^x \), and \( i > 0 \), a gift of cash will always be advantageous.

In the preceding analysis, \( \pi(1,x) \) was assumed to be equal to \( (1+i)^x \). Stated differently, it was assumed that the donor's after-tax rate of return, \( j_k \), was equal to the donee's after-tax rate of return on cash, \( i \). If \( j_k < i \), where \( j_k \) has the same value for all \( k \), then

\[ \frac{\pi(1,d)}{(1+i)^d} < \frac{\pi(1,g)}{(1+i)^g}. \]

Therefore, \( q \) will still be less than \( r \) and \( s \), and \( q \) will be relatively smaller.
In the case of cash, an inter vivos gift is always advantageous if $j_k < i$. If $j_k > i$, no simple rule exists. A decision is found by evaluating $D_2$ and $D_{10}$ for all possible values of $g$ and then comparing these present values with $D_1$.

Stock

Introduction.--Chapter II developed a set of equations for four general cases involving the transfer of securities. The equations are applicable to bonds as well as stocks if the bonds' fair market value and basis never exceed their par or liquidation value. The four general cases result from the following conditions: (1) either the donee will sell the stock at some time in the future or the donee will never sell the stock and (2) either there is adequate cash to pay transfer taxes or part of the stock must be sold to generate cash. Obviously, all securities will be sold at some future time, but if that date is far enough in the future, the effect of capital gains and losses when discounted to time zero will have an immaterial effect on the total present values.

The following definition of the four general cases will be used for organizational structure in the development of this chapter: (1) cash is available to pay any taxes and the stock will never be sold; (2) cash is available to pay taxes and the stock will be sold at some time after the date of death; (3) stock must be sold to pay taxes and the stock will never be sold except when necessary to pay taxes; and (4) stock must be sold to pay taxes and the remaining stock will be sold at some time after the date of death. Due to the fact that the subject matter of this
dissertation is gifts versus estate transfer, sales before the date of
death were not relevant.

Case 1.--If cash is available to pay taxes and the stock will never
be sold, a decision can be made by evaluating equation S2 for an estate
transfer and comparing this present value to the present values obtained
by evaluating equations S9 and S16 for all possible gift dates. As will
be shown, a gift will be advantageous in most cases; however, determining
the optimum year requires evaluating equations S2, S9, and S16.

The following analysis will demonstrate that if cash is available to
pay taxes and the stock will never be sold, and if the value of the stock
at death exceeds the value of the stock at time zero by at least the
amount that the donor can earn on cash after taxes, and if the stock does
not pay dividends, then a gift at time zero will always be advantageous.

The preceding does not state that time zero is the optimal gift year,
only that a gift at time zero is better than an estate transfer. Again,
a reader uninterested in the details of the mathematical proof of the
preceding statement can ignore the following without loss of content.

Under what circumstances will a gift of stock and cash be advanta-
geous? Assuming the stock does not pay any dividends, S2 becomes

\[ E = \frac{(1-\beta)}{(1+i)^d} \left[ V_d + M(1,d) \right]. \]

Letting \( \delta = \alpha + \frac{\beta - \alpha}{(1-\beta)\pi(g+1,\theta)} \), S9 becomes
\[ G = \frac{V_d}{(1+i)^d} + \frac{1}{(1+i)^g} \left[ \frac{M_{1,g} - V_g \delta}{1 + \delta} \right]. \]

\( M \) must be large enough to pay gift taxes of

\[ D_{15}. \]

\[ \alpha V_g. \]

and estate taxes of

\[ D_{16}. \]

\[ (\beta - \alpha) V_g + \beta C_e \pi(g+1,d) \]

\[ \pi(g+1,d) = C_e. \]

Solving for \( C_e \) gives

\[ D_{17}. \]

\[ C_e = \frac{(\beta - \alpha) V_g}{(1 - \beta) \pi(g+1,d)}. \]

Let \( M \) be equal to \( D(16 + 17) \):

\[ D_{18}. \]

\[ M = \alpha V_g + \frac{(\beta - \alpha) V_g}{(1 - \beta) \pi(g+1,d)}. \]

then

\[ D_{19}. \]

\[ M = \delta V_g. \]

Substituting \( \delta V_g \) for \( M \) in \( D14 \) and \( D15 \) gives

\[ D_{20}. \]

\[ E = \frac{(1 - \beta) \left[ V_d + \delta V_g \pi(1,d) \right]}{(1+i)^d}. \]
and

\[ G = \frac{V_d}{(1+i)d} + \frac{1}{(1+i)g} \left[ \frac{\delta V g \pi(1,g) - \delta V g}{1 + \delta} \right]. \]

Will a gift be advantageous at \( g = 0 \)? At \( g = 0 \), \( \pi(1,g) = 1 \) and \( G \) is

\[ G = \frac{V_d}{(1+i)d}. \]

A gift will be advantageous if \( G > E \):

\[ \frac{V_d}{(1+i)d} > \frac{(1-\beta)\delta\pi(g+1,d)}{(1+i)d}. \]

Rearranging gives

\[ \frac{V_d}{V_g} > \frac{(1-\beta)\delta\pi(g+1,d)}{\beta}. \]

Substituting for \( \delta \) and rearranging gives

\[ \frac{V_d}{V_g} > 1 + \frac{\alpha}{\beta} \left[ (1-\beta)\pi(g+1,d) - 1 \right]. \]

If \( V_d = V_g \pi(g+1,d) \), rearranging gives

\[ (g+1,d) > \frac{1 - \frac{\alpha}{\beta}}{1 - \frac{\alpha(1-\beta)}{\beta}}. \]
This inequality will be valid for all cases except $\beta = 0$, which would not be relevant.

A gift of nondividend-paying stock and cash adequate to pay gift and estate taxes will be advantageous if $g + 3 < d$ and $D26$ is valid. As can be seen from $D27$, $D26$ will be valid in most cases. If $V_d < Vg\pi(g+1,d)$, it would be illogical for the donor to hold the security unless large capital gains taxes would have to be paid before the stock could be invested more advantageously.

In the event $g + 3 > d$, $D26$ becomes

$$D28.\quad \frac{V_d}{V_g} > 1 + \alpha + \frac{\alpha}{\beta} \left[(1-\beta)\pi(g+1,d) - 1\right],$$

and $D27$ becomes

$$D29.\quad \pi(g+1,d) > 1.$$

Again, in most situations, the gift will be advantageous.

Case 2.--If cash is available to pay taxes and the donee plans to sell the stock at some time after death, the optimal decision can be found by evaluating equations $S17$, $S19$, and equations $S20$, $S22$, $S27$, and $S29$ for all possible gift dates and choosing the largest present value available. This will require a large amount of computation, but no simple rule of thumb can be readily derived for this case due to the complexities introduced by the capital gains taxes with three possible bases: the donor's original basis, the donee's basis if there is an inter vivos gift, and the donee's basis if there is an estate transfer.
Case 3.—If stock must be sold to pay taxes and the stock will never be sold except when necessary to pay taxes, the optimal decision can be found by evaluating equations S34, S37, S50, S53, and S62 for all possible gift dates and choosing the largest present value available. In some instances, enough cash will be accumulated from dividends to pay gift and estate taxes without selling stock at points in time after time zero. Thus, for some gift dates, what initially was Case 3 will become Case 1. Again, no simple rule of thumb for making a decision can be given.

Case 4.—If stock must be sold to pay taxes and the donee plans to sell the stock at some time after death, the optimal decision can be found by evaluating equations S27, S31, S64, S65, S66, and S67 for all possible gift dates and choosing the largest present value available. In some instances, enough cash will be accumulated from dividends to pay gift and estate taxes without selling stock at points in time after time zero. Thus, for some gift dates, what initially was Case 4 will become Case 2. Again, no simple rule of thumb for making decisions can be given.

Raw Land

In certain limited cases, the equations developed for marketable securities will be applicable to raw land. They will be applicable if the land has a positive cash flow, for example, rent income exceeds payments for property taxes; and the appropriate capital gains tax rate is equal to one half the marginal income tax rate for both donor and donee. The equations developed for marketable securities are limited to a small transfer and in many cases will not be directly applicable to a large transfer such as will normally be encountered in the case of raw land.
Chapter II developed a model for two of many possible types of raw land transfers: (1) a gift of cash and land where the land will never be sold and (2) a gift of cash and land where the donee will sell the land after the expected date of death. These models were designed to accept any size transfer and to permit mortgages and principal payments, negative cash flows, and use two income tax rates for donor and donee, one for ordinary income or loss and one for capital gains. The models were based on the assumption that the donor would always transfer adequate cash to cover any negative cash flow. This would, in many cases, be accomplished by mortgaging the land.

If the donee will never sell the land, an optimal decision can be found by evaluating equation L7 for the estate transfer present value and comparing this to the present value for a gift in the different possible years, which is obtained by evaluating equation L13 for each possible year. If the donee will sell the land at some time after death, the optimal decision can be found by evaluating equations L21, L22, L23, and L24 for all possible dates of gift or sale and choosing the highest present value attainable.

Some Examples

Closely Held Stock

Consider a rather common estate which will consist primarily of cash and closely held stock. The following assumptions will be made: (2) the stock pays no dividends but its fair market value increases 10 percent per year; (2) the parent's after-tax rate of return on cash is 4\% \text{ percent}; (3) the parent's basis is 5 percent of the fair market value at t=0;
(4) the parent's marginal estate tax rate is 70 percent; (5) the parent's average estate tax rate is 60 percent; (6) the parent's marginal income tax rate is 55 percent; (7) the child's marginal income tax rate is 25 percent; and (8) the estate's marginal income tax rate is 30 percent.

This example was processed by the computer program, which simulates the four general cases for marketable securities. Each of the four cases was processed for a marginal gift tax rate of 32 percent and 50 percent and imputed interest rates of 3 percent, 4 percent, 5 percent, 6 percent and 7 percent. The donor's date of death was assumed to be $t = 8$. In the cases where cash and stock were transferred, the date of sale was assumed to be $t = 10$, and the cash at $t = 0$ was set at $\$4$ for $\$1$ fair market value of stock. Although $\$4$ seems to be an unnecessarily high figure, it is close to the minimum necessary to avoid the sale of stock in the later years.

All of these different possibilities resulted in the generation of 900 different present values. The analysis presented in Table III includes a small fraction of the total output.

Recall that Case 1 is where there is adequate cash to pay taxes and the donee will never sell the stock; Case 2 is where there is adequate cash to pay taxes but the donee plans to sell the stock at some point in time after death, either the donee can sell the stock in the future or the donor can sell the stock and make a cash gift; Case 3 is where there is not adequate cash to pay taxes and the donee will never sell the stock except to pay taxes; in this case, three of the possible ways of obtaining cash were developed in Chapter II—the donor sells a fraction of the stock, the donor borrows enough to pay taxes and transfers the stock
### TABLE III

**CLOSELY HELD STOCK**

**3% IMPUTED INTEREST RATE**

<table>
<thead>
<tr>
<th>CASE</th>
<th>Estate Present Value</th>
<th>Optimal Gift Present Value</th>
<th>Optimal Year</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>32% marginal gift tax rate</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>no stock sold</td>
<td>1.8549</td>
<td>2.9542</td>
<td>0</td>
</tr>
<tr>
<td>Case 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>donor sells</td>
<td>1.9895</td>
<td>2.1995</td>
<td>0</td>
</tr>
<tr>
<td>donee sells</td>
<td>1.9895</td>
<td>3.0120</td>
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<td>Case 3</td>
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<td>.4065</td>
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<td>donor borrows to pay taxes</td>
<td>not possible</td>
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</tr>
<tr>
<td>donee pays gift tax</td>
<td>.4065</td>
<td>.5545</td>
<td>0</td>
</tr>
<tr>
<td>Case 4</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>donor sells enough to pay taxes</td>
<td>.4971</td>
<td>.7281</td>
<td>0</td>
</tr>
<tr>
<td>donor sells all</td>
<td>.4971</td>
<td>.3901</td>
<td>0</td>
</tr>
<tr>
<td>donor borrows to pay taxes</td>
<td>not possible</td>
<td></td>
<td></td>
</tr>
<tr>
<td>donee pays gift tax</td>
<td>.4971</td>
<td>.5734</td>
<td>0</td>
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<tr>
<td><strong>50% marginal gift tax rate</strong></td>
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<tr>
<td>Case 1</td>
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<td>donor sells</td>
<td>1.9895</td>
<td>2.4698</td>
<td>5</td>
</tr>
<tr>
<td>donee sells</td>
<td>1.9895</td>
<td>3.2982</td>
<td>0</td>
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<td>Case 3</td>
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<td></td>
</tr>
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<td>donor sells enough to pay taxes</td>
<td>.4605</td>
<td>.7972</td>
<td>0</td>
</tr>
<tr>
<td>donor borrows to pay taxes</td>
<td>not possible</td>
<td></td>
<td></td>
</tr>
<tr>
<td>donee pays gift tax</td>
<td>.4605</td>
<td>.5306</td>
<td>0</td>
</tr>
<tr>
<td>Case 4</td>
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<tr>
<td>donor sells enough to pay taxes</td>
<td>.4971</td>
<td>.8283</td>
<td>0</td>
</tr>
<tr>
<td>donor sells all</td>
<td>.4971</td>
<td>.5403</td>
<td>5</td>
</tr>
<tr>
<td>donee borrows to pay taxes</td>
<td>not possible</td>
<td></td>
<td></td>
</tr>
<tr>
<td>donee pays gift tax</td>
<td>.4971</td>
<td>.5513</td>
<td>0</td>
</tr>
</tbody>
</table>
subject to the loan, or the donee assumes responsibility for payment of gift taxes and after the transfer sells adequate stock to pay the gift taxes; and Case 4 is where there is not adequate cash to pay taxes and the donee plans to sell the stock at some point in time after death, in this case taxes can be provided for by the three methods used for Case 3 and, in addition, the donor can sell all of the stock to pay the taxes and then make a cash gift.

Only data for the 3 percent imputed interest rate are presented. Inspection of the data generated showed that the optimal decision year was not affected by the variation in the imputed rate.

The numbers under the heading "Estate Present Value" represent the highest estate transfer present value. There are two possibilities, the estate sells stock or the donee sells stock. The numbers under the heading "Optimal Gift Present Value" represent the highest present value of any gift year, and the numbers under the heading "Optimal Year" are the gift years in which the optimal present gift occurred.

In Case 1, a gift at time zero is the optimal decision. In Case 2, a gift in year zero of the stock is the optimal decision. In Case 3, the optimal decision is for the donor to sell enough stock to generate cash for taxes and make a gift of the stock to the donee at time zero. In Case 4, the optimal decision is for the donor to sell adequate stock to pay taxes and give the remaining stock to the donee who will sell it at some date after death. The alternative of having the donor borrow cash and transfer stock subject to a loan was not possible in this example because of the low donor's basis. A loan exceeding the donor's basis
would be treated as a sale if the stock were transferred and the model developed in Chapter II does not permit this condition.

It is at first startling to note that the donee's present value is higher for a 50 percent marginal gift tax rate than for a 32 percent marginal gift tax rate. In all instances of the analysis in the dissertation, we are dealing with the last and highest portion of the estate. The uniform rate schedule for estate taxes is cumulative and the tax rates increase as the total amount transferred increases. (See Appendix B.) As a result, if the marginal gift tax rate is less than the marginal estate tax rate, the donor must retain adequate cash after paying the gift taxes and making the gift to pay the future estate taxes. The future estate taxes will be equal to the marginal estate tax rate times the sum of the amount of the gift plus the cash retained to pay the estate tax plus the gift tax paid if death occurs within three years of the date of death. The preceding tax is reduced by a credit equal to the gift tax paid. What occurs is that the beneficial effect of not losing income on gift taxes paid is more than offset by the estate tax on the cash retained to pay the estate tax. The implication of the preceding is that in some circumstances, it will be advantageous to make a series of gifts which will place the donor in the highest marginal estate tax bracket.

There are two main reasons why a gift will be advantageous in year 0. The appreciation on the security from time zero and the date of death is not subjected to the transfer tax, and the gift taxes paid are not subject to a transfer tax.
In planning for the estate in this example, it would be advantageous in all instances to make gifts prior to death. The large amount of cash needed to avoid having to sell stock would suggest that action should be taken to assure the estate's liquidity. If there are adequate cash constraints to make it possible, gifts should be made up to the point where the gift tax rate is equal to the highest marginal estate tax rate. Otherwise, cash retained to pay taxes will also be subject to the estate tax. The potential benefits from inter vivos gifts are extremely large. In this example, the donee's present value with a gift will be from 46 percent to 73 percent greater than with an estate transfer.

**High Basis Marketable Securities**

Consider an estate which will consist primarily of cash and marketable securities in which the donor's basis is 90 percent of their fair market value at time zero. The following assumptions will be made: (1) the stock pays a dividend of 5 percent of its fair market value and the fair market value increases 3 percent each year; (2) the donor's after-tax rate of return is 2.475 percent; (3) the donor's marginal income tax rate is 70 percent; (4) the donee's marginal income tax rate is 25 percent; (5) the estate's marginal income tax rate is 70 percent; (6) the marginal estate tax rate is 70 percent; and (7) the average estate tax rate is 60 percent.

This example was processed by the computer program which simulates the four general cases for marketable securities. Each of the four cases was processed for a marginal gift tax rate of 32 percent and 50 percent, and imputed interest rates of 3 percent, 4 percent, 5 percent,
6 percent, and 7 percent. The donor's date of death was assumed to be \( t=8 \). In the cases where cash and stock were transferred, the date of sale was assumed to be \( t=10 \), and the cash at \( t=0 \) was set at $4. All of these different possibilities resulted in the generation of 900 different present values. The data presented in Table IV presents only a small fraction of the total output.

In this example, the optimum decision is always a gift at \( t=0 \). Again, the gift is extremely advantageous. The donee's present value is 47 percent to 70 percent greater if an inter vivos transfer is made instead of an estate transfer. Inspection of the data generated showed that the optimal decision year was not affected by the variation in the imputed interest rate. The main factors which make a gift advantageous are exclusion of gift taxes from the gross estate and the fact that the appreciation in the stock and the after-tax dividends are not subject to the estate tax.

In planning for the estate in this example, it would be advantageous in all instances to make gifts prior to death. Again, a large amount of cash will be required to avoid selling stock, but, in this case, the estate is fairly liquid and the capital gains taxes that would be incurred are small relative to the example of closely held stock.

**Advantageous Conditions**

Consider the following situation where, (1) the marginal gift tax rate is 32 percent; (2) the marginal estate tax rate is 70 percent; (3) the donor's marginal income tax rate is 70 percent; (4) the donee's marginal income tax rate is 14 percent; (5) the estate's marginal income
<table>
<thead>
<tr>
<th>CASE</th>
<th>Estate Present Value</th>
<th>Optimal Gift Present Value</th>
<th>Optimal Year</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>32% marginal gift tax rate</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 1</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>no stock sold</td>
<td>1.5223</td>
<td>2.4155</td>
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<td>Case 2</td>
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<tr>
<td>donor sells</td>
<td>1.5691</td>
<td>2.1087</td>
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<td>donee sells</td>
<td>1.5691</td>
<td>2.4447</td>
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</tr>
<tr>
<td>Case 3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>donor sells enough to pay taxes</td>
<td>0.3611</td>
<td>0.5434</td>
<td>0</td>
</tr>
<tr>
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<td>0.3611</td>
<td>0.4263</td>
<td>0</td>
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<tr>
<td>Case 4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>donor sells enough to pay taxes</td>
<td>0.3780</td>
<td>0.5556</td>
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<tr>
<td>donor sells all</td>
<td>0.3780</td>
<td>0.4150</td>
<td>0</td>
</tr>
<tr>
<td>donor borrows to pay taxes</td>
<td>0.3780</td>
<td>0.5301</td>
<td>0</td>
</tr>
<tr>
<td>donee pays gift tax</td>
<td>0.3780</td>
<td>0.4358</td>
<td>0</td>
</tr>
</tbody>
</table>

| **50% marginal gift tax rate** |                      |                            |              |
| Case 1                      |                      |                            |              |
| no stock sold               | 1.5223               | 2.7395                     | 0            |
| Case 2                      |                      |                            |              |
| donor sells                 | 1.5691               | 2.4314                     | 0            |
| donee sells                 | 1.5691               | 2.7704                     | 0            |
| Case 3                      |                      |                            |              |
| donor sells enough to pay taxes | 0.3611           | 0.6274                     | 0            |
| donor borrows to pay taxes  | 0.3611               | 0.6075                     | 0            |
| donee pays gift tax         | 0.3611               | 0.4137                     | 0            |
| Case 4                      |                      |                            |              |
| donor sells enough to pay taxes | 0.3780           | 0.6422                     | 0            |
| donor sells all             | 0.3780               | 0.4785                     | 0            |
| donor borrows to pay taxes  | 0.3780               | 0.5617                     | 0            |
| donee pays gift tax         | 0.3780               | 0.4234                     | 0            |
tax rate is 70 percent; (6) the stock's fair market value increases 6 percent per year and the stock pays a dividend equal to 10 percent of its fair market value at the beginning of the year; (7) the donor's basis is 10 percent of the fair market value at \( t = 0 \); (8) the imputed interest rate is 8 percent; (9) the donor dies in year 8; and (10) the stock is sold in year 10. Based on the preceding, 80 present values were generated. Only the optimal and estate values are shown below.

TABLE V

ADVANTAGEOUS CONDITIONS

<table>
<thead>
<tr>
<th>CASE</th>
<th>Estate Present Value</th>
<th>Optimal Gift Present Value</th>
<th>Optimal Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>no stock sold</td>
<td>1.1354</td>
<td>2.6429</td>
</tr>
<tr>
<td>Case 2</td>
<td>donor sells</td>
<td>1.2264</td>
<td>2.0195</td>
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<tr>
<td></td>
<td>donee sells</td>
<td>1.2264</td>
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<td>Case 3</td>
<td>donor sells enough to pay taxes</td>
<td>.3026</td>
<td>.5084</td>
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<tr>
<td></td>
<td>donor borrows to pay taxes</td>
<td>.3026</td>
<td>.5916</td>
</tr>
<tr>
<td></td>
<td>donee pays gift tax</td>
<td>.3026</td>
<td>.4027</td>
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<td>Case 4</td>
<td>donor sells enough to pay taxes</td>
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<td>.5312</td>
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<td></td>
<td>donor sells all</td>
<td>.3346</td>
<td>.2953</td>
</tr>
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<td></td>
<td>donor borrows to pay taxes</td>
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<td>donee pays gift tax</td>
<td>.3346</td>
<td>.4209</td>
</tr>
</tbody>
</table>

In this example, the optimum decision is always a gift in year zero. The choice of a gift results in the donor receiving from 68 percent to 133 percent more than the choice of an estate transfer. Although this
example was chosen to demonstrate extremely advantageous conditions, recall that in the two preceding examples, a gift gave a 40 percent to 70 percent increase in present value over the estate transfer.

Disadvantageous Conditions

Consider the following situation where: (1) the marginal gift tax rate is 32 percent; (2) the marginal estate tax rate is 40 percent; (3) the donor's marginal income tax rate is 14 percent; (4) the donee's marginal income tax rate is 70 percent; (5) the security has a constant fair market value of 1 and yields 8 percent; (6) the estate's marginal income tax rate is 14 percent; (7) the average estate tax rate is 39 percent, the donor's after-tax rate of return in cash is 6.88 percent; (8) the donor's basis is .1; (9) the donor's December 31, 1976 fair market value is $1; (10) the imputed interest rate is 10 percent; and (11) the donor is expected to die in one year and, if the donee sells the stock, it will be at t = 2. Based on the preceding, 26 present values were generated. Only the optimal and estate values are shown in Table VI.

In Case 1, cash is available to pay taxes and the stock will never be sold; the optimal decision is a gift in year zero. In Case 2, cash is available to pay taxes and the stock will be sold at some time after death; the optimal decision is for the donor to immediately sell the stock and make a cash gift to the donee. In Case 3, stock must be sold to pay taxes but the remaining stock will never be sold; the optimum decision is for the donor to transfer the security through the estate. This occurs because the donor and estate have a low income tax rate and there is little appreciation which will escape the estate tax. In Case 4,
stock must be sold to pay taxes and the donee will sell the stock at some time after death; the optimum decision is for the donor to sell all the stock immediately and give the cash to the donee at the time zero.

### TABLE VI

**DISADVANTAGEOUS CONDITIONS**

<table>
<thead>
<tr>
<th>Case</th>
<th>Estate Present Value</th>
<th>Optimal Gift Present Value</th>
<th>Optimal Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>no stock sold</td>
<td>2.9243</td>
<td>2.9772</td>
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<tr>
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<td>donee sells</td>
<td>2.9243</td>
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<td>donor sells enough to pay taxes</td>
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<td>.4301</td>
<td>1</td>
</tr>
<tr>
<td>donor sells all</td>
<td>.5547</td>
<td>.5698</td>
<td>0</td>
</tr>
<tr>
<td>donor borrows to pay taxes</td>
<td>.5547</td>
<td>.3424</td>
<td>1</td>
</tr>
<tr>
<td>donee pays gift tax</td>
<td>.5547</td>
<td>.3185</td>
<td>1</td>
</tr>
</tbody>
</table>

It should be noted that in this example, the gift tax rate and estate tax rate are extremely low. As a result, the donor's low income tax rate made it advantageous for the donor to sell even though by doing so the added basis from gift or estate taxes was lost. In all three cases where a gift is advantageous, the advantage is small. In Case 3, the advantage of a low estate income tax and the increased basis from high December 31,
1976 basis substitution make the estate transfer preferable. However, again the advantage is small.

When this study was initiated, it was presumed incorrectly that the problem facing a practitioner would be to identify those situations when a gift would be preferable and that only when gift tax rates were low would it be advantageous to do so. The preceding examples would indicate that the higher the gift tax rate the more advantageous the gift and that the problem is not to identify advantageous gift situations but to identify the few instances where the estate transfer is preferable.

If the estate's marginal income tax rate is low, then an estate transfer may be preferable. More important, if the donor owns securities which have a basis that is lower than the security's December 31, 1976 fair market value, transferring the security through the estate will result, in some instances, in the security having a higher basis than if it were transferred by gift, particularly if the estate and gift tax rates are low.

Probabilistic Date of Death and Economic Projections

Consider an example where the date of death has a probability of .1 in each of the next ten years and the donee's marginal income tax rate will be either 14 percent, 30 percent, 50 percent or 70 percent, with a probability of .25 for each. Assume that stock will be sold only to pay taxes. Also assume the following: (1) the donor's marginal income tax rate is 10 percent, the marginal gift tax rate is 40 percent, the imputed interest rate is 1 percent, the marginal estate tax rate is 60 percent, the stock pays a dividend of $.01 per year, the market value of the stock
increases $.10 per year. Based on the preceding, 158 pages of computer output were generated and about 880 present values were generated. Only a small fraction of this data is presented.

TABLE VII

COMPARATIVE PRESENT VALUES: GIFT VERSUS ESTATE, AN ILLUSTRATION

Gift when $t$ is equal to: | Expected Present Value of (Gift/Estate):
---|---
$0$ | $.90427 / .63912
$1$ | $.90334 / .63912
$2$ | $.85945 / .63912
$3$ | $.83580 / .63912
$4$ | $.80107 / .63912
$5$ | $.76625 / .63912
$6$ | $.73169 / .63912
$7$ | $.69752 / .63912
$8$ | $.66393 / .63912
$9$ | $.65452 / .63912
$10$ | $.64622 / .63912

The estate expectation was .63912, the gift expectation was .90427 for a gift at time zero, .90334 for a gift at the end of year one, and decreased steadily to year 10. A gift at time zero resulted in the donee always receiving at least 10 percent more than a decision to pass by the estate; there is a 70 percent probability that the donee will receive 40 percent more, and there is a 57.5 percent probability that the donee will receive more than 50 percent more. This example is similar to the
illustration presented at the beginning of this chapter. Tables VII and VIII present the relevant data.

**TABLE VIII**

**PROBABILITY A GIFT PRESENT VALUE WILL EXCEED AN ESTATE TRANSFER PRESENT VALUE: AN EXAMPLE**

<table>
<thead>
<tr>
<th>Expected Present Value of Gift at $t = 0 (Gift/Estate)</th>
<th>Probability of Occurrence</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\geq 1$</td>
<td>1.000</td>
</tr>
<tr>
<td>$\geq 1.1$</td>
<td>1.000</td>
</tr>
<tr>
<td>$\geq 1.2$</td>
<td>0.700</td>
</tr>
<tr>
<td>$\geq 1.3$</td>
<td>0.700</td>
</tr>
<tr>
<td>$\geq 1.4$</td>
<td>0.700</td>
</tr>
<tr>
<td>$\geq 1.5$</td>
<td>0.575</td>
</tr>
<tr>
<td>$\geq 1.6$</td>
<td>0.000</td>
</tr>
</tbody>
</table>

**Large Cash Transfer**

Consider an estate which will consist primarily of cash. The donor is willing to give the donee $3,100,000 net of all taxes either by inter vivos gift or estate transfer. Also, assume the following conditions:
1. the donor will die in six years;
2. the donor has made $500,000 in taxable prior transfers;
3. the donor's taxable estate, exclusive of the $3,100,000, will be $1,000,000;
4. the donor has a 60 percent marginal income tax rate;
5. the donee has a 30 percent marginal income tax rate;
6. the donor's after-tax rate of return on cash is 6 percent and the donee's imputed interest rate is 3 percent.

The computer program generated the presented value of each possible decision. The highest present value was $2,202,249 for a gift in year 3.
The lowest present value was $1,590,987 for an estate transfer. The present value increased during the first three years and then decreased to the date of death. The optimum decision is a gift exactly three years and one day before death.

Raw Land Transfer

Land not sold.—Consider an estate which consists primarily of cash and raw land. The donor is willing to give the donee $3,100,000 in cash and the raw land net of all taxes either by inter vivos gift or estate transfer. Also assume the following conditions: (1) the land has a fair market value of $400,000 at time zero; (2) the fair market value of the land will increase $100,000 per year; (3) the donee will not sell the land; (4) the land has a negative cash flow of $50,000 per year, not including a $10,000 per year principal payment on an original mortgage of $300,000; (5) the donor's marginal income tax rate is 60 percent; (6) the donee's marginal income tax rate is 30 percent; (7) the donor's after-tax rate of return on cash is 6 percent; (8) the donee's imputed interest rate is 3 percent; (9) the donor has made $500,000 in taxable prior transfer; and (10) the donor's taxable estate exclusive of the $3,100,000 in cash and the land is $1,000,000.

The present values generated by the computer were similar to the previous example in relative magnitude. A gift in year three gave a present value of $2,512,199. The estate transfer present value was $1,764,026. Although only a few final numbers were generated by the computer in this example, the number of computations was extremely large.
Failure of Congressional Intent


Your committee believes that it is desirable to reduce the disparity of treatment between lifetime and deathtime transfers through the adoption of a single unified estate and gift tax rate schedule providing progressive rates based on cumulative lifetime and deathtime transfers. However, your committee is retaining part of the incentives for lifetime transfers. Thus, the provisions of present law under which the amount of gift tax is not included or "grossed up" in the transfer tax base are continued, except in the case of gifts made within three years of date of death. In addition, the annual gift tax exclusion of $3,000 per donee is continued.

The advantage of avoiding a transfer tax on the appreciation which might accrue between the time of a gift and the donor's death represents a further incentive for lifetime transfers.

The estate tax provisions relating to transfers in contemplation of death have caused substantial problems for executors, beneficiaries, and the Internal Revenue Service. The presumption that gifts made within 3 years of death are in contemplation of death has caused considerable litigation concerning the motives of decedents in making gifts. Your committee believes that this problem should be eliminated by requiring the inclusion of all such gifts in the gross estate without having to attempt to ascertain the motives of the decedent. Under a unified transfer tax system, this requirement would not affect the tax imposed upon most estates in a major way because the gift taxes paid would be allowed as a credit in determining the net estate tax due.

Since the gift tax paid on a lifetime transfer which is included in a decedent's gross estate is taken into account both as a credit against the estate tax and also as a reduction in the estate tax base, substantial tax savings can be derived under present law by making so-called "deathbed gifts" even though the transfer is subject to both taxes. To eliminate this tax avoidance technique, the committee believes that the gift tax paid on transfers made within 3 years of death should in all cases be included in the decedent's gross estate. This "gross-up" rule will eliminate any incentive to make deathbed transfers to remove an amount equal to the gift taxes from the transfer tax base.1

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This report states that there are three advantages to lifetime transfer: (1) exclusion of gift taxes if the gift is made more than three years prior to death, (2) not paying taxes on the appreciation which occurs between the date of death and the date of the gift, and (3) the $3,000 per recipient per year exclusion. This report states that the "deathbed gift" problem has been eliminated. These statements are not true.

The committee report ignores the potential saving which can occur in avoidance of income taxes both on dividend or interest income and capital gains by making inter vivos gifts. If the donor is in a high marginal income tax bracket and the donee is in a low marginal income tax bracket, substantial savings can occur by reduced income taxes although these benefits are reduced by the loss of imputed interest on the gift tax paid from the date of the gift to the date of death. That inter vivos gifts of cash or cash equivalents will frequently be advantageous has been shown above.

The fact that there is a minimum tax on the capital gains deduction and that the alternative capital gains tax computation is limited to $50,000 gain occurring after 1969, makes the realization of capital gains in the amount of $50,000 or less over a period of years extremely advantageous.

The law has not eliminated the value of deathbed gifts and further has introduced the prospect of deathbed sales. In the case of an estate transfer, the average estate tax rate is used instead of the average gift tax rate paid on the gift. A donee who receives stock by gift has a basis equal to the donor's basis plus the average gift tax for the calendar
quarter paid on the gift times the percentage of the gift that is appreciation. In all instances, the average estate tax rate is less than the marginal estate tax rate. Thus, just prior to death a donor can give a stock with a low basis to the donee and, as a result, the donee will have stock with a higher basis than if it had been received by an estate transfer. The preceding will be true in most cases, but would possibly not be true if the estate were large and prior gifts were minimal.

In some cases, a deathbed gift can be advantageous if the property will be sold by the estate or by the heir. The deathbed gift will tend to be most advantageous when the decedent has a low basis and the gift tax rate is higher than the average estate tax rate. For example, consider a $10 million taxable estate which consists of $5 million of prior transfer, $3 million of cash, and $1 million IBM stock in which the decedent has a basis of $100,000. Also assume the decedent, the estate, and the heir have a taxable income of $200,000. This example ignores the alternative capital gains tax, maximum tax, and income averaging. If the donor makes a deathbed gift, the stock will have a basis of $730,000.\(^2\) If the stock is transferred by the estate, the stock will

\[ \frac{100,000 + 700,000 \times (1,000,000 - 100,000)}{1,000,000} = 730,000. \]

\(^2\)In the case of a gift, the donee's basis is the donor's basis plus a fraction of the gift tax paid. The fraction being the fair market value of the gift minus the donor's basis divided by the fair market value of the gift. The gift tax will be 70 percent of the gift or $700,000. The donee's basis will be:
have a basis of $645,382. In both cases, the sum of estate and gift
taxes will be the same. The difference in basis will increase the total
capital gain by $84,618, from this a 50 percent capital gains deduction
will result in an increase in taxable income of $42,309. The donee will
pay income taxes on this amount at a 70 percent rate. The extra income
taxes will be $29,616, which is the value of the deathbed gift. If the
example had assumed the donee had a low taxable income, the savings would
be greater since the donee could sell the stock before other assets
passed out of the estate and increased his taxable income. Thus, a death-
bed gift could also have a timing effect as well as an increased basis.

A donor whose estate consists mostly of appreciated capital gain
property can reduce his taxes by selling a portion of those assets. The
advantage occurs because the capital gain taxes will reduce the gross
estate. These savings could be substantial if the estate will have to
recognize considerable capital gains in generating cash to pay estate
taxes. The donor who needs cash should also consider transferring stock
subject to a loan just prior to death.

Consider a $3,000,000 estate which consists solely of IBM stock in
which the donor has a basis of $300,000. Assume the donor's taxable
income in the year is $25,000. His estate tax would be $1,290,800. If
he sells $2,000,000 of stock just before death, he will recognize a

3 In the case of an estate transfer, the donee's basis is computed in
exactly the same manner as the gift transfer except instead of using
$700,000 the gift tax paid the average estate tax paid on the gift will
be used. The total taxable estate is $10,000,000, the total estate tax
will be $6,069,800 disregarding credits. The average estate tax paid on
the IBM stock will be $606,980. The donee's basis will be:

$$\frac{100,000 + 605,980 \times \left(1,000,000 - 100,000\right)}{1,000,000} = 645,382.$$
capital gain of $1,800,000. His extra income tax will be $630,590. His minimum tax will be $87,706. These taxes would reduce his estate to $2,281,704. The estate tax would be $918,835. Total taxes would be $1,637,131. If the stock was transferred through the estate, $2,000,000 of stock was sold, and the estate's taxable income was $25,000, the estate would have a capital gain of $1,025,520. The estate would pay an income tax of $718,454. The estate would pay a minimum tax of $23,030. For the estate transfer, total taxes would be $2,032,284. Total potential savings will be $395,153, or 13.2 percent of the total estate.
CHAPTER IV

CONCLUSIONS AND RECOMMENDATIONS

FOR FUTURE RESEARCH

Conclusions

Initial Objectives

The primary objective of this research was to design a model which yields as a product the donee's present value of: (1) property net of taxes if an inter vivos transfer is made, and (2) property net of taxes if the transfer is made by death. This was accomplished in Chapter II. Computer programs were written which simulate the models developed in Chapter II. In Chapter III, typical examples were presented, decision criteria were defined in terms of the equations developed in Chapter II, and in the case of cash, and marketable securities and cash, some general decision rules were derived. In addition, Chapter III examined congressional intent and found the law lacking.

Decision Criteria

In most instances, an inter vivos transfer is preferable to an estate transfer. The optimal decision year in most cases will be time zero or three years and one day before death, assuming stable input variables. Contrary to what one might expect, the value of inter vivos gifts tends to increase rather than decrease as the marginal gift tax rate increases. This research began with the idea of identifying situations where inter vivos transfers are advantageous. It ended with
hunting for disadvantageous conditions. Due to the fact that most real life situations, which contemplate inter vivos gifts, the marginal estate tax rate plus the donor's marginal income tax rate will exceed the donee's marginal income tax rate, an inter vivos gift will be advantageous. One notable exception to this rule is when the donor has property which was acquired before December 31, 1976 and the utilization of special basis treatment for such property will reduce anticipated capital gains taxes. Also, the estate might have a low marginal income tax rate which could be utilized. For example, both the donor and donee might have a 70 percent marginal income tax rate, but the estate will have a marginal income tax rate of 14 percent. By having the estate recognize income, lower income taxes would be paid.

Although the inter vivos transfer will be in most cases preferable, determining the optimal transfer year requires determining present values from the model's equations.

Limitations and Literature Search

It is imperative that in using the models developed in Chapter II a practitioner note that the model assumes one donee and an estate consisting of cash and one other asset. For example, rather than sell a low basis stock to generate cash, the optimum decision might be to sell a high basis stock and transfer a low basis stock. The model for a transfer to a child does not consider the effects of the decreased marital deduction in the event the child has another parent who will also be a donee. The model assumes that gifts included in the gross estate are valued at the gift valuation.
As of June 1977, little had been written specifically about the subject matter of this dissertation. The articles which have been written pay little attention to the subject except to note that the benefits of inter vivos transfers have been decreased and that evaluating these factors is complex. A model for making decisions under the law prior to the 1976 Tax Reform Act was developed by Seagle and Petrie.¹ This model minimized the total of estate and gift taxes and considered only a limited number of variables. Rivers and Crumbley developed a limited model for the new law which minimizes estate and gift taxes but this model has a limited number of variables.² Specifically, it does not consider income taxes in a comprehensive manner. The lack of current references is due to the newness of the topic. The lack of prior studies is most likely due to the fact that under the prior law gifts were so advantageous that the real constraint was the donor's willingness to transfer a large fraction of his assets.

**Failure of Congressional Intent**

In most cases, inter vivos gifts are advantageous. The models developed in Chapter II permit a practitioner to evaluate all relevant variables and make an optimal decision. This will necessitate in most cases a large amount of computation, but this is inherent in the


complexity of the new law. The legal profession has succeeded in writing a law so complex that most of them will find it necessary to extend their academic backgrounds into the quantitative area before they will fully understand what they have written.

The new law has eliminated the litigation which resulted from gifts in contemplation of death. However, it will now be advantageous, as shown in Chapter III, to make deathbed gifts to give the donee a higher basis and to make deathbed sales to generate cash and reduce the total estate by the income tax liability. The committee report which accompanied the new law did not indicate an awareness of these possibilities and indirectly implied that, except for the $3,000 per year exclusion, inter vivos gifts would have limited value. Recall that the examples of a closely held corporation and high basis marketable securities give the donee a 46 percent to 73 percent increase in present value under a gift transfer as compared to an estate transfer.

Continuous Estate Planning

Due to the complexities of the new law and the fact that death no longer washes away potential capital gains, a tax practitioner will have to monitor continuously an estate. In particular, severe bear markets will provide an advantageous opportunity for making gifts and will provide an opportunity to offset gains against losses and thus obtain an estate with a higher basis. If the donor owns property which he can foresee will increase in value at an above average rate, it will be advantageous to transfer the property before the increase is reflected in current market values. Changes in the client's health away from the
norm for his age will necessitate changes or possible alteration of
gifts.

Recommendations for Future Research

This dissertation has addressed only the simple aspects of decision
making for gifts under the Tax Reform Act of 1976. Analyses similar to
Chapter II need to be performed on depreciable assets, ordinary income
property such as inventory, property subject to the special estate valu-
ation rules for land, and the complexities of the new law as it affects
generation skipping trusts.

Except for cash and raw land, this dissertation has considered only
the transfer of a small amount of marketable securities. A model for
transfers large enough such that the marginal gift tax rate, the marginal
estate tax rate, the donor's marginal income tax rate, and the donee's
marginal income tax rate change needs to be developed. Note that a large
transfer could result in a drop in the highest marginal estate tax rate.
For example, the gift taxes paid could lower the total estate from
$5,000,000 to $4,000,000, thus reducing the marginal estate tax rate from
70 percent to 69 percent.

The general problem of what is the best sequence for various types
of gifts needs to be explored. Many estates are going to encounter a
cash shortage. A model to ascertain the best method of obtaining this
cash is needed. This problem is tied to the gift sequence problem. A
general model that will cover all types of assets and will yield as an
output the proper sequence of gifts and specify the optimal method of
obtaining cash while minimizing all taxes based on an imputed interest
rate, a probability density function for the date of death, and a probability density function for the possible projections of variables through the date of death is needed.

The models developed in Chapter II are only useful if one has as an input the variables they require. There is a vast area of empirical research needed which will form a basis for predicting future market values and imputed interest rates that need to be explained.
APPENDIX A

INFORMAL GLOSSARY OF SYMBOLS FOR MARKETABLE SECURITIES

α  Current marginal gift tax rate

β  Highest marginal estate tax rate

ζ  Fraction of stock that must be sold to pay gift, capital gain, and future estate taxes

ζ_g+3>d  Same as ζ except the gift will occur within three years of the date of death. Under these circumstances, a larger fraction will have to be sold since any gift taxes paid will be included in the taxable estate.

μ  Fraction of stock the estate must sell to pay estate and capital gains taxes.

Σ(x,y,i)  Present value of child's dividends net of income tax from time x to time y.

Σ(x,y,p)  The amount of dividends net of taxes plus interest at the parent’s after-tax rate of return on cash from time x (0 in most cases), to time y, the date of gift or death.

π(x,y)  The amount $1 at time x would become at time y if interest were compounded annually at the parent's after-tax rate of return on cash.

bp  Parent's current basis for determining capital gain or loss

bd  Basis if stock passes through estate (see S18)

bg  Basis for inter vivos transfer where the parent pays the gift tax (see S21)

bc  Basis for inter vivos gift where parent passes stock subject to a loan (see S47)

g  Cash donee will receive along with stock for an inter vivos gift in year g.

gg+3>d  Cash donee will receive along with stock where gift taxes are included in gross estate because death occurs within three years of the date of death.
$C_s$ Cash donee will receive if donor sells all stock and makes a cash gift.

$C_{s+3>d}$ Same as $C_s$ except donor dies within three years of the date of death.

$r_i$ Donee's after-tax imputed interest rate

$r_{k}$ Dividends in year $k$ paid on the fraction of one share of stock represented by $V_0$

$r_j$ Parent's after-tax rate of return on cash in year $l$.

$L$ Loan necessary to pay gift taxes and future estate taxes when stock is transferred subject to a loan.

$V$ Number of shares of stock which have a fair market value of $1$ at time, $t = 0$.

$V_0$ Fair market value of $V$ at time $t = 0$

$V_d$ Fair market value of $V$ at the date of death

$V_g$ Fair market value of $V$ at the date of gift

$V_s$ Proceeds after sales commission, if any, from the sale of $V$ in year $s$.

$\ell_p$ Parent's marginal income tax rate in year $k$

$\ell_c$ Child's marginal income tax rate in year $k$

$\ell_e$ Estate's marginal income tax rate in year $d$
### APPENDIX B

#### TABLE IX

**UNIFIED ESTATE AND GIFT TAX**

<table>
<thead>
<tr>
<th>Taxable Estate</th>
<th>Tax Rate</th>
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