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NATURAL THERMAL STRATIFICATION IN TANKS

Phase 1 Final Report

by

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ABSTRACT

This report describes how solar system tanks fail to stratify, a new solar system control strategy that allows stratification, a one-dimensional analytical model of thermally stratified tanks, experimental measurement of thermal stratification in tanks, correlation of experimental measurements with empirical constants in the analytical model, and a procedure for designing thermally stratified tanks.

Failure to stratify is explained in terms of the critical Richardson number. The key to the new control strategy is to avoid a Richardson number that decreases during solar collection. The analytical model is an approximate solution based on assumptions that 1) the solution is a function of elevation and time, only, 2) plug flow exists, 3) flow rate is constant, 4) the cross-sectional areas of the tank and tank wall are constant 5) there is a step change of inlet temperature, 6) there is heat transfer between the tank wall and the water, and 7) thermal losses from the tank are negligible. Empirical constants in the theory are determined by adjusting them until the best least-squares fit with experimental data is obtained and correlating the constants with the Fourier and Richardson numbers.

The new control strategy allows tanks to stratify and reduces the average collector operating temperature by as much as 20°C. The result is up to 20% improvement in system performance. The analytical model and correlation of its empirical constants allow system designers to select from several alternatives the tank configuration that provides the best stratification.

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1. INTRODUCTION

1.1 Contents of This Report

The purpose of this report is to show how thermal storage tanks in solar systems fail to stratify, to show how to modify the system so that the benefits of stratification can be obtained, and to provide a procedure for designing the system so that stratification can be obtained. Although the report is written primarily for solar systems, much of the material can be applied to load management systems.

This report consists of five main sections and three appendices. Section 1: Introduction describes the contents of the report and contains a review of literature on thermally stratified storage. It also tells how systems fail to stratify and describes a new collection strategy that allows stratification to occur. Section 2: Analytical Model develops a one-dimensional analytical model of stratification and defines a stratification index by which the performance of tanks can be compared. Section 3: Description of Experiments describes the flow loop, the tanks, the instrumentation, and what experiments were performed. Section 4: Experimental Results describes what results were obtained and how they relate to designing stratified tanks. Section 5: Conclusions and Recommendations highlights specific design features that can improve or degrade stratification and makes recommendations for future work.

Appendix A: Designing Stratified Tanks is intended to be a stand-alone article that could be submitted to a trade magazine or journal. Appendix B: Summary of Data is a tabulation of the important parameters and results for each data point. Appendix C: Stratification Index Tables is a tabulation of stratification index as a function of heat capacity ratio, Fourier number, and Richardson number. Also included in Appendix C is a computer program that can be used to calculate stratification index.

1.2 Literature Review

Theory

Thermal stratification is closely related to hydrodynamic stability and buoyancy effects which have a considerable amount of literature. Monographs by Chandrasekhar [1] and Turner [2] indicate that the Richardson number Ri plays an important role in the formation and growth of the thermocline.

$$Ri = \frac{\beta g L \Delta T}{u^2} \quad (1.1)$$

where g = gravitational acceleration
 β = volumetric thermal expansion coefficient of the fluid

L = depth of the fluid
 ΔT = top-to-bottom temperature difference
u = inlet velocity

Reference [1 and 2] indicate that mixing occurs for Richardson numbers less than the critical value of 0.25. A paper by Lorentz and Howard [3] also helped to understand the mechanisms for the growth of the thermocline. A heat transfer textbook [4] provides an estimate of the spread of the thermocline in a motionless liquid.

There have been several computer models developed for predicting performance of thermally stratified storage [5-9]. The paper by Cabelli [9] includes a 1-dimensional analytical model which accounts for thermal losses from the tank but does not account for the thermal mass of the tank wall.

Lunde [24]* has compared stratified systems with well mixed systems and has shown that a stratified system with several "turnovers" of storage per day behaves as though the tank were completely mixed. He also claims that for one storage "turnover" per day the advantages of stratification are offset by poorer heat transfer in the collector. But, as we point out later in this report, a minor plumbing change can raise the flowrate in each collector to a rate that is close to the conventional flow rate.

Papers by Veltkamp [10] and Rademaker [11] have shown that the collection strategy plays an important role in the utilization of the benefits of stratification. Veltkamp states that thermally stratified storage can increase the output of real systems by 20% or more and that conventional systems utilize collector flowrates that are three times too large. The prospect of a 20% performance improvement is the main incentive for the work that led to this report.

Experiments

The work of Phillips and Pate [12] showed that viscous entrainment is an important mechanism in the development of the thermocline. Their paper also presented a computer model of stratification.

Lavan and Thompson [13] did a detailed experimental study of stratified domestic water tanks and developed an empirical model of tank efficiency as a function of Reynolds number, Grashof number, and height-to-diameter ratio. They did not study the effect of Richardson number on tank performance. Their data show that tank efficiency levels off for height-to-diameter ratios greater than 3 or 4, and they concluded that the optimum height-to-diameter ratio was in that range.

Loehrke, et. al. [14-16] have developed an inlet diffuser that introduces a fluid into the tank at an elevation corresponding to the fluid temperature. This allows the solar collector to operate with a variable outlet temperature, but as we show later in this report a constant outlet temperature is more desirable.

*We are grateful to Professor McGarity for pointing out Lunde's analysis.

Sandia Laboratory [17-18] has experimented with a high temperature (about 600°F) storage system. They have identified three of the causes of thermocline degradation as mixing caused by the diffuser, vertical conduction in the metal tank walls, and thermal diffusion from the hot layer to the cold layer.

Sliwinski, et. al. [19] conducted experiments on a thermally stratified tank and related the results to the Richardson number. Their experiments confirm theories that there is a critical Richardson number of about 0.25 below which mixing occurs.

A particularly dramatic example of the effect of the critical Richardson number can be found in [20]. Although the authors of [20] were apparently unaware of the importance of the Richardson number we calculated the Richardson number from their data. The section that follows explains how systems fail to stratify.

How Systems Fail to Stratify

Examination of Figure 1.1 (Figure 8 of reference [20]) shows that the Forest City Dillon Apartments storage tank is stratified when startup occurs at 09:11 in the morning. At 09:43 the tank remains stratified, but at 10:10 the shape of the profile changes as the tank begins to mix. By 11:09 the tank is completely mixed, and it remains mixed for the rest of the day.

At 09:11 the pump draws 10°C water from the bottom of the tank. Since the flowrate is nearly constant regardless of the rate energy is delivered by the collector and at 09:11 the collector delivers very little energy, the water is returned to the tank only slightly warmer than 10°C.* The return water being denser than the 40°C water at the top of the tank sinks to the bottom with relatively little mixing. In this manner the temperature at the bottom of the tank increases while the remainder of the temperature profile changes little. The Richardson number at 09:11 is 0.55.

The curve at 09:43 shows that the bottom temperature has risen to 24.5°C but the remainder of the temperature profile has not changed significantly. At this time the Richardson number of 0.28 is larger than the critical value of 0.25. Since the return temperature is about 25.5°C, the return water sinks and continues to warm the bottom of the tank. The Richardson number which is proportional to the top-to-bottom temperature difference continues to decrease.

At 10:10 the ΔT has decreased to 12°C, and the Richardson number has decreased to 0.22. As Figure 1.1 shows, the tank has begun to mix. By 11:09 the tank is nearly isothermal which ensures a Richardson number less than about 0.06. The tank can not be restratified until the pump is turned off and a ΔT develops. Other figures found in [20] gave similar results.

*It should be noted that the return temperature indicated for 09:11 at the top of Figure 1.1 is higher than it should be. The return temperature sensor was sensing the temperature at the top of the tank before the pump was turned on. At 09:11 the sensor had not stabilized to the true return temperature.

Figure 1.1. Temperature Profile Measurements as a Function of Time.

Source of Data: M. J. Kennedy, S. J. Sersen, and S. M. Rossi (Vitro Laboratories, Silver Spring, Md.), "Comparison of Liquid Solar Thermal Storage Subsystems in the National Solar Data Network," ASME Paper Number 80/WA/SOL-36 (November 1980).

The cause of the problem is clearly the collection strategy. The collection strategy causes the top-to-bottom ΔT (and hence, the Richardson number) to decrease during the morning and has an excessively high flow rate. The high flow rate appears as a squared factor in the denominator of the Richardson number.

Although the collection strategy used at the Forest City Dillon Apartments is the same strategy that has been widely recommended for solar systems [21], we have shown for the first time in print that such a strategy is incompatible with thermal stratification in tanks. Therefore, we recommend changing the collection strategy in the following three ways:

1) Decrease the flowrates so that no more than one tank volume passes through the collector or collector heat exchanger per day. This part of the new strategy has been recognized by others [10, 11, 24].

2) Vary the flowrate so that the water returns to the top of the tank at a constant temperature. This can be done by using a centrifugal pump and installing a temperature-sensing valve similar to an automobile thermostat at the outlet of the collector. An antifreeze system requires a second temperature sensing valve at the outlet of the collector heat exchanger.

3) Adjust the collector outlet temperature so that it is equal to the required end-use temperature plus a few degrees for thermal losses and heat exchange processes. Thus, the collector always operates at the lowest practical temperature which may be as much as 40°F less than the average operating temperature with the conventional strategy. This change in the operating strategy is responsible for the improvement in performance attributed to thermal stratification, but it can not be implemented until the first two changes have been accomplished.

Several observations can be made about the effect of the new strategy on the system.

1) Pumping power requirements are much less than for the conventional strategy because the flowrates are smaller by a factor of five or more.

2) Because the collector raises the water to operating temperature in one pass instead of several, a larger number of collectors can be plumbed in series than with the conventional strategy. This means that there will be fewer parallel circuits, and the flow distribution between parallel circuits will be better. Moreover, the flow in each collector will be comparable to the flow with the conventional strategy. This overcomes Lunde's objections to stratification [24].

3) The temperature of the water returning from the load to the bottom of the tank will be nearly constant in most cases. For some systems, however, a temperature-sensing valve in the return line may be necessary. This valve, which controls the temperature at the bottom of the tank, should open when the returning water is colder than the set point.

4) A small amount of leakage in the temperature-sensing valves will allow them to sense the temperature more accurately when they are closed.

5) The manifold described in [14-16] is of no use because the water returns from the collector at a constant temperature equal to the hottest temperature in the tank.

6) The new collection strategy depends on maintaining a sharp thermocline between the hot and cold water in the tank. The remainder of this report describes the theory and experiments that were done and how the results can be used to design stratified tanks.

2. ANALYTICAL MODEL

In this section we derive an approximate expression for the temperature in the tank as a function of position and time. We assume the following:

- 1) The solution is a function of elevation and time, only.
- 2) Plug flow is assumed. The theory does not apply to regions near the inlet and outlet where mixing and other 3-dimensional effects occur.
- 3) The flow enters the tank at a constant rate.
- 4) The cross-sectional areas of the tank and the tank wall are constant as a function of elevation. Vertical cylindrical tanks are allowed by this assumption which also allows generalization to tanks with noncircular bases. Horizontal cylindrical tanks are excluded from consideration by this assumption.
- 5) The tank is subject to a step change of temperature such that cold water enters the bottom of the tank during discharging or hot water enters the top of the tank during charging.
- 6) Heat transfer between the fluid and the tank wall is allowed. The amount of heat contained in the tank wall is small compared to the amount of heat contained in the fluid.
- 7) There are no thermal losses from the tank.

Also in this section we define a stratification index S such that a perfectly stratified, lossless tank has $S = 1$ and a completely mixed, lossless tank has $S = 0$. The stratification index allows easy comparison between theory and experiment and between different operating conditions. As it is presently defined the stratification index requires the fifth assumption above.

2.1 One-Dimensional Theory

Development of Theory

The one-dimensional theory describing thermal stratification was developed from first principles. Figure 2.1 is a schematic of the tank with control volume 1 representing the water from elevation x to $x + dx$. Energy balances on the control volumes yield two coupled partial differential equations in elevation x and time t . The equation for control volume 1 (the water) is:

$$k_1 A_1 \frac{\partial^2 T_1}{\partial x^2} - \rho_1 C_{p1} A_1 v \frac{\partial T_1}{\partial x} - \rho_1 C_{p1} A_1 \frac{\partial T_1}{\partial t} = -hL(T_2 - T_1) \quad (2.1)$$

Figure 2.1. Control volumes for theory

The equation for the control volume 2 (the tank wall) is:

$$k_2 A_2 \frac{\partial^2 T_2}{\partial x^2} - \rho_2 C_{p2} A_2 \frac{\partial T_2}{\partial t} = h \ell (T_2 - T_1) \quad (2.2)$$

where k = thermal conductivity
 A = cross-sectional area
 T = temperature
 ρ = density
 C_p = heat capacity
 v = velocity of water in the tank
 x = elevation above tank bottom for discharging or depth below water level for charging
 t = time
 h = film coefficient
 ℓ = circumference of tank
subscript 1 refers to the water, and
subscript 2 refers to the tank wall.

The initial temperature of the water and the tank wall is uniformly T_s . At time $t = 0$ the inlet temperature changes instantaneously to T_i . A third boundary condition states that no heat can flow past the inlet $x = 0$. These boundary conditions are expressed mathematically as:

$$\begin{aligned} T_1(x, 0) &= T_2(x, 0) = T_s \\ T_1(0, t) &= T_2(0, t) = T_{in} \text{ for } t \geq 0 \\ \frac{\partial T_1(0, t)}{\partial x} &= \frac{\partial T_2(0, t)}{\partial x} = 0 \end{aligned} \quad (2.3)$$

Because of the coupling of equations 2.1 and 2.2 through the film coefficient term $h \ell (T_2 - T_1)$, no solution for the complete set of equations could be found. Instead, the equations are decoupled by setting $h = 0$ in equation 2.1 and an approximate solution for nonzero h will be developed later in this section. Equation 2.1 was solved by Laplace transforms for uniform cross-sectional area A and $h = 0$, to yield:

$$\theta = \frac{1}{2} \left[1 + \operatorname{erf} \left(\frac{\frac{x}{L} - \frac{t}{t_r}}{2 \sqrt{\frac{x}{L}} \sqrt{\frac{t}{t_r}}} \right) \right] \quad (2.4)$$

where

$$t' = \frac{\rho_1 A_1 L C_{p1}}{\rho_1 A_1 v C_{p1}} \quad = \text{time constant}$$

$$F_o = \frac{k_1 t'}{\rho_1 C_{p1} L^2} \quad = \text{Fourier number}$$

$$\theta = \frac{T - T_{in}}{T_s - T_{in}} \quad = \text{normalized fluid temperature}$$

L = tank height

Modified Theory

Two modifications of equation 2.4 improve its usefulness. 1) Since the experiment measures the effect of the heat contained in the tank walls as well as the heat contained in the water, the effect should be included in the time constant. Therefore, a modified time constant t^* is defined.

$$t^* = \frac{\sum_{i=1}^n \rho_i A_i L C_{pi}}{\rho_1 A_1 v C_{p1}} \quad (2.5)$$

2) The effect of mixing at the inlet and outlet can be accounted for by including an empirical constant c in the solution. Adding c to the t/t' term in equation 2.4 causes the thermocline to broaden as it does in the mixing regions at the inlet and outlet, but adding c to t/t' has the side effect of changing the position of the thermocline by an amount equal to c . Therefore, if c is added to t/t' to broaden the thermocline, c must also be added to x/L to bring the thermocline back to its original position. The two c 's in the numerator are of opposite sign and cancel yielding equation 2.6.

$$\theta = \frac{1}{2} \left[1 + \operatorname{erf} \left(\frac{\frac{x}{L} - \frac{at}{t^*}}{2 \sqrt{F_o} \sqrt{\frac{at}{t^*} + c}} \right) \right] \quad (2.6)$$

where $a = t^*/t'$

Wall Heat Capacity Approximation

The effect of wall heat capacity is approximated by assuming that T_1 in Figure 2.1 has the functional form of equation 2.6 and solving equation 2.2 to calculate the temperature difference $T_2 - T_1$. The heat flux at the wall which is proportional to $T_1 - T_2$ is integrated to determine a small addition ΔT_1 to the water temperature T_1 .

Two simplifications were necessary to solve the equations. The first is to neglect conduction in the x -direction within the tank wall, that is to set $k_2 = 0$ in equation 2.2. The effect of conduction along the wall is believed to be small compared to the effect of wall heat capacity. To affect the water temperature via the conduction path, the heat must suffer a film resistance as it passes from the water to the wall, a resistance as it conducts along the wall, and another film resistance as it passes from the wall back to the water. By contrast, heat that is initially in the tank wall needs to pass through only one film resistance to affect the water temperature.

The second simplification is to assume that the denominator in the error function of equation 2.6 is a constant.

$$\theta = \frac{1}{2} \left[1 + \operatorname{erf} \left(\frac{\frac{x}{L} - \frac{at}{t^*}}{c} \right) \right] \quad (2.7)$$

where $c \approx 2\sqrt{F_0} \sqrt{\frac{at}{t^*} + c}$

Examination of equation 2.6 shows that for $x/L = 1$, $\theta = 1$ for at/t^* less than about 0.9, and $\theta = 0$ for at/t^* greater than about 1.1. The value of c is typically between 0.2 and 10. In the worst case the value of $\sqrt{at/t^* + c}$ differs from a constant by only about $\pm 5\%$ for $0.9 \leq at/t^* \leq 1.1$. Therefore, the approximation of equation 2.7 is a good one. Solution of equation 2.2 subject to the two simplifications gives the following result for the temperature difference $T_2 - T_1$:

$$\frac{T_2 - T_1}{T_s - T_{1n}} = \frac{1}{2} e^{\frac{\left(\frac{HC^2}{2a}\right)^2 + \frac{2x}{L} \left(\frac{HC^2}{2a}\right)}{C^2} - \frac{Ht}{t^*}} \left[\operatorname{erf} \left(\frac{\frac{x}{L} + \frac{HC^2}{2a}}{C} \right) - \operatorname{erf} \left(\frac{\frac{x}{L} + \frac{HC^2}{2a} - \frac{at}{t^*}}{C} \right) \right] \quad (2.8)$$

where $H = \frac{h^* t^*}{\rho_2 C_{p2} A_2}$ is the normalized film coefficient.

Equation 2.8 allows an approximate value for heat flux $\dot{q} dx = h l (T_2 - T_1) dx$ shown in Fig. 2.1 to be calculated. The accuracy of the approximation improves as the heat capacity of the wall becomes smaller.

The last step is to compute the small addition δT_1 to the water temperature due to the heat flux $\dot{q}$. The rate of change of temperature in control volume 1 due to the heat flux is:

$$\frac{d(\delta T_1)}{dt} = \frac{\dot{q}}{\rho_1 A_1 C_{p1}} \quad (2.9)$$

Replacing the stationary control volume 1 by control volume 1' that moves along with the fluid is convenient for integrating equation 2.9. This is accomplished by replacing the coordinate x by $x' + vt$ where x' is the coordinate that travels with the fluid. The result of replacement of x by $x' + vt$, integration, and replacement of x' by $x - vt$ is:

$$\Delta = \frac{1}{2} H \left(\frac{a-1}{a} \right) \left\{ e^{\frac{\left(\frac{HC^2}{2a} \right)^2 + \frac{2x}{L} \left(\frac{HC^2}{2a} \right)}{C^2} - \frac{Ht}{t^*} \left(\frac{x}{L} + \frac{HC^2}{2a} \right)} \left[\operatorname{erf} \left(\frac{\frac{x}{L} + \frac{HC^2}{2a}}{C} \right) - \operatorname{erf} \left(\frac{\frac{x}{L} + \frac{HC^2}{2a} - \frac{at}{t^*}}{C} \right) \right] \right. \\ \left. + \frac{C}{\sqrt{\pi}} \left[e^{-\left(\frac{x^2}{L^2 C^2} + \frac{Ht}{t^*} \right)} - e^{-\left(\frac{\left(\frac{x}{L} - \frac{at}{t^*} \right)^2}{C^2} \right)} \right] \right\} \quad (2.10)$$

where $\Delta = \frac{\delta T_1}{T_s - T_{in}}$ is the normalized addition to the water temperature.

Therefore, the total normalized water temperature is approximately

$$\delta_T = \theta + \Delta \quad (2.11)$$

The constants c in equation 2.6 and H in equation 2.10 are empirical and must be determined by experiment. Equations 2.6, 2.10, and 2.11 are plotted in Figure 2.1 to show the shapes of the curves and the effect of wall heat capacity.

2.2 Stratification Index

We have defined a stratification index that allows comparison of different tanks. In defining the stratification index it is convenient to work with the normalized tank outlet temperature θ_o defined as:

$$\theta_o = \frac{T_o - T_{in}}{T_s - T_{in}} \quad (2.12)$$

where T_o is the tank outlet temperature as a function of time.

Therefore, a measure of the amount of heat removed in one time constant is:

$$I = \frac{1}{t^*} \int_0^{t^*} \theta_o dt \quad (2.13)$$

For perfectly stratified lossless tank $I = 1$.

A completely mixed tank is the opposite of a perfectly stratified tank. A lossless completely mixed tank has an exponential decay of outlet temperature so that $I = 1 - e^{-1} = 0.632$.

The stratification index S is defined so that it will range from 0 for a completely mixed lossless tank to 1 for a perfectly stratified tank.

$$S = \frac{I - 0.632}{0.368} \quad (2.14)$$

Negative values of S are possible if there is an excessive thermal loss or a direct short circuit from the tank inlet to the tank outlet.

Figure 2.2. Comparison of Equations 2.6, 2.10, and 2.11.

Carrying the integral of equation (2.13) beyond t^* until θ becomes negligible results in a measure of amount of energy removed from the tank. If the integral is carried out sufficiently far it should equal unity minus the normalized thermal losses from the tank. This energy balance integral is used as a check on the accuracy of the experiments.

3. DESCRIPTION OF EXPERIMENTS

This section describes the equipment used to perform the experiments and the operation of the equipment during the performance of the experiments. The equipment consists of a flow loop, tanks, and instruments which are discussed in detail below.

3.1 Flow Loop

The flow loop is shown schematically in Figure 3.1. It consists of two pumps, three flowmeters, heaters, and the control valves necessary to conduct the experiments.

A gear pump driven by a variable-speed motor is used to provide flows of 0-7.5 ℓ /min (0-2 gpm). The larger pump is a variable-displacement pump driven by a constant-speed motor. It provides flows up to 75 ℓ /min (20 gpm). Bypass and shut-off valves provide fine control of the flowrate of the larger pump. The three flowmeters are described in Section 3.3.

Six 5-kW heaters and two 1-kW heaters provide a total of 32-kW of electric heat. Smaller amounts of heat can be provided by disconnecting some of the heaters. A safety circuit shuts off power to the heaters and sounds an alarm if flow is interrupted or the temperature at the heat outlet exceeds a preset limit.

Complete control over the flow is provided by ball valves. Pump suction can be taken from any tank in the loop and directed to any tank in the loop or down the drain. The flow in any test tank can be upward for discharge tests or downward for charge tests except in the flow visualization tank. Either 3000 ℓ (800 gal) tank can supply the water used in the experiments.

A typical discharge test requires the following operations which are controlled by the valves:

- 1) Fill the supply tank with cold water.
- 2) Fill the test tank with water and circulate the test tank water through the heater to raise it to the required initial temperature.
- 3) Circulate cold water from the supply tank through the pipes and back to the supply tank with the heaters off to cool the pipes and flowmeters.
- 4) Fill and empty the calibration tank several times to check the flowmeter calibration.
- 5) Circulate cold water from the supply tank to the bottom of the test tank to do the test. The outflow from the test tank goes to the drain. It may be necessary to add tap water to the supply tank during the test.
- 6) Repeat step 4 to check the flowmeter for drift.

Figure 3.1. Schematic of Loop

Interrupted discharge tests were also performed. The interrupted discharge tests are similar to the normal discharge tests except that after 50% of the tank volume has been discharged the tank is allowed to stand idle for approximately five hours before discharging is allowed to go to completion.

Two types of quiescent mode tests were performed on the 3000- ℓ tank. In the first the fully charged tank was allowed to stand idle for 93 hours while the temperatures within the tank were measured. The second type of quiescent mode test was an interrupted discharge test in which the tank was allowed to stand idle for 40 hours while the temperatures were measured.

The operations for a charge test are similar to those of a discharge test except that the supply tank is initially hot, the test tank is initially cold, and the test tank is filled from the top.

3.2 Tanks

Five tanks were used for the tests. They are a 190 ℓ (50 gal) cylindrical tank, a 3000 ℓ (800 gal) cylindrical tank, a 3000 ℓ rectangular tank, a flow visualization tank, and a calibration tank.

The 190 ℓ tank shown in Figure 3.2 is approximately the size and shape of a domestic water heater. It has an inside diameter of 39.4 cm (15.5 in) and a height of 157.5 cm (62 in) for an L/d ratio of 4.0. The steel wall is 0.63-cm (0.25 in) thick. The tank is covered with 15 cm (6 in) of fiberglass insulation. Six instrumentation fittings are installed at each of the 8%, 21%, 50%, 79%, and 92% elevation levels in the tank for a total of 30 fittings. Ten thermocouples are installed in each of the 8%, 21%, 79%, and 92% elevation levels for a total of 40 thermocouples. This 3-dimensional thermocouple array is shown in Figure 3.3.

Four one-inch pipe couplings are available for input and output connections. The top and bottom covers each have a coupling in the center, one coupling is located in the side centered 5.7 cm (2.25 in) above the bottom cover, and one coupling is located in the side centered 5.7 cm below the top cover. The baffle and diffuser configurations shown in Figure 3.4 were tested in the 190 ℓ tank.

The 3000 ℓ cylindrical tank shown in Figure 3.5 is approximately the size used for single-family residence space heating applications. It has a diameter of 183 cm (6 ft) and a height of 122 cm (4 ft) for an L/d ratio of 0.601 based on actual water depth. The tank wall consists of 3 mm (0.125 in) fiberglass staves held in place by tensioning bands. Because the rubber sealant failed to seal the staves to the bottom, a 1 mm (0.040 in) PVC liner was installed. The tank is insulated with 15 cm of fiberglass. An array of 36 thermocouples shown in Figure 3.6 is installed in the tank.

The inlet at the bottom of the 3000-liter cylindrical tank consists of a 2-inch bulkhead fitting in the wall 15 cm (6 in) above the bottom of the tank. Two 90° elbows form an S-bend inside the tank so the height of the inlet nozzle can be adjusted. Nozzles ranging from 2.5-10 cm (1-4 in) were used to study the effects of different inlet velocities. A plastic mesh serves as a flow restrictor to ensure uniform flow across the nozzle diameter. The flow restrictor was necessary for the larger nozzles.

Figure 3.2. Photograph of 190-l Tank
with Insulation Partly
Removed

Figure 3.3. Three-dimensional Thermocouple Array in a 190-l Tank. Dimensions are in centimeters. Each of the four elevations contains ten thermocouples for a total of 40 thermocouples.

Figure 3.4. Baffle and Diffuser Configurations Tested in the 190-1 Tank.

d. Single Horizontal Baffle

e. Dual Radial-flow Diffusers

Figure 3.4 continued

Figure 3.5. Photograph of 3000-l Tank.

The outlet at the top of the 3000-liter cylindrical tank consists of a two-inch bulkhead fitting in the wall 112.5 cm (44.25 in) above the bottom of the tank. A pipe conducts the flow by gravity to a drain. No special nozzles were connected to the outlet.

A 3000 l (800 gal) rectangular tank shown in Figure 3.7 was used for supply water for the tests. The tank is constructed of 2 x 6 lumber, 1/2-inch plywood, and a 40-mil thick PVC liner. Inside dimensions are 120 cm wide x 213 cm long x 120 cm deep (4 ft x 7 ft x 4 ft). Although no stratification tests on this tank were done, such tests could be performed in the future.

The flow visualization tank is a 61 cm (2 ft) cube of 6.3 mm (0.25 in) thick clear acrylic plastic. No insulation was installed so that the flow of dyed water could be observed. One rake of thermocouples spaced 6.3 mm apart provided temperature measurement although the principal means of observation was visual. Two types of inlets were used. One consisted of a 12.7 mm (0.5 in) O.D. copper tube bent to project the water horizontally. The height of the inlet could be adjusted. The second inlet is a 10 cm (4 in) diameter radial-flow diffuser located at the bottom of the tank. No outlet was provided because the purpose of the flow visualization experiments was to study the mixing zone at the bottom of the tank.

Unlike the other tanks, the flow visualization tank was not connected to the flow loop. Instead, the gear pump was removed from the flow loop and used to pump potassium permanganate-dyed water from a 19 l (5 gal) can into the flow visualization tank.

The calibration tank consists of the flow visualization tank set on a scale. Rubber hoses connected to the flow loop serve as fill and suction lines.

3.3 Instrumentation

Instruments consists of flow measurement equipment, temperature measurement equipment, and a datalogger. A 35 mm camera was also used to photograph the flow visualization experiments.

Flow Measurements

The main flow instruments are a Brooks turbine flowmeter capable of measuring 0.75 - 7.5 l/min (0.2 - 2 gpm), a Ramapo impact flowmeter capable of measuring 1.5 - 15 l/min (0.4 - 4 gpm), and a Ramapo impact flowmeter capable of measuring 7.5 - 75 l/min (2 - 20 gpm). These instruments were installed in parallel legs of the flow loop with shutoff valves and 10 pipe diameters of straight pipe upstream and five pipe diameters of straight pipe downstream. Because the strain gauges in the Ramapo meters are temperature sensitive it was necessary to carefully stabilize the flowmeter temperature before each test. The procedure for stabilizing the temperature is described in Section 3.1. In addition, the Ramapo meters must be mounted with the stem downward to ensure that the strain gauges are completely submerged in water.

Figure 3.7. Sketch of 3000-l Rectangular Tank.

To ensure that the flowmeter did not drift during testing, we checked its calibration before and after each test using the stopwatch-and-weight method. No tests were invalidated because of flowmeter drift.

Temperature Measurements

Temperature measurements include the thermocouple arrays, test tank inlet and outlet, and heater inlet and outlet. Except as noted, all thermocouples are type E (chromel-constantan) with a 1.6 mm (1/16 in) stainless steel sheath that is electrically insulated from the thermocouple junction. These thermocouples have the highest voltage output of any standard type and have a time constant of about 7 seconds.

The thermocouple arrays used in the 190 l and 3000 l tanks are shown in Figures 3.3 and 3.6. The thermocouple rake used in the flow visualization tank differed from the other thermocouples by having bare junctions for faster time response. The time constant for the bare-junction thermocouples is of the order of 0.1 second.

Two types of temperature measurement are used at the test tank inlet and outlet. The first is a thermocouple giving a direct temperature reading on the datalogger. The second is an 8-junction thermopile giving a temperature differential reading on the datalogger. The four junctions on each side of the thermopile are spaced apart to measure the average temperature difference between the inlet and outlet pipes. In addition, flow mixers are used at the inlet and outlet. For the 1-inch pipe fittings, the mixer consists of an 17 mm (11/16 in) sharp-edged orifice about one pipe diameter upstream of the thermocouples. For the 3000-l tank outlet the mixer consists of a weir and two right-angle bends immediately upstream of the thermocouples.

The heater inlet and outlet thermocouples are used for adjusting the heater. The heater outlet temperature is also coupled to the safety system.

Datalogger

The datalogger is a Kaye Instruments Digistrip II. A sample of its output is shown in Table 3.1. In addition to its automatic datalogging function it converts electrical signals to engineering units. Some of the later tests were tape recorded so that further data reduction could be done by a computer.

Another datalogger function is the heater safety control. If the flowrate drops below its limit or the heater outlet temperature rises above its limit, the datalogger automatically shuts off the heater, sounds an alarm, and logs the event.

3.4 Experiments

Flow Visualization

A series of experiments were performed in which cold (20°C) water dyed with potassium permanganate was injected into a tank containing hot

Table 3.1. Sample of Data Logger Output

1112 100 200 300	STRATIFICATION MEASUREMENT AND ANALYSIS														
	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15
	THERMOCOUPLE ARRAY NO. 1				01-12								HR1-IN	HR2-IN	T-PILE
	THERMOCOUPLE ARRAY NO. 2				01-12								HR1-OUT	HR2-OUT	FLOW
	THERMOCOUPLE ARRAY NO. 3				01-12								TOP	BOT	FLOW, 4-4FLOWS-30
11:37:00	17.2C	17.3C	17.2C	19.7C	31.0C	44.1C	45.6C	45.9C	46.0C	45.9C	45.6C	45.8C	20.2C	15.0C	7.381M 0.608M
	17.2C	17.1C	18.1C	19.6C	28.5C	43.3C	46.0C	46.2C	46.2C	46.1C	46.0C	46.1C	21.6C	15.6C	45.9C 0.5G
	16.6C	17.2C	17.0C	19.5C	29.8C	43.2C	45.8C	45.8C	46.3C	46.3C	46.1C	45.9C	33.0C	15.6C	2.970G 0.7071
11:42:00	17.1C	17.2C	17.0C	19.2C	26.3C	41.2C	45.6C	45.9C	46.0C	45.9C	45.6C	45.7C	20.2C	14.9C	7.386M 0.610M
	17.1C	17.1C	17.9C	18.0C	24.5C	40.4C	45.9C	46.2C	46.2C	46.1C	46.1C	46.0C	21.6C	15.6C	45.9C 0.5G
	16.6C	17.0C	17.4C	18.9C	25.6C	39.9C	45.7C	45.8C	46.3C	46.3C	46.1C	45.9C	33.0C	15.5C	2.965G 0.7071
11:47:00	16.9C	17.0C	16.9C	18.4C	22.9C	37.3C	45.3C	45.9C	45.9C	45.9C	45.6C	45.8C	20.2C	14.9C	7.387M 0.610M
	16.9C	16.9C	17.7C	18.4C	21.8C	35.9C	45.6C	46.1C	46.2C	46.1C	46.0C	46.1C	21.6C	15.6C	45.9C 0.5G
	16.3C	16.9C	17.1C	18.3C	22.3C	35.9C	45.1C	45.8C	46.3C	46.3C	46.0C	46.0C	33.0C	15.5C	2.962G 0.7071
11:52:00	16.8C	16.9C	16.7C	18.2C	20.8C	32.6C	43.9C	45.9C	45.9C	45.9C	45.5C	45.8C	20.1C	14.9C	7.383M 0.610M
	16.9C	16.8C	17.4C	18.1C	20.1C	31.2C	44.0C	46.0C	46.2C	46.1C	46.1C	46.1C	21.6C	15.5C	45.9C 0.5G
	16.3C	16.7C	17.0C	17.9C	20.4C	30.8C	43.6C	45.7C	46.2C	46.3C	46.0C	45.9C	33.0C	15.4C	2.956G 0.7071
11:57:00	16.7C	16.8C	16.6C	17.8C	19.4C	27.9C	41.6C	45.8C	45.9C	45.8C	45.5C	45.7C	20.1C	14.8C	7.398M 0.609M
	16.8C	16.7C	17.3C	17.8C	19.1C	26.8C	41.5C	45.6C	46.2C	46.1C	45.9C	45.9C	21.5C	15.5C	45.8C 0.4G
	16.2C	16.6C	16.6C	17.7C	19.3C	26.6C	40.8C	45.6C	46.2C	46.2C	46.0C	45.9C	32.9C	15.4C	2.959G 0.6325
12:02:00	16.6C	16.7C	16.5C	17.6C	18.8C	24.3C	38.1C	45.4C	45.9C	45.8C	45.4C	45.7C	20.0C	14.8C	7.406M 0.609M
	16.6C	16.6C	16.9C	17.5C	18.6C	23.3C	37.9C	44.1C	46.1C	46.1C	46.0C	45.9C	21.5C	15.4C	45.8C 0.5G
	16.1C	16.5C	16.6C	17.4C	18.6C	22.9C	37.2C	45.1C	46.2C	46.2C	46.0C	45.8C	32.9C	15.4C	2.962G 0.7071
12:07:00	16.4C	16.6C	16.4C	17.3C	18.4C	21.6C	33.6C	44.1C	45.8C	45.8C	45.4C	45.7C	20.1C	14.8C	7.403M 0.609M
	16.6C	16.5C	16.8C	17.3C	18.3C	21.1C	33.3C	41.6C	46.1C	46.0C	45.9C	45.9C	21.5C	15.4C	45.8C 0.5G
	16.0C	16.4C	16.4C	17.1C	18.2C	20.8C	32.7C	44.0C	46.1C	46.2C	46.0C	45.9C	32.9C	15.3C	2.955G 0.7071
12:12:00	16.4C	16.4C	16.3C	17.2C	18.1C	20.0C	29.3C	41.8C	45.8C	45.8C	45.4C	45.7C	20.0C	14.8C	7.417M 0.610M
	16.4C	16.4C	16.8C	17.2C	18.0C	19.8C	28.6C	38.3C	46.0C	46.0C	45.9C	46.0C	21.5C	15.4C	45.8C 0.5G
	15.9C	16.4C	16.2C	17.0C	17.9C	19.6C	28.1C	41.7C	45.9C	46.2C	45.9C	45.9C	32.9C	15.3C	2.950G 0.7071
12:17:00	16.3C	16.4C	16.2C	16.3C	17.8C	19.1C	25.4C	38.6C	45.5C	45.8C	45.4C	45.7C	20.0C	14.7C	7.406M 0.610M
	16.4C	16.4C	16.8C	16.9C	17.7C	19.0C	25.0C	33.7C	45.4C	46.0C	45.9C	45.9C	21.5C	15.3C	45.8C 0.5G
	15.9C	16.4C	16.1C	17.0C	17.6C	18.8C	24.4C	38.4C	45.4C	46.2C	45.9C	45.6C	32.9C	15.3C	2.965G 0.7071
12:22:00	16.2C	16.3C	16.1C	16.3C	17.6C	18.5C	22.4C	34.3C	44.4C	45.7C	45.4C	45.7C	20.0C	14.7C	7.421M 0.609M
	16.3C	16.3C	16.6C	16.8C	17.4C	18.5C	22.2C	29.5C	44.1C	45.9C	45.9C	45.9C	21.4C	15.3C	45.8C 0.4G
	15.7C	16.2C	16.1C	16.6C	17.4C	18.3C	21.7C	34.4C	44.0C	46.1C	45.9C	45.8C	32.9C	15.3C	2.953G 0.6325

(60°C) water. The behavior of the jet was photographed, and in some cases thermocouple data was recorded. Both types of inlet nozzles were tested at several different flowrates. The data obtained from these experiments is primarily qualitative rather than quantitative.

190-Liter Tank

A total of 27 experiments were performed on the 190-liter tank. Tank configurations that were tested were 1) uninsulated dip tube, 2) insulated dip tube, 3) single horizontal baffle, 4) dual concentric baffle, 5) side inlet and outlet, 6) dual radial-flow diffusers with 1.9 cm (0.75 in) diffuser height, and 7) dual radial-flow diffusers with 0.95 cm (0.375 in) diffuser height. Charge, discharge, and interrupted discharge tests were performed. Flowrates were varied from 0.75 - 45 l/min (0.2-12 gpm) and the temperature difference $T_g - T_{in}$ was varied from 10 to 40°C. The Richardson number varied from about 0.2 to over 500. These runs are tabulated in Appendix B.

3000-Liter Tank

Eight discharge tests were performed on the 3000-liter cylindrical tank. Low and high values used in the tests for of each three independent variables are:

Richardson number R_i : 1.0 - 22.

Fourier number F_o : 0.00045 - 0.0018

Normalized inlet elevation δ/L : 0.0925 - 0.162

The tests completed to date are tabulated in Appendix B.

4. EXPERIMENTAL RESULTS

Results of experiments fall into six general classifications which demonstrate the validity of the theory developed in Section 2. The quiescent mode tests of the 3000-l tank show that the thermocline spreads according to previously developed theories based on thermal conductivity [4]. This finding is essential to the success of the theory developed in Section 2. Another result of the quiescent mode tests is that thermal losses from the tank are sufficiently small that they can be neglected for charge and discharge tests.

Flow visualization tests show that the Kelvin-Helmholtz instability is the principal mixing mechanism at the inlet. The Kelvin-Helmholtz instability is governed by the Richardson number [1,2].

Tests with dip tubes and vertical baffles show that such items degrade stratification. A prudent tank designer should avoid using dip tubes or vertical baffles. Therefore, the inability of the theory developed in Section 2 to account for heat transfer across a dip tube or vertical baffle does not limit its usefulness. The best stratification was obtained for configurations having simple inlet and outlet diffusers that can be modeled by the theory.

Three-dimensional thermocouple array measurements show that temperatures in the tank at any time are nearly constant in any horizontal plane (except near the inlet and outlet). That means that temperature is a function of elevation x and time t only, and a one-dimensional theory is sufficient.

One series of experiments corroborates evidence [19] of a critical Richardson number. For Richardson numbers less than about 0.31-0.48 the theory developed in Section 2 is not valid.

Finally, the empirical constants c and H in Equation 2.11 were measured and correlated with the Fourier and Richardson numbers. This correlation will allow tank designers to choose between alternative designs.

4.1 Tests of 3000 Liter Tank in Quiescent Mode

Two types of quiescent tank tests were performed. In the first type of test a fully-charged 3000 l cylindrical tank was observed during a cool-down test. Figure 4.1 shows the progress of a 93-hour cool-down tests for an initial temperature of 60°C and an ambient temperature of 20°C. Each temperature plotted is the average of three points in a horizontal plane. As Figure 4.1 shows the tank cools uniformly except near the bottom which cools more rapidly.

Conduction through the side wall insulation is believed to be the mechanism for the uniform cooling. The bottom is believed to cool more rapidly because of conduction through the bottom insulation.

An alternative explanation for the more rapid cooling at the bottom --a thin layer of water losing heat through the wall and sinking to the bottom--was rejected. The alternative explanation would not allow a significant amount of cooling at the top of the tank and would establish a thermo-

Figure 4.1. Temperature Profiles for a Quiescent Mode Test from a Fully Charged Condition of the 3000-l Tank.

cline that grows with time near the bottom. Since neither of these phenomena is present in Figure 4.1 the alternative explanation for the rapid cooling near the bottom was rejected.

The total heat content of the tank is plotted on a logarithmic scale as a function of time in Figure 4.2. The plot gives a thermal time constant of 0.00298 hr^{-1} . A rough calculation based on 14 cm (5.5 in) - thick fiberglass insulation ($R = 19^\circ\text{F}\cdot\text{hr}\cdot\text{ft}^2/\text{Btu}$) gives a time constant $UA/\rho V C_p = 0.00109 \text{ hr}^{-1}$. The rough calculation neglects heat loss through the 2" x 6" lumber supports beneath the tank, losses at the pipe connections, areas where the insulation is compressed, gaps in the insulation, evaporative loss from the tank vent, and possible dampness of the insulation. Since many of the heat losses are unavoidable, the calculations point out the difficulty of maintaining the rated resistance of the insulation. Nevertheless, the tank is well insulated, and the loss of energy from the tank during charging and discharging tests is negligible (less than 2.3% for an 8-hour test).

The second type of quiescent tank test involved discharging the fully-charged 3000-l cylindrical tank halfway and observing the spread of the thermocline. Results are compared with one-dimensional thermal diffusion theory adapted from reference [4] in Figure 4.3. The theory agrees reasonably well with the experiment in the bottom half of the tank but diverges from the experiment in the top half. The reason for the divergence is that the theory neglects the thermal losses from the tank. The largest thermal loss occurs at the top of the tank where the difference between the water temperature and ambient temperature is greatest. (The thermal loss that is insignificant for an 8-hour charge or discharge test becomes significant in the 40-hour quiescent mode test). Agreement in the bottom half of the tank is better because the difference between the water temperature and the ambient temperature is small there.

The theory [4] also predicts the growth and broadening of the thermocline with reasonable accuracy as shown in Figure 4.3. Since the theory developed in Section 2 of this report is closely related to [4], failure of [4] to represent the growth and broadening of a quiescent thermocline would cast doubt on the theory developed in Section 2.

4.2 Flow Visualization Tests

A series of flow visualization experiments in which cold dyed water was injected into hot water show that the Kelvin-Helmholtz instability is the primary mechanism of mixing at the the inlet. Figure 4.4 is two typical photographs of the inlet jet. Individual eddies that are characteristic of the Kelvin-Helmholtz instability can be seen. As the bulk of the cold water falls to the bottom of the tank it tends to spread and entrain warmer water in the stream but it is not completely dissipated. Additional Kelvin-Helmholtz eddies can be seen at the shear-flow interface between the cold water and the warm water as the cold water flows along the bottom.

Figure 4.2. Energy Loss from the Quiescent Tank.
Plotted Points are Experiments. Line
is a Least-Squares Fit.

Figure 4.3. Quiescent Mode Test of a Partially Discharged Tank. Plotted Points are Experiment. Curves are Theory.

(a) Nozzle Approximately 15 cm Above Bottom of Tank

(b) Nozzle Approximately 2.5 cm Above Bottom of Tank

Figure 4.4. Flow Visualization Tests Showing Evidence of Kelvin-Helmholtz Instability

4.3 190-Liter Tank Tests

Dip Tubes and Vertical Baffles

Results of discharge tests for the dual concentric baffle, uninsulated dip tube, insulated dip tube, and side inlet and outlet configurations are shown in Figure 4.5. The effect of heat transfer across the dip tubes and dual concentric baffles can be seen in the slope of the curves from normalized time 0.3 to 0.8. Cold water flowing in the dip tube cools the hot water in the tank before the hot water is discharged in a discharge test. The dual concentric baffle configuration which has the largest area for heat transfer has the most rapid decrease in its discharge temperature and, consequently, the poorest performance.

The uninsulated dip tube has better performance than the dual concentric baffles because of its smaller surface area. Insulating the dip tube with 3.2 cm (1.25 in) of closed-cell foam rubber results in another substantial increase in performance. Performance of the side inlet and outlet configuration is better than that of the others because it does not have the same loss mechanism.

The stratification index (equation 2.14) was calculated for the Figure 4.5 data and tabulated below. Other details regarding these runs can be found in Appendix B.

Table 4.1. Comparison of Discharge Tests of Configurations Having Vertical Baffles

<u>Run Identification</u>	<u>Configuration</u>	<u>Stratification Index, S</u>
Nov. 19, 1980	Side Inlet and Outlet	0.75
Aug. 29, 1980	Insulated Dip Tube	0.78
Aug. 07, 1980	Uninsulated Dip Tube	0.65
Oct. 03, 1980	Dual Concentric Baffles	0.56

The table shows that the stratification index for the side inlet and outlet configuration is slightly less than the stratification index for the insulated dip tube configuration. However, Figure 4.5 shows that the side inlet and outlet configuration performance is actually better than the insulated dip tube performance because the outlet temperature remains higher until it begins to drop off. The low stratification indices of the poorer performing configurations are closely related to their rapid discharge temperature drop off. Thus, the stratification index (particularly when it is calculated from experimental data) is not sensitive to minor differences in performance, but it can indicate major differences.

Similar results obtained from the charging tests shown in Figure 4.6 are tabulated below.

Figure 4.5. Discharge Tests of 190-1 Tank at 7.5 l/min.
 $\Delta T = 40^{\circ}\text{C}.$

Figure 4.6. Charge Tests of 190-*l* Tank at 0.75 *l*/min.
 $\Delta T = 40^{\circ}\text{C}$.

Table 4.2. Comparison of Charge Tests of Configurations Having Vertical Baffles

<u>Run Identification</u>	<u>Configuration</u>	<u>Stratification Index, S</u>
Sept. 09, 1980	Side Inlet and Outlet	0.74
Sept. 03, 1980	Insulated Dip Tube	0.71
Aug. 22, 1980	Uninsulated Dip Tube	0.45
Oct. 06, 1980	Dual Concentric Baffles	0.49

Best Stratification

In addition to testing the configurations already mentioned we tested configurations with a single horizontal baffle and dual radial-flow baffles shown in Figure 3.4. Although all three configurations shown in Figure 4.7 performed well in discharge tests, the best stratification was obtained with the dual radial-flow diffusers. Stratification indices for the discharge tests are tabulated below.

Table 4.3. Comparison of Discharge Tests of Configurations Having the Best Stratification

<u>Run Identification</u>	<u>Configuration</u>	<u>Stratification Index, S</u>
Nov. 19, 1980	Side Inlet and Outlet	0.75
Jul. 08, 1980	Single Horizontal Baffle	0.79
Nov. 04, 1980	Dual Radial-Flow Diffusers	0.84

Similar results are shown in Figure 4.8 for charge tests. Because the charge tests take place at a much lower flowrate than the discharge test flowrate, the thermocline spreads more. Stratification indices for the charge tests are tabulated below. The high value of S for the dual radial-flow diffusers for the charge tests compared to the discharge test is probably due to experimental errors.

Table 4.4. Comparison of Charge Tests of Configurations Having the Best Stratification

<u>Run Identification</u>	<u>Configuration</u>	<u>Stratification Index, S</u>
Sept. 09, 1980	Side Inlet and Outlet	0.74
Jul. 14, 1980	Single Horizontal Baffle	0.76
Oct. 27, 1980	Dual Radial-Flow Diffusers	0.88

Figure 4.7. 7.5 l/min Discharge Tests of Three 190-l Tank Configurations. $\Delta T = 40^{\circ}\text{C}$.

Figure 4.8. 0.75 l/min Charge Tests of Three 190-l Tank Configurations. $\Delta T = 40^{\circ}\text{C}$.

4.4 Three Dimensional Thermocouple Array Measurements

Typical thermocouple array results during the passage of the thermocline are shown in Table 4.5 for the 190 liter tank. The data shown in Table 4.5 include each temperature measurement, the mean temperature at each elevation, and the standard deviation at each elevation. Elevations are 8%, 29%, 71%, and 92% of total water depth. The largest standard deviation shown is 2.01°C, and most temperature measurements have a standard deviation less than 1°C. Since temperature gradients of about 2°C/cm are present in the thermocline, the temperature is nearly uniform in a horizontal plane, and a one-dimensional theory is sufficient to describe the thermocline, except near the inlet and outlet.

Table 4.6 shows the passage of the thermocline at elevations (14%, 42%, 62%, and 84% of the total water depth) in the 3000-l tank. The largest standard deviation for the thermocouple measurements in a horizontal plane is 3.52°C. Since temperature gradients of about 1.8°C/cm are present in the thermocline, the standard deviation of 3.52°C corresponds to a vertical displacement of about 1.9 cm. The data of Table 4.6 (October 27, 1981) were chosen because that run had a relatively high flow rate, a small Richardson number, and a large vertical dimension for the inlet--conditions that make a one-dimensional flow difficult to achieve. Therefore, the temperature is nearly uniform in a horizontal plane, and a one-dimensional theory is sufficient to describe the thermocline in the 3000-l tank, except near the inlet and outlet.

4.5 Evidence of a Critical Richardson Number

A series of experiments was conducted in which the Richardson number was varied by varying the flow rate. Figure 4.9 shows that the stratification index drops off sharply as the Richardson number becomes less than 0.31-0.48. This corroborates other experimental evidence [19] of a critical Richardson number of about 0.30.

4.6 Evaluation of Empirical Constants

For most of the experimental runs equation 2.11 was fitted to the experimental measurements by adjusting the parameters a , c , and H until the best least-squares fit was obtained [22]. The adjusted values of a , c , and H are tabulated in Appendix B and are considered as the measured values of a , c , and h . A typical fit of equation 2.11 to the experimental data is shown in Figure 4.10.

The functional relationship of c to the independent variables F_0 , R_i , and δ/L was determined by plotting measured values of c versus each independent variable on either log-log paper or semi-log paper and determining the best least-squares linear fit. The relationship determined for c is approximately

$$c = 2.44 \times 10^{-5} F_0^{-1.5} \exp(-0.0134 R_i) \quad (4.1)$$

Table 4.5. Temperature Measurements During Passage of the Thermocline in the 190-1 Tank. August 7, 1980 data.

Time, minutes	Thermocouples, °C										Average	Standard Deviation
	1	2	3	4	5	6	7	8	9	10		
$\frac{x}{L} = 0.08$												
0	60.2	60.3	59.8	60.3	61.0	60.4	60.2	59.9	60.6	60.1	60.3	0.34
1	47.1	47.3	48.3	47.9	50.6	47.5	46.6	47.4	48.7	47.7	47.9	1.12
2	42.6	42.8	43.4	43.1	44.6	43.2	42.7	42.8	44.4	43.2	43.3	0.69
3	39.8	39.9	40.8	40.4	40.4	40.2	39.4	39.8	41.1	40.2	40.2	0.51
4	37.6	36.8	38.3	37.9	38.4	38.2	37.4	37.6	38.3	37.8	37.8	0.50
5	35.8	36.1	36.8	36.1	36.7	36.4	36.0	35.8	37.2	36.1	36.2	0.46
$\frac{x}{L} = 0.29$												
5	58.3	58.2	59.1	58.5	57.4	59.3	57.6	57.2	56.9	58.4	58.1	0.80
6	42.6	41.3	43.4	42.3	41.1	45.4	41.7	43.2	40.9	43.7	42.6	1.40
7	37.9	37.5	38.5	37.8	36.7	37.8	37.4	37.0	37.6	37.7	37.6	0.50
8	34.4	35.8	36.4	34.9	34.4	35.6	35.1	34.8	35.2	35.2	35.2	0.62
9	33.4	33.0	34.3	33.3	32.3	32.2	32.9	31.8	33.5	33.3	33.0	0.73
$\frac{x}{L} = 0.71$												
13	57.2	56.4	57.0	57.1	56.4	57.1	57.1	57.2	57.3	57.2	57.0	0.33
15	55.4	51.4	55.1	55.2	55.4	55.2	55.1	55.2	55.2	55.2	54.8	1.21
17	42.8	36.6	42.1	42.2	43.7	43.3	41.9	42.7	41.7	43.2	42.0	2.01
19	34.9	32.3	34.3	34.6	34.3	34.8	34.3	34.7	34.5	34.9	34.4	0.76
21	31.6	30.3	31.1	31.2	31.3	31.4	31.2	31.5	31.4	31.5	31.2	0.37
23	29.7	28.8	29.5	29.6	28.7	29.6	29.5	29.6	29.6	29.6	29.4	0.36
25	28.6	27.9	28.2	28.2	27.5	28.4	28.3	28.4	28.5	28.4	28.2	0.32
$\frac{x}{L} = 0.92$												
15	58.5	58.6	58.6	58.6	58.6	58.6	58.6	58.6	58.3	58.6	58.6	0.10
17	57.6	57.7	57.6	57.4	57.6	57.5	57.4	57.6	57.3	57.6	57.5	0.13
20	54.8	54.7	54.8	54.8	55.0	54.8	54.7	54.8	54.4	54.8	54.8	0.15
23	40.1	40.2	40.6	40.3	41.0	40.5	39.9	40.3	39.4	40.2	40.2	0.42
26	32.2	32.2	32.3	32.3	32.4	32.3	32.2	32.3	31.8	32.1	32.2	0.17
29	29.5	29.6	29.6	29.6	29.7	29.6	29.4	29.5	29.2	29.3	29.5	0.16
32	25.9	25.4	25.7	25.7	25.7	25.7	25.7	25.8	25.9	25.8	25.7	0.14

Table 4.6. Temperature Measurements During Passage of the Thermocline in the 3000-ℓ Tank.
October 27, 1981 data.

Time, minutes	Thermocouples, °C			Average, °C	Standard Deviation
	P1	P2	P3		
$\frac{x}{L} = 0.14$					
1	49.1	49.1	49.1	49.1	0
2	47.4	48.5	45.0	47.0	1.79
3	44.8	44.6	41.9	43.8	1.62
4	41.9	41.6	40.3	41.3	0.85
5	40.2	38.2	38.5	39.0	1.08
6	37.7	36.9	37.4	37.3	0.40
7	35.4	35.2	33.4	34.7	1.10
8	33.8	33.3	33.4	33.5	0.26
9	32.4	32.1	31.4	32.0	0.51
$\frac{x}{L} = 0.42$					
15	49.0	48.8	48.7	48.8	0.15
16	47.4	47.5	47.9	47.6	0.26
17	46.3	45.8	45.3	45.8	0.50
18	43.9	41.4	43.5	42.9	1.34
19	39.6	40.6	41.8	40.7	1.10
20	36.8	34.4	34.5	35.2	1.36
21	33.7	32.2	33.9	33.3	0.93
22	31.3	3.08	31.3	31.1	0.29
$\frac{x}{L} = 0.62$					
27	48.8	47.7	48.8	48.4	0.64
28	46.8	45.2	48.3	46.8	1.55
29	46.7	41.5	47.3	45.2	3.19
30	44.3	39.1	45.8	43.1	3.52

Table 4.6. (Continued)

Time, minutes	Thermocouples, °C			Average, °C	Standard Deviation
	P1	P2	P3		
31	40.9	36.3	42.4	39.9	3.18
32	37.1	32.8	38.9	36.3	3.13
33	34.7	31.2	34.9	33.6	2.08
34	30.9	29.3	33.4	31.2	2.07
35	39.9	27.8	31.4	29.7	1.81
39	48.2	48.4	49.0	48.5	0.42
40	47.6	47.7	48.9	48.1	0.72
41	46.6	45.9	48.3	46.9	1.23
42	44.8	43.8	47.3	45.3	1.80
43	41.8	40.9	45.2	42.6	2.27
44	38.1	37.8	43.1	39.7	2.98
45	35.6	35.6	39.6	36.9	2.31
46	32.6	32.2	36.8	33.9	2.55
47	30.2	30.3	33.8	31.4	2.05
48	29.4	28.8	31.5	29.9	1.42
49	26.8	26.5	29.5	27.6	1.65

Figure 4.9. Stratification Index as a Function of Richardson Number for 190-l Tank with Side Inlet and Outlet.

Figure 4.10. Typical Fit of Equation 2.11 to Experimental Data.

The measured values of c are compared to equation 4.1 in Figure 4.11.

An unexpected result is that the independent variable δ/L does not appear in equation 4.1. Examination of the data for the 3000- ℓ tank suggests that there may be a relationship between c and δ/L , but no such relationship could be found for the 190- ℓ tank. Equation 4.1 should not be expected to yield extremely accurate results because of the scatter in the experimental data used to derive it. In most cases the computed value of c is within a factor of 3.5 of its measured value. Equation 4.1 does not give good results for Richardson number larger than about 500, nor should it be used for Richardson numbers smaller than 1.

The importance of equation 4.1 is that it gives the trend of c as functions of the Fourier and Richardson numbers. Thus, although a tank designer might not be able to accurately predict the performance of a particular tank design, he can select the better of two designs.

No relationship between H and the independent variables could be determined. Therefore, an average value for H was calculated. The average was calculated by taking the antilogarithm of the arithmetic mean of the logarithm of H . Such an average is skewed less than the ordinary arithmetic mean is by a few points that differ substantially from the average. The average value of H is:

$$H = 7.7 \quad (4.2)$$

In most cases the value of H given by equation 4.2 is within a factor of 2 of the measured value. In any case, the accuracy of H is less important than the accuracy of c because H appears only in the small increment (equation 2.10) that is added to equation 2.6 to obtain the normalized temperature (equation 2.11).

The discrepancy between the measured value of a and the value of $a = t^*/t'$ is believed to be related to integrated errors in the flow measurement. Therefore, we made no attempt to correlate the measured value of a with the independent parameters, and a tank designer should use the definition $a = t^*/t'$ in his calculations.

A comparison of the theory (equation 2.11) including the correlations for c and H (equations 4.1 and 4.2) with experimental measurements is shown in Figure 4.12. The figure shows that the theory yields a good estimate of tank temperatures not only at the outlet ($x/L = 1$), but also in the interior of the tank ($x/L = 0.277, 0.486, \text{ and } 0.763$). Values of c used in the interior of the tank are half of the value of c used for $x/L = 1.0$ because the water undergoes only one mixing process at the tank inlet to reach the interior of the tank, but the water undergoes two mixing processes at the inlet and outlet before it leaves the tank.

Figure 4.11. Comparison of Equation 4.1 to Experimentally Measured Values of c .

Figure 4.12. Comparison of Analytical Model (Equations 2.11, 4.1, and 4.2) with November 3, 1981 Experiment.

5. CONCLUSIONS AND RECOMMENDATIONS

In this section we conclude that the one-dimensional theory predicts the performance of stratified thermal storage tanks with reasonable accuracy. A procedure for designing stratified systems based on the theory was developed and is given in Appendix A of this report.

Since our present work does not cover all situations that would be useful for solar and energy conservation systems, we present some recommendations for future work.

5.1 Conclusions

1. There is a critical value of Richardson number of about 0.31-0.48 below which rapid mixing occurs. Good stratification can be obtained if the Richardson number for the tank is substantially above its critical value. This is reasonably close to the theoretical value of 0.25 [1,2], and it corroborates other experimental measurements [19].

2. This is the most important conclusion in this report. If stratification is to be maintained in solar systems the conventional collection strategy can not be used. The conventional collection strategy guarantees that the Richardson number will decrease to less than its critical value and that stratification will be destroyed.

A collection strategy that maintains stratification is given below. Minor equipment modifications are required to implement the new strategy.

a) Allow no more than one tank volume per day to pass through the collector or collector heat exchanger. This rule has recently been recognized by others [10,11].

b) Vary the flowrates through the collector and collector heat exchanger so that the temperature of the water returned to the tank is constant. This can be done by using centrifugal pumps and installing valves similar to an automobile thermostat in the collector and collector heat exchanger outlets.

c) Adjust the temperature settings on the valves for Rule 2 so that water is returned to the tank at the minimum temperature necessary to satisfy the load plus a few degrees allowance for thermal losses and heat exchange processes. This rule is made possible by the first two, and it is chiefly responsible for the improved performance that can be achieved with a stratified system. The average collector operating temperature can be as much as 20°C (40°F) cooler with the new strategy than with the old strategy.

3. Dip tubes and vertical baffles degrade stratification due to heat transfer from the hot side of the vertical boundary to the cold side. Designers of stratified tanks should not use dip tubes or vertical baffles (such as labyrinth baffle schemes). If using a dip tube is unavoidable, the dip tube should be small to minimize its heat transfer surface, and it should be heavily insulated with waterproof insulation.

4. The principal cause of mixing at the inlet and outlet is the Kelvin-Helmholtz instability which occurs at the shear-flow interface between the hot and cold water. If cold water is introduced into the tank above a layer of hot water, the bulk of the cold water will fall through the hot water. Mixing occurs in the waves of the Kelvin-Helmholtz instability as entrainment of hot water in the cold water and of cold water in the hot water. Additional Kelvin-Helmholtz mixing occurs as the cold water flows along the bottom beneath a layer of hot water.

5. A one-dimensional theory is sufficient to describe the performance of a thermally stratified tank, except in the mixing regions near the inlet and outlet. Such a theory has been developed and is summarized in equation 2.11. Equation 4.1 gives the empirical constant c as a function of Fourier and Richardson number accurate to a factor of 3.5 for $1 \leq R_i \leq 500$. Increasing the Fourier number decreases c , and increasing the Richardson number decreases c . Although no functional dependence of c on δ/L was found, such a dependence may exist. No dependence of the empirical constant H upon any of the test parameters was found. Instead, H given by equation 4.2 is approximately constant and is accurate to a factor of 2.

6. Tall tanks give the best performance. To obtain the best performance (that is, the sharpest thermocline) one should make the denominator of the error function in equation 2.10 small. If the denominator is called D , then equations 2.10 and 4.1 give:

$$D^2 = \frac{a_i}{t^*} F_0 + 2.44 \times 10^{-5} F_0^{-0.5} \exp(-0.0134 R_i) \quad (5.1)$$

Taking the derivative of D^2 with respect to R_i implies that the Richardson number should be as large as possible. In other words, the tank height L and the top-to-bottom temperature difference ΔT should be made large, and the inlet velocity u should be made small. Since equation 4.1 is not valid for Richardson number greater than 500, $R_i = 500$ is a practical upper limit.

Taking the derivative of D^2 with respect to F_0 gives the following relationship for minimum D^2 :

$$F_0 = \left[\frac{1.22 \times 10^{-5} \exp(-0.0134 R_i)}{a_i/t^*} \right]^{2/3} \quad (5.2)$$

For $R_i = 500$ and $a_i/t^* \approx 1$ the value of the Fourier number that minimizes D is about 6×10^{-6} . In practice such a small Fourier number is difficult to achieve, but the Fourier number can be made small by making the tank height L large.

Reference [23] shows that thermal losses begin to increase rapidly as the tank's height-to-diameter ratio increases past about 4. Reference [13] also indicates that a height-to-diameter ratio of about 4 gives best performance. Therefore, the height-to-diameter ratio should not exceed 4, but the tank should be as tall as possible within that limit.

7. Relatively simple inlet and outlet diffusers give the best performance. The best performance was obtained with dual radial-flow diffusers that are nothing more than a circular plate. See Figure 3.4. Adequate performance can be obtained with a side inlet and outlet.

8. Massive tank walls are detrimental to stratification. The heat contained in the tank wall can enter the water only after the thermocline has passed. Furthermore, the temperature of the heat contained in the wall is degraded by the film resistance H . Most of the heat contained in the wall can be extracted only at temperatures below the minimum usable temperature. See equation 2.10 and Figure 2.2.

5.2 Recommendations

The following recommendations for future work are made without regard to their likelihood of being funded. They represent the steps that would be necessary to improve the accuracy of the theory and develop it for other types of solar and energy conservation systems.

1) Do additional testing of the present tank configurations to improve the accuracy of the correlations in equations 4.1 and 4.2. The correlation for H (equation 4.2) may be sufficiently accurate as it is, but an improved correlation for c is sought. In particular, it may be possible to find a relationship between δ/L and c .

2) Develop the theory for and measure the performance of horizontal cylindrical tanks. Although the present theory covers a useful range of vertical tanks, it does not apply to tanks where the cross sectional area is not constant with respect to elevation. Developing the theory for horizontal cylindrical tanks would be useful because many large solar systems use one-of-a-kind horizontal cylindrical tanks. To use cut-and-try methods of achieving stratification in those tanks is impractical because of cost. Extending the theory would allow inexpensive, reliable stratification of these tanks.

3) Examine the limits of applicability of the empirical correlations. One such limitation ($1 \leq Ri \leq 500$) has already been discussed in this report. Can the limitation on Richardson number be relaxed? Are there limitations on other independent parameters? The present theory does not apply to situations where isotherms within the tank are not horizontal planes. Nonhorizontal isotherms are most likely to affect tanks with a low height-to-diameter ratio, one of the most desirable tank shapes. Can the present theory be modified to account for mildly non-planar isotherms? What are the limits of applicability of the modified theory?

4) Develop the theory for and measure performance of cold storage tanks. Although stratified cold storage tanks are very desirable, the theory was developed for fluids with a constant coefficient of thermal

expansion. Thus, the present theory can not be used for cold storage where the thermal expansion coefficient is not only variable but changes sign at 4°C. A modified theory would be applicable to cold-side storage for solar-powered airconditioning and for load management.

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APPENDIX A:
DESIGNING STRATIFIED TANKS

INTRODUCTION

Thermal stratification is the maintenance of a sharp separation between hot and cold water in the same tank. This article tells how to design a solar system to maintain stratification and to take advantage of the performance improvement due to stratification.

According to researchers the performance of active solar systems using thermally stratified storage is substantially better than the performance of systems using mixed storage. As much as 20% better performance has been reported [1]. Despite this, most liquid-based systems remain unstratified. This article tells how tanks fail to stratify, how to redesign the system to take advantage of stratification, and how to design the tank to minimize the mixing of hot water at the top with colder water at the bottom.

Argonne National Laboratory has investigated ways of achieving thermal stratification in tanks. The conclusions of their report [2] are:

- 1) The most important parameter governing stratification is the Richardson number $R_i = g\beta\Delta T/u^2$. When the tank's Richardson number is greater than the critical value of about 0.25, the tank will stratify with hot water at the top and colder water at the bottom. But when the tank's Richardson number is less than about 0.25, the contents of the tank mix, and stratification is destroyed.
- 2) The conventional solar collection strategy guarantees the destruction of stratification by ensuring that the Richardson number decreases while the collector operates. The Argonne report describes a new collection strategy that allows stratification to occur.
- 3) Dip tubes and vertical baffles degrade stratification. Heat is exchanged between the hot water at the top of the tank and the cold water flowing in the dip tube or baffle, thus cooling the top of the tank and warming the bottom.
- 4) Most of the mixing occurs near the inlet and outlet as dense cold water flows under a layer of lighter warm water or warm water flows over a layer of water. The mixing phenomenon is called the Kelvin-Helmholtz instability and is closely related to the waves that form when wind blows over a lake or ocean. The parameter that governs the mixing and growth of the waves is the Richardson number.
- 5) A 1-dimensional theory is sufficient to describe the temperature at any point in the tank except in the mixing regions near the inlet and outlet. Such a theory was developed under the following assumptions:
 - a) The water temperature is a function of elevation and time, only.

- b) The flow in the tank is plug flow (that is, the same velocity and direction everywhere) except in the mixing regions near the inlet and outlet. The mixing can be accounted for by an empirical constant.
 - c) The flow rate does not vary with time.
 - d) The cross sectional areas of the water and the tank walls are constant with respect to elevation.
 - e) There is a step change of inlet temperature at the beginning of the charge or discharge process.
 - f) Heat transfer between the water and the tank walls is allowed. The heat transfer coefficient must be measured experimentally.
 - g) Thermal losses from the tank are neglected.
- 6) Tall tanks provide the best stratification, but the height-to-diameter ratio should not exceed about 4 to 1 because of thermal losses.
- 7) Relatively simple inlet and outlet diffusers such as the dual radial-flow configuraton shown in Figure 1 provide the best stratification.

HOW TANKS FAIL TO STRATIFY

Monitoring of the Forest City Dillon apartment in Washington, DC by the National Solar Data Network has provided the time and space history of a real tank using the conventional collection strategy [3] shown in Figure 2.

The tank was stratified at the 09:43 startup. At 09:43 the tank remains stratified, but at 10:10 the shape of the profile changes as the tank begins to mix. By 11:09 the tank is completely mixed, and it remains mixed for the rest of the day.

At 09:11 the pump draws 10°C water from the bottom of the tank. The Richardson number is 0.55. Since the flowrate is nearly constant regardless of the rate, energy is delivered by the collector and at 09:11 the collector delivers very little energy, the water is returned to the tank only slightly warmer than 10°C.* The return water being denser than the 40°C water at the top of the tank sinks to the bottom with relatively little mixing. In this manner the temperature at the bottom of the tank increases while the remainder of the temperature profile changes little.

The curve at 09:43 shows that the bottom temperature has risen to 24.5°C but the remainder of the temperature profile has not changed significantly. At this time the Richardson number of 0.28 is larger than the

*It should be noted that the return temperature indicated for 09:11 at the top of Figure 2 is higher than it should be. The return temperature sensor was sensing the temperature at the top of the tank before the pump was turned on. At 09:11 the sensor had not stabilized to the true return temperature.

Figure 1. Dual Radial-flow Diffusion Configuration.

Figure 2. Temperature Profile Measurements as a Function of Time.

Source of Data: M. J. Kennedy, S. J. Sersen, and S. M. Rossi (Vitro Laboratories, Silver Spring, Md.), "Comparison of Liquid Solar Thermal Storage Subsystems in the National Solar Data Network," ASME Paper Number 80/WA/SOL-36 (November 1980).

critical value of 0.25. Since the return temperature is about 25.5°C, the return water sinks and continues to warm the bottom of the tank. The Richardson number which is proportional to the top-to-bottom temperature difference continues to decrease.

At 10:10 the top-to-bottom temperature difference has decreased to 12°C, and the Richardson number has decreased to 0.22. As Figure 1.1 shows, the tank has begun to mix. By 11:09 the tank is nearly isothermal which ensures a Richardson number less than about 0.06. The tank can not be re-stratified until the pump is turned off and a temperature difference develops. Other figures found in [3] gave similar results.

The cause of the failure to stratify is the collection strategy. The collection strategy ensures that the top-to-bottom temperature difference (and hence, the Richardson number) will decrease during the day, and the collection strategy has an excessively high flow rate. The high flow rate appears as a squared factor in the denominator of the Richardson number.

Even if the top-to-bottom temperature difference could be maintained, the high flowrate through the collector ensures that the temperature difference will be less than the full degree of stratification that should be maintained. With the usual rules for sizing storage relative to the collector [4] a fully charged water tank will be 20-30°C hotter at the end of a sunny day than a completely discharged tank. For complete stratification the top of the tank should be the same 20-30°C hotter than the bottom of the tank. Figure 2 shows that at 12:10 hrs. the return temperature from the collector heat exchanger (40°C) is only 3°C hotter than the bottom of the tank. The 3°C temperature difference would produce a negligible improvement in performance compare to the full 20-30°C of stratification.

Clearly the control strategy causes failure to stratify by putting cool water into the top of the tank in the morning and by having a collector flowrate that is too high. The knowledge of how-failure to stratify occurs suggests a remedy.

NEW COLLECTION STRATEGY

Three rules defining a new collection strategy which will allow stratification have been developed. The rules are:

1) Assuming a diurnal cycle, allow no more than one tank volume per day to pass through the solar collector or collector heat exchanger, and similarly, allow no more than one tank volume per day to be delivered to the load. This rule, which has recently been recognized by others [1,5], makes it possible to achieve the full 20-30°C of stratification. The reduced flowrate provides a secondary benefit of reduced parasitic pumping power.

Because the collector raises the water to operating temperature in one pass instead of several, a larger number of collectors can be plumbed in series than with the conventional strategy. This means that there will be fewer parallel circuits, and the flow distribution between parallel circuits will be better. The new system schematic is compared to the conventional schematic in Figure 3.

(a) Conventional System

(b) Modified System

Figure 3. Schematic of Conventional System (a) Compared to Schematic of Modified System (b).

2. Vary the flowrate through the collector and collector heat exchanger so that the temperature of the water returned to the top of the tank is constant.

The flowrate through the collector can be controlled by using a centrifugal pump and installing a thermostatic valve similar to an automobile thermostat that opens when the collector output rises to the desired temperature. In an antifreeze system two thermostatic valves are required, one at the collector outlet and one at the heat-exchanger-to-tank outlet. This minor system change ensures that cool water will not be put into the top of the tank and maintains the high Richardson number necessary for stratification. A small amount of leakage will improve the valve's ability to sense temperature when it is closed.

The temperature of the water returning from the load to the bottom of the tank will be nearly constant in most cases. For some systems, however, a temperature-sensing valve in the return line may be necessary. This valve, which controls the temperature at the bottom of the tank, should open when the returning water is colder than the set point.

3. Adjust the temperature setting of the collector outlet valve so that the water is returned to the tank at the minimum temperature necessary to satisfy the load plus a few degrees allowance for heat exchange processes and thermal losses.

The first two rules must be implemented before this rule can be implemented, but this rule is responsible for the improved performance that can be achieved with stratification. The collectors in a stratified system can be expected to operate at an average temperature that is about 20°C (40°C) cooler than the average temperature for a mixed system. The resulting benefits to the system will be larger if a flatplate collector is used than if a vacuum tube or concentrating collector is used.

One alternative configuration that was suggested but does not give the full benefits of stratification is to recirculate a portion of the collector flow back to the collector inlet using a temperature sensitive three-way valve. Such a scheme would allow the traditional high flowrate through the collector and give a constant tank inlet temperature. Unfortunately, the recirculation methods also raises the collector inlet temperature and, therefore, the average collector temperature. This causes a lower collector efficiency than would be the case if the three collection strategy rules had been followed rigorously.

This method of stratifying the tank can not be used with coil-in-tank heat exchangers. For example, a coil-in-tank collector heat exchanger is located near the bottom of the tank. As it heats the bottom of the tank the Richardson number decreases, and the tank will mix when the critical Richardson number is reached. There is no known way of controlling the flowrate inside the tank to prevent mixing if a coil-in-tank heat exchanger is used.

SUMMARIZED THEORY

An understanding of the theory developed at Argonne National Laboratory [2] is not necessary for designing systems with stratified tanks. The im-

portant point is that there is a measure of "goodness" of stratification called the stratification index S which is determined by the heat capacity ratio a , the Fourier number F_o , and the Richardson number R_i . The stratification index is defined so that $S = 1$ for a perfectly stratified lossless tank, and $S = 0$ for a completely mixed tank. The theory is summarized below so that the reader can determine how much detail is required for his needs.

The theory relates the temperature T in the tank at any elevation x and time t to the Fourier number F_o , the heat capacity ratio a , a constant c describing the mixing that occurs near the inlet and outlet, a time constant t^* , and the normalized heat transfer coefficient H . The relationship is:

$$\frac{T - T_{in}}{T_s - T_{in}} = \theta + \Delta \quad (1)$$

where

$$\theta = \frac{1}{2} \left[1 + \operatorname{erf} \left(\frac{\frac{x}{L} - \frac{at}{t^*}}{2\sqrt{F_o} \sqrt{\frac{at}{t^*}} + c} \right) \right] \quad (2)$$

$$\Delta = \frac{1}{2} H \left(\frac{a-1}{a} \right) \left\{ e^{\frac{\left(\frac{HC^2}{2a} \right)^2 + \frac{2x}{L} \left(\frac{HC^2}{2a} \right)}{c^2} - \frac{Ht}{t^*}} \left(\frac{x}{L} - \frac{HC^2}{2a} \right) \left[\operatorname{erf} \left(\frac{\frac{x}{L} + \frac{HC^2}{2a}}{c} \right) \right. \right. \\ \left. \left. - \operatorname{erf} \left(\frac{\frac{x}{L} + \frac{HC^2}{2a} - \frac{at}{t^*}}{c} \right) \right] + \frac{c}{\sqrt{\pi}} \left[e^{-\left(\frac{x^2}{L^2 c^2} + \frac{Ht}{t^*} \right)} - e^{-\left(\frac{\frac{x}{L} - \frac{at}{t^*}}{c} \right)^2} \right] \right\} \quad (3)$$

$$c = 2\sqrt{F_o} \sqrt{\frac{at}{t^*}} + c \quad (4)$$

$$F_o = \frac{k_l t^*}{\rho_l c_{pl} L^2 a} = \text{Fourier number} \quad (5)$$

$$a = \frac{\sum_{i=1}^n \rho_i A_i C_{pi} L}{\rho_1 A_1 C_{p1} L} \quad (6)$$

$$\tau^* = \frac{La}{v} = \text{time constant} \quad (7)$$

Figure 4 shows the tank outlet temperature plotted against time (that is, equation (1) at $x = L$). Both time and temperature have been normalized. One quirk of the temperature normalization is that for a charging process the cold initial temperature is at the top of the graph and the warmer temperatures are at the bottom of the graph. Thus, the plotted curve is the same whether the tank is being charged or discharged. In the paragraphs that follow a discharging process will be referred to, but with minor modifications the same words could apply to a charging process. Also plotted in Figure 4 are the two components that comprise the outlet temperature. One component is the water in the tank (Equation 2) and the other is the heat stored in the tank wall (Equation 3).

As Figure 4 shows the heat does not leave the wall until after the thermocline has passed. Furthermore, when the heat does leave the tank wall, its temperature is less than the minimum usable temperature.

Figure 5 compares the performance of a tank with a massive wall ($a = 1.1$) to the performance of a light wall ($a = 1.025$). As Figure 5 shows, the tank with the light wall maintains a high outlet temperature longer than the tank with the massive wall does. Therefore, a tank with a light wall provides better stratification than a tank with a massive wall does. System designers should avoid using massive tank walls.

The mixing constant c and the normalized heat transfer coefficient H and empirical constants. The Argonne National Laboratory report [2] measured and correlated them with the Fourier and Richardson numbers. The correlations are:

$$c = 2.44 \times 10^{-5} F_o^{-1.5} \exp(-0.0134 R_i) \quad (8)$$

$$H = 7.7 \quad (9)$$

Extreme accuracy should not be expected of Equations (8) and (9) nor is it required. Equation (8) gives the value of c accurate to within a factor of about 3.5 in most cases for Richardson numbers between 1 and 500. Equation (9) gives H accurate to within a factor of 2. The value of equations (8) and

Figure 4. Comparison of Equations 1, 2, and 3.

Figure 5. Comparison of Tanks Having High and Massive Walls

(9) is that they preserve the overall trends of the Fourier and Richardson numbers. Thus, a designer can use equations (8) and (9) to estimate the performance of several tank designs and choose the one that gives the best performance.

In Figure 5 the area under the curve from $t/t^* = 0$ to $t/t^* = 1$ is a good measure of stratification. In mathematical terms the area is the integral.

$$I = \int_0^1 \frac{(T - T_{in})}{(T_s - T_{in})} d \frac{t}{t^*} \quad (10)$$

For a perfectly stratified tank without thermal losses the area $I = 1$. The disadvantage of equation (10) is that for a completely mixed tank $I = 0.632$. A more meaningful measure of stratification would be equal to 1 for a perfectly stratified tank and equal to 0 for a completely mixed tank. Such a measure is called the stratification index S and is defined in equation (11).

$$S = \frac{I - 0.632}{0.368} \quad (11)$$

Stratification indices have been calculated for various values of heat capacity ratio a , Fourier number F_0 , and Richardson number R_i . Results are tabulated in Appendix C. A computer program written in Applesoft* BASIC to calculate the stratification index is also listed in Appendix C.

HOW TO DESIGN A STRATIFIED SYSTEM

This section is titled "How to Design a Stratified System" instead of "How to Design a Stratified Tank" because the entire system is involved in the design process. A design example is included to illustrate the step-by-step process.

Example: A solar space heating system must heat air returning from the building at 18°C (64°F) to the 41°C (106°F) discharge temperature. The liquid-to-air load heat exchanger has an effectiveness $\epsilon_L = 0.67$. Thermal loss from storage (including piping to the load heat exchanger) will be less than 2% in 12 hours. The tank is located in a 16°C (61°F) basement. The collector performance is given as:

*Applesoft is a trademark of Apple Computer, Inc.

$$\eta_c = 0.78 - (1.384 \text{ W/m}^2 \cdot ^\circ\text{C})(T_{CC} - T_a)/I$$

measured at a flow rate of $0.0142 \text{ kg/s} \cdot \text{m}^2$ ($10.44 \text{ lb/hr} \cdot \text{ft}^2$). Piping losses increase the effective loss coefficient from 1.384 to $1.412 \text{ W/m}^2 \cdot ^\circ\text{C}$. The collector area is 44.6 m^2 (480 ft^2) comprised of 20 collector panels. The collector heat exchanger has an effectiveness $\epsilon_c = 0.80$.

Step 1: Modify the system schematic to include thermostatic valves and more collectors in series

Example: The conventional system as shown in Figure 3 has 10 parallel branches of two collector panels in the collector array. The new system schematic has only five parallel branch, but each branch has four collectors in series. The modified system schematic calls for centrifugal pumps and thermostatic valves in the collector array outlet and the heat exchanger outlet to the storage tank. Temperatures used in this design procedure are marked on the modified schematic.

Step 2: Determine the temperature requirements of the load

Example: The statement of the example gives the required discharge temperature as $T_{LH} = 41^\circ\text{C}$ and the return air temperature as $T_{LC} = 18^\circ\text{C}$.

Step 3: Calculate the top and bottom tank temperature

Example: A capacity rate $\dot{W}_L = \dot{m}_L C_{pL}$ of the water to the load equal to the capacity rate $\dot{W}_A = \dot{m}_A C_{pA}$ of the air will be assumed for this example, but the formula can be applied to unequal capacity rates. "min ($\dot{W}_A, \dot{W}_L$)" means the smaller of $\dot{W}_A$ or $\dot{W}_L$.

a: Calculate the top and bottom storage temperatures neglecting thermal losses

$$\begin{aligned} T_{SH} &= T_{LC} + \frac{\dot{W}_A (T_{LH} - T_{LC})}{\epsilon_L \min(\dot{W}_A, \dot{W}_L)} \\ &= 18^\circ\text{C} + (1) \frac{41 - 18^\circ\text{C}}{0.67} = 52.3^\circ\text{C} (126^\circ\text{F}) \end{aligned}$$

$$T_{SC}' = T_{SH}' - \frac{W_A}{W_L} (T_{LH} - T_{LC})$$

$$= 52.3 - (1) (41 - 18^\circ\text{C}) = 29.3^\circ\text{C} (85^\circ\text{F})$$

- b. Increase the tank temperature by a small amount ΔT to compensate for about 12 hours of thermal loss. The thermostatic valve setting for the heat exchanger outlet is T_{SH}

The example gives the thermal loss $\ell = 0.02$ in 12 hours. The ambient temperature for the storage tank T_{as} is 16°C .

$$\begin{aligned}\Delta T &= \ell (T_{SH}' - T_{as}) \\ &= 0.02 (52.3 - 16^\circ\text{C}) = 0.7^\circ\text{C}\end{aligned}$$

$$\begin{aligned}T_{SH}' &= T_{SH}' + \Delta T \\ &= 52.3 + 0.7 = 53^\circ\text{C} (127^\circ\text{F}) \text{ valve setting}\end{aligned}$$

$$\begin{aligned}T_{SC}' &= T_{SC}' + \Delta T \\ &= 29.3 + 0.7 = 30^\circ\text{C} (86^\circ\text{F})\end{aligned}$$

Step 4: Calculate the collector inlet and outlet temperatures T_{CC} and T_{CH}
The thermostatic valve at the collector outlet should be set a T_{CH} .

In this example the capacity rate $W_C = \dot{m}_C C_{pC}$ of the collector is equal to the capacity rate $W_S = \dot{m}_S C_{pS}$ of the collector heat exchanger-to-storage loop, but the formula can be used for unequal capacity rates. " $\min (W_S, W_C)$ " means the smaller of W_S or W_C .

$$T_{CH} = T_{SC}' + \frac{W_S (T_{SH}' - T_{SC}')}{\epsilon_C \min (W_S, W_C)}$$

$$= 30^\circ\text{C} + (1) \frac{53 - 30^\circ\text{C}}{0.80} = 58.8^\circ\text{C} (138^\circ\text{F}) \text{ valve setting}$$

$$T_{CC} = T_{CH} - \frac{W_S}{W_C} (T_{SH}' - T_{SC}')$$

$$= 58.8 - (1) (53 - 30^\circ\text{C}) = 35.8^\circ\text{C} (96^\circ\text{F})$$

Step 5: Roughly estimate the amount of energy the collector can collect on a sunny day. Assume the energy incident on the collector has the form.

$$I = I_0 \cos \left(\frac{t - t_n}{t_o} \right)$$

where

$$I_0 = 1000 \text{ W/m}^2 \text{ (317 Btu/hr} \cdot \text{ft}^2\text{)}$$

$$t_o = 13751 \text{ s (3,820 hr)}$$

a: Convert collector efficiency from inlet temperature T_{CC} format to average temperature $T_{avg} = (T_{CC} + T_{CH})/2$ format and to outlet temperature T_{CH} format.

$$\begin{aligned} \eta_{avg} &= \left(\frac{\frac{\dot{m}C_P}{A_c}}{\frac{\dot{m}C_P}{A_c} - \frac{F_R U_L}{2}} \right) \left[F_R \tau \alpha - F_R U_L \left(\frac{T_{avg} - T_a}{I} \right) \right] \\ &= \left[\frac{\left(0.0142 \frac{\text{kg}}{\text{s} \cdot \text{m}^2} \right) \left(4184 \frac{\text{J}}{\text{kg} \cdot ^\circ\text{C}} \right)}{(0.0142)(4184) - \frac{1.384}{2} \frac{\text{W}}{\text{m}^2 \cdot ^\circ\text{C}}} \right] \left[0.78 - \left(1.384 \frac{\text{W}}{\text{m}^2 \cdot ^\circ\text{C}} \right) \left(\frac{T_{avg} - T_a}{I} \right) \right] \\ &= 0.7392 - \left(1.4003 \frac{\text{W}}{\text{m}^2 \cdot ^\circ\text{C}} \right) \left(\frac{T_{avg} - T_a}{I} \right) \end{aligned}$$

$$\begin{aligned}
\eta_H &= \left(\frac{\frac{\dot{m}C_p}{A_c}}{\frac{\dot{m}C_p}{A_c} - F_R U_L} \right) \left[F_R \tau \alpha - F_R U_L \left(\frac{T_{CH} - T_a}{I} \right) \right] \\
&= \left[\frac{(0.0142)(4184)}{(0.0142)(4184) - 1.384} \right] \left[0.78 - 1.384 \left(\frac{T_{CH} - T_a}{I} \right) \right] \\
&= 0.7986 - \left(1.4170 \frac{W}{m^2 \cdot ^\circ C} \right) \left(\frac{T_{CH} - T_a}{I} \right)
\end{aligned}$$

b. Calculate turn-on, turn-off time $|t_{on/off} - t_n|$ based on $\eta_H = 0$

In this example a late winter ambient temperature $T_a = -10^\circ C$ ($14^\circ F$) is assumed.

$$\begin{aligned}
|t_{on/off} - t_n| &= t_o \cos^{-1} \left[\frac{(F_R U_L)_H (T_{CH} - T_a)}{(F_R \tau \alpha)_H I_o} \right] \\
&= (1351s) \cos^{-1} \left[\frac{\left(1.4170 \frac{W}{m^2 \cdot ^\circ C} \right) (58 - (-10^\circ C))}{(0.7986)(1000 \text{ W/m}^2)} \right] \\
&= 19917s \text{ (5.532 hr)}
\end{aligned}$$

c: The energy collected Q is approximately the integral of incident energy I times collector area A times collector efficiency η_{avg} (average temperature format). The sine functions argument $(t_{off} - t_{noon})/t_o$ has units of radians.

$$\begin{aligned}
Q &= 2A_c \left[(F_R \tau \alpha)_{\text{avg}} I_o t_o \sin \left(\frac{t_{\text{off}} - t_n}{t_o} \right) - (F_R U_L)_{\text{avg}} (T_{\text{avg}} - T_a) (t_{\text{off}} - t_n) \right] \\
&= (2)(44.6 \text{ m}^2) \left[(0.782) \left(1000 \frac{\text{W}}{\text{m}^2} \right) (13751 \text{ s}) \sin \left(\frac{19917}{13751} \right) \right. \\
&\quad \left. - \left(1.4003 \frac{\text{W}}{\text{m}^2 \cdot ^\circ\text{C}} \right) (47.3 - (-10^\circ\text{C})) (19917 \text{ s}) \right] \\
&= 818 \text{ MJ } (0.776 \text{ MBtu})
\end{aligned}$$

Step 6: Calculate the minimum tank size

$$\begin{aligned}
(mC_p)_1 &= \frac{Q}{T_{\text{SH}} - T_{\text{SC}}} \\
&= \frac{818 \text{ MJ}}{53 - 30^\circ\text{C}} = 35.6 \text{ MJ}/^\circ\text{C} \text{ (18930 Btu}/^\circ\text{F)} \\
V &= \frac{(mC_p)_1}{\rho C_{p1}} \\
&= \frac{35.6 \text{ MJ}/^\circ\text{C}}{\left(1000 \frac{\text{kg}}{\text{m}^3} \right) \left(4184 \frac{\text{J}}{\text{kg} \cdot ^\circ\text{C}} \right)} = 8.50 \text{ m}^3 \text{ (2245 gal)}
\end{aligned}$$

This volume is $0.191 \text{ m}^3/\text{m}^2$ ($4.67 \text{ gal}/\text{ft}^2$) which is considerably larger than the volume specified by the conventional rule of thumb. Unlike the conventional sizing method which is based only on collector size, the new sizing method is based on both the collector size and the temperature requirements of the load.

Step 7: Select tank dimensions and materials, and calculate the heat capacity ratio a.

Assume a water depth $L = 1.5 \text{ m}$ (5 ft) the tank diameter is:

$$d = \left[\frac{4V}{\pi L} \right]^{1/2}$$

$$= \left[\frac{(4)(8.50 \text{ m}^3)}{(\pi)(1.5\text{m})} \right]^{1/2} = 2.686 \text{ m (8 ft 10 in)}$$

The tank will be made of 1.2 cm (0.5 inch) thick plywood with a 1mm (40 mil) thick PVC liner.

$$C_{p2} = 2720 \text{ J/kg} \cdot ^\circ\text{C}$$

$$\rho_2 = 550 \text{ kg/m}^3$$

Neglect the heat capacity of the liner because it is very thin compared to the wood.

$$(mC_p)_2 = \pi \left[\frac{d^2}{2} + Ld \right] s_2 \rho_2 C_{p2}$$

$$= \pi \left[\frac{(2.686 \text{ m})^2}{2} + (2.68 \text{ m})(1.5 \text{ m}) \right] (0.012\text{m}) \left(550 \frac{\text{kg}}{\text{m}^3} \right) \left(2720 \frac{\text{J}}{\text{kg} \cdot ^\circ\text{C}} \right)$$

$$= 0.43 \text{ MJ}/^\circ\text{C}$$

$$a = \frac{(mC_p)_1 + (mC_p)_2}{(mC_p)_1}$$

$$= \frac{35.6 + 0.43 \text{ MJ}/^\circ\text{C}}{35.6} = 1.012$$

Step 8: Choose dimensions for radial-flow diffusers that will give a Richardson number of 100-300 at the maximum flow rate, and calculate the Fourier number.

- a. Calculate maximum flow rate.

$$\begin{aligned}
 q_{\max} &= \frac{A_c [F_{R\alpha\tau}]_{\text{avg}} I_o - (F_R U_L)(T_{\text{avg}} - T_a)]}{\rho_l C_{pl} (T_{\text{SH}} - T_{\text{SC}})} \\
 &= \frac{(44.6 \text{ m}^2) \left[(0.7892)(1000 \text{ W/m}^2) - (1,4003 \frac{\text{W}}{\text{m}^2 \cdot ^\circ\text{C}})(47.3 - (-10^\circ\text{C})) \right]}{(1000 \frac{\text{kg}}{\text{m}^3})(4184 \frac{\text{J}}{\text{kg} \cdot ^\circ\text{C}})(53 - 30^\circ\text{C})} \\
 &= 0.000329 \text{ m}^3/\text{s} \text{ (5.21 gpm)}
 \end{aligned}$$

This maximum flow rate is about half of the flow rate specified by the conventional rule of thumb.

- b. Assume a Richardson number and calculate the maximum inlet velocity.

For this example assume $R_i = 300$.

$$\begin{aligned}
 u_{\max} &= \left[\frac{g \beta \Delta T}{R_i} \right]^{1/2} \\
 &= \left[\frac{(9.8 \text{ m/s}^2)(0.00043^\circ\text{C}^{-1})(1.5 \text{ m})(53 - 30^\circ\text{C})}{300} \right]^{1/2} \\
 &= 0.022 \text{ m/s} \text{ (0.87 in/s)}
 \end{aligned}$$

- c. Choose a diffuser diameter approximately 0.25 ~ 0.33 times the tank diameter. See Figure 1 for an illustration of a radial-flow diffuser.

For this example assume a diffuser diameter $d_d = 0.762 \text{ m}$ (2.5 ft).

- d. Calculate the spacing δ between the diffuser and the top or bottom of the tank.

$$\delta = \frac{q_{\max}}{\pi d_d u_{\max}}$$

$$= \frac{0.000329 \text{ m}^3/\text{s}}{(\pi)(0.762 \text{ m})(0.022 \text{ m/s})} = 0.00625 \text{ m (0.25 in)}$$

- e. Calculate the Fourier number

$$F_o = \frac{\pi k_i d^2}{4 \rho_l c_{pl} q_{\max}}$$

$$= \frac{(\pi) (0.638 \frac{\text{W}}{\text{m} \cdot ^\circ\text{C}}) (2.686 \text{ m})^2}{(4) (1000 \frac{\text{kg}}{\text{m}^3}) (4184 \frac{\text{J}}{\text{kg} \cdot ^\circ\text{C}}) (1.5 \text{ m}) (0.000329 \text{ m}^3/\text{s})}$$

$$= 1.75 \times 10^{-3}$$

Step 9: Estimate the stratification index S

Interpolate the tables in Appendix C to get the stratification index for the values of a , F_o , and R_i that have been calculated.

For this example $S = 0.923$. If the designer has the option of choosing one of several tank designs, the design having the best stratification index can be selected.

SUMMARY

The design procedure given above is relatively simple and can result in an overall performance improvement of up to 20%. At the present time it applies to vertical cylindrical or rectangular tanks for storing hot water. It does not apply to horizontal cylindrical tanks because the cross sectional area of such tanks is a function of elevation, nor does it apply to cold storage tanks because the volumetric coefficient of thermal expansion for such tanks is not constant. Several design recommendations deserve special emphasis. They are listed in order of their importance.

- 1) Use the modified schematic and the new collection strategy described in this article. Failure to do so will cause an otherwise properly designed system to fail to stratify.
- 2) Use a tank height-to-diameter ratio of 4, if possible. If the space available does not allow such a tall tank, use the tallest tank possible.
- 3) Use light weight tank walls. The amount of heat stored in the tank wall degrades the overall tank performance.
- 4) Insulate the tank properly. The thermal loss from the tank should not exceed 2% in 12 hours for a diurnal cycle.
- 5) Avoid using dip tubes or vertical baffles. Heat transfer across the walls of the dip tube or vertical baffle cools the top of the tank, heats the bottom of the tank, and degrades stratification. If using a dip tube is unavoidable, make the dip tube small to minimize its surface area and insulate it heavily with waterproof insulation.
- 6) Use the dual radial-flow baffles described in this article, if possible. They provide a smooth, low velocity entrance and exit for the water.

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NOMENCLATURE

A	= area, m^2
a	= heat capacity ratio, dimensionless
C	= approximation to denominator of error function, dimensionless
c	= mixing parameter, dimensionless
C_p	= heat capacity, $J/kg \cdot ^\circ C$
d	= tank diameter, m
d_d	= diffuser diameter, m
Fo	= Fourier number, dimensionless
$F_R U_L$	= thermal loss coefficient for collector, $W/m^2 \cdot ^\circ C$
$F_R \tau \alpha$	= optical parameter for collector, dimensionless
g	= gravitational constant, 9.81 m/s^2
H	= normalized heat transfer coefficient, dimensionless
I	= 1) integral related to stratification index, dimensionless 2) solar insolation in the plane of the collector, W/m^2
k	= thermal conductivity, $W/m \cdot ^\circ C$
L	= tank height, m
$\dot{L}$	= thermal loss rate
m	= mass, kg
$\dot{m}$	= mass flow rate, kg/s
Q	= thermal energy, J
q	= flow rate, m^3/s
R_i	= Richardson number, dimensionless
S	= stratification index, dimensionless
s	= thickness, m
T	= temperature, $^\circ C$

NOMENCLATURE (Continued)

t	= time, s
t*	= time constant, s
u	= inlet velocity, m/s
V	= tank volume, m ³
v	= vertical velocity in the tank, m/s
W	= capacity rate, J/s·°C
x	= elevation above tank inlet for tank discharging or depth below tank inlet for tank charging, m
β	= volumetric coefficient of thermal expansion, °C ⁻¹
Δ	= portion of normalized tank temperature deriving from the tank walls, dimensionless
ΔT	= 1) top-to-bottom temperature difference in tank, °C 2) a small temperature difference, °C
δ	= spacing between the diffuser plate and the top or bottom of the tank, m
ε	= heat exchanger effectiveness, dimensionless
η	= solar collector efficiency, dimensionless
π	= 3.14159265
ρ	= density, kg/m ³
θ	= portion of normalized tank temperature deriving from the water, dimensionless

Subscripts

1	water in the tank
2	tank wall
A	air
a	outdoor ambient condition
as	ambient condition beside storage
avg	average
c	1) based on collector inlet temperature 2) collector
CC	cold side of collector
CH	hot side of collector
H	based on collector outlet temperature
i	ith component of the tank. For example, i = 1 refers to the water, and i = 2 refers to the tank wall
in	inlet condition
L	load
LC	cold side of load
LH	hot side of load
max	maximum
n	noon
o	baseline condition
on/off	collector turn on or turn off condition
s	storage side of collector heat exchanger
S	before the beginning of charging or discharging
SC	cold side of thermal storage
SH	hot side of thermal storage
w	water

APPENDIX B:
SUMMARY OF EXPERIMENTAL DATA

Exp. Date	07-08-80	07-14-80	08-07-80	08-22-80	08-29-80
Flow (l/m)	7.681	0.737	7.725	0.734	7.721
V(l)	190	190	190	190	190
Chg./Dischg.	D	C	D	C	D
Conf'g'tion	HB	HB	DT	DT	IDT
Ts(°C)	60.20	23.65	61.08	23.20	59.90
Tin(°C)	23.19	62.30	23.22	57.96	22.90
t*(hrs.)	0.449	4.716	0.449	4.734	0.449
a(calc)	1.086	1.086	1.086	1.086	1.086
F _O	9.93x10 ⁻⁵	1.0x10 ⁻³	9.93x10 ⁻³	1.0x10 ⁻³	9.93x10 ⁻⁵
R _i					
δ/L					
L/D	4	4	4	4	4
Re	586.48	55.84	586.48	55.62	586.48
μ'/μm	1.47	1.47	1.47	1.47	1.47
$\int_0^1 \theta d\left(\frac{t}{t^*}\right)$	0.920	0.910	0.884	0.796	0.929
$\int_0^t \theta d\left(\frac{t}{t^*}\right)$	0.996	0.993	0.965	0.925	0.985

Measured

S
a
C
H

Exp. Date	09-03-80	09-09-80	10-03-80	10-06-80	10-23-80
Flow (l/m)	0.704	0.742	7.566	0.753	2.050
V(l)	190	190	190	190	190
Chg./Dischg.	C	C	D	C	D
Conf'g'tion	IDT	SIO	CB	CB	RF
Ts(°C)	23.48	22.70	60.15	23.70	62.35
Tin(°C)	61.95	60.49	23.05	61.74	20.56
t*(hrs.)	4.904	4.716	0.459	4.275	0.447
a(calc)	1.086	1.086	1.086	1.086	1.086
F _o	1.085x10 ⁻³	1.043x10 ⁻³	1.015x10 ⁻⁴	9.455x10 ⁻⁴	9.89x10 ⁻⁵
R _i		444.09			208.27
δ/L		0.045			6.048x10 ⁻³
L/D	4	4	4	4	4
Re	53.69	55.84	573.71	61.59	589.11
μ/μ _m	1.47	1.47	1.47	1.47	1.47
$\int_0^1 \theta_d \left(\frac{t}{t^*} \right)$	0.893	0.903	0.838	0.813	0.98
$\int_0^t \theta_d \left(\frac{t}{t^*} \right)$	0.956	0.951	0.959	1.00	1.03
Measured					
S					0.95
a					1.057
C					0.0127
H					18.453

Exp. Date	10-27-80	10-28-80	10-28-80	10-28-80	10-30-80
Flow (l/m)	0.291	2.025	2.023	0.464	1.058
V(l)	190	190	190	190	190
Chg./Dischg.	C	D	D	C	D
Conf'g'tion	RF	RF	RF	RF	RF
Ts(°C)	22.17	40.44	51.88	21.65	61.45
Tin(°C)	47.84	21.59	21.43	39.96	16.50
t*(hrs.)	3.144	0.452	0.454	0.464	0.870
a(calc)	1.086	1.086	1.086	1.086	1.086
F _O	6.95x10 ⁻⁴	1.00x10 ⁻⁴	1.00x10 ⁻⁴	4.37x10 ⁻⁴	9.93x10 ⁻⁴
R _i	6359	96.47	156.15	1784.56	842.74
δ/L	6.048x10 ³	6.048x10 ⁻³	6.048x10 ⁻³	6.048x10 ⁻³	6.048x10 ⁻³
L/D	4	4	4	4	4
Re	83.76	582.59	580.02	133.13	302.68
μ/μ _m	1.47	1.47	1.47	1.47	1.47
$\int_0^1 \theta d\left(\frac{t}{t^*}\right)$	0.98	0.98	0.95	0.96	0.96
$\int_0^t \theta d\left(\frac{t}{t^*}\right)$	1.07	1.02	1.00	1.03	0.98
Measured					
S	0.95	0.95	0.86	0.89	0.89
a	1.011	1.056	1.103	0.994	1.00
C	7.590	13.322	0.473	12.115	2.026
H	3.540	5.187	11.112	98.28	119.35

Exp. Date	10-31-80	10-31-80	11-04-80	11-19-80	11-24-80
Flow (l/m)	4.039	4.005	2.020	2.114	7.620
V(l)	190	190	190	190	190
Chg./Dischg.	D	D	D	D	D
Conf'g'tion	RF	RF	RF	RF	RF
Ts(°C)	51.59	40.49	61.62	63.60	61.85
Tin(°C)	16.29	16.92	17.32	21.45	19.76
t*(hrs.)	0.857	0.870	0.454	0.433	0.436
a(calc)	1.086	1.086	1.086	1.086	1.086
F _o	1.89x10 ⁻⁴	1.92x10 ⁻⁴	1.00x10 ⁻⁴	9.58x10 ⁻⁵	9.65x10 ⁻⁵
R _i	650.72	441.90	227.84	4.204	798.66
δ/L	6.048x10 ⁻³	6.048x10 ⁻³	6.048x10 ⁻³	0.045	1.00x10 ⁻³
L/D	4	4	4	4	4
R _e	307.27	302.68	580.02	608.16	603.97
μ/μ _m	1.47	1.47	1.47	1.47	1.47
$\int_0^1 \theta d\left(\frac{t}{t^*}\right)$	0.96	0.96	0.94	0.91	0.94
$\int_0^t \theta d\left(\frac{t}{t^*}\right)$	1.02	1.03	0.99	0.95	0.98
Measured					
S	0.89	0.89	0.84	0.76	0.84
a	1.088	1.071	1.111	1.101	1.045
C	0.0216	0.0137	0.312	21.453	10.003
H	12.372	15.068	10.660	1.207	5.561

Exp. Date	05-28-81	05-28-81	05-28-81	05-28-81
Flow (l/m)	7.922	14.49	22.76	30.57
V(l)	190	190	190	190
Chg./Dischg.	D	D	D	D
Conf'g'tion	SIO	SIO	SIO	SIO
Ts(°C)	64.4	63.6	63.9	67.9
Tin(°C)	21.5	22.7	17.4	16.1
t*(hrs.)	0.438	0.239	0.153	0.114
a(calc)	1.086	1.086	1.086	1.086
FO	9.68×10^{-5}	5.29×10^{-5}	3.38×10^{-5}	2.52×10^{-5}
Re	3.65	1.12	0.48	0.31
δ/L	0.045	0.045	0.045	0.045
L/D	4	4	4	4
Re	601.2	1101.8	1721.1	2309.9
μ/μ_m	1.47	1.47	1.47	1.47
$\int_0^1 \theta d\left(\frac{t}{t^*}\right)$	0.93	0.93	0.93	0.88
$\int_0^t \theta d\left(\frac{t}{t^*}\right)$				
Measured				
S	0.81	0.81	0.80	0.68
a				
C				
H				

Exp. Date	05-29-81	05-28-81
Flow (l/m)	34.56	46.93
V(l)	190	190
Chg./Dischg.	D	D
Conf'g'tion	SIO	SIO
Ts(°C)	61.20	61.0
Tin(°C)	18.80	18.27
t*(hrs.)	0.100	0.074
a(calc)	1.086	1.086
F _O	2.22x10 ⁻⁵	1.635x10 ⁻⁵
R _i	0.20	0.11
δ/L	0.045	0.045
L/D	4	4
R _e	2633.3	3558.5
μ/μ _m	1.47	1.47
$\int_0^1 \theta d\left(\frac{t}{t^*}\right)$	0.89	0.89
$\int_0^t \theta d\left(\frac{t}{t^*}\right)$		
Measured		
S	0.68	0.68
a		
C		
H		

Exp. Date	10-14-81	10-26-81	10-27-81	10-28-81	11-03-81
Strategy Pt.	B	C	G	E	A
Flow (l/m)	11.398	44.963	46.428	11.341	11.559
V(l)	3000	3000	3000	3000	3000
Chg./Dischg.	D	D	D	D	D
Conf'g'tion	SIO	SIO	SIO	SIO	SIO
Ts(°C)	47.42	44.97	48.99	57.91	62.55
Tin(°C)	17.11	14.48	17.12	15.08	15.73
t*(hrs.)	4.056	7.030	0.997	4.077	3.988
a(calc)	1.0089	1.0089	1.0089	1.0089	1.0089
F _o	1.84×10^{-3}	4.68×10^{-4}	4.53×10^{-4}	1.85×10^{-3}	1.81×10^{-3}
R _i	1.052	1.026	1.006	22.645	23.67
δ/L	0.0925	0.0925	0.162	0.162	0.0925
L/D	0.615	0.615	0.615	0.615	0.615
R _e	205	824	852	208	213
μ/μ _m	1.47	1.47	1.47	1.47	1.47
$\int_0^1 \theta d\left(\frac{t}{t^*}\right)$	0.93	0.94	0.92	0.92	0.95
$\int_0^t \theta d\left(\frac{t}{t^*}\right)$	0.95	0.98	0.99	0.97	0.98
Measured					
S	0.81	0.84	0.78	0.78	0.86
a	1.067	1.074	1.072	1.070	1.102
C	0.601	5.15	9.51	0.830	0.328
R	0.444	2.88	3.93	7.27	14.79

Exp. Date	11-06-81	11-12-81	11-17-81
Strategy Pt.	D	F	H
Flow (l/m)	45.504	11.542	45.864
V(l)	3000	3000	3000
Chg./Dischg.	D	D	D
Conf'g'tion	SIO	SIO	SIO
Ts(°C)	61.09	46.11	60.98
Tin(°C)	14.47	15.06	14.39
t*(hrs.)	1.017	4.007	1.009
a(calc)	1.0089	1.0089	1.0089
F _O	4.62x10 ⁻⁴	1.82x10 ⁻³	4.58x10 ⁻⁴
R _i	22.056	1.051	21.689
δ/L	0.0925	0.162	0.162
L/D	0.615	0.615	0.615
Re	835	212	842
μ/μ _m	1.47	1.47	1.47
$\int_0^1 \theta_d \left(\frac{t}{t^*} \right)$	0.94	0.94	0.93
$\int_0^t \theta_d \left(\frac{t}{t^*} \right)$	0.99	0.98	0.99
Measured			
S	0.84	0.84	0.81
a	1.064	1.116	1.071
C	2.67	0.213	4.72
H	4.23	11.51	4.45

APPENDIX C:

STRATIFICATION INDEX TABLES

The tables in this appendix give the stratification index S as a function of heat capacity ratio a , Fourier number F_0 , and Richardson number R_i . Following the tables is a computer program to calculate the stratification index written in Applesoft* BASIC.

$a = 1$						
FOURIER NUMBER	RICHARDSON NUMBER					
	1	3	10	30	100	300
1.E-5	0.865	0.867	0.873	0.889	0.930	0.977
3.E-5	0.897	0.898	0.903	0.915	0.945	0.979
1.E-4	0.922	0.923	0.926	0.935	0.956	0.978
3.E-4	0.936	0.937	0.939	0.945	0.959	0.970
1.E-3	0.936	0.936	0.937	0.940	0.947	0.951
3.E-3	0.913	0.913	0.913	0.915	0.917	0.919
1.E-2	0.857	0.857	0.857	0.857	0.858	0.859

$a = 1.025$						
FOURIER NUMBER	RICHARDSON NUMBER					
	1	3	10	30	100	300
1.E-5	0.849	0.851	0.856	0.871	0.907	0.942
3.E-5	0.878	0.879	0.883	0.894	0.920	0.943
1.E-4	0.900	0.901	0.904	0.911	0.928	0.942
3.E-4	0.912	0.913	0.915	0.919	0.930	0.938
1.E-3	0.912	0.912	0.913	0.916	0.921	0.924
3.E-3	0.893	0.893	0.893	0.894	0.896	0.898
1.E-2	0.842	0.842	0.842	0.842	0.843	0.843

*Applesoft is a trademark of Apple Computer, Incorporated.

$a = 1.05$

FOURIER NUMBER	RICHARDSON NUMBER					
	1	3	10	30	100	300
1.E-5	0.835	0.837	0.842	0.855	0.886	0.909
3.E-5	0.862	0.863	0.866	0.876	0.896	0.909
1.E-4	0.881	0.882	0.884	0.890	0.902	0.909
3.E-4	0.891	0.891	0.893	0.896	0.903	0.908
1.E-3	0.891	0.891	0.892	0.894	0.898	0.900
3.E-3	0.875	0.875	0.876	0.877	0.878	0.880
1.E-2	0.829	0.829	0.829	0.830	0.830	0.831

$a = 1.075$

FOURIER NUMBER	RICHARDSON NUMBER					
	1	3	10	30	100	300
1.E-5	0.825	0.826	0.831	0.843	0.870	0.887
3.E-5	0.849	0.850	0.853	0.861	0.878	0.888
1.E-4	0.866	0.866	0.868	0.873	0.883	0.888
3.E-4	0.874	0.874	0.875	0.878	0.884	0.887
1.E-3	0.874	0.875	0.875	0.877	0.880	0.882
3.E-3	0.861	0.862	0.862	0.863	0.864	0.865
1.E-2	0.819	0.820	0.820	0.820	0.821	0.821

$a = 1.1$

FOURIER NUMBER	RICHARDSON NUMBER					
	1	3	10	30	100	300
1.E-5	0.817	0.818	0.823	0.834	0.860	0.875
3.E-5	0.840	0.841	0.844	0.851	0.867	0.876
1.E-4	0.856	0.856	0.858	0.862	0.871	0.876
3.E-4	0.863	0.863	0.864	0.867	0.872	0.875
1.E-3	0.864	0.864	0.865	0.866	0.869	0.870
3.E-3	0.853	0.853	0.853	0.854	0.855	0.856
1.E-2	0.813	0.813	0.813	0.813	0.814	0.814

LIST

```

5  DIM A(0),RI(0),FO(0):I = 0:J =
    0:K = 0
10  REM MAIN PROGRAM TO CALCULAT
    E STRATIFICATION INDEX
15  A$ = ""
20  INPUT "HEAT CAPACITY RATIO ";
    A$: IF LEN (A$) = 0 THEN END

25  FO$ = ""
30  A(0) = VAL (A$): INPUT "FOURI
    ER NUMBER ";FO$: IF LEN (FO
    $) = 0 THEN END
35  RI$ = ""
40  FO(0) = VAL (FO$): INPUT "RIC
    HARDSON NUMBER ";RI$: IF LEN
    (RI$) = 0 THEN END
50  RI(0) = VAL (RI$):H = 7.7
60  X = 1
70  PI = 4 * ATN (1)
80  DT = .025
90  GOSUB 20000
100 PRINT "STRATIFICATION INDEX
    = ";S
110 PRINT
120 GOTO 20
10000 REM CALCULATE ERROR FUNCT
    ION ERF(ZZ)
10001 REM M. ABRAMOWITZ AND I.
    STEGUN
10002 REM HANDBOOK OF MATHEMATI
    CAL FUNCTIONS
10003 REM NATIONAL BUREAU OF ST
    ANDARDS, WASHINGTON, D.C.
10004 REM P. 299, NOV 1970
10010 IF ABS (ZZ) > 4.4 THEN ER
    F = SGN (ZZ): RETURN
10020 ERF = 1 / (1 + .3275911 * ABS
    (ZZ))
10030 ERF = (((((1.061405429 * ERF
    - 1.453152027) * ERF + 1.42
    1413741) * ERF - .204496736)
    * ERF + .254829592) * ERF
10040 ERF = SGN (ZZ) * (1 - ERF *
    EXP ( - ZZ * ZZ))
10050 RETURN
15000 REM CALCULATE NORMALIZED
    TEMPERATURE

```

```

15010 CCAP = 2 * SQR (FO(J) * (1
      + C))
15020 C2 = CCAP * CCAP
15030 HC = H * C2 / 2 / A(I)
15040 ZZ = (X + HC) / CCAP: GOSUB
      10000
15050 D = ERF:ZZ = (X + HC - A(I)
      * T) / CCAP: GOSUB 10000
15060 D = (D - ERF) * (X + HC) *
      EXP (HC * (HC + 2 * X) / C2
      - H * T)
15070 D = (D + CCAP * ( EXP ( - X
      * X / C2 - H * T) - EXP ( -
      ((X - A(I) * T) / CCAP) ^ 2
      ))) / SQR (PI)) * H * (A(I)
      - 1) / A(I) / 2
15080 ZZ = (X - A(I) * T) / 2 / SQR
      (FO(J) * (A(I) * T + C)): GOSUB
      10000
15090 TH = (1 + ERF) / 2 + D
15100 RETURN
20000 REM TRAPEZOIDAL INTEGRATI
      ON OF NORMALIZED TEMPERATURE

20010 C = 2.44E - 5 * FO(J) ^ -
      1.5 * EXP ( - .0134 * RI(K)
      )
20020 S = 0:T = 0:DT = .025: GOSUB
      15000
20030 T0 = TH
20040 FOR Y = 1 TO 40
20050 T = DT * Y: GOSUB 15000
20060 S = S + (T0 + TH) * DT / 2:
      T0 = TH
20070 NEXT Y
20080 S = (S - .63212056) / .3678
      7944
20090 RETURN
]

```