**APPROXIMATIONS OF GAMMA CROSS SECTIONS
FOR FAST NUCLEAR REACTORS**

by

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ABSTRACT

This report shows a method to approximate a P_1 scattering solution for the flux in a fast reactor, using an isotropic, but not a diagonal-transport-approximation scattering matrix. Presented are flux errors relative to a P_1 solution for different levels of transport approximation in an EBR-II type of core surrounded by a stainless steel reflector. Problems associated with the use of the method are also presented.

I. INTRODUCTION

For calculation of the neutron environment in Experimental Breeder Reactor II (EBR-II), the diagonal-transport approximation (P_0^T) with S_4 angular quadrature gives satisfactory results.¹ For calculation of the gamma environment, however, scattering cross sections of a higher order must be used.¹ Use of those cross sections increases calculation time, requirements for computer storage of cross sections, and dependence on anisotropic-transport codes. Therefore, we investigated the possibility of making gamma calculations with an "extended" diagonal-transport approximation that takes into account the effects of higher-order scattering cross sections other than those on the diagonal. This approximation is called the "full-transport" approximation here and is symbolized by P_0^F . Determined were the spatial variations of cross sections for full-transport approximation that would simulate a solution based on P_1 scattering, but would retain an isotropic scattering matrix. Also investigated were solutions generated using a current- and area-weighted average of these spatially dependent cross sections.

II. THEORY

The cross sections for the full-transport approximation are derived from the one-dimensional Boltzmann equation for steady state, with the scattering cross section and flux (in the scattering integral) expanded in Legendre polynomials:

$$\mu \frac{\partial \varphi_g(x, \mu)}{\partial x} + \Sigma_t^g(x) \varphi_g(x, \mu) = \sum_{g'} \sum_{\ell=0}^{\infty} \frac{2\ell+1}{2} \Sigma_{s\ell}^{g' \rightarrow g}(x) P_{\ell}(\mu) \varphi_{g'}^{\ell}(x),$$

where

$\varphi_g(\mathbf{x}, \mu)$ = angular flux at position $\mathbf{x}$ and angle μ for energy group g ,

$\varphi_g^t(\mathbf{x})$ = t th moment of the angular flux at position $\mathbf{x}$ for energy group g ,

$\Sigma_1^g(\mathbf{x})$ = macroscopic total cross section at position $\mathbf{x}$ for energy group g ,

$\Sigma_{gt}^{g' \rightarrow g}(\mathbf{x})$ = t th moment of the macroscopic scattering cross section at position $\mathbf{x}$ for scattering from energy group g' to energy group g ,

and

$P_t(\mu)$ = t th Legendre polynomial.

Substituting into the above equation the Legendre expansion for the flux, multiplying by the Legendre polynomial $P_n(\mu)$, and integrating from -1 to $+1$ give

$$\begin{aligned} \frac{\partial}{\partial \mathbf{x}} \left[\frac{\mathbf{x}}{r} \left(\frac{r}{2} + 1 \right) \frac{\partial \varphi_g^t(\mathbf{x})}{\partial \mathbf{x}} \right] + \int_{-1}^1 P_1(\mu) P_t(\mu) P_n(\mu) d\mu + \Sigma_1^g(\mathbf{x}) \varphi_g^t(\mathbf{x}) \delta_{t,n} \\ - \sum_{g'} \Sigma_{gt}^{g' \rightarrow g}(\mathbf{x}) \varphi_{g'}^t(\mathbf{x}) \delta_{t,n} = 0, \end{aligned}$$

where

$$\delta_{t,n} = \text{Kronecker delta function} = \begin{cases} 1, & t = n \\ 0, & t \neq n \end{cases}$$

The equations for $n = 0$ and $n = 1$, with the summation on t going from zero to one, are the consistent P_1 equations. They are

$$\frac{\partial \varphi_g^0(\mathbf{x})}{\partial \mathbf{x}} + \left[\Sigma_1^g(\mathbf{x}) - \Sigma_{g0}^{g \rightarrow g}(\mathbf{x}) \right] \varphi_g^0(\mathbf{x}) - \sum_{g' \neq g} \Sigma_{g0}^{g' \rightarrow g}(\mathbf{x}) \varphi_{g'}^0(\mathbf{x}) = 0$$

and

$$\frac{\partial}{\partial \mathbf{x}} \left[\frac{\mathbf{x}}{r} \frac{\partial \varphi_g^0(\mathbf{x})}{\partial \mathbf{x}} \right] + \left[\Sigma_1^g(\mathbf{x}) - \Sigma_{g0}^{g \rightarrow g}(\mathbf{x}) \right] \varphi_g^0(\mathbf{x}) - \sum_{g' \neq g} \Sigma_{g1}^{g' \rightarrow g}(\mathbf{x}) \varphi_{g'}^1(\mathbf{x}) = 0$$

These equations can be made equivalent to the corresponding equations for isotropic scattering if the following redefinitions for the total and the scattering cross sections are made

$$\left[\Sigma_{\downarrow}^E(\kappa) \right]^{\text{Full}} = \Sigma_{\downarrow}^E(\kappa) - \Sigma_{n1}^{E \rightarrow E}(\kappa) = \sum_{E' \neq E} \Sigma_{n1}^{E' \rightarrow E}(\kappa) \frac{q_{E'}^{\downarrow}(\kappa)}{q_E^{\downarrow}(\kappa)},$$

$$\left[\Sigma_{n0}^{E \rightarrow E}(\kappa) \right]^{\text{Full}} = \Sigma_{n0}^{E \rightarrow E}(\kappa) - \Sigma_{n1}^{E \rightarrow E} = \sum_{E' \neq E} \Sigma_{n1}^{E' \rightarrow E}(\kappa) \frac{q_E^{\downarrow}(\kappa)}{q_{E'}^{\downarrow}(\kappa)},$$

and

$$\left[\Sigma_{n0}^{E' \rightarrow E}(\kappa) \right]^{\text{Full}} = \Sigma_{n0}^{E' \rightarrow E}(\kappa)$$

The above equations define the full transport approximation

The cross section for the diagonal transport approximation is obtained by ignoring the first moment of the downscatter cross sections, hence,

$$\left. \begin{aligned} \left[\Sigma_{\downarrow}^E(\kappa) \right]^{\text{Diagonal}} &= \Sigma_{\downarrow}^E(\kappa) - \Sigma_{n1}^{E \rightarrow E}(\kappa) = \Sigma_{\downarrow}^E(\kappa) - \mu_E \Sigma_{n0}^{E \rightarrow E} \\ \left[\Sigma_{n0}^{E \rightarrow E}(\kappa) \right]^{\text{Diagonal}} &= \Sigma_{n0}^{E \rightarrow E}(\kappa) - \Sigma_{n1}^{E \rightarrow E}(\kappa) = \Sigma_{n0}^{E \rightarrow E}(\kappa) - \mu_E \Sigma_{n0}^{E \rightarrow E}(\kappa), \end{aligned} \right\} \quad (1)$$

where $\mu_E \equiv \mu$ is the average angle of scatter for ingroup scattering. The full transport cross sections can be written in the same format as the diagonal transport cross sections, but $\mu_E \equiv \mu$ is replaced by a space dependent weighted mean angle of scatter ($\langle \mu(\kappa) \rangle$)

$$\left. \begin{aligned} \left[\Sigma_{\downarrow}^E(\kappa) \right]^{\text{Full}} &= \Sigma_{\downarrow}^E(\kappa) - \langle \mu(\kappa) \rangle \Sigma_{n0}^{E \rightarrow E} \\ \left[\Sigma_{n0}^{E \rightarrow E}(\kappa) \right]^{\text{Full}} &= \Sigma_{n0}^{E \rightarrow E} - \langle \mu(\kappa) \rangle \Sigma_{n0}^{E \rightarrow E} \end{aligned} \right\} \quad \begin{array}{l} (2) \\ \text{(Contd.)} \end{array}$$

where

$$\left. \begin{aligned} \langle \mu(x) \rangle &= \sum_{g'} \bar{\mu}_{g' \rightarrow g} \left(\frac{\Sigma_{g0}^{g' \rightarrow g}(x)}{\Sigma_{g0}^{g \rightarrow g}(x)} \right) \frac{\varphi_{g'}^1(x)}{\varphi_g^1(x)} \\ &= \bar{\mu}_{g \rightarrow g} + \sum_{g' \neq g} \bar{\mu}_{g' \rightarrow g} \left(\frac{\Sigma_{g0}^{g' \rightarrow g}(x)}{\Sigma_{g0}^{g \rightarrow g}(x)} \right) \frac{\varphi_{g'}^1(x)}{\varphi_g^1(x)} \end{aligned} \right\} \quad \begin{array}{l} \text{(Contd.)} \\ (2) \end{array}$$

and $\bar{\mu}_{g' \rightarrow g}$ is the average angle of scatter for scattering from group g' to group g .

There are three difficulties to the use of full-transport cross sections:

(1) A current is needed, so we have to have some estimate of the current before we can do the problem; (2) the current is spatially dependent in a homogeneous region, so the full-transport cross sections will be heterogeneous in a homogeneous region; (3) problems arise when the spatial ingroup current is zero.

To circumvent the limitations imposed by space dependency of the cross sections, the average cross sections for the full-transport approximation ($\langle P_0^F \rangle$) were also investigated. The region-dependent average transport cross section for the full-transport approximation is defined as

$$\langle \Sigma_t^g \rangle^{\text{Full}} = \frac{\int_r [\Sigma_t^g(r)]^{\text{Full}} \varphi_g^1(r) r \, dr}{\int_r \varphi_g^1(r) r \, dr} \quad (3)$$

The full-transport ingroup scattering cross section also has to be modified to preserve the removal cross section.

III. MODEL DESCRIPTION AND METHOD OF SOLUTION

The study was made for an idealized, one-dimensional, homogeneous cylindrical fast-reactor core of the EBR-II type surrounded by a homogeneous stainless steel reflector (see Fig. 1). The core-smeared number densities were generated by smearing a two-dimensional (X-Y) description of an actual EBR-II core loading. The core fuel enrichment was 66% (63% ^{235}U and 3% ^{239}Pu). The number densities used in the calculation are shown in Table I.

Fig. 1. Model Description

TABLE I. Smeared Densities Used for Model

Core		Reflector	
Element	Density ^a	Element	Density ^a
²³⁵ U	4.72×10^{-3}	Fe	4.71×10^{-2}
²³⁸ U	2.55×10^{-3}	Ni	5.91×10^{-3}
²³⁹ Pu	2.16×10^{-4}	Cr	1.30×10^{-2}
Fe	1.84×10^{-2}	Na	4.80×10^{-3}
Ni	2.43×10^{-3}		
Cr	4.91×10^{-3}		
Na	1.00×10^{-2}		
O	1.88×10^{-3}		
Mo	4.60×10^{-4}		
Nb	2.40×10^{-6}		
Zr	3.76×10^{-4}		
U5FP1 ^b	4.00×10^{-2}		
U5FP2 ^c	3.13×10^{-3}		
U5FP3 ^d	1.29×10^{-4}		

^aIn units of cm⁻¹/barn, 1 barn = 1×10^{-28} m².^bA rapidly saturating ²³⁵U lumped fission-product chain.^cA slowly saturating ²³⁵U lumped fission-product chain.^dA nonsaturating ²³⁵U lumped fission-product chain.

The transport calculations were done using the one-dimensional transport code ANISN.² There were 34 mesh points in the core and 26 in the reflector. The mesh spacing was about 1 cm, except near boundaries, where it was about 0.5 cm. The gamma-source distribution was generated using gamma-production cross sections generated from the POPOP4 library³ and with the POPOP4 code.⁴ The neutron-flux eigenvalue was calculated with S₄ angular quadrature and a 29-group diagonal-transport isotropic neutron-transport cross-section set generated by the MC² code⁵ from ENDF/B cross sections. Fixed-source gamma-transport calculations were done using S₄ angular quadrature and a 20-energy-group cross-section set generated by the MUG code.⁶

IV. DIFFICULTIES IN CALCULATING FULL-TRANSPORT CROSS SECTIONS

If the spatial currents needed in the full-transport method are calculated near a reversal in the in-group current (ϕ_g^1), the spatially dependent correction term in the full-transport approximation may become large and oscillatory. Two examples of effects of spatially dependent singularity are shown in Table II. In the group-2 example, the ratio of the transport cross section for the full-transport approximation to the isotropic total cross section is between 0.95 and 0.98 for all reflector mesh points except 41, where the ratio is 45.3. Also shown in Table II is a singularity effect in group 19. These effects of spatially dependent singularities were circumvented by using interpolation based on nearest neighbors (or extrapolation near boundaries) in the cross section. In the group-2 example, the interpolated value of 0.962 was used as the true ratio instead of 45.3.

TABLE II. Examples of Singularities in Full-transport Cross Section

Group 2 (7 → 8 MeV)		Group 19 (0.1 → 0.2 MeV)	
Mesh Point (Reflector)	$\frac{\Sigma_{Tr}^{Full^a}}{\Sigma_{Tot}}$	Mesh Point (Reflector)	$\frac{\Sigma_{Tr}^{Full^a}}{\Sigma_{Tot}}$
38	0.967	36	0.704
39	0.967	37	0.506
40	0.978	38	0.021 (1.03) ^b
41	45.3 (0.962) ^b	39	-2.15 (1.57) ^b
42	0.951	40	7.08 (2.10) ^b
43	0.956	41	2.64
44	0.956	42	2.03
		43	1.94
		44	1.93
		45	1.93
$\left(\frac{\Sigma_{Tr}^{Diagonal^c}}{\Sigma_{Tot}}\right)_{Refl} = 0.973$		$\left(\frac{\Sigma_{Tr}^{Diagonal^c}}{\Sigma_{Tot}}\right)_{Refl} = 0.846$	

^aFull transport, using P_1 currents.^bInterpolated values (see text).^cDiagonal transport.

The question arose as to what criteria should be used to judge when and spatially how far interpolation should be performed. As seen in Table II, the group-2 example was fairly straightforward, but in the group-19 example, it was not clear as to how far on either side of the obviously bad value at mesh point 39 interpolated values should be used. In cases such as this, interpolated values were used over wide enough spatial ranges to provide reasonably smooth transitions of the values for transport cross section. In the calculation using the model, nine (out of 1200) transport cross sections for the full-transport approximation were smoothed out because of abrupt discontinuities in cross section.

When there is not a current reversal, full-transport cross sections can be unrealistic if an in-group current is small relative to the out-of-group currents. This effect was noted in the model calculation for energies below 0.2 MeV (groups 19 and 20) in certain regions. Table III gives an example of this effect. The table shows the ratios of full-transport-approximation transport cross section to total cross section for group 20 in a fraction of the reflector region. No interpolation scheme seemed reasonable for any of the points. In fact, some regions had a large number of negative full-transport-approximation transport cross sections. This problem was circumvented by using the diagonal-transport-approximation transport cross section, thereby ignoring the current-related correction term, in the energy groups and regions where this effect was seen. This procedure was used in group 19 in the core and group 20 in the reflector. Using diagonal-transport cross sections in these groups will not lead to significant errors, because diagonal transport is a reasonable assumption for the core (see Fig. 2) and absorption, not scattering, is the dominant effect in group 20 in the reflector.

TABLE III. Example of Effect of Relatively Small
In-group Currents on Full-transport
Cross Section

Group 20 (0.01 → 0.1 MeV)		Group 20 (0.01 → 0.1 MeV)	
Mesh Point (Reflector)	$\frac{\Sigma_{Tr}^{Full^a}}{\Sigma_{Tot}}$	Mesh Point (Reflector)	$\frac{\Sigma_{Tr}^{Full^a}}{\Sigma_{Tot}}$
35	-0.073	42	-0.558
36	-0.101	43	-0.241
37	0.083	44	-0.307
38	-0.163	45	-0.273
39	0.443	46	0.001
40	0.299	47	-0.328
41	-0.400	48	0.037
$\left(\frac{\Sigma_{Tr}^{Diagonal^b}}{\Sigma_{Tot}}\right)_{Refl}$		0.998	

^aFull transport, using P_1 currents.

^bDiagonal transport.

Fig. 2
Relative Error in Total Gamma
Flux: $P_0 S_4$ and $P_0^T S_4$ vs $P_1 S_4$

The smoothed space-dependent full-transport-approximation cross sections were used in all full-transport calculations and were also used when the average full-transport cross sections were calculated.

V. RESULTS

Since the full-transport approximation can at best only approximate a solution based on P_1 scattering, one has to ensure that the P_1 solution for the problem under consideration is adequate. Figure 3 shows the error in the total gamma flux from a P_1S_4 calculation relative to that from a P_3S_8 calculation of the model calculation. The error is defined as

$$\frac{\text{Flux}(P_3S_8) - \text{Flux}(P_1S_4)}{\text{Flux}(P_3S_8)} \times 100.$$

The maximum relative error in the total flux is about 1%, whereas the maximum relative error in any groupwise flux is less than 2%. Hence, for the problem under consideration, a P_1 solution is adequate.

Fig. 3. Relative Error in Total Gamma Flux: P_1S_4 vs P_3S_8

the reflector because of gamma transport from the source-rich core to the source-poor reflector (see Figs. 5 and 6).

Since anisotropic scattering cross sections are important only to the solution for flux where anisotropic fluxes exist,⁷ the solution for flux in the reflector is more dependent on the higher-order moments of the scattering cross section than the solution for flux in the core. Thus, when the higher-order moments of the scattering are totally ignored, as for the isotropic cross sections, and almost totally ignored--except for the in-group P_1 scattering cross section ($\Sigma_{g1}^{P_1 \rightarrow g}$)--as for the diagonal-transport-approximation cross sections, the relative error in the reflector should be larger.

Fig. 4. Radial Distribution of Total Gamma Source in Core and Reflector

Fig. 5. Angular Gamma Flux for Group 12 at Midcore and Midreflector

Fig. 6
Angular Gamma Flux for Group 17
at Midcore and Midreflector

As was mentioned in Sec. II, the full-transport cross sections are functions of an estimated current for the calculation. The currents for the model problem, calculated using P_1 cross sections and diagonal-transport cross sections, are shown in Figs. 7 and 8 for the representative groups 12 and 17. The values are reasonably similar, and hence currents from a diagonal-transport gamma solution could be used, without introducing much additional error, in determining the full-transport-approximation cross sections.

Fig. 7. Radial Distribution of Group-12 Gamma Currents in Core and Reflector

Fig. 8. Radial Distribution of Group-17 Gamma Currents in Core and Reflector

In the model calculation, average full-transport-approximation cross sections were calculated using currents generated from a diagonal-transport solution. Two examples of the differences between isotropic, diagonal-transport, and full-transport-approximation cross sections are shown in Figs. 9 and 10 for energy groups 12 and 17. Plotted are the constant values of total isotropic cross section (P_0) and diagonal-transport-approximation transport cross section (P_0^T), the spatially dependent full-transport-approximation transport cross section (P_0^F), and the full-transport-approximation transport cross section averaged with diagonal-transport currents ($P_0^{F'}$).

Fig. 9

Group-12 Gamma Macroscopic Total Cross Sections

Fig. 10

Group-17 Gamma Macroscopic Total Cross Sections

In the core and reflector regions, the diagonal-transport-approximation transport cross section was smaller than the total cross section, because $\bar{\mu}_{g \rightarrow g}$ is positive (see Eq. 1). In those regions, however, the full-transport-approximation transport cross section normally was smaller than either the total or diagonal-transport-approximation transport cross section, because the values for $\bar{\mu}_{g \rightarrow g}$ are positive (see Eq. 2) and, in the model problem, most of the currents were positive. At some mesh points for some energy groups, however, the presence of negative currents caused the full-transport-approximation transport cross sections to be larger than the diagonal-transport-approximation transport and total cross sections.

In addition, near the core-reflector interface in both the core and reflector regions, where the flux is more anisotropic, the full-transport-approximation transport cross sections are strongly spatially dependent. Table IV shows the energy dependence of the isotropic, diagonal-transport, and current-weighted average full-transport-approximation cross sections.

In several groups, the average full-transport-approximation transport cross section was larger than the maximum spatial value. This was due to the fact that in the averaging scheme (see Eq. 3), there were current reversals.

TABLE IV. Energy Dependence of Different Cross Sections

Group	Upper Energy, MeV	Core			Reflector		
		P_0^a	$P_0^T{}^b$	$\langle P_0^F \rangle, P_0^T{}^c$	P_0	P_0^T	$\langle P_0^F \rangle, P_0^T$
1	10.0	0.229	0.225	0.225	0.183	0.178	0.178
2	8.0	0.223	0.219	0.224	0.183	0.179	0.176
3	7.0	0.221	0.219	0.219	0.184	0.182	0.153
4	6.5	0.220	0.216	0.215	0.186	0.183	0.166
5	6.0	0.220	0.217	0.214	0.189	0.186	0.169
6	5.5	0.221	0.218	0.213	0.193	0.189	0.171
7	5.0	0.221	0.217	0.211	0.197	0.192	0.177
8	4.5	0.221	0.216	0.219	0.201	0.195	0.182
9	4.0	0.224	0.217	0.207	0.208	0.201	0.188
10	3.5	0.230	0.221	0.206	0.219	0.209	0.188
11	3.0	0.237	0.228	0.207	0.231	0.220	0.197
12	2.6	0.246	0.234	0.210	0.244	0.230	0.199
13	2.2	0.264	0.246	0.212	0.266	0.245	0.203
14	1.8	0.296	0.265	0.220	0.297	0.262	0.208
15	1.35	0.370	0.314	0.242	0.352	0.288	0.210
16	0.9	0.519	0.440	0.292	0.430	0.339	0.198
17	0.6	0.811	0.703	0.468	0.520	0.394	0.229
18	0.4	1.990	1.800	3.425	0.682	0.461	5.274
19	0.2	7.440	7.274	7.274	1.228	1.046	1.762
20	0.1-0.01	95.325	95.267	103.430	63.616	63.549	63.549

^aIsotropic cross section.

^bDiagonal-transport cross section.

^cCurrent-weighted average full-transport cross section.

Figures 11-16 compare the relative errors in the gamma flux with the flux from a P_1S_4 solution for energy groups 12 and 17 and the total flux. The flux differences in the core (Figs. 11, 13, and 15), already relatively small, even when isotropic or diagonal-transport-approximation cross sections are used, are further reduced when any one of the full-transport schemes is used. The major benefits of the full-transport method are seen in the reflector region (Figs. 12, 14, and 16), where rather large relative errors are considerably reduced. In group 12, for example, the maximum relative error in the reflector (Fig. 12) is -16.7% for the isotropic (P_0) calculation, -12.5% for the diagonal-transport (P_0^T) calculation, 1.9% for the full-transport calculation using diagonal-transport currents (P_0^F, P_0^T), -1.1% for the full-transport calculation using P_1 currents (P_0^F, P_1), and -1.1% for the full-transport calculation using average full-transport cross sections and diagonal-transport currents ($\langle P_0^F \rangle, P_0^T$). The reduction in the relative flux errors, especially in the reflector region, with a full-transport calculation using P_1 currents as compared to the reduction in relative flux error with a full-transport calculation using diagonal-transport currents is much smaller than the reduction in relative error when going from a diagonal-transport calculation to a full-transport calculation.

Fig. 11. Errors in Calculated Gamma Flux in Core Relative to Results from P₁S₄ Solution: Energy Group 12

Fig. 12. Errors in Calculated Gamma Flux in Reflector Relative to Results from P₁S₄ Solution: Energy Group 12

Fig. 13
Errors in Calculated Gamma Flux
in Core Relative to Results from
P₁S₄ Solution: Energy Group 17

Fig. 14. Errors in Calculated Gamma Flux in Reflector Relative to Results from P_1S_4 Solution: Energy Group 17

Fig. 15. Errors in Calculated Gamma Flux in Core Relative to Results from P_1S_4 Solution: Total Flux

Fig. 16
Errors in Calculated Gamma Flux
in Reflector Relative to Results
from P_1S_4 Solution: Total Flux

Table V compares relative calculation time and cross-section storage requirements to those for a P_1S_4 calculation. It is seen that the pointwise full-transport calculations (P_0^F) take more time and need more storage than a P_1S_4 calculation. The average full-transport calculation ($\langle P_0^F \rangle$) requires about 96% of the calculation time (or essentially the same calculational time) as a P_1S_4 calculation, but the cross-section storage requirements are 50% less.

TABLE V. Calculation Times and Cross-section Storage Requirements Relative to Those for a P_1S_4 Calculation

Calculation Type	Averaging Currents	Relative Time	Relative Cross-section Storage Needs
P_1S_4	-	1.00	1.0
P_0^F	P_1	1.20	15.0
P_0^F	P_0^T	1.24	15.0
$\langle P_0^F \rangle$	P_0^T	0.96	0.5
P_0^C	-	0.89	0.5
P_0	-	0.92	0.5

VI. CONCLUSIONS

Gamma fluxes calculated using the full-transport isotropic cross sections described in this report can be very close to those from a P_1 solution. However, care must be taken in using the full-transport-approximation cross sections, because pointwise and regionwise failures may occur. In the model calculation, few modifications were made to the full-transport cross sections, but this may not always be the case. The number of modifications is dependent on the relative magnitudes and shapes of the assumed currents.

The use of full-transport cross sections requires a current, preferably the current that would be obtained upon solution of the problem. However, this limitation is not serious if the calculations are to be done for a system such as EBR-II. The EBR-II reactor changes core loadings frequently, but the relative changes are minor. Hence, the current from a previous loading could be used as the current for the next calculation. The error introduced by using this current estimate would probably be negligible compared to the relative error resulting from the use of diagonal-transport cross sections. If only an isotropic capability is available, an approximate P_1 solution can be obtained by running two successive isotropic calculations (a diagonal-transport calculation for the currents and a full-transport calculation for the flux), since the currents obtained from a diagonal-transport solution are sufficiently accurate for evaluating the full-transport cross sections.

The use of the average full-transport cross sections reduces the amount of computer cross-section storage needed to do a calculation. The

maximum relative flux errors are larger than the relative errors obtained when pointwise full-transport cross sections are used, but are still small in comparison to those for a diagonal-transport solution. If a calculation has a large number of mesh points, average full-transport cross sections could be used where they are reasonably constant (such as for regions of nearly isotropic flux), and pointwise full-transport cross sections could be used where there is a strong spatial dependence (such as for regions of highly anisotropic flux).

REFERENCES

1. D. Meneghetti and W. B. Loewenstein, "EBR-II Physics Experience," *Proc. Int. Symp. Physics of Fast Reactors*, Tokyo, Oct 16-19, 1973, Vol. 1, p. 175 (1973).
2. W. W. Engle, *ANISN, A One-dimensional Discrete Ordinates Transport Code with Anisotropic Scattering*, K-1693, Oak Ridge National Laboratory (Mar 1967).
3. W. E. Ford III, *The POPOL Library of Neutron Induced Secondary Gamma-ray Yield and Cross Section Data*, CTC-42, Oak Ridge National Laboratory (Sept 1970).
4. W. E. Ford III and D. H. Wallace, *POPOL, A Code for Converting Gamma-ray Spectra to Secondary Gamma-ray Production Cross Sections*, CTC-12, Oak Ridge National Laboratory (May 1969).
5. B. J. Toppel, A. L. Rago, and D. M. O'Shea, *M², A Code to Calculate Multigroup Cross Sections*, ANL-7318 (June 1967).
6. J. R. Knight and P. R. Mynatt, *MH1, A Program for Generating Multigroup Photon Cross Sections*, CTC-17, Oak Ridge National Laboratory (Jan 1970).
7. R. V. Meghrebian and D. K. Holmen, *Transport Theory*, Chapter 1: Reactor Analysis, McGraw-Hill, New York (1960).