PLANE-STRAIN STRESS INTENSITY FACTORS FOR CRACKED HEXAGONAL SUBASSEMBLY DUCTS

by

H. J. Petroski, J. L. Glazik, and J. D. Achenbach

BASE TECHNOLOGY

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Reactor Analysis and Safety Division

October 1977

*Department of Civil Engineering, Northwestern University
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H. J. Petroski, J. L. Glazik, and J. D. Achenbach

ABSTRACT

Plane-strain stress intensity factors for a pressurized hexagonal subassembly duct with a crack in a corner or midflat are presented in convenient graphical form for representative LMFBR hexcan dimensions. Corner-crack calibrations based on several different models of the round hexcan corner are determined first in order to bound the stress intensity factor. A subsequent finite-element analysis of a uniformly pressurized hexcan with a corner crack gives accurate data for the stress intensity factor from which a weight function for this geometry may be constructed.

The effects of different numbers of cracks, different locations for cracks, and different loading modes are discussed briefly, and some comments are made on the application of linear elastic fracture mechanics to cracked hexagonal ducts that have suffered a high degree of irradiation embrittlement.

I. INTRODUCTION

Liquid Metal Fast Breeder Reactor (LMFBR) subassembly ducts have cross sections that are regular hexagons with rounded corners, as shown in Fig. 1. These "hexcans" are typically made of stainless steels, Type 304 or 20% cold-worked Type 316, which are tough, ductile materials under ordinary service conditions. Thus, flaws, scratches, and cracks, which invariably are introduced during fabrication, transportation, handling, and service, and which are small enough to escape detection, are not generally considered to pose a threat to the integrity of hexcans. However, the prolonged exposure to a fast-neutron environment that these ducts can experience is now known to alter greatly the mechanical properties of stainless steel. The high-toughness, low-ductility material of fresh hexcans loaded into an LMFBR core is expected to become a low-toughness, low-ductility material at the goal fluences \(10^{27} \text{n/m}^2\) of current reactor designs.
Although the degree of ductility loss has been monitored for stainless steels exposed to fluences approaching such limits, the degradation in the ability of the material to resist unstable crack propagation (the "fracture toughness" of the material) is not currently known. Experience with at least one highly irradiated hexcan, however, does indicate that the toughness may be reduced to a level at which fast, brittle fracture is a problem of concern in LMFBR Safety Analysis. The abnormal overpressurization resulting from certain interactions within a subassembly or the rupture of one or more fuel pins may be sufficient to overload an otherwise subcritical crack in an embrittled hexcan. It is to such situations that the results of the present report are addressed.

From the point of view of fracture mechanics, there are three adverse effects of exposure to high fast-neutron fluences:

1. Reduced ductility
2. Reduced fracture toughness
3. Increased yield strength around inlet temperatures.

Brittle fracture becomes an especially acute problem when the yield strength of the material is raised to such an extent that the stress intensity at the tip of a crack reaches the critical value at net section stresses below the yield stress. Under such circumstances, linear elastic fracture mechanics is applicable, and its applicability implicitly will be assumed throughout this report. In other words, the failure criteria developed here are intended to apply only when they would indicate failure at lower load levels than some ultimate-tensile-strength or strain-limit criterion.

The effect of a crack in a linear elastic material is potentially to introduce a large stress gradient and thus greatly intensify the stresses at the tip of the crack. A mathematical singularity of order $r^{-1/2}$ in the stress, where $r$ is the distance measured from the crack tip, is characteristic of linear elastic materials, and the strength of this singularity, which is proportional to the loading, is known as the stress intensity factor, $K_I$. Thus, if the hexcan material is ductile, sufficient plastic yielding may be able to occur around the crack tip and thus preclude the possibility of unstable crack propagation. If, on the other hand, the material is brittle, the stress intensity cannot be relieved to any great extent by yielding, and fast fracture will occur when the stress intensity exceeds some critical value characteristic of the material. This characteristic value is the fracture toughness, $K_{IC}$.

Since the stress intensity factor associated with a crack depends in nonlinear ways on geometry, and since the hexcan geometry is complex, especially in the critical corner regions, it is necessary to "calibrate" cracks in hexcans for various crack sizes; i.e., $K_I$ must be determined as a function of crack size for a fixed hexcan geometry and mode of loading. These calibrations are customarily presented in the form of plots of dimensionless stress.
intensity factor versus crack size, and such plots are the principal result of this report. These calibrations may be readily used to check the safety of a postulated crack, if the critical stress intensity factor $K_{IC}$ is known.

Since the hexcan corner is a region of high stress levels and large stress gradients, even in the absence of flaws, the problem of a cracked hexcan corner appears to be a worst-case problem, at least in the case of overpressurization. The plane-strain problem of a long, shallow crack in such a geometry (see Fig. 2) is one that might model a scratch introduced during manufacture or assembly, and the solution to this problem will provide a basis on which to build solutions for more complex crack geometries. However, even such an idealized problem presents formidable analytical obstacles because of the finite thickness and difficult geometry of the actual corner. Therefore, an exact analysis is not considered possible at present and a finite-element analysis of the true geometry or an analytical solution to a more idealized problem is a natural course of action. This report presents the results of calculations of stress intensity factors for the plane-strain problem by means of various methods.

The following paragraphs provide an outline of the report.

In Sec. II the concept of stress intensity factor is made mathematically precise, and the conventional notation is introduced. The important principle of superposition of linear-elasticity solutions is also discussed in the context of its important application in the calculation of stress intensity factors.

The uniform pressure loading of a hexcan is considered in Sec. III, and the corner and midflat sections are identified as critical. The stress distributions across these sections are calculated from straight- and curved-beam theory. Explicit results are presented for two hexcan designs. Although only uniform loading is treated here, Sec. XIII shows how stress intensity factors associated with other loadings may be derived from this base case by methods explained in this report.

Section IV uses standard calibrations from a handbook of stress intensity factors to construct calibration curves for hexcan midflat cracks.
Handbook calibrations are not adequate for cracks in hexcan corners, however, and some comments relevant to modeling the round corner are presented in Sec. V. Cracked rings, C-shaped fracture-toughness specimens, and cracked infinite strips are discussed in relation to their ability to model the compliance of the hexcan.

The strip calibration is compared with the straight-beam calibration in Sec. VI, where the inadequacy of the straight-beam handbook calibration to model the curved-beam hexcan-corner stress distribution is demonstrated. The experimentally based C-specimen calibration is presented in Sec. VII, where it is seen to agree well with the strip calibration, thus confirming the validity of the latter.

The important and powerful technique of calculating stress intensity factors by a weight-function procedure is introduced in Sec. VIII. This technique is essentially a method of superposition, which builds up the desired solution from known results for a basic problem involving the geometry of interest and a simple loading. In the case of the cracked hexcan, since there do not exist any solutions on which to build directly, some results of Grandt for cracked rings have been used in conjunction with some assumptions about the crack shape.

Bounds on the stress intensity factors for corner cracks are presented in Sec. IX. These are based on the models discussed in the previous sections.

Section X reports the finite-element technique used to determine the stress intensity factor for a corner crack, and it is seen to give results within the bounds reported earlier.

Sections XI and XII consider the effects of cracks in different locations under different loading conditions and of different numbers of corner cracks. A conservative analysis based on a single corner crack should generally be considered in any fracture-mechanics analysis of cracked ducts.

A weight function based on the finite-element results of Sec. X for corner cracked hexcans is presented in Sec. XIII, and it is seen capable of giving results as good as those obtained by finite-element analysis.

Some comments on the application of linear elastic fracture mechanics to cracked hexcan are made in Sec. XIV, and some of the conclusions of the report are summarized in Sec. XV.
II. THE CONCEPT OF STRESS INTENSITY FACTOR*

When a linear elastic body contains a sharp crack, the stress field at the tip of the crack possesses a mathematical singularity of order $r^{-1/2}$ (where $r$ is the distance from the crack tip), and the strength of this singularity is known as a stress intensity factor. Three stress intensity factors are associated with a crack, one for each mode of crack opening: $K_I$, which characterizes the stress field tending to open the crack symmetrically in tension; $K_{II}$, which characterizes the stress field tending to displace the crack faces with an in-plane shear; and $K_{III}$, which characterizes the stress field tending to skew the crack faces by an antiplane shear. The three basic modes of crack surface displacement are shown in Fig. 3.

Generally the first mode, that of pulling the faces of the crack apart symmetrically, leads to unstable crack propagation, and this report is restricted to stress intensity factors of mode one.

When an elastic body, such as the cracked plate in Fig. 4, has dimensions that are large compared to the crack length $2a$, the mathematical problem of a crack in an infinite sheet is considered to accurately model the stresses near the crack tip. The uniform stress field $\sigma$ at infinity gives rise to the plane-strain stress field, which, near the crack tip, may be represented by the singular terms alone:

$$
\begin{align*}
\sigma_x &= \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left( 1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right); \\
\sigma_y &= \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left( 1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right); \\
\tau_{xy} &= \frac{K_I}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2}; \\
\sigma_z &= \nu(\sigma_x + \sigma_y); \\
\tau_{yz} = \tau_{zx} &= 0,
\end{align*}
$$

*The development of this section follows closely that of Ref. 5.
where \( v \) is Poisson's ratio and the polar coordinates \((r, \theta)\) are defined in the plane of symmetry \((x, y)\) according to Fig. 4. The associated displacement field near the crack tip is given by:

\[
\begin{align*}
  u_x &\approx \frac{K_I}{G} \sqrt{\frac{r}{2\pi}} \cos \frac{\theta}{2} \left(1 - 2v + \sin^2 \frac{\theta}{2}\right); \\
  u_y &\approx \frac{K_I}{G} \sqrt{\frac{r}{2\pi}} \sin \frac{\theta}{2} \left(2 - 2v - \cos^2 \frac{\theta}{2}\right); \\
  u_z &= 0,
\end{align*}
\]

where \( G \) is the shear modulus. If \( E \) represents Young's modulus, then \( G = E/[2(1 + v)] \).

Since only two variables, the stress field \( \sigma \) and the half-crack length \( a \), characterize the plane elastic problem of Fig. 4, dimensional considerations lead to the conclusion that \( K_I \) is proportional to \( \sigma \) and to \( a \) to the one-half power. If \( C \) is the constant of proportionality,

\[
K_I = C\sigma\sqrt{a}.
\]  

(3)

Since \( C = \sqrt{\pi} \) for the crack in a uniformly loaded infinite sheet, it is customary to normalize the stress intensity factor with respect to \( \sigma\sqrt{\pi} \) and to work with the dimensionless stress intensity factor \( Y \):

\[
Y = \frac{K_I}{\sigma\sqrt{\pi}a}.
\]  

(4)

In finite bodies, the value of \( C \) in Eq. 3, and hence of \( Y \) in Eq. 4, depends on the dimensions of the body as well as the crack length and loading, and the determination of this coefficient for configurations of interest for LMFBR safety analysis is the principal goal of this report. The results will be presented in the form of plots of appropriately normalized stress intensity factors versus crack lengths for specific loading conditions and geometry. Such plots are known as K-calibrations.

According to linear elastic fracture mechanics, the crack in the sheet of Fig. 4 will not propagate in an unstable manner until the product \( \sigma\sqrt{a} \) reaches a critical value. This obviously can be achieved in one of two ways: (1) by overloading the sheet, i.e., by increasing \( \sigma \) beyond some critical value for a particular crack length, or (2) by introducing a crack that is larger than some critical size for a specified loading. The first situation may arise by some
abnormally high accident loading; the second may occur after an initially subcritical crack has grown to a critical size due to some mechanism such as stable, subcritical fatigue-crack propagation due to prolonged service under fluctuating loading conditions.

The criterion for unstable crack propagation may be expressed in terms of a critical value of $K_I$, which is designated $K_{IC}$ and is known as the fracture toughness of the material. This quantity is believed to be an independent material property and may be measured by prescribed tests on standardized specimens. When the fracture toughness is known, the allowable loading for a given crack size is given by

$$\sigma_c = \frac{K_{IC}}{Y\pi a},$$

while the largest crack size that may be tolerated at a given load $\sigma$ is given by

$$a_c = \frac{1}{\pi} \left( \frac{K_{IC}}{Y\sigma} \right)^2.$$

Since the principle of superposition applies in linear elastic fracture mechanics, the stress intensity factor may be computed simply by loading the crack faces with the negative of the stress distribution that would exist across the crack plane in an unflawed body of the same overall dimensions and loaded in the same way as the cracked body of interest. This follows because the solution to the problem of the unflawed body would be nonsingular and, hence, would have associated with it a zero stress intensity factor. Adding the two solutions would cancel the stresses on the crack faces, satisfying the boundary conditions of a stress-free crack. Figure 5 illustrates the principle of superposition.

If there were some crack-face pressure due to the existence of a pressurizing medium filling the crack, the total stress intensity factor would then be the sum of the $K_I$ associated with the unflawed-body stress distribution and that associated with the uniform pressure loading.
III. STRESS DISTRIBUTIONS IN UNIFORMLY PRESSURIZED HEXCANS

When a hexcan is uniformly loaded, only one-twelfth of the duct need be analyzed. Figure 6 shows the parameters characterizing such a segment, and Fig. 7 defines the positive directions for the bending moment $M$ and normal force $N$ involved in the curved- and straight-beam theories herein employed.

![Figure 6: One-twelfth of a Hexcan](image)

![Figure 7: Force and Coordinate Systems](image)

Table I gives the dimensions of two typical LMFBR hexcan designs, hereinafter designated $E$ and $F$. Note that the mean corner radius, $R = r + (h/2)$ (see Fig. 8), has been used to specify the dimensionless corner.

**TABLE I. Representative Hexcan Dimensions and Parameters**

<table>
<thead>
<tr>
<th>Hexcan $E$, mm (in.)</th>
<th>Hexcan $F$, mm (in.)</th>
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<tbody>
<tr>
<td>$p$</td>
<td>$p$</td>
</tr>
<tr>
<td>28.07 (1.105)</td>
<td>55.05 (2.1675)</td>
</tr>
<tr>
<td>$h$</td>
<td>$h$</td>
</tr>
<tr>
<td>1.02 (0.040)</td>
<td>3.05 (0.120)</td>
</tr>
<tr>
<td>$r$</td>
<td>$r$</td>
</tr>
<tr>
<td>1.78 (0.070)</td>
<td>4.45 (0.175)</td>
</tr>
<tr>
<td>$R$</td>
<td>$R$</td>
</tr>
<tr>
<td>2.29 (0.090)</td>
<td>5.975 (0.235)</td>
</tr>
<tr>
<td>$e$</td>
<td>$e$</td>
</tr>
<tr>
<td>0.0384 (0.00150)</td>
<td>0.132 (0.00520)</td>
</tr>
<tr>
<td>$L$</td>
<td>$L$</td>
</tr>
<tr>
<td>15.19 (0.598)</td>
<td>29.21 (1.150)</td>
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**Ratios**

<table>
<thead>
<tr>
<th>$h/p$</th>
<th>$r/h$</th>
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<th>$Q^a$</th>
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<tr>
<td>0.036</td>
<td>1.745</td>
<td>2.25</td>
<td>0.636</td>
</tr>
<tr>
<td>0.055</td>
<td>1.458</td>
<td>1.96</td>
<td>0.593</td>
</tr>
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</table>

$^aQ = e/(r + h)$ is the ratio of inner to outer radius of the hexcan corner.
radius $R/h$. (Sometimes it is more convenient to work with the ratio $r/h$.)

Since this ratio is approximately 2 for the duct designs E and F, the assumption of a linear stress distribution across the rounded corner section would result in maximum bending stresses more than 15% below that calculated assuming the hyperbolic stress distribution of curved-beam theory.\(^7\) Figure 9 demonstrates the underestimate of the stress by straight-beam theory at the inside corner of the hexcan, the most critical location for a crack. Hence, the stress distribution in the corner will be taken to be hyperbolically varying through the thickness of the duct wall. Elastic-equilibrium considerations, including the use of Castigliano's theorem to calculate $M_A$ for this statically indeterminate problem,\(^7\) give the following force and moment distributions acting on a duct of length $B$, where $B$ is measured normal to the hexagonal cross section:

\[
\begin{align*}
N &= r p B + (A - r) B p \cos \varphi \quad \text{on AO} \\
M &= M_A - (A - r) (1 - \cos \varphi) p R B \\
N &= \varphi p B \\
M &= M_A - [(A - r) (1 - \cos \varphi) p R + p L x - \frac{1}{2} p x^2] B \\
\end{align*}
\]

where $0 \leq \varphi \leq \pi = \pi/6$ on AO, $0 \leq x < L$ on OB, $p$ is the internal pressure, and

\[
\begin{align*}
R &= r + (h/2) \\
L &= (p - r) \tan \gamma; \\
\bar{A} &= r + [(p - r) / \cos \gamma]; \\
M_A &= \left\{ (A - r) \left[ (\mu - \frac{L}{h} \cos \gamma) R + \frac{L}{h} \sin \gamma \left( 1 - \frac{R}{\varepsilon} \right) \right] \\
&\quad + \frac{L}{3} + \frac{r \eta}{h} \right\} B p / \mu\quad \text{on OB,}
\end{align*}
\]

\(^7\) (Contd.)
\[ \mu = \frac{\eta}{\eta e} + \frac{L}{L} \]

\[ e = \frac{h^2}{12R} \left[ 1 + \frac{4}{15} \frac{h}{2R} \right] \]

\[ I = \frac{h^3}{12} \]

It follows that the membrane force \( N \) is constant through the flat portion of the duct, and that the ratio of \( N_A = ABp \) to the membrane force \( N_B \) is

\[ \frac{N_A}{N_B} = \frac{1}{\rho} + \left( 1 - \frac{1}{\rho} \right) \frac{1}{\cos \eta} \]  \( \tag{10} \)

For the reference hexcans, this gives the force at A to be about 14% above that at B. The magnitude of the bending moments, however, can differ by a factor of two from corner to midflat, and the variation of the bending moment around a duct with an \( h/\rho \) equal to that of an \( F \) duct is shown in Fig. 10. In this figure, points A, B, and O are, respectively, the corner, the midflat, and the point at which the round corner section joins the flat. Since different normalizing parameters are used, Fig. 10 is distorted in the horizontal direction. Nevertheless, the figure does show that the midflat moment is relatively insensitive to corner radii \( r/h \) below about 6, while the corner moment changes by almost a factor of two over that same range.
Table II gives the dimensionless values
\[ n = \frac{N}{pBp} \quad m = \frac{M}{p^2 Bp} \]  

of the total normal force \( N \) and bending moment \( M \) at the sections identified as critical in Fig. 10, viz., the corner \( A \) and the midflat \( B \) of Fig. 1.

<table>
<thead>
<tr>
<th>Hexcan</th>
<th>Corner ( n_A )</th>
<th>Corner ( m_A )</th>
<th>Midflat ( n_B )</th>
<th>Midflat ( m_B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>1.145</td>
<td>0.102</td>
<td>1.000</td>
<td>-0.056</td>
</tr>
<tr>
<td>F</td>
<td>1.142</td>
<td>0.099</td>
<td>1.000</td>
<td>-0.057</td>
</tr>
</tbody>
</table>

\( \alpha_n = N/(pBp); \quad \alpha_m = M/(p^2 Bp) \).

For internal pressurization of a hexcan, the inside corner and the outside midflat are in tension. For external pressurization, the maximum tensile stresses occur at the outside corner and inside midflat. These latter are not numerically as great as the values corresponding to internal pressurization, because of the compressive membrane force that acts in the external-pressurization case. Stress distributions for the more severe internal pressurization will therefore be calculated.

In computing the total tensile-stress distribution in a section, the tension due to bending is superimposed on that due to the membrane force \( N \) to give the total tensile stress. For straight beams, the familiar formula
\[ \sigma = \frac{My}{I} \]  
gives a linear stress distribution, such as that shown by the solid line in Fig. 9. For the curved corner of the hexcan, however, to compute the stresses due to bending in the hexcan corner, we will use the hyperbolic distribution (also shown in Fig. 9) given by
\[ \sigma = \frac{My}{Ae(R - e - y)}, \]  
where \( A \) is the cross-sectional area of the beam (numerically equal to \( h \) for a unit length of hexcan section).

The tensile-stress distributions through the duct wall at sections \( A \) and \( B \) are therefore given by
\[
\frac{\sigma_A}{p} = \frac{\rho}{h} n_A + \frac{\rho^2 y}{he(R - e - y)} m_A; \\
\frac{\sigma_B}{p} = \frac{\rho}{h} n_B + \frac{12\rho^2 y}{h^3} m_B,
\]

where \( y \) is positive when measured toward the center of the hexcan from the local neutral axis associated with bending. This axis coincides with the centroidal axis at the midflat.

The maximum tensile stresses at the critical corner and midflat sections are given by

\[
\begin{align*}
\left( \frac{\sigma_A}{p} \right)_{\text{max}} &= \frac{\rho}{h} n_A + \frac{\rho^2 (h + 2e)}{eh(2R + h)} m_A; \\
\left( \frac{\sigma_B}{p} \right)_{\text{max}} &= \frac{\rho}{h} n_B - \frac{6\rho^2}{h^2} m_B,
\end{align*}
\]

where the upper signs correspond to internal pressure \((p > 0)\) and the lower to external pressure \((p < 0)\). For external pressurization, because of a larger projected area on which the pressurizing medium acts, the normal-force terms are increased by a factor \( h/p \), but this does not appreciably change the total stress distribution given by Eqs. 15.

Table III summarizes the magnifications of the pressure these maximum tensile stresses represent.

<table>
<thead>
<tr>
<th>Hexcan</th>
<th>Corner ( \sigma/p )</th>
<th>Midflat ( \sigma/p )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Inside ( \sigma/p )</td>
<td>Outside ( \sigma/p )</td>
</tr>
<tr>
<td>E</td>
<td>256 (144)</td>
<td>284 (229)</td>
</tr>
<tr>
<td>F</td>
<td>579 (374)</td>
<td>130 (94)</td>
</tr>
</tbody>
</table>

*Parenthetical entries represent external-pressurization data.

Hence, under internal (external) pressurization, the inside (outside) corner stresses are about twice (one and one-half times) the outside (inside) midflat stresses. In all cases, the maximum stress occurs in the corner.

For the purpose of calculating stress intensity factors, it will be convenient to have an expression for the stress distribution through the thickness of the hexcan corner as a function of distance \( x \) from the inside corner.
(see Fig. 8). Since \( y + e = (h/2) - x \), it follows from Eqs. 14 that in the corner of the hexcan the circumferential tensile stress distribution may be put in the form, with \( \zeta = x/h \),

\[
\frac{\sigma}{p} = f(\zeta) = \frac{c_1 + c_2 \zeta}{c_3 + \zeta},
\]

where the values of the constants are as given in Table IV for the reference ducts.

<table>
<thead>
<tr>
<th>Hexcan</th>
<th>( c_1 )</th>
<th>( c_2 )</th>
<th>( c_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>1012.63</td>
<td>-2038.34</td>
<td>1.75</td>
</tr>
<tr>
<td>F</td>
<td>373.06</td>
<td>-730.37</td>
<td>1.46</td>
</tr>
</tbody>
</table>

The stress distribution represented by Eq. 16 for hexcan F is plotted as the solid line in Fig. 9. It is seen to be in good agreement with finite-element results. The inadequacy of the straight-beam stress distribution is also shown in this figure.

Hexcan sections loaded nonuniformly will, of course, have different stress distributions than those computed above. If such unsymmetrical stress distributions are computed in a similar manner, however, stress intensity factors may also be computed in ways similar to those outlined below.

When the hexcan has a crack, the stress distribution is, naturally, different from that described above. If we neglect the effect of pressure \( p \) on the crack faces, these faces are stress-free and Eq. 16 is no longer directly applicable. However, this stress distribution, calculated for an unflawed specimen, may be used to determine the stress intensity factor associated with a flawed specimen through the principle of superposition, which applies in linear elasticity. This principle, as outlined in Sec. II, states that any two solutions to the equations of equilibrium may be added to give a third solution to the equations.

Thus, the problem of a crack in a hexcan corner may be considered to be the sum of two elasticity solutions: (1) the solution to the problem of an unflawed hexcan loaded with internal pressure \( p \), and (2) the solution to the problem of a cracked hexcan loaded on its crack faces with stresses equal and opposite to those stresses calculated in problem (1) to act across the plane where the crack is now located. Since the stress field in problem (1) at the location of the crack tip is nonsingular, no stress intensity factor is associated with that solution. Therefore, the stress intensity factor calculated in problem (2) is the total stress intensity factor for the cracked hexcan loaded by internal pressure \( p \). Thus, in all problems discussed below, the crack faces are simply loaded with the stress distribution of Eq. 16 and the corresponding stress intensity factors are calculated.
IV. CALIBRATIONS OF STRESS INTENSITY FACTOR
FOR MIDFLAT CRACKS

The stress intensity factor $K_I$ associated with long, shallow midflat cracks (at section B in Fig. 1; see Fig. 2) may be approximated by assuming the loading of the section to be represented by the superposition of a uniformly distributed membrane stress and a linearly varying bending stress, as suggested in Fig. 11. Since the pressure is orders of magnitude smaller than the maximum tensile stresses to which it gives rise (see Table III), one should not expect too much error in $K_I$ to result from ignoring the pressure loading transmitted to the crack faces through the pressurizing medium when compared to the loading due to the bending moment $M$ and membrane force $N$. Since the normal force $N$ does not vary across the flat width, and since the bending moment $M$ is almost constant near the midflat (see Fig. 10), one would expect the handbook superposition technique to give good results for midflat cracks.

The stress intensity factor for a crack of depth $a$ in a hexcan midflat may then be taken as

$$K_I = K_{IM} + K_{IN},$$

where $K_{IM}$ and $K_{IN}$ are the stress intensity factors associated with the bending moment $M$ and the normal force $N$, respectively. Handbook values for these $K_I$ are of the form

$$K_{IM} = \sigma_M \sqrt{\pi a F_M(a/h)};$$

$$K_{IN} = \sigma_N \sqrt{\pi a F_N(a/h)},$$

where

$$\sigma_M = \frac{6M}{h^2};$$

$$\sigma_N = \frac{N}{h},$$
and where the values of the functions $F_M$ and $F_N$ are given to within 0.5% by

$$
F_M = \sqrt{\frac{\tan C}{C}} \left[ \frac{0.923 + 0.199 (1 - \sin C)}{\cos C} \right]^4;
$$

$$
F_N = \sqrt{\frac{\tan C}{C}} \left[ \frac{0.752 + 2.02 (a/h) + 0.37 (1 - \sin C)}{\cos C} \right]^3,
$$

where $C = \pi a/(2h)$.

For an externally cracked hexcan loaded by internal pressure, $K_I$ is given by the curves in Fig. 12, based on the above calibrations.

Although the functions $F_M$ and $F_N$ in the forms of Eq. 20 are indeterminate as $a \to 0$, the limiting values for the normalized stress intensity factors $K_I/p\sqrt{a}$ associated with vanishingly small cracks may be determined from the well-known expression

$$
K_I = 1.122 \sigma \sqrt{\pi a}
$$

for a crack in a half-space, using for $\sigma$ the maximum stresses given in Table III. These limiting values have been used in completing the $K_I$ calibration curves for vanishingly small cracks.

Since the maximum midflat stresses due to external pressurization are always lower than those due to internal pressurization, and since hexcan internals would add to the duct strength, the stress intensity factors associated with internal midflat cracks will not be explicitly presented here. A conservative analysis would result from using the $K_I$ of Fig. 12 for internal midflat cracks in externally pressurized hexcans.

Because of the significant magnification of the maximum tensile stresses due to the curvature of the hexcan corner (see Fig. 9), handbook calibrations, such as those made above for midflat cracks, would not be expected to give reliable results for cracks in the hexcan corner. Such handbook calibrations have been made, however, for comparison with more accurate computations, and these comparisons are presented in Sec. VI below.
V. MODELING A CRACKED HEXCAN CORNER

Using standard handbook $K_1$ calibrations for a cracked straight beam subjected to tension and bending would not be expected to give very accurate results for the cracked hexcan corner, because such calibrations completely ignore the significant curvature of the true geometry. Jones\textsuperscript{9} has compared such handbook-based calibrations for three-point bend specimens with finite-element results for sections of cylinders with $Q = 0.95$, $0.9$, and $0.8$, and the handbook results give progressively poorer estimates of $K_1$ as $Q$ decreases. The indicated trend is that such straight-beam calibrations for the $Q = 0.6$ hexcan corner would underpredict the true $K_1$.

However, a handbook calibration for a cracked infinite strip loaded by a point force (see Fig. 13) may be integrated with the stress distribution of Eq. 16 acting on the crack faces, and this would be expected to give one calibration for the hexcan corner. This calibration would be expected to overestimate the actual stress intensity factor, however, because the stiffening effects of hexcan flats and corner curvature are totally ignored.

Figure 14a shows, to scale, the details of one corner of the hexcan of Fig. 1., including the location of the crack whose associated stress intensity factor is desired. The hexcan segment ABCDEFA may be thought of geometrically as a segment of a cracked ring of the same radius and thickness, but beyond the common segment ABCDEFA the two geometries are obviously different, and even where the geometries are congruent, the stress distributions are not equivalent.

However, if the stress intensity factor is calculated by loading the crack faces with the stress distribution that would act across the same plane in an uncracked body, as described above, then one might expect the local stress field around the crack to be similar in both the hexcan and ring segments. That this is not exactly the case may be seen by the following argument for the case of two rings, one with a single radial crack, and one with two diametrically opposite radial cracks.

The two rings are illustrated in Fig. 14. The singly cracked ring is superimposed on the hexcan corner in Fig. 14a, and the symmetrical doubly cracked ring is represented by one of its halves in Fig. 14b. If the identical cracks at A are loaded with a uniform crack-face pressure $p$, the associated stress intensity factors should be the same if the far crack did not influence the behavior within the apparently similar sectors ABCDEFA.

Fig. 14

\textit{Cracked Infinite Strip Loaded by Point Force.}
Grandt$^3$ has reported calibrations of the stress intensity factor for rings like those of Fig. 14 for inner- to outer-radius ratios $Q = 0.5$ and 0.8. These are presented in Fig. 15, and the divergent curves indicate that the presence of a second crack has an effect that is significant for the thicker ring and cannot be ignored for larger cracks in the thinner ring. The hexcan corner, for which $Q \approx 0.6$, would appear to be thick enough so that the geometry outside sections AB and EF should not be ignored.

Furthermore, the compliance of the rings that are congruent with a hexcan corner would not be expected to be the same as that of a hexcan corner, and a crack in a hexcan corner should be expected to open up more freely than a crack in a ring. However, as one approximation, stress intensity factors have been calculated for hexcan corner cracks by interpolating information available for cracked rings. The procedure is described in Sec. VIII below. Results for doubly cracked rings would be expected to model the hexcan corner crack more accurately because of the greater compliance of that configuration over the singly cracked ring.

The hexcan corner might also be modeled by a sector of a cylinder loaded as shown in Fig. 14c. Collocation results have been studied by Underwood et al.,$^{10}$ for this configuration; these authors have provided an analytical expression for the stress intensity factor, which they found depends upon the ratio of load eccentricity to specimen thickness, $e/h$, as well as the usual ratio of crack depth to specimen thickness, $a/h$:

$$\frac{K_I B \sqrt{h}}{P_c} = F_c \left( \frac{a}{h}, \frac{e}{h} \right). \quad (22)$$
This calibration is reported to be applicable to geometries with \( Q \) ranging from 0.4 to 0.7 and for \( 0.3 < \frac{e}{h} < 1.6 \) and \( 0.2 < \frac{a}{h} < 0.6 \). The calibration is given below in Sec. VII, where its application to the hexcan is made, but one would expect that the compliance of the C-shaped specimen, like that of the infinite strip, would exhibit more flexibility than the cracked hexcan corner. Nevertheless, the results based on these specimens indicate an upper bound to the true hexcan \( K_1 \); those based on the ring, because of its greater stiffness than the hexcan corner, are expected to bound the true \( K_1 \) from below.

VI. STRIP CALIBRATION FOR CORNER CRACKS

One approach for obtaining \( K_1 \) calibrations for corner-cracked hexcans is to use a handbook calibration for a cracked infinite strip of unit thickness whose crack faces are loaded by concentrated forces \( P \) as shown in Fig. 13. For a crack of length \( a \), with \( P \) acting a distance \( x \) from the crack mouth, the stress intensity factor is given by

\[
K_1^P = \frac{2P}{\sqrt{\pi a}} F_P \left( \frac{x}{a}, \frac{a}{h} \right),
\]

where, letting \( \zeta = \frac{x}{h} \) and \( c = \frac{a}{h} \),

\[
F_P(\zeta/c, c) = \frac{3.52[1 - (\zeta/c)]}{(1 - c)^{3/2}}
\]

\[
- \frac{4.35 - 5.28(\zeta/c)}{(1 - c)^{1/2}} + \frac{1.30 - 0.30(\zeta/c)^{3/2}}{[1 - (\zeta/c)^2]^{1/2}}
\]

\[
+ 0.83 - 1.76(\zeta/c) \left[ 1 - \left( 1 - \frac{\zeta}{c} \right) c \right].
\]

The stress intensity factor for the hexcan corner may then be calculated by loading the crack with \( P = \sigma(x)dx \) and integrating over the crack length:

\[
K_1 = \int_0^a K_1^P \frac{\sigma(x)dx}{P}
\]

or

\[
\frac{K_1}{P\sqrt{\pi a}} = \frac{2}{\pi c} \int_0^c F_P(\zeta/c, c)f(\zeta)d\zeta.
\]
where \( f(\zeta) \) is given by Eq. 1c for the hexcan corner. This integration has been carried out, and the results are presented in Fig. 16.

![Graph showing hexcan corner stress distributions](image)

The handbook calibration based on the straight-beam superposition principle illustrated in Fig. 11 is also plotted in Fig. 16. This latter calibration underestimates the stress intensity factor for shallow corner cracks, when compared with the calibration based on the more accurate infinite-strip model loaded with the hexcan-corner stress distribution as given by curved-beam theory.

Since a hexcan is expected to provide more resistance than a strip to the opening of a corner crack, the infinite-strip calibration is expected to be conservative. That is, the actual stress intensity factors for corner-cracked hexcans should be less than those given in Fig. 16 by the strip calibration curves.

VII. C-SPECIMEN CALIBRATIONS

The C-shaped specimens of Fig. 14c may be loaded by the forces \( P_c \) at the offset \( e/h \) in such a way that the normal force \( N_c \) and bending moment \( M_c \) transmitted across the crack plane are in the same ratio as those in a hexcan corner. For the C-shaped specimen, across the crack plane

\[
\frac{M_c}{N_c} = \frac{h}{2} (1 + 2 \frac{e}{h}) .
\]
and for the hexcan corner, $M/N$ is shown in Table V. If the C specimen is taken to have the same $Q$ and $h$ as the hexcan corner, the $M/N$ ratios will be the same if and only if
\[
\frac{e}{h} = \frac{1}{h} \frac{M}{N^2} - \frac{1}{2},
\]
the values of which are also given in Table V. Since $N_c = P_c$ and $N = n_p B_p$, the normal forces, and therefore the stresses, are equal in the hexcan and the C specimen if and only if $P_c = n_p B_p$. Then the C-specimen calibration, Eq. 7, takes the form
\[
\frac{K_I}{p \sqrt{\pi a}} = \frac{n_p}{\sqrt{\pi h^3}} F_c \left( \frac{a}{h} \frac{e}{h} \right),
\]
where the function $F_c$ is given by
\[
F_c \left( \frac{a}{h} \frac{e}{h} \right) = \left[ 1.411 + 33.68(a/h) - 104.9(a/h)^2 
+ 221.8(a/h)^3 - 125.4(a/h)^4 \right] (e/h) 
+ [6.447 - 48.17(a/h) + 277.8(a/h)^2 - 575.0(a/h)^3 + 469.3(a/h)^4].
\]

<table>
<thead>
<tr>
<th>Table V. Parameters for C-specimen Calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hexcan</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>E</td>
</tr>
<tr>
<td>F</td>
</tr>
</tbody>
</table>

This calibration is plotted in Fig. 17, and, within the range $0.2 \leq a/h \leq 0.6$ for which it is applicable, it demonstrates that the straight-strip calibrations of Sec. VI, plotted again in Fig. 17, give trends of the $K_I$ behavior that are consistent with the experimental data on which the C-specimen calibration is based. These C-specimen data would reflect the true stress distribution existing across the crack plane, while the infinite-strip calibration is based on (approximate) curved-beam theory.
VIII. RING CALIBRATIONS

As discussed in Sec. V, a cracked hexcan corner might also be modeled as a cracked ring having the same ratio \( Q \) of inner to outer radius. If the crack faces of the ring are loaded with the stress distribution that exists through the uncracked wall of a hexcan corner, the corresponding \( K_I \) values should bound those of the hexcan from below. The weight-function principle associated with Rice\(^1\) and Bueckner\(^2\) provides a convenient technique for determining the \( K_I \) values associated with the hexcan stress distribution from those of Grandt,\(^3\) given in Fig. 15, for the ring loaded by uniform crack-face pressure.

This principle states that, if the crack-face displacement \( u \) and the stress intensity factor \( K \) are known as functions of crack length \( a \) for any one symmetrical load system acting on a linear elastic body in plane strain, then the stress intensity factor \( K_I \) associated with any other symmetrical load system on the same body may be determined by

\[
K_I = \int_0^a \sigma(x) h(a, x) \, dx
\]  

(31)

where \( \sigma(x) \) is the stress distribution with which \( K_I \) is associated and where the weight function

\[
h(a, x) = \frac{H}{K} \frac{\partial u(a, x)}{\partial a}
\]  

(32)
is computed from the crack-face displacement $u$ of the reference problem, which is a function of both the crack length $a$ and the distance $x$ from the crack mouth. The material constant $H$ is equal to $E/(1 - \nu^2)$ for plane strain.

Petroski and Achenbach\textsuperscript{13} recently have developed a simple technique by which the weight function may be determined from the known stress intensity factor $K$ alone without any a priori knowledge of the displacement $u$. They have obtained excellent results for cracked strips, holes, and rings by using the simple representation

$$u(a, x) = \frac{\sigma_0}{H \sqrt{2}} \left[ 4F \left( \frac{a}{L} \right) a^{1/2}(a - x)^{1/2} + G \left( \frac{a}{L} \right) a^{-1/2}(a - x)^{3/2} \right],$$

(33)

where $\sigma_0$ and $L$ are a characteristic stress and length of the problem, and $F$ is the dimensionless stress intensity factor $K/\sigma_0(\pi a)^{1/2}$. Since Eq. 31 yields an identity for the reference case when $K = K_I$, the function $G$ has been computed in Ref. 13 to be given by

$$G \left( \frac{a}{L} \right) = \left[ I_1(a) - 4F \left( \frac{a}{L} \right) a^{1/2} I_2(a) \right] a^{1/2}/I_3(a),$$

(34)

with

\[
\begin{align*}
I_1(a) &= \pi \sigma_0 \sqrt{2} \int_0^a \left[ F \left( \frac{a}{L} \right) \right]^2 a \, da; \\
I_2(a) &= \int_0^a \sigma(x) (a - x)^{1/2} \, dx; \\
I_3(a) &= \int_0^a \sigma(x) (a - x)^{3/2} \, dx.
\end{align*}
\]

(35)

The important advantage of this technique is that $K_I$ may be computed from simple quadratures involving only the stress $\sigma(x)$ and the dimensionless stress intensity factor $F$ of the reference problem, which for the ring has been taken as that corresponding to uniform crack-face pressure.

The normalized stress intensity factor $F$ was represented as a fifth-order polynomial fit to the $K$ calibrations due to Grandt,\textsuperscript{3} which are shown in Fig. 15, for this reference problem. The resulting polynomial suffices to determine the function $G$ from Eqs. 34 and 35, and this determination is simplified by the fact that $\sigma(x) = \sigma_0$, a constant. By interpolating between the derived values of $G$ for the rings, we can determine values of the function $G$ for the cases $Q = 0.59$ and $Q = 0.64$ corresponding, respectively, to the
F and E hexcans. Fitting polynomials to these enables a determination of the weight function $h$ in Eq. 32 from Eq. 33. Then it is simple to use this weight function with the hexcan stress distribution of Eq. 16 in Eq. 31 to calculate hexcan $K_I$ values. Calibrations for $K_I$ based on the weight-function principle are presented in Figs. 18 and 19. The upper curve is based on Grandt's $K$ for two cracks, the lower on his $K$ for one crack. As explained in Sec. V above, the two-crack case is believed to more accurately bound the true hexcan corner $K_I$ from below.

The calculations based on results for doubly cracked rings and C-shaped fracture-toughness specimens are believed to provide bounds on the true stress intensity factor, because these models of the hexcan corner include the effects of curvature and the compliance of the hexcan. The C-specimen and closely agreeing infinite-strip model, both being more flexible than a hexcan corner, are expected to bound the true $K_I$ from above. The doubly cracked ring, on the other hand, providing much more resistance to crack opening than exists in the hexcan, is expected to bound the true values of $K_I$ from below.

These bounds, and the strip calibration that falls between them, are plotted in Figs. 20 and 21 to fix limits on $K_I$ for the hexcan corner. Although the curves identified as bounds on the true $K_I$ calibration for a cracked hexcan
corner diverge considerably for cracks greater than half the thickness, it is believed that such large cracks cannot be reasonably postulated to exist in subassembly corners for a linear elastic analysis. For cracks up to 40% of the wall thickness, the lower bound is only 25% below the upper one, and a conservative analysis could be based on the higher curve, or the curve based on the strip could be used.

These elementary models have served as a means for getting a feel for the behavior of a cracked hexcan corner before performing more complex and costly analyses. The next section discusses a finite-element technique that has been applied to the same problem.
X. FINITE-ELEMENT ANALYSIS

One method of determining $K_I$, the strength of the stress singularity at the crack tip, is the finite element method. In recent years, much use has been made of the finite-element method to describe mathematical singularities in elastic bodies. Due to the high stress gradient, a large number of ordinary finite elements surrounding the singular point are needed to obtain acceptably accurate solutions. The error decreases with element size as $O(\sqrt{h})$ rather than as $O(h^2)$ when no singularity is present. The use of very small ordinary elements near the crack tip is obviously a costly and inefficient procedure.

For this reason, several special elements containing the proper singularity have been recently formulated. These "singular" elements are based on the asymptotic near-crack-tip displacement field. This procedure requires the formulation of a complicated element-stiffness matrix that can be easily coupled to conventional elements. Coarse meshing near the crack tip gives very accurate results.

Benzley\textsuperscript{14} presented an analysis of crack problems in elastic bodies using is parametric quadrilateral elements with a singular displacement field near the crack tip. A finite-element computer code based on this work, CHILES,\textsuperscript{15} was acquired through the Argonne Code Center to treat the cracked-hexcan problem. The CHILES computer program is a two-dimensional-solid, finite-element code which calculates the state of strain at the tip of a crack in either a plane-stress, plane-strain, or axisymmetric geometry. Linear isotropic stress-strain material properties are used, and small-strain theory is assumed. Isoparametric quadrilateral finite elements are employed, and compatibility between singular and ordinary elements is maintained to ensure monotonic convergence.

The version of CHILES acquired is suitable for use on the CDC 6600 computer. Repunching of the program, conversion to double precision, and some other programming changes were necessary for implementation on the IBM 370/195. The code was subsequently checked out with the supplied problem as well as other test problems.

In linear elastic fracture mechanics, a crack in a brittle material becomes unstable as $K_I$ approaches $K_{IC}$, the critical stress intensity factor. For brittle materials, $K_{IC}$, which is a material property, can be measured in the laboratory with an accuracy of about $5\%$. It has not been determined if this accuracy can be achieved with highly irradiated materials, since testing must be performed remotely. Nevertheless, calculations made with CHILES appear to be well within this accuracy.

The dimensions of the LMFBR hexcan design chosen for analysis are those of hexcan F given in Table I. Preliminary to determining the stress intensity factors associated with a uniformly pressurized hexcan having a
corner flaw is the selecting of a suitable finite-element mesh. Since a large stress intensification is expected in the corners of even unflawed hexcans, particular attention must be given to the spatial discretization of these regions. Several numerical experiments were done varying the number and, therefore, the size of the elements both circumferentially and through the thickness of the hexcan corner. Finite-element results showing the stress distribution through the corner are presented in Fig. 9. The meshing chosen for 1/12th of the hexcan is shown in Fig. 22. Half the hexcan must be modeled in this fashion so that the problem of a crack in a single corner can be solved. The total grid for the half hexcan consists of 1700 elements with 3762 degrees of freedom.

Crack-tip singular elements were placed at various positions through the hexcan corner to represent different depths of crack penetration. Stress intensity factors for hexcan F with a crack in one corner subjected to uniform internal pressure were calculated and are shown in nondimensional form in Fig. 23. The previously determined estimates and bounds on $K_I$ derived from the simpler models discussed above are repeated in this figure.

Based on other test problems in which the behavior is predominantly bending, the finite-element results are believed to be extremely accurate (within a few percent), especially for smaller crack lengths, and they tend to underestimate the true solution. The curve predicted using the infinite-cracked-strip model is a very good approximation for cracks that penetrate deeper than 20% of the hexcan wall and gives a conservative estimate of the stress intensity factors for more shallow cracks. The C-specimen model is
also a fairly good approximation and gives consistently conservative values over its range of applicability. On the other hand, the cracked-ring model offers entirely too much resistance to crack opening and does not appear to be a reasonable approximation for the hexcan corner.

XI. EFFECT OF NONUNIFORM LOADING

To address questions of integrity, one must identify worst-case problems, so that cracks and flaws may be postulated to exist in the most unfavorable locations in the hexcan wall. Where these locations are depends on loading conditions, and how a hexcan is loaded depends on what abnormal event is postulated. Figure 24 illustrates three possible loading modes of concern in safety analysis. Figure 24a illustrates a uniform overpressurization of the hexcan section; Figs. 24b and 24c represent localized pressure pulses which may arise from the escape of fission gas from a failed fuel pin. The complex reaction on the hexcan wall opposite the pin failure is simply represented by its resultant. The time-independent case will be considered in this first look at the problem.

![Fig. 24. Uniform and Nonuniform Hexcan Loadings](image)

Figure 25 shows finite-element results for the mode of deformation (amplified for clarity) of an unflawed hexcan under uniform pressure. As expected, the maximum radial deflection occurs at the midflats. The small outward radial deflections that occur at the corners are also predicted by the

![Fig. 25](image)
The elastic deformation of a hexcan loaded at two opposite midflats is shown (exaggerated, but to scale) in Fig. 26. Although the corners adjacent to a loaded midflat are not as severely strained as the midflat, those remote from the load are (i.e., the remote corners undergo a greater change of curvature), and thus a greater peak tensile stress occurs in them.

The stresses due to uniform pressure loading, as in Fig. 24a, have been discussed in Sec. III, where it was shown for hexcan F that the maximum tensile hoop stress $\sigma$ occurs at an inside corner and represents a 250-times magnification of the pressure $p$; the maximum stress at a midflat is only about half that value. For the loading illustrated in Fig. 24b, which may be further idealized as a pair of equal and opposite forces $P$ per unit axial length $B$ of the hexcan section, the maximum stresses occur at the midflat under the load, but the peak tensile stresses at corners remote from the load are only about 15% lower.

The loading illustrated in Fig. 24c again causes maximum stresses at the corner to be significantly higher than the maximum midflat stresses, and this case will not be considered further here. The effect of distributing a load over a finite portion of the hexcan wall is to slightly reduce the maximum stresses under the load, so that the concentrated load represents a conservative model that is to be favored for its simplicity. Moving the concentrated load away from the midflat generally increases corner stresses and reduces midflat stresses.

Since the maximum hoop stresses have relative maxima at corner and midflat sections under these loading cases, it is conservative to assume that any flaws that might exist in a hexcan wall are located at these critical sections. A common flaw expected in subassembly ducts is an axial scratch that is long compared to the hexcan thickness (see Fig. 2), and such a flaw could be modeled as a crack that penetrates to a depth $a$ in a hexcan wall of thickness $h$ and is oriented perpendicular to the maximum tensile stresses.

Two such cracks are illustrated in Fig. 2. Since the cracks are long compared to the dimension $h$, a plane-strain elasticity problem results, in which the singular stress distribution around a crack tip is measured by the stress intensity factor $K_I$, which has been seen to be a function of crack size, hexcan geometry, and loading. As long as the $K_I$ associated with a particular crack is below some critical value $K_{IC}$, that crack is stable. For very shallow
cracks, the stress intensity factor may be estimated as \( K_I = 1.12\sigma_{\text{MAX}}\sqrt{a} \), where \( \sigma_{\text{MAX}} \) is the maximum tensile stress that would exist at the section were there no crack present.

Then it follows that, for equally deep but shallow cracks at the inside corner and outside midflat of a hexcan, the more critical crack will be that in the corner when the hexcan is uniformly pressurized as in Fig. 24a, whereas the midflat crack is slightly more critical under loading conditions like that shown in Fig. 24b. To compare larger cracks of unequal sizes, it is necessary to have \( K_I \) values as functions of crack size at both locations.

For uniform pressurization, the \( K_I \) calibration for midflat cracks shown in Fig. 12 and that for corner cracks shown in Fig. 23 may be used to compare the relative importance of the two crack locations. To more readily compare the effects of different crack lengths at different locations for this loading case, the stress intensity factors have been expressed in the dimensionless form \( K_I/p\sqrt{h} \) rather than the more conventional \( K_I/p\sqrt{a} \) and plotted together in Fig. 27. This figure shows that, for a uniformly pressurized hexcan, an inside corner crack is always worse than an outside midflat crack of the same depth. A corner crack whose depth is only 10\% of the wall thickness, for example, has a \( K_I \) in this case equal to that of a 30\% deep midflat crack.

The case of a cracked hexcan loaded at opposite midflats must be calibrated before a similar comparison of midflat and corner cracks can be made.

Since the strip model has been seen to give a good estimate of the hexcan-corner \( K_I \) calibration for uniform pressure, it can be expected to give an equally good estimate of \( K_I \) for nonuniform loading. The midflat crack may be calibrated, as before, by handbook calibrations for straight beams.

If the technique outlined in Sec. III is used, it is easy to calculate the stress distribution through a hexcan corner remote from the two midflats under concentrated loads \( \Phi \):

\[
\frac{\sigma hB}{p} = \frac{46.33 - 99.35\zeta}{1.46 + \zeta}.
\]

(36)

It also follows that the midflats under the loads are subject to a pure bending moment equal in magnitude to \( M/hP = 6.12 \).
Using the cracked-strip model of Sec. VI with the stress distribution (Eq. 36) for the corner crack and the straight-beam model of Sec. IV for the midflat crack enables a comparison of the cases.

Figure 28 shows the resulting calibrations, and it demonstrates that the worst location for a crack in this case is directly under a lead, but the same size of crack at remote corners may be only slightly less severe. A midflat crack whose depth is 10% of the hexcan wall and a 15% deep corner crack, for example, have the same $K_I$. Since inside corners are less accessible to inspection than outside midflats, however, one might have to assume larger cracks in corner locations. Such considerations could make a corner crack at least as important as a midflat one in this case also. Furthermore, the critical stress intensity factor $K_{IC}$ could be lower in the corner region due to additional coldworking there.

XII. EFFECT OF DIFFERENT NUMBERS OF CORNER CRACKS

The analysis based on simple models does not consider the effects of having more than one hexcan corner cracked. Grandt's analysis of cracked rings, however, provides some insight into the multiple-crack problem. Figure 15 shows that, when loaded by a uniform crack-face pressure, a ring with two diametrically opposite radial cracks exhibits higher stress intensity factors than a ring with only one crack. The effect is much more pronounced for rings with lower ratios of inner to outer radii. The behavior of the hexcan with two diametrically opposite corner cracks is expected to be similar. The correctness of this assumption can be checked by applying the CHILES code to the problem.

In addition to the half-hexcan grid used to generate the stress intensity factors for the one-corner cracked hexcan, a quarter-hexcan grid was created to solve the problem of two corner cracks diametrically opposite each other, and a 1/12th-hexcan grid (illustrated in Fig. 22) was constructed for the problem of a hexcan with six corner cracks. The results from the code calculations for these problems are shown in Fig. 29. The one- and two-crack hexcans exhibit the same type of behavior as Grandt's cracked rings (see Fig. 15). The difference in stress intensity factor is small for shallow cracks, as for the thin cracked rings, with greater differences appearing for larger cracks. The gross deformations of the one- and two-crack cases are perpendicular to the plane of the two cracks.
Naturally, the stress intensity factors for the two-crack case are higher, since the cracks can open more in that direction. The six-crack hexcan, however, deforms uniformly, much like the unflawed hexcan, and the effect of this mode of deformation is to restrain the opening of any one crack. Even so, the difference between the stress intensity factor for the one- and six-crack cases is less than 7% for cracks that are 40% of the hexcan thickness and smaller. The similarity of results for smaller cracks in all three cases is due to the thinness of the hexcan wall compared to the distance between corners. This distance appears to be large enough so that effects or one corner from the others are minor.

The results presented indicate that the determination of the worst-case problem (i.e., one, two, or six cracks) is not particularly important for the hexcan-F geometry studied. For the smaller cracks, which are of major interest, the differences in stress intensity factors are not especially significant for this geometry.

A comparison of the different modes of deformation discussed in the previous section is also helpful in understanding the effects of having more than one hexcan corner cracked. The deformation of a uniformly pressurized hexcan with two opposite corner cracks would deform in a mode with twofold symmetry like that of Fig. 26, whereas a pressurized hexcan with only one corner cracked would offer more resistance to opening and have only one plane of symmetry: the crack plane. A hexcan with all six corners cracked would deform in a sixfold symmetric mode more like that of the uncracked hexcan illustrated in Fig. 25. Thus, for example, one would expect two opposite corner cracks to be a worse case than a single crack. These observations have already been quantified by the finite-element results reported above and illustrated in Fig. 29.

XIII. A WEIGHT FUNCTION FOR HEXCAN CORNER CRACKS

The finite-element results shown in Fig. 23 may be used to construct a weight function for hexcan corner cracks. Such a tool would enable one to study the effects of a variety of hexcan loadings on the stress intensity factor $K_I$ without further resort to the more costly finite-element analyses.
Table VI shows the finite-element results, which provide the function $F$ and the values of the function $G$, which are derived from Eqs. 34 and 35. These suffice to construct the weight function (Eq. 32) from Eq. 33 as follows:

$$h(a, x) = \frac{1}{F \sqrt{2\pi a}} \frac{\partial}{\partial a} \left[ 4Fa^{1/2}(a - x)^{1/2} + Ga^{-1/2}(a - x)^{3/2} \right]. \quad (37)$$

Further details of computation are given in Ref. 13.

<table>
<thead>
<tr>
<th>$\frac{a}{h}$</th>
<th>$F$ (Finite-element results)</th>
<th>$G$ (From Eq. 34)</th>
<th>$\frac{a}{h}$</th>
<th>$F$ (Finite-element results)</th>
<th>$G$ (From Eq. 34)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>287$^a$</td>
<td>-126.5</td>
<td>0.4</td>
<td>276.6</td>
<td>339.8</td>
</tr>
<tr>
<td>0.1</td>
<td>248</td>
<td>-27.9</td>
<td>0.5</td>
<td>314.7</td>
<td>623.3</td>
</tr>
<tr>
<td>0.2</td>
<td>245</td>
<td>45.2</td>
<td>0.6</td>
<td>373.8</td>
<td>1122.6</td>
</tr>
<tr>
<td>0.3</td>
<td>254.3</td>
<td>156.8</td>
<td>0.7</td>
<td>460.4</td>
<td>1976.9</td>
</tr>
</tbody>
</table>

$^a$Limiting value for an edge crack in a half-space.

To illustrate the efficacy of a weight function so constructed, the problem of a corner-cracked hexcan loaded by concentrated forces at opposite midflats was considered. This problem was solved independently by the finite-element method as described in Sec. X and by the weight-function technique of Sec. VIII. The stress distribution (Eq. 36) was used in Eq. 31 with the weight function derived from Table VI.

Figure 30 compares the results. The weight-function and finite-element results are essentially identical, and this provides validation of the technique of Sec. VIII. The figure also shows the results of using the straight-strip calibration in conjunction with the stress distribution of Eq. 36. For reasonable sizes of cracks the strip is seen again to provide a good estimate, but the weight-function technique is to be preferred for its accuracy.
XIV. COMMENTS ON THE APPLICATION OF LEFM TO CRACKED HEXCANS

Whether an actual crack in an irradiated hexcan will propagate unstably depends on what the fracture toughness $K_{IC}$ of the hexcan material is when the pressure $p$ is applied. This number is not presently available for highly irradiated hexcan material, which typically for $E$ cans is Type 304 and for $F$ cans 20% coldworked Type 316 stainless steel.

The fracture toughness of unirradiated stainless steel is extremely high, and the material is very ductile. Hence, yielding occurs at stresses well below the stress corresponding to $K_{IC}$, and an ultimate stress or strain criterion of failure is applicable. After prolonged irradiation (fluences of the order of $10^{27}$ n/m$^2$, $E > 0.1$ MeV), however, the hexcan material is expected to exhibit the following deleterious changes in its mechanical properties:

1. The yield strength increases (at lower temperatures).
2. The ductility decreases.
3. The fracture toughness decreases.

Quantitative data on items 1 and 2 are reported in the Nuclear Systems Materials Handbook, which is updated as new data become available. However, the decrease in fracture toughness of the subassembly materials has not been monitored, and it will probably be years before such data become available in the range of fluences of interest.

Table VII compares geometrically similar cracks in the hexcan designs $E$ and $F$. Although the normalized $K_I/p\sqrt{h}$ is over twice as great in $E$ as in $F$ hexcans, the $K_I/p$ values are comparable when the geometric factor is eliminated. Since the larger $F$ design may be expected to be subject to larger energy releases, fracture considerations may indeed have to be more of a concern in these ducts.

<table>
<thead>
<tr>
<th>Hexcan Type</th>
<th>$K_I/p\sqrt{h}$</th>
<th>$K_I/p, \sqrt{m}/(\sqrt{\text{in.}})$</th>
<th>$K_I/\sigma_{max}, \sqrt{m}/(\sqrt{\text{in.}})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>332.57</td>
<td>10.60 (66.51)</td>
<td>0.0183 (0.115)</td>
</tr>
<tr>
<td>F</td>
<td>146.50</td>
<td>8.90 (50.75)</td>
<td>0.0348 (0.198)</td>
</tr>
</tbody>
</table>

If one assumes that 20% coldworked Type 316 stainless steel can suffer irradiation damage such that its fracture toughness drops to around 220 MPa$\cdot$cm$^{1/2}$ (20 ksi$\cdot$\sqrt{in.}), then a crack in the corner of an $F$ hexcan could
propagate unstably when $p = 2.72$ MPa (394 psi), since the corresponding maximum hoop stress is about $\sigma_{\text{max}} = 690$ MPa (100,000 psi), by Table VII, and this is below the yield strength under certain combinations of irradiation conditions and test conditions. For example, the yield strength of 20% cold-worked Type 316 stainless steel irradiated to a fluence of $3 \times 10^{26}$ n/m$^2$ ($E > 0.1$ MeV) near 370°C (700°F) and tested at or below the irradiation temperature at a strain rate of $3 \times 10^{-5}$ s$^{-1}$ is about 830 MPa (120,000 psi).

The ASTM criterion for plane-strain conditions to apply is that the size of the plastic zone at the tip of a crack should be much smaller than the smallest dimension of the specimen. The plastic zone size is assumed to be ignorable if the quantity

$$\xi = 2.5\left(\frac{K_{\text{IC}}}{\sigma_y}\right)^2,$$

where $\sigma_y$ is the material yield stress, and is smaller than the uncracked ligament. For the above example, $\xi \approx 1.76$ mm (0.07 in.), which is about half the thickness of the F duct. Thus, for a crack $h/10$ deep (0.3 mm or 0.012 in.) the uncracked ligament would be sufficiently large to accommodate a plastic zone and maintain plane-strain conditions, thus justifying the analysis by linear elastic fracture mechanics carried out in this report.

It must be reemphasized, however, that the critical material property, the fracture toughness, $K_{\text{IC}}$, is not known at this time for the condition of interest. Therefore, no firm conclusions about the strength or safety of flawed hexcans can be drawn here.

XV. CONCLUSIONS

Plane-strain stress intensity factors for cracked hexagonal subassembly ducts subject to uniform pressurization have been determined. A crack in the unique hexcan-corner geometry was modeled in several different ways to estimate the true hexcan $K_I$, and a subsequent finite-element analysis proved that a simple strip model provides the best first estimate.

A weight-function technique enables one to obtain results with as good an accuracy as finite-element results, but with considerably less computational effort. This technique and the finite-element method have corroborated each other in the analysis of a hexcan loaded nonuniformly by concentrated loads at opposite midflats.

The effects of cracks at different locations have been explored. The corner crack is much more severe than that of a similar midflat crack in a uniformly pressurized hexcan. Although the midflat crack can give rise to the greatest $K_I$ under certain loading conditions, the corner crack appears to be
at least an equally important case to consider because of greater uncertainties associated with flaw size and properties of materials at the corner location.

If more than one corner of a hexcan should be cracked simultaneously, the stress intensity factor will be a function of the number of cracks. The $K_I$ associated with two opposite corner cracks has been determined to be the worst case, but the increase in stress intensity over the one-crack case is not significant for cracks of moderate size.

The two hexcan designs considered—-the 1-mm (40-mil)-wall hexcan $E$ and the 3-mm (120-mil)-wall hexcan $F$—are geometrically similar. However, geometrically similar cracks in these two designs should not be expected to behave similarly. Besides the different material and irradiation conditions of the two hexcans, the potential for larger energy releases in the larger can and the ability of the greater wall thickness to contain plastic yielding could make unstable crack propagation possible in $F$ hexcans even though it would not occur in an equivalently cracked smaller $E$ hexcan.

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Dr. S. H. Fistedis, manager of the Engineering Mechanics Program, was responsible for initiating these fracture-mechanics investigations for LMFBR safety analysis, and he continues to support these efforts with encouragement and interest.
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