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FLUIDELASTIC INSTABILITIES IN TUBE BUNDLES
EXPOSED TO NONUNIFORM CROSSFLOW

by

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Materials and Components Technology Division

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NOMENCLATURE

c	Prism height
$C_D (C_L)$	Steady drag (lift) coefficient
$C'_{Dj} (C'_{Lj})$	Periodic fluctuating drag (lift) coefficient for the j th cylinder
d	Prism depth
D	Diameter of a cylinder ($D = 2R$)
f	Oscillation frequency
f_f	Natural frequency in fluid
f_v	Reference natural frequency in vacuum
g_j	Fluid-force component in the x-direction of the j th cylinder
h_j	Fluid-force component in the y-direction of the j th cylinder
ℓ	Cylinder length
m_j	Cylinder mass per unit length of cylinder j
N	Number of cylinders in an array
P	Pitch
t	Time
T	Period
u_j	Displacement of the j th cylinder in the x-direction
U	Gap flow velocity in a tube array
U_e	Equivalent uniform flow velocity
U_m	Mean flow velocity
U_r	Reduced flow velocity ($U_r = U_m/fD$ or U/fD)
v_j	Displacement of the j th cylinder in the y-direction
x, y, z	Cartesian coordinates
$\alpha_{jk}, \beta_{jk}, \sigma_{jk}, \tau_{jk}$	Added-mass coefficients
$\alpha'_{jk}, \beta'_{jk}, \sigma'_{jk}, \tau'_{jk}$	Fluid-damping coefficients
$\alpha''_{jk}, \beta''_{jk}, \sigma''_{jk}, \tau''_{jk}$	Fluid-stiffness coefficients

$\bar{\alpha}_{jk}, \bar{\beta}_{jk}, \bar{\sigma}_{jk}, \bar{\tau}_{jk}$	Added-mass matrices
$\bar{\alpha}'_{jk}, \bar{\beta}'_{jk}, \bar{\sigma}'_{jk}, \bar{\tau}'_{jk}$	Fluid-damping matrices
$\bar{\alpha}''_{jk}, \bar{\beta}''_{jk}, \bar{\sigma}''_{jk}, \bar{\tau}''_{jk}$	Fluid-stiffness coefficients
ζ_f	Damping ratio in flow
ζ_j	Damping ratio
ζ_j, η_j	Damping ratio in vacuum
ζ_v	Reference damping ratio
ρ	Fluid density
ω	Circular frequency ($\omega = 2\pi f$)
ω_v	Reference natural frequency in radians in vacuum
ω_j, Ω_j	Natural frequency in radians of the j th cylinder in vacuum ($\omega_j, \Omega_j = 2\pi f_{vj}$)
$\phi_n(z), \phi(z)$	Orthonormal function of the n th mode
ψ	Flow velocity distribution function
Subscripts	
j, k	Cylinder number j, k ($j, k = 1$ to N)
N	Number of cylinders

CREDITS

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- Fig. 9 "Unsteady Lifts and Wakes of Oscillating Rectangular Prisms," Y. Nakamura and T. Mizota, Journal of Engineering Mechanics Division, ASCE, Vol. 101 (EM6), pp. 855-871, 1975, Figures 3 and 4. Permission granted by American Society of Civil Engineering.
- Fig. 10 "Aeroelastic Instability of Rectangular Cylinders in a Heaving Mode," K. Washizu, A. Ohya, Y. Otsuki, and K. Fujii, Journal of Sound and Vibration, Vol. 59, No.2, pp. 195-210, 1978, Figures 11 and 13. Permission granted by Academic Press Inc.
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- Fig. 13 "Investigation of the Aerodynamic Instability of a Square-Section Cylinder by Forced Oscillation," P. W. Bearman and S. C. Luo, Journal of Fluids and Structures, Vol 2, pp. 161-176, 1988, Figure 3. Permission granted by Academic Press Inc.

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ABSTRACT

A mathematical model is presented for fluidelastic instabilities in tube bundles exposed to nonuniform crossflow to develop an equivalent flow velocity. The model incorporates all motion-dependent fluid forces based on the unsteady flow theory. Published fluid-force coefficients are compiled and examined; all fluid-force coefficients are approximately independent of the reduced flow velocity when it is large but are strongly dependent on reduced flow velocity when it is lower. The unsteady flow model is used to show that the mode-weighted averaging method is valid for large reduced flow velocity, but that it is incorrect at lower reduced flow velocity. In the lower reduced flow velocity range, an iterative procedure is needed to establish the critical flow velocity. The results from the unsteady flow model are compared with those of other mathematical models. A general procedure is recommended to predict the equivalent flow velocity under different situations.

I. INTRODUCTION

Fluidelastic instabilities of tube bundles in crossflow have been studied extensively since the early 1970s. Published experimental and analytical studies deal with the following issues: (1) determination of the critical flow velocity; (2) nondimensional parameters governing the instability threshold; (3) mathematical models for the instability threshold; (4) stability constants for the Connors type correlation; (5) fluid-force components affecting the instability; (6) fluid damping; and (7) practical application of various empirical correlations. Several reviews on this subject are available (Chen 1987, Paidoussis 1987, and Weaver and Fitzpatrick 1987).

Empirical correlations of the threshold flow velocity for fluidelastic instability have been developed for tubes subjected to uniform crossflow. However, in practical applications, the tube arrays are usually subjected to nonuniform crossflow along the axial direction. Consequently, the design analyst is confronted with the problem of defining an equivalent uniform

crossflow velocity for use in computing the reduced flow velocity. The general practice has been to compute an equivalent uniform velocity as

$$U_e^2 = \frac{\int_0^{\ell} U^2(z) \phi_m^2(z) dz}{\int_0^{\ell} \phi_m^2(z) dz}, \quad (1)$$

where $U(z)$ is the axial distribution of crossflow velocity, $\phi_m(z)$ is the mode of interest, ℓ is the tube length, and z is the axial distance parameter. This relationship was proposed by several investigators (Pettigrew et al. 1977, Franklin and Soper 1977, and Connors 1978) and has been extensively used in practical applications.

Only a few studies have been conducted to verify the applicability of Eq. (1) for fluidelastic instability (see Table 1). Connors (1978) tested a square array in water, a U-bend model in air with half of the span subjected to flow, and two-span tube arrays (a tube row, a square array, and a triangular array) subjected to reversed air flow (two-span arrays were subjected to flow in a different direction). His results show that Eq. (1) is applicable in all cases, with a maximum deviation of approximately 15% attributed to variations in test setups. On the contrary, Franklin and Soper (1977) tested triangular arrays in a wind tunnel and showed that the theory may be conservative or nonconservative, depending on the location of the flow and the percentage of the span in the flow. Similarly, Waring and Weaver (1987) performed tests in air and in water; they found that theory does not agree well with experimental data. In both papers (Franklin and Soper 1977 and Waring and Weaver 1987), they pointed out the necessity of systematic experiments to evaluate the effects of nonuniform flow velocity distribution.

Chen (1983) and Chen and Jendrzejczyk (1983) presented a general theory for fluidelastic instability of tube bundles in crossflow, in which added mass, fluid damping, and fluid stiffness are represented in terms of fluid-force coefficients. Stability is then studied in terms of these coefficients. Among other things, Chen and coworkers demonstrated two basic instability mechanisms: (1) a velocity mechanism and (2) a displacement mechanism. With the velocity mechanism, the dominant fluid force is proportional to the tube velocity. This mechanism is frequently dominant for small values of reduced flow velocity corresponding to liquid flows, whereas the displacement mechanism is often dominant for large values of reduced flow velocity corresponding to gas flows. Chen and Jendrzejczyk (1983) also investigated the validity of Eq. (1) as a method for computation of an equivalent uniform flow velocity. Their results can be summarized as follows:

Table 1. Experimental Studies of Fluidelastic Instability in Nonuniform Flow

Investigators	Fluid Medium	Type of Tube Arrays and Number of Spans	Flow Condition	Comparison with Theory
Franklin & Soper 1977	Air Flow	Triangular arrays, single span	50% and 70% in flow	The theory is 21% higher when 70% of the span measured from the cantilevered, fixed end is in flow; and it is nonconservative when 50% of the blockage is applied at the root.
Connors 1978	1. Air flow 2. Water flow 3. Air flow	1. U-bend model, single span 2. Square array, single span 3. Tube row, two spans Square array, two spans Triangular array, two spans	1. 50% in flow 2. 50% in flow 3. Reversed flow	Agreement is good.
Waring & Weaver 1987	Air flow	1. Rotated triangular arrays, single span 2. Rotated triangular arrays, two and three spans	1. 33% to 100% in flow 2. One span in flow	1. The theory is conservative when the flow is centered on the span; it overpredicts the effects of flow location. 2. The theory does not agree well with the data for multi-span tubes.

- At high reduced flow velocities, fluid-force coefficients are independent of the reduced flow velocity, and an equivalent flow velocity defined by Eq. (1) can be obtained theoretically.
- At low reduced flow velocities, fluid-force coefficients are functions of reduced flow velocity, and the equivalent flow velocity defined by Eq. (1) is not valid theoretically.

The results of Chen and Jendrzejczyk (1983) demonstrated that Eq. (1) is not valid for small values of reduced flow velocity. Nevertheless, Eq. (1) may be a reasonable approximation in practical application. Because information about the fluid-force coefficient is very limited, design analysts urgently need an alternate approach that will allow them to evaluate the fluidelastic instability potential of a given heat exchanger tube bundle. Application of Eq. (1) together with stability diagrams developed from ideal laboratory tests provides such an approach.

The main objective of the present study is to evaluate the applicability of the mode-weighted averaging method in computing an equivalent flow velocity for nonuniform flow conditions in practical applications. Specifically, this work includes the following:

- Performing an analytical study to assess the validity of the mode-weighted averaging method.
- Conducting an experimental study to obtain the data required to compute an equivalent flow velocity for nonuniform flow conditions.
- Developing a design guide for applications in practical cases.

This report summarizes the analytical results.

II. AN UNSTEADY FLOW THEORY

A. Motion-dependent Fluid Forces for Tube Arrays in Crossflow

Consider an array of N circular cylinders oscillating in a flow, as shown in Fig. 1. The axes of the cylinders are parallel to the z -axis. The subscript j is used to denote variables associated with cylinder j . The displacement components of cylinder j are u_j and v_j , and the fluid force components are g_j and h_j . The motion-dependent fluid forces are (Chen 1983)

Fig. 1. An Array of Tubes in Nonuniform Crossflow.

$$g_j = - \sum_{k=1}^N \left\{ \left[\bar{\alpha}_{jk} \frac{\partial^2 u_k}{\partial t^2} + \bar{\alpha}'_{jk} \frac{\partial u_k}{\partial t} + \bar{\alpha}''_{jk} u_k \right] + \left[\bar{\sigma}_{jk} \frac{\partial^2 v_k}{\partial t^2} + \bar{\sigma}'_{jk} \frac{\partial v_k}{\partial t} + \bar{\sigma}''_{jk} v_k \right] \right\} \quad (2)$$

and

$$h_j = - \sum_{k=1}^N \left\{ \left[\bar{\tau}_{jk} \frac{\partial^2 u_k}{\partial t^2} + \bar{\tau}'_{jk} \frac{\partial u_k}{\partial t} + \bar{\tau}''_{jk} u_k \right] + \left[\bar{\beta}_{jk} \frac{\partial^2 v_k}{\partial t^2} + \bar{\beta}'_{jk} \frac{\partial v_k}{\partial t} + \bar{\beta}''_{jk} v_k \right] \right\} . \quad (3)$$

Note that $\bar{\alpha}_{jk}$, $\bar{\sigma}_{jk}$, $\bar{\tau}_{jk}$, and $\bar{\beta}_{jk}$ are elements of the added-mass matrix, $\bar{\alpha}'_{jk}$, $\bar{\sigma}'_{jk}$, $\bar{\tau}'_{jk}$, and $\bar{\beta}'_{jk}$ are elements of the fluid-damping matrix, and $\bar{\alpha}''_{jk}$, $\bar{\sigma}''_{jk}$, $\bar{\tau}''_{jk}$, and $\bar{\beta}''_{jk}$ are elements of the fluid stiffness matrix.

Experimental data have shown that the fluid-damping and fluid-stiffness matrices are a function of the reduced flow velocity. Equations (2) and (3) can be written in terms of dimensionless force coefficients:

$$\begin{aligned} \bar{\alpha}_{jk} &= \rho \pi R^2 \alpha_{jk} , & \bar{\alpha}'_{jk} &= - \frac{\rho U^2}{\omega} \alpha'_{jk} , & \bar{\alpha}''_{jk} &= -\rho U^2 \alpha''_{jk} \\ \bar{\sigma}_{jk} &= \rho \pi R^2 \sigma_{jk} , & \bar{\sigma}'_{jk} &= - \frac{\rho U^2}{\omega} \sigma'_{jk} , & \bar{\sigma}''_{jk} &= -\rho U^2 \sigma''_{jk} \\ \bar{\tau}_{jk} &= \rho \pi R^2 \tau_{jk} , & \bar{\tau}'_{jk} &= - \frac{\rho U^2}{\omega} \tau'_{jk} , & \bar{\tau}''_{jk} &= -\rho U^2 \tau''_{jk} \\ \bar{\beta}_{jk} &= \rho \pi R^2 \beta_{jk} , & \bar{\beta}'_{jk} &= - \frac{\rho U^2}{\omega} \beta'_{jk} , & \bar{\beta}''_{jk} &= -\rho U^2 \beta''_{jk} , \end{aligned} \quad (4)$$

where U is flow velocity, ρ is fluid density, R is the cylinder radius, and ω is the radian frequency. Using Eqs. (2), (3), and (4) yields

$$\begin{aligned}
g_j = & -\rho\pi R^2 \sum_{k=1}^N \left(\alpha_{jk} \frac{\partial^2 u_k}{\partial t^2} + \sigma_{jk} \frac{\partial^2 v_k}{\partial t^2} \right) \\
& + \frac{\rho U^2}{\omega} \sum_{k=1}^N \left(\alpha'_{jk} \frac{\partial u_k}{\partial t} + \sigma'_{jk} \frac{\partial v_k}{\partial t} \right) \\
& + \rho U^2 \sum_{k=1}^N \left(\alpha''_{jk} u_k + \sigma''_{jk} v_k \right)
\end{aligned} \tag{5}$$

and

$$\begin{aligned}
h_j = & -\rho\pi R^2 \sum_{k=1}^N \left(\tau_{jk} \frac{\partial^2 u_k}{\partial t^2} + \beta_{jk} \frac{\partial^2 v_k}{\partial t^2} \right) \\
& + \frac{\rho U^2}{\omega} \sum_{k=1}^N \left(\tau'_{jk} \frac{\partial u_k}{\partial t} + \beta'_{jk} \frac{\partial v_k}{\partial t} \right) \\
& + \rho U^2 \sum_{k=1}^N \left(\tau''_{jk} u_k + \beta''_{jk} v_k \right) .
\end{aligned} \tag{6}$$

α_{jk} , σ_{jk} , τ_{jk} , and β_{jk} are called added-mass coefficients; α'_{jk} , σ'_{jk} , τ'_{jk} , and β'_{jk} are called fluid-damping coefficients; and α''_{jk} , σ''_{jk} , τ''_{jk} , and β''_{jk} are called fluid-stiffness coefficients.

B. Analysis

The in-vacuum variables associated with the tube motion in the x-direction are displacement u_j , mass per unit length m_j , modal damping ζ_j , and natural frequency ω_j . The corresponding variables in the y-direction are v_j , m_j , η_j , and Ω_j . The modal function of the tube vibrating in vacuum and in fluid is $\phi(z)$,

$$\frac{1}{l} \int_0^l \phi^2(z) dz = 1 , \tag{7}$$

where l is the length of the tubes. Let

$$u_j(z, t) = a_j(t)\phi(z) \quad \text{and} \quad v_j(z, t) = b_j(t)\phi(z) , \quad (8)$$

where $a_j(t)$ and $b_j(t)$ are functions of time only. Assume that the flow velocity distribution is given by

$$U(z) = U_m \psi(z) . \quad (9)$$

The equations of motion for tube arrays are (Chen and Jendrzejczyk 1983)

$$\begin{aligned} \frac{d^2 a_j}{dt^2} + 2\zeta_j \omega_j \frac{da_j}{dt} + \omega_j^2 a_j + \frac{\rho \pi R^2}{m_j} \sum_{k=1}^N (\alpha_{jk} \frac{d^2 a_k}{dt^2} + \sigma_{jk} \frac{d^2 b_k}{dt^2}) \\ - \frac{\rho U_m^2}{m_j \omega_j} \sum_{k=1}^N (\alpha_{jk}^d \frac{da_k}{dt} + \sigma_{jk}^d \frac{db_k}{dt}) - \frac{\rho U_m^2}{m_j} \sum_{k=1}^N (\alpha_{jk}^e a_k + \sigma_{jk}^e b_k) = 0 , \end{aligned} \quad (10)$$

$$\begin{aligned} \frac{d^2 b_j}{dt^2} + 2\eta_j \Omega_j \frac{db_j}{dt} + \Omega_j^2 b_j + \frac{\rho \pi R^2}{m_j} \sum_{k=1}^N (\tau_{jk} \frac{d^2 a_k}{dt^2} + \beta_{jk} \frac{d^2 b_k}{dt^2}) \\ - \frac{\rho U_m^2}{m_j \omega_j} \sum_{k=1}^N (\tau_{jk}^d \frac{da_k}{dt} + \beta_{jk}^d \frac{db_k}{dt}) - \frac{\rho U_m^2}{m_j} \sum_{k=1}^N (\tau_{jk}^e a_k + \beta_{jk}^e b_k) = 0 , \end{aligned}$$

where

$$\begin{aligned} \alpha_{jk}^d &= \frac{1}{\ell} \int_0^\ell \alpha'_{jk} \phi^2 \psi^2 dz , & \alpha_{jk}^e &= \frac{1}{\ell} \int_0^\ell \alpha''_{jk} \phi^2 \psi^2 dz , \\ \sigma_{jk}^d &= \frac{1}{\ell} \int_0^\ell \sigma'_{jk} \phi^2 \psi^2 dz , & \sigma_{jk}^e &= \frac{1}{\ell} \int_0^\ell \sigma''_{jk} \phi^2 \psi^2 dz , \\ \tau_{jk}^d &= \frac{1}{\ell} \int_0^\ell \tau'_{jk} \phi^2 \psi^2 dz , & \tau_{jk}^e &= \frac{1}{\ell} \int_0^\ell \tau''_{jk} \phi^2 \psi^2 dz , \\ \beta_{jk}^d &= \frac{1}{\ell} \int_0^\ell \beta'_{jk} \phi^2 \psi^2 dz , & \beta_{jk}^e &= \frac{1}{\ell} \int_0^\ell \beta''_{jk} \phi^2 \psi^2 dz . \end{aligned} \quad (11)$$

Let ω_v be the reference circular frequency, which may be the natural frequency of a particular cylinder in vacuum. For the dimensionless

parameters

$$\tau = \omega_v t, \quad U_v = \frac{U}{f_v D} \quad (f_v = \frac{\omega_v}{2\pi}), \quad \text{and} \quad \gamma_j = \frac{\rho \pi R^2}{m_j}, \quad (12)$$

Equations (10) become

$$\begin{aligned} \ddot{a}_j + \gamma_j \sum_{k=1}^N (\alpha_{jk} \ddot{a}_k + \sigma_{jk} \ddot{b}_k) \\ + 2\zeta_j \left(\frac{\omega_j}{\omega_v}\right) \dot{a}_j - \frac{\gamma_j}{\pi} U_v^2 \left(\frac{\omega_v}{\omega}\right) \sum_{k=1}^N (\alpha_{jk}^d \dot{a}_k + \sigma_{jk}^d \dot{b}_k) \\ + \left(\frac{\omega_j}{\omega_v}\right)^2 a_j - \frac{\gamma_j}{\pi^3} U_v^2 \sum_{k=1}^N (\alpha_{jk}^e a_k + \sigma_{jk}^e b_k) = 0, \\ \ddot{b}_j + \gamma_j \sum_{k=1}^N (\tau_{jk} \ddot{a}_k + \beta_{jk} \ddot{b}_k) \\ + 2\eta_j \left(\frac{\Omega_j}{\omega_v}\right) \dot{b}_j - \frac{\gamma_j}{\pi^3} U_v^2 \left(\frac{\omega_v}{\omega}\right) \sum_{k=1}^N (\tau_{jk}^d \dot{a}_k + \beta_{jk}^d \dot{b}_k) \\ + \left(\frac{\Omega_j}{\omega_v}\right)^2 b_j - \frac{\gamma_j}{\pi^3} U_v^2 \sum_{k=1}^N (\tau_{jk}^e a_k + \beta_{jk}^e b_k) = 0, \end{aligned} \quad (13)$$

where the dot denotes differentiation with respect to τ . Fluid-damping and fluid-stiffness coefficients are functions of the reduced flow velocity U_r ($U_r = U/fD$). The frequency of oscillation, $f = \omega/2\pi$, is different from the reference frequency f_v .

The stability of a cylinder array is determined from Eqs. (13). The nondimensional parameters in Eqs. (13) are γ_j , ω_j/ω_v , Ω_j/ω_v , ζ_j , η_j , U_v , α_{jk} , σ_{jk} , τ_{jk} , β_{jk} , α_{jk}^d , σ_{jk}^d , τ_{jk}^d , β_{jk}^d , α_{jk}^e , σ_{jk}^e , τ_{jk}^e , and β_{jk}^e . Therefore, the critical flow velocity can be written in a functional form as

$$U_v = F(\gamma_j, \omega_j/\omega_v, \Omega_j/\omega_v, \zeta_j, \eta_j, \alpha_{jk}, \sigma_{jk}, \tau_{jk}, \beta_{jk}, \alpha_{jk}^d, \sigma_{jk}^d, \tau_{jk}^d, \beta_{jk}^d, \alpha_{jk}^e, \sigma_{jk}^e, \tau_{jk}^e, \beta_{jk}^e). \quad (14)$$

These parameters can be divided into the following groups:

- **Mass Ratio (γ_j):** This represents the ratio of displaced mass of fluid to the cylinder mass. In most practical applications, all cylinders are identical; therefore

$$\gamma_j = \gamma_v, \quad j = 1, 2, 3, \dots, n. \quad (15)$$

- **Detuning in Frequency (ω_j/ω_v and Ω_j/ω_v):** Frequency variations of different cylinders in an array and of the same tube in the two directions can affect stability. For an in-tune cylinder array,

$$\omega_j = \Omega_j = \omega_v, \quad j = 1, 2, 3, \dots, n. \quad (16)$$

- **Detuning in Damping (ζ_j and η_j):** Detuning in damping can also affect stability. For a cylinder array without damping variation,

$$\zeta_j = \eta_j = \zeta_v, \quad j = 1, 2, 3, \dots, n. \quad (17)$$

- **Added-mass Coefficients (α_{jk} , σ_{jk} , τ_{jk} , and β_{jk}):** These coefficients depend only on tube arrangement. For a given tube array, they are by definition constants.
- **Effective Fluid-damping Coefficients (α_{jk}^d , σ_{jk}^d , τ_{jk}^d , and β_{jk}^d):** These coefficients depend on the flow velocity distribution function $\psi(z)$, the tube mode shape $\phi(z)$, and the fluid-damping coefficients α'_{jk} , σ'_{jk} , τ'_{jk} , and β'_{jk} . Fluid-damping coefficients are a function of the reduced flow velocity U_r for small U_r and are approximately constant for large U_r .
- **Effective Fluidelastic-stiffness Coefficients (α_{jk}^e , σ_{jk}^e , τ_{jk}^e , and β_{jk}^e):** These coefficients are functions of flow velocity distribution function $\psi(z)$, tube mode shape $\phi(z)$, and fluidelastic-stiffness coefficients α''_{jk} , σ''_{jk} , τ''_{jk} , and β''_{jk} . In general, fluidelastic-stiffness coefficients are functions of the reduced flow velocity U_r ; however, for large U_r , they are very weak functions of U_r and can be considered as constants.

For a cylinder array in which all cylinders are identical, having the same natural frequency and damping in both directions, using Eqs. (13) and (15)-(17) yields

$$\begin{aligned}
& \ddot{a}_j + \gamma_v \sum_{k=1}^N (\alpha_{jk} \ddot{a}_k + \sigma_{jk} \ddot{b}_k) \\
& + 2\zeta_v \dot{a}_j - \frac{\gamma_v}{\pi} U_v^2 \left(\frac{\omega_v}{\omega} \right) \sum_{k=1}^N (\alpha_{jk}^d \dot{a}_k + \sigma_{jk}^d \dot{b}_k) \\
& + a_j - \frac{\gamma_v}{\pi} U_v^2 \sum_{k=1}^N (\alpha_{jk}^e a_k + \sigma_{jk}^e b_k) = 0 ,
\end{aligned}
\tag{18}$$

$$\begin{aligned}
& \ddot{b}_k + \gamma_v \sum_{k=1}^N (\tau_{jk} \ddot{a}_k + \beta_{jk} \ddot{b}_k) \\
& + 2\zeta_v \dot{b}_j - \frac{\gamma_v}{\pi} U_v^2 \sum_{k=1}^N \left(\frac{\omega_v}{\omega} \right) (\tau_{jk}^d \dot{a}_k + \beta_{jk}^d \dot{b}_k) \\
& + b_j - \frac{\gamma_v}{\pi} U_v^2 \sum_{k=1}^N (\tau_{jk}^e a_k + \beta_{jk}^e b_k) = 0 ,
\end{aligned}$$

and using Eqs. (14)-(17) yields

$$\begin{aligned}
U_v = F(\gamma_v, \zeta_v, \alpha_{jk}, \sigma_{jk}, \tau_{jk}, \beta_{jk}, \alpha_{jk}^d, \sigma_{jk}^d, \tau_{jk}^d, \beta_{jk}^d, \\
\alpha_{jk}^e, \sigma_{jk}^e, \tau_{jk}^e, \beta_{jk}^e) .
\end{aligned}
\tag{19}$$

Equations (13) and (18) are the coupled equations of motion based on the unsteady flow theory.

III. FLUID-FORCE COEFFICIENTS

For a given tube array, the critical flow velocity is determined by the flow velocity distribution and fluid-force coefficients. In this section, various fluid-force coefficients will be considered.

Motion-dependent fluid forces acting on a tube array can be measured by oscillating a tube and measuring the fluid-force components acting on the oscillating tube and the surrounding tubes. In experiments, this can be done in two steps, by oscillating a tube in a tube array in quiescent fluid and then in crossflow.

A. Cylinder Oscillating in Quiescent Fluid

Let the fluid force components acting on cylinder j because cylinder k is moving with small displacement components $u_k(t)$ and $v_k(t)$ be $g_j(t)$ and $h_j(t)$, respectively. Furthermore, assume that cylinder k is oscillating in a steady-state harmonic motion, i.e.,

$$u_k(t) = \bar{u}_k \sin \omega t \quad \text{and} \quad v_k(t) = \bar{v}_k \sin \omega t . \quad (20)$$

The fluid forces acting on cylinder j are

$$g_j(t) = \bar{g}_j \sin(\omega t + \phi_j) \quad \text{and} \quad h_j(t) = \bar{h}_j \sin(\omega t + \psi_j) . \quad (21)$$

The added-mass and fluid-damping matrices are

$$\begin{aligned} \bar{\alpha}_{jk} &= \frac{\bar{g}_j \cos \phi_j}{\omega^2 \bar{u}_k} , & \bar{\sigma}_{jk} &= \frac{\bar{g}_j \cos \phi_j}{\omega^2 \bar{v}_k} , \\ \bar{\tau}_{jk} &= \frac{\bar{h}_j \cos \psi_j}{\omega^2 \bar{u}_k} , & \bar{\beta}_{jk} &= \frac{\bar{h}_j \cos \psi_j}{\omega^2 \bar{v}_k} , \end{aligned} \quad (22)$$

and

$$\begin{aligned} \bar{\alpha}'_{jk} &= -\frac{\bar{g}_j \sin \phi_j}{\omega \bar{u}_k} , & \bar{\sigma}'_{jk} &= -\frac{\bar{g}_j \cos \phi_j}{\omega \bar{v}_k} , \\ \bar{\tau}'_{jk} &= -\frac{\bar{h}_j \sin \psi_j}{\omega \bar{u}_k} , & \bar{\beta}'_{jk} &= -\frac{\bar{h}_j \sin \psi_j}{\omega \bar{v}_k} . \end{aligned} \quad (23)$$

Fluid added-mass and fluid-damping coefficients can be calculated. The values obtained by this method correspond to the application of the unsteady flow theory. Various investigators have used this technique to determine fluid coupling in quiescent fluid--Moretti and Lowery (1976) and Lubin et al. (1977) for ideal fluid and Chen et al. (1977) for viscous fluid.

B. Cylinder Oscillating in Crossflow

Let cylinder k be excited in the y -direction; its displacement in the y -direction is given by

$$v_k = \bar{v}_k \exp(i\omega t) , \quad (24)$$

where $i = \sqrt{-1}$. The fluid force acting on cylinder j in the x -direction is given by

$$g_j = \frac{1}{2} \rho U^2 (c_{jk} \cos \psi_{jk} + i c_{jk} \sin \psi_{jk}) \bar{v}_k , \quad (25)$$

where c_{jk} is the fluid-force amplitude and ψ_{jk} is the phase angle between the fluid force and the cylinder displacement. These values are measured experimentally.

Using Eqs. (5) and (24), we can also write the fluid-force component as

$$g_j = (\rho \pi R^2 \omega^2 \sigma_{jk} + \rho U^2 \sigma''_{jk} + i \rho U^2 \sigma'_{jk}) \bar{v}_k . \quad (26)$$

Comparing Eqs. (25) and (26) yields

$$\sigma''_{jk} = \frac{1}{2} c_{jk} \cos \psi_{jk} - \frac{\pi^3}{2 U_r^2} \sigma_{jk} , \quad \sigma'_{jk} = \frac{1}{2} c_{jk} \sin \psi_{jk} , \quad (27)$$

where U_r is the reduced flow velocity ($U_r = \pi U / \omega R$). Other coefficients can be obtained in a similar manner.

As shown in Eqs. (27), the fluid-stiffness and added-mass coefficients are lumped together. To separate these two coefficients, additional information is needed. In practice, it generally is assumed that the added-mass coefficient can be calculated on the basis of those obtained in quiescent fluid.

Measurements of fluid forces on oscillating cylinders in crossflow have been made by several investigators.

- Chen and Jendrzejczyk (1981) measured the response of cylinders in crossflow; from these data, the diagonal terms of the fluid-damping matrix can be calculated.
- Tanaka and his colleagues (1981 and 1982) made extensive measurements of fluid forces acting on cylinders in crossflow. All fluid force coefficients can be obtained on the basis of their data for several arrangements.
- Goyder and Teh (1984) measured the force acting on a vibrating cylinder. These data can be used to obtain all the diagonal terms of fluid force matrices.
- Jendrzejczyk and Chen (1987) studied the motion-dependent fluid forces for a tube row.

The data obtained by Tanaka are the most complete. These data correspond to the general unsteady flow theory for small cylinder motion.

C. Fluid-damping and Fluid-stiffness Coefficients for Tube Arrays

Figure 2 shows the tube arrays for which the unsteady fluid forces were measured by Tanaka and his colleagues (Tanaka and Takahara 1981, Tanaka et al. 1982). The values of α'_{jk} , β'_{jk} , σ'_{jk} , τ'_{jk} , α''_{jk} , β''_{jk} , σ''_{jk} , and τ''_{jk} are given in Tables 2-4 and Figures 3-8.

D. Fluid-force Coefficients for Other Structural Components

Since the fluid-force coefficients for tube arrays are very limited, such coefficients in the literature for other structural components have been evaluated to study their characteristics, in particular their dependence on the reduced flow velocity (Keulegan and Carpenter 1958, Nakamura and Mizota 1975, Washizu et al. 1978, Bearman and Graham 1979, Bearman and Luo 1988). Nakamura and Mizota (1975) measured the unsteady lifts acting on oscillating prisms as a function of reduced flow velocity for three height-to-depth (d/c) ratios, 1:1, 1:2, and 1:4. In their measurements, the reduced flow velocity is based on the magnitude of the prism depth c ($U_r = U/c$). The prism displacement in the lift direction and the unsteady lift force are given by

Fig. 2. Tube Row and Square Array.

Table 2. Fluid-damping and Fluid-stiffness Coefficients
for a Tube Row with $P/D = 1.33$

U_r	α'_{11}	α'_{12}	β'_{11}	β'_{12}	σ'_{12}	τ'_{12}
1.5	.000	0.000	-.523	.000	2.228	.804
2.0	.148	-.141	-.297	-.209	1.167	.611
3.0	.418	-.281	-.544	-.524	.667	.344
4.0	.918	-.498	-.679	-.546	.207	.373
5.0	1.256	-.536	-.538	-.442	0.000	.395
6.0	.993	-.246	-.468	-.354	-.079	.379
7.0	.655	-.063	-.471	-.290	-.106	.329
8.0	.337	.022	-.467	-.247	-.138	.275
9.0	.052	.059	-.446	-.216	-.129	.230
10.0	-.079	.076	-.423	-.188	-.100	.205
12.0	-.171	.092	-.351	-.156	-.067	.165
15.0	-.225	.103	-.241	-.126	-.063	.136
18.0	-.235	.100	-.177	-.099	-.061	.091
20.0	-.229	.095	-.146	-.082	-.056	.065
25.0	-.220	.090	-.113	-.041	-.048	.046
30.0	-.201	.087	-.102	-.028	-.046	.038
40.0	-.175	.078	-.094	-.016	-.038	.031
50.0	-.153	.074	-.091	-.014	-.038	.026
60.0	-.141	.069	-.087	-.009	-.034	.022
70.0	-.135	.064	-.085	-.007	-.031	.020
80.0	-.129	.060	-.085	-.007	-.031	.018
90.0	-.126	.057	-.083	-.004	-.032	.016
100.0	-.123	.054	-.083	-.004	-.033	.016
150.0	-.120	.044	-.083	-.004	-.035	.012

U_r	α''_{11}	α''_{12}	β''_{11}	β''_{12}	σ''_{12}	τ''_{12}
1.5	-.334	-1.118	-.656	.819	3.569	-5.355
2.0	-.126	-.513	-.306	.360	1.589	-2.855
3.0	.145	-.247	-1.114	.128	.052	-1.065
4.0	.007	-.048	-.807	-.007	-.175	-.476
5.0	-.655	.493	-.477	.021	-.136	-.293
6.0	-1.080	.607	-.257	.061	-.073	-.226
7.0	-1.163	.538	-.116	.075	-.020	-.192
8.0	-1.124	.451	-.049	.083	.021	-.160
9.0	-1.024	.387	-.033	.086	.092	-.134
10.0	-.905	.336	-.033	.089	.128	-.112
12.0	-.709	.277	-.033	.089	.128	-.073
15.0	-.543	.233	-.023	.088	.135	-.028
18.0	-.442	.200	-.015	.099	.164	.046
20.0	-.402	.186	-.011	.105	.177	.063
25.0	-.327	.164	.009	.117	.208	.066
30.0	-.287	.150	.024	.120	.225	.066
40.0	-.246	.131	.044	.123	.247	.062
50.0	-.224	.122	.057	.125	.271	.061
60.0	-.210	.113	.061	.126	.280	.058
70.0	-.200	.109	.064	.125	.295	.056
80.0	-.190	.105	.066	.126	.300	.054
90.0	-.184	.104	.068	.126	.309	.052
100.0	-.180	.102	.069	.126	.314	.051
150.0	-.174	.096	.071	.127	.334	.045

Table 3. Fluid-damping and Fluid-stiffness Coefficients for a Square Array with $P/D = 1.33$

U_r	α'_{11}	α'_{12}	α'_{13}	α'_{14}	β'_{11}	β'_{12}	β'_{13}	β'_{14}	σ'_{15}	τ'_{15}
1.5	-1.021	.471	-.564	-.549	-.994	-.557	.414	.092	2.561	.933
2.0	-.604	.521	-.544	-.469	-1.124	-.439	.272	.103	1.682	1.205
2.5	-.853	.524	-.549	-.518	-1.406	-.401	.209	.131	1.303	1.269
3.0	-.350	.539	-.556	-.524	-1.465	-.381	.412	.090	.919	1.141
3.5	.000	.517	-.602	-.559	-1.245	-.390	.324	-.258	.602	.940
4.0	.061	.485	-.667	-.552	-1.320	-.082	.323	-.496	.356	.834
5.0	.101	.359	-.679	-.461	-1.166	.305	.355	-.594	.042	.685
6.0	.149	.249	-.638	-.408	-1.031	.340	.357	-.495	-.082	.593
7.0	.162	.179	-.606	-.352	-.901	.220	.335	-.423	-.135	.531
8.0	.159	.103	-.598	-.272	-.818	.144	.283	-.376	-.138	.481
10.0	.169	.026	-.587	-.082	-.671	.108	.231	-.324	-.124	.429
12.0	.094	-.010	-.493	.010	-.552	.092	.222	-.310	-.117	.402
15.0	.103	-.037	-.409	.043	-.457	.072	.218	-.310	-.109	.374
20.0	.125	-.061	-.333	.060	-.385	.062	.195	-.307	-.100	.334
25.0	.153	-.070	-.293	.047	-.329	.056	.175	-.293	-.101	.301
30.0	.179	-.080	-.265	.039	-.292	.051	.151	-.280	-.096	.271
35.0	.197	-.087	-.246	.061	-.254	.048	.136	-.249	-.089	.245
40.0	.211	-.094	-.246	.093	-.221	.045	.113	-.201	-.078	.227
50.0	.215	-.100	-.246	.129	-.167	.040	.088	-.156	-.059	.201
60.0	.206	-.093	-.247	.159	-.138	.035	.066	-.156	-.038	.187
80.0	.181	-.073	-.240	.195	-.091	.029	.032	-.178	.009	.173
100.0	.154	-.063	-.236	.200	-.063	.023	.022	-.190	.060	.168

U_r	α''_{11}	α''_{12}	α''_{13}	α''_{14}	β''_{11}	β''_{12}	β''_{13}	β''_{14}	σ''_{15}	τ''_{15}
1.5	10.165	2.796	2.594	-3.037	9.672	-1.632	-2.527	2.500	-.493	-.126
2.0	5.246	1.501	.467	-1.734	5.459	-.948	-1.346	1.402	-.506	.518
2.5	3.597	1.026	.179	-1.146	2.741	-.659	-.692	.505	-.651	.482
3.0	2.560	.564	.196	-.752	1.126	-.315	-.135	-.044	-.782	.385
3.5	1.590	.322	.159	-.428	.284	.203	.058	-.280	-.837	.352
4.0	1.190	.175	.094	-.124	.059	.470	.058	-.326	-.805	.345
5.0	.609	.038	-.081	.082	.107	.210	.025	-.164	-.602	.310
6.0	.357	.006	-.218	.172	.168	.086	.062	-.030	-.425	.346
7.0	.176	.026	-.279	.257	.204	.040	.101	.028	-.292	.360
8.0	.067	.057	-.280	.313	.204	.024	.112	.040	-.185	.377
10.0	-.112	.074	-.276	.353	.178	0.000	.094	.056	-.085	.361
12.0	-.249	.074	-.223	.309	.148	-.015	.074	.070	-.043	.338
15.0	-.375	.067	-.177	.268	.110	-.022	.049	.050	-.012	.325
20.0	-.438	.058	-.105	.234	.088	-.035	.006	.028	.032	.301
25.0	-.478	.069	-.041	.228	.086	-.042	-.023	.052	.049	.271
30.0	-.490	.083	.007	.227	.119	-.049	-.047	.101	.072	.244
35.0	-.504	.099	.038	.215	.142	-.053	-.059	.163	.092	.229
40.0	-.510	.113	.062	.202	.159	-.057	-.070	.219	.107	.211
50.0	-.515	.135	.097	.186	.177	-.062	-.080	.248	.132	.195
60.0	-.516	.153	.130	.160	.184	-.065	-.080	.248	.153	.181
80.0	-.528	.164	.161	.113	.175	-.068	-.078	.235	.180	.167
100.0	-.540	.173	.184	.065	.172	-.071	-.066	.225	.196	.157

Table 4. Fluid-damping and Fluid-stiffness Coefficients for a Square Array with $P/D = 2.0$

U_r	α'_{11}	α'_{12}	α'_{13}	α'_{14}	β'_{11}	β'_{12}	β'_{13}	β'_{14}	σ'_{15}	τ'_{15}
6.0	.820	-.116	-.100	.056	-.514	-.525	.095	-.373	.108	.138
8.0	.797	-.247	-.010	.127	-.371	-.249	.097	-.249	-.050	.155
10.0	.486	-.189	-.007	.077	-.287	-.124	.089	-.177	-.044	.127
15.0	.314	-.267	-.009	.043	-.186	.062	.064	-.102	-.010	.085
20.0	.288	-.271	-.009	.029	-.140	.062	.046	-.073	.002	.063
30.0	.249	-.249	-.009	.019	-.095	.029	.018	-.049	.015	.049
40.0	.219	-.224	-.008	.011	-.074	.018	-.003	-.036	.019	.042
50.0	.189	-.192	-.007	.006	-.057	.009	-.010	-.029	.019	.039
70.0	.143	-.140	-.005	.001	-.046	.001	-.002	-.020	.015	.035
100.0	.119	-.098	-.002	-.001	-.039	0.000	.008	-.016	.001	.033

U_r	α''_{11}	α''_{12}	α''_{13}	α''_{14}	β''_{11}	β''_{12}	β''_{13}	β''_{14}	σ''_{15}	τ''_{15}
6.0	.491	-.790	.059	.342	.255	.071	-.022	-.203	-.333	.220
8.0	-.324	-.213	.046	.076	.117	.056	-.012	-.051	-.136	.056
10.0	-.421	-.045	.026	0.000	.078	.085	-.015	-.015	-.070	-.002
15.0	-.474	.104	.039	.026	.058	.065	-.026	-.002	-.031	-.043
20.0	-.561	.192	.046	.029	.069	-.011	-.031	-.005	-.027	-.053
30.0	-.635	.284	.050	.033	.084	-.040	-.036	-.014	-.022	-.057
40.0	-.685	.330	.049	.031	.084	-.057	-.032	-.019	-.021	-.056
50.0	-.712	.361	.047	.027	.078	-.064	-.026	-.020	-.023	-.056
70.0	-.740	.385	.044	.019	.080	-.070	-.022	-.020	-.031	-.056
100.0	-.752	.393	.039	.011	.085	-.070	-.015	-.017	-.040	-.055

Fig. 3. Fluid-damping Coefficients for a Row of Tubes with $P/D = 1.33$.

Fig. 4. Fluid-stiffness Coefficients for a Row of Tubes with $P/D = 1.33$.

Fig. 5. Fluid-damping Coefficients for a Square Array with $P/D = 1.33$.

Fig. 5. (Cont'd) Fluid-damping Coefficients for a Square Array with $P/D = 1.33$.

Fig. 6. Fluid-stiffness Coefficients for a Square Array with $P/D = 1.33$.

Fig. 6. (Cont'd) Fluid-stiffness Coefficients for a Square Array with $P/D = 1.33$.

Fig. 7. Fluid-damping Coefficients for a Square Array with $P/D = 2.0$.

Fig. 7. (Cont'd) Fluid-damping Coefficients for a Square Array with $P/D = 2.0$.

Fig. 8. Fluid-stiffness Coefficients for a Square Array with $P/D = 2.0$.

Fig. 8. (Cont'd) Fluid-stiffness Coefficients for a Square Array with $P/D = 2.0$.

$$u_j(t) = \bar{u}_j \sin(\omega t) , \quad (28)$$

$$g_j(t) = \frac{1}{2} \rho U^2 c |C'_{Lj}| \sin(\omega t + \psi_{Lj}) .$$

In the above equations, the inertia force has been removed. Using Eqs. (5) (damping and stiffness terms only, because the inertia was excluded) and (28) yields

$$\alpha'_{jj} = \frac{1}{2\eta} \left(\frac{c}{d}\right) |C'_{Lj}| \sin \psi_{Lj} , \quad (29)$$

$$\alpha''_{jj} = \frac{1}{2\eta} \left(\frac{c}{d}\right) |C'_{Lj}| \cos \psi_{Lj} ,$$

where

$$\eta = \bar{u}/d .$$

Figure 9 shows the values C'_{Lj} and ψ_{Lj} as functions of the reduced flow velocity. As the reduced flow velocity increases, both C'_{Lj} and ψ_{Lj} practically become a constant that is independent of the reduced flow velocity. Therefore, α'_{jj} and α''_{jj} are independent of U_r when U_r is large.

Similar tests were conducted by Washizu et al. (1978) for two height-to-depth ratios, 1:2 and 1:4. On the basis of their results and Eq. (5), the fluid-force coefficients are

$$\alpha'_{jj} = \frac{1}{2\eta} \left(\frac{c}{d}\right) C'_{LR} \quad \text{and} \quad \alpha''_{jj} = \frac{1}{2\eta} \left(\frac{c}{d}\right) C'_{LI} . \quad (30)$$

The values of C'_{LR} and C'_{LI} are given in Fig. 10 for different oscillation amplitudes $\bar{u}_j$. Figure 10 shows that at higher reduced flow velocity, the fluid-force coefficients are approximately independent of the reduced flow velocity.

Keulegan and Carpenter (1958) studied the fluid forces acting on a circular cylinder in oscillating flow. The flow velocity is $U_m \sin(\omega t)$. The drag force coefficient C_D is given as a function of the period parameter $U_m T/D$. Note that the period parameter in oscillating flow actually

Fig. 9. Magnitude and Phase Angle of the Unsteady Lift Forces on Prisms (from Nakamura and Mizota 1975).

Fig. 10. C'_{LR} and C'_{LI} as a Function of U_r (from Washizu et al. 1978).

corresponds to the reduced flow velocity in a steady flow. Figure 11 shows that when the period parameter is large, the drag coefficient is almost a constant.

Bearman and Graham (1979) measured the drag forces acting on cylindrical bodies in oscillations similar to those studied by Keulegan and Carpenter (1958). The cylindrical bodies included a two-dimensional, sharp-edged, flat plate; a circular cylinder; a square-section cylinder; and a diamond-section cylinder. The drag force coefficients are given in Fig. 12 as a function of the period parameter. At the higher values of the period parameter, the drag force coefficient approaches the value for steady-state flow $\bar{C}_D$ and is almost independent of the period parameter. The ratio $C_D/\bar{C}_D$ is also given in Fig. 12. As U_m/TD increases, $C_D/\bar{C}_D$ approaches one.

Bearman and Luo (1988) measured the fluctuating lift and drag for a square-section cylinder forced to oscillate transverse to an air stream. Typical lift and drag force coefficients as a function of the reduced flow velocity are given in Fig. 13. As the reduced flow velocity increases, both lift and drag force coefficients reach steady values, which correspond approximately to those for a stationary square section, and the flow appears to have reached a quasi-steady state.

The fluid force coefficients obtained by various investigators for different structural components indicate that at high reduced flow velocity, the fluid force coefficients are independent of the reduced flow velocity. This is true for tube arrays as well as for other structural shapes.

At high reduced flow velocity, the effect of reduced flow velocity on fluid-force coefficients becomes less important. This can be examined by considering the velocity of the structures. Let the displacement of the structure be

$$u = \bar{u} \sin(\omega t) . \quad (31)$$

The ratio of the flow velocity to the structural velocity is

$$r = \frac{\bar{U}}{\bar{u}\omega} = \frac{U_r}{2\pi(\bar{u}/D)} . \quad (32)$$

If r is large, the structural velocity is expected to have no significant effect. For example, when the structural displacement is small, i.e., $\bar{u}/D = 0.1$, the value of r will be more than 30, if U_r is larger than 20. In this case, fluid-force coefficients will be independent of U_r . Equation (32)

Fig. 11. Variation of Drag Coefficient of Cylinders
(from Keulegan and Carpenter 1958).

Fig. 12. Variation of Drag Coefficient C_D and the Ratio of C_D and $\bar{C}_D$ (the Steady-state Value of C_D) as a Function of Keulegan-Carpenter Number (from Bearman and Graham 1979).

Fig. 13. Fluctuating Lift and Drag as a Function of U_r (from Bearman and Luo 1988).

shows that r depends on the amplitude $\bar{u}/D$. This has been verified by Bearman and Luo (1988); they showed that fluid force coefficients can be plotted as a function of $U_r/(\bar{u}/D)$ (Fig. 14). When this ratio is larger than about 80, all fluid force coefficients are independent of $U_r/(\bar{u}/D)$.

IV. QUALITATIVE RESULTS

Some general conclusions can be derived by considering several specific cases. This will be illustrated by several examples.

A. High Reduced Flow Velocity

For large values of reduced flow velocity, all fluid-force coefficients are independent of U_r . Therefore, Eqs. (11) can be written as follows:

$$\begin{aligned} \alpha_{jk}^d &= \alpha'_{jk} \cdot \mu, & \alpha_{jk}^e &= \alpha_{jk} \cdot \mu, \\ \sigma_{jk}^d &= \sigma'_{jk} \cdot \mu, & \sigma_{jk}^e &= \sigma_{jk} \cdot \mu, \\ \tau_{jk}^d &= \tau'_{jk} \cdot \mu, & \tau_{jk}^e &= \tau_{jk} \cdot \mu, \\ \beta_{jk}^d &= \beta'_{jk} \cdot \mu, & \beta_{jk}^e &= \beta_{jk} \cdot \mu, \end{aligned} \quad (33)$$

where

$$\mu = \frac{1}{\ell} \int_0^{\ell} \phi^2 \psi^2 dz. \quad (34)$$

At large values of U_r , fluid inertia can be neglected; then Eqs. (18) may be written

$$\begin{aligned} \ddot{a}_j + 2\zeta_v \dot{a}_j + a_j - \frac{\gamma_v}{3} \mu U_v^2 \left(\frac{\omega_v}{\omega} \right) \sum_{k=1}^N (\alpha'_{jk} \dot{a}_k + \sigma'_{jk} \dot{b}_k + \alpha_{jk} a_k + \sigma_{jk} b_k) &= 0 \\ \ddot{b}_j + 2\zeta_v \dot{b}_j + b_j - \frac{\gamma_v}{3} \mu U_v^2 \left(\frac{\omega_v}{\omega} \right) \sum_{k=1}^N (\tau'_{jk} \dot{a}_k + \beta'_{jk} \dot{b}_k + \tau_{jk} a_k + \beta_{jk} b_k) &= 0. \end{aligned} \quad (35)$$

Fig. 14. C_L' versus U_r ($\bar{u}/D$) for Various Amplitude Ratios (from Bearman and Luo 1988).

In Eqs. (35), the variables controlling the system stability are the parameters ζ_v and $\gamma_v \mu U_v^2 (\omega_v/\omega)$. These two terms contribute to system damping; the modal damping of a mode can be written as

$$\zeta_f = \zeta_v - C \gamma_v \mu U_v^2 \left(\frac{\omega_v}{\omega} \right), \quad (36)$$

where C depends on the fluid-force coefficients. The instability will occur if $\zeta_f = 0$; i.e.,

$$\mu U_v^2 = \frac{\zeta_v}{C \gamma_v} \left(\frac{\omega}{\omega_v} \right). \quad (37)$$

At large U_r , the oscillation frequency ω is approximately equal to ω_v on the basis of experimental data. Therefore

$$\frac{U_e}{f_v D} = \left(\frac{1}{C} \right)^{0.5} \left(\frac{2\pi \zeta_v m_v}{\rho D^2} \right)^{0.5}, \quad (38)$$

where

$$U_e = U_m \left(\frac{1}{l} \int_0^l \phi^2 \psi^2 dz \right)^{1/2}. \quad (39)$$

Equations (38) and (39) show that

- at high U_r , the critical reduced flow velocity is approximately proportional to the half-power of the mass damping parameter, and
- at high U_r , an equivalent value of flow velocity can be defined in accordance with the mode-weighted averaging method.

This means that the general practice in defining an equivalent flow velocity based on the mode-weighted averaging technique is acceptable as long as the fluid-force coefficients are approximately independent of U_r . As demonstrated in Section III, the fluid-force coefficients are approximately independent of U_r if U_r is large (approximately larger than 20). Therefore, for $U_r > 20$, the mode-weighted averaging technique is acceptable. For tube arrays in light fluid, such as heat exchanger tubes subjected to air flow, the generally used technique is acceptable.

B. Constrained Modes

Consider a particular cylinder j moving in the x -direction in a rigid array. The equation of motion is [see Eq. (10)]:

$$\begin{aligned} \left(1 + \frac{\rho\pi R^2}{m_j} \alpha_{jj}^d\right) \frac{d^2 a_j}{dt^2} + \left(2\zeta_j \omega_j - \frac{\rho U_m^2}{m_j \omega} \alpha_{jj}^d\right) \frac{da_j}{dt} \\ + \left(\omega_j^2 - \frac{\rho U_m^2}{m_j} \alpha_{jj}^e\right) a_j = 0 . \end{aligned} \quad (40)$$

The critical flow velocity can be determined by setting the damping term equal to zero; i.e.,

$$\frac{U_m}{fD} = \left(\frac{4\pi}{\frac{1}{\ell} \int_0^\ell \alpha_{jj}^d \phi^2 \psi^2 dz} \right)^{0.5} \left(\frac{\omega_j}{\omega} \right)^{0.5} \left(\frac{2\pi \zeta_j m_j}{\rho D^2} \right)^{0.5} , \quad (41)$$

where

$$\omega = \left[\left(\omega_j^2 - \frac{\rho U_m^2}{m_j} \alpha_{jj}^e \right) / \left(1 + \frac{\rho\pi R^2}{m_j} \alpha_{jj}^d \right) \right]^{0.5} , \quad (42)$$

$$\alpha_{jj}^e = \frac{1}{\ell} \int_0^\ell \alpha_{jj}'' \phi^2 \psi^2 dz .$$

Note that α_{jj}^d and α_{jj}'' are functions of U/fD and that U is a function of z . In solving Eq. (41), an iterative method is required to find the critical flow velocity. In addition, the oscillation frequency $f = \omega/2\pi$ depends on flow velocity U_r as well as flow velocity distribution $\psi(z)$ through the integration associated with the fluid stiffness coefficient α_{jj}'' . For uniform flow,

$$\frac{U_m}{fD} = \left[\left(\frac{4\pi}{\alpha_{jj}^d} \right) \left(\frac{\omega_j}{\omega} \right) \left(\frac{2\pi \zeta_j m_j}{\rho D^2} \right) \right]^{0.5} , \quad (43)$$

$$\omega = \left[\left(\omega_j^2 - \frac{\rho U_m^2}{m} \alpha_{jj}'' \right) / \left(1 + \frac{\rho\pi R^2}{m_j} \alpha_{jj}^d \right) \right]^{0.5} . \quad (44)$$

From Eqs. (41)-(44), an equivalent flow velocity can be defined as follows:

$$U_e = U_m \left[\left(\frac{\frac{1}{\ell} \int_0^\ell \alpha'_{jj} \phi^2 \psi^2 dz}{\alpha'_{jj}} \right) \left(\frac{\omega_j^2 - \frac{\rho U_m^2}{m_j} \alpha''_{jj}}{\omega_j^2 - \frac{\rho U_m^2}{m_j} \frac{1}{\ell} \int_0^\ell \alpha''_{jj} \phi^2 \psi^2 dz} \right) \right]^{0.5} . \quad (45)$$

Equation (45) is different from Eq. (39); therefore, the mode-weighted averaging method defined in Eq. (39) is not valid for this case. Equation (45) can be reduced to the mode-weighted averaging method if the following condition is satisfied:

$$\left(\omega_j^2 - \frac{\rho U_m^2}{m_j} \alpha''_{jj} \right) \alpha'_{jj} \int_0^\ell \phi^2 \psi^2 dz = \left(\omega_j^2 - \frac{\rho U_m^2}{m_j} \frac{1}{\ell} \int_0^\ell \alpha''_{jj} \phi^2 \psi^2 dz \right) \int_0^\ell \alpha'_{jj} \phi^2 \psi^2 dz . \quad (46)$$

In general, Eq. (46) is not satisfied; thus, the mode-weighted averaging method is not generally valid.

Note that in Eqs. (45) and (46), the integrals involving α'_{jj} and α''_{jj} should be evaluated at the critical reduced flow calculated from Eq. (41) for nonuniform flow, while the integrals not involving α'_{jj} and α''_{jj} should be calculated from Eq. (43) for uniform flow. It is not possible to calculate the equivalent flow velocity without knowing the critical flow velocity for both cases.

Equations (41) and (42) can be simplified by using the following nondimensional parameters.

$$U_r = \frac{U_m}{fD} ,$$

$$\gamma = \frac{\pi \rho R^2}{m_j} , \quad (47)$$

$$\delta = \frac{2\pi \zeta_j m_j}{\rho D^2} = \frac{\pi^2}{2} \frac{\zeta_j}{\gamma} .$$

Using Eqs. (41), (42), and (47) yields

$$U_r = \left[\frac{4\pi}{\frac{1}{\ell} \int_0^{\ell} \alpha_{jj}' \phi^2 \psi^2 dz} \left(\frac{\omega_j}{\omega} \right) \delta \right]^{0.5}, \quad (48)$$

$$\frac{\omega}{\omega_j} = \left[1 - \gamma \left(\frac{\omega}{\omega_j} \right)^2 \left(\frac{U_r^2}{\pi} \right) \left(\frac{1}{\ell} \right) \int_0^{\ell} \alpha_{jj}'' \phi^2 \psi^2 dz \right]^{0.5} / (1 + \gamma \alpha_{jj})^{0.5}. \quad (49)$$

Equation (49) can be solved for ω/ω_j ,

$$\frac{\omega}{\omega_j} = \left[1 + \gamma \alpha_{jj} + \gamma \left(\frac{U_r^2}{\pi} \right) \left(\frac{1}{\ell} \right) \int_0^{\ell} \alpha_{jj}'' \phi^2 \psi^2 dz \right]^{-0.5}. \quad (50)$$

Substituting Eq. (50) into Eq. (48) yields

$$U_r = \left\{ \frac{4\pi\delta}{\frac{1}{\ell} \int_0^{\ell} \alpha_{jj}' \phi^2 \psi^2 dz} \left[1 + \gamma \alpha_{jj} + \gamma \left(\frac{U_r^2}{\pi} \right) \left(\frac{1}{\ell} \right) \int_0^{\ell} \alpha_{jj}'' \phi^2 \psi^2 dz \right] \right\}^{0.5}. \quad (51)$$

For a uniform flow,

$$\frac{\omega}{\omega_j} = \left[1 + \gamma \alpha_{jj} + \gamma \left(\frac{U_r^2}{\pi} \right) \alpha_{jj}'' \right]^{-0.5} \quad (52)$$

$$U_r = \left\{ \frac{4\pi\delta}{\alpha_{jj}'} \left[1 + \gamma \alpha_{jj} + \gamma \left(\frac{U_r^2}{\pi} \right) \alpha_{jj}'' \right] \right\}^{0.5}.$$

C. Mode-weighted Averaging Method Based on the Published Mathematical Models

Various mathematical models for fluidelastic instability of tube arrays have been developed by different investigators. These models can be divided into three types: quasi-static, quasi-steady, and unsteady flow theories. The

differences and similarities of these theories have been discussed by Chen (1987).

1. Quasi-static Flow Models

The fluid dynamic characteristics of cylinders oscillating in a flow are equal, at any instant of time, to the characteristics of the same stationary cylinders whose configuration is identical to the actual instantaneous configuration. The fluid forces depend on u and v only. Therefore, only fluid-stiffness forces are considered, and they are determined on the basis of the deviation from a reference state of steady flow. Typical examples are those of Connors (1970) and Blevins (1974). Based on the quasi-static flow model, fluid-stiffness coefficients are independent of the reduced flow velocity; therefore, the mode-weighted averaging method is acceptable. In other words, when a quasi-static flow model is adequate in describing the instability phenomenon in tube arrays in crossflow, the mode-weighted averaging method is acceptable.

2. Quasi-steady Flow Models

The fluid dynamic characteristics of cylinders moving in flow are equal, at any instant in time, to the characteristics of the same cylinders moving with constant velocities equal to the actual instantaneous values. The fluid forces depend on the cylinder configuration and are proportional to the cylinder motion. In this case, the fluid-stiffness coefficients are independent of the reduced flow velocity, but the fluid-damping coefficients are functions of the reduced flow velocity.

In cases in which the instability is of the fluid-stiffness-controlled type, the dominant fluid force is the fluid-stiffness force. On the basis of the quasi-steady flow theory, the fluid-stiffness coefficients are independent of the reduced flow velocity; thus, the mode-weighted averaging method is applicable. On the other hand, if the instability is of the fluid-damping-controlled type, the fluid-damping force is dominant. Fluid-damping forces are functions of the reduced flow velocity, according to the quasi-steady flow theory; therefore, the mode-weighted averaging method is not necessarily valid.

3. Unsteady Flow Theory

In the unsteady flow theory, fluid-damping and fluid-stiffness coefficients are generally functions of the reduced flow velocity, and the mode-weighted averaging method is generally not valid. At high reduced flow velocity, this method is applicable because the fluid-force coefficients are independent of the reduced flow velocity.

V. NUMERICAL EXAMPLES

A series of examples is presented in this section to illustrate the effect of nonuniform flow.

A. High Reduced Flow Velocity

At high reduced flow velocity, fluid-force coefficients are approximately independent of U_r . As demonstrated in Section IV, the critical flow velocity and equivalent flow velocity are given by Eqs. (38) and (39),

$$\frac{U_e}{f_v D} = \left(\frac{1}{C}\right)^{0.5} \left(\frac{2\pi \zeta_v m_v}{\rho D^2}\right)^{0.5}, \quad (53)$$

$$U_e = U_m \left(\frac{1}{\ell} \int_0^{\ell} \phi^2 \psi^2 dz \right)^{0.5}. \quad (54)$$

The mode-weighted averaging technique is applicable, and Eq. (54) defines the equivalent uniform flow velocity for nonuniform flow.

Let us consider several examples shown in Fig. 15.

1. A Single-span Tube Array Partially Subjected to Flow (Fig. 15a)

For a single-span tube array simply supported at both ends and subjected to flow at the left end portion ℓ_1 , the two functions in Eq. (54) are

$$\begin{aligned} \phi(z) &= \sqrt{2} \sin \frac{\pi z}{\ell}, & 0 < z < \ell \\ \psi(z) &= 1, & 0 < z < \ell_1 \\ &= 0, & \ell_1 < z < \ell. \end{aligned} \quad (55)$$

Substitution of Eq. (55) into Eq. (54) yields

$$\frac{U_e}{U_m} = \left[\frac{\ell_1}{\ell} - \frac{1}{2\pi} \sin\left(\frac{2\pi \ell_1}{\ell}\right) \right]^{0.5}. \quad (56)$$

Fig. 15. Tubes Subjected to Nonuniform Crossflow.

The values of U_e/U_m as a function of ℓ_1/ℓ are given in Table 5.

Table 5. Ratio of U_e/U_m as a Function of ℓ_1/ℓ

$\frac{\ell_1}{\ell}$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$\frac{U_e}{U_m}$	0.08	0.22	0.39	0.55	0.71	0.83	0.92	0.98	1.0	1.0

2. A Continuous Tube Array with Equal Span Subjected to Flow at One Span Only (Fig. 15b)

Consider the lowest mode of a K-span tube array. $\phi(z)$ and $\psi(z)$ are given by

$$\begin{aligned}\phi(z) &= \sqrt{2} \sin \frac{K\pi z}{\ell}, & 0 < z < \ell, \\ \psi(z) &= 1, & 0 < z < \frac{\ell}{K}, \\ &= 0, & \frac{\ell}{K} < z < \ell.\end{aligned}\tag{57}$$

Using Eqs. (54) and (57) yields

$$\frac{U_e}{U_m} = \left(\frac{1}{K}\right)^{0.5}.\tag{58}$$

In this case, for the first mode the ratio of U_e/U_m depends only on the number of spans. Because of the particular mode shape, the flow can be at any span; i.e., the critical flow velocity does not depend on the location of the span where the flow crosses it. On the basis of Eq. (58), for a 100-span ($K = 100$) tube array with one span in flow, U_e is equal to $0.1U_m$ for a completely submerged tube array.

For a multiple-span tube array, the critical flow velocity is not necessarily associated with the lowest mode and is generally dependent on the flow velocity distribution and tube span arrangement. For the particular condition given in Fig. 15b, it is fairly easy to show that the lowest

critical flow velocity is associated with the lowest mode. For other cases, one is generally required to evaluate the critical flow velocity of all modes and then determine the most critical mode.

3. A Single-Span Tube Array Subjected to Linearly Varying Flow (Fig. 15c)

The two functions $\phi(z)$ and $\psi(z)$ are

$$\phi(z) = \sqrt{2} \sin \frac{\pi z}{\ell} , \quad 0 < z < \ell , \quad (59)$$

$$\psi(z) = z/\ell , \quad 0 < z < \ell .$$

Substituting Eq. (59) into Eq. (54) yields

$$U_e/U_m = 0.532 . \quad (60)$$

4. A Single-Span Tube Array Subjected to Nonuniform Flow (Fig. 15d)

In Fig. 15d, $\phi(z)$ and $\psi(z)$ are

$$\begin{aligned} \phi(z) &= \sqrt{2} \sin \frac{\pi z}{\ell} , \quad 0 < z < \ell \\ \psi(z) &= \frac{2z}{\ell} , \quad 0 < z < \frac{\ell}{2} , \\ &= 2\left(1 - \frac{z}{\ell}\right) , \quad \frac{\ell}{2} < z < \ell . \end{aligned} \quad (61)$$

Substituting Eqs. (55) into (48) yields

$$\frac{U_e}{U_m} = 0.732 . \quad (62)$$

The above examples illustrate the effect of nonuniform flow. In most cases, the equivalent uniform flow velocity can be calculated easily.

B. General Cases

The critical flow velocity for a row of three tubes normal to the flow has been calculated for several cases to evaluate the equivalent uniform flow velocity for the data given in Table 2. The tubes under consideration are CuNi tubes with 0.75 in. O.D., 0.652 in. I.D., 30 in. length, and Young's modulus of 18,000,000 psi. The fundamental natural frequency in vacuum is 63.62 Hz. The modal damping ratio in vacuum is assumed to be 0.5%, 2%, or 5%. The tubes are subjected to external flow only, and there is no fluid inside the tubes.

Figure 16 shows the critical reduced flow velocity as a function of mass-damping parameter for three damping values. The mass-damping parameter is based on the in-vacuum values; that is, all added masses and fluid damping are excluded from the mass and damping parameter. For each case, the damping value is fixed, and the tube mass is a constant; therefore, the variation of the mass-damping parameter is due to the change in fluid density. Two values of the reduced flow velocities are presented; the solid line is based on the natural frequency in vacuum of 63.62 Hz, and the dotted line is based on the oscillation frequency at instability, which is a function of system parameters.

We note that the two critical reduced flow velocities are the same at high values of mass-damping parameter and different at low values. This is due to the variation of the oscillation frequency at instability, which depends strongly on fluid density. Oscillation frequencies at instability are given in Fig. 17 for the three damping values. As the mass-damping value decreases, the oscillation frequency also decreases. At a fixed mass-damping value, the larger the damping, the smaller the oscillation frequency.

The critical flow velocities for the three damping values given in Fig. 16 are replotted in Fig. 18 (a) with the in-vacuum frequency and (b) with the oscillation frequency at instability. We note that, with the in-vacuum frequency, the critical reduced flow velocity at a fixed mass-damping parameter differs for different sets of damping and mass. The critical velocity decreases with increasing damping value. On the other hand, with the oscillation frequency at instability, the instability curves for the three sets of damping collapse in one curve. Although one might prefer to have a single instability curve, the oscillation frequency is not known a priori; without the knowledge of the oscillation frequency at instability, it is difficult to use this instability map.

From the above results, we recognize that at low values of mass-damping parameter, even for uniform flow, considerable difficulty exists in presenting the stability curves. This is one reason why scatterings of data exist at low

Fig. 16. Critical Reduced Flow Velocities as a Function of Mass-damping Parameter.

Fig. 17. Oscillation Frequencies at Instability.

(a)

(b)

Fig. 18. Critical Flow Velocities for Three Damping Values (a) with the In-vacuum Frequency and (b) with the Oscillation Frequency at Instability.

values of mass-damping parameter. In many cases, no effort was made to differentiate the two different frequencies.

Figure 19 shows the critical flow velocities for three flow velocity distributions: (1) uniform flow, (2) half of the span subjected to uniform flow, and (3) linearly varying flow. The critical flow velocities are calculated for a tube row consisting of three tubes, as discussed earlier. Figure 19a is based on the in-vacuum frequency, and Fig. 19b is based on the oscillation frequency at instability. The instability curves for the three cases are different because of the different flow velocity distributions.

Once the critical reduced flow velocities for the nonuniform flow and uniform flows are calculated, the equivalent flow velocity based on the mode-weighted averaging method given in Eq. (1) can be calculated. The results are presented in Fig. 20 (a) with the in-vacuum frequency and (b) with the oscillation frequency at instability. The mode-weighted averaging method provides a reasonable approximation for evaluating the equivalent flow velocity for nonuniform flow distributions considered in Fig. 19.

In constructing an instability map, different sets of parameters can be used, including in-vacuum values, in-fluid (quiescent fluid) values, or in-flow values. In-vacuum values are the easiest ones to determine, in-flow values are the most difficult to calculate, and many sets of in-fluid values are associated with different modes. From a practical point of view, in-vacuum parameters appear to be the most suitable for use. In particular, Fig. 20a demonstrates that the equivalent flow velocity calculated from the mode-weighted averaging method with in-vacuum parameters correlates very well with the velocity for the uniform flow case. We have shown that the mode-weighted averaging method is applicable for large values of mass-damping parameter. The numerical examples presented here show that the mode-weighted averaging method is also applicable at low values of mass-damping parameter if in-vacuum parameters are used.

VI. CONCLUDING REMARKS

In this report, the critical flow velocity for tube arrays subjected to nonuniform flow is studied on the basis of the unsteady flow theory. A key element of the unsteady flow theory is the fluid-force coefficients, including fluid added-mass, fluid-damping, and fluid-stiffness coefficients. These fluid-force coefficients can now be evaluated only from experimental data available in the literature. When the fluid-force coefficients for unsteady flow are known, the critical flow velocity for tube arrays subjected to crossflow can be calculated.

Fig. 19. Critical Reduced Flow Velocities for Three Flow Velocity Distributions (a) with the In-vacuum Frequency and (b) with the Oscillation Frequency at Instability.

Fig. 20. Equivalent Critical Reduced Flow Velocities for Nonuniform Flow (a) with the In-vacuum Frequency and (b) with the Oscillator Frequency at Instability.

The analytical models in the literature for fluidelastic instability of tube arrays can be divided into three groups, i.e., quasi-static flow theory, quasi-steady flow theory, and unsteady flow theory. The first two theories have the limitation that they are applicable only in a certain parameter range. We have discussed the role of nonuniform flow with respect to the three different models. In general, at large mass-damping parameters, the three models give similar results; however, at low mass-damping parameter values, only the unsteady flow model will provide accurate results.

A characteristic of the fluid-force coefficients is that for large values of reduced flow velocity, all fluid-force coefficients are practically constant. This is true for tube arrays as well as other structural shapes. On the basis of this characteristic, we concluded that the mode-weighted averaging method is applicable to evaluate the equivalent flow velocity for nonuniform flow. No matter which mathematical model is used in the evaluation, similar results are obtained.

At low values of mass-damping parameter, the situation is much more complicated. The system parameters can be defined under different circumstances. The stability curves differ for different sets of parameters. The equivalent flow velocity for nonuniform flow calculated from the mode-weighted averaging method also depends on the parameters used. The numerical examples presented in this report demonstrate that if in-vacuum parameters are used, the mode-weighted averaging method for nonuniform flow will provide a reasonable approximation.

We will perform tests to verify the analytical results presented in this report. Further evaluations of the model and the mode-weighted averaging method will be made when experimental data become available. For practical applications, we now recommend use of the mode-weighted averaging method to calculate the equivalent flow velocity with in-vacuum parameters.

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