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ONE-DIMENSIONAL LEAKAGE-FLOW VIBRATION INSTABILITIES

by

T. M. Mulcahy

Materials and Components Technology Division

September 1987

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ABSTRACT

Simple boundary conditions, pressure losses, and channel geometries necessary for the unstable, rigid-body translational vibrations of the wall of a one-dimensional leakage-flow channel are identified. General expressions for the flow damping and stiffness forces acting on the vibrating channel wall are derived and specific results are given for channels with wall friction, point pressure losses, sharp-edged constrictions, and diverging or converging widths. The minimum conditions necessary for dynamic and static (divergence) instability were found to be an upstream point pressure loss and a diverging channel width with a finite-length throat region, respectively.

I. INTRODUCTION

The main coolant flow paths through the components of energy conversion and utilization systems often parallel each other from one relatively constant-pressure plenum to another. However, the flow paths and plenums are rarely completely sealed from each other, because the designs must allow for removal or expansion of components. Thus, leakage flow through the pressure boundaries and along the components penetrating the boundary is not uncommon. When the elastic response of either the penetrating component or the pressure boundary opening can interact with and alter the leakage flow, the conditions for leakage-flow vibration instabilities are present.

Leakage-flow instabilities are really a subset of fluidelastic instabilities for which a constant difference in pressure is maintained between the entrance and exit of flow channels that are usually narrow and constricted, but always open. Thus, the class of leakage-flow vibration instabilities does not include components subject to a constant external velocity (e.g., airplane wings) or constant internal velocity (e.g., fire hoses), or even conditions where a seal or valve closes and completely blocks the flow. Many energy system component designs have suffered from leakage-flow vibration instabilities. Examples from reactor systems range in size from fuel or control rods forming annular leakage-flow paths with their surrounding shrouds, through coolant piping or shrouds penetrating pressure boundaries, and up to large thermal liner shells subject to the leakage flow that bypasses the core [1-3].

The avoidance of leakage-flow vibration instabilities is difficult because the existence and type of excitation mechanism is very sensitive to the local geometry of the leakage-flow path responsible for creating the pressure losses, especially entrance, exit, and internal-constriction losses. Structural dynamics is important since higher natural frequencies are usually associated with higher critical flowrates or pressure drops, and the mode shape is crucial in determining the effectiveness of the excitation mechanism; a pressure fluctuation distribution that peaks at the antinode of a vibration mode and is in phase with the vibration (velocity or displacement) is more likely to lead to a vibration instability. Since the ability to predict critical flow rates for instability without substantial testing is poor, full-scale model tests are usually done to qualify suspect geometries. Some rules of thumb for design exist [2], but a better understanding of the excitation mechanisms is needed for guiding component testing and the development of better design rules.

Recently [4], closed-form solutions characterizing the self-excitation of elastically mounted rigid rods in very narrow annular regions have made clear the importance of the channel entrance and exit configurations in determining the existence of instability phenomena. In particular, entrance constrictions and diffusers at the exit can lead to instabilities, whereas exit constrictions are stabilizing. However, even the simple annular geometry adopted in Ref. 4 is too complicated to permit a thorough investigation of the excitation sources associated with fluid viscosity, constrictions, or losses in the middle of a leakage flow channel, because closed-form solutions are difficult to obtain. Since circumferential squeeze flow due to center body motion in the annular region is significant only for relatively long annuli [5], the excitation mechanisms for leakage flow between two flat and parallel surfaces can be expected to be similar to those of an annular region for many practical configurations. Thus, closed-form solutions are determined here for the flow in a one-dimensional channel that includes wall friction and an intermediate constriction and losses. The objective of the parameter study is to identify more of the conditions that may lead to leakage-flow vibration instabilities.

II. MODEL PROBLEM

Even for the simple case of one-dimensional leakage flow, the channel geometry and wall vibration modes must be limited to obtain closed-form solutions that are useful for investigating sources of leakage-flow instabilities. The simplest of model problems was chosen. The flow channel definition includes entrance, exit, and intermediate constrictions and losses, as well as wall friction. Also, the channel walls are assumed to be rigid and elastically mounted such that their vibration occurs in a single mode, which causes a uniform, harmonic variation in the channel width. With a priori knowledge of the vibration mode, an unsteady-flow problem with prescribed harmonic motions of the channel walls can be solved for the fluid forces on the walls. These same forces can be used to investigate the self-excitation of the channel walls.

Dynamic instability (unbounded vibration amplitudes) can only occur if the net system damping goes to zero; coupled-mode flutter, for instance, is not possible because a system with only a single degree of freedom is being considered. Static divergence (unbounded average displacements) can occur if the total stiffness or the natural frequency goes to zero. Thus, sources that promote dynamic instabilities will produce fluid forces on the vibrating channel walls which are in phase with the channel wall velocity dh/dt , while sources that promote static divergence will produce forces in phase with the wall displacement h . For harmonic motion, a force out of phase with the acceleration d^2h/dt^2 is in phase with the displacement. (A list of nomenclature is given in Appendix I.)

A. Definition of the Unsteady-Flow Problem

One-dimensional leakage flow is assumed to pass through a channel formed by two parallel walls that are prescribed to oscillate at small amplitudes, such that the flow channel width H is constant in space but periodic in time:

$$H = \bar{H} + h$$

and

(1)

$$h = \bar{h} \sin \omega t ,$$

where the amplitude of motion $\bar{h}$ is assumed to be small in comparison to the mean channel width $\bar{H}$, and the frequency of wall vibration is denoted by ω . The leakage flow is driven through the channel by the constant difference between the pressure P_U in the plenum upstream and the pressure P_D in the plenum downstream of the channel. As shown in Fig. 1, the geometry of the channel includes the possibilities of having sharp-edged flow constrictions located at the entrance, exit, and one intermediate position along the total channel length L , with mean widths $\bar{H}_U$, $\bar{H}_D$, and $\bar{H}_I$, respectively. Also, the intermediate constriction, from which axial distance x is measured, breaks the flow channels into right and left channels which may have different lengths, L_R and L_L , as well as different mean widths, $\bar{H}_R$ and $\bar{H}_L$, respectively.

The one-dimensional (uniform) flow velocities assumed to exist in the right and left flow channels are denoted by U_R and U_L , respectively. The uniform velocities in the entrance, exit, and intermediate constrictions are denoted by U_U , U_D , and U_I , respectively. Since $h/\bar{H} \ll 1$, the flow velocity anywhere in the leakage flow channel is assumed to consist of a mean component $\bar{U}$ and a fluctuating component u , such that

*In general, a bar over a symbol (e.g., $\bar{H}$) denotes a constant with respect to time.

Fig. 1. One-dimensional leakage flow channel

$$U = \bar{U} + u ; \quad u/\bar{U} \ll 1 . \quad (2)$$

A similar assumption is made for the pressure:

$$P = \bar{P} + p ; \quad p/\bar{P} \ll 1 . \quad (3)$$

The pressure drop across each of the constrictions is assumed to be proportional to the dynamic pressure of the flow through the constriction, with loss coefficients of K_U , K_D , and $K_I + k_I$. Since constriction loss coefficients are not available for unsteady flow, in general, and expanding or contracting constrictions, in particular, the entrance, intermediate, and exit loss coefficients K_U , K_I , and K_D , respectively, are assumed to remain constant at their steady-flow values. However, to model a previously analyzed constriction with no steady or fluctuating pressure recovery [4], the loss coefficient of the intermediate constriction is augmented by a variable part k_I , linearly dependent upon h . For small motion of the channel walls, one would not expect the loss coefficients to vary much from their steady-state values.

The fluid viscosity is assumed to create, on the walls of the channel, shear stresses that oppose the flow velocity and have the same magnitude as

would occur in steady-state turbulent flow:

$$\tau = 1/2 \rho U^2, \quad (4)$$

where c is a known constant determined from the Darcy-Weisbach equation. Again, information for characterizing wall shear stresses in unsteady flow is lacking, but the approximation of constant c should be reasonable for small fluctuations in U .

B. Integrals of the Linearized Governing Equations

Conservation of mass requires that the net mass flow rate into the control volume, of width H and infinitesimal length in the flow direction, must equal the rate of increase of the mass in the control volume due to the wall motion, or

$$\frac{\partial U}{\partial x} = -\frac{1}{H} \frac{dH}{dt}. \quad (5)$$

The continuity equation (5) assumes the fluid is incompressible. Conservation of momentum requires that the change in linear momentum of the particles in the control volume equal the forces applied to the particles in the control volume, or

$$\frac{\partial P}{\partial x} - 2 \frac{\tau}{H} = \rho \left(\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} \right). \quad (6)$$

With τ given by Eq. (4) and H prescribed, the continuity and momentum relations (5) and (6) constitute a set of quasi-linear partial differential equations for U and P .

Linearized solutions to the governing equations (5) and (6) are sought by separating them into steady-state and fluctuating parts, after substitution of relations (1-4) and the retention of only first-order terms in h , u , and p . This procedure was previously applied [4] to a two-dimensional annular region surrounding a vibrating center body. The familiar steady-state equations are

$$\frac{\partial \bar{U}}{\partial x} = 0 \quad (7)$$

and

$$\frac{\partial \bar{P}}{\partial x} = - \frac{c\rho \bar{U}^2}{\bar{H}} . \quad (8)$$

The equations for the fluctuating components are

$$\frac{\partial u}{\partial x} = - \frac{1}{\bar{H}} \frac{dh}{dt} \quad (9)$$

and

$$-\bar{H} \frac{\partial p}{\partial x} - h \frac{\partial \bar{P}}{\partial x} - 2c\rho \bar{U}u = \rho \bar{H} \frac{\partial u}{\partial t} - \rho \bar{U} \frac{dh}{dt} . \quad (10)$$

Solutions for U , and then P , can be obtained by direct integration of Eqs. (7)-(10):

$$\bar{U} = \bar{U}_c , \quad (11)$$

$$u = - \frac{dh}{dt} \left(\frac{x}{\bar{H}} \right) + u_c(t) , \quad (12)$$

$$\bar{P} = - \frac{c\rho \bar{U}^2}{\bar{H}} x + \bar{P}_c , \quad (13)$$

$$\begin{aligned} p = & \rho \bar{U}^2 \left(c \frac{h}{\bar{H}} \right) \frac{x}{\bar{H}} + 2\rho \bar{U}c \left(\frac{1}{2} \frac{dh}{dt} \frac{x}{\bar{H}} - u_c \right) \frac{x}{\bar{H}} \\ & + \rho \bar{H} \left(\frac{1}{2} \frac{d^2 h}{dt^2} \frac{x}{\bar{H}} - \frac{du_c}{dt} \right) \frac{x}{\bar{H}} + \rho \bar{U} \frac{dh}{dt} \frac{x}{\bar{H}} + p_c(t) \end{aligned} \quad (14)$$

where $\bar{U}$ and $\bar{P}_c$ are constants and u_c and p_c are functions of time (h , dh/dt , and d^2h/dt^2) that are determined by the boundary conditions.

As expected, constant mean velocities independent of x occur in the right and left half of the channels, since each half of the channel width is uniform, and the variation in the mean pressure due to friction forces on the channel walls is piecewise linear. Also, the four constants of integration for the mean values, $\bar{U}_L$, $\bar{U}_R$, $\bar{P}_{cL}$, and $\bar{P}_{cR}$, are determined by the boundary

conditions on the mean velocities and pressures at the ends of each half of the channels.

C. Sources of Damping and Self-Excitation

Since several terms in expression (14) for the pressure fluctuations are not associated with the functions u_t and p_t , but are functions of h and its time derivatives, then sources of damping and self-excitation other than the boundary conditions are possible. To understand the other possibilities, consider first the sources of the velocity fluctuation terms given in expression (12).

The existence of the first and only spatially dependent term in expression (12) does not depend upon wall friction, because it must exist regardless of the boundary conditions and the mean flow velocity (including $\bar{U} \rightarrow 0$) to satisfy continuity requirements. In words, as the channel opens ($dh/dt > 0$), some fluid must be sucked into both ends of the channel to fill the increasing volume; conversely, some fluid must be expelled as the channel closes ($dh/dt < 0$). This "breathing action" results in a "squeeze-flow" velocity distribution which is piecewise linear in x and proportional to dh/dt .

The location in the channel where the squeeze-flow velocity distribution is zero depends upon the boundary conditions and other parameters, such as the length and width of each half of the channel. For instance, if the downstream end of the channel is very constricted compared to the upstream end, then most of the squeeze flow required by continuity will enter at the upstream end and the zero location of the squeeze-flow distribution will be near the downstream end; squeeze-flow fluctuations opposing the mean flow velocity will be minimal for $dh/dt > 0$. The extent of the velocity fluctuations opposing the mean velocity for $dh/dt > 0$ is increased by increasing the upstream constriction. The term in expression (12) that determines the zero location of the squeeze-flow velocity fluctuation will be part of u_t and proportional to dh/dt . In general, u_t represents "uniform" velocity fluctuations that are different in the right and left channels (u_{tr} and u_{tL}). The u_t values are determined by the boundary conditions that specify the relations between the velocity and pressure fluctuations at the ends of each half of the channel.

The fluctuating components of pressure come from several sources which can be explained by examination of Eqs. (10) and (14) while remembering that the mean pressure gradient, given by Eq. (8), is negative. The first term in expression (14) for p , or the second from the left in Eq. (10), is associated with fluctuations in the pressure gradient caused by the wall friction and motion. Essentially, as the channel is made larger ($h > 0$), the flow area over which the pressure acts is made larger. Therefore a smaller pressure gradient (drop) is needed to equilibrate the wall shear stresses due to the constant mean velocity $\bar{U}$. Since this part of the pressure fluctuation is proportional to h , the wall friction alone has the potential to contribute to instability of the wall motion by static divergence or stabilization of the motion by increasing the vibration frequency.

The second term in expression (14) for the pressure fluctuations, or the third term from the left in Eq. (10), is associated with fluctuations in the pressure gradient caused by fluctuations in the velocity and the associated wall shear stresses. Since the squeeze-flow part of the velocity fluctuation is proportional to dh/dt , the combined effect of the wall friction and squeeze-flow has the potential to create pressure (and net force) fluctuations in phase with the wall velocity and to contribute to the dynamic instability of the wall vibrations or to create out-of-phase fluctuations which would further damp vibrations.

The third and fourth terms in the pressure fluctuation expression (14), which come from the terms on the right-hand side of Eq. (10), are both related to fluctuations in the pressure gradients caused by acceleration of the fluid in the channel. The third term defines the pressure gradient decrease (drop) associated with positive "local" accelerations of the squeeze flow and the uniform flow acceleration (du_t/dt) required by the boundary conditions, while the fourth term prescribes the decrease (drop) associated with positive "convective" accelerations of the squeeze flow. Since the squeeze flow fluid accelerations are proportional to both dh/dt and d^2h/dt^2 , the potential exists for the squeeze flow alone to contribute to both the static and dynamic instability or stabilization of the wall motion.

The only term in the pressure fluctuation expression (14) that is independent of x is the "uniform" pressure fluctuation p_t , which generally will be a different function of time (p_{tR} and p_{tL}) in the left and right halves of the channel. Depending upon the boundary conditions, p_t and u_t terms in Eq. (14) may be functions of h and its derivatives that promote unstable motion of the channel wall. Constrictions at the entrance and variable-efficiency diffusers at the exit of annuli are known [4,6] to create strong self-excitation mechanisms. Also, exit constrictions can produce significant amounts of stabilizing damping. Similar boundary condition mechanisms can be expected to occur for the one-dimensional channel that will dominate any of the other potential sources associated with wall friction or the squeeze-flow discussed above. These other sources may lead to self-excitation when the boundary conditions do not create strong damping or excitation mechanisms. However, the potential and relative strength of any of the sources can only be determined by investigation of specific boundary conditions.

D. Boundary Conditions and Solutions

The boundary conditions for the governing equations (7-10) in each half of the channel in Fig. 1 are controlled by the constrictions and losses that are located at the ends of either half of the channel. The three constrictions are assumed to be very short, having lengths which are diminishingly small in comparison to the length of the channel, and the pressure losses produced across any of the constrictions are assumed to be representable in the form

$$P_A + \frac{1}{2} \rho U_A^2 = P_B + \frac{1}{2} \rho U_B^2 + \frac{1}{2} K \rho U^2, \quad (15)$$

where the subscripts A and B denote the total pressure and velocities immediately up- and downstream, respectively, of the constriction. The loss coefficient K is referenced to the total flow velocity U through the constriction. As discussed previously, the pressure loss relation (15) is an obvious generalization of a steady-flow energy equation. By separating the P and U terms of relation (15) into the steady and fluctuating components of relations (2) and (3), and by retaining only first-order terms, separate linearized relations for the steady and fluctuating components are generated:

$$\bar{P}_A + \frac{1}{2} \rho \bar{U}_A^2 = \bar{P}_B + \frac{1}{2} \rho \bar{U}_B^2 + \frac{1}{2} K \rho \bar{U}^2, \quad (16)$$

$$P_A + \rho \bar{U}_A u_A = P_B + \rho \bar{U}_B u_B + \rho K \bar{U} u + \frac{1}{2} \rho k \bar{U}^2. \quad (17)$$

The use of the variable loss coefficient $K + k$ will be limited to the intermediate constriction, and then only variations proportional to the gap size of that constriction will be considered:

$$k = 2k_I(h/\bar{H}_I). \quad (18)$$

Again, there is no guidance in the literature for characterization of loss relations for modulating constrictions, but the loss coefficient chosen in Eq. (18) models a modulating constriction without any downstream pressure recovery that was employed extensively in the study of annuli [4].

Evaluation of the loss relations (16) and (17) at each of the constrictions provides three equations to solve for the four constants of integration $\bar{U}_R$, $\bar{P}_{cR}$, $\bar{U}_L$, and $\bar{P}_{cL}$, associated with the steady components of flow in expressions (11) and (13), and three equations for evaluation of the functions u_{tR} , p_{tR} , u_{tL} , and p_{tL} , associated with the fluctuating components of the flow in expressions (12) and (14). Note that in evaluating the loss relations (16) and (17) at the entrance and exit constrictions, the total velocity and the fluctuating pressures in the plenums are by definition zero:

$$\bar{U}_U = u_U = p_U = 0 \quad (19)$$

and

$$\bar{U}_D = u_D = p_D = 0 . \quad (20)$$

The fourth equation necessary for the unique solution of the 4 constants of integration and the 4 functions of integration are the steady and fluctuating parts of the continuity relation

$$U_{AB}H_A = U_BH_B . \quad (21)$$

Continuity determines the relationship between the flow velocity U_B in the constriction of width H_B with the flow velocity U_{AB} at the end of the adjoining half of the channel with width H_A . By expanding the terms in the continuity equation in the steady and fluctuating parts given in Eqs. (1) and (2), and by retaining only first-order terms, the steady and fluctuating parts of the continuity equation are obtained:

$$\bar{U}_{AB} = (\bar{H}_B/\bar{H}_A)\bar{U}_B \quad (22)$$

and

$$u_{AB} = (\bar{H}_B/\bar{H}_A)u_B + (h/\bar{H}_A)(1 - \bar{H}_B/\bar{H}_A)\bar{U}_B , \quad (23)$$

where $\bar{U}_{AB}$ and u_{AB} are the steady and fluctuating parts of U_{AB} . Using the continuity relations (22) and (23) in combination with the general integrals (11) and (12), the velocities in either part of the channel or any constriction can be determined in terms of the velocities in any other part of the channel or constriction.

There are several ways to solve for the constants of integration, but the method chosen (see Appendix II) was to first express $\bar{U}_L$, $\bar{U}_R$, $\bar{U}_U$, and $\bar{U}_D$ in terms of $\bar{U}_I$, and u_{tL} , u_{tR} , u_U , and u_D in terms of u_I , by using Eqs. (22) and (23). Then three steady and three fluctuating pressure loss relations were generated by evaluating Eqs. (16) and (17) at each constriction.

The three steady equations enabled the direct determination of the unknowns $\bar{U}_I$, $\bar{P}_{cL}$, and $\bar{P}_{cR}$. In particular, the mean velocities in the different halves of the channels and the constrictions could all be expressed in the form

$$\bar{U}_A = \beta_A \bar{U}_S , \quad (24)$$

where the mean velocity for the reference channel width $\bar{H}_S$ is

$$\bar{U}_S^2 = 2(\bar{P}_U - \bar{P}_D)/\rho R_1, \quad (25)$$

$\beta_A = \bar{H}_S/H_A$ is evaluated by substituting for the channel or constriction width of interest (e.g., $\beta_L = \bar{H}_S/\bar{H}_L$), and

$$R_1 = \beta_I^2 K_I + \beta_U^2 K_U + \beta_D^2 K_D + 2(\beta_R^2 K_R + \beta_L^2 K_L). \quad (26)$$

Note that the friction stress coefficients, defined by the shear stress relation (4), have been combined into a loss coefficient in each half of the channel: $K_R = c_R L_R / \bar{H}_R$, for instance.

Since the three equations for the fluctuating components of the flow included the unknown derivative du_I/dt , two of the equations were used to eliminate the other unknowns p_{tL} and p_{tR} from the third equation. The resulting ordinary differential equation was found to have the following particular solution for u_I :

$$u_I/\bar{U}_I = T_2 \Omega_S \frac{d\eta}{dt} - T_1 \eta, \quad (27)$$

where $\eta = h/\bar{H}_S$ is a reduced displacement and $\Omega_S = \omega L/\bar{U}_S$ is a reduced frequency. The values

$$T_1 = [-R_2 R_4 + R_1 R_5 + R_1 R_3 V_S^2]/[R_2^2 + R_1^2 V_S^2]$$

and (28)

$$T_2 = [R_2 R_5 + (R_2 R_3 + R_1 R_4) V_S^2]/[R_2^2 + R_1^2 V_S^2]$$

have been expressed in terms of the reduced velocity $V_S = 1/\Omega_S$, and

$$R_2 = \beta_R \gamma_R + \beta_L \gamma_L,$$

$$R_3 = [K_D \beta_D^2 (\beta_I - \beta_D) + K_U \beta_U^2 (\beta_I - \beta_U)]$$

$$+ [K_R \beta_R^2 (2\beta_I - 3\beta_R) + K_L \beta_L^2 (2\beta_I - 3\beta_L)] + \beta_I^3 k_I, \quad (29)$$

$$\begin{aligned}
R_4 &= \gamma_R [K_D \beta_D^2 - \beta_R (\beta_I - \beta_R) + K_R \beta_R^2] \\
&\quad - \gamma_L [K_U \beta_U^2 + \beta_L (\beta_I - \beta_L) + K_L \beta_L^2] , \\
R_5 &= \frac{1}{2} [\beta_R \gamma_R^2 - \beta_L \gamma_L^2] ,
\end{aligned} \tag{29}$$

(Contd.)

where, for example, $\gamma_R = L_R/L$. Once u_I has been determined, the fluctuating velocities in the other constrictions and either half of the channel can be determined from velocity expressions (11) and (12) and the continuity equations (22) and (23):

$$u_A/\bar{U}_A = u_I/\bar{U}_I + \left(\frac{\beta_I - \beta_A}{\beta_A} \right) h/\bar{H}_A - (x/\bar{H}_A) \frac{dh}{dt}/\bar{U}_A , \tag{30}$$

where the subscript A can be L, R, U, or D. Of course, $x = -L_L$ for the upstream subscript $A = U$, and $x = +L_R$ for the downstream subscript $A = D$. Also, the functions of integration u_{tL} , u_{tR} , p_{tL} , p_{tR} , and, therefore, the pressure in each half channel can be found by substituting the expression for u_I into Eq. (14) (see Appendix II). However, the net force on the moving channel wall is of interest for calculating fluid added mass and damping, not the pressure distribution.

E. Fluctuating Fluid Forces and Instability Conditions

The fluctuating fluid forces on the moving channel wall were not obtained by integrating the explicit solutions for fluctuating pressures (given in Appendix II as A16) over the channel length. Rather, more general expressions (14) were used, after eliminating u_{tL} and u_{tR} by using Eq. (30). Then Eq. (27) for $u_I/\bar{U}_I$ and the solutions for p_{tL} and p_{tR} (given in Appendix II as A8 and A11) were substituted to obtain the components of the fluid force 180° out of phase with the wall velocity and acceleration

$$F = -C_D M \omega (dh/dt) - C_M M (d^2h/dt^2), \tag{31}$$

where $M = \rho(\bar{H}_L L_L + \bar{H}_R L_R) = \rho \bar{H}_V L$ is the mass of the fluid in the channel,

$$C_D = (L/\bar{H}_V)(L/\bar{H}_S) V_S [-S_1 T_2 + S_2 T_1 + S_4] \tag{32}$$

is the fluid damping coefficient, and

$$C_M = (L/\bar{H}_V)(L/\bar{H}_S)[-S_2 T_2 + S_5 + (S_3 - S_1 T_1) V_S^2] \quad (33)$$

can be interpreted as a mass or negative stiffness coefficient since $d^2h/dt^2 = -\omega^2 h$ in the analysis. The parameters that do not depend upon V_S are

$$\begin{aligned} S_1 &= [K_D \beta_D^2 + \beta_R^2 (K_R - 1)] \gamma_R - [K_U \beta_U^2 + \beta_L^2 (K_L + 1)] \gamma_L, \\ S_2 &= \frac{1}{2} (\beta_R \gamma_R^2 - \beta_L \gamma_L^2), \\ S_3 &= [K_D \beta_D^2 (\beta_I - \beta_D) - \beta_R^2 (\beta_I - \beta_R) + \frac{1}{2} K_R \beta_R^2 (2\beta_I - 3\beta_R)] \gamma_R \\ &\quad - [K_U \beta_U^2 (\beta_I - \beta_U) + \beta_L^2 (\beta_I - \beta_L) + \frac{1}{2} K_L \beta_L^2 (2\beta_I - 3\beta_L)] \gamma_L, \\ S_4 &= [K_D \beta_D^2 - \frac{1}{2} \beta_R \beta_I + \frac{2}{3} K_R \beta_R^2] \gamma_R^2 + [K_U \beta_U^2 + \frac{1}{2} \beta_L \beta_I + \frac{2}{3} K_L \beta_L^2] \gamma_L^2, \\ S_5 &= \frac{1}{3} (\beta_R \gamma_R^3 + \beta_L \gamma_L^3). \end{aligned} \quad (34)$$

The T_1 and T_2 parameters, which are functions of V_S , have already been given in Eqs. (28) and (29).

The equation of motion for the one-dimensional flow-induced vibrations of a wall with structural mass M_W mounted on a spring with stiffness K_W and a dashpot with damping C_W can be written as

$$(M_W + M C_{M0}) \frac{d^2 h}{dt^2} + (C_W + M C_D \omega) \frac{dh}{dt} + (K_W + C_K M \omega^2) h = 0, \quad (35)$$

where the fluid forces of Eq. (31) normally on the right side of an equation of motion have been combined with the structural parameters on the left side. Also, the fluid force $C_M M d^2 h/dt^2$ of Eq. (31) has been partitioned into added mass and stiffness components by using the identity $d^2 h/dt^2 = -\omega^2 h$. In particular, the added mass factor C_{M0} is defined as the value of C_M at zero velocity, which is independent of ω . It follows that the added stiffness is

$$C_K = C_{M0} - C_M, \quad (36)$$

which is a function of V_S .

Dynamic instability occurs when the system damping is negative, or

$$C_W < -MC_D\omega . \quad (37)$$

Apparently, a necessary condition for dynamic instability is a negative fluid damping coefficient C_D , which is a function of Ω_S or V_S as given in Eq. (32).

Divergence or static instability will occur when the system stiffness and frequency in Eq. (35) are less than or equal to zero, or

$$K_W + \frac{\bar{U}_S^2}{L^2} M \Omega_S^2 C_K \leq 0 , \quad \Omega_S \rightarrow 0 . \quad (38)$$

By recognizing that C_{M0} is independent of Ω_S but C_M is not, one sees that the limit $\Omega_S^2 C_K$ is the same as the limit of $-\Omega_S^2 C_M$. When Eq. (33) is used to evaluate Eq. (38), the structural stiffnesses that produce static divergence are

$$K_W \leq \rho \bar{U}_S^2 (L/\bar{H}_S) (S_3 R_1 - S_1 R_3) / R_1 . \quad (39)$$

Since $R_1 > 0$ and a realistic K_W requires the right side of Eq. (39) to be finite and positive, it can be shown that

$$S_3 R_1 > S_1 R_3 \quad (40)$$

is a necessary condition for divergence. A critical velocity for divergence can be calculated from Eq. (39), but its dependence on the definition of the added mass and stiffness partitioning of Eq. (35) must be recognized. Other partitioning of C_M may predict different critical velocities; however, the necessary conditions given in expression (40) will be the same for any partitioning. In particular, for $\Omega^2 C_M$ to remain finite as $\Omega \rightarrow 0$ or $V_S \rightarrow \infty$, C_M must go to infinity. When the necessary condition of expression (40) is satisfied, the component of the fluid force of Eq. (31) in phase with the acceleration become positively infinite, which will result in a zero system frequency no matter what partitioning is chosen.

Once the general expressions for the fluid forces and necessary conditions for instability are known, the different combinations of boundary condition, fluid viscosity, and geometry can be investigated to determine which have the capability to produce self-excited flow-induced vibrations.

III. CASE STUDIES

Many interesting and practical boundary condition configurations are contained in the model problem solution of the previous section. However, the more practical boundary conditions mask some of the sources of self-excitation, because the sources do not combine linearly when one or more are present. Therefore some simple cases of idealized boundary conditions are considered first, followed by more practical examples. Since the conditions for self-excitation depend upon the specific details of the boundary conditions, not all conditions can be demonstrated. However, enough cases are given to identify the general conditions that should be avoided and those which should be incorporated in the design of a leakage-flow path.

Before consideration of specific cases, a general discussion of the parameter ranges is appropriate. Assuming that the channel width selected to normalize the other channel widths, $\bar{H}_S$, is some measure of the average width of the total channel irrespective of the widths of the sharp-edged constrictions, and the width of any part of the channel does not exceed another part by a factor of ten, then $1 \leq \beta_A < 10$ for all locations. Clearly, $0 \leq (\gamma_L, \gamma_R) \leq 1$ for any lengths of the channel parts. The steady-state loss coefficients for constrictions are dependent upon their β value as well as their shape (sharp, rounded, etc.), but typically they will be in the range $0 < (K_U, K_I, K_D) < 3$. When there are no constrictions, the K_U , K_D , K_I represent inlet, outlet, and contraction (or expansion) loss coefficients, respectively, which are less than or equal to one. Whereas steady-state loss coefficient selection can be based on reliable data [7], information for the k_I of a modulating orifice is not available. The choice of one k_I will be rationalized in the discussion of the case in which it is employed.

The losses due to channel wall friction depend upon the Reynolds number of the flow, N_R , and the roughness of the channel, at least for steady flow conditions (the only conditions for which information is available). When the flow is turbulent ($N_R \geq 10^3$), the wall coefficient of friction, which is one-quarter of the Darcy-Weisbach or Moody friction factor, will be a constant in the range of $0.0025 \leq c \leq 0.025$ depending upon surface roughness. Thus, for $L/\bar{H}$ in the range of 0 to 100, the turbulent loss coefficients are in the range $0 < (K_L, K_R) < 2.5$.

Usually, the reduced velocity, or pressure drop, is the main independent variable. It can be expressed in the form $V_S = \bar{U}_S / (2\pi\bar{U}_f)$, where $\bar{U}_f = L/T$ and T is the period of the wall oscillation. The $\bar{U}_f$ can be interpreted as the mean flow velocity at which a fluid particle entering the channel at the beginning of a vibration cycle will just be leaving the channel at the end of the cycle. Therefore, if the selected $\bar{U}_S$ is a measure of the mean flow velocity in the channel, then V_S is a measure of the potential for interaction between the flow and wall motion. When $\bar{U}_S \gg \bar{U}_f$, the fluid will pass through the channel before the structure can respond. The total flow velocity will appear constant to the structure, and only divergent instabilities will be possible. When $\bar{U}_S \ll \bar{U}_f$, many cycles of motion will occur before a fluid

particle passes through the channel. The fluid forces will be similar to those for a completely still fluid ($\bar{U}_S = 0$), and self-excitation is not of concern. Therefore, interaction can be expected when $\bar{U}_S/U_f = \bar{U}_S/(fL)$ is in the range of 0.1-10.0, or $0 < V_S < 2$. Because of its direct physical interpretation, $\bar{U}_S/\bar{U}_f$ is used as the independent variable (abscissa) in presenting graphical results.

A. Case A - Uniform Wall Friction with Idealized Inlets and Outlets

The simplest leakage-flow channel that incorporates wall friction is a uniform-width channel with a uniform distribution of wall friction, no constrictions, and lossless inlets and outlets. In terms of the parameters of Fig. 1, all the channel and constriction widths are $\bar{H}$, including $\bar{H}_S$ and $\bar{H}_V$, and $L_R = L_L = L/2$. With the total wall friction loss defined as $K_f = cL/\bar{H}$, these assumptions lead to the independent parameters in Table 1. The corresponding dependent parameters defining the flow are given in Table 2.

Table 1. Independent Parameters

	Case					
	A	B	C	D	E	F
β_L	1	1	1	1	1	β
β_R	1	1	1	1	1	1
β_U	1	1	1	1	1	β
β_I	1	1	1	β	β	1
β_D	1	1	1	1	1	1
β_V	1	1	1	1	1	$2\beta/[1+\beta+\delta(\beta-1)]$
γ_L	1/2	$(1-\delta)/2$	1/2	$(1-\delta)/2$	0	$(1-\delta)/2$
γ_R	1/2	$(1+\delta)/2$	1/2	$(1+\delta)/2$	1	$(1+\delta)/2$
K_L	$K_f/2$	0	$K_f/2$	0	0	0
K_R	$K_f/2$	0	$K_f/2$	0	0	0
K_U	0	0	K_U	0	0	1/2
K_I	0	K_I	0	K	$(\beta^2-1)/\beta^2$	0
K_D	0	0	K_D	0	0	1
k_I	0	0	0	0	$-(\beta-1)/\beta^3$	0

Table 2. Dependent Parameters

	Case					
	A	B	C	D	E	F
S_1	-1	-1	$\frac{K_D - K_U}{2} - 1$	-1	0	$\frac{-3\beta^2(1-\delta)}{4}$
S_2	0	$\delta/2$	0	$\delta/2$	1/2	$\frac{[(1+\delta)^2 - \beta(1-\delta)^2]}{8}$
S_3	0	0	0	$-(\beta-1)$	0	$\frac{3\beta^2(\beta-1)(1-\delta)}{4}$
S_4	$\frac{K_f}{6}$	$-\delta/2$	$\frac{(K_D + K_U)}{4} + \frac{K_f}{6}$	$\frac{-\beta\delta}{2}$	$\frac{1-\beta}{2}$	$\frac{[(1+\delta)^2 + \beta(\beta+1)(1-\delta)^2]}{8}$
S_5	$\frac{1}{12}$	$\frac{(1+3\delta^2)}{12}$	$\frac{1}{12}$	$\frac{(1+3\delta^2)}{12}$	$\frac{1}{3}$	$\frac{[(1+\delta)^3 + \beta(1-\delta)^3]}{24}$
R_1	$2K_f$	$2K_f$	$K_U + K_D + 2K_f$	$\beta^2 K$	β^2	$\frac{\beta^2}{2} + 1$
R_2	1	1	1	1	1	$\frac{[(1+\delta) + \beta(1-\delta)]}{2}$
R_3	$-K_f$	0	$-K_f$	0	0	$\frac{\beta^2(1-\beta)}{2}$
R_4	0	0	$\frac{(K_D - K_U)}{2}$	$-(\beta-1)$	$2-\beta$	$\frac{[1 + \delta + (1-\delta)\beta(\frac{\beta}{2} - 1)]}{2}$
R_5	0	$\frac{\delta}{2}$	0	$\frac{\delta}{2}$	$\frac{1}{2}$	$\frac{[(1+\delta)^2 - \beta(1-\delta)^2]}{8}$

1. Conditions for dynamic instability

The damping coefficient found by evaluation of Eq. (32) is

$$C_D = \frac{1}{6} (L/\bar{H})^2 K_f V_s [1 - 6V_s^2 / (1 + 4K_f^2 V_s^2)] , \quad (41)$$

which is a relatively complex function of the reduced velocity in the channel, $V_s = \bar{U}/(\omega L)$, and the turbulent-flow loss coefficient, K_f . For $K_f > 1.3$, C_D monotonically increases from zero with increased V_s , as shown in Fig. 2. (Note that the reduced velocity used in Fig. 2 is $\bar{U}_f = \bar{U}/(fL)$ instead of $V_s = \bar{U}_f/(2\pi)$.) For $1.3 > K_f > 1.23$, C_D increases from zero to a maximum and

Fig. 2. Case A for various wall friction loss coefficients, K_f

then dips to a positive minimum as V_S is increased. But after the minimum, C_D increases monotonically as V_S increases. However, the minimum and maximum values are not very different, as shown in Fig. 2 for $K_f = 1.29$. For $0 < K_f < 1.23$, C_D increases to a positive maximum and then decreases monotonically as V_S increases. In other words, C_D becomes negative for $V_S^2 > 1/(6 - 4K_f^2)$, and $K_f < 1.23$ is the necessary condition for dynamic instability that is satisfied by many channels in turbulent flow.

The positive variation of C_D at very small V_S is not realistic because the flow will be laminar in much of this range. The damping coefficient C_D will not decrease to zero as $V_S \rightarrow 0$ but will attain a finite positive value [5].

Clearly, an increase in the turbulent-flow loss coefficient K_f in Eq. (41) produces an increase in positive C_D . However, since C_D becomes directly proportional to $K_f V_S$ at small values of these parameters, negative damping and dynamic instability are either promoted or suppressed by increased friction losses, depending on the magnitude of the loss. For instance, an increase of K_f from 0.1 to 0.4 increases the magnitude of the negative damping, as can be seen in Fig. 2, but an increase from 0.4 to 0.8 decreases the magnitude. Thus, an increase in the uniform friction losses along the channel may not always provide a means to stabilize vibrations.

2. Increased stiffness

According to Eq. (40), divergence will not occur for any velocity and parameter combination. This can be seen from the expression for the fluid stiffness coefficient obtained from Eq. (33):

$$C_K = C_{M0} - C_M = 2(L/\bar{H})^2 K_f^2 V_S^4 / (1 + 4K_f^2 V_S^2), \quad (42)$$

where $C_{M0} = (1/12)(L/\bar{H})^2$. An increase in either V_S or K_f increases the fluid stiffness coefficient and the system frequency.

B. Case B - Point Loss with Idealized Inlets and Outlets

Case A shows that the magnitude of uniform wall friction may be a factor affecting the existence of dynamic instabilities. Now the effect of loss distribution will be investigated by analyzing a point loss, K_I , at different locations in a uniform channel that is otherwise the same as for Case A. For these assumptions, the independent parameters in Table 1 can be derived, where $\delta = (L_R - L_L)/L$ is defined to locate the point loss. Note that $-1 \leq \delta \leq 1$, where $\delta = 1$ and $\delta = -1$ correspond to an entrance location with $L_R = L$ and an exit location with $L_L = L$, respectively. The corresponding dependent parameters defining the flow are given in Table 2. Choosing $K_I = 2K_f$ will give the same mean velocity and pressure drop as in Case A.

The damping coefficient found by evaluation of Eq. (32) is

$$C_D = (L/\bar{H})^2 \frac{K_I V_S}{(1 + K_I^2 V_S^2)} \left[\left(\frac{\delta}{2}\right)^2 - \left(\frac{\delta}{2}\right) K_I V_S^2 \right]. \quad (43)$$

Evidently the magnitude of C_D will increase for large V_S ; however, satisfaction of the conditions for dynamic instability, Eq. (37), now depends more upon the location δ than upon the magnitude of the loss K_I . For downstream locations ($\delta < 0$), a central location ($\delta = 0$), and upstream locations ($\delta > 0$), the damping coefficient C_D is greater than zero, equal to zero, or less than zero when $V_S^2 > \delta/(2K_I)$, respectively. In other words, upstream losses can produce negative flow damping and promote conditions for dynamic instability, downstream losses always produce positive flow damping and promote dynamic stability, and no flow damping is produced when the loss is located at the center of the channel.

1. Effects of point loss location on dynamic instability

The monotonic increase of positive C_D with V_S , which occurs for any downstream location ($\delta < 0$) and K_I , is illustrated in Fig. 3 for $K_I = 1.0$ at several locations: $\delta = -0.33$, -0.66 , and -1.0 . Obviously, for the same reduced velocity, the damping coefficient is larger for a loss located farther

Fig. 3. Case B for loss coefficient $K_I = 1.0$ and various point loss locations δ

downstream. In fact, the exit location ($\delta = -1.0$) produces more positive damping than any upstream location at any reduced velocity. Thus, the exit is the optimum loss location to maximize positive flow damping and suppress dynamic instabilities.

Upstream loss locations ($\delta > 0$) can produce both positive and negative flow damping for any loss K_I , depending on the magnitude of V_S . As shown in Fig. 3 for $K_I = 1.0$ and $\delta = 0.33, 0.66$, and 1.0 , C_D initially increases to a positive maximum and then monotonically decreases to negative values with increased V_S . Furthermore, the positive damping produced by upstream losses is not always insignificant. In fact, the upstream location δ_U will produce more positive damping than the downstream locations $\delta_D > -\delta_U$ for a limited range of reduced velocities: $0 < V_S^2 < (\delta_U + \delta_D)/(2K_I)$. Among upstream locations, the magnitude of the positive damping produced depends upon location in two ways. Locating the same loss farther upstream increases the reduced velocity $V_S^2 = \delta/(2K_I)$ at which the transition to negative damping first occurs, and it also increases the positive damping at the reduced velocities below the transition. In other words, for reduced velocities below that of the transition at the entrance location, $V_S^2 < 1/(2K_I)$, the production of positive damping and the suppression of a dynamic instability by an upstream loss are optimized by locating the loss at the entrance to the channel ($\delta = 1.0$). However, for $V_S^2 > 1/(2K_I)$, negative flow damping is produced at every upstream location.

The upstream loss location that produces the most negative flow damping is a function of V_S , because both the transition reduced velocity and $-dC_D/dV_S$ increase when the same loss is located farther upstream (see Fig. 3). As a result, the intermediate location of $\delta = K_I V_S^2$ produces the most negative damping for $V_S^2 < 1/K_I$, whereas the entrance location ($\delta = 1.0$) produces the most negative damping for $V_S^2 > 1/K_I$. In other words, the entrance is not the most susceptible location for dynamic instabilities, unless $V_S^2 > 1/K_I$.

2. Effects of point loss magnitude on dynamic instability

Whereas in Case A the very existence of a negative damping coefficient required distributed friction $K_f < 1.23$, in Case B the magnitude of the point loss K_I only affects the range of V_S over which negative damping occurs. Also in contrast to Case A, larger K_I always increases the magnitude of the negative damping that occurs for upstream losses at a given reduced velocity, mainly by decreasing the transition reduced velocity at which negative damping begins, $V_S^2 = \delta/(2K_I)$. This is shown in Fig. 4, where the lower branches of the curves have nearly identical negative slopes for various values of $K_I > 0.5$ at the entrance loss location ($\delta = 1$).

Fig. 4. Case B for point loss location $\delta = -1$ (upper branches) and $\delta = +1$ (lower branches) with various loss coefficients

As in Case A, a larger point loss K_I increases the positive flow damping magnitude for all downstream loss locations at any reduced velocity, as demonstrated by the upper-branch curves for an exit location ($\delta = -1.0$) in Fig. 4. Also, increases occur at those upstream loss locations having reduced velocities below the decreasing transition value of $V_S^2 = \delta/(2K_I)$. Curves for $K_I = 8$, which is not a practical value for a point loss, are included in Fig. 4 to demonstrate that a limit is being approached on the amount of positive or negative flow damping that can be produced by increasing K_I . For very large K_I , Eq. (43) gives the limit $C_D = (L/\bar{H})^2(\delta/2)V_I$.

Although upstream losses have been shown to produce substantial positive flow damping for some conditions, the usefulness of the conditions is marginal. For typical $K_I \sim 1$, the required condition $V_S^2 > 0.5\delta$ is difficult to satisfy in most practical situations. Therefore, for purposes of design, increases in the magnitude of the point loss K_I must be assumed to promote dynamic instabilities for all upstream loss locations and to suppress them only for downstream locations.

3. No static divergence

According to Eq. (40), divergence will not occur for any velocity and parameter combination. This can be seen from the expression for the fluid mass coefficient obtained from Eq. (33):

$$C_M = C_{M0} + \frac{1}{2} \frac{(L/\bar{H})^2 \delta K_I V_S^2}{(1 + K_I^2 V_S^2)} \left(1 + \frac{\delta}{2} K_I\right) \quad (44)$$

where $C_{M0} = (1/12)(L/\bar{H})^2$. Since C_M monotonically increases or decreases from zero, depending upon the sign and magnitude of δK_I , but remains finite as $V_S \rightarrow \infty$, the system frequency will not become zero for any V_S .

C. Case C - Entrance, Exit, and Wall Friction Losses

The idealized Cases A and B make clear that the distribution of flow losses in a leakage-flow channel has an important influence on dynamic instabilities, especially entrance and exit losses. Now a more realistic uniform-width channel is analyzed; point losses K_U and K_D are assumed to exist at the entrance and exit, respectively, and the channel walls may create uniformly distributed friction losses. However, cases with no wall friction losses are of practical interest, because the entrance and exit losses often dominate the wall friction losses when $L/\bar{H}$ is small. The independent parameters given in Table 1 for Case C define a uniform channel with entrance, exit, and uniform wall friction losses. The corresponding dependent flow parameters are given in Table 2. Note that $\bar{H}_S = \bar{H}_V = \bar{H}$ has been chosen.

The damping coefficient found by evaluation of Eq. (32) is

$$C_D = \frac{1}{4} (L/\bar{H})^2 V_S \left[\frac{2}{3} K_f + K_D + K_U - \frac{(K_D - K_U - 2)[-2K_f + (K_D - K_U)(K_U + K_D + 2K_f)]V_S^2}{1 + (K_U + K_D + 2K_f)^2 V_S^2} \right]. \quad (45)$$

The dependence of C_D on the parameters K_D , K_U , and K_f is too complex to discuss in general, but for the case of no wall friction ($K_f = 0$),

$$C_D = \frac{1}{4} (L/\bar{H})^2 V_S (K_D + K_U) \frac{[1 - 2(K_U - K_D - 2K_U K_D)V_S^2]}{1 + (K_U + K_D)^2 V_S^2}. \quad (46)$$

Clearly, for $K_f = 0$,

$$V_S^2 > \frac{1}{2(K_U - K_D - 2K_U K_D)}, \quad (47)$$

and

$$K_U > K_D + \frac{2K_D^2}{1 - 2K_D}, \quad (48)$$

C_D will become exceedingly more negative with increased V_S , after first increasing to a positive maximum at smaller V_S . This variation is similar to that of the lower-branch curves shown in Fig. 4. However, according to Eq. (48), negative flow damping and dynamic instability cannot occur for just any entrance and exit losses; K_U must be larger than K_D and K_D cannot exceed 0.5. Since $K_D = 1$ for a square-edged outlet with no pressure recovery and K_U has a maximum value of 0.5 for a square-edged inlet, then dynamic instability will only be possible for a very well-designed diffuser outlet with $K_D < 0.25$.

For the common geometry of a square-edged inlet and outlet with wall friction in the channel, the damping coefficient of Eq. (45) reduces to

$$C_D = \frac{1}{6} (L/\bar{H})^2 V_S \left[\left(K_f + \frac{9}{4} \right) + \frac{\left(\frac{3}{4} - K_f \right) V_S^2}{\frac{4}{9} + \left(1 + \frac{4}{3} K_f \right)^2 V_S^2} \right]. \quad (49)$$

Inspection of the terms in the bracket shows that negative C_D and dynamic instability cannot be produced by the addition of wall friction losses K_f ; in fact, the flow damping becomes more positive.

D. Case D - A Constriction in a Channel with an Ideal Entrance and Exit

Case C makes clear that most uniform-width channels will not be susceptible to dynamic instabilities. Essentially, entrance losses for uniform-width channels are not large enough, in comparison to practical outlet losses, to cause a dynamic instability. However, relatively larger entrance pressure losses, or losses at any other location, can occur in nonuniform-width flow channels that have a local reduction in the channel width which increases the local flow velocity.

A single sharp-edged constriction at an arbitrary location in an otherwise uniform-width channel with a lossless entrance and exit is chosen to explore the effects of channel contractions, because it is the simplest geometry that increases the local flow velocity and pressure loss. In particular, all the channel widths in Fig. 1 are defined to be the same, $\bar{H}$, and all the pressure losses are assumed to be zero except those of the intermediate constriction, which has a width of $\bar{H}_I$ and a loss of $K_I = K$. Also, the loss K is assumed to be independent of β_I , although this is not realistic.

If one chooses $\bar{H}_S = \bar{H}$ and defines the location of the constriction by $\delta = (L_R - L_L)/L$, then $\beta_I = \bar{H}/\bar{H}_I = \beta$ and the independent and dependent parameters of Tables 1 and 2 can be derived. According to Eq. (32), the damping coefficient is

$$C_D = (L/\bar{H})^2 \frac{\beta^2 K V_S^2}{1 + \beta^4 K^2 V_S^2} \left[\left(\frac{\delta}{2}\right)^2 - \frac{\delta}{2} \beta^2 K V_S^2 - (\beta - 1) \left(\frac{\delta}{2} \beta^2 K + 1\right) V_S^2 \right], \quad (50)$$

where V_S is the reduced velocity in the flow channel of width $\bar{H}$. Negative damping will occur for

$$V_S^2 > \delta / [2\beta^2 K + (\beta - 1)(2\beta^2 K + 4/\delta)], \quad (51)$$

when

$$\delta > \frac{-2}{\beta^3 K} (\beta - 1) \geq -\frac{0.3}{K}. \quad (52)$$

This is a slightly larger range than the $\delta > 0$ range of the point loss in Case B. Of course, the results of Case B can be obtained by substituting $\beta = 1$ in Eqs. (50) to (52).

1. Comparison with the fluid damping produced by a point loss ($\beta = 1$)

The similarity between the fluid damping coefficient produced by the point loss of Case B and the constriction of Case D can be seen by

substituting $K_I = \beta^2 K$ into Eq. (43) of Case B and comparing the result with Eq. (50), assuming that K is the same for any β . The quantity $\beta^2 K$ may be interpreted as an "equivalent point loss" which accounts for the velocity amplification of the constriction. Except for the last term in the brackets of Eq. (50), which includes the $(\beta - 1)$ multiplier, Eqs. (50) and (43) with $K_I = \beta^2 K$ are identical. Thus the effects of a sharp-edged constriction on the flow damping are not due only to the velocity amplification of a point loss; a constriction effect also exists. However, the qualitative variations of C_D with respect to V_S , δ , and $\beta^2 K$ are very similar to those for the point loss of Case B; compare Figs. 3 and 4 with Figs. 5 and 6. In contrast, the quantitative differences can be large, especially for large contraction ratios, β .

For downstream constriction locations $\delta < -2/(\beta^2 K)$, not only is the positive damping coefficient increased by the velocity amplification of the constriction, where $K \rightarrow \beta^2 K$ for $\beta \neq 1$, but the contraction effect of the last term in Eq. (50) further increases the damping coefficient to levels sometimes significantly greater than those of the point loss alone. Noting the differences in the scales of Figs. 3 and 5, compare the C_D of the exit ($\delta = -1.0$) point loss to the C_D of the exit constriction with $\beta = 4.0$ and the same loss coefficient $K = 1.0$. Also, Fig. 6 shows that because of the contraction effect, the magnitude of C_D does not become independent of the constriction at large $\beta^2 K$, as it became independent of the point loss K_I in Fig. 4. Perhaps this is best seen in the limit of Eq. (50) for large $\beta^2 K$,

Fig. 5. Case D for a sharp-edged constriction with loss coefficient $K_I = 1.0$, contraction ratio $\beta = 4.0$, and various point loss locations δ

Fig. 6. Case D for sharp-edged constrictions at $\delta = +1.0$ (lower branches) and $\delta = -1.0$ (upper branches) with various contraction ratios, β

$$C_D = -(L/\bar{H})^2 v_s \beta \frac{\delta}{2}, \quad (53)$$

where a clear linear dependence on β exists.

For the remaining downstream locations, $-2/(\beta^2 K) < \delta < 0$, the contraction effect of the last term in Eq. (50) reverses and acts to reduce the damping coefficient. Thus, the flow damping of a constriction is less than that of a point loss ($\beta = 1$) for these locations. In fact, as shown in Fig. 5 for $\delta = 0$, where velocity amplification has no effect, the contraction effect may produce negative damping.

For upstream constriction locations ($\delta > 0$), the contraction effect always acts to decrease the damping coefficient. Therefore, the positive damping produced by an upstream constriction is not increased as much as might be expected based on velocity amplification alone. For example, the maximum of the entrance ($\delta = 1.0$) loss curve in Fig. 4 with $K_I = 1.0$ is similar to that of the entrance constriction curve in Fig. 6 with $\beta = 10$ and $K = 1.0$, or an equivalent point loss of $K_I = 100$. However, the transition reduced velocity is decreased and the magnitude of the negative damping at any larger reduced velocity is increased by both the velocity amplification and the contraction effects. Therefore, the differences relative to Case B for the

point loss can be significant. Observe the nearly factor of 30 decrease in the transition reduced velocity for the $\beta = 10$ curve in Fig. 6 relative to that of the $K_I = 1.0$ curve in Fig. 4. Of course, this reduction is apparent in Eq. (51), where both the velocity amplification terms with $\beta^2 K$ and the contraction effect terms with $(\beta - 1)$ can be identified. The fact that the increased magnitudes of negative damping do not depend solely on the decrease in the transition reduced velocity, as they nearly do for the point loss of Case B, can be seen by comparing Figs. 4 and 6, but this linear dependence on β in Eq. (53) is more revealing.

2. The absence of a static divergence instability

The similarity of the added mass coefficient for Case D,

$$C_M = C_{M0} + \frac{1}{2} \frac{(L/\bar{H})^2 \delta \beta^2 K V_S^2}{(1 + \beta^4 K^2 V_S^2)} \left[\left(1 + \frac{\delta}{2} \beta^2 K\right) - (\beta - 1)(2\beta^4 K^2 V_S^2 / \delta - 1) \right], \quad (54)$$

to the C_M for the point loss of Case B can be seen by substituting $K_I = \beta^2 K$ into Eq. (44). Again assuming that K is the same for any β and $\beta^2 K$ is an equivalent point loss coefficient which accounts for the velocity amplification of the constriction, the contraction effect can be identified with the last term in Eq. (54) with the $(\beta - 1)$ multiplier. Without the last term in Eq. (54), the variation of C_M with V_S would be the same as for a point loss with a loss coefficient of $\beta^2 K$: a monotonic increase ($\delta > 0$) or decrease ($\delta < 0$) to finite values as $V_S \rightarrow \infty$ and no static divergence. However, the contraction effect of the last term in Eq. (54) quickly produces negative C_M values for $\delta > 0$ and $V_S^2 > \delta / (2\beta^4 K^2)$. Therefore, a channel with a sharp-edged constriction is even less likely to experience a static divergence instability than a channel with a point loss. The static instability condition of Eq. (40), $(\beta - 1)\beta^2 K < 0$, shows the same trend.

The absence of conditions that promote a static divergence instability for an entrance constriction is somewhat surprising, because channels with widths that increase linearly in the flow direction are known [1-3] to be susceptible to the static divergence condition and an entrance constriction is an extreme example of a channel whose width diverges in the flow direction.

E. Case E - An Entrance Constriction and an Exit with No Pressure Recovery

The existence of a dynamic instability for an annular (two-dimensional) region with a specific type of entrance constriction and a square-edged exit ($K_D = 1$) has been demonstrated [4]. Equivalent conditions will be assumed in Case E, as a practical example of a sharp-edged constriction in a one-dimensional channel. In particular, no losses are assumed to occur for the flow entering the throat of the entrance constriction, and no pressure recovery is assumed to occur for the flow entering the channel from the outlet of the constriction.

A constriction loss coefficient of $(\beta^2 - 1)/\beta^2 - 2[(\beta - 1)/\beta^3](h/\bar{H}_I)$ is required for equal total pressure in the left end of the channel and in the throat of the intermediate constriction for a one-dimensional channel governed by the loss relation of Eq. (15), $\beta_I = \beta = \bar{H}/\bar{H}_I$, and $L_L = 0$. Thus, according to Eqs. (16)-(18), both a fluctuating loss coefficient,

$$k_I = -(\beta - 1)/\beta^3, \quad (55)$$

and a steady loss coefficient,

$$K_I = (\beta^2 - 1)/\beta^2, \quad (56)$$

must be defined to model this specific type of inlet constriction. With the assumptions above, the independent and dependent parameters of Tables 1 and 2 can be derived, and according to Eq. (32), the damping and mass coefficients are

$$C_D = \frac{1}{2} (L/\bar{H})^2 \beta^2 V_S \frac{\frac{1}{2} - \beta^2(\beta - 2)V_S^2}{1 + \beta^4 V_S^2} \quad (57)$$

and

$$C_M = C_{M0} + \frac{1}{4} (L/\bar{H})^2 \beta^2 (2\beta - 3) \frac{V_S^2}{1 + \beta^4 V_S^2}. \quad (58)$$

Again, V_S is the reduced velocity in the channel of width $\bar{H}$.

Clearly, negative C_D will occur for $\beta > 2$ and $V_S^2 > 1/[2 - 2\beta^2(\beta - 2)]$. Also, as for the more general constriction of Case D, dynamic instability will occur for sufficiently large V_S , because $C_D \rightarrow -(L/\bar{H})^2(\beta - 2)V_S/2$ as $V_S \rightarrow \infty$, and static divergence will not occur because C_M remains finite for all V_S . As expected, these trends are the same as those described for an annular region [4] with the same type of constriction.

F. Case F - Channels with Converging or Diverging Widths

The single sharp-edged constrictions of Cases D and E are extreme examples of channels with widths that monotonically converge or decrease in the flow direction. Other channels having the same overall contraction ratio, but with smoother convergences, also can be expected to increase local pressure losses and create fluid damping which suppresses or promotes dynamic instabilities, depending on the location of the narrow part of the channel.

In Case F, the channel of Fig. 1 is studied with no sharp-edged constrictions, $\bar{H}_L \neq \bar{H}_R$, a square entrance having $K_U = 0.5$, a square exit having $K_D = 1.0$, and otherwise frictionless walls, including $K_I = 0.0$. The channel described represents a piecewise linear approximation to a relatively short channel with converging or diverging widths, depending on whether $\bar{H}_R/\bar{H}_L$ is less or greater than one. Clearly, the exact shape of smoothly converging or diverging widths cannot be modeled; however, the assumption of no pressure loss at the intermediate junction of the two parts of the channel having different widths implies that a smoothly converging or diverging width is being modeled. Also, different spatial gradients for a smooth variation in channel width can be crudely modeled by varying $\delta = \gamma_R - \gamma_L$. Essentially, the length of the narrowest part of the channel, which shall be called the throat region, can be specified. For instance, a larger fraction of the total length of a channel is subjected to the higher velocity flow of the throat region when δ is made smaller in a channel with a diverging width. Conversely, the geometry of a sharp-edged entrance constriction can be modeled as $\delta \rightarrow 1$.

To study the same effects that sharp-edged entrance or exit constrictions produced at different contraction ratios in Cases D and E, the reference width $\bar{H}_S = \bar{H}_L$ is selected for the channel with converging widths and $\bar{H}_S = \bar{H}_R$ for those with diverging widths. Thus all channels with converging widths will have the same reduced velocity at the entrance, $\bar{U}_L/(\omega L)$, and $\bar{U}_R/(\omega L)$ will be the same at the exit of all the channels with diverging widths. The parameters given in Table 1 are for diverging channel widths. The corresponding dependent parameters are given in Table 2, but they are obviously too complex to be used to derive closed-form solutions that would be useful in investigating the general effects of parameter variations. Instead, numerical evaluations were made for selected values of the independent parameters.

1. Flow damping increases for longer throat regions

The trends in C_D for channels with converging widths are clear, as can be seen in Fig. 7, which presents results for $\delta = -0.75$, or $L_R/L = 0.125$, and $\delta = 0.5$, or $L_R/L = 0.75$. As for the sharp-edged exit constriction of Case D, smaller constrictions with larger contraction ratios, $\beta = \bar{H}_L/\bar{H}_R$, can produce significantly larger C_D at the same reduced velocity, $\bar{U}_L/(fL)$. The qualitative effect of changing the length of the throat region is similar; a larger δ or L_R/L can produce a significantly larger C_D for the same $\bar{H}_L/\bar{H}_R$ and reduced velocity. Interestingly, the latter trend implies that a sharp-edged exit constriction has an even smaller C_D for the same $\bar{H}_L/\bar{H}_R$, K , and $\bar{U}_L/(fL)$. Comparison of the upper branches of the Case D curves in Fig. 6 with the corresponding curves in Fig. 7 substantiates this trend, even though the channel containing the sharp-edged constriction has no entrance loss. The comparisons above do not account for the increase in the loss coefficient that accompanies a decrease in the size of a sharp-edged constriction, at least during steady state flow. To assess this practical implication, the C_D curves for Case F in Fig. 7(a) with $\beta = 6.0$ are reproduced in Fig. 8(a) for comparison with the C_D curves created by a sharp-edged exit constriction in a

channel having the same contraction ratio, a more realistic loss coefficient of $K_I = 2.5$, and the same square-shaped entrance ($K_U = 0.5$) assumed in Case F. Apparently, channels with smoothly converging widths and a finite-length throat region may produce more flow damping than a sharp-edged constriction with the same overall contraction ratio β .

Fig. 7. Case F for channels with converging widths and throat lengths of (a) $L_R/L = 0.125$ and (b) $L_R/L = 0.75$, with various contraction ratios $\beta = \bar{H}_L/\bar{H}_R$

Fig. 8. Comparisons of fluid damping coefficient C_D for channels with sharp-edged constrictions and two channels with (a) converging and (b) diverging widths and finite-length throat regions. All are at the same contraction ratio of $\beta = 6.0$

The trends in C_D for channels with diverging widths are not as clear as for channels with converging widths, as can be seen in Fig. 9, which presents results for $\delta = 0.75$, or $L_L/L = 0.125$, and $\delta = 0.5$, or $L_L/L = 0.75$. Two of the qualitative trends in C_D are the same as those of the sharp-edged entrance constriction given in Fig. 6; an increase in the contraction ratio $\beta = \bar{H}_R/\bar{H}_L$ decreases the transition reduced velocity at which negative damping is first produced and increases the negative damping at a larger reduced velocity. However, the maximum amount of positive damping produced for a channel with a diverging width and a finite-length throat region is not independent of β , as it was for the sharp-edged entrance constriction of Fig. 6. In fact, the length of the throat region, L_L/L , significantly affects all the trends in C_D .

An increase in L_L/L , or a decrease in δ , increases the maximum value of positive C_D , the transition reduced velocity, and the rate $|dC_D/dV_S|$ at which negative damping is produced at larger reduced velocities. Further, at larger β , the increases in positive damping and $|dC_D/dV_S|$ can be disproportionately larger than the increase in the transition velocity. This can be seen by comparing the slope of the C_D curves for the same β in Figs. 9(a) and 9(b), and knowing that their slopes remain approximately the same for $C_D < -0.2$. A

Fig. 9. Case F for channels with diverging widths and throat lengths of (a) $L_L/L = 0.125$ and (b) $L_L/L = 0.75$, with various contraction ratios, $\beta = \bar{H}_R/\bar{H}_L$

better comparison can be made in Fig. 8(b), which shows a larger range of negative C_D values for the two curves with $\beta = 6.0$. The main purpose of Fig. 8(b) is to show that channels with diverging widths and finite-length (L_L/L) throat regions can produce more negative damping at practical reduced velocities than a sharp-edged constriction, with a realistic $K = 2.5$, located at the entrance of an otherwise uniform channel having a square exit with $K_D = 0.5$. Thus, the shape of the throat region in a channel with diverging widths should be considered in determining susceptibility to dynamic instability. In particular, some channels with smoothly diverging widths are more susceptible than a sharp-edged entrance constriction.

2. Conditions for static divergence instabilities

The trends in the added mass coefficient are well represented in Fig. 10, where C_M curves are presented for the single contraction ratio of $\beta = 6.0$. Results for several geometries are included: both smoothly converging ($\bar{H}_S = \bar{H}_L$) and diverging ($\bar{H}_S = \bar{H}_R$) channel widths with different δ , $K_D = 1.0$ and $K_U = 0.5$; an upstream constriction ($\bar{H}_S = \bar{H}_R = \bar{H}_L$) with $K_I = 2.5$ and $K_D = 1.0$; and a downstream constriction ($\bar{H}_S = \bar{H}_R = \bar{H}_L$) with $K_I = 2.5$ and $K_U = 0.5$. Channels with converging widths are seen to produce C_M values that are relatively independent of δ and become more negative as the reduced velocity is increased. Although not shown, larger contraction ratios also increase the amount of negative damping. Thus, flow through channels with

Fig. 10. Comparison of C_M for channels with $\beta = 6.0$ and throat regions of different lengths, including sharp-edged entrance and exit constrictions

converging widths stiffens the fluid-structure system. Also, the suppression of static divergence instability is more effective at larger contraction ratios and reduced velocities, but it is nearly independent of the spatial gradient of the converging channel widths, as measured by the length of the throat region, L_R/L or δ .

Channels with diverging widths are seen in Fig. 10 to produce positive C_M values which monotonically increase at larger reduced velocities and smaller δ , except for the sharp-edged entrance constriction ($\delta = 1.0$) where C_M increases to a finite positive maximum as $V_r \rightarrow \infty$. In contrast to channels with converging widths, the magnitude of C_M is very sensitive to the spatial gradient of the diverging channel width, or the length of the throat region L_L/L . As for converging channel widths, significant increases in C_M (not shown in Fig. 10) can be produced by increases in the contraction ratio β . Thus, flow through a channel with a diverging width softens the fluid-structure system; and a static divergence instability is promoted by larger contraction ratios, higher reduced velocities, and longer throat regions. In fact, for a static divergence instability to exist, a channel must have diverging widths and a finite-length throat region.

The conclusion that channels with diverging widths promote static divergence instabilities is confirmed by evaluation of Eq. (39) for susceptible structural stiffnesses:

$$K_W \leq \frac{3}{2} \rho U_R^2 \left(\frac{L}{H_R} \right) \frac{\beta^2 (\beta - 1)(1 - \delta)}{\beta^2 + 2} . \quad (59)$$

Clearly, the minimum structural stiffness required to avoid a static instability increases as either $\bar{U}_R$ or β is increased and δ is decreased. Also, a static instability is not predicted for a channel with a divergent geometry formed by a sharp-edged entrance constriction ($\delta = 1$), in agreement with Fig. 10 and Case D.

IV. SUMMARY

Approximate solutions for the unsteady velocities and pressures in the one-dimensional flow channel shown in Fig. 1 have been determined for a constant pressure drop $P_U - P_D$ and a harmonic variation in the width of the channel. The magnitude of the harmonic variation was assumed to be a small fraction of the mean width and the same at every location in the channel. The unsteady pressure losses due to the entrance, exit, constrictions, channel junction, and wall friction were specified by using quasi-steady approximations: the losses were assumed proportional to the local unsteady dynamic pressure, but the loss coefficients for steady, turbulent flow were used. General expressions for the resultant fluid forces on the vibrating walls were calculated and used to assess the structural instabilities of a one-dimensional system (oscillator) consisting of the rigid channel wall mounted on springs and constrained to move in rigid-body translations. In particular, simple cases of the flow channel geometry were studied to identify the elementary pressure loss and boundary conditions that produce the negative flow damping or reductions in stiffness necessary for the fluid-structure system to become dynamically or statically (divergence) unstable, respectively.

A. Idealized Flow Channels

Several case studies were made for a channel of uniform width with a lossless entrance and exit. The presence of a single point loss upstream of the channel midpoint was shown to be the only factor necessary to produce negative flow damping; downstream locations always produced positive flow damping. Some upstream locations, the entrance in particular, were able to produce relatively large amounts of positive flow damping at small reduced velocities, but these velocities are usually exceeded in practical situations. Otherwise, the entrance and exit locations maximized the production of negative and positive flow damping, respectively. Generally, larger losses produced larger magnitudes of positive or negative flow damping. However, no combination of location or magnitude of the single point loss produced conditions for static divergence, although some limited softening of the fluid-structure system stiffness was found.

The effectiveness of the point loss was found to be enhanced by combining it with a point reduction in the flow channel area; in other words, by creating a sharp-edged constriction. The flow damping produced by sharp-edged constrictions and that produced by point losses were shown to have qualitatively similar dependences on their location and the reduced velocity in the flow channel. However, the magnitude of the flow damping for small sharp-edged constrictions was much larger than that of the point loss, owing in part to the local velocity amplification created by the constricted flow area. In general, smaller constrictions created larger positive or negative flow damping, but the constriction size also determined which downstream locations and reduced velocities produced negative flow damping. As for a point loss, conditions for a static divergence instability did not exist. This was somewhat surprising because a sharp-edged entrance constriction is a special case of a channel with a width that increases in the flow direction, and a channel with a linearly divergent width is known [1-3] to be susceptible to the static divergence instability.

In another case, the effects of uniform wall friction in the idealized channel were studied. For long flow channels with large slenderness (length/width) ratios and rough walls, positive flow damping was predicted which increased with increases in wall friction or reduced velocities. For small slenderness ratios and little wall friction, negative flow damping was predicted for large reduced velocities. This alone would not have been of concern, because the unrealistic lossless exit of the channel was believed responsible for the negative flow damping. However, a range of wall friction was found where increased friction decreased the flow damping. Thus, increases in the wall friction may not always aid in suppressing a dynamic instability. As might be expected, increased wall friction of any magnitude was found to further stiffen the fluid-structure system and thus always promote static stability.

The results for the idealized flow channel provide an understanding of the features of the boundary conditions and flow losses that produce dynamic instabilities, and, therefore, they can be used in design to avoid features that cause instabilities. However, if a channel design contains a feature which alone produces negative flow damping and one which alone produces positive flow damping, then the conditions for instability can only be determined by studying the case that has both features because of the

nonlinear nature of the unsteady flow. Of course, if the effects of one feature are dominant, then it will control the instability conditions. In practical flow channels, competing features are usually present.

B. More Practical Flow Channels

A case study of a uniform-width channel with arbitrary entrance and exit losses showed that negative damping and the potential for dynamic instabilities were not possible unless the exit loss was very small - like those produced by a well-designed diffuser. The addition of wall friction of any magnitude was found to increase the flow damping in a uniform channel with a square entrance and exit, in contrast to its sometimes destabilizing effects in a uniform channel with a lossless entrance and exit. Clearly, the entrance losses and wall friction losses of practical uniform-width channels are too small in comparison to the exit losses to produce negative damping and conditions for dynamic instability. Static divergence instabilities were not found, nor were they expected, because none of the features of this flow channel were found to promote static divergence instabilities in the case studies where they were the only source of pressure loss.

On the basis of previously obtained results for an annular region [4] and the results for the one-dimensional channel summarized above, the addition of a sufficiently small, sharp-edged constriction at the entrance of an otherwise uniform one-dimensional channel with a square exit was expected and found to provide the additional upstream pressure losses necessary to produce negative damping and conditions for dynamic instability. The variations in the fluid damping and stiffness with reduced velocity and constriction size were found to be similar to those of the two-dimensional annular region [4], in a case study involving equivalent one-dimensional boundary conditions. In particular, a 50% reduction in channel flow area at the constriction was required to produce negative flow damping.

In the last case study, piecewise linear approximations to channels with smoothly converging or diverging widths were found to produce larger magnitude positive or negative flow damping when a larger fraction of the length of the channel was exposed to the higher velocity flow in the narrowest part of the channel - the throat region. In particular, some channels with smoothly diverging widths were found to be more susceptible to dynamic instabilities than channels with sharp-edged entrance constrictions. Also, channels with diverging widths and finite-length throat regions were found to be necessary conditions for the existence of static divergence instabilities.

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APPENDIX I. NOMENCLATURE

c	Wall shear stress coefficient
C_D	Fluid damping coefficient
C_K	Added stiffness coefficient
C_M	Fluid mass coefficient
C_{MO}	Added mass coefficient: C_M (at $V_S = 0$)
C_W	Wall structural damping
f	Frequency (cycles/sec): $\omega/(2\pi)$
F	Fluid force on wall
h	Harmonic variation in channel width
$\bar{h}$	$ h $
H	Channel width: $\bar{H} + h$
$\bar{H}$	Mean channel width
$\bar{H}_V$	An average $\bar{H}$ based on channel volume: $(\bar{H}_L L_L + \bar{H}_R L_R)/L$
k	Variable loss coefficient
K	Constant loss coefficient
K_L, K_R, K_f	Wall friction loss coefficients: $c_L L_L / \bar{H}_L$, $c_R L_R / \bar{H}_R$, $cL / \bar{H}$
K_W	Wall structural stiffness
L	Channel length
M	Fluid mass in channel
M_W	Wall structural mass
N_R	Reynolds number
p	Fluctuating component of pressure
P	Pressure: $\bar{P} + p$
$\bar{P}$	Mean pressure
P_U, P_D	Static pressure in the upstream and downstream plenums
$R_1, S_1, T_1, \dots$	Solution parameters

t	Time
T	Period of wall oscillation: $1/f$
u	Fluctuating component of flow velocity
U	Flow velocity: $\bar{U} + u$
$\bar{U}$	Mean flow velocity
$\bar{U}_f$	Channel transit velocity: L/T
v	$u_I/\bar{U}_I$
V_S	Reduced velocity: $\bar{U}_S/\omega\bar{H}_S = 1/\Omega_S$
x	Distance from the intermediate constriction
α_A	Channel width ratio: $\bar{H}_I/\bar{H}_A$
β_A	Channel width ratio: $\bar{H}_S/\bar{H}_A$
γ_R, γ_L	Length ratios: $L_R/L, L_L/L$
δ	$\gamma_R - \gamma_L = (L_R - L_L)/L$
δ_V	Viscous penetration depth: $(2\nu/\omega)^{1/2}$
η	Reduced displacement: $h/\bar{H}_S$
ν	Kinematic viscosity
ρ	Fluid mass density
τ	Wall shear stress
ω	Frequency (rad/sec): $2\pi f$
Ω_S	Reduced frequency: $\omega L/\bar{U}_S = 1/V_S$

Subscript References

U,I,D	Upstream, intermediate, and downstream constrictions
L,R	Left and right part of the channel
S	Normalization parameters

A,B	Subscripts which may be U, I, D, L, or R
t	Identifies a time function of integration
c	Identifies a constant of integration

APPENDIX II: DERIVATION DETAILS

The continuity equations (22-23) and the velocity expressions (11-12) can be combined to determine the velocities in either part of the channel and in the upstream and downstream constrictions as a function of the unknown velocities $\bar{U}_I$ and u_I in the intermediate constriction. In particular,

$$\bar{U}_A = \alpha_A \bar{U}_I \quad (A1)$$

and

$$u_A = -\alpha_A \left(\frac{dh}{dt} / \bar{H}_I \right) x + \alpha_A [u_I + (1 - \alpha_A) \bar{U}_I (h / \bar{H}_I)] , \quad (A2)$$

where $\alpha_A = \bar{H}_I / \bar{H}_A$ and the subscript A is set equal to R or L for the right or left half of the channel, respectively. For the constrictions,

$$\bar{U}_B = \alpha_B \bar{U}_I \quad (A3)$$

and

$$u_B = \alpha_B [u_I + (1 - \alpha_B) \bar{U}_I (h / \bar{H}_I) - (x_B / \bar{H}_I) \frac{dh}{dt}] , \quad (A4)$$

where $x_D = L_R$ and $x_U = L_L$ for subscripts B = D and B = U, respectively.

Evaluation of the boundary conditions (16-18) at the intermediate constriction by means of Eqs. (A1-A4) provides

$$(\bar{P}_{cL} - \bar{P}_{cR}) / \rho \bar{U}_S^2 = \frac{1}{2} (\beta_R^2 - \beta_L^2) + \frac{1}{2} K_I \beta_I^2 \quad (A5)$$

and

$$\begin{aligned} (p_{tL} - p_{tR}) / \rho \bar{U}_S^2 &= (\beta_R^2 - \beta_L^2 + K_I \beta_I^2) v + k_I \beta_I^3 \eta \\ &+ [\beta_R^2 (\beta_I - \beta_R) - \beta_L^2 (\beta_I - \beta_L)] \eta , \end{aligned} \quad (A6)$$

where $\bar{U}_S$ is a selected reference velocity associated with the channel width $\bar{H}_S$; $\beta_R = \bar{H}_S / \bar{H}_R$, for instance; $v = u_I / \bar{U}_I$; and $\eta = h / \bar{H}_S$. At the upstream

constriction, the same type of evaluation of the boundary conditions (16, 17, 19) provides

$$(\bar{P}_{cL} - \bar{P}_U)/\rho \bar{U}_S^2 = -\frac{1}{2} \beta_L^2 - \frac{1}{2} K_U \beta_U^2 - K_L \beta_L^2 \quad (A7)$$

and

$$P_{tL}/\rho \bar{U}_S^2 = -E_1 v - E_2 \Omega_S \dot{v} - E_3 \eta - E_4 \Omega_S \dot{\eta} - E_5 \Omega_S^2 \eta'', \quad (A8)$$

where $\Omega_S = \omega L/\bar{U}_S$, $K_L = C_{LL}/H_L$, $(\dot{}) = d()/d(\omega t)$, $\ell_L = 1/\gamma_L$, and

$$\begin{aligned} E_1 &= 2K_L \beta_L^2 + \beta_L^2 + K_U \beta_U^2, \\ E_2 &= \beta_L \ell_L, \\ E_3 &= K_L \beta_L (2\beta_I - 3\beta_L) + \beta_L^2 (\beta_I - \beta_L) + K_U \beta_U^2 (\beta_I - \beta_U), \\ E_4 &= \ell_L [\beta_L (\beta_I - \beta_L) + K_L \beta_L^2 + K_U \beta_U^2], \\ E_5 &= \frac{1}{2} \ell_L^2 \beta_L. \end{aligned} \quad (A9)$$

At the downstream constriction, the same type of evaluation of the boundary conditions (16, 17, 20) provides

$$(\bar{P}_{cR} - \bar{P}_D)/\rho \bar{U}_S^2 = -\frac{1}{2} \beta_R^2 + \frac{1}{2} K_D \beta_D^2 + K_R \beta_R^2 \quad (A10)$$

and

$$P_{tR}/\rho \bar{U}_S^2 = -G_1 v - G_2 \Omega_S \dot{v} - G_3 \eta - G_4 \Omega_S \dot{\eta} - G_5 \Omega_S^2 \eta'', \quad (A11)$$

where $K_R = C_{RR}/\bar{H}_R$,

$$G_1 = -2K_R \beta_R^2 + \beta_R^2 - K_D \beta_D^2 ,$$

$$G_2 = -\beta_R \ell_R ,$$

$$G_3 = -K_R \beta_R^2 (2\beta_I - 3\beta_R) + \beta_R^2 (\beta_I - \beta_R) - K_D \beta_D^2 (\beta_I - \beta_D) , \quad (A12)$$

$$G_4 = \ell_R [-\beta_R (\beta_I - \beta_R) + K_R \beta_R^2 + K_D \beta_D^2] ,$$

$$G_5 = \frac{1}{2} \ell_R^2 \beta_R ,$$

and $\ell_R = L_R/L$.

Equations (A5, A7, A10) constitute a system of three equations which can be solved for the three steady-state unknowns $\bar{U}_I$, $\bar{P}_{cR}$, and $\bar{P}_{cL}$. Expressions (24-26) give $\bar{U}_I$, while

$$\bar{P}_{cL}/\rho \bar{U}_S^2 = \bar{P}_U/\rho \bar{U}_S^2 - \frac{1}{2} \beta_L^2 - \frac{1}{2} K_U \beta_U^2 - K_L \beta_L^2$$

and (A13)

$$P_{cR}/\rho \bar{U}_S^2 = \bar{P}_D/\rho \bar{U}_S^2 - \frac{1}{2} \beta_R^2 + \frac{1}{2} K_D \beta_D^2 + K_R \beta_R^2 .$$

Equations (A6, A8, A11) constitute a system of three equations which can be solved for the three unknown functions v , p_{cR} , and p_{cL} . Eliminating the fluctuating pressure functions in Eq. (A6) by using Eqs. (A8) and (A11) gives a linear ordinary differential equation for v :

$$R_2 \Omega_S^2 v + R_1 v = R_5 \Omega_S^2 \eta + R_4 \Omega_S \dot{\eta} - R_3 \ddot{\eta} , \quad (A14)$$

where

$$R_1 = E_1 - G_1 + \beta_R^2 - \beta_L^2 + K_I \beta_I^2 ,$$

$$R_2 = E_2 - G_2 ,$$

$$R_3 = E_3 - G_3 + \beta_R^2(\beta_I - \beta_R) - \beta_L^2(\beta_I - \beta_L) + \beta_I^3 k_I , \quad (A15)$$

$$R_4 = -E_4 + G_4 ,$$

$$R_5 = -E_5 + G_5 .$$

Substitution of Eqs. (A9) and (A12) in Eq. (A15) gives the expanded expressions (26, 29) for the parameters R_1 to R_5 . Only the particular (steady-state) solution of Eq. (A14), given in Eqs. (26-29), is of interest for steady-state motion of the channel wall.

With u_I and $\bar{U}_I$ known, u_{tR} can be determined from Eq. (A2) at $x = 0$, and p_{tL} and p_{tR} can be determined from Eqs. (A8) and (A11). With all four of the functions of integration known, the pressure in the channel is found with Eq. (14) to be

$$p = -c_{dR} m \omega \frac{dh}{dt} - c_{mR} m \frac{d^2 h}{dt^2} \quad x \geq 0 , \quad (A16)$$

$$p = -c_{dL} m \omega \frac{dh}{dt} - c_{mL} m \frac{d^2 h}{dt^2} \quad x < 0 ,$$

where $m = \rho \bar{H}_V \rho (\bar{H}_L L_L + \bar{H}_R L_R) / L$. Also, with $V_S = \bar{U}_S / \omega L$ and $\gamma_R = L_R / L$, for instance,

$$\begin{aligned} c_{dR} &= (L/\bar{H}_V)(L/\bar{H}_S) V_S [G_1 T_2 - G_2 T_1 + G_4] \\ &\quad + \{2K_R \beta_R^2 T_2 - \beta_R \gamma_R T_1 + b_R(\beta_I - 2\beta_R) \gamma_L\} x / L_R - \{K_R \beta_R^2 \gamma_R\} x^2 / L_R^2 ; \\ c_{mR} &= (L/\bar{H}_V)(L/\bar{H}_S) [(G_1 T_1 V_S^2 + G_2 T_2 - G_3 V_S^2 + G_5) - \{\frac{1}{2} \beta_R \gamma_R^2\} x^2 / L_R^2] \\ &\quad + \{\beta_R \gamma_R T_2 + 2K_R \beta_R^3 T_1 V_S^2 - K_R \beta_R^2 (2\beta_I - 3\beta_R) V_S^2\} x / L_R . \end{aligned} \quad (A17)$$

The expressions for c_{dL} and c_{mL} can be found from Eqs. (A16) and (A17) for c_{dR} and c_{mR} , respectively, by letting the subscript $R \rightarrow L$ and $(G_1, \dots, G_5) \rightarrow (E_1, \dots, E_5)$.

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