A REVIEW OF PARAMETERIZATIONS OF MICROPYHICAL PROCESSES IN CLOUDS FOR APPLICATION IN MODELS OF REGIONAL ATMOSPHERIC DEPOSITION

by

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ABSTRACT

The literature on parameterizations of cloud microphysical processes was reviewed to examine the theoretical basis of those parameterizations and to evaluate their applicability to regional models. The constant distribution functions of hydrometeors assumed in previous work were found to be unrealistic. We obtained new parameterizations for cloud microphysical processes (i.e., condensation, evaporation, autoconversion, and accretion) by a multiple regression on mixing ratios of water vapor, cloud water, and rainwater. The solution fields used were derived from simulations of a cloud model that incorporates sophisticated treatments of microphysical processes and cloud dynamics. The new parameterizations were found to give similar results and to be computationally more efficient when they were compared with those currently available through use of a Lagrangian air-parcel model under variable atmospheric conditions.

1 INTRODUCTION

The motivation for this study came from the perceived need for more-efficient parameterizations of cloud microphysical processes for simulation of cloud acidification. Such computationally efficient parameterizations are highly desirable for use in regional models applicable to mesoscale clouds whose dimensions characteristically range from 10 km to 100 km.

Parameterization of cloud microphysical processes was initiated by Kessler (1967, 1969) and has been explored thereafter by many investigators, including Cotton (1972), Berry and Reinhardt (1974a, 1974b, 1974c, 1974d), Yau and Austin (1979), and Lin et al. (1983). In general, the parameterizations were derived by assuming that hydrometeors could be distributed according to a simple density function and that bulk terminal velocities of cloud water and rainwater could be expressed as functions of their respective mixing ratios. Formulas based on these assumptions, however, may lead to improper distribution of hydrometeors and unrealistic dynamic fields within the life cycle of a cloud system. For example, assuming an exponential size distribution for rainwater (Kessler, 1969; Lin et al., 1983) may result in incorrect precipitation predictions during various stages of cloud development because of oversimplified representations of cloud
droplets and raindrops as functions of cloud and rainwater mixing ratios. Numerical studies by Lee et al. (1986) show that variations in the chemical and physical properties of aerosols associated with particular air mass types can produce clouds that are different in terms of drop-size distributions. These distributions are critical for the initiation of rain by collision and coalescence.

Evolution of the cloud droplet spectrum depends on the microphysical and chemical properties of aerosols and on the dynamic processes that give rise to the general conditions for cloud formation. Bulk dynamic analysis (Kessler, 1969; Yau and Austin, 1979; Lin et al., 1983) or determination of the quasi-saturation state of moisture content (Asai, 1965) may produce a cloud water content quite different from that calculated from droplet spectral evolution. Assuming that the effective fall velocity depends only on rainwater content and height (Kessler, 1969) may oversimplify the complicated nature of cloud and precipitation physics and lead to inaccurate calculations of precipitation reaching the surface. Indeed, the bulk terminal velocity of rainwater depends on the rainwater mixing ratio, the density of air, and the shape of the raindrop spectrum that evolves during cloud development.

It appears highly desirable to construct parameterizations using data obtained from laboratory, field, or numerical experiments under less restrictive conditions. We developed new parameterizations of cloud microphysical processes based on fields of data from simulations using a sophisticated cloud model that treats the physics of aerosols, clouds, and precipitation. The cloud model computes the evolution of drop spectra, considering dynamic processes such as vertical advection, dynamic entrainment, and lateral eddy mixing, and microphysical processes such as activation of cloud condensation nuclei (Fitzgerald, 1973; Hänel, 1976), diffusional growth of droplets, and stochastic collection of liquid drops (Berry, 1967). The system's moisture content is differentiated into water vapor, cloud droplets, and raindrops. The ice phase is not examined. Cloud droplets are defined as drops smaller than 50 \( \mu m \) in radius; larger drops are defined as raindrops. The rates for condensation, evaporation, autoconversion, and accretion are determined in terms of bulk parameters.

In Sec. 2, the cloud microphysics parameterizations currently available are examined as to how well they conform to the theoretical bases of cloud and precipitation physics and as to their applicability to regional models. In Sec. 3, the treatments of cloud dynamics and microphysics incorporated into the cloud model are theoretically described, and mathematical formulations are given for the rates of condensation and evaporation of cloud and rain and for the rates of autoconversion and accretion. In Sec. 4, the initial and boundary conditions, computational procedures, and new formulas obtained from optimization methods are presented. Finally, in Sec. 5, the rates produced by the new parameterizations are compared with those from more-detailed simulations, and the behaviors of different formulas are examined with the aid of a Lagrangian air-parcel model.
The distribution of atmospheric moisture is governed by dynamic processes that create the general conditions for the formation of clouds and precipitation and by microphysical processes that control the growth of individual precipitation particles from the gas phase by condensation or deposition and from smaller cloud particles by collision and coalescence. The two types of processes interact strongly because an upward motion originally attributable to dynamic processes creates atmospheric conditions suitable for microphysical processes to proceed, while simultaneously being modified by the latent heat released by condensation and the drag of particles. The dynamic and microphysical processes proceed concurrently in nature, even though strong disparities exist in time scales. The time scales associated with dynamic processes are typically much greater than those for cloud microphysics. Therefore, computational economy must also be considered in numerical simulations of the distribution of moisture over a region.

Atmospheric moisture occurs as vapor, liquid, or ice. The liquid and ice phases can be further differentiated by size, opacity, and shape of the precipitation particles. Although size is the significant difference between cloud droplets and raindrops, size, opacity, and shape can all be considered in classifications of ice-phase particles. Microphysical interactions occur among the many types of liquid or solid water particles in clouds. These interactions can be categorized as condensation of water vapor to form cloud droplets, evaporation of cloud droplets, conversion of cloud droplets to raindrops, evaporation of raindrops, sublimation of water vapor to form ice crystals, glaciation, freezing of raindrops, accretional growth of ice crystals, melting of ice crystals, and sublimation of ice crystals.

Most of the parameterization formulas for cloud microphysical processes in current use were derived under the assumption that the distribution functions for liquid or ice particles are either given or provided from observations (Marshall and Palmer, 1948; Gunn and Marshall, 1958). Widely used by many investigators, including Kessler (1967), is the Marshall-Palmer distribution (Marshall and Palmer, 1948), an empirical formula defined as:

\[ n = n_o e^{-\lambda d} \Delta d \]

where:

- \( d \) = particle diameter,
- \( n \) = number concentration for sizes between \( d \) and \( d + \Delta d \),
- \( n_o \) = constant, and
- \( \lambda \) = constant.
2.1 LIQUID-VAPOR INTERACTIONS

Asai (1965) presents a saturation adjustment scheme in order to properly distribute the excess water vapor due to supersaturation into cloud droplets and cloud air so that the air in the cloud remains exactly saturated. His formula can be written as:

\[
\delta Q_c = (Q_v - Q_s) \left[ 1 + \frac{L_v^2 Q_s}{c_v R_v T^2} \right]^{-1}
\]

where:

- \( Q_c \) = liquid water mixing ratio produced from condensation,
- \( Q_v \) = actual water vapor mixing ratio in the cloud,
- \( Q_s \) = saturation water vapor mixing ratio,
- \( L_v \) = latent heat of evaporation,
- \( c_v \) = specific heat of water vapor,
- \( R_v \) = gas constant for water vapor, and
- \( T \) = absolute temperature.

Condensation warms the cloud environment because of the release of latent heat, and the saturation value increases in response to the higher temperature. Therefore, the calculation must be iterated for accuracy.

Because the supersaturation ratio in clouds rarely exceeds 1% and usually is smaller than about 0.1%, the saturation adjustment scheme is widely used by modelers (e.g., Asai and Kasahara, 1967; Ogura and Takahashi, 1971). However, studies by Lee et al. (1980) show that characteristic differences in the cloud condensation nuclei spectra with air mass type can produce quite different droplet distributions, which are critical for the initiation of rain by collision and coalescence. Therefore, effects related to the chemical composition and physical properties of aerosols should be considered. Kessler (1967) presents a parameterized relationship for the rate of condensation based on a prescribed profile of vertical velocity to produce liquid water as a function of height. The obvious limitation in his formula is that updraft velocity profiles are often vastly different from one cloud to another. The condensation function is therefore arbitrary.

Condensation results in rapid growth of drops to the size of average cloud drops, say 10 \( \mu \)m in radius; however, growth rates become progressively too slow for millimeter-sized drops to be produced by condensation alone. Cloud particles have been shown to grow rapidly to precipitation size by collection (Twomey, 1966; Brazier-Smith et al., 1972; Neiburger et al., 1974) and by a three-phase process (Koenig, 1971). The collection
process occurs when falling larger drops overtake and capture smaller droplets. The three-phase, or Bergeron, process occurs because drops remain liquid at temperatures below freezing while ice crystals exist. Because the equilibrium vapor pressure over ice is lower than that over water at the same temperature, there is a strong gradient of vapor density away from the liquid drops toward the ice crystals. Water is rapidly transferred from the evaporating drops to the ice crystals, which quickly grow large compared with the supercooled drops.

For computational purposes, the conversion of cloud droplets to raindrops can be computed in two steps: autoconversion and accretion. Autoconversion is assumed to occur when large cloud drops, say 50 μm in radius, are produced by condensation and collection. Kessler (1967, 1969) proposes the following formula for the rate of autoconversion:

\[
\frac{dQ}{dt} = G(Q - \alpha),
\]

where \( \alpha \) is the threshold value below which cloud conversion does not occur, and \( G \) is a numerical constant. Kessler suggests 0.0005 g g\(^{-1}\) and 10\(^{-3}\) s\(^{-1}\) for \( \alpha \) and \( G \), respectively. Kessler assumes that the autoconversion rate is a linear function of \( Q_c \) and can be initiated when \( Q_c \) exceeds its threshold value. Berry (1968), Cotton (1972), and Berry and Reinhardt (1974d) propose other formulas for autoconversion based on numerical studies of droplet spectral evolution. Berry’s formula is based on computations of the flux of water passing a raindrop 40 μm in radius. Simpson and Wiggert (1969) propose a slightly different formula based on the analysis of water flux passing a larger raindrop 100 μm in radius. Silverman and Glass (1973) found that the choice of minimum precipitation particle size and the distribution parameters for the assumed cloud droplet spectrum are not important. Cotton’s formula for autoconversion rate is time-dependent and varies widely as a function of spectral properties. The formulas of Berry (1968) and Cotton (1972) cannot be used directly in regional models where the parameters associated with the spectral properties are not readily available. Berry and Reinhardt (1974d) propose an advanced formula, which is a modified version of Berry’s (1968) formula:

\[
\frac{dQ}{dt} = A Q_o^{2/3} [1 - (B/Q_o)^{4/3}] [1 - C/Q_o]^{1/3},
\]

where:

\( Q_o \) = mass of average cloud drop,

\( A = \) constant (5.5 \( \times \) 10\(^{-12}\)),

\( B = \) constant (2.5 \( \times \) 10\(^{-9}\)), and

\( C = \) constant (2.04 \( \times \) 10\(^{-9}\)).

Equation 2-4 also requires knowledge of the distribution function for cloud droplets.
Accretion of cloud droplets by raindrops is due solely to hydrodynamic capture between drops. Kessler (1967, 1969) proposes a formula for the accretion rate as a function of cloud water and rainwater content, and a linear collision efficiency. His formula can be written as:

\[
\frac{dQ_r}{dt} = 928.26EQ_c(p_aQ_r)^{0.875},
\]

(2-5)

where \( E \), for which he suggests a value of unity, is the linear collection efficiency, and \( p_a \) is the air density. His formula is based on the assumption that raindrops are distributed according to an inverse exponential law as given by Eq. 2-1. Scott (1982) modified Kessler's formula by reducing the linear collection efficiency value from 1.0 to 0.65. Lin et al. (1983) derived a formula for the rate of accretion of cloud water that considered the increase in terminal velocity with height. With suggested values for collection efficiency and parameters for the raindrop spectrum, their formula can be simplified to:

\[
\frac{dQ_r}{dt} = 2316.4Q_c(p_aQ_r)^{0.95}(\frac{\rho_o}{\rho_a})^{0.5},
\]

(2-6)

where \( \rho_o \) is the air density at the surface. Note that all the formulas for accretion rates are derived under the assumption that the raindrop spectrum can be described by a simple function and that the distribution function is invariable during cloud development.

The evaporation rate of raindrops is much smaller than that of cloud droplets. Ogura and Takahashi (1971) present a formula for evaporation of raindrops based on the assumption that the raindrop spectrum follows the Marshall-Palmer distribution during evaporation. With the ventilation term, the equation can be written as:

\[
\frac{dQ_r}{dz} = \frac{F[(Q_v/Q_s) - 1](p_aQ_r)^{0.525}}{\rho_a[5.4 \times 10^5 + (4.1 \times 10^6/e_{ws})]},
\]

(2-7)

where \( F \) is the ventilation coefficient, and \( e_{ws} \) is the saturation vapor pressure in millibars with respect to water. The ventilation coefficient can be computed as:

\[
F = 1.6 + 7.6 \times 10^{-4}V^{1.5},
\]

(2-8)

where \( V \) is the terminal velocity of raindrops, described as:

\[
V = 3120(p_aQ_r)^{0.125}.
\]

(2-9)

Orville and Kopp (1977) present a parameterized formula for the rate of rainwater evaporation based on concepts of particle growth by water vapor diffusion. While their formula contains many variables and coefficients associated with the characteristic nature of raindrop distributions, the formula can be simplified by
incorporating suggested values for parameters associated with the raindrop spectrum to give (Lin et al., 1983):

\[
\frac{dQ_r}{dt} = \frac{0.503\left(\frac{Q_r}{Q_s} - 1\right) [1.56\left(\frac{Q_r}{Q_a}\right)^{0.5} + 157.8\left(\frac{Q_r}{Q_a}\right)^{0.25}\left(\frac{Q_r}{Q_a}\right)^{0.725}]}{\rho_a \left(\frac{L_v}{\kappa R v T^2} + \frac{1}{\rho_D D_{sw}}\right)},
\]

(2-10)

where \( \kappa \) is the thermal conductivity of air, and \( D_{sw} \) is the diffusivity of water vapor in air. Rutledge and Hobbs (1983) also present a formula similar to that derived by Orville and Kopp (1977) for the evaporation rate of rainwater. However, the formulas cannot properly reflect the effect of spectral evolution on rainwater evaporation, as the raindrop spectra vary with time and height during the storm period and depart significantly from Marshall-Palmer size distributions (Silverman and Glass, 1973). Furthermore, as raindrops evaporate, atmospheric humidity increases and some of the raindrops become cloud droplets. Therefore, the formulas should allow for conversion of some rain to cloud water through evaporation.

Parameterization formulas for warm-cloud microphysical processes are compared in Lagrangian air-parcel calculations and presented in Sec. 5. The results of the literature review can be summarized as follows:

1. Rates for cloud microphysical interactions have been derived under the assumption that the size distribution functions are given.

2. Rates for condensation or evaporation of cloud water have been computed for a quasi-saturated state.

3. Rates for autoconversion of cloud water have been expressed as functions of cloud water content and spectral properties not readily available in regional models.

4. Rates for accretion of cloud water have been expressed as functions of cloud water content, rainwater content, and collection efficiency that varies from one formula to another.

5. Rates for evaporation of rainwater have been expressed as functions of various dynamic and thermodynamic variables.

6. Parameterization formulas must be applied with caution because the spectral evolution of both cloud droplets and raindrops may depart significantly from the assumed spectra such as the Marshall-Palmer distribution.
2.2 INTERACTIONS WITH THE ICE PHASE

The parameterization problem becomes even more complicated with the inclusion of the ice phase, which is crucial in the life cycle of convective storms (Danielsen et al., 1972; Wisner et al., 1972; English, 1973). Ice particles can be differentiated into cloud ice, snow, graupel, and hail, depending on structural configuration. Their spectra evolve in a complex cloud system with multiphase water substances and strong convective instability. Not surprisingly, few parameterizations of ice-particle evolution are available. Furthermore, existing formulas have limited applicability to regional models because they were derived mostly under the assumption that the various types of ice particles have specified size distributions and that certain relevant microscale parameters, such as terminal velocity and collision efficiency of different types of ice particles, are not readily available from most of the macroscale models.

The rates for deposition of water vapor to form cloud ice or sublimation of cloud ice have been investigated by, among others, Byers (1965), Thorpe and Mason (1966), Ogura and Takahashi (1971), Stephens (1979), and Lin et al. (1983). By combining the sublimation formula presented by Mason (1953, 1971) with the Marshall-Palmer size distribution, Ogura and Takahashi (1971) derived a formula as functions of air density, water vapor mixing ratio, saturation mixing ratio of water vapor, and saturation vapor pressure over a plane ice surface. Lin et al. (1983) present formulas having different expressions, depending on the forms of ice particles, with increased complexity of the characteristic parameters being associated with their size distribution and terminal velocity. However, the formulas can be simplified substantially by inserting suggested values for microscale parameters (Beard and Pruppacher, 1971; Lin et al., 1983) and then rearranged to compare closely with the formula proposed by Ogura and Takahashi (1971).

Lin et al. (1983) interpret the aggregation processes, either from cloud ice to snow or from snow to graupel, as similar to the autoconversion concepts proposed by Kessler (1967), with the factor G in Eq. 2-3 being modified to reflect the dependency of crystal structure on temperature. They also propose a variety of accretional growth formulas for snow and graupel interacting with all types of hydrometeors that are based on a geometric sweep-out concept integrated over all sizes for an assumed particle spectrum similar to the Marshall-Palmer distribution as given in Eq. 2-1. However, a numerical analysis by Pitter (1977) shows that collision efficiencies for water drops colliding with ice plates are much smaller than unity, which is a significant departure from the geometric collision efficiency, and that the maximum drop size collected by ice particles is about 5 μm in radius. For these reasons, the formulas in Lin et al. (1983) may overpredict accretional growth for snow and graupel.

Glaciation — the transition from a liquid to a solid phase — can take place when the air temperature drops below freezing. Rates have been proposed by many scientists, including Ogura and Takahashi (1971), Wisner et al. (1972), Hales and Easter (1982), and Lin et al. (1983). Ogura and Takahashi (1971) present a simple formula in which the rate of production of ice crystals is proportional to the rainwater mixing ratio. The formula proposed by Hales and Easter (1982) considers the temperature dependency as well, with the rate of freezing increasing as the temperature decreases below the freezing point and becoming infinite when the temperature drops to -10°C. The glaciation time
assumed by both Ogura and Takahashi (1971) and Hales and Easter (1982) is within the observed value of about 100 s (Koenig, 1963). The formula derived by Wisner et al. (1972) depends on raindrop properties and temperature, which may vary significantly (Barklie, 1959). If values suggested by Lin et al. (1983) are used, the formula by Wisner et al. (1972) can be expressed in terms of air density, temperature, and rainwater mixing ratio.

The melting process, which takes place when the air temperature rises above the melting point, has been studied by Mason (1956, 1971), Wisner et al. (1972), Hales and Easter (1982), and Lin et al. (1983), among many. The theoretical analysis by Mason (1956) shows that the rate of melting of hailstones varies significantly with crystal density, but not with crystal structure. Wisner et al. (1972) include the effect of liquid water accretion on hail or snow and integrate over the Marshall-Palmer size distributions. With the aid of suggested values by Lin et al. (1983), the melting rate can be expressed in terms of macroscale parameters, that is, air density, temperature, snow or graupel mixing ratio, water vapor mixing ratio, saturation mixing ratio of water vapor over plane ice, and the rates of accretion of cloud water and rainwater. However, uncertainties arise because the spectral evolution may not follow the Marshall-Palmer distribution law. Hales and Easter (1982) propose a simple formula in which the melting rate increases linearly with the increase in snow mixing ratio. A temperature dependency is also included in their formula so that the melting process is augmented as the temperature increases. They assume that all snow melts if the temperature is higher than 5°C.

Orville and Kopp (1977) interpret the Bergeron-Findelsen process as a mass transfer from cloud water to ice crystals by deposition and riming. They use the empirical formula of Koenig (1971) to represent the depositional growth mechanism, along with a simple term for accretion of cloud water. However, applying their formula to macroscale simulations is difficult without knowledge of the characteristic properties of ice crystals. Hsie et al. (1980) present a modified formula based on the growth of an ice crystal 50 μm in radius. The formula can easily be used in macroscale cloud simulations. However, because the 50-μm particles may not adequately represent the size distribution, which varies considerably throughout the stages of cloud development, reservations are necessary.

The results of the review of parameterizations associated with cold-cloud microphysical processes can be summarized as follows:

1. Rates have been derived under the assumption that the particle size distribution functions for the various types of ice crystals are given.

2. Rates for deposition or sublimation have been derived by integrating the growth rate, which may only be applied to a specified ice-crystal form, over all size ranges of an assumed ice crystal spectrum.

3. Rates for autoconversion, either from cloud ice to snow or snow to graupel, have been formulated in a manner similar to that for autoconversion of cloud water to rainwater.
4. Accretion rates have been based on the concept of geometrical sweep-out and on the assumption that the particles follow the distributions of the Marshall-Palmer type. Uncertainties exist because the collection efficiencies between various types of hydrometeors are much smaller than unity and because the specified size distributions may be significantly in error for some stages of cloud development.

5. Both the glaciation and melting rates have been expressed in simple forms in terms of macroscale parameters. Uncertainties arise from the specification of the spectral evolution and from inadequate knowledge of the behavior of ice crystals.

6. Rates for three-phase processes are formulated in terms of cloud water mixing ratio, temperature, and the many hypothetical spectral properties of ice crystals.
Developing parameterizations of microphysical processes by optimization methods requires many data, either from observations or from numerical analyses. For this study, the data set was provided from numerical calculations with a warm-cloud model that treats cloud dynamics and sophisticated microphysics of liquid-vapor interactions. Section 3 presents the theoretical descriptions used in the cloud model.

The microphysical processes associated with development of clouds and rain are mainly condensation, evaporation, autoconversion, and accretion, as shown in Fig. 1. The ice phase is neglected in this study. Cloud droplets and raindrops are described, the former in nine size categories ranging from 1.0 μm to 40.32 μm in radius and the latter in nine larger sizes (Table 1). This subdivision generally agrees with that of Berry and Reinhardt (1974a, 1974b, 1974c, 1974d), who studied spectral evolution resulting from the stochastic collection process and found a 50-μm radius to be an appropriate dividing point between cloud droplets and raindrops.

The governing equation for the diffusional growth rate of a drop with radius $r_i$ can be expressed as:

$$
\frac{dr_i}{dt} = \left[ s - \frac{2\sigma M_w}{\rho_w RT_i} + \frac{\Delta \rho_n r_n^3}{\rho_w (r_i^3 - r_n^3)} \right] \left[ \frac{L_v \rho_w}{\kappa T} \left( \frac{L_v}{RT} - 1 \right) + \frac{\rho_w R}{D M e_s} \right],
$$

where:

- $s = \text{supersaturation ratio}$,
- $M_w = \text{molecular weight of water}$,
- $e_s = \text{saturation vapor pressure over pure liquid water at temperature } T$,
- $R = \text{universal gas constant}$,
- $\sigma = \text{surface tension of aqueous solution against air}$,
- $r_n = \text{aerosol radius}$,
- $\rho_n = \text{bulk density of aerosols}$, and
- $\rho_w = \text{density of water}$. 

### Section 3: CLOUD MICROPHYSICS AND DYNAMICS
FIGURE 1  Schematic Diagram of Microphysical Interactions among Hydrometeors

TABLE 1  Size Categories for Hydrometeors

<table>
<thead>
<tr>
<th>Hydrometeor</th>
<th>Bin Number</th>
<th>Median Radius (μm)</th>
<th>Fall Velocity (cm s⁻¹)ᵃ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cloud</td>
<td>1</td>
<td>1.00</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.59</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2.52</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>4.00</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>6.35</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>10.08</td>
<td>1.25</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>16.00</td>
<td>3.12</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>25.40</td>
<td>7.66</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>40.32</td>
<td>17.86</td>
</tr>
<tr>
<td>Rain</td>
<td>10</td>
<td>64.00</td>
<td>38.10</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>101.59</td>
<td>73.33</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>161.27</td>
<td>129.15</td>
</tr>
<tr>
<td></td>
<td>13</td>
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<td>214.65</td>
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<td>1625.00</td>
<td>900.00</td>
</tr>
<tr>
<td></td>
<td>18</td>
<td>2580.32</td>
<td>990.00</td>
</tr>
</tbody>
</table>

ᵃValues are computed at 1000 mb.
Equation 3-1 is similar to those of Howell (1949) and Fitzgerald (1974), except for the treatment of the solute effect. The hygroscopicity of aerosols \( \beta_n \) is determined from the following empirical formula (Hanel, 1972):

\[
\beta_n = B_o + B_1 \left( \frac{m_n}{m_w} \right)^{1/2} + B_2 \left( \frac{m_n}{m_w} \right) + B_3 \left( \frac{m_n}{m_w} \right)^{3/2},
\]  
(3-2)

where the \( B \)'s are constants, and \( m_n \) and \( m_w \) are the aerosol mass and the water uptake of the aerosol deposit, respectively.

The time evolution of the droplet number density as a result of condensation is governed by the spray equation (Arnason and Greenfield, 1972):

\[
\frac{\partial n_i}{\partial t}_c = - \frac{3}{\partial r_i} \left( n_i \frac{\partial r_i}{\partial t} \right).
\]  
(3-3)

Condensation and evaporation rates for cloud droplets and raindrops are computed using Eqs. 3-1 through 3-3. When the supersaturation ratio is greater than zero, the cloud droplets and raindrops grow by diffusion. Evaporation occurs when the air is unsaturated. Cloud condensation and evaporation rates (g g\(^{-1}\) s\(^{-1}\)), \( CC \) and \( EC \) in Fig. 1, can be expressed as:

\[
CC = \frac{\rho_w}{\rho_a} \sum_{i=1}^{I} v_i \frac{3}{\partial r_i} \left( n_i \frac{\partial r_i}{\partial t} \right), s > 0, \text{ and }
\]  
(3-4)

\[
EC = - \frac{\rho_w}{\rho_a} \sum_{i=1}^{I} v_i \frac{3}{\partial r_i} \left( n_i \frac{\partial r_i}{\partial t} \right), s < 0,
\]  
(3-5)

where \( I \) is the number of cloud droplet bins, and \( v_i \) is the volume of a droplet in the size bin \( i \). Likewise, the condensation and evaporation rates (g g\(^{-1}\) s\(^{-1}\)) for rain can be expressed as:

\[
CR = \frac{\rho_w}{\rho_a} \sum_{i=I+1}^{J} v_i \frac{3}{\partial r_i} \left( n_i \frac{\partial r_i}{\partial t} \right), s > 0, \text{ and }
\]  
(3-6)

\[
ER = - \frac{\rho_w}{\rho_a} \sum_{i=I+1}^{J} v_i \frac{3}{\partial r_i} \left( n_i \frac{\partial r_i}{\partial t} \right), s < 0,
\]  
(3-7)

where \( J \) is the number of cloud droplet and raindrop bins combined.
The time evolution of the droplet number density as a result of coagulation can be written in discrete form as:

\[
\frac{dn_i}{dt} = -n_i \sum_{j=1}^{J} K_{ij} n_j - K_{ii} n_i^2 + \sum_{j=1}^{i} K_{ij} n_i n_j \left( \frac{v_{i+1} - v_i - v_j}{v_{i+1} - v_i} \right) \]

\[
+ \sum_{j=1}^{i-1} K_{i-1,j} n_{i-1} n_j \left( \frac{v_j}{v_i} - \frac{v_{i-1} + v_j}{v_i} \right) \left( \frac{1}{v_i} \right). \tag{3-8}
\]

The autoconversion rate, defined as the total mass transfer rate from the largest cloud droplet to the smallest raindrop by condensation and coagulation, can be expressed in discrete form as:

\[
AU = \frac{\rho_w}{\rho_a} v_i \left[ -n_i \sum_{j=1}^{I} K_{ij} n_j - K_{ii} n_i^2 \right. \\
+ \sum_{j=1}^{I} K_{ij} n_i n_j \left( v_r + v_j \right) \left( \frac{v_{i+1} - v_i - v_j}{v_{i+1} - v_i} \right) \left( \frac{1}{v_i} \right) \left. \right] \tag{3-9}
\]

\[
- \frac{\rho_w}{\rho_a} v_i \frac{3}{\partial r_I} \left( n_i \frac{dr}{dt} \right) .
\]

In a similar way, the accretion rate can be expressed as:

\[
AC = \frac{\rho_w}{\rho_a} \sum_{k=1}^{I} v_k \left[ -n_k \sum_{j=1}^{J} K_{kj} n_j \right] \tag{3-10}
\]

In Eqs. 3-8 to 3-10, \( K \) denotes the collection kernel, which is defined as:

\[
K_{ij} = \pi (r_i + r_j)^2 E_{ij} (V_i - V_j) . \tag{3-11}
\]

The values of \( E \) are subject to considerable uncertainty. Brownian coagulation is most important for particles of about one micrometer or smaller, whereas turbulence and gravity control coagulation for particles larger than a few micrometers (Pruppacher and Klett, 1980). The collection kernel for Brownian coagulation is neglected for now because the smallest cloud droplets are usually larger than the modal size because of Brownian diffusion. In general, the collision efficiency for two drops of similar size is large because of aerodynamic capture (wake effect); however, the probability of producing a larger drop approaches zero because of the small relative terminal velocity between two nearly identical drops (Schotland, 1957; Neiburger, 1967).
The collision efficiency \( E \) is determined from a formula proposed by Neiburger et al. (1974). Berry (1967) recognizes the convenience of having an analytical expression for the collection efficiency and presents a formula that fits the values given in Shafrir and Neiburger (1963) and Neiburger (1967) fairly well. Later, Scott and Chen (1970) developed somewhat less complicated formulas to represent the collision efficiency. Neither Berry nor Scott and Chen include wake effects for nearly equal drops. The formula used here is less complicated, represents nearly equal drops better, and can easily be changed to fit improved values of \( E \). The effect of electric fields on collision efficiency (Semonin and Plumlee, 1966) is not considered here. The terminal velocities of cloud droplets and raindrops are computed using the formulas proposed by Beard (1974). The terminal velocity increases with altitude as well as with drop size.

The cumulus cloud model exercised here consists of cloudy and clear regions (Asai and Kasahara, 1967; Lee, 1986): the inner cylindrical column with radius \( a \) corresponds to the cloudy region, and the outer concentric annular column with radius \( b \) corresponds to the surrounding clear region. The areal ratio, defined by \( \sigma^2 = a^2/b^2 \), represents the ratio of the cloud area to the whole cross-sectional area of the model. Interactions between cloudy and clear regions take place through the net buoyancy force, which depends on the difference in virtual temperature between the cloudy and clear regions, the amount of liquid water produced during cloud development, dynamic entrainment, and lateral eddy mixing at the boundary between the cloudy and clear regions. The equations for vertical velocity \( w \), temperature \( T \), water vapor mixing ratio \( q \), and droplet number density \( n_i \) can be written as:

\[
\frac{\partial w}{\partial t} = -w \frac{\partial w}{\partial z} + \frac{2}{a} (w - \bar{w})u - \frac{2u|w|}{2(1 - \sigma^2)} (w - w') + \frac{T_v - T_{vo}}{T_{vo}} g - Q_g , \quad (3-12)
\]

\[
\frac{\partial T}{\partial t} = -w \frac{\partial T}{\partial z} + \frac{2}{a} (T - \bar{T})u - \frac{2u|w|}{a(1 - \sigma^2)} (T - T') - \frac{C_p}{C_p} w + \frac{L_v}{C_p} M , \quad (3-13)
\]

\[
\frac{\partial q}{\partial t} = -w \frac{\partial q}{\partial z} + \frac{2}{a} (q - \bar{q})u - \frac{2u|w|}{a(1 - \sigma^2)} (q - q') - M , \quad (3-14)
\]

\[
\frac{\partial n_i}{\partial t} = -(w - V_i) \frac{\partial n_i}{\partial z} + \frac{n_i w \partial a}{\rho_a} + \frac{2}{a} (n_i - \bar{n}_i)u - \frac{2u|w|}{a(1 - \sigma^2)} (n_i - n_i') \\
+ \left( \frac{\partial n_i}{\partial t} \right)_e + \left( \frac{\partial n_i}{\partial t} \right)_c \quad (3-15)
\]
for the cloudy region, and

\[
\frac{\partial T'}{\partial t} = -w' \frac{\partial T'}{\partial z} - \frac{2a^2}{a(1 - c^2)} (T' - T)u + \frac{2\mu_o \sigma^2 |w|}{a(1 - c^2)^2} (T - T') - \frac{\rho}{c_p} \frac{\partial T'}{\partial t} + \frac{L_v}{c_p} M',
\]

(3-16)

\[
\frac{\partial q'}{\partial t} = -w' \frac{\partial q'}{\partial z} - \frac{2a^2}{a(1 - c^2)} (q' - q)u + \frac{2\mu_o \sigma^2 |w|}{a(1 - c^2)^2} (q - q') - M', \quad \text{and}
\]

(3-17)

\[
\frac{\partial n_i}{\partial t} = -w' \frac{\partial n_i}{\partial z} + \frac{n_i w' \partial \rho_a}{\rho_a} \frac{\partial M}{\partial z} - \frac{2a^2}{a(1 - c^2)} (n_i - \tilde{n}_i)u + \frac{2\mu_o \sigma^2 |w|}{a(1 - c^2)^2} (n_i - \tilde{n}_i)
\]

\[
+ \left( \frac{\partial n_i}{\partial t} \right)_e + \left( \frac{\partial n_i}{\partial t} \right)_c
\]

(3-18)

for the clear region, where primed and tilde variables denote the values in the clear region and at the cloud perimeter, respectively, and:

subscript \(i\) = drop size bin,

\(c_p\) = specific heat of dry air,

\(M\) = rate of condensation,

\(g\) = acceleration of gravity,

\(u\) = entrainment coefficient, and

\(T_v\) = virtual temperature of cloud, and

\(T_{vo}\) = virtual temperature of total modeled region.

If the radial velocity \(u\) at the perimeter of the cloud is less than zero, entrainment is occurring, and any variable defined at the cloud perimeter will carry the environmental values. If \(u\) is greater than zero, detrainment is occurring, and any tilde variable will carry the cloud values. The terms \(\left( \frac{\partial n_i}{\partial t} \right)_e\) and \(\left( \frac{\partial n_i}{\partial t} \right)_c\) in Eqs. 3-15 and 3-18 represent changes in droplet concentration due to microphysical processes of condensation and coagulation, respectively. In Eqs. 3-12 to 3-14, 3-16, and 3-17, the first terms represent the vertical advection, the second terms the dynamic entrainment needed to satisfy mass continuity between the cloud and the environment, and the third terms the lateral eddy mixing at the perimeter of the cloud. The fourth and fifth terms in Eq. 3-12 represent the buoyancy and the liquid water drag, respectively. The first two terms in Eqs. 3-15 and 3-18 represent vertical advection.
The liquid water mixing ratio $Q$ is expressed as:

$$Q = \frac{4\pi}{3} \frac{\rho_w}{\rho_a} \sum_{i} n_i r_i^3.$$  \hspace{1cm} (3-19)

The diagnostic equations for mass continuity and for the relationships of virtual temperatures and of vertical velocities between cloudy and surrounding clear areas can be written as:

$$\frac{2}{a} u + \frac{1}{\rho} \frac{3}{dz} (\rho w) = 0 ,$$  \hspace{1cm} (3-20)

$$\sigma^2 T_v + (1 - \sigma^2) T_v' = T_{v0} , \text{ and}$$  \hspace{1cm} (3-21)

$$\sigma^2 w + (1 - \sigma^2) w' = 0 .$$  \hspace{1cm} (3-22)

In Eq. 3-20, $\rho$ denotes the average air density of the total area, including the cloudy and clear regions. In this hydrostatic model, the integral properties of the system cloud-plus-environment, such as total water substance ($q + Q$) and moist static energy ($c_p T + gz + L_v$), are conserved approximately, if the amount of precipitation reaching the ground surface can be ignored. The conservation laws are significantly disturbed when compensating downward motions in the environment are ignored because no dynamic entrainment can be considered.
The initial soundings for thermal and moisture structures are presented in Table 2. Two simulations were made using two slightly different conditions in moisture content, as shown in cases A and B. Case B is designed to reduce the evaporation of raindrops as they fall through the subcloud layer. In the subcloud layer, the temperature decreases dry adiabatically, and the relative humidity increases, reaching about 95% at the cloud base. Dry adiabatic lapse rates are most frequent in the subcloud regions (Dugan, 1973). A dry inversion layer is assumed at 500 mb in order to inhibit extensive cloud development. Cloud formation and dissipation mostly occur between the subcloud and inversion layers, where the temperature and relative humidity decrease initially, with a lapse rate of about 5-6°C km⁻¹ and about 5% km⁻¹, respectively. An impulse of the type \( w = \sin[w(z - z_0)/z_0] \) is assumed in the lower portion of the cloudy region (900-2100 m from the surface) in order to initiate cloud development. \( z_0 \) (= 900 m) is the height of the initial cloud base, and the maximum vertical velocity of 1 m s⁻¹ occurs at 1500 m.

Initially, the droplet number density is set to be 968 cm⁻³ at the surface and to decrease with height with the decrease in air density. The modal size and concentration are 1.0 μm in radius and 560 cm⁻³, respectively, at the surface. Aerosol characteristics are assumed to be uniform with respect to chemical composition throughout the size range; 70% by weight of the aerosol is assumed to consist of solubles. The aerosol content in droplets is determined initially under the assumption that the condensation nuclei are in equilibrium with respect to the partial pressure of water vapor (Lee et al., 1980). Therefore, the initial aerosol radius \( r_n \) is determined from:

\[
s - \frac{2aM_w}{\rho_w BR} + \frac{8 \rho_n r_n^3}{\rho_w (r_n^3 - r_n^3) n} = 0 .
\]  

TABLE 2 Initial Soundings Chosen for the Present Computations

<table>
<thead>
<tr>
<th>Air Pressure (mb)</th>
<th>Air Temperature (°C)</th>
<th>Relative Humidity (%)</th>
<th>Case A</th>
<th>Case B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000.0</td>
<td>29.0</td>
<td>80.0</td>
<td>90.0</td>
<td></td>
</tr>
<tr>
<td>900.0</td>
<td>20.0</td>
<td>95.0</td>
<td>95.0</td>
<td></td>
</tr>
<tr>
<td>850.0</td>
<td>16.2</td>
<td>92.5</td>
<td>92.5</td>
<td></td>
</tr>
<tr>
<td>700.0</td>
<td>5.5</td>
<td>90.0</td>
<td>90.0</td>
<td></td>
</tr>
<tr>
<td>600.0</td>
<td>-0.5</td>
<td>80.0</td>
<td>85.0</td>
<td></td>
</tr>
<tr>
<td>500.0</td>
<td>-5.0</td>
<td>75.0</td>
<td>80.0</td>
<td></td>
</tr>
<tr>
<td>400.0</td>
<td>0.0</td>
<td>60.0</td>
<td>70.0</td>
<td></td>
</tr>
</tbody>
</table>
In Eq. 4-1, \( \delta_n \) and \( \rho_n \) are set at 0.20 and 1.85 g cm\(^{-3} \), respectively, to conform closely to the chemical properties of continental background aerosols (Hänel, 1976).

Values for air pressure, temperature, relative humidity, water vapor mixing ratio, air density, and virtual temperature at each vertical grid point are determined by interpolating initial soundings. The air pressure \( p \) at \( z_k \) is computed using the hydrostatic equation:

\[
p_k = p_{k-1} \exp\left[-g(z_k - z_{k-1})/(R_a \bar{T})\right], \tag{4-2}
\]

where:

\( k \) = the vertical grid index,
\( R_a \) = gas constant for dry air, and
\( \bar{T} \) = average temperature between levels of \( k \) and \( k - 1 \).

The temperature \( T \) and relative humidity \( \xi \) are linearly interpolated to give:

\[
T_k = T_{k-1} + \frac{T_j - T_{j-1}}{\log(p_j/p_{j-1})} \log(p_k/p_{k-1}), \tag{4-3}
\]

\[
\xi_k = \xi_{k-1} + \frac{\xi_j - \xi_{j-1}}{\log(p_j/p_{j-1})} \log(p_k/p_{k-1}). \tag{4-4}
\]

In Eqs. 4-3 and 4-4, the subscript \( j \) denotes the index of the level at which the initial soundings are measured. With \( p \), \( T \), and \( \xi \) values at level \( k \), the water vapor mixing ratio \( q \), air density \( \rho_a \), and supersaturation ratio \( s \) are determined from:

\[
q_k = \frac{3800 \xi_k}{p_k} \exp\left[A(\frac{1}{T_0} - \frac{1}{T_k})\right], \tag{4-5}
\]

\[
\rho_{a_k} = \frac{p_k}{(R_a T_k)}, \tag{4-6}
\]

\[
s_k = \xi_k - 1 \tag{4-7}
\]

and with \( q_k \), the virtual temperature \( T_v \) at level \( k \) is computed from:

\[
T_v = T_k (1 + 0.61 q_k). \tag{4-8}
\]

In Eq. 4-5, \( A (= 5460) \) and \( T_0 (= 273.15) \) are constants.
Adiabatic processes of dynamic variables and of drop concentrations are computed first to produce intermediate values. At this point, values for water vapor mixing ratio $q$, supersaturation ratio $s$, cloud water mixing ratio $Q_c$, and rainwater mixing ratio $Q_r$ are stored in a disk file for use in optimization calculations. Next, nonadiabatic effects due to condensation and droplet spectral evolution by coagulation are calculated. However, direct application of a finite-difference scheme to solve Eq. 3-3 encounters a rather severe stability difficulty. An alternative method is used; that is, Eq. 3-1 is used to compute the growth rates, and the time step for condensation is then determined from the maximum growth rate and the size interval. The growth rates are updated to predict changes in the radii defining the movable boundaries of the size intervals composing the spectrum. Condensation produces additional liquid water, reduces the water vapor mixing ratio, and warms the cloud environment. Therefore, the values for temperature, water vapor mixing ratio, and supersaturation ratio are adjusted accordingly to fully reflect the nonadiabatic effects. These stepwise computations for condensational growth proceed during the time step for dynamic parameters; the transition from this Lagrangian framework to an Eulerian one is made by interpolation. The spectral evolution by coagulation is computed using Eq. 3-8.

If $s > 0$, the macroscale condensation rates for cloud water and rainwater are computed using Eqs. 3-4 and 3-6, respectively. If not, the evaporation rates are computed for cloud water and rainwater using Eqs. 3-5 and 3-7. The autoconversion rates are determined by condensation as well as by coagulation. The autoconversion and accretion rates are computed using Eqs. 3-9 and 3-10, respectively. Finally, the macroscale terminal velocities for cloud water $V_c$ and rainwater $V_r$ are computed as:

\[
V_c = \sum_{i=1}^{I} n_i v_i V_i / \sum_{i=1}^{I} n_i v_i , \quad \text{and} \quad (4-9)
\]

\[
V_r = \sum_{i=1}^{I+1} n_i v_i V_i / \sum_{i=1}^{I+1} n_i v_i . \quad (4-10)
\]

At each time step, 12 parameters are stored in a disk file for use in optimization processes. As described before, four dynamic parameters ($q$, $s$, $Q_c$, and $Q_r$) are stored after adiabatic processes only, and the macroscale rates of $(dQ_c/dt)_\text{condensation}$, $(dQ_r/dt)_\text{evaporation}$, $(dQ_r/dt)_\text{condensation}$, $(dQ_r/dt)_\text{evaporation}$, $(dQ_r/dt)_\text{autoconversion}$, $(dQ_r/dt)_\text{accretion}$, $V_c$, and $V_r$ are stored after the microphysics computations. The number of data stored for each parameter is determined by the system. The only limitations are to exclude data smaller than the threshold values to avoid computational underflow.

The data from the two simulations (cases A and B) are used to derive the first-order rates as functions of bulk variables from multiple regression. The rates can be written as:

\[
\frac{dQ_c}{dt}_\text{condensation} = e^{-4.926} q_c^{0.417} (Q_v - Q_s)^{0.537} , \quad (4-11)
\]
\[
\frac{dQ_c}{dt}_{\text{evaporation}} = -e^{-6.153} Q_c^{0.743} (Q_s - Q_v)^{0.323}, \quad (4-12)
\]

\[
\frac{dQ_r}{dt}_{\text{condensation}} = e^{-2.167} Q_r^{0.490} (Q_v - Q_s)^{1.129}, \quad (4-13)
\]

\[
\frac{dQ_r}{dt}_{\text{evaporation}} = -e^{-6.601} Q_r^{0.420} (Q_s - Q_v)^{0.746}, \quad (4-14)
\]

\[
\frac{dQ_r}{dt}_{\text{autoconversion}} = e^{0.363} Q_r^{2.184} (Q_v - Q_s)^{0.173}, \quad s > 0, \quad (4-15)
\]

\[
\frac{dQ_r}{dt}_{\text{autoconversion}} = e^{-2.824} Q_r^{1.878} (Q_s - Q_v)^{0.076}, \quad s < 0, \quad (4-16)
\]

\[
\frac{dQ_r}{dt}_{\text{accretion}} = e^{2.025} Q_r^{1.029} Q_r^{1.042}, \quad (4-17)
\]

\[
V_{\text{cloud}} = e^{2.562} Q_c^{0.236} (1000\rho_a)^{-2.175}, \quad \text{and} \quad (4-18)
\]

\[
V_{\text{rain}} = e^{10.90} Q_r^{0.604} (1000\rho_a)^{0.466}. \quad (4-19)
\]

In Eqs. 4-11 to 4-19, the units are in g g\(^{-1}\) for \(Q_v, Q_s, Q_c, \) and \(Q_r\), in g cm\(^{-3}\) for \(\rho_a\), in g g\(^{-1}\)s\(^{-1}\) for all dQ/dt rates, and in cm s\(^{-1}\) for terminal velocities. In Eq. 4-11, the minimum value for \(Q_c\) is set at 10\(^{-6}\) g g\(^{-1}\).
5 COMPARISON STUDIES

Computations were carried out to compare the rates produced by our regression parameterizations with those produced by the detailed model simulations, and also to compare the behavior of different formulas from the literature with a Lagrangian air-parcel model.

The cloud microphysical process rates computed using our parameterized formulas were compared with those obtained from model simulations with a cloud model that treats processes of dynamics and detailed microphysics, including spectral evolution caused by condensation and coagulation. The cloud model computes changes in moisture content, before and after spectral evolution caused by condensation, evaporation, and coagulation. It then saves those values, along with bulk dynamic parameters at each dynamic-scale time step, to produce the macroscale rates associated with the cloud microphysical processes. Our new parameterizations are summarized in Table 3. Rates

<table>
<thead>
<tr>
<th>Rate</th>
<th>Data Source</th>
<th>Correlation Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Condensation of cloud droplets</td>
<td>Eq. 4-11</td>
<td>Eq. 3-4</td>
</tr>
<tr>
<td>Evaporation of cloud droplets</td>
<td>Eq. 4-12</td>
<td>Eq. 3-5</td>
</tr>
<tr>
<td>Condensation of raindrops</td>
<td>Eq. 4-13</td>
<td>Eq. 3-6</td>
</tr>
<tr>
<td>Evaporation of raindrops</td>
<td>Eq. 4-14</td>
<td>Eq. 3-7</td>
</tr>
<tr>
<td>Autoconversion of cloud droplets</td>
<td>Eq. 4-15, s&gt;0</td>
<td>Eq. 3-9</td>
</tr>
<tr>
<td></td>
<td>Eq. 4-16, s&lt;0</td>
<td>Eq. 3-9</td>
</tr>
<tr>
<td>Accretion of cloud droplets</td>
<td>Eq. 4-17</td>
<td>Eq. 3-10</td>
</tr>
<tr>
<td>Terminal velocity of cloud droplets</td>
<td>Eq. 4-18</td>
<td>Eq. 4-9</td>
</tr>
<tr>
<td>Terminal velocity of raindrops</td>
<td>Eq. 4-19</td>
<td>Eq. 4-10</td>
</tr>
</tbody>
</table>
for condensation and evaporation of cloud and rain are computed using Eqs. 3-4 to 3-7. The autoconversion and accretion rates are computed using Eqs. 3-9 and 3-10, respectively. The macroscale terminal velocities for cloud water and rainwater are computed using Eqs. 4-9 and 4-10. Finally, the corresponding rates for microphysical processes and terminal velocities in terms of bulk parameters are computed using parameterized formulas as presented in Eqs. 4-11 to 4-19.

In Figs. 2-10, the scatter diagrams compare the rates produced from regression parameterizations with those from detailed formulations. The correlation coefficients for the two data sets are given in Table 3. In general, the rates computed from the regression formulas compare closely with the detailed treatments. The correlation coefficients range from about 0.72 for the rates of condensation of rain to almost 1.0 for the rates of accretion of cloud water. High correlations are found for the rates of accretion, autoconversion, and terminal velocity of cloud water, whereas lower correlations are found for the rates of condensation and evaporation of rainwater. The high correlation for accretion rates was expected because those rates are determined mainly by collision and coalescence processes.

The behavior of the parameterization formulas proposed by different investigators is examined using a simple cloud model. An air parcel saturated at 283 K and located at the 1000-mb level initially is assumed to rise adiabatically with a vertical velocity of 1 m s⁻¹. Part of the resulting excess water vapor will condense to form a cloud, and the rest will stay in the cloud air because the saturation value increases as a result of the increase in temperature caused by release of the latent heat of condensation. The saturation adjustment scheme of Asai (1965) as given in Eq. 2-2 and our parameterized formula as given in Eq. 4-11 are used to compare evolution of the cloud water mixing ratio (see Fig. 11). Both formulas accurately predict changes in cloud water mixing ratio due to condensation following the saturation line. However, our formula performs better in terms of computational economy; Eq. 2-2 requires saturation adjustment through iteration within the macroscale time step.

The condensation rate for rainwater is one to two orders of magnitude smaller than that for cloud water under similar atmospheric conditions. Figure 12 presents the evolution of rainwater mixing ratio due to condensation. No cloud water condensation is considered in this case. The formula proposed by Ogura and Takahashi (1971), Eq. 2-7, and our formula, Eq. 4-13, are used for comparison. The rainwater mixing ratio remains small during the first five minutes of cloud development and increases rapidly thereafter. The difference between the two predictions is small. The \( \frac{dQ_r}{dt} \) \text{condensation} remains small until the cloud accumulates enough water vapor to become supersaturated by a few percent, which explains the time lag during the initial stages of cloud development.

In Figs. 13 and 14, the changes in cloud water mixing ratio due to autoconversion and accretion are presented, respectively. In both cases, the cloud water mixing ratios are assumed to be 1.0 g kg⁻¹ initially. For autoconversion rates, the following three formulas are compared: Eq. 2-3 (Kessler, 1967); Eq. 2-4 (Berry and Reinhardt, 1974d); and Eq. 4-15. The Kessler formula reduces the water content by 45% in 30 min compared with the very small changes made by Berry and Reinhardt. Our formula has an intermediate result, that is, a 12% reduction after 30 min of cloud development. For accretion rates, formulas by Scott (1982, Eq. 2-5, with \( E = 0.65 \)); Lin et al. (1983,
FIGURE 2 Scatter Diagram for Regression and Detailed Parameterizations of Condensation Rates of Cloud Water

FIGURE 3 Scatter Diagram for Regression and Detailed Parameterizations of Evaporation Rates of Cloud Water
FIGURE 4 Scatter Diagram for Regression and Detailed Parameterizations of Condensation Rates of Rainwater

FIGURE 5 Scatter Diagram for Regression and Detailed Parameterizations of Evaporation Rates of Rainwater
FIGURE 6 Scatter Diagram for Regression and Detailed Parameterizations of Autoconversion Rates When the Cloud is Supersaturated

FIGURE 7 Scatter Diagram for Regression and Detailed Parameterizations of Autoconversion Rates When the Cloud is Unsaturated
FIGURE 8 Scatter Diagram for Regression and Detailed Parameterizations of Accretion Rates

FIGURE 9 Scatter Diagram for Regression and Detailed Parameterizations of Terminal Velocities of Cloud Water
FIGURE 10 Scatter Diagram for Regression and Detailed Parameterizations of Terminal Velocities of Rainwater

FIGURE 11 Comparison of the Evolution of Cloud Water Mixing Ratio due to Condensation Using Our Optimized Formula and That of Asai (1965)

Eq. 2-6); and Eq. 4-17 are compared. Our results agree closely with those of Scott, while Lin et al. predict somewhat lower values because of their faster rate. Most of the cloud water can be transformed into rainwater within 20 min.

Finally, the combined effects of condensation, autoconversion, and accretion on cloud water and rainwater mixing ratios are examined. In this case, both cloud water and rainwater coexist in the cloud. The results are presented in Figs. 15 and 16 for the changes in cloud water and rainwater, respectively, with time. The curves with open squares in both figures show the changes made, considering the cloud microphysical processes involved in formation of cloud water by condensation and of rainwater by autoconversion and accretion. During the initial stages of cloud development, the cloud water mixing ratio increases rapidly, following closely the adiabatic line (compare with the curve with open circles in Fig. 15), while the increase in the rainwater mixing ratio remains negligible (see the first 10-min portion of the curve with open squares in Fig. 16). Thereafter, the combined processes of autoconversion and accretion start to control the distribution of water substances. Cloud water reaches a maximum value after 14 min, decreases rapidly during the next 11 min, and decreases very slowly thereafter, whereas rainwater content starts to increase rapidly.
The findings from the comparison studies are as follows:

1. The rates produced from the regression parameterizations compare favorably with those from sophisticated cloud simulations. The correlation coefficients range from 0.72 to almost 1.0. Very high correlation occurs when the rates can be determined solely by collectional processing of hydrometeors.

2. The new formula for the rate of cloud water condensation closely follows a moist adiabatic line and performs better in terms of computational economy.

3. The new formula for the rate of rainwater condensation compares closely with other formulas; the rate is smaller than that for cloud water condensation by one order of magnitude.

4. Large discrepancies are found in cloud water evolution when different formulas for the autoconversion rates are applied.

5. The accretion rates compare closely when the assumed collection efficiencies in other formulas are smaller than one.
Fig. 14 Comparison of the Evolution of Cloud Water Mixing Ratio due to Accretion Using Our Optimized Formula and Those of Scott (1982) and Lin et al. (1983)

Fig. 15 Examination of the Relative Contribution of the Parameterized Formulas on the Evolution of the Cloud Water Mixing Ratio
FIGURE 16 Examination of the Relative Contribution of the Parameterized Formulas on the Evolution of Rainwater Mixing Ratio
6 CONCLUSIONS

The literature on parameterizations of cloud microphysical processes was reviewed to examine the theoretical bases of those parameterizations and to evaluate their applicability to regional models. New parameterizations were produced by multiple regression upon the solution fields derived from simulations of a cloud model incorporating sophisticated microphysics.

The currently available rates for cloud microphysical interactions were generally derived under the assumption that the size distribution functions for various hydrometeors are given. Such parameterizations must therefore be applied with caution because the spectral evolution of various types of hydrometeors in reality varies significantly during the stages of cloud development. Uncertainties exist in assigning values for aerodynamic properties such as the bulk collection efficiency, and the growth processes for various types of ice crystals are not well enough known for accurate multiphase cloud-microphysics parameterizations.

The new parameterizations, in general, compare favorably with those currently available and are more efficient and applicable to regional models. The largest discrepancies occur in the autoconversion rates, whereas the accretion rates agree closely when the assumed collection efficiencies in other formulas are smaller than unity.
REFERENCES


Hanel, G., 1976: The properties of atmospheric aerosol particles as a function of relative humidity at thermodynamic equilibrium with the surrounding moist air. Advances in Geophysics, 19, 73-188.


Scott, B.C., 1982: Predictions of in-cloud conversion rates of \( \text{SO}_2 \) to \( \text{SO}_4 \) based upon a simple kinetic and kinematic storm model. J. Atmos. Environ., 16, 1735-1752.


APPENDIX

NOMENCLATURE
NOMENCLATURE

A  constant
AC accretion rate
AU autoconversion rate, or the total mass transfer rate from the largest cloud droplet to the smallest raindrop by condensation and coagulation
B  constant
B₁ constant
B₂ constant
B₃ constant
B₀ constant
C  constant
CC cloud condensation rate
CR rain condensation rate
Dₜ diffusivity of water vapor in air
E  linear collection efficiency
EC cloud evaporation rate
ER rain evaporation rate
F  ventilation coefficient
G  numerical constant
I  number of cloud droplet bins
J  number of cloud droplet and raindrop bins combined
K  collection kernel
Lᵥ latent heat of evaporation
M  rate of condensation
Mₜ molecular weight of water
Q  liquid water mixing ratio
Qₑ liquid water mixing ratio produced from condensation
Qₒ mass of average cloud drop
Qᵣ rainwater mixing ratio
Qₛ saturation water vapor mixing ratio
Qᵥ actual water vapor mixing ratio in cloud
R  universal gas constant
Rₐ gas constant for dry air
Rᵥ gas constant for water vapor
T  absolute temperature
NOMENCLATURE (Cont'd)

\( \bar{T} \) average temperature between levels \( k \) and \( k - 1 \)

\( T_0 \) constant

\( T_v \) virtual temperature of cloud

\( T_{vo} \) virtual temperature of total modeled region

\( V \) terminal velocity of raindrops

\( V_c \) macroscale terminal velocities for cloud water

\( V_r \) macroscale terminal velocities for rainwater

\( a \) cloud radius

\( c_v \) specific heat of water vapor

\( c_p \) specific heat of dry air

\( d \) particle diameter

\( e_s \) saturation vapor pressure over pure liquid water at temperature \( T \)

\( e_{ws} \) saturation vapor pressure with respect to water

\( g \) acceleration of gravity

\( k \) vertical grid index

\( m_n \) aerosol mass

\( m_w \) water uptake of aerosol deposit

\( n \) number concentration for sizes between \( d \) and \( d + \Delta d \)

\( n_i \) droplet number density in size bin \( i \)

\( n_0 \) constant

\( p \) air pressure

\( q \) water vapor mixing ratio

\( r \) droplet radius

\( r_n \) aerosol radius

\( s \) supersaturation ratio

\( t \) time

\( u \) radial velocity at cloud perimeter

\( v_i \) volume of a droplet in size bin \( i \)

\( w \) vertical velocity

\( z \) vertical coordinate

\( \alpha \) threshold value below which cloud conversion does not occur

\( \beta_n \) hygroscopicity of aerosols

\( \kappa \) thermal conductivity of air
NOMENCLATURE (Cont’d)

\( \lambda \)  
constant

\( \mu \)  
entrainment coefficient

\( \xi \)  
relative humidity

\( \rho \)  
average air density of total area

\( \rho_a \)  
air density

\( \rho_n \)  
bulk density of aerosols

\( \rho_o \)  
air density at surface

\( \rho_w \)  
density of water

\( \sigma \)  
surface tension of aqueous solution against air

\( (\text{dn}_i/\text{dt})_e \)  
time evolution of droplet number density

\( \text{dr}_i/\text{dt} \)  
diffusional growth rate of drop with radius \( r_i \)

\( \text{dQ}_r/\text{dt} \)  
autoconversion rate
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