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A REVIEW OF TWO-PHASE FLOW-INDUCED VIBRATION

by

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CONTENTS

	<u>Page</u>
ABSTRACT.....	1
I. INTRODUCTION.....	1
II. ADDED MASS AND DAMPING.....	4
III. CROSSFLOW-INDUCED VIBRATION.....	11
IV. AXIAL-FLOW-INDUCED VIBRATION.....	30
V. CLOSING REMARKS.....	38
ACKNOWLEDGMENTS.....	39
REFERENCES.....	40

FIGURES

	<u>Page</u>
1 Effective density for two-phase flow as a function of void fraction.....	5
2 Two-phase flow damping coefficient.....	8
3 A group of circular cylinders in fluid.....	9
4 Wake behind a cylinder: (a) water flow ($U = 0.4$ m/s), (b) oscillatory wake in two-phase flow ($U = 0.4$ m/s, $\alpha = 0.06$), (c) steady wake in two-phase flow ($U = 0.4$ m/s, $\alpha = 0.2$).....	16
5 Time histories of lift and drag forces for $\alpha = 70\%$	17
6 RMS values of lift and drag forces as a function of gap velocity and void fraction.....	18
7 The rms accelerations of a cylinder as a function of reduced flow velocity U/fD : (a) in-line direction; (b) crossflow direction.....	21
8 Vibration characteristics.....	23
9 Stability map.....	25
10 Flow patterns in axial flow as a function of void fraction	33
11 Typical longitudinal spatial correlation of fluid pressure.....	34
12 Typical peripheral spatial correlation of fluid pressure	35
13 Variance of vibration strain and two-phase flow Reynolds number Re	37

TABLES

	<u>Page</u>
1 Tests on two-phase crossflow-induced vibration.....	12

NOMENCLATURE

<u>Symbol</u>	<u>Description</u>
A	Channel area
$C_{Dj} (C_{Lj})$	Steady drag (lift) coefficient for j th cylinder
$C'_{Dj} (C'_{Lj})$	Periodic fluctuating drag (lift) coefficient for the j th cylinder
C_m	Added-mass coefficient
C_N, C_T	Drag coefficient
C_s, C_{sj}	Viscous damping coefficient of a structure
C_t	Two-phase flow damping coefficient
C_v	Viscous damping coefficient
D	Diameter of cylinder
$E_j I_j, EI$	Flexural rigidity of cylinder
f	Oscillation frequency
f_f	Natural frequency in fluid
f_v	Natural frequency in vacuum
g	Fluid force component
g_j	Fluid force component in the x direction of the j th cylinder
g'_j	Fluctuating fluid force component in the x direction of the j th cylinder
h_j	Fluid force component in the y direction of the j th cylinder
h'_j	Fluctuating fluid force component in the y direction of the j th cylinder
l	Cylinder length
m	Cylinder mass per unit length
m_a	Added mass
m_j	Mass per unit length of cylinder j
M_d	Displaced mass of fluid or mass of fluid inside a pipe
N	Number of cylinders in an array
P	Pitch
Q	Flow rate
Re	Reynolds number
t	Time
T	Axial tension, transverse pitch
u	Cylinder displacement
u_j	Displacement of the j th cylinder in the x direction

<u>Symbol</u>	<u>Description</u>
U	Flow speed, either free stream velocity or gap flow velocity in a tube array
U_r	Reduced flow velocity ($= U/fD$)
v_j	Displacement of the j th cylinder in the y direction
x, y, z	Cartesian coordinates
W	Mass flow rate
α	Void fraction
$\bar{\alpha}_{jk}, \bar{\beta}_{jk}, \bar{\sigma}_{jk}, \bar{\tau}_{jk}$	Added mass matrices
$\bar{\alpha}'_{jk}, \bar{\beta}'_{jk}, \bar{\sigma}'_{jk}, \bar{\tau}'_{jk}$	Fluid damping matrices
$\alpha''_{jk}, \beta''_{jk}, \sigma''_{jk}, \tau''_{jk}$	Fluid stiffness matrices
δ_m	Scruton's number ($= 2\pi m \zeta / \rho D^2$)
ζ	Damping ratio
ζ_f	Damping ratio in fluid
ζ_t	Damping ratio in two-phase flow
ζ_v	Damping ratio in vacuum
μ	Absolute viscosity
ν	Kinematic viscosity
ρ	Fluid density
χ	Mass quality
ω	Circular frequency ($= 2\pi f$)
$\Omega_{DJ} (\Omega_{Lj})$	Circular frequency associated with parameter in the drag (lift) direction
$\phi_{Dj} (\phi_{Lj})$	Phase angle associated with parameter in the drag (lift) direction

Subscripts

$D (L)$	Drag (lift) direction
f	Parameters related to fluid
g	Gas phase
j, k	Cylinder number j, k ($j, k = 1$ to N)
l	Liquid phase
s	Parameters related to structure
v	Parameters measured in vacuum

CREDITS

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- Fig. 1 "Damping and Hydrodynamic Mass of a Cylinder in Simulated Two-Phase Flow," L. N. Carlucci, ASME J. Mech. Des. 102, 597-602, 1980, Fig. 10. Permission granted by the American Society of Mechanical Engineers.
- Fig. 2 "Experimental Studies of Damping and Hydrodynamic Mass of a Cylinder in Confined Two-Phase Flow," L. N. Carlucci and L. D. Brown, J. Vibration, Acoust., Stress and Reliability in Des. 105, 83-89, 1982, Fig. 8. Permission granted by the American Society of Mechanical Engineers.
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Fig. 13

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A REVIEW OF TWO-PHASE FLOW-INDUCED VIBRATION

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ABSTRACT

Two-phase flow exists in many shell-and-tube heat exchangers and power generation components. The flowing fluid is a source of energy that can induce small-amplitude subcritical oscillations and large-amplitude dynamic instabilities. In fact, many practical system components have experienced excessive flow-induced vibrations. To prevent unacceptable flow-induced vibration, we must understand excitation mechanisms, develop analytical and experimental techniques, and provide reliable design guidelines. Thus, we are conducting a comprehensive program to study structural vibration in components subjected to two-phase flow. This report reviews the current understanding of vibration of circular cylinders in quiescent fluid, crossflow, and axial flow, with emphasis on excitation mechanisms, mathematical models, and available experimental data. A unified theory is presented for cylinders oscillating under different flow conditions. Based on the theory, future research needs are outlined.

I. INTRODUCTION

Two-phase flow exists in a wide variety of engineering systems, including power, heat transfer, process, and transport systems. For example, in a typical U-tube steam generator, the exit flow in the U-bend region is composed of a water and steam mixture. A flowing two-phase fluid is a source of energy that can induce and sustain structural vibration and instability. In fact, two-phase flow can induce oscillations that are very similar to those of single-phase flow and produce detrimental effects comparable to those of single-phase flow.

Based on the direction of flow with respect to the major axis of structural components, two-phase flow-induced vibration problems can be divided into two groups: axial flow, in which the flow direction is parallel to the structural axis; and crossflow, in which the flow direction is perpendicular to the structural axis. Both axial-flow- and crossflow-induced vibrations have been studied in past decades. A state-of-the-art review of axial flow, with emphasis on circular rods, was reported by Shin (1978), and a review of vibrations of circular cylinders in two-phase crossflow was reported by Hara (1987b). This report reviews two-phase flow-induced vibrations for crossflow as well as axial flow. The objective is to summarize the excitation mechanisms, mathematical models, response characteristics, design considerations, and future research needs.

Two models are frequently used for two-phase flow: the homogeneous flow model and the separated flow model. The homogeneous flow model is used in most cases for flow-induced vibration. If the cross sectional area of a channel is A and the cross sectional area occupied by the gas and liquid phases are A_g and A_l respectively, where the subscripts g and l indicate gas and liquid respectively, then the void fraction α is given by

$$\alpha = \frac{A_g}{A}, \quad (1)$$

$$1 - \alpha = \frac{A_l}{A}.$$

The mass flow rate is represented by W and will be the sum of the individual phase flow rates W_g and W_l . Thus, the mass quality (also referred to as mass dryness fraction) χ is defined as

$$\chi = \frac{W_g}{W_g + W_l}, \quad (2)$$

$$1 - \chi = \frac{W_l}{W_g + W_l}.$$

The mean velocity of an individual phase is denoted by U and the density by ρ ; thus

$$U_g = \frac{W_g}{\rho_g A_g},$$

$$U_l = \frac{W_l}{\rho_l A_l}, \quad (3)$$

$$\frac{U_g}{U_l} = \left(\frac{\chi}{1 - \chi} \right) \left(\frac{\rho_l}{\rho_g} \right) \left(\frac{1 - \alpha}{\alpha} \right).$$

The mean flow velocity of the mixture U is given by

$$U = \frac{W_g + W_l}{\rho A}, \quad (4)$$

where the mixture density is defined by

$$\rho = (1 - \alpha)\rho_l + \alpha\rho_g . \quad (5)$$

The Reynolds number is defined by

$$Re = \rho U / \mu \quad (6)$$

where D is cylinder diameter and the dynamic viscosity μ is determined by McAdams correlation (Collier 1981):

$$\frac{1}{\mu} = \frac{\chi}{\mu_g} + \frac{1 - \chi}{\mu_l} . \quad (7)$$

Based on the homogeneous two-phase flow model and theory of elasticity, the dynamic fluid/structure interaction can be described by the same equation as that for single-phase flow (Chen 1987):

$$[M]\{\ddot{Q}\} + [C]\{\dot{Q}\} + [K]\{Q\} = \{G\} \quad (8)$$

or

$$[M_s + M_f]\{\ddot{Q}\} + [C_s + C_f]\{\dot{Q}\} + [K_s + K_f]\{Q\} = \{G\} \quad (9)$$

where $\{Q\}$ is the generalized structural displacement; $[M]$ is the mass matrix, including structural mass $[M_s]$ and added mass $[M_f]$; $[C]$ is the damping matrix, including structural damping $[C_s]$ and fluid damping $[C_f]$; $[K]$ is the stiffness matrix, including structural stiffness $[K_s]$ and fluid stiffness $[K_f]$; and $\{G\}$ represents the other excitation forces, including vortex shedding, turbulence, and acoustic noises.

In general, M , C , K , and G are functions of Q , $\dot{Q}$, and $\ddot{Q}$; therefore, a complete solution is rather difficult to obtain. Fortunately, in most practical situations, one can ignore all nonlinear terms that make M , C , K , and G dependent on structural response. However, even in a linear case, it is still not possible to obtain M_f , C_f , K_f , and G for most situations.

Once the two-phase fluid mass, damping, stiffness, and excitation are known, Eq. (9) can be solved in a straightforward manner. The major task is to obtain these fluid matrices and excitation forces for various flow conditions. For small-amplitude oscillations, M_f and C_f are important for

structures oscillating in a quiescent fluid; in cross and axial flows, M_f , C_f , K_f , and G are important. In this report, examination of the various excitation mechanisms is based on the consideration of these fluid-force matrices.

It should be noted that Eqs. (8) and (9) are approximate for structural response in two-phase flow. All fluid effects are incorporated in M_f , C_f , K_f , and G . In most cases, the equations are acceptable. However, in some situations, they may not be adequate. Their applicability and limitations are not well known at present. Modeling these fluid effects in two-phase flow and establishing the range of applicability of the models are topics for future research studies.

II. ADDED MASS AND DAMPING

The added mass and damping for a circular cylinder oscillating in two-phase flow has been studied based on the homogeneous model [Carlucci (1980), Carlucci and Brown (1983), Hara and Kohgo (1982) and (1986), Pettigrew and Tromp (1985)]. When such a circular cylinder has a displacement value u , the fluid force g acting on it is given by the equation

$$g = -m_a \frac{\partial^2 u}{\partial t^2} - C_v \frac{\partial u}{\partial t}, \quad (10)$$

where m_a is the added mass and C_v is the fluid damping.

The added mass m_a of a single cylinder in two-phase flow is given by

$$m_a = C_m M_d, \quad (11)$$

where C_m is the added-mass coefficient and M_d is the displaced mass of fluid. Experimental data for a single cylinder in an annular region indicate that the added-mass coefficient for two-phase flow is the same as that for single-phase flow. A summary of the results for single-phase flow has been reported [Chen and Chung (1976)].

The added mass for two-phase flow is calculated from the homogeneous model; i.e., it is equal to the displaced volume of fluid multiplied by the effective fluid density ρ . Theoretical and experimental studies have been performed to quantify ρ . Typical experimental data are shown in Fig. 1 [Carlucci (1980)], where the ratio of the effective density to the single-phase flow density ρ_g is plotted as a function of void fraction α .

Several theories for the effective density have been summarized by Schumann (1981):

Fig. 1. Effective density for two-phase flow as a function of void fraction [from Carlucci (1980), with permission]

$$\begin{aligned}
(a) \quad \rho &= (1 - \alpha)\rho_l + \alpha\rho_g , \\
(b) \quad \rho &= \rho_l(1 - \alpha)/(2\alpha + 1) , \\
(c) \quad \rho &= \rho_l(1 - \alpha)(1 + 2\alpha)/(1 + 4\alpha - 2\alpha^2) .
\end{aligned} \tag{12}$$

The first of these equations represents the volumetrically averaged mixture density; the other two are approximate virtual density formulas. The theoretical results obtained from Eq. (12a) are presented in Fig. 1 to facilitate comparison with the experimental data. According to Eq. (12a), for a homogeneous two-phase mixture without slip, the effective density decreases linearly with the void fraction α ; at low values of α , it agrees well with the experimental data. For larger α , the mixture density does not agree well with the data; the effective density is less than the averaged density. The other two theories represented in Eq. (12) correlate well with experimental data for the middle range of α , but do not agree well with the data obtained when α is close to 0 or 1.

There are several reasons for the discrepancies between the theory and experimental data:

- **Nonhomogeneity:** The gas phase is not uniformly distributed in two-phase flow. For example, in the bubbly flow regime, the gas bubbles may stay close to the vibrating cylinder and reduce the effective density. In addition, at void fractions of 60 to 80 percent, the flow regime is essentially annular and the central portion of the flow passage is occupied largely by the gas phase.

- **Compressibility:** Gas phase compressibility affects the inertia effect.

- **Slip Between the Phases:** The two phases of fluid are not moving at the same speed. Schumann's theory (1981) predicts that if there is slip between phases, the effective density is lower than the mixture density. The detailed mechanism(s) of how added mass causes deviation from the mixture theory has not been established.

Although fluid damping in a two-phase flow was studied experimentally by Carlucci and Brown (1983), Hara and Kohgo (1982, 1986), and Pettigrew et al. (1985), the damping mechanisms operating in two-phase flow are not well understood. Experimental studies have been performed for a single cylinder in an annular region and for tube arrays. Mathematically, the viscous damping coefficient C_v for two-phase flow is represented by

$$C_v = C_v' + C_t , \tag{13}$$

where C_v' is the damping coefficient attributed to the viscous effect and C_t is the two-phase flow damping coefficient.

C_v' can be calculated on the basis of methods derived for single-phase flow [Chen et al. (1976)]. However, when doing so, the effective fluid density and viscosity should be used. The effective fluid density based on the mixture theory is given in Eq. (5) and the mixture viscosity according to McAdams is given in Eq. (7) [Collier (1981)]. There is no analytical expression for C_t . The most complete experimental data, those of Carlucci and Brown (1980), are given in Fig. 2. With Fig. 2, the two-phase damping coefficient C_t can be calculated based on C_v' .

In addition to the mixture theory, a mathematical model was developed by Hara and Kohgo (1986) to calculate the added mass and damping for a single cylinder oscillating in two-phase flow. The bubble flow is modeled as gas columns parallel to the vibrating cylinder. The gas columns are assumed to have neither mass nor rigidity. Small-amplitude oscillations of the cylinder and gas columns in fluid are then solved analytically. The added mass is based on the ideal flow theory and the damping on linearized viscous flow theory. Analytical and experimental results agree reasonably well.

The experimental technique for measuring the added mass and fluid-damping in two-phase flow is the same as that for single-phase flow. The natural frequencies and modal damping ratio in vacuum and in two-phase flow can be measured separately; these values are represented by f_v and f_f , and ζ_v and ζ_f . If the mass of the cylinder per unit length is m and the viscous damping coefficient in vacuum is C_s , the added mass and fluid damping are given by

$$m_a = m \left(\frac{f_v^2}{f_f^2} - 1 \right),$$

and

$$C_v = C_s \left[\frac{\zeta_f}{\zeta_v} \left(\frac{m + m_a}{m} \right)^{0.5} - 1 \right].$$

(14)

This technique has been used for a single cylinder [Chen and Chung (1976)].

Although a single added mass m_a and damping coefficient C_v are sufficient to characterize the effect of two-phase flow on a single cylinder, the characterization of an array of N cylinders requires $4N^2$ added masses and $4N^2$ fluid damping coefficients. For example, consider an array of N cylinders as shown in Fig. 3. The axes of the cylinders are parallel to the z axis. The subscript j is used to denote variables associated with cylinder j . The displacement components of cylinder j are u_j and v_j and the fluid force components are g_j and h_j . Mathematically, the fluid force components associated with cylinder motion can be written [Chen (1987)] as follows:

Fig. 2. Two-phase flow damping coefficient [from Carlucci and Brown (1982), with permission]

(a) A GROUP OF CIRCULAR CYLINDERS

(b) FLUID FORCE AND CYLINDER DISPLACEMENT COMPONENTS

Fig. 3. A group of circular cylinders in fluid

$$g_j = - \sum_{k=1}^N \left[\left(\bar{\alpha}_{jk} \frac{\partial^2 u_k}{\partial t^2} + \bar{\alpha}'_{jk} \frac{\partial u_k}{\partial t} \right) + \left(\bar{\sigma}_{jk} \frac{\partial^2 v_k}{\partial t^2} + \bar{\sigma}'_{jk} \frac{\partial v_k}{\partial t} \right) \right],$$

and

(15)

$$h_j = - \sum_{k=1}^N \left[\left(\bar{\tau}_{jk} \frac{\partial^2 u_k}{\partial t^2} + \bar{\tau}'_{jk} \frac{\partial u_k}{\partial t} \right) + \left(\bar{\beta}_{jk} \frac{\partial^2 v_k}{\partial t^2} + \bar{\beta}'_{jk} \frac{\partial v_k}{\partial t} \right) \right],$$

where $\bar{\alpha}_{jk}$, $\bar{\sigma}_{jk}$, $\bar{\tau}_{jk}$, and $\bar{\beta}_{jk}$ are added mass matrices and $\bar{\alpha}'_{jk}$, $\bar{\sigma}'_{jk}$, $\bar{\tau}'_{jk}$, and $\bar{\beta}'_{jk}$ are fluid-damping matrices. These results are the same as those for single-phase flow. However, at this time, no analytical or experimental data are available for two-phase flow. It is expected that the added-mass matrices can be calculated on the basis of the potential flow theory, which has been successfully used for single-phase flow, provided the effective fluid density is used. Two-phase fluid damping remains a difficult parameter to predict or measure for the case of a cylinder array.

It is apparent that even for a single cylinder oscillating in two-phase flow, the added mass and damping for different flow regimes are not well understood. For a general case of cylinder arrays, there are no data available for the added-mass and fluid-damping matrices. Although tube arrays have been tested [e.g., Pettigrew et al. (1985)], no attempts have been made to obtain the coupling effect. Thus, the coupling effect is a subject that requires further study if one wishes to predict the response of a cylinder array in two-phase flow.

Studies of added mass and damping have considered small oscillation amplitudes. In single-phase flow, as long as the oscillation amplitude is less than approximately 0.4 times the cylinder diameter, the linear theory given in Eqs. (10) and (15) is applicable [Skop et al. (1976)]. Data for both small- and large-amplitude oscillations in two-phase flow are lacking.

Once the added-mass and fluid-damping matrices are known, the response of cylinders oscillating in a two-phase fluid can easily be calculated by using Eq. (9). In a stationary fluid, the equation of motion is

$$[M_s + M_f]\{\ddot{Q}\} + [C_s + C_f]\{\dot{Q}\} + [K_s]\{Q\} = \{G\}. \quad (16)$$

In a single-phase fluid, M_f and C_f are symmetric matrices. In a two-phase fluid, the characteristics of M_f and C_f are not known.

The interaction of two-phase flow with a vibrating cylinder is dependent on the different flow regimes. In addition, cylinder oscillations may affect phase distribution. The detailed physics of the interaction of cylinders and two-phase flow is extremely complicated. Much more detailed studies,

including flow visualization, are needed to understand the interaction process.

III. CROSSFLOW-INDUCED VIBRATION

Two-phase crossflow-induced vibration of circular cylinders has recently been studied in several countries. As shown in Table 1, with the exception of the tests by Axisa et al. (1984, 1985, 1986), all tests were performed for air-water flow. The main objective of these studies has been to determine the response of cylinder arrays in two-phase flow with the emphasis on identifying the critical flow velocity. Several investigations have been conducted to measure the fluid excitation forces: Hara (1982), for a single cylinder; and Nakamura et al. (1982) and Taylor et al. (1986), for cylinder arrays.

Void Fraction

Typically, the volume ratio β of the two phase flow is established by metering the two types of fluid individually prior to mixing at the test section. The assumption of a homogeneous mixture implies that the two phase velocities are the same, in which case, the volume flow ratio (β) and the void fraction (α) are the same. Several studies include measurements of void fraction upstream or downstream of the bundle as well as inside the bundle.

Axisa et al. (1984) measured the void fraction at five locations along the cylinder axis, but only at the upstream and downstream of the cylinder array. The void fraction is determined as a function of mass quality χ . In general, the flow is not homogeneous across the test section. For a fixed mass quality at low flow velocities, the local void fraction is smaller at the center of the test section than at a location close to the wall. As flow velocity increases, the void fraction is fairly uniform across the test channel. At low flow velocity, the averaged spatial void fraction $\bar{\alpha}$ is smaller than the homogeneous void fraction α , but the difference between $\bar{\alpha}$ and α decreases with increasing flow velocity. Measurements of these values at upstream and downstream of the tube bundle are similar and tests in air-water and steam-water flows display similar trends.

Remy (1982) measured the void fraction between the fourth and fifth rows of a square array with $P/D = 1.44$. Results show the very well-organized characteristics of the flow. The void fraction exhibits a periodic variation with the same period as the cylinder pitch. Furthermore, the variations change with flow velocity. At low velocities, a maximum void fraction occurs at the gap of the array. But at large flow velocities, there are two maxima, one in the gap and the other in the wake of the cylinders. Although void fraction within an array of cylinders varies, the averaged void fraction value in a section is the same as the corresponding values upstream or downstream of the bundles.

Table 1. Tests on two-phase crossflow-induced vibration

Authors	Fluid	Flow Regimes or Void Fraction	Test Section	Instrumentation	Measured Parameters
Axisa et al. (1984)	Air and water, Steam and water	0, 91%-100%	Square array, P/D = 1.44	γ-ray densitometers, Accelerometers, Strain gauges	Void fraction, Tube response, Damping
Axisa et al. (1985)	Steam and water	0, 85%-100%	Square array, Triangular array, P/D = 1.44	Strain gauges, Accelerometers	Tube displacement, Damping
Axisa et al. (1986)	Air and water, Steam and water	0, 50%-100%	Straight tubes: Single tube, Tube row, Square array U-Tubes: Square array, P/D = 1.44	γ-ray densitometers, Accelerometers, Strain gauges	Void fraction, Tube displacement, Damping
Goyder (1987)	Air and water	0, 50%-80% and 100%	Triangular array P/D = 1.25	Load cells	Lift and drag forces
Hara (1982,1984)	Air and water	Bubbles	Single tube	Strain gauges, Pressure trans- ducers, Accelerometers	Tube displacement, Lift and drag forces
Hara (1987a)	Air and water	Bubble, Max. Re = 3.25×10^4	Tube row, P/D = 1.33	Accelerometer	Tube response, Damping $\delta_m = 3.09$ to 4.72
Heilker and Vincent (1980)	Air and water	0, 50%-87%	Triangular array, P/D = 1.36; Square array, P/D = 1.50	Accelerometers, Strain gauges,	Tube displacement, Damping

Table 1. Tests on two-phase crossflow-induced vibration (Contd.)

Authors	Fluid	Flow Regimes or Void Fraction	Test Section	Instrumentation	Measured Parameters
Nakamura et al. (1982)	Air and water	Bubble and slug flow	Square and rotated square array, P/D = 1.47	Strain gauges	Lift and drag forces
Nakamura and Fujita (1987)	Air and water	Bubble and slug flow	Square array, P/D = 1.42, One flexible cylinder in a rigid tube array	Strain gauges	Tube displacement, Damping
Pettigrew and Gorman (1973,1978)	Air and water	0 to 95%, $Re = 10^4 - 10^5$	Triangular array, Rotated tri- angular array, Square array, Rotated square array, P/D = 1.47	Strain gauges	Tube displacement, Damping
Pettigrew et al. (1985)	Air and water	5 to 98%	Triangular and Square array, P/D = 1.47	Strain gauge	Tube displacement, Damping
Remy (1982)	Air and water	Bubble	Square array, P/D = 1.44	Accelerometers, Optical fiber probe	Void fraction, Damping, Tube response
Taylor et al. (1986)	Air and water	0 to 95%, $Re = 1.12 \times 10^4$ to 1.24×10^5	Tube row, P/D = 1.5 and 3.0	Force transducer	Drag and lift forces

Added Mass and Damping

The added mass of an array of cylinders in crossflow has not been studied in detail. Although this is true for both single- and two-phase flows, two-phase flow data are especially limited. For practical purposes, there are no complete sets of data for added-mass matrices of an array of cylinders. The results obtained when Remy (1982) tested a flexible cylinder in a rigid square array can be used to calculate only the self-added mass $\bar{\alpha}_{jj}$ and $\bar{\beta}_{jj}$ [see Eq. (15)]. Other investigators performed tests on a flexible cylinder array [Axisa et al. (1984) and Pettigrew et al. (1985)] but only the dominant coupled modes were considered. Therefore, the individual elements of the added-mass matrices are not considered by Axisa and Pettigrew. In order to obtain the added-mass matrices, motion-dependent fluid forces associated with the cylinder motion must be measured. No test results have been reported in the literature.

If a cylinder is oscillating in two-phase flow, damping can be associated with different sources. Mathematically, it is convenient to separate the damping into several components:

$$\zeta = \zeta_s + \zeta_f + \zeta_t \quad (17)$$

where ζ is the total damping ratio, ζ_s is the structural damping ratio, ζ_f is the fluid viscous damping ratio for single-phase flow, and ζ_t is the two-phase-flow damping ratio.

Fluid damping of single-phase flow consists of two parts: viscous damping and flow-velocity-dependent damping. Calculation of the viscous damping of two-phase flow, as discussed in Section II, can be based on the same method as single-phase flow. The flow-velocity-dependent damping can be calculated by the same method developed for a single cylinder in single-phase flow [Chen (1987)]:

$$\begin{aligned} \zeta_f &= \frac{C_D}{\pi^2} \left(\frac{M_d}{m + m_a} \right) \left(\frac{U}{fD} \right) \quad \text{for drag direction,} \\ &= \frac{C_D}{2\pi^2} \left(\frac{M_d}{m + m_a} \right) \left(\frac{U}{fD} \right) \quad \text{for lift direction,} \end{aligned} \quad (18)$$

where M_d is the displaced mass of fluid, f is oscillation frequency, and C_D is the drag coefficient. Equation (18) is applicable for an isolated cylinder at high reduced flow velocity.

Axisa et al. (1984) used Eq. (18) to obtain the two-phase flow damping for a square array in air-water and steam-water two-phase flows. The results agree reasonably well with those measured for a single cylinder in quiescent fluid [Carlucci and Brown (1982)].

For an array of cylinders, the two-phase flow damping effect can also be described by Eq. (15), where $\bar{\alpha}'_{jk}$, $\bar{\sigma}'_{jk}$, $\bar{\tau}'_{jk}$, and $\bar{\beta}'_{jk}$ are attributed to viscous effects, flow-velocity-dependent components, and two-phase flow effects. The details for a cylinder array in crossflow are not well understood at this time.

Fluid Excitation Forces

Fluid excitation forces acting on circular cylinders were measured by Hara (1982) for a single cylinder, and by Nakamura et al. (1982), Taylor et al. (1986), and Goyder (1987) for an array of cylinders. In addition, fluid forces have also been computed from cylinder responses subjected to crossflow [Pettigrew and Gorman (1973)].

Hara measured the unsteady lift and drag forces and the pressure fluctuation on a stationary cylinder immersed in a crossflow. The test, performed in an air-water two-phase flow, shows that the existence of vortex shedding for a single cylinder in crossflow depends on the void fraction. For α less than about 10%, visualization of flow and measurement of fluid pressure show the existence of vortex shedding in two-phase flow [see Fig. (4b)]. However, as α increases, the wavy motion of the wakes disappears [Fig. (4c)].

The characteristics of the pressure fluctuations and fluctuating drag and lift forces also depend on the void fraction. For small α , where vortex shedding exists, the pressure fluctuation and fluid forces vary insignificantly with α and are similar to those of single-phase flow. Furthermore the lift force is narrow-banded and the drag force is less periodic. Although for small α , the bubbles impinging on the cylinder may produce a larger force compared with those for single-phase flow, the dominant frequency is that attributed to vortex shedding because Karman vortex shedding remains the dominant mechanism. On the contrary, when α is large, the motion of air bubbles in the flow may be random and the impinging frequency may be proportional to α and flow velocity. The lift and drag forces are of the same magnitude and contain high-frequency components. For large α , vortex shedding disappears and the drag and lift forces depend strongly on the void fraction.

Goyder (1987) measured the fluctuating drag and lift forces of a triangular array with a P/D of 1.25. Figures 5 and 6 show the time histories for $\alpha = 70\%$, and rms values as a function of flow velocity, respectively. Note that the peak amplitude of the drag force is much larger than that of the lift force. Furthermore, whereas the drag force exhibits peaks and is strongly asymmetric, the lift force is symmetric. The rms values of lift and drag forces are approximately proportional to the velocity and are clearly dependent on the void fraction α .

Fig. 4. Wake behind a cylinder: (a) water flow ($U = 0.4$ m/s),
(b) oscillatory wake in two-phase flow ($U = 0.4$ m/s, $\alpha = 0.06$),
(c) steady wake in two-phase flow ($U = 0.4$ m/s, $\alpha = 0.2$) [from
Hara (1982), with permission]

Fig. 5. Time histories of lift and drag forces for $\alpha = 70\%$ [from Goyder (1987), with permission]

Fig. 6. RMS values of lift and drag forces as a function of gap velocity and void fraction [from Goyder (1987), with permission]

Nakamura et al. (1982) tried to characterize the excitation force and found that the characteristics of the forces depend on the flow regime. The cross spectral density of the excitation force $\phi_{gg}(z, z', \omega)$ is considered to be the product of $R_1(z, \omega)$, determined by the position, and $R_2(z - z', \omega)$, the component that gives the correlation of two points:

$$\phi_{gg}(z, z', \omega) = R_1(z, \omega) R_2(z - z', \omega) . \quad (19)$$

$R_1(z, \omega)$ is determined by the strength of the two-phase fluid force at each point. $R_2(z - z', \omega)$ is a function of distance. If the shapes of the power spectrum of excitation force are spatially homogeneous, $R_1(z, \omega)$ can be written

$$R_1(z, \omega) = R_1^*(z) R_1''(\omega) , \quad (20)$$

where $R_1''(\omega)$ is normalized such that

$$\frac{1}{2\pi} \int_0^{\infty} R_1''(\omega) d\omega = 1 . \quad (21)$$

Nakamura et al. (1982) assumed that $R_2''(z - z', \omega)$ is 1 and $R_1^*(z)$ is a constant. Based on these assumptions, the following characteristics are noted.

- Excitation forces in two-phase flow can be classified according to flow pattern. In slug flow, liquid slugs flow intermittently at a rate of several times a second. The exciting force is mainly due to the bumping of liquid slugs against the cylinder, resulting in a large drag force and a lift force about 40% of the drag force. In bubble flow, the excitation force is reduced and its magnitude is equal to about 30 to 50% of the slug flow.
- The power spectra of excitation forces for bubble and slug flows are similar. In both cases, the frequency components at low frequency are high (1-10 Hz) and decrease with frequency. Cylinder location and arrangement do not have a significant effect on the power spectra.
- The rms drag and lift forces in a slug flow are independent of the superficial gas velocity and are proportional to the superficial liquid velocity. (Superficial-phase velocity is defined as the velocity the phase would have if it flowed alone through the flow channel). Taylor et al. (1986) obtained similar results for a tube row with a P/D of 1.5 and 3.0. For α from 20 to 80%, the spectra show a dominance of the drag force over the lift force. For low values of α , the spectra are almost identical to those of single-phase flow.

In single-phase flow, the power spectral density of the fluid excitation force can be normalized by the dynamic pressure head multiplied by the cylinder length ℓ and cylinder diameter D , i.e., as shown by Taylor et al. (1986) and Chen and Jendrzeczyk (1986):

$$\phi_{gg}(s) = \left[\frac{\phi_{gg}(\omega)}{\frac{1}{2} \rho U^2 D \ell} \right] \left(\frac{U}{D} \right). \quad (22)$$

However, for two-phase flow, a nondimensional group of parameters has not been found to collapse the two-phase data.

In spite of the difficulty of normalizing the spectra, Pettigrew and Gorman (1978) use Eq. (22) to estimate the spectra of the fluid force coefficients in practical applications. The coefficients are inferred from tube response to two-phase flow.

Structural Response

The vibration of cylinders in two-phase flow is similar to that in single-phase flow. Oscillations can be divided into three groups: vortex-excited oscillations, subcritical oscillations, and dynamic instability.

The response of a single cylinder in two-phase flow depends on the void fraction. Typical responses of a cylinder in the drag and lift direction obtained during a detailed study by Hara (1984) are given in Fig. 7. Single-phase flow can induce large in-line oscillations when $U/fD \approx 2.1$. When the void fraction is 5%, the response amplitude is significantly reduced for U_r from 0.5 to 2.5; in particular, at $U_r = 2.1$, the amplitude is reduced to about 25% of that in water flow. As the void fraction increases, the response amplitude increases. When α is large, the oscillation amplitude is very large compared with that in water flow ($\alpha = 0$).

The response in the crossflow direction of a single cylinder is different from the in-line response. For $\alpha \leq 20\%$, vibration amplitude increases almost linearly with U_r up to $U_r = 4.5$. For $U_r > 4.5$, vibration amplitude decreases for $\alpha < 10\%$ and is almost a constant value for larger α , e.g., $\alpha = 20\%$. Figure 7 shows that for $U_r > 4.5$ the oscillation amplitude is drastically reduced by injecting a certain amount of air bubbles into the flow. For $U_r < 4.5$, however, the two-phase flow-induced vibration is much stronger than that excited by vortex shedding in water flow.

Based on data of Hara (1984), it may be concluded that the excitation mechanisms for a single cylinder are the buffeting forces due to the random motion of air bubbles in the flow and vortex shedding that generates a narrow-banded force on the cylinder at low α . Furthermore, a small amount of air bubbles reduces the in-line and crossflow lock-in resonances and a large amount of air bubbles excites large cylinder oscillations.

Fig. 7. The rms accelerations of a cylinder as a function of reduced flow velocity U/fD : (a) in-line direction; (b) crossflow direction [from Hara (1984), with permission]

In a detailed study of tube rows subjected to crossflow, Hara (1987a) determined various vibrational characteristics and divided them into four types with respect to the effect of air bubbles on vibration. These are shown in Fig. 8 for four regions on the $\delta_m - U_r$ map and are described as follows:

- (A) When the void fraction is small, air bubbles excite random vibration, and the vibration amplitude increases with the void fraction.
- (B) At high reduced flow velocity and large mass-damping parameter, air bubbles have little effect on cylinder response, but for $\alpha = 20$ to 30%, vibration amplitude is reduced.
- (C) A small amount of air bubbles (about 2%) destabilizes cylinder vibration when U_r is slightly smaller than the critical value. However, at $\alpha = 30$ to 40%, the vibration is suppressed somewhat.
- (D) At high U_r , when the cylinder array is in a fluidelastic instability region, air bubbles have little effect on cylinder oscillations.

These results were obtained for tube rows. It is not certain the same conclusions can be made for other cylinder arrays.

For a cylinder array in two-phase flow below a certain critical flow velocity, oscillation amplitude increases with flow velocity. No major differences are observed in the lift and drag direction, although amplitudes may differ by a factor of two from one cylinder to another. The motion is excited by turbulence and bubbles. An array contains many coupled modes. With turbulence and bubble excitations, there is no clear coherence or phase angle between different cylinders. These characteristics are similar to the single-phase flow. However, the cylinder response at low flow velocity is higher than in single-phase flow and, at high flow velocities, the opposite is true. This trend is similar to that of a single cylinder in two-phase flow [Hara (1984)].

In subcritical vibration, the increase in cylinder response is generally related to a power of the flow velocity. In single-phase flow, the power is approximately equal to 2. However, in two-phase flow, the cylinder response is proportional to approximately the first power of the mass flux. Although Pettigrew and Gorman (1973) found the exponent to vary between 0.9 and 1.3, Pettigrew et al. (1985) show that the exponent decreases with α , starting from 2 at $\alpha = 0$, decreasing to 1 at $\alpha = 0.20$, and to a little smaller than 1 for $\alpha > 0.2$. The discrepancy between the single- and two-phase flow can probably be attributed to the bubble excitations. It is possible that turbulence forces are less correlated at higher mass fluxes and thus reduce the vibration response.

Fig. 8. Vibration characteristics (σ = vibration amplitude, α = void fraction) [from Hara (1987a), with permission]

In two-phase flow, the vibration within an array and the vibration of different arrays are also not the same as in single-phase flow. For example, Pettigrew and Gorman (1973) found that in two-phase flow the upstream tubes vibrate most and for a fixed P/D and various classical tube layouts, the rotated square arrays vibrate most, the rotated triangle and square arrays vibrate least, and the triangular arrays vibrate moderately. In addition, the preferred direction of motion is in the flow direction.

A cylinder array in two-phase flow will become unstable when the flow velocity is increased to a threshold value. Axisa et al. (1984) found that there is a fairly clearly defined critical flow velocity below which cylinder displacement is increasing with flow velocity raised to a certain power. Above the critical flow velocity the coherence and phase angle between various pairs of cylinders are well defined. In contrast, Pettigrew et al. (1985) found that the instability thresholds are well defined at lower and higher α but less well defined at the intermediate values of void fractions. In any case, both groups use the following stability criteria to correlate their data:

$$\frac{U}{fD} = K \left(\frac{2\pi\zeta_m}{2\rho D} \right)^{0.5}, \quad (23)$$

where U and ρ are given in Eqs. (4) and (5). Pettigrew et al. (1985) and Axisa et al. (1985), respectively, determined that K is equal to 4.0 and 7.2

Hara (1987a) summarizes the published data for tube arrays as well as his own data in Fig. 9, where the broken lines are the theoretical results obtained by Chen and Jendrzejczyk (1983) in water flow. When the void fraction is small, the critical flow velocity increases with the void fraction. The mass-damping parameter also increases with the void fraction owing to the reduction in two-phase flow density defined by the component $(1 - \alpha)\rho_l$. For a relatively large mass-damping parameter ($\delta_m = 3$ to 5), the critical flow velocity in two-phase flow is different from that for water flow.

In liquid flow, there exist two critical flow velocities: intrinsic critical flow velocity and excited critical flow velocity. The former is larger than the latter by as much as 30% [Chen and Jendrzejczyk (1987) and Hara (1987a)]. In two-phase flow, no such hysteresis is observed.

Mathematical Models for Two-Phase Crossflow-Induced Vibration

The analysis of an array of cylinders oscillating in two-phase flow has been performed based on the incorporation in the beam equation of some fluid effects [Pettigrew and Gorman (1973) and Axisa et al. (1984)]. No analysis has been made for coupled modes of multiple cylinders. A model based on coupled modes for single-phase flow is applied here to two-phase flow.

Fig. 9. Stability map: Δ, ∇ : Pettigrew et al. (1985); $\bullet$: Axisa et al. (1984); $\diamond$: Yoshikawa and Hara (1985); $*$: Tanaka and Takahara (1981); [from Hara (1987a), with permission]

An array of N cylinders subjected to a crossflow is shown in Fig. 3. The axes of the cylinders are parallel to the z axis and flow is parallel to the x axis. The subscript j is used to denote variables associated with cylinder j . The variables associated with cylinder motion in the x direction are flexural rigidity $E_j I_j$, cylinder mass per unit length m_j , the structural damping coefficient C_{sj} , and displacement u_j . The equation of motion for cylinder j in the x direction is

$$E_j I_j \frac{\partial^4 u_j}{\partial z^4} + C_{sj} \frac{\partial u_j}{\partial t} + m_j \frac{\partial^2 u_j}{\partial t^2} = g_j, \quad j = 1, 2, 3, \dots, N, \quad (24)$$

where g_j is the fluid force, including motion-dependent fluid forces and fluid excitation forces. Similarly, the equation of motion in the y direction is

$$E_j I_j \frac{\partial^4 v_j}{\partial z^4} + C_{sj} \frac{\partial v_j}{\partial t} + m_j \frac{\partial^2 v_j}{\partial t^2} = h_j, \quad j = 1, 2, 3, \dots, N, \quad (25)$$

where v_j and h_j are cylinder displacement and fluid-force component in the y direction, respectively.

The fluid force components g_j and h_j are given as follows [Chen (1987)]:

$$\begin{aligned} g_j = - \sum_{k=1}^N & \left[\left(\bar{\alpha}_{jk} \frac{\partial^2 u_k}{\partial t^2} + \bar{\alpha}'_{jk} \frac{\partial u_k}{\partial t} + \bar{\alpha}''_{jk} u_k \right) + \left(\bar{\sigma}_{jk} \frac{\partial^2 v_k}{\partial t^2} \right. \right. \\ & \left. \left. + \bar{\sigma}'_{jk} \frac{\partial v_k}{\partial t} + \bar{\sigma}''_{jk} v_k \right) \right] \\ & + \frac{1}{2} \rho U^2 DC_{Dj} + \frac{1}{2} \rho U^2 DC'_{Dj} \sin(\Omega_{Dj} t + \phi_{Dj}) + g'_j, \end{aligned} \quad (26)$$

and

$$\begin{aligned} h_j = - \sum_{k=1}^N & \left[\left(\bar{\tau}_{jk} \frac{\partial^2 u_k}{\partial t^2} + \bar{\tau}'_{jk} \frac{\partial u_k}{\partial t} + \bar{\tau}''_{jk} u_k \right) + \left(\bar{\beta}_{jk} \frac{\partial^2 v_k}{\partial t^2} \right. \right. \\ & \left. \left. + \bar{\beta}'_{jk} \frac{\partial v_k}{\partial t} + \bar{\beta}''_{jk} v_k \right) \right] \\ & + \frac{1}{2} \rho U^2 DC_{Lj} + \frac{1}{2} \rho U^2 DC'_{Lj} \sin(\Omega_{Lj} t + \phi_{Lj}) + h'_j. \end{aligned} \quad (27)$$

Note that $\bar{\alpha}_{ij}$, $\bar{\sigma}_{ij}$, $\bar{\tau}_{ij}$, and $\bar{\beta}_{ij}$ are added mass matrices; $\bar{\alpha}'_{ij}$, $\bar{\sigma}'_{ij}$, $\bar{\tau}'_{ij}$, and $\bar{\beta}'_{ij}$ are fluid damping matrices; and $\bar{\alpha}''_{ij}$, $\bar{\sigma}''_{ij}$, $\bar{\tau}''_{ij}$, and $\bar{\beta}''_{ij}$ are fluid stiffness matrices. C_{Dj} (C_{Lj}) is the steady drag (lift) coefficient, C'_{Dj} (C'_{Lj}) is the fluctuating drag (lift) coefficient, Ω_{Dj} (Ω_{Lj}) is the circular frequency of periodic flow excitation in the drag (lift) direction, ϕ_{Dj} (ϕ_{Lj}) is the corresponding phase angle with respect to a particular fluid-force component, and g'_j (h'_j) represents other fluctuating drag (lift) forces such as turbulent buffeting forces.

These equations are identical to those for single-phase flow. However, various fluid-force coefficients are different from those for single-phase flow.

Substituting Eqs. (26) and (27) into Eqs. (24) and (25) yields

$$\begin{aligned} E_j I_j \frac{\partial^4 u_j}{\partial z^4} + C_{sj} \frac{\partial u_j}{\partial t} + m_j \frac{\partial^2 u_j}{\partial t^2} + \sum_{k=1}^N \left(\bar{\alpha}_{jk} \frac{\partial^2 u_k}{\partial t^2} + \bar{\sigma}_{jk} \frac{\partial^2 v_k}{\partial t^2} \right) \\ + \sum_{k=1}^N \left(\bar{\alpha}'_{jk} \frac{\partial u_k}{\partial t} + \bar{\sigma}'_{jk} \frac{\partial v_k}{\partial t} \right) + \sum_{k=1}^N (\bar{\alpha}''_{jk} u_k + \bar{\sigma}''_{jk} v_k) \\ = \frac{1}{2} \rho U^2 D C_{Dj} + \frac{1}{2} \rho U^2 D C'_{Dj} \sin(\Omega_{Dj} t + \phi_{Dj}) + g'_j \end{aligned} \quad (28)$$

and

$$\begin{aligned} E_j I_j \frac{\partial^4 v_j}{\partial z^4} + C_{sj} \frac{\partial v_j}{\partial t} + m_j \frac{\partial^2 v_j}{\partial t^2} + \sum_{k=1}^N \left(\bar{\tau}_{jk} \frac{\partial^2 u_k}{\partial t^2} + \bar{\beta}_{jk} \frac{\partial^2 v_k}{\partial t^2} \right) \\ + \sum_{k=1}^N \left(\bar{\tau}'_{jk} \frac{\partial u_k}{\partial t} + \bar{\beta}'_{jk} \frac{\partial v_k}{\partial t} \right) + \sum_{k=1}^N (\bar{\tau}''_{jk} u_k + \bar{\beta}''_{jk} v_k) \\ = \frac{1}{2} \rho U^2 D C_{Lj} + \frac{1}{2} \rho U^2 D C'_{Lj} \sin(\Omega_{Lj} t + \phi_{Lj}) + h'_j. \end{aligned} \quad (29)$$

Equations (28) and (29) are the equations of motion for cylinder j in an array of cylinders subjected to crossflow. In a group of N cylinders, there are $2N$ equations of motion.

The response of an array of cylinders can be calculated fairly easily from Eqs. (28) and (29) if the various fluid forces in the equations of motion are known. It is straightforward to show that Eqs. (28) and (29) can be reduced to the standard form given in Eq. (9). A complete analysis of a cylinder array in crossflow requires knowledge of all the fluid forces.

Unfortunately, at this time, complete information is not available for specific cylinder arrays. This suggests that, for practical applications, simplifications or assumptions must be made to solve the problem.

Cylinders Immersed in Fluidized Beds

So far discussions have been limited to two-phase flow consisting of two fluid components. Another important problem of two-phase crossflow-induced vibration is the fluidized bed. Heat is extracted from fluidized bed reactors by tube arrays immersed in the bed.

Whereas heat transfer in fluidized beds has received considerable attention, very little work has been reported on fluid forces and associated tube vibration and wear. The limited number of studies reported in the open literature have all been published subsequent to 1980 [Kennedy et al. (1981), Turner and Irving (1982), Hosny and Grace (1983, 1984), and Grace and Hosny (1985)]. These investigations are useful because they provide insights into the characteristics of fluid forces acting on tube arrays in fluidized beds. Some of the essential characteristics determined from these studies can be summarized as follows:

Flow Regimes

In a fixed bed, up to the condition of incipient fluidization, the tubes are subjected to an effective buoyancy force only. As the flow velocity increases to the bubbling regime, the tubes are subjected to a drag force in the downstream direction as the bubble approaches and to a force in the upstream direction as the bubble leaves the tube. Once the flow velocity is increased to the point where the flow is in the turbulent regime, the magnitude of the fluid force is reduced. The bubbling regime is of particular interest because the fluid force encountered has a maximum magnitude which is difficult to quantify.

Tube Array vs. Single Tube

Fluid forces depend on tube arrangement and spacing. Fluid forces acting on a tube array are smaller than those acting on the same tube in isolation. In general, the fluid force is largest on the upstream tube and smallest on the downstream tube. In addition, the peak frequency of the fluid forces for tubes in the middle of a tube array is higher because bubble splitting by the tube array results in a greater frequency of bubble-induced pulses. In a tube array, vertical components are also substantially larger than the corresponding horizontal components.

Effects of System Parameters

Tube Shape: Finned and unfinned tubes were tested by Grace and Hosny (1985). The rms fluid force acting on a finned tube approaches the force acting on a bare tube of the same volume at low flow velocities and the force

acting on a bare tube of diameter equal to the outer diameter at high flow velocities. Only one type of finned tube was tested. The effect of other types of fins is not known.

Bed Depth: The rms fluid forces increase with bed depth. This is attributed to the tendency of larger bubbles in the deeper beds to cause larger forces. In addition, in a shallower bed, the largest fluid force may occur in the middle of a tube array.

Particle Size and Density: At low flow velocities, particle size does not have any significant effect on fluid forces, whereas at high flow velocities, particle size plays an important role. Fluid forces are also found to increase with particle density.

There are four basic excitation mechanisms for tube arrays subjected to single-phase crossflow: vortex shedding, fluidelastic instability, turbulent buffeting, and acoustic resonance. In fluidized beds, in addition to these four excitation mechanisms, the excitation force attributed to the bubbles represents an important mechanism. In the literature relating to forces on tubes in fluidized beds, the significance of the four basic mechanisms has not been examined carefully.

(1) Vortex Shedding: It appears that no systematic investigations have been made to study the role of vortex shedding in fluidized beds or its effect on tube response. In fluidized beds, tube spacings are generally large. Consequently, at high flow velocities, in particular in the dilute phase, vortex shedding could be an important excitation mechanism.

(2) Fluidelastic Instability: Because of the relatively large tube spacing in fluidized beds, the critical flow velocity at which large tube oscillations (instability) occur is expected to be high. However, no analyses or experimental studies have been made to quantify the safety margin.

(3) Turbulent Buffeting: Random fluid forces in fluidized beds have been identified as an excitation mechanism. However, the determination of force magnitude, frequency spectra, and the nondimensional parameters required to quantify the characteristics of the turbulent buffeting forces remain incomplete.

(4) Acoustic Resonance: Particles and tubes immersed in a fluid medium will reduce the effective speed of sound [Chen (1987)]. Therefore, the natural frequencies of acoustic modes in fluidized beds will be smaller than those for the corresponding single-phase flow without tubes. This effect will increase the chance of acoustic resonance and associated tube oscillations. In particular, in a large fluidized bed, acoustic resonance can be a potential problem.

From a review of the published data and a consideration of the potential excitation mechanisms, it may be concluded that, although the published data

provide insights into the dynamic characteristics of the fluid forces acting on tube arrays in fluidized beds, a systematic study is lacking and urgently needed.

IV. AXIAL-FLOW-INDUCED VIBRATION

Axial-flow-induced vibration problems can be divided into those attributable to internal flow and those attributable to external flow. Two-phase flow in pipes is a typical internal flow problem; nuclear fuel pins subjected to axial two-phase flow is a typical external flow problem. Both internal and external flow problems have been studied in the last 15 years.

Internal Flow

Pipes conveying a two-phase fluid have been studied by Hara (1973), Hara et al. (1974), Hara (1977), Hara (1980), Yih and Griffith (1970), and Pettigrew and Paidoussis (1975). The dynamic responses considered include subcritical vibration, parametric resonance, and flutter.

The equation of motion for a pipe conveying a two-phase fluid is given by Hara (1977) as

$$EI \frac{\partial^4 u}{\partial z^4} + (m + M_d) \frac{\partial^2 u}{\partial t^2} + 2M_d U \frac{\partial^2 u}{\partial z \partial t} + M_d U^2 \frac{\partial^2 u}{\partial z^2} + \left(\frac{\partial M_d}{\partial t} + U \frac{\partial M_d}{\partial z} \right) \left(\frac{\partial u}{\partial t} + U \frac{\partial u}{\partial z} \right) = g(z, t) \quad (30)$$

where the symbols are given in the nomenclature. Using the appropriate boundary conditions and the equation of motion, the internal flow problem is reduced to Eq. (9).

Hara (1973, 1977) considered the two-phase flow as periodic flow consisting of water slugs periodically distributed in the pipe and moving at a constant speed. Using this flow model, Hara (1980) obtained a system of Mathieu's equations from Eq. (30). Parametric resonance occurs if the ratio of the frequencies of water slugs and pipe natural frequency is 2, 1, 2/3, In addition, resonance occurs when the ratio is 1, 1/2, 1/3, When the ratio is equal to 1, parametric excitation and resonance results in large oscillations. The theoretical result is verified by experimental data. In general, vibration amplitude depends on the two-phase flow pattern and velocity; oscillation amplitude is largest for a piston flow. The excitation is attributed to periodic changes of the mass of the pipe/fluid system, which, in turn, results in periodic changes in system mass, centrifugal force, and the Coriolis force of liquid slugs in the pipe.

Pettigrew and Paidoussis (1975) performed tests on a cylinder immersed in an annulus two-phase flow. In the subcritical flow velocity range, the vibration response is fairly severe and broadbanded. The motion is excited by random turbulence associated with two-phase flow.

External Flow

Studies of cylinders submerged in two-phase flow are focused on nuclear fuel pins. Investigations have been reported for a single cylinder by Gorman (1971), Harris and Holland (1971), Harris and Ledwidge (1971), Harris and Holland (1973), Hara (1975a) and (1975b), Hara and Yamashita (1978), and Hara (1980); for multiple cylinders by Winsbury and Ledwidge (1973), Dallavalle et al. (1973), and Gorman (1975); and for simulated fuel bundles by Cedolin et al. (1971), and Hayes (1982). A survey of this problem was reported by Shin (1978).

The equation of motion of a single cylinder in axial two-phase flow is still not well established. In the past, the Bernoulli-Euler beam equation of motion was used [Hara (1975a) and Gorman (1975)] and the fluid-force components associated with two-phase flow, including added mass, fluid damping, and fluid stiffness, were not accounted for consistently. Using the model for single-phase flow, Shin (1978) modified the Chen and Wambsganss (1972) equation of motion for single-phase flow for application to two-phase flow. The modified equation is as follows:

$$\begin{aligned}
 EI \frac{\partial^4 u}{\partial z^4} + \eta I \frac{\partial^5 u}{\partial t \partial z^4} + m_a \left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial z} \right)^2 u \\
 - \left[\gamma T_0 + \frac{1}{2} C_T \frac{m_a U^2}{D} G(z,t) + \frac{1}{2} (1 - \gamma) C_T' m_a U^2 \right] \frac{\partial^2 u}{\partial z^2} \\
 + \frac{1}{2} C_N \frac{m_a U C_N}{D} \left(\frac{\partial u}{\partial t} + U \frac{\partial u}{\partial z} \right) + C_v \frac{\partial u}{\partial t} + m \frac{\partial^2 u}{\partial t^2} = g(z,t) , \quad (31)
 \end{aligned}$$

where η is the structural damping coefficient, C_T and C_N are tangential and normal drag coefficients, respectively, C_T' is the drag coefficient at the free end, T_0 is the initial axial tension, and other symbols are listed in the nomenclature. In Eq. (31), $\gamma = 0$ for a free-end, whereas for both ends fixed, $\gamma = 1$. Equation (31) is an approximate equation of motion for a single cylinder in two-phase flow. Once the fluid-force coefficients are known it can be solved in a straightforward manner.

The characteristics of two-phase flows in the vicinity of PWR steam generator tube support plates were investigated in a qualitative manner by Bates and Stewart (1979). The model test section consists of a 2 x 2 tube

bundle fitted with prototypic support plates enclosed in a clear lucite flow housing. Figure 10 shows the flow patterns for various values of void fraction. Observations of both the in-operation test assembly and the high-speed motion picture evidence reveal that very definite two-phase flow regimes could be distinguished for different values of void fraction:

$\alpha = 0.1 - 0.3$ -- Low turbulence, bubbly flow, with uniform-size bubbles

$\alpha = 0.4 - 0.7$ -- More turbulent churning flow

$\alpha = 0.8 - 0.9$ -- More separated, oscillatory chugging or plug flow

$\alpha = 0.95$ -- Annular flow with the liquid phase concentrated on the wetted surfaces.

The driving force in two-phase flow originates from turbulence and bubble impacts. In addition, the time-varying virtual mass along the cylinder gives rise to the potential for parametric excitation. The phenomenon is the same as that for internal flow. Experiments and analysis of parametric excitation were performed by Hara (1980). Parametric resonance occurs when the ratio of the natural frequency of the cylinder to the void slug frequency is equal to 0.5, 1, and 1.5, and resonance occurs when the ratio is equal to 1, 2, 3. The analysis can be based on Eq. (31). Hara simplified the analysis by neglecting the effect of drag forces. His experimental and analytical results agree reasonably well [Hara (1980)].

For subcritical vibration, it is important to characterize the flow field. Gorman (1971) studied the pressure field acting on the surface of a cylinder in an annulus. To eliminate the system noises, two pairs of diametrically opposite pressure transducers were used. The longitudinal and peripheral correlation, and power spectra of pressure fluctuations have been studied under different flow conditions. Figure 11 shows the longitudinal spatial correlation vs. transducer separation for two values of void fraction. The correlation is found to be a damped cosinusoidal curve,

$$A(z, z') = \exp(-\beta|z - z'|) \cos \delta(z - z') . \quad (32)$$

Equation (32) is different from the exponential function for single-phase flow. Figure 12 shows the typical peripheral spatial correlation. It is similar to a cosinusoidal distribution. Gorman (1971) suggested the following correlation:

$$B(\theta, \theta') = \cos (\theta - \theta') - \lambda \sin 2|\theta - \theta'| \quad (33)$$

Fig. 10. Flow patterns in axial flow as a function of void fraction [from Bates and Stewart (1979), with permission]

Fig. 11. Typical longitudinal spatial correlation of fluid pressure
[from Gorman (1971), with permission]

Fig. 12. Typical peripheral spatial correlation of fluid pressure [from Gorman (1971), with permission]

where λ may be selected to best fit the curve associated with each void fraction.

Based on Gorman's study (1971), several conclusions can be made:

- The mathematical model for single-phase flow is applicable for two-phase flow provided proper fluid parameters are used.
- There is good agreement between the shape of the measured displacement, the shape of the displacement predicted by the random vibration theory, and the measured boundary-layer pressure fluctuations. The vibration amplitude in two-phase flow is higher than that in single-phase flow because of higher peripheral spatial correlation of pressure fluctuations.
- Damping coefficients associated with the first mode were found to be about four times as high as for liquid flow.

The effect of void fraction on cylinder response is of particular interest. Detailed studies have been performed by Harris and Holland (1973) and Winsbury and Ledwidge (1973). The cylinder response is found to be proportional to $\alpha^n(1 - \alpha)$, mass flux, and other parameters. The values of n are given by Winsbury and Ledwidge (1973):

$$\begin{aligned}
 0.16 < \alpha \leq 0.30, & \quad n = 1.3 \\
 0.30 < \alpha < 0.70, & \quad n = 2.6 \\
 0.70 \leq \alpha \leq 0.94, & \quad n = 5.2.
 \end{aligned} \tag{34}$$

Equation (34) is based on their own data and the results of others. For a given mass flux, the maximum response occurs for $0.80 < \alpha < 0.88$.

Hara (1980) studied the relation between vibration response and the Reynold's number, which is defined as $(1 + Q_A/Q_L)DU_L/\nu_L$, where Q_A and Q_L are the flow rate of air and water, respectively. A typical result is shown in Fig. 13. When Re is smaller than 6×10^4 , vibration is small; when $Re > 8.5 \times 10^4$, vibration is large.

The cylinder response for a simulated fuel rod in both a two-phase and boiling flow were measured by Hayes (1982). The results show that the boiling tests and two-phase inlet tests resulted in higher vibration than that obtained in single-phase flow. For a constant mass flux, the fuel rod vibration peaks at a particular void fraction in both, boiling and adiabatic, two-phase flows.

Fig. 13. Variance of vibration strain and two-phase flow Reynolds number Re [from Hara (1980), with permission]

V. CLOSING REMARKS

The effect of two-phase flow is much more complicated than that of single-phase flow. This is attributed to the fluctuations of flow rate, density and pressure gradients, as well as oscillations due to compressibility of the fluid. The passage of bubbles is accompanied by microhydraulic impacts, which produce fluctuation forces in two-phase flow. Therefore, in addition to the pressure fluctuations encountered in single-phase flow, the microhydraulic impacts associated with two-phase flow are very important.

The mechanisms of two-phase flow-induced vibration include subcritical vibration excited by turbulence and bubble impacts, parametric resonance associated with the periodic variation of mass flux, vortex-induced vibration for small void fractions, and large amplitude oscillation of fluidelastic instability. Reported studies of these phenomena for axial- and crossflow indicate that progress has been made. However, investigations that are much more systematic are needed. Several topical areas are important:

- Flow Field: Flow velocity distribution in heat transfer equipment, distribution of bubbles, mathematical characterization of flow patterns and effect of system parameters on flow field.
- Fluid Excitation Forces: Power spectral density of fluid excitation forces, axial and peripheral correlation of fluid pressure fluctuations, and effect of void fraction on fluid excitation forces.
- Motion-Dependent Fluid Forces: Added mass, fluid damping, and fluid stiffness as a function of void fraction and flow rate for different cylinder arrays as well as different oscillation frequencies.
- Cylinder Response: Mathematical models that incorporate all fluid effects, simplified equations for different excitation mechanisms, and response as a function of system parameters.
- Design Guides: Relationship between two-component (e.g., air-water) two-phase flow and single-component (e.g., steam-water) two-phase flow, design guides for predicting cylinder response, acceptance criteria, and design modification techniques.

All of the above topics are very important and must be studied to resolve the issues of two-phase flow-induced vibration and to provide the basis for the development of prediction methods and design guides. Argonne National Laboratory is planning a comprehensive research program to look into this subject. A two-phase air-water loop is being designed. Experiments are expected to begin in FY 1988. A series of tests will be performed to study fluid excitation force, cylinder response, and motion-dependent fluid forces.

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