FLOW-INDUCED VIBRATION OF THE SSME LOX POSTS:
ADDITIONAL ISSUES

by

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ABSTRACT

A preliminary assessment of the SSME LOX post vibration problem was performed recently. This report discusses additional issues raised during discussions among Rocketdyne, MSFC/NASA, and ANL and not covered in the earlier report.

I. INTRODUCTION

A preliminary assessment of the SSME LOX post vibration problem was presented in a recent report [1]. Based on the scoping calculations and experiment, fluidelastic instability and turbulent buffeting have been identified as the potential excitation mechanisms that can cause detrimental vibration of the SSME LOX posts. Both theoretical and experimental investigations on the subject are being conducted at the Rocketdyne Division of Rockwell International and at Argonne National Laboratory. The objectives of these studies are to understand the vibrational characteristics and to develop techniques to avoid detrimental vibrational effects with the overall objective of improving engine life.

Two series of tests are being performed at Rocketdyne Division:

- In the dynamic load study, various post array configurations are being tested in air flow. The objective of this series of tests is to determine the existence of fluidelastic instability, the effects of various parameters and possible fixes.

- In the second series of tests, a solid-wall hot gas manifold (SWHGM) is tested for air flow. The purpose of this test series is to experimentally evaluate techniques to improve the flow uniformity in the manifold and reduce the LOX post loads [2].

Argonne National Laboratory is working for Marshall Space Flight Center of NASA to participate in the evaluation of Rocketdyne's test programs. The purpose of this report is to summarize additional issues raised during discussions among Rocketdyne, MSFC/NASA, and ANL.
II. A MATHEMATICAL MODEL FOR FLOW-INDUCED VIBRATION OF THE SSME LOX POSTS

A mathematical model for LOX post vibration is presented in Ref. 1. A brief summary of the model is presented here to facilitate the discussion of the additional issues raised.

An array of \( N \) LOX posts subjected to a cross-flow is shown in Fig. 1. The axes of the posts are parallel to the \( z \) axis and flow is parallel to the \( x \) axis. The subscript \( j \) is used to denote variables associated with post \( j \). The variables associated with post motion in the \( x \) direction are flexural rigidity \( E_{j}I_{j} \), post mass per unit length \( m_{j} \), structural damping coefficient \( C_{s_{j}} \), and displacement \( u_{j} \). The equation of motion for post \( j \) in the \( x \) direction is

\[
E_{j}I_{j} \frac{\partial^{4} u_{j}}{\partial z^{4}} + C_{s_{j}} \frac{\partial u_{j}}{\partial t} + m_{j} \frac{\partial^{2} u_{j}}{\partial t^{2}} = g_{j}, \quad j = 1, 2, 3, \ldots, N
\] (1)

where \( g_{j} \) is the fluid forces including motion-dependent fluid forces and fluid excitation forces. Similarly, the equation of motion in the \( y \) direction is

\[
E_{j}I_{j} \frac{\partial^{4} v_{j}}{\partial z^{4}} + C_{s_{j}} \frac{\partial v_{j}}{\partial t} + m_{j} \frac{\partial^{2} v_{j}}{\partial t^{2}} = h_{j}, \quad j = 1, 2, 3, \ldots, N
\] (2)

where \( E_{j}I_{j}, C_{s_{j}}, v_{j}, \) and \( h_{j} \) are flexural rigidity, structural damping coefficient, post displacement, and the force per unit length in the \( y \) direction.

Mathematically, the fluid force components can be divided into two groups:

**Fluid Excitation Forces**

\[
g_{j} = \frac{1}{2} \rho U^{2} D C_{D_{j}} + \frac{1}{2} \rho U^{2} D C_{D_{j}}' \sin(\Omega_{D_{j}} t + \Phi_{D_{j}}) + g_{j}'
\] \hspace{1cm} (3)

and

\[
h_{j} = \frac{1}{2} \rho U^{2} D C_{L_{j}} + \frac{1}{2} \rho U^{2} D C_{L_{j}}' \sin(\Omega_{L_{j}} t + \Phi_{L_{j}}) + h_{j}'
\]

where \( \rho \) is the fluid density, \( U \) is the flow velocity, \( D \) is the post diameter, \( C_{D_{j}} \) (\( C_{L_{j}} \)) is the steady drag (lift) coefficient, \( C_{D_{j}}' \) (\( C_{L_{j}}' \)) is the fluctuating drag (lift) coefficient, \( \Omega_{D_{j}} \) (\( \Omega_{L_{j}} \)) is the circular frequency of periodic flow excitation in the drag (lift) direction and \( \Phi_{D_{j}} \) (\( \Phi_{L_{j}} \)) the
(a) A GROUP OF CIRCULAR CYLINDERS

(b) FLUID FORCE AND CYLINDER DISPLACEMENT COMPONENTS

Fig. 1. Schematic of a Group of LOX Posts in Crossflow
corresponding phase angle, and \( g'_j \) (\( h'_j \)) is the random fluctuating drag (lift) force.

**Motion-dependent Fluid Forces**

\[
g_j = - \sum_{k=1}^{N} \left\{ \left[ \alpha_{jk} \frac{\partial^2 u_k}{\partial t^2} + \alpha'_{jk} \frac{\partial u_k}{\partial t} + \alpha''_{jk} u_k \right] + \left[ \sigma_{jk} \frac{\partial^2 v_k}{\partial t^2} + \sigma'_{jk} \frac{\partial v_k}{\partial t} + \sigma''_{jk} v_k \right] \right\},
\]

and

\[
h_j = - \sum_{k=1}^{N} \left\{ \left[ \tau_{jk} \frac{\partial^2 u_k}{\partial t^2} + \tau'_{jk} \frac{\partial u_k}{\partial t} + \tau''_{jk} u_k \right] + \left[ \beta_{jk} \frac{\partial^2 v_k}{\partial t^2} + \beta'_{jk} \frac{\partial v_k}{\partial t} + \beta''_{jk} v_k \right] \right\}.
\]

Note that \( \alpha_{ij} \), \( \sigma_{ij} \), \( \tau_{ij} \), and \( \beta_{ij} \) are added mass matrices, \( \alpha'_{ij} \), \( \sigma'_{ij} \), \( \tau'_{ij} \), and \( \beta'_{ij} \) are fluid damping matrices, and \( \alpha''_{ij} \), \( \sigma''_{ij} \), \( \tau''_{ij} \), and \( \beta''_{ij} \) are fluid stiffness matrices.

Substituting Eqs. 3 and 4 into Eqs. 1 and 2 yields

\[
E_j \left\{ \frac{\partial^4 u_j}{\partial x^4} + C_j \frac{\partial^2 u_j}{\partial t^2} + m_j \frac{\partial^2 u_j}{\partial t^2} + \sum_{k=1}^{N} \left( \alpha_{jk} \frac{\partial^2 u_k}{\partial t^2} + \sigma_{jk} \frac{\partial^2 v_k}{\partial t^2} \right) \right\}
\]

\[
+ \sum_{k=1}^{N} \left( \alpha'_{jk} \frac{\partial u_k}{\partial t} + \sigma'_{jk} \frac{\partial v_k}{\partial t} \right) + \sum_{k=1}^{N} \left( \alpha''_{jk} u_k + \sigma''_{jk} v_k \right) \right\} = \frac{1}{2} \rho U^2 DC_{Dj} + \frac{1}{2} \rho U^2 DC_{Lj} \sin(\alpha_{Dj} + \phi_{Dj}) + g'_j
\]

and

\[
E_j \left\{ \frac{\partial^4 v_j}{\partial x^4} + C_j \frac{\partial^2 v_j}{\partial t^2} + m_j \frac{\partial^2 v_j}{\partial t^2} + \sum_{k=1}^{N} \left( \tau_{jk} \frac{\partial^2 u_k}{\partial t^2} + \beta_{jk} \frac{\partial^2 v_k}{\partial t^2} \right) \right\}
\]

\[
+ \sum_{k=1}^{N} \left( \tau'_{jk} \frac{\partial u_k}{\partial t} + \beta'_{jk} \frac{\partial v_k}{\partial t} \right) + \sum_{k=1}^{N} \left( \tau''_{jk} u_k + \beta''_{jk} v_k \right) \right\} = \frac{1}{2} \rho U^2 DC_{Dj} + \frac{1}{2} \rho U^2 DC_{Lj} \sin(\alpha_{Lj} + \phi_{Lj}) + h'_j.
\]
Equations 5 and 6 are the equations of motion for post \( j \) in an array of posts subjected to a crossflow. In a group of \( N \) posts, there are \( 2N \) equations of motion.

If all posts are of the same length and have the same type of boundary conditions in the \( x \) and \( y \) directions, the modal functions for posts vibrating in the \( x \) and \( y \) directions will be the same; thus let

\[
u_j(z,t) = \sum_{m=1}^{\infty} a_{jm} \phi_m(z) \tag{7}\]

and

\[
v_j(z,t) = \sum_{m=1}^{\infty} b_{jm} \phi_m(z),
\]

where \( \phi_m(z) \) is the \( m \)th orthonormal function of the cylinder in vacuum. Assume that the flow-velocity distribution is given by

\[
U(z) = \bar{U} \psi(z). \tag{8}
\]

Using Eqs. 5, 6, 7, and 8 yields

\[
\frac{d^2 a_{jm}}{dt^2} + 2v_{jm} \omega v_{jm} \frac{da_{jm}}{dt} + \omega^2 v_{jm} a_{jm} 
\]

\[
+ \frac{1}{m_j} \sum_{k=1}^{N} \left( \dot{a}_{jk} \frac{d^2 a_{km}}{dt^2} + \ddot{a}_{jk} \frac{d^2 b_{km}}{dt^2} \right)
\]

\[
+ \frac{1}{m_j} \sum_{k=1}^{N} \left( \dot{a}_{jkm} \frac{da_{km}}{dt} + \ddot{a}_{jkm} \frac{db_{km}}{dt} \right)
\]

\[
+ \frac{1}{m_j} \sum_{k=1}^{N} \left( \dddot{a}_{jkm} a_{km} + \dddot{b}_{jkm} b_{km} \right) \tag{9}
\]

\[
= \frac{1}{2m_j} \rho D C_{Djm} \ddot{U}^2 + \frac{1}{2m_j} \rho D C_{Djm} \ddot{U}^2 \sin(\Omega_{Dj} + \Phi_{Dj}) + \frac{1}{m_j} g_{jm}
\]

and
\[
\frac{d^2 b_{jm}}{dt^2} + 2\zeta_{vj m} \omega_{vj m} \frac{db_{jm}}{dt} + \omega^2_{vj m} b_{jm} \\
+ \frac{1}{m_j} \sum_{k=1}^{N} \left( \tau_{km} \frac{d^2 a_{km}}{dt^2} + \beta_{jk} \frac{d^2 b_{km}}{dt^2} \right) \\
+ \frac{1}{m_j} \sum_{k=1}^{N} \left( \tau_{jk m} a_{km} + \beta_{jm} b_{km} \right) \\
+ \frac{1}{m_j} \sum_{k=1}^{N} \left( \tau_{jk m} \omega_{vj m} \frac{da_{km}}{dt} + \beta_{jm} \omega_{vj m} \frac{db_{km}}{dt} \right) \\
+ \frac{1}{m_j} \sum_{k=1}^{N} \left( \tau_{jk m} a_{km} + \beta_{jm} b_{km} \right)
\]

\[
= \frac{1}{2m_j} \rho D L c_{L j m} \tilde{u}^2 + \frac{1}{2m_j} \rho D C_{L j m} \tilde{u}^2 \sin(\Omega_{Lj} + \phi_{Lj}) + \frac{1}{m_j} h_{jm},
\]

where

\[
\tilde{\alpha}_{jm} = \frac{1}{x} \int_{0}^{l} \tilde{\alpha}_{jm} \psi^2 dz, \quad \tilde{\alpha}_{jm} = \frac{1}{x} \int_{0}^{l} \tilde{\alpha}_{jm} \psi^2 dz,
\]

\[
\tilde{\sigma}_{jm} = \frac{1}{x} \int_{0}^{l} \tilde{\sigma}_{jm} \psi^2 dz, \quad \tilde{\sigma}_{jm} = \frac{1}{x} \int_{0}^{l} \tilde{\sigma}_{jm} \psi^2 dz,
\]

\[
\tilde{\tau}_{jm} = \frac{1}{x} \int_{0}^{l} \tilde{\tau}_{jm} \psi^2 dz, \quad \tilde{\tau}_{jm} = \frac{1}{x} \int_{0}^{l} \tilde{\tau}_{jm} \psi^2 dz,
\]

\[
\tilde{\beta}_{jm} = \frac{1}{x} \int_{0}^{l} \tilde{\beta}_{jm} \psi^2 dz, \quad \tilde{\beta}_{jm} = \frac{1}{x} \int_{0}^{l} \tilde{\beta}_{jm} \psi^2 dz,
\]

\[
C_{Dj m} = \frac{1}{x} \int_{0}^{l} C_{Dj m} \psi^2 dz, \quad C_{Lj m} = \frac{1}{x} \int_{0}^{l} C_{Lj m} \psi^2 dz,
\]

\[
C_{Dj m} = \frac{1}{x} \int_{0}^{l} C_{Dj m} \psi^2 dz, \quad C_{Lj m} = \frac{1}{x} \int_{0}^{l} C_{Lj m} \psi^2 dz,
\]

and \( \omega_{vj m} \) and \( \zeta_{vj m} \) are the circular frequency and modal damping ratio of the \( m \)th modes of post j cylinder in vacuum.
The responses of an array of posts can be calculated fairly easily from Eqs. 9 and 10 if the various fluid force coefficients in the equations are known. Equations 9 and 10 can be written in the standard form

\[ [M]{\ddot{Q}} + [C]{\dot{Q}} + [K]{Q} = {G} \]  \hspace{1cm} (12)

or

\[ [M_s + M_f]{\ddot{Q}} + [C_s + C_f]{\dot{Q}} + [K_s + K_f]{Q} = {G}, \] \hspace{1cm} (13)

where \([M]\) is the mass matrix including structural mass \([M_s]\) and added mass \([M_f]\); \([C]\) is the damping matrix, including structural damping \([C_s]\) and fluid damping \([C_f]\); \([K]\) is the stiffness matrix, including structural stiffness \([K_s]\) and fluid stiffness \([K_f]\); and \([G]\) is the fluid excitation forces, including vortex shedding, turbulence, acoustic noise, etc. Note that the fluid matrices \([M_f, C_f, K_f]\) are related to the various matrices given in Eq. 4: \(\sigma_{jk}, \sigma_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}, \bar{\sigma}_{jk}, \bar{\sigma}_{j',k}.

III. ADDITIONAL ISSUES IN LOX POST VIBRATION

A. Definition of the Critical Flow Velocity

The critical flow velocity is defined as the flow velocity above which large post oscillations occur. Mathematically, this is described as follows: Let the displacement of a particular location of the posts be

\[ u(t) = a \exp(\lambda + i\omega)t. \] \hspace{1cm} (14)

The stability of the posts is determined by \(\lambda\), which is a function of flow velocity: If \(\lambda < 0\), the post motion is damped; if \(\lambda > 0\), the post displacement increases with time. \(\lambda = 0\) is the condition that separates the stable and unstable regions. Therefore, the critical flow velocity can be determined from the condition \(\lambda = 0\).

There are a number of methods that have been used to define the critical flow velocity in laboratory tests and practical equipment tests. These are discussed below:

(1) Sensory Observations - Post vibration amplitudes are determined visually or auditorially. When an array can be viewed from the end or at a particular section, the amplitude increase can generally be detected visually. If, in addition, post impacting results; a distinctive loud noise associated with the impacting is readily audible. This method does not
provide a precise determination of the critical flow velocity in most cases, because it is subjective and requires a fair amount of engineering judgment and experience.

(2) Vibration Amplitude vs. Flow Velocity - The rms amplitude or peak amplitude is plotted as a function of flow velocity. The flow velocity at which the posts experience a rapid increase in response is defined as the critical flow velocity. Using this definition, different investigators define the critical flow velocity in different manners. For example, Weaver and El-Kashlan [3] define the threshold flow velocity as the point on the curve where there is a sudden change in slope, while Soper [4] defines the intersection with the velocity axis of the tangent to that portion of the curve which is rapidly rising as the critical flow velocity.

(3) Vibration Amplitude vs. Flow - The critical flow velocity is defined as the velocity at which the threshold displacement is first exceeded. Values of 2 to 2.5 percent of post diameter have been suggested [5]. Once the threshold amplitude is established, it is straightforward to determine the critical flow velocity. This method is attractive for practical applications but it is not theoretically correct.

(4) Frequency Response Data - When a post array is subjected to a flow, there are many natural frequencies of coupled modes. At flow velocities below the threshold value, turbulent buffeting or other excitation source is the dominant excitation mechanism. It excites, in general, a broad range of coupled frequencies. On the other hand, the vibration at instability will typically be at a well-defined, single frequency corresponding to a particular instability mode. Therefore, the critical flow velocity can be defined as the flow velocity at which the response PSD changes from a relatively broad-band spectrum to a narrow-band spectrum. This method is accurate in general. However, difficulty arises for light fluid in which the natural frequencies of coupled modes are in a narrow band as well as for the case in which either there is a gradual transition from broad band to narrow band spectra or instability is too abrupt such that it results in impacting with broad-band spectra above the critical flow velocity.

There is no single method that can be used to predict the critical flow velocity precisely in all cases. According to the definition of the critical flow velocity associated with $\lambda = 0$, the combination of the following two methods appears most useful: vibration amplitude vs. flow velocity and frequency response data. In practical applications, from the vibration amplitude-flow velocity curve, the critical flow velocity can be established approximately. With the information of frequency response data at different flow velocities near the critical region, the critical flow velocity then can be determined more precisely. In light fluids, the
critical flow velocity is associated with high values of $U_r$ ($U_r = U/fD$; $f$ = post natural frequency) and it can generally be determined from the amplitude velocity curve alone. The response spectra will be used to verify the existence of instability. In heavy fluids, the frequency spectra can be used as the primary tool in determining the critical flow velocity.

B. Detuning

Note that LOX posts in different rows are of different lengths, i.e., dynamic characteristics of LOX posts vary according to their position. The frequency variation of LOX posts in vacuum is called detuning. Once an array of LOX posts is submerged in a fluid, the LOX posts are coupled by the fluid. Therefore, in defining detuning, the natural frequencies of each individual post in vacuum (practically in air) must be employed.

In general, the detuning of an array increases the critical flow velocity [6]. Fluid-damping-controlled instability is predominantly attributed to the motion of the post itself; i.e., the controlling parameter is the diagonal terms of the fluid damping matrix $C_f$ in Eq. 13 or $\tilde{C}_{ij}$ and $\tilde{C}_{jj}$ in Eqs. 5 and 6. Coupling with the neighboring posts is not a necessary condition for this type of instability to occur. Therefore, the effect of detuning is not very significant. On the contrary, fluid-stiffness-controlled instability is attributed to the coupling effect and the controlling parameters are the off-diagonal terms of the fluid-stiffness matrix $K_f$ in Eq. 13, or $\tilde{K}_{jk}$ and $\tilde{K}_{jk}$ in Eqs. 5 and 6. In this case, detuning plays a more significant role.

The effect of detuning on the fluid-stiffness-controlled instability was demonstrated by Southworth and Zdravkovich [7] for a row of cylinders. In a wind-tunnel test, they obtained the critical flow velocity of a row of in-tune cylinders at about $U_r = 45$. However, when only one of the cylinders in a row is flexible, they do not observe instability for $U_r$ up to 100. This is consistent with the theoretical prediction [6]. Tests also have been done for rows with three adjacent tubes flexible, and alternating tubes flexible. At a given flow velocity, tube response is largest for all tubes flexible and smallest for one tube flexible. This also illustrates that detuning tends to stabilize the system.

In a water-loop test, Chen and Jendrzejczyk [8] demonstrate the effect of detuning on fluid-damping-controlled instability. They show that an elastic tube surrounded by rigid tubes in water flow can lose stability; the motion is predominantly in the lift direction. This agrees with the theory that a single elastic tube in a square array can lose stability by fluid-damping force in the lift direction.

Other experimental data [4,9,10,11] basically agree with those of Southworth and Zdravkovich [7] as well as analytical results [12]. However,
Weaver and Lever [11] show that tests on a rotated triangular array produce an increase in critical flow velocity of up to 46% for a 3% difference in frequency and no significant effect for a frequency difference greater than 10%. Larger detuning might cause the critical mode with the lowest critical flow velocity to change to some other mode. For a particular mode, detuning is expected to be beneficial in stabilizing the tubes.

A series of tests with the difference in streamwise and transverse frequencies ranging from 6.3 to 57% for a rotated triangular array with a pitch ratio of 1.375 was conducted by Weaver and Koroyannakis [13]. They found that the critical reduced flow velocity based on the lower frequency was increased only slightly over the symmetric case, being about 20% higher than that for tubes with identical stiffness in the transverse and streamwise directions. The effect is essentially independent of the difference in frequency and "direction" of the lower frequency relative to flow. Note that the results are applicable for the particular tube arrangement only. For different tube arrays, the effect will not be the same. For example, for a tube row in water flow, the lowest critical flow velocity is associated with the out-of-phase mode in the lift direction [6]; in this case, an increase in the natural frequency in the drag direction will have little effect on the critical flow velocity.

Based on the analytical and experimental results, it may be concluded that the critical flow velocity for an in-tune post array is generally smaller than that of a detuned post array. Therefore, in the evaluation of the critical flow velocity, it is conservative to assume that all LOX posts have the same length as those in Row 13.

C. Nonuniform Flow Distribution

Flow velocity is generally not uniform in the axial direction along the post or perpendicular to the post axis. Most of the published experimental data are obtained for uniform flow. In practice, nonuniform flow distribution has to be considered.

(1) Nonuniform Flow in the Transverse Direction

Gorman [14] considered the effect of open tube lanes on instability of tubes adjacent to these lanes and found that there was no evidence of local triggering of instabilities. Connors [15] shows that the skimming flows created in the vicinity of inlet-nozzle impingement plates can cause instability; the critical flow velocity depends on tube pattern and spacing and on the clearance between the tube array and the wall. In a large tube array, the tubes will not become unstable at the same time; this is attributed to the nonuniform flow distribution as well as other effects. In practical applications, it is difficult to assess the effect of the nonuniform flow in the transverse direction, but it is reasonable to
consider an equivalent uniform-flow case with the flow velocity being the maximum flow of the nonuniform case.

(2) Nonuniform Flow in the Axial Direction

Empirical correlations are developed for the case in which the entire post length is subjected to the same flow velocity. In many structural components or experiments, the flows are not uniform. A general practice is to reduce the general case of nonuniform flow to the ideal case of uniform flow. An equivalent uniform flow velocity is defined by (see Eqs. 8-11)

\[
\frac{U_e^2}{\rho} = \frac{\int U(z)^2 \phi_m^2(z) dz}{\int \phi_m^2(z) dz} = \frac{\bar{U}^2}{\rho} \frac{\int \psi(z) \phi_m^2(z) dz}{\int \phi_m^2(z) dz}
\]

where \( \phi_m(z) \) is the mth orthonormal function of the posts. Equation 12 has been used by various investigators [16, 17, 18].

The validity of Eq. 15 can be examined as follows. The various fluid force coefficients \( \bar{a}_{jk}, \bar{a}'_{jk}, \bar{a}''_{jk}, \bar{\sigma}_{jk}, \bar{\sigma}'_{jk}, \bar{\sigma}''_{jk}, \bar{\beta}_{jk}, \bar{\beta}'_{jk}, \bar{\beta}''_{jk}, \bar{\tau}_{jk}, \bar{\tau}'_{jk}, \bar{\tau}''_{jk} \), and \( \bar{\tau}''_{jk} \) given in Eq. 4 can be written as [6]

\[
\begin{align*}
\bar{a}_{jk} & = \rho \pi R^2 a_{jk} , \\
\bar{a}'_{jk} & = \frac{\rho U^2}{\omega} a'_{jk} , \\
\bar{a}''_{jk} & = \rho U^2 a''_{jk} , \\
\bar{\sigma}_{jk} & = \rho \pi R^2 \sigma_{jk} , \\
\bar{\sigma}'_{jk} & = \frac{\rho U^2}{\omega} \sigma'_{jk} , \\
\bar{\sigma}''_{jk} & = \rho U^2 \sigma''_{jk} , \\
\bar{\beta}_{jk} & = \rho \pi R^2 \beta_{jk} , \\
\bar{\beta}'_{jk} & = \frac{\rho U^2}{\omega} \beta'_{jk} , \\
\bar{\beta}''_{jk} & = \rho U^2 \beta''_{jk} , \\
\bar{\tau}_{jk} & = \rho \pi R^2 \tau_{jk} , \\
\bar{\tau}'_{jk} & = \frac{\rho U^2}{\omega} \tau'_{jk} , \\
\bar{\tau}''_{jk} & = \rho U^2 \tau''_{jk} ,
\end{align*}
\]

where \( \rho \) is fluid density, \( U \) is flow velocity, \( R \) is post radius, and \( \omega \) is oscillation circular frequency. The nondimensional coefficients include added mass coefficients \( a_{jk}, \sigma_{jk}, \beta_{jk}, \) and \( \tau_{jk} \), fluid damping coefficients, \( a'_{jk}, \sigma'_{jk}, \beta'_{jk}, \) and \( \tau'_{jk} \), and fluid stiffness coefficients \( a''_{jk}, \sigma''_{jk}, \beta''_{jk}, \) and \( \tau''_{jk} \). These coefficients depend on the reduced flow velocity \( U_r \) (= \( U/ED; f = \omega/2\pi \)). The added mass coefficients generally can be considered as independent of \( U_r \). However, fluid damping and fluid stiffness coefficients are functions of \( U_r \). For \( U_r < 10 \), they vary with \( U_r \) significantly and are difficult to characterize. However, for large \( U_r \) (\( U_r > 20 \)), they are almost independent of \( U_r \).

At high reduced flow velocity \( U_r \), based on Eqs. 9, 10, 11, and 16, an equivalent flow velocity defined as given in Eq. 15 is applicable. Once the equivalent flow velocity is determined, the critical flow velocity can be calculated as if the flow is uniform. However, at low reduced flow
velocity, no such equivalent flow velocity can be determined. Using the concept of the equivalent flow velocity is generally not applicable.

In determining the stability of LOX posts, nonuniformity of the flow velocity distribution in the transverse direction and axial direction must be considered. The flow impinging on the 13th row of LOX posts is expected to be fairly nonuniform. The significance cannot be evaluated until the flow velocity distribution is determined from tests [2].

The reduced flow velocity for LOX posts is about 32 [1]. At the value of $U_r$, the fluid damping and fluid stiffness coefficients can be considered as constants. Therefore, the equivalent flow velocity defined in Eq. 15 can be applied.

D. Upstream Turbulence

Upstream turbulence can affect the critical flow velocity. Wind-tunnel experiments [7,9] have shown that turbulence produces a shift in the initiation of fluidelastic instability to higher flow velocities. Gorman [19] carried out tests in water for typical heat-exchanger tubes. He found that the existence of upstream grids and screens had no appreciable effect on the critical liquid approach velocity. However, other wind-tunnel experiments [18] have shown that turbulence tends to reduce the critical flow velocity. A water-tunnel test has been used to resolve the discrepancy [8]; turbulence can stabilize or destabilize the tube array, depending on the characteristics of the turbulence. This conclusion is verified by Soper in his wind-tunnel tests [20]. In practical situations, the turbulence characteristics are not known; it is difficult to account for the effect of turbulence. The effect of turbulence on the critical flow velocity is not caused by turbulent buffeting, but by the change of $C_F$ and $K_F$ in Eq. 13 with turbulence characteristics.

Evaluation of the effect of turbulence on the critical flow velocity requires detailed measurements of the turbulence characteristics in the transfer ducts. Turbulence is also the main mechanism causing random vibration of LOX posts. Again, the characterization of the turbulence is needed for predicting post response due to turbulent buffeting.

E. Post Location

In a post array, the post response depends on its location. In liquid flow, the upstream cylinder row is usually the critical one. Experiments in water [8,20,21] have shown that the upstream tubes are most susceptible to instability. The concept of "prison bars" in the upstream has been proposed on the basis of this observation [22].

Tube responses in gas flow have been investigated for different arrangements [3,9]. For in-line arrays ($60^\circ$ and $90^\circ$), the first three rows
in the upstream might be the critical ones; for out-of-line arrays (30° and 45°), the first two rows might lose stability at the lowest flow velocity. The critical row is shifted in the direction of flow as the pitch ratio becomes larger.

In LOX post array, the natural frequency of the posts in the 13th row is the smallest. Therefore, it is expected that these posts will be the most critical ones; i.e., they become unstable first and are also subjected to high intensity turbulent flow.

F. Scruton's Number (Mass-Damping Parameter)

Scruton's number $\delta_s$ is given by

$$\delta_s = \frac{2\pi m \zeta}{\rho D^2},$$  \hspace{1cm} (17)

where $m$ is the mass of LOX post per unit length, $\zeta$ is damping ratio of LOX posts, $D$ is LOX post outside diameter, and $\rho$ is fluid density. The significance of simulation for mass and damping depends on the values of $\delta_s$. For large $\delta_s$, because the fluid force coefficients given in Eq. 16 are independent of $U_x$, the mass ratio $m/\rho D^2$ and damping $\zeta$ can be combined as a single parameter $\delta_s$ [6]. In this case, the simulation of the individual values of $m/\rho D^2$ and $\zeta$ is not very important. At lower values of $\delta_s$, one has to simulate not only the value of Scruton's number, but also the individual values of mass ratio ($m/\rho D^2$) and damping ($\zeta$), since the two parameters cannot generally be combined as a single parameter.

Scruton's number for the SSME LOX posts is about 11. In this range, simulation of the individual values of $m/\rho D^2$ and $\zeta$ in the Scruton's number is not very important in predicting the critical flow velocity.

G. Interaction of Turbulent Buffeting and Fluidelastic Instability

Fluidelastic instability of an array of cylinders was first studied by Connors [23] based on the displacement mechanism. Relative motion of the individual cylinders is a necessary condition for instability; this type of instability is now referred to as fluid-stiffness controlled instability [6]. Because of this requirement, a question has been raised frequently regarding the effect of turbulent buffeting on fluidelastic instability—more specifically, whether fluidelastic instability can occur without the small post vibration caused by turbulent buffeting. This can be answered from the equations of motion given in Eq. 13.

Fluidelastic instability is caused by $C_f$, $K_f$, or a combination of both. The right side of the equation is associated with excitation forces including turbulent buffeting. Fluidelastic instability can occur regardless of the magnitude of $G$. Therefore, except in special situations
in which the fluid excitation forces have caused relatively large oscillations, the fluid excitation force $G$ does not affect the critical flow velocity.

H. Post Arrangement

The LOX posts are subjected to flow coming from different angles and with different flow velocity. In the evaluation of LOX posts, these effects have to be taken into account. Although the LOX posts are arranged in a particular pattern; the posts may be considered as in-line or staggered arrays, depending on the flow direction.

It has been demonstrated by several investigators [4,9,16] that the critical flow velocities vary with post pattern and post pitch. In addition, in the air flow (high $\delta_g$), for a given post pitch-to-diameter ratio, the critical flow velocity for in-line arrays is smaller than that of staggered arrays [4]. This suggests that the most critical region is expected to be in the region in which the flow impinges on the posts as if they are in-line arrays. Based on this consideration, the flow direction has to be considered in addition to flow velocity.

I. Swirlers

Four swirlers wrap around each LOX post. Their effect on post response is not quantitatively known. In general, these swirlers are beneficial to suppress vibration:

- They reduce the correlation length of vortex shedding; this is one of the techniques to suppress lock-in oscillations.
- The effect of swirlers on the critical flow velocity is expected to be similar to that of fins on cylinder, which tend to stabilize the cylinders (increasing the critical flow velocity and decreasing the oscillation amplitude at instability) [6].

Results from tests on plain LOX posts are conservative based on the consideration of fluidelastic instability and vortex shedding. Mathematically, these effects correspond to the decrease of fluctuating force coefficients $C_{fLj}$ and $C_{fDj}$ in Eq. 3 and change of motion-dependent fluid force coefficients given in Eq. 16.

J. Mathematical Models

As discussed in Ref. 1, there are three types of mathematical models: the quasi-static model, the analytical model, and the general semi-analytical model. The differences in these models are discussed here.
(1) **Quasi-Static Model**

The equation of motion is given by

\[
[M_s + M_f] \ddot{Q} + [C_s] \dot{Q} + [K_s + K_f] Q = 0
\]

(18)

Only the fluid inertia and fluid stiffness are included and fluid damping is not considered. The instability is of the stiffness controlled type. Based on this model, a single flexible post among an array of rigid posts will not become unstable. In general, this model is applicable for some specific conditions only.

(2) **Analytical Model**

The equation of motion is given by

\[
[M_s + M_f] \ddot{Q} + [C_s + C_f] \dot{Q} + [K_s] Q = 0
\]

(19)

Fluid stiffness is not included in the equation of motion; i.e., the instability is of the damping controlling type. Based on this model, a single post within an array of rigid posts can lose stability. This model is applicable for the lower values of \( \delta_s \). At large \( \delta_s \), the model predicts that the critical reduced flow velocity is proportional to the first power of \( \delta_s \); this does not agree with the experimental data.

(3) **General Semi-Analytical Model**

The equation of motion includes both fluid damping and fluid stiffness in addition to fluid inertia as given in Eq. 13. This model can predict both fluid-damping and fluid-stiffness controlled types of instability.

Three different approaches can be used to obtain the fluid force coefficients in the different mathematical models: experimental method, numerical techniques and analytical methods. Each approach has its advantage and disadvantage. The three approaches should be considered in the development of the mathematical models.

**IV. CONCLUDING REMARKS**

An evaluation of the vibration of the SSME LOX posts requires data on the structural parameters as well as flow parameters. At this time, the detailed flow field is not known. Tests on SWHGM by Rocketdyne are designed to obtain such data. Therefore, the data for SWHGM are important and will be very useful in the assessment and modifications of the SSME LOX posts. It is recommended that tests should be conducted to obtain all necessary parameters needed for the evaluation of the vibration of LOX posts.
Rocketdyne's tests on dynamic loads are complementary to the SWHGM. Most of the issues raised can be resolved using this channel flow test; it is more economical and convenient to obtain the data. In addition, any suggested modifications can be verified in the channel.

The central issue in predicting the critical flow velocity is the fluid force coefficients. This remains a difficult issue. Argonne National Laboratory, under the sponsorship of NASA/Lewis, is undertaking a study to develop techniques and to conduct tests to obtain fluid force coefficients. This effort, in conjunction with Rocketdyne's test programs, is considered essential in the understanding of the vibration of the SSME LOX posts and in the development of techniques to avoid detrimental flow-induced vibration.

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