SIMPLE CONDUCTION MODEL WITH
PHASE CHANGE FOR FUEL PIN

by

W. L. Chen, M. Ishii,
and M. A. Groimes

APPLIED TECHNOLOGY

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Reactor Analysis and Safety Division

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## NOMENCLATURE

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<tr>
<td>$C_c$</td>
<td>Lumped parameter for cladding, sec</td>
</tr>
<tr>
<td>$C_c = (R_i^2 - R_F^2)\alpha C_{pc}$</td>
<td></td>
</tr>
<tr>
<td>$C_F$</td>
<td>Lumped parameter for fuel, sec</td>
</tr>
<tr>
<td>$C_F = R_F F C_{PF}$</td>
<td></td>
</tr>
<tr>
<td>$C_F^*$</td>
<td>Lumped parameter for fuel in the fuel-melting phase, sec</td>
</tr>
<tr>
<td>$C_F^* = R_F F C_{PF}$</td>
<td></td>
</tr>
<tr>
<td>$C_p$</td>
<td>Specific heat, cal g$^{-1}$ C$^{-1}$</td>
</tr>
<tr>
<td>$C_{PF}$</td>
<td>Fuel specific heat evaluated at the average temperature $T_{av} = \frac{1}{2}(T_F + T_{fm})$, cal g$^{-1}$ C$^{-1}$</td>
</tr>
<tr>
<td>$h$</td>
<td>Coolant overall heat-transfer coefficient, cal C$^{-1}$ cm$^{-2}$ sec$^{-1}$</td>
</tr>
<tr>
<td>$h_{gap}$</td>
<td>Gap heat-transfer coefficient, cal C$^{-1}$ cm$^{-2}$ sec$^{-1}$</td>
</tr>
<tr>
<td>$k$</td>
<td>Thermal conductivity, cal cm$^{-1}$ sec$^{-1}$ C$^{-1}$</td>
</tr>
<tr>
<td>$L$</td>
<td>Latent heat, cal g$^{-1}$</td>
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<tr>
<td>$Q$</td>
<td>Lumped parameter for heat-generation rate, °C</td>
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<tr>
<td>$Q = R_F q 2h'$</td>
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<tr>
<td>$q$</td>
<td>Axial volumetric heat-generation rate, cal cm$^{-3}$ sec$^{-1}$</td>
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<td>$q_N$</td>
<td>Normalized axial power distribution</td>
</tr>
<tr>
<td>$R$</td>
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<tr>
<td>$r$</td>
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<tr>
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<tr>
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<td>Time, sec</td>
</tr>
<tr>
<td>$t_d$</td>
<td>Dryout time, sec</td>
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<tr>
<td>$t_{FM}$</td>
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</tr>
<tr>
<td>$t_{fm}$</td>
<td>Complete melting time of cladding, sec</td>
</tr>
<tr>
<td>$t_p$</td>
<td>Starting melting time of cladding, sec</td>
</tr>
<tr>
<td>$t_{PF}$</td>
<td>Starting melting time of fuel, sec</td>
</tr>
<tr>
<td>$z$</td>
<td>Axial position, cm</td>
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<tr>
<td>$z_{FM}$</td>
<td>Volumetric concentration fraction of liquid fuel</td>
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<tr>
<td>$\rho$</td>
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<tr>
<td>$f$</td>
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### Subscripts
- $c$: Cladding
- $F$: Fuel
- $sp$: Cladding melting point
- $fp$: Fuel melting point
- $Na$: Sodium
- $Sat$: Saturated condition
- $s$: Steady state
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ABSTRACT

A simple conduction model with phase change has been developed for the transient analysis of a fuel pin based on average properties and lumped-parameter techniques. The transient behavior of fuel and cladding can be accurately described by simple analytical expressions that agree with conventional numerical approaches for undercooling transient analysis. If it be assumed that the heat-transfer resistance between the fuel and cladding remains the same for both steady-state and transient periods, the phase-change problem for fuel and cladding melting can be significantly simplified. By using the predetermined average overall heat-transfer coefficient across a fuel pin in the steady-state period, the average transient fuel and cladding temperatures can be formulated analytically. For loss of flow at constant power, the start of melting and complete melting for both the fuel and cladding can be estimated with considerable accuracy.

1. INTRODUCTION

A simple conduction model with phase change for transient analysis of a fuel pin can be formulated based on the average properties and lumped-parameter techniques. In contrast to conventional numerical approaches, the transient behavior of the fuel and cladding are expressed by simple analytical formulas. The unique assumption made here is that the heat-transfer resistance between the fuel and cladding remains constant throughout both the steady-state and transient-state periods. Therefore, the complexity of the nonlinear conduction problem can be significantly reduced so that the average fuel and cladding temperatures can be obtained analytically. To further clarify the complex conduction problem with phase change in both fuel and cladding regions, the analysis is divided into the following four phases:

1. Transient phase before the cladding melting.
2. Cladding-melting phase.
3. Transient phase before the fuel melting.


Detailed development of formulations will be discussed according to the above categories in the next section.

For many reactor-transient analyses, a simple tool is desired to obtain the basic information about fuel and cladding behavior with phase changes. The present simple conduction model not only provides histories of average fuel and cladding temperatures, but also gives the timings of fuel and cladding melting. At present, this model is being used in the companion study for analyzing cladding motion and cladding-blockage formation in the subassembly of a reactor core in the case of a loss-of-coolant accident.1 Computing times and computer storages are greatly saved by this method.

Some sample calculations were conducted for a case in which the fuel and cladding behavior of a fuel pin after coolant voiding were evaluated at a constant power. The start of melting and complete melting for both the fuel and cladding are illustrated in Sec. III. The general formulations presented in this report can be useful, not only for hypothetical reactor-accident analyses, but also for interpretation of certain prototype in-pile experiments or design of nuclear reactors.

II. FORMULATIONS

To examine the fuel and cladding behavior of a fuel pin after the core is voided, the analysis can be simplified by the following basic assumptions.

1. An averaging method is applied to the fuel and cladding temperatures throughout the entire analysis, and the average fuel and cladding temperatures are defined as follows:

\[
\langle T_F \rangle = \frac{1}{\pi R_F^2} \int_0^{R_F} T_F 2\pi r \, dr
\]

and

\[
\langle T_C \rangle = \frac{1}{\pi (R_C^2 - R_F^2)} \int_{R_F}^{R_C} T_C 2\pi r \, dr.
\]

If the symbol \(\langle \rangle\) is dropped in the following analysis, the notations \(T_F\) and \(T_C\) denote the average fuel and cladding temperatures, respectively.

2. No void is presently considered in the fuel region; \(r = 0\) denotes the centerline of the fuel region.
3. Constant thermal properties of fuel and cladding are used.

4. A nonuniform, axial, volumetric heat-generation rate $q$ is considered.

5. The gap distance between the cladding and fuel regions is ignored in the formulation. However, the gap heat-transfer coefficient $h_{\text{gap}}$ is used for taking into account the heat-transfer resistance at the gap.

6. Axial heat conduction in the fuel rod is neglected.

7. Perfect contact between cladding and sodium is assumed before the dryout of cladding in view of very high heat-transfer coefficients of boiling sodium or liquid sodium. (The overall heat-transfer coefficient $h$ approaches $\infty$.) As soon as the cladding surface dries out, the heat flux across the boundary between cladding and sodium vapor becomes negligibly small; thus it is assumed that at dryout $h = 0$.

8. If the heat-transfer resistance across the fuel pin is assumed to remain the same throughout the steady-state and transient periods, the average overall heat-transfer coefficient between fuel and cladding $h'$ can be evaluated from the steady-state condition. According to the steady-state analysis, the average overall heat-transfer coefficient $h'$ is given by (see Appendix A)

$$h' = \frac{1}{\frac{R_F}{4k_F} + \frac{1}{h_{\text{gap}}} + \frac{R_C - R_F}{2k_c}}.$$

For convenience, the lumped parameters are defined as

$$C_F = \frac{R_F \rho_F C_{pF}}{2h'},$$

$$C_c = \frac{(R_c^2 - R_F^2) \rho_c C_{pc}}{2R_F h'},$$

and

$$Q = \frac{R_F q}{2h'},$$

in which all the fuel properties are evaluated at the maximum steady-state fuel temperature and all the cladding properties are evaluated at the cladding melting temperature $T_{cp}$. However, in the fuel-melting phase, the algebraic mean fuel temperature is used to calculate the fuel specific heat $C_{pF}$ (as will be discussed in the subsequent analysis). The quantity $C_F'$ denotes $C_F$ evaluated by substituting the new fuel specific heat $C_{pF}'$, such as

$$C_F' = \frac{R_F \rho_F C_{pF}'}{2h'}.$$
A. **Transient Phase before the Cladding Melting** \((t_d < t < t_p)\)

The transient integral energy balance of the fuel and cladding based on the average property are given as follows (see Appendix B):

\[
\frac{dT_F}{dt} = -(T_F - T_c) + Q; \tag{1}
\]

\[
\frac{dT_C}{dt} = T_F - T_c - \frac{h R_c}{h' R_F} (T_c - T_{Na}), \tag{2}
\]

where \(h\) is the coolant heat-transfer coefficient.

If constant heat-generation rate is assumed at a given axial position and the core is completely voided, Eq. 2 can be reduced to

\[
\frac{dT_C}{dt} = T_F - T_c. \tag{3}
\]

This set of equations is solved for the above-discussed particular case in which the nonuniform heat-generation rate \(\dot{q}\) is a function of axial position only. With the initial conditions (see Eqs. A.3 and A.4 in Appendix A)

\[
T_F = T_{F0} = T_{Sat} + C_i \dot{q} \text{ at } t = t_d \tag{4}
\]

and

\[
T_c = T_{C0} = T_{Sat} + C_2 \dot{q} \text{ at } t = t_d, \tag{5}
\]

where \(T_{Sat}\) = saturated sodium temperature at \(t = t_d\), and

\[
C_i = \frac{R_F^2}{2} \left\{ \frac{1}{4k_c} + \frac{1}{h R_c} + \frac{1}{h_{gap} R_F} + \frac{1}{k_c} \ln \frac{R_c}{R_F} \right\};
\]

\[
C'_i = \frac{R_F^2}{2} \left\{ \frac{1}{h R_c} + \frac{1}{2k_c} - \frac{R_c^2}{R_F^2 - R_F^2} \frac{1}{k_c} \ln \frac{R_c}{R_F} \right\},
\]

the solutions for fuel and cladding temperatures are

\[
T_F = T_{F0} + \frac{Q}{C_F + C_C} (t - t_d) + \frac{C_F C_c}{(C_F + C_C)^2} (T_{F0} - T_{C0})
\]

\[
\times \left\{ \exp \left[ \frac{C_F + C_C}{C_F C_c} (t - t_d) \right] - 1 \right\}. \tag{6}
\]
and

$$T_c = T_{c0} + \frac{Q}{C_F + C_c}(t - t_d) - \frac{C_F^2}{(C_F + C_c)^2(T_{F0} - T_{c0})} \times \left\{ \exp \left[ -\frac{C_F + C_c}{C_FC_c}(t - t_d) \right] - 1 \right\}. \quad (7)$$

B. Cladding-melting Phase \((t_p < t \leq t_m)\)

If the average cladding temperature is assumed to remain at the melting point during the entire cladding-melting phase, in view of the high conductivity of cladding material, or \(T_c = T_{cp}\), the gap heat-transfer coefficient \(h_{gap}\) may increase due to the cladding melting. However, changes in total \(h'\) may be small because the fuel resistance is the dominant factor. The energy equations for the fuel and cladding are (see Appendix C)

$$\frac{dT_F}{dt} = -(T_F - T_{cp}) + Q \quad (8)$$

and

$$\frac{d(R_p)'}{dt} = \frac{2RT_{F0}h'}{\rho_c L_c}(T_F - T_{cp}). \quad (9)$$

with initial conditions \(T_F = T_F(t_p), T_c = T_{cp},\) and \(R_p = R_F,\) where \(L_c\) is cladding latent heat and \(R_p\) is the moving front of the melting cladding.

Case 1. For \(t_p - t_d \leq C_FC_c/(C_F + C_c),\) the initial condition for the fuel temperature can be approximated by (see Eq. B.13 in Appendix B)

$$T_F(t_p) = T_{F0}. \quad (10)$$

The solutions for Eqs. 8 and 9 are

$$T_F = T_{F0} + (T_{cp} - T_{c0}) \left[ 1 - \exp \left( -\frac{t - t_p}{C_F} \right) \right] \quad (10)$$

and

$$\frac{R_p'}{2} - \frac{R_F'}{2} = \frac{R_Fh'(T_{F0} - T_{c0})}{\rho_c L_c} \left\{ t - t_p - \frac{C_F(T_{cp} - T_{c0})}{T_{F0} - T_{c0}} \left[ 1 - \exp \left( -\frac{t - t_p}{C_F} \right) \right] \right\}. \quad (11)$$
Case 2. For $t_p - t_d > C_F C_c / (C_F + C_c)$, the initial condition for the fuel temperature can be approximated by (see Eq. B.15 in Appendix B)

$$T_F(t_p) = T_F(0) - \frac{C_F C_c}{(C_F + C_c)^2}(T_F(0) - T_{c0}) + \frac{Q}{C_F + C_c}(t_p - t_d).$$

The solutions for Eqs. 8 and 9 are

$$T_F = T_{cp} + (T_F(0) - T_{c0})\left[1 - \frac{C_F}{C_F + C_c}\exp\left(-\frac{t - t_p}{C_F}\right)\right]$$

and

$$\frac{R_p^2 - R_F^2}{2} = \frac{R_p h'(T_F(0) - T_{c0})}{\rho_c l_c} \left\{t - t_p - \frac{C_F}{C_F + C_c}\left[1 - \exp\left(-\frac{t - t_p}{C_F}\right)\right]\right\}.$$ 

C. Transient Phase before the Fuel Melting ($t_m < t < t_{pF}$)

By neglecting the cladding motion in view of the smaller thermal inertia of the cladding and the high thermal resistance within fuel, we can approximate the energy equations for both fuel and cladding by

$$C_F \frac{dT_F}{dt} = -(T_F - T_c) + (T_F(0) - T_{c0})$$

and

$$C_c \frac{dT_c}{dt} = T_F - T_c.$$ 

with initial fuel and cladding temperatures at $t = t_m$ given by $T_F = T_F(t_m)$ and $T_c = T_{cp}$. $T_F(t_m)$ can be evaluated from Eq. 10 or 12 at $t = t_m$. The fuel and cladding temperatures are

$$T_F = T_F(t_m) + \frac{T_F(0) - T_{c0}}{C_F + C_c} \left\{t - t_m\right\}$$

$$+ \left[\frac{T_F(t_m) - T_{cp}}{T_F(0) - T_{c0}} - \frac{C_c}{C_F + C_c}\exp\left(-\frac{C_F + C_c}{C_F C_c}(t - t_m)\right) - 1\right].$$
\[ T_c = T_{cp} + \frac{T_{F0} - T_{co}}{C_F' + C_c} (t - t_m) \]

\[ + C_F' \left[ \frac{T_F(t_m) - T_{cp}}{T_{F0} - T_{co}} - \frac{C_c}{C_F' + C_c} \right] \left\{ 1 - \exp \left[ - \frac{C_F' + C_c}{C_F' C_c} (t - t_m) \right] \right\}. \] (17)

D. **Fuel-melting Phase** \( (t_pF < t \leq t_Fm) \)

Assume that the fuel temperature stays at the melting point \( T_Fp \) during the fuel-melting period and that the motion of cladding can be neglected. The energy equations for the fuel and cladding at this phase are

\[ \frac{d\alpha_{Fm}}{dt} = \frac{2h'}{R_{Fp} F L_F} \left[ -(T_{Fp} - T_c) + (T_{F0} - T_{co}) \right] \] (18)

and

\[ \frac{dT_c}{C_c dt} = T_{Fp} - T_c. \] (19)

with initial conditions

\[ \alpha_{Fm} = 0 \text{ at } t = t_pF; \]

\[ T_c = T_c(t_{Fp}) \text{ at } t = t_pF. \]

where \( \alpha_{Fm} \) is the volumetric concentration of liquid fuel and \( T_c(t_{Fp}) \) is the cladding temperature at \( t = t_pF \), which can be evaluated from Eq. 17.

The solutions for Eqs. 18 and 19 are

\[ \alpha_{Fm} = \frac{2h'}{R_{Fp} F L_F} \left\{ (T_{F0} - T_{co}) (t - t_pF) \right. \]

\[ - C_c [T_{Fp} - T_c(t_{Fp})] \left[ 1 - \exp \left( - \frac{t - t_pF}{C_c} \right) \right] \} \] (20)

and

\[ T_c = T_{Fp} - [T_{Fp} - T_c(t_{Fp})] \exp \left[ - \frac{t_pF}{C_c} (t - t_pF) \right]. \] (21)
The fuel- and cladding-temperature histories are completed during the period from the cladding dryout to the completion of fuel melting. The moving front of cladding melting in the cladding region can be estimated from Eq. 11 or 13. The volumetric concentration of melted liquid fuel can be calculated from Eq. 20. The dryout time \( t_d \) is provided at any axial position of a fuel pin. Other timings of stages can be obtained in the following way:

1. The starting time of cladding melting, \( t_p \), can be estimated from Eq. 7 by letting \( T_c = T_{cp} \) (cladding melting point).
2. The complete time of fuel melting, \( t_m \), is obtained from Eq. 11 or 13 by putting \( R_p = R_c \) (outer radius of cladding).
3. The starting time of fuel melting, \( t_{pF} \), is computed from Eq. 16 by substituting \( T_F = T_{Fp} \) (fuel melting point).
4. The complete time of fuel melting, \( t_{pF} \), is calculated from Eq. 20 by assuming \( \alpha_{Fm} = 1 \) (100% fuel melted).

All the above-mentioned formulas can be simplified by linearizing the nonlinear exponential term:

\[
e^{-\eta} = \begin{cases} 
1 - \eta & \text{for small argument of } \eta \\
0 & \text{for large argument of } \eta.
\end{cases}
\]

In other words, each original nonlinear formula can be estimated by two linear-approximation formulas that intersect each other in the period of interest. A detailed discussion of the linearizing approximation is included in the appendixes. The resulting simplified formulas are summarized in Table 1. Column 3 of Table 1 provides formulas for average fuel and cladding temperatures, and column 2 gives the timings of various transients in a fuel pin. All the computed results here were based on the linearized formulas shown in Table 1.

III. RESULTS AND CONCLUSION

The sample calculations presented here were based on the simple conduction model with phase change as summarized in Table 1. Radii for the fuel rod used were: \( R_F = 0.254 \) cm (outer radius of fuel region) and \( R_C = 0.292 \) cm (outer radius of cladding region). Fuel and cladding properties used were:

\[
\begin{align*}
\rho_F &= 9.964 \text{ g/cm}^3, \ C_{pF} = 0.096 \text{ cal/g} \cdot \text{C}, \ k_F = 5.375 \times 10^{-1} \text{ cal/s} \cdot \text{C} \cdot \text{cm}, \\
L_F &= 64.76 \text{ cal/g}, \ \rho_C = 7.363 \text{ g/cm}^3, \ C_{pc} = 0.176 \text{ cal/g} \cdot \text{C}, \ k_c = 0.07963 \text{ cal/sec} \cdot \text{C} \cdot \text{cm}, \text{and } L_C = 67.82 \text{ cal/g}.
\end{align*}
\]

Sample calculations were conducted for both R-5 and L-2 test cases. For Test R-5, the length of the fuel section of a fuel rod was 91.44 cm, and the maximum linear power was 0.3461 kW/cm or 10.55 kW/ft. For Test L-2, the length of the fuel section of a fuel rod was

<table>
<thead>
<tr>
<th>Phase</th>
<th>Timing</th>
<th>Average Heat and Mass Transfer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Phase Before Conduction</td>
<td>$t_y$ (time starting of solid melting)</td>
<td>$t_i = t_y + t_{o} - t_{f}$ (for $t_o &gt; t_f$)</td>
</tr>
<tr>
<td>Transition Phase Between Conduction and Condensation</td>
<td>$t_y$ (time starting of solid melting)</td>
<td>$t_i = t_y + t_{o} - t_{f}$ (for $t_o &gt; t_f$)</td>
</tr>
<tr>
<td>Condensation Phase</td>
<td>$t_y$ (time starting of solid melting)</td>
<td>$t_i = t_y + t_{o} - t_{f}$ (for $t_o &gt; t_f$)</td>
</tr>
<tr>
<td>Final Phase</td>
<td>$t_y$ (time starting of solid melting)</td>
<td>$t_i = t_y + t_{o} - t_{f}$ (for $t_o &gt; t_f$)</td>
</tr>
</tbody>
</table>
The normalized axial power distribution $q_N$ was described by the following chopped-cosine expression:

$$q_N = \cos[\pi \xi (Z^* - 0.5)],$$

where $Z^* (=Z/Z_F)$ is a relative position of the fuel section of a fuel pin and $\xi$ is a constant given by

$$\xi = \begin{cases} 0.459 & \text{for Test R-5} \\ 0.4769 & \text{for Test L-2.} \end{cases}$$

Operational parameters were: saturated sodium temperature, $T_{\text{Sat}} = 898.9^\circ\text{C}$; cladding melting point, $T_{\text{cp}} = 1426.7^\circ\text{C}$; fuel melting point, $T_{\text{fp}} = 2760^\circ\text{C}$; coolant overall boiling heat-transfer coefficient, $h = 10^{10}$ cal cm$^{-2}$ sec$^{-1}$ C (approximating the constant-temperature boundary condition); gap heat-transfer coefficient, $h_{\text{gap}} = 0.23899$ cal/sec-cm$^{-2}$ C ($1761$ Btu/hr-ft$^2$-F).

Time histories for the cladding dryout were provided by taking mean values of SAS results as shown in Figs. 1 and 2 for both tests. However, the dryout line can also be readily calculated by the method illustrated in Ref. 2.

![Fig. 1](image1.png)

**Fig. 1.** Comparison of Results from Simple Conduction Model and SAS Code for Test R-5 beyond Voiding. Axial-position envelope for (1) sodium voiding, (2) cladding dryout, (3) cladding begins melting, (4) cladding 95% melting, and (5) cladding 100% melting. ANL Neg. No. 764-57-75.

![Fig. 2](image2.png)

**Fig. 2.** Comparison of Results from Simple Conduction Model and SAS Code for Test L-2 beyond Voiding. Axial-position envelope for (1) cladding dryout, (2) cladding 95% melting, and (3) cladding 100% melting. ANL Neg. No. 764-57-76.

Figures 1 and 2 compare results of the simple conduction model and SAS code$^{155}$ beyond voiding for Tests R-5 and L-2, respectively. Figure 1 indicates that the agreement between the simple conduction model and SAS code for the starting time of cladding melting is good. Note that only 95% cladding melting was reported in the SAS results for both Tests R-5 and L-2. The agreement between 100% cladding melting from simple conduction model and 95% cladding melting from SAS code is qualitatively good.
For Test R-5, the analytical solution of the simple conduction model was compared with the numerical results of the THTB code because the THTB code is one of verified conduction computer programs. The number of nodes used in calculations with the THTB code were 24 for the fuel and 6 for the cladding. For the simple conduction model it was assumed that the temperature of fuel remained at the fuel melting point during the fuel-melting period. To make a fair comparison, the complete time of fuel melting from the THTB code in Table II was chosen as the time when the temperature at the outer nodal point of fuel reached its melting point.

Physically, the outer portion of the fuel region becomes a two-phase mixture (mixture of molten and solid fuel); some portion of fuel is already completely melted, and the temperature rises much beyond the melting temperature, as shown in Fig. 3b. At this stage, fuel would be mobilized and begins to eject. For sequential accident analysis, the complete time of fuel melting of the simple conduction model is physically realistic. The reason is that, in a THTB type of analysis, superheating of molten fuel during the melting process is permitted. Therefore, the time of complete melting in the simple model is underestimated in comparison with THTB code, and it is considered to be closer to the fuel-mobilization time. Table II shows the good

![Fig. 3. Typical Temperature Profiles of THTB Result ANL Neg. No. 5616-1-31.](image)

**Table II.** Comparison in Melting Timing from Simple Conduction Model and THTB Code

<table>
<thead>
<tr>
<th>Relative Axial Position of a Fuel Pin</th>
<th>Complete Melting Time&lt;sup&gt;a&lt;/sup&gt; of Cladding, sec</th>
<th>Complete Melting Time&lt;sup&gt;a&lt;/sup&gt; of Fuel, sec</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Simple Conduction Model</td>
<td>THTB</td>
</tr>
<tr>
<td>0.0</td>
<td>8.744</td>
<td>8.200</td>
</tr>
<tr>
<td>0.1</td>
<td>5.971</td>
<td>5.570</td>
</tr>
<tr>
<td>0.2</td>
<td>4.609</td>
<td>4.150</td>
</tr>
<tr>
<td>0.3</td>
<td>3.936</td>
<td>3.795</td>
</tr>
<tr>
<td>0.4</td>
<td>3.380</td>
<td>3.295</td>
</tr>
<tr>
<td>0.5</td>
<td>3.046</td>
<td>2.975</td>
</tr>
<tr>
<td>0.6</td>
<td>2.866</td>
<td>2.705</td>
</tr>
<tr>
<td>0.7</td>
<td>2.789</td>
<td>2.695</td>
</tr>
<tr>
<td>0.8</td>
<td>2.955</td>
<td>2.700</td>
</tr>
<tr>
<td>0.9</td>
<td>2.503</td>
<td>2.800</td>
</tr>
<tr>
<td>1.0</td>
<td>1.504</td>
<td>2.900</td>
</tr>
</tbody>
</table>

<sup>a</sup>The time is measured from the start of the boiling.
agreement between the results from the simple conduction model and THTB code. Here the maximum difference between the predicted time of fuel mobilization from comparisons is only 0.54 sec. Comparison of the results of computation of all timings using the simple conduction model with those of the THTB code clearly demonstrates the power of lumped-parameter techniques and its extreme simplification in a nonlinear conduction problem with phase change.

The objective of this analysis was to provide simple formulations to obtain general information about fuel and cladding leading into the transient-phase analysis. For sequential accident analysis, the present model is useful without depending on the conventional numerical analysis, particularly when multichannel analyses are necessary. The modeling of the sequential accident analysis is significantly simplified by predetermining the timings of events of fuel and cladding behaviors. At present, the simple conduction model is being incorporated into the development of a model for cladding motion and cladding-blockage formation in the subassembly of the reactor core.1

The solution of the simple conduction model for the incipient melting time and the complete melting time of the cladding of fuel pins in Tests R-5 and L-2 was compared with the numerical solution obtained from the SAS code,5,5 and the agreement between both solutions is excellent. For Test R-5, the analytical solution of the simple conduction model for the complete melting times for both cladding and fuel was further compared with numerical solution calculated from the THTB code6 at various axial positions of the fuel pin. The comparison shows good agreement. The present lumped-parameter model for a fuel pin has been developed to be used in an analysis of multichannel cladding motions in a loss-of-flow accident.
APPENDIX A
Solution for Steady-state Conduction

Consider the cylindrical coordinate system and notation shown in Fig. A.1. The plane \( Z = 0 \) denotes the bottom of a fuel pin. No void fraction is considered in the fuel region, and the line \( r = 0 \) denotes the center of fuel region. The gap heat-transfer coefficient \( h_{\text{gap}} \) is introduced for taking into account the thermal resistance at the boundary between the fuel and cladding regions, and the gap distance is ignored in the formulation. Constant thermal conductivities are assumed. A nonuniform axial volumetric heat-generation rate \( \dot{q} \) is considered, i.e., \( \dot{q} = \dot{q}(z) \), but the axial conduction is neglected. At any given axial position \( z \), the steady-state conduction equations for fuel and cladding can be written as

\[
\frac{k_F}{r} \frac{\partial}{\partial r} \left[ r \frac{\partial T_F(r)}{\partial r} \right] + \dot{q}(z) = 0, \quad 0 \leq r \leq R_F,
\]

and

\[
\frac{k_C}{r} \frac{\partial}{\partial r} \left[ r \frac{\partial T_C(r)}{\partial r} \right] = 0; \quad R_F \leq r \leq R_C,
\]

with the boundary conduction

\[
k_C \frac{\partial T_C}{\partial r} = -h(T_C - T_{\text{Na}}) \quad \text{at} \quad r = R_C;
\]

\[
-k_F \frac{\partial T_F}{\partial r} = -k_C \frac{\partial T_C}{\partial r} = h_{\text{gap}}(T_F - T_C) \quad \text{at} \quad r = R_F.
\]

where \( h \) is the coolant heat-transfer coefficient, \( T \) is temperature, \( k \) is thermal conductivity, and the subscripts \( F, C \), and \( Na \) denote fuel, cladding, and sodium, respectively.

Fig. A.1
Cylindrical Coordinate System and Notation.
ANL Reg. No. 960-75-05.
The steady-state fuel and cladding temperatures are obtained by solving the above system of equations:

\[ T_F = T_{Na} + \frac{\dot{q} R_F^2}{2} \left[ \frac{1}{h R_c} + \frac{1}{h_{\text{gap}} R_F} + \frac{1}{2 k_F} \left( \frac{1}{R_F^2} - \frac{1}{R_c^2} - \ln \frac{R_c}{R_F} \right) \right] \]  

(A.1)

and

\[ T_c = T_{Na} + \frac{\dot{q} R_F^2}{2} \left( \frac{1}{h R_c} + \frac{1}{k_c} \ln \frac{R_c}{R} \right). \]  

(A.2)

The average fuel and cladding temperatures can be reformulated from Eqs. A.1 and A.2 as

\[ \langle T_F \rangle = \frac{1}{\pi R_F^2} \int_0^{R_F} T_F 2\pi r \, dr \]

\[ = T_{Na} + \frac{\dot{q} R_F^2}{2} \left( \frac{1}{4 k_F} + \frac{1}{h R_c} + \frac{1}{h_{\text{gap}} R_F} + \frac{1}{k_c} \ln \frac{R_c}{R_F} \right) \]  

(A.3)

and

\[ \langle T_c \rangle = \frac{1}{\pi (R_c^2 - R^2)} \int_{R_F}^{R_c} T_c 2\pi r \, dr \]

\[ = T_{Na} + \frac{\dot{q} R_F^2}{2} \left( \frac{1}{4 k_F} + \frac{1}{h R_c} - \frac{R_c^2 - R_F^2}{2 k_c \frac{R_c^2}{R_F^2}} + \frac{1}{k_c} \ln \frac{R_c}{R_F} \right). \]  

(A.4)

where \( \langle \rangle \) denotes the average temperature. Combining Eqs. A.3 and A.4 gives

\[ \langle T_F \rangle - \langle T_c \rangle = \frac{\dot{q} R_F^2}{2} \left[ \frac{1}{4 k_F} + \frac{1}{h_{\text{gap}} R_F} + \frac{1}{k_c} \left( \frac{R_c^2}{2} - \frac{R_c^2}{R_c^2 - R_F^2} \ln \frac{R_c}{R_F} \right) \right] \]

\[ = \frac{\dot{q} R_F^2}{2} \left( \frac{1}{4 k_F} + \frac{1}{h_{\text{gap}} R_F} + \frac{1}{2 k_c} \frac{R_c - R_F}{R_F} \right). \]  

(A.5)
Here we have used the approximation

$$
\frac{1}{2} + \frac{R_c^2}{R_c^2 - R_F^2} \ln \left( \frac{R_c}{R_F} \right) = \frac{1}{2} + \frac{R_c}{R_c - R_F} \left( \frac{1}{2} - \frac{R_F - R_c}{4R_c} + \ldots \right)
$$

$$
\left( \frac{R_c - R_F}{R_F} - \frac{(R_c - R_F)^2}{2R_F^2} + \ldots \right)
$$

$$
= \frac{3}{4} \frac{R_c - R_F}{R_F} \left( \frac{2}{3} - \frac{1}{3} \frac{R_c - R_F}{R_F} \right) = \frac{1}{2} \frac{R_c - R_F}{R_F}.
$$

(A.6)

For the steady-state condition, the heat flux is known:

$$
k_F \frac{\partial T_F}{\partial r} \bigg|_{R_F} = -h'(T_F - T_c) = \frac{\dot{q} R_F}{2},
$$

(A.7)

where $h'$ is the average overall heat-transfer coefficient. The average overall heat-transfer coefficient can be determined as

$$
h' = \frac{1}{\frac{R_F}{4k_F} + \frac{1}{R_c - R_F} + \frac{1}{h_{\text{gap}}} + \frac{1}{2k_c}}.
$$

(A.8)
APPENDIX B

Solution for Transient Simulation

If the heat-transfer resistance across the fuel pin remains the same throughout the steady-state and transient periods, the average overall heat-transfer coefficient $h'$ is constant in the entire analysis, and the formulation of transient fuel and cladding temperatures can be significantly simplified. All the thermal properties are constant, and the use of average values over the temperature range of the analysis is suggested. We define

$$C_F = \frac{R_F \rho F C_{pF}}{2h'}, \quad C_c = \frac{(R_c - R_F) \rho_c C_{pC}}{2R_F h'}, \quad Q = \frac{R_c q}{2h'},$$

where $\rho$ is density, $C_p$ is specific heat, and $h'$ is average overall heat-transfer coefficient and its value is determined by Eq. A.8. We omit $\langle \rangle$ symbol for average property.

The transient energy equations of the fuel and cladding based on the average property and lumped-parameter techniques are

$$\frac{dT_F}{dt} = -(T_F - T_c) + Q \quad \text{(B.1)}$$

and

$$\frac{dT_c}{dt} = T_F - T_c - \frac{h R_c}{h' R_F} (T_c - T_{Na}), \quad \text{(B.2)}$$

where $h$ is coolant overall heat-transfer coefficient.

If a constant heat-generation rate is assumed at a given axial position and the core is completely voided, Eq. B.2 can be reduced to

$$\frac{dT_c}{dt} = T_F - T_c. \quad \text{(B.3)}$$

Combination of Eqs. B.1 and B.3 yields

$$\frac{d}{dt}(C_F T_F + C_c T_c) = Q. \quad \text{(B.4)}$$

If the dryout time $t_d$ for the cladding surface of a fuel pin is known at a given axial position $Z$, Eq. B.4 can be integrated for any time $t > t_d$:

$$C_F T_F + C_c T_c = Q(t - t_d) + C_F T_{F0} + C_c T_{c0}. \quad \text{(B.5)}$$
where $T_{F0}$ and $T_{c0}$, the steady-state temperatures for fuel and cladding respectively, can be calculated from Eqs. A.3 and A.4 by assuming the sodium temperature be saturated at $t = t_d$.

Equations B.1 and B.3 can be rewritten as

$$\frac{dT_F}{dt} + \left( \frac{1}{C_F} + \frac{1}{C_c} \right)T_F = \frac{Q}{C_F C_c} t + \frac{T_{F0}}{C_c} + \frac{T_{c0}}{C_F} \left( 1 - \frac{t_d}{C_c} \right) \quad (B.6)$$

and

$$T_c = T_{c0} + \frac{Q}{C_c} (t - t_d) - \frac{C_F}{C_c} (T_F - T_{F0}) \quad (B.7)$$

Let

$$\alpha = \frac{1}{C_F} + \frac{1}{C_c}$$

$$\beta = \frac{Q}{C_F C_c}$$

and

$$\nu = \frac{T_{F0}}{C_c} + \frac{T_{c0}}{C_F} + \frac{Q}{C_F} \left( 1 - \frac{t_d}{C_c} \right).$$

Equation B.6 can be rewritten as

$$\frac{dT_F}{dt} + \alpha T_F = \beta t + \nu. \quad \text{(B.8)}$$

The solution for Eq. B.8 is

$$T_F = C e^{\alpha t} + \frac{\beta}{\alpha} + \frac{1}{\alpha} \left( \gamma - \frac{\beta}{\alpha} \right).$$

Since $T_F = T_{F0}$ at $t = t_d$,

$$C \left[ T_{F0} + \frac{\beta}{\alpha} + \frac{1}{\alpha} \left( \gamma - \frac{\beta}{\alpha} \right) \right] \exp(\alpha t_d) \quad \text{(B.9)}$$

(Contd.)
It is also noticed that $Q = \frac{qR_F}{2h'} = T_{F_0} - T_c0$ and the transient average fuel temperature can be expressed as

$$T_F = T_{F_0} + \frac{Q}{C_F + C_c}(t - t_d)$$

\[+ \frac{C_F C_c}{(C_F + C_c)}(T_{F_0} - T_c0)\left\{\exp\left[\frac{C_F + C_c}{C_F C_c} (t - t_d)\right] - 1\right\}, \quad (B.10)\]

the transient average cladding temperature can be also written as

$$T_c = T_{c0} + \frac{Q}{C_F + C_c}(t - t_d)$$

\[- \frac{C_c}{(C_F + C_c)}(T_{F_0} - T_{c0})\left\{\exp\left[\frac{C_F + C_c}{C_F C_c} (t - t_d)\right] - 1\right\}. \quad (B.11)\]

Equations B.10 and B.11 can be simplified by the linear-approximation method. At $t = t_d$, the derivative of the cladding temperature is calculated as

$$\left.\frac{dT_c}{dt}\right|_{t = t_d} = \frac{Q}{C_c}.$$  

The cladding temperature can be approximated by a linear relation at the time near $t_d$ as

$$T_c = T_{c0} + \frac{Q}{C_c}(t - t_d). \quad (B.12)$$

For a large argument of $t$, Eq. B.11 can be linearly approximated by

$$T_c = T_{c0} + \frac{Q}{C_F + C_c}\left(\frac{C_c}{C_F + C_c} + (t - t_d)\right). \quad (B.13)$$

intersection of Eqs. B.12 and B.14 is at $t = t_d \quad C_F C_c (C_F + C_c)$. The transient average fuel temperature can be also approximated as follows:
Case 1. For \( t < t_d \cdot C_F/C_c \), we approximate that
\[
\exp\left[ -\frac{C_F + C_c}{C_F C_c} (t - t_d) \right] \cdot 1 - \frac{C_F + C_c}{C_F C_c} (t - t_d)
\]
and
\[ Q = T_F - T_C. \]

Then the fuel temperature can be reduced into the form
\[ T_F = T_{F_0}. \]  \hfill (B.14)

Case 2. For \( t - t_d > C_F/C_c/(G_F + C_c) \), the fuel temperature can be approximated by neglecting the exponential term from Eq. B.10, so that
\[
T_F = T_{F_0} \cdot \frac{C_F + C_c}{(G_F + C_c)^2 (T_{F_0} - T_C) + \frac{Q}{G_F + C_c} t_d}. \]  \hfill (B.15)

Figure B.1 shows the relationship between the original average temperatures of fuel and cladding and their approximation formulas (Eqs. B.12-B.15).
APPENDIX C

Cladding-melting Problem

1. Start of Cladding Melting

The starting time of the cladding melting (tp) can be easily estimated. As soon as the cladding temperature reaches its melting point (T_melting), the cladding begins to melt. Using Eqs. 7 and 9, the starting time of the cladding melting (tp) can be obtained as follows:

\[ t_p - t_d = \frac{T_{cp} - T_{co}}{T_{F0} - T_{co}} C_c \quad \text{for} \quad t_p - t_d \geq \frac{C_F C_c}{C_F + C_c} \] (C.1)

\[ t_p - t_d = \left[ \frac{T_{cp} - T_{co}}{T_{F0} - T_{co}} \cdot \frac{C_F^2}{(C_F + C_c)^2} \right] (C_F + C_c) \quad \text{for} \quad t_p - t_d < \frac{C_F C_c}{C_F + C_c} \] (C.2)

Because the cladding is thin, an excellent estimation of the starting time is achieved by using the average cladding temperature. The fuel temperatures can be calculated from either Eq. B.10 or approximated by Eqs. B.14 and B.15.

2. Completion of Cladding Melting

Figure C.1 shows the typical temperature profiles during the cladding-melting period. The moving front of the melting cladding is described by \( R_p (R_F < R_p < R_e) \). The energy equations for both fuel and cladding are

\[ \frac{\partial(T_F)}{\partial t} = \frac{2}{R_F \cdot \rho_F \cdot C_{pf}} \frac{k_F}{R_F} \frac{\partial T_F}{\partial r} \bigg|_{R_F} + \frac{\hat{q}}{\rho_F C_{pf}} \] (C.3)

![Fig. C.1](image_url)

Typical Temperature Profiles during Cladding-melting Period. ANL. Neg. No. 2938-75-71.)
and
\[
\frac{\partial(T_c)}{\partial t} = -\frac{2R_F}{R_p^2 - R_F^2} \frac{k_c}{\rho_c^2 C_{pc}} \frac{\partial T_c}{\partial r} \bigg|_{R_F} + \frac{2R_p}{R_p^2 - R_F^2} \frac{k_c}{\rho_c^2 C_{pc}} \frac{\partial T_c}{\partial r} \bigg|_{R_p}.
\]  
(C.4)

The boundary conditions for the cladding melting are known as
\[
-k_c \frac{\partial T_c}{\partial r} \bigg|_{R_p} = \rho_c L_c \frac{dR_p}{dt},
\]  
(C.5)

\[
k_F \frac{\partial T_F}{\partial r} = k_c \frac{\partial T_c}{\partial r} = -h'(\langle T_F \rangle - \langle T_c \rangle),
\]  
(C.6)

and
\[
T_c \bigg|_{R_p} = T_{cp},
\]  
(C.7)

where \( L_c \) is the latent heat of the cladding. Suppose that \( \langle T_c \rangle = T_{cp} \); by omitting \( \langle \cdot \rangle \), we have
\[
C_F \frac{dT_F}{dt} = -(T_F - T_{cp}) + Q
\]  
(C.8)

and
\[
\frac{\rho_c L_c}{2R_F h'} \frac{d(R_p)^2}{dt} = T_F - T_{cp},
\]  
(C.9)

where
\[
C_F = \frac{R_F^0 C_p F}{2h'}
\]

and
\[
Q = \frac{R_F^0 q}{2h'}.
\]

**Case 1.** \( t_p - t_d - C_F C_c/(C_F + C_c) \). Assume that \( T_F = T_{F_0} \) at \( t = t_d \) (most simplified case). The solution for Eq. C.8 can be obtained as
\[
T_F = T_{F_0} + (T_{cp} - T_{co}) \left[ 1 - \exp \left( \frac{t - t_p}{c_F} \right) \right].
\]  
(C.10)
Substituting Eq. 13 into Eq. 12 gives

\[ \frac{\rho_c L_c}{2RFh'} \frac{d(R_p^l)^2}{dt} = TF_0 - Tcp + (Tcp - Tc_0) \left[ 1 - \exp \left( \frac{t - tp}{c_F} \right) \right]. \quad (C.11) \]

By knowing that \( R_p = RF \) at \( t = tp \), the solution for Eq. C.11 is

\[ \frac{(R_p^l - RF)c_c L_c}{2RFh'(TF_0 - Tc_0)} = t - tp - \frac{c_F(Tcp - Tc_0)}{(TF_0 - Tc_0)} \left[ 1 - \exp \left( \frac{t - tp}{c_F} \right) \right]. \quad (C.12) \]

For a small-time approximation, Eqs. C.10 and C.12 can be written as

\[ TF = TF_0 + (Tcp - Tc_0) \frac{t - tp}{c_F}; \]

\[ R_p^l - RF = \frac{2RFh'}{\rho_c L_c} (TF_0 - Tcp)(t - tp). \quad (C.13) \]

On the other hand, for a large-time approximation, Eqs. C.10 and C.12 can be reduced to

\[ TF = TF_0 + (Tcp - Tc_0); \]

\[ R_p^l - RF = \frac{2RFh'}{\rho_c L_c} ((TF_0 - Tcp)(t - tp) - c_F(Tcp - Tc_0)) \quad (C.14) \]

Intersection of Eqs. C.13 and C.14 yields

\[ t - tp = CF. \]

Therefore, the approximation formulas for the cladding-melting moving-boundary histories are

\[ t - tp = \frac{(R_p^l - RF)c_c L_c}{2RFh'(TF_0 - Tcp)} \quad \text{for} \quad t - tp \leq CF; \quad (C.15) \]

\[ t - tp = \frac{(R_p^l - RF)c_c L_c}{2RFh'(TF_0 - Tcp)} + \frac{(Tcp - Tc_0)}{c_F(TF_0 - Tc_0)} \quad \text{for} \quad t - tp > CF. \quad (C.16) \]

In fact, the cladding-melting process is completed as soon as \( R_p \) (the cladding-melting front) reaches \( R_c \) (the outer radius of the fuel pin). The completion time of the cladding melting \( (t_m) \) can be estimated as follows:
\[ t_m = t_p + \frac{(R_c^i - R_c^f)\rho_c L_c}{2R_{Fh}^\prime (T_{F0} - T_{cp})} \] for \( t_m - t_p \leq CF; \quad \text{(C.17)} \]

\[ t_m = t_p + \frac{(R_c^i - R_c^f)\rho_c L_c}{2R_{Fh}^\prime (T_{F0} - T_{cp})} + \frac{T_{cp} - T_{c0}}{T_{F0} - T_{c0}} \] for \( t_m - t_p > CF. \quad \text{(C.18)} \]

**Case 2.** \( t_p - t_d > CF; \quad (C_F + C_c). \) From Eq. B.15, we have the initial fuel temperature at \( t = t_p: \)

\[ T_F = T_{F0} - \frac{C_F C_c}{(C_F + C_c)^2} (T_{F0} - T_{c0}) + \frac{Q}{C_F + C_c} (t_p - t_d) \] for \( t_p - t_d > \frac{C_F C_c}{C_F + C_c}. \quad \text{(C.19)} \]

The solution for Eq. C.8 can be obtained as

\[ T_F = T_{cp} + (T_{F0} - T_{c0}) \left[ 1 - \frac{C_F}{C_F + C_c} \exp \left( \frac{t - t_p}{C_F} \right) \right]. \quad \text{(C.20)} \]

In similar manner, Eq. C.20 can be approximated as

\[ T_F = T_{cp} + \frac{T_{F0} - T_{c0}}{C_F + C_c} \left[ C_c + t - t_p \right] \] for \( t - t_p \leq CF \quad \text{(C.21)} \]

and

\[ T_F = T_{F0} + (T_{cp} - T_{c0}) \] for \( t - t_p > CF. \quad \text{(C.22)} \]

Substituting Eq. C.20 into Eq. C.10 yields

\[ \frac{\rho_c L_c}{2R_{Fh}^\prime} \frac{d(R_p^i)}{dt} = (T_{F0} - T_{c0}) \left[ 1 - \frac{C_F}{C_F + C_c} \exp \left( \frac{t - t_p}{C_F} \right) \right]. \quad \text{(C.23)} \]

with the initial condition \( R_p = R_F \) at \( t = t_p. \) The solution for Eq. C.23 is

\[ \frac{R_p^i - R_F^i}{2} = \frac{R_{Fh}^\prime}{\rho_c L_c} (T_{F0} - T_{c0}) \left[ t - t_p - \frac{C_F^i}{C_F + C_c} \left[ 1 - \exp \left( \frac{t - t_p}{C_F} \right) \right] \right]. \quad \text{(C.24)} \]

In similar manner, Eq. C.24 can be approximated by the following formulas:

1. For small time, \( t - t_p > CF: \)

\[ \frac{R_p^i - R_F^i}{2} = \frac{R_{Fh}^\prime}{\rho_c L_c} (T_{F0} - T_{c0}) \frac{C_c}{C_F + C_c} (t - t_p). \quad \text{(C.25)} \]
2. For large time, $t - t_p > C_F$:

$$\frac{R_p - R_F}{2} = \frac{R_F h'}{\rho_c L_c (T_{F_0} - T_{c_0})} \left( t - t_p - \frac{C_F}{C_F + C_c} \right). \tag{C.26}$$

The completion time of the cladding melting ($t_m$) can be estimated by letting $R_p = R_c$ in Eqs. C.25 and C.26:

$$t_m = t_p + \frac{(R_c - R_F) \rho_c L_c}{2 R_F h'(T_{F_0} - T_{c_0})} \frac{C_F + C_c}{C_c} \quad \text{for} \quad t_m - t_p < C_F \tag{C.27}$$

$$t_m = t_p + \frac{(R_c - R_F) \rho_c L_c}{2 R_F h'(T_{F_0} - T_{c_0})} \frac{C_F}{C_F + C_c} \quad \text{for} \quad t_m - t_p > C_F. \tag{C.28}$$
APPENDIX D

Fuel-melting Problem

1. Start of Fuel Melting

This is the stage from complete melting of the cladding to the start of the fuel melting. During this time interval, the cladding motion starts and the thickness of the cladding may change. However, if we assume that the cladding relocation has secondary effects on the fuel temperature, in view of the smaller thermal inertia of cladding and large thermal resistance within fuel, the first-order approximations for energy balances for fuel and cladding are

\[
\frac{\delta^2 T_F}{\delta t^2} = \frac{\kappa_F}{R_F} \cdot \frac{\delta T_F}{\delta r} \cdot \frac{k_F}{2r} \left| \frac{\delta T_F}{\delta r} \right| \cdot \frac{\dot{q}}{\varepsilon_F C_p F} \quad (D.1)
\]

and

\[
\frac{\delta^2 T_c}{\delta t^2} = \frac{2R_F}{(R_c - R_F)} \cdot \frac{k_c}{R_c C_p} \cdot \frac{\delta T_c}{\delta r} \cdot \frac{k_c}{2r} \left| \frac{\delta T_c}{\delta r} \right| \cdot \frac{\dot{q}}{R_F} \quad (D.2)
\]

The boundary conditions are

\[
k_F \cdot \frac{\delta^2 T_F}{\delta r^2} - k_c \frac{\delta T_c}{\delta r} = h' \left( T_F - T_c \right) \text{ at } r = R_F.
\]

If the cladding motion and thickness of the molten cladding are known from a cladding-relocation analysis, then Eq. D.2 can be modified to take those effects into account. However, in such cases, a simple analytical solution may not be obtained.

Over the wide, high fuel-temperature range between the fuel temperatures at the end of cladding melting \( T_F(t_m) \) and the fuel melting temperature \( T_{FP} \), the average specific heat \( C_{pF} \) used in previous cases has to be adjusted. Mean fuel specific heat at the mean temperature between \( T_F(t_m) \) and \( T_{FP} \) is used in the analysis at this stage.

Redefine

\[
C_{pF}^* = \frac{\frac{1}{T_{FP}} - \frac{1}{T_F(t_m)}}{\frac{1}{T_{FP}} - \frac{1}{T_F(t_m)}}
\]
If $\dot{q}$ is a function of axial position only and $R_F\dot{q}$, $\phi h^i = T_{F0} - T_{c0}$, Eqs. D.1 and D.2 can be rewritten by dropping the symbol \( \langle \rangle \):

$$C_F \frac{dT_F}{dt} = -(T_F - T_c) + (T_{F0} - T_{c0}) \tag{D.3}$$

and

$$C_c \frac{dT_c}{dt} = T_F - T_c \tag{D.4}$$

The initial fuel and cladding temperatures at $t = t_m$ are (see Eqs. C.10 and C.20)

$$T_F(t_m) = T_{F0} + (T_{cp} - T_{c0}) \left[ 1 - \exp\left( \frac{t_m - t_p}{C_F} \right) \right] \text{ for } \frac{T_{cp} - T_{c0}}{T_{F0} - T_{c0}} \leq \frac{C_F}{C_p + C_c};$$

$$T_F(t_m) = T_{F0} + (T_{F0} - T_{c0}) \left[ 1 - \frac{C_F}{C_F + C_c} \exp\left( \frac{t_m - t_p}{C_F} \right) \right] \text{ for } \frac{T_{cp} - T_{c0}}{T_{F0} - T_{c0}} > \frac{C_p}{C_p + C_c}; \tag{D.5}$$

with

$$T_c(t_m) = T_{cp}.$$

Combination of Eqs. D.3 and D.4 gives

$$\frac{d}{dt}(C_F T_F + C_c T_c) = T_{F0} - T_{c0}. \tag{D.6}$$

Integration of Eq. D.7 for $t \geq t_m$ yields

$$C_F T_F + C_c T_c = (T_{F0} - T_{c0})(t - t_m) + C_F T_F(t_m) + C_c T_{cp} \tag{D.8}$$

and

$$T_c = T_{cp} + \frac{C_F}{C_c} [T_f(t_m) - T_F] + (T_{F0} - T_{c0}) \frac{t - t_m}{C_c}. \tag{D.9}$$
Substituting Eq. D.9 into Eq. D.7 gives

$$\frac{dT_F}{dt} + \left(\frac{1}{C_c} + \frac{1}{C_F'}\right)T_F = \frac{T_{F0} - T_{co}}{C_F' \cdot C_c} \cdot t + \left[\frac{T_{cp}}{C_F} + \frac{T_F'(t_m)}{C_c}\right] \cdot \frac{T_{F0} - T_{co}}{C_F} \left(1 - \frac{t_m}{C_c}\right).$$

(D.10)

Therefore, the solution for Eq. D.10 is

$$T_F = T_F'(t_m) + \frac{1}{C_F' + C_c} \left((T_{F0} - T_{co})(t - t_m)\right) + C_c\left[T_F'(t_m) \cdot T_{cp} - \frac{C_c}{C_F' + C_c} (T_{F0} - T_{co})\right] \left[1 - \exp\left[-\frac{C_F' + C_c}{C_F' \cdot C_c} (t - t_m)\right]\right].$$

(D.11)

Equation D.9 can be rewritten as

$$T_c = T_{cp} + \left(\frac{T_{F0} - T_{co}}{C_F' + C_c}\right) \cdot \frac{t - t_m}{C_F' + C_c} + \frac{C_F'}{C_F' + C_c} \left[T_F(t_m) - T_{cp}\right]$$

$$+ \frac{C_c}{C_F' + C_c} (T_{F0} - T_{co}) \left[1 - \exp\left[-\frac{C_F' + C_c}{C_F' \cdot C_c} (t - t_m)\right]\right].$$

(D.12)

Based on the linear expansion of the exponential term, the time when the exponential term becomes negligible is given by $t \cdot t_m = C_F' C_c / (C_F' + C_c)$. The transient average fuel temperature can be also approximated as follows.

**Case 1.** For $t \cdot t_m = C_F' C_c / (C_F' + C_c)$, the fuel temperature can be approximated by

$$T_F = T_F'(t_m) - \frac{1}{C_F'} [T_{F0}(t_m) - T_{cp} - T_{F0} + T_{co}(t - t_m)].$$

(D.13)

**Case 2.** For $t \cdot t_m > C_F' C_c / (C_F' + C_c)$, the fuel temperature can be approximated by neglecting the exponential term from Eq. D.11

$$T_F = T_F'(t_m) + \frac{T_{F0} - T_{co}}{C_F' + C_c} (t - t_m)$$

$$- \frac{C_c}{C_F' \cdot C_c} \left[T_F(t_m) - T_{cp} - \frac{C_c}{C_F' + C_c} (T_{F0} - T_{co})\right].$$

(D.14)
The fuel melting starts at $t = t_{pF}$ as soon as the fuel temperature reaches the fuel-melting temperature $T_{Fp}$. The starting time of fuel melting, $t_{pF}$, can be obtained as (see Eqs. D.13 and D.14):

a. If $t_{pF} - t_{m} = C_{F}C_{c} (C_{F} + C_{c})$

or

$$T_{F}(t_{m}) = \frac{C_{c}}{C_{F} + C_{c}} |T_{F}(t_{m}) - T_{cp} - T_{Fp} + T_{co}| - T_{Fp}$$

then

$$t_{pF} - t_{m} = \left\{ \frac{T_{Fp} - T_{F}(t_{m})}{T_{Fp} - T_{cp} - T_{F}(t_{m}) + T_{cp}} \right\} C_{F}$$

(D.15)

b. If $t_{pF} - t_{m} > C_{F}C_{c} (C_{F} + C_{c})$

or

$$T_{F}(t_{m}) = \frac{C_{c}}{C_{F} + C_{c}} |T_{F}(t_{m}) - T_{cp} - T_{Fp} + T_{co}| - T_{cp}$$

then

$$t_{pF} - t_{m} = (C_{F} + C_{c})$$

$$T_{Fp} - T_{F}(t_{m}) + \frac{C_{c}}{C_{F} + C_{c}} \left[ T_{F}(t_{m}) - T_{cp} - \frac{C_{c}}{C_{F} + C_{c}} (T_{Fp} - T_{co}) \right]$$

$$\frac{T_{Fp} - T_{F}(t_{m})}{T_{Fp} - T_{co}}$$

(D.16)

2. Completion of Fuel Melting

Suppose the temperature of the fuel stays at the fuel melting temperature $T_{Fp}$ during the period from the start to the completion of the melting. Energy equations of the fuel and cladding at this stage are

$$\frac{\partial^{2} T_{F}}{\partial t^{2}} = - \frac{\partial h}{\partial R_{F} F_{c} C_{pF}} ((T_{F} - T_{c})^{2} + \frac{\partial l_{F}}{\partial F_{c} C_{pF}} \frac{L_{F}}{C_{pF}} \frac{\partial \delta F_{m}}{\partial t}$$

(D.17)
and

\[
\frac{d(T_c)}{dt} = \frac{\Delta R}{R_i^2 - R_F^2} \cdot \frac{h}{\kappa C_p} (\langle T_F \rangle - \langle T_{c0} \rangle) 
\]  

(D.18)

where \( \Delta P \) is latent heat of the fuel and \( \alpha_{Fm} \) is the volumetric concentration of liquid fuel. If \( T_{Fp} \) is constant, i.e., \( \frac{d(T_F)}{dt} = 0 \), and \( \dot{q} \) is a function of axial position only, Eqs. D.17 and D.18 can be rewritten by dropping the symbol \( \tau \) as

\[
\frac{R_F - R_i \cdot d\alpha_{Fm}}{2h'} \cdot \langle T_{Fp} - T_{c0} \rangle + \langle T_{F0} - T_{c0} \rangle 
\]  

(D.19)

and

\[
C_c \frac{dT_c}{dt} = T_{Fp} - T_{c0} 
\]  

(D.20)

The initial condition for \( \alpha_{Fm} \) is

\[
\alpha_{Fm} = 0 \text{ at } t = t_{pF} 
\]

The initial conditions of the cladding temperature can be approximated from Eq. D.12 as

\[
T_c(t_{pF}) = T_{cp} + \frac{1}{C_c} |T_F(t_m) - T_{cp}|(t_{pF} - t_m) \text{ for } t_{pF} - t_m \leq \frac{C_F C_c}{C_F + C_c} 
\]  

(D.21)

and

\[
T_c(t_{pF}) = T_{cp} + \left( T_{F0} - T_{c0} \right) \frac{t_{pF} - t_m}{C_F + C_c} + \frac{C_F}{C_F + C_c} \left[ T_F(t_m) - T_{cp} \right. 
\]

\[ 
\left. - \frac{C_c}{C_F + C_c} (T_{F0} - T_{c0}) \right] \text{ for } t_{pF} - t_m > \frac{C_F C_c}{C_F + C_c} 
\]  

(D.22)

The solution for Eq. D.20 is

\[
T_c = T_{Fp} - |T_{Fp} - T_c(t_{pF})| \exp \left( \frac{t - t_{pF}}{C_c} \right) 
\]  

(D.23)
and the solution for Eq. D.19 is

$$\frac{R_F^2 F L_F}{\chi'} \alpha_{Fm} = (T_{F_0} - T_{C_0})(t - t_{pF}) - \left[ T_{F_p} - T_c(t_{pF}) \right] C_c \left[ - \exp \left( \frac{t - t_{pF}}{C_c} \right) \right].$$  \hfill (D.24)

In similar manner, the approximation formulas for Eq. D.18 are:

a. For $t < t_{pF} + C_c$ the small-time approximation formula is

$$\alpha_{Fm} = \frac{h'}{2 R_F^2 F L_F} \left[ (T_{F_0} - T_{C_0} - T_{F_p} + T_c(t_{pF}))(t - t_{pF}) \right].$$  \hfill (D.25)

b. For $t > t_{pF} + C_c$, the large-time asymptotic formula is

$$\alpha_{Fm} = \frac{h'}{2 R_F^2 F L_F} \left[ (T_{F_0} - T_{C_0})(t - t_{pF}) - \left[ T_{F_p} - T_c(t_{pF}) \right] C_c \right].$$  \hfill (D.26)

At $t = t_{Fm}$, $\alpha_{Fm}$ is equal to unity. The completion time of the fuel melting ($t_{Fm}$) can be estimated as follows (see Eqs. D.25 and D.26):

$$t_{Fm} = \frac{R_F^2 F L_F}{\chi' \left[ (T_{F_0} - T_{C_0} - T_{F_p} + T_c(t_{pF})) \right]} \text{ for } t_{Fm} < t_{pF} + C_c$$  \hfill (D.27)

and

$$t_{Fm} = \frac{R_F^2 F L_F}{\chi' (T_{F_0} - T_{C_0})} + \frac{C_c \left[ T_{F_p} - T_c(t_{pF}) \right]}{T_{F_0} - T_{C_0}} \text{ for } t_{Fm} > t_{pF} + C_c.$$  \hfill (D.28)
REFERENCES


