DEVELOPMENT OF PLACE-VALUE NUMERATION CONCEPTS IN CHINESE CHILDREN: AGES 3 THROUGH 9

DISSERTATION

Presented to the Graduate Council of the University of North Texas in Partial Fulfillment of the Requirements

For the Degree of

DOCTOR OF PHILOSOPHY

By

Sy-Ning Chang, B.A., M.ED.
Denton, Texas
August, 1995
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This investigation examined Chinese children's development of place-value numeration concepts from ages 3 through 9, compared the development of place-value understanding of these Chinese children with that of American and Genevan children whose performances had been described in the literature, and examined the influence of adult assistance during Chinese children's performances on some of the place-value tasks. A standard interview method was adopted. Tasks and procedures were adapted from several cognitive studies in the place-value domain. The subjects were 98 children (14 for each age level, equally divided as to sex) randomly selected from two schools in Taipei, Taiwan. The 98 interviews were videotaped and transcribed into both Chinese and English.

The findings indicated that Chinese children's performances in a variety of place-value tasks highly suggested a developmental progression in the understanding of the common place-value numeration system; that all children in the studies cited--Chinese, American, and Genevan--probably go through the same developmental sequence in comprehending the place-value numeration system, but that the Chinese apparently formed the base-10 conceptual structure at earlier age levels than did the American and Genevan children; and that the Chinese 5-, 6-, 7- and 8-year-olds benefited the most from adult assistance in some place-value tasks.
Findings further indicated that the inability to create the hierarchical structure of numerical inclusion (part-whole numerical relations) was a universal cognitive limitation common to all Chinese, American, and Genevan younger children in their attempt to comprehend the place-value numeration system; that structures of Chinese spoken-number words seemingly had influences on children's construction of place-value understanding; and that adult assistance during a child's performance in some place-value tasks involved a sort of "scaffolding" process that led the child in a direction that enabled him/her to solve problems which would have been beyond his/her unassisted efforts.

Implications for instructional strategies are made for both American and Chinese teachers. Suggestions for further research are also discussed.
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CHAPTER 1

INTRODUCTION

Background of the Problem

Quantity is indigenous to the whole world; thus, numerical knowledge becomes a natural domain for human mental functioning (Klein & Starkey, 1988). As a result of their daily experiences, 22-week-old infants are capable of discriminating exact numbers of items when the number of a given item is under four and presented visually (Starkey & Cooper, 1980). During the years of preschool, children are able to orally count sets of items and form their cardinality (Gelman & Gallistel, 1978). The average 4-year-old child can count up to 9 objects without error, and the 5-year-old can enumerate up to 20; the 6-year-old can reach about 28 (Ginsburg, 1982). At the time of entrance to school, the majority of children have informal, counting-based means to handle numbers (Baroody & Others, 1983; Ginsburg, 1982; Rea & Reys, 1970) and are in the process of constructing the “next-by-one” mental number line (Resnick, 1983). For them, the fundamental relationships between numbers are units. Number 16 means 16 ones and is seen as the next number following 15 on their mental number line.

To reduce memory demands and to increase counting efficiency when larger quantities are involved, children are introduced to a base-10/place-value numeration system in school. This system is a formal and powerful tool for presenting numbers and for working with arithmetic
algorithms (Baroody et al., 1983; Barr, 1978). In this system, number is the process of using combinations of any of the 10 digits (0-9); the value of a given digit in a multi-digit numeral depends on both its face and place values; the value of the multi-digit numeral is the sum of the face and place values for each digit; the value of each place increases progressively by multiples of 10 from the rightmost position to the left (Ashlock, 1978; Hinrichs, Yurko, & Hu, 1981; Labinowicz, 1985; Ross, 1986).

Understanding place value requires children’s coordination of two mental number lines: the “next-by-one” and the “next-by-ten.” This concept of a set of sets and the understanding of part-whole numerical relations is fundamental (Ashlock, 1978; C. Kamii & Joseph, 1989; Labinowicz, 1985; Resnick, 1983; Ross, 1986; Ronshausen, 1978; Ross, 1990; Smith, 1973).

For example, the number 16 represents a composite of 1 ten and 6 ones because 10 ones can be equated as one 10 and shown with a single digit. Children who do not construct place-value concepts diminish their ability in a wide range of mathematical operations, such as addition, subtraction, multiplication, and division (Baroody et al., 1983; C. Kamii & Joseph, 1989).

Although the conventional place-value system is very economical, this economy itself may be a source of learning difficulty for children (Labinowicz, 1985). Most teachers and educators agree that the understanding of place value is the most important, yet difficult, mathematical task in the early school years (Baroody et al., 1983; Easley, 1980). Research literature also reveals school children’s poor

Many of these studies suggested that the difficulty in place-value understanding is related to the limitations in children’s cognitive development. C. Kamii (1986) believed that place value is a problem of conceptual abstraction. C. Kamii and Joseph (1989) asserted that 6-and 7-year-olds find it impossible to construct the “next-by-ten” mental number line (multi-unit conceptual structure) while they are still working on their “next-by-one” mental number line (unitary conceptual structure). Likewise, they believed that children could not create the hierarchical structure of numerical inclusion before their thought processes become reversible, approximately at the age of 7 or 8 (C. Kamii & DeClark, 1985). Ross (1990) concluded that not until the age 8 or 10 does the understanding of numerical part-whole relations become operational.

On the other hand, some cross-cultural studies, such as Fuson and Kwon (1992a, 1992b), Hong (1989), Miller and Stigler (1987), Miura (1987), Miura, et al. (1988), Miura & Okamoto (1989), pointed out that the
differences between American and Asian children’s achievement in place-value tasks are the function of numerical language. Although Arabic numerals are used internationally and their written marks are the same, the verbal forms employed by Asian and Western populations vary to a great extent. Asian languages, such as Burmese, Chinese, Korean, Japanese, and Thai are based on Ancient Chinese and have a regular named-value system for spoken (two-digit) numerals. When a number is spoken, both face- and place-values of that number word are named in the sequence of the digits (Fuson & Kwon, 1991). For example, the number 23 in Chinese is said as “two-ten-three” (see Table 1). The parts of a spoken number are given from the largest (leftmost) to the smallest (rightmost) in expressing its multi-units, namely, hundreds, tens, and ones (Hatano, 1982). The standard oral name for a multi-digit number is the same as its grouping number name and corresponds exactly to its written form (Ashlock, 1978). Additionally, in Chinese, simply by prefixing “di” to a cardinal number, its ordinal term is formed (Baroody, 1993). For example, in Chinese, people use the term “ten-six” to indicate 16 ones; however, when people want to express the cardinal number, using the same digits, they prefix it with a “di” - “di ten-six.” Because Asian spoken number names clearly show their base-10 and place-value structures, the generation of number words is transparent and systematic. Therefore, this arrangement tends to support children in fostering a view of multi-digit numbers as composites of different multi-units (Baroody, 1990), which is a prerequisite to place-value understanding.
Table 1

Chinese and English Oral Number Words 1 to 110

<table>
<thead>
<tr>
<th>Written marks</th>
<th>Chinese</th>
<th>English</th>
<th>Positional base-10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>one</td>
<td>one</td>
<td>one</td>
</tr>
<tr>
<td>2</td>
<td>two</td>
<td>two</td>
<td>two</td>
</tr>
<tr>
<td>3</td>
<td>three</td>
<td>three</td>
<td>three</td>
</tr>
<tr>
<td>4</td>
<td>four</td>
<td>four</td>
<td>four</td>
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<td>5</td>
<td>five</td>
<td>five</td>
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<td>6</td>
<td>six</td>
<td>six</td>
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<tr>
<td>7</td>
<td>seven</td>
<td>seven</td>
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<tr>
<td>8</td>
<td>eight</td>
<td>eight</td>
<td>eight</td>
</tr>
<tr>
<td>9</td>
<td>nine</td>
<td>nine</td>
<td>nine</td>
</tr>
<tr>
<td>10</td>
<td>ten</td>
<td>ten</td>
<td>one zero</td>
</tr>
<tr>
<td>11</td>
<td>ten one</td>
<td>eleven</td>
<td>one one</td>
</tr>
<tr>
<td>12</td>
<td>ten two</td>
<td>twelve</td>
<td>one two</td>
</tr>
<tr>
<td>13</td>
<td>ten three</td>
<td>thirteen</td>
<td>one three</td>
</tr>
<tr>
<td>14</td>
<td>ten four</td>
<td>fourteen</td>
<td>one four</td>
</tr>
<tr>
<td>15</td>
<td>ten five</td>
<td>fifteen</td>
<td>one five</td>
</tr>
<tr>
<td>16</td>
<td>ten six</td>
<td>sixteen</td>
<td>one six</td>
</tr>
<tr>
<td>17</td>
<td>ten seven</td>
<td>seventeen</td>
<td>one seven</td>
</tr>
<tr>
<td>18</td>
<td>ten eight</td>
<td>eighteen</td>
<td>one eight</td>
</tr>
<tr>
<td>19</td>
<td>ten nine</td>
<td>nineteen</td>
<td>one nine</td>
</tr>
<tr>
<td>20</td>
<td>two ten</td>
<td>twenty</td>
<td>two zero</td>
</tr>
<tr>
<td>30</td>
<td>three ten</td>
<td>thirty</td>
<td>three zero</td>
</tr>
<tr>
<td>40</td>
<td>four ten</td>
<td>forty</td>
<td>four zero</td>
</tr>
<tr>
<td>50</td>
<td>five ten</td>
<td>fifty</td>
<td>five zero</td>
</tr>
<tr>
<td>60</td>
<td>six ten</td>
<td>sixty</td>
<td>six zero</td>
</tr>
<tr>
<td>70</td>
<td>seven ten</td>
<td>seventy</td>
<td>seven zero</td>
</tr>
<tr>
<td>80</td>
<td>eight ten</td>
<td>eighty</td>
<td>eight zero</td>
</tr>
<tr>
<td>90</td>
<td>nine ten</td>
<td>ninety</td>
<td>nine zero</td>
</tr>
<tr>
<td>100</td>
<td>one hundred</td>
<td>one hundred</td>
<td>one zero zero</td>
</tr>
<tr>
<td>101</td>
<td>one hundred zero one</td>
<td>one hundred one</td>
<td>one zero one</td>
</tr>
<tr>
<td>110</td>
<td>one hundred one ten</td>
<td>one hundred ten</td>
<td>one one zero</td>
</tr>
</tbody>
</table>
Many European languages, including English, have irregular named-value number-word systems between 10 and 99, although they are regular for the values of 100 to 1000 (Fuson & Kwon, 1991). In English, the naming of multi-digit numbers does not articulate the value of tens and ones (Jones & Thornton, 1993). Such teens as 11 and 12 hide their place-value structure altogether (Baroody, 1993). A reversal in the teen words results in children’s spell-as-heard error in writing these numerals (Baroody et al., 1983; Charbonneau & John-Steiner, 1988). The two different forms of ten (-teen in the first decade and -ty in the remaining decades) do not explicitly name “ten” (Fuson, 1990a); even some adults have no idea that the “-teen” and “-ty” in number words indicate “ten” (Fuson, 1986).

Because of the obfuscation of the underlying 10 structure in English numerical language (see Table 1), conflicts between counting and place value may arise for English-speaking children, who tend to maintain a unitary conceptual structure for two-digit numbers (Bednarz & Janvier, 1982; Cobb & Wheatley, 1988; Fuson, 1986; M. Kamii, 1982; C. Kamii & DeClark, 1985; C. Kamii, 1986; C. Kamii & Joseph, 1989; Labinowicz, 1985; Miura, 1987; Miura et al., 1988; Miura & Okamoto, 1989; Resnick, 1983; Ross, 1986, 1989, 1990.)

Statement of the Problem

Asian or Asian-American children generally outperform their American counterparts in mathematics during school years (Fuson, 1990a; Fuson & Kwon, 1992a, 1992b; Song & Ginsburg, 1987; Stevenson, Lee, & Stigler, 1986; Stigler, Lee, & Stevenson, 1987; Tsang, 1988). These
differences are, in part, due to oral English number words from 10 to 99 that impede American children's recognition of the place-value numeration system, the basis for subsequent arithmetic operations. From a very early age, children are actively seeking sense in their quantitative world. The way each culture organizes numerical meaning and the associated verbal, symbolical representation shapes the way children make sense of the numerical world. Educators and parents need to know when and how the balance of and the connection between invention (personal creation of numerical meaning) and convention (socially established numerical system) come about. If educators and parents knew about these variables, they could facilitate to a greater degree children's place-value understanding. A cross-cultural, research-based description regarding Asian children's development in place-value concepts across a wide age range might contribute to a more focused picture for both educators and parents.

Purposes of the Study

The four-fold purposes of the study were to (a) describe the development of place-value numeration concepts in Chinese children, ages 3 through 9; (b) compare the development of place-value understanding of Chinese-speaking children with that of English-speaking children, the latter which has been described in the literature; (c) examine the influence of adult assistance, such as verbal prompts, questions, and demonstrations, during Chinese children's performances on place-value tasks; and (d) formulate alternatives that will assist young children in their construction of place-value concepts.
Research Questions

The following questions guided this study toward the accomplishment of its purposes:

1. How do Chinese children perform place-value tasks at different age levels, 3 through 9?

2. Through what developmental sequences of place-value understanding do Chinese children go?

3. Do Chinese children go through the same developmental course of place-value understanding as English-speaking children do?

4. Do Chinese children have the same cognitive limitations when forming their conceptual structure of place value as that described in the literature which dealt with English-speaking children?

5. What is the age level at which the majority of Chinese-speaking children demonstrate their understanding of the place-value numeration system? What does the literature say about the age level at which English-speaking children reach understanding?

6. How does adult assistance facilitate Chinese children’s performances on place-value tasks at the different age levels?

Significance of the Study

Cross-cultural research on children’s mathematical development has the potential to (a) enhance the understanding of the processes concerning the natural order of children’s mathematical development, (b) increase the understanding of cultural influences on children’s mathematical development, and (c) guide in the formation of developmentally appropriate
goals, expectations, and educational practices for assisting children in achieving up to their mathematical capabilities (New, 1993).

First, a review of the literature reveals that the current cross-culture studies on children's place-value understanding have been limited to a specific age, such as preschoolers, kindergartners, or primary school children. Therefore, Asian children's developmental processes of place-value understanding across a wide age range remain unclear.

Secondly, the place-value tasks used in previous studies were confined to only one aspect of children's place-value concepts, such as digit-correspondence, dual meaning of the word ten, regrouping in adding or subtracting, among others. An overall description of Asian children's understanding of place value was needed. Therefore, by using a variety of place-value tasks in the process of sampling their understanding, the solidarity of their conceptualization could be determined.

Thirdly, mathematical learning is a fertile field in which one can track the interplay between language and children's cognitive development. Much imagery can be evoked by appropriate language (Bishop, 1985; Scholnick, 1988). Most children usually demonstrate a well-grounded understanding of language by age 5 (Labinowicz, 1985). Would mathematical understanding come at about the same age level? Or would this come earlier for Chinese children than for American children inasmuch as the former grasp their numerical language naturally as a part of their overall language development. This acquisition meaningfully reflects the structures of the place-value numeration system. The answer could possibly
be found only from a study that includes children representing all age levels of early childhood.

Fourthly, some studies (Jones & Others, 1992; Wood, Bruner, & Ross, 1976) showed that adult assistance increased young children’s performances on (numerical) problem solving. Would adult assistance have the same effectiveness at all age levels of Chinese children’s performances on place-value tasks? For helping children’s understanding of the place-value numeration system, this question is worthy of study.

In addressing the above four components, the purpose of the present study is to elucidate place-value understandings before and after an adult’s assistance among Chinese children in the age ranges of 3 to 9, as indicated by performances in a variety of place-value tasks. To examine the possible influence that Chinese number words have on Chinese children’s formation of place-value concepts, Chinese children’s performances on the different place-value tasks are compared with that of American peers whose performances have been described in the literature.

Definition of Terms

The following terms are defined for this study:

Place-value numeration concepts, in our base-10 numeration system, pertain to the following principles:

1. The same numeral represents different quantitative values by virtue of the place in which it is found (Charlesworth & Lind, 1990).

2. The value of each place increases by a power of 10 with respect to the units’ (the rightmost) place (Baroody, 1989).
3. A consistent one/ten ratio between adjacent values can be applied in computational processes (Fuson, 1990b; Smith, 1973).

4. A multi-digit number can be partitioned in many different ways, such as one-to-one collection, canonical base-10, and noncanonical base-10, whereby the parts are equal to the whole (Miura, 1987; Ross, 1986).

5. Regardless of the place, intrinsic values are indicated by the use of individual digits, zero to nine (Labinowicz, 1985).

6. Zero, as a place holder, means that there is no quantitative value in the place it holds (Charlesworth & Lind, 1990).

The place-value tasks administered in this study are designed to test the place-value numeration concepts defined previously.

A unitary conceptual structure refers to one-to-one representation of number words, such as using 26 unit blocks to present the number 26 (Ross, 1986; Miura, 1987).

A canonical base-10 presentation reflects a standard place-value partitioning, such as using two 10-block bars and six unit blocks to represent the number 26, with no more than nine units in the one’s place (Miura, 1987; Ross, 1986, 1989).

A noncanonical base-10 presentation manifests a more flexible place-value grouping, such as using 1 ten block bar and 16 unit blocks to represent the number 26; naturally, there are more than 9 units in the one’s position (Miura, 1987; Ross, 1986, 1989).

The multi-unit conceptual structure includes both canonical and noncanonical base-10 representation.
Limitations

This study was limited to 98 children randomly selected from one public elementary school in Taipei City, Taiwan, and one private early childhood program near there. These students were from families of differing socioeconomic backgrounds. Taipei City, being the capital of the Republic of China, has families different from those found in other cities, especially with regard to parents’ levels of education, attitudes toward education, levels of income, availability of resources, and attitudes on how to educate children. However, these differences are diminishing because of the relative smallness of Taiwan, the convenience of transportation, and the goal of universal education in Taiwan.

Assumptions

The following assumptions were made for this study:

1. When interviewed by a researcher experienced in early childhood education and trained in research methods, children will reveal their understanding of place-value concepts. Even mistakes made in responses help reveal their current understanding or misconception.

2. Using a variety of place-value tasks adapted from other studies, the interviewer will elicit work modes and responses from the children that will reveal their understanding of place-value concepts.

3. Some assistance from the interviewer, such as prompts, questions, and demonstration, will be used to keep children’s level of motivation high, to facilitate their thinking, and to clarify the particular objective of a task.
4. The interviewer will recognize that the understanding of place-value concepts at a lower age level is different qualitatively and quantitatively from that found at a higher age level.

5. Since the structures of Chinese oral number words reflect more accurately the place-value numeration system than does English notation, the interviewer will look for the differences between English- and Chinese-speaking children’s conceptualization of place-value numeration.

Summary

This study is designed to provide a comprehensive description concerning the development of Chinese children’s place-value concepts from ages 3 through 9 and the relationship between adult assistance and children’s place-value understanding. Furthermore, the results are to be compared to the literature on American children’s performance of place-value tasks and conceptual structures regarding two-digit numbers. Both Piaget’s constructive theory and Vygotsky’s social-cultural theory on children’s cognitive development form the theoretical foundation of this study.

A review of related literature on children’s recognition, understanding, and application of place-value concepts, including cross-cultural comparison, is presented in Chapter 2. In Chapter 3, a methodology applied to this study is described. Chapter 4 provides a detailed analysis of data, followed by conclusions and implications in Chapter 5.
CHAPTER 2
REVIEW OF RELATED LITERATURE

Although living in relative isolation, many cultures throughout history have tended to group collections of objects naturally by tens simply because the first “calculator” used by man was fingers (Labinowicz, 1985). These 10 fingers have taught him counting and increased the scope of numbers (Dantzig, 1935). Gradually, this systematic grouping by tens became the basis of the base-10/place-value numeration system in which the same principles are used for producing and extending numbers indefinitely. For example, in this system, number is the process of using combinations of any of the 10 digits (0-9). The value of each place, such as units, tens, hundreds, and thousands, and so on, increases progressively by powers of 10 from the rightmost to the left. Therefore, the value of a given digit in a multi-digit numeral is a product of its face and place values, and the value of a multi-digit numeral is the sum of the face and place values for the digits that make up that numeral (Ashlock, 1978; Hinrichs et al., 1981; Labinowicz, 1985; Ross, 1986).

Children’s Understanding of Place-value Numeration System

The base-10/place-value numeration system as used by adults is an efficient way for communicating quantity, but for most school children and other youngsters, its complexity is not easily grasped. In the past decades, cognitive psychology has greatly increased the extent of our understanding of children’s mathematical thinking. Also, in view of American children’s
low mathematical achievement (which may be the result of a fragile understanding of the place-value numeration system), some researchers have recently focused more specifically on the difficulties and sequence of place-value understanding and have suggested divergent stage models applicable to children’s formation of place-value concepts.

**Baroody and Others Study**

In Baroody et al.’s (1983) study, 78 primary school (K-3) children from four schools were individually tested in a standardized interview in which 23 tasks were administered. In the place-value tasks, children were asked to point out the ones, tens, hundreds, and thousands of multi-digit numbers. The results (see Table 2) showed that neither the kindergartners nor the first graders could recognize the hundreds and thousands in multi-digit numbers. Only 6% of the first graders were able to recognize the ones and tens, and none of the kindergartners could do so. However, 71% of the second graders and all of the third graders successfully pointed out the ones and tens in multi-digit numbers; as to the recognition of hundreds and thousands of multi-digit numbers, 57% of the second graders and 95% of the third graders correctly identified the places. An increase in place-value understanding occurred at each grade level. However, the overall results of the 23 tasks indicated that most first and second graders and some third graders had an imprecise understanding of the repetitive pattern of the place-value numeration system at the three-digit level.
Table 2

Percentages of Children Who Correctly Performed the Tasks of Place-Value and Numeral Reading in the Study of Baroody and Others (1983)

<table>
<thead>
<tr>
<th>Tasks</th>
<th>K</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Place value--ones and tens</td>
<td>0</td>
<td>6</td>
<td>71</td>
<td>100</td>
</tr>
<tr>
<td>Place value--hundreds and thousands</td>
<td>0</td>
<td>0</td>
<td>57</td>
<td>95</td>
</tr>
<tr>
<td>Reading one-digit numerals</td>
<td>92</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Reading teen numerals</td>
<td>72</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Reading two-digit numerals</td>
<td>36</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Reading three-digit numerals</td>
<td>4</td>
<td>39</td>
<td>86</td>
<td>100</td>
</tr>
<tr>
<td>Reading four-digit numerals</td>
<td>0</td>
<td>11</td>
<td>43</td>
<td>95</td>
</tr>
</tbody>
</table>

The Fourth Mathematics Assessment of the National Assessment of Educational Progress

The fourth assessment in mathematics was conducted by the Educational Testing Service (ETS) in 1986. Subjects for the fourth assessment were a representative national sample of third-grade, seventh-grade, and eleventh-grade students. The results showed that about 65% of the third graders were able to solve successfully the tasks involving grouping by 10, identifying the tens digit, and figuring out the number which is 10 more than a given number. However, when dealing with similar tasks having place values beyond tens, the proportion of subjects who successfully performed on the tasks fell below 50%; however, about 75% of the seventh graders could do those tasks. Approximately 84% of the
third-grade students successfully performed two-digit addition that involved regrouping. The percentage for seventh graders on the same problems was 95. Seventy percent of the third graders were able to solve two-digit subtraction that involved regrouping, but the percentage for three-digit subtraction items that involved borrowing dropped to 50. The proportions of the seventh graders who correctly solved two- and three-digit subtraction that involved regrouping were 94% and 85%, respectively. What caused the difficulty that these third graders experienced with subtraction when moving from two-digit problems to those having three digits? According to Kouba, Brown, Carpenter, Lindquist, Silver, and Swafford (1988), the difficulty may have resulted, in part, from the children’s lack of understanding of place value and the relationship between place value and subtraction.

**Cauley’s Study**

In Cauley’s (1988) study, 42 second graders and 48 third graders were interviewed individually. First, each child completed a pretest consisting of 10 two- and three-digit subtraction problems of varying difficulty. From these two groups, 34 children identified as procedurally proficient were asked about their procedures, their understanding of regrouping, and their conservation of the minuend in the processes of three subtraction problems. Thirty-five percent of these procedurally proficient children conserved the minuend; however, only half of them also knew the dynamic of place value during the regrouping procedure. The results suggested that second and third graders’ procedural proficiency on two- or
three-digit subtraction involving borrowing did not guarantee conceptual understanding of place value in the numeration system.

M. Kamii's study

During individual interviews, 80 children between the ages of 4 to 9, were asked by M. Kamii (M. Kamii, 1980, 1982, cited in C. Kamii & DeClark, 1985) to make correspondences between the individual digits in the numeral 16 and a collection of 16 objects. From the data, five levels of interpreting a two-digit numeral were found.

Level 1

For children at Level one, neither digit in numeral 16 has anything to do with quantity, yet both are linked to objects in the real world in which they are found. For example, the 6 can stand for Channel 6.

Level 2

Children at this level try to make some correspondence between the number symbols (1 or 6) they have written and something else on the paper that might be quantitative, such as the number of colors used in writing numeral 16.

Level 3

Although number symbols can stand for quantities of objects represented, children at this level believe that the two-digit number cannot be dissected into the number's individual digits. For example, when a two-digit number is broken down into its written parts, the number disappears.
Level 4

For children at this level, a two-digit numeral (16) consistently stands for the totality of the objects represented (16 ones), but each of the individual digits is interpreted by its face value. For example, the numeral 1 in number 16 means one object and “6” means six objects.

Level 5

Children at this level understand that the individual digits (1 and 6) make up a two-digit numeral (16) and that the quantities of the individual digits are determined by their face and place values (10 and 6).

The results of M. Kamii’s (1980, 1982, cited in C. Kamii & DeClark, 1985) studies showed that 87% of 7-year-olds are at Levels 3 and 4. Eight-year-olds often talked about ones, tens, and hundreds, but only 18% of them indicated that the 1 in number 16 stands for 10 objects (see Table 3). Forty-two percent of 9-year-olds indicated that the 1 in number 16 represents 10 of the 16 objects.

Harvin (1984), Ross (1986), Silvern and C. Kamii (1988, cited in C. Kamii and Joseph, 1989) also conducted studies in which the digit-correspondence task, similar to that used by M. Kamii (1980, 1982, cited in C. Kamii & DeClark, 1985), was administered. The percentages of children who reached M. Kamii’s Level 5 at different age levels are shown in Table 3. The results of the studies, except for Harvin’s, indicated that the majority of third graders’ and many fourth graders’ place-value understanding were still partial, incomplete, and fragile.
Resnick’s Stage of Decimal Knowledge

After reviewing research literature, Resnick (1983) identified three main stages in the development of decimal knowledge.

Table 3

<table>
<thead>
<tr>
<th></th>
<th>Grade</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>M. Kamii</td>
<td>0</td>
<td>13</td>
<td>18</td>
<td>42</td>
</tr>
<tr>
<td>Harvin</td>
<td>35</td>
<td>28</td>
<td>11</td>
<td>100</td>
</tr>
<tr>
<td>Ross</td>
<td>20</td>
<td>33</td>
<td>53</td>
<td>67</td>
</tr>
<tr>
<td>Silvern and C. Kamii</td>
<td>7.5</td>
<td>29</td>
<td>35</td>
<td></td>
</tr>
</tbody>
</table>

Stage 1

In Stage 1, children partition two-digit numerals into tens and ones and assume that there are no more than 9 units of a given place. For example, number 54 is 5 tens plus 4 ones.

Stage 2

In the second stage, children recognize the possibility of multiple partitioning of two-digit numerals. More than 9 units of a given place are allowed, without changing the total value of the two-digit number. For example, number 54 can be partitioned as 4 tens plus 14 ones or 3 tens plus 24 ones without changing its total value as 54 ones.
Stage 3

Children in this stage apply the part-whole scheme to written algorithms of subtraction and addition, such as borrowing and carrying. This application makes the computational procedures sensible to children.

In Resnick’s stage models, the development of decimal number knowledge can be seen as the successive elaboration of the part-whole scheme for numbers; number is subject to special regroupings under control of the part-whole scheme.

C. Kamii’s Study

One hundred children in grades 1 to 5 were interviewed individually by C. Kamii (1986) in Geneva, Switzerland. First, the subjects were asked to estimate and then count the larger quantity of chips spontaneously. All the first graders and most of the second, third, fourth, and fifth graders counted them by ones. Only a few children in the fourth grade counted by tens spontaneously. Then the school children were asked to count these chips by tens. There were four levels of response for the task.

Level 1

When asked to count by tens, the response from children at the lowest level was “no idea how.”

Level 2

When asked to count by tens, children in this category easily “made heaps of tens, but without conservation of the whole” (C. Kamii, 1986, p. 82). They either answered there are “7” (heaps of ten) chips instead of 70
chips or indicated 70 chips by counting by ones. They were unable to think about ones and tens simultaneously.

Level 3

When asked to count by tens, children at this level first counted out 10 chips and left them in a heap. Then they counted out another 10 chips, making separate group, and, as they said “twenty,” they combined the second group with the first group. They continued the same process until all the chips were counted. These children seemed to be able to count by tens, but in actuality these children did not “separate the whole into parts” (C. Kamii, 1986, p. 82). In reality, they counted by ones.

Table 4

Percentages of Children’s Responses at Each Grade Level When Asked to Count by Tens in C. Kamii’s (1986) Study

<table>
<thead>
<tr>
<th>Level</th>
<th>Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. No idea how</td>
<td></td>
</tr>
<tr>
<td></td>
<td>33</td>
</tr>
<tr>
<td>2. Made heaps of tens, but without conservation of the whole</td>
<td></td>
</tr>
<tr>
<td></td>
<td>29</td>
</tr>
<tr>
<td>3. Counted by ten, but not separate the whole into parts</td>
<td></td>
</tr>
<tr>
<td></td>
<td>38</td>
</tr>
<tr>
<td>4. Think about ones and tens at the same time</td>
<td></td>
</tr>
<tr>
<td></td>
<td>39</td>
</tr>
</tbody>
</table>
Level 4

Children in this category made separate heaps of 10 first and then counted the heaps to determine the total quantity of chips. These children could think about ones and tens at the same time. This indicated that they had constructed a system of tens on the system of ones.

The results indicated (see Table 4) that first graders were still working on the system of ones. When the chips were physically separated into groups of 10, most of the first graders suggested that it was better to mix them up before counting. The construction of a system of tens (Level 4) appears for the first time in the second grade.

Ross's Study

In Ross’s studies (1986, 1989) of 60 students in second grade through fifth grade, students were individually administered the digit-correspondence tasks adapted from M. Kamii’s studies (1980, 1982, cited in C. Kamii & DeClark, 1985). Based on data from the study and findings from related research, Ross (1986, 1989) proposed a five-stage model of how children interpret two-digit numerals.

Stage 1

In this stage, children interpret a two-digit numeral as the whole amount it represents. They assign no meaning to the individual digits of the numeral as to face or place values.

Stage 2

Children at this stage recognize the positional property of two-digit numerals; for example, the digit on the right is in the “ones place,” and the
digit on the left is in the “tens place.” However, their knowledge of the individual digits does not include the quantities indicated by each digit.

**Stage 3**

Children interpret each digit by its face value. The tens digit is seen to represent one quantity of objects, and the ones digit represents another. They are unable to see that the number represented by the tens digit is a multiple of 10.

**Stage 4**

In this transitional stage, the tens digit is interpreted as representing sets of 10 objects; however, the knowledge is not fully developed, and the performance is unreliable.

**Stage 5**

At the highest level, children understand that the individual digits in a two-digit numeral stand for two different partitions—namely, tens and ones—and that two-digit numerals can be partitioned in nonstandard ways.

Among these 60 children, no second grader reached the understanding stage (Stage 5). The percentages, representing performance of students in grades 3, 4, and 5, who reached Stage 5, were 13, 47, and 47, respectively. Ross (1990) concluded that since the value of a multi-digit numeral is the sum of face- and place-values for each digit, an understanding of numerical part-whole relations is the prerequisite to reaching the understanding stage (Stage 5). It takes a period of several years for children to gradually develop the part-whole understanding; typically, it is not achieved until age 8 or 10.
Based on these studies, a developmental sequence in children’s place-value understanding has emerged. To achieve the understanding of place-value understanding, children needed time to construct the sequence of abstractions individually—from the formation of unitary conceptual structure to the construction of multi-unit (canonical base-10 and noncanonical base-10) conceptual structure, and then to the reflection of the multi-unit conceptual structure in written arithmetic. Although teaching on place-value numeration beginning at age 6 is common, this practice seems to have very little influence. Children’s fragile understanding of the place-value system through ages 6 to 9 or 10 may reflect the constraints of children’s cognitive development. All the conclusions of these studies are apparently supported by the research of Piaget and his collaborators: the universal processes in children’s cognitive development and the ongoing constructive intellectual development.

Piaget’s Theory

Influenced by his early training and work as a biologist, Piaget believed that an individual’s behavior or ways of thinking enable him/her to adapt to the environment in more satisfactory ways (Thomas, 1992). The techniques of adaptation are called schemes by Piaget. Schemes have two forms: sensorimotor and cognitive (Brainerd, 1978). On the intellectual level, schemes are cognitive structures and are repeatedly applied to organize environmental stimuli perceived by the organism into groups in accordance with common characteristics (Wadsworth, 1979). A child’s actions on objects and interaction with people cause that scheme to change
quantitatively and qualitatively. Assimilation is the process of taking in events by matching the perceived characteristics of those events to the child’s available schemes. When lacking an adequate match, the child either creates a new scheme or modifies an existing scheme to allow the assimilation of the events that otherwise would not be comprehensible (Brainerd, 1978). This is the process of accommodation. Adaptation is a balance between assimilation and accommodation, which Piaget termed as equilibrium (Piaget, 1963). When equilibrium is not achievable, the child is motivated to further assimilation or accommodation. In this way, a child’s cognitive growth and development proceed; his/her original schemes gradually change into more sophisticated adultlike schemes (Wadsworth, 1979); his/her schemes proliferate with age but are tightly integrated and interdependent as a coordinated whole (Brainerd, 1978).

Four Stages of Cognitive Development

Although Piaget recognized the continuity of cognitive development, he was also able to identify distinct developmental stages (Piaget & Inhelder, 1969). Each stage groups similar qualitative changes into many schemes that occur during the same period of development (Tanner & Inhelder, 1956, cited in Berk, 1991). A child’s development in one stage partly explains development in the following stage.

The Sensorimotor Stage

The span of the sensorimotor begins at birth and ends, roughly, at 18 months of age. At this time, schemes are primarily sensorimotor. Without
symbolic function and language, children at this level construct all their
cognitive structures by means of a sensory-motor coordination of actions.

The Preoperational Stage

The period from 2 to 7 years of age is one of development
accompanied by the appearance of cognitive schemes and characterized by
the development of language and rapid conceptual development. However,
the thoughts of children at this level are egocentric. They explain the world
in terms of how it appears to them.

The Concrete Operational Stage

The ages 7 to 11 represent a developmental stage when a child relies
on concrete objects to assist his/her logical thinking. Children at this level
begin to conserve. They understand that as long as nothing is added or
removed, things still may have the same length, weight, amount, and
volume, even though the form of the things has been changed. They also
develop their concepts of number, relationships, and processes.

The Formal Operational Stage

Twelve years and up is a time when children can think in terms of
abstraction. Children at this level succeed in freeing themselves from the
concrete and they transform reality by means of internalized actions. The
processes of decentering make it possible for him/her to reason correctly,
form hypotheses, and draw conclusions based on truths that are only
possible. Piaget believed that the four developmental stages occur in a
fixed order; however, the age range associated with each stage varies from
person to person (Berk, 1991).
Language and Cognitive Development

According to Piaget, the majority of children at around 2 years of age begin to use spoken words to represent objects; by the age of 4 or 5, they demonstrate a good grasp of the spoken language, including an increasing vocabulary and the use of grammatical rules (Wadsworth, 1979). The acquisition of spoken language speeds up and widens children's cognitive development because thinking is able to occur by the internalization of action through representation rather than the concreteness of sensorimotor thought. Nevertheless, studies of deaf mutes show that these children's logical development proceeds in the same sequences as typical children, but at a slower rate (Piaget & Inhelder, 1969). For Piaget, language development can facilitate children's cognitive development, but is not necessary for cognitive development; the construction of sensorimotor schemes is well developed before language development and thus seems to be a prerequisite to language development (Wadsworth, 1979).

Three Types of Knowledge

By acting on their environment, children construct their own knowledge. Piaget and Inhelder (1969) describe three main types of knowledge: physical knowledge, logico-mathematical knowledge, and social knowledge. Children discover physical knowledge through actions that are associated with the physical properties of objects (Wadsworth, 1979); children invent logico-mathematical knowledge by coordinating the relationships they created earlier between/among objects (C. Kamii, 1982); because of its arbitrary nature and its relation to people, children learn
social and conventional knowledge, such as law, language, holidays, by interacting with other people (C. Kamii, 1982). According to C. Kamii (1986), both written and spoken number words belong to social knowledge; however, the numerical concepts behind them are logico-mathematical knowledge.

**Vygotsky’s Theory**

Like Piaget, Vygotsky’s theory of development admits the internal maturation in development (Vygotsky, 1986), and he views that children construct the content of their mind by engaging themselves in activities (Vygotsky, 1978). By placing children and adolescents in problem-solving situations (Vygotsky-blocks problem) and by analyzing reactions based on their solutions, Vygotsky (1986) found three progressive stages of concept formation.

**Three Stages of Concept Formation**

**Putting Things in an Unorganized Heap**

The child in the earliest stage groups disparate objects into an unorganized heap, and only by chance is this activity a true reflection of his/her perception. Vague and unstable syncretic images play the role of concepts in this first stage of the child’s development.

**Putting Things Together in Complexes**

At the arrival of puberty, the child unites his/her mind and objects, not only by subjective impressions, but also by the concrete and factual bonds (complexes) existing among the objects. A bond of this sort is discovered through direct experience. Therefore, at this time the child
moves away from egocentrism and starts thinking objectively. The complexes developing at this stage “have a functional equivalence with real concepts” (Vygotsky, 1986, p. 112).

**Thinking in Real Concepts**

This new concept formation first appears before complex thinking has run the full course of its development. Complex thinking, the second stage, unifies scattered impressions. In real concept thinking, abstraction (separation) is as important as generalization (unification). Therefore, synthesis and analysis are closely intertwined in the final stage of intellectual development. By early adolescent, the child begins to view things in terms of synthesized and analyzed concepts.

The results of Vygotsky’s (1986) study showed that the development of conceptual thought is along two main lines. The first line of development is the formation of complexes in which the child groups diverse objects on the basis of maximum similarity. The second line is the formation of abstraction, based on single-characteristic selections. However, “a functional use of the word, or any other sign, as means of focusing one’s attention, selecting distinctive features and analyzing and synthesizing them, that plays a central role in concept formation” (Vygotsky, 1986, p. 106).

**The Social and Cultural Roots of Knowledge**

While admitting the internal maturation in cognitive development, Vygotsky (1978) believes a child’s cognitive development, no matter what kind of knowledge is involved, has its cultural and social roots. First, he
stresses that a child’s cognitive functioning occurs twice and on two levels. The first level exists on the social plane in which the child interacts with people; the second level is within the child after he/she internalizes the results of social interaction (Vygotsky, 1978). In other words, cognitive function is able to be carried out in collaboration with people as well as instigated by the individual. To be effective, the social communication factor in cognitive development should take place in the child’s zone of proximal development, which is “the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers” (Vygotsky, 1978, p. 86).

Secondly, Vygotsky (1978) believes that because of some cultural artifacts, such as language, number, and writing, created and used by human society for group thinking, children are able to make a jump from the sensory to the perceptual (Luria, 1976). For example, in the early stages of cognitive development, children deal with quantities in a spontaneous and perceptual way. Instead of counting objects, they perceive small quantities immediately. By using mediators, such as numerical language, this primitive form of operating quantity becomes more sophisticated and efficient as children develop (Kozuline, 1990). Using socially meaningful mediators, individuals interact with their representations of the world rather than interacting with the world directly. Numeration systems, linguistic signs, discourses, and other human behavior are some of these socially
constructed representations (Saxe & Posner, 1983). For Vygotsky (1978), child development involves appropriation of the cultural artifacts of the surrounding cultural setting. One of the cultural artifacts of widespread importance in thinking is language, because the capability of representing abstract objects and events by language is essential to thought (Sinclair, 1976). As a cognitive tool, specific language systems may result in specific intellectual approaches. In other words, different language forms may support different conceptual structures (Fuson, 1990a; Fuson & Kwon, 1991).

Children’s Place-Value Development and Oral Numerical Language

For acquiring the capability of counting, children first notice sound patterns in spoken number words that they have heard around them (Ashlock, 1978). Based on these sound patterns, they internally create some rules. Then children apply these rules to generate their number word sequences (Sinclair, 1976). However, oral numerical languages vary in the extent to which they obscure or emphasize features of the place-value numeration system (Miller & Stigler, 1987); thus, the degree to which the place-value structures are reflected in number-name formation has an impact on the development of counting. In other words, counting develops at rates in one culture that are different from the rates found in another culture (Miller & Stigler, 1987; Resnick, 1989). Counting is essential in the early development of number concept. Through its use, children can assimilate and develop their understanding of quantity (Van de Walle, 1990) and the place-value numeration system (Hong, 1989). The characteristics of
a culture’s particular numbering system may be a factor for explaining the
differences in mathematical achievement among children of different
cultures (Miura, 1987; Miura et al., 1988; Miura & Okamoto, 1989).

Recently, some cross-cultural research was conducted in order to
answer whether different languages produce different cognitive structures
and developmental rates in mathematics.

**Miura et al.’s Studies**

Based on the belief that numbers are tied to language, Miura and
colleagues (Miura, 1987; Miura et al., 1988; Miura & Okamoto, 1989) tried
to examine the possibility that different cognitive organizations of number
resulted from differences in national language characteristics.

When interviewed individually, Asian first graders and kindergartners
and American first graders were asked to construct two-digit numerals by
using base-10 blocks in two different ways. In Miura and Okamoto’s
(1989) study, the digit-correspondence tasks, similar to M. Kamii’s (1982),
were also administered to assess children’s place-value understanding. The
results (see Table 5) showed that Asian language speakers are more likely
than English speakers to use the canonical and noncanonical base-10
representations for constructing numbers concretely. On the other hand,
English speakers seemed to prefer using a collection of ones to represent
numbers. Compared with their English-speaking peers, Asian children also
showed a greater capability in being able to construct two different
representations for each two-digit numeral. For both U. S. and Asian
children, an understanding of place value correlated positively with the use
of canonical base-10 representations and correlated negatively with the use of one-to-one collections.

These findings suggest that Asian children, even in kindergarten, see two-digit numerals as a composite of tens and ones. However, American children comprehend two-digit numerals as representing whole quantities.

### Table 5

**Percentages of Correct Constructions of Two-Digit Numerals for Each Country in Miura et al.'s (1988) Study**

<table>
<thead>
<tr>
<th>Category</th>
<th>American-1</th>
<th>Chinese-1</th>
<th>Japanese-1</th>
<th>Korean-1</th>
<th>Korean-k</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Trial 1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>One-to-one</td>
<td>91</td>
<td>10</td>
<td>18</td>
<td>6</td>
<td>59</td>
</tr>
<tr>
<td>Canonical</td>
<td>8</td>
<td>81</td>
<td>72</td>
<td>83</td>
<td>34</td>
</tr>
<tr>
<td>Noncanonical</td>
<td>1</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>7</td>
</tr>
<tr>
<td><strong>Trial 2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>One-to-one</td>
<td>10</td>
<td>43</td>
<td>59</td>
<td>56</td>
<td>33</td>
</tr>
<tr>
<td>Canonical</td>
<td>71</td>
<td>16</td>
<td>12</td>
<td>9</td>
<td>48</td>
</tr>
<tr>
<td>Noncanonical</td>
<td>19</td>
<td>41</td>
<td>29</td>
<td>35</td>
<td>19</td>
</tr>
</tbody>
</table>

### Miller and Stigler's Study

In Miller and Stigler's (1987) study, the developmental courses of counting in two different languages (Chinese and English) were compared. Forty-eight preschoolers (16 for each age group: 3, 4, and 5) from the United States and 48 (same age groups) from Taiwan participated in two counting tasks: abstract counting and object counting that was accomplished during a 15-minute individual session. Based on the data, an
analysis of specific error types that children made in counting was conducted. The findings seem to indicate the effects of linguistic structure on children's acquisition of a number system.

When asked to count as high as they could in the absence of objects, at all age levels the Chinese subjects could count higher than the Americans. Chinese 3-, 4-, and 5-year-olds could generally count to 47, 50, 100, respectively. However, American 3-, 4-, and 5-year-olds could generally count to only 22, 43, and 73, respectively. American children made many more counting errors than their Chinese counterparts did. The most common error among the Americans was the skipping of numbers. American children made this error much more often than did the Chinese children. The data reflect the fact that young American children essentially count by rote until they reach numbers in the 20s because of the irregularity of teens in English. Nonstandard numbers, such as number "forty-twelve," were produced by American children at all age levels, but none were made by Chinese children. This suggests that the structure of English number-naming limits induction of the underlying rules for number words generation.

An error, common and found in both countries, was decade error. The difficulty of making the transition between decades indicated that coordinating the incrementing of two series--decades and units--was a problem for children, independent of the language in which they were counting. In fact, the only error that Chinese children were more likely to produce than Americans was incorrectly counting by tens. This, perhaps,
resulted from the way Chinese produce decade notation by prefixing the term Ten with a unit number, followed by the word for the number of ones. It was expected that there would be an increase in the likelihood that children would confuse incrementing decade and unit values in counting. For example, when counting in Chinese, after counting from “ten-nine” (19) to “two-ten” (20), the next number is “two-ten-one” (21). But about 12 percent of the Chinese children tended to count by tens, such as two-ten (20), three-ten (30), and four-ten (40).

The data from object counting revealed a picture of gradually expanding competence. As children mature, they can count an increasingly larger set to determine the numerosity. Although Chinese children show the same developmental pattern, they are more advanced along this course when compared with their American counterparts. When objects were arranged randomly, Chinese 3-, 4-, and 5-year-olds could count 42, 64, 69 objects, respectively; American children, ages 3 to 5, could count 40, 44, 57 objects, respectively. One of the findings of this study is that the oral number names in English and Chinese differ in the support they give children in comprehending the base-10 structure underlying it.

**Fuson and Kwon’s Studies**

Thirty-six Korean first graders, in Fuson and Kwon’s (1992a) study, were given addition problems with sums of 10, single-digit addition problems with sums between 10 and 18, and single-digit subtraction problems with minuends between 10 and 18 to solve before they had received instructions on problems that involved numbers larger than 10.
These children rapidly and correctly solved 95%, 85%, and 75% of all three kinds of problems, respectively. Almost two thirds of the children solved the problems above 10 by using addition and subtraction recomposition methods structured around 10 or known facts. The recomposition up-to-ten method for addition involves breaking up one or more addends in order to form a ten related to the final sum. In subtraction, recomposition methods include down-over-ten and subtract-from-ten. On the other hand, a known fact required that children give an answer immediately and make the claim that they knew that answer already. With regard to the question “How did Korean young children solve these kinds of problems, using procedures they never learned before, by efficiently using recomposition methods around 10?”, this accomplishment was probably due to the regular name-ten Korean number words for numbers between 10 and 20.

Based on the data, three developmental sequences followed by Korean children were suggested: counting all, counting on, and recomposition around 10. According to Fuson and Kwon (1992a), when compared with Korean children, most American children lack linguistic support for the ten-structured method to solve addition and subtraction above 10, because, in part, English number words, between 10 to 20, do not name the ten and ones explicitly. When using the recomposition ten-structured methods, there is an extra step for English-speaking children: change each number word between 10 to 20 into one ten and some ones.
In another study, Fuson and Kwon (1992b) examined 72 Korean second and third graders' understanding of two- and three-digit addition and subtraction, particularly their ability to explain the trading procedures in the addition and subtraction problems. These Korean children were first asked to solve two- and three-digit addition and subtraction problems; then they discussed the correctness or wrongness of previously solved problems presented in the individual interview. At the time of interviewing, the second-grade Korean children had not yet learned how to solve three-digit problems as a part of their schooling.

The second graders correctly solved 94% of addition problems with regrouping; the percentage for the third graders was 98%. For subtraction problems with regrouping, the second graders accurately solved 94% of the two-digit problems, and 78% of the three-digit problems; the percentages for the third graders were 100% and 93%, respectively. All the second and third graders identified the second position as the "tens" place. All the third graders and 86% of the second graders identified the third position as "hundreds." Every child correctly identified the trade between the ones and tens columns as a traded ten. Ninety-two percent of the third graders identified the traded "1" written in the hundred column on addition problems as one hundred, and the percentage for the second graders who correctly made identification was 47.

When compared with those findings, noted in the literature on the performance of American children, the Korean children showed exceptional competence in two- and three-digit addition and subtraction, and they
solved these problems on the basis of their quantitative understanding of multi-digit numbers. Whereas, for American second and third graders, who correctly carried out two- and three-digit addition and subtraction, only 88% of them knew the equal exchange between the ones and tens columns, and only a discouraging 24% of them understood the equal exchange between the tens and hundreds columns (Cauley, 1988). Most of the U. S. second graders who built only concatenated single-digit conceptual structures saw multi-digit numerals as though they are single-digit numbers placed beside each other (Fuson, 1990a). These differences, in part, result from the diverse influences of English and Korean languages that tend to support Korean children’s recognition of the values of the ones, tens, and hundreds places in multi-digit numerals.

Children’s Cognitive Performance With Adult Guidance

According to Vygotsky (1956, cited in Rogoff & Wertsch, 1984), differences occur between a child’s cognitive performance when unassisted and when performance was undergirded with the help of leading questions, examples, and demonstrations which take place in the child’s zone of proximal development. Some studies were conducted to examine this differences.

Wood, Bruner, and Ross’s Study

In Wood, Bruner, and Ross’s (1976) study, 30 children, 10 each from ages 3, 4, and 5, were tutored by an adult in the task of constructing a pyramid from complex, interlocking constituent blocks. The tutoring involved verbal instructions and demonstrations that enabled a child to
solve a problem, carry out a task, or achieve a goal that probably was beyond his unassisted efforts.

Unsurprisingly, the results of the study (see Table 6) showed that 5-year-olds did better in the task; the 3-year-olds needed more help; and the 5-year-olds were apparently ready to accept tutoring. The rate of success for receiving instructions increased steadily between the ages of 3 and 5. Three-year-old children successfully carried out 18% of the adult verbal instructions; the percentage for 4-year-olds was 40%; and the rate for 5-year-olds, 57%. The children's successful performances after the adult's demonstrations were 40%, 63%, and 80% for the 3-, 4-, and 5-year-olds, respectively.

Table 6
Percentages of Successes at Each Age Group With Adults "Showing" and "Telling" Assistance in Wood, Bruner, and Ross's (1976) Study

<table>
<thead>
<tr>
<th>Age</th>
<th>Showing succeeds</th>
<th>Telling succeeds</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>40</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>63</td>
<td>40</td>
</tr>
<tr>
<td>5</td>
<td>80</td>
<td>57</td>
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</tbody>
</table>

Jones et al.'s Study

In the study of Jones et al. (1992), 41 first graders from two classrooms at a university laboratory school were under 12 preservice mathematics teachers' tutoring for 8 weeks during each of 2 semesters. Children usually worked in pairs with one of the tutors. Occasionally,
whole- and small-group instruction lessons were taught regarding multi-
digit numbers and place value. One-to-one assessment data indicated that
on common assessment items, the average correct response rate increased
from 65 % to 78 % by mid-year and that students adopted more flexible
approaches to solving numerical problems.

The Pilot Study

In 1993 a pilot study compared the acquisition of place-value
numeration concepts between American and Chinese children from first
grade through fourth grade and examined the influences of adult assistance
in children's performance of place-value tasks. The 20 American subjects, a
group of 8 girls and 12 boys, were enrolled in a 5-week summer camp in a
private school in Texas, U. S. A. The second group was 20 Chinese
subjects--6 girls and 14 boys--who enrolled in an 8-week summer
enrichment program in a private educational institute in Yung Ho, Taiwan,
R. O. C. The majority of both American and Chinese subjects were from
families having either a middle or upper-middle socioeconomic status
(SES). The subjects constituted a convenience sample. The digit-
correspondence tasks, adapted from Silvern and C. Kamii's (1988, cited in
C. Kamii & DeClark, 1985) study, were administered during the interview
sessions.

The results (see Table 7) showed that Chinese school children were
more likely to know the meaning of each individual digit in two-digit
numerals than were their American peers. There were significant
differences among grade levels of American school children. However, for
Chinese subjects, there were no significant differences among grade levels. The percentage of Chinese first graders correctly performing the tasks was 100. It was judged that Chinese children gain an understanding of the place-value system earlier than American children. When a child performed a task incorrectly and when some leading questions were given by the interviewer, the percentages of successful performance of both American and Chinese children increased. First, results of the study indicated that Chinese spoken number names, which clearly show their base-10 and place-value structures, support children in fostering a view of multi-digit numbers as composites of different multi-units. Second, results indicated that adult assistance would

<table>
<thead>
<tr>
<th>Country and grade</th>
<th>A-1</th>
<th>C-1</th>
<th>A-2</th>
<th>C-2</th>
<th>A-3</th>
<th>C-3</th>
<th>A-4</th>
<th>C-4</th>
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<tr>
<td>Without leading questions</td>
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<tr>
<td>Knew that “6” meant 6 “1” meant 10</td>
<td>20</td>
<td>100</td>
<td>40</td>
<td>80</td>
<td>43</td>
<td>100</td>
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<td>With leading questions</td>
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<td>60</td>
<td>100</td>
<td>80</td>
<td>100</td>
<td>86</td>
<td>100</td>
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</table>

n = 5 for each age group of each country.
facilitate both American and Chinese children's performances in place-value tasks.

Summary

Research on American and Asian children's place-value understanding indicated developmental sequences. With age, children's conceptual structure transforms from the unitary to the canonical base-10 and, finally, the noncanonical base-10. The noncanonical base-10 conceptual structure is more flexible and necessary for understanding the regrouping and renaming algorithms.

Asian number words have named-value underpinnings and have a structure that clearly emphasizes grouping by tens. From their counting experience, Asian children are more aware of the ones, tens, and hundreds in multi-digit numerals and see them as composites of these units. Consequently, Asian children's multi-unit conceptual structure—namely, the canonical base-10 and noncanonical base-10—first appears at an early age, around age 5. English number words between 10 to 100, on the other hand, do not underscore the place value structure, and some numbers obscure it altogether, such as in the case of the teens. With a lack of linguistic support, English-speaking children tend to form the unitary conceptual structure for representing multi-digit numbers. Roughly half of American fourth graders do not use the multi-unit conceptual structure. Compared with Asian children, American children need a prolonged period for constructing their multi-unit conceptual structure.
Language strongly influences children's understanding of the place-value numeration system. Social communication between children and adults or more competent peers also offers assistance that helps children gain numerical understanding or problem-solving skills.

In sum, there is a universal process and sequence for children’s development of place value, but the developmental rate varies by cultures, in part due to the characteristics both of the numerical language that a cultural community uses and the assistance from adults or helpful peers.

The many studies reviewed in this chapter used tasks to compare children’s numerical understanding between different countries, different age levels, and with or without adult assistance. However, no study has been conducted to trace a wide range of Chinese children’s place-value understanding before and after adult assistance, as indicated by performances in a variety of place-value tasks, and also to compare the children’s performances with that of English-speaking peers, whose performances have been described in the literature.
CHAPTER 3
METHODOLOGY

The studies reviewed in Chapter 2 examined the relationships between children's conceptual development in place-value principles and associated numerical language and the influences of adult assistance. These studies not only inspired the present study, but also became foundational in this investigation.

However, some points remained unclear. First, although some cross-cultural studies found that Asian children reached place-value understanding much earlier than did English-speaking children, no study traced the age levels at which Asian youngsters manifest their initial place-value concepts or the developmental sequences that Asian children go through.

Secondly, whereas some studies showed that adult assistance increased children's performances in place-value tasks, no study answered the questions indicating at what age levels Asian youngsters benefited the most from adult assistance.

Thirdly, the place-value tasks used in the previous studies regarding Asian children's place-value understanding were confined to only one aspect of children's place-value concepts. No study examined different aspects of Asian children's place-value concepts by administering a variety of place-value tasks during interviews.
In addressing the above three components, the purpose of the present study was to elucidate place-value understandings before and after an adult’s assistance among Chinese children in the age ranges of 3 to 9 as indicated by performances in a variety of place-value tasks. To examine the possible influence that Chinese spoken number words have on Chinese children’s formation of place-value concepts, Chinese children’s performances on the different place-value tasks have been compared with that of American and Genevan peers whose performances have been described in the literature.

The Current Study

This study, utilizing structured interview methodology, has been conducted to (a) describe the development of place-value numeration concepts in Chinese children ages 3 through 9; (b) compare the development of place-value understanding of Chinese children with that of American and Genevan children whose performances have been described in the literature; (c) examine the influence of adult assistance, such as verbal prompts, questions, and demonstrations during Chinese children’s performances on place-value tasks; and (d) formulate alternatives that will assist young children in their construction of place-value concepts.

Definition of interview

An interview “is a face-to-face interpersonal role situation in which one person, the interviewer, asks a person being interviewed, the respondent, questions designed to obtain answers pertinent to the research problem” (Kerlinger, 1986, p. 441). To find out about the thinking that
underlies children’s responses to questions, Piaget gradually developed a clinical interview method (Labinowicz, 1985). The clinical interview was flexible, open-ended, and adaptable to individual situations, and was often used when other methods were impossible or inadequate (Kerlinger, 1986). However, Piaget’s clinical interview method has been subjected to criticism because it is not applied in the same way to all children, as a test is (Berk, 1991). During a clinical interview, the questions, their sequences, and their wordings are in the hands of the interviewer. Therefore, “variations in responses may be due to the manner of interviewing rather than the real differences in the way subjects think about a certain topic” (Berk, 1991, p. 45). Nevertheless, a standardized, structured interview in which an identical set of questions is asked each child can eliminate the weakness of nonstandardized interviews (Berk, 1991).

According to Bruner (1973), verbal reports given by interviewees could be less than accurate and not sufficient to make generalizations about children’s concept attainment. Regularities in children’s decision making that are reflected in children’s behaviors when they grapple with a problem “might provide the basis for making inferences about the processes involved in learning or attaining a concept” (p. 135). Therefore, some manipulative problem-solving tasks regarding place-value concepts should be included in the interview.

**Five Components of an Individual Interview**

According to Labinowicz (1985), the interview method has five components: initiating an interview, questioning, waiting and listening,
responding with acceptance and encouragement, and recording and analyzing the interviews.

Initiating an interview. During this very important stage, rapport between the interviewer and the child needs to be established by the following procedures: asking nonthreatening personal questions, such as the child’s age, name, and favorite subjects; introducing materials that are to be employed and providing opportunity for the child’s free exploration; limiting the viewing and listening audience; and orienting the child to the interview situation. “Good interviews are those in which the subjects are at ease and talk freely about their points of view” (Bogdan & Biklen, 1992, p. 97).

Questioning. Because children often know more than what they actually reveal in their responses, the interviewer can encourage children to consider the task further through the use of some probing questions. The answers often provide the interviewer with a better understanding with regard to the child’s thinking in the problem context. “Particulars and details will come from probing questions that require an exploration” (Bogdan & Biklen, 1992, p. 98).

Waiting and listening. By providing sufficient pause between questions, children are allowed to have enough time to interpret questions and elaborate further on their responses (Long & Ben-Hur, 1991). “Good interviewers need to display patience” (Bogdan & Biklen, 1992, p. 101).

Responding with acceptance and encouragement. Children’s responses should be acknowledged as being heard by the interviewer who
nods his/her head and utters, “Uhuh” or “Hmm” in a friendly way. By remaining nonjudgmental and encouraging a child’s further elaboration as he/she responds, the interviewer manifests a respect for the child’s thinking and curiosity.

**Recording and analyzing interviews.** There are several ways to record an interview, such as jotting down notes, audiotaping, and videotaping. Because of its potential to give interviewers the option of complete attention to the interviewing process, videotaping provides the most complete documentation of an interview.

Based on these interviewing principles, this study has adopted the standardized structured interview in which children are to be given a variety of place-value tasks to solve. At the same time, an attempt is made to fathom the reasoning behind the elicited responses, which are noted by a trained interviewer.

**The Subjects**

The subjects consisted of 98 Chinese children. There were 14 children (7 boys, 7 girls) at ages 3 ($M = 3.7$, $r = 3.2$ to 3.9); 4 ($M = 4.5$, $r = 4.2$ to 4.8); 5 ($M = 5.6$, $r = 5.2$ to 5.8); 6 ($M = 6.5$, $r = 6.1$ to 6.9); 7 ($M = 7.4$, $r = 7.2$ to 7.8); 8 ($M = 8.5$, $r = 8.2$ to 9.0); and 9 ($M = 9.6$, $r = 9.2$ to 10.0) (see Table 8). The 3-, 4-, and 5-year-old subjects were enrolled in a private early childhood program in Taipei, Taiwan; the subjects from ages 6 to 9 were enrolled in an elementary school in Taipei, Taiwan. The two schools were selected because their students represent various socioeconomical backgrounds. The breakdown of the 98 children was as
follows: 1% were from families having a low socioeconomic status (SES); 17.3% were from families having a lower-middle SES; 54.1% were from families having a middle SES; 25.5% were from families having an upper-middle SES; and 2% were from families having a high SES. The subjects selected in this study were randomly selected from each school’s enrollment lists. Permission to interview subjects was obtained by means of letter to the subjects’ parents (Appendix A).

Table 8

Descriptions of Subjects in the Present Study

<table>
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<td>6.5</td>
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<td>6.1-6.9</td>
<td>7.2-7.8</td>
<td>8.2-9</td>
<td>9.2-10</td>
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</tbody>
</table>

n = 14 for each age group.

Tasks

In September and October 1994, 98 individual structured interviews were conducted in Taipei, Taiwan, by the researcher. The four tasks
administered in the individual interview (counting, digit-correspondence, representation of number, single- and multi-digit addition and subtraction) were adapted from the studies of Miller and Stigler (1987), Silvern and C. Kamii’s study (1988, cited in C. Kamii and Joseph, 1989), Miura (1987), and Fuson and Kwon (1992a, 1992b).

Procedures for Collection of Data

Subjects were videotaped during the individual sessions. Meanwhile, the interviewees’ responses for questions were jotted down quickly by the interviewer. The interviewer was the researcher, who also is a native Chinese speaker.

Before being interviewed, each child was led to a quiet room. On the way to the room, the interviewer asked the child his/her name; then she introduced herself and oriented the child to the interview by explaining its purpose:

I am going to be a teacher and someday teach children mathematics. In order to know more about the ways children think about numbers, I will ask you some questions about numbers. You just feel free to tell me what you think. There will be no right or wrong answers to these questions.

When the child was seated, the interviewer asked the child some personal questions, such as his/her age, grade, and favorite pastime after school. Due to the relative youth of the subjects, the interviewer did not ask the 3-, 4-, and 5-year-olds about their pastimes coming after school hours.

During the rapport time, the child was also asked about his/her informal mathematical knowledge and experiences. For example, when does he/she use numbers; when did he/she first learn numbers, and who
taught him/her; has he/she worked on the abacus before and at what age; did he/she recognize the place value of each column of beads on the abacus; did he/she ever use money to buy things; and could he/she tell the value of a collection of ten- and one-dollar coins? Due to the relative youth of the subjects, the interviewer asked the 3-, 4-, and 5-year-olds only about their experiences and knowledge having to do with money and about the person who taught them to count.

The four place-value tasks, were then administered. Tasks were given in a fixed order: counting, digit-correspondence, representation of number, single- and multi-digit addition and subtraction. If a child in the younger age groups showed any sign of being unable to attack a certain task, even after the probing questions had been given, the interview for that item was terminated, and the child was encouraged to try the next task. For example, the majority of the 3- and 4-year-olds could not correctly recognize the two-digit numerals used in Task 2 (digit-correspondence task) and Task 3 (representation of a two-digit number), namely “16” and “32”. For this reason, Task 2, 3, and 4 (addition and subtraction tasks), for which number recognition was prerequisite, were not administered to these 3- and 4-year-olds.

Counting Tasks

In order to observe how the child generated numbers, how he/she counted a collection of objects spontaneously, and how he/she counted by tens, the interviewer allowed the child to participate in the following three counting tasks: rote counting, object counting, and counting by tens.
Rote counting. First, the interviewer said, “You know what counting is, don’t you? One, two, three, and so on.” Then the interviewer asked, “By the way, how high can you count? Can you count to ten, a hundred, a thousand, ten thousand, or hundred thousand?” If the answer from the child was one of the single- or two-digit numbers, or “no idea,” or if the child showed reluctance, he/she was prompted by “1, 2, 3, . . .” or “Let’s count together (the interviewer stopped at 3) and see who counts more.” The child was permitted to count until he/she stopped. When the child stopped, two prompts were used to encourage him/her to go on. The child was asked, “What comes after (the last number counted)?” If the child did not continue his/her counting, the interviewer repeated the last three numbers counted. Rote counting was stopped if the child did not go on after these prompts. If the child’s response was hundreds, for example, the researcher said, “OK, let’s start with 87 and continue counting out loud.” If he/she successfully reached 121, the interviewer stopped the counting and stated, “Let’s stop with 121 because you did a good job.”

Once the criteria had been reached, the interview moved to object counting. If the child claimed that he/she could count to hundreds, but went to 109, the interviewer prompted twice, “What comes after 109?” “107, 108, 109, and then . . .” If the child did not continue his/her counting or was making errors, the rote counting was terminated at this point, and the child’s response was noted as “up to two-digit numbers only.” The same procedure was repeated when the researcher worked with other children who stated that they could count up to one of the other categories, namely,
four-, five-, and six-digit numbers. The counting ranges were “987-1021,” “9987-10021,” and “99987-100021,” respectively.

Object counting. With this task, the focus was on ways in which the children spontaneously counted a collection of objects. First, the interviewer randomly placed 78 identical poker chips on the table and asked, “You see these chips? Let’s count how many chips are here.” The majority of the Chinese children in the pilot study had a tendency to count objects with their eyes only; to avoid this, the interviewer reminded the subjects, “By the way, there are a lot of chips; you may move them as you count.”

Counting by tens. If the child had already counted the chips by tens during the object counting, this item was omitted. However, if the child failed to count by tens in the previous task, the same 78 chips were again randomly arranged on the table. The interviewer stated, “You know how to count the chips in your own way. Let’s try to count them by tens.” If the child counted out 10 chips and left them in a group, then counted out another 10 chips, making them a separate group, and said “twenty” as he combined the second group to the first heap, as did the subjects at Level 3 in C. Kamii’s (1986) study, the interviewer asked the child to come back and count the groups again to make sure of the total quantity of chips. By asking the child to recount the chips, the interviewer was able to see if the child could think about ones and tens at the same time, as in Level 4 in C. Kamii’s (1986) study. This item of the counting tasks was not administered to the 3- and 4-year-olds because of its difficulty.
Digit-Correspondence Task

The digit-correspondence task focused on the meanings children attributed to each digit of a two-digit numeral. Sixteen identical chips were placed on the table. The interviewer stated, "There are some chips in front of you. Would you count them to make sure how many chips are here?"

The child then was shown the number 16 written on a card. The interviewer asked, "What's the number?" If the child was unable to recognize the numeral 16 correctly, Task 2 was terminated at this point. Otherwise, the interviewer reminded the child that the numeral 16 stands for these 16 chips in front of him/her; the interviewer then circled the "6" and asked, "Do you see this part?" "What does it mean?" "Can you show me by using these chips?" After the subject's response, the interviewer then circled the "1" and asked, "And this part, what does it mean?" "Can you show me by using these chips?" If the child showed only one chip, the interviewer pointed to the remaining 9 chips and prompted, "What about these?" "Is this the way it's supposed to be?" "Or, is there something strange?" If after probes were given, the child insisted that the numeral 1 in number 16 stands for one instead of ten, some leading questions regarding this task were given at the end of the primary interview in order to avoid contamination of the child's responses on Tasks 2, 3, and 4.

Representation of Two-Digit Number

This task was administered to reveal children's cognitive representation of number, such as one-to-one collection, canonical base-10, and noncanonical base-10 (Miura, 1987).
**Trial 1.** A set of base-10 blocks was introduced, and children had opportunities to explore a collection of unit blocks and 10-block bars. The equivalence of a 10-block bar and 10 unit blocks was also pointed out by the interviewer, "If you line up the separate 10 unit blocks, they will be the same as one 10-block bar." After the child found this equivalence by comparing 10 unit blocks with one 10-block bar, he/she was shown a card on which the number 32 was written. The interviewer asked, "Do you see the numeral written on the card?" "What's the number?" If the child was unable to recognize numeral 32 correctly, Task 3 was terminated at this point. Otherwise, the interviewer asked, "Will you show the number by using both these 10-block bars and the unit blocks?"

**Trial 2.** Soon after Trial 1, the child was reminded of the equivalence of the 10-block bar and unit blocks. Then, he/she was shown the first representation of the number 32 and was asked, "On the first trial you built the number 32 this way." "Can you show me the number 32 another way by using the blocks and bars?"

If the child could not represent number 32 in two different ways, some demonstrations were given and probes were made at the end of the primary interview to avoid contamination of the child's responses on Task 4.

**Addition and Subtraction Tasks**

These tasks gave the researcher an opportunity to observe how a child applied his/her conceptual understandings of the place-value system to written algorithms. First, the child was asked, "Do you know how to solve
adding and subtracting problems?" Then, the single-, two-, three-, and four-digit addition and subtraction problems (8 + 5, 12 - 5, 27 + 58, 65 - 27, 394 + 241, 535 - 253, 4258 + 5831, 4083 - 1253) were presented in horizontal form on cards. The interviewer stated, "Do you know how to solve these problems?" Depending on the child’s answer with respect to how many digits he/she could add or subtract, either the adding or the subtracting (or both) was (were) given and then solved in written form by the child. If the child show an inability or reluctance to solve even the one-digit addition problem (8 + 5), the task was terminated at this point.

To find the way in which the child solved the single-digit addition problem, the interviewer stated, "Well done. How did you get the answer? Did you get it by counting all, or counting up from one number or by separating one number in order to add one numeral to ten? Or did you not need to think about it because you already knew the answer when you saw the problem?" The same procedures were used to find the way in which the child solved the single-digit subtraction, 12 - 5, such as counting down, counting up, taking away, recomposition around 10, and known fact.

As to the child’s understanding of the equivalencies between tens and ones, hundreds and tens, thousands and hundreds, and ten-thousands and thousands, the subject was probed by the interviewer by asking some related questions. For example, in an addition problem such as 394 + 241 = 635, the interviewer first circled the "6" in numeral 635 and asked, "We know 3 plus 2 equals 5. How did you get a ‘6’ here?" If the child answered, "Nine plus 4 equals 13, but we can only write down ‘3’ in the place. The ‘1’ needs
to be carried to the next place. That's why we got '6' here," the interviewer asked, "So, how many does the '1' that you carried to the next place stand for?" If a wrong answer was given by the child, such as 1 or 10, the interviewer reminded the child to check from the rightmost place and to see what place the '1' was carried to. For an older child who was able to accomplish all the four tasks, the interview was terminated at this point, and the researcher expressed her thanks for the child's cooperation during the interview session. For a younger child who was unable to perform Task 2 or (and) Task 3, some follow-up demonstrations and questions relating to the task(s) were given at the end of the interview.

Follow-Up Questions for Task 2 (Digit-Correspondence)

For children who could recognize the numerals in 16 in the Digit-Correspondence Task but insisted that the "1" in number 16 stood for one instead of ten, the follow-up questions were administered to lead and help the child to think in the right direction. First, the interviewer placed one group of 10 poker chips and one group of 6 chips in front of the child. Then, while showing the card on which the numeral 16 was written to the child, the interviewer stated that number ten-six means a group of 10 chips (pointing to the group of 10 chips) and a group of 6 chips (pointing to the group of 6 chips) go together. The interviewer then circled the numeral 6 in number 16 and reminded the subject that the numeral 6 stands for the group of 6 chips. After the reminding, she circled the numeral 1 in number 16 and asked, "What does the numeral 1 stand for?" For older children, the interviewer stated that we also write the number for 16 chips this way: 16 =
10 + 6. The interviewer then underlined the numeral 6 in number 16 and the "6" on the other side of the arithmetic sentence, and said, "The numeral 6 stands for 6 chips." Then, the numeral 1 in number 16 was underlined, and the interviewer asked the child, "So, how many chips does the numeral 1 indicate?"

Follow-Up Demonstrations for Task 3 (Representation of Two-Digit Number)

For children who could recognize numeral 32 in Task 3 but failed to construct number 32 in two different ways, the interviewer showed and introduced base-10 blocks to the child. The equivalence of 10 units and a 10-block bar was pointed out by the interviewer. Then the interviewer demonstrated how the blocks and the bars could be used for constructing number 22 in one-to-one and base-10 constructions. After the demonstrations were given, the child was asked to construct number 32 in another way different from his/her first construction.

Data Analysis

Each child’s oral and behavioral responses on videotapes were transcribed in English and then were coded on coding sheets designed for this data (Appendix B). After the procedure of data coding, which involved assigning numerical values to non-numeric categories of a variable (Hinkle, Wiersma, & Jurs, 1988), different procedures of data analysis were used for different items of the task (a) to illuminate Chinese children’s acquisition of place-value concepts at different age levels, (b) to compare the performance differences at different age levels, and (c) to reveal the effects of adult
assistance on each age level. Also, this coding made it possible to compare the data with that found in the literature regarding performance and conceptual structure among comparable American and Genevan children.

Counting Tasks

**Rote counting.** First, the child’s counting errors were sorted into eight types: no error, mixing up numbers, skipping numbers, repeating numbers, decade errors, skipping numbers and decade errors, repeating numbers and decade errors, and skipping and repeating numbers and decade errors. A numeral from 0 to 7 was assigned to each error type, respectively. The percentages of each error type at each age level were calculated. The child’s ability for generating numbers was assigned to the following categories and coded: (1) single-digit number, (2) two-digit number, (3) three-digit number, (4) four-digit number, (5) five-digit number, and (6) over six-digit number. The percentages of children’s capabilities for number generating at each age level were calculated. The differences between age levels and the oral counting categories were analyzed by using the chi-square distribution.

**Object counting.** The child’s ability and manners of counting a collection of objects were noted and recorded. The number of objects that was counted by a child was recorded as his ability to count objects. According to the method of grouping objects, such as no idea how; by ones, twos, threes, fours, or fives; by combining different approaches but none based on ten; or by tens when he/she counted, a numeral from 0 to 7 was assigned to the child’s behavior, respectively. For example, if the child
could count by twos, his/her response for this item was coded as “2.” If the child counted objects by ones sometimes, by twos at another time, or by fives, his collective responses for this item was coded as “6.”

The percentages of children’s spontaneous counting by using different ways of grouping objects at each age level were calculated. The differences between age levels and the object counting categories were analyzed by using the chi-square distribution.

**Counting by tens.** Adapted from C. Kamii’s categories of counting by tens (C. Kamii, 1986), each child’s response was categorized and coded as follows: (0) no idea how; (1) making groups of 10 and leaving a group of 8 and counting each group as “1”; (2) making groups of 10 and leaving a group of 8 and counting each group as “10”; (3) making groups of 10 and leaving a group of 8 and counting each group by adding “10,” including the last group of 8 objects; (4) making groups of 10 and leaving a group of 8 and counting each group of 10 by adding “10,” and counting each object of the last group of 8 by adding ones; (5) making groups of 10 and leaving a group of 8 and counting each group of 10 by adding “10,” and counting the last group of 8 objects by adding “8.” The percentages of children in each of the categories at each age level were calculated. The differences between age levels and the counting by tens categories were analyzed by using the chi-square distribution.

**Digit-correspondence task**

Regarding children’s recognition and interpretation of a two-digit number, each child’s response, before being given some leading questions,
was assigned to one of the following six categories: (0) no recognition of either numeral in number 16; (1) recognized only numeral 1 in number 16; (2) recognized only numeral 6 in number 16; (3) recognized both numerals in number 16 but saw them in reverse order; (4) recognized both numerals in number 16 in the correct order but interpreted them by their face values only; (5) recognized both numerals in number 16 in the correct order, saw them as a two-digit number, but interpreted them only by the face values; and (6) recognized both numerals in number 16 in the correct order, saw them as a two-digit number, and interpreted the digits by both their face and place values. In correspondence with the categories assigned to the child, his/her response was coded as “0,” “1,” “2,” “3,” “4,” “5,” or “6.” The percentages of children in each of the categories at each age level were calculated. The differences between age levels and the digit-correspondence categories were analyzed by using the chi-square distribution.

After a child received some follow-up questions, his/her responses were coded and analyzed again in the same way. The increment of correct responses between the performances in which the leading questions were not given and those in which the leading questions were given, was examined to find at which age level children benefited the most from adult assistance on the task.

Representation of Two-Digit Number

Trial 1. According to the type of response the child made, a numeral was assigned. The one-to-one representation was coded as “1,” the
canonical base-10 representation as "2," and the noncanonical base-10 representation as "3." The numeral 0 was assigned to the children who would not demonstrate or who had failed on Trial 1. Young children who were not tested on this task were coded as "8." The percentages of children in each of the categories at each age level were calculated. The differences between age levels and the number representation categories were analyzed by using the chi-square distribution.

**Trial 2.** The same data coding and analysis procedures in Trial 1 were used in Trial 2.

After the follow-up demonstrations were given by the interviewer, the child’s responses on two trials were coded and analyzed in the same ways as in Trial 1 and Trial 2. The increment of correct responses between the performances in which the demonstrations were not given and in which the demonstrations were given was examined to find at which age level children benefited the most from an adult assistance on the task.

**Addition and Subtraction tasks**

One of the numerals 1, 2, 3, or 4 was assigned to a child, corresponding to his highest capability to correctly solve the single-, two-, three-, and four-digit addition problems. For example, if a child could solve all the single-, two-, three-, and four-digit additions correctly, his/her ability to solve addition was coded as "4." For children who had no idea how to do addition, the numeral 0 was assigned to them. For young children who were not tested on the task, a code "8" was assigned to them. The percentages of children’s abilities to solve addition exercises composed of different digits
for each age level were calculated. The differences between age levels and
the abilities to solve multi-digit adding problems were analyzed by using
the chi-square distribution. The same coding and analyzing procedures
were used on children’s abilities for solving subtraction problems.

Based on Fuson and Kwon’s (1992a, 1992b) categories of solution
procedures by Korean children for single-digit addition, the child’s
solutions for single-digit addition, in which the sum was over 10, were
categorized as no idea how, procedure unclear, counting all one by one,
counting onward, recomposition around 10, and known fact. Then, one of
the following numerals—0, 1, 2, 3, 4, or 5—corresponding to the respective
categories, was assigned to the category in which the child’s solution
belonged. If a child’s strategy for solving the one-digit adding problem in
which the sum was over 10 was such that the interviewer was unable to
include it to one of these categories, his/her response was coded as 7
(Others). As to children’s solutions for the one-digit subtraction problem
whose minuend was over 10, there were seven categories, coded as follows:
(0) no idea how, (1) procedure unclear, (2) counting downward, (3)
counting up, (4) taking away (5) recomposition around 10, (6) known fact
and (7) others. For young children who were not tested on the single-digit
addition and subtraction, the code “8” was assigned. The percentages of
children in each category at each age level for each problem were
calculated. The differences between age levels and the solution categories
was analyzed by using the chi-square distribution.
Based on the child’s discussion of his/her solution for two-, three-, and four-digit addition and abstraction problems, the child’s understanding of the equivalence among digits during carrying or borrowing procedures was assigned to the following categories and coded: (0) no idea; (1) “1” means “1”; (2) “1” always means “10” (concatenated single-digit conceptual structure); (3) “1” can mean 10, 100, 1000, 10000 with interviewer’s reminder; and (4) “1” can mean 10, 100, 1000, 10000 without the interviewer’s reminder, respectively. For young children who were not tested on this item of task, the numeral 8 was assigned. The percentages of children in each category at each age level were calculated. The differences between age levels and the understanding categories were analyzed by using the chi-square distribution.

Also, children’s informal mathematical knowledge was coded to compare with children’s performances on the place-value tasks. A child’s ability to count money was coded as: (0) no idea how; (1) one-dollar coin; (2) ten-dollar coin; (3) both one- and ten-dollar coins; and (7) others. As to the children’s highest ability to recognize the place-value on an abacus, one of the numerals from 0 to 6 was assigned to represent no recognition, ones, tens, hundreds, thousands, ten thousands, and hundred thousands, correspondingly. The 3-, 4-, and 5-year-olds who were not tested on this item were given a code of “8.”

In addition to some quantitative data, some qualitative data obtained from the interviews were used to illustrate the strategies used by Chinese
children at different developmental stages and for each place-value task, as listed in Chapter 4.

Summary

A standardized interview method, with emphasis on uncovering a child's mental processes, when he/she was dealing with place-value tasks was adopted for this study. A sample of 98 children from ages 3 through 9 was randomly selected from two schools in Taipei, Taiwan. To eliminate the sex influence in the resulting data, half of the subjects were male, the other half, female. Tasks and procedures were adapted from several cognitive studies in the place-value domain (see Table 9). Data collection modes included interviewing children, observing their actions and modes of expression during the interview sessions, videotaping interviews, and transcribing children's oral and behavioral responses. The sets of data collected were analyzed both quantitatively and qualitatively in order to answer the research questions.

Table 9

Summary of Tasks Used in the Present Study

<table>
<thead>
<tr>
<th>Task</th>
<th>Age groups</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Counting</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Oral counting</td>
<td>3 - 9</td>
<td>Miller &amp; Stigler (1987)</td>
</tr>
<tr>
<td>Object counting</td>
<td>3 - 9</td>
<td>C. Kamii (1986)</td>
</tr>
<tr>
<td>Counting by ten</td>
<td>5 - 9</td>
<td>C. Kamii (1986)</td>
</tr>
<tr>
<td>3. Representation 2-digit numbers</td>
<td>5 - 9</td>
<td>Miura (1987)</td>
</tr>
<tr>
<td>4. Addition and subtraction</td>
<td>5 - 9</td>
<td>Fuson &amp; Kwon (1992a, b)</td>
</tr>
</tbody>
</table>
CHAPTER 4
ANALYSIS OF DATA AND FINDINGS

This study investigated Chinese children's development of place-value numeration concepts from ages 3 through 9, compared the development of place-value understanding of Chinese-speaking children with that of American and Genevan children whose performances had been described in the literature, and examined the influence of adult assistance during Chinese children's performances on some of the place-value tasks. This chapter is designed to present the analysis of data and a review of the findings as they are related to the purposes of the study. In addition, the sources of children's numerical understanding, such as scholastic formal numerical learning and informal numerical knowledge, are included as a background for a comparison of the objects' responses to the tasks. A summary concludes the chapter.

Standardized interviews of the designated children were completed during a 3-week period in October 1994. Ninety-eight interviews were conducted. For both the elementary school children and the 5-year-olds, each interview lasted approximately 25 minutes. In contrast, however, the interviews for the 3-and 4-year-olds, lasted only approximately 10 minutes each. The interview protocols were adapted from studies by Miller and Stigler (1987), C. Kamii (1986), Silvern and C. Kamii (1988, cited in C.
Kamii & Joseph, 1989), Miura (1987), and Fuson and Kwon (1992a, 1992b) and was described in Chapter 3.

Sources of Children’s Numerical Understanding

Scholastic Formal Numerical Teaching

In Taiwan, the academic year begins in September, and entrance into school is determined by age as of September 1, much as it is in the United States. However, compared with the United States, the educational policy is more centralized in Taiwan (Stigler et al., 1987). Except for kindergarten, the curriculum for all schools in Taiwan is specified in detail by the Ministry of Education. According to a fixed curriculum, textbooks are published by the ministry. Every school in Taiwan adopts the same set of textbooks. The weekly teaching schedule for each grade level in a school is specified by teachers at the beginning of a semester.

Taiwanese kindergartens usually include 5-, 4-, and, sometimes, 3-year-old children. Kindergartens are not regarded as a part of the elementary school. Each kindergarten may have its own curriculum and textbooks, and both are based on its educational philosophy. The weekly teaching schedule at each age level in a kindergarten is not prefixed. Teachers formulate their own teaching schedules, which are governed by the children’s learning.

For this study, the 98 formal interviews were conducted at the time children began their fall semester, about 3 weeks into the school year. Basically, the teachers’ current mathematical teaching at each age level of
the elementary school appeared to be almost the same. This was not true for the kindergarten levels, ages 3, 4, and 5.

According to discussions of the teachers' arithmetical teaching with the researcher, the numerical instruction at the time of the interviews consisted of a progression of number concepts and skills. Most of the 3-year-olds were learning oral counting from 1 to 10, object counting from 1 to 5, and numeral recognition from 1 to 5; a majority of the 4-year-olds were learning oral counting up to 20, object counting up to 10, and numeral recognition up to 10; and a majority of the 5-year-olds were learning oral counting up to 100, object counting up to 50, and numeral recognition up to 70. The first graders were learning one-digit addition in which the sum was not over 10 and one-digit subtraction in which the minuend was not over 10. The second graders were learning two-digit addition which involved regrouping, one-digit subtraction in which the minuend was over 10, and two-digit subtraction in which no regrouping was involved. The third graders were reviewing two-digit addition and/or subtraction where regrouping/renaming was involved. This review was preparatory to learning three-digit addition and subtraction, which would come later in the semester. The fourth graders were learning three-digit addition and subtraction, and both operations involved regrouping.

**Informal Numerical Knowledge**

Regarding designated children's informal numerical knowledge, some questions were asked at the beginning of the interviews. The following data are summary and analyses of the children's responses.
Do You Remember Who Taught You Numbers?

When asked this question, 42% of the children’s answers indicated parents or grandparents; 36% of the children’s answers referred to teachers in school. However, 9% of the children answered that they learned numbers by themselves, sometimes by watching video tapes or reading books. Only 6% of the children said that their older siblings taught them about numbers. Most of the children told the interviewer that they first learned about numbers at or around the ages of 3, 4, or 5.

When Do You Use Numbers?

The 6-, 7-, 8-, and 9-year-old children’s responses to this question were varied but emphasized the role of school mathematics. Sixty-three percent of the children said that they used numbers when they were in a mathematics class, were solving mathematical problems, were taking examinations, and were doing homework. Seven percent of the children thought that they used numbers when they were buying things. Another 7% of the children mentioned that they used numbers when they were counting things. Subjects also mentioned other occasions when numbers were used: teaching younger siblings about numbers, 5%; writing numerals, 5%; telling times and days, 2%; making a phone call, 2%; playing games, 2%; working with an abacus, 1%; singing songs, 1%; and drawing pictures, 1%. Around 4% of the children had no idea when they used numbers.
Do You Usually Buy Things by Yourself, or Do Your Parents Do It for You?

About 35% of the children expressed the fact that their parents buy things for them or accompany them when they buy something; 42% said that sometimes they buy what they need, but sometimes their parents do it for them; only 17% of the children (all of them are 6-, 7-, 8-, or 9-year-olds) answered that they usually buy things by themselves.

Can You Count This Money?

When the children were asked to count a collection of money (3 ten-dollar coins and 5 one-dollar coins), their responses clustered around the following four categories. An example response pattern is given for each category.

**Category 1.** Jiann-Meng, 3 years and 11 months, had no idea how to count a collection of money pieces.

*Interviewer (I):* (Three ten-dollar coins and 5 one-dollar coins were shown.) This is some money. Can you count out the total amount of this money?

*Jiann-Meng (J):* (silent)

**Category 2.** Ssu-Han, 4 years and 7 months, counted all the one- and ten-dollar coins as though each had the value of one dollar.

*Ssu-Han:* (She moved and counted the coins one at a time.) 1, 2, 3, 4, 5, 6, 7, 8 dollars.

**Category 3.** Yi-Wen, 5 years and 7 months, was unable to add the denominations of one-and ten-dollar coins although she knew the value of each coin.
Yi-Wen (Y): 10, 11, 12, 13, 14, 15.
I: Good. Let's count them again. (One ten-dollar coin was pointed to.) How much is it?
Y: 10.
I: (Another ten-dollar coin was pointed to.) How much are these two coins?
Y: 20.
I: (The last ten-dollar coin was pointed to.) How much are these three coins?
Y: 30.
I: (The 5 one-dollar coins were pointed to one by one.) How much are these five coins?
Y: 1, 2, 3, 4, 5.
I: How much is the total amount of these coins?
Y: 5 dollars.

Category 4. When asked to count a collection of money pieces, Shiou-Hao, 7 years and 8 months, counted the money by moving the pieces and adding the value of the money one at a time.

Shiou-Hao: (He moved one coin at a time.) 10, 20, 30, 31, 32, 33, 34, 35.

When asked to count a collection of money pieces, Wei-Chen, 6 years and 11 months, counted the money by looking at the money a second time and then correctly gave the amount of the money.

Wei-Chen: (He looked at the coins for a while.) 35.

A summary of the children's responses on the money counting task is shown in Table 10. Except for two 3-year-olds, who had no idea how to count the money, all the 3- and 4-year-olds tended to count the collection of one- and ten-dollar coins as a group of ones; however, some of them recognized that there were both one- and ten-dollar coins. At age 5,
children began counting the one- and ten-dollar coins separately as a first step; then they added them together. Half of the 5-year-olds successfully counted the collection of money pieces. Almost all the 6-, 7-, 8-, and 9-year-olds counted out the total amount of the money correctly. Collectively, the children's individual performances on money counting among age levels were significant, $X^2 (18, N = 98) = 110.13, p < .05$.

Table 10

Responses for Counting a Collection of One- and Ten-Dollar Coins (By Number of Children)

<table>
<thead>
<tr>
<th>Category</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. No idea how</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Counting any coin as one dollar</td>
<td>12</td>
<td>14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Counting one- and ten-dollar coins separately, but not adding together</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Counting by adding one- and ten-dollar coins</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$n = 14$ for each age level.

chi-square = 110.13; df = 18; $p < .00000$.

Have You Ever Learned How to Use an Abacus? Can You Show Me the Numbers 1, 20, 300, 4000, 50000, and 600000 on an Abacus?

The elementary school children in Taiwan learn how to use an abacus beginning in the fourth grade. When asked to pull or push beads on an abacus that stand for the numbers 1, 20, 300, 4000, 50000, and 600000, all
the fourth graders demonstrated their understanding of place value on an abacus. An example interview is given.

Interviewer (I): (An abacus was shown.) Do you know what it is?  
Yi-Jei (9 years and 11 months): An abacus.  
I: Have you ever learned how to use it?  
Y: I am learning it right now because teachers at my school teach kids how to use an abacus from fourth grade on.  
I: So, for 3 weeks you have been learning on it?  
Y: Yes.  
I: Can you push a bead on this abacus that stands for number 1?  
Y: (In the lower section, he correctly pushed up the topmost bead in the ones column.)  
I: How about 20?  
Y: (He correctly pushed up the topmost two beads in the tens column.)  
I: How about 300?  
Y: (He correctly pushed up the three topmost beads in the hundreds column.)  
I: How about 4000?  
Y: (He correctly pushed up all four beads in the thousands column.)  
I: How about 50000?  
Y: (In the upper section he correctly pulled down the only bead in the ten thousands column.)  
I: How about 600000?  
Y: (In the lower section he correctly pushed up the topmost lower bead and, in the upper section, pulled down the only bead in the hundred thousands column.)

The results of the children's responses are summarized in Table 11. Of the forty-two 6-, 7-, and 8-year-olds, 29 (69%) of them claimed that they had not studied on an abacus. Among the other 13 children, who were more or less familiar with the work on an abacus, 4 of them could not demonstrate their understanding of place values on it; 3 of them knew the
hundreds place, 4 of them knew the ten thousands place; 2 of them knew place value on an abacus up to hundred thousands. Children’s knowledge associated with place values on an abacus was significantly different among age levels, $X^2 (9, N = 56) = 51.63, p < .05$.

Table 11

Understanding of Place-Values on an Abacus (By Number of Children)

<table>
<thead>
<tr>
<th>Category</th>
<th>Age</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6</td>
</tr>
<tr>
<td>1. No idea</td>
<td>11</td>
</tr>
<tr>
<td>2. Hundreds</td>
<td>1</td>
</tr>
<tr>
<td>3. Ten thousands</td>
<td>2</td>
</tr>
<tr>
<td>4. Hundred thousands</td>
<td></td>
</tr>
</tbody>
</table>

$n = 14$ for each age level.

chi-square = 51.63; df = 9; $p < .00000$.

Development of Place-Value Numeration Concepts

The first purpose of the present study was to describe Chinese children’s development of place-value numeration concepts. The following questions were asked in order to redefine this purpose: How do Chinese children perform counting and place-value tasks at different age levels, 3 through 9? Through what developmental sequences of place-value understanding do Chinese children go?

The second purpose of the present study was to compare the development of the place-value understanding of Chinese children with that of American and Genevan children whose performances have been
described in the literature. Formulated as research questions, this purpose was redefined. Do Chinese children go through the same developmental courses of place-value understanding as do American and Genevan children? Do Chinese children have the same cognitive limitations when forming their conceptual structures of place value as those described in the literature that dealt with American and Genevan children? What is the age level at which the majority of Chinese children demonstrate their understanding of the place-value numeration system? What does the literature say about the age level at which the majority of the American and Genevan children reach this understanding?

The third purpose of the present study was to examine the influence of adult assistance during Chinese children’s performances on place-value tasks. The research question, parallel to the purpose, asked: How does adult assistance facilitate Chinese children’s performances on place-value tasks at the different age levels?

As a means of answering the research questions, videotaped interviews were transcribed in both Chinese and English, coded, and analyzed. This section is organized by the task. Following each task are representative interviews illustrating the levels of responses. Then a summary table shows the various levels of place-value understanding, ages 3 to 9. The data were analyzed by using the chi-square distribution. Then the results were compared to the developmental data of American children and-- in two cases--Genevan children. In the tasks of digit-correspondence
and number representation, the children's performances before and after adult assistance are represented.

**Oral Number Generation**

Oral counting was administered to observe how the children generated numbers orally and to ascertain their understanding regarding the rules for generating number names in the place-value numeration system. This task was adapted from Miller and Stigler's (1987) study. The original study was concerned with children 3 to 5 years of age. In the present study, an adaptation was made so that some children, who were beyond age 5, were given a counting range instead of counting from one.

**Interview Strategy**

After being questioned as to how high he/she could count, the child, who may have been reluctant to count or who claimed that he/she could count only up to one or two digits, was given an opportunity to count as high as he/she could. For the child who said that he/she could count up to hundreds, the counting range for him/her was 87 to 121. When the child counted up to 121, the interviewer stopped the counting process arbitrarily because the criteria had been reached. The counting ranges for thousands, ten thousands, and hundred thousands were 987 to 1021, 9987 to 10021, and 99987 to 100021, respectively. Therefore, for the 3-, 4-, and 5-year-olds, who generally counted lower than 121, the average of counting peaks was calculated because it represented true counting. Most 6-, 7-, 8-, and 9-year-old children, who could count to hundreds, thousands, ten thousands,
or hundred thousands, had their counting stopped arbitrarily; therefore, the average of counting limits for these age levels was not calculated.

**Performance**

Six levels for children's oral number generation were formed. A transcript of an interview with a child at each level of oral number generation demonstrates the number knowledge typical at that level.

**Single-digit.** Jer-Lun, 3 years and 10 months, was able to count orally from 1 to 6. Thus, his responses was classified as single-digit.

Interviewer (I): Let's count 1, 2, 3, together?
Jer-Lun (J): 1, 2, 3, 4 (pause).
I: Let's count it again.
J: 1, 2.
I: What comes after 2?
J: The sun.
I: Does the sun come after number 2?
J: (silent)
I: Let's count 1, 2, 3.
J: What you counted was wrong.
I: I did wrong. So, can you show me how to count?
J: Count what?
I: 1, 2, 3, 4.
J: I don't know how to count.
I: (A card on which the number 16 was written was shown.) What's the number?
J: 1.
I: Good.
J: 1, 2, 3, 4, 5.
I: What comes after 5?
J: (silent)
I: You counted 1, 2, 3, 4, 5 very well.
I: Good. What comes after 6?
J: (He played with the chips.)
Two-digit (counting 10s but less than 121). The oral counting of Ying-Wei, 3 and 10 months, who demonstrated her oral counting ability up to 29, was classified as two-digit.

Interviewer (I): Do you know how to count 1, 2, 3, 4?
Ying-Wei (Y): (She nodded her head positively.)
I: Let's count together.
Y: 1, . . . , 29 (pause).
I: What comes after 29?
Y: (silent)
I: 27, 28, 29 and then what?
Y: (silent)

Shyang-Yi, 5 years and 5 months, was able to count up to 119, but was not able to reach 121; therefore, his capability on oral counting was also classified as two-digit.

Interviewer (I): Let's count 1, 2, 3 loudly, together, and see who counts more? (The interviewer stopped oral production on "3.")
Shyang-Yi (S): (He counted successively from 1 through 119.) I don't know how to count the numbers after 119.
I: 117, 118, 119 and then what?
S: I don't know.
I: You counted very well.

Three-digit (counting 100s but less than 1021). Pey-Ying, 6 years and 5 months, counted numbers between 87 and 121 fluently. Her oral counting capability was classified as three-digit.

Interview (I): How high can you count?
Pey-Ying (P): I can count to two or three hundred.
I: Let's start with 87 and continue counting out loud.
P: (She counted fluently and consecutively from 87 through 121.)
I: Let's stop with 121. Very good, you can count up to 121.
Although Yen-Ling, 6 years and 3 months, was able to count numbers up to 1000, he could not count up to 1021. His oral counting capability was also classified as three-digit.

Interviewer (I): Do you know how high you can count?
Yen-Lin (Y): 1000.
I: Let’s start with 987 and continue counting out loud.
Y: 987, 988, 989, (pause) 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000 (pause).
I: What comes after 1000?
Y: (He shook his head negatively.)
I: 998, 999, 1000, and then ....
Y: (He shook his head negatively.)

Four-digit (counting 1000s but less than 10021). Tzong-Horng, 9 years and 11 months, was capable of counting up to 1021. His responses on oral counting, thus, was classified as four-digit.

Interview (I): Do you know how high you can count?
Tzong-Horng (T): Probably hundreds.
I: How about thousands?
T: Yes, I can, but I count it very slowly.
I: Ten thousands?
T: Probably not, because I have never done it before.
I: Is it O.K. for you to try to count up to ten thousands?
T: No. I am afraid that counting this high must be very tiring.
I: Let’s start with 987 and continue counting out loud.
T: (He counted consecutively from 987 through 1021.)
I: Let’s stop with 1021. You did a good job.

Five-digit (counting 10000s but less than 100021). Chih-Yin, 8 years and 5 months, exhibited her capability on oral counting up to 10021. Her performance was classified five-digit.

Interviewer (I): Do you know how high you can count? Tens? Hundreds? Thousands?
Chih-Yin (C): What did you mean “count”?
I: For example, you count 1, 2, 3, 4, \ldots until you can’t continue.
C: I can count.
I: How about thousands?
C: Yes, I can.
I: How about ten thousands?
C: Yes.
I: Let’s start with 9987 and continue counting out loud.
C: (She counted successively from 9987 through 10021.)
I: Let’s stop with 10021. You did a good job.

Six-digit (counting up to 100021). Jih-Yuan, 7 years and 8 months, was able to count up to 99999. His oral counting was much slower because of the series of long numbers he needed to recite.

Interviewer (I): Do you know how high you can count? Hundreds?
Jih-Yuan: (He nodded his head positively.)
I: How about thousands?
J: (He nodded his head positively.)
I: How about ten thousands?
J: (He nodded his head positively.)
I: Let’s start with 9987 and continue counting out loud.
J: 9987, 9988, 9989, 9990, 9991, 9992, 9993, 9994, 9995, 9996, 9997, 9998, 9999 (pause).
I: What comes after 9999?
J: (He counted consecutively from 10000 through 10021.)
I: Can you count up to hundred thousands?
J: Yes.
I: Let’s start with 99987.
J: 99987, 99988, 99989, 99990 (pause).
I: What comes after 99990?
J: 99991, 99992, 99993, 99994 (pause).
I: What comes after 99994?
J: 99995, 99996, 99997, 99998, 99999 (pause).
I: What comes after 99999?
J: 100000 (pause).
I: What comes after 100000?
J: (He counted successively from 100001 through 100021.)
I: Let’s stop with 100021. You did a very good job.

The results on children’s oral counting are summarized in Table 12. Two (14%) of the fourteen 3-year-olds counted to 6; the remainder (86%) of the 3-year-old group counted up to two-digit numbers, ranging from 10 to 39. The average oral counting number for 3-year-olds was 18. For 4-year-olds, all of the 14 children counted up to two-digit numbers, ranging from 12 to 100. The average oral counting number for 4-year-olds was 39. Twelve (86%) of the fourteen 5-year-olds counted up to two-digit numbers, ranging from 69 to 119. Two (14%) of the 5-year-olds were able to count up to a three-digit number—121. The average number to which they were able to count was 106.

Table 12
Performance on Oral Counting (By Number of Children)

<table>
<thead>
<tr>
<th>Category</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single-digit (1 - 10)</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Two-digit (11 - 120)</td>
<td>12</td>
<td>14</td>
<td>12</td>
<td>7</td>
<td>6</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Three-digit (121 - 1020)</td>
<td>2</td>
<td>7</td>
<td>7</td>
<td>8</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Four-digit (1021 - 10020)</td>
<td></td>
<td>4</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Five-digit (10021 - 100020)</td>
<td>1</td>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Six-digit (up to 100021)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

n = 14 for each age level.
chi-square = 131.89; df = 30; p < .0000.
About half of the 6- and 7-year-olds counted to hundreds. The majority (93%) of 8-year-olds were able to count to/over hundreds, such as thousands. The majority of 9-year-olds (93%) counted to thousands or ten thousands.

The data associated with oral counting successes showed that the older the children, the more competent they were in oral counting. The differences among age groups were significant, $X^2(30, N = 98) = 131.89, p < .05$.

Comparison

The results of Miller and Stigler’s (1987) study showed the developmental progression in counting skills among subjects from two different languages groups (Chinese and English). When asked to count as high as they could in the absence of objects, at all age levels, the Chinese subjects could count higher than the Americans. Based on averages, Chinese 3-, 4-, and 5-year-olds could count approximately up to 47, 50, 100, respectively. American 3-, 4-, and 5-year-olds could count approximately up to only 22, 43, and 73, respectively.

The Chinese subjects in the present study could count orally up to 18 at age 3; 39 at age 4; and 106 at age 5, on average. According to the results of the present study, the Chinese 3- and 4-year-olds did not perform as well in counting as the 3- and 4-year-old Chinese children featured in Miller and Stigler’s (1987) study; but they performed within the same ranges as did the American children in Miller and Stigler’s study. However, the Chinese 5-year-olds’ average oral counting number in the present study
was 106, a level almost equal to the performance of 5-year-old Chinese children in Miller and Stigler's study, but much higher than that of the American 5-year-olds in the same study.

**Oral Counting Errors**

Based on the children's performances in the oral counting tasks, an analysis of the kinds of counting errors that children made may suggest the presence of acquisition problems intrinsic to the task of learning oral counting.

**Performance**

Children's error types in oral counting could be classified according to the following six categories: no error; mixing up numbers; skipping numbers; decade errors; skipping and decade errors; and skipping, repeating, and decade errors. Some examples are as follows:

**Mixing up numbers.** Jiann-Ling, 4 years and 8 months, orally produced numbers that were not in accordance with the rules in the place-value numeration system. Thus, his error in oral counting was classified in the category of mixing up numbers.

Interviewer (I): Let's count 1, 2, 3, together.
Jiann-Ling (J): 1, 2, 3, 4, 5, 6, 7 (pause).
I: What comes after 7?
J: (silent)
I: 5, 6, 7 and then what?
J: (It was very difficult to hear his sounds as he counted.) 7, . . . .
I: Good job, but try it again. Let's count loudly this time.
J: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, (skipping 14), 15, 12, 15, 20, 16, 17, 18, 19, 20.
Skipping numbers. When Jia-Hsin, 3 years and 10 months, was counting from 1 to 20, she skipped the number between 11 and 13. Her error was classified in the category skipping numbers.

Interviewer (I): Do you know how to count 1, 2, 3?
Jia-Hsin (J): Yes.
I: Let’s try it.
J: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, (skipping 12), 13, 14, 15, 16, 17, 18, 19, 20.

Decade errors. When Chich-Fang, 6 years and 3 months, was counting, he counted by tens between 100 and 200. He verbalized 300 after 209. This sort of error was classified as decade error.

Interviewer (I): How high can you count?
Chich-Fang (C): I can count to 100.
I: Let’s start with 87 and continue counting out loud.
C: 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 110, 120, 130, 140, 150, 160, 170, 180, 190, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 300.

Skipping numbers and decade errors. When Bor-Yi, 4 years and 4 months, counted numbers, he skipped some numbers within some decades, but the order of the numbers in the decades was not mixed. He also made some decade errors. Therefore, his errors, a combination of two sorts of errors, were classified in the category skipping numbers and decade errors.

Interviewer (I): Let’s count 1, 2, 3, loudly, together.
Bor-Yi (B): 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 19, 20, 21, 22, 23, 29, 40.

Skipping numbers, repeating numbers and decade errors. When Jia-Liang, 4 years and 3 months, was counting, he skipped and repeated some
numbers. Also, he counted 30 after 39. His combined errors were classified in this category.

Interviewer (I): Let's count 1, 2, 3 loudly, together?
Jia-Liang (J): 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20.
I: What comes after 20?
I: And then comes what number?
J: 22, 23, 24, 34, 35, 36, 37, 38, 39, 30.

A summary of oral counting errors the children made during the interviews is shown in Table 13. Children in the present study produced counting errors 27% of the time. Eighty-six percent of the 3-year-olds did not make any counting errors because of their very limited capability for number generating. Due to the extended counting pattern and higher expectations, 9 (65%) of the 4-year-olds made a variety of oral counting errors, such as mixing up numbers, decade errors, skipping numbers and decade errors, and skipping numbers, repeating numbers, and decade errors. These varied errors were found in only three age groups: 3, 4, and 5 in the present study. Except for one 5-year-old, the only kind of error made by the 5-, 6-, 7-, and 8-year-olds was decade errors. Therefore, errors on oral counting were most likely to occur at the beginning of a decade (decade transition). As was expected, the percentage for making decade errors decreased as the ages increased. At age 9, no decade error was made by any of the children. As with oral counting errors, the older the children, the fewer errors that were made, indicating a gradually expanding oral counting
competence. The differences among age groups were significant, $X^2 (30, N = 98) = 43.75, p = .05$.

Table 13

**Error Types for Oral Counting (By Number of Children)**

<table>
<thead>
<tr>
<th>Error type</th>
<th>Age</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3</td>
</tr>
<tr>
<td>1. No error</td>
<td>12</td>
</tr>
<tr>
<td>2. Mixing up numbers</td>
<td></td>
</tr>
<tr>
<td>3. Skipping numbers</td>
<td>1</td>
</tr>
<tr>
<td>4. Decade errors</td>
<td></td>
</tr>
<tr>
<td>5. Skipping numbers and decade errors</td>
<td>1</td>
</tr>
<tr>
<td>6. Skipping numbers repeating numbers</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$n = 14$ for each age level.

chi-square = 43.75; df = 30; $p = .05$.

**Comparison**

In Miller and Stigler’s (1987) study, the American 3-, 4-, and 5-year-old subjects made counting errors on 85% of the trials, as opposed to 50% for the Chinese subjects in their study. In the present study, the Chinese 3-, 4-, and 5-year-olds produced counting errors on 41% of the attempts.

According to Miller and Stigler (1987), the most common error among the Americans was in skipping numbers; approximately 60% of the American children produced it. American children produced this error much more often than did the Chinese counterparts (18%). In the present
study, only 12% of the Chinese 3-, 4-, and 5-year-olds skipped numbers when they counted.

Nonstandard numbers were produced by 20% of the American 3-, 4-, and 5-year-olds in Miller and Stigler's (1987) study, but none were produced by Chinese children. In the present study, no nonstandard numbers were verbalized by the Chinese children.

The results of Miller and Stigler's (1987) study revealed that the only error that Chinese children were more likely to make than were the Americans was incorrectly counting by tens approximately 11% for the Chinese as opposed to 2% for the Americans. In the present study, 16% of the Chinese children, ages 3 through 9, made mistakes by counting tens (98, 99, 100, 110, 120, 130), hundreds (998, 999, 1000, 1101), or thousands (98, 99, 100, 1000), mostly at counting transition points.

According to Miller and Stigler (1987), a common error found in both countries was that of decade error: approximately 41% for the Americans, and 28% for the Chinese. In the present study, 48% of the errors that Chinese 3-, 4-, and 5-year-olds made were decade transition errors, including mistakenly counting by tens and hundreds.

Evidently, American 3-, 4-, and 5-year-olds made more various kinds of oral counting errors than did the Chinese children in the present study. The Chinese 5-year-olds tended to make more errors at the decade transition.
Object Counting

When a child was asked to count a collection of objects—78 chips—the strategy the child spontaneously used to group and to count the objects could be observed. The focus was on a child’s spontaneous counting by tens, which might reveal his/her multi-unit conceptual structure, a prerequisite for understanding the place-value numeration system. The task was adapted from C. Kamii’s (1986) study.

Interview Strategy

The interviewer randomly placed 78 poker chips on the table and asked children to count how many chips were there by actually moving or grouping the chips.

Performance

When asked to count 78 chips, children’s ways of grouping and counting the chips clustered around the following categories: by ones; by twos; by fours; by fives; by combining ones, twos, fours, and fives; and by tens. A representational example for each category is as follows:

By ones. When asked to count the 78 chips on the table, Yin-Wei, 3 years and 10 months, moved and counted the chips one by one.

Interviewer (I): (78 poker chips were shown.) Do you see these chips? Let’s count how many chips are here.
Ying-Wei (Y): (She moved and counted the chips by ones.) 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20 (pause).

By twos. Pey-Ying, 6 years and 5 months, grouped and counted the 78 chips by twos.
Interviewer: (78 poker chips were shown.) See these chips? Let’s count how many chips are here. By the way, there are a lot of chips; you may move them as you count.

Pey-Ying: (She counted the chips by twos.) 2, . . . , 48, (pause), 70, . . . , 98.

By fours. Yen-Lin, 6 years and 3 months, grouped the 78 chips by fours. Actually, he counted the chips by adding ones.

By fives. Pey-Tyng, 8 years and 5 months old, grouped and counted the 78 chips by fives.

I: (78 poker chips were shown.) See these chips. Let’s count how many chips are here. There are a lot of chips. You may move them as you count.

P: (She moved and counted the chips by fives.) 5, 10, 15, . . . , 75, 78.

By combining ones, twos, fours, fives, or sixes, but not based on ten. Wei-Jou, 8 years and 2 months, counted the chips by ones sometimes, by twos at another time, or by fours, fives, and sixes, but the ways he grouped the chips were not based on ten.

I: (78 poker chips were shown.) See these chips. Let’s count how many chips are here. There are a lot of chips. You may move them as you count.

W: (He counted the chips alternatively by adding one chip, two chips, four chips, five chips, or six chips.) 80.

By tens. At first, Jiun-Wei, 6 years and 11 months, grouped the chips alternatively by twos, by fours, by sixes, or by tens. However, when he was getting tired, he grouped and counted the chips by tens.

Interviewer: (78 poker chips were shown.) See these chips. Let’s count how many chips are here? By the way, there are a lot of chips; you may move them as you count.
Jiun-Wei: (He counted the chips by ones, twos, fours, sixes, and tens.) 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 46, 50, 60, 70, 72, 74, 76, 77.

Wei-Chen, 6 years and 11 months, counted the 78 chips by grouping them into tens, and then went back and counted the total number of chips.

Interviewer: (78 poker chips were shown.) See these chips? Let’s count how many chips are here. By the way, there are a lot of chips; you may move them as you count.

Wei-Chen: (He moved and counted the chips by twos until he reached “10.” Then he left them in a group. Then he counted out another 10 chips, put them in a separate group. He repeated the same procedure five more times, leaving a group of 8 chips. He went back and counted them and said 10, 20, 30, 40, 50, 60, 70, 78.)

Table 14 shows the results of object counting in the present study. It was not surprising that all of the 3-, 4-, and 5-year-olds and about two thirds of the 6- and 7-year-olds counted the chips by ones. The percentage for the children from ages 3 to 9 who counted the chips by ones was about 69. Except for counting by ones, the technique of counting by twos was the denomination most often used. Of the 98 children, 14 % counted by twos because it was a faster way to count. Counting by tens appeared at the age 6 for the first time. About 9 % of the children counted by tens. One 6-year-old preferred grouping chips by fours, and one-8-year-old preferred grouping and counting by fives. Five (5 %) of the 98 children counted by ones sometimes, by twos at another time, or by fives, and so on.

Counting by ones was the most frequently used technique for children from ages 3 through 7. Gradually, as age levels increased, the children moved their preference in object counting to multiples, such as by twos, by
fours, by fives, but not any system based on tens. Only a few children realized that the counting by tens was an easy, fast, and accurate way to count a large quantity of objects; counting by tens was employed spontaneously by fewer than 21% of the older children. Therefore, although the children were able to grasp the number generation rules to multi-digit numbers and to verbalize number words over two-, three-, or four-digits, their initial preference for counting and grouping objects was by ones. There were significant differences among age groups on the techniques they used to count a large quantity of chips, $X^2 (30, N = 98) = 58.37, p < .05$.

Table 14

Strategies for Grouping Objects (By Number of Children)

<table>
<thead>
<tr>
<th>Age</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. By one</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>8</td>
<td>9</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>2. By two</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. By four</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. By five</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. In combination</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. By ten</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$n = 14$ for each age level.

chi-square = 58.37; df = 30; $p < .002$.

Comparison

Because there was no comparable American study, performances by the Chinese children in the present study were compared to performances by
children in C. Kamii's (1986) Geneva study. The results of Kamii's study (see Table 15) showed that when asked to count a large quantity of chips, the majority of Genevan 6-, 7-, 8-, and 10-year-olds counted by ones. Consistent with Kamii's study, the Chinese 3-, 4-, 5-, 6-, and 7-year-olds in the present study also showed a strong preference for using the counting-by-ones technique to count a large number of chips. However, the Chinese 8- and 9-year-olds tended to count the chips by multiples, such as twos, fours, fives, but not tens. Similarly, in Kamii's study, one third of the 8-year-olds and more than half of the 9-year-olds counted chips by twos or by other strategies rather than by tens, any of which might be considered as a faster way of counting by ones. It was interesting to find that about half of the 9-year-olds in both studies had a tendency to count by twos. In Kamii's study, the strategy of counting by tens first appeared in the 9-year-olds. For Chinese children in the present study, counting by tens appeared at age 6 for the first time, much earlier than for the Genevan children.

Table 15
Percentages of Genevan Children's Strategies for Grouping Objects in Kamii's (1986) study

<table>
<thead>
<tr>
<th>Age</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. By ones</td>
<td>100</td>
<td>94</td>
<td>71</td>
<td>32</td>
<td>72</td>
</tr>
<tr>
<td>2. By twos</td>
<td>6</td>
<td>19</td>
<td>45</td>
<td>22</td>
<td></td>
</tr>
<tr>
<td>3. By others</td>
<td>10</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. By tens</td>
<td></td>
<td></td>
<td></td>
<td>14</td>
<td>5</td>
</tr>
</tbody>
</table>
The two studies suggested that children's numerical knowledge develops by constructing the system of ones first and moves only gradually to constructing the system of tens. For a long period, children work on a system of ones and then move from ones to tens slowly. In the two studies, about half of the 9-year-olds were still spontaneously counting a collection of chips by twos instead of by tens.

Counting by Tens

The focus of this task was to reveal children's construction of the system of ones (unitary conceptual structure) and tens (multi-unit conceptual structure) and to see if the child could think about ones and tens at the same time. This task was adapted from C. Kamii's (1986) study, which was done in Geneva. Although the present study and the Kamii study were similar, there were some adaptations in the former. For example, the Chinese children were asked to count chips by making individual groups of tens instead of being asked to count chips by tens. In some cases, if a child seemed to count out the chips by tens, but mentally counted them out by one (see the interview examples), the interviewer asked him/her to go back and count the groups again to confirm the child's capability of thinking about ones and tens at the same time.

Interview Strategy

If a child had already counted the chips by tens in the previous task - object counting, this item was omitted. If a child failed to count by tens in the task of object counting, the same 78 chips were again randomly arranged on the table and the child was requested to count them by tens.
The counting-by-tens task was not administered to the 3- and 4-year-olds because of its difficulty.

When responding to the task of counting by tens, a child first counted out 10 chips and left them in a group; when an additional 10 chips were counted and made into a separate group, he/she said “Twenty.” He/she seemed to count by ones. Therefore, in order to see if the child actually could handle ones and tens at one time, the child was asked to come back and count the groups of 10 and the 8 ones again to make sure of the total quantity of chips.

**Performance**

Adapted from Kamii’s categories of counting by tens (C. Kamii, 1986), and in accordance with the children’s responses in the present study, five categories were selected: (1) no idea how; (2) making groups of 10, leaving a group of 8, and counting each group as a separate “10”; (3) making groups of 10, leaving a group of 8, and counting each group by adding “10,” including the last group of 8 chips; (4) making groups of 10, leaving a group of 8, and counting each group of 10 by adding “10,” and counting each chip from the group of 8 by adding ones; and (5) making groups of 10, leaving a group of 8, and counting each group of 10 by adding “10,” and counting the last group of 8 chips by adding “8.” Children qualifying for either Category 4 or 5 demonstrated a cognitive capability to think ones and tens simultaneously.
No idea how. Tzong-Ting, 5 years and 2 months, responded to the counting-by-tens task by grouping the 78 chips into several lines of fives. Apparently, he had no idea about counting by tens.

Interviewer (I): (78 chips were randomly arranged on the table.) You know how to count these chips by ones. Now, let’s count them by tens.
Tzong-Ting (T): (He seemed very puzzled.)
I: Can you make a group when you count to 10, and make another group when you count to another 10.
T: (He counted the chips by one and made several lines of five.)

Counting seven groups of 10 chips each and one group of 8 chips as a separate 10. When responding to this task, Ming-Hwa, 5 years and 11 months, first grouped the chips into seven groups of 10 and one group of 8. But when asked to go back and count the groups again, she counted the groups of 10 and the one group of 8 by saying “10” eight times.

Interviewer (I): (78 chips were randomly arranged on the table.) You know how to count these chips by ones. Now, let’s count them by tens.
Ming-Hwa (M): (She moved and counted chips by ones until she reached “10.” She left them in a group. Then she counted out another 10 chips and put them in a separate group. She repeated the same procedure five more times. A group of 8 chips was left.)
I: Let’s go back and count them one more time to make sure of the total quantity. (She pointed to seven groups of 10 and one group of 8.)
M: 10, 10, 10, 10, 10, 10, 10, 10.
I: What do you have when you have one group of 10 chips plus the other one group of 10 chips?
M: 10.

Counting seven groups of 10 chips each and one group of 8 chips by adding tens. Perng-Yow, 5 years and 8 months, when asked about counting
chips by tens, grouped them into seven groups of 10 and one group of 8 and counted both the former and latter groups by adding 10 each time.

Interviewer (I): (78 chips were randomly arranged on the table.) You know how to count these chips by ones. Now, let's count them by tens.
Perng-Yow (P): (He moved and counted chips by ones until he reached "10." He left them in a group. Then he counted out another 10 chips and put them in a separate group. He repeated the same procedure five more times until he reached "70"; a group of 8 chips was left.)
I: Let's go back and count them one more time to make sure of the total quantity. (She pointed to seven groups of 10 and one group of 8.)
P: 10, 20, 30, 40, 50, 60, 70, 80.

Counting seven groups of 10 by adding tens and counting the last group of 8 by adding ones. In doing the counting-by-ten task, Pey-Ying, 6 years and 5 months, grouped the 78 chips into seven groups of 10 and one group of 8. When counting the groups of 10, she added 10 each time. When she counted the group of 8, she added "1" each time to the decade.

Interviewer (I): (78 chips were randomly arranged on the table.) You know how to count these chips by ones. Now, let's count them by tens.
Pey-Ying (P): (She moved and counted chips by twos until she reached "10." Then, she left them in a group and said "10." Then she counted out another 10 chips, put them in a separate group, and said "Twenty." She repeated the same procedure five more times until she reached "70." After she counted the remaining 8 chips, she said "78.")
I: Let's go back and count them one more time to make sure of the total quantity. (She pointed to seven groups of 10 and one group of 8.)
P: 10, 20, 30, 40, 50, 60, 70, 71, 72, 73, 74, 75, 76, 77, 78.
Counting seven groups of 10 by adding tens and counting the last group of 8 by adding 8. Sheau-Lin, 6 years and 11 months, grouped the 78 chips into seven groups of 10 and one group of 8. She counted them by adding “10” to the sum seven times, and after saying “70,” mentally added 8 and said “78.”

Interviewer (I): You know how to count these chips by ones and twos. Now, let’s count them by tens.
Sheau-Ling (S): (She moved and counted chips by twos until she reached “10.” Then she left them in a group and said “10.” Then, she counted out another 10 chips, put them in a separate group, and said “20.” She repeated the same procedure five more times until she reached “70.” After she counted the remaining 8 chips, she said “78.”)

I: Let’s go back and count them one more time to make sure of the total quantity. (She pointed to seven groups of 10 and one group of 8.)
S: 10, 20, 30, 40, 50, 60, 70, 78.

The children’s performances on the structured counting-by-tens task, including the performances of the children who spontaneously counted chips by tens in the object-counting task, are summarized in Table 16. Only a few children at ages 5, 6, and 7 could not perform the structured counting-by-ten task. All the 8- and 9-year-olds knew how to count a collection of 78 chips by tens. The 21% of the 5-year-olds who made groups of tens without conservation of the whole and 14% of the 5-year-olds who were not able to think about ones and tens at the same time were expected. These performances indicated that the 5-year-olds were still in the process of constructing system of tens. The children who reached either Category 4 or 5 could think about ones and tens at the same time; however, the children
who reached Category 5 were more advanced than those in Category 4. Naturally, the percentages for the children having this kind of multi-unit conceptual structure were, respectively, 50, 86, 93, 100, and 100 for the age groups 5 through 9.

Table 16

Ways of Responding When Asked to Count 78 Chips by Tens (By Number of Children)

<table>
<thead>
<tr>
<th>Category</th>
<th>Age</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. No idea how</td>
<td>2</td>
</tr>
<tr>
<td>2. Counting 8 groups as separate 10</td>
<td>3</td>
</tr>
<tr>
<td>3. Counting 8 groups by adding 10</td>
<td>2</td>
</tr>
<tr>
<td>4. Counting 7 groups by adding 10, the last group by adding 1</td>
<td>7</td>
</tr>
<tr>
<td>5. Counting 7 groups by adding 10, the last group by adding 8</td>
<td>4</td>
</tr>
</tbody>
</table>

n = 14 for each age level.

chi-square = 50.96; df = 16; p = .00002.

When structured to count with groups of tens, most were successful as early as age 6. About half of the 5-year-olds could not coordinate tens and ones at the basic level. The developmental course in constructing unitary, multi-unit conceptual structures progresses with age. The differences among ages were significant, $X^2 (16, N = 70) = 50.96, p < .05.$
Comparison

In the Kamii (1986) study that was cited in the previous section, the Genevan children’s abilities of making groups of ten with conservation of the whole (thinking about ones and tens at the same time) was exhibited for the first time at age 7 (see Table 4). The percentages for the Genevan 6-, 7-, 8-, and 9-year-olds who demonstrated this multi-unit conceptual structure were 0, 39, 71, and 36, respectively. However, the data in the present study showed that the Chinese children who demonstrated an ability to work with the ones and tens simultaneously appeared as early as age 5. The percentages for the 6-, 7-, 8-, and 9-year-olds who demonstrated the multi-conceptual structure were 86, 93, 100, and 100, respectively. The results of the two studies showed that when asked to count a collection of chips in group of tens, the Chinese children knew how to count them by using multi-unit structure at an earlier age than did the Genevan children.

Digit-Correspondence Task

The digit-correspondence task focused on the meanings children attributed to each digit of a two-place numeral. Children’s responses in this task revealed their place-value understanding at the ones and tens places. Although adapted from Silvern and Kamii’s study (Silvern & Kamii, 1988, cited in C. Kamii & Joseph, 1989), some of the changes in the digit-correspondence task resulted in extensions to the present study. For example, in order to test the effects of adult assistance in the children’s performances on the task, at the end of the interview, some leading
questions were given to the children who were unable to interpret the value of the numeral at the tens place.

**Interview Strategy**

Sixteen identical chips were placed on the table. The interviewer asked the child to count how many chips there were. Then the child was shown the number 16 written on a card and asked what the number was. If the child was unable to recognize the numeral 16 correctly, this task was terminated. Otherwise, the interviewer circled the two numerals separately—but not simultaneously—in number 16 and asked the child to explain the meanings of the two numerals and to represent the numerals by using the chips. If the child showed six chips for the numeral 6 and only one chip for the numeral 1, the interviewer pointed to the remaining nine chips and asked the child to tell why there were nine chips left. After the probes were given and if the child insisted that the numeral 1 in number 16 stood for one instead of ten, some additional leading questions were given later, at the end of the primary interview.

During this follow-up instructional period at the end of the primary interview, the interviewer arranged 16 chips into one group of 10 and one group of 6; then she showed the card on which the numeral 16 was written to the child and stated that number ten-six means a group of 10 chips (pointing to the group of 10 chips) and a group of 6 chips (pointing to the group of 6 chips) go together; and that people sometimes write the number for the 16 chips in the following way: 16 = 10 + 6. The interviewer then
circled the two numerals separately in number 16 and asked the child about the quantity of each numeral.

**Performance Before Leading Questions Were Given**

Based on the children’s responses on this task, and before some leading questions were given, the following hierarchical categories were formulated, along with interview examples.

*No recognition of either numeral in number 16.* When asked to recognize the number 16, Bor-Ting, 3 years and 6 months, kept silent.

Interviewer (I): (A card on which the number 16 was written was shown.) What’s the number?
Bor-Ting (B): (silent)

*Recognized only numeral 1 in number 16.* Hsin-Yi, 3 years and 1 month, could only recognized numeral 1 in number 16. When asked to give the value of the numeral 6 in number 16, she kept silent.

Interviewer (I): (A card on which the number 16 was written was shown.) What’s the number?
Hin-Yi: (She pointed to the numeral 1 in number 16.) 1.
I: (The numeral 6 in number 16 was circled.) What’s this number?
H: (silent)

*Recognized both numerals in number 16 in the correct order but saw them as two individual digits.* Feng-Hwa, 4 years and 10 months, recognized both numerals in number 16, but she did not differentiate them by their place-values.

Interviewer (I): You counted very well. (A card on which the number 16 was written was shown and 16 chips were shown.) What’s the number?
Feng-Hwa (F): 1, 6.
Recognized both numerals in number 16 and saw it as a two-digit number but in reverse order. Ying-Ying, 4 years and 1 months, could read number 16 (ten-six in Chinese) in reverse order as 60 (six-ten in Chinese).

Interviewer (I): (A card on which the number 16 was written was shown, and 16 chips were shown.) What’s the number? Ying-Ying (Y): 60.

Recognized both numerals in number 16 in the correct order but interpreted them only by the face values. Yi-Wen, 5 years and 7 months, recognized number 16 as a two-place number and saw them in correct order; however, she interpreted the two numerals in number 16 only by their face values.

Interviewer (I): (A card on which the number 16 was written was shown, and 16 chips were shown.) What’s the number? Yi-Wen (Y): 16.
I: So, the number 16 stands for these 16 chips.
I: (The numeral 6 in number 16 was circled.) Do you see this part? What does it mean?
Y: 6.
I: Can you show me by using the chips?
Y: (6 chips were moved out.)
I: (The numeral 1 in number 16 was circled.) And this part, what does it mean? Can you show me by using the chips?
Y: (1 chip was moved out.)
I: (She pointed to the remaining 9 chips.) What about these chips? If we said the number 16 stands for 16 chips, the numeral 6 means 6 chips, and the numeral 1 means 1 chip, why are there 9 chips that are not included?
Y: Because there was no numeral 9 on the card.

Recognized both numerals in number 16 in the correct order and interpreted the digits by both their face and place values. Yen-Lin, 6 years
and 3 months, saw the number 16 as a two-place number, read it in correct order, and interpreted the two numerals in number 16 by both their face and place values.

Interviewer (I): (16 chips were randomly arranged on the table.)
There are some chips in front of you. Would you count them to make sure how many chips are here?
Yen-Lin (Y): (He looked at the chips.) 16.
I: Good. (A card on which the number 16 was written was shown.)
What’s the number?
Y: 16.
I: (The numeral 6 in number 16 was circled.) Do you see this part?
What does it mean?
Y: 6.
I: Can you show me by using the chips?
Y: (He moved 6 chips out.)
I: (The numeral 1 in number 16 was circled.) And this part, what does it indicate?
Y: 10. (10 chips were moved out.)

Children’s responses before leading questions were given are summarized in Table 17. Before some leading questions were given, no 3-year-olds and few (29%) 4-year-olds were able to read a two-digit number correctly. All but one 5-year-old and all 6-year-olds were able to do it. However, only 14% of the 5- and 6-year-olds could interpret a two-digit number by both its face and place values. About one sixth of them interpreted the individual digits in number 16 by their face value only. The majority of the 7-, 8-, and 9-year-olds could interpret the numerals in number 16 by both their face and place values. The percentages for 7-, 8-, and 9-year-olds who exhibited the place-value understanding when associated with a two-digit number were 71, 93, and 100, respectively. At
about age 7, the majority of children not only comprehended the quantity represented by a whole numbers but also had an understanding of the meaning of the individual digits (places). The performance differences among age levels before the leading questions were given were significant, \(X^2 (30, N = 98) = 143.94, p < .05.\)

Table 17

**Performance on Digit-Correspondence Task Before Leading Questions Were Given (By Number of Children)**

<table>
<thead>
<tr>
<th>Category</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. No recognition of either numerals</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>2. Recognized numeral 1 only</td>
<td></td>
<td>8</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Saw numerals as individual digits</td>
<td></td>
<td>4</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Saw numerals in reverse order</td>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Interpreted them by face values</td>
<td>4</td>
<td>11</td>
<td>12</td>
<td>4</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Interpreted them by face and place values</td>
<td>2</td>
<td>2</td>
<td>10</td>
<td>13</td>
<td>14</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(n = 14\) for each age level.
chi-square = 143.9; \(df = 30;\) \(p < .0000.\)
Performance after Leading Questions Were Given

To children who claimed that the numeral 1 in number 16 stood for one chip (Category 5) during the interview, the interviewer gave some leading questions at the end of the primary interview. After the questions were given, the children were retested. The children's responses to the questions were retabulated under the same categories.

Interpreted the two numerals in number 16 only by their face values. Although being led to the correct way of responding to the digit-correspondence task, Chi-Wei, 6 years and 1 month, insisted that the numeral 1 in number 16 indicated one chip.

Interviewer (I): (16 chips were arranged into one group of 10 and one group of 6.) One group of 10 chips and one group of 6 chips together are 16 chips.
I: Therefore, we also write the number for 16 chips in this way: 16 = 10 + 6. (The two numerals 6 in the arithmetic sentence were underlined.) The numeral 6 stands for 6 chips. (The numeral 1 in the number 16 in the arithmetic sentence was underlined.) How many chips does the numeral 1 indicate?
Chi-Wei: One chip.

Interpreted the numerals in number 16 by both their face and place values. After being led to the right way of thinking about a two-place number, Pey-Yuh, 7 years and 2 months, interpreted the numeral 1 in number 16 by both its face and place values.

Interviewer (I): (16 chips were arranged into one group of 10 and one group of 6.) One group of 10 chips and one group of 6 chips together are 16 chips.
I: Therefore, we also write the number for 16 chips in this way: 16 = 10 + 6. (The two numerals 6 in the arithmetic sentence were underlined.) The numeral 6 stands for 6 chips. (The numeral 1 in the
number 16 in the arithmetic sentence was underlined.) How many chips does the numeral 1 represent?

Pey-Yuh: 10.

Children’s responses after leading questions were given are summarized in Table 18. After the extending questions were given, the number of children who correctly interpreted the numeral 1 in number 16 as one “10” were one (7 %), four (29 %), six (43 %), four (29 %), and one (7 %) for ages 4, 5, 6, 7, and 8, respectively. Three age groups of children who benefited the most from adult assistance in the digit-correspondence task were ages 5, 6, and 7. After receiving extending questions, about half the 5- and 6-year-olds and all the 7-, 8-, and 9-year-olds exhibited the place-value understanding when presented with a two-digit number. This suggests that at about age 7, the children had developed the understanding of the meaning of the individual digits up to the tens place; however, different questions used to test children for this kind of understanding may produce different results. The responding differences among age levels were still significant after the leading questions were given, $X^2 (30, N = 98) = 123.74, p < .05$.

**Comparison**

In Silvern and C. Kamii’s study (Silvern & C. Kamii, 1988, cited in Kamii & Joseph, 1989), the percentages for English-speaking 7-, 8-, and 9-year-olds who interpreted the numeral 1 in number 16 by both its face and place values were 7.5, 29, and 35; the majority of 7-, 8-, and 9-year-olds were unable to interpret a two-digit number as a composition of ones and tens. Respectively, about 7.5, 4, and 6 of the 7-, 8-, and 9-year-olds in
Silvern and C. Kamii's study pointed out that the numeral 1 in number 16 stood for 10 but moved only one chip out.

Table 18
Performance on Digit-Correspondence Task After Leading Questions Were Given (By Number of Children)

<table>
<thead>
<tr>
<th>Category</th>
<th>Age</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3</td>
</tr>
<tr>
<td>1. No recognition of either numerals</td>
<td>2</td>
</tr>
<tr>
<td>2. Recognized numeral 1 only</td>
<td>8</td>
</tr>
<tr>
<td>3. Saw numerals as individual digits</td>
<td>4</td>
</tr>
<tr>
<td>4. Saw numerals in reverse order</td>
<td>1</td>
</tr>
<tr>
<td>5. Interpreted them by face values</td>
<td>3</td>
</tr>
<tr>
<td>6. Interpreted them by face and place values</td>
<td>1</td>
</tr>
</tbody>
</table>

n = 14 for each age level.
chi-square = 123.74; df = 30; p < .0000.

Compared with the results of the Silvern and C. Kamii's (1988) study, the Chinese children in the present study, even before adult assistance was given, demonstrated an understanding of the meanings of the individual numerals in two-digit numbers as early as age 5. The percentages for the 7-, 8-, and 9-year-olds who demonstrated the place-
value understanding in the digit-correspondence task were 71, 93, and 100. At age 7, the majority of Chinese children understood the meaning of individual digits in number 16.

**Representation of Two-digit Number**

When constructing a two-digit number by using a collection of base-10 blocks, a child’s basic conceptual representations of two-digit numbers can be revealed. This task was not administered to the 3-, and 4-year-olds because of its difficulty. Although based on the studies of Miura and colleagues (Miura, 1987; Miura et al., 1988; Miura & Okamoto, 1989), some adaptations were made for the present study. For example, in testing the effectiveness of adult assistance, the children were not given any coaching or practice exercises before the task; however, for the children who failed to construct a two-digit number in two different representations, some demonstrations were given at the end of the primary interview and rescored.

**Interview Strategy**

**Trial 1.** A set of base-10 blocks was introduced. The equivalence of a 10-block bar and 10 unit blocks was also pointed out by the interviewer. After a card on which the number 32 was written was shown, the interviewer asked the child what the number was. If the child was unable to recognize numeral 32 correctly, the task was terminated. Otherwise, the interviewer asked the child to represent the quantity of the number by using both the 10-block bars and the unit blocks.
Trial 2. Soon after Trial 1, the child was reminded of the equivalence of the 10-block bar and unit blocks. Then he/she was shown or reminded of his/her first representation of the number 32. The interviewer asked the child to show the number 32 another way by using the blocks and bars.

For the children who failed to represent number 32 in two different ways, some demonstrations were given at the end of the primary interview. First, the equivalence of 10 units and a 10-block bar was pointed out again by the interviewer. Then the interviewer demonstrated how the unit blocks and the 10-block bars could be used for constructing number 22 in one-to-one collection and base-10 constructions. After the demonstrations were given, the child was asked to construct number 32 in two different ways or in another way different from his/her first trial before demonstrations were given.

Performance Before Demonstrations Were Given

Examples of the children’s ways of constructing the number 32 usually corresponded to four categories that were formulated in Miura’s (1987) study.

No idea how. When asked to construct the number 32 by using the blocks on the first trial, Tzong-Ting, 5 years and 2 months, could not make the construction correctly because he was unable to use 10-block bars and unit blocks at the same time.

Interviewer (I): (One of the 10-block bars was shown.) This is a 10-block bar. How many does it stand for?
Tzong-Ting (T): (He counted the divisions.) 10.
I: If you line up the separate 10 unit blocks, they will have the same length as the 10-block bar. Right?
T: (Ten unit blocks were lined up alongside the 10-block bar.)
I: Do they have the same length?
T: (Nodding head positively.) There are 20 unit blocks.
I: (A card on which the number 32 was written was shown.)
What’s the number?
T: 32.
I: Will you show me the number 32 by using these 10-block bars and the unit blocks.
T: (Four 10-block bars were moved out.)
I: We are making number 32. How many blocks are here?
T: (He counted the blocks again.) I don’t know.

Chi-Wei, 6 years and 1 months, constructed number 32 by using 32 unit blocks on his first trial; but he had no idea how to construct “32” another way on his second trial.

Interviewer (I): (One 10-block bar was shown.) This is a 10-block bar. It stands for 10 units. (Ten unit blocks were lined up alongside the 10-block bar.) If you line up the separate 10 unit blocks, they will have the same length as the 10-block bar. Right?
Chi-Wei (C): (He nodded his head positively.)
I: (A card on which the number 32 was written was shown.)
What’s the number?
C: 32.
I: Will you show me the number 32 by using both these 10-block bars and the unit blocks.
C: (He tried to use unit blocks to make the shapes of numeral 32.)
I: Let’s find the same quantity of blocks that equals number 32.
C: (He counted out 32 unit blocks.)
I: Good job. You used 32 unit blocks to equal number 32. As we mentioned earlier, 10 unit blocks are equal to one 10-block bar; therefore, can you show me the number 32 another way by using these blocks (pointing to the 10-block bars)?
C: (He shook his head negatively.)
**One-to-one representation.** Yi-In, 7 years and 5 months, constructed number 32 by moving out 32 unit blocks on her first trial.

Interviewer (I): (One 10-block bar was shown.) How many units does the 10-block bar stand for?  
Yi-In (Y): (She counted them by ones.) 10.  
I: If you line up the separate 10 unit blocks, they will be the same length as the 10-block bar. (Ten unit blocks were lined up alongside the 10-block bar.) Right?  
Y: (She compared the line of 10 unit blocks with the 10-block bar.) Yes.  
I: (A card on which the number 32 was written was shown.) What’s the number?  
Y: 32.  
I: Will you show me the number 32 by using both these 10-block bars and the unit blocks.  
Y: (She moved out 32 unit blocks.)

**Canonical base-10 representation.** Pey-Shiun, 8 years and 3 months old, used three 10-block bars and two unit blocks to represent number 32.

The canonical base-10 representation is standard in the place-value numeration system.

Interviewer (I): (One 10-block bar was shown.) How many does the 10-block bar stand for?  
Pey-Shiun (P): 10 liter.  
I: How many does this 10-block bar stand for?  
P: 10.  
I: So, if you line up the separate ten unit blocks, they will be the same as the 10-block bar, right?  
P: (Ten unit blocks were lined up alongside one 10-block bar.) Right, they are the same.  
I: (A card on which the number 32 was written was shown.) What’s the number?  
P: 32.
I: Will you show me the number 32 by using these 10-block bars and the unit blocks.
P: (She moved three 10-block bars and two unit blocks.)

**Noncanonical base-10 representation.** When Hann-Chen, 8 years and 5 months old, constructed number 32, he used two 10-block bars and twelve unit blocks on his first trial. Since the noncanonical base-10 representation is more flexible, he needed to regroup and rename.

Interviewer (I): (One 10-block bar was shown.) How many units does this 10-block bar stand for?
Hann-Chen (H): 10.
I: If you line up the separate 10 unit blocks, will they be the same as the 10-block bar? (Ten unit blocks were lined up alongside one 10-block bar.)
H: (He nodded his head positively.)
I: (A card on which the number 32 was written was shown.) What’s the number?
H: 32.
I: Will you show me the number 32 by using these 10-block bars and the unit blocks.
H: (He moved two 10-block bars and twelve unit blocks.)

Summary results before the follow-up demonstrations were given are shown in Tables 19. On the first trial, two of the 5-year-olds had no idea how to construct number 32; more than half (57%) of 5-year-olds constructed 32 by using the one-to-one collection structure. The majority of 6-, 7-, 8-, and 9-year-olds represented “32” by the canonical structure, the percentages being 57, 79, 93, and 100, respectively.

On the second trial, the number of children who did not know another way to construct number 32 increased to 13; 8 were 5-year-olds, 2 were 6-year-olds, and 3 were 7-year-olds. That more than half of the 5-year-olds
could not construct “32” in two different ways suggests a less developed mental facility with number quantity at this age level. The number of children who used the one-to-one collection and noncanonical base-10 representations for another way to represent “32” increased.

Table 19
Performance on Representation of a Two-Digit Number Before Demonstrations Were Given (By Number of Children)

<table>
<thead>
<tr>
<th>Category</th>
<th>Age</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td><strong>Trial 1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. No idea how</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. One-to-one collection</td>
<td>8</td>
<td>2</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Canonical base-10</td>
<td>4</td>
<td>8</td>
<td>11</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>4. Noncanonical base-10</td>
<td>4</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Trial 2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. No idea how</td>
<td>8</td>
<td>2</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. One-to-one collection</td>
<td>2</td>
<td>5</td>
<td>5</td>
<td>11</td>
<td>6</td>
</tr>
<tr>
<td>3. Canonical base-10</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Noncanonical base-10</td>
<td>1</td>
<td>6</td>
<td>5</td>
<td>3</td>
<td>8</td>
</tr>
</tbody>
</table>

* n = 14 for each age level.
* Trial 1: chi-square = 43.22; df = 12; p = .00002.
* Trial 2: chi-square = 36.34; df = 12; p = .00003.

The differences between the first trial and the second trial revealed that, except for the 5-year-olds, the majority (71%) of Chinese 6-, 7-, 8-, and 9-year-olds preferred constructing “32” by using canonical base-10 representation more than other representations on the first trial. On the second trial, the majority of children, ages 6 to 9, had a tendency to
construct "32" by using either one-to-one collection (41 %) or noncanonical base-10 representations (33 %). Except for the majority (79 %) of 8-year-olds who used the one-to-one collection to represent "32," about half of the 6-, 7-, and 9-year-olds used the noncanonical base-10 representation on the second trial.

A developmental progression in the representation of a two-digit number was found. The differences among age levels were significant on both trials: the first trial, $X^2 (12, N = 70) = 43.22, p < .05$; and the second trial, $X (12, N = 70) = 36.34, p < .05$.

Performance After Demonstrations Were Given

For the children who failed to construct "32" in two different representations, some demonstrations were given at the end of the primary interview. Their individual performances were classified in one of the same four categories that were mentioned previously.

The example of Tzong-Ting mentioned earlier is cited here again to describe how a child who first had no idea how to construct "32" was able to show "32" by two different representations after adult assistance was given.

Interviewer (I): (The equivalence between 10 unit blocks and one 10-block bar was reintroduced.) You see, first, I can make the number 22 this way. (She counted 22 unit blocks.) And, because the 10 unit block equals to one 10-block bar, I also can make number 22 another way like this. (She moved two 10-block bars and two unit blocks.) Can you use both kinds to make the number 32? (The card on which the number 32 was written was shown.)

Tzong-Ting (T): (Four 10-block bars were moved out as was the case before.)

I: These blocks equal number 40, right?
T: (He took away one 10-block bar and put two unit blocks in.)
I: Good job. We mentioned earlier that the 10 unit blocks are equal to one 10-block bar. Can you show me another way to make "32"?
T: (He counted 32 unit blocks.)

Summary results after follow-up demonstrations were given are shown in Table 20. After some demonstrations were given, the two 5-year-olds who failed on the early first trial constructed "32" by using the canonical representations. On the second trial, the 13 children who failed on the earlier second trial, two (15%) of them, who were 5- and 6-year-old children, still had no idea how to construct "32" another way. Of the other 11 children (85%) who, with adult assistance, successfully constructed number 32 in ways different from the way used on the first trial, seven (50%) were 5-year-olds; one (7%) was 6-year-olds, and three (21%) were 7-year-olds. All three age groups benefited from adult assistance in the number-representation task.

On the first trial, after demonstrations were given, the canonical base-10 representations was still the favorite way for the Chinese children to construct a two-digit number; about 74% of the tested children used this method. On the second trial, 50% of the children used the one-to-one collections, and 34% of the children used the noncanonical base-10 representations in constructing "32." After some adult assistance was given, the differences among age levels were still significant on both two trials: the first trial, $X^2 (12, N = 70) = 32.96, p < .05$; and the second trial, $X^2 (12, N = 70) = 26.34, p < .05$. 
Table 20

Performance on Representation of a Two-Digit Number After Demonstrations Were Given (By Number of Children)

<table>
<thead>
<tr>
<th>Category</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Trial 1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. One-to-one collection</td>
<td>8</td>
<td>2</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Canonical base-10</td>
<td>6</td>
<td>8</td>
<td>11</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>3. Noncanonical base-10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4</td>
</tr>
<tr>
<td><strong>Trial 2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. No idea how</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. One-to-one collection</td>
<td>6</td>
<td>5</td>
<td>7</td>
<td>11</td>
<td>6</td>
</tr>
<tr>
<td>3. Canonical base-10</td>
<td>6</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Noncanonical base-10</td>
<td>1</td>
<td>6</td>
<td>6</td>
<td>3</td>
<td>8</td>
</tr>
</tbody>
</table>

\( n = 14 \) for each age level.

**Trial 1**: chi-square = 32.96; df = 8; \( p = .00006 \).

**Trial 2**: chi-square = 26.34; df = 12; \( p = .01 \).

Comparison

In Miura et al.'s (1988) study (see Table 21), 91% of the American 6-year-olds used one-to-one representations on the first trial when constructing a two-digit number; 8% of them used the canonical base-10 representations; and only 1% used the noncanonical base-10 representations. On the second trial, 10% of the American 6-year-olds, used the one-to-one representations; 71% of them used the canonical base-10 representations; and 19% used the noncanonical base-10 representations.
Table 21
Percentages of Correct Constructions of Two-Digit Numbers for American Children in Miura et al.'s (1988) Study and Chinese Children in the Present Study

<table>
<thead>
<tr>
<th>Category</th>
<th>Country and age</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>American -- 6</td>
<td></td>
<td>Chinese -- 6</td>
</tr>
<tr>
<td>Trial 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>One-to-one collection</td>
<td>91</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>Canonical base-10</td>
<td>8</td>
<td>57</td>
<td></td>
</tr>
<tr>
<td>Noncanonical base-10</td>
<td>1</td>
<td>29</td>
<td></td>
</tr>
<tr>
<td>Trial 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No idea how</td>
<td>14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>One-to-one collection</td>
<td>10</td>
<td>36</td>
<td></td>
</tr>
<tr>
<td>Canonical base-10</td>
<td>71</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>Noncanonical base-10</td>
<td>19</td>
<td>43</td>
<td></td>
</tr>
</tbody>
</table>

On the first trial, and before follow-up demonstrations were given, the percentages of Chinese 6-year-olds in the present study (see Table 19) who used the representations one-to-one, canonical base-10, or noncanonical base-10 were 14, 57, and 29, respectively. On the second trial, the percentages for no idea how, one-to-one, canonical base-10, and noncanonical base-10 representations were 14, 36, 7, and 43.

On the first trial, the 6-year-old Chinese children in the present study obviously preferred using the canonical base-10 representations more than did their American peers in Miura et al.'s (1988) study, the latter tended to use the one-to-one collection. On the second trial, the Chinese 6-year-olds seemingly preferred using either one-to-one collection representations or
noncanonical base-10 representations more so than did Miura’s American 6-
year-olds, who had a tendency to use the canonical base-10 representations.

On the first trial, Chinese children preferred canonical representation for a two-digit number, and this was different from the one the American children preferred (one-to-one collection). On the second trial, the Chinese 6-year-olds were able to use more noncanonical base-10 representations for a two-digit number than the American 6-year-olds did.

In the present study, two Chinese 6-year-olds could not perform the task on the second trial without adult assistance, but none of the American 6-year-olds failed. In Miura et al.’s (1988) study, children were given some coaching, when needed, during practices, but in the present study, this kind of assistance from an adult was given only after the first or second trial was failed. As stated before, this assistance was given to see the effects of adult assistance on children’s place-value tasks.

Addition and Subtraction

This task focused on children’s procedural proficiency with the one-, two-, three-, and four-digit addition and subtraction algorithms which involved regrouping. This task was given only to the children ages 5 to 9 because it was beyond what most 3- and 4-year-olds could handle mentally. This task was adapted from two of Fuson and Kwon’s studies (1992a, 1992b).

Interview Strategy

First, a child was asked as to the number of places he/she could handle when working with adding and subtracting problems. Then the
single-, two-, three-, and four-digit addition and subtraction problems (8 + 5, 12 - 5, 27 + 58, 65 - 27, 394 + 241, 535 - 253, 4258 + 5831, 4083 - 1253) were presented in horizontal form on cards (two problems per card). The interviewer repeated the above question again. Depending on the child's answer with respect to how many digits he/she could add or subtract, either the adding or the subtracting (or both) was (were) given and then solved in written form by the child. If the child showed an inability or reluctance to solve even the one-digit addition problem (8 + 5), the task was terminated. Some children mistakenly added numbers on the subtraction problem during their solving procedures, or vice versa. The interviewer asked them to check the sign of the problem one more time to make sure what kind of problem it was. Because the children's ability to solve the adding and subtracting problems was the focus of the present study instead of the sign recognition, this minor problem did not enter into the scoring scheme.

Performance

A summary of the results on performances regarding addition and subtraction problems can be found in Table 22. On addition problems, except for five of the 5-year-olds and one of the 6-year-olds, all were able to solve the one-digit problem whose sum was over ten; in another words, about two thirds of the 5-year-olds and 6-year-olds solved it correctly. All the 7-year-olds solved the problems having (or over) two digits. The majority (93%) of the 8-year-olds solved the problems having three digits. All the 9-year-olds solved the problems having four digits.
Table 22
Performance on Adding and Subtracting Problems (By Number of Children)

<table>
<thead>
<tr>
<th>Category</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Addition</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Unable or failed to do one-digit addition whose sum was over 10</td>
<td></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>2. One-digit (Sum over 10)</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>3. Two-digit</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Three-digit</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Four-digit</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Subtraction</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Unable or failed to do one-digit subtracting problems whose minuend was over 10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. One-digit, the subtrahend over 10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Two-digit</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Three-digit</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Four-digit</td>
<td></td>
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</tr>
</tbody>
</table>

n = 14 for each age level.

Addition: chi-square = 118.77; df = 16; p < .00000.
Subtraction: chi-square = 96.63; df = 16; p < .00000.

On subtraction problems, twelve (86%) of the 5-year-olds, five (36%) of the 6-year-olds, and five (36%) of the 7-year-olds failed to solve the one-digit problem in which the minuend was over 10. About half of the
6-year-olds solved the problem. For 7-year-olds, four (29%) of them solved the one-digit problem whose minuend was over 10; five of them (36%) solved the problems having (or over) two digits. The majority (86%) of the 8-year-olds solved the three-digits problem. All the 9-year-olds solved the four-digit problem easily.

Evidently, children know how to solve adding problems at an earlier age than that for subtracting. All the 7-year-olds knew how to do regrouping in the two-digit adding problem. However, the age level at which all the children knew how to regroup on the two-digit subtracting problem was 8.

With the progression of age, children’s number operation capability associated with adding and subtracting increased. For the Chinese children in the present study, for example, at age 9, they all were competent in adding and subtracting problems up to four digits. The performances among age groups in adding and subtracting problems were different significantly: for addition, $X^2 (16, N = 70) = 118.77, p < .05$; and for subtraction, $X^2 (16, N = 70) = 96.63, p < .05$.

Comparison

The results that came out of the fourth mathematics assessment, conducted by the National Assessment of Educational Progress (Kouba et al., 1988) showed that approximately 84% of the 8-year-old American students successfully performed two-digit addition that involved regrouping. About 70% of the 8-year-olds were able to solve two-digit
subtraction that involved regrouping, but the percentage of students who did three-digit subtraction items that involved regrouping dropped to 50.

All the Chinese 8-year-olds in the present study solved the two-digit adding and subtracting problems that involved regrouping. About 86% of them solved the three-digit subtracting problem that involved regrouping. Compared with Kouba et al.’s (1988) study, the Chinese 8-year-olds in the present study outperformed their American counterparts on two- or three-digit adding and subtracting regrouping problems.

Solution for One-Digit Addition and Subtraction

The focus of this task is on the conceptual structures a child revealed when applying numerical knowledge to the procedures used with one-digit addition with a sum over 10, and one-digit subtraction whose minuend was over 10. The task was adapted from Fuson and Kwon’s (1992a) study using Korean children.

Interview Strategy

To find the way in which the child solved the single-digit addition problem, the interviewer asked the child how the answer was obtained. Possible strategies were counting all, counting up from one number, separating one number in order to add one number to 10, and known fact. The same procedures were used to find the way in which the child solved single-digit subtraction, such as counting down, counting up, taking away, subtracting from 10, and known fact. When a child got a wrong answer in one of the adding or subtracting problems, the interviewer still asked them about how he/she got the answer. If the strategy the child used was one of
the possible solutions, his/her solution was assigned to the category to
which it belonged even though the answer was not correct. Some examples
are provided.

Performance

The children's solutions for a one-digit adding problem whose sum
was over 10 were described first. Four categories were formed.

No idea how. Perng-Yu, 5 years and 11 months, claimed that she
had no idea how to solve the problem, $8 + 5$.

Interviewer: (One-digit adding and subtracting problems written on a
card was shown.) Do you know how to solve one-digit adding
problems?
Perng-Yu: I don't know how.

Counting onward. Chun-Jen, 6 years and 5 months, solved the one-
digit adding problem by counting up from one number.

Interviewer: Well done. How did you get the answer on $8 + 5 = 13$.
Chun-Jen: I held up five fingers and said 9, 10, 11, 12, and 13.

Recomposition around 10. Hann-Chen, 8 years and 5 months, solved
the one-digit problem by separating one number into two, adding one of the
two to the other addend to get 10, and finally, adding the remaining digit to
the subtotal--10.

Interviewer: Well done. Let's see how you got the answer on $8 + 5 = 13$.
Hann-Chen: I separated number 8 into two numbers: 5 and 3. I
then added the 5 to the addend (5) and got 10. There was 3 left;
so, I added it to 10. Then I got 13.
Known fact. When Pinn-Yi, 9 years and 1 month, saw the problem, 8 + 5, she already knew the answer. There was no need for her to calculate the problem.

Interviewer: Well done. Let’s see how you got the answer on 8 + 5 = 13. Did you get it by counting up, or by separating one number in order to add one numeral to 10? Or there was no need to think about it because you already knew the answer when you saw the problem? Pinn-Yi: I already knew the answer when I saw the problem.

The children’s solutions for the one-digit subtracting problem whose minuend was over 10 is described as follows. There were seven categories.

No idea how. Wei-Guu, 5 years and 10 months, did not know the meaning of subtraction and how to solve it.

Interviewer (I): Well done. Do you know how to solve the one-digit subtracting problem, 12 - 5?
Wei-Guu (W): What did you mean 12 minus 5?
I: You take 5 from 12.
W: (He thought for a while and tried to use his fingers.) I don’t know how.

Procedure unclear. Nae-Tsyrr, 6 years and 6 months, got the right answer on the one-digit subtracting problem, but her procedure for solving the problem was not clear.

Interviewer (I): How did you get the answer on the problem 12 - 5 = 7?
Nae-Tsyrr (N): (silent)
I: Was it right because there were seven fingers needed to go from 5 to 12?
N: Right.
I: Or did you use another way to find the answer?
N: I used another way. (She held up five fingers and then folded them one by one and said 3, 4, 5, 6, 7.)
Taking away. Yu-Ling, a 6-year-old kindergartner, solved the problem 12 - 5 by taking away 5 circles from 12 circles she drew.

Interviewer: Do you know how to solve the one-digit subtracting problem, 12 - 5?
Yu-Ling: I drew 12 circles first and then crossed out 5 circles. There were 7 circles left.

Counting downward. Ruey-Yen, 6 years and 7 months, solved the one-digit subtracting problem, 12 - 5, by counting downward 5 numbers from 12.

Interviewer: How did you get the answer on the problem 12 - 5 = 7?
Ruey-Yen: (He held up five fingers.) I remembered the number 12 and then counted five fingers backward: 11, 10, 9, 8, 7.

Counting up. Chun-Hsien, 9 years and 3 months old, got the answer on the problem, 12 - 5, by counting up 7 numbers from 5.

Interviewer: How did you get the answer on 12 - 5 = 7?
Chun-Hsien: First, I thought 5 + ? = 12. Then I figured out 5 + 7 = 12. Because when I count 6, 7, 8, 9, 10, 11, 12, there are seven numbers.

Recomposition around 10. When Yuh-Ru, 8 years and 4 months, solved the problem, 12 - 5, she used a way called recomposition around 10.

Interviewer: How did you get 12 - 5 = 7?
Yuh-Ru: The 2 is less than the 5 and cannot be subtracted from; so, I borrowed one 10 from the tens place. Then, 10 minus 5 equals 5; 5 plus the left 2 equals 7.

Known fact. Chih-Yin, 8 years and 5 months, claimed that the answer for the problem, 12 - 5, was already in her mind the moment she saw the problem.

Interviewer: How did you solve the problem 12 - 5 = 7.
Chih-Yin: Because 7 + 5 = 12.
A summary of the results is shown in Table 23. To solve the one-digit adding problem whose sum was over 10, a plurality (49%) of the children, ages 5 through 9, solved it by counting onward. All these children counted upward from the larger number. About two thirds of the 5-, 6-, and 7-year-olds used the counting-onward strategy. About four fifths of the 8-

Table 23

Solutions for One-Digit Adding and Subtraction Problems That Involved Regrouping (By Number of Children)

<table>
<thead>
<tr>
<th>Category</th>
<th>Age</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5</td>
</tr>
<tr>
<td><strong>Addition (8 + 5 = 13)</strong></td>
<td></td>
</tr>
<tr>
<td>1. No idea how</td>
<td>4</td>
</tr>
<tr>
<td>2. Counting onward</td>
<td>9</td>
</tr>
<tr>
<td>3. Recomposition</td>
<td>1</td>
</tr>
<tr>
<td>around 10</td>
<td></td>
</tr>
<tr>
<td>4. Known fact</td>
<td>1</td>
</tr>
<tr>
<td><strong>Subtraction (12 - 5 = 7)</strong></td>
<td></td>
</tr>
<tr>
<td>1. No idea how</td>
<td>11</td>
</tr>
<tr>
<td>2. Unclear</td>
<td>1</td>
</tr>
<tr>
<td>3. Taking away</td>
<td>1</td>
</tr>
<tr>
<td>4. Counting downward</td>
<td>2</td>
</tr>
<tr>
<td>5. Counting up</td>
<td></td>
</tr>
<tr>
<td>6. Recomposition</td>
<td>1</td>
</tr>
<tr>
<td>around 10</td>
<td></td>
</tr>
<tr>
<td>7. Known fact</td>
<td>1</td>
</tr>
</tbody>
</table>

n = 14 for each age level.

Addition: chi-square = 39.61; df = 12; p = .00008.

Subtraction: chi-square = 54.88; df = 24; p = .00032.
and 9-year-olds moved toward ten-structured strategy and known fact. Thirteen percent of the children, primarily 8- and 9-year-olds, solved the problem by known fact. These children gave the answer the moment they saw the problem. No children solved the adding problem by counting all.

Based on the data of the present study, a developmental sequence on children's adding ability was found. Children added the two one-digit numbers whose sum was over 10 by using the unitary counting structure, such as counting onward; using the multi-unit structure, including the division of a number so that the separating would make it convenient for one number to be added to 10. The majority of the 5-, 6-, and 7-year-olds tended to use their unitary cognitive structure to solve the adding problem. However, at age 8, most of the children preferred using their multi-unit structure to solve the problem. The differences between age levels were significant, $X^2 (12, N = 98) = 39.61, p < .05$.

To solve the one-digit subtracting problem whose minuend was over 10 (12 -5), a plurality (47 %) of the children, ages 5 through 9, used the method of recomposition around 10. About half of the 7-year-olds and almost all of the 8- and 9-year-olds employed the ten-structured method. The other solutions for solving the subtracting problem were known fact, about 6 %; counting downward, about 6 %; taking away, about 4 %; and counting up, about 1 %. The 5- and 6-year-olds either were not able to solve the subtracting problem or solved it by using a variety of methods.

Also, a developmental sequence for children's ability in solving a one-digit subtracting problem whose minuend was over 10 was found.
Children’s solutions ranged widely: using the unitary counting structure, such as taking away, counting downward and counting up; using the multi-units structure, such as recomposition around 10; and finally, using known fact. At age 6, children preferred using unitary cognitive structure to solve a subtracting problem. However, half of the 7-year-olds employed multi-unit structures to solve it. The differences among age levels were significant, $X^2 (24, N = 70) = 54.88$, $p < .05$. It was interesting to find that children had a tendency to use more multi-unit structures to solve a one-digit subtracting problem whose minuend was over 10 than they did in the one-digit adding problem whose sum was over 10.

**Comparison**

Although a direct comparison using prior research was not possible, the Cobb and Wheatly (1988) study was judged to be close enough to the present study to be used as a guide; the adding problem used in the two studies was different. In the Cobb and Wheatly (1988) study, 14 American second graders were interviewed early in the school year. When asked to solve the problem, $16 + 9$, nine (64 %) of the fourteen second graders counted onward to get the answer.

In the present study, when solving the problem, $8 + 5$, nine (64 %) of the fourteen second graders counted onward to reach the answer, 13. When solving the two-digit adding problem, $27 + 58$, about 71 % of the second graders used the ten-structured solution. In comparing the two studies, the researcher found that when subjects solved one-digit adding or subtracting problems whose sums or minuends were over 10, the Chinese second
graders preferred using the unitary counting strategy, just as the American peers did. The Chinese subjects used unitary counting in spite of the fact that they knew how to solve the two-digit adding problem by using the ten-structured strategy.

**Equivalencies between Places**

When the interviewer asked the children about regrouping procedures that were used and the values that were exchanged in the multi-digit adding and subtracting problems, their understanding of the equivalencies among the places of ones, tens, hundreds, thousands, and ten thousands were revealed. This task was adapted from Fuson and Kwon’s (1992b) study and Cauley’s (1988) study.

**Interview Strategy**

When a child carried out his/her adding and subtracting techniques on the problems, the interviewer asked the child about the regrouping procedures that were made and the values that were exchanged. For example, in an addition problem, such as $394 + 241 = 635$, the interviewer first circled the “6” in number 635 and stated that 3 plus 2 equals 5. Then the interviewer asked the child how he/she got a ‘6’ here. If the child answered, “9 plus 4 equals 13, but we can only write down ‘3’ in the tens place; the ‘1’ needs to be carried to the next place; that is why we got ‘6’ here. The interviewer asked, “So, how many does the ‘1’ that you carried to the next place stand for?” If a wrong answer was given by the child, such as 1 or 10, the interviewer reminded the child to check from the rightmost place and to see what place the ‘1’ was carried to.
This task was not administered to 3-, 4-, and 5-year-olds because of its difficulty. One thing that should be noted was that for the children who were able to solve problems up to two digits, the interviewer could only test their understanding of the equivalence between ones and tens.

**Performance**

Based on the children’s responses, six categories were formed.

**No idea how.** Chich-Fang, 6 years and 3 months, did not understand how to do carrying when there were more than 9 ones in the ones place.

Interviewer: How did you get the answer on the problem 27 + 58 = 75?
Seven plus 8 equals 15, right? You wrote down the “5.” Where was the “1”?
Chich-Fang: (He wrote down a “1” between 7 and 5. The answer became 715.)

The “1” that was carried or borrowed always means 1. When being asked about what the “1” he carried to the tens place stood for, Dyi-Ju, 7 years and 9 months, answered that the “1” meant “1.”

Interviewer (I): (The numeral 1 which was written above the numeral 2 in the number 27 was circled.) Why did you write a little “1” above numeral 2 in the problem 27 + 58 = 95?
Dyi-Ju (D): Seven plus 8 equals 15. I can’t write down both numerals 1 and 5 in the same place; so I carried the numeral 1 to the next place.
I: What did this “1” actually stand for?
D: 1.
I: Check its place value again to make sure of the value that the “1” stood for.
D: It was in the tens place.
I: So what did this “1” actually stand for?
D: 1.
The "1" that was carried or borrowed means 10 in the two-digit problems. Sheau-Lin, 6 years and 11 months, knew the "1" that was carried to the tens place meant "10."

Interviewer (I): (The interviewer circled the numeral 2 in number 27 and the numeral 5 in number 58.) In the problem $27 + 58 = 85$, you see that $2 + 5 = 7$. How did you get 8?
Sheau-Lin (S): Seven plus 8 equals 15; I carried the "1" to the next place.
I: So the "1" you carried actually stands for what?
S: 10.

The "1" that was carried or borrowed always means 10 even in three- or four-digit problems. Sheau-Wen, 7 years and 1 months, was able to solve the three-digit adding and subtracting problems correctly. However, she thought, no matter where the "1" was, all the "1" stood for was 10.

Interviewer (I): (The numeral 1 which was written above the numeral 2 in the number 27 was circled.) In the problem $27 + 58 = 85$, why did you write a little "1" above numeral 2.
Sheau-Wen (S): Because $7 + 8 = 15$; the "1" need to be carried to the next place.
I: What does it stand for?
S: 10.
I: (The numeral 5 in number 65 and the numeral 7 in number 27 were circled.) In the problem $65 - 27 = 38$, the 5 is smaller than the 7 and cannot be subtracted from. How did you do it?
S: I borrowed one 10. $10 - 7 = 3$; $3 + 5 = 8$. $6 - 1 = 5$; $5 - 2 = 3$.
I: (The numeral 1 which was written under the numeral 2 in the number 241 was circled.) In the problem $394 + 241 = 635$, why did you write a little "1" under numeral 2?
S: Because $9 + 4 = 13$, the "1" needs to be carried to the next place.
I: What does it stand for?
S: 10.
I: Can you check the place where the "1" was located?
S: The place of hundreds.
I: So, what does it stand for?
S: 1.
I: One hundred?
S: I do not know.

The regrouped “1” can mean 10, 100, 1000, 10000 with interviewer’s reminder. Wen-Chieh, 8 years and 4 months, stated that the “1” that was carried to the hundreds place in a three-digit problem meant “10.” After the interviewer reminded her to check what place the “1” was located, she changed her answer to “100.”

Interviewer (I): (The numeral 3 in number 394 and the numeral 2 in number 241 were circled.) In the problem, 394 + 241 = 635, you see that 3 + 2 = 5. How did you get “6”?
Wen-Chieh: Because 9 + 4 = 13, the “1” ten needed to be carried to the next place; so, 1 + 2 + 3 = 6.
I: How many did the “1” actually stand for?
W: 10.
I: Check where the “1” was located.
W: It was in the hundreds place.
I: So, how many did the “1” stand for?
W: 100.

The regrouped “1” can mean 10, 100, 1000, 10000 without the interviewer’s reminder. Yi-Yunn, 9 years and 10 months, solved all the problems correctly. When asked about the exchanges she made between places, she knew all the values exchanged between places without the interviewer’s reminder.

Interviewer (I): (The numeral 3 in number 394 and the numeral 2 in number 241 were circled.) In the problem, 394 + 241 = 635, you see that 3 + 2 = 5. How did you get “6”?
Yi-Yunn (Y): Because 9 + 4 = 13, the “1” needed to be carried to the next place.
I: How many did the “1” actually stand for?
Y: 10. (She immediately corrected her answer to 100.)
I: In the problem $535 - 253 = 282$, the 3 in number 535 is less than the 5 in number 253 and cannot be subtracted from. How many did you need to borrow?
Y: 100.
I: In the problem $4083 - 1253 = 2830$, the 0 in number 4083 is less than the 2 in number 1253 and cannot be subtracted from. How many did you need to borrow?
Y: 1000.
I: Good. (The numeral 4 in number 4258 and the numeral 5 in number 5831 were circled.) In the problem, $4258 + 5831 = 10089$, you see that $4 + 5 = 9$. How did you get "0"?
Y: Because $2 + 8 = 10$. The "1" needed to be carried to the next place.
I: So, how many did the "1" stand for?
Y: 1000. Then, $1 + 4 + 5 = 10$. The "1" also needed to be carried to the next place.
I: How many did the "1" you carried actually stand for?
Y: 10000.

The results are summarized in Table 24. All the 5-year-olds and about four fifths of the 6-year-olds could not do the regrouping or did not know the exchange values between one and tens in one-digit adding or subtracting problems which involved regrouping. About two thirds (9) of the 7-year-olds knew the exchanged values between ones and tens, or between tens and hundreds, or between hundreds and thousands without the interviewer's prompts. However, six of the nine 7-year-olds were able only to solve adding or subtracting problems up to two-digit numbers. Thus, their understanding of the exchange of values between tens and hundreds was not revealed. Except for one 8-year-old, about half of the 8- and 9-
year-olds knew the exchanged values between the places without prompts; the other half knew the equivalencies between places with prompts.

Table 24

Understanding of the Exchanges Among Places When Doing Addition and Subtraction (By Number of Children)

<table>
<thead>
<tr>
<th>Category</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. No idea</td>
<td>14</td>
<td>11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. 1 always means 1</td>
<td></td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. 1 means 10 in two-digit problems</td>
<td>2</td>
<td>6</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. 1 always means 10 even in three- or four-digit problems</td>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. 1 can mean 10, 100, 1000, 10000 with reminder</td>
<td>1</td>
<td>6</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. 1 can mean 10, 100, 1000, 10000 without reminder</td>
<td>3</td>
<td>6</td>
<td>7</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

n = 14 for each age level.
chi-square = 99.76; df = 20; p < .00000.

According to the data, a developmental sequence was found. As age increased, children’s understanding of the equivalence between places extending from ones and tens, to ones and tens and hundreds, to ones and tens and hundreds and thousands, and to ones and tens and hundreds and
thousands and ten thousands. The differences among age groups were significant, $X^2 (20, N = 70) = 99.76, p < .05$.

**Comparison**

In Cauley’s (1988) study, 90 second and third graders who were interviewed completed multi-digit subtracting problems that involved regrouping. Thirty-four (38%) of them demonstrated procedural proficiency with subtraction algorithm and were asked about the values that were exchanged during borrowing. Among these 34 children, only about 18% knew the values that were exchanged.

In the present study, of the 28 second and third graders, eighteen (64%) of them were able to solve the two- or three-digit subtraction problems which involved regrouping. Among the 18 children, 61% knew the values that were borrowed from the next places.

Compared with the American peers in Cauley’s (1988) study, Chinese second and third graders in the present study were less likely to know the subtraction algorithm when there was no understanding of the values that were exchanged. Chinese children, at age 7 and 8, were able to apply their place-value understanding in two- or three-digit additions and subtractions earlier than the American peers did.

**Summary**

Based on children’s responses in the present study, place-value understanding develops in a hierarchical fashion. In the oral counting task, the average oral counting numbers for 3-, 4-, and 5-year-olds were 18, 39, and 106, respectively. About half of the 6- and 7-year-olds counted to
hundreds. Almost all of the 8-year-olds were able to count to/over hundreds, such as thousands. The majority 9-year-old counted to thousands or ten thousands. As to children's oral counting errors, the majority of 3-year-olds did not make any counting errors because of their very limited capability for number generating. To extend a counting pattern, two thirds of the 4-year-olds made a variety of oral counting errors. Except for one 5-year-old, the only kind of errors made by the 5-, 6-, 7-, and 8-year-olds were decade errors; the percentage for making decade errors decreased as the ages increased. At age 9, no counting errors were made by the children.

In the object-counting task, all of the 3-, 4-, and 5-year-olds and about two thirds of the 6- and 7-year-olds counted the chips by ones. Counting by ones was the most frequently used technique for children ages 3 through 7. Gradually, the 8- and 9-year-olds moved their preference in object counting to multiples, such as by twos, by fours, by fives, and finally by tens.

When structured to count with groups of tens, about half of the 5-year-olds could not coordinate tens and ones at the basic level. Others were successful as early as age 6. All the 8- and 9-year-olds knew how to count a collection of 78 chips by tens.

In the digit-correspondence task, before some leading questions were given, no 3-year-olds and only a few 4-year-olds were able to read a two-digit number correctly. All but one 5-year-old and all 6-year-olds were able to read it successfully. However, only one sixth of the 5- and 6-year-olds could interpret a two-digit number by both its face and place values. The majority of the 7- and 8-year-olds and all 9-year-olds could interpret the
numerals in number 16 by both their face and place values. After receiving extended questions, about half the 5- and 6-year-olds and all the 7-, 8-, and 9-year-olds exhibited place-value understanding when associated with a two-digit number. The three age groups who benefited the most from adult assistance in the digit-correspondence task were ages 5, 6, and 7.

In the number-construction task, two of the 5-year-olds had no idea how to construct number 32 on the first trial; more than half of 5-year-olds constructed “32” by using the one-to-one collection structure. The majority of 6-, 7-, 8-, and 9-year-olds represented “32” by the canonical base-10 structure. On the second trial, eight 5-year-olds, two 6-year-olds, and three 7-year-olds did not know another way to construct number 32. The majority of children ages 6 to 9 had a tendency to construct “32” by using either a one-to-one collection or noncanonical base-10 representation. Except for the 8-year-olds, about half of the 6-, 7-, and 9-year-olds used the noncanonical base-10 representation on the second trial. After some demonstrations were given, the two 5-year-olds who failed on the earlier first trial constructed “32” by using the canonical representation. On the second trial, most of the thirteen 5-, 6-, and 7-year-olds who failed on the earlier second trial, successfully constructed, after adult assistance, the number 32 in ways different from the way used on the first trial. However, one 5-year-old and one 6-year-old were not able to meet criteria, even after assistance was given. The three age groups of children benefited from adult assistance in the number-representation task.
On adding problems, about two thirds of the 5-year-olds and 6-year-olds correctly solved the one-digit problem whose sum was over 10. All the 7-year-olds solved the problems to or over two digits. The majority of the 8-year-olds solved the problems having three digits. All the 9-year-olds solved the problem, up to four digits.

On subtracting problems, 12 of the 5-year-olds could not solve the one-digit problem in which the minuend was over 10. About half of the 6-year-olds solved the problems in which the minuend was over 10. For 7-year-olds, over one third of them solved the problems up to or over two digits. The majority of the 8-year-olds solved the three-digit problem. All the 9-year-olds solved the four-digit problem easily.

To solve a one-digit adding problem whose sum was over 10, two thirds of the 5-, 6-, and 7-year-olds used the counting-onward strategy. About four fifths of the 8- and 9-year-olds moved toward ten-structured strategy and known fact. To solve the one-digit subtracting problem whose minuend was over 10, the 5- and 6-year-olds either were not able to solve the subtracting problem or solved it by using a variety of unitary methods. Half of the 7-year-olds and almost all the 8- and 9-year-olds employed the ten-structured method.

In the task of equivalence between digits, all the 5-year-olds and about four fifths of the 6-year-olds could not do the regrouping or did not know the exchange values between one and tens in one-digit adding or subtracting problems which involved regrouping. Two thirds of the 7-year-olds knew the exchange values between ones and tens. Half of the 8- and 9-
year-olds knew the exchanged values between the places without prompts; the other half knew the equivalencies between places with prompts.

After being compared with the American and Genevan children whose performances on similar place-value tasks were described in the literature, the Chinese children demonstrated at an earlier age level the mastery of place-value tasks. Despite the different age levels for achieving place-value tasks, all the American, Chinese, and Genevan children went through the same developmental sequence in obtaining an understanding of the place-value numeration system.
CHAPTER 5

CONCLUSIONS AND IMPLICATIONS

Summary

The four objectives of the study were to (a) describe the development of place-value numeration concepts in Chinese children ages 3 through 9; (b) compare the development of place-value understanding of Chinese children with that of American and Genevan children, the latter having been described in the literature; (c) examine the influence of adult assistance, such as verbal prompts, questions, and demonstrations during Chinese children’s performances on place-value tasks; and (d) formulate alternatives that will assist young children in their construction of place-value concepts.

The subjects were 98 Chinese children. There were 14 children (7 boys, 7 girls) for each age level, 3 through 9. The 3-, 4-, and 5-year-old subjects were enrolled in a private early childhood program in Taipei, Taiwan; the older subjects, ages 6 through 9, were enrolled in an elementary school in Taipei, Taiwan. The two schools were selected because their students represented various social and economical backgrounds, ranging from low to high in socioeconomic status. The subjects selected in this study were randomly selected from each school’s enrollment lists.

A standardized interview method with emphasis on uncovering a child’s mental processes when he/she was dealing with place-value tasks was adapted for this study. Tasks and procedures were adapted from
several cognitive studies in the place-value domain. Data collection modes included interviewing children, observing their actions and modes of expression during the interview sessions, videotaping interviews, and transcribing and coding children’s oral and other behavioral responses. The sets of data collected were analyzed both quantitatively and qualitatively in order to answer the research questions.

Research Questions

The first purpose of the present study was to describe Chinese children’s development of place-value numeration concepts. The following questions were asked in order to redefine this purpose: How do Chinese children perform counting and place-value tasks at different age levels, 3 through 9? Through what developmental sequences of place-value understanding do Chinese children go?

The second purpose of the present study was to compare the development of place-value understanding of Chinese children with that of American and Genevan children whose performances have been described in the literature. Formulated as research questions, this purpose was redefined. Do Chinese children go through the same developmental courses of place-value understanding as do American and Genevan children? Do Chinese children have the same cognitive limitations when forming their conceptual structures of place value as that described in the literature that dealt with American and Genevan children? What is the age level at which the majority of Chinese children demonstrate their understanding of the place-value numeration system; what does the literature say about the age
level at which the majority of the American and Genevan children reach understanding?

The third purpose of the present study was to examine the influence of adult assistance during Chinese children’s performances on place-value tasks. The research question, parallel to the purpose, asked: How does adult assistance facilitate Chinese children’s performances on place-value tasks at the different age levels?

Conclusions

The present study yielded five primary conclusions that were drawn from the analyzed collected data. First, the Chinese children’s performances in a variety of tasks indicated a developmental progression in understanding the common place-value numeration system. Second, after comparing the Chinese children’s place-value understanding with that of American and Genevan children whose performances were delineated in the literature, it was judged that all children go through the same developmental sequence in comprehending the place-value numeration system. Third, it appeared that the inability to create the hierarchical structure of numerical inclusion (part-whole numerical relations) was a universal cognitive limitation common to all younger children in their attempt to comprehend the place-value numeration system. Fourth, based on the comparisons, the Chinese apparently formed the base-10 conceptual structure at earlier age levels than did the American and Genevan children. The structures of Chinese spoken number words evidently had influences on children’s construction of place-value understanding. Fifth, in the present study, adult
assistance during a child’s performance in some place-value tasks involved a sort of “scaffolding” process that led the child in a direction that enabled him/her to solve problems that would have been beyond his/her unassisted efforts.

**Developmental Sequence**

A final summary of children’s performances in each task regarding place-value understanding is in Table 25. The children’s responses gave evidence that children at different ages performed place-value tasks differently and that they revealed a gradual developmental progression in accomplishing the place-value tasks.

**Learning Conventional Representations Orally and Graphically and Constructing a Unitary Cognitive Structure**

To understand the place-value numeration system, a child has to first become engaged in the process of learning the conventional representations orally and then to do so graphically. In the present study, all the 3- and 4-year-olds recited some number names and recognized some single-digit numerals; however, this learning was basically memorization. They had no idea about ones and tens. Thus, a two-digit number, for them, was only two numerals placed side by side; there was no difference to a person who read the number from right to left. Being built on their ever-extending oral- and object-counting abilities, unitary thinking was formed.

**Inducing the Rules for Generating Two-Digit Numbers and Recognizing the Positional Property of Two-Digit Numerals**
Along with their advanced oral counting ability and expanded numeral recognition, some of the 5- and 6-year-olds began inducing the rules for generating two-digit numbers up to 100 and recognizing that

Table 25

Summary of the Children's Performances on the Tasks. (By Percentages of Children at an Age Level)

<table>
<thead>
<tr>
<th>Task</th>
<th>Age</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3</td>
</tr>
<tr>
<td>1. Money counting</td>
<td></td>
</tr>
<tr>
<td>Counting all coins as one-dollar</td>
<td>86</td>
</tr>
<tr>
<td>Adding one- and ten-coins together</td>
<td>57</td>
</tr>
<tr>
<td>2. Oral counting</td>
<td></td>
</tr>
<tr>
<td>10s to 120</td>
<td>86</td>
</tr>
<tr>
<td>100s to 1020</td>
<td></td>
</tr>
<tr>
<td>10000s</td>
<td></td>
</tr>
<tr>
<td>3. Counting errors</td>
<td></td>
</tr>
<tr>
<td>Decade errors</td>
<td></td>
</tr>
<tr>
<td>No error</td>
<td>86</td>
</tr>
<tr>
<td>4. Counting objects</td>
<td></td>
</tr>
<tr>
<td>By ones</td>
<td>100</td>
</tr>
<tr>
<td>By twos, fours, fives etc.</td>
<td></td>
</tr>
<tr>
<td>5. Counting by tens</td>
<td></td>
</tr>
<tr>
<td>Not tested</td>
<td>--</td>
</tr>
<tr>
<td>Unable to do it</td>
<td></td>
</tr>
<tr>
<td>Counted by tens</td>
<td></td>
</tr>
</tbody>
</table>
Table 25 -- (continued)

<table>
<thead>
<tr>
<th>Task</th>
<th>Age</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3</td>
</tr>
<tr>
<td>6. Digit-correspondence</td>
<td></td>
</tr>
<tr>
<td>Before adult assistance</td>
<td></td>
</tr>
<tr>
<td>Unable to read number 16</td>
<td>100</td>
</tr>
<tr>
<td>Interpreted the digits in 16</td>
<td></td>
</tr>
<tr>
<td>Interpreted the digits in 16 by their face value only</td>
<td></td>
</tr>
<tr>
<td>Interpreted the digits in 16 by their face and place values</td>
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<tr>
<td>After adult assistance</td>
<td></td>
</tr>
<tr>
<td>Not tested</td>
<td>--</td>
</tr>
<tr>
<td>Still interpreted the digits in 16 by their face value only</td>
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<tr>
<td>Interpreted the digits in 16 by their face and place values</td>
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<td>7. Constructing number 32</td>
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</tr>
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<td>Before adult assistance</td>
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<td>Trial 1</td>
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</tr>
<tr>
<td>One-to-one collection</td>
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<tr>
<td>Canonical base-10</td>
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<tr>
<td>Trial 2</td>
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</tr>
<tr>
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<tr>
<td>Canonical base-10</td>
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Table 25 -- (continued)

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<td>8. Constructing number 32</td>
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<td>Trial 1</td>
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<tr>
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<td>9. Addition</td>
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<td>1-digit (sum over 10)</td>
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<td>10. Subtraction</td>
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<td>86</td>
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<tr>
<td>1-digit (the minuend over 10)</td>
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<td>64</td>
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Table 25 -- (continued)

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<td>11. Solution for one-digit addition sum over 10</td>
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<tr>
<td>One-structured</td>
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<tr>
<td>Ten-structured</td>
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<td>12. Solution for one-digit subtraction minuend over 10</td>
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<td>Not tested</td>
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</tr>
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<td>Ten-structured</td>
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<td>13. Equivalence between places</td>
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<tr>
<td>Not tested</td>
<td>--</td>
</tr>
<tr>
<td>No idea</td>
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<tr>
<td>Ones and tens</td>
<td>71</td>
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<tr>
<td>Tens and hundreds and thousands with reminder</td>
<td>43</td>
</tr>
<tr>
<td>Tens and hundreds and thousands without reminder</td>
<td>43</td>
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</table>

different places in a two-digit number stand for different values: ones and tens. The positional property of two-digit numerals was being recognized at the ages of 5 or 6. However, a great number of them could not grasp the precise meaning of the different digits in two-digit numerals. They also
were unable to realize that the value of a two-digit number was the sum of the values of ones and tens.

Refining Unitary Conceptual Structure and Initiating Ten-Structured Thinking

While some children were in the process of refining their unitary conceptual structure at age 5, other children had begun initiating their thinking in terms of tens. However, the majority of the children who tried working on a system of tens experienced some cognitive limitations, such as lacking of conservation and reversibility. This, therefore, limited their handling of ones and tens simultaneously. These two seemingly inherent systems were thus independent of each other. Therefore, the majority of them felt more secure in using a unitary conceptual structure (system of ones). Their problem-solving and numerical operations honestly reflected the one-to-one conceptual structure time after time.

Including Ones to Tens

At age 6, about half of the children were able to work on the multi-unit conceptual structure simply because of their capability to think of ones and tens simultaneously. Yet it was still difficult for some of them to tell the precise meaning of individual digits in a two-digit numeral without adult assistance. Consequently, their ability to apply the early place-value understanding to one-digit addition or subtraction involving regrouping was extremely limited.
Recognizing the Precise Meaning of the Individual Digits in Two-Digit Numerals and the Value of a Two-Digit Number as the Sum of the Values of Ones and Tens

Along with even more expanded numeral knowledge and a more functional kind of thinking at age 7, the children's understanding of numerical part-whole relations became operational. With this understanding, the majority of 7-year-olds recognized the precise meaning of the individual digits in two-digit numerals and the value of a two-digit number as the sum of the values of ones and tens; they also extended the number-generating rules up to thousands.

Refining the Ten-Structured Cognitive Structure and Applying the Understanding of Equivalence between Ones and Tens to Two-Digit Written Arithmetic

The majority of the 7-year-olds were in the process of refining their ten-structured thinking. After they had gained a better understanding of the equivalence of ones and tens, they were better prepared to apply this part-whole schema to two-digit written arithmetic involving regrouping.

Extending the Multi-Unit Conceptual Structure to Written Arithmetic Associated Three or Four Digits

At age 8, children extended their multi-unit thinking to hundreds and then applied it to three-digit written arithmetic. For the 9-year-olds, their multi-unit conceptual structure was applied to written arithmetic beyond the place of thousands; the majority of the 9-year-olds extended the number generating rules up to ten thousands. The children's place-value
understanding was increasingly stable and complete at ages 8 and 9. The majority of the 8-year-olds and almost all the 9-year-olds accomplished all the place-value tasks with confidence.

When the Chinese children's place-value understanding in the present study is compared with that of American and Genevan children described in the literature, a similar developmental progression can be found. Performances by American, Chinese, and Genevan children indicated that all the children in the three groups exhibited a developmental progression in achieving place-value understanding. For example, both C. Kamii's (1986) Geneva study and the present research suggest that children's numerical conceptual structure develops gradually, from being able to think only in terms of ones to thinking in tens. The two systems first appear to be independent of each other, but, later, the child begins to see an alliance between the two. As development advances, simultaneous thinking in terms of ones and tens becomes increasingly sophisticated.

In sum, the children's understanding of the place-value numeration system was a gradual process. Although the age levels at which individuals reached a developmental stage were variable, all children progressed through a similar sequence in obtaining place-value understanding.

In addition, two interesting things were found incidentally. First, about four fifths of the Chinese 5- and 6-year-olds in the present study, before adult assistance were unable to understand that the value of a two-digit number was the sum of the values of the ones and the tens. However, over half of the 5-year-olds and almost all the 6-year-olds were able to
comprehend the values of one- and ten-dollar coins and to count a collection of one- and ten-dollar coins correctly. The 5- and 6-year-olds in the present study were better able to think by ones and tens simultaneously when the task represented a practical, everyday life experience (Issacs, 1966).

Second, in the task of representation of a two-digit number, about six 9-year-olds, eleven 8-year-olds, five 3-year-olds, and four 6-year-olds used the canonical base-10 conceptual structure to represent number 32 on the first trial, but they went back to using the one-to-one structure to construct "32" on the second trial. Compared to age peers who used the noncanonical structure on the second trial, these children were less likely to solve adding and subtracting problems which involved regrouping by using the ten-structured method or to reach an understanding of equivalencies between digits. Seemingly, the more flexible conceptual structure, such as noncanonical structure, in which more than 9 units can be in any place facilitates children's ability to solve addition and subtraction that involved regrouping.

Cognitive Limitation

In addition to a common developmental sequence found in American, Chinese, and Genevan children's development of place-value understanding, a pervasive cognitive limitation had also been found in the younger children's performances on place-value tasks. The recognizable result in cognitive limitation in the development of a place-value understanding is the inability to create the hierarchical structure of numerical inclusion (the understanding of part-whole number relation). For
example, in both Miller and Stigler’s (1987) study and the present study, the common and major oral counting error for both the Chinese and American children was decade transition error; the majority of Chinese 5-, and 6-year-olds in the present study and the majority of American 8 year-olds in Silvern and C. Kamii’s (1988) study were unable to find that the value of a two-digit number was the sum of the values of the ones place and the tens place; and all the American, Chinese, and Genevan children first were able to work on a system of ones, then to think in terms of either tens or ones (the two were independent of each other), and finally, to work on the two systems together.

Evidently, the understanding of the part-whole relationship of number (the ability to create the hierarchical structure of numerical inclusion) was an integral factor for the development of place-value understanding. The ability to think in terms of a system of ones and a system of tens simultaneously requires children’s conservation and reversibility.

**Linguistic Influence**

Even though they had a similar developmental sequence and a like cognitive limitation in understanding the place-value numeration system, the age levels at which American, Chinese, and Genevan children reached a given developmental stage differed.

Compared with Miller and Stigler’s (1987) study, the majority of the Chinese 5-year-olds in the present study were able to create some rules and to apply them to generate their number word sequences beyond 100; however, the American 5-year-olds in Miller and Stigler’s study could only
generate numbers up to 73. Chinese children seem to induce the number-generating rules from their spoken numerical language much earlier than do American children.

Except for one 5-year-old, the only oral counting error made by Chinese 5-, 6-, 7-, and 8-year-olds in the present study was the decade transition error. It indicated that the majority of Chinese children from age 5 had grasped the ten-structured rules necessary to generate number sequence. However, in Miller and Stigler’s (1987) study, the American 5-year-olds still made variant kinds of oral counting errors, such as nonstandard numbers and skipping numbers.

Compared to C. Kamii’s Geneva (1986) study, the present study showed that, when asked to count a collection of chips in groups of tens, half of the Chinese 5-year-olds and five sixths of the Chinese 6-year-olds knew how to count them by using multi-unit structure earlier than did the Genevan children, who could not use this structure until around 8 years of age.

In the present study, the Chinese 5-year-olds demonstrated their initial ten-structured cognitive structure. The age levels paralleled the age levels at which children demonstrated a good grasp of the spoken language. Based on this finding (although some other cultural factors, such as school experiences, parental attributions, societal expectations, etc., might have affected the different performances by the American, Chinese, and Genevan children), language is the main factor associated with the Chinese children’s early construction of a multi-unit conceptual structure. This may suggest
that the regular named-values associated with Chinese spoken number words assist children in inducing the rules of number generation in the place-value numeration system.

Additionally, in the Silvern and C, Kamii's (1988) study, some American children pointed out that the "1" in number 16 stood for 10, but actually moved only one chip. This kind of response never happened in the present study. Also, compared to Miura et al.'s (1988) study, Chinese children preferred canonical base-10 representation for a two-digit number, and this was different from the one the American children preferred (one-to-one collection). Evidently, young Chinese children mentally organize numbers as structures of tens and ones. This multi-unit conceptual structure may be influenced by the Chinese spoken numerical language, which supports children in fostering a view that two-digit numbers are the compositions of ones and tens. In English, the numbers from 10 to 99 do not articulate the value of tens and ones. Lacking a numerical language system that incorporates place value, English-speaking children see numbers as collections of units.

In sum, Chinese children, some 5- and 6-year-olds and the majority of the 7-year-olds in the present study, demonstrated their place-value understanding associated with two-digit numbers. The 8- and 9-year-olds had mastered the processes of extending the multi-unit conceptual structure to three- and four-digit numbers. For American and Genevan children, place-value understanding associated with two- or three-digit numbers was fragile, incomplete, and unstable for the majority of 8- and 9-year-olds. The
spoken number words, whether articulating the place-value structure or not, can be one of the explanations.

**Effect of Adult Assistance**

Based on the results before and after the interviewer's assistance, which was given in the digit-correspondence and the number-representation tasks, the effects of adult assistance were apparent, especially for the age groups 5, 6, and 7. As described in the previous sections, the 5-, 6-, and 7-year-olds, with their well-grounded understanding of language, understood the base-10 rules for generating numbers and the positional property of two-digit numerals. They had begun to initiate and form the ten-structured numerical thinking. During the processes of constructing and refining their multi-unit cognitive structure, the 5-, 6-, and 7-year-olds benefited the most from adult assistance in the number-representation and the digit-correspondence tasks. It seemed that the adult assistance given during a child's performance in place-value tasks involved a sort of "scaffolding" process that led the child in a right direction and then enabled him/her to solve a problem that would be beyond his/her unassisted efforts.

**Implications**

The analysis of data in the present study holds implications for several areas of early childhood education. First, implications for a place-value curriculum for children of different ages in different countries can be drawn from the findings of the present study. Second, suggestions for further research on the study of young children's construction of place-value understanding also can be made based on the findings.
Implication for Education

The findings of the present study hold implications for instructional strategies and topics for American, Chinese, and Genevan children.

Implication for Chinese Teachers

The following educational implications are made for Chinese teachers.

1. Although children go through a common developmental sequence to acquire place-value understanding, there are individual differences. Developmentally appropriate instructions on numerical learning are needed to facilitate the learning of children who are at different developmental levels.

2. In understanding the place-value numeration system, children first form a cognitive structure; then they use it in problem-solving; and, finally, they apply it to written arithmetic. Numerical teaching in school should be based on the same order. Otherwise, children will lack a connection between a symbol system and understanding.

3. Children, especially the younger ones, are interested in and are motivated by the learning activities that push children into a practical and everyday life context, which is undergirded by meaningfulness. Learning activities that represent situations in everyday life should be designed and administered in numerical teaching. For example, the “school store” and the “savings bank” could be instrumental in introducing the child to the real world.

4. The regular named-value structure of Chinese spoken number
words tends to support children's construction of a multi-unit conceptual structure. Verbalization and interaction between children and children or between children and adults should be an integral part of children's numerical learning.

5. Although the Chinese spoken number words were able to assist in children's place-value understanding for younger children, who are in the process of grasping a good understanding in language, some manipulative and concrete materials should be adopted and used in numerical instruction.

6. Since children cognitively benefit from social communication with competent peers or adults, cooperative numerical learning should be included in the curriculum.

7. The evaluation techniques related to young children's numerical understanding should not be paper and pencil only. Some alternative assessments on children's numerical understanding should be included, such as observing, questioning, interviewing, and using the results from problem-solving tasks, including evaluation of hands-on activities.

Implications for American Teachers

The implications for Chinese teachers also hold for American teachers' numerical instructions. Also, there is additional implication for American teachers because of the lack of a regular named-ten structure in English spoken number words between 11 to 99.

The Chinese tens words, such as one-ten-one (11), one-ten-two (12), . . . , nine-ten-nine (99), could be introduced and used as words to tell the meaning of the English spoken number words between 10 to 99. It might
support English-speaking children in fostering a view of two-digit numbers as composites of ones and tens and in understanding the precise meaning of individual digits in two-digit numerals. This kind of understanding can facilitate American children’s understanding of equivalencies between places, which is a prerequisite for addition and subtraction involving regrouping.

**Implications for Further Research**

The researcher suggests two changes for replicating the present study. First, although different kinds of rapport-building strategies were tried during the interviewing, a few children seemed a bit uncomfortable in talking with the researcher. The elimination of any kind of uncomfortableness, which may weaken an interview, is desirable. A suggestion is made for future investigation: After selecting the subjects for an interview, there should be an ample time arrangement, whereby the interviewer and the interviewee will have the opportunity to know each other in situations other than interviewing. Second, because children are able to use their numerical knowledge in an everyday context, future investigation should also be undertaken by employing observation methods to collect data regarding children’s place-value understanding.

The following recommendations for further research are based on the results of the present study:

1. Future study should be conducted either to support or refute the developmental sequences of place-value understanding proposed in the present study.
2. Future longitudinal studies should be conducted to describe changes in children’s understanding of the place-value numeration system.

3. Future research is recommended to study the linguistic influences on children’s place-value understanding by including three groups of subjects: Chinese children, American children, and Chinese-American children who are being raised in a Chinese family that speaks English.
APPENDIX A

PERMISSION LETTER AND CONSENT FORM
Dear Parents,

I will be conducting a research project that is designed to study how children think about the place-value numeration system. I request permission for your child to participate. This study consists of a thirty-minute session where children will do counting, construct a two-digit numeral by using base-10 blocks, and do some multi-digit addition and subtraction problems (for older children) and talk about the strategies they used with these problems. Each child will be invited to go to a quiet room to be interviewed. At the beginning of the interview session, he or she will be informed that in these questions there is no right or wrong answer. This is done in order to minimize children’s anxiety.

Interview will be conducted by me and videotaped by my research assistant. Children’s responses will be reported as group results only. Individual taped responses will be used as examples of the scoring procedure, but the children will not be identified by last name. At the study conclusion, videotapes will be retained by me. These tapes may be viewed by the child’s teachers, and some may be shown to groups when the study is presented to students, teachers, and at professional conferences. To preserve confidentiality, only first names will be used to identify children.

Your decision whether or not to allow your child to participate will in no way affect your child’s standing in his or her class/school. At the conclusion of the study, a summary of group results will be made available to all interested parents and teachers. Should you have any questions or desire further information, please feel free to call me at 934-4096. Thank you in advance for your cooperation and support.

Sincerely,

Sy-Ning Chang

This project has been reviewed by University of North Texas Committee for the protection of human subjects (Phone: 1-817-565-3940).
Please indicate whether or not you wish to have your child participate in this project by checking one of the options below and returning this consent form to your child’s teacher as quickly as possible.

( ) I do grant permission for my child to participate in this project.
( ) I do not grant permission for my child to participate in this study.

Date __________________ Children’s Name ____________________

Parent/Guardian’s Signature ________________________________
APPENDIX B

CODING SHEETS
1. _____ No.

2. 3 4 5 6 7 8 9 Age

3. 1 2 Sex
   
   1-F 2-M

4. 1 2 3 4 5 SES
   
   1-Low 2-Lower-middle 3-Middle
   4-Upper-middle 5-High

5. 0 1 2 3 4 5 6 7 8 A child's favorite pastime
   
   0-Housework 1-Playing musical instrument 2-Reading 3-Video games
   4-Sports 5-Play 6-Watching TV
   7-Others 8-Not tested

6. 0 1 2 3 4 5 6 7 8 Instances of using numbers
   
   0-No idea 1-Counting objects 2-Telling time 3-Using money 4-Teaching other youngsters numbers
   5-In math class and doing math work and homework 6-Taking exam 7-Others 8-Not tested

7. 0 1 2 3 4 5 6 8 Knowing the Highest Place Value on an Abacus
   
   0-No 1-Ones 2-Tens 3-Hundreds
   4-Thousands 5-Ten thousands 6-Hundred thousands 8-Not tested
Experiences of Using Money

0-No 1-With parents 2-Sometimes 3-Usually
8-Not tested

Recognizing Money

0-No 1-One dollar 2-Ten dollars 3-Both
one- and ten-dollar coins

Adding money

0-No 1-One dollar 2-Ten dollars 3-Both
one- and ten-dollar coins 7-Others

Task 1: Counting

Subtask 1: Oral Counting (Ordinal Sequence)

Ability to generate numbers

1-Single digit 2-Two-digit 3-87 to 121
4-987-1021 5-9987-10021 6-99987-100021

Error type

0-No 1-Mixing up numbers 2-Skipping
numbers 3-Repeating numbers 4-Decade
errors 5-Skipping numbers and decade errors
6-Repeating numbers and decade errors
7-Skipping numbers, repeating numbers, and
decade errors

Subtask 2: Object counting

The child counted _____ objects.
Strategy for grouping objects
0-No idea 1-By 1 2-By 2 3-By 3 4-By 4 5-By 5 6-Combination (not based on 10) 7-By ten

Item 3: Counting by Ten
0-No idea how 1-Making groups of 10 and leaving a group of 8 and counting each group as “1” 2-Making groups of 10 and leaving a group of 8 and counting each group as “10” 3-Making groups of 10 and leaving a group of 8 and counting each group by adding “10”, including the group of 8 objects 4-Making groups of 10 and leaving a group of 8 and counting each group of 10 by adding “10”, and counting the group of 8 by adding “1” 5-Making groups of 10 and leaving a group of 8 and counting each group of 10 by adding “10”, and counting the group of 8 by adding “8” 7-Others 8-Not tested 9-Successful, not tested

Task 2: Digit-Correspondence

Before the cue questions were given:
A child’s recognition and interpretation of
A child's recognition and interpretation of a two-digit number - 16
0-No recognition of either numeral
1-Recognized only numeral "1"
2-Recognized only numeral "6"
3-Recognized both numerals but saw them in reverse order
4 - Recognized both numerals in correct order but interpreted them by their face-values
5-Recognized both numerals in correct order, saw them as a two-digit number, but interpreted it by the face-values of the digits
6-Recognized both numerals in correct order, saw them as a two-digit number, and interpreted the digits by both their face- and place-values
8-Not tested

After the cue questions were given:
12-2. 0 1 2 3 4 5 6 8 9
them by their face-values  5-Recognized both numerals in correct order, saw them as a two-digit number, but interpreted it by the face-values of the digits  6-Recognized both numerals in correct order, saw them as a two-digit number, and interpreted the digits by both their face- and place-values  8-Not tested  9-Successful, not tested

13-1. 0 1 2 3 4 5 8

Task 3: Representation of Two-Digit Number - 32

A child's recognition of a two-digit number 0-No recognition of either numeral 1-Recognized only numeral “3” 2-Recognized only numeral “2” 3-Recognized both numerals but saw them in reverse order 4- Recognized both numerals in correct order but saw them by their face-values only 5-Recognized both numerals in correct order and saw them by both their face- and place-values  8-Not tested
Before the demonstrations were given

13-2. 0 1 2 3 8  
Trial 1  
0-No idea how 1-One-to-one representation 2-Canonical base-10 representation 3-Noncanonical base-10 representation 8-Not tested

13-3. 0 1 2 3 8  
Trial 2  
0-No idea how 1-One-to-one representation 2-Canonical base-10 representation 3-Noncanonical base-10 representation 8-Not tested  
Representation of Two-Digit Number-32

After the demonstrations were given

13-4. 0 1 2 3 8 9  
Trial 1  
0-No idea how 1-One-to-one representation 2-Canonical base-10 representation 3-Noncanonical base-10 representation 8-Not tested  
9-Successful, not tested

13-5. 0 1 2 3 8 9  
Trial 2  
0-No idea how 1-One-to-one representation 2-Canonical base-10 representation 3-Noncanonical base-10 representation 8-Not tested
9-Successful, not tested

Task 4: Addition and Subtraction

14-1. 0 1 2 3 4 8 A child’s highest ability to solve adding problems
0-No idea how 1-One-digit, the sum over ten 2-Two-digit 3-Three-digit 4-Four-digit 8-Not tested

14-2. 0 1 2 3 4 8 A child’s highest ability to solve subtracting problems
0-No idea how 1-One-digit, the subtrahend over ten 2-Two-digit 3-Three-digit 4-Four-digit 8-Not tested

14-3. 0 1 2 3 4 5 7 8 A child’s solution for single-digit addition
0-No idea how 1-Unclear 2-Counting all one by one 3-Counting onward 4-Recomposition around ten 5-Known fact 7-Others 8-Not tested

14-4. 0 1 2 3 4 5 6 7 8 A child’s solution for single-digit subtraction
0-No idea how 1-Unclear 2-Counting downward 3-Counting up 4-Taking away 5-Recomposition around ten 6-Known fact 7-Others 8-Not tested
14-5. 0 1 2 3 4 8

A child's understanding of the equivalence between digits

0-No idea  1-1 means 1  2-Concatenated single-digit conceptual structure (1 always means 10)  3-Understands that 1 can mean 10, 100, 1000, 10000 with reminder  4-Understand that 1 can mean 10, 100, 1000, 10000 without reminder  8-Not tested
REFERENCES


