MAGNETO-OPTICAL AND CHAOTIC ELECTRICAL PROPERTIES OF n-InSb

DISSERTATION

Presented to the Graduate Council of the University of North Texas in Partial Fulfillment of the Requirements

For the Degree of

DOCTOR OF PHILOSOPHY

By

Xiang-Ning Song, B.S., M.S.
Denton, Texas
December, 1991
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Song, Xiang-Ning, Magneto-Optical and Chaotic Electrical Properties of n-InSb. Doctor of philosophy (Physics), December, 1991, 150 pp., 2 tables, 35 illustrations, bibliography, 80 titles.

High resolution magneto-optical techniques have been used to study donor impurities in n-InSb. As a result, a new kind of the optical transition between magneto-donor states in InSb, assisted by the emission of longitudinal optic phonons, has been observed and described. The phonon-assisted excitations allow the unique opportunity to study high excited states of an electron subjected to simultaneous Coulomb and magnetic field interactions. High excited states of the magneto-donor system were seen up to principal quantum number \( n = 13 \), providing information on the excited states of a system which simulates the hydrogen atom in gigantic magnetic fields. A three-level \( \mathbf{P} \cdot \mathbf{F} \) model was used to derive the energy eigenvalues for the donor in the presence of a magnetic field and impurity potential. The magneto-donor states have been described variationally, taking into account the narrow energy gap and the spin-orbit interaction of the band structure of InSb.

In addition, the chaotic electrical properties of n-InSb have been studied experimentally and theoretically. Both self-generated and driven oscillations were experimentally investigated in InSb in the presence of a magnetic field at the liquid helium temperatures. Various types of oscillations and chaotic behavior were observed, including Hopf bifurcation to a simple periodic
oscillation, quasiperiodic oscillation, period-doubling bifurcation scenario, and period-having bifurcation. The prediction of the Feigenbaum number $\delta$ of $\approx 4.62$ from the data was shown to be in good agreement with the theoretically predicted value of $\delta = 4.669$. Nonlinear dynamic methods, such as phase portraits, power spectra, correlation function, Lyapunov exponent spectrum, were applied in the characterization of the periodic and chaotic oscillations. The observed chaotic attractor was characterized by the Lyapunov spectrum $(+,0,-)$. The simulation based on the dielectric relaxation of the longitudinal and transverse field coupled with the magneto-donor impact ionization also provided a qualitative description of the dynamic behavior in InSb.
ACKNOWLEDGEMENTS

I gratefully thank D. G. Seiler and W. Zawadzki for collaboration on the research work and helpful comments. I acknowledge the support of this work by the National Science Foundation under grant No. DMR-8617823.
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CHAPTER 1

INTRODUCTION

This thesis investigation concerns the optical and nonlinear
electrical properties of n-InSb. Two specific areas have been
studied. First is the magneto-optical study of magneto-donors, and
second is the nonlinear dynamic study of nonlinear and chaotic
oscillations in InSb. The magneto-optical study of InSb provides a
physical picture of the magneto-donor levels, which has an
important impact on the physical model of nonlinear and chaotic
oscillations. Thus, the subjects discussed in this thesis connect
the discipline of semiconductor physics with the field of nonlinear
dynamics. Both of these fields are currently of great interest.

1.1 Magneto-Optical Study of InSb

Magneto-donor (MD) states in semiconductors, since their
discovery in the magnetic freeze-out effect by Keyes and Sladek\(^1\)
and the pioneering theoretical description of Yafet et al.\(^2\), have
been the subject of sustained experimental and theoretical
interest. The effect of a magnetic field on shallow donor states is
particularly important in narrow gap materials with small
effective masses \(m^*\) since, in the absence of the field, the donors
are ionized even at low temperatures and can not be observed.
Important progress in the magneto-optical investigations of
shallow donors has been achieved by Kaplan\(^3\), who used the 
photoconductive detection technique to observe both low-energy 
transitions between MD states belonging to the same Landau 
subband, as well as those occurring between MD states belonging to 
adjacent Landau subbands. The latter is sometimes referred to as 
"donor-shifted cyclotron resonance." Similar MD transitions 
related to spin resonance\(^4\) and combined cyclotron-spin resonance\(^5\) 
have also been observed in InSb. Magneto-donor investigations have 
been used to determine the static dielectric constant of a 
material\(^6\) and its pressure dependence\(^7\), to identify the chemical 
nature of impurities\(^8\), to study screening properties of the 
electron gas\(^9\), to investigate the metal-nonmetal transition\(^10\), 
etc. Recently, magneto-optical and magneto-transport 
investigations proved to be useful in determining the positions of 
donors in modulation-doped two-dimensional GaAs-GaAlAs 
structures\(^11,12\), which is important for device applications.

The importance of the magneto-donor system goes beyond 
semiconductor physics, however, since a magneto-donor imitates the 
hydrogen atom in giant magnetic fields. The problem of an electron 
subjected to simultaneous Coulomb and magnetic field interactions 
is characterized by the parameter \(\gamma = (\hbar e B/m^*)/2Ry^*\), where \(Ry^* = 13.6(m^*/m_0)(1/\kappa_0)\) eV, and \(\kappa_0\) is the static dielectric constant. The 
value of \(\gamma\) is of the order of \(10^{-5}\) for the hydrogen atom in vacuum. 
In narrow gap semiconductors, however, \(\gamma\) can attain values of 100 or 
more for available magnetic field strengths. In the experiments 
the typically \(\gamma\) value is \(\approx 25\), which corresponds to the hydrogen
atom in vacuum subjected to a magnetic field of $\approx 10^7$ T. The above scaling allows one to transpose magneto-donor behavior to that of the hydrogen atom in gigantic magnetic fields.

The problem of atoms in extremely large magnetic fields has attracted a great deal of attention in recent years both in atomic physics and astrophysics. The reason is that white dwarf stars can produce magnetic fields of $10^3$ T and accreting neutron stars fields of the order of $10^6 - 10^8$ T. As a consequence, the optical spectra of atoms in such magnetic fields have been observed in the spectra of these stars. For example, very good agreement between all observed spectral features and the computed wavelengths of stationary transitions of the hydrogen atom in magnetic fields $1.5 - 3.5 \times 10^4$ T has been found in the white dwarf GrW+70°8L47. By virtue of scaling laws the energies of heavier atoms can also be gained from those of the hydrogen atom at scaled values of magnetic field. This allows one to study, for example, Fe XXVI in gigantic fields, which appears to be prevailing in the vicinity of X-ray pulsars. The ionization thresholds of the hydrogen atom associated with higher Landau levels is of current interest in astrophysical investigations.

The importance of the magneto-Coulomb system in astrophysics and atomic physics has motivated considerable theoretical work concerned with the behavior of the hydrogen atom at arbitrary magnetic fields, cf. Rösner et al. The corresponding theoretical work related to magneto-donors and magneto-acceptors has been
Magneto-optical transitions in semiconductors with a change in the Landau quantum number $\Delta n = 2$ and $\Delta n = 3$ are possible due to the intricacies of the band structure, such as warping and the lack of inversion symmetry\textsuperscript{20,21}, and they have been observed in InSb.\textsuperscript{20,22} The corresponding donor-shifted resonances have been observed by Grisar et al.\textsuperscript{20} The effect of optic phonons on free electron transitions between Landau levels (breaking of the selection rules) has been predicted by Bass and Levinson\textsuperscript{23} and first observed by Enck et al.\textsuperscript{24} Transitions to very high Landau states (up to $n = 23$) have been seen by Goodwin and Seiler.\textsuperscript{25} Phonon-assisted spin-flip transitions have been observed and described by Zawadzki et al.\textsuperscript{26} The effect of optic phonons on optical transitions between MD states has been observed by Kaplan and Wallis\textsuperscript{27} in the form of resonant polaron behavior in donor-shifted cyclotron resonance, and by McCombe and Wagner\textsuperscript{5} in donor-shifted combined resonance.

In this dissertation a new kind of optical transition between MD states assisted by the emission of optic phonons is presented.\textsuperscript{28,29} Photoconductive detection and magnetic field modulation have been used to obtain well resolved magneto-optical data. Phonon-assisted excitations provide a unique opportunity to investigate high excited states of the magneto-donor system (up to principal quantum number $n = 13$).
1.2 Nonlinear Electrical Properties of InSb

Oscillatory instabilities in semiconductors due to nonlinear effects have been observed and studied for the past three decades. A review of oscillatory instabilities in semiconductors with the nonlinear generation-recombination kinetics, coupled to charge-transport processes and to Maxwell's equations for the electromagnetic fields, can be found in Schöll's book. Very recently, the attention is concentrated on chaotic phenomena in semiconductors. The concept of chaos introduces a new means for analyzing and understanding these nonlinear semiconductor systems, and even helps to establish the physical origin of the oscillations and nonlinear behavior. Semiconductors, with their generation-recombination kinetics and easily tunable parameter, provide an ideal testing ground for low-dimensional models of dynamic systems with chaotic states.

Chaotic behavior in both discrete mappings and differential equations of systems exhibits an extreme sensitivity to initial conditions (a positive Lyapunov characteristic exponent). A closely related feature is the broad-band component in the power spectrum and the strange attractor in the phase portrait for certain values of a system parameter. One of the basic reasons for the interest in chaos stems from the fact that there are relatively few, but well-defined "universal" routes from an orderly regime of behavior to chaos.
Spontaneous or periodic externally driven oscillations and chaotic behavior related to impact ionization of impurity or defect levels have been studied experimentally in a number of semiconductors: n-GaAs,\textsuperscript{33,34,35,36} p-Ge,\textsuperscript{37,38,39,40,41} high purity n-InSb,\textsuperscript{42,43} and n-Si.\textsuperscript{44} Various types of periodic oscillations and chaotic behavior were observed in these experiments including Hopf bifurcation to a simple periodic oscillation, quasiperiodic oscillation, period-doubling bifurcation (Feigenbaum scenario), Ruelle-Takens-Newhouse scenario, and intermittency.

In this dissertation, both the self-generated and driven oscillations are experimentally investigated in InSb in the presence of a magnetic field at the liquid helium temperature. Various types of oscillations and chaotic behavior were observed, including Hopf bifurcation to a simple periodic oscillation, quasiperiodic oscillation, period-doubling bifurcation scenario, and period-having bifurcation. The methods of nonlinear dynamics, such as phase portraits, power spectra, correlation function, Lyapunov exponent spectrum, were applied in the characterization of the periodic and chaotic oscillations. In addition, a model based on the dynamic Hall electric field coupled with the carrier concentration is discussed.\textsuperscript{45}

This thesis is divided into five chapters. Chapter 2 describes the experimental apparatus and methods used to perform measurements on magneto-optical and nonlinear dynamic properties in n-InSb. Chapter 3 presents the theoretical description and experimental results of magneto-optical transitions between
magneto-donor states. Chapter 4 compares a simple nonlinear
dynamic model and experimental results for the periodic and the
chaotic oscillations in InSb. Chapter 5 is the conclusion of this
research and suggestions for future possible experimental and
theoretical work.
CHAPTER 2

EXPERIMENTAL METHODS

In this chapter the various parts of the experimental methods and apparatus used to obtain the results of magneto-donor optical properties and nonlinear dynamic properties of n-InSb discussed in Chapter 3 and Chapter 4 are described. The description is divided into three parts: Section 2.1 presents the InSb sample characteristics including sample preparation, electrical contacts, low temperature environment, and special treatment of the sample for magneto-optical and nonlinear dynamic studies. Section 2.2 discusses how the magneto-optical measurements were carried out, with particular emphasis on the excitation light source, photoconductivity detection method, and signal processing techniques. Section 2.3 describes the method for nonlinear dynamic measurements including the acquisition of longitudinal and Hall voltages, digitized time sequences of nonlinear oscillations, power spectra, and phase portraits for both the self-generated and driven nonlinear oscillations in the n-type InSb extrinsic semiconductor.

2.1 Sample Characteristics

a. Sample Preparation

The n-InSb samples used in this investigation were obtained from Cominco American, Inc. The high-purity samples had the
following specifications at 77K: an extrinsic electron carrier concentration of $9 \times 10^{13}$ cm$^{-3}$ and an electron mobility of $7 \times 10^5$ cm$^2$ (V·s)$^{-1}$.

Samples were cut into rectangular bars using a Servomat spark-gap cutting machine. The thickness of the samples ranged from 0.1 to 0.4 mm, width from 0.6 to 0.9 mm, and length from 5 to 10 mm. A low speed polishing wheel, micro cloth pad, and 3.5 μm aluminum oxide grit were used to lap the sample to the final desired thickness. The surfaces were polished using 0.3 μm grit to minimize the surface damage. The residual wax left on surfaces of samples was removed using benzene. The samples were then etched in a 2 percent bromine-methanol solution for 20 to 40 seconds to remove any remaining surface damage. After etching the samples were immediately washed using methanol to stop the etching process.

b. Sample Contacts and Mounting

Ohmic electrical contacts were made to the InSb samples using pure indium as solder and 60 μm gold wires as electric leads. The sample had total of six contacts: two current contacts, two resistance contacts, and Hall contacts at opposite sides as shown in Fig. 2.1. These contacts were Ohmic throughout the magnetic field and temperature range employed in this study. This can be seen from Fig. 2.2 and 2.3, which show the typical current-logitudinal voltage and current-Hall voltage characteristics for different magnetic fields, respectively.
Figure 2.1: Schematic diagram of the sample geometry used to perform magneto-optical and nonlinear dynamical measurements of n-InSb.
Incident CO$_2$ Laser Radiation
Figure 2.2: Voltage-current characteristics of n-InSb for different magnetic field at temperature 4.2K. The $V_L$ was measured from resistance contacts.
Figure 2.3: Hall voltage-current characteristics of n-InSb for different magnetic field at temperature 4.2K.
If there is a p-n junction (i.e. a barrier potential) caused by non-Ohmic contacts, the I-V characteristic curves should show strong asymmetric character due to the p-n junction electric behavior. However, the current-voltage curves from resistance and Hall leads show the symmetric character about the origin of zero current and zero voltage. On the other hand, the I-V curves show the strong nonlinear behavior even for the Ohmic contacts in the presence of magnetic field. Thus, this nonlinear behavior represents the sample's I-V characteristics. Especially, Fig. 2.3 shows the regime of negative differential conductivity

$$\sigma_{\text{diff}} = \frac{dj}{dE} < 0,$$  \hspace{1cm} (2.1)

where the current density $j = \text{current/cross section of sample}$, electric field $E = \text{Hall voltage/sample width}$. These nonlinear characteristics and negative differential conductivity are associated with the spatial fluctuation of the electric field and the carrier concentration and will be described in Chapter 4.

The samples were mounted to a piece of electrically insulating board using low temperature thermal grease. The leads of sample were soldered to electrical connections on the holder. Six low capacitance miniature coaxial cables were attached on the holder with one side connected to the sample and the other side to output and/or input BNC's.
c. Low Temperature Environment

All measurements reported in this dissertation were performed at low temperatures (1.8K to 100K). The low temperature apparatus used in this study was a Janis Research cryogenic optical supervaritemp dewar. The InSb sample was immersed either in a liquid helium bath, or surrounded by flowing helium gas vaporized by a small heater located in the dewar. The temperature was monitored by a calibrated Lake Shore Cryogenics carbon-glass resistor. Measurements at 4.2 K were obtained from boiling He at the atmospheric pressure. Temperatures lower than 4.2K were attained by decreasing the pressure above the liquid helium in the sample chamber. A resistance heater was used to achieve temperature greater than 4.2K. The accuracy of temperature was ± 0.05K.

d. Optical Window and Shielding

In order to perform the investigation on the optical properties of the sample, the low temperature dewar was equipped with a ZnSe or KRS5 optical window. The optical window is mounted in the tail section of the Janis dewar. For the nonlinear dynamic studies (under dark conditions), the shielding of InSb samples from external visible light and 300K blackbody radiation becomes important. The reason is the following: The sample chamber, consisting of a small optical window with a relatively large
cavity, approximates an ideal blackbody cavity. A photon entering this cavity from the optical window has a very small probability of escaping through the optical window. The external visible light (photon energy about 2 eV) and 300 K radiation (central radiation photon energy in the intensity-wavelength curve at about 0.1 eV based on Wien's displacement law) can ionize the shallow donors in the sample. For nonlinear dynamic studies these external ionization process directly effect the dynamics of the InSb system. The shielding of InSb samples from external radiation was carried out in two ways: (i) directly covering the InSb sample with the low temperature tape on the sample holder. (ii) sealing the optical window at the 77 K stage with multiple layers of Al foil.

2.2 Magneto-Optical Measurement Techniques

A schematic diagram of experimental apparatus used in the magneto-optical studies of InSb shown in Fig. 2.4. A constant current was impressed across the cooled InSb sample, which was subjected to both a constant magnetic field and CO₂ laser pulse illumination. The resulting photoconductive signal was then measured. Either a Keithley Model 220 programmable constant current source or Hewlett-Packard constant current source was used to obtained the most of the experimental results. The current passing through sample was carefully maintained at Ohmic condition by checking I-V Ohmic relation.
Figure 2.4: Schematic diagram of experimental apparatus used to perform photoconductivity measurements.
(cf. Fig. 2.2, the Ohmic regime is located at very low current). The
Janis Research cryogenic optic dewar system was also equipped with a
superconducting solenoid, surrounding the cooled sample core area,
capable of producing dc magnetic fields as high as 12 Tesla. For
all measurements reported in Chapter 3 the Faraday configuration
was used, where Faraday configuration is defined as the direction
of light propagation parallel to the magnetic field direction. The
optical arrangement and signal processing techniques performed on
the photoconductivity response and are described in the remainder
of this section based on Fig 2.4.

a. Excitation Laser Source

The optical excitation source used for the magneto-donor
studies was a Apollo Lasers Inc CO$_2$ laser in capable of power
output up to 150 Watts/line. The CO$_2$ laser was an axial-flowing
gas, electric discharge, water-cooled continuous-wave laser. The
grating tunable CO$_2$ laser provided the output with single mode and
single line wavelengths from about 9.1 $\mu$m to 11 $\mu$m correspondence
to photon energies of about 113 meV to 136 meV.

The laser beam was first condensed and collimated using a II-VI
Inc. ZnSe 3:1 beam condenser/collimator. A Princeton Applied
Research variable light chopper was placed at the focal point of a
4 inch ZnSe lens designed for anti-reflection at 10.6 $\mu$m. The beam
was then collimated using another 4 inch focal length ZnSe lens.
The cw CO\textsubscript{2} laser was mechanically chopped into 20-\textmu sec-wide pulses with a low duty cycle in order to prevent lattice-heating effects. The chopped laser light was then passing through an Edinburgh Instruments wedge-coated germanium attenuator in capable of varying continuously the laser intensity over 3 orders of the magnitude. The laser radiation was then focused on the sample using a 12 inch focal length ZnSe lens.

b. Signal Processing Techniques

Boxer averaging and lock-in amplifier techniques were employed to obtain derivative-like spectra.\textsuperscript{48,49} The voltage \( V(t) \) corresponding to photoconductivity or/and magnetoresistance was measured from either the resistance or current contacts. The signal then passed through differential and/or wide band amplifiers, fed into PARC Boxcar Averager, and went into PARC Lock in amplifier. Spectra were finally recorded using a Honeywell X-Y recorder.

The details of electronic configuration were the following: A He-Ne laser beam passed through the same chopper as CO\textsubscript{2} laser beam was detected using a Si photodiode. The pulse was used to trigger the boxcar sampling unit. The trigger pulse was carefully set to occur before CO\textsubscript{2} laser pulse. In order to do phase detection with lock-in amplifier, a 22 Hz small (\( \pm 0.1 \text{T} \)) ac magnetic field was superimposed on the larger dc field using a modulation coils. The Boxcar sampling rate, triggered by the He-Ne laser pulses was made
much greater than Nyquist criterion \(^{50}\) to preserve the modulation signal. In order to obtain a sufficient signal-to-noise ratio, a pulse repetition rate (300 Hz) exceeding the modulation frequency (22 Hz) of magnetic field by a factor of 14 was chosen. A schematic display of the electric and magnetic signal as function of time is shown in Fig. 2.5. The top part of Fig. 2.5 shows the modulation magnetic field \(B_m(t)\) superimposed on the linearly increasing dc magnetic field. The photoconductivity of sample corresponding to a laser pulses with a repetition rate 300Hz are shown in trace (b) without ac modulation magnetic field and in trace (c) with ac field \(B_m(t)\). In trace (c) the photoconductive voltage carries the modulation signal. The output of the Boxcar, which performs sampling the same rate as the laser pulses, carries the essential information of modulated photoconductive voltage along with noise, as shown in the part (d) of Fig. 2.5. The lock-in amplifier, when set to 22 Hz, phase sensitively detects the 22-Hz component of the Boxcar output signal and demodulates it to a dc voltage. If there are no nonlinear changes of the ac amplitude from the output of the Boxcar, the reference frequency and the output signal of Boxcar are in phase. Thus, the dc output of the lock-in amplifier will be essential constant, as shown in part (e) of Fig. 2.5. Otherwise, the first or second derivative of photoconductive vs. magnetic field will be obtained, as described below.

To observe all of the interesting features of the photocondutivity signal, the dc magnetic field was slowly swept. Care was taken to sweep B slowly to avoid losing any information
Figure 2.5: Schematic diagram of time dependence of the magnetic and electric signals appearing at various components of the apparatus. The magneto resistance is a function of magnetic field. The output of the Boxcar carries the essential information about the dependence of the resistance on B along with noise. The lock-in amplifier when set to 44 Hz phase sensitively detects the 44 Hz component of the Boxcar output signal and demodulates it to dc voltage. If a linear relation exists between magneto-resistance and B, the dc output of the lock-in amplifier will be constant. If the photoconductivity causes nonlinear changes the amplitude of ac part of Boxcar output, the result will be like Fig. 2.6.
A. Magnetic Field

B. Photovoltaic Conductive Voltage $B_m = 0$

C. Photovoltaic Conductive Voltage $B_m \neq 0$

D. Boxcar Sampling Oscilloscope Output

E. Lock-in Amplifier Output

$t$(msec)
contained in the photoconductivity signal, i.e. the field sweeping through any features of the signal must be slower than the rate at which the post-detection low pass filter of the lock-in amplifier can respond. Fig. 2.6 gives the schematic display of the output of the interesting features of the signal from Boxcar and lock-in amplifier. The top part of Fig. 2.6 shows the small sinusoidal modulation of magnetic field, coinciding with the top part of Fig. 2.5. If there are any nonlinear changes to the photoconductive voltage, the Boxcar will output the signal curve shown in part (b) of Fig. 2.6. By setting the lock-in amplifier to 22Hz, the first-derivative photoconductive vs. magnetic field is obtained, as shown in the part (c) of Fig. 2.6. By setting the lock-in amplifier to the second harmonic frequency (44Hz) of modulation and 90 degree phase difference with the setting phase of first derivative signal, the second-derivative type spectra is obtained, as shown in part (d) of the Fig. 2.6.

Photoconductive measurements provide a sensitive means of detecting small changes in absorption. All spectra shown in Chapter 3 represent the second derivative of the photoconductive response versus magnetic field. An example of the sensitivity and resolution of the modulation lock-in technique is shown in Fig. 2.7. The top trace of Fig. 2.7 shows the Boxcar output without modulation magnetic field and the detailed structure is not clear. The bottom part of Fig. 2.7 shows second-derivative type spectra using the techniques described aboved. A well-resolved doublet structure appears, which is obscured in the boxcar trace.
Figure 2.6: Schematic diagram of signal acquisition from Boxcar and lock-in amplifier with a small sinusoidal modulation magnetic field superimposed on a slowly sweeping large dc magnetic field.
Figure 2.7: Photoconductive response of n-type InSb vs magnetic field obtained at 5 K using a CO$_2$ laser wavelength of 10.83 $\mu$m (second derivative with respect to magnetic field). The final Landau states are indicated. The primes refer to phonon-assisted transitions.
BOXCAR AVERAGER DATA

T=5K
\( \lambda=10.61 \, \mu m \)

SECOND DERIVATIVE, FROM LOCK-IN AMPLIFIER

MAGNETIC FIELD (T)
2.3 Nonlinear Dynamical Measurement Techniques

Fig. 2.8 shows a schematic diagram of the experimental apparatus, which was used for the nonlinear dynamical measurements. In this experiment the magnetic field was transverse to the sample current direction, i.e. the standard Hall orientation. The InSb sample was shielded from outside illumination, a constant current was applied, and the resulting longitudinal voltage $V_L(t)$ and transverse Hall voltage $V_H(t)$ were simultaneously measured. The voltage response $V_L(t)$ and $V_H(t)$ from resistance and Hall contacts were fed into a buffer, which had very high input impedance and low output impedance to isolate the instrument from affecting the properties of sample. Tektronix 502 and PAR 114 ac coupled differential amplifiers were used to amplify $V_L(t)$ and $V_H(t)$ and cut off low frequency signals ($< 100 \text{ Hz}$) to avoid low frequency noise problems. The time derivative signals of $V_L(t)$ and $V_H(t)$ were generated using a low-noise electronic differentiator. The following procedures were then used to obtain measurements of the nonlinear dynamic behavior of the samples electrical properties:

(i) The power spectra offers a clear method for identifying the period-doubling route to chaos and was used to construct bifurcation diagram. In this experiment a Hewlett-Packard 3585A spectrum analyzer was used to record the oscillatory and/or chaotic spectral behavior at any desired current, or electric field, and or magnetic field. This spectrum analyzer is a swept receiver that
provides a visual display of amplitude versus frequency. It shows how energy is distributed as a function of frequency, displaying the absolute value of the Fourier components of a given waveform.

(ii) Time sequences acquisition provided phase portraits important for showing the physical state of systems with fixed points, limit cycles, and strange attractors. The digital time sequences were obtained using a Tektronix 7D20 programmable digitizer, and then stored in the memory of IBM PC. The evolution of the phase portrait could also be obtained using a video camera to record the experimental phase portrait constructed on Tektronix oscilloscope.

Similar experiments of nonlinear oscillations were also performed using an ac voltage source in series with the dc voltage source. The voltage drive consisted of a dc component and a sinusoidal modulation part [i.e. $V_{dc} + V_{ac} \sin(\omega t)$] that biased the sample, and the resulting reponse $V_L(t)$ and $V_H(t)$ were measured. The dc voltage source was a low noise 10 V battery with the voltage varied using a 20-turn wire potentiometer. A Tektronix 505 signal generator was used in series with the battery to produce a time periodic voltage of the form $V_{dc} + V_{ac} \cdot \sin(\omega t)$. The signal generator and one current contact of sample were grounded together. Typically the d.c. voltage was less than 5 Volts, the a.c. amplitude less than 4 Volts, and the driving frequency less than 100 kHz.
Figure 2.8: Schematic diagram of experimental apparatus used to perform the nonlinear dynamical measurements.
3.1 Introduction

Magneto-donor (MD) states in semiconductors, since their discovery in the magnetic freeze-out effect by Keyes and Sladek\textsuperscript{1} and the pioneering theoretical description of Yafet et al.\textsuperscript{2}, have been the subject of sustained experimental and theoretical interest. The effect of a magnetic field on shallow donor states is particularly important in narrow gap materials with small effective masses $m^*$ since, in the absence of the field, the donors are ionized even at low temperatures and can not be observed. Important progress in the magneto-optical investigations of shallow donors achieved by Kaplan,\textsuperscript{3} who used the photoconductive detection technique to observe both low-energy transitions between MD states belonging to the same Landau subband, as well as those occurring between MD states belonging to adjacent Landau subbands. The latter is sometimes referred to as "donor-shifted cyclotron resonance." Similar MD transitions related to spin resonance\textsuperscript{4} and combined cyclotron-spin resonance\textsuperscript{5} have also been observed in InSb. Magneto-donor investigations have been used to determine the static dielectric constant of a material\textsuperscript{6} and its pressure dependence,\textsuperscript{7} to identify the chemical nature of impurities,\textsuperscript{8} to study screening properties of the electron gas,\textsuperscript{9} to investigate the
metal-nonmetal transition,\textsuperscript{10} etc. Recently, magneto-optical and magneto-transport investigations proved to be useful in determining the positions of donors in modulation-doped two-dimensional GaAs-GaAlAs structures,\textsuperscript{11,12} which is important for device applications.

The importance of the magneto-donor system goes beyond semiconductor physics, however, since a magneto-donor imitates the hydrogen atom in large magnetic fields. The problem of an electron subjected to simultaneous Coulomb and magnetic field interactions is characterized by the parameter \[ \gamma = \left( \frac{\hbar e B}{m^*} \right) / 2Ry^*, \]
where \( Ry^* = 13.6 \left( \frac{m^*/m_0}{1/\kappa_0} \right) \text{ eV}, \) and \( \kappa_0 \) is the static dielectric constant. The value of \( \gamma \) is of the order of \( 10^{-5} \) for the hydrogen atom in vacuum. In narrow gap semiconductors, however, \( \gamma \) can attain values of 100 or more for available magnetic field strengths. In the experiments the typical \( \gamma \) value is \( \simeq 25 \), which corresponds to the hydrogen atom in a vacuum subjected to a magnetic field of \( \simeq 10^7 \) T. The above scaling allows one to transpose magneto-donor behavior to that of the hydrogen atom in gigantic magnetic fields.

The problem of atoms in extremely large magnetic fields has attracted a great deal of attention in recent years both in atomic physics and astrophysics.\textsuperscript{13} The reason is that white dwarf stars can produce magnetic fields of \( 10^3 \) T and accreting neutron stars fields of the order of \( 10^6 - 10^8 \) T. As a consequence, the optical spectra of atoms in such magnetic fields have been observed in the spectra of these stars. For example, very good agreement between all observed spectral features and the computed wavelengths of
stationary transitions of the hydrogen atom in magnetic fields 1.5 - 3.5 x 10^4 T has been found in the white dwarf GrW+70°8L47. By virtue of scaling laws the energies of heavier atoms can also be gained from those of the hydrogen atom at scaled values of magnetic field. This allows one to study, for example, Fe XXVI in gigantic fields, which appears to be prevailing in the vicinity of X-ray pulsars. The ionization thresholds of the hydrogen atom associated with higher Landau levels is of current interest in astrophysical investigations.

The importance of the magneto-Coulomb system in astrophysics and atomic physics has motivated considerable theoretical work concerned with the behavior of the hydrogen atom at arbitrary magnetic fields, cf. Rosner et al. The corresponding theoretical work related to magneto-donors and magneto-acceptors has been reviewed by Zawadzki. Magnetooptical transitions in semiconductors with a change in the Landau quantum number Δn = 2 and Δn = 3 are possible due to the intricacies of the band structure, such as warping and the lack of inversion symmetry, and they have been observed in InSb. The corresponding donorshifted resonances have been observed by Grisar et al. The effect of optic phonons on free electron transitions between Landau levels (breaking of the selection rules) has been predicted by Bass and Levinson and first observed by Enck et al. Transitions to very high Landau states (up to n = 23) have been seen by Goodwin and Seiler. Phonon-assisted spin-flip transitions have been observed and described by Zawadzki et al. The effect of optic
phonons on optical transitions between MD states has been observed by Kaplan and Wallis\textsuperscript{27} in the form of resonant polaron behavior in donor-shifted cyclotron resonance, and by McCombe and Wagner\textsuperscript{5} in donor-shifted combined resonance.

In Section 3.2 the theory of shallow donors in InSb is developed with four parts: [A]. One-band effective-mass approximation. It gives the characteristic properties of hydrogenic donors in InSb in the presence of a magnetic field. [B]. Three-level $\mathbf{\mathbf{P}} \cdot \mathbf{P}$ model. This is used to derive the energy eigenvalues for a donor in the presence of a magnetic field and impurity potential. This reduces to the eigenvalues of free electrons in the conduction band when the impurity potential is set to zero. [C]. Magnetodonor energies. They are obtained using the two-parameter trial functions in the 3-level $\mathbf{\mathbf{P}} \cdot \mathbf{P}$ model. [D]. Selection rules.

In Section 3.3 the experimental results are presented and discussed. Excellent agreement between theory and experiment is obtained for transitions up to principal quantum number $n = 13$.

3.2 Theory for Magnetodonor States

A. One-Band Effective Mass Approximation

InSb crystal belongs to the groups III-V semiconductor, which is formed by an ordered bonding of the individual atoms to form the crystal structure. The bonding is attributed to the valence
electrons of In and Sb atoms which pair up with valence electrons of adjacent atoms to form covalent bonds. When Sb of the group V atom in InSb is replaced by one of higher valency, e.g. substitution of Sb by Te, the Sb$^+$ ion is replaced substitutionally by a Te$^+$ ion (Te has six valence electrons) and a free electron. Such an impurity (Te) is called a donor impurity and gives rise to donor levels near the conduction band of InSb. When an electron is trapped by an ionized donor, it orbits around the donor much like the situation in hydrogen. This type of donor is often called as hydrogenic donor, i.e. a donor impurity in a semiconductor host has one protonic core and one bound electron. Physical speaking, a shallow donor in a semiconductor host is analogous to a hydrogen atom because both of them have only one electron to contribute to the electronic energy states. In such cases the energy levels and wave functions for the bound electron of shallow donor can be obtained by the correspondence of energies and wave functions for a hydrogen atom. However, the electron has an effective mass $m_\text{e}^*$ (much smaller than electron mass), and the nucleus has a charge $e/\kappa_\text{e}$, the normal protonic charge screened by the large dielectric constant $\kappa_\text{o}$ of the semiconductor medium.

In order to define all quantities and to stress the analogy between the magneto-donor and the hydrogen atom in a magnetic field, the one-band effective mass approximation $^2$ is described below. The initial MD Hamiltonian reads (neglecting spin),
where \( \text{curl} \, \vec{A} = \vec{B} \) is the magnetic field (\( \vec{B} \parallel \vec{z} \) is taken). The effective mass \( m_0^* \) and the static dielectric constant \( \kappa_0 \) account for the presence of the semiconducting medium. Using the symmetric gauge \( \vec{A} = \left[ -By/2, Bx/2, 0 \right] \), the Hamiltonian becomes

\[
H = \frac{\mathbf{p}^2}{2m_0^*} + \frac{\hbar \omega_c}{2} L_z + \frac{1}{8} m_0^* \omega_c^2 (x^2 + y^2) \left( x^2 + y^2 \right)^{-2} - \frac{e^2}{\kappa_0 r},
\]

where \( \omega_c = eB/m_0^* \) is the cyclotron frequency and \( \hbar L_z = (xp_y - yp_x) \).

Introducing the effective Rydberg \( \text{Ry}^* = m_0^* e^4 / 2 \hbar^2 \kappa_0^2 \) as a unit of energy, and the effective Bohr radius \( a_B^* = \kappa_0^2 / m_0^* e^2 \) as a unit of length, one obtains

\[
\frac{H}{\text{Ry}^*} = -\nabla^2 + \gamma L_z + \frac{\gamma^2}{4} (x^2 + y^2) - 2 \frac{2}{r}
\]

where \( x, y, z, \) and \( r \) are dimensionless. The characteristic parameter

\[
\gamma = \frac{\hbar \omega_c}{2 \text{Ry}^*} = \left( \frac{a_B^*}{L} \right)^\frac{1}{2}
\]

measures the relative strength of the magnetic and the Coulomb interactions. The magnetic length is \( L = \sqrt{\hbar/eB} \).
A comparison of the magnetic field dependence of the hydrogenic parameters are given in Table I. At zero magnetic field the effective Rydberg $R_y^*$ of the donor bound electron in InSb is very small. This means that there are effectively no bound electrons (i.e. all donors are ionized) even at liquid helium temperature in the absence of magnetic field due to the multiple absorption of acoustic phonons (phonon energy $\approx 0.5$ meV at 5K). The very large effective Bohr radius $a_B^*$ represents the fact that the electron is very loosely bound on the donor site in InSb. The cyclotron resonance energy $\hbar \omega_C$ of the bound electron in InSb is much larger than the bound electron in a hydrogen atom due to the fact that the magnetic interaction on the donor electron is much stronger than that in the hydrogen atom. The physical meaning of characteristic parameter $\gamma$ can be understood by looking at equations (3.3) and (3.4). Since $\hbar \omega_C$ and $R_y^*$ represents the magnetic interaction and the screened Coulomb interaction, respectively, $\gamma$ is the ratio of the magnetic and Coulomb interaction. Although the Hamiltonians for the magneto-donor and the hydrogen atom in a magnetic field have the same form which depends on $\gamma$ as the only parameter, the values of $\gamma$ in the two cases are usually very different. For example, for the hydrogen atom in a magnetic field of $B = 8$ T, one has $\gamma \approx 3.68 \times 10^{-4}$, which is the regime of the Zeeman effect, and thus the Zeeman effect is introduced by considering the magnetic interaction as a perturbation on Coulomb interaction in the hydrogen atom. On the other hand, in InSb at 8 T the value of $\gamma$ is 56.9 due to the small effective mass and large
Table I: Magnetic field dependence of hydrogenic parameters. The symbols $Ry^*$, $a_B^*$, $\hbar \omega_c$, $L$, and $\gamma$ refer to the effective Rydberg, effective Bohr radius, cyclotron energy, magnetic length, and the effective field, respectively.

<table>
<thead>
<tr>
<th>Unit</th>
<th>H atom</th>
<th>InSb donor</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Ry^*$</td>
<td>13.60 eV</td>
<td>0.65 meV</td>
<td>0</td>
</tr>
<tr>
<td>$a_B^*$</td>
<td>0.53 Å</td>
<td>658.60 Å°</td>
<td>0</td>
</tr>
<tr>
<td>$\hbar \omega_c$</td>
<td>1.00 meV</td>
<td>73.97 meV</td>
<td>8 T</td>
</tr>
<tr>
<td>$L$</td>
<td>87.31 Å°</td>
<td>87.31 Å°</td>
<td>8 T</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>3.68x10^{-5}</td>
<td>56.90</td>
<td>8 T</td>
</tr>
</tbody>
</table>
dielectric constant. The magnetic interaction in InSb is thus much stronger than the Coulomb interaction between the electron and the donor atom core. In this situation it is not possible to treat the magnetic interaction as a perturbation. Historically, there have been three ways to treat the problem for arbitrary $\gamma$: (1) the variational procedure, (2) the adiabatic approximation (based on a separation of the transverse and parallel motions at high magnetic fields), and (3) expansion techniques (in which the wave function is expanded in a set of known functions). These methods have been reviewed by Zawadzki. Here, the variational procedure is used, which can be generalized for the case of narrow gap semiconductors.

B. Three-level $\mathbf{p} \cdot \mathbf{p}$ Model

Although the one-band effective mass approximation gives the basic properties of hydrogenic donor in InSb in the presence of a magnetic field and essential features of narrow-gap semiconductors, it fails to interpret the magneto-optical results of the shallow donor in InSb. The reason is the following: The criterion for the one-band effective mass approximation is that the electron energy, as counted from the bottom of the band, must be small compared to the energy gap between the interested bands and all other bands. In InSb this requirement is not met because the donor energies, attached to corresponding Landau subbands (cf. Section C.), can rise quite high above the bottom of the conduction band.
In order to account for the influence of the band structure on the magneto-donor energies, a three-level \( \vec{P} \cdot \vec{P} \) approach is used and described in this section for the interpretation of the magneto-optical properties of shallow donors in InSb [cf. Refs. 51 and 52]. The band structure used in the three-level model is shown in Fig. 3.1. The band structure is centered at the \( \Gamma \) point (\( k = 0 \), i.e. center of the Brillouin zone) [cf. Ref 53]. The \( \Gamma^c \) level (conduction band) is separated by the energy gap \( \epsilon_g \) from the two-fold degenerate \( \Gamma^v \) level (light and heavy hole valence band), which is split off by the spin-orbit interaction \( \Delta \) from the \( \Gamma^v \) level (spin-orbit split valence band). The main idea is that it incorporates explicitly the \( \Gamma^c \), \( \Gamma^v \), and \( \Gamma^v \) bands, neglecting the influence of all other bands. The initial Hamiltonian for the problem reads

\[
H = \frac{1}{2m_0} \vec{p}^2 + V_0(\vec{r}) + \frac{\hbar}{4m_0c^2} \left( \vec{\sigma} \times \vec{V} \right) V_0 \cdot \vec{P} + \mu_B \vec{B} \cdot \vec{\sigma} + U(\vec{r}),
\]

where \( \vec{P} = \vec{p} + e\vec{A} \) is the kinetic momentum, \( e \) is the electron charge, \( \vec{A} \) is the vector potential of the magnetic field \( \vec{B} \), \( m_0 \) is the free electron mass, \( \mu_B \) is the Bohr magneton, the spin-orbit and Pauli terms are written in the standard notation, \( V_0 \) is the periodic potential of the lattice, and

\[
U(\vec{r}) = -\frac{e^2}{\kappa_0 r}
\]

is the donor (slowly-varying) potential (i.e. the Coulomb potential
Figure 3.1: Three-level model for energy band structure of InSb near the $\Gamma$ point of the center of Brillouin zone. The $\Gamma_6^C$, $\Gamma_8^V$, and $\Gamma_7^V$ refer to the conduction band, light hole valence band, and spin-orbit split valence band, respectively.
of the donor core). Since the donor potential and magnetic field are incorporated in the Hamiltonian (3.5), the Hamiltonian is not periodic and its eigenstates are not in the form of the Bloch functions. The solutions of the eigenenergy problem is expected in the form

$$\psi(\vec{r}) = \sum_{l} f_{1}(\vec{r}) u_{1}(\vec{r})$$  \hspace{1cm} (3.7)$$

where the summation runs over the energy bands, $f_{1}$ are the slowly-varying envelope functions, and $u_{1}$ are the Luttinger-Kohn periodic amplitudes satisfying the band-edge eigenenergy equation

$$\left\{ \frac{1}{2m_{o}} \vec{p}^{2} + V_{o} + H_{s-o} \right\} u_{1} = \epsilon_{1o} u_{1}$$  \hspace{1cm} (3.8)$$

where $\epsilon_{1o}$ is the edge energy for the 1th band. Equation (3.8) is free electron eigenvalue equation in the absence of magnetic field and donor potential. Applying the Hamiltonian operator (3.5) on wave function (3.7), assuming that the vector potential $\vec{A}$, the impurity potential $U(\vec{r})$, and the envelope function $f_{1}(\vec{r})$ are slowly varying in space, and using (3.8) the result is in the form

$$\sum_{l} \left[ \left\{ -E + \frac{1}{2m_{o}} \vec{p}^{2} + \epsilon_{1o} + U \right\} s_{l'} 1' + \frac{1}{m_{o}} \vec{p}_{l'} 1' \cdot \vec{p} + \mu_{B} \vec{B} \cdot \vec{p}_{l'} 1' \right] f_{1} = 0$$  \hspace{1cm} (3.9)$$

where $l' = 1, 2, 3, ...$ and
\[ \bar{p}_{1,1} = \langle u_1 | \bar{p} + \frac{\hbar}{4m_0c^2} \left( \mathbf{\sigma} \times \nabla \mathbf{v}_o \right) | u_1 \rangle. \] (3.10)

In order to obtain equation (3.9), integration over a unit cell has been applied and the slowly varying function \( \mathbf{A}(\mathbf{r}) \), \( U(\mathbf{r}) \), and \( f_1(\mathbf{r}) \), considered as constants among unit cell, have been taken out from the integral. Specifying the periodic Luttinger-Kohn functions for the three levels in question and neglecting all other bands, the infinite set (3.9) reduces to eight coupled differential equations. In addition the orbital and spin free-electron terms are neglected since they give very small contributions for narrow-gap materials. The resulting set can be solved by substitution, expressing the \( f_1, \ldots, f_8 \) functions by \( f_1 \) and \( f_2 \). The result is

\[
\begin{bmatrix}
-(E - U)(E - U + \epsilon_g) (E - U + \epsilon_g + \Delta) \\
\kappa^2 \left( E - U + \epsilon_g + \frac{2}{3} \Delta \right) P^2 \pm \frac{1}{3} \kappa^2 \hbar eB\Delta
\end{bmatrix}
\begin{Bmatrix}
\{ f_2 \} \\
\{ f_1 \}
\end{Bmatrix} = 0
\] (3.11)

where \( \kappa = -(i/m_0)(S|p_x|X) \) is the interband matrix element of momentum and \( \Delta \) is the spin-orbit energy. The plus and minus signs correspond to spin-down and spin-up states, respectively. Equation (3.11) presents a generalization of Kane's \(^{54}\) and Bowers and Yafet's \(^{55}\) equations for conduction electrons in the simultaneous presence of a magnetic field and an electric potential. To arrive at the form (3.11), the commutators \([ \bar{p}, U \]) have been neglected. They give rise to small corrections to the energies [cf. Ref. 51],
but are negligible for the measurements presented here.

The equations given in (3.11) account not only for the existence of magnetic field, but also for the intricacies of the InSb band structure (i.e., the conduction band, light-hole valence band, and spin-orbit split valence band) [cf. Fig. 3.1]. Now the eigenvalue problem of equation (3.11) can be solved using the variational procedure. If \( E - \langle U \rangle \ll \epsilon_g + 2\Delta/3 \), which is the case for InSb, one can neglect the corresponding terms in Eq. (3.11) and solve the resulting quadratic equation for the energies. The total energy is obtained in the form (in Ry* units)

\[
E = -\left(\frac{\epsilon_g}{2} - \langle U \rangle\right) + \left[\left(\frac{\epsilon_g}{2} - \langle U \rangle\right)^2 + \epsilon_g \left(\langle K \rangle + \langle U \rangle \pm \frac{\Delta}{2\Delta + 3\epsilon_g}\right)\right]^{1/2} \tag{3.12}
\]

where

\[
K = -\nabla^2 - i\gamma \frac{\partial}{\partial \phi} + \frac{\rho^2}{4} \tag{3.13}
\]

and the last term under the square root gives the effective spin splitting. The quantities \( \langle K \rangle \) and \( \langle U \rangle \) are variational averages of the kinetic energy (3.13) and potential energy \( U = -2/\sqrt{\rho^2 + z^2} \) (in cylindrical coordinates), respectively. The energy gap \( \epsilon_g \) and the spin-orbit energy \( \Delta \) in (3.12) should be expressed in Ry* units. The effective mass at the band edge resulting from the above theory is
Writing the effective spin-splitting term in the standard form

\[
\frac{1}{m^*_o} = \frac{2\hbar^2}{3} \epsilon_g \left( \frac{2\Delta + 3\epsilon_g}{\epsilon_g} \right).
\]  

(3.14)

the spin-splitting factor \( g^*_o \) at the band edge is obtained in the form of

\[
\pm \gamma \frac{\Delta}{2\Delta + 3\epsilon_g} = \pm \frac{1}{2} \frac{\mu_B g^*_o B}{\mu_B g^*_o B},
\]  

(3.15)

Equation (3.12) is the total energy of a magneto-donor state, derived by assuming that the electron motion in a magneto-donor state is identical to the free-electron motion in a magnetic field and taking into account the effect of nonparabolic conduction band. The free-electron energies can also be obtained from the same model by putting donor potential \((U) = 0\) and kinetic energy \((K) = (2n+1)\gamma\). In Section 3.3 the upper bound to the energy shift between the Landau level transitions and corresponding magneto-donor level transitions is obtained by making these changes from the same equation.

C. Magneto-Donor Energies
From (3.14) it is clear that the theoretical derivation of the effective mass $m^*_o$ is difficult due to the unknown $\kappa^2$ term of matrix element. For the spin-splitting factor $g^*_o$ (function of $m^*_o$) from (3.16) the same difficulty is encountered. In practice one uses the experimental values for $m^*_o$ and the spin-splitting factor $g^*_o$. Thus the evaluation of the MD energy amounts to the calculation of the trial averages $\langle K \rangle$ and $\langle U \rangle$ for a given state and a minimization of the energy given in (3.12). For large $\gamma$ values the trial functions should be based on exact solutions of free electron in a magnetic field. Following Ref. 52, the two parameter trial functions are chosen in the form (in cylindrical coordinates),

$$f_{NM\beta} = A_{NM} e^{iM\phi} \eta^2 e^{-\frac{\eta}{2}} L_N^m(\eta) P_\beta(2\pi$$  \hspace{1cm} (3.17)

where $N$ is principle quantum number, $M$ quantizes the projection of the angular momentum on the magnetic field direction, and $\beta$ quantizes the motion parallel to the magnetic field, $\eta = \gamma \rho^{1/2}$, the normalization factor for the $(\rho, \phi)$ part is $A_{NM} = [\gamma N! \Gamma(N + M)!2\pi]^{1/2}$, $L$ is the first variational parameter, $L_N^m$ are the associated Laguerre polynomials, and $m = |M|$. The variables $\rho$ and $z$ are dimensionless (in units of $a_B^*$). The quantum numbers can take the following values: $N = 0, 1, 2, \ldots$; $M = \ldots -2, -1, 0, 1, 2, \ldots$; $\beta = 0, 1, 2, \ldots$. The $z$-parts of the trial functions for $\beta = 0$ and $\beta = 1$ are
\begin{equation}
\psi_0 = \left( \frac{\gamma b^2}{2\pi} \right)^{1/4} e^{-\frac{\gamma b^2 z^2}{4}},
\end{equation}

and

\begin{equation}
\psi_1 = \left( \frac{\gamma b^2}{2\pi} \right)^{1/4} z e^{-\frac{\gamma b^2 z^2}{4}},
\end{equation}

where \( b \) is the second variational parameter. The function \( \psi(z) \) for \( \beta = 2 \) can be found in Refs. 52 and 56. The physical significance of the variational parameters \( b \) and \( a \) is related to the size of the wavefunction in the directions of parallel and perpendicular to the magnetic field, respectively.

The above functions are generalizations of the Yafet, Keyes, and Adams\(^2\) ground state \( \psi_{000} \). For large \( \gamma \) values a magneto donor state described by \( N M \beta \) "belongs" to the Landau subband described by the quantum number \( n = N + (M + m)/2 \), i.e. its energy is somewhat lower than the \( n \)th Landau level. Thus, at high fields one deals with "ladders" of magneto donor states "attached" to each Landau subband. The ladders arise due to different \( \beta \) values and from the fact that the Coulomb potential lifts the degeneracy of the Landau levels related to the \( N \) and \( M \) values. Using the above variational functions one can now calculate trial integrals for the kinetic and potential energies. The kinetic energy integrals are best done in cylindrical coordinates, while the potential energy integrals in spherical coordinates. For arbitrary \( N, M, \) and \( \beta = 0, 1 \) one arrives at the following expressions:
\[ \langle K \rangle = \frac{3}{2} (2N + m + 1) \left[ \lambda + \frac{\lambda}{\epsilon} \right] + \gamma M + \gamma \lambda \frac{2\beta + 1}{4} \quad (3.20) \]

\[ \langle U \rangle = -2 \frac{N! (N + m)!}{(2N + m)!} \frac{\gamma\lambda}{2\pi} \left[ -2\epsilon \right]^\beta \times \]

\[ \frac{\partial \beta}{\partial \epsilon} \left[ (1-\epsilon)^{-m} \sum_{j=0}^{N} \frac{\Gamma \left[ 2N - 2j + \frac{1}{2} \right]}{(N - j)! j! (m + j)!} \right] \times \]

\[ \frac{\partial^2 m}{\partial \epsilon^2} \left[ (1-\epsilon)^{m} + 2j \frac{d^m}{d\epsilon^m} \left( \epsilon^{2N + m} \frac{d^{2N}}{d\epsilon^{2N}} \frac{D(\epsilon)}{(1-\epsilon)^{1/2}} \right) \right] \quad (3.21) \]

where \( \lambda = b^2 \) and \( \epsilon = b^3 / b^3 \) are new variational parameters, and

\[ D(\epsilon) = \ln \left[ \frac{1 + \sqrt{1 - \epsilon}}{1 - \sqrt{1 - \epsilon}} \right]. \quad (3.22) \]

The expressions for \( \beta = 2 \) can be found in Refs. 52 and 56.

For InSb, typical values are \( \gamma > 30 \). In this case the transverse motion is controlled by the magnetic field and the transverse magneto-donor radius becomes equal to the longitudinal radius \( L \) (cf. Ref. 2). In the notation of this section this means \( \lambda = 1 \) and \( \epsilon = \lambda \). This is equivalent to using the one-parameter trial wavefunctions first proposed by Wallis and Bowlden. The above expressions for the sum \( \langle K \rangle + \langle U \rangle \) are equivalent to the corresponding Wallis and Bowlden's expressions, since the same trial functions are used.

Equation (3.21) becomes particularly simple for the \((0m0)\) states, which belong to the corresponding \( n = m \) Landau subbands. One obtains
\[ \langle K \rangle_{0m0} = \gamma (2m + 1) + \frac{1}{4} \gamma \lambda \quad (3.23) \]

and

\[ \langle U \rangle_{0m0} = -\frac{2}{m!} \sqrt{\frac{\gamma \lambda}{2\pi}} \frac{d^m}{d\lambda^m} \left[ \lambda^m \frac{D(\lambda)}{\sqrt{1-\lambda}} \right], \quad (3.24) \]

where \( D(\lambda) \) is defined in (3.22).

From the equations (3.20) - (3.24) one can see that the kinetic energies for any given magneto-donor states are simple calculations, but not the potential energies. The potential energies become mathematically more complicated if all of quantum number \( N, M, \beta \) are not zero. In order to solve this difficulty, a calculation package VAX UNIX MACSYMA has been used. The MACSYMA (MAC's Symbolic MAnipulation System) is a large computer programming system written in LISP (Mnl) used for performing symbolic and numeric mathematical manipulation. It permits one to write programs for transforming symbolic expressions. The calculated results of magneto-donor states reported here are all obtained using MACSYMA.

The energies of the (0m0) states are very close to the energies of the respective magneto-donor ground states, attached to the corresponding Landau subband. In order to illustrate this point, the lowest magneto donor states attached to the \( n = 2 \) Landau subband in the parabolic-band approximation, i.e. taking the total energy \( \langle E \rangle = \langle K \rangle + \langle U \rangle \) is used. For the (020) state

\[ \langle E \rangle_{020} = 5\gamma + \frac{1}{4} \gamma \lambda - \sqrt{\frac{\gamma \lambda}{2\pi}} \left[ \frac{(3\lambda^4 - 8\lambda + 8) D(\lambda)}{4(1-\lambda)^{5/2}} + \frac{3(\lambda - 2)}{(1-\lambda^3)} \right]. \quad (3.25) \]
For the (110) state

\[
\langle E \rangle_{110} = 5\gamma + \frac{1}{4} \gamma\lambda + \frac{7\lambda^2 - 30\lambda^2 - 24\lambda - 16}{8 (1 - \lambda)^{\frac{3}{2}}} D(\lambda) + \frac{7\lambda^2 - 4\lambda + 12}{4 (1 - \lambda)^{\frac{3}{2}}}
\]

(3.26)

For the (200) state

\[
\langle E \rangle_{200} = 5\gamma + \frac{1}{4} \gamma\lambda - \frac{\gamma\lambda}{2\pi} x \left[ \frac{(41\lambda^4 - 112\lambda^3 - 240\lambda^2 - 128\lambda + 64) D(\lambda) - 23\lambda^3 + 42\lambda^2 - 8\lambda + 48}{32 (1 - \lambda)^{\frac{9}{2}}} \right]
\]

(3.27)

Minimizing the above energies, the corresponding binding energies

\[ E_{NM/3}^b = \frac{5\gamma}{2} - E_{NM/3} \] (in Ry* units) can be calculated. For \( \gamma = 56.9 \) (corresponding to \( B = 8 \) T in InSb) the calculated result is shown in Fig. 3.2. As explained below, the magneto-donor levels, attached to the corresponding Landau levels \( n = 0 \) and \( n = 2 \), are calculated at a magnetic field 8 Tesla. The energy is counted from the bottom of the conduction band at the zero field. The transitions (in solid lines) shown in Fig. 3.2 were observed and the data will be presented in following section (cf. Fig. 3.12). The energies of \( E_{200}^b = 4.094 \), \( E_{110}^b = 3.994 \), and \( E_{020}^b = 3.905 \) are nearly the same. This is true also for magneto-donor states attached to higher Landau subbands, where the differences are even less.

Consequently, the energies of the (0m0) states and minimizing conditions are calculated up to \( m = 13 \), identifying them with those of the ground magneto-donor states belonging to the \( n = m \) Landau
subbands. The results of the calculated binding energies (in Ry* units)

\[ E_{b,m0}^b = \gamma (2m + 1) - E_{0m0}^{\text{var}} , \]  

with the use of the variational procedure (based on Eq.s 3.23 and 3.24) for the standard energy band (i.e., taking \( E = \langle \mathcal{K} \rangle + \langle U \rangle \)) is shown in Fig. 3.3. It shows the binding energies of investigated magneto donor levels versus characteristic parameter \( \gamma \) up to \( m = n = 13 \). These results represent a significant progress since Yafet et.al.\(^2\) and Wallis and Bowlden's\(^56\) work, who stopped at \( n = 2 \) level (i.e. magneto-donor states attached to the Landau level \( n = 2 \)). It can be seen that the binding energies decrease with increasing Landau number \( n = m \). This can be understood qualitatively by observing that higher magneto-donor states have progressively larger radii, so that the corresponding Coulomb energies (which are responsible for the electron binding) are progressively smaller. The binding energies are also plotted on a logarithmic scale \( \left( E_{b,m0}^b \right. \) versus \( \gamma \) \) and shown in Fig. 3.4. It can be seen that the binding energies versus \( \gamma \) appear as almost straight lines, which means that \( E_{b,m0}^b \) is to a good approximation a power function of \( \gamma \) or magnetic field (recall \( \gamma \equiv \gamma(B) \)). The approximate expression can be written in the form

\[ E_{b,m0}^b = 10^j \gamma^j , \]  

(3.29)
where constants $i$ and $j$ can be derived using Fig. 3.4. For the energy of the magneto-donor (0130) state the $i$ and $j$, for example, are $-0.4915$ and $0.4581$, respectively. Checking $\gamma = 10$ Eq. (3.29) gives binding energy 0.926 for (0130) state. Comparing with the exact value 0.94, the error is less than 2%.

In the actual comparison of the theory with experiment on InSb the magneto-donor energies are calculated using Eq. (3.12), which results from the three-level model with the approximation that $E - \langle U \rangle \ll \epsilon_g - 2\Delta/3$. This is a simpler procedure than the one used for calculating the Landau level energies, since the latter includes higher band contributions to $k^2$ terms. For this reason the magneto-donor binding energies are first calculated using Eq. (3.12) and the same model for the Landau level energies. This amounts to putting $\langle U \rangle = 0$ and $\langle K \rangle_{0m0} = \gamma (2m + 1)$ in Eq. (3.12). Once the magneto-donor binding energies are calculated, they are subtracted from the Landau level energies computed using the Pidgeon-Brown scheme. This gives the theoretical energies of the optical transitions between magneto-donor states. These procedures can be shown more clearly using Fig. 3.5. In Fig. 3.5 $E_{0m0}$ is the energy for the magneto donor transition given by Eq. (3.12), $E_L$ is the corresponding energy of Landau level transition given by Eq. (3.12) putting $\langle U \rangle = 0$, and $E^b_{000}$ and $E^b_{0m0}$ are the binding energies given by Eq. (3.28). Thus the transition energy of magneto-donor is

$$E_{0m0} = E^b_{000} + E_L - E^b_{0m0} = E_L + \delta(n=m), \quad (3.30)$$

where $\delta(n=m)$ is
Figure 3.2: Energy-level diagram for magneto-donor levels attached to $n = 0, 2$ Landau levels in the presence of magnetic field $B = 8$ Tesla.
ENERGY (meV)

B = 8T

n = 0

n = 2

(211) (111) (021)

(230) (110) (020)

(200) (010) (000)
Figure 3.3: Calculated binding energies of magneto-donor states attached to the consecutive Landau subbands $n = 0, 1, 2, \ldots, 13$ versus the parameter $\gamma$. 
Figure 3.4: Calculated binding energies of magneto-donor states attached to the consecutive Landau subbands $n = 0, 1, 2, \ldots, 13$ versus the parameter $\gamma$ (for the parabolic energy band) in the logarithmic plots.
\[ E(R^*_\gamma) = \frac{\hbar \omega_c}{2R^*_\gamma} \]
Figure 3.5: Energy-level diagram for magneto-donor transition between magneto donor states and corresponding transition between Landau Levels.
\[ E^b_{\text{omo}} \rightarrow (\text{omo}) \]

\[ (\text{omo}) \rightarrow E^b_{\text{000}} \]

\[ n = m \]

\[ n = 0 \]

\[ (000) \]
$$\delta(n=m) = E^b_{\text{coo}} - E^b_{\text{omo}}$$ (3.31)

the shifted energy from Landau-level energy. These calculations are used in Section 3.3.

D. Selection Rules

Since the trial functions (3.15) of appropriate symmetry have been chosen, the selection rules for magneto-optical transitions between magneto donor states can be calculated. The main selection rules for a spherical energy band are quoted in Table II (cf. Ref. 52). They can be understood by keeping in mind that the magneto donor problem with a scalar effective mass possesses a cylindrical symmetry. This symmetry is preserved in the Dingle gauge for the vector potential and it is reflected in the trial wavefunctions given in Eq. (3.17). When the spin is included, the projection of the total electron momentum on the magnetic field direction $j_z = M \pm 1/2$ is a good quantum number. One can deduce the selection rules of Table II remembering that the photons of $\sigma^L, \sigma^R,$ and $\pi$ polarizations carry the values of $j_z = +1, -1,$ and 0, respectively. Complications of the band structure, such as nonsphericity and the lack of inversion symmetry, result in additional selection rules $^{20,21}$ and are discussed in the next section.

The effect of optic phonons on free electron transitions between Landau levels, i.e. an emission or absorption of optic
Table II: Selection rules for the main magneto-optical transitions between donor states at high magnetic fields for various light polarizations. The symbols $s$, $M$, and $n$ refer to the spin, angular momentum, and Landau quantum numbers, respectively. The symbol $\beta$ is the number related to the quantized motion parallel to the magnetic field.

<table>
<thead>
<tr>
<th>Polarization</th>
<th>$\Delta s$</th>
<th>$\Delta M$</th>
<th>$\Delta n$</th>
<th>$\Delta \beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_L$</td>
<td>0</td>
<td>+1</td>
<td>+1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>+1</td>
<td>0</td>
<td>0</td>
<td>$\pm 1$</td>
</tr>
<tr>
<td>$\sigma_R$</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>$\pm 1$</td>
</tr>
<tr>
<td>$\pi$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\pm 1$</td>
</tr>
<tr>
<td></td>
<td>-1</td>
<td>+1</td>
<td>+1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>+1</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
</tr>
</tbody>
</table>
phonons breaks the selection rules for the free-electron magneto-optical transitions, has been predicted by Bass and Levinson\textsuperscript{24} and first observed by Enck et al.\textsuperscript{23} This allows one to observe higher harmonics of cyclotron resonance, i.e. transitions with $\Delta n > 1$. Below the phonon-assisted magneto-optical transitions between magneto-donor states are theoretically described using the wavefunctions given in Eq. (3.17).

A phonon-assisted magneto-optical transition is a two-quantum process, in which photon absorption is simultaneously accompanied by an emission of an optic phonon. The electron-photon and electron-phonon interactions simultaneously perturb the system. Although they can be introduced into the initial Hamiltonian (3.5) with the separated forms like standard perturbation forms, the first-order process are not considered. The transition probability of such a second-order process is given by

\[ W_{if} = \frac{2\pi}{\hbar} | M_{if} |^2 \delta (E_f - E_i) \]  \hspace{1cm} (3.32)

where the matrix element is

\[ M_{if} = \sum_l \frac{(f | H_R | l) (l | H_L | i)}{E_l - E_i} + \sum_{l'} \frac{(f | H_L | l') (l' | H_R | i)}{E_{l'} - E_i}. \]  \hspace{1cm} (3.33)

The summation is over intermediate states and $H_R$ and $H_L$ denote the electron-photon and electron-phonon interactions, respectively. The electron-photon interaction form can be derived by this way: In
the three-level $\vec{p} \cdot \vec{P}$ scheme the kinetic momentum $\vec{P}$ in the initial Hamiltonian is replaced by introducing radiation term $e\vec{A}^\ast$. The kinetic momentum is now $\vec{P} + e\vec{A}^\ast$. It leads to the electron-photon interaction term: $H_R = (e/m_0^*)\vec{A}^\ast \cdot \vec{P}$, where $\vec{A}^\ast$ is the vector potential of the radiation. The electron-phonon interaction is based on the Fröhlich Hamiltonian:\textsuperscript{23} $H_L = (C/q) \left[ b_q \exp(i\vec{q} \cdot \vec{P}) - b_q^\dagger \exp(-i\vec{q} \cdot \vec{P}) \right]$, where $C_e = (2\pi\hbar\omega_L/V) (1/\kappa_\infty - 1/\kappa_0)$. The quantities $\kappa_\infty$ and $\kappa_0$ are the high-frequency and static dielectric constants, $\vec{q}$ is the phonon wavevector, $\hbar\omega_L$ is the phonon energy, and $b_q$ and $b_q^\dagger$ are the phonon annihilation and creation operators, respectively.

Equations (3.32) and (3.32) describe both the free-electron and magneto-donor phonon-assisted transitions. For magneto-donor transitions the matrix elements of $H_R$ give the selection rules quoted in Table II. This means that the nonvanishing $H_R$ matrix elements have strict conditions. The transitions Eq. (3.33) exist for arbitrary final state if and only if the $H_L$ matrix elements are all nonvanishing for any intermediate states. Since the summation Eq. (3.33) is over all intermediate states, it is always possible to find one nonvanishing $H_R$ matrix elements if all $H_L$ matrix elements are nonvanishing. Thus, the $H_L$ matrix elements (for phonon emission) are the main consideration. One can consider the matrix elements of $H_L$ in the first term of Eq. (3.33), beginning with the ground magneto-donor state (000) and ending with the state (NM/9). The one-parameter trial functions is used, i.e. put in Eq. (3.17) $a = 1$. The calculation of the matrix elements can be separated into an
integral over $z$ involving the term $\exp(-i q z)$, and the corresponding integral over $(\rho,\phi)$ is:

$$
(N_M|H_L|000)=C \int P_\beta(zb) e^{-i q z} P_\rho dz. \quad (3.34)
$$

The integral (3.34) over $z$ is nonzero for a state of arbitrary $\beta$. For $\beta = 0$ and 1 this is seen directly from Eqs. (3.18) and (3.19). In other words the integral over $z$ couples the ground state to a state of arbitrary $\beta$. In Eq. (3.34) $C$ is the coefficient, $F$ is the integral over $\phi$ and $\rho$

$$
F = \int \psi_{NM} e^{-i q x - i q y} \psi_{00} \rho d\rho d\phi, \quad (3.35)
$$

where $\psi_{NM}$ is the $(\rho,\phi)$ part of the trial functions (3.17), can be done explicitly, as shown below.

Writing $q_x x + q_y y = q_\perp \rho \cos \phi$, where $q_\perp^2 = q_x^2 + q_y^2$, and counting the angle $\phi$ from the vector $\vec{q} = (q_x, q_y)$, the integral over $\phi$ is

$$
\frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-i M\phi - i q \rho \cos \phi} d\phi = i^m J_m (-q \rho) \quad (3.36)
$$

where $J_m$ is the Bessel function and $m = |M|$, as before. The remaining integral over $\rho$, containing the Bessel function, the Laguerre polynomial, the exponential and the power function, can also be done analytically. The final result is
\[ F = (-1)^N (-i)^m \left[ \frac{N!}{(N + m)!} \right] t^2 \int_0^1 e^{-t} t^{N + m} (t) \]  

(3.37)

where \( t = L^2 q \sqrt{2} \). The result shown in Eq. (3.37) is similar to that obtained for the phonon-assisted free-electron transitions with the use of the Landau (asymmetric) gauge for \( \bar{A} \) (cf. Refs. 23 and 24). This is not surprising since, by putting \( L = 1 \) in the wavefunctions (3.17), the transverse magneto-donor motion has been tacitly assumed identical to the free-electron motion, and the final result for the transition probability should be gauge invariant.

The above results show that the Frohlich electron-phonon interaction can couple the ground magneto-donor state (000) to an arbitrary magneto donor state (NM\( \beta \)). In other words, an optic phonon emission breaks the magneto-optical selection rules for the magneto-donor transitions, as it does for the free-electron transitions. The physical interpretation is that a momentum transfer due to the phonon emission breaks the cylindrical magneto-donor symmetry, and the angular momentum selection rules of Table II cease to be valid.

The matrix element given in (3.33) can also be calculated analytically for the two-parameter magneto-donor functions given in (3.17), allowing for the variational adjustment of the transverse motion (the corresponding integrals can be found in Prudnikov et al.'s work.58 The final result is qualitatively similar to (3.37), but it now differs quantitatively from that for the phonon-assisted free-electron transitions.
The delta function in (3.30) involves the total energies of the initial and final states (including photons and phonons). They are $E_i = E_i^{MD} + \hbar \omega$ and $E_f = E_f^{MD} + \hbar \omega_L$. Energy conservation requires that $E_f - E_i = 0$, which yields the resonance condition

$$\hbar \omega = E_f^{MD} - E_i^{MD} + \hbar \omega_L.$$  (3.38)

In the limit of $B \to 0$ the difference $E_f^{MD} - E_i^{MD}$ is practically zero, so that the phonon-assisted magneto-donor transition energies should converge to $\hbar \omega = \hbar \omega_L$, in a similar fashion to the phonon-assisted free-electron transitions between Landau levels. This is indeed observed experimentally, as will be shown in the Results and Discussion section.

It should be finally mentioned that in InSb-type semiconductors the probability of spin-flip transitions directly induced by optic phonons is low (although nonvanishing, cf. Ref. 59). For this reason, in phonon-assisted spin-flip transitions (cf. the transition $2' -$ in Figs. 3.7) the spin reversal is likely to occur due to the spin-photon interaction rather than the spin-phonon part of the matrix element (3.33) (cf. Ref. 26).

3.3 Results and Discussion

In this section the results of the magneto-optical experiments carried out on n-type InSb extrinsic semiconductor is reported and discussed. The presentation runs parallel to the theoretical
discussion already given in above section. The experimental results of the magneto-donor investigation in InSb sample are presented in two ways: (i) The derivative-like magneto-optical spectra (i.e. second derivative of the photoconductive response with respect to magnetic field). Both transitions between magneto-donor states and free-electron transitions between Landau levels caused by optical and/or phonon-assisted excitation are observed and presented. (ii) The fan chart presentation of the experimental data and theoretical fitting. The fundamental tool for the identification of magneto-optical transitions is to plot the photon energies of a group of spectral features as a function of magnetic field; because the energies of different transitions in a given series tend toward the same energy (such as the energy gap for interband transitions) as the magnetic field is reduced to zero, and the energies separate, or fan out, as the magnetic field increases. A plot of such transition energies is known as a "fan chart" and many examples will be given throughout this Chapter. A comparison of a given group of experimental features for different laser wavelengths, for example, with such a theoretical fan chart will often reveal the zero-magnetic-field intercept energy, which is an important clue to the origin of the transition or group of transitions. Also, a comparison of the energy spacing between adjacent levels with previously measured cyclotron resonance energies, for example, provide an important clue to the type of transition (interband or intraband).
It is important to first identify the transitions between magneto-donor states and the free-electron transitions between Landau levels using experimental methods. In Fig. 3.6 the observed peaks exhibit a doublet structure. The temperature dependence of this structure is shown in the inset of Fig. 3.6 and allows the identification of the origin of each peak. The higher-field peaks are due to free-electron transitions and the lower field peaks are related to the corresponding transitions between magneto-donor levels, because at higher temperatures thermal excitations depopulate the ground magneto-donor state, populate the lowest Landau state, and reverse the relative heights of the peaks in the doublets. This assignment also concides with the theoretical prediction based on equation (3.12), resulting that the transition energy between Landau levels is lower than corresponding transition between magneto-donor states. Thus the peak caused by the lower energy transition appears at the higher-field side due to the resonance requiring a higher field to fit a fixed photon energy (given CO2 laser wavelength).

Figure 3.7 shows the energies of the observed free-electron and magneto-donor transitions as a function of the magnetic field strength, together with the energies calculated using the Pidgeon and Brown model.57 It can be seen that some transitions correspond to simple (converging to $\hbar \omega = 0$ at $B = 0$) and others to phonon-assisted (converging to $\hbar \omega = \hbar \omega_L$ at $B = 0$) excitations. This correlation of the theory with experiment served as a basis of the transition assignments. The free-electron transitions $0^+ \rightarrow 2^+, 0^+$
\( \rightarrow 2^- \), and \( 0^+ \rightarrow 3^+ \) (where \( n^+ \) and \( n^- \) denote the Landau levels \( n \) with spin-up and spin-down, respectively) and the associated magneto-donor transitions for these levels have been observed previously by Grisar et al.\(^{20} \) For the used field orientation \( \vec{B} \parallel (111) \) and the light polarization \( \vec{E} \perp \vec{B} \) the \( 2\omega_C \) (\( 0^+ \rightarrow 2^+ \)) and the \( 2\omega_C + \omega_S \) (\( 0^+ \rightarrow 2^- \)) transitions are allowed due to the inversion asymmetry of InSb.\(^{21} \) The observed transition \( 3\omega_C \) is, however, not allowed for this field orientation. The same difficulty was encountered by Favrot et al.,\(^{22} \) who observed the transition \( 2\omega_C \) for \( \vec{B} \parallel (001) \), which is not allowed. The current (not confirmed) interpretation is that such forbidden transitions become allowed by the assistance of shallow impurities in the same way as the phonon-assisted ones.\(^{60} \) Judging by the doublet structure shown in Figure 3.6, the corresponding donor-shifted transitions are allowed for the same reasons.

Figure 3.8 shows the derivative-like data for transitions to higher final states for which the distinction between free-electron and magneto-donor excitations was presently possible. Because of the momentum transfer caused by the phonon emission, the selection rules are broken and the phonon assistance provides a unique opportunity to investigate high excited states of the magneto-Coulomb system. As a result, the transitions to final magneto-donor states attached to the Landau subbands \( n \) up to \( n = 13 \) have been observed. The observed and calculated energies of Landau level and magneto-donor transitions are shown in Figures 3.9 and 3.10. The transitions labelled by the primed numbers are the phonon-assisted excitations. The solid lines for the Landau level
transitions in Figs. 3.9 and 3.10 were calculated using the Pidgeon-Brown model with the following band parameters: \( \epsilon_g = 235.2 \text{ meV} \), \( E_p = 23.2 \text{ eV} \), \( \Delta = 0.803 \text{ eV} \), \( \gamma_1 = 3.25 \), \( \gamma_2 = -0.2 \), \( \gamma_3 = 0.9 \), \( \kappa = -1.3 \), \( F = -0.2 \), \( q = 0.0 \), and \( N_x = -0.55 \). This corresponds to the band-edge values of \( m_0^* = 0.0136 \) and \( g_e^* = -51.1 \). These parameters describe well also other magneto-optical data taken on InSb (cf. Ref. 61).

The phonon-assisted Landau level transition energies were obtained by adding the LO optic phonon energy \( \hbar \omega_{LO} = 24.4 \text{ meV} \) to the calculated free-electron energies.

As indicated in the inset of Figure 3.9, the shift between the Landau level and magneto-donor transitions is due to the fact that, with increasing \( n \) the binding energies of the corresponding magneto donor states become smaller. The dashed lines for the magneto-donor transitions in Figs. 3.9 and 3.10 have been calculated by adding the corresponding shifts: \( \delta (n=m) = E_{b_{oo}}^b - E_{b_{omo}}^b \) to the Landau level transition energies. This procedure has been discussed in the previous section. In order to get the best fit to the data we used the value \( R_y^* = 0.65 \text{ meV} \). With the above quoted value of \( m_0^* \) it corresponds to the static dielectric constant \( \kappa_0 = 16.9 \), which is very reasonable (cf. discussion in Ref. 62). The agreement between experiment and theory is very good (it is seen more clearly in Figure 3.11 for the 2' + transition at one laser wavelength).

The highest resolution data revealed two additional resonances on the lower field side, as shown in Figure 3.12 for the \( 0^+ \rightarrow 2^+ \) phonon-assisted excitations. The additional resonances are present
in the phonon-assisted as well as the ordinary magneto-donor transitions. This suggests that the resonances are due to additional selection rules caused by the band structure complications rather than the phonons. The transitions shown in Figure 3.11 have been identified as originating from the magneto-donor ground state \((000^+)\) to excited magneto-donor states associated with the \(2^+\) Landau subband. The corresponding selection rules have been calculated by Wlasak.\(^6^3\) The main transition \((000^+ \rightarrow 200^+)\) [or the transition \((000)^+ \rightarrow (020^+)\) having almost the same energy, cf. discussion in section 3.2 C] is possible due to inversion asymmetry. The weaker transitions are possible due to band warping. The best theoretical fit to the middle transition has been obtained for the final magneto-donor state \((230)\) (the calculated binding energy is \(E_b = 1.80\) meV at \(B = 8\) T), but one cannot exclude the states \((220)\) (\(E_b = 2.07\) meV) and \((210)\) (\(E_b = 2.31\) meV) as possible final states. For the lowest field transition in Figure 3.12 the best fit is obtained for the final magneto-donor state \((221)\) (\(E_b = 0.626\) meV), but again one cannot exclude the nearby states \((111)\) (\(E_b = 0.657\) meV) and \((021)\) (\(E_b = 0.675\) meV). All of the above binding energies have been calculated for \(B = 8\) T. The weaker resonances require further investigation concerning their dependence on light polarization, orientation of the magnetic field, and precise assignment of the final magneto-donor states.

The theoretical description presented above has aimed to treat consistently the high excited magneto-donor states in a workable fashion, which required the four simplifications listed below. In
order to arrive at the simple nonparabolic formula given in Eq. (3.14) for the variational magneto donor energies from the $\mathbf{p} \cdot \mathbf{P}$ theory (3.9) within the three-level model, the commutators $[p, U]$ have been neglected and $E_{\text{min}}(U) < \epsilon_{g} + 2\Delta/3$ has been assumed. This approximation is known to affect the ground state (000) and can lead to a few percent error in the energy estimations (it lowers the ground state, increasing the binding energy at high fields, cf. Kaplan's work\textsuperscript{64}).

The central-cell corrections due to the short-range component of the donor potential have been neglected. This, again, affects mostly the magneto-donor ground state energy. The exact chemical nature of the donors in InSb samples of this experiment is not known. The energies of the ground magneto-donor states (N00) have been identified with the slightly higher energies of the (0m0) states (for $m = N = 2, 3, 4, \ldots$), since the latter are much easier to calculate. This also introduces a few percent error in the energy estimations. On the other hand, whether the phonon-assisted transitions occur to the final (N00) or (0m0) magneto-donor states is not really known.

In the energy estimations the one-parameter variational functions of the Wallis and Bowlden\textsuperscript{56} type, which are valid for the $\gamma > 10$ range of the magnetic field intensities, have been used. This criterion is not well satisfied for the transitions to the highest magneto-donor levels, which occur at lower magnetic fields (cf. Fig. 3.10). This leads to slight underestimations of the binding magneto-donor energies at low fields.
All of the above approximations amount to a few percent of the binding energies. The latter are small compared to the transitions energies observed here, so that a high theoretical precision is not necessary. The accepted values of the effective Rydberg for donors in InSb range from 0.6 to 0.7 meV and the overall good fit to the data with the value $Ry^* = 0.65$ meV should be considered a success.
Figure 3.6: Photoconductive response of n-InSb versus magnetic field obtained at 5K using a CO$_2$ laser wavelength of 10.83 μm (second derivative with respect to magnetic field). The final Landau states are indicated. The primes refer to phonon-assisted transitions. The insert shows magneto-optical spectrum obtained at a higher temperature.
T = 14K

T = 5K

$\lambda = 10.83 \, \text{\mu m}$

MAGNETIC FIELD (T)
Figure 3.7: Energies of the observed magneto-donor and Landau-level transitions versus magnetic field (solid dots). The solid lines indicate the calculated energies of Landau level transitions for simple excitations (converging to $\hbar \omega = 0$), and phonon-assisted excitations (converging to $\hbar \omega = \hbar \omega_L$).
Figure 3.8: Photoconductive response versus magnetic field showing the Landau level - magneto-donor doublet structure for higher quantum numbers.
\( \lambda = 10.59 \mu m \)

\( \vec{B} II (111) \)

PHOTOCONDUCTIVE RESPONSE

MAGNETIC FIELD (T)

1.6  1.8  2.0  2.2  2.4  2.6  2.8
Figure 3.9: Energies of the free electron (solid circles) and magneto-donor (open circles) transitions versus magnetic field for the Landau quantum numbers 1 - 4. Primed numbers indicate phonon-assisted transitions. The solid lines were calculated using the Pidgeon-Brown model and the dashed lines were obtained by adding the calculated donor shifts. The insert shows a schematic diagram of the free electron (solid arrow) and associated magneto-donor (dashed arrow) transitions.
Figure 3.10: Energies of the free electron (solid circles) and magneto-donor (open circles) transitions versus magnetic field for the Landau quantum numbers 5 - 13. The solid lines were calculated using the Pidgeon-Brown model and the dashed lines were obtained by adding the calculated donor shifts.
Figure 3.11: High resolution magneto-optical spectrum for the phonon-assisted free electron $0^+ \rightarrow 2^+$ transition (solid arrow), and the $(000^+) \rightarrow (200^+), (000^+) \rightarrow (230^+), (000^+) \rightarrow (211^+)$ MD transitions (from left to right). The calculated field positions for the transitions are indicated by the arrows (see text).
$T = 3.5 \text{ K}$

$\lambda = 10.61 \mu\text{m}$

MAGNETIC FIELD (T)

PHOTOCONDUCTIVE RESPONSE

$7.7$ $7.9$ $8.1$ $8.3$ $8.5$

$2^{-1}1+$

$2^{-3}0+$

$2^{+}$
4.1 Introduction

Oscillatory instabilities in semiconductors due to nonlinear effects have been observed and studied for the past three decades. A review of oscillatory instabilities in semiconductors with nonlinear generation-recombination dynamics, coupled to charge-transport processes and to Maxwell's equations for the electromagnetic fields, can be found in Ref. 32. Very recently, attention has been concentrated on chaotic phenomena in semiconductors. The concept of chaos introduces a new means for analyzing and understanding these nonlinear semiconductor systems, and helps to establish the physical origin of the oscillations and nonlinear behavior. Semiconductors, with their generation-recombination kinetics and easily tunable parameters, provide an ideal testing ground for low-dimensional models of dynamic systems with chaotic states. In order to understand some basic concepts of nonlinear dynamics, related to the characterization of an experimental system, are now briefly presented.

Period-Doubling Route to Chaos. As the experimental control parameter $\lambda$ is varied the experimental system undergoes a pitchfork bifurcation with twice the period of the original motion. As $\lambda$ is changed further, the system again bifurcates to periodic
oscillations with twice the period of the previous oscillation. One outstanding feature of this scenario is that the critical values of $\lambda$ at which successive period doublings occur obey the universal scaling rule, namely:

$$\delta = \lim_{n \to \infty} \frac{\lambda_n - \lambda_{n-1}}{\lambda_{n+1} - \lambda_n} = 4.6692016 .$$ \text{(4.1)}

This process will accumulate at a critical value of the control parameter, after which the system becomes chaotic. In this thesis the bifurcation parameter (i.e. the control parameter) in numerical simulation was the magnetic field $B$. In the experiment both the current and the magnetic field have been used as control parameters.

**Phase Portraits.** The phase portraits are essentially two-dimensional reconstructions of the phase space flow of the simulated or experimental dynamic system. Thus, phase portraits express the dynamic properties of a nonlinear system in a geometric way. In particular, those features which are related to chaoticity arise when the orbits in the phase space converge to an object which is neither a fixed point nor a limit cycle, but rather a strange attractor.

**Power Spectra and Autocorrelation Function.** The power spectra and autocorrelation function are related to the Fourier analysis. Such Fourier analysis is useful for obtaining the frequency components and the power distribution as a function of frequency. In particular, an important property of the low-
dimensional dynamic system is that chaotic temporal behavior is accompanied by the presence of a broadband noise in the power spectrum [see, e.g., Ref. 66]. While the power spectrum shows a broad band of frequencies, the autocorrelation function tends to zero as time delay tends to infinite, i.e., the chaotic signal is only correlated with its recent past.

Let \( s(t) \) denote a signal that is a function of time, and let \( \{s_\alpha\} \alpha = 1, 2, \ldots, N \) denote a discrete set of measured values at the times \( t_\alpha = \alpha \tau, \tau > 0, \alpha = 1, \ldots, N \). The quantity \( \tau \) represents the length of the sampling interval. Define \( \mathcal{S}_\beta \) as the discrete Fourier transform of the \( N \) measurements \( \{s_\alpha\} \):

\[
\mathcal{S}_\beta = \sum_{\alpha=1}^{N} s_\alpha e^{i2\pi \alpha \beta / N},
\]

The power spectrum is defined as the plot of \( |\mathcal{S}_\beta|^2 \) as a function of the frequency \( f_\beta = \beta / N \tau \).

\[
|\mathcal{S}_\beta|^2 = \sum_{\gamma=1}^{N} C_\gamma e^{i2\pi \gamma \beta / N},
\]

The relation between the autocorrelation function \( C_\gamma \) and the power spectrum \( |\mathcal{S}_\beta|^2 \) is given by [see, e.g., Ref. 67]

\[
C_\gamma = \frac{1}{N} \sum_{\beta=1}^{N} |\mathcal{S}_\beta|^2 e^{-i2\pi \gamma \beta / N},
\]
Equations (4.3) and (4.4) show that the autocorrelation function and the power in the $\beta$th frequency component form a discrete transform pair.

Lyapunov Characteristic Exponents. A main feature of a chaotic state of nonlinear system is its the sensitive dependence on initial conditions. The sensitive dependence on initial conditions can be characterized by Lyapunov characteristic exponents. The Lyapunov exponents describe the average, exponentially fast divergence or convergence of nearby orbits in phase space. Systems with the exponential orbital divergence strongly amplify even negligible differences in the initial condition and the observer's predictive ability is rapidly lost. The typical system containing at least one positive Lyapunov exponent is chaotic.\textsuperscript{68}

Spontaneous externally driven oscillations and chaotic behavior have been studied experimentally in a number of semiconductors: n-GaAs,\textsuperscript{33-36} p-Ge,\textsuperscript{37-41} high purity n-InSb,\textsuperscript{42,43} and n-Si.\textsuperscript{44} Various types of periodic oscillations and chaotic behavior were observed in these experiments including Hopf bifurcation to a simple periodic oscillation, quasiperiodic oscillation, period-doubling bifurcation (Feigenbaum scenario), Ruelle-Takens-Newhouse scenario, and intermittency.

In this chapter the theoretical description and experimental studies of the nonlinear oscillations and chaotic behavior in n-InSb are presented and discussed. In section 4.2 a model based on the dynamic Hall electric field coupled with the carrier
The longitudinal electric field, the transverse Hall electric field, and the carrier concentration are dynamic variables with the evolution described by three coupled nonlinear ordinary differential equations of the autonomous type. In section 4.3 the experimental results are presented and discussed, including an experimental evidence of period-doubling route to chaos, phase portraits, power spectra, correlation function, and the characteristic Lyapunov exponents for the characterization of oscillations.

4.2 Theory of Chaotic Oscillations in InSb

4.2.1 The Model

In order to construct a simple dynamic model for carrier generation and recombination in InSb, the following approximations are made: (1) Only the ground magneto-donor level is considered, i.e. the excited magneto-donor states are neglected. (2) The conduction band is considered as quasi-continuous, i.e. the splitting of the conduction band into discrete Landau levels is neglected. (3) The carrier concentration is uniform and the electron temperature is constant across the InSb sample. Then, if the InSb sample is biased with a constant current source, the externally applied current density $\vec{j}_0$ is composed of the conduction current density $\vec{j}$ and the displacement current density $\frac{\partial E}{\partial t}$. 
where $\epsilon$ is the electric permittivity, $\overline{E}$ is the electric field, and the "dot" denotes the time derivative. In the presence of the magnetic field the conduction current density with accuracy to $B^2$ terms is given by [see, e.g., Ref. 69]:

$$\overline{j} = \epsilon n \overline{E} - \frac{\mu_B B}{2} \times \overline{E} + \epsilon n \mu_B B (\overline{B} \cdot \overline{E}) ,$$

where $n$ is the carrier concentration and $\mu$ is the drift mobility of the carriers. The drift mobility is defined by $\mu = \frac{r_m e}{m^*}$, where $r_m$ is the mean momentum relaxation time in the absence of magnetic field and $e$ is the elementary charge. The effective mass of the carriers is denoted by $m^*$. The magnetic field dependence of the Hall mobility is described by:

$$\mu_B = \frac{\mu}{1 + \frac{\mu^2 B^2}{4}} ,$$

where the mobility is reduced in the presence of the magnetic field [see, e.g., Ref. 70]. In this model the direction of the external current density is given by $\overline{j} = (j_0, 0, 0)$ and the magnetic field direction is given by $\overline{B} = (0, B, 0)$. From Eqs. (4.5) and (4.6) one obtains the dynamic equations for the electric field components:

$$\epsilon \dot{E}_L = j_0 - \epsilon \mu_B (E_x + \mu B E_z) ,$$

$$\epsilon \dot{E}_H = \epsilon \mu_B (\mu B E_x - E_z) ,$$
where $E_L$ is the longitudinal electric field and $E_H$ is the transverse electric field (i.e. Hall field). Setting $\vec{E} = 0$ the steady state solution of Eqs. (4.8) and (4.9) is

$$E_H^* = \mu E_L^*,$$  \hspace{1cm} (4.10)

$$j_0 = e \mu n^* E_L^*,$$ \hspace{1cm} (4.11)

where the " * " is used to denote steady state and $E_H^*$ is the classical Hall field.

For constant carrier concentration $n$ the equations (4.8) and (4.9) describe a harmonic oscillator. However, self-sustained chaotic oscillations can be generated when $n$ behaves as a third dynamic variable. Using the simplest generation and recombination kinetics $^7$ leads to the rate equation

$$\dot{n} = g_T (N_{ED} - n) - rn(n + N_A) + g_I n (N_{ED} - n), \hspace{1cm} (4.12)$$

where $g_T$ is the thermal generation coefficient, $N_{ED} = N_D - N_A$ is the effective donor concentration, $N_D$ is the donor concentration, $N_A$ is the acceptor concentration, $r$ is the recombination coefficient, and $g_I$ is the impact ionization coefficient. The generation and recombination terms in Equation (4.12) have the following physical meaning: (i) The first term is the thermal generation rate, i.e. the increase of carriers due to the thermal ionization of magneto-donors. (ii) The second term represents the is recombination rate, i.e. the loss of carriers caused by the capture of free electrons on
the ionized donor site. (iii) The last term is the impact ionization rate, i.e. the carriers that are ionized by the inelastic collision between the free electrons and the magneto-donors. With two terms containing $n^2$, Eq. (4.12) is nonlinear and Equations (4.8), (4.9), (4.12) form a three-dimensional nonlinear coupled differential system.

The parameters used in the simulations of the dynamic electrical behavior of InSb were selected as follows. The mobility was directly measured via the static Hall effect and estimated to be $\mu = 16 \text{ m}^2/\text{Vs}$. The current density $j_0$ was set experimentally at $10^{-5} \text{ A/m}^2$ using a constant current source. The value for the thermal generation coefficient $g_T$ was taken from the work by Littler et al., with $g_T = 2 \times 10^{-7} \text{ sec}^{-1}$. The magnetic field $B$ was a control parameter, and the permittivity $\epsilon = 16.9 \epsilon_0$ was obtained from the magneto-donor studies presented in Chapter 3. The field dependence of the recombination coefficient was modeled by

$$r(E) = r_0 \left( \frac{v_o}{v_o + \mu_B E} \right)^{3/2} \text{ m}^3 \text{ s},$$

where $r_0 = 10^{-14} \text{ m}^3 \text{ s}$, $v_o = 2500 \mu_B \text{ m/s}$, and $E = \sqrt{E^2_L + E^2_H} \text{ V/m}$. The field dependence of the impact ionization coefficient was modeled by

$$q_I(E) = q_0 \frac{\alpha(E)}{1 + \epsilon^8(E)} \text{ m}^3 \text{ s},$$

where
\[
\alpha(E) = \frac{\sqrt{0.5\mu BE/v_{th}}}{1 + \sqrt{0.5\mu BE/v_{th}}}, \quad (4.15)
\]
\[
\beta(E) = \frac{v_{th} - 0.5\mu BE}{2v_0 + \frac{1}{3}\mu BE}, \quad (4.16)
\]

The parameter values in Eqs. (4.14-16) are: \( g_0 = 2.0 \times 10^{-16} \text{ m}^3 \text{ s} \) and \( v_{th} = 1.25 \times 10^5 \mu \text{B m/s} \).

The results of the calculation show complex oscillations, both periodic and chaotic. Fig. 4.1 shows the phase portraits of the time derivative of the longitudinal electric field \( E_L(t) \) versus \( E_L \) and the transverse electric field \( E_H(t) \) versus \( E_L(t) \). The bifurcation parameter here is the magnetic field. For \( B < 0.1 \text{ T} \) the system quickly decays to a stable equilibrium. At \( B = 0.1 \text{ T} \) a Hopf bifurcation occurs (see Fig. 4.1a), and the system resides on a stable limit cycle. At \( B = 0.38 \) the system undergoes a period-doubling bifurcation (see Fig. 4.1b). For higher field values the system displays complex behavior with multiple periods and complicated structure of the power spectrum. At \( B = 0.5 \text{ T} \) one observes a transition to a chaotic attractor (Fig. 4.1c). This observation has been confirmed by the numerical calculation of the system Lyapunov exponents. The Lyapunov spectrum was obtained using Kostelich et al.'s algorithm. The results were \( \lambda_{\text{max}} = 70.8 \), \( \lambda_0 = -0.52 \), \( \lambda_{\text{min}} = -350.4 \). The signature of the spectrum and the phase plots give strong evidence for the existence of a chaotic attractor. Additionally, the information dimension \( d_I \) was obtained using the Kaplan-Yorke conjecture as \( d_I = 2.2 \). Fig. 4.2
illustrate the time series varying in the chaotic regime. In Fig. 4.2 (a), (b), and (c) represent longitudinal field $E_L(t)$, transverse field $E_H(t)$, and carrier concentration $n(t)$ versus $t$, respectively. Note that there is no apparent periodicity in each of these figures. The three-dimensional phase portraits, reconstructed using chaotic time series of $E_L(t)$, $E_H(t)$, and $n(t)$, are shown in Fig. 4.3. In Fig. 4.3 the part (a) is plotted with $(n, E_H, E_L)$ as dynamic variable and the part (b) is plotted with $(E_H, n, E_L)$ as dynamic variables. The relationship between the simulated and experimental temporal evolution will be discussed in the results section.

4.3 Results and Discussion

Current-Voltage Characteristics

Typical current-voltage curves are shown in Fig. 4.4, where the voltage $V_L$, obtained from a pair of side contacts on the rectangular-shaped sample, is plotted as a function of sample bias current. These traces show the symmetric character of the current-voltage characteristics about the origin of zero current and zero voltage. It is seen that the magnetic field strongly influences the $I$-$V$ characteristics. These $I$-$V_L$ curves, as well as the Hall coefficient data are consistent with the impurity impact ionization mechanism. The application of a sufficiently high electric field, accelerates the free electrons in the conduction band so that they have enough energy to ionize through impact shallow impurities.
Figure 4.1: Phase portraits of time derivative longitudinal electric field $E_L$ vs. $E_L$ (left column) and Hall electric field $E_H$ vs. $E_L$ (right column) for the following magnetic field $B$ in unit Tesla: (a) 0.1, (b) 0.38, and (c) 0.5.
Figure 4.2: Chaotic time series at $B = 0.5$ T: (a) Longitudinal electric field $E_L$ vs. time, (b) Hall electric field $E_H$ vs. time, and (c) Carrier concentration $n$ vs. time.
(a) $E_L \left( \frac{V}{m} \right)$

(b) $E_H \left( \frac{V}{m} \right)$

(c) $n \left( \frac{m^2}{m^2} \right)$

$t(s)$
Figure 4.3: Three-dimensional phase portraits at $B = 0.5$ T: (a) $(n, E_H, E_L)$, (b) $(E_H, n, E_L)$. 
STRANGE ATTRACTOR

(a)

(b)
Consequently, the free electron concentration \( n \) increases with increasing the electric field as shown in Fig. 4.5, and the resistance decreases giving the characteristic shape seen in Fig. 4.4. The mobility is observed to decrease with increasing the electric field in this region as shown in Fig. 4.5. Regions marked "a", "b", "c", and "d" in Fig. 4.4 and 4.5 correspond to regions of oscillatory and chaotic behavior in the sample. The saturation voltage \( V_B \) is also shown in Fig. 4.4. The spontaneous nonlinear oscillations occur in the pre-saturation regime.

Fig. 4.6 shows the typical current-Hall voltage curves at a fixed temperature 4.2 K and various magnetic fields. These plots show strong nonlinearity. In particular, there is a regime in which there exists three values of current corresponding to a single value of the Hall voltage. This is the negative differential conductivity (NDC) regime, which is associated with current-controlled instabilities.\(^{32}\) This is exactly the regime in which the chaotic oscillations are observed in this investigation of InSb. These nonlinearities of \( I-V_L \) and \( I-V_H \) coincide with Hüpper and Schöll's model\(^ {45}\) (see last section) based on the dielectric relaxation of the Hall field and the applied longitudinal field coupled with the donor impact ionization, which was developed in the previous section.

**Oscillations and Chaotic Behavior in the Power Spectra**

As mentioned in the introduction section, the semiconductors possess easily tunable control parameters. In this study the
Figure 4.4: $I-V_L$ curves showing characteristic behavior of impact ionization of the shallow donor levels at a lattice temperature of 4.2 K. The nonlinear behavior is much more pronounced at 1.09 T (10.9 kG) because the shallow donor impurities are split further away from the conduction band edge. The saturation voltage $V_B$ is shown. The letters "a", "b", "c", and "d" mark significant regions of oscillatory or chaotic behavior as shown in Figs. 4.7-4.10.
Figure 4.5: The dependence of free electron concentration $n$ and mobility $\mu$ versus applied electric field $E_L$ for a magnetic field of 1.09 T (10.9 kG) and a lattice temperature of 4.2 K.
Figure 4.6: $I-V_H$ characteristics of n-InSb for various magnetic field and a lattice temperature of 4.2 K. The $s$-type negative differential conductivity is shown.
steady state electric field, the magnetic field, and the current
density were the control parameters for InSb. Power spectra were
obtained with a spectrum analyzer (spectral range: 20 Hz-40 MHz),
for various values of electric field, a fixed magnetic field of 10.9
kG, and a lattice temperature of 4.2 K. The frequency bifurcation
cascade is shown in Fig. 4.7, where the various observed frequencies
are plotted versus longitudinal electric field ($E^*_L$) over the range
620-690 mV/cm. At $E^*_L$ equal to $\approx$ 620-630 mV/cm, a spontaneous
frequency at 200 kHz appears. This oscillatory behavior was seen
earlier in InSb by Phelan and Love$^{75}$ and Haslett and Love$^{76}$ who
also observed oscillatory behavior at a frequency of $\approx$ 200 kHz, but
no chaotic properties were observed. In the experiment discussed
here, however, the nonlinearities were strong enough to produce
frequency doubling, tripling, etc. at higher fields (630-650
mV/cm). For example, a further increase of the electric field
produced period-doubling bifurcations until the system became
chaotic at region "a". After a small increase in $E_L$, the system
again became chaotic at region "b". Quasiperiodic behavior was
observed between regions "b" and "c". Outside of the region "c" the
system displayed period-halving bifurcations, leading once more to
chaotic behavior at region "d".

Figure 4.8 shows the power spectrum for various electric
fields from $E^*_L \approx$ 650-667 mV/cm up to the region "a". The period-
doubling route to chaos is evident in the observed bifurcations. At
region "a" for $E^*_L = 666.48$ mV/cm, chaotic behavior is seen as
evidenced by the broad band and/or lack of distinct narrow band
oscillatory frequencies. From this data the estimation of Feigenbaum’s number is

\[ \delta_n = \frac{E^{*}_n - E^{*}_{n+1}}{E^{*}_{n+2} - E^{*}_{n+1}} \]

\[ = \frac{664.07 - 663.47}{664.20 - 664.07} \approx 4.62, \]  

(4.17)

which is in good agreement with theoretical value \( \delta = 4.669 \).

The power spectra for various E-fields and the regions "a", "b", and "c" are shown in Fig. 4.9. Both regions "a" and "b" show chaotic behavior characterized by the broad-band power spectra. Following region "b", one mode \( f_1 \) appears and even shows period doubling (see \( E^{*}_L = 669.92 \text{ mV/cm} \)). In addition, a second apparently incommensurate mode \( f_2 \) appears, giving rise to quasiperiodic behavior (seen best at \( E^{*}_L = 671.12 \text{ mV/cm} \)). Finally, the mode \( f_2 \) dominates after the quasiperiodic regime.

Figure 4.10 shows another of the interesting aspects of the data, i.e., regular period-halving bifurcation is observed. First, the \( f/16 \) components appear, followed by \( f/8 \), then \( f/4 \). At slightly higher E-fields, the \( f/6 \) components appear, then \( f/3 \), and finally chaotic behavior in region "d". The critical parameter values for the electric field at which successive period halvings occur again approach Feigenbaum’s number, but obeying the relation

\[ \delta_n = \frac{E^{*}_{n-1} - E^{*}_n}{E^{*}_{n-2} - E^{*}_{n-1}} \]
The phenomenon of period-halving bifurcations is well known mathematically (see Ref. 77 for period-halving bifurcations and Ref. 78 and 79 for general pitchfork bifurcations).

Fig. 4.11 shows the convergence of the experimental $\delta_n$ sequence to the Feigenbaum number, where dashed line indicates the universal Feigenbaum number and solid squares are experimental data. The experimental sequence approaches the Feigenbaum number to within 1% due to the high resolution in the period doubling, with the period-2$^6$ bifurcation included.

Phase Portraits of Auto-Oscillations

Fig. 4.12 shows the waveforms of $E_L(t)$, phase portraits of the time derivative $E_L(t)$ versus $E_L(t)$ and the Hall field $E_H(t)$ versus the longitudinal field $E_L(t)$, and the power spectra of $E_L(t)$ for various magnetic fields (i.e. $B$ as the control parameter) and a fixed current density $j_0 = 0.25 \text{ A/cm}^2$ and a lattice temperature 1.8 K. For magnetic field $B = 0.6 \text{ T}$ there is only a single periodic oscillation. For the magnetic field at 0.8 T the power spectra exhibits maxima at frequencies that are multiples of the 30 kHz fundamental frequency. This is the frequency-locked, period-two oscillatory state [see, e.g., Ref. 80] because the two fundamental frequencies, associated with the motion about the large and small circles in the phase portraits, are commensurate. A further
Figure 4.7: Frequency bifurcation diagram showing the complex oscillatory and chaotic behavior observed at 1.09 T (10.9 kG) and 4.2 K in n-InSb. Regions "a", "b", and "d" are regions exhibiting chaotic behavior. Representative power spectra are shown in Figs. 4.8-4.10 for particular values of electric field.
Figure 4.8: Plots of the power spectrum for various electric field values at 1.09 T and 4.2 K. Evidence for a period-doubling bifurcation route to chaos is present. Region a shows chaotic behavior.
Figure 4.9: Plots of power spectrum for various electric fields between regions "a" and "c" at 1.09T and 4.2 K. At 671.12 mV/cm quasiperiodic behavior is observed. Chaotic behavior occurs in regions "a" and "b". At increasing fields above region "b" a switching from one mode $f_1$ to another mode $f_2$ takes place.
Figure 4.10: Plots of power spectrum for various electric fields between regions "c" and "d" at 1.09 T and 4.2 K. Region d shows chaotic behavior. A period-halving bifurcation apparently takes place near region "d".
10dB

POWER SPECTRUM (Relative Units)

FREQUENCY (kHz)

0 100 200 300

(f/3 2f/3)

(f/6)

(f/8)

(f/16)

(mV/cm)

684.85

680.07

678.16

677.69

677.58

677.09

675.42

100 200 300
Figure 4.11: Plot of Feigenbaum number approach. The dashed line indicates the Feigenbaum number 4.6692016. The solid squares are the experimental value. The symbol n here notes the period-n bifurcation.
Feigenbaum Num. Approach

Feigenbaum Num.
increase of magnetic field generates various frequency-locked states; for example, there is period-three oscillation state at \( B = 1.07 \) T. Later, the system becomes chaotic oscillation (i.e., the phase portraits show chaotic trajectories and power spectra shows a broad band of frequencies at \( B = 1.8 \) T). Fig. 4.13 shows the measurement of the transverse field. The \( E_H(t) \) was measured simultaneously with the longitudinal field. The waveforms, the phase portraits of the time derivative \( E_H(t) \) versus \( E_H(t) \), and power spectra are seen to have basically the same nonlinear oscillation behavior as the longitudinal field, as shown in Fig. 4.13.

Fig. 4.14 shows the normalized autocorrelation function versus time delay for the chaotic state of the InSb dynamic system at \( B = 1.8 \) T, where the top trace is the autocorrelation function of the longitudinal field and the bottom trace is the autocorrelation function of Hall field corresponding to Fig. 4.12c and Fig. 4.13c, respectively. The autocorrelation functions have a peak value at the origin and drop off rapidly with increasing time delay. Comparing Fig. 4.14 with Fig. 4.12d and 4.13d, one sees that the characteristics of the autocorrelation functions are consistent the broad band behavior of frequencies of Fourier spectra at chaotic states.

Phase portraits of Driven Oscillations

For samples driven with a periodic voltage source, the results of the experimental observation are shown in Fig. 4.15 and Fig.
4.16. In Fig. 4.15 a period doubling cascade to transient chaotic oscillation was recorded using the current density as control parameter, where the driving frequency $f = 100 \text{ kHz}$ is close the frequency of the non-driven system. With the increasing current density a period-doubling bifurcation to $f/2$ frequency occurs, and then a transition of the system takes place to the chaotic oscillation at $j_0 = 0.2621 \text{ A/cm}^2$. Following the chaotic oscillation, the period-three window appears, as shown in the part (d) of Fig. 4.15. Fig. 4.16 shows the oscillation behavior at higher current densities. At $j_0 = 0.3545 \text{ A/cm}^2$ a period-seven bifurcation occurs. The system exhibits complex behavior with multiple periods and acquires a chaotic attractor at $j_0 = 0.3626 \text{ A/cm}^2$. Following the chaotic oscillation, the system resides on a period-three oscillation $f/3$ (as seen at the part (c) of Fig. 4.16). After the period-three window, the system again displays a transition to the chaotic attractor (as seen at the part (d) of Fig. 4.16).

Discussion of Results

The phase portraits, power spectra, and autocorrelation functions have been used to characterize the nonlinear dynamic system of InSb extrinsic semiconductor biased by either a dc current source or a periodic driven voltage source. The general strategy of this approach is to extract characteristics of the nonlinear system from the experimental time series, bypassing the detailed knowledge of the underlying dynamics. For example, the
Figure 4.12: Waveform of $E_L(t)$, phase portrait of time derivative $E_L(t)$ vs. $E_L(t)$, phase portrait of $E_H(t)$ vs. $E_L(t)$, and power spectrum of $E_L(t)$ for a fixed $j_0 = 0.25$ A/cm$^2$ and $T = 1.8$K and increasing magnetic fields: (a) $B = 0.6$ Tesla; (b) $B = 0.8$ Tesla; (c) $B = 1.07$ Tesla; (d) $B = 1.8$ Tesla.
Figure 4.13: Waveform of $E_H(t)$, phase portrait of time derivative $E_H(t)$ vs. $E_H(t)$, and power spectrum of $E_H$ for a fixed $j_0 = 0.25$ A/cm$^2$ and $T = 1.8$K and increasing magnetic field: (A) $B = 0.6$ Tesla; (B) $B = 0.8$ Tesla; (C) $B = 1.07$ Tesla; (D) $B = 1.8$ Tesla.
Figure 4.14: Normalized autocorrelation function of the chaotic signal for a fixed $j_0 = 0.25 \text{ A/cm}^2$, $T = 1.8\text{K}$, and $B = 1.8$ Tesla: (a) longitudinal electric field $E_L$; (b) Hall electric field $E_H$. 
Figure 4.15: Waveform of $E_L(t)$, phase portrait of time derivative $E_L(t)$ vs. $E_L(t)$, phase portrait of $E_H(t)$ vs. $E_L(t)$, and power spectrum of $E_L$ for a fixed $B = 0.8$ Tesla and $T = 1.8K$ and increasing current density: (a) $j_0 = 0.1728$ A/cm$^2$; (b) $j_0 = 0.2189$ A/cm$^2$; (c) $j_0 = 0.2621$ A/cm$^2$; (d) $j_0 = 0.2796$ A/cm$^2$. 
Figure 4.16: Waveform of $E_L(t)$, phase portrait of time derivative $E_L(t)$ vs. $E_L(t)$, phase portrait of $E_H(t)$ vs. $E_L(t)$, and power spectrum of $E_L$ for a fixed $b = 0.8$ Tesla and $T = 1.8K$ and increasing current density: (a) $j_0 = 0.3545 \, \text{A/cm}^2$; (b) $j_0 = 0.3626 \, \text{A/cm}^2$; (c) $j_0 = 0.3938 \, \text{A/cm}^2$; (d) $j_0 = 0.4693 \, \text{A/cm}^2$. 
phase portraits obtained experimentally give geometric information about the structure of attractors (periodic and chaotic) for the dynamic system. Further qualitative and quantitative characterization of the dynamic behavior in InSb was obtained by determining the Lyapunov exponents from the experimental time series. For the periodic oscillations the typical Lyapunov exponents are $\lambda_1 = 0.014$, $\lambda_2 = -0.8$, and $\lambda_3 = -1.0$ [i.e., the limit cycle is characterized, as it should be, by the Lyapunov spectrum $(0,-,-)$]. For the chaotic oscillations the typical Lyapunov exponents were estimated as $\lambda_{\text{max}} = 0.031$, $\lambda_0 = -0.00062$, and $\lambda_{\text{min}} = -0.045$ [i.e., the strange attractor is characterized, as it should be, by the Lyapunov spectrum $(+,0,-)$]. Positive Lyapunov exponents characterize of the InSb dynamic system to have a sensitive dependence on initial conditions. In addition, the information dimension $d_I$ of the system in the chaotic states was estimated to be $d_I = 2.6$.

The results of the experiment is now compared to the numerical simulation of the system described by Eqs. 4.8-4.12. Fig. 4.1 of the simulation when compared with the experimental Fig. 4.12 shows remarkable similarities between the theory and the experiment. For various magnetic field, both the phase portraits of the simulation and the experiment display single periodic oscillations, period-doubling bifurcations, and a transition to a chaotic attractor. The chaotic time series of Fig. 4.2 also has similar feature to Fig. 4.12c obtained from the experiment. In addition, the information dimension $d_I = 2.2$ from the simulation is close to $d_I = 2.6$ obtained
from the experiment. Thus, the simulation gives a qualitative characterization of the dynamic behavior in InSb. However, three limitations in the quantitative characterization of the dynamic behavior in InSb by the simulations were seen: (i) The self-sustained frequency from the simulation is in the region $\sim 1kHz - 10kHz$. On the other hand, the measured frequency in InSb is in the region $\sim 10kHz - 200kHz$, (ii) The carrier concentration $n \sim 10^{12} /m^3$ is far from the experimental value $n \sim 10^{20} /m^3$, and (iii) In the simulation the current density is set at $j_0 = 10^{-5} A/m^2$ in contrast to the experimental value $j_0 \approx 10^3 A/m^2$. However, both $n$ and $j_0$ values in the simulation and in the experiment differ by the same order of magnitude. In addition, in order to obtain chaotic solutions which simulated the experimental behavior, the thermal generation rate of carriers had to be set low. This suggests that the traditional form of the thermal generation process does not apply to InSb or that additional sources of carrier generation need to be included in the model. Finally, the analytical forms of coefficients of the impact ionization and the recombination are not well-known. Thus, future work should involve a more complex model involving excited states of the impurity and modified coefficients.
CHAPTER 5

CONCLUSIONS

In this thesis the investigation of the magneto-donor states and chaotic oscillations in InSb have been presented. The principal result of the magneto-donor work is the observation and theoretical description of a new kind of the optical transition between magneto-donor states in InSb, assisted by the emission of longitudinal optic phonons. The observation of the doublet structure due to magneto-donor and Landau-level transitions for high excited states of the system have been made possible by combining the photoconductive detection with magnetic field modulation. The phonon-assisted excitations allow the unique opportunity to study high excited states of an electron subjected to simultaneous Coulomb and magnetic field interactions. High excited states of the magneto-donor system were seen up to principal quantum number \( n = 13 \), providing information on the excited states of a system which simulates the hydrogen atom in gigantic magnetic fields. A three-level \( \vec{p} \cdot \vec{P} \) model was used to derive the energy eigenvalues for the donor in the presence of a magnetic field and impurity potential. The magneto-donor states have been described variationally, taking into account the narrow energy gap and the spin-orbit interaction of the band structure of InSb. Since the system simulates the hydrogen atom in gigantic fields, the results are of the direct interest to semiconductor
physics, atomic physics, and astrophysics.

In addition, an experimental study and theoretical description of the nonlinear dynamic system of InSb was presented. Both self-generated and driven oscillations were experimentally investigated in InSb in the presence of a magnetic field at the liquid helium temperatures. Various types of oscillations and chaotic behavior were observed, including Hopf bifurcation to a simple periodic oscillation, quasiperiodic oscillation, period-doubling bifurcation scenario, and period-having bifurcation. The prediction of the Feigenbaum number \( \delta \) of \( \approx 4.62 \) from the data was shown to be in good agreement with the theoretically predicted value \( \delta = 4.669 \).

Nonlinear dynamic methods, such as phase portraits, power spectra, correlation function, Lyapunov exponent spectrum, were applied in the characterization of the periodic and chaotic oscillations. From the phase portraits the system displayed the expected behavior for a nonlinear dynamic system, i.e. limit cycles and a chaotic attractor for various control parameters. The observed chaotic attractor was confirmed by the calculation of the system Lyapunov exponents from the experimental time series. The typical Lyapunov exponents are \( \lambda_{\text{max}} = 0.031, \lambda_0 = -0.00062, \) and \( \lambda_{\text{min}} = -0.045 \), i.e. the strange attractor is characterized, as it should be, by the Lyapunov spectrum \((+,0,-)\). The positive Lyapunov exponents characterize the dynamic system of InSb to possess a sensitive dependence on the initial conditions. In addition, the information dimension \( d_I \) of the system in the chaotic states was
estimated to be 2.6.

The simulation based on the dielectric relaxation of the longitudinal and transverse field coupled with the magneto-donor impact ionization provided a qualitative description of the dynamic behavior in InSb, including Hopf bifurcation to a periodic oscillation, period-doubling bifurcation, and a transition to a chaotic attractor. The information dimension $d_I = 2.2$ from the simulation is close to the value of $d_I = 2.6$ from the experiment.
BIBLIOGRAPHY


28. C. L. Littler, W. Zawadzki, M. R. Loloee, X. N. Song, and D. G.


