THE FIXED V. VARIABLE SAMPLING INTERVAL SHEWHART X-BAR CONTROL CHART IN THE PRESENCE OF POSITIVELY AUTOCORRELATED DATA

DISSERTATION

Presented to the Graduate Council of the
University of North Texas in Partial
Fulfillment of the Requirements

For the Degree of

DOCTOR OF PHILOSOPHY

By

Martha M. Harvey, B.A., M.S.

Denton, Texas

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This study uses simulation to examine differences between fixed sampling interval (FSI) and variable sampling interval (VSI) Shewhart X-bar control charts for processes that produce positively autocorrelated data. The influence of sample size (1 and 5), autocorrelation parameter, shift in process mean, and length of time between samples is investigated by comparing average time (ATS) and average number of samples (ANSS) to produce an out of control signal for FSI and VSI Shewhart X-bar charts. These comparisons are conducted in two ways: control chart limits pre-set at $\pm 3\sigma_x / \sqrt{n}$ and limits computed from the sampling process.

Proper interpretation of the Shewhart X-bar chart requires the assumption that observations are statistically independent; however, process data are often autocorrelated over time. Results of this study indicate that increasing the time between samples decreases the effect of positive autocorrelation between samples. Thus, with sufficient time between samples the assumption of independence is essentially not violated. Samples of size 5 produce a faster signal than samples of size 1 with both the FSI and VSI Shewhart X-bar chart when positive autocorrelation is present. However, samples of size 5 require the same time when the data are independent, indicating that this effect is a result of autocorrelation.

This research determined that the VSI Shewhart X-bar chart signals increasingly faster than the corresponding FSI chart as the shift in the process mean increases. If the process is likely to exhibit a large shift in the mean, then the VSI technique is recommended. But the faster signaling time of the VSI chart is undesirable when the process is operating on target. However, if the control limits are estimated from process samples, results show that when the process is in control the ARL for the FSI and the ANSS for the VSI are approximately the same, and exceed the expected value when the limits are fixed.

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CHAPTER I

INTRODUCTION

Natural variation is inherent in any process, no matter how well designed. When a manufacturing process operates with only random variation, explainable by random chance, then the process is said to be "in control." A manufacturing process that contains variability unexplained by random chance is said to be "out of control," and analysts hunt for a reason or "assignable cause" for the unexplained variation.

An out of control process is expensive. Products manufactured while the process is out of control may not meet minimum requirements and thus must be designated as scrap or sold at a reduced rate. Additionally, the profit margin may be reduced if the out of control process causes an over consumption of raw materials. Ultimately, the end product of an out of control process is unacceptable. Prior to the development of statistical process control (SPC), the unacceptable product was not discovered until final inspection. This inspection process was the last procedure prior to marketing. The inferior product was sold at a discount if it was usable, re-worked if possible, or scrapped. Little attempt was made to discover the cause of the inferior product and improve the process itself. This type of quality control (the inspection of finished product) is being replaced by statistical process control where the manufacturing process itself is monitored to make corrections before a large quantity of the inferior product is produced (Wardell, Moskowitz, and Plante, 1991).

The control chart is used in industry to differentiate between natural variation and variation with an assignable cause. It provides a dynamic record of the process and is used to detect changes in a critical process variable to avoid manufacture of an unacceptable product. Control limits for the chart are based on the probability that a large number of the statistics of interest would fall within these limits if the process were operating in control. Samples are taken from the process, the statistics of interest are computed, and then these statistics are plotted on the chart in time sequence. When a statistic falls beyond the control limits, the process is considered to be out of control. As a result, the production process is stopped, steps are taken to find the assignable cause, the necessary corrections are made, and the process is brought back into control.

In spite of many new designs, the Shewhart X-bar control chart, designed in 1931, is still the most popular control chart for monitoring a process mean (Saniga and Shirland, 1977; Modarress, 1989; Wardell, Moskowitz and Plante, 1991). This is primarily due to the ease with which its implementation can be explained and mastered by non-technical personnel on the assembly floor. Also, the Shewhart X-bar chart is intuitively obvious in its analysis and thus easily explained in industry to those without training in statistics. The traditional Shewhart X-bar charts use samples taken from the process at evenly spaced or fixed sampling intervals (FSI), and the means of these samples are plotted to determine if the process remains within acceptable limits (in control). After each sample is taken, the decision is made to either continue the process for a fixed interval of time or to stop the process. The expected length of time for the process to operate without the control chart signaling that a problem exists is called the average run length (ARL). If the process is in control, it would be desirable to set the control limits in order for the ARL to be long so that the number of false alarms is small. However, if the process is out of control, the ARL should be short so that less product will be manufactured before the problem is discovered and corrected.

With a traditional Shewhart X-bar control chart, the statistic of interest from each sample is plotted, and if the plotted statistic is anywhere within the designated control limits, the decision is made to continue the process to the next sampling point. This type of sampling scheme, based on a pre-determined time interval, is the fixed sampling interval technique. In 1988, Reynolds, Arnold, Amin, and Nachlas proposed a new technique called a variable sampling interval (VSI) control scheme, in which the length of time before the next sample is taken depends on the location of the value of the current sample in the sequence. If the value of the sample mean indicates that the process is operating on target, it is reasonable to wait a longer time than usual before taking the next sample. If, however, the value of the sample mean indicates that the process is operating within control limits but close to the limits, it is reasonable to suggest an increase in the frequency of the sampling rate. This allows potential out of control situations to be detected and corrected more quickly than with the FSI techniques (Reynolds, Arnold, Amin, and Nachlas, 1988).

These charts, whether using FSI or VSI techniques, are designed under the assumption that the process being plotted produces data that are independent and normally distributed (Shewhart, 1931). However, in actual industrial operations, it is frequently the case that the observations are autocorrelated (Box, Jenkins, and MacGregor, 1974; Vasilopouos and Stamboulis, 1978; Wardell, Moskowitz and Plante, 1991). Loosely defined, autocorrelation is present whenever data carry some dependency from the previous value. This autocorrelation can make the results obtained from traditional control chart analysis misleading: with positive autocorrelation the chart may indicate an out of control situation when in fact the process is operating within limits; with negative autocorrelation the chart may fail to signal when a significant deviation from the process mean occurs (Maragah, 1989). When the process data are independent, it is known that VSI techniques signal an out of control condition more quickly than FSI (Reynolds, Amin, Arnold, and Nachlas,1988).

If a process is suspected of autocorrelation, specialized computer software will allow identification of the type and amount. The limits of the control chart can then be adjusted to compensate for the autocorrelation, and the control chart will operate with the same probabilities as if used with independent data (Wardell, Moskowitz, and Plante, 1991). However, a process is often assumed to produce independent data when, in fact, the data are actually autocorrelated. This undetected autocorrelation presents potentially expensive consequences. An illustrative hypothetical example of the importance of the appropriate use of the control chart and the role which autocorrelation might play in its use are presented below.

For example, assume that at a rate of 40 per minute a certain facility manufactures cans of soup with chunky chicken and vegetable ingredients. The Shewhart X-bar control chart is employed to monitor the mean weight per can. Samples composed of 5 consecutive cans are taken from the line every 20 minutes, and the finished containers are weighed. These weights are averaged, and the mean (x-bar) is plotted on the control chart. If the plot indicates an out of control situation, then the process is making either heavier or lighter product than appropriate. Lighter product may yield a proportion of cans that is less than the guaranteed analysis on the label. As a result, this product cannot be sold except to company employees or others aware of the lighter weights, at less than the cost to manufacture. Heavier product, on the other hand, involves the cost of additional ingredients and may also incur an increased microbial risk because of the difficulty in uniform and adequate penetration in the heat sterilization process.

The costs associated with changes from a target value (mean) make it desirable to use the control chart to detect changes as quickly as possible. If the fill process is producing independent and normally distributed mean weights for the cans, then one can expect an "assignable cause" or change in the mean fill weight of the cans whenever the process signals. If the fill weights are autocorrelated, then the out of control signal may occur because of the autocorrelation rather than a deviation in mean weight. This out of control signal institutes a search for the cause of the change in the fill process. Looking for a problem when one does not exist is time consuming and expensive, both in labor costs and down-time on the assembly line. Alternatively, failure to detect a problem when one does exist is potentially even more expensive; for example, customer dissatisfaction and/or discrepancies in labeling specifications can be extremely costly.

Such a process as the above hypothetical soup manufacturing process is, in fact, likely to be autocorrelated because of the gravity-feed nature of the fill process. In spite of the mixing equipment, the ingredients are likely to be concentrated in the bottom of the batch's mixing container, therefore filling the cans with increasingly lighter mixture, or with less and less heavy material. The autocorrelation present in this process will cause the control chart to signal that a problem exists when the cans are actually being filled with an appropriate weight of soup. The economic impact of autocorrelation is the cost (in manpower time and production loss) of searching for this non-existent shift in the mean rather than working on the true process problem.

While process data are frequently correlated, the FSI and the VSI operations previously described depend on the assumption of independent observations (Shewhart, 1931; Grant and Leavenworth, 1988; and Montgomery, 1985). The assumption of data independence when it does not exist is error prone and costly (Goldsmith and Withefield, 1961; Johnson and Bagshaw,1974; Vasilopoulos and Stamboulis, 1978). Therefore, the development of methodologies that allow for the assumption of correlated data makes a valuable contribution to statistical quality control.

Purpose, Problem, and Significance

This dissertation investigates the use of Shewhart X-bar charts with fixed versus variable sampling interval techniques when the critical variable in the process produces positively autocorrelated data. Specifically, this dissertation seeks to determine if the use of the newer variable sampling interval (VSI) techniques will reduce the time required for a Shewhart X-bar control chart to detect a deviation in the target mean when monitoring a process producing positively autocorrelated data. The VSI technique is considered superior to the FSI if it will yield an out of control signal faster when the process mean has shifted from the target and a slower out of control signal in the absence of a shift. With increasing economic constraints, it is important that process control be accurately monitored to decrease the number of defectives and scrap. Faster detection of an out of control status shuts down production to allow for a process correction that results in a cost savings (Saccucci, Amin, and Lucas,1991).

If autocorrelation is suspected, several methods are available to determine its type and strength (Box and Jenkins, 1976). After the characteristics of the autocorrelation are determined, Alwan and Roberts (1988) and Montgomery and Friedman (1988) propose methods for adjusting for autocorrelation using time series models. Alwan and Roberts (1988) suggest two new charts: first, the common cause chart that uses fitted values for detection of specific corrective action rather than simply a signal that some action is needed, and second, the special cause chart that uses residuals from a time series model to form the chart. Ermer (1980) proposed a Dynamic Data System approach to overcome problems of autocorrelation by using a stochastic time series approach to modify the Shewhart X-bar control chart in order to make it appropriate for use in the continuous production processes used extensively in metal and chemical industries. Baik, Reynolds, and Arnold (1991) determined that the control limits of the Shewhart X-bar control chart could be adjusted in order to allow for autocorrelation, but only after the specific characteristics of the autocorrelation are known.

This dissertation assumes that positive autocorrelation is unknowingly present and that the Shewhart X-bar chart, due to its popularity, is being used to monitor deviation in the process mean. In this case, the presence of autocorrelation can make the interpretation of the chart erroneous because the assumption of independence is violated with autocorrelated data; thus the Shewhart X-bar chart does not signal accurately (Alwan and Roberts, 1988; Ermer, 1980; Maragah, 1989). It is the intent of this work to make recommendations for the use of the Shewhart X-bar chart in practice, to evaluate the impact of current research streams in literature to practice, and to extend the research on interpretation and understanding of the

Shewhart X-bar control chart in the presence of positively autocorrelated data, including both the traditional FSI and the newer VSI sampling techniques.

CHAPTER II

PRIOR RESEARCH

Essential to the reader's appreciation of this work is an understanding of the traditional Shewhart X-bar control chart, the newer variable sampling interval (VSI) techniques when used with the Shewhart X-bar chart, applicability of the Shewhart X-bar control chart when data is autocorrelated, and finally, the use of VSI techniques with autocorrelated data. Below is a literature review of these areas as they are applicable to this work.

Shewhart X-bar Control Chart

The control chart is a statistical device used to distinguish between natural variation in a process and variation due to a specific cause or problem. The Shewhart X-bar chart (1931) was designed to detect changes in the process mean and is one of the most widely used control charts in industry (Amin, 1987; Saniga and Shirland, 1977;

Modarress, 1989). The chart is designed with a central line representing the target mean μ_o and two control lines usually located at $\mu_o \pm 3\sigma_x$ (see Appendix B, Figure 1). The location of these control limits is based on the probability that a sample mean would, by chance, be so large or so small as to fall outside these limits with a known frequency when the process mean is in fact equal to μ_0 (in control). The use of what are referred to as 3-sigma limits is built on the manufacturer's acceptance of a risk of .0027 of having the sample mean fall beyond the limits, assuming the process is in control. The computed sample means are plotted independently across time at fixed sampling intervals (FSI), and if one of these means falls outside the control lines, the process is considered to be out of control and a signal is given. Shewhart X-bar charts are simple to use and give quick indication of a large shift in the process mean (Baik, Reynolds, and Arnold, 1991; Champ and Woodall, 1987). For small shifts, however, additional warning lines (Page, 1955) and/or runs rules (Weiler, 1953 and Moore, 1958) can be added to make the chart more sensitive.

Variable Sampling Intervals

The concept of variable sampling intervals (see Appendix B, Figure 2) is an extension of the continuous sampling plans for product inspection. Dodge (1943) introduced a plan (CSP-1) which alternated between 100% inspection and inspection of only a fraction of the products selected at random intervals. Modifications (CPS-2 and CPS-3) were made by Dodge and Torrey (1951) which allowed the random inspection process to continue rather than reverting to 100% inspection whenever a defective item was found. Lieberman and Solomon (1955) introduced multi-level inspection plans that decreased the problems involved with jumping from one inspection level to another. Their plan allowed for many levels of inspection; 100% inspection when the product quality was poor, and decreased inspection when quality was good.

This concept in acceptance sampling, determining the time between samples according to the results of the current sample, was carried over into statistical process control by Arnold (1970). Arnold utilized variable sampling intervals (VSI) in a study monitoring water quality in streams by taking more samples when the quality was poor and fewer samples when the quality was good. Crigler (1973) formulated an economically optimal sampling plan using the Markovian technique developed by Arnold (1970). Crigler and Arnold (1979, 1986) and Smeach and Jernigan (1977) extended the work with variable sampling intervals between samples, but not until Hui (1980) was this concept specifically applied to control charts.

Reynolds, Amin, Arnold, and Nachlas (1988) found that, when used with independent data, adding the VSI technique improved the signaling time of the Shewhart X-bar chart. Then, Reynolds, Amin, and Arnold (1990a) extended the work and found that adding VSI consistently improved the performance of the Cumulative Sum (CUSUM) chart under the same assumptions. All of their work is based on samples of size 1 and the assumption that individual samples are independent. They defined the average time to signal (ATS) and average number of samples to signal (ANSS) to replace the average run length (ARL) which is used with FSI techniques as a measure of performance.

Control Charts Used with Correlated Data

In traditional statistical quality control, the state of statistical control is identified with a process generating independent and identically distributed random variables (Shewhart, 1931; Grant and Leavenworth, 1988; and Montgomery, 1985). Much work is being devoted to the effects produced when this assumption fails to be true. Goldsmith and Withefield (1961) studied the effect of correlated data on the CUSUM chart. Johnson and Bagshaw (1974) and Bagshaw and Johnson (1975) studied the effects of serial correlation on the CUSUM chart using first order autoregressive and first order moving average models. They concluded that the CUSUM chart fails to signal appropriately when used with data which are not independent. Vasilopoulos and Stamboulis (1978) utilized a time series approach with a second order autoregressive process to incorporate dependent data into standard control chart methodology. They confirmed that if correlated data were present but not recognized, the false alarm rate of the Shewhart X-bar control chart was increased. Abraham and Kartha (1978, 1979) connected the Geometric Moving Average (GMA) chart and time series forecasting whereas Kartha and Abraham (1979) examined by simulation the effect of serial correlation on the ARL of CUSUM charts. Alwan and Roberts (1988) illustrated the use of statistical modeling of time series effects using auto-regressive integrated moving average (ARIMA) models and the application of standard control chart procedures to the residuals from these models. The study of the residuals makes it possible to isolate the common causes creating the departures from control because after the process is modeled, the residuals will be independent even though the individual data were not. Wardell, Moskowitz, and Plante (1991) compared the ARL performance of the Shewhart chart and the Exponentially Weighted Moving Average (EWMA) chart to the Common-Cause Control chart (CCC) and the Special-Cause Control chart (SCC) proposed by Alwan and Roberts (1988) using the ARMA(1,1) model. Wardell, Moskowitz, and Plante (1991) did not consider VSI in their work. They found the standard deviation of the process based upon a prior knowledge of the correlation (using formulas from Box and Jenkins, 1976), and used this analytically derived value to set the control limits. Their results showed that the CUSUM chart was robust in the presence of data correlation whereas the Shewhart chart rarely performed as well as the other charts with correlated data.

Maragah (1989) studied the effect of autocorrelation on two charts, both of which incorporated fixed sampling intervals: the Shewhart X-bar chart and the EWMA chart. He set the control limits retrospectively using the first 25 data points, but he did not attempt to calculate the time to signal or run lengths. Using only the first order autoregressive model AR(1), he determined that autocorrelation can cause control charts to signal more frequently if the autocorrelation is positive and less frequently given negative autocorrelation. If positive autocorrelation causes the chart to signal, then the chart is indicating some change in the process when indeed none exists; if negative autocorrelation is retarding the signaling process, then the process may be unknowingly operating out of control.

VSI Techniques with Correlated Data

Baik, Reynolds, and Arnold (1991) used a Markov chain representation to determine how to design control charts in the presence of autocorrelation when the distribution of observations of sample size 1 is normal. They determined that a two sampling interval control chart tends to perform similarly to a three or more sampling interval control chart and that asymmetric control charts do not always seem to have shorter detection time. Therefore, they recommend a symmetric two sampling interval control chart. They employed both fixed and variable sampling interval techniques with an autoregressive model of order 1.

Summary

Although considerable research was conducted on the Shewhart X-bar chart, on variable sampling interval techniques, and on correlated data, work integrating these three areas is not easily found. Baik, Reynolds, and Arnold (1991) integrate all three areas, but their work was still unpublished at the time of this dissertation. Baik, et al. altered their control limits in order that the in control values of the ATS and ANSS were close to the theoretical ARL of 370.4. However, in practice the control limits are usually set by estimating the process parameters based on calculations obtained using the first 25 samples (Montgomery, 1985; Grant and Leavenworth, 1988). In order to determine the usefulness of the new VSI techniques with the Shewhart X-bar control chart in practice, the effect, if any, of calculating the control limits needs to be determined. The Shewhart X-bar chart is still the most popular chart for monitoring the process mean (Saniga and Shirland, 1977; Modarress, 1989; Wardell, Moskowitz and Plante, 1991), even though research has shown the CUSUM chart to be more efficient in detecting small shifts (Lucas, 1976) and in handling autocorrelated data (Wardell, Moskowitz, and Plante, 1991). The CUSUM chart is more mathematically complex and thus more difficult to explain and implement. Therefore, this dissertation investigates the effects of the VSI technique using Shewhart X-bar chart on positively autocorrelated data because of its continuing popularity in practice.

The autoregressive model of order one was chosen because of its applicability in manufacturing (Baik, Reynolds, and Arnold, 1991). Metal manufacturing, such as steel making operations, and certain chemical processes have autoregressive properties due to their continuous production schemes (Ermer, 1980; Alwan and Roberts, 1988). As automatic measurement of process items in time sequence makes the taking of measurements easier, measurements are taken more frequently. One result of this improved data collection is that autocorrelation, if it exists within the process, becomes more obvious (Baik, Reynolds, and Arnold, 1991). Thus the concept of autocorrelation is of increasing interest to industry. The term "autoregressive," loosely defined, implies that each data item carries some dependency from the previous item. If this dependency can be isolated and removed from the process, then the Shewhart X-bar control chart will yield accurate results. If dependency cannot be removed from the process, then a model incorporating this dependency must be utilized in order to prevent the Shewhart chart from indicating an out of control situation when the process is still on target or from neglecting to signal when the process mean has drifted.

It is the intent of this dissertation to determine if results from prior research on VSI techniques could/should be used in industrial practice. This interest motivated research questions in several areas where industrial practice differs from the methodology of the prior research. No published work was found incorporating samples of size 5, yet this sample size is the industry standard (Montgomery, 1985; Grant and Leavenworth, 1988).

Another area lacking previous investigation is the use of intervals (or times) between samples. In the production process, each manufactured item is seldom inspected. Samples are taken at times determined by the manufacturer (FSI) or by the process (VSI). Prior research assumed that each item was sampled because only the interval length of 1 is examined. The influence of increasing interval length (increasing the amount of time between sampling) is not addressed in the literature. This dissertation investigates the influence of increasing interval lengths on the Shewhart X-bar control chart by using both fixed and variable sampling techniques. Prior research is available on autocorrelated data using FSI methods because many industrial processes produce this type of data, but the studies are limited to samples of size 1 and control limits that are set from theoretical parameters. It is of interest to see if some pattern is evident when FSI and VSI techniques are used on autocorrelated data with samples of size 5, control limits calculated from the process samples, and interval lengths greater than 1 between samples. Increasing the interval length when autocorrelation is present in the process motivates the question of whether increasing interval length creates the effect of decreasing the autocorrelation present in the sampled data. A detailed theoretical discussion addressing the above questions is available in the next chapter.

It is of primary economic concern to a manufacturer to obtain a signal as quickly as possible if the manufacturing process is out of control. The out of control situation occurs when the process mean has shifted from its desired or target value and is reflected in a signal by the Shewhart X-bar chart. Thus, prior research on the effectiveness of the VSI technique has compared the behavior of the VSI technique under the influence of various shifts in the target mean of the process to that of the FSI. The VSI technique is considered superior to the FSI in practice if it has a slower average time to signal when the process is in control and a faster ATS when the process is out of control. Prior research, using data that are assumed independent, determined the ATS to be faster for the VSI techniques. It would not be an advantage to receive the faster out of control signal if the process is, in fact, operating in control, and the signal is caused by autocorrelation in the process data rather than by a deviation of the process mean. Because prior research indicates that the Shewhart X-bar variable sampling interval control chart is faster to signal than the corresponding fixed sampling interval chart when samples of size 1 are independent over time, (Reynolds, Amin, Nachlas, and Arnold, 1988) this dissertation seeks to discover, by simulation, if the behavior pattern of the ATS is similar when the process under consideration produces positively autocorrelated data.

CHAPTER III

THEORETICAL FRAMEWORK

Industrial applications of statistical process control frequently utilize the Shewhart X-bar control chart (Amin, 1987; Saniga and Shirland, 1977; Modarress, 1989). The usefulness of the chart depends on its ability to signal quickly and accurately when the process drifts from the target mean. VSI techniques improve the time to signal for independent data (Reynolds, Amin, Arnold, and Nachlas, 1988). Baik, Reynolds, and Arnold (1991) showed that VSI improved the time to signal for some AR(1) data, but the control limits were set based on standard normal data, and every data point was considered a sample of size 1.

In actual industrial processes, the production is sampled as it is manufactured (Montgomery, 1985; Grant and Leavenworth, 1988). Only a portion of the production is sampled, and these samples are taken at either fixed or variable intervals. Samples of sizes larger than 1 are frequently utilized to obtain more accurate representations by reducing variation through averaging. This work explores samples of size 1 because prior work examining FSI and VSI techniques with and without autocorrelation used this size, and of size 5 because it is the industry standard (Grant and Leavenworth,1988; Montgomery, 1985).

At the beginning of the production process, the control limits for the Shewhart X-bar chart are established. The usual procedure for estimating these limits is to use approximately the first 25 samples, measuring both the mean and the range of each sample, to find x-double bar as an estimate for the process mean and r-bar as the statistic used in calculating the estimate of the process standard deviation. X-double bar is defined as the average of all of the x-bars which are the calculated mean averages of each sample, and R-bar is defined as the average of all of the ranges of each sample. (More detailed definitions are available in the following chapter.) These estimates are used to set the control limits, assuming that the process is in control at this time.

The use of samples of size 1 and pre-set control limits by Baik, Reynolds, and Arnold (1991) and Wardell, Moskowitz, and Plante (1991) restricts the application of their work due to the deviations from actual practice (Maragah, 1989; Grant and Leavenworth,1988; Montgomery, 1985). Therefore this dissertation investigates the effect of FSI versus VSI techniques on the performance of the Shewhart X-bar control chart when statistics from samples of size 1 and size 5 are used to calculate estimates for the mean and standard deviation of a process. These estimates are then used to set the control limits as in industrial applications. Values of the ARL/ANSS and the ATS obtained from these charts with calculated control limits are compared to the results using the Shewhart X-bar charts with control limits set before sampling at $\pm 3\sigma_x/\sqrt{n}$.

Research Question #1: Autocorrelation

The autoregressive model is denoted by the abbreviation AR followed by a number within parentheses which gives the degree of the autoregressive component. Thus, the first order autoregressive model is denoted by AR(1). The general nonseasonal Box and Jenkins autoregressive model of order one is

$$X_t = \delta + \phi X_{t-1} + a_t \qquad (3.1)$$

where $\delta = \mu(1-\phi)$, X_t = observation at time t, ϕ = the auto-regressive parameter between consecutive terms, μ = the mean of the process, and a_t represents the noise or error term. These noise terms are independent, normally distributed, random variables with mean 0, and standard deviation σ_a. For the AR(1) process, the autocorrelation between the terms X_t and X_{t-j} is ϕ^j for $j \geq 1$. Without loss of generality, μ is considered here to be zero which gives a general model,

$$X_t = \phi X_{t-1} + a_t \tag{3.2}$$

Note that the commonly assumed independent case occurs when $\phi = 0$. The level of ϕ must be in the interval (-1,1) exclusive in order for the autoregressive process to meet the constraints of stationarity (Box and Jenkins, 1976). All parameters of the generated data are assumed known.

The standard deviation of the autocorrelated process depends on the particular autocorrelation structure and is described by

$$\sigma_x = \sqrt{\frac{1}{1 - \phi^2}} \ \sigma_a \tag{3.3}$$

where σ_a is the standard deviation of the error terms. Formula (3.3) is used in prior research to determine the standard deviation of the process, but in order to do so the autocorrelation parameters must be known at the time of sampling. In this dissertation, knowledge of the presence or amount of autocorrelation is not assumed. The standard deviation is estimated in this work based on the average range of the sampled observations. In equation (3.3), it is observed that as the absolute value of the autocorrelation parameter ϕ increases, the standard deviation (or variance) of the observations increases. The variance of the model increases without bound as ϕ approaches a value of 1. This increasing variance motivates one of the questions of this dissertation because if the variance of the process increases as the autocorrelation parameter increases, then the observations from such a process would go out of control more quickly due to their increased variance. This leads to a faster signal from the control chart. If the process is truly out of control, this faster signal is advantageous. But if the signal is caused or aided by the autocorrelation, the faster signal is actually a false indication of a shift in the mean when, in fact, one does not exist. Thus, research question #1 is formulated: Does the ATS on the Shewhart X-bar control chart decrease as the autocorrelation in the process increases?

Research Question #2: Length of the Sampling Interval

In prior work, the length of time between samples was taken as 1 (Reynolds, Arnold, Amin, and Nachlas, 1988; Baik, Reynolds, and Arnold, 1991; Alwan and Roberts, 1988). That is, every observation or manufactured item was sampled. This is not the case in practice because sampling is expensive, both in time required for employees to take the samples and in production units for destructive sampling techniques. This dissertation seeks to explore, by the surrogate of skipping 10 to 50 items between samples, the more likely process of sampling over time. Larger intervals were originally planned but required such excessive CPU time that they were non-feasible. These time intervals are still small for some production models but reflect possible numbers for large item manufacture. For example, if 480 items are manufactured within an 8 hour shift, then 10 items are manufactured every 10 minutes. As a result of the economic impact of an out of control process, the manufacturers might decide that 10 minutes is the longest length of time that they are willing to let the process continue without knowledge of its accuracy. The 10 minute time-frame is then chosen as the Fixed Sampling Interval (FSI). An illustration of the FSI technique can be seen in Appendix B, Figure 1. Items are taken from the production line in sequential order after each 10 minutes according to the desired sample size.

The correlation of X_t and X_{t-j} was identified in the previous section as being ϕ^j for $j \geq 1$. Thus, as the time between observations increases, the correlation between the observations in question decreases. For example, if the autocorrelation parameter $\phi = 0.9$, then for consecutive samples of size 1 (n=1, L=1) the autocorrelation is $(0.9)^1$. If the interval length is increased to L=10 and the sample size remains at n=1, then the autocorrelation between consecutive samples will be $(0.9)^{10} = 0.3487$. The autocorrelation between samples of size 1 will continue to decrease as the interval length increases. Maragah (1989) found that the number of control chart signals within a given number of observations increases as the positive autocorrelation increases. This implies that the ATS is slower for a given autocorrelation value as the time between samples increases. Prior work was done with samples of size 1 taken consecutively (Reynolds, Arnold, Amin, and Nachlas, 1988; Baik, Reynolds, and Arnold, 1991; Maragah, 1991) because this scenario is tractable. The research being conducted here uses samples of both size 1 and size 5. Although the exact autocorrelation between consecutive samples of size 5 is not known, it is possible to surmise relationships. For example, if the samples with n=5 and L=1 are composed of observations 1,2,3,4,5 and observations 6,7,8,9,10, then the autocorrelation between various members of these two different samples (inter-sample) can be computed. The highest inter-sample autocorrelation lies between observations 5 & 6. All other inter-sample autocorrelations will be less. The autocorrelation between observations 1 & 6, 2 & 7, 3 & 8, 4 & 9, and 5 & 10 is ϕ^5 whereas the autocorrelation between observations 1 & 7, 2 & 8, 3 & 9, 4 & 10 is ϕ^6, and so on, until the smallest autocorrelation value at the farthest pair, observations 1 & 10, is computed to be ϕ^9. If $\phi = 0.6$, then the autocorrelation values in the example range from $(.6)^1$ to $(.6)^9 = 0.0101$. Given these autocorrelations, it is intuitively seen that the autocorrelation between the sample means should be in between the maximum autocorrelation value of 0.6 and the minimum value of 0.0101. If, however, the interval length between samples is L = 10, then the observations might be 1,2,3,4,5 and 16,17,18,19,20. Assuming the same autocorrelation value of ϕ=0.6, the autocorrelation between individual members of the samples now ranges from a maximum of $(.6)^{10} = 0.0060$ to a minimum of $(.6)^{19} = 0.0001$.

Although the exact values are unknown, the increasing interval length can be expected to decrease the autocorrelation between samples regardless of sample size. Thus, research question #2 is defined: Does the ARL/ANSS on a Shewhart X-bar control chart increase as the length of the interval between samples increases when positive autocorrelation is present?

Research Question #3: Sample Size

Shewhart (1931) originally recommended samples of size 4 for his chart, but size 5 has become more frequently used. While it is difficult to argue if this change to size 5 is due to statistical properties or ease of hand calculation, it is a documented industry standard (Grant and Leavenworth, 1988; Montgomery, 1985). In spite of this, prior theoretical investigation has been conducted on samples of size 1 because of the mathematical intractability of larger sample size (Reynolds, Arnold, Amin, and Nachlas, 1988; Baik, Reynolds, and Arnold, 1991; Maragah, 1991; Wardell, Moskowitz, and Plante, 1991). To avoid these complications, the investigations of this dissertation are conducted through simulation. Therefore, the more commonly used samples of size 5 can be utilized to determine the impact of the larger sample size on the VSI technique when used with the Shewhart X-bar control chart. Samples of size 1 are also used to allow a comparison to prior work and to avoid confounding with research questions about interval length.

Again, the correlation of X_t and X_{t-j} was identified in the previous section as being ϕ^j for $j \geq 1$, and it is established that the autocorrelation between samples of size 1 is ϕ. The exact autocorrelation between the means of samples of size 5 is unknown, although enough information is available from which a general idea of the behavior can be surmised. As an example, if observations 1,2,3,4,and 5 comprise the first sample, then the autocorrelation ϕ between any two observations within the sample (intra-sample) can be computed. There are four elements of the sample with correlation ϕ^1 (observations 1 & 2, 2 & 3, 3 & 4, 4 & 5), three elements of the sample with correlation ϕ^2 (observations 1 & 3, 2 & 4, 3 & 5), two elements of the sample with correlation ϕ^3 (observations 1 & 4, 2 & 5), and one element of the sample with correlation ϕ^4 (observations 1 & 5). The identical relationship is present in every other sample of five consecutive observations within this process. It is possible that when the intra-sample observations are averaged, the autocorrelation of the mean of a sample of size 5 may be greater than or equal to the original autocorrelation ϕ and thus greater than or equal to the autocorrelation present in a sample of size 1. If this relationship exists, then the samples of size 5 might cause the Shewhart X-bar control chart to signal more quickly. This rationale motivated research question #3: Does the use of samples of size 5 instead of samples of size 1 increase the effect of positive autocorrelation on the Shewhart X-bar control chart?

Research question #4: Shift in the Process Mean

As a product is manufactured, the Shewhart X-bar chart is used to monitor any changes in some chosen mean statistic such as weight, diameter, or length. The desired value is designated the target mean, and any deviation from this target mean is referred to as a shift in the process mean. The effect on the ATS and the ARL/ANSS of specific shifts in the process mean is known when the process produces independent observations (Reynolds, Arnold, Amin, and Nachlas, 1988).

This dissertation investigates the effect on the ATS and the ARL/ANSS of the Shewhart X-bar control chart of a shift in the process mean of $\delta = 0.0$, 1.0, 2.0, and 3.0 standard deviations from the original target when the process produces autocorrelated data. Shifts between 0.0 and 4.0 were utilized with independent observations in several of the previously cited works (Reynolds, Amin, Arnold, and Nachlas, 1988; Saccucci, Amin, and Lucas, 1991), but a shift as large as 4.0 standard deviations seemed excessive for the investigations because the ARL/ANSS is 2.0 at a shift of 3.0 and ARL/ANSS values lower than 2.0 seem of questionable value in establishing a pattern of behavior for the ARL/ANSS. It was anticipated that the same effect would be found with autocorrelated data -- that is, the greater the shift of the process mean from the target value, the faster the Shewhart X-bar control chart will indicate an out of control signal with either the FSI or the VSI technique. Thus, research question #4 is the following: Do increasing shifts in the process mean cause a lower ATS for the Shewhart X-bar control chart when the process data is positively autocorrelated?

Research question #5: The VSI Technique

For the VSI technique (see Appendix B, Figure 2), two sampling intervals were shown to be just as effective and much simpler than additional intervals (Saccucci, Amin, and Lucas, 1990). Work by Reynolds, Arnold, Amin, and Nachlas (1988) showed that with independent data, choosing the two sampling intervals between zero times the fixed interval and twice the fixed interval yields the lowest average time to signal for all shifts in the target mean. Their work was done using the short sampling interval as .1*L, and the long sampling interval as 1.9*L, where L = the fixed interval length. It is known that two sampling intervals as far apart as possible are optimal for the AR(1) model (Baik, Reynolds, and Arnold, 1991). The minimum interval might be the limiting value to the time required to actually take the required sample or to manufacture an item. At time zero, the shorter interval is automatically selected. This is not unreasonable because "start-up" processes frequently require more monitoring than "on-going" processes. The maximum value might be the amount of time personnel are willing to let the process run without sampling even when the process is in control. Every statistic of process manufacture cannot be monitored on a control chart, and personal intervention or inspection at times to be determined within an individual process is certainly valuable.

In this dissertation, intervals are established so that the probability of an observation falling in the interval designated I_1 equals the probability that the observation falls in interval I_2,

$$P(I_2) = P(I_1) \tag{3.4}$$

when the process is on target ($\mu = \mu_0$). The intervals are defined as follows:

$$\begin{aligned} I_1 &= (\mu_0+3\sigma_x\ ,\ \mu_0+k\sigma_x) \cup (\mu_0-k\sigma_x\ ,\ \mu_0-3\sigma_x) \\ I_2 &= (\mu_0+k\sigma_x\ ,\ \mu_0-k\sigma_x) \end{aligned} \tag{3.5}$$

where k is determined to be 0.6766 in the following manner.

$$P[\text{long}] = P[\text{short}] \text{ when } \mu = \mu_0$$

$$P[-k < Z < k] = P[-3<Z<-k \text{ or } k<Z<3] \qquad (3.6)$$

$$P[0<Z<k] = P[k<Z<3]$$

$$2P[0<Z<k] = .5 - P[Z>3]$$

$$P[0<Z<k] = 0.2493$$

$$\text{therefore, } k = 0.6766.$$

This value of k allows a long sampling interval with approximately the same frequency as the short sampling intervals.

Reynolds, Arnold, Amin, and Nachlas (1988) showed that the ATS for the VSI chart is lowest when the two sampling intervals are chosen as 0.1 and 1.9 times the sampling interval used in the FSI case. A chart with greater than 1.9 times the FSI implies that the probability of the longer sampling interval is less than the probability of the shorter interval when the process is in control. Thus, the sampling intervals for this work were chosen as $d_1 = 0.1*L$

and $d_2 = 1.9*L$ observations where L = length of the fixed sampling interval.

The variable sampling interval technique was used on the Shewhart X-bar control chart both for processes that produce independent observations and for those which produce autocorrelated data. In each different research effort, however, only samples of size 1, sampling intervals of length 1, and control limits set by theoretical parameters were used. Results showed that the VSI technique made the chart give an out of control signal faster in all cases; that is, the ATS was lower, regardless of the ARL/ANSS. Yet each of the prior studies employed at least one technique not available in most industrial settings. This dissertation seeks to determine if the VSI technique should be used in practice. Therefore, research question #5 asks the following: Do VSI techniques produce a lower ATS than FSI techniques on the Shewhart X-bar control chart with positively autocorrelated data when control limits are calculated from process observations?

Limitations and Key Assumptions

Simulation is used in this work due to the analytical intractability when using samples of size 5 and interval lengths, in the presence of autocorrelation, between samples greater than 1 (Reynolds, Arnold, Amin, and Nachlas, 1988; Baik, Reynolds, and Arnold, 1991; Maragah, 1991; Wardell, Moskowitz, and Plante, 1991). Representative values are used in any simulation, and this implies that assumptions are made in establishing a pattern. Only positive autocorrelation is studied in this dissertation because negative autocorrelation is seldom seen in industrial manufacturing processes (Baik, Reynolds, and Arnold, 1991). It was necessary to limit the autocorrelation parameter to several values in order to keep the total number of simulations manageable. The values chosen (ϕ=0.0, ϕ=0.3, ϕ=0.6, ϕ=0.9) are those of Baik, Reynolds, and Arnold (1991) to allow comparisons. The four autocorrelation parameter values give sufficient information to establish a pattern for the results.

The shifts in the process mean are incremented in jumps of 1.0 from the target value. This increment is large, and it does not allow information about specific behavior (only the pattern of behavior) for small shifts. The choice of the Shewhart X-bar control chart limits information about small shifts in the process mean because it is known to be insensitive to small shifts.

The sample sizes are limited to 1 and 5, and while it would be interesting to know the effect of different sample sizes, prior research is dominated by samples of size 1, and the industrial standard is a sample of size 5.

In this work, the interval length is the number of observations between selected samples and is a surrogate for time between samples. Whereas lengths of 1, 10, 20, and 50 are considered, numerous other possibilities exist. The assumption is made that the patterns established with the increasing interval length will continue as the interval length continues to increase.

Several other assumptions are germane to this work. An IMSL subroutine (a published collection of mathematical and statistical computer procedures) was used to generate autocorrelated data sets of great length (2.5 million in several simulations). While the statistics of the generated data were checked for shorter runs, the continuation of the correct parameters must be assumed. The capabilities of the random number generator were presumed adequate for the simulation requirements. The AR(1) model was assumed to have noise or error terms which were random and normally distributed. Every effort was made to check these assumptions, but their existence must be recognized.

Research Questions

The above section contains a discussion of the motivation for the research questions in this dissertation. For clarity, the five research questions proposed in this study are restated below:

(1) Does the ATS on the Shewhart X-bar control chart decrease as the autocorrelation in the process increases?

(2) Does the ARL/ANSS on a Shewhart X-bar control chart increase as the length of the interval between samples increases when positive autocorrelation is present?

(3) Does the use of samples of size 5 instead of samples of size 1 increase the effect of positive autocorrelation on the Shewhart X-bar control chart?

(4) Do increasing shifts in the process mean cause a lower ATS for the Shewhart X-bar control chart when the process data are positively autocorrelated?

(5) Do VSI techniques produce a lower ATS than FSI techniques on the Shewhart X-bar control chart with positively autocorrelated data when control limits are calculated from process observations?

An important distinction for the interpretation of the results of this dissertation is the difference between the Average Time to Signal (ATS) and the Average Run Length (ARL) or Average Number of Samples to Signal (ANSS). The ARL and ANSS refer to the number of samples taken from the process before one of the sample statistics triggers an out of control signal from the control chart. ATS refers to the number of items that are manufactured before the control chart puts a stop to the manufacturing process by signaling an out of control situation. Thus it is possible, and in fact does occur, that the ATS is quicker for the VSI technique when the ANSS is larger than the ARL for the FSI technique. For example, if the fixed sampling interval is chosen as every 10 items, then if the FSI charts signal on the average with a run length of 4, the ARL is 4 and the number of manufactured items (ATS) is 40. The VSI technique may take 5 samples on the average before signaling (ANSS = 5), but if 2 long sampling intervals are used (20 items manufactured) and 3 short intervals (1 item each), then only 23 items have been manufactured when the VSI chart signals; thus, its average time to signal (ATS) is 23.

CHAPTER IV

METHODOLOGY

Simulation of Observations

For the simulation, data points were generated using the IMSL, Inc. subroutine RNARM which allows pre-determined correlation. The RNARM subroutine uses randomly generated input data that is normally distributed with mean $\mu=0$ and $\sigma^2=1$. The data generated by this method were plotted /fitted using four Shewhart X-bar control charts: two fixed sampling interval charts (one with control limits fixed and the other with limits calculated from estimates obtained from the process observations) and two variable sampling interval charts (one with control limits fixed and the other with calculated limits). The shift in the process mean was introduced after the control limits were calculated from data with a mean set at the target value (if fixed control limits were used, the data comprising the first 25 samples were ignored). The introduced shifts were examined in increments of 1.0 from the target mean of $\mu = 0.0$ to determine the change in the ATS and ANSS when a shift in the process occurred. Shifts between 0.0 and 4.0 were utilized in several works, but most prior work considers shifts between 0.0 and 3.0. The time required for the data to produce an out of control signal was measured for both FSI and VSI charts and these times compared for the same data. The FSI and VSI comparisons were replicated a large number of times and at different levels of correlation with different changes in the mean and different sampling interval lengths. Both fixed control limit charts and calculated control limit charts were subjected to this process.

Shewhart X-bar chart with Fixed Sampling Intervals

For the FSI chart, the sampling interval (L) was set at lengths of 10, 20, and 50. These values were chosen because the short sampling interval of the VSI technique must be one-tenth of each fixed interval length, and the VSI long interval must be 1.9 times the fixed interval length, and fractional values are not possible in practice. Of course, other choices were possible but a longer interval increases the number of observations beyond the capability of the simulation. The fixed interval, regardless of its length, was chosen so that a comparison could be made with the VSI chart whenever possible. For the charts with the fixed control limits, the first data points (observations) were ignored and control limits were fixed using $\pm 3\sigma/\sqrt{n}$. Where applicable, the first data points were used to calculate the control limits for both the FSI and VSI chart. The exact number of points required to determine the control limits was set by the sample size times interval length for the particular replication. As the data points were generated, samples of size $n=1$ or size $n=5$ were taken every L points. The sample average was computed for each replication.

As an example, if the sample size $n = 1$, and interval length $L = 10$, then

$$\overline{\overline{X}} = \frac{\sum_{t=10}^{250} \overline{X}_t}{25} \quad , \quad for \quad t = 10,20,30...250 \quad where \ x_t = \overline{x}_t \tag{3.7}$$

where X-double bar is the mean average of all the individual 25 sample means, and the average range, R-bar, (which was calculated as a moving range only for sample size of one) of the process is

$$\overline{R} = \frac{\sum_{t=20}^{250} R_t}{24} \ , \ where \ R_t = |x_t - x_{t-10}| \ for \ t = 20,30,...,250. \tag{3.8}$$

For samples of size 5, the range R is the difference between the maximum and minimum value within the sample, and R-bar was calculated using

$$\overline{R} = \sum_{t=l}^{25L} R_t \quad where \ R_t = \max\{sample\} - \min\{sample\} \tag{3.9}$$

The usual Shewhart control chart limits were computed using

$$UCL = \overline{\overline{X}} + 3\,\frac{\overline{R}}{d_2\sqrt{n}} \qquad (3.10)$$

$$LCL = \overline{\overline{X}} - 3\,\frac{\overline{R}}{d_2\sqrt{n}} \qquad (3.11)$$

and

n=1	n=2	n=5
d_2=1.128	d_2=1.128	d_2=2.326

where d_2 is a subgroup size factor (Grant and Leavenworth, 1988; Montgomery,1985). Note that because of the moving range utilized with subgroups of size 1, the d_2 factor was computed on the two terms within the moving range.

After the control limits were set, the remaining data were plotted/fitted using the Shewhart chart FSI techniques. A Fortran program was used to recognize an out of control condition and to determine the following: first, the time to signal, TS, (the length of time from the start of the process to the time where the chart signals); and second, the number of samples to signal, NSS, (the number of samples taken from the start of the process until the chart signals). The time required will equal the number of samples to signal since this chart used the FSI that was defined to be one time unit. Repetitions of this procedure generated an average time to signal, ATS, and average number of samples to signal, ANSS.

The parameter (ϕ) of the AR(1) model was set at levels of ϕ=0.0, ϕ=0.3, ϕ=0.6, and ϕ=0.9 degrees of correlation. Negative correlation between successive observations is considered rare in practical situations and was omitted in this work (Baik, Reynolds, and Arnold, 1991). On the Shewhart X-bar chart, the control limits were chosen as $\pm 3\sigma_x/\sqrt{n}$ because of their use in prior work (Wardell, Moskowitz, and Plante, 1991; Baik, Reynolds, and Arnold, 1991; and Reynolds, Amin, and Arnold, 1988). An observation generated by the IMSL subroutine with a correlation of $\phi = 0.0$ has a probability of .0027 of plotting beyond the control limits of ±3, that is, beyond 3 standard deviations from the mean. The expected value of the length of time elapsed before an observation falls beyond 3 standard deviations is the reciprocal of the probability that this event will occur.

$$E[ARL] = 1/p \tag{3.12}$$

$$E[ARL] = 1/.0027$$

$$= 370.37 \approx 370.4$$

For each different AR(1) model, the mean was incremented in units of 1.0 to simulate a shift in the process mean from the target value of 0.0 to a value of 3.0. The ATS was measured for each different model, and this ATS determined which chart signaled faster at which level of correlation for different interval lengths and sample sizes. The entire procedure, starting with newly computed control limits, was replicated for each parameter change.

<u>Shewhart X-bar chart with Variable Sampling Intervals</u>

The control limits were both fixed and calculated, as applicable, for the VSI charts using the FSI techniques described above. Using this methodology, the limits were identically the same for both FSI and VSI charts. After the limits were set by sampling the appropriate number of data points, the variable interval approach was immediately employed for the remaining data points. The shorter sampling interval was always used to calculate the first x-bar from the first sample after the control limits were set. If the first x-bar fell in I_1 the shorter sampling interval was used, and another sample was taken after the next fixed sampling length. If x-bar fell in I_2 the longer sampling interval was used next, and the process was allowed to continue an extended time. A modification of the Fortran program designed for the FSI was used to recognize an out of control condition and to determine: first, the time to signal, TS, (the length of time from the start of the process to the time where the chart signals), and second, the number of samples to signal, NSS, (the number of samples taken from the start of the process until the chart signals). Repetitions of this procedure generated an average time to signal, ATS, and an average number of samples to signal, ANSS. The entire procedure, starting with newly computed control limits, was replicated for each parameter change.

The number of replications for all of the simulations was originally chosen to be 10,000. This number is much larger than used in prior work, but preliminary results (see Appendix B, Figure 3) indicated that the variation in ARL was large with less than 5,000, and 10,000 was chosen for additional accuracy. Because of the unexpected excessive computer time required for the large number of observations necessary for the chart to determine an out of control signal, some of the simulations were limited to 5,000 replications. The results (indicated in Appendix B, Figure 3) show that at 5,000 replications the ARL begins to show decreased variability.

CHAPTER V

SIMULATION RESULTS

Two distinct approaches for computation of the control limits were used in this dissertation. The first involved both FSI and VSI techniques with the control limits pre-set at $\pm 3\sigma_x / \sqrt{n}$. The second group used control limits that were calculated from estimates obtained from the first 25 samples of the data. Comparisons of the results, for varying interval lengths, correlation coefficients, sample sizes, and shifts in the process mean are presented below. Note that in many instances where values were too widespread for graphic clarity using a linear scale, the logarithmic scale is also presented.

Influence of Interval Length

FSI with Fixed Control Limits

For data points generated with zero correlation ($\phi =$ 0.0), interval length has no effect on the ARL for the sampling process for either samples of size 1 or 5 (see Appendix A,Tables 1 and 5 and Appendix B, Figures 20a-h). This is expected because the data are independent and normally distributed with a mean of 0 and a standard deviation of 1. At the autocorrelation level of $\phi = 0.3$, the ARL values begin to show a slight downward trend (see Appendix A, Table 2 and Appendix B, Figures 20a-h) indicating that for samples of size 1 the ARL is decreasing with an increase in length from 1 to 10 between samples. After the interval length of 10, no further changes are noted as a result of increasing interval length at this level of correlation. For samples of size 5 (see Appendix A, Table 6 and Appendix B, Figures 24a-h), increasing the interval length from 1 to 10 shows less decrease than that noted for samples of size 1. The results at $\phi = 0.6$ for samples of both size 1 and 5 (see Appendix A, Tables 3 and 7) indicate a larger decrease in ARL between L=1 and L=10 particularly for shifts of 0.0 and 1.0 but no discernable difference when interval length is increased to L=20 or L=50.

At a correlation of $\phi = 0.9$, the difference between interval lengths of L=1 and L=10 is more definitive for samples of size 1 than for samples of size 5 (see Appendix A, Tables 4 and 8, and Appendix B, Figures 20b-h) with a lesser decreasing trend evident for interval lengths greater than 10. For samples of size 5, this trend is less pronounced (see Appendix A, Table 8, and Appendix B, Figures 24e-h).

When the sample size is 1, the ARL decreases because consecutive observations have the highest autocorrelation (based on the autocorrelation formula ϕ^j between X_t and X_{t-j} for $j \geq 1$). Thus, as the interval length increases, the probability of finding a value as highly autocorrelated to X_t as X_{t-j} decreases, and after the interval length is in excess of L=10, the probabilities are not changing and the variability in the ARL decreases. With a sample size of 5, the data are smoothed by averaging, and therefore less variability is noted between L=1 and L=10, and none is found with increasing interval length after the L=10.

VSI with Fixed Control Limits

For data points generated with zero correlation (ϕ=0.0, normally distributed, μ=0 , σ=1), increasing the interval length from 10 to 50 does not change the ANSS (average number of samples to signal) for samples of size 1 or 5 (see Appendix A, Tables 9 and 13,and Appendix B, Figures 22a-h and Figures 26a-h). The first cluster on each graph indicates the lack of movement in ANSS with changing interval length for correlation of ϕ=0.0. The other clusters refer to other correlations. For samples of size 1 and correlations of ϕ=0.3 and ϕ=0.6 (see Appendix A, Tables 10 and 11), the increasing interval length produces an increase in the ANSS for no shift and, of lesser degree, for a shift of 1. This trend reverses, and the ANSS exhibits a decrease for the larger shifts in the process mean. For samples of size 5 with correlation of ϕ=0.3 or ϕ=0.6 (see Appendix A, Tables 14 and 15, and Appendix B, Figures 26a-h), neither of the results indicated above for samples of size 1 are present. The values for the ANSS change very little until the interval length of L=50, at which time a slight decrease in ANSS is indicated for all shifts except δ=3.0. At a correlation level of ϕ=0.9 (see Appendix A, Tables 12 and 16, and Appendix B, Figures 22g,h and 26g,h), the ANSS decreases for all shifts when the interval length increases, but the rate of change decreases for samples of size 5.

FSI with Calculated Control Limits

For a small portion of the simulations (18/5000), the number of observations required for the chart to produce an out of control signal exceeded 2 million for samples of size 1 when the control limits were calculated from the process samples and the correlation ϕ=0.0 with an interval length of 10 and a shift δ=0.0. Thus, the results are listed as greater than 2754.1 (see Appendix A, Table 17 and Appendix B, Figures 21a-h) with the number of replications in which the 2 million data points were exceeded without an out of control signal being generated given in Appendix A, Table 65. For a correlation of ϕ=0.0, interval length has little or no effect regardless of the shift in the target mean, as is expected for the independent data. However, when the correlation increases to $\phi=0.3$, increasing the interval length from L=1 to L=10 causes the ARL to increase dramatically for all shifts, from an ARL = 182.9 to an ARL > 2385.5 for no shift. When the interval length increases from L=10 to L=20 or 50, the increase in the ARL continues but is less pronounced, particularly for shifts of $\delta=2.0$ and $\delta=3.0$ (see Appendix A, Tables 18 and Appendix B, Figures 21a-h). This trend continues for autocorrelations of $\phi=0.6$ and $\phi=0.9$ (see Appendix A, Tables 19 and 20). It appears that, for positively autocorrelated data, as the interval length between samples increases, the effect of the autocorrelation is decreasing, and the ARL is approaching that for independent data.

When the sample size is increased to 5, the number of observations required to cause the control chart to signal decreases to approximately 300,000, allowing exact values for the ANSS to be obtained. Changes in the interval length have no effect (see Appendix A,Tables 21-22 and Appendix B, Figures 25a-h) on the ANSS until the correlation reaches approximately $\phi \geq 0.6$ (see Appendix A, Tables 23 and 24), at which time the ANSS decreases very slightly from L=1 to L=10 and then displays noticeably decreased variability. The dramatic effect of increasing interval length is not seen when samples of size 5 are used due to the smoothing effect of the averaging.

VSI with Calculated Control Limits

The number of observations required for the VSI chart to produce an out of control signal for samples of size 1 exceeded 800,000 for 61 of the 5000 replications when the correlation ϕ=0.0 with an interval length of L=10 and shift of δ=0.0. The results are listed as greater than 2152.4 (see Appendix A, Table 25), with the number of replications that exceeded the data set displayed in Appendix A, Table 65. The large numbers required to generate an ANSS limited the results for longer interval lengths, particularly for the shifts of 0.0 and 1.0 (see Appendix A, Tables 25, 26,27, and 28), but the results obtained were sufficient to show the comparison to those obtained with FSI with calculated limits. For samples of size 1, the increase in interval length from L=10 to L=20 causes an increase in ANSS but to a lesser degree than the comparable FSI (see Appendix B, Figures 23a-h). For samples of size 5, the interval length has little influence over the ARL (see Appendix A, Tables 29-32 and Appendix B, Figures 27a-h) until the interval length reaches L=50 at which time a slight decrease in the ANSS can be seen.

There is some indication that the increasing interval length is causing a decrease in the autocorrelation effect for the VSI technique but less evidence is available due to the excessive computer time required for the longer interval lengths.

Influence of Correlation Parameter on ARL/ANSS and ATS

The ARL/ANSS is comparable for the FSI and VSI for samples of size 1 and 5 for all shifts in the process mean for the correlation coefficient $\phi \leq 0.6$ (see Appendix A, Tables 1, 2, 3, 5, 6, 7, 9, 10, 11, 13, 14, 15, 17, 18, 19, 21, 22, 23, 25, 26, 27, 29, 20, 31 and Appendix B, Figures 8a-d, 9a-d, 10a-d, 11a-d). When the correlation becomes approximately $\phi = 0.9$, the ANSS for the VSI technique generally exceeds the ARL for the FSI technique with the exception of the interval lengths of 50 (see Appendix A, Tables 4, 8 ,12, 16, 20, 24, 28, and 32).

The ATS (average time to signal) allows a better comparison for FSI and VSI techniques. The time required for the VSI technique to signal, that is, to indicate that a shift in the mean has occurred, is for every comparison of correlation, shift, interval length, and sample size less than the time required for the FSI technique to signal (see Appendix A, Tables 33-64 and Appendix B, Figures 12a-d, 13a-d, 14a-d, 15a-d, 16a-d).

Influence of Sample Size on ARL/ANSS and ATS

FSI and VSI with Fixed Control Limits

With samples of size 1, the ARL/ANSS of the FSI and VSI techniques with pre-set control limits are fairly consistent and similar although a slight decrease in run length was noted with increasing interval length (see

Appendix A, Tables 1-4, 9-12 and Appendix B, Figures 28a-e). For some treatment combinations with samples of size 1, the results are expressed as inequalities because the run length exceeded the number of points in a small percentage of the cases and was not determined exactly. However, when the sample size is increased to 5, the time required for the chart to signal is greatly reduced. The ARL/ANSS for both techniques indicate noticeably decreased variability with increasing interval length for sample sizes of 5 (see Appendix A, Tables 5-8, 13-16 and Appendix B, Figures 28a-e). The ATS for the VSI is shorter than for FSI in all cases for the interval length of 10 (see Appendix B, Figures 30a-e). But the advantage of the VSI over the FSI in ATS is increasingly evident as the correlation increases. The ATS using FSI with samples of size 5 is longer than the ATS using the same FSI technique but with samples of size 1 when $\phi=0.0$ with no shift (see Appendix B, Figures 30a-b) because the ARL/ANSS remains the same and when samples of size 5 are used, the actual number of manufactured items is increased by a factor of 5. However, when correlation increases to

$\phi=0.3$ with no shift, the charts signal so much faster that the size 5 samples have an ATS which is less than that for samples of size 1.

FSI and VSI with Calculated Control Limits

With samples of size 1, the ARL/ANSS of the FSI and VSI techniques were difficult to obtain due to the excessive number of observations required for the chart to find an out of control value (see Appendix A, Tables 17-20, 25-28). The ARL/ANSS were steady when the interval length was increased for all shifts at zero correlation but increased quickly when the interval length was increased at correlations greater than zero (see Appendix B, Figures 29a-f). The samples of size 5 possess the unchanging values over increasing interval lengths not evidenced in samples of size 1 for both the FSI and the VSI techniques (see Appendix A, Tables 21-24, 29-32 and Appendix B, Figures 29a-f). With calculated limits, the ATS for the size 5 samples is much less than for size 1 samples even with zero correlation and zero shift (see Appendix B, Figures 31a-h).

Influence of the Shift in the Process Mean

The ARL is a function of both the correlation ϕ and the shift δ. When the sample size is 1 for FSI with pre-set limits (see Appendix B, Figures 4a-b, 5a-b, 6a-b, 7a-b), the spread of the ARL values, when the shift δ=0.0, is over 300 units (from 371.5 at ϕ=0.0 to 14.4 at ϕ=0.9). When the shift is increased to 1.0, the ARL spread decreases to approximately 30 (from 43.7 at ϕ=0.0 to 12.8 at ϕ=0.9) and continues to decrease for shifts of 2.0 and 3.0. The reduction in the spread of the ARL values as the shift increases is more drastic when the sample size is 5 (see Appendix B, Figures 4c-d, 5c-d, 6c-d, 7c-d).

The ARL for the FSI technique with pre-set limits, n=1, L=1, (see Appendix B, Figures 4a-b, 5a-b) has a similar spread in its values when compared to the ANSS for the VSI techniques. The spread in ARL values when the sample size is 5 (see Appendix B, Figures 4c-d, 5c-d) is comparable to the ARL's generated by samples of size 1.

Important information is gained in this work about the interaction between the autocorrelation ϕ present in the process and the degree of shift δ in the process mean. The ARL was graphed with both linear and logarithmic scale to aid in visually interpreting the interaction. However, the exact value where the lines cross is not available, and care must be exercised in judgement. For samples of size 1 (see Appendix B, Figures 4b, 5b), when using the FSI with pre-set limits, the equilibrium point (where all correlations have approximately the same ARL) occurs at approximately δ=1.8, but for the VSI with pre-set limits, the equilibrium point is approximately δ=2.4 . When the control limits are calculated from sample data, the interaction point is more complex, and no equilibrium point for all four correlation values exists. FSI simulations (see Appendix B, Figure 6b) with correlations of ϕ=0.0 and ϕ=0.9 do not reach an equilibrium point until the shift is almost 3.0, but correlations of ϕ=0.6 and ϕ=0.9 cross at approximately δ = 2.0. For VSI with calculated control limits (see Figure 7b), the complexity of the interaction increases, but equilibrium points between pairs of correlations (ϕ=0.0

and ϕ=0.9, ϕ=0.0 and ϕ=0.3) occur at smaller shift values than for the FSI graphs.

For samples of size 5, interactions between the shift δ and the correlation ϕ are seen in Figures 4d, 5d, 6d, and 7d of Appendix B. When FSI with pre-set limits are used, correlations of ϕ=0.0, ϕ=0.6, and ϕ=0.9 have an equilibrium point at approximately δ= 1.6, but processes with autocorrelation coefficients of ϕ=0.3 interact with other correlations at various shift values. The equilibrium point for the VSI techniques is at approximately δ = 1.6 for all correlations studied.

Fixed versus Calculated Limits

The ARL/ANSS using fixed control limits (pre-set at $\pm 3\sigma_x / \sqrt{n}$) for both the FSI and VSI were easily simulated by generating data sets of up to 100,000 observations at interval length L=10 and 500,000 observations for L=50. For samples of size 1, the ARL/ANSS for the FSI and VSI techniques required large numbers of observations using control limits calculated by finding estimates from the ranges and means of the first 25 samples. When using these calculated limits, the ARL/ANSS requires over 800,000 points for approximately 1% of the replications, and over 2 million in 0.36%. These excessively long run lengths made the computer time required to find the exact ARL/ANSS for the longer interval lengths non-feasible.

The differences in the behavior of both FSI and VSI charts for fixed versus calculated control limits is profound. Using limits of $\pm 3\sigma_x/\sqrt{n}$ (pre-set), the autocorrelation causes both FSI and VSI charts to signal faster (for all correlation values) with shifts of δ=0 or δ=1 (see Appendix A, Tables 1-4, 9-12). The FSI chart signals slower with δ=2 or δ=3 when the interval length is L=1 and faster than with a zero correlation when $L > 1$. However, this is not true when the treatment is $\phi = 0.9$ with δ=3. The VSI continues to signal faster for all interval lengths with the exception of a slight decrease in signaling time for $\phi = 0.9$, $\delta = 3$, $L = 1$.

If the control limits are calculated, the FSI and VSI interactions are very similar. Neither of the techniques has an equilibrium point involving all four correlations, but for correlations greater than zero the equilibrium point occurs approximately at δ=1.6 for both FSI and VSI when the limits are calculated from the sample data.

Correlations of $0.0 < \phi < 0.3$

The findings of this work indicated that the value at which autocorrelation becomes a significant factor might occur at a correlation of less than those originally planned in the study. Table 66 (Appendix A) gives values for the additional work which was pursued in an attempt to pin-point the exact value at which autocorrelation becomes influential. The small simulation study indicates that autocorrelation influences the ARL/ANSS at levels as low as ϕ=0.01.

Correlations of $0.6 < \phi < 0.9$

During the analysis of the data of this dissertation, it was noticed that the decreasing trend of the ARL/ANSS, with increasing autocorrelation in both techniques, reverses in the vicinity of ϕ=0.9 with the shift in the process mean between δ=1.0 and δ=2.0. However, it is important to note that this reversal is not present in the ATS. A small number of additional computer runs were made in order to more closely determine the autocorrelation value at which this reversal occurs. These results can be seen in Appendix A, Tables 67 and 68, and show that the first indication of a reversal occurs between ϕ=0.6 and ϕ=0.7 when a shift in the target mean is less than δ=1.0. This additional study emphasizes the increasing advantage of the VSI technique in terms of the amount of time, as opposed to ARL/ANSS, required to indicate that a shift has occurred.

CHAPTER VI

CONCLUSIONS

This dissertation investigated differences between the fixed sampling interval and the variable sampling interval Shewhart X-bar control chart in the presence of autocorrelated data. Specifically, the investigation sought to determine if the newer VSI technique would result in a superior ATS (average time to signal) when the Shewhart X-bar chart is used on a process that produces autocorrelated data. The VSI technique is considered superior to the FSI if it will yield an out of control signal faster when the process mean has shifted from the target and a slower out of control signal in the absence of a shift.

This work assumed that autocorrelation was unknowingly present in the process and that the Shewhart X-bar chart, due to its popularity, was being used to monitor deviations in the process mean. In this case, the presence of autocorrelation can make the interpretation of the chart erroneous because the assumption of independence is violated with autocorrelated data; thus the Shewhart X-bar chart does not signal accurately (Alwan and Roberts, 1988; Ermer, 1980; Maragah, 1989). Positive autocorrelation causes the chart to signal that an out of control situation exists when one does not, and negative autocorrelation retards the signaling process (Maragah, 1989). This dissertation investigated the effects of various levels of positive autocorrelation because negative autocorrelation is seldom seen in industrial processes (Baik, Reynolds, and Arnold, 1991). The economic impact of positive auto-correlation is the cost in manpower, production time, and production loss of searching for a non-existent shift in the mean rather than working on true process problems.

This dissertation is a simulation study using the autoregressive model of order 1, AR(1) to describe the autocorrelated data because data that can be described by this model are frequently found in industrial processes (Baik, Reynolds, and Arnold, 1991). The control limits were both pre-set at $\pm 3\sigma_x/\sqrt{n}$ and determined by estimates obtained by sampling the process data. These two different methods of obtaining control limits, the first consistent with previous research methodology in order to allow confirmation of the results of this dissertation, and the second, consistent with industrial practice, are used with all control chart variations. A Fortran program was created to recognize an out of control condition and to determine the following: first, the time to signal, TS, (the length of time from the start of the process to the time where the chart signals); and second, the number of samples to signal, NSS, (the number of samples taken from the start of the process until the chart signals). Repetitions of this procedure generated an average time to signal, ATS, and an average run length (ARL) for the FSI or average number of samples to signal (ANSS) for the VSI. The implications of these simulations on the research questions posed in this dissertation are discussed individually in the following sections.

Research question #1:

> Does the ATS on the Shewhart X-bar control chart decrease as the autocorrelation in the process increases?

The AR(1) model was used in this study, and the autocorrelation within this model was increased from $\phi=0.0$ to $\phi=0.9$ in increments of 0.3. These values were chosen to allow comparison with the work of Baik, Reynolds, and Arnold (1991). These comparisons were limited to samples of size 1 and interval length of 10. It is known (see Chapter III) that the variation within the AR(1) model is described by

$$\sigma_x = \sqrt{\frac{1}{1-\phi^2}}\ \sigma_a \qquad (3.3)$$

where σ_x is the standard deviation of the error terms. Thus, it is known that the variance (standard deviation is the square root of the variance) of the process will increase as the autocorrelation parameter increases. With increased variance, a process should more quickly produce a data value that is beyond the control limits of a particular control chart. This means that with increasing autocorrelation, the ATS for the Shewhart X-bar control chart could be expected to decrease. If the limits for the control chart are pre-set at $\pm3\sigma_x$, where $\sigma_x = 1$ because of the assumption of normally distributed data, then the increased variance of the data (which is unknowingly autocorrelated by the assumptions of this dissertation), is not incorporated into the control limits. Thus, a much faster ATS is expected (the control limits do not change but the variance of the process increases) and found. If, however, the control limits are set using estimates from the range and x-double bar, then the increased variance of the autocorrelated data is incorporated into the control limits, and the limits exhibit increased variability. Because the control limits and the process data both have increased variability, the ATS, although still decreasing with increasing autocorrelation, is greater than for the corresponding control chart with fixed limits.

For a chart with calculated control limits, the ATS for zero correlation (ϕ=0.0), zero shift (δ=0.0), and samples of size five (n=5) was 6475.5 for the FSI chart (see Table 53) and 6093.5 for the VSI chart (see Table 61). This means that the production is stopped 382 items sooner if VSI techniques are used with the given parameters. If the correlation is increased to ϕ=0.3, the ATS is reduced to 489.0 (see Table 54) for the FSI chart and 378.0 (see Table 62) for the VSI chart. Thus, a small amount of autocorrelation will cause the Shewhart X-bar control chart to signal out of control (when, because of the simulation procedure, it is known that the process is in control) 5986.5 items sooner for the FSI and 5715.5 items sooner if VSI techniques are used. By the definition of "superior performance" given earlier, the VSI technique is not superior to the FSI in this case because the process mean is on target and it would be undesirable to stop a process more quickly when it is operating in control. If autocorrelation is considered a primary problem in the production process, this disadvantage of the VSI technique, of stopping an in control process more quickly , might be overlooked. However, if for a specific manufacturing process, stopping the production process is more costly than the autocorrelation between manufactured units, then this shorter time to signal is not advantageous. The specific evaluation of these problems is the emphasis of current research in economic control chart analysis and is unique to each different manufacturing situation.

This work confirms the findings of prior works that autocorrelation can be a decisive factor in the interpretation of the Shewhart X-bar control chart. However, the findings of this dissertation do not agree with those of Reynolds, Arnold, Amin, and Nachlas (1988) for an autocorrelation parameter of $\phi=0.9$. They found that the ATS increased for correlations of $\phi=0.9$ whereas this study found a continuing pattern of decreasing ATS with increasing autocorrelation. Maragah (1989) found that the number of out of control signals in 200 observations continued to increase for an increasing autocorrelation parameter. The findings of this study are consistent with the findings in Maragah's (1989) research in that a faster time to signal supports his increased number of out of control signals for increased autocorrelation but extends the ability to apply these results because Maragah did not consider the influence of shifts from the target mean.

Research question #2:

> Does the ARL/ANSS on a Shewhart X-bar control chart increase as the length of the interval between samples increases when positive autocorrelation is present?

Interval lengths between samples of more than one unit were not found in the literature search and thus were considered important for investigation. Industry seldom samples each item as it is manufactured because sampling is expensive, both in the time required for employees to take the samples and in the possible destructive sampling techniques. Usually an interval is allowed between samples based on the unique requirements of the specific manufacturing process. This dissertation sought to explore the more practical process of sampling over time by the surrogate of skipping 10 to 50 items between samples.

The autocorrelation between observations X_t and X_{t-j} was identified as ϕ^j for $j \geq 1$. Thus, the autocorrelation between observations decreases as the time between observations increases. This implies that if the ARL/ANSS on the Shewhart X-bar control chart increases as the autocorrelation in the process increases, the ARL/ANSS will be longer for a given autocorrelation value as the time between samples of size 1 increases. However, one emphasis in this dissertation was on samples of size 5, and it was established earlier that the exact autocorrelation relationship between these larger samples is unknown.

The results of this study indicated that increasing the interval length, all other parameters held constant, causes the ARL/ANSS to increase. One interpretation of this result is that the effect of the autocorrelation decreases as the time between samples increases. This interpretation is reasonable because it is consistent with the known theoretical relationships between single autocorrelated observations discussed earlier in this dissertation. Although excessively long data sets made the memory storage requirements and the CPU time demands for a study of interval lengths greater than 50 non-feasible, a pattern of increasing ARL/ANSS is established. The decreasing influence of autocorrelation in the process with increasing time between samples indicates that, given sufficient time between samples, the assumption of independence upon which the accurate interpretation of the Shewhart X-bar chart depends is essentially not violated. Therefore, manufacturers would determine at what interval length any autocorrelation in their process becomes insignificant; if that sampling interval meets the economic constraints of their production process, manufacturers would proceed to use the Shewhart X-bar chart with the assurance that it is operating with its stated probabilities.

Research question #3:

> Does the Shewhart X-bar control chart signal faster with samples of size 5 than with samples of size 1 when positive autocorrelation is present?

Samples of size 5 are the industry standard (Grant and Leavenworth, 1988; Montgomery, 1985), yet prior research was conducted using samples of size 1 (Reynolds, Arnold, Amin, and Nachlas, 1988; Baik, Reynolds, and Arnold, 1991; Maragah, 1991; Wardell, Moskowitz, and Plante, 1991). It was proposed (see pages 39-40) in this dissertation that the intra-sample autocorrelation should be stronger for samples of size 5 than for samples of size 1. If this is true, then it was expected that using samples of size 5 rather than samples of size 1 would cause the corresponding Shewhart X-bar control charts to have a lower average time to signal.

For small or medium autocorrelation parameters (below the vicinity of ϕ=0.9), this dissertation shows that when using the Shewhart X-bar chart, samples of size 5 should be utilized whenever economically feasible because a much faster ATS is obtained with the larger samples size. The faster ATS is obtained regardless of whether the control limits are pre-fixed according to theoretical parameters or established through estimates based on the process observations.

The ARLs were extremely long for samples of size 1 when using control limits calculated from the simulated process observations. These long ARLs occur because of the use of the moving range estimator (although theoretically a poor estimator, it is the only choice available) in the size 1 samples. The small but finite possibility of obtaining a large range is greater when utilizing samples of size 5 than for samples of size 1, but this possibility is compensated for by the increasing value of both d_2 and n. The effect can be seen in

$$\hat{\sigma}_{\bar{x}} = \frac{\bar{R}}{d_2\sqrt{n}} \tag{6.1}$$

where both d_2 and n increase simultaneously (d_2 increases from a value of 1.128 to 2.326, and n from 1 to 5) when sample size is changed from 1 to 5. The results of the simulation showed that in 10,000 trials the differences in the values obtained for the range when samples of size 1 were used were of large magnitude. Therefore, single item sampling, due to the control limits that are calculated from estimates based on single units, can be misleading in industrial applications because the control limits may be influenced by one or two single observations which are the extremes of the process. These extremes will cause the control limits to be so wide that observations which actually deviate more than desired from the target will fail to cause the chart to signal, or so narrow that observations that are actually within acceptable bounds will cause the chart to signal. In contrast, when samples of size 5 were used, smaller values for R-bar and values of x-double bar more indicative of the true parameters were obtained. These smaller values resulted in narrower control limits, and out of control observations were more quickly found, yielding smaller ARLs.

The economic impact of larger sample sizes must be considered by the manufacturer, both in the possible additional time required to obtain and in the additional expense of possible product damage. If destructive sampling is necessary, then the faster time to signal with samples of size 5 needs careful weighing against the increased expense.

Research question #4:

Do increasing shifts in the process mean cause a lower ATS for the Shewhart X-bar control chart for process data that are positively autocorrelated?

Shewhart X-bar control charts are used to monitor changes in the calculation of an arithmetic mean such as weight or diameter or length. The desired value is called the target mean, and any deviation from this value is referred to as a shift. This dissertation investigated shifts from the target mean, assumed to be 0.0, to a value of 3.0 in increments of 1.0.

The ARL/ANSS, and thus the ATS, are a function of both the shift δ and the autocorrelation ϕ. Shift and autocorrelation are confounding variables, and it is not prudent to separate their effects when drawing conclusions. The ARL, as a function of both shift and autocorrelation, was graphed using both linear and logarithmic scales in an attempt to visually pin-point the relationship between shift and autocorrelation (see Appendix B, Figures 4-7, all parts). The lines cross, indicating an equilibrium point, and although it is not possible to determine the exact crossing points, it is possible to determine that shift plays a strong role in reducing the ARL/ANSS and ATS. It is possible to state that for zero correlation, the ARL/ANSS and ATS are reduced by an increasing shift, but the effect of shift is confounded by the presence of autocorrelation.

Research question #5:

> Do VSI techniques produce a lower ATS than FSI techniques on the Shewhart X-bar control chart with positively autocorrelated data when control limits are calculated from process observations?

Average time to signal refers to the number of items that are manufactured before the control chart puts a stop to the manufacturing process by signaling an out of control situation. Because the ATS can be quicker for the VSI technique even when the ANSS is larger than the ARL for the FSI technique (for a specific example, see page 50), it is necessary to consider ATS as an issue separate from ARL/ANSS. The ATS is less for the VSI Shewhart X-bar chart than for the corresponding FSI chart (see Appendix A, Tables 33-64, and 66-68; Appendix B, Figures 12-19, all parts) for each fixed shift, autocorrelation parameter, sample size, and interval length. In each group of graphs, the linear scale is used to show the large difference between the ATS at $\phi=0.0$ and $\phi>0.0$. However, the logarithmic scale is necessary for comparison of all values for $\phi>0.0$. These figures show that the VSI technique becomes progressively faster than the FSI to signal as the shift in the process mean increases for any given level of autocorrelation. For example, in Figure 13b as the shift increases (the horizontal axis), the difference between the two bar graph heights becomes greater. This implies that the VSI technique shows its greatest advantage and should be employed for any process where a large shift in the mean is expected to occur. That is, if the process is operating on target, no shift in the mean has occurred; then the quicker time to signal for the VSI technique is actually a disadvantage, and the FSI technique will allow a larger number of items to be manufactured before stopping the process. Within both FSI and VSI charts, various patterns are evident for the ATS but are different for each technique. This is an indication that the recent work establishing specialized control charts for specific situations, such as the Common Cause Chart and the Special Cause Chart, is valuable (see, for example, Wardell, Moskowitz, and Plante, 1991 or Alwan and Roberts, 1988).

Which control chart is better is an economic issue. However, in many cases the VSI chart is preferable because its disadvantage at zero autocorrelation (when the process is producing independent data) is small versus a large advantage at autocorrelation value greater than zero (when the process is producing autocorrelated data). This advantage is a numerical advantage based on absolute numbers rather than on the cost of errors.

Calculated Control Limits

Prior published research was conducted with pre-set control limits and a sample size of 1 because this approach is theoretically tractable. These past studies, while valuable, do not reflect the more common industrial practice where control limits are calculated from sample data. It was originally planned to limit this dissertation to calculated control limits only, but the huge discrepancy in the results obtained in this simulation as compared to prior work with pre-set limits made it necessary to include a study of fixed limits in order to validate the simulation results. In these simulations, the control limits were calculated using the typical industrial practice of using estimates obtained from the first 25 samples. The results show that when the process is in control, the theoretical value of 370.4 for the ARL using fixed limits is far exceeded for both FSI and VSI when the limits are calculated (ARL >2406). If the process is in control, we do not want the production stopped; therefore, this result, using estimates from process observations to calculate the control limits, is advantageous. Although calculating the limits with samples of size 5 allows the process to continue longer when there is no shift in the mean, the chart with calculated limits (zero autocorrelation) stops the process (ARL/ANSS $\approx$ 5) when there has been a shift almost as quickly as the pre-set limit charts (ARL/ANSS $\approx$ 4.5). There is very little difference in the ARL/ANSS as the autocorrelation increases between the FSI versus VSI charts with calculated limits. The Tables of Appendix B contain the specific numbers. As an example, Table 22 compared to Table 30 shows that for autocorrelation ϕ=0.3, the ATS for no shift is 32.6

compared to the ANSS of 32.4 while for shifts of 1.0, both techniques yield an ARL/ANSS of 3.5.

In practical applications, the additional complexity of the VSI technique is an important consideration, and it is questionable whether the VSI technique is warranted when the process produces independent observations and the mean is on target. This is because the VSI signals faster than the FSI that the (in control) process is out of control. If the entire control chart sampling process is automated, thus requiring no hand calculations, then this complexity is not a consideration, and the VSI is superior to the FSI when a shift exists in the mean, particularly a large shift. If the process is suspected of positive autocorrelation or of a potentially large shift in the process mean, then any additional problems of the VSI technique are outweighed by its faster signaling time.

Results from this study indicated that the effect of autocorrelation causes the Shewhart X-bar control chart to signal much faster even though no change in the mean occurs in the general region between $0.0 \leq \phi \leq 0.3$. As a result of this finding, an investigation of the influence of correlation between $\phi=0.0$ and $\phi=0.3$ was performed. Results from this extended investigation indicate that the effect of autocorrelation is present at correlation levels as low as $\phi=0.01$ (see Appendix A, Table 66). This would indicate that the VSI technique should always be considered in practice since autocorrelation levels this small may exist in many applications.

In spite of the findings in this dissertation and by prior researchers, industry does not commonly use the VSI technique. While it is not within the scope of this work to investigate why industry has not adopted this technique, it is appropriate to speculate about this lack of adoption because it is pertinent in making recommendations for industry. The cost involved in implementing the newer VSI charts includes the cost of training employees and may also include the capital cost of new automated monitoring equipment. Therefore, it is necessary to balance the improvement in process control against the added complexity and expense. It is possible that VSI charts are not commonly used because these costs are poorly understood by academia and well understood by the industries that must incur the expense. Alternatively, the understanding of the application, limitations, and advantages of the VSI techniques is a current research topic, and industry may not posses full knowledge of its advantages.

While this dissertation provides further information on the advantages of VSI charts, it also indicates that the process of investigating its use in practical applications is incomplete. Further research is needed in the use of control charts in practice (calculated limits), both with independent data and with correlated data. Prior research has not explored this area due to the difficulties of obtaining analytical proof. Even using simulations, research in this area is limited because of its computer intensive nature. The required size of the data set, the large number of replications needed, and the large number of treatment combinations make huge demands on computer CPU time and memory resources. However such work is important to establish criteria for the application of theory to practice.

APPENDIX A

TABLES

FIXED SAMPLING INTERVALS WITH FIXED LIMITS:ARL

ϕ = Correlation Sample size = 1 δ = Shift in mean L = Interval length

TABLE 1: $\phi = 0.0$

δ \ L	1	10	20	50
0.0	371.5	376.2	367.4	369.8
1.0	43.7	43.3	43.6	44.0
2.0	6.4	6.3	6.3	6.3
3.0	2.0	2.0	2.0	2.0

TABLE 2: $\phi = 0.3$

δ \ L	1	10	20	50
0.0	241.2	239.7	236.2	237.2
1.0	38.9	35.5	35.2	35.4
2.0	6.9	5.9	5.9	5.8
3.0	2.3	2.0	2.0	2.0

TABLE 3: $\phi = 0.6$

δ \ L	1	10	20	50
0.0	75.0	60.6	60.8	60.9
1.0	26.9	18.0	17.7	17.5
2.0	7.6	4.8	4.8	4.7
3.0	2.9	2.0	2.0	2.0

TABLE 4: $\phi = 0.9$

δ \ L	1	10	20	50
0.0	14.4	5.6	5.3	5.2
1.0	12.8	4.8	4.4	4.3
2.0	9.1	3.4	3.0	2.9
3.0	5.6	2.3	2.1	2.0

FIXED SAMPLING INTERVALS WITH FIXED LIMITS:ARL

ϕ = Correlation Sample size = 5 δ= Shift in mean L=Interval length

TABLE 5: $\phi = 0.0$

δ \ L	1	10	20	50
0.0	369.1	371.9	370.5	370.2
1.0	4.5	4.5	4.5	4.6
2.0	1.1	1.1	1.1	1.1
3.0	1.0	1.0	1.0	1.0

TABLE 6: $\phi = 0.3$

δ \ L	1	10	20	50
0.0	41.9	41.7	41.1	41.6
1.0	3.6	3.5	3.5	3.6
2.0	1.3	1.2	1.2	1.2
3.0	1.0	1.0	1.0	1.0

TABLE 7: $\phi = 0.6$

δ \ L	1	10	20	50
0.0	7.6	7.2	7.2	7.1
1.0	3.1	2.8	2.8	2.8
2.0	1.3	1.3	1.3	1.3
3.0	1.0	1.0	1.0	1.0

TABLE 8: $\phi = 0.9$

δ \ L	1	10	20	50
0.0	2.2	1.9	1.9	1.9
1.0	2.0	1.8	1.8	1.8
2.0	1.7	1.6	1.5	1.5
3.0	1.4	1.3	1.3	1.2

VARIABLE SAMPLING INTERVALS WITH FIXED LIMITS:ANSS

ϕ = Corrleation   Sample size = 1 δ = Shift in mean   L = Interval length

TABLE 9: $\phi = 0.0$

δ \ L	10	20	50
0.0	372.4	366.0	369.3
1.0	44.2	43.5	43.3
2.0	6.3	6.3	6.2
3.0	2.0	2.0	1.0

TABLE 10: $\phi = 0.3$

δ \ L	10	20	50
0.0	201.8	232.1	238.6
1.0	31.5	34.1	35.7
2.0	6.4	6.0	5.9
3.0	2.2	2.1	1.0

TABLE 11: $\phi = 0.6$

δ \ L	10	20	50
0.0	49.7	55.5	60.9
1.0	17.0	16.8	17.9
2.0	5.8	5.2	1.0
3.0	2.6	2.2	1.0

TABLE 12: $\phi = 0.9$

δ \ L	10	20	50
0.0	7.5	6.5	5.8
1.0	6.5	5.5	1.0
2.0	4.7	4.0	1.0
3.0	3.2	1.0	1.0

VARIABLE SAMPLING INTERVALS WITH FIXED LIMITS:ANSS

ϕ = Correlation Sample size = 5 δ = Shift in mean L = Interval length

TABLE 13: $\phi = 0.0$

δ \ L	10	20	50
0.0	363.8	370.3	368.4
1.0	4.6	4.5	4.5
2.0	1.1	1.1	1.1
3.0	1.0	1.0	1.0

TABLE 14: $\phi = 0.3$

δ \ L	10	20	50
0.0	41.3	41.6	40.8
1.0	3.6	3.6	3.5
2.0	1.2	1.2	1.2
3.0	1.0	1.0	1.0

TABLE 15: $\phi = 0.6$

δ \ L	10	20	50
0.0	7.4	7.3	7.3
1.0	2.9	2.9	2.2
2.0	1.3	1.3	1.1
3.0	1.0	1.0	1.0

TABLE 16: $\phi = 0.9$

δ \ L	10	20	50
0.0	2.1	2.0	1.9
1.0	1.9	1.9	1.7
2.0	1.7	1.6	1.4
3.0	1.3	1.3	1.2

CALCULATED LIMITS FIXED SAMPLING INTERVALS:ARL

ϕ = Correlation Sample size = 1 δ = Shift in Mean L = Interval length

(See Table 65 for superscripted values)

TABLE 17: $\phi = 0.0$

δ \ L	1	10	20	50
0.0	>2406.0[1]	>2754.1[2]		
1.0	260.8	242.7	241.1	
2.0	14.1	14.2	14.4	14.3
3.0	2.7	2.6	2.6	2.7

TABLE 18: $\phi = 0.3$

δ \ L	1	10	20	50
0.0	182.9	>2385.5[3]		
1.0	39.1	269.0	>272.5[4]	>280.3[5]
2.0	5.9	18.6	17.0	19.0
3.0	2.0	3.1	3.1	3.2

TABLE 19: $\phi = 0.6$

δ \ L	1	10	20	50
0.0	24.6	>1830.9[6]		
1.0	14.1	361.4	>451.2[7]	>553.6[8]
2.0	4.8	34.3	46.5	38.4
3.0	2.0	6.0	6.3	6.2

TABLE 20: $\phi = 0.9$

δ \ L	1	10	20	50
0.0	6.7	114.7		
1.0	6.1	89.6	>480.3[9]	
2.0	4.6	35.2	>131.8[10]	>258.1[11]
3.0	3.1	14.7	39.9	>68.3[12]

CALCULATED LIMITS FIXED SAMPLING INTERVALS:ARL

ϕ = Correlation Sample size = 5 δ = Shift in Mean L = Interval length

TABLE 21: $\phi = 0.0$

δ \ L	1	10	20	50
0.0	432.8	431.7	433.2	431.6
1.0	5.0	5.1	5.1	5.1
2.0	1.1	1.1	1.1	1.1
3.0	1.0	1.0	1.0	1.0

TABLE 22: $\phi = 0.3$

δ \ L	1	10	20	50
0.0	32.4	32.6	33.3	33.3
1.0	3.5	3.5	3.4	3.4
2.0	1.1	1.2	1.1	1.1
3.0	1.0	1.0	1.0	1.0

TABLE 23: $\phi = 0.6$

δ \ L	1	10	20	50
0.0	6.4	6.0	6.3	6.1
1.0	2.9	2.7	2.7	2.7
2.0	1.3	1.3	1.3	1.3
3.0	1.0	1.0	1.0	1.0

TABLE 24: $\phi = 0.9$

δ \ L	1	10	20	50
0.0	2.0	1.8	1.8	1.8
1.0	1.9	1.8	1.7	1.7
2.0	1.7	1.5	1.4	1.5
3.0	1.4	1.3	1.3	1.2

CALCULATED LIMITS VARIABLE SAMPLING INTERVALS:ANSS

ϕ = Correlation Sample size = 1 δ = Shift in Mean L = Interval length

(See Table 65 for superscripted values)

TABLE 25: $\phi = 0.0$

δ \ L	10	20	50
0.0	>2152.4[13]		
1.0	221.5		
2.0	14.3	14.6	
3.0	2.7	2.6	

TABLE 26: $\phi = 0.3$

δ \ L	10	20	50
0.0	1142.8[14]		
1.0	177.4	249.9	
2.0	15.7	17.3	15.8
3.0	3.6	3.3	3.1

TABLE 27: $\phi = 0.6$

δ \ L	10	20	50
0.0	911.6[15]		
1.0	204.5	253.1	
2.0	28.8	28.8	28.9
3.0	8.3	7.1	6.2

TABLE 28: $\phi = 0.9$

δ \ L	10	20	50
0.0	89.4		
1.0	66.7	171.1	
2.0	41.2	79.1	85.4
3.0	21.0	38.6	43.2

CALCULATED LIMITS VARIABLE SAMPLING INTERVALS:ANSS
Correlation = ϕ Sample size = 5 δ = Shift in Mean L = Interval length

TABLE 29: $\phi = 0.0$

δ \ L	10	20	50
0.0	425.7	420.4	
1.0	5.1	5.2	2.6
2.0	1.1	1.1	1.0
3.0	1.0	1.0	1.0

TABLE 30: $\phi = 0.3$

δ \ L	10	20	50
0.0	32.4	32.6	32.8
1.0	3.5	3.5	2.2
2.0	1.1	1.0	1.0
3.0	1.0	1.0	1.0

TABLE 31: $\phi = 0.6$

δ \ L	10	20	50
0.0	6.0	6.1	6.1
1.0	2.9	2.8	2.1
2.0	1.3	1.1	1.1
3.0	1.0	1.0	1.0

TABLE 32: $\phi = 0.9$

δ \ L	10	20	50
0.0	1.9	1.9	1.8
1.0	1.9	1.8	1.7
2.0	1.6	1.6	1.4
3.0	1.3	1.3	1.2

FIXED SAMPLING INTERVALS WITH FIXED LIMITS: ATS

Φ = Correlation   Sample size = 1 δ = Shift in Mean   L = Interval length

TABLE 33: $\Phi = 0.0$

δ \ L	1	10	20	50
0.0	371.5	4138.2	7715.4	18859.8
1.0	43.7	476.3	915.6	2244
2.0	6.4	69.3	132.2	321.3
3.0	2.0	22	42	102

TABLE 34: $\Phi = 0.3$

δ \ L	1	10	20	50
0.0	241.2	2636.7	4960.2	12097.2
1.0	38.9	390.5	739.2	1805.4
2.0	6.9	64.9	123.9	295.8
3.0	2.3	22	42	102

TABLE 35: $\Phi = 0.6$

δ \ L	1	10	20	50
0.0	75.0	666.6	1276.8	3105.9
1.0	26.9	198.0	371.7	892.5
2.0	7.6	52.8	100.8	239.7
3.0	2.9	22	42	102

TABLE 36: $\Phi = 0.9$

δ \ L	1	10	20	50
0.0	14.4	61.6	111.3	265.2
1.0	12.8	52.8	92.4	219.3
2.0	9.1	37.4	63.0	147.9
3.0	5.6	25.3	44.1	102

FIXED SAMPLING INTERVALS WITH FIXED LIMITS: ATS

Φ = Correlation Sample size = 5 δ = Shift in Mean L = Interval length

TABLE 37: $\Phi = 0.0$

δ \ L	1	10	20	50
0.0	1845.5	5578.5	9262.5	20361.0
1.0	22.5	67.5	112.5	253
2.0	5.5	16.5	27.5	60.5
3.0	5.0	15	25	55

TABLE 38: $\Phi = 0.3$

δ \ L	1	10	20	50
0.0	210.5	625.5	1027.5	2288.0
1.0	18.0	52.5	87.5	198.0
2.0	6.0	18	30	66.0
3.0	5.0	15	25	55

TABLE 39: $\Phi = 0.6$

δ \ L	1	10	20	50
0.0	38.0	108	180.0	390.5
1.0	15.5	42	70.0	154.0
2.0	6.5	19.5	32.5	71.5
3.0	5.0	15	25	55

TABLE 40: $\Phi = 0.9$

δ \ L	1	10	20	50
0.0	11.0	28.5	47.5	104.5
1.0	10.0	27	45.0	99.0
2.0	8.5	24	37.5	82.5
3.0	7.0	19.5	32.5	66.0

VARIABLE SAMPLING INTERVALS WITH FIXED LIMITS: ATS

Φ = Correlation Sample size = 1 δ = Shift in Mean L = Interval length

TABLE 41: $\Phi = 0.0$

δ \ L	10	20	50
0.0	3734.7	7156.0	16619.3
1.0	304.4	571.0	1377.9
2.0	16.2	28.2	62.7
3.0	2.3	3.7	5.0

TABLE 42: $\Phi = 0.3$

δ \ L	10	20	50
0.0	1918.8	4356.3	11043.0
1.0	203.1	436.5	1129.8
2.0	17.7	29.1	64.0
3.0	2.9	4.2	5.0

TABLE 43: $\Phi = 0.6$

δ \ L	10	20	50
0.0	389.5	886.2	2407.6
1.0	99.7	200.9	533.4
2.0	18.9	31.5	5.0
3.0	4.6	6.5	5.0

TABLE 44: $\Phi = 0.9$

δ \ L	10	20	50
0.0	35.8	63.4	139.7
1.0	30.1	51.5	5.0
2.0	19.1	31.6	5.0
3.0	10.6	2.0	5.0

VARIABLE SAMPLING INTERVALS WITH FIXED LIMITS: ATS

Φ = Correlation Sample size = 5 δ = Shift in Mean L = Interval length

TABLE 45: $\Phi = 0.0$

δ \ L	10	20	50
0.0	5097.3	8715.1	16438.2
1.0	23.8	29.1	47.1
2.0	1.4	2.4	5.4
3.0	1.0	2.0	5.0

TABLE 46: $\Phi = 0.3$

δ \ L	10	20	50
0.0	491.7	803.3	1693.8
1.0	21.1	29.4	52.8
2.0	1.8	2.9	6.1
3.0	1.0	1.0	5.0

TABLE 47: $\Phi = 0.6$

δ \ L	10	20	50
0.0	67.8	104.4	214.8
1.0	18.8	27.3	32.5
2.0	3.5	5.1	6.4
3.0	1.2	2.2	5.0

TABLE 48: $\Phi = 0.9$

δ \ L	10	20	50
0.0	11.4	16.7	31.3
1.0	9.9	14.3	24.7
2.0	7.1	9.9	15.6
3.0	4.2	6.4	9.3

CALCULATED LIMITS FIXED SAMPLING INTERVALS:ATS

Φ = Correlation Sample size = 1 δ = Shift in Mean L = Interval length

TABLE 49: Φ = 0.0

δ \ L	1	10	20	50
0.0	>2406.0	>30295.1		
1.0	260.8	2669.7		
2.0	14.1	156.2	302.4	729.3
3.0	2.7	28.6	54.6	137.7

TABLE 50: Φ = 0.3

δ \ L	1	10	20	50
0.0	182.9	>26240.5		
1.0	39.1	2959.0	5722.5	
2.0	5.9	204.6	357.0	969.0
3.0	2.0	34.1	42.0	163.2

TABLE 51: Φ = 0.6

δ \ L	1	10	20	50
0.0	24.6	>20139.9		
1.0	14.1	3975.4	9475.2	
2.0	4..8	378.4	976.5	1943.1
3.0	2.0	66.0	42.0	316.2

TABLE 52: Φ = 0.9

δ \ L	1	10	20	50
0.0	6.7	1261.7		
1.0	6.1	985.6	>10086.3	
2.0	4.6	387.2	>2767.8	
3.0	3.1	161.7	>823.2	3432.3

CALCULATED LIMITS FIXED SAMPLING INTERVALS:ATS

Φ = Correlation Sample size = 5 δ = Shift of Mean L = Interval length

TABLE 53: $\Phi = 0.0$

δ \ L	1	10	20	50
0.0	2164.0	6475.5	10805.0	23738.0
1.0	25.0	76.5	127.5	280.5
2.0	5.5	16.5	27.5	60.5
3.0	5.0	15.0	25.0	55.0

TABLE 54: $\Phi = 0.3$

δ \ L	1	10	20	50
0.0	162.0	489.0	832.5	1831.5
1.0	17.5	52.5	85.0	187.0
2.0	5.5	18.0	27.5	60.5
3.0	5.0	15.0	25.0	55.0

TABLE 55: $\Phi = 0.6$

δ \ L	1	10	20	50
0.0	32.0	90.0	157.5	335.5
1.0	14.5	40.5	67.5	148.5
2.0	6.5	19.5	32.5	71.5
3.0	5.0	15.0	25.0	55.0

TABLE 56: $\Phi = 0.9$

δ \ L	1	10	20	50
0.0	10.0	27.0	45.0	99.0
1.0	9.5	27.0	42.5	93.5
2.0	8.5	22.5	35.0	82.5
3.0	7.0	19.5	32.5	66.0

CALCULATED LIMITS VARIABLE SAMPLING INTERVALS:ATS
Φ = Correlation Sample size = 1 δ = Shift in Mean L = Interval length

TABLE 57: $\Phi = 0.0$

δ L	10	20	50
0.0	>26568.5		
1.0	2069.2		
2.0	54.1	118.4	180.7
3.0	3.5	4.9	

TABLE 58: $\Phi = 0.3$

δ L	10	20	50
0.0	>13330.6		
1.0	1579.0	4709.0	
2.0	57.2	128.8	260.4
3.0	5.5	7.7	13.8

TABLE 59: $\Phi = 0.6$

δ L	10	20	50
0.0	>9851.5		
1.0	1722.0	4705.8	
2.0	115.9	252.3	664.7
3.0	16.7	24.6	52.2

TABLE 60: $\Phi = 0.9$

δ L	10	20	50
0.0	483.2		
1.0	334.9	2347.2	
2.0	201.7	813.0	2838.8
3.0	63.6	265.9	986.6

CALCULATED LIMITS VARIABLE SAMPLING INTERVALS:ATS
Φ = Correlation Sample size = 5 δ = Shift in Mean L = Interval length

TABLE 61: $\Phi = 0.0$

δ \ L	10	20	50
0.0	6093.5	10163.8	
1.0	28.1	36.7	19.2
2.0	1.5	2.4	5.0
3.0	1.0	2.0	5.0

TABLE 62: $\Phi = 0.3$

δ \ L	10	20	50
0.0	378.0	619.0	1328.7
1.0	20.9	29.1	23.7
2.0	2.8	2.2	5.2
3.0	2.0	2.0	5.0

TABLE 63: $\Phi = 0.6$

δ \ L	10	20	50
0.0	52.4	82.5	172.6
1.0	18.3	26.9	32.0
2.0	3.3	2.9	6.6
3.0	1.2	2.2	5.0

TABLE 64: $\Phi = 0.9$

δ \ L	10	20	50
0.0	9.7	15.0	27.2
1.0	8.9	13.2	23.7
2.0	6.6	9.5	15.1
3.0	4.0	6.3	9.2

Table 65

Number of data points	Number of times exceeded
(1) 800,000	42 out of 5000 replications
(2) 2,000,000	18 out of 5000 replications
(3) 800,000	45 out of 5000 replications
(4) 800,000	7 out of 5000 replications
(5) 800,000	61 out of 5000 replications
(6) 500,000	47 out of 10000 replications
(7) 700,000	11 out of 5000 replications
(8) 2,500,000	142 out of 5000 replications
(9) 500,000	12 out of 10000 replications
(10) 200,000	15 out of 10000 replications
(11) 600,000	22 out of 5000 replications
(12) 100,000	10 out of 5000 replications
(13) 200,000	15 out of 10000 replications
(14) 200,000	89 out of 10000 replications
(15) 300,000	62 out of 5000 replications

Table 66: $0.0 < \phi < 0.3$ Interval length L=10 Sample size n=5

	FSI ARL	FSI ATS	VSI ANSS	VSI ATS
Φ=.001	436.8	6552.0	433.7	6448.1
Φ=.005	419.2	6288.0	428.8	6146.6
Φ=0.01	378.3	5674.5	390.0	5551.3
Φ=0.1	163.0	2445.0	158.6	2136.5

Table 67: Correlation $\phi = 0.7$ Interval length L=10 Sample size n=5

	FSI ARL	FSI ATS	VSI ANSS	VSI ATS
δ=0.0	3.9	58.5	4.0	31.3
δ=1.0	2.5	37.5	2.6	15.9
δ=2.0	1.4	21.0	1.4	4.4
δ=3.0	1.1	16.5	1.1	2.9

Table 68: Correlation $\phi = 0.8$ Interval length L=10 Sample size n=5

	FSI ARL	FSI ATS	VSI ANSS	VSI ATS
δ=0.0	2.7	40.5	2.8	18.3
δ=1.0	2.2	33.0	2.2	12.9
δ=2.0	1.5	22.5	1.5	5.8
δ=3.0	1.2	18.0	1.2	3.0

APPENDIX B

ILLUSTRATIONS

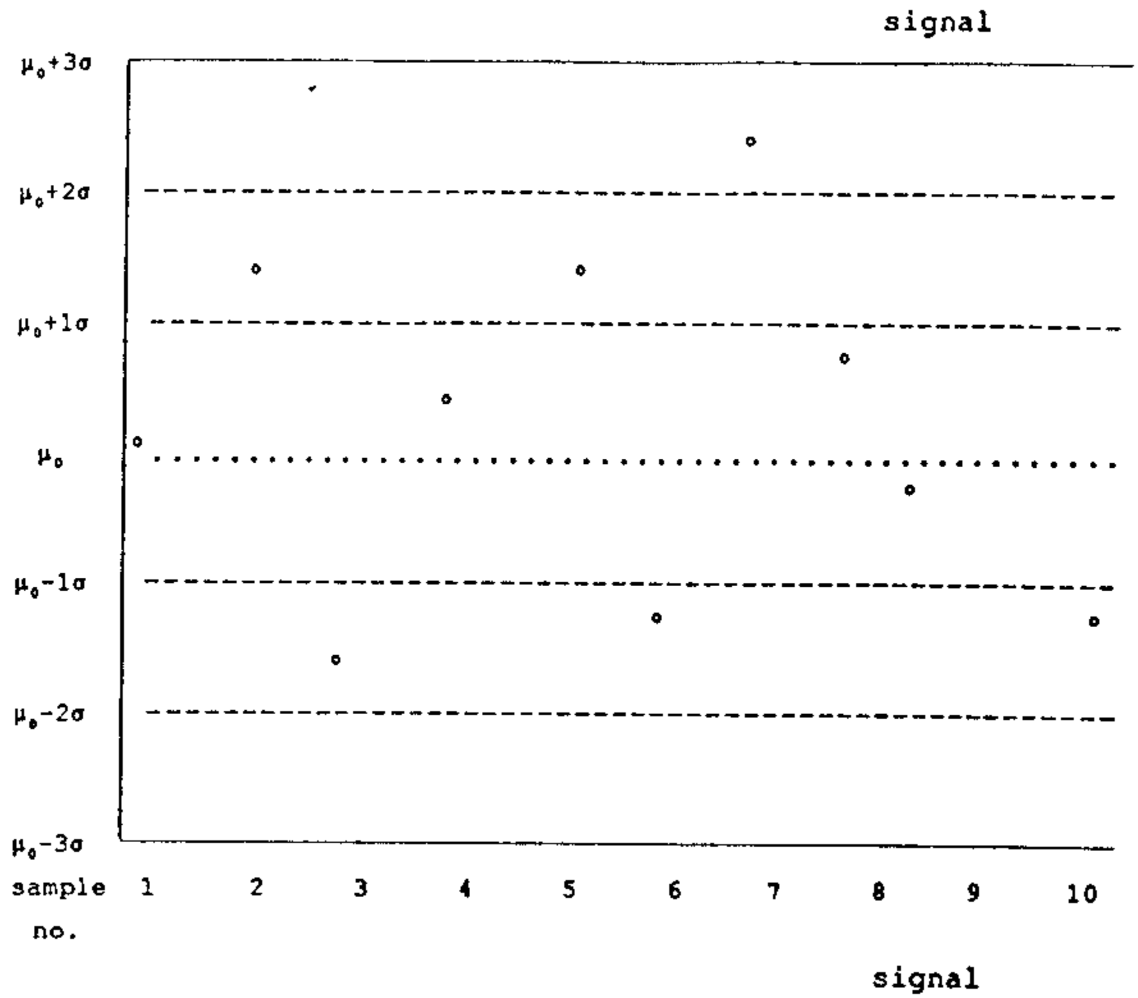


FIGURE 1: Shewhart X-bar Control Chart with FSI

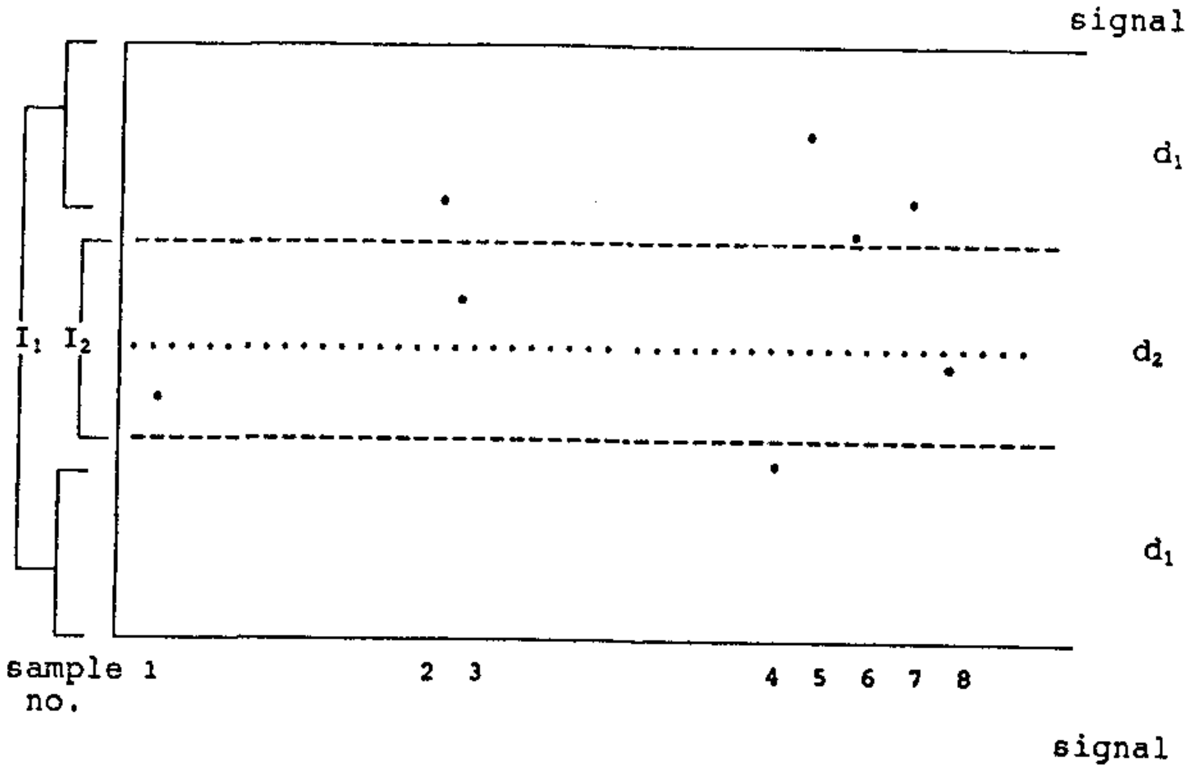


FIGURE 2: Shewhart X-bar Control Chart with VSI

Number of Replications v. ARL

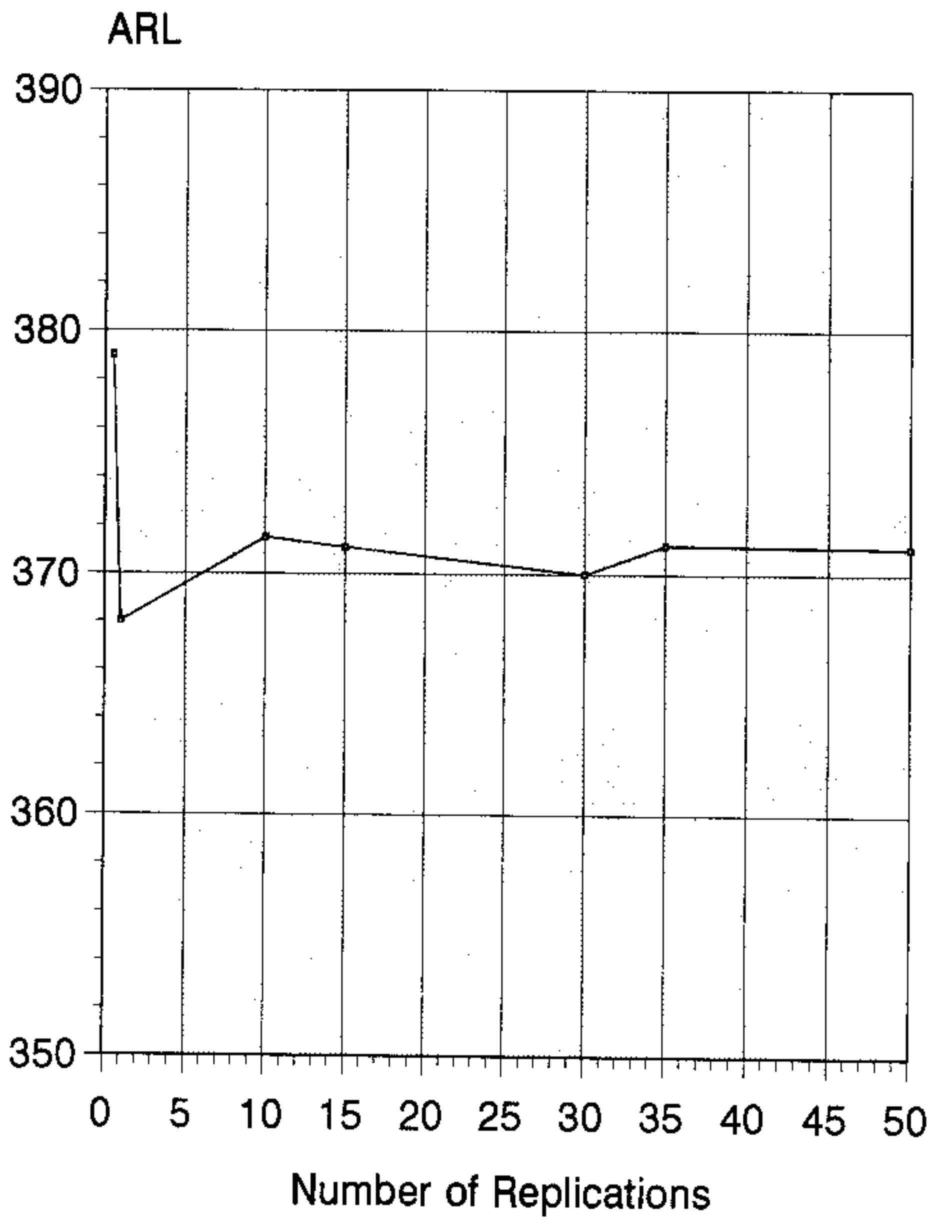


Figure 3: Number of replications v. ARL
used to determine the minimum number of replications

ARL for FSI Shewhart X-bar Chart
with Fixed Control Limits
All Correlations (including shifts)

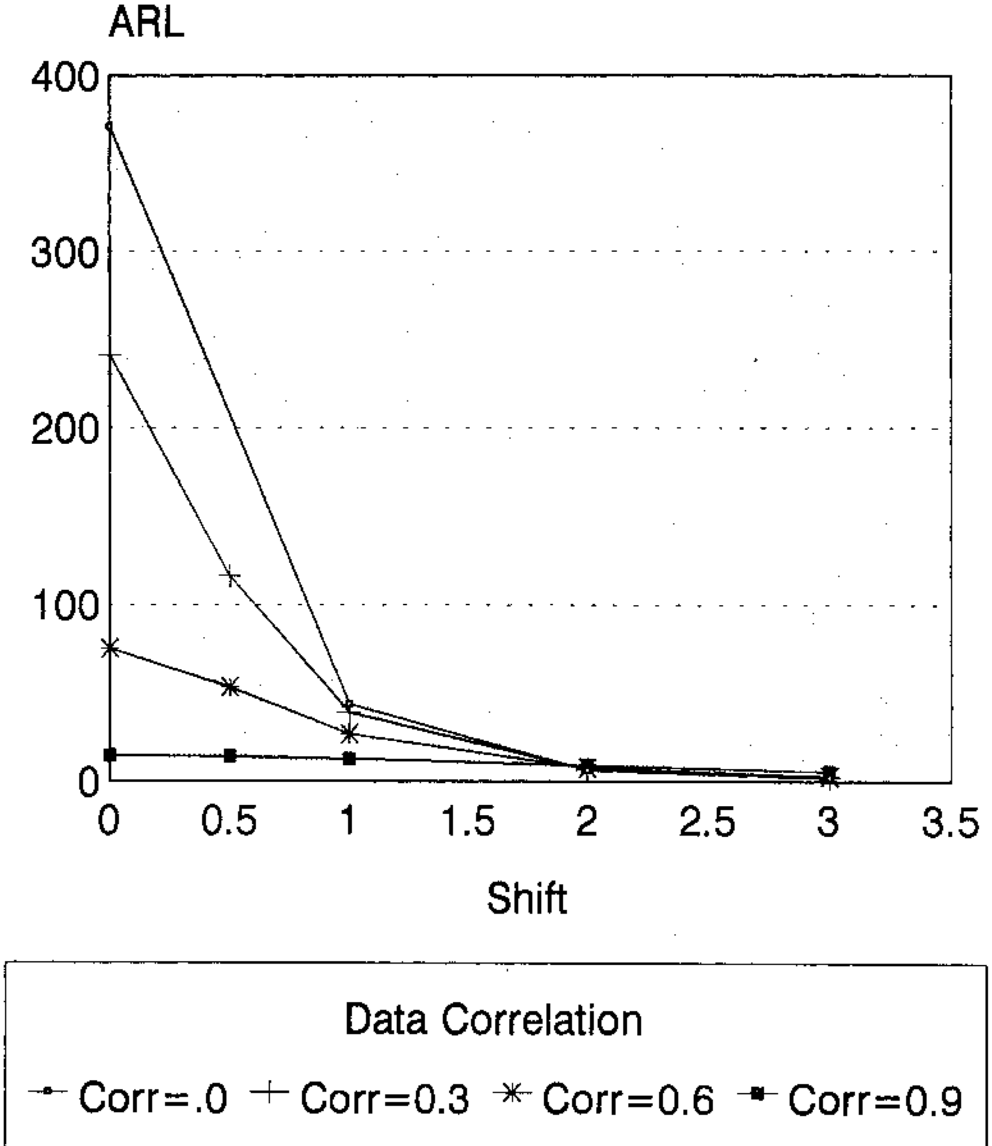


Figure 4a: A comparison of ARLs of FSI with control limits pre-set
Sample size: n = 1 Interval length: l = 1
y axis: linear scale

ARL for FSI Shewhart X-bar Chart
with Fixed Control Limits
All Correlations (including shifts)

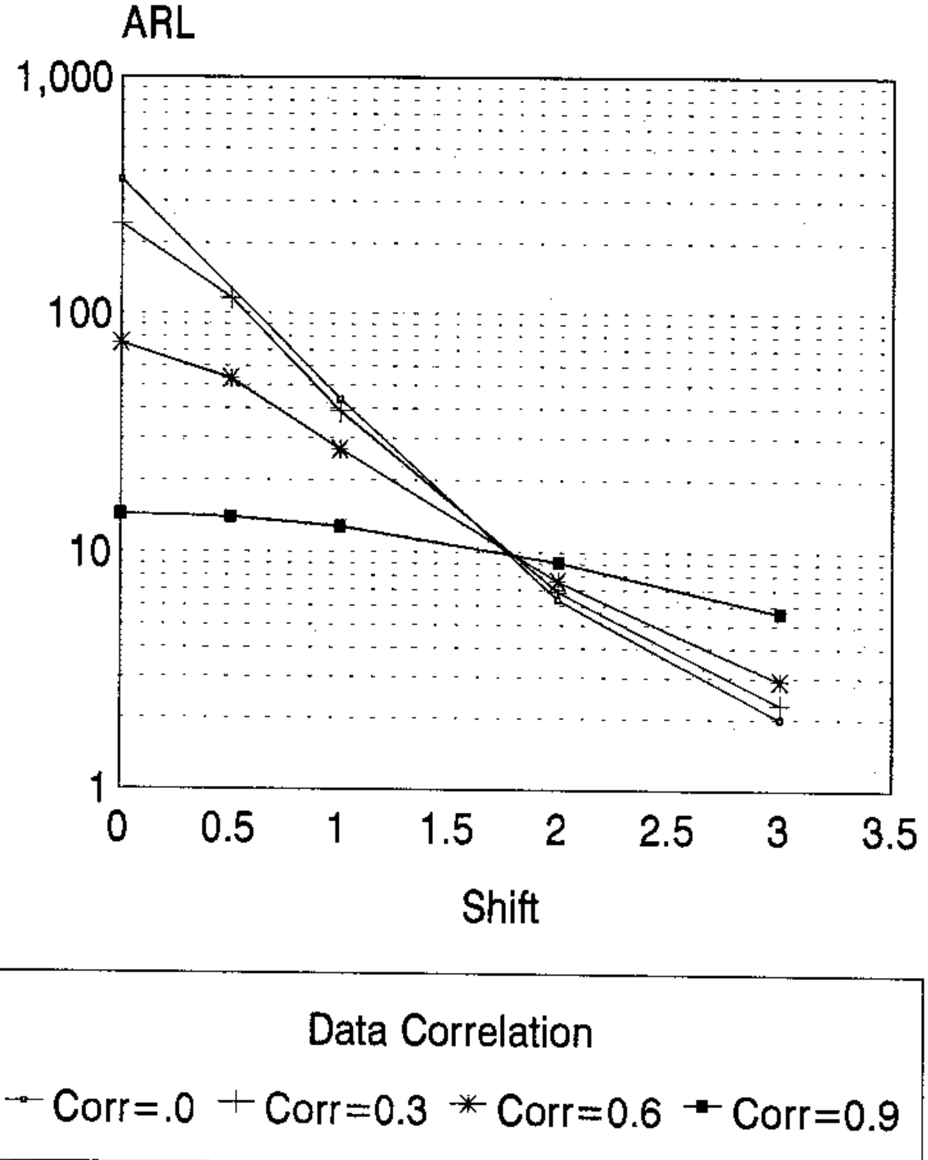


Figure 4b: A comparison of ARLs of FSI with control limits pre-set
Sample size: n=1 Interval length: l = 1
y axis: logarithmic scale

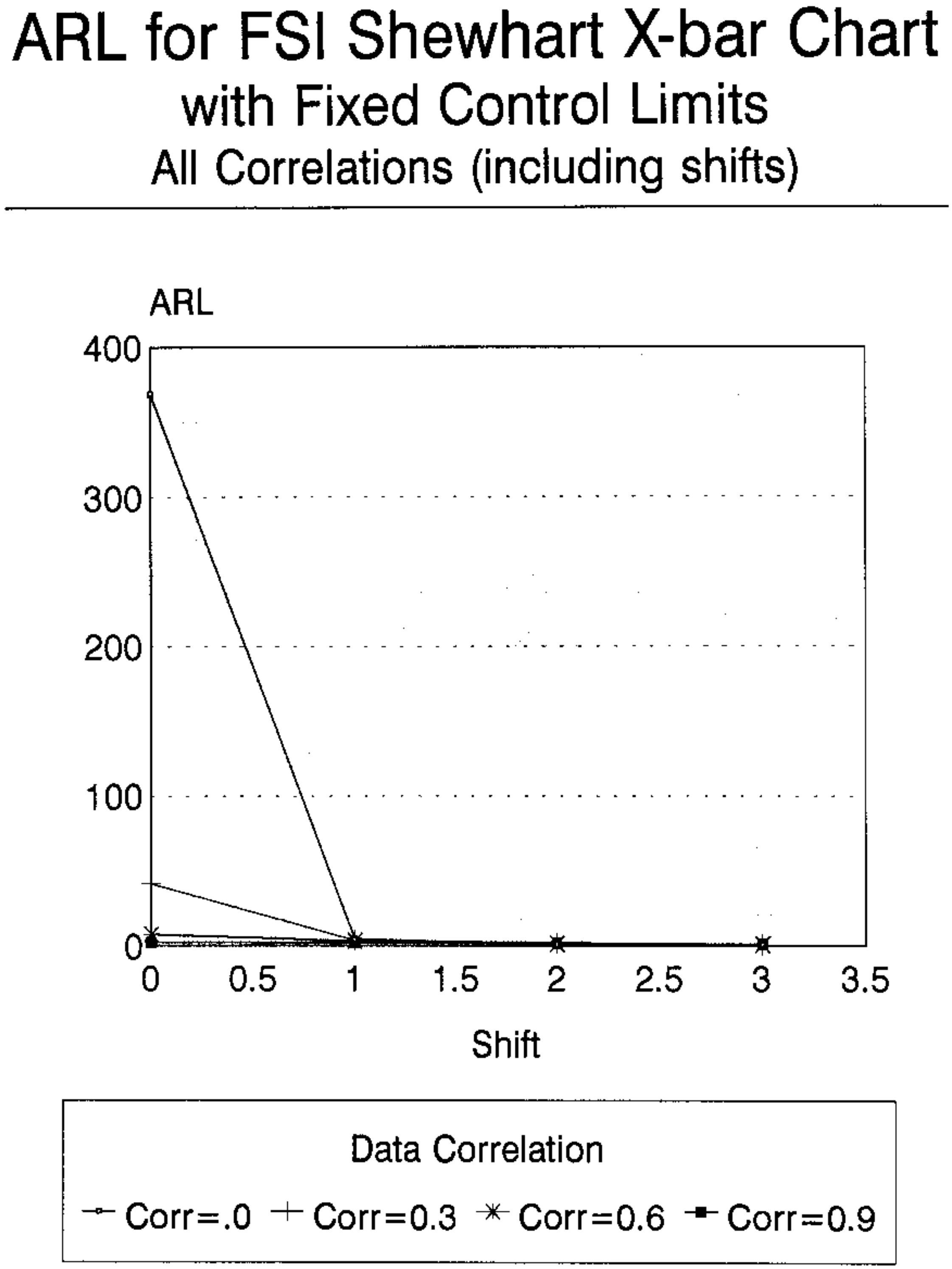


Figure 4c: A comparison of ARLs of FSI with control limits pre-set
Sample size: n = 5 Interval length: l = 1
y axis: linear scale

ARL for FSI Shewhart X-bar Chart
with Fixed Control Limits
All Correlations (including shifts)

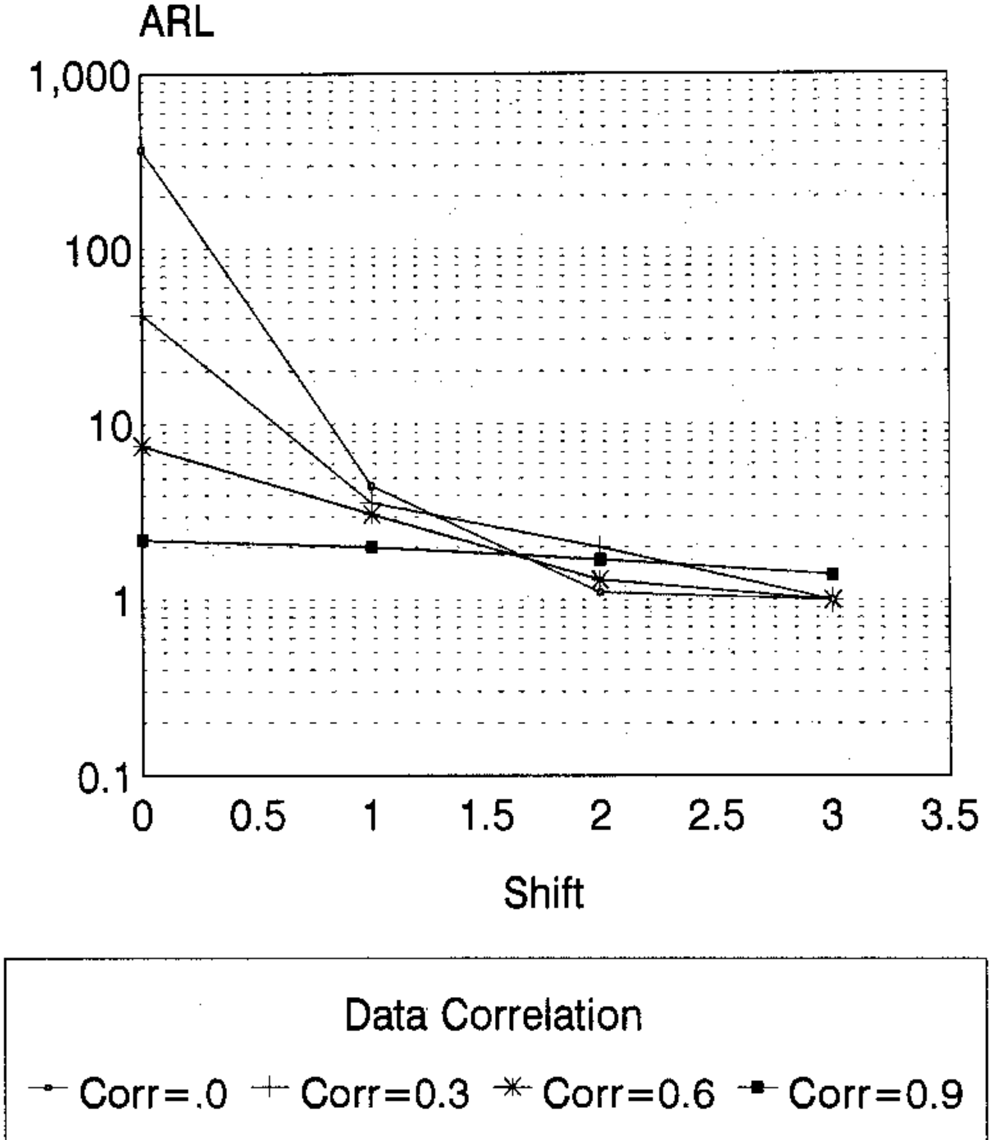


Figure 4d: A comparison of ARLs of FSI with control limits pre-set
Sample size: n = 5 Interval length: l = 1
y axis: logarithmic scale

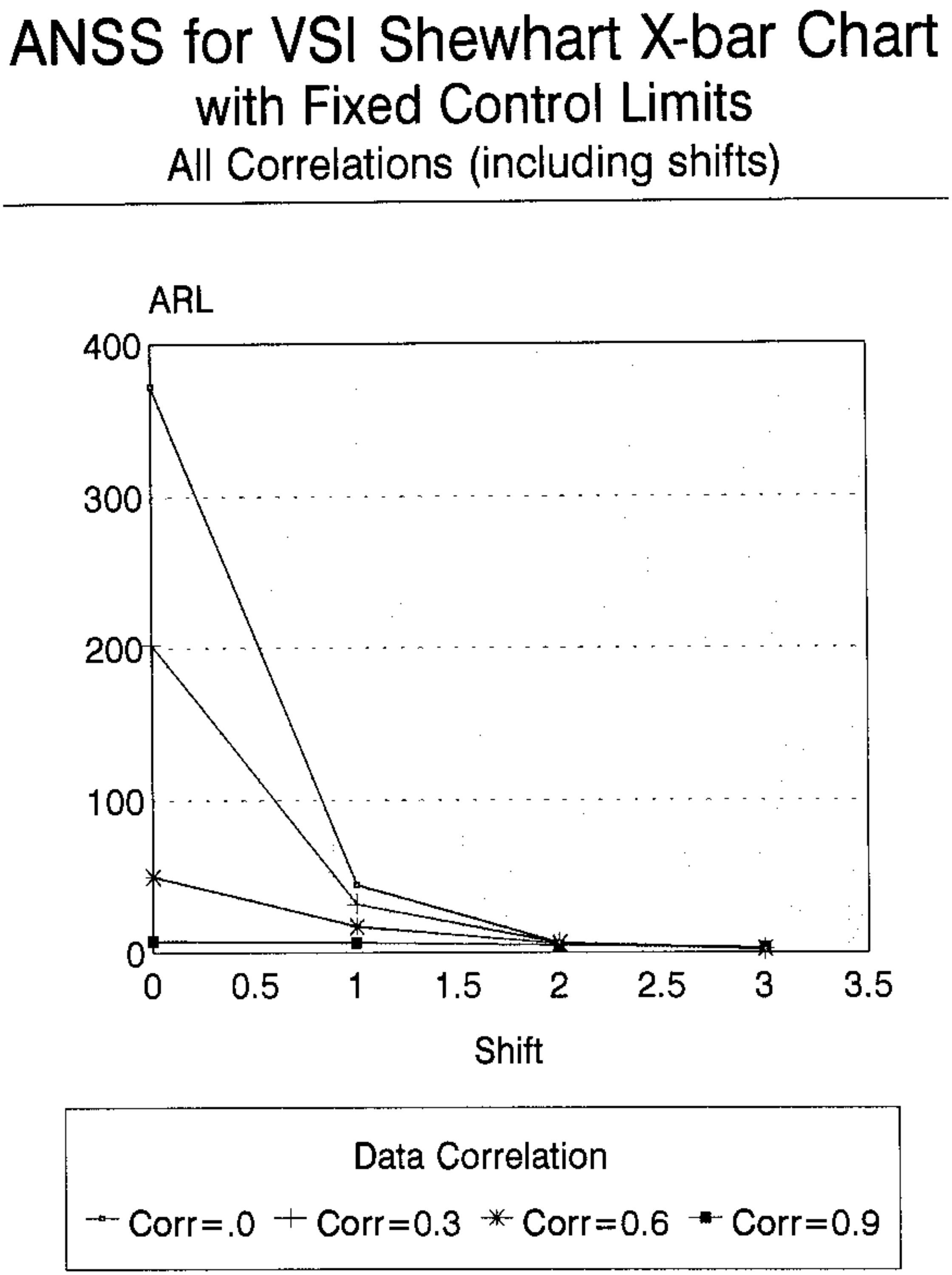


Figure 5a: A comparison of ANSS of VSI with control limits pre-set
Sample size: n = 1 Interval length: I = 10
y axis: linear scale

ANSS for VSI Shewhart X-bar Chart
with Fixed Control Limits
All Correlations (including shifts)

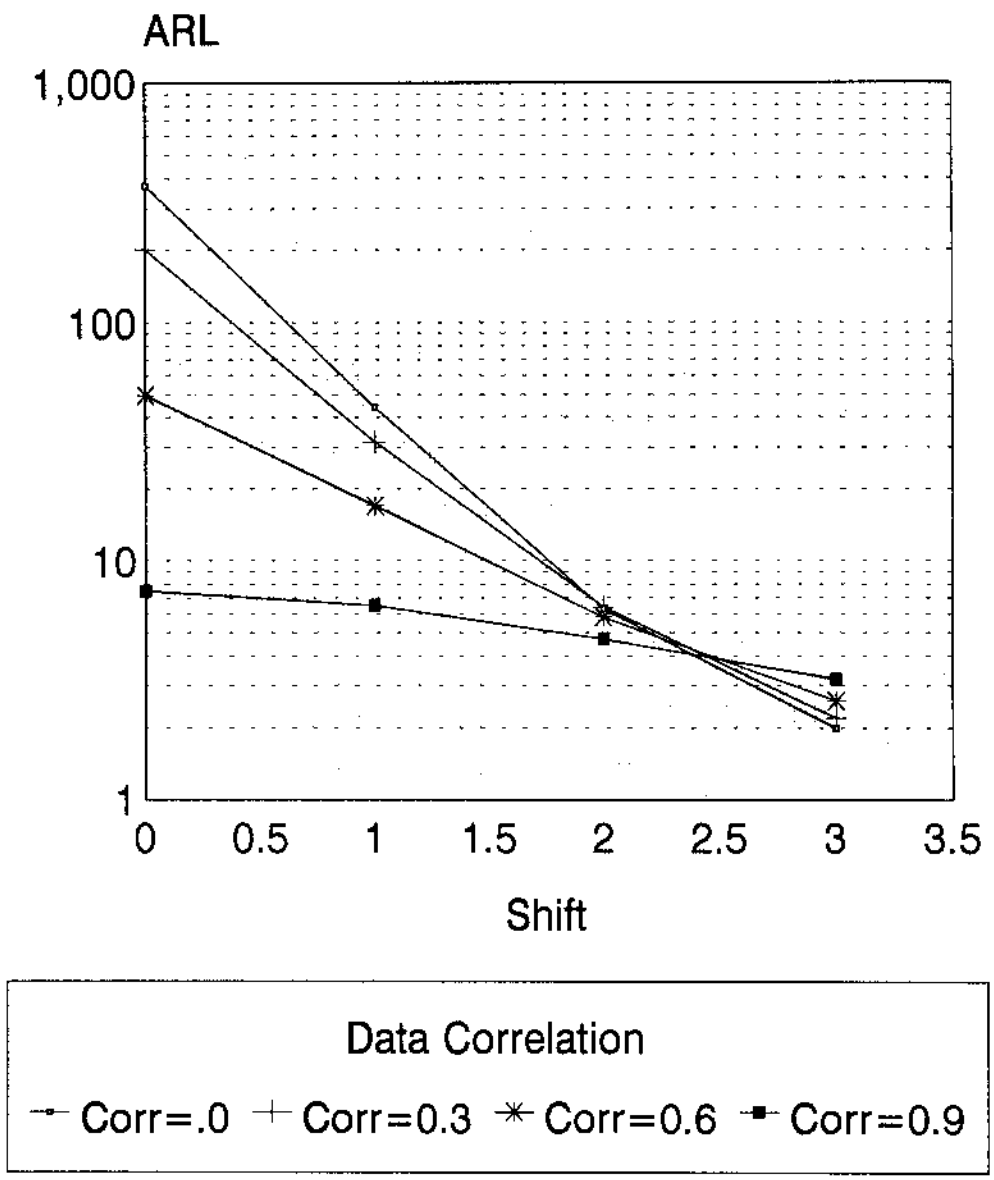


Figure 5b: A comparison of ANSS of VSI with control limits pre-set
Sample size: n = 1 Interval length: L = 10
y axis: logarithmic scale

ANSS for VSI Shewhart X-bar Chart
with Fixed Control Limits
All Correlations (including shifts)

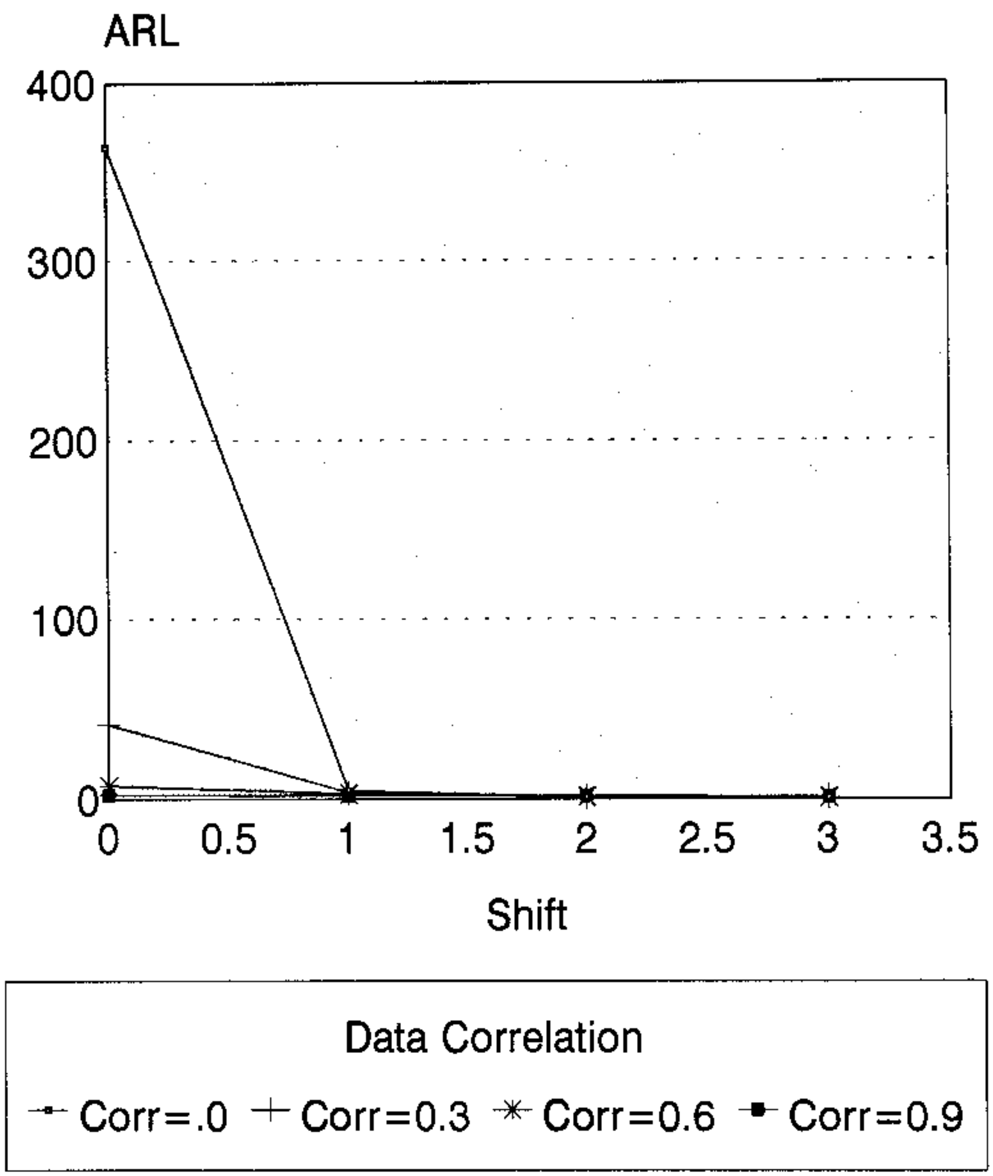


Figure 5c: A comparison of ANSS of VSI with control limits pre-set
Sample size: n = 5   Interval length: I = 10
y axis: linear scale

ANSS for VSI Shewhart X-bar Chart
with Fixed Control Limits
All Correlations (including shifts)

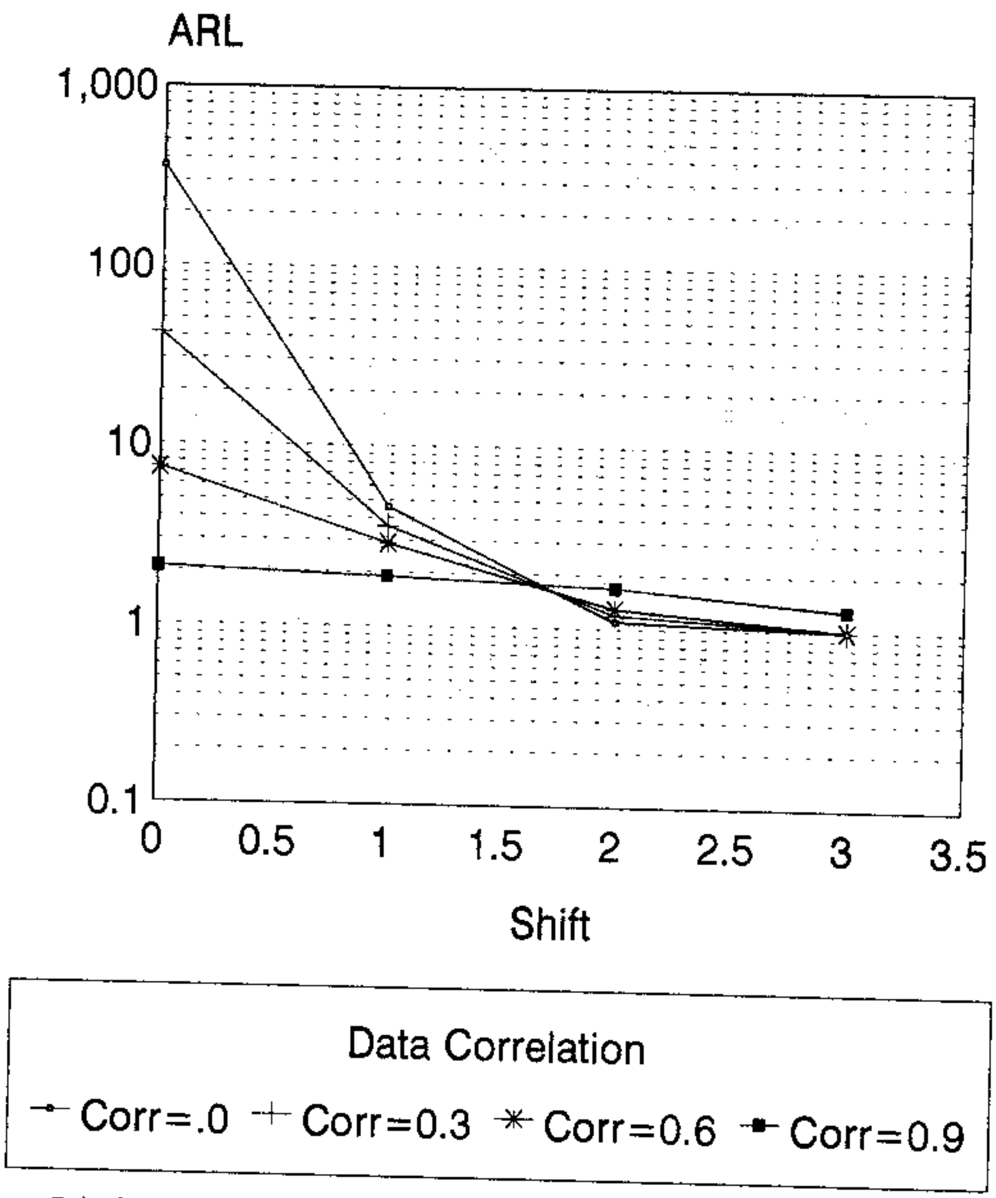


Figure 5d: A comparison of ANSS of VSI with control limits pre-set
Sample size: n = 5 Interval length: l = 10
y axis: logarithmic scale

ARL for FSI Shewhart X-bar Chart
with Calculated Control Limits
All Correlations (including shifts)

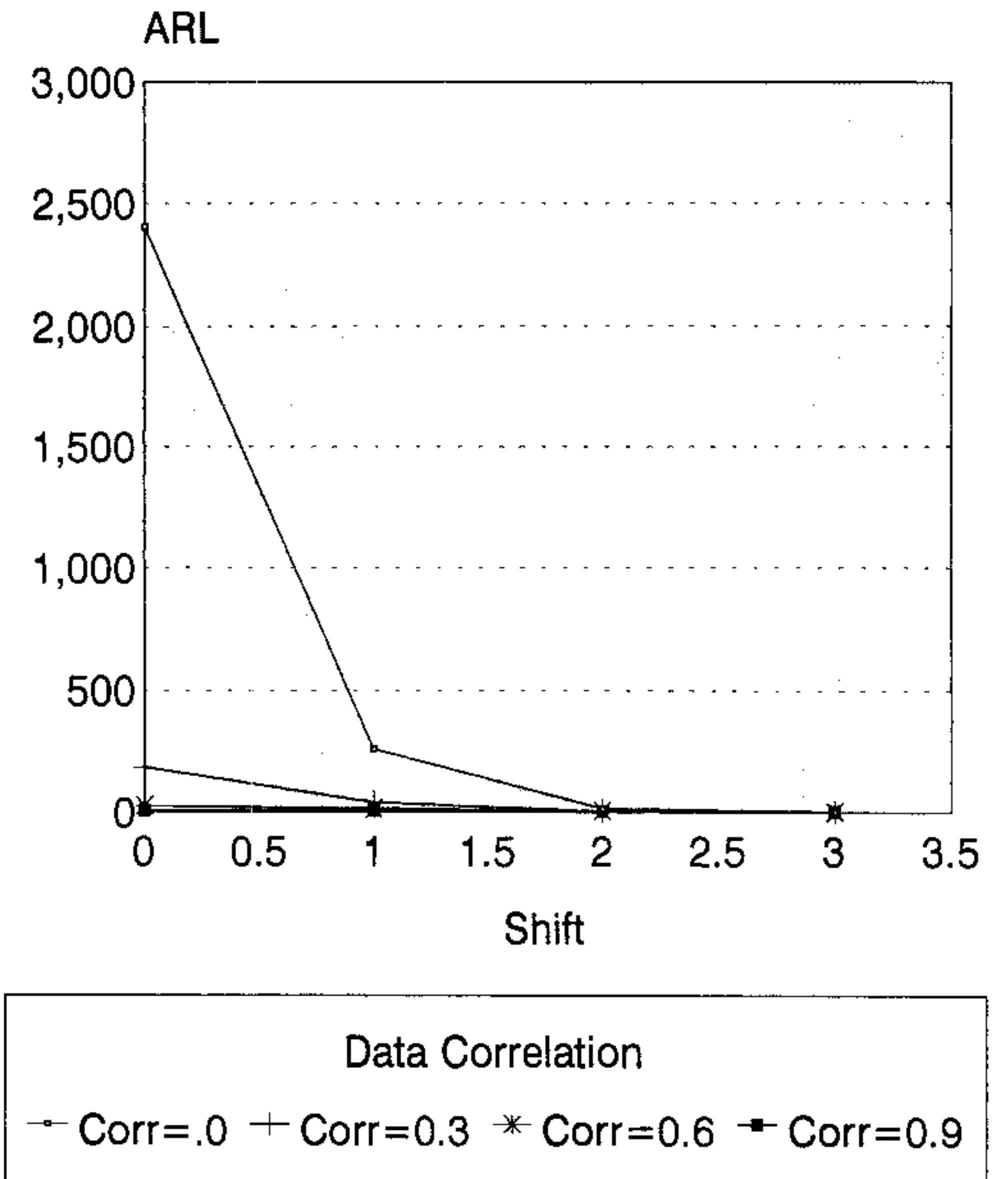


Figure 6a: A comparison of ARLs of FSI with control limits calculated from sample data
Sample size: n = 1 Interval length: l = 1
y axis: linear scale

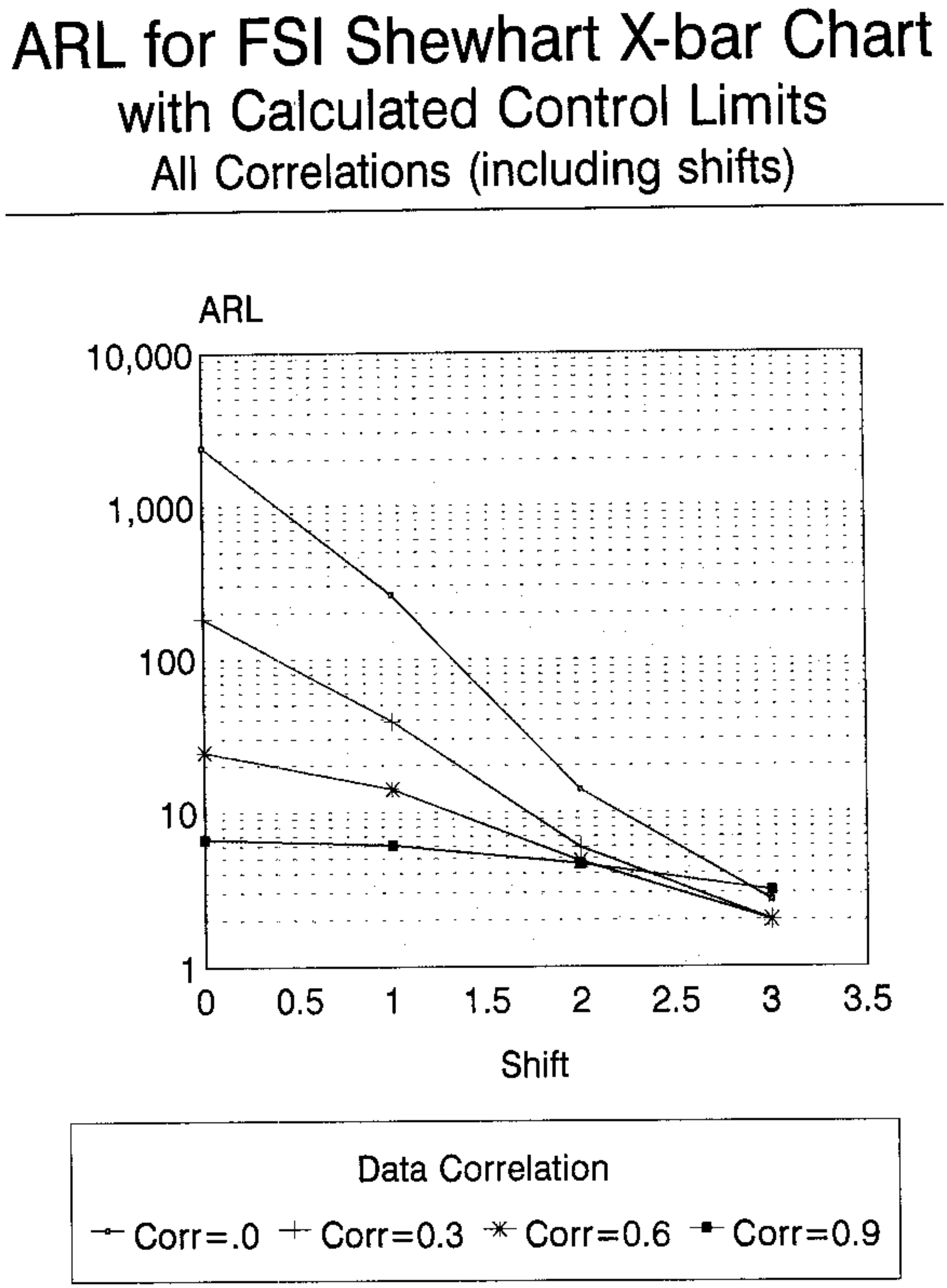


Figure 6b: A comparison of ARLs of FSI with control limits calculated from sample data
Sample size: n = 1 Interval length: I = 1
y axis: logarithmic scale

ARL for FSI Shewhart X-bar Chart
with Calculated Control Limits
All Correlations (including shifts)

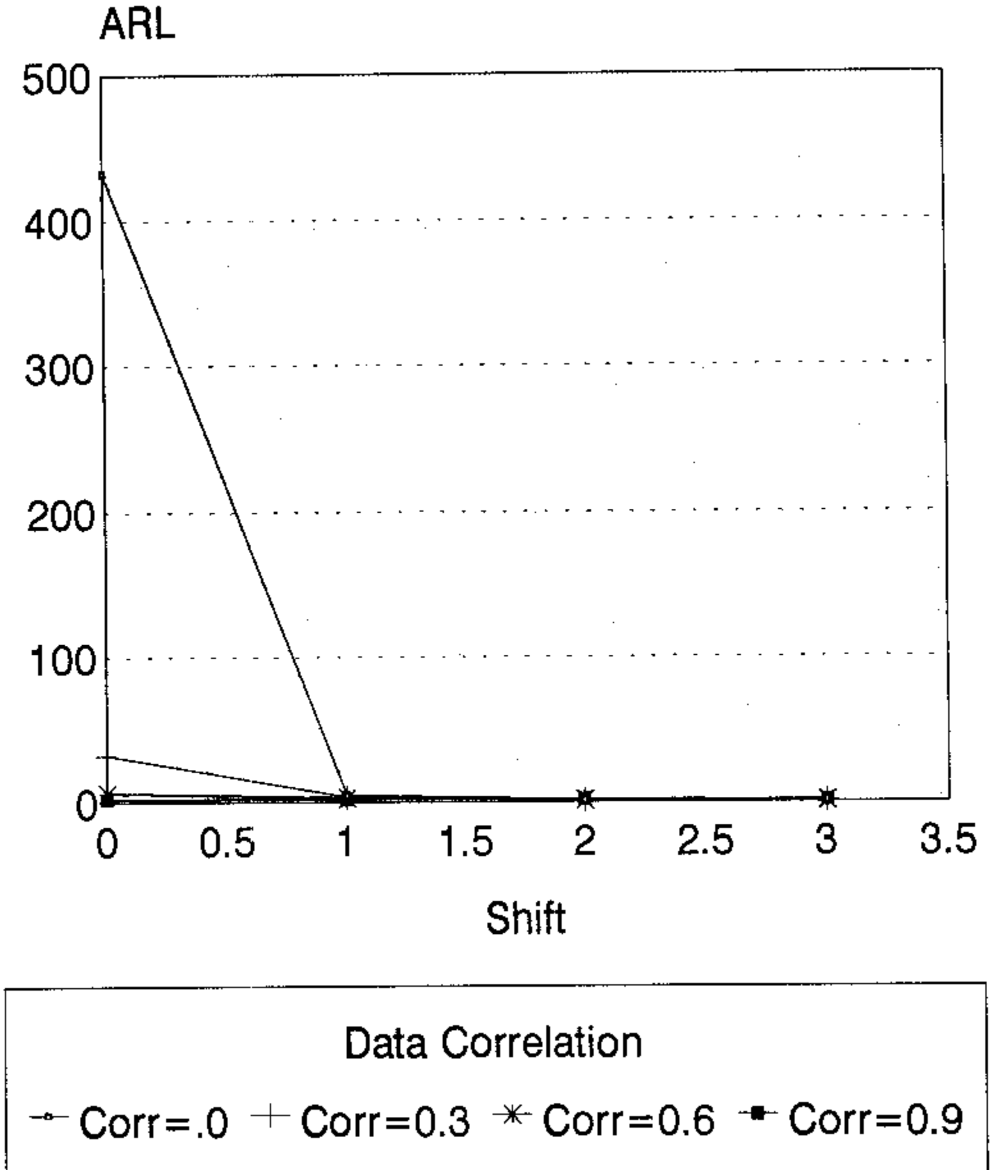


Figure 6c: A comparison of ARLs of FSI with control limits calculated from sample data
Sample size: n = 5 Interval length: I = 1
y axis: linear scale

ARL for FSI Shewhart X-bar Chart
with Calculated Control Limits
All Correlations (including shifts)

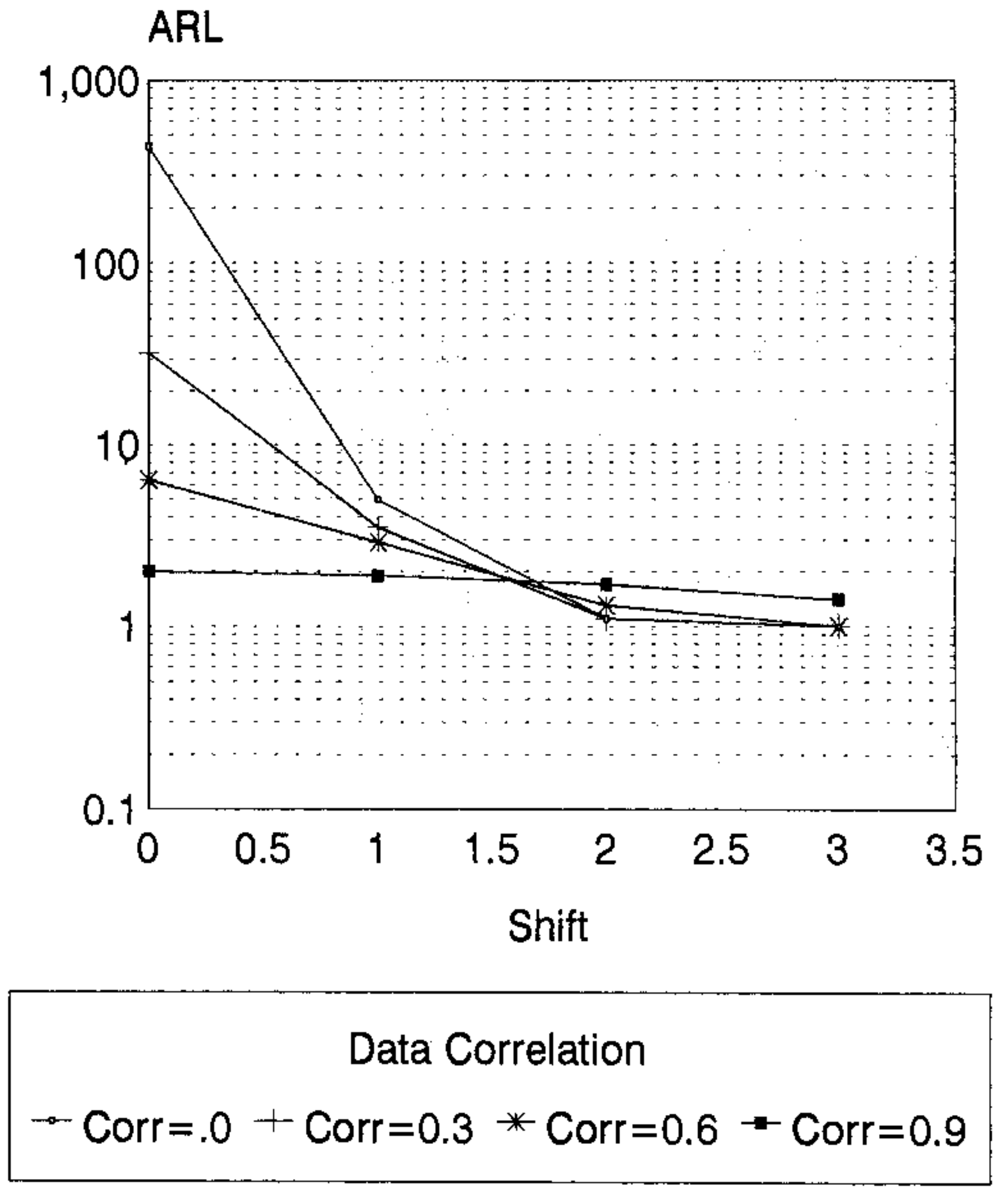


Figure 6d: A comparison of ARLs of FSI with control limits calculated from sample data
Sample size: n = 5 Interval length: l = 1
y axis: logarithmic scale

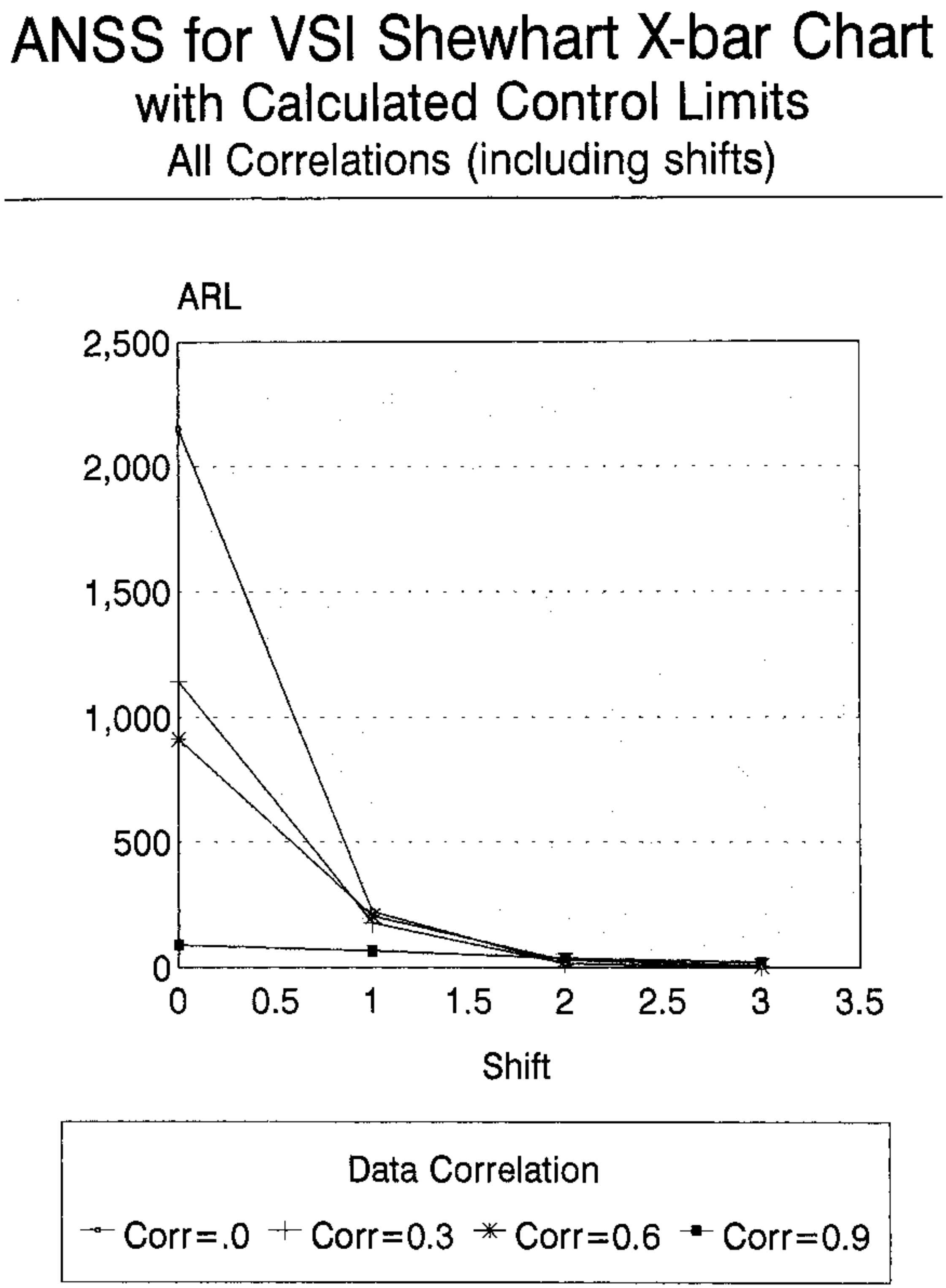


Figure 7a: A comparison of ANSS of VSI with control limits calculated from sample data
Sample size: n = 1   Interval length: I = 10
y axis: linear scale

ANSS for VSI Shewhart X-bar Chart
with Calculated Control Limits
All Correlations (including shifts)

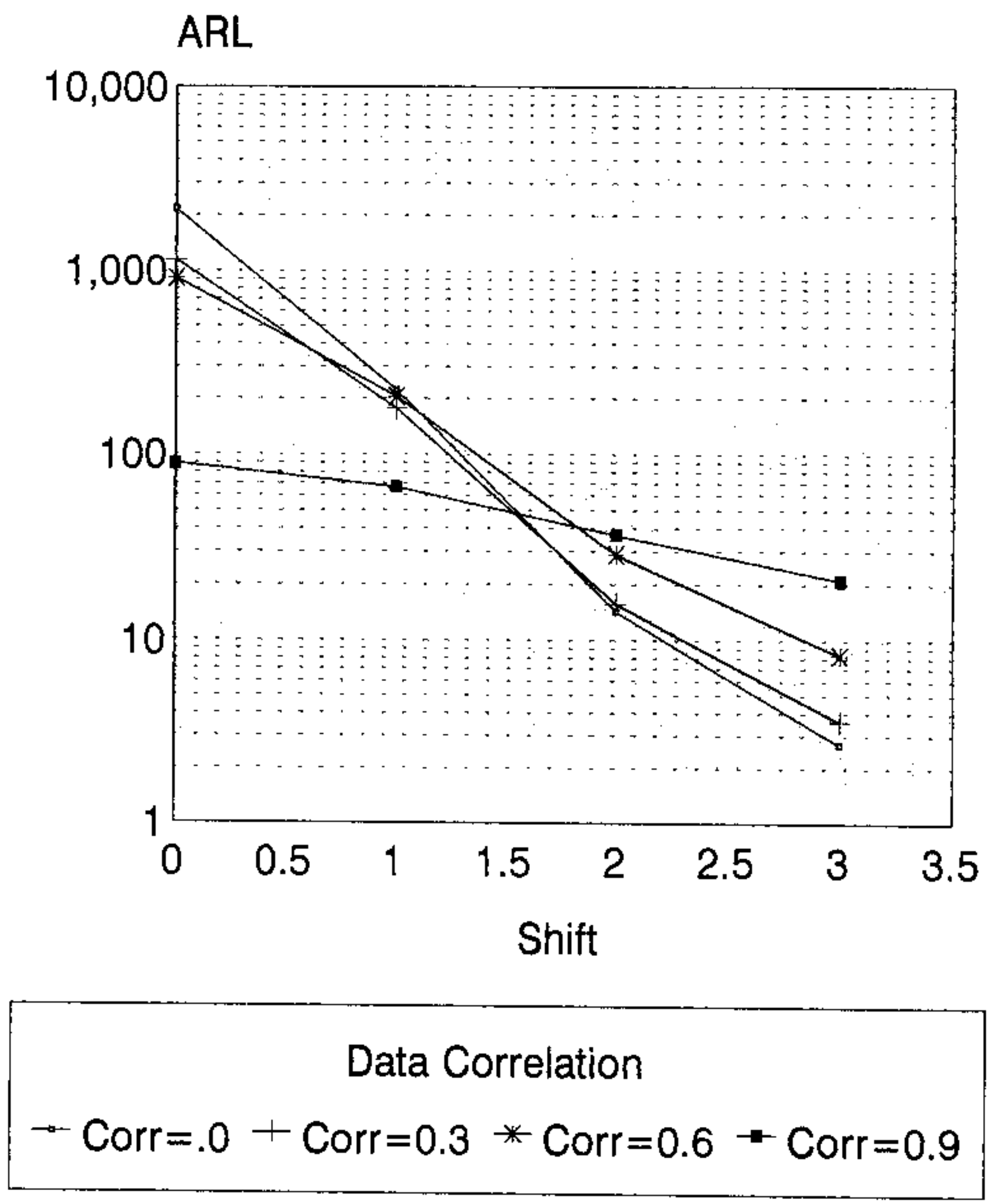


Figure 7b: A comparison of ANSS of VSI with control limits calculated from sample data
Sample size: n = 1 Interval length: I = 10
y axis: logarithmic scale

ANSS for VSI Shewhart X-bar Chart
with Calculated Control Limits
All Correlations (including shifts)

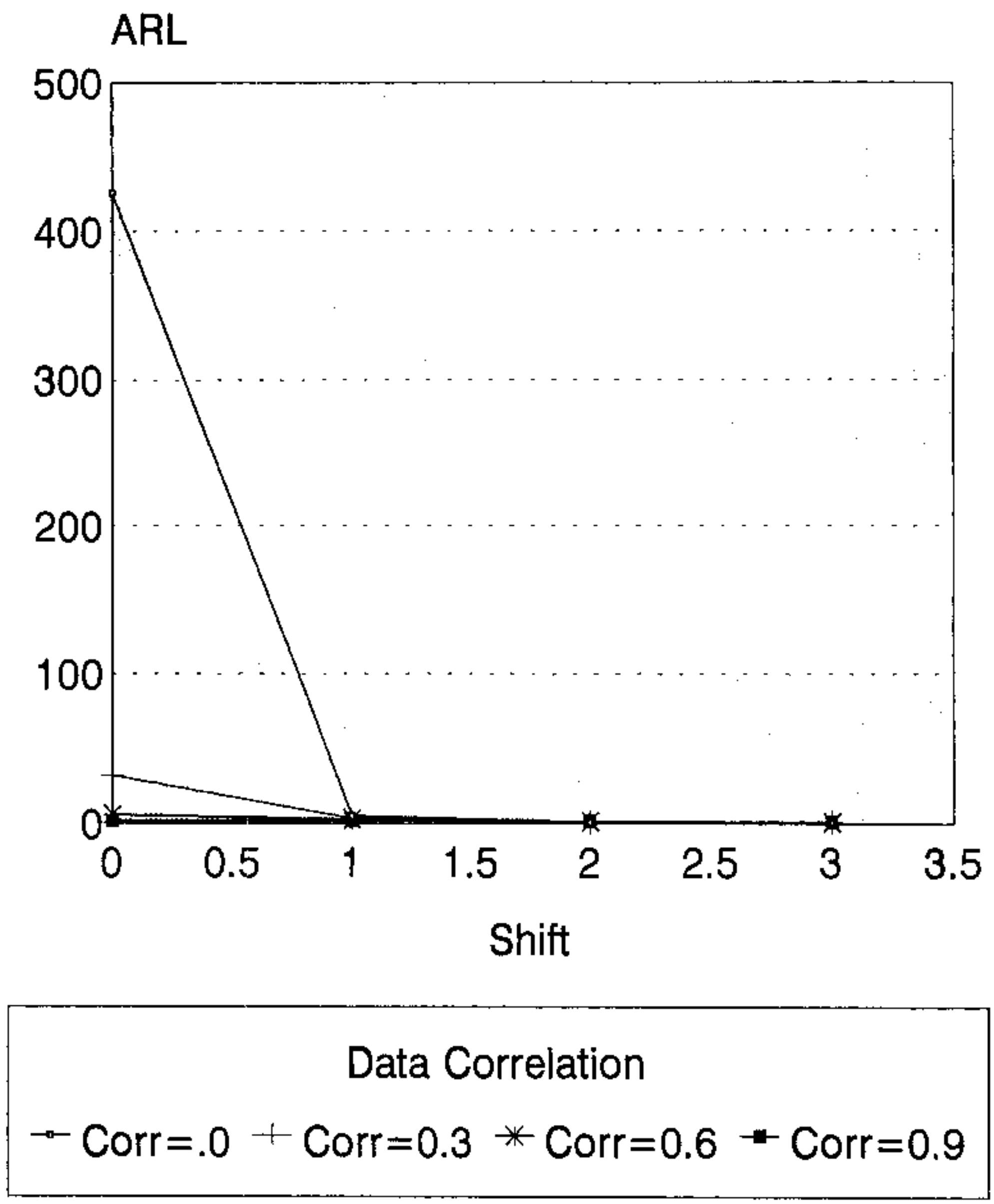


Figure 7c: A comparison of ANSS of VSI with control limits calculated from sample data
Sample size: n = 5 Interval length: I = 10
y axis: linear scale

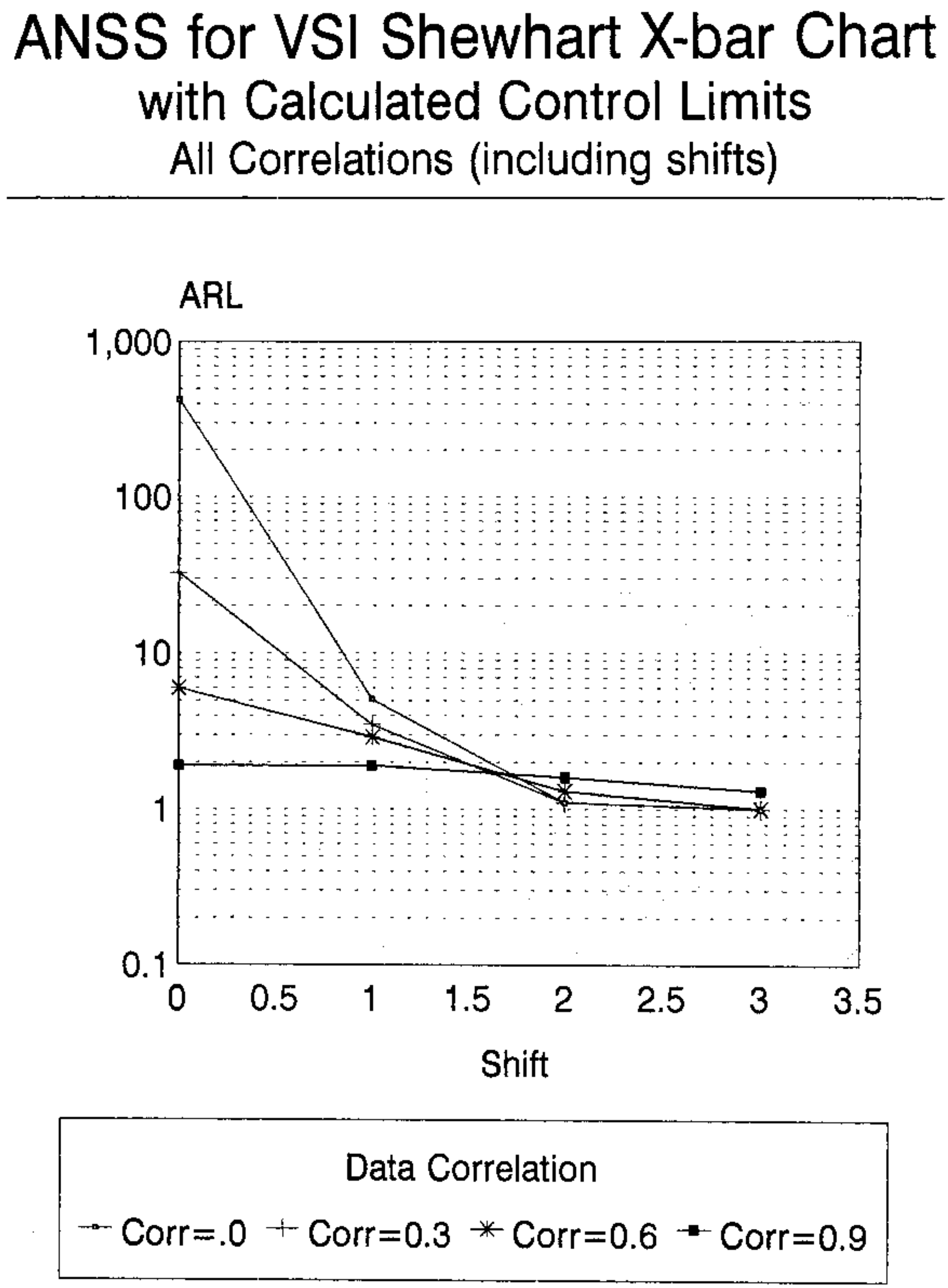


Figure 7d: A comparison of ANSS of VSI with control limits calculated from sample data
Sample size: n = 5 Interval length: I = 10
y axis: logarithmic scale

ARL for FIXED SAMPLING INTERVALS
with FIXED CONTROL LIMITS

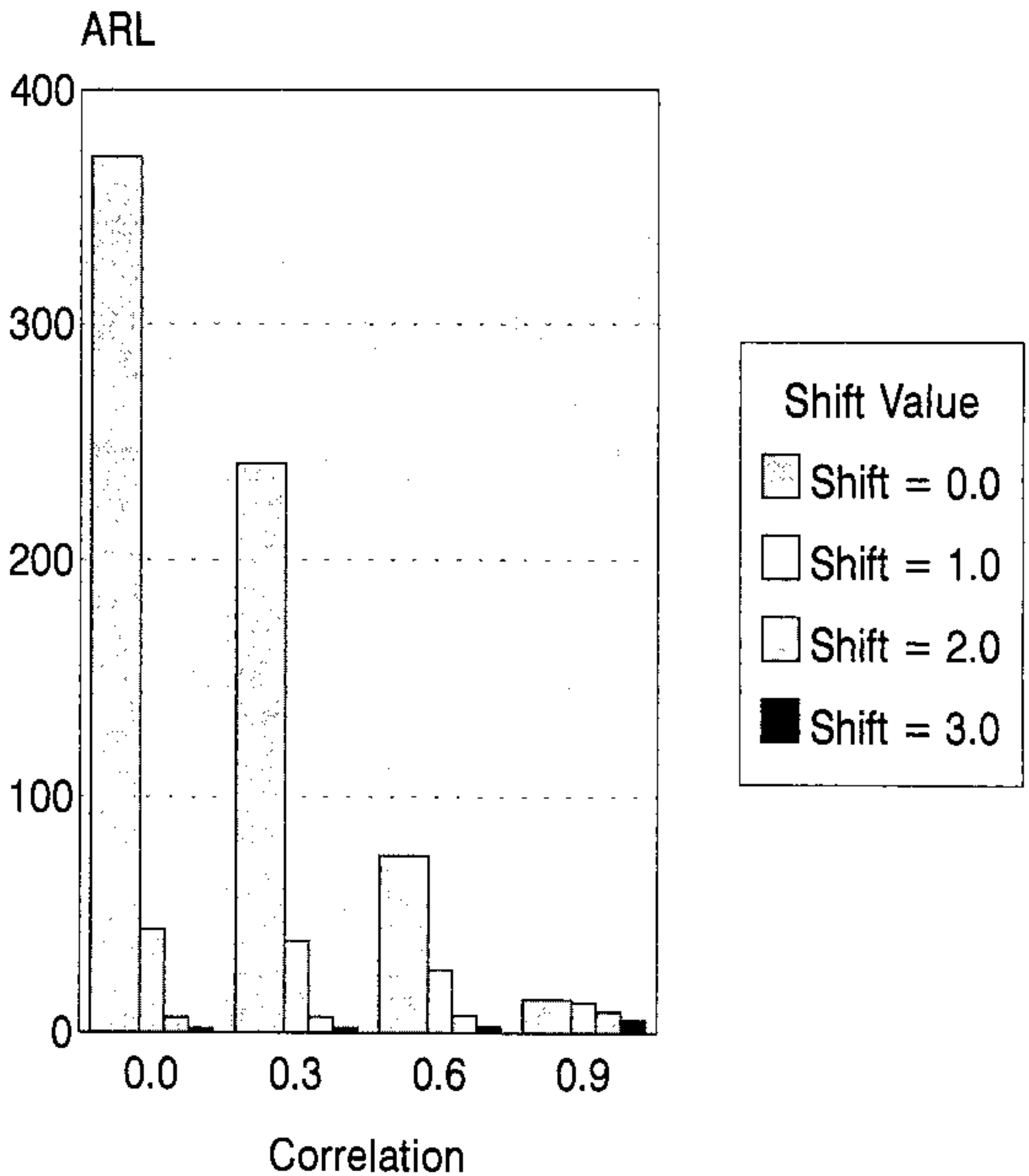


Figure 8a: A comparison of FSI with control limits pre-set
Sample size: n = 1 Interval length: l = 1
y axis: linear scale

ARL for FSI Shewhart X-bar Charts
with Fixed Control Limits

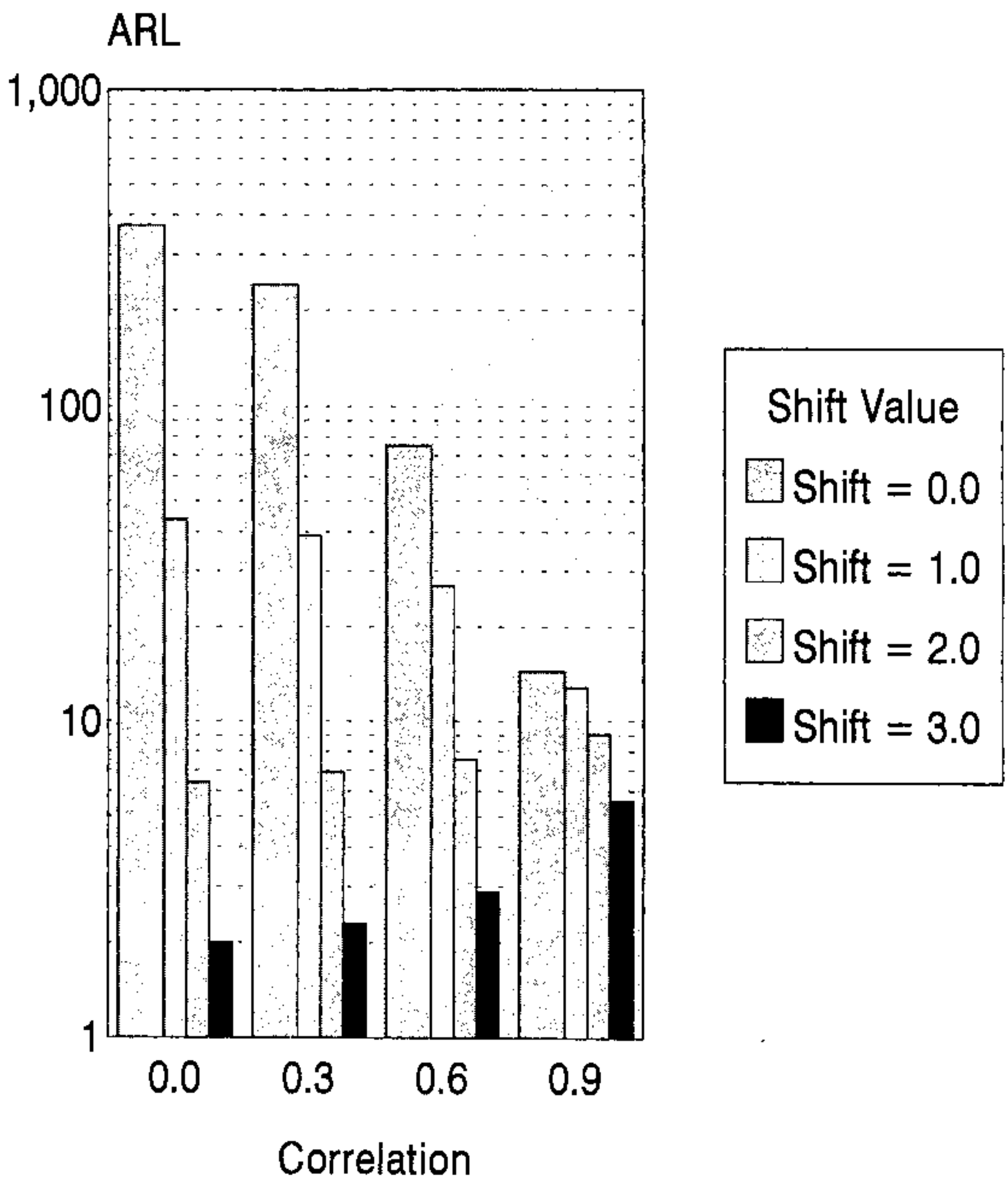


Figure 8b: A comparison of average run length for samples of size 1 taken consecutively from observations generated with indicated correlation
Sample size: n=1 Interval length L=1 y axis: logarithmic scale

ARL for FSI
with Fixed Control Limits

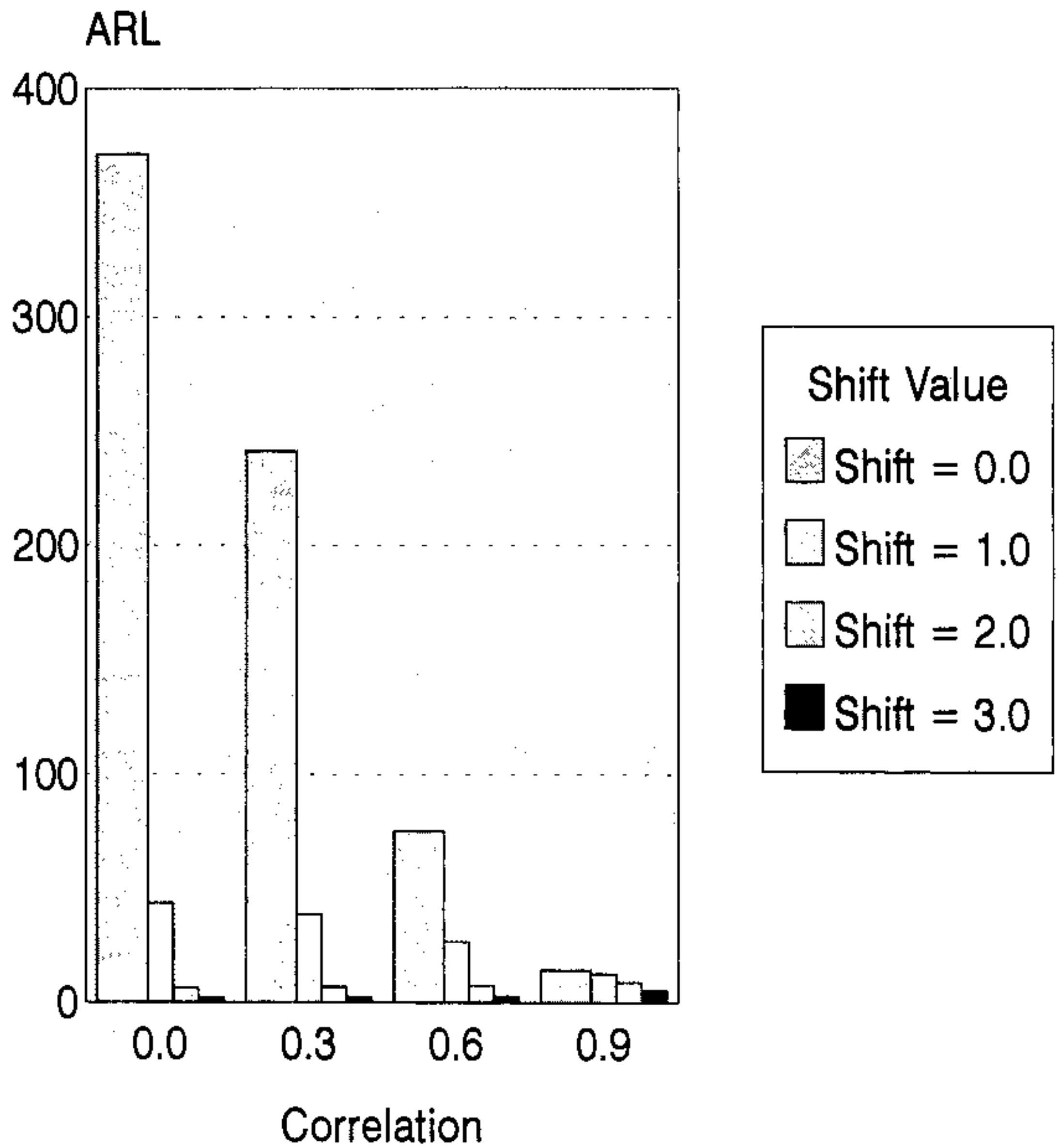


Figure 8c: ARLs with control limits pre-set
Sample size: n = 5 Interval length: L = 1
y axis : linear scale

ARL for FSI
with Fixed Control Limits

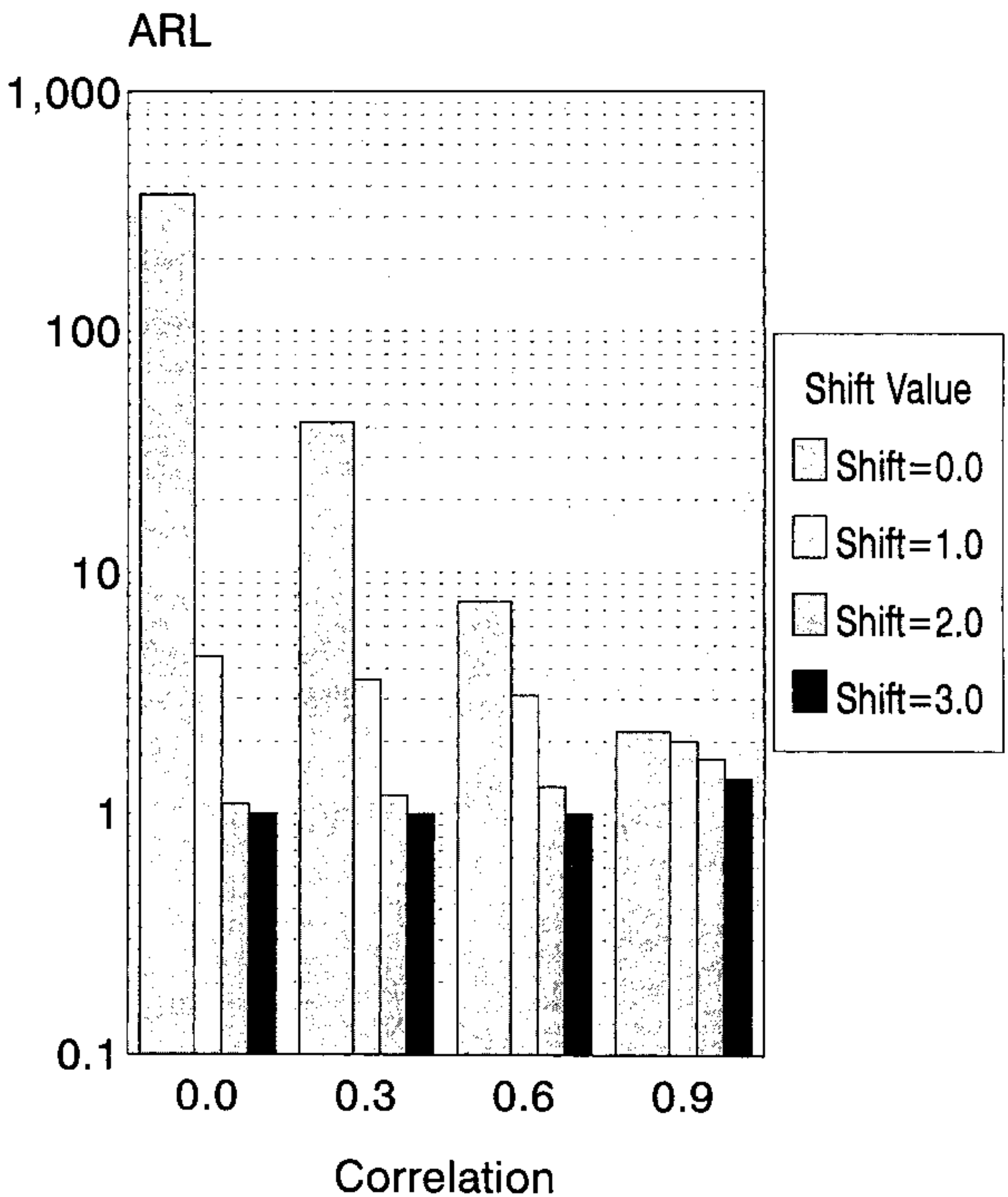


Figure 8d: A comparison of average run length for samples of size 5 taken consecutively from observations generated with indicated correlation
Sample size: n = 5 Interval length: L = 1 y axis: logarithmic scale

ANSS for VARIABLE SAMPLING INTERVALS
with FIXED CONTROL LIMITS

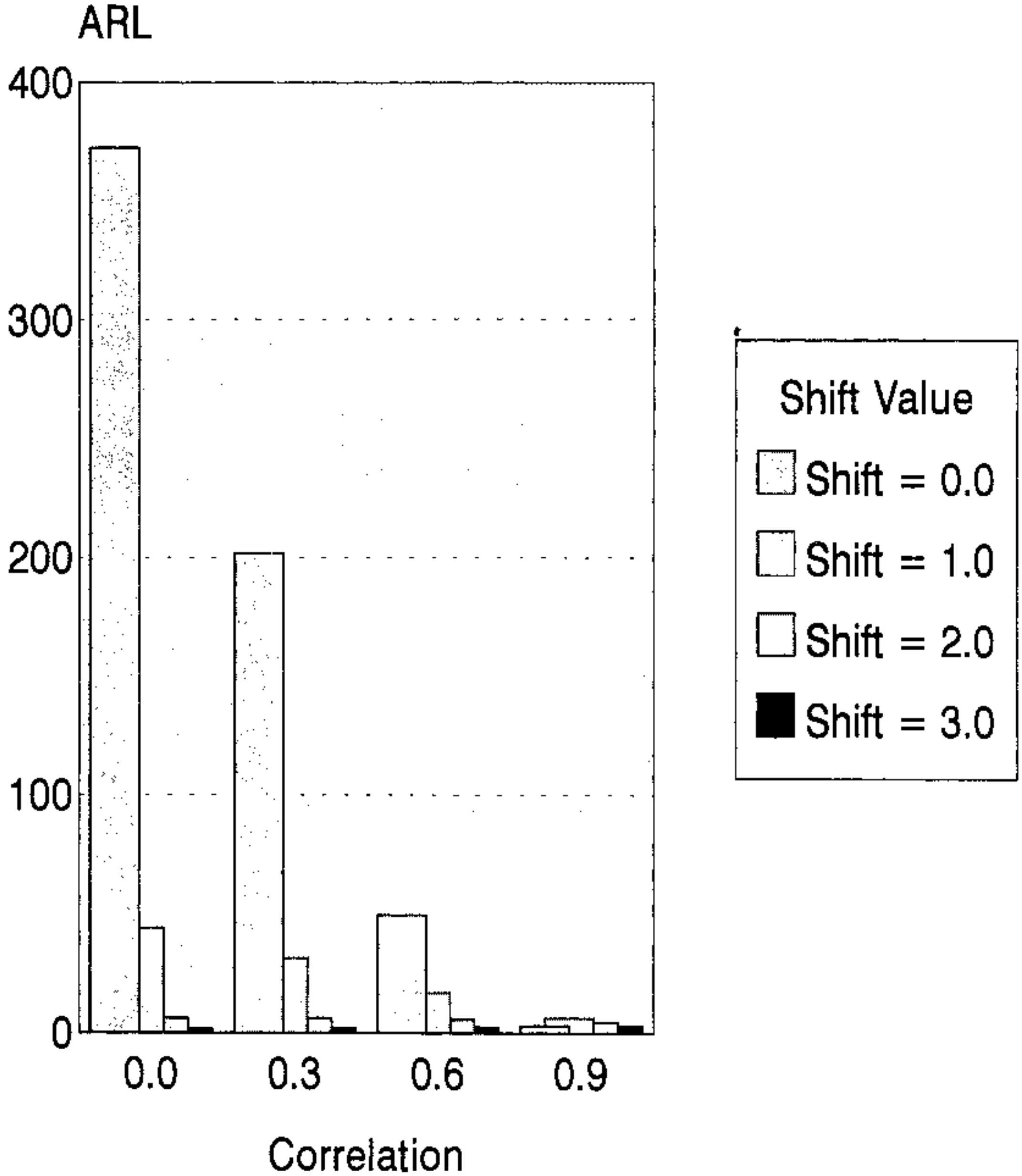


Figure 9a: A comparison of VSI with control limits pre-set
Sample size: n = 1 Interval length: l = 10
y axis: linear scale

ANSS for VARIABLE SAMPLING INTERVALS
with FIXED CONTROL LIMITS

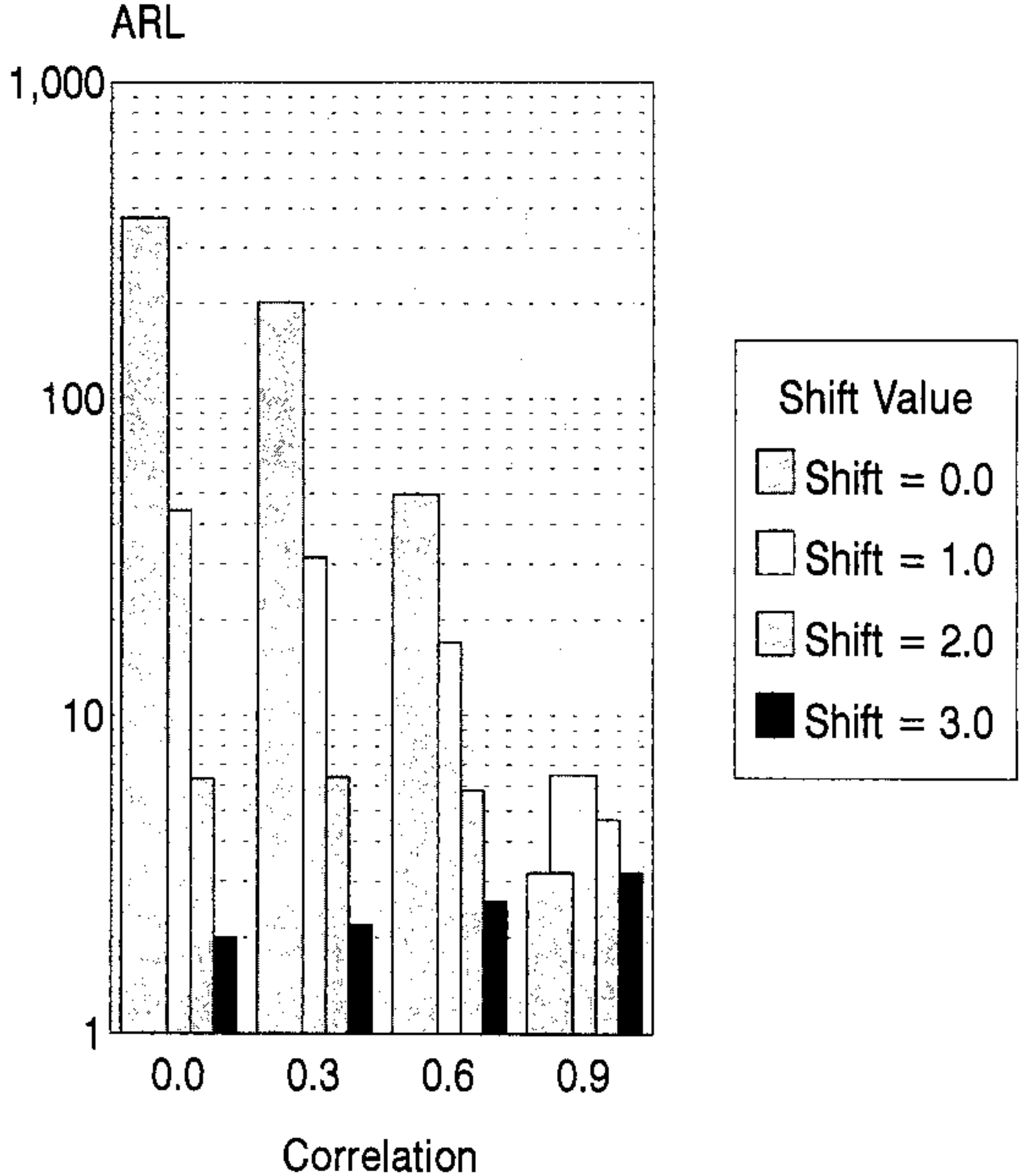


Figure 9b: A comparison of VSI with control limits pre-set
Sample size: n = 1 Interval length: I = 10
y axis: logarithmic scale

ANSS for VSI
with Fixed Control Limits

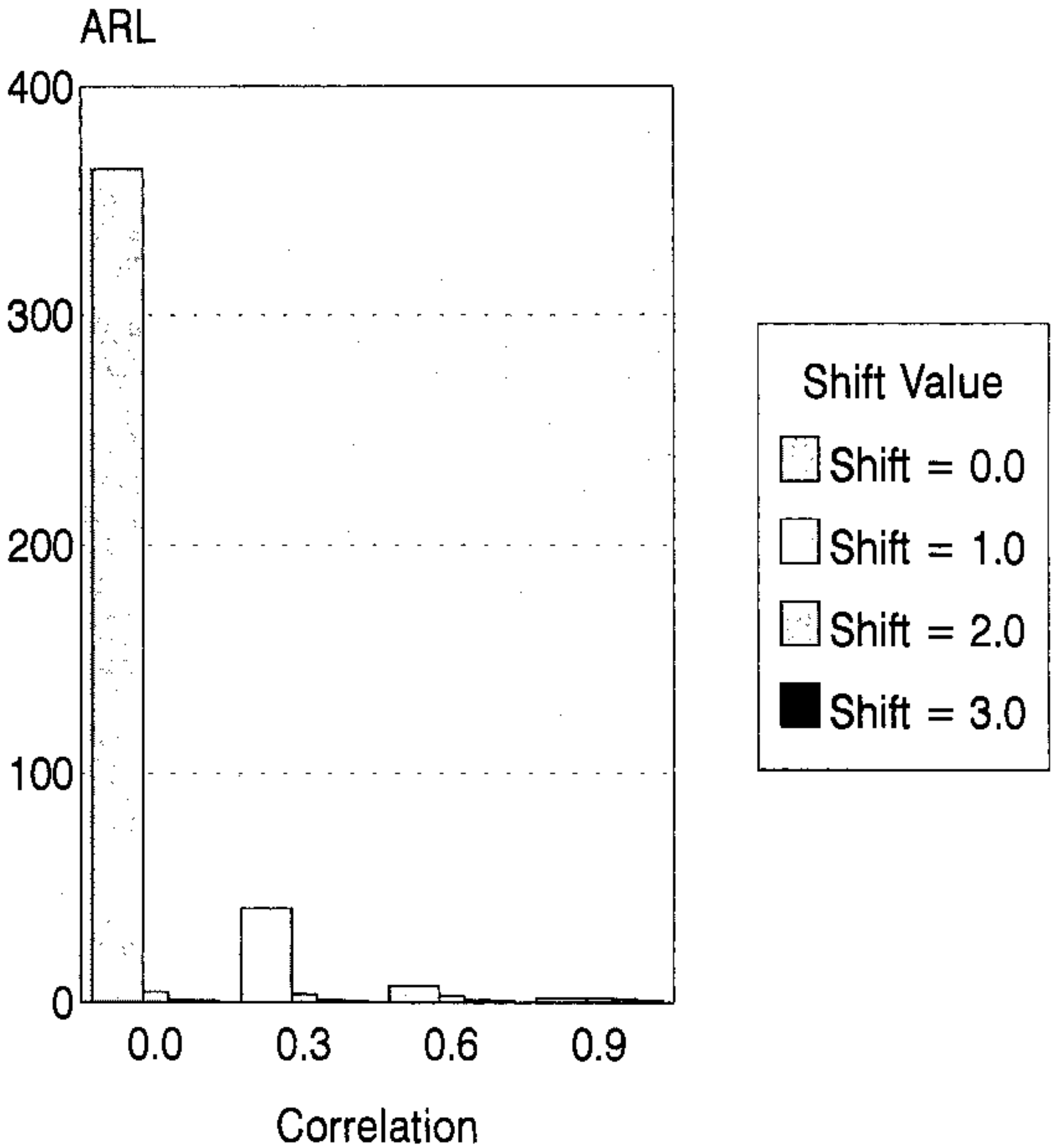


Figure 9c: ANSS with control limits pre-set
Sample size: n = 5 Interval length: I = 10
y axis : linear scale

ANSS for VSI
with Fixed Control Limits

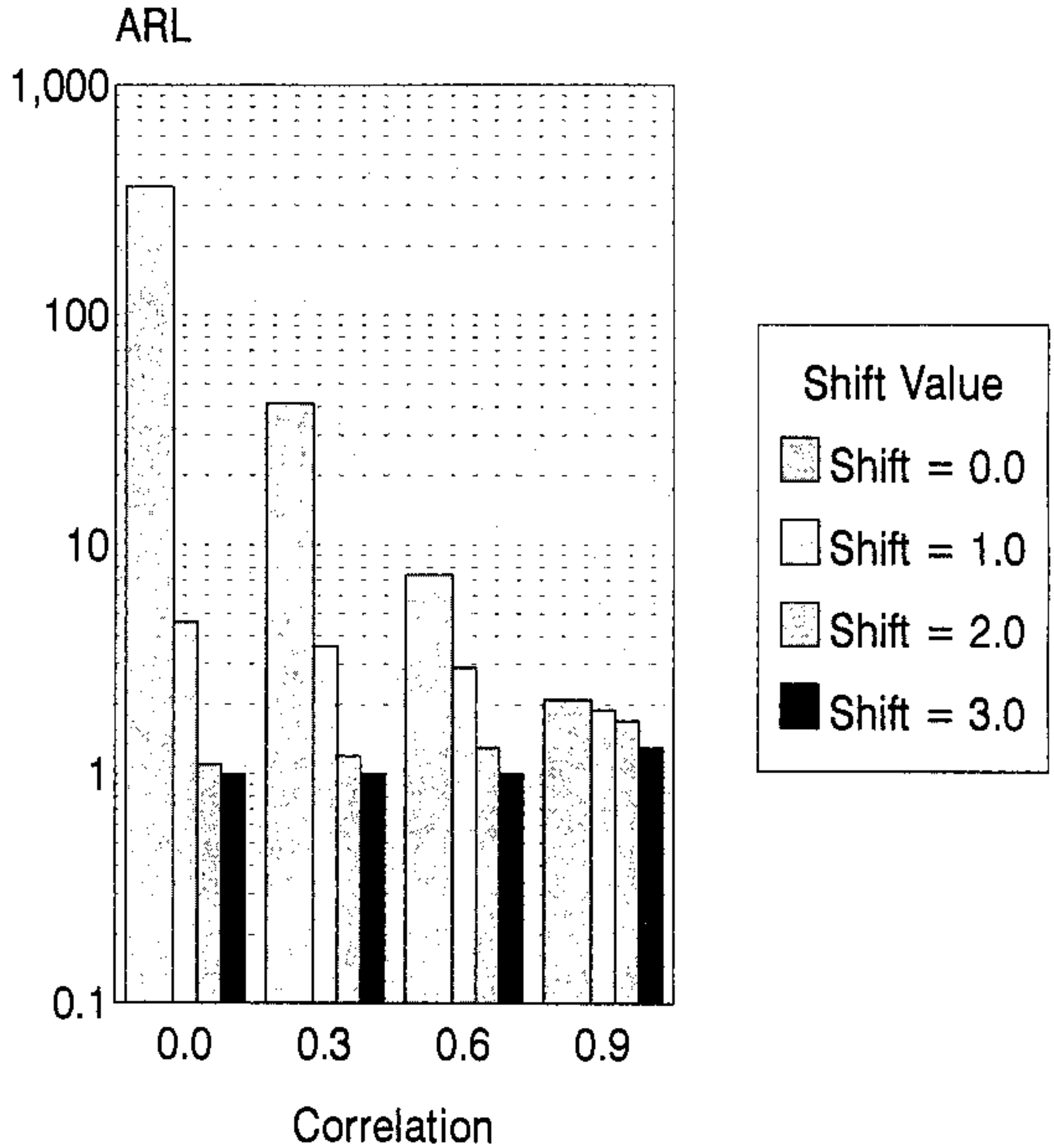


Figure 9d: ANSS with control limits pre-set
Sample size: n = 5 Interval length: l = 10
y axis : logarithmic scale

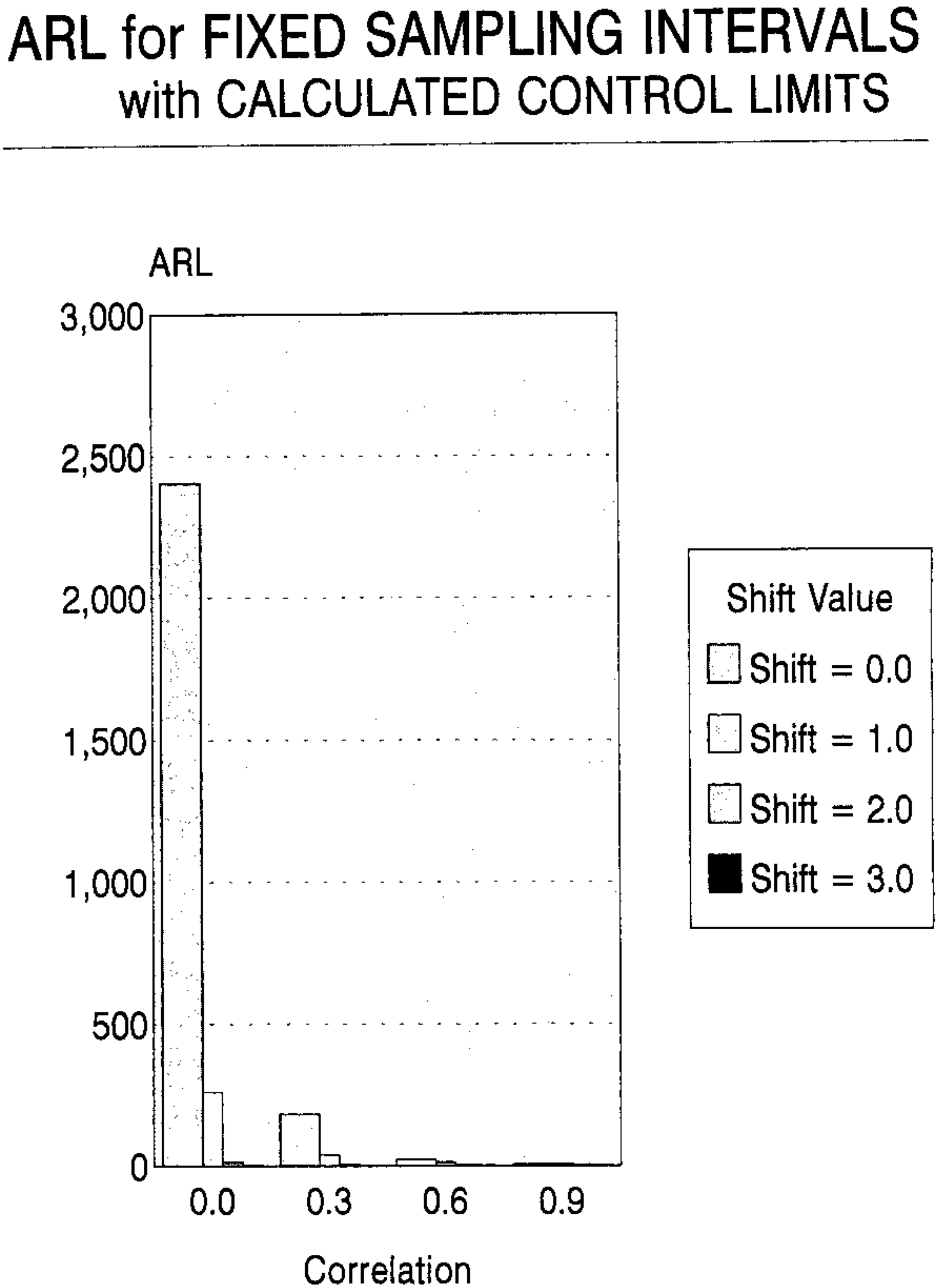


Figure 10a: A comparison of FSI with control limits calculated from sample data
Sample size: n = 1 Interval length: l = 1
y axis: linear scale

ARL for FIXED SAMPLING INTERVALS
with CALCULATED CONTROL LIMITS

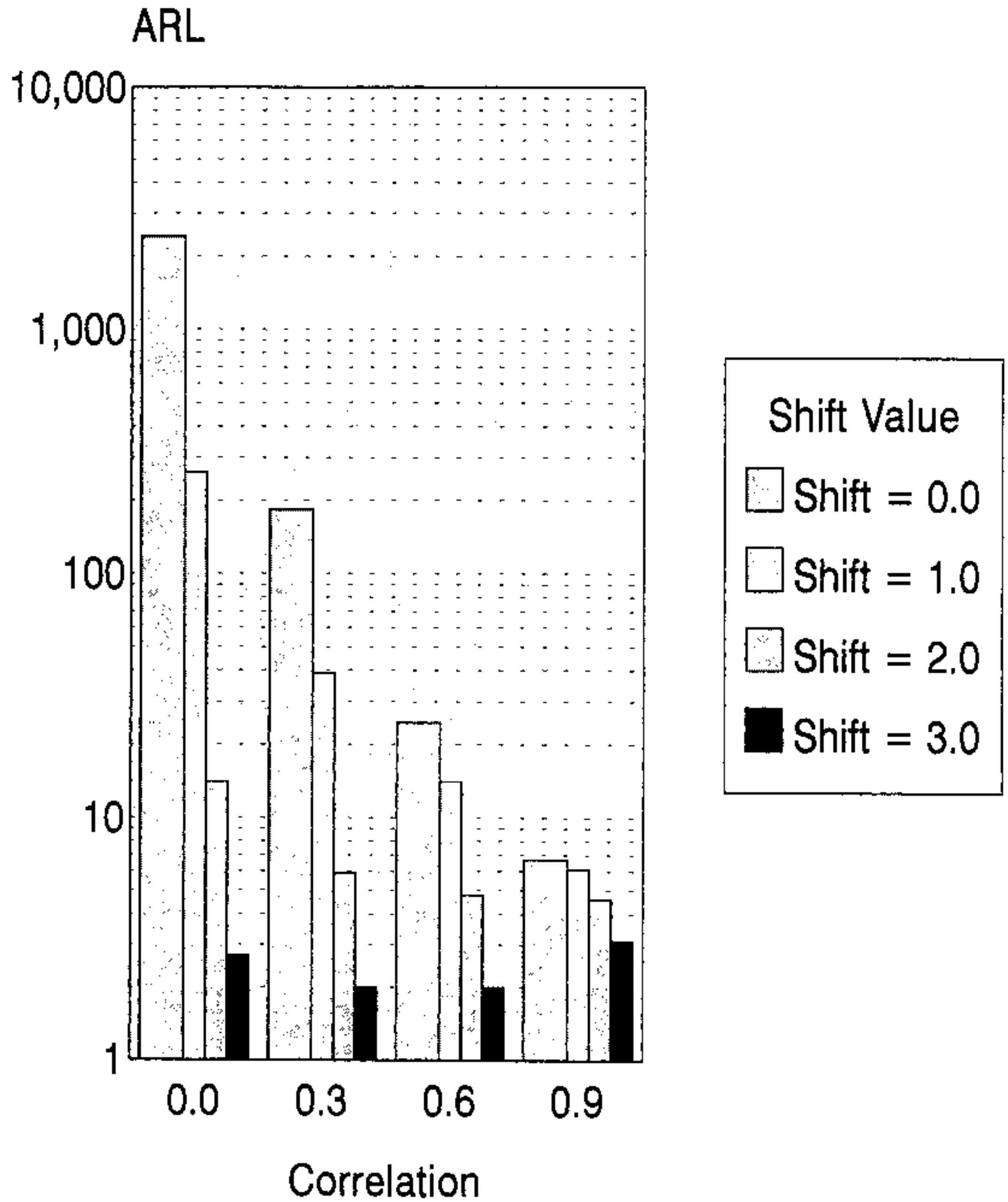


Figure 10b: A comparison of FSI with control limits calculated from sample data
Sample size: n = 1 Interval length: I = 1
y axis: logarithmic scale

ARL for FSI
with Calculated Control Limits

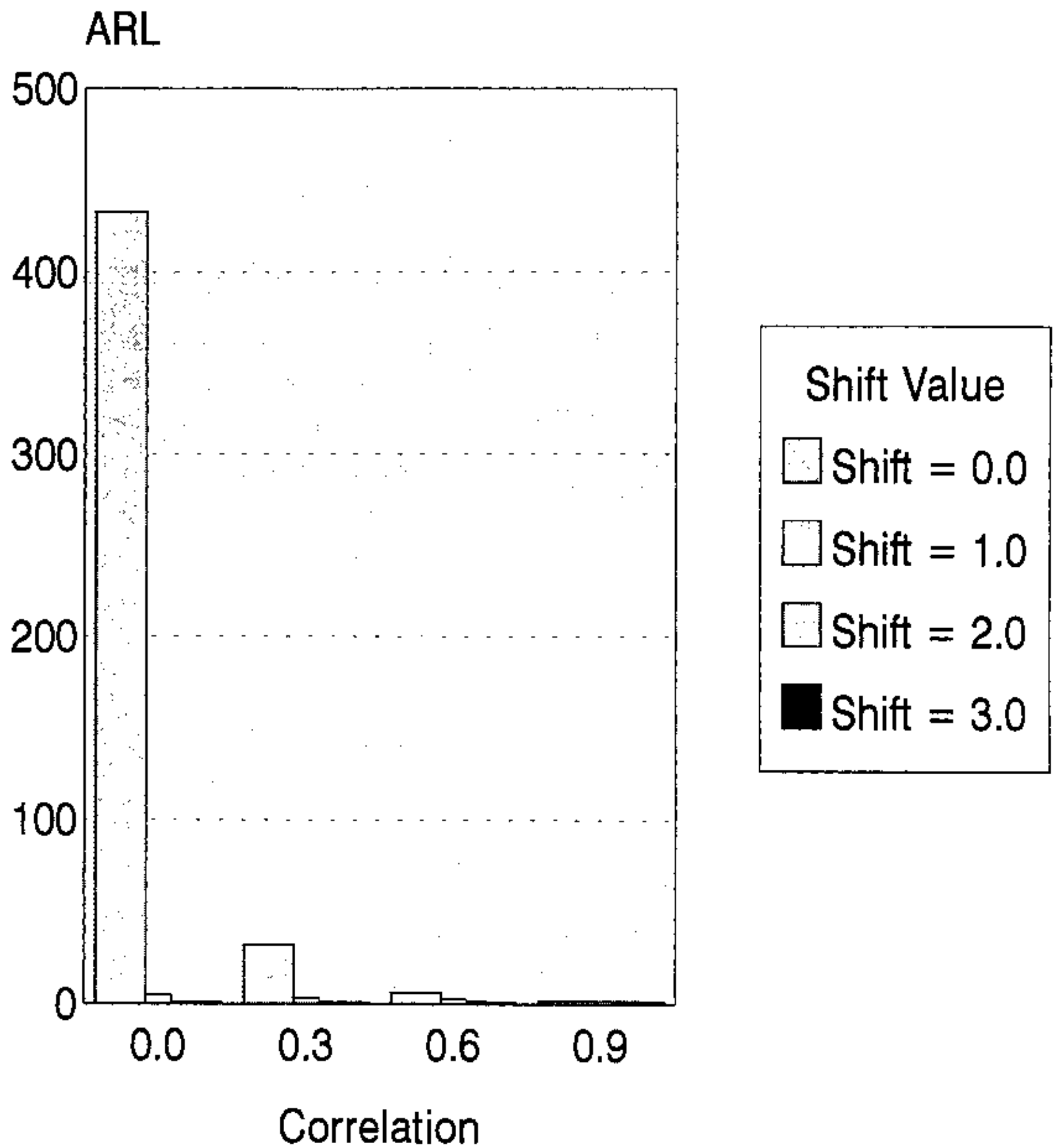


Figure 10c: ARLs with control limits calculated from sample data
Sample size: n = 5 Interval length: L = 1
y axis : linear scale

ARL for FSI
with Calculated Control Limits

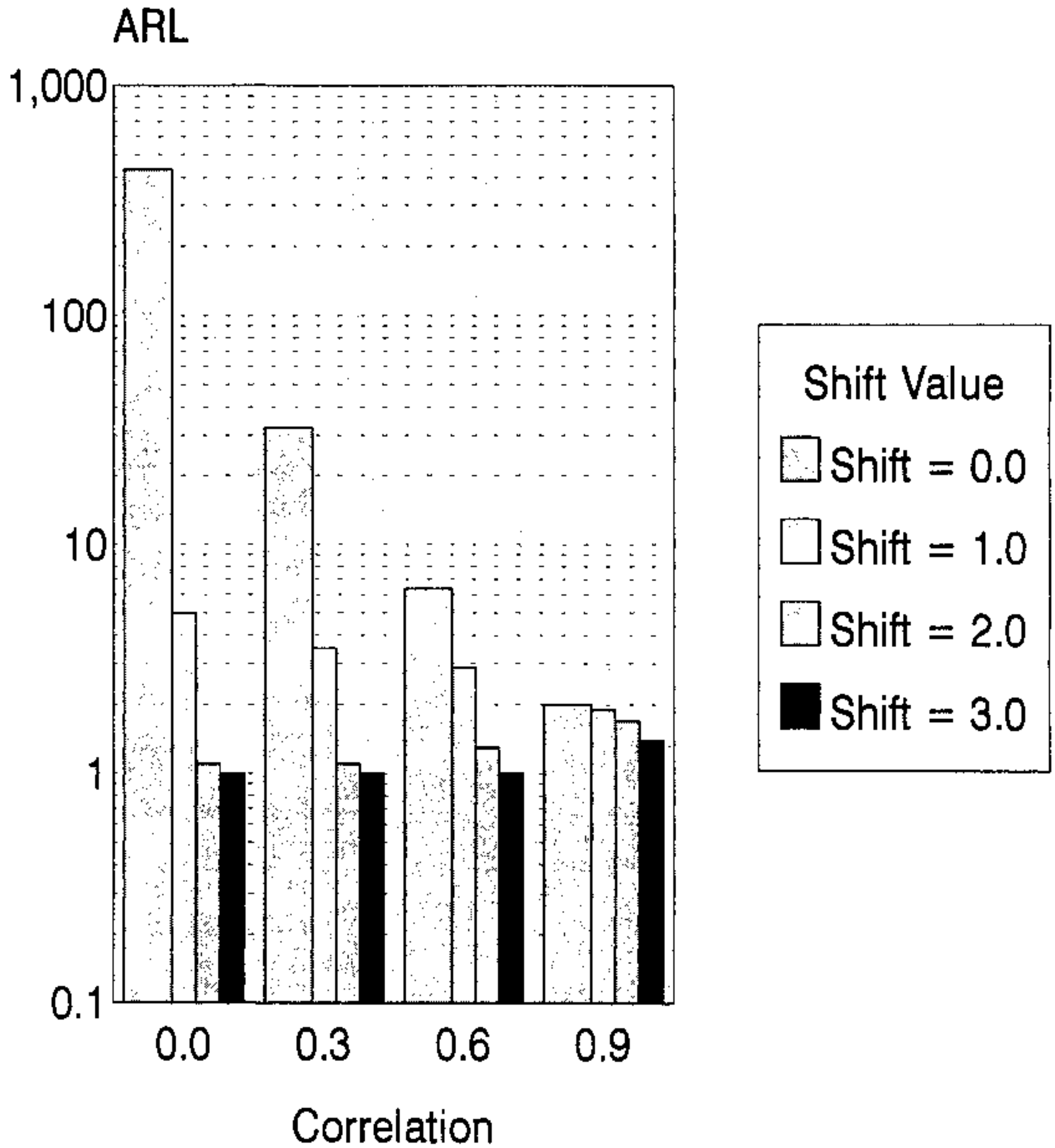


Figure 10d: ARLs with control limits calculated from sample data
Sample size: n = 5 Interval length: L = 1
y axis : logarithmic scale

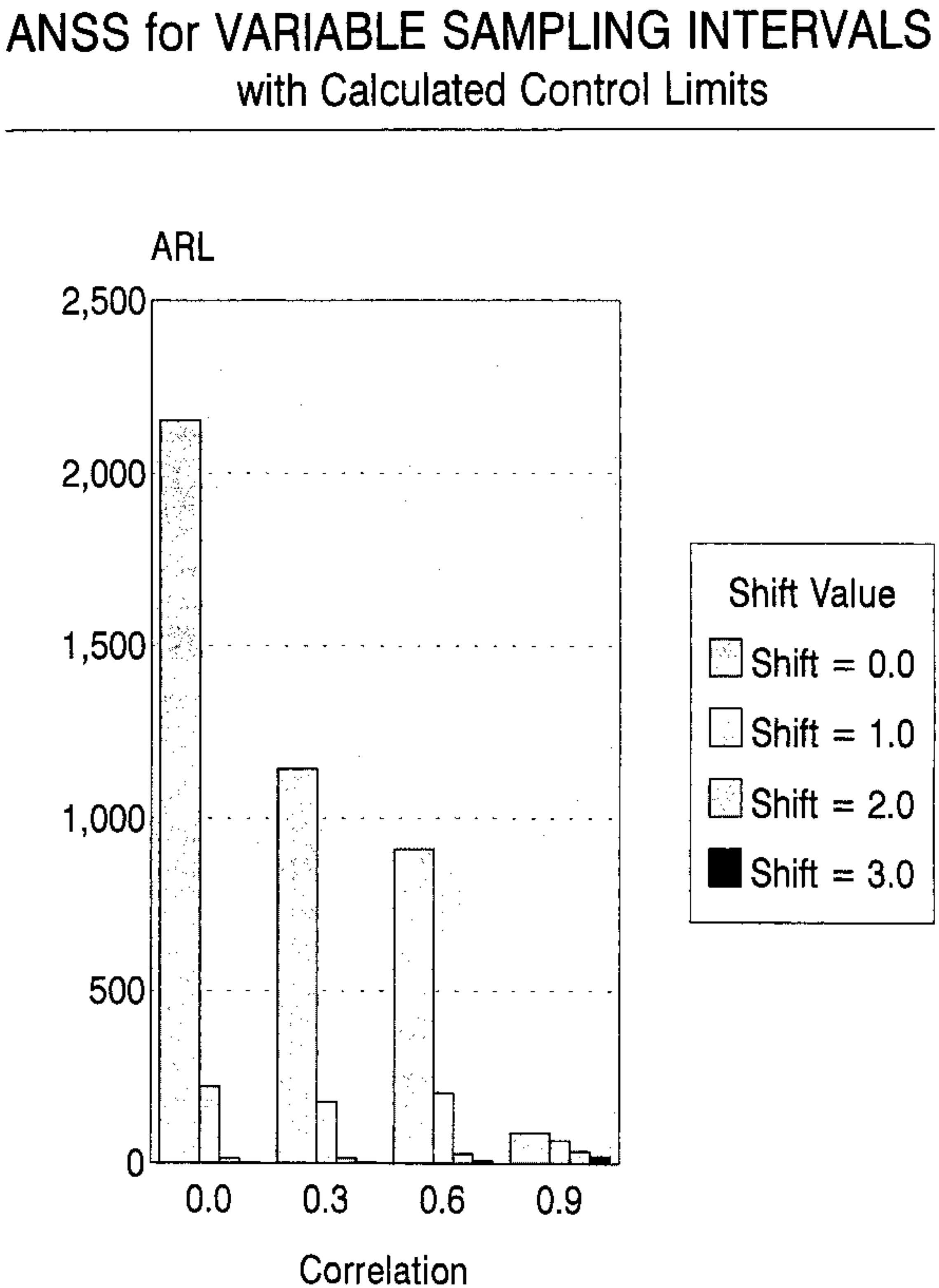


Figure 11a: A comparison of VSI with control limits calculated from sample data
Sample size: n = 1 Interval length: l = 10
y axis: linear scale

ANSS for VARIABLE SAMPLING INTERVALS
with Calculated Control Limits

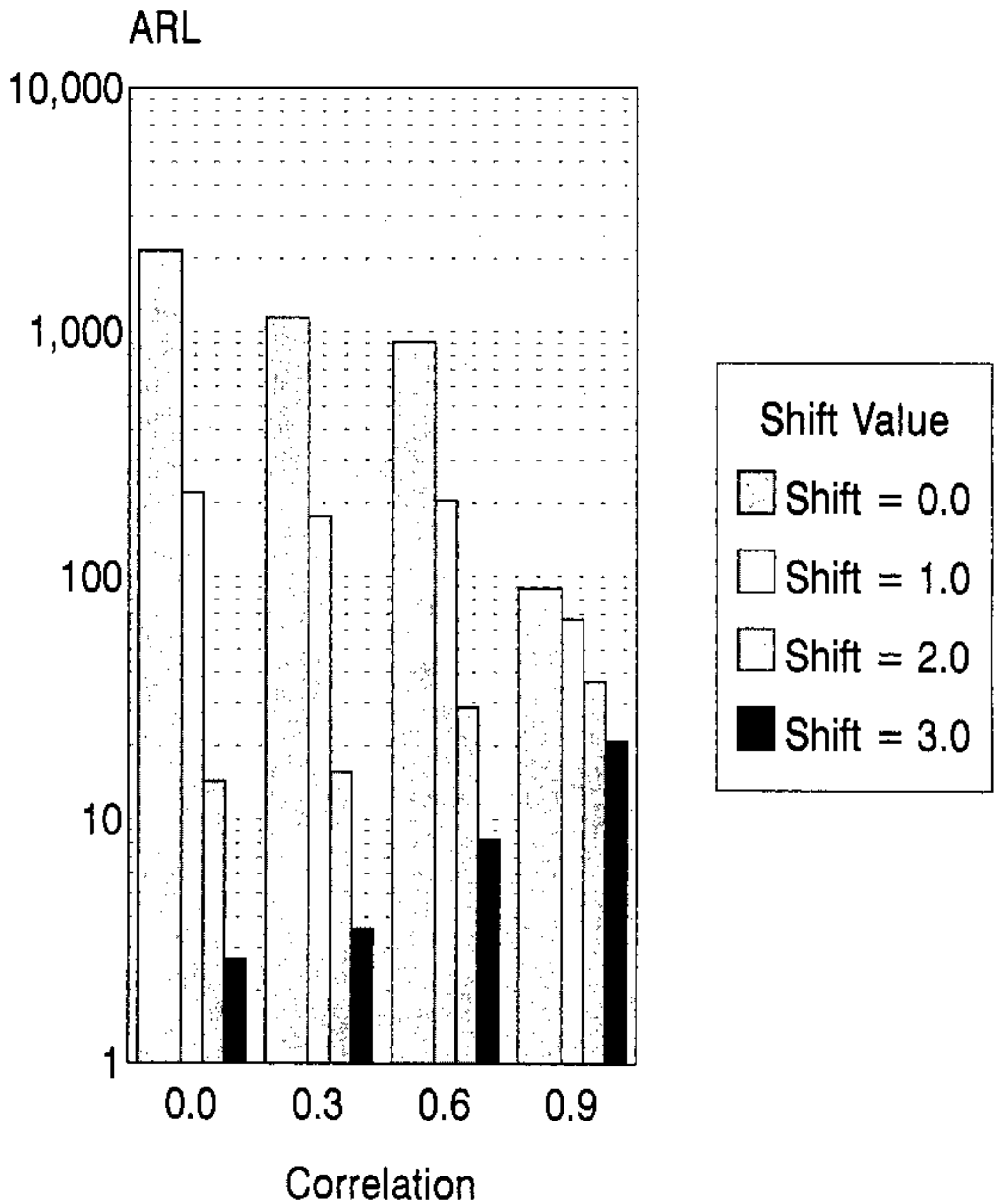


Figure 11b: A comparison of VSI with control limits calculated from sample data
Sample size: n = 1 Interval length: I = 10
y axis: logarithmic scale

ANSS for VARIABLE SAMPLING INTERVALS
with Calculated Control Limits

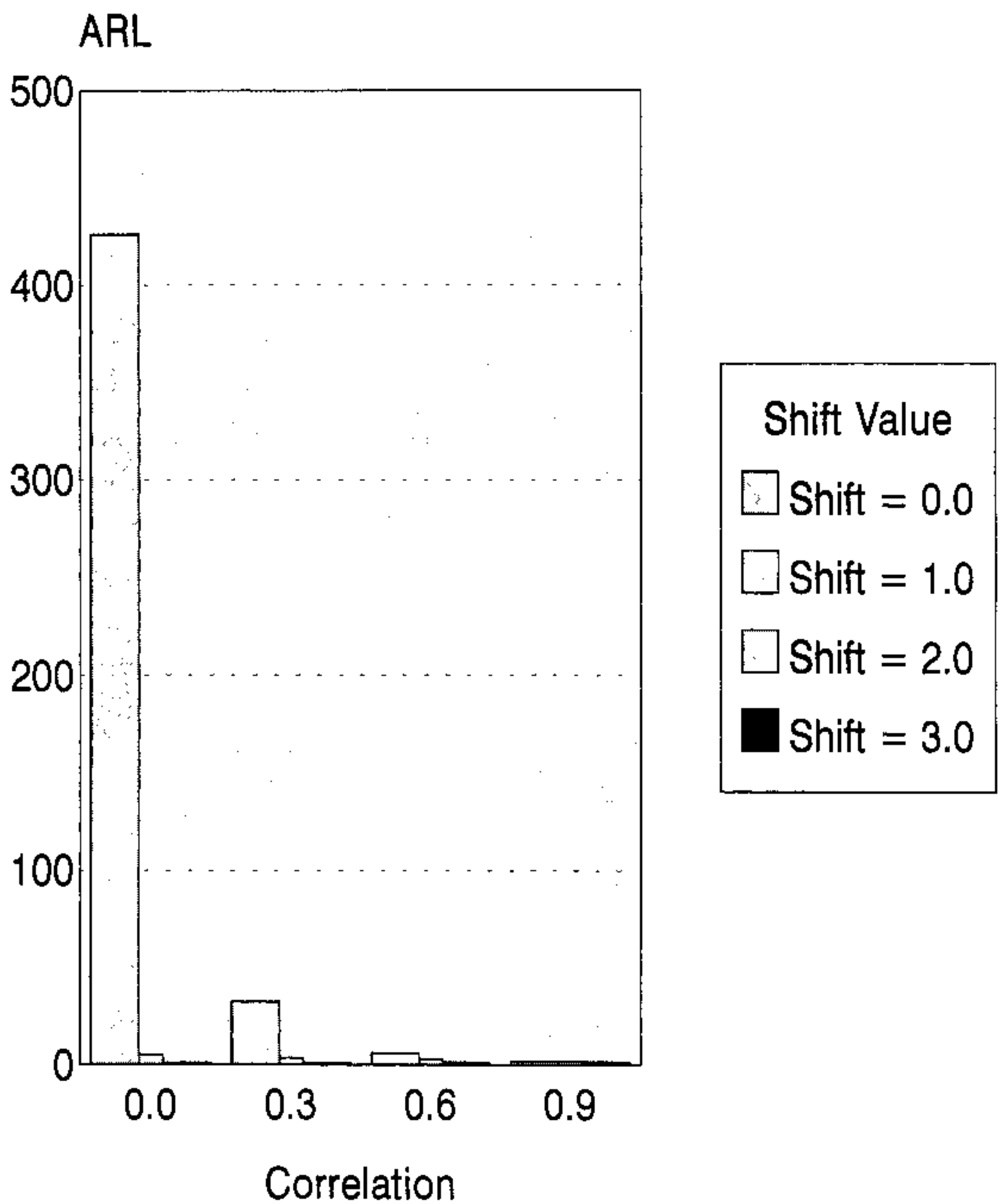


Figure 11c: A comparison of VSI with control limits calculated from sample data
Sample size: n = 5 Interval length: I = 10
y axis: linear scale

ANSS for VARIABLE SAMPLING INTERVALS
with Calculated Control Limits

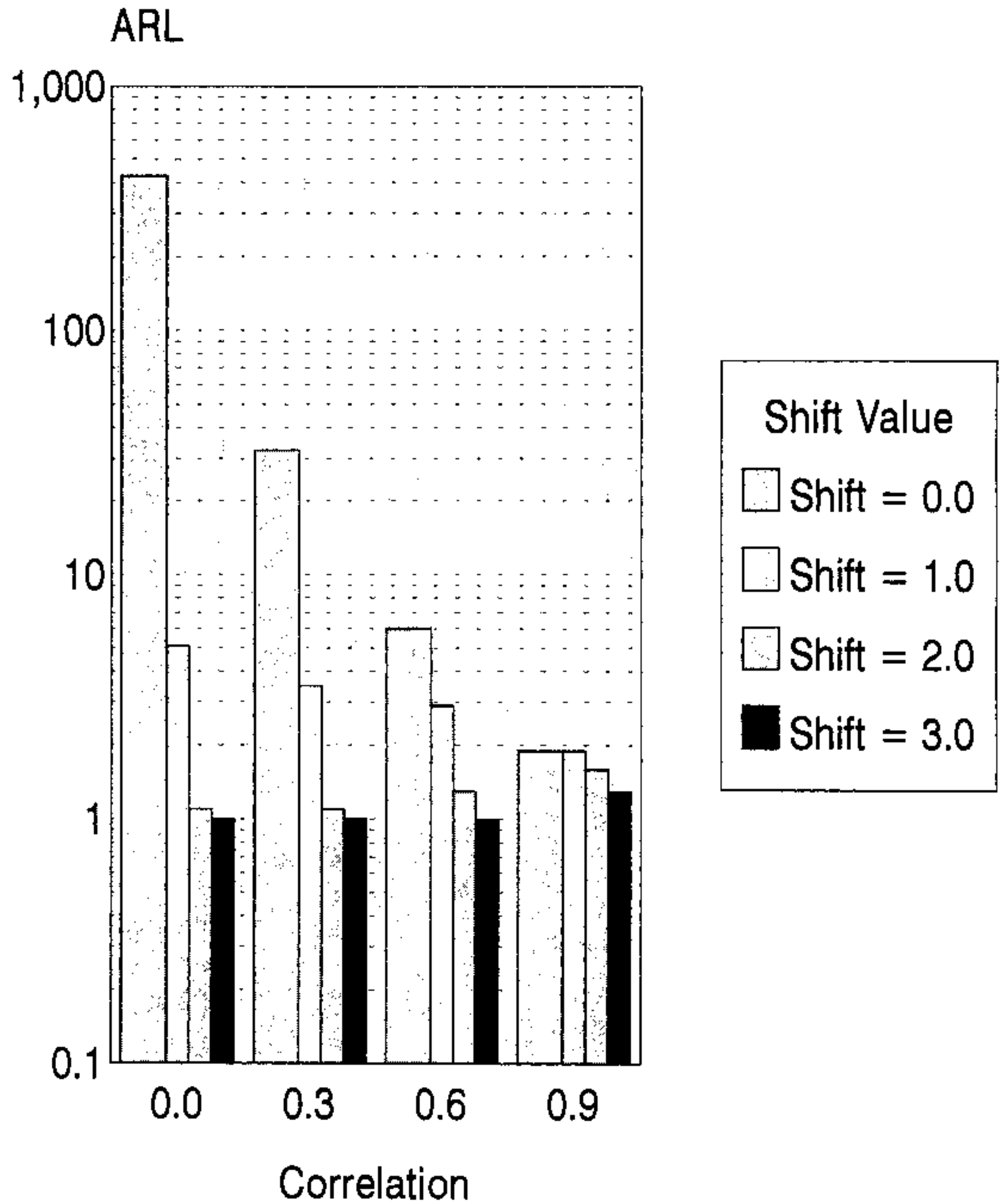


Figure 11d: A comparison of VSI with control limits calculated from sample data
Sample size: n = 5 Interval length: l = 10
y axis: logarithmic scale

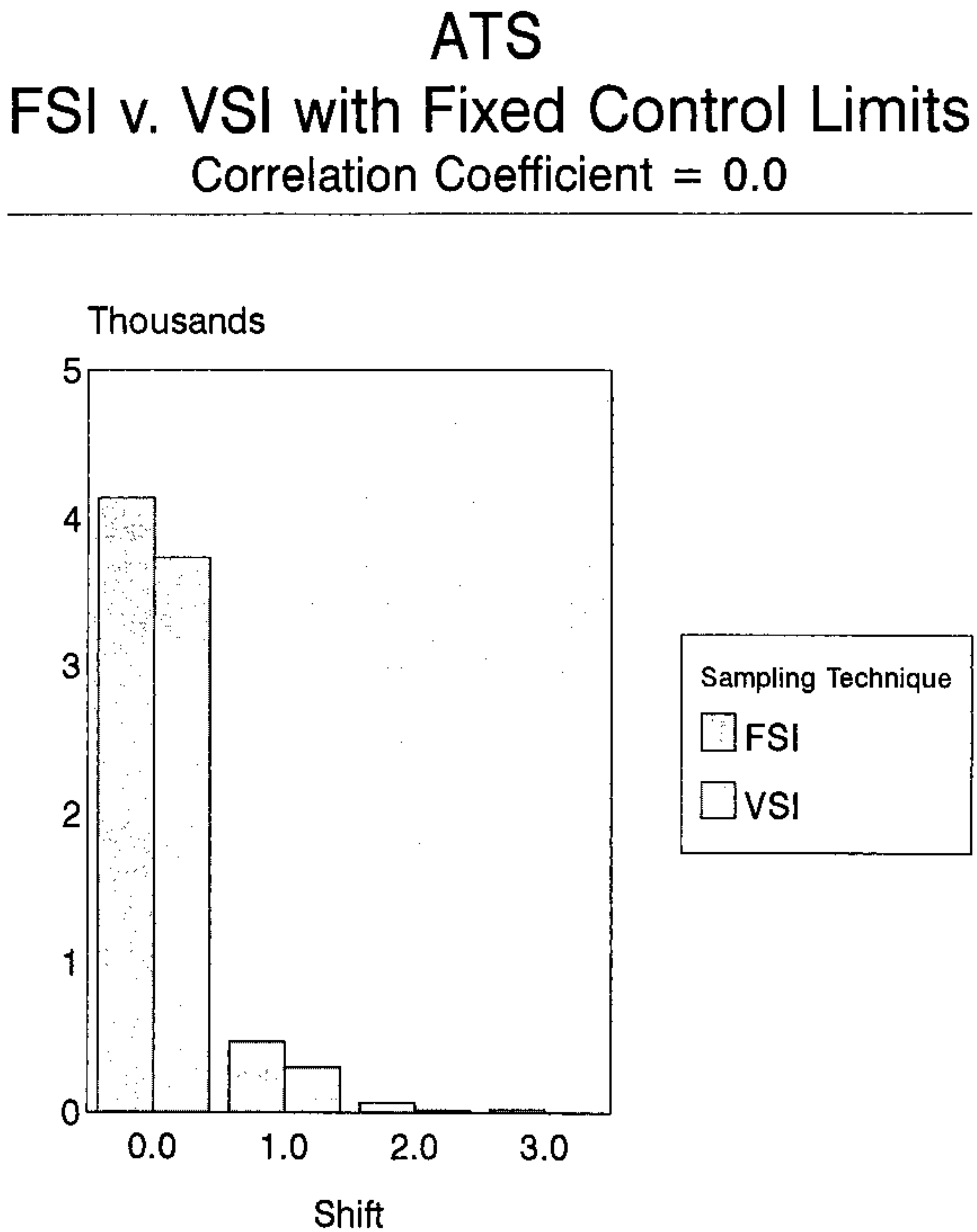


Figure 12a: A comparison of FSI v. VSI with pre-set control limits
Sample size: n = 1 Interval length: I = 10
y axis: linear scale

ATS
FSI v. VSI with Fixed Control Limits
Correlation Coefficient = 0.0

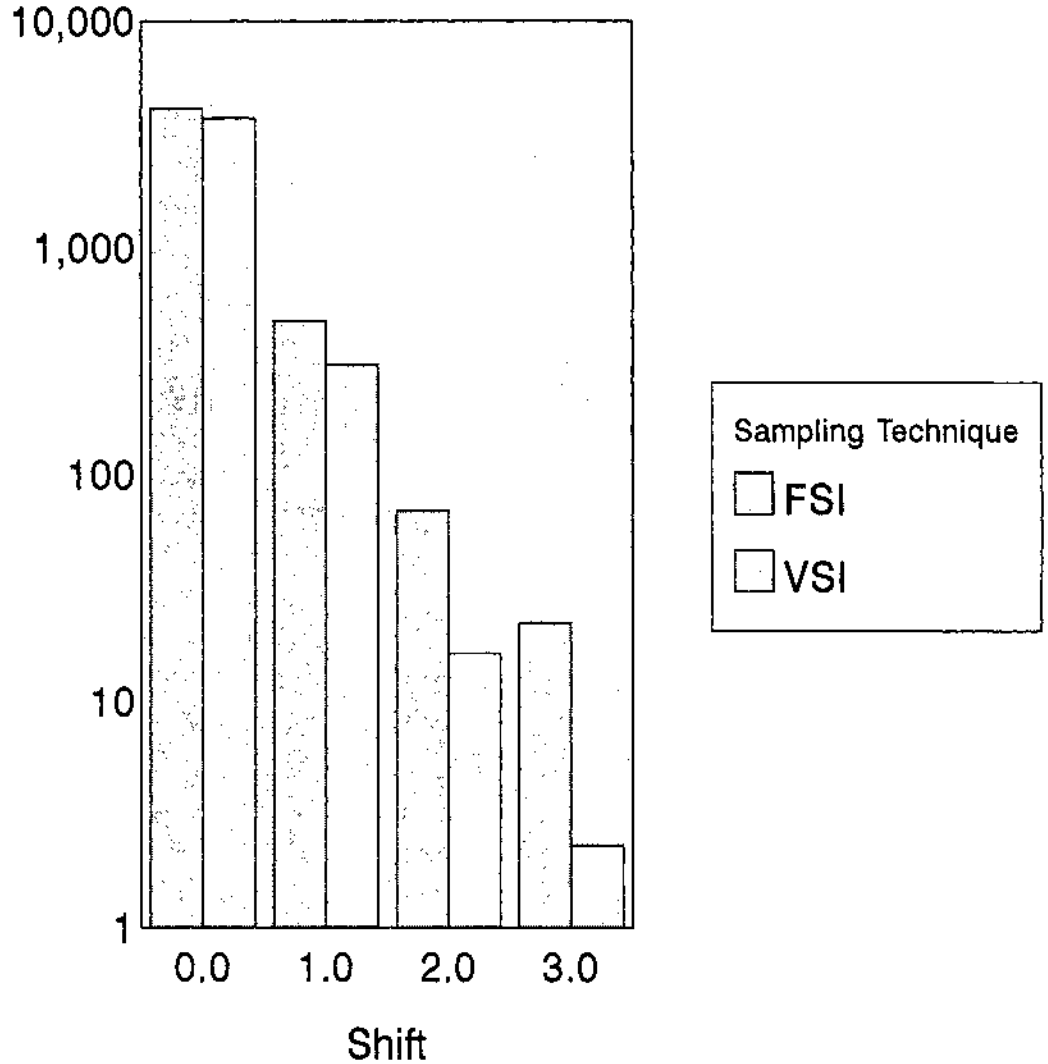


Figure 12b: A comparison of FSI v. VSI with pre-set control limits
Sample size: n = 1 Interval length: l = 10
y axis: logarithmic scale

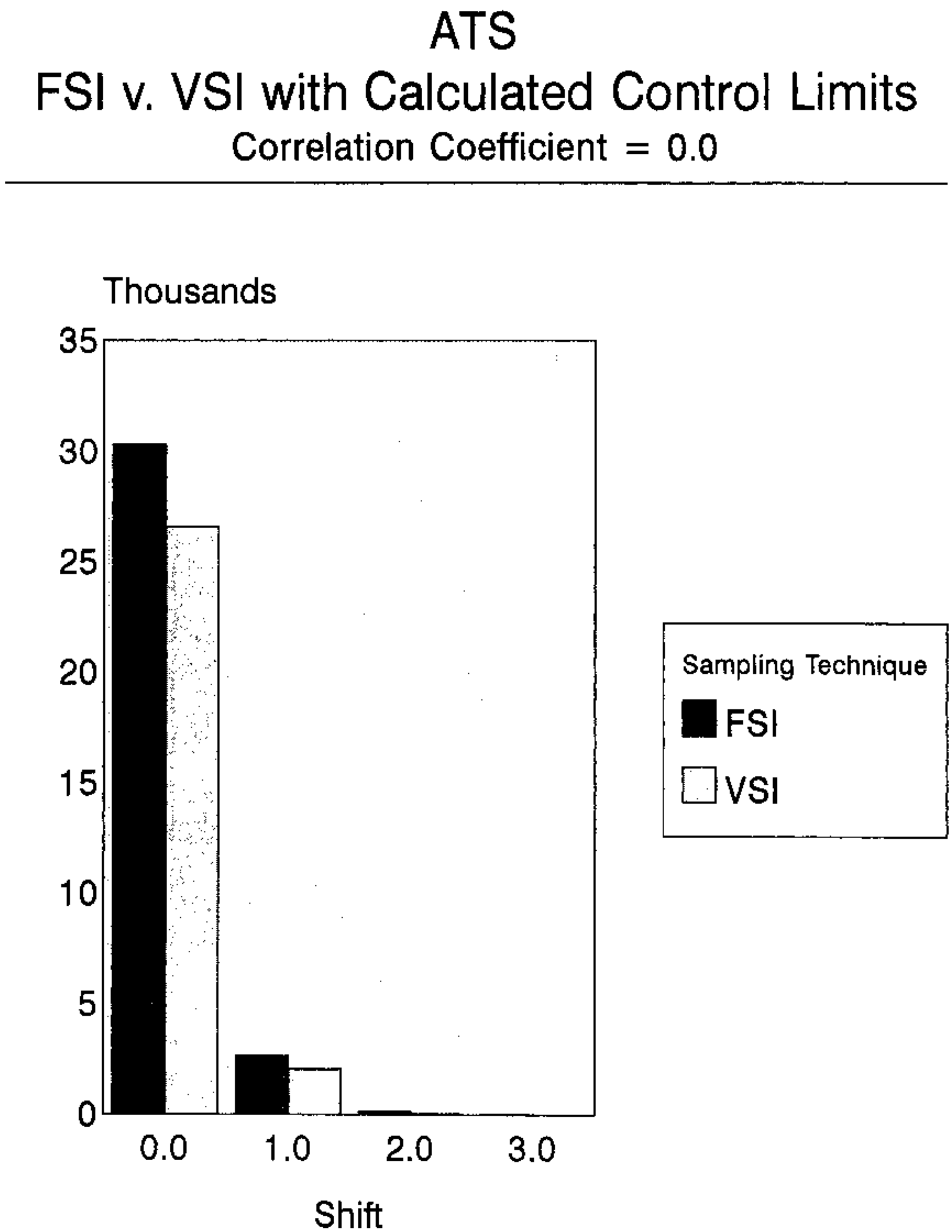


Figure 12c: A comparison of FSI v. VSI with control limits calculated from sample data
Sample size: n = 1 Interval length: I = 10
y axis: linear scale

ATS
FSI v. VSI with Calculated Control Limits
Correlation Coefficient = 0.0

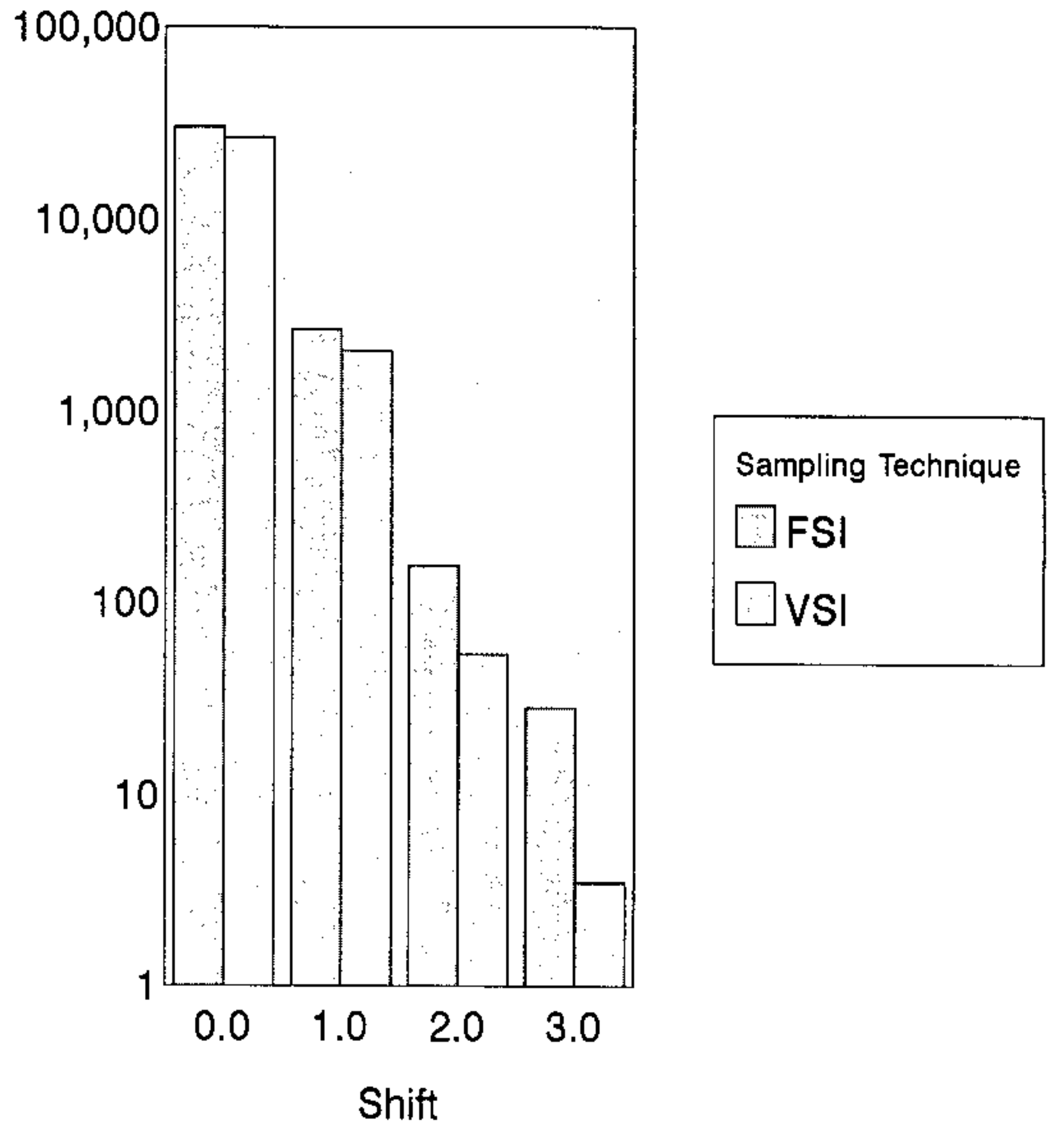


Figure 12d: A comparison of FSI v. VSI with control limits calculated from sample data
Sample size: n = 1 Interval length: I = 10
y axis: logarithmic scale

ATS

FSI v. VSI with Fixed Control Limits

Correlation Coefficient = 0.3

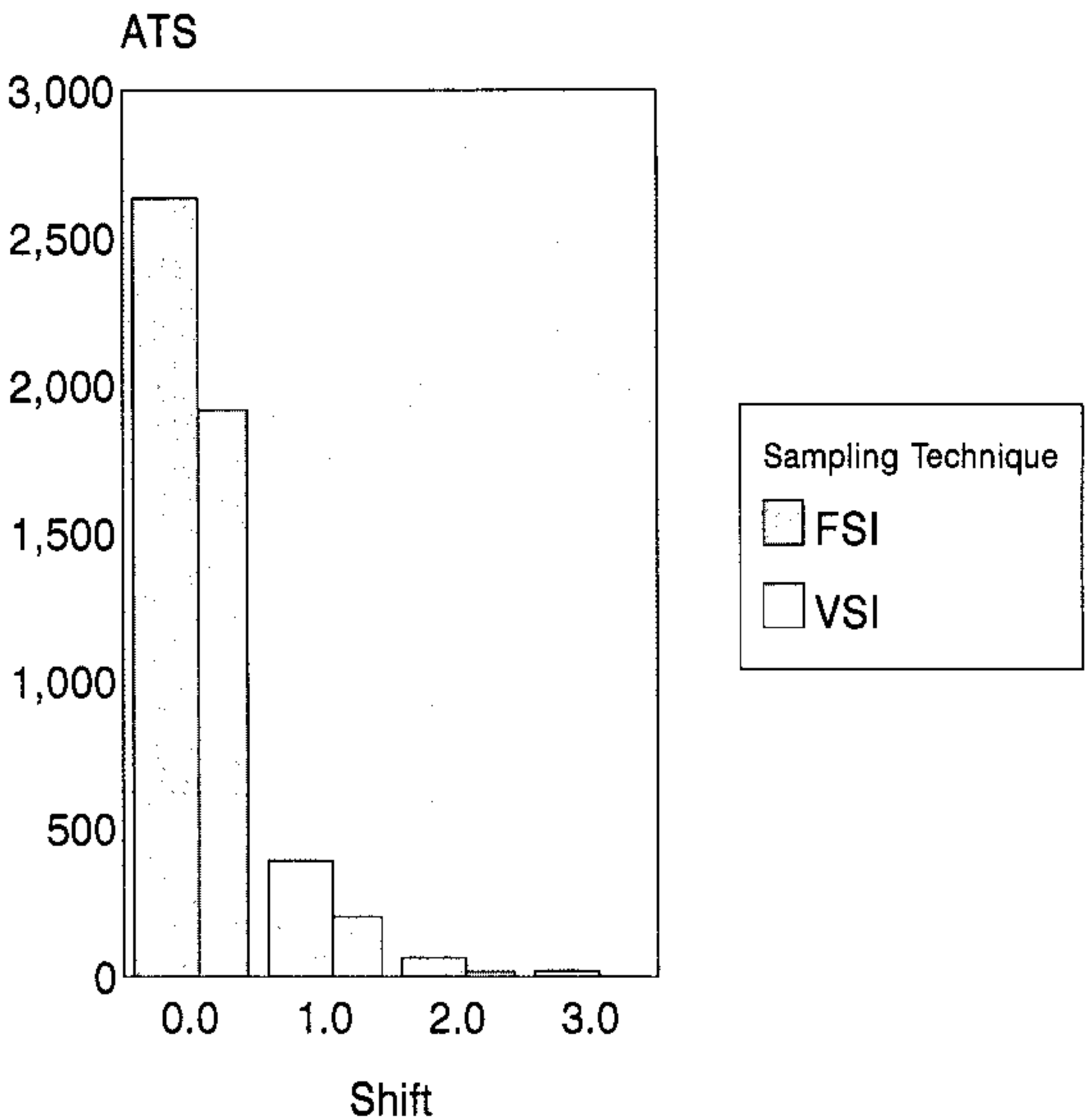


Figure 13a: A comparison of ATS for FSI v. VSI with pre-set control limits
Sample size: n = 1 Interval length: I = 10
y axis: linear scale

ATS
FSI v. VSI with Fixed Control Limits
Correlation Coefficient = 0.3

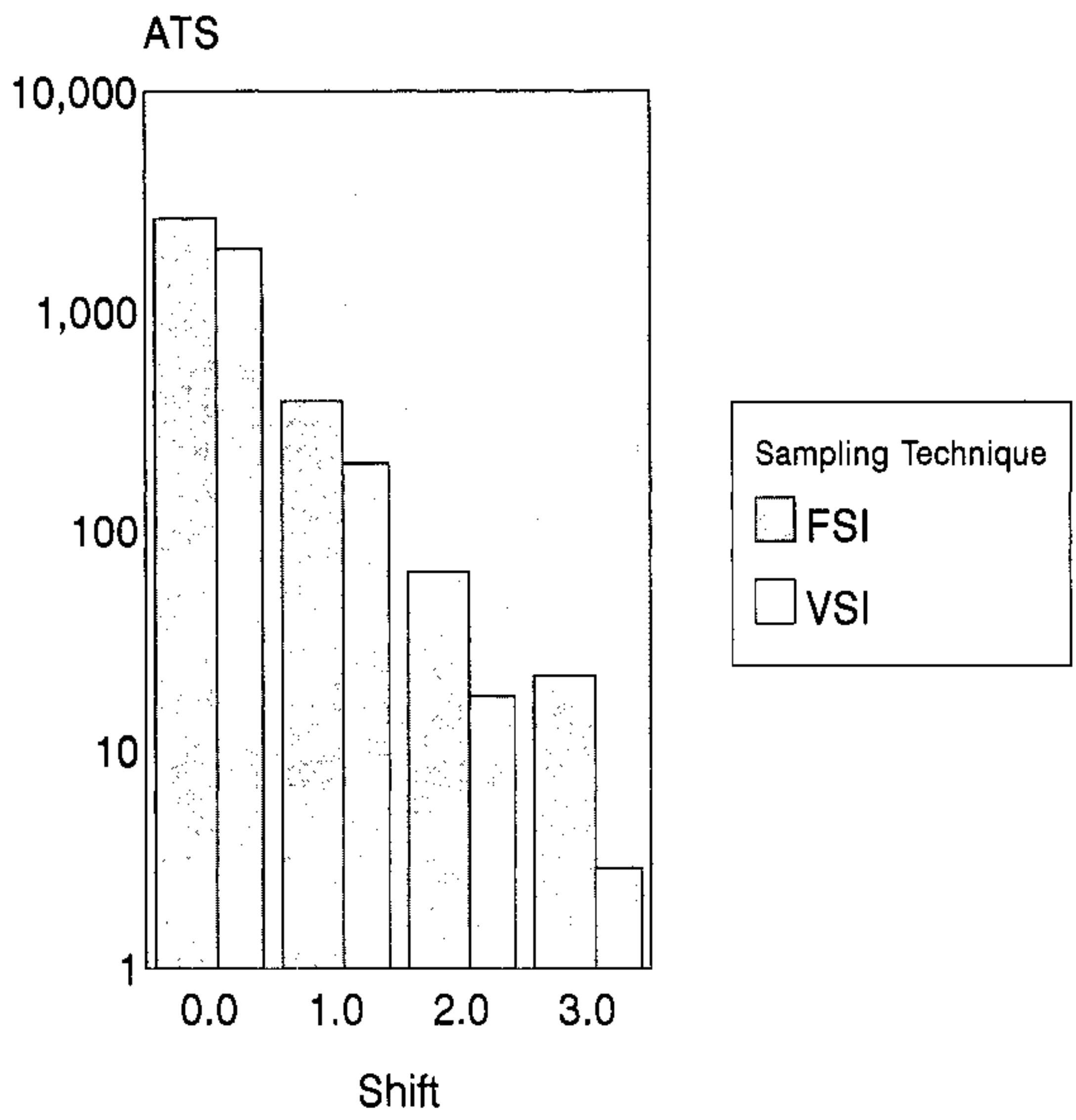


Figure 13b: A comparison of ATS for FSI v. VSI with pre-set control limits
Sample size: n = 1 Interval length: l = 10
y axis: logarithmic scale

ATS
FSI v. VSI with Calculated Control Limits
Correlation Coefficient = 0.3

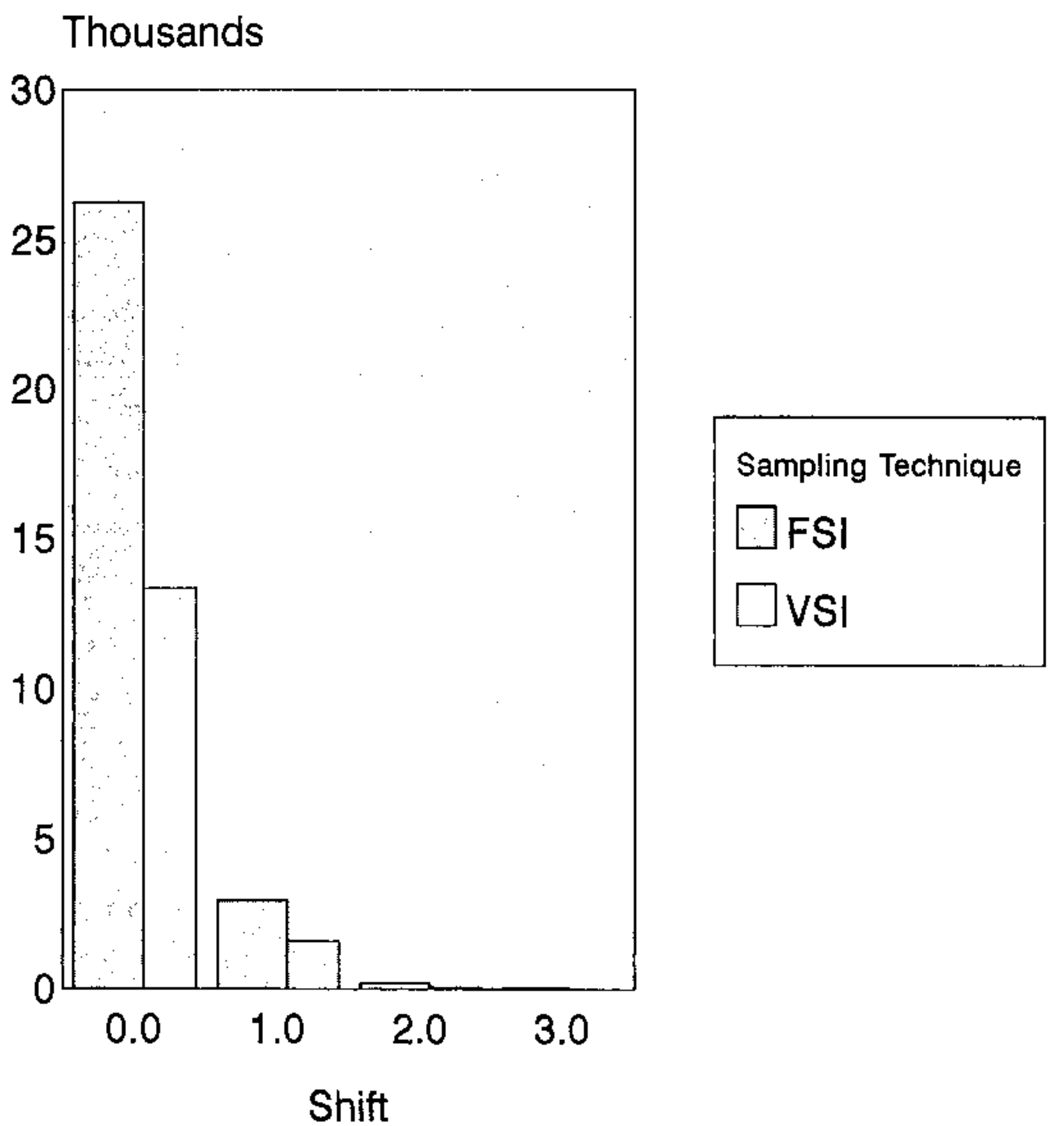


Figure 13c: A comparison of FSI v. VSI with control limits calculated from sample data
Sample size: n = 1 Interval length: I = 10
y axis: linear scale

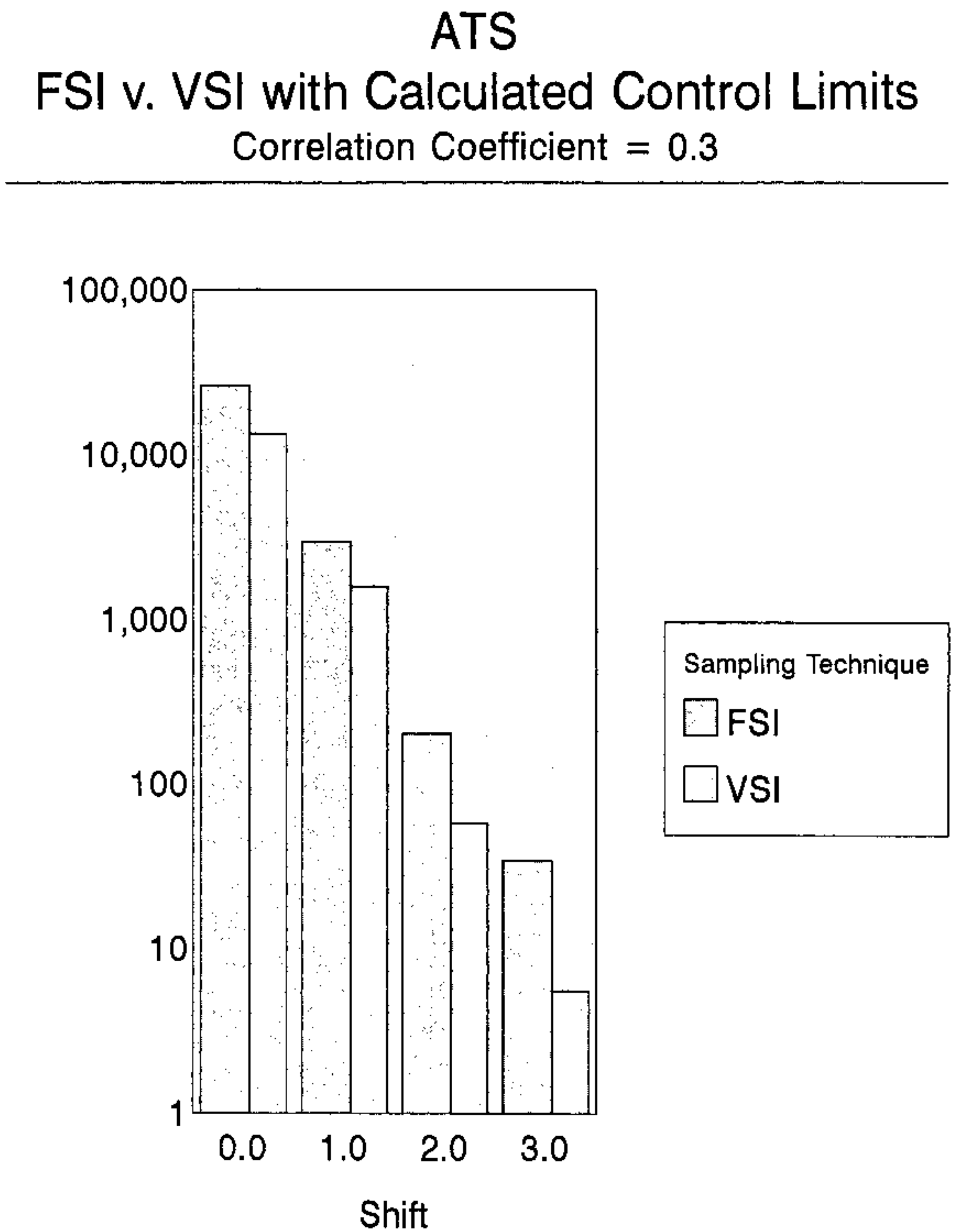


Figure 13d: A comparison of FSI v. VSI with control limits calculated from sample data
Sample size: n = 1 Interval length: I = 10
y axis: logarithmic scale

ATS
FSI v. VSI with Fixed Control Limits
Correlation Coefficient = 0.6

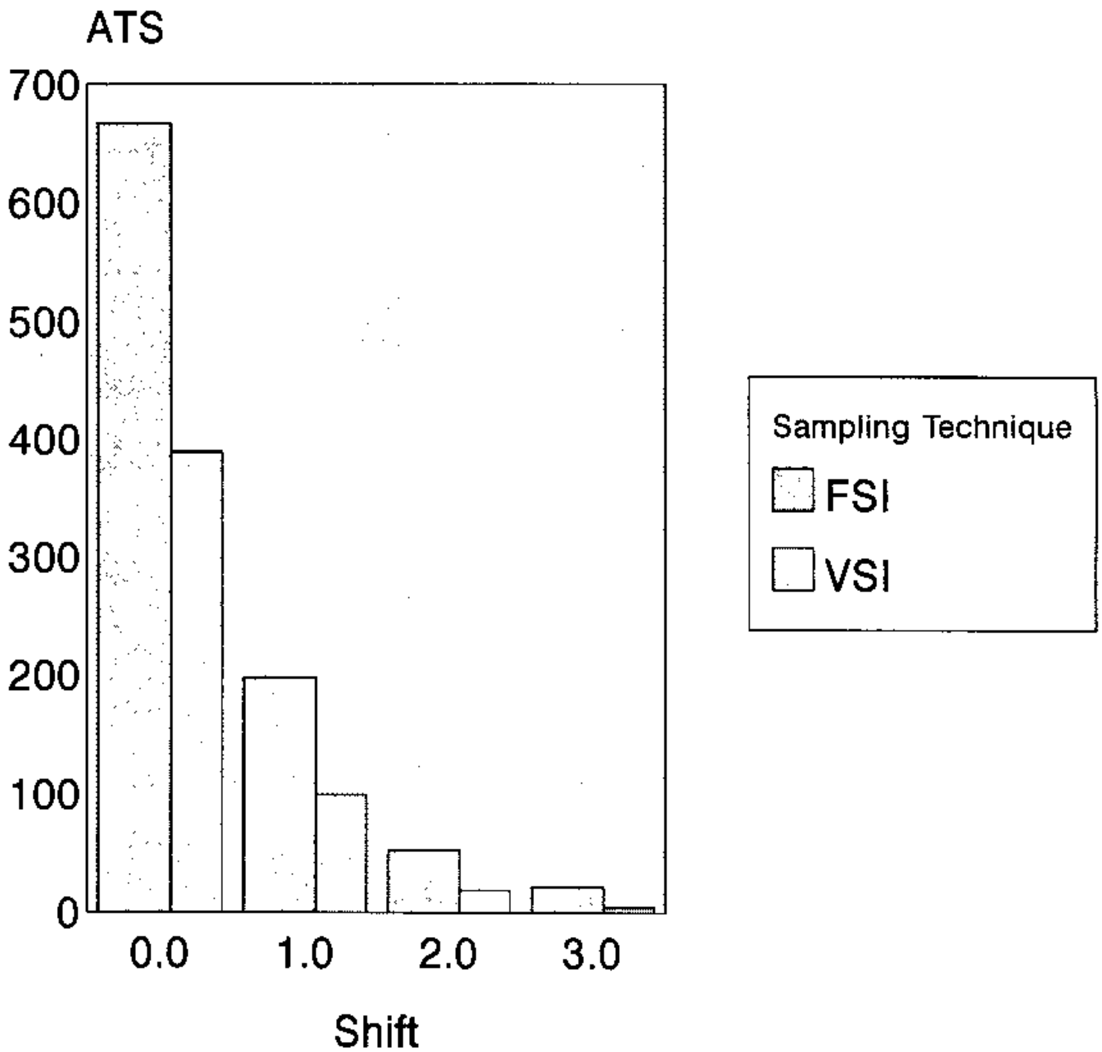


Figure 14a: ATS for FSI v. VSI with pre-set control limits
Sample size: n = 1 Interval length: l = 10
y axis: linear scale

ATS
FSI v. VSI with Fixed Control Limits
Correlation Coefficient = 0.6

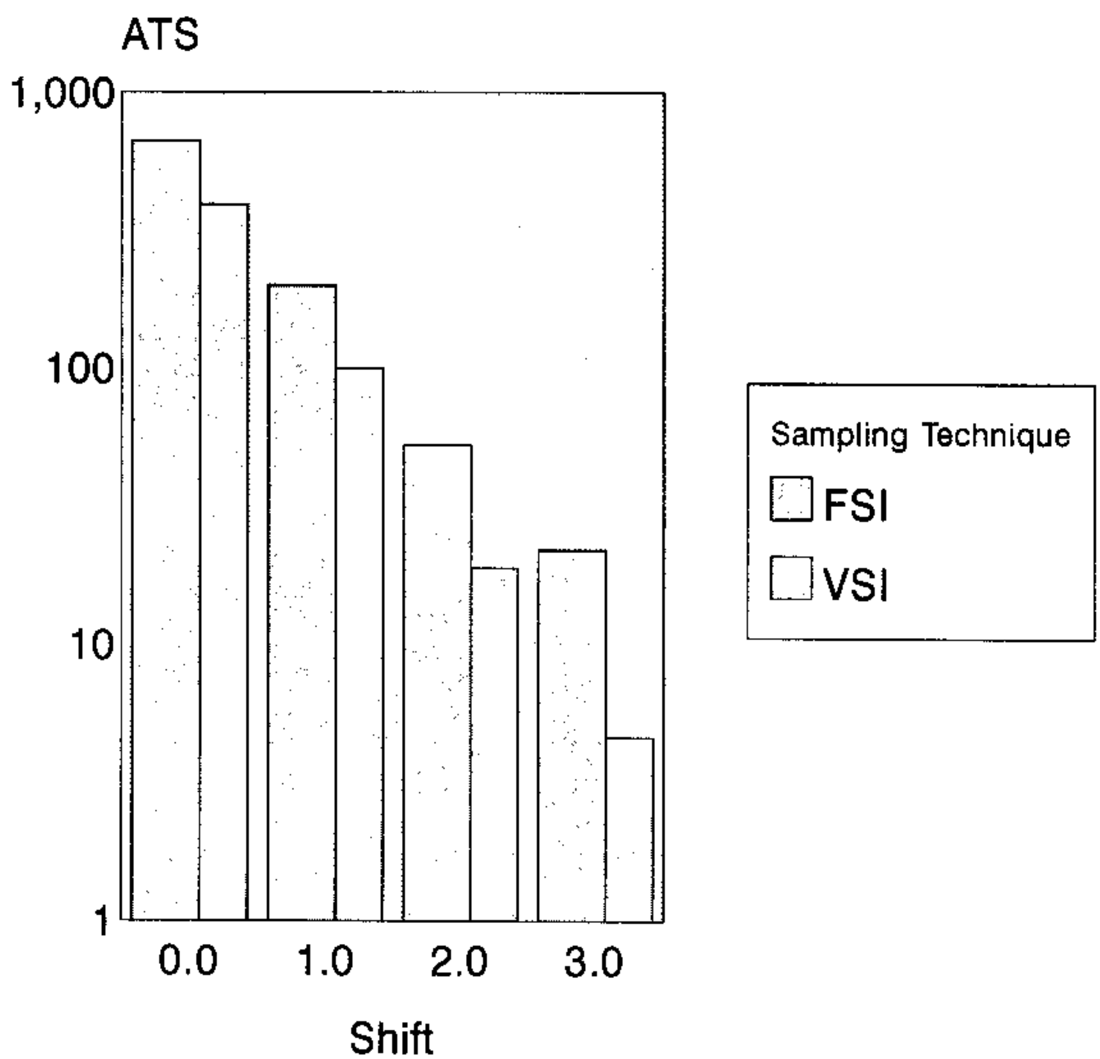


Figure 14b: ATS for FSI v. VSI with pre-set control limits
Sample size: n = 1   Interval length: I = 10
y axis: logarithmic scale

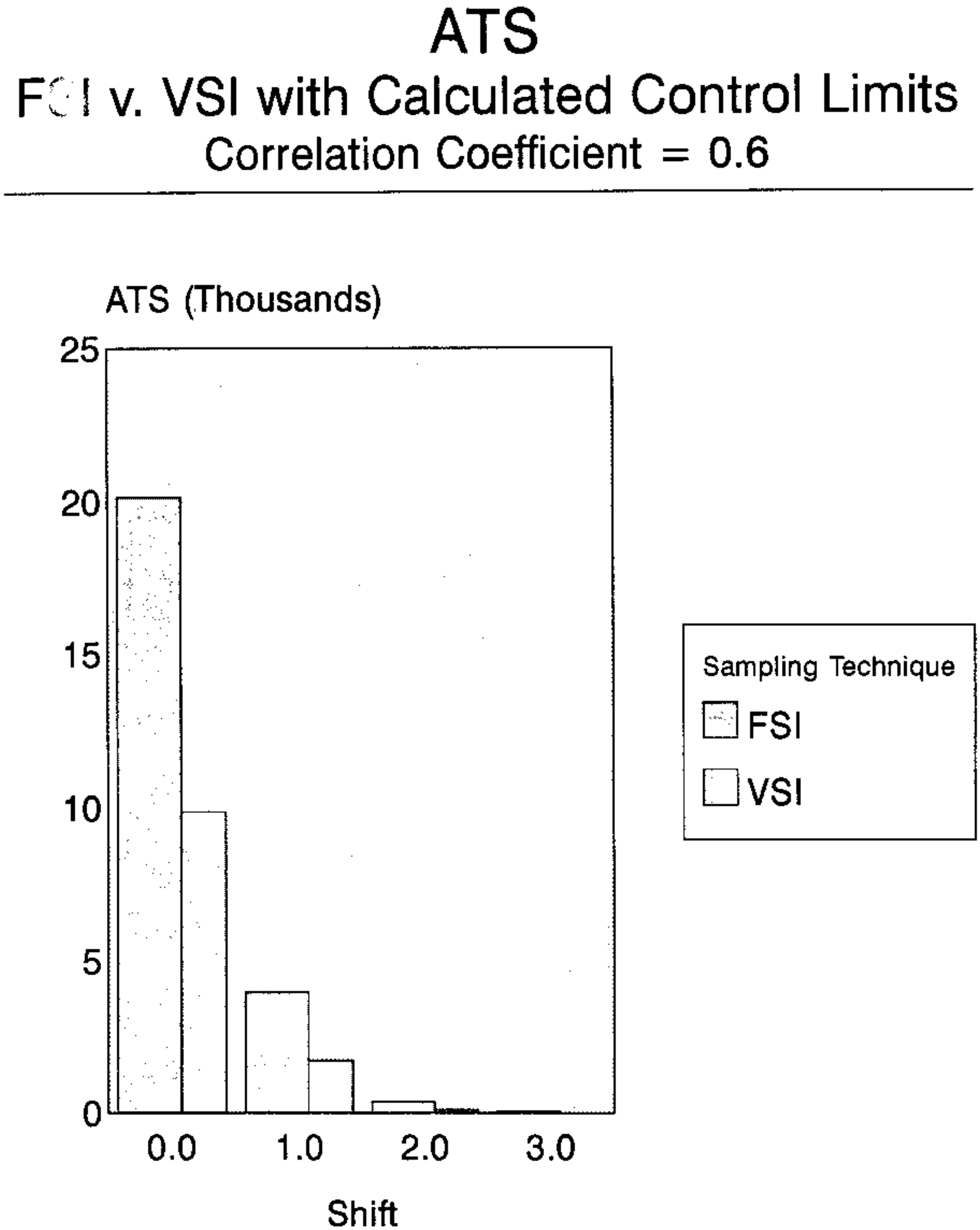


Figure 14c: ATS for FSI v. VSI with control limits calculated from sample data
Sample size: n = 1 Interval length: l = 10
y axis: linear scale

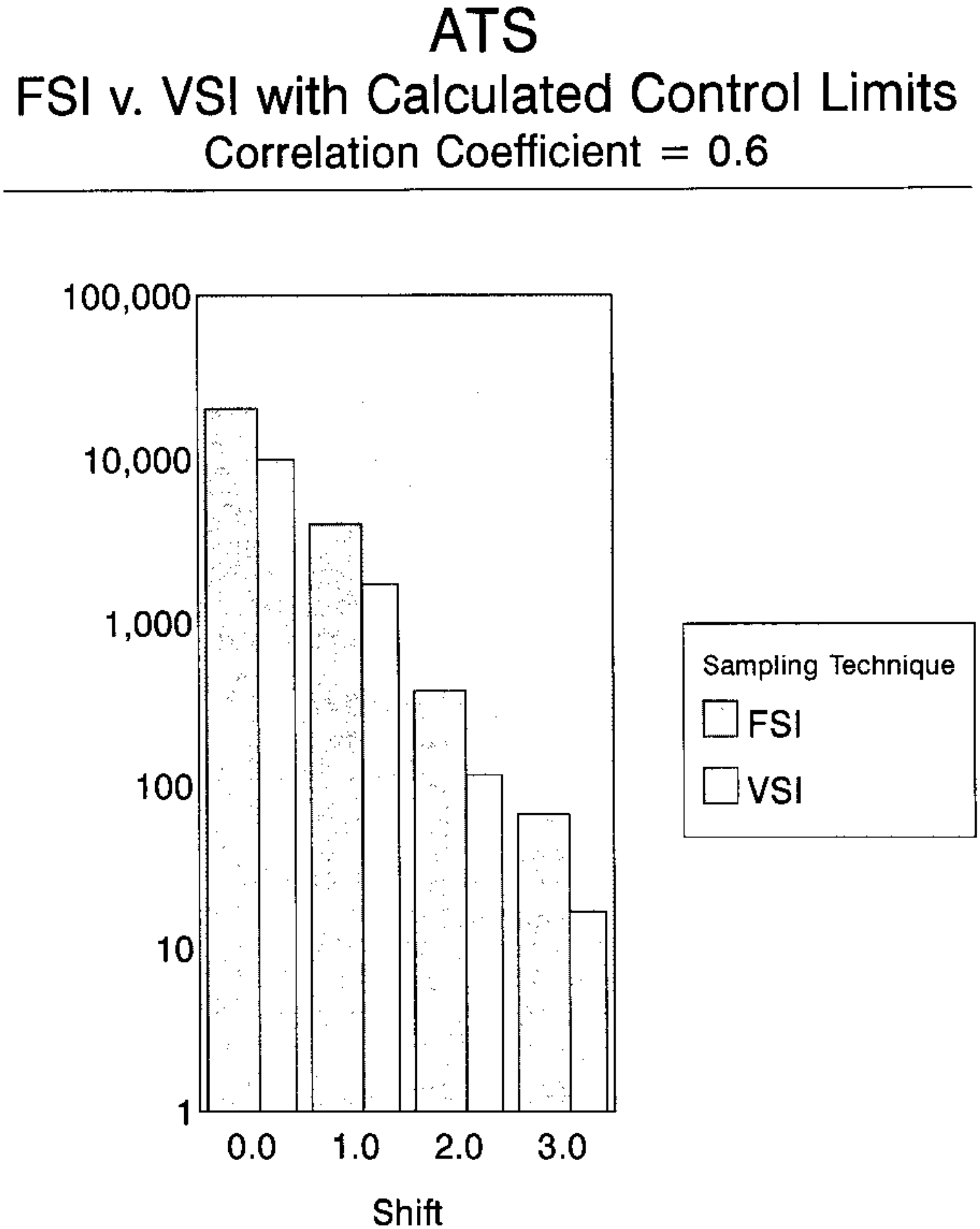


Figure 14d: ATS for FSI v. VSI with control limits calculated from sample data
Sample size: n = 1 Interval length: I = 10
y axis: logarithmic scale

ATS

FSI v. VSI with Fixed Control Limits

Correlation Coefficient = 0.9

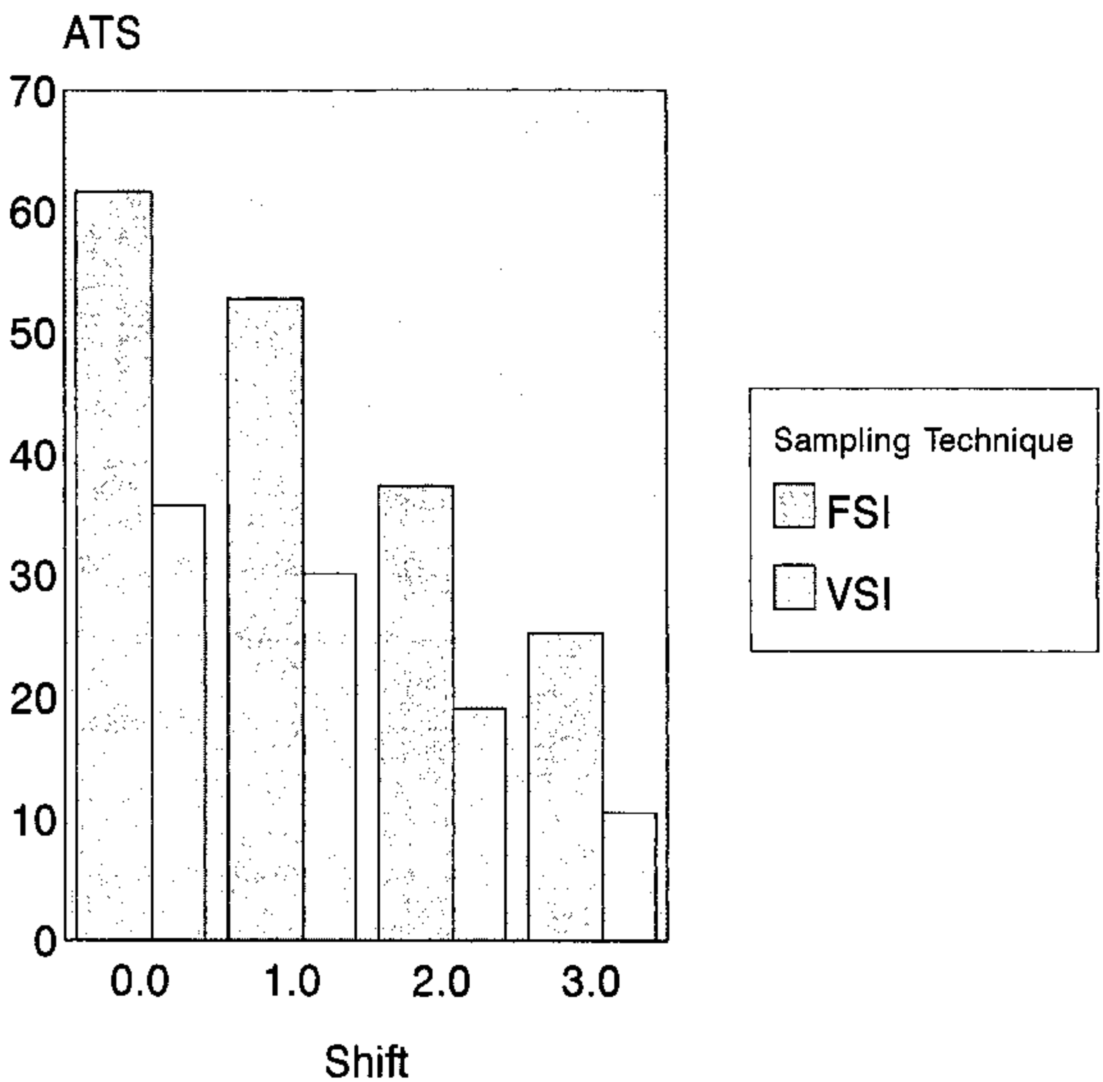


Figure 15a: ARL for FSI v. VSI with pre-set control limits
Sample size: n = 1 Interval length: l = 10
y axis: linear scale

ATS
FSI v. VSI with Fixed Control Limits
Correlation Coefficient = 0.9

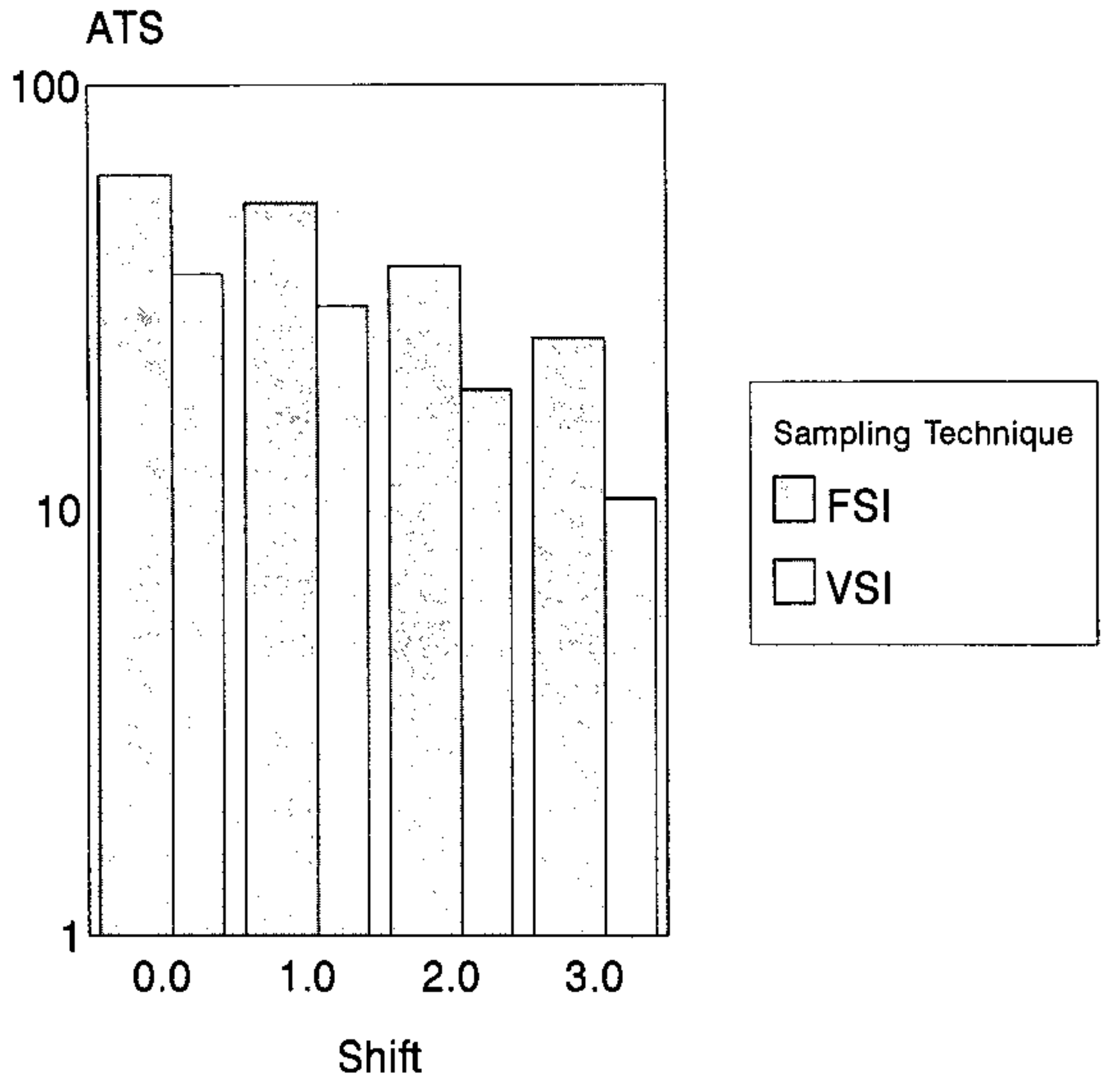


Figure 15b: ARL for FSI v. VSI with pre-set control limits
Sample size: n = 1 Interval length: l = 10
y axis: logarithmic scale

ATS
FSI v. VSI with Calculated Control Limits
Correlation Coefficient = 0.9

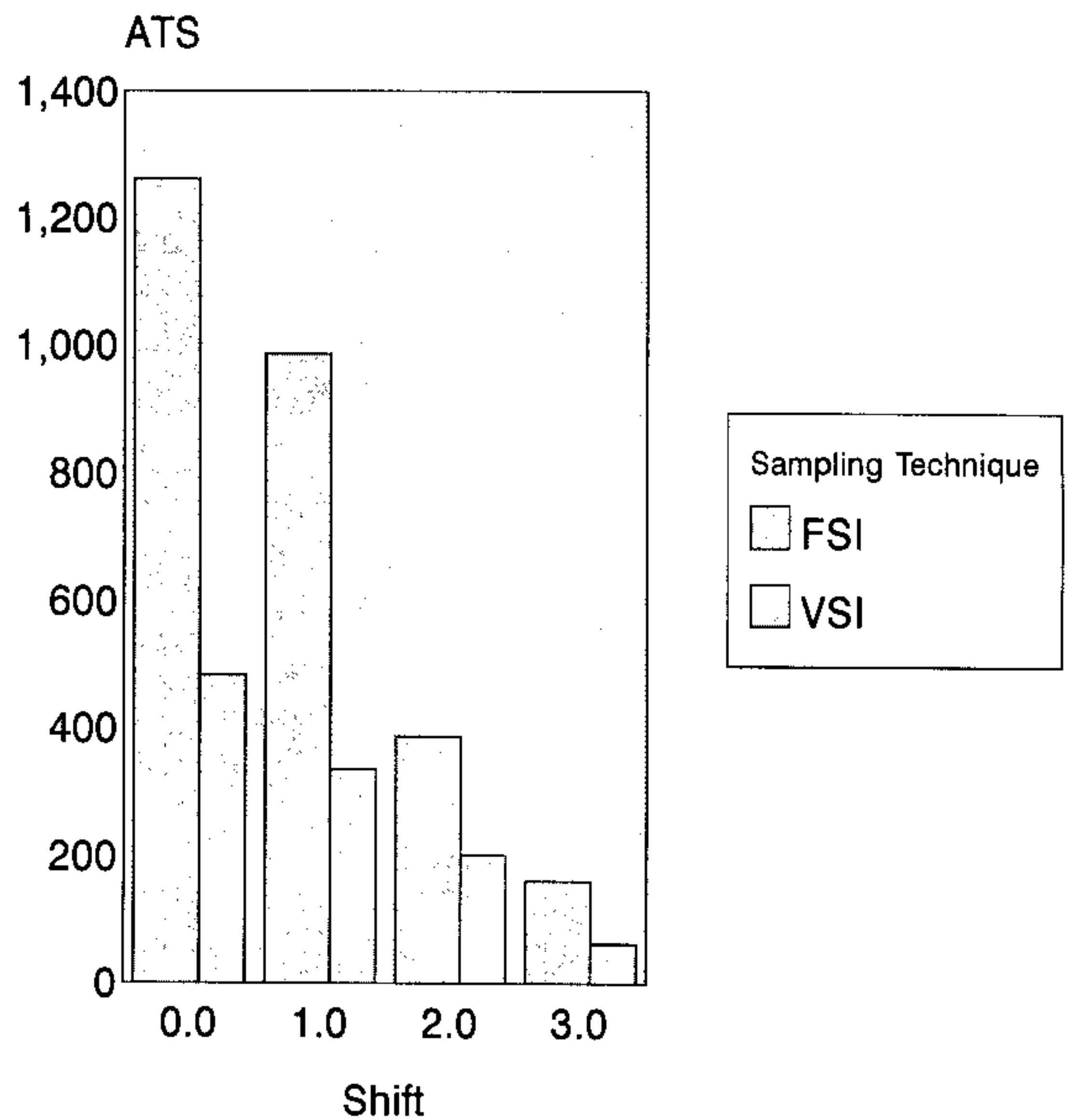


Figure 15c: ARL for FSI v. VSI with control limits calculated from sample data
Sample size: n = 1 Interval length: l = 10
y axis: linear scale

ATS
FSI v. VSI with Calculated Control Limits
Correlation Coefficient = 0.9

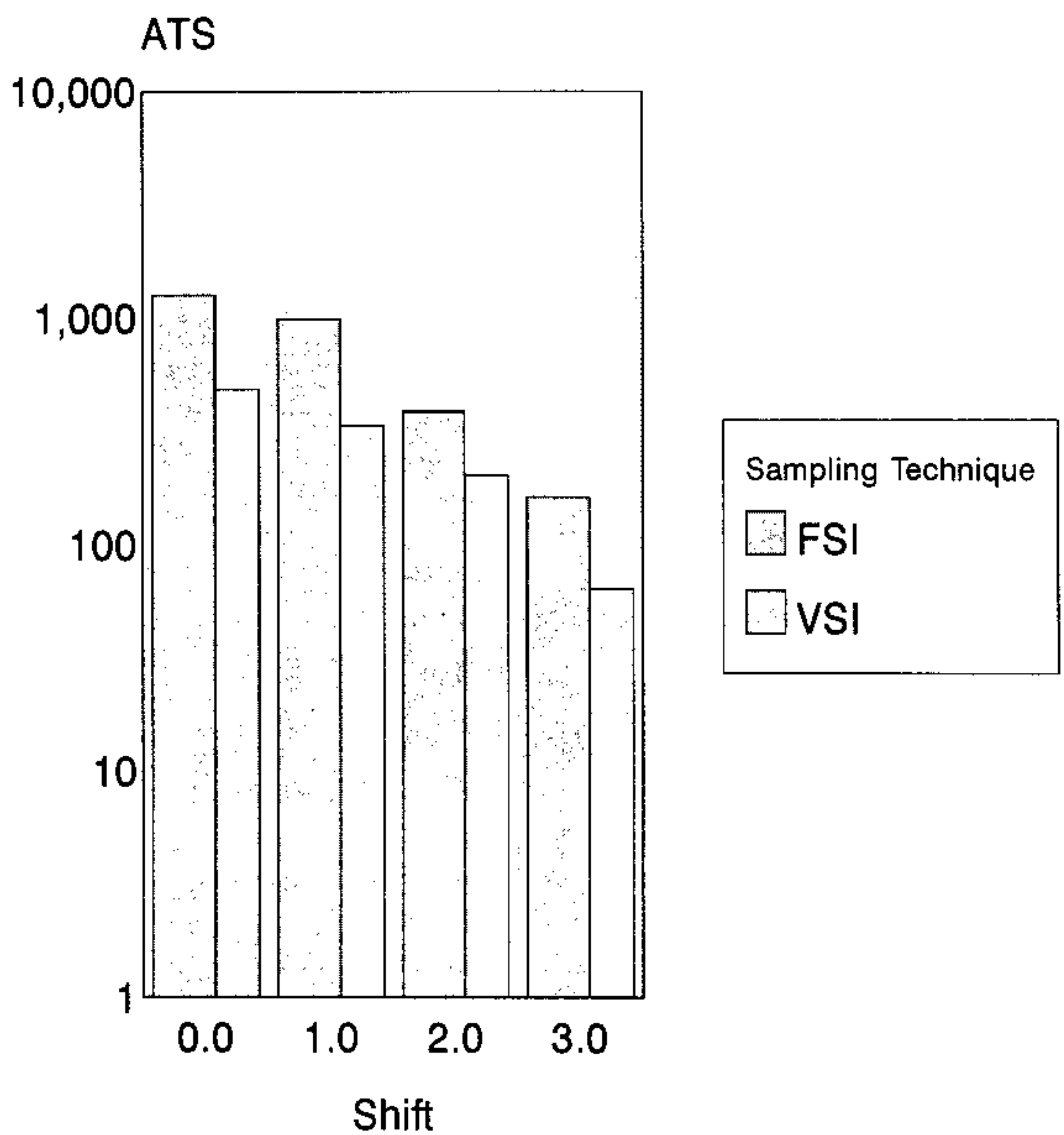


Figure 15d: ARL for FSI v. VSI with control limits calculated from sample data
Sample size: n = 1 Interval length: I = 10
y axis: logarithmic scale

ATS
FSI v. VSI with Fixed Control Limits
Correlation Coefficient = 0.0

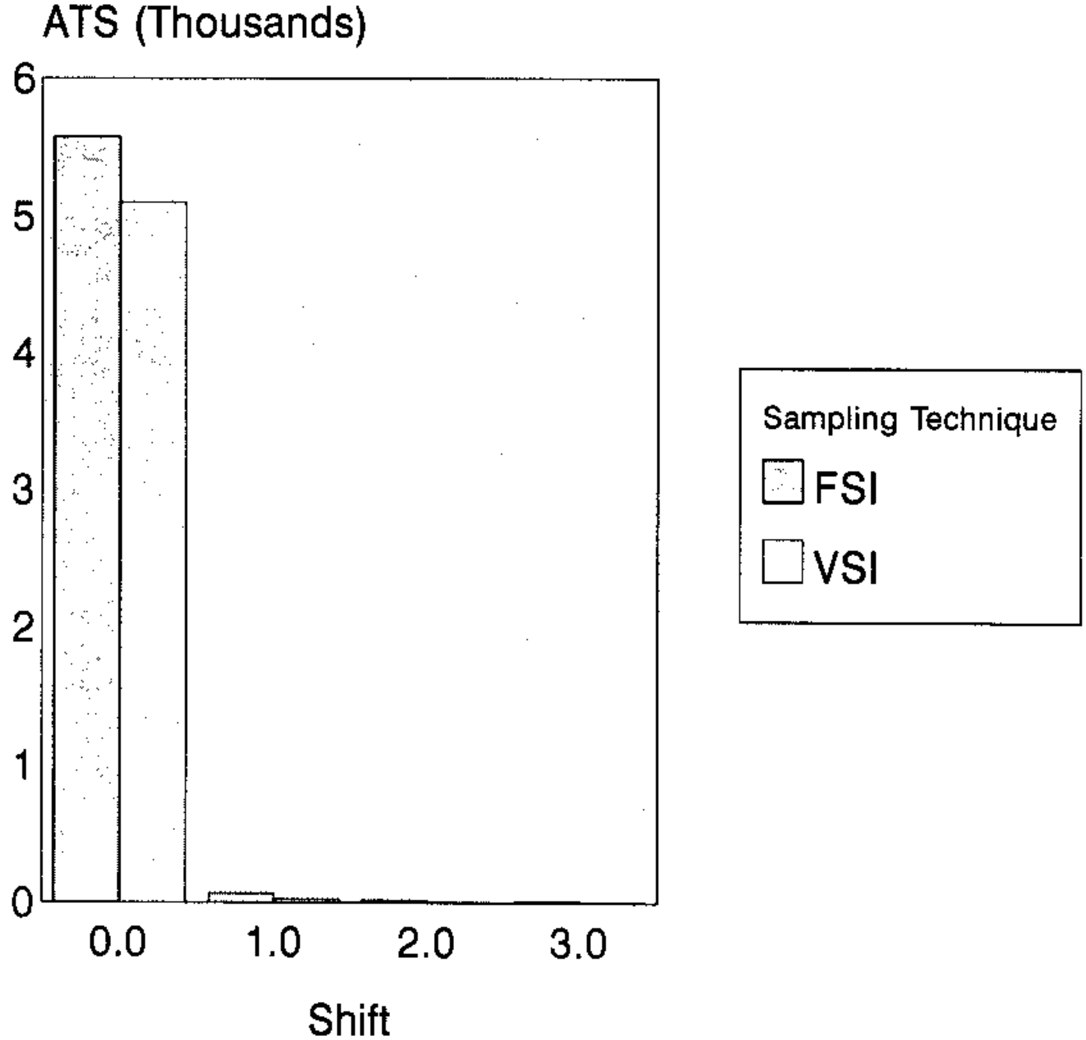


Figure 16a: ATS for FSI v. VSI with pre-set control limits
Sample size: n = 5 Interval length: I = 10
y axis: linear scale

ATS
FSI v. VSI with Fixed Control Limits
Correlation Coefficient = 0.0

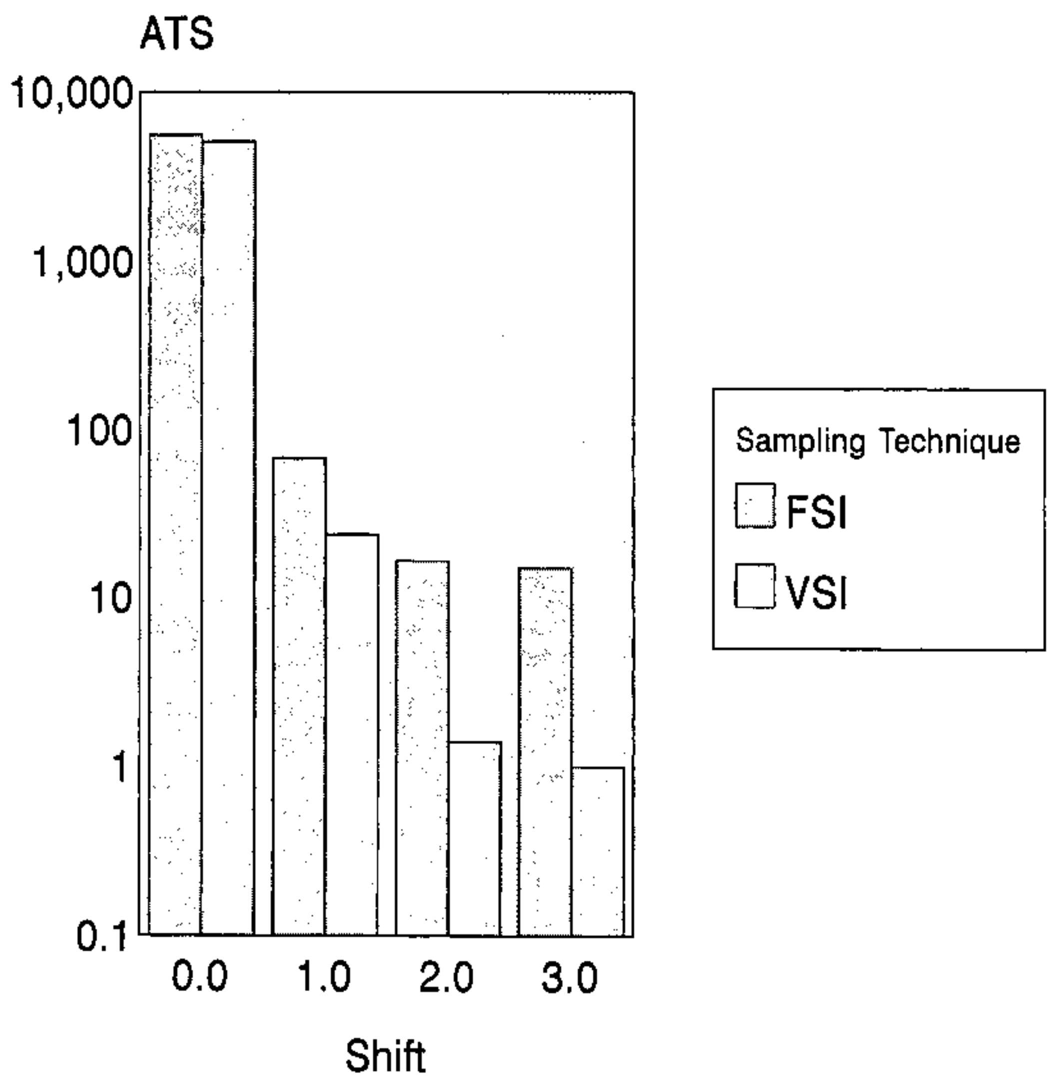


Figure 16b: ATS for FSI v. VSI with pre-set control limits
Sample size: n = 5 Interval length: I = 10
y axis: logarithmic scale

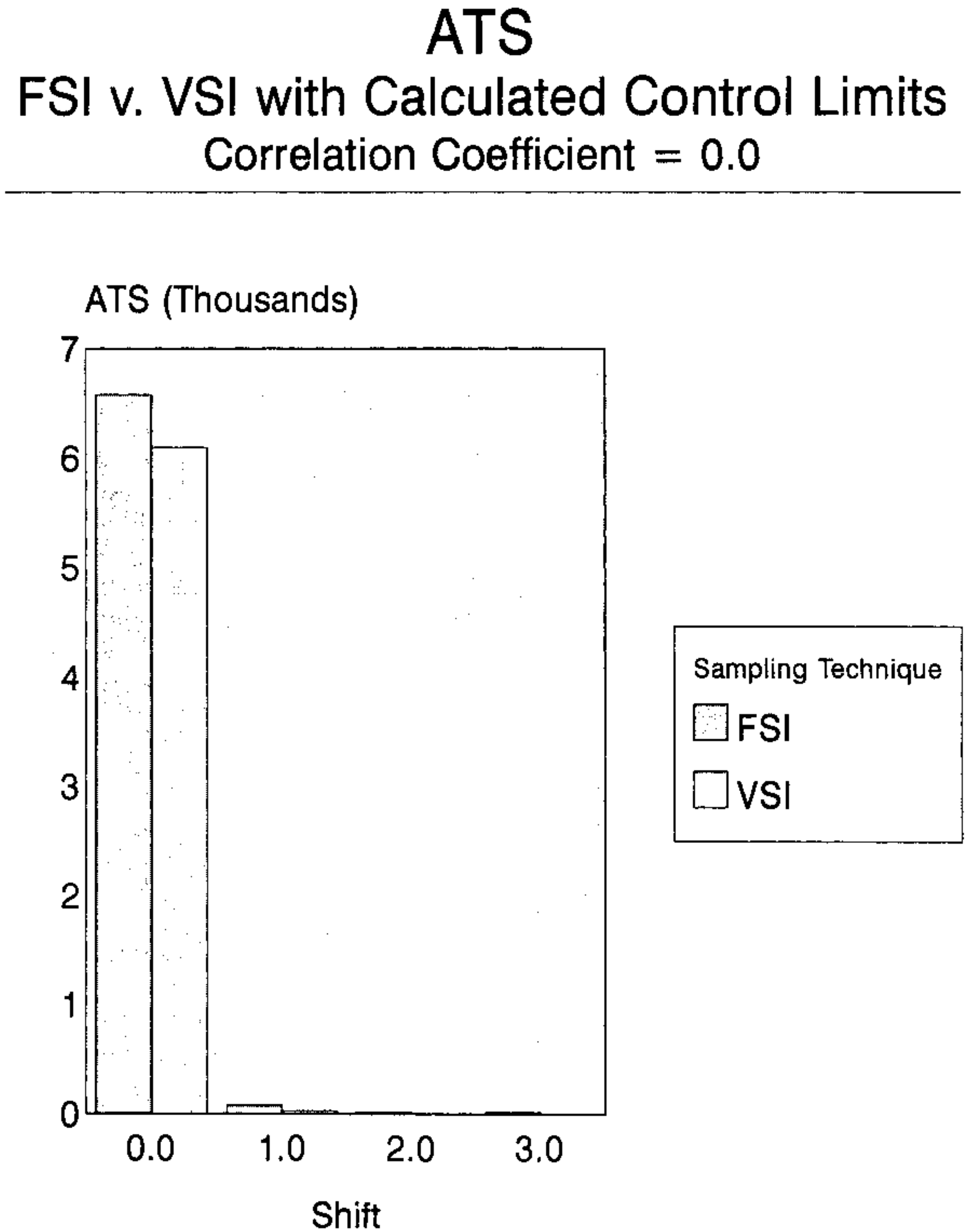


Figure 16c: ATS for FSI v. VSI with control limits calculated from sample data
Sample size: n = 5 Interval length: I = 10
y axis: linear scale

ATS
FSI v. VSI with Calculated Control Limits
Correlation Coefficient = 0.0

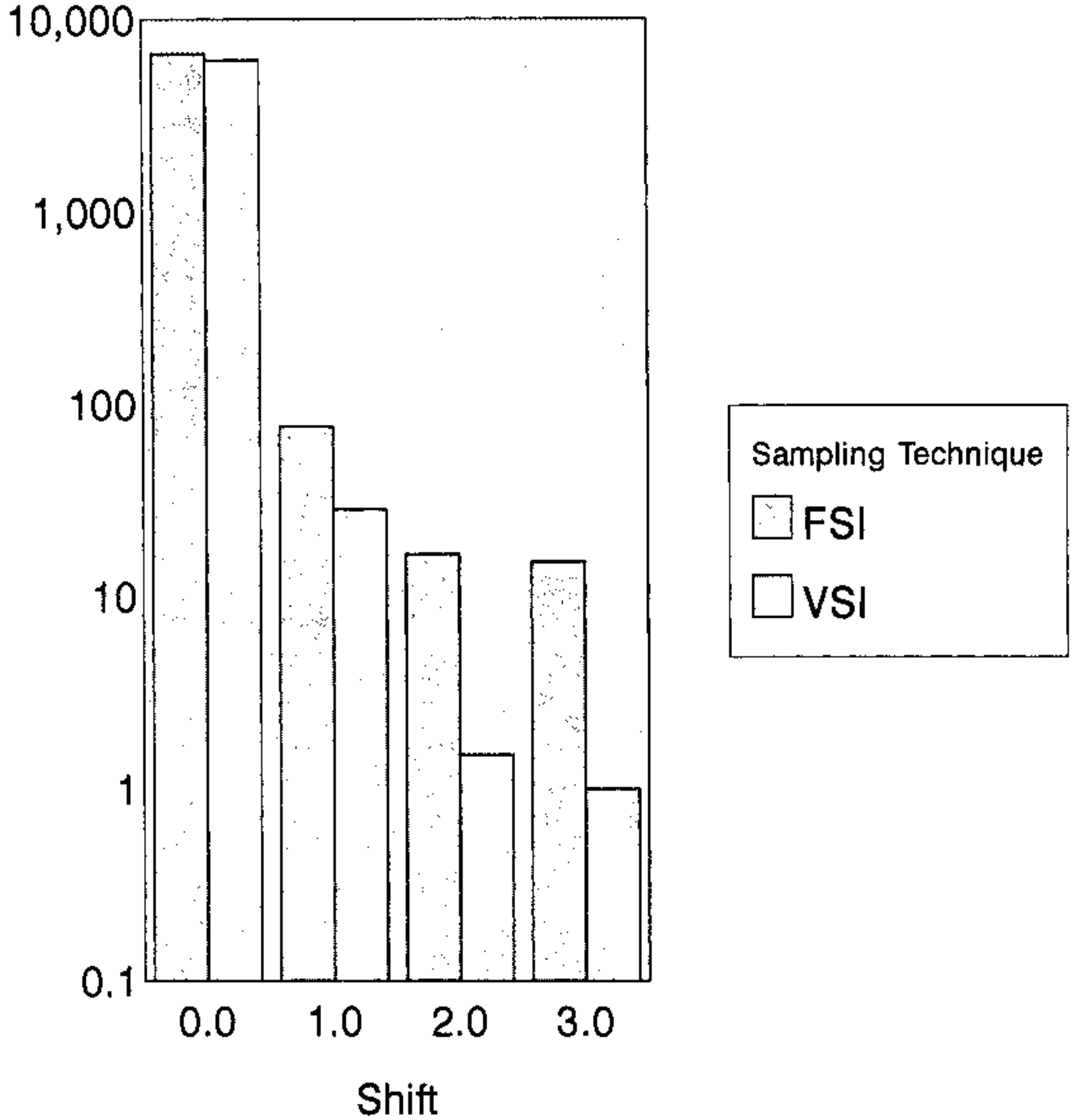


Figure 16d: ATS for FSI v. VSI with control limits calculated from sample data
Sample size: n = 5 Interval length: I = 10
y axis: logarithmic scale

ATS
FSI v. VSI with Fixed Control Limits
Correlation Coefficient = 0.3

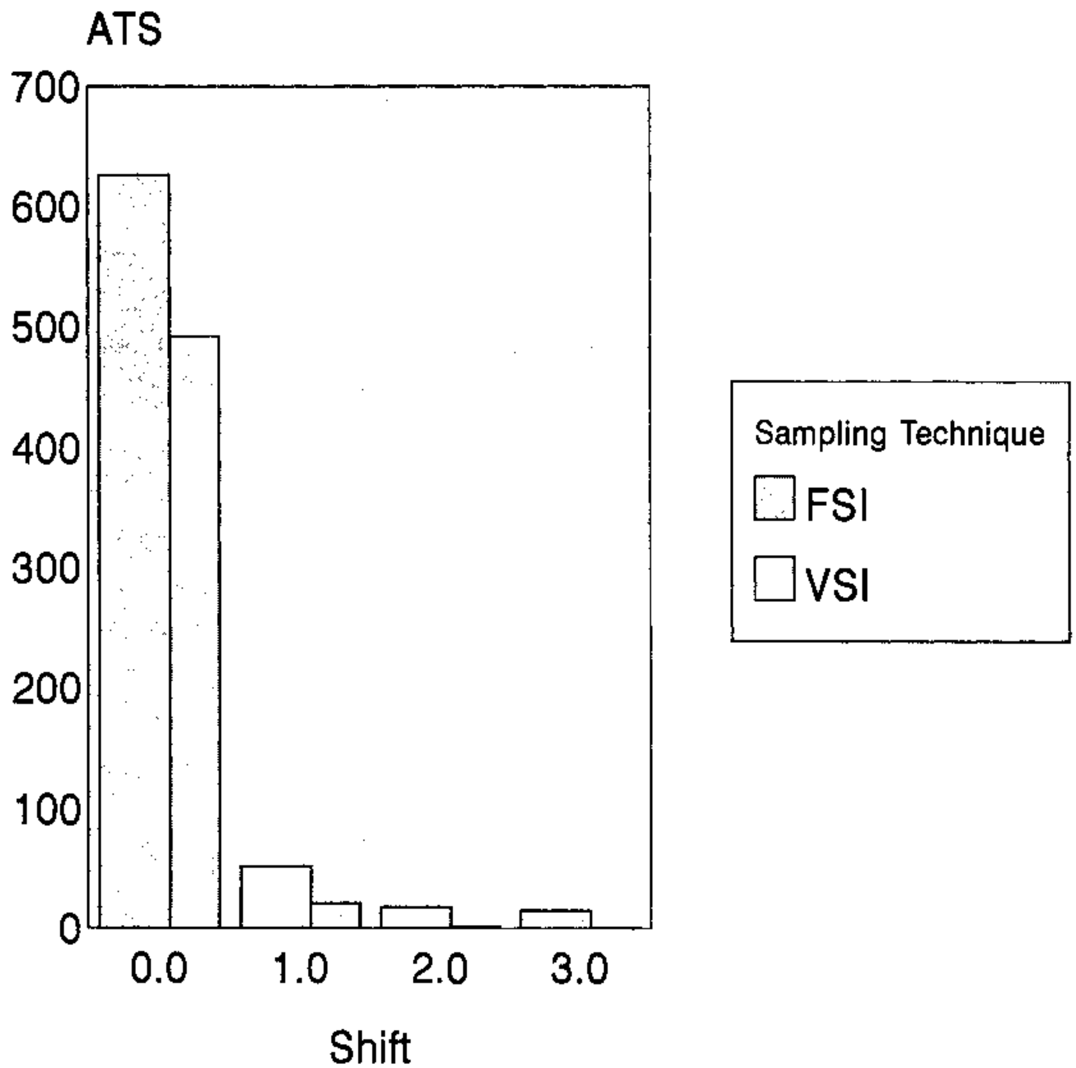


Figure 17a: ATS for FSI v. VSI with pre-set control limits
Sample size: n = 5 Interval length: l = 10
y axis: linear scale

ATS
FSI v. VSI with Fixed Control Limits
Correlation Coefficient = 0.3

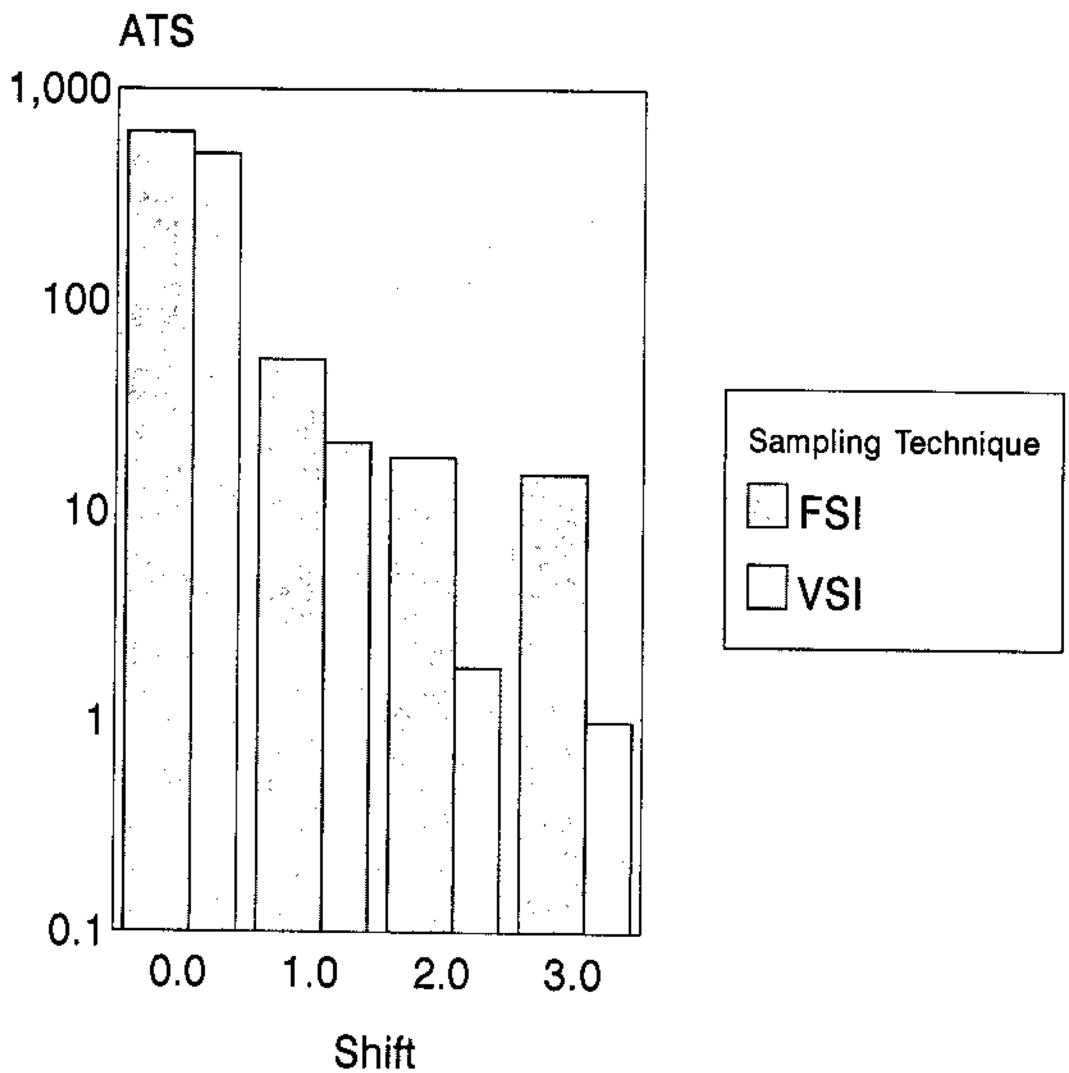


Figure 17b: ATS for FSI v. VSI with pre-set control limits
Sample size: n = 5   Interval length: I = 10
y axis: logarithmic scale

ATS
FSI v. VSI with Calculated Control Limits
Correlation Coefficient = 0.3

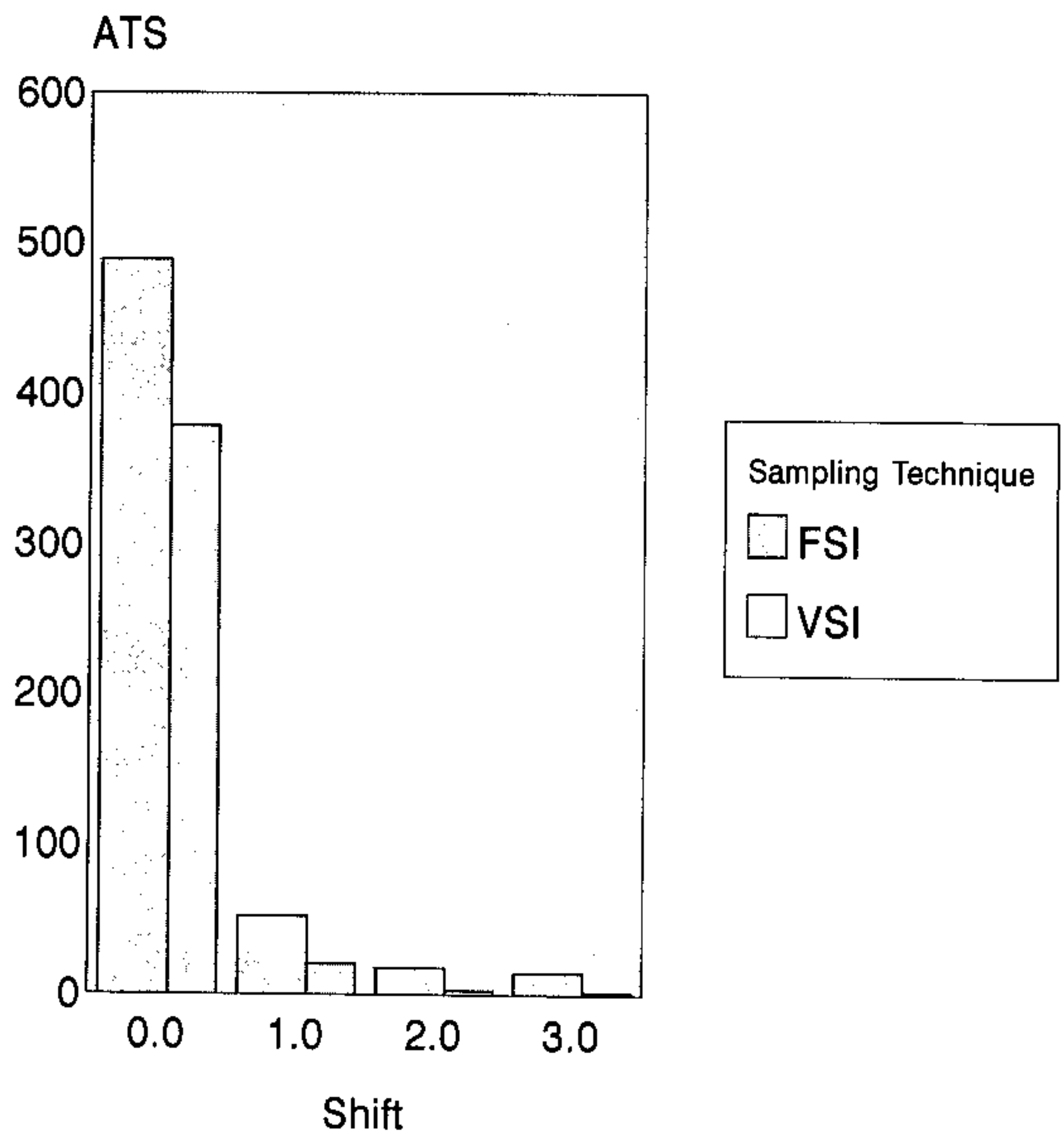


Figure 17c: ATS for FSI v. VSI with control limits calculated from sample size
Sample size: n = 5 Interval length: l = 10
y axis: linear scale

ATS
FSI v. VSI with Calculated Control Limits
Correlation Coefficient = 0.3

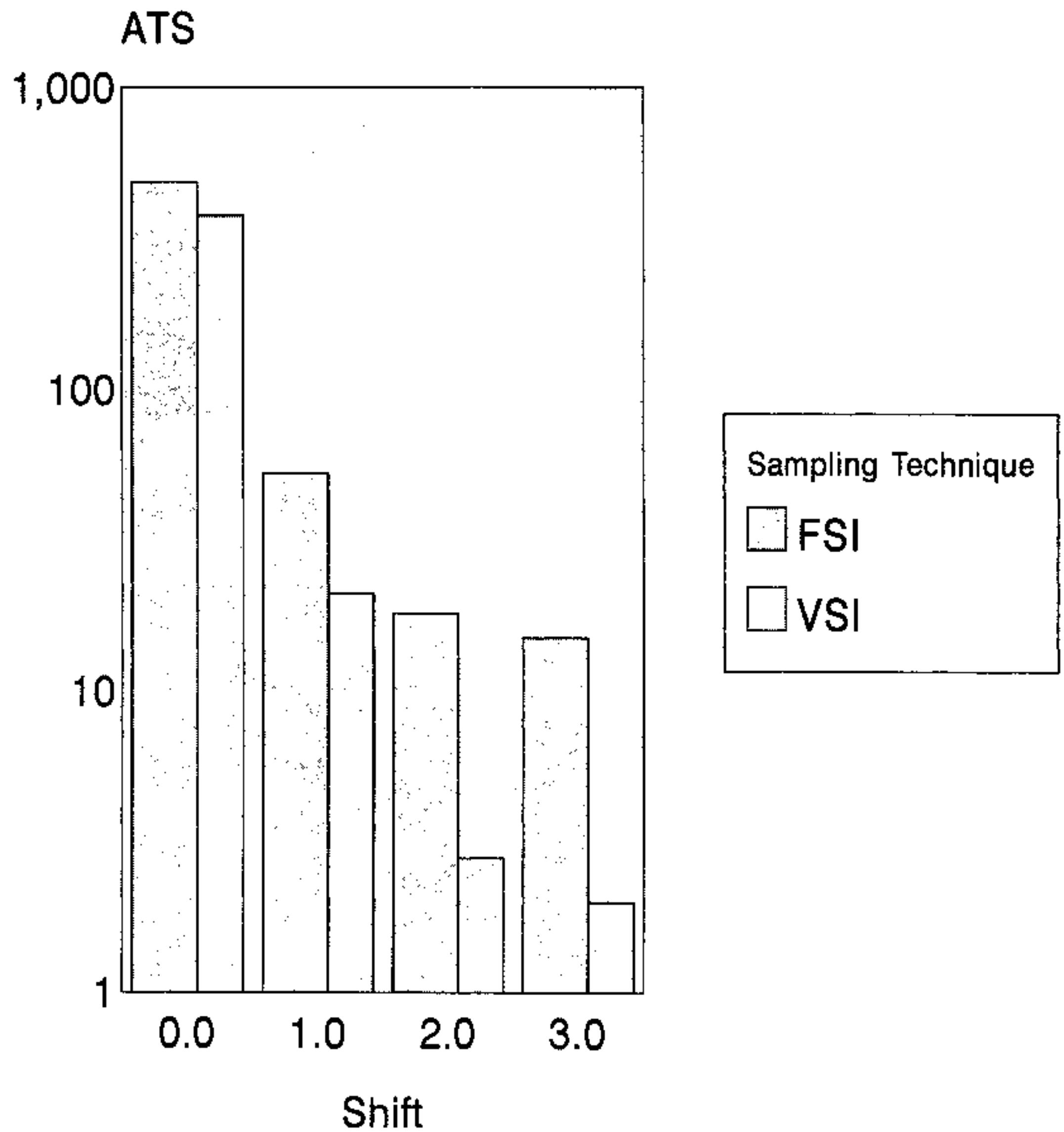


Figure 17d: ATS for FSI v. VSI with control limits calculated from sample size
Sample size: n = 5   Interval length: l = 10
y axis: logarithmic scale

ATS
FSI v. VSI with Fixed Control Limits
Correlation Coefficient = 0.6

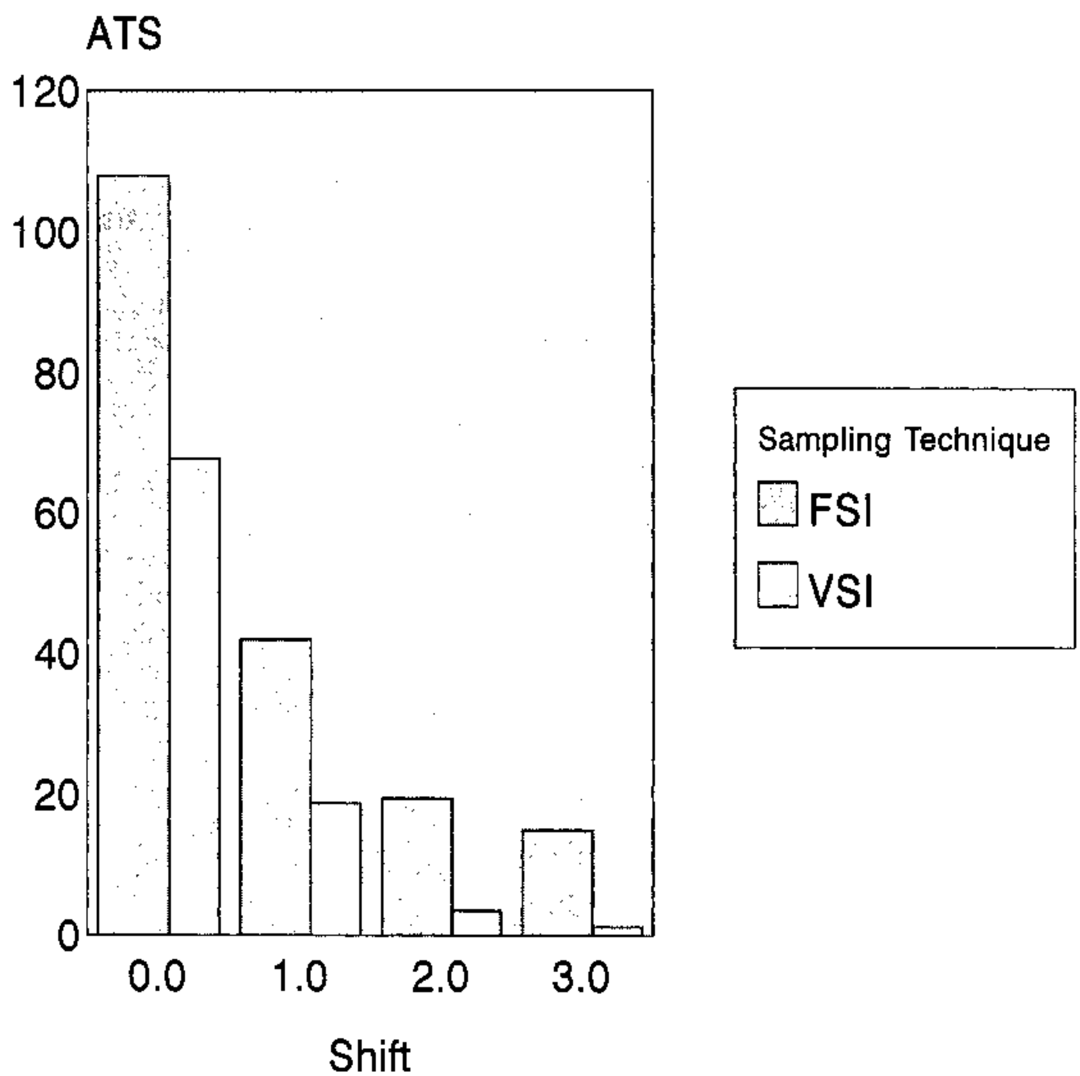


Figure 18a: ATS for FSI v. FSI with pre-set control limits
Sample size: n = 5 Interval length: l = 10
y axis: linear scale

ATS
FSI v. VSI with Fixed Control Limits
Correlation Coefficient = 0.6

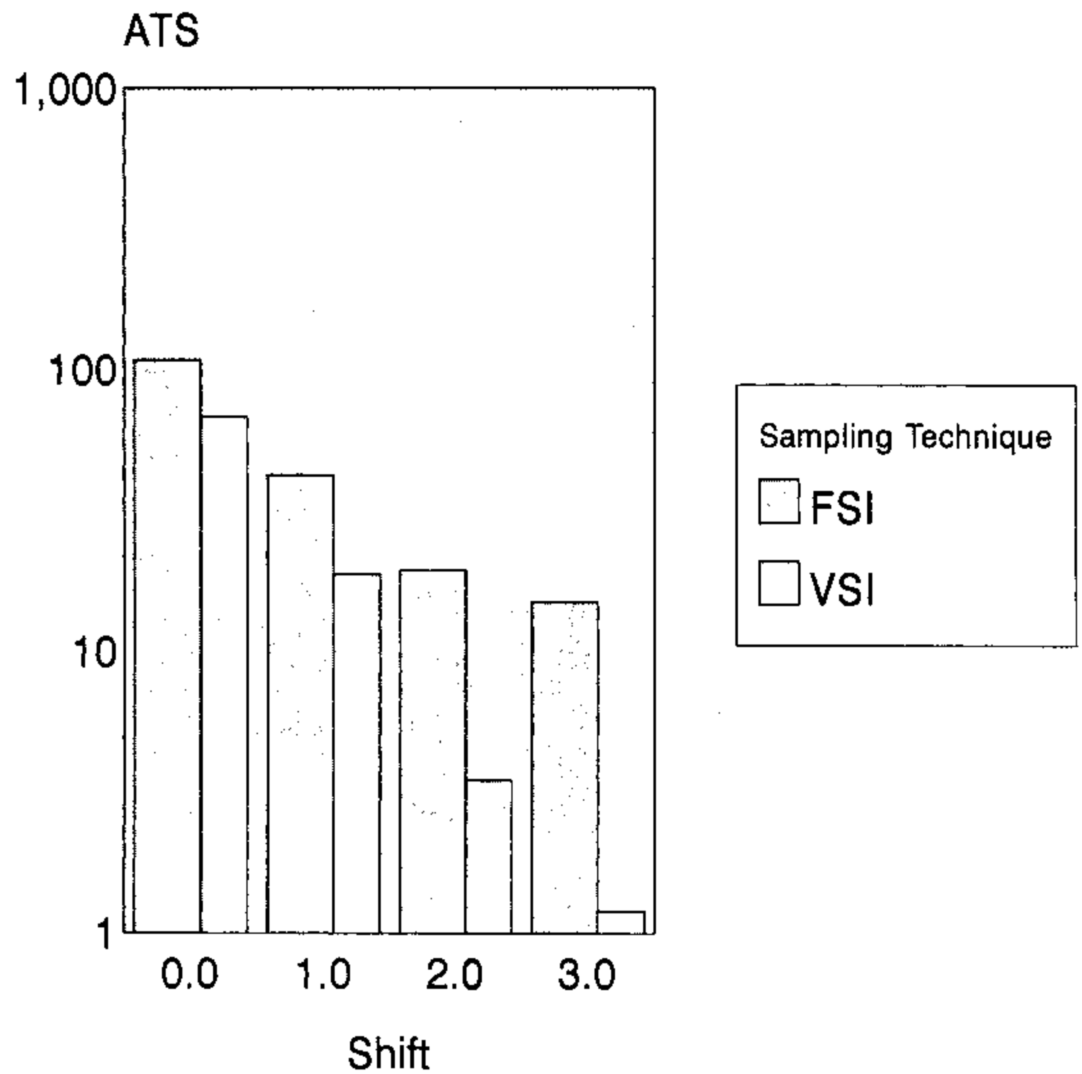


Figure 18b: ATS for FSI v. FSI with pre-set control limits
Sample size: n = 5 Interval length: I = 10
y axis: logarithmic scale

ATS
FSI v. VSI with Calculated Control Limits
Correlation Coefficient = 0.6

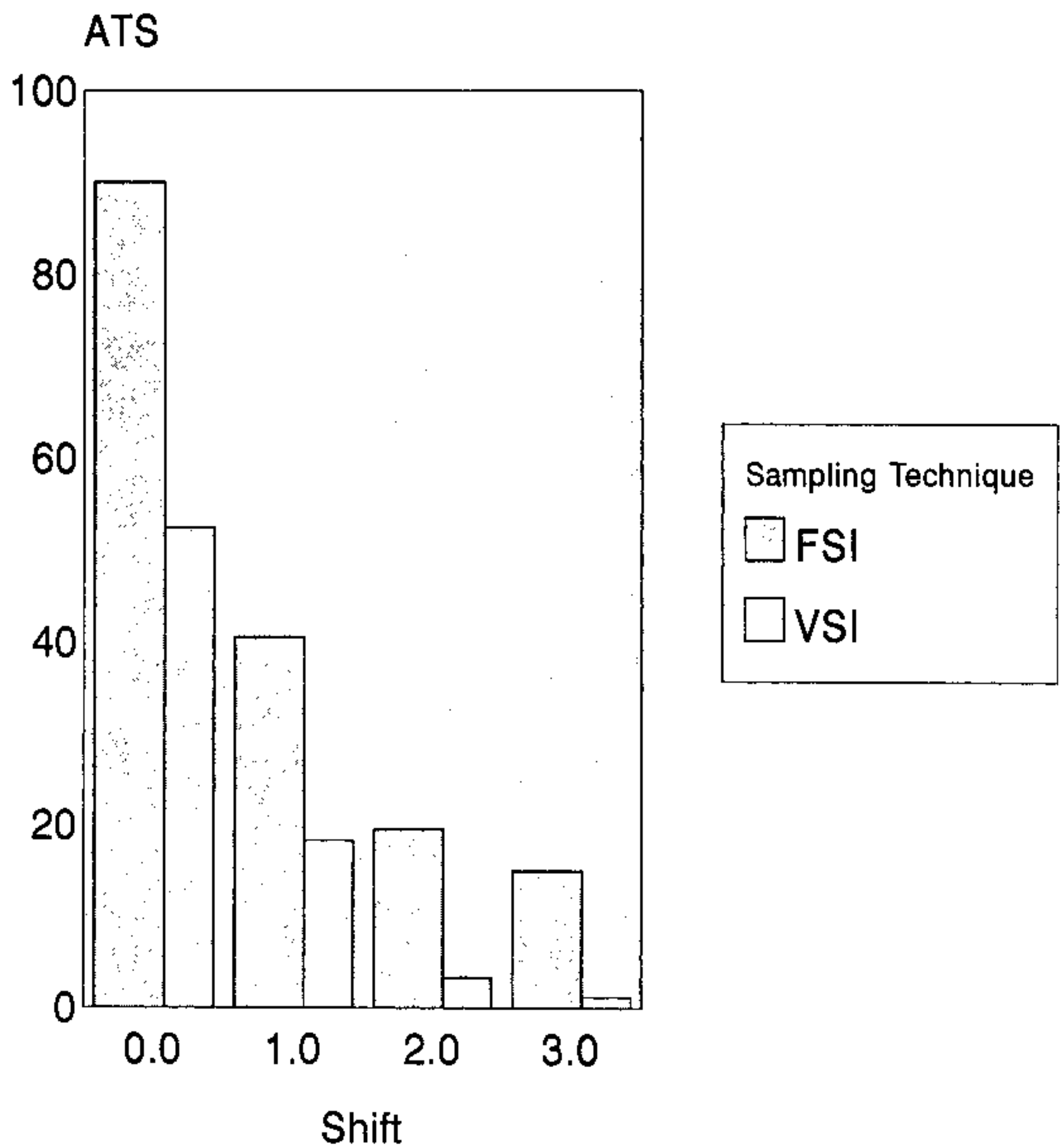


Figure 18c: ATS for FSI v. FSI with control limits calculated from sample data
Sample size: n = 5 Interval length: l = 10
y axis: linear scale

ATS
FSI v. VSI with Calculated Control Limits
Correlation Coefficient = 0.6

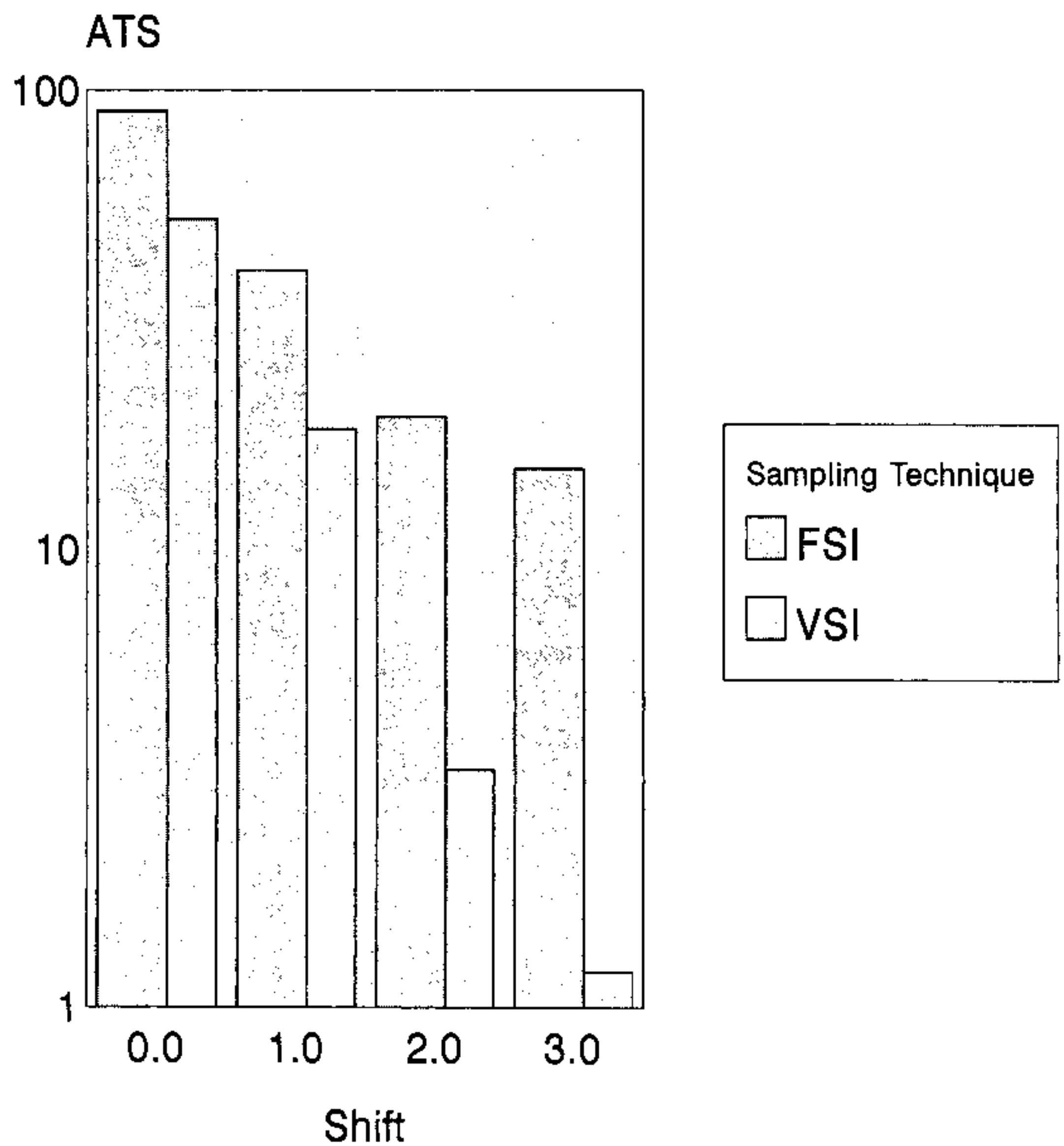


Figure 18d: ATS for FSI v. FSI with control limits calculated from sample data
Sample size: n = 5 Interval length: l = 10
y axis: logarithmic scale

ATS
FSI v. VSI with Fixed Control Limits
Correlation Coefficient = 0.9

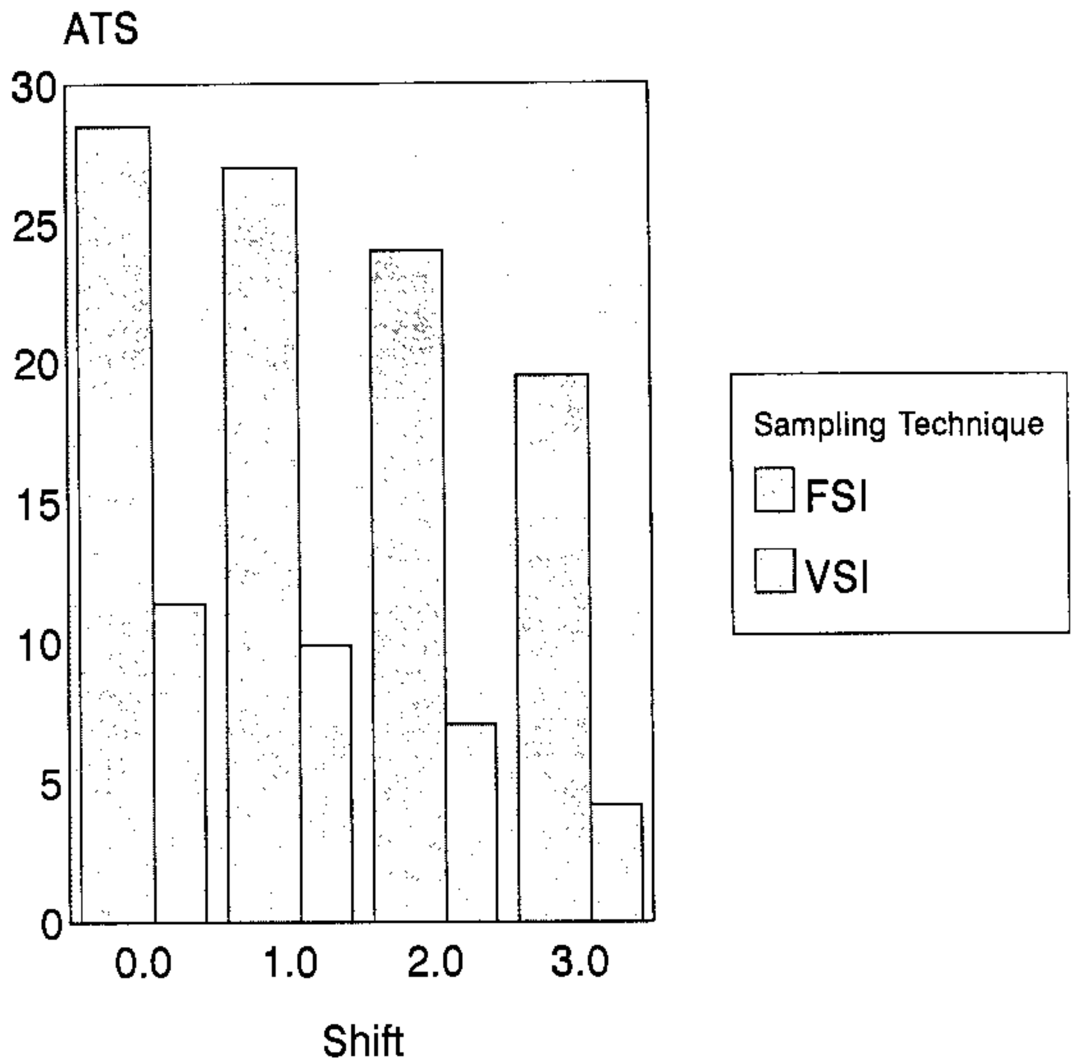


Figure 19a: ATS for FSI v. VSI with pre-set control limits
Sample size: n = 5 Interval length: l = 10
y axis: linear scale

ATS
FSI v. VSI with Fixed Control Limits
Correlation Coefficient = 0.9

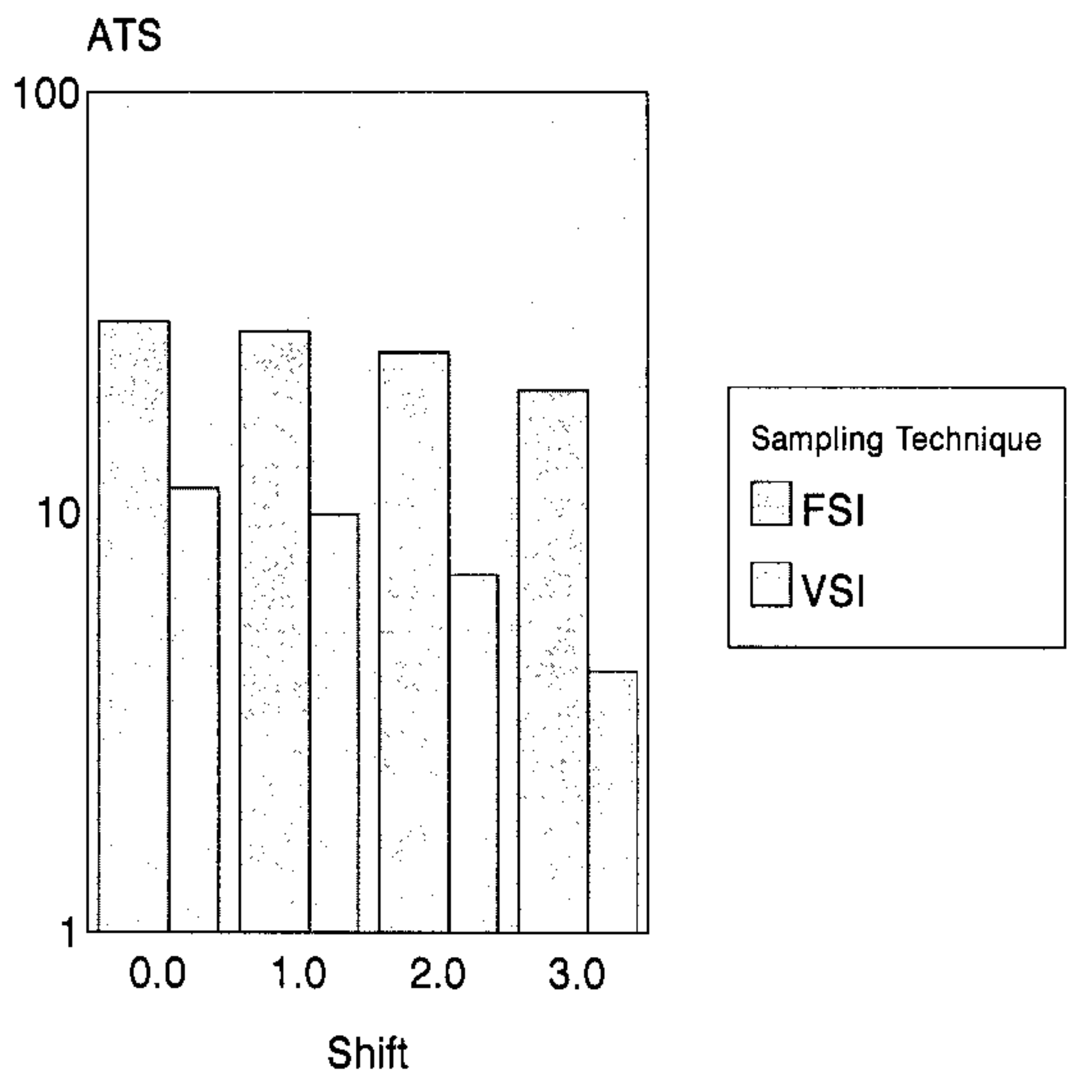


Figure 19b: ATS for FSI v. VSI with pre-set control limits
Sample size: n = 5 Interval length: l = 10
y axis: logarithmic scale

ATS
FSI v. VSI with Calculated Control Limits
Correlation Coefficient = 0.9

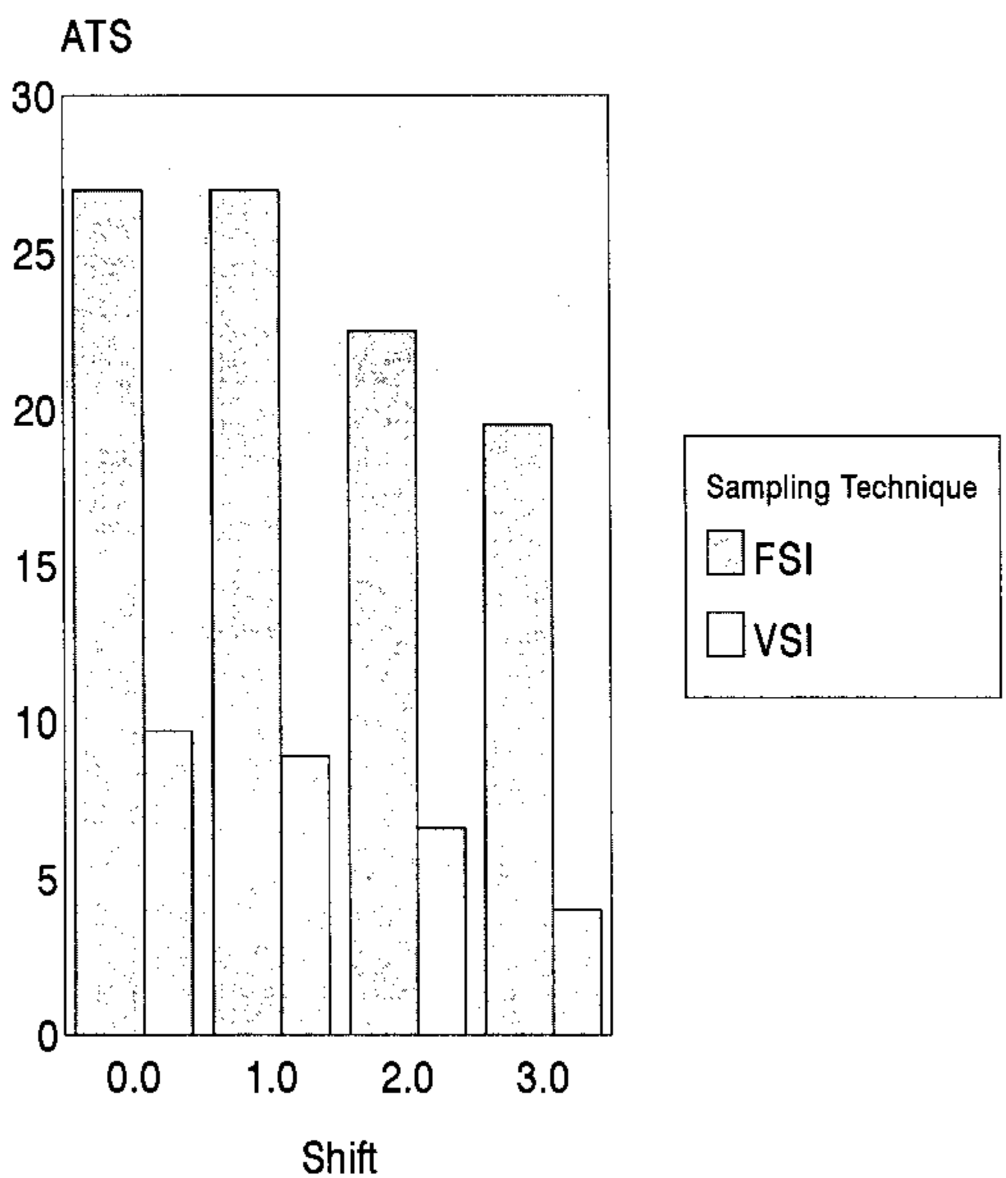


Figure 19c: ATS for FSI v. VSI with control limits calculated from sample data
Sample size: n = 5 Interval length: I = 10
y axis: linear scale

ATS
FSI v. VSI with Calculated Control Limits
Correlation Coefficient = 0.9

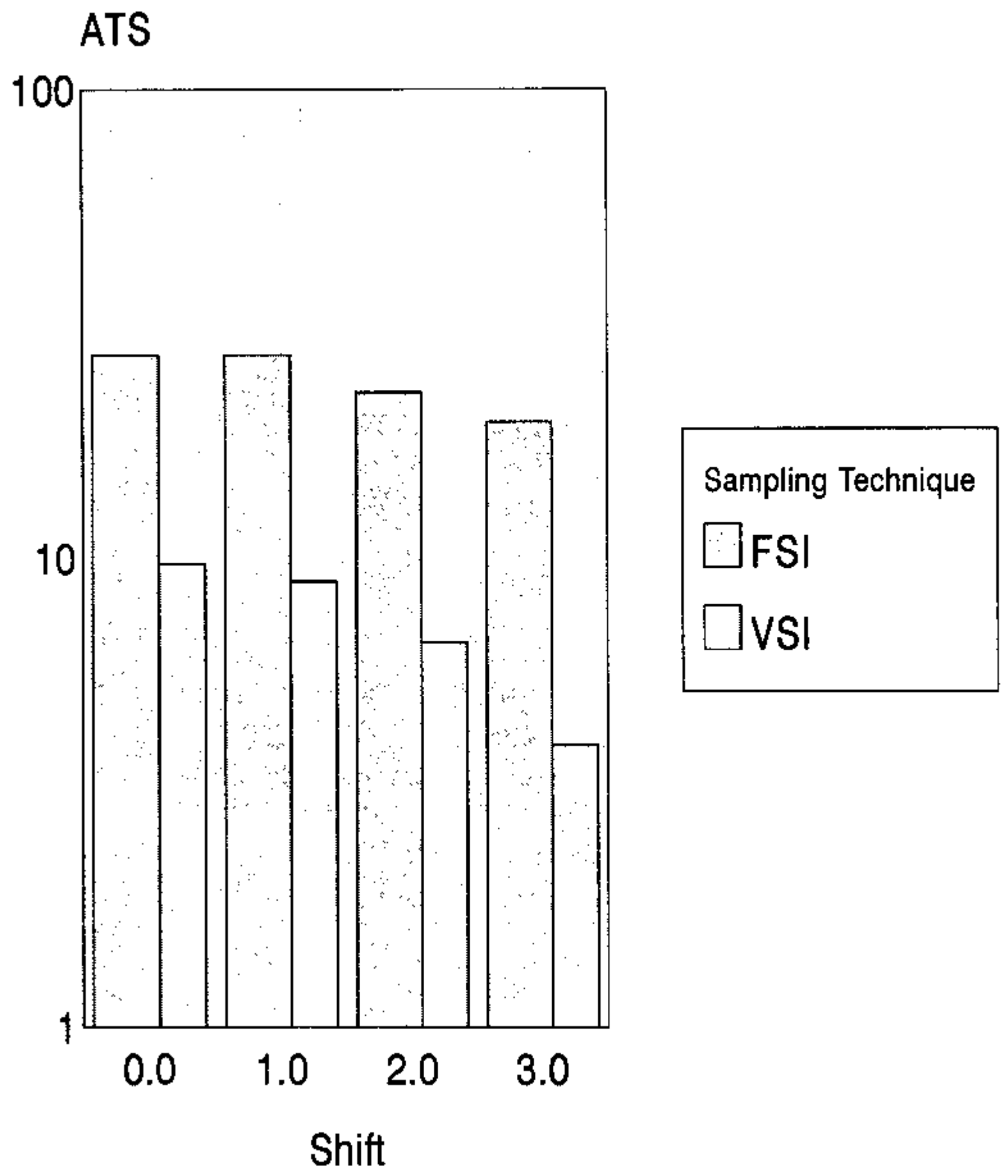


Figure 19d: ATS for FSI v. VSI with control limits calculated from sample data
Sample size: n = 5 Interval length: I = 10
y axis: logarithmic scale

INTERVAL LENGTH v. ARL
FSI with Fixed Limits
Shift in Target Mean = 0.0

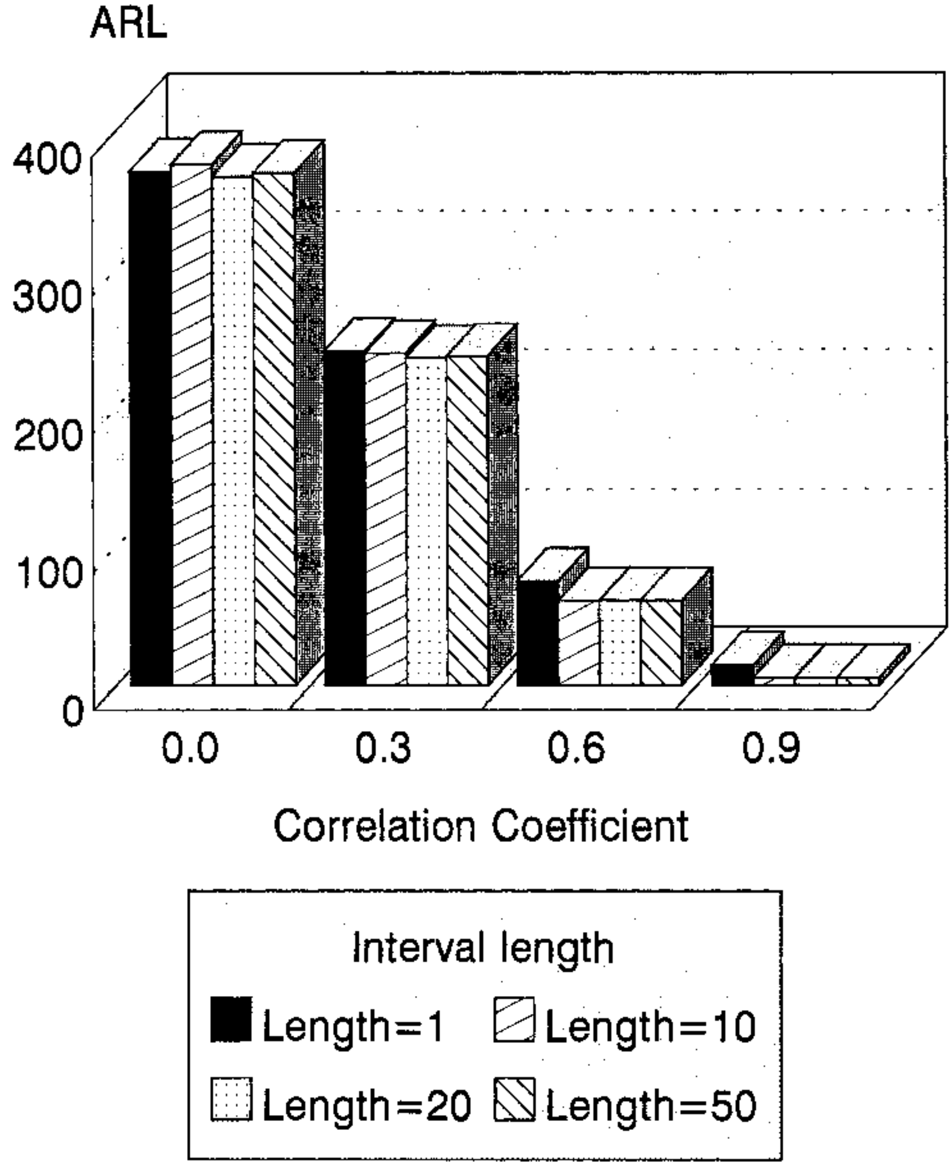


Figure 20a: Effect of interval length on ARL for FSI with pre-set limits
Sample size = 1 Shift = 0.0
y axis: linear scale

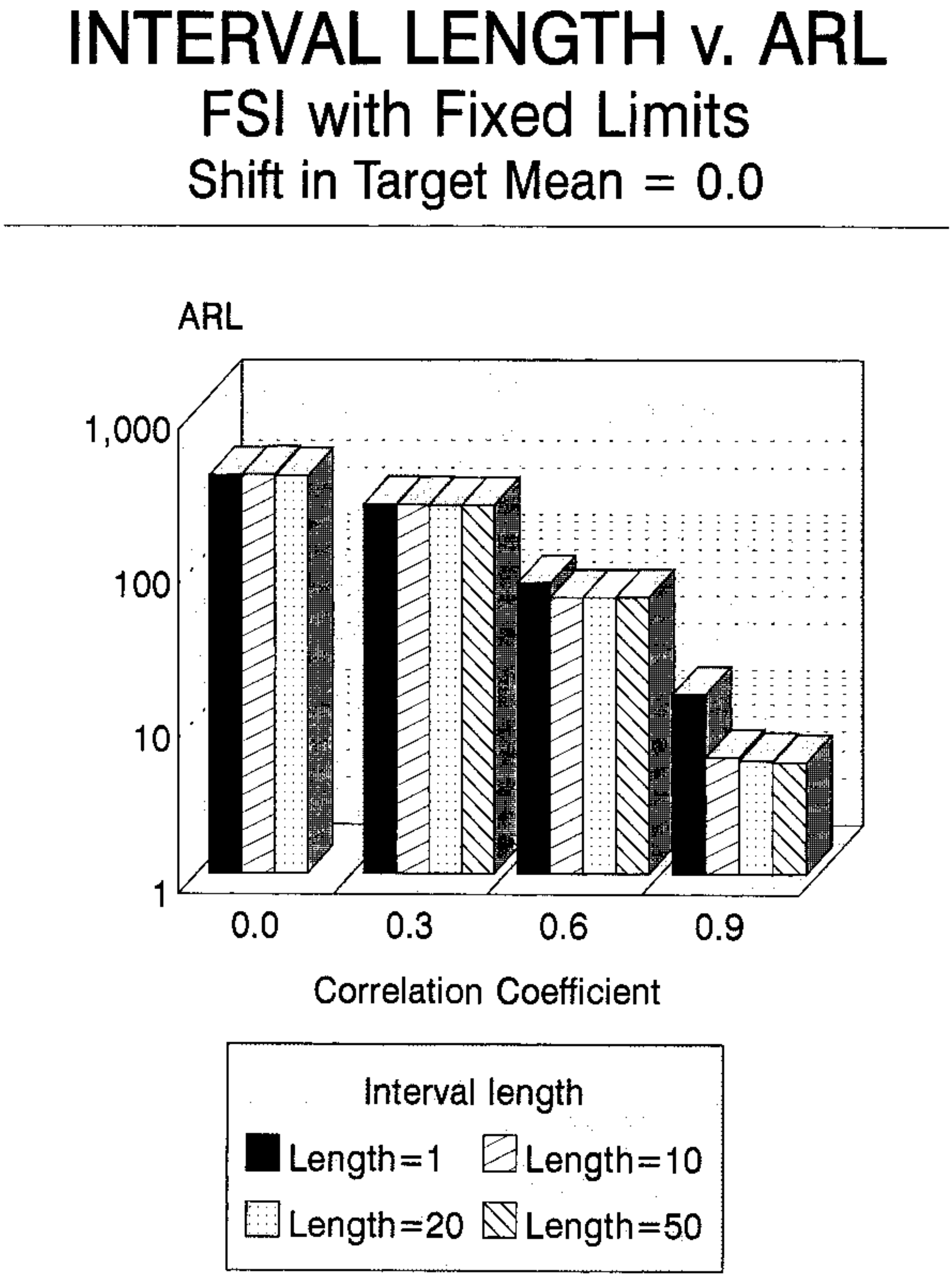


Figure 20b: Effect of Interval length on ARL for FSI with pre-set limits
Sample size = 1 Shift = 0.0
y axis: logarithmic scale

INTERVAL LENGTH v. ARL
FSI with Fixed Limits
Shift in Target Mean = 1.0

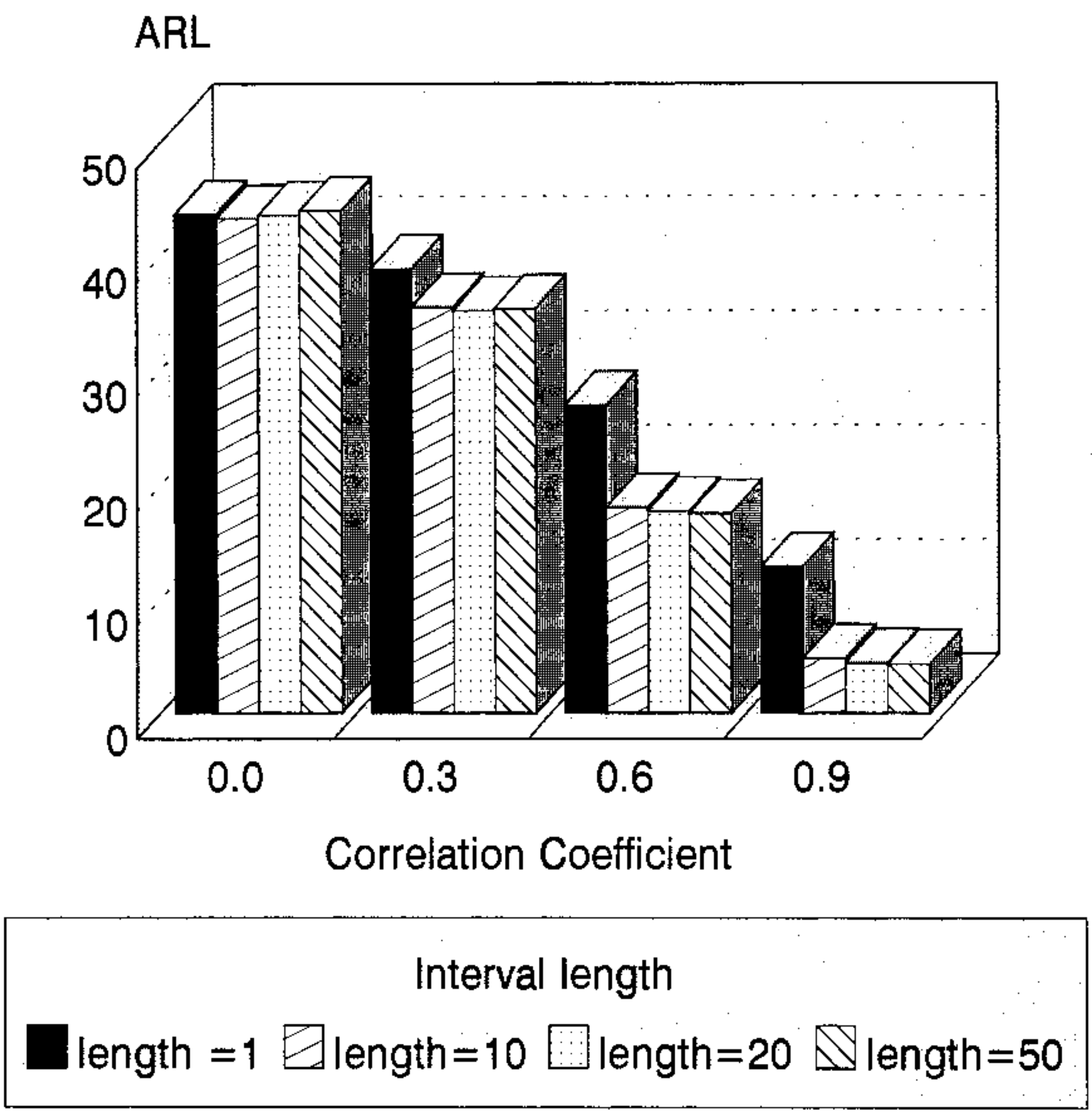


Figure 20c: Effect of Interval length on ARL
Sample size = 1 Shift = 1.0
y axis: linear scale

INTERVAL LENGTH v. ARL
FSI with Fixed Limits
Shift in Target Mean = 1.0

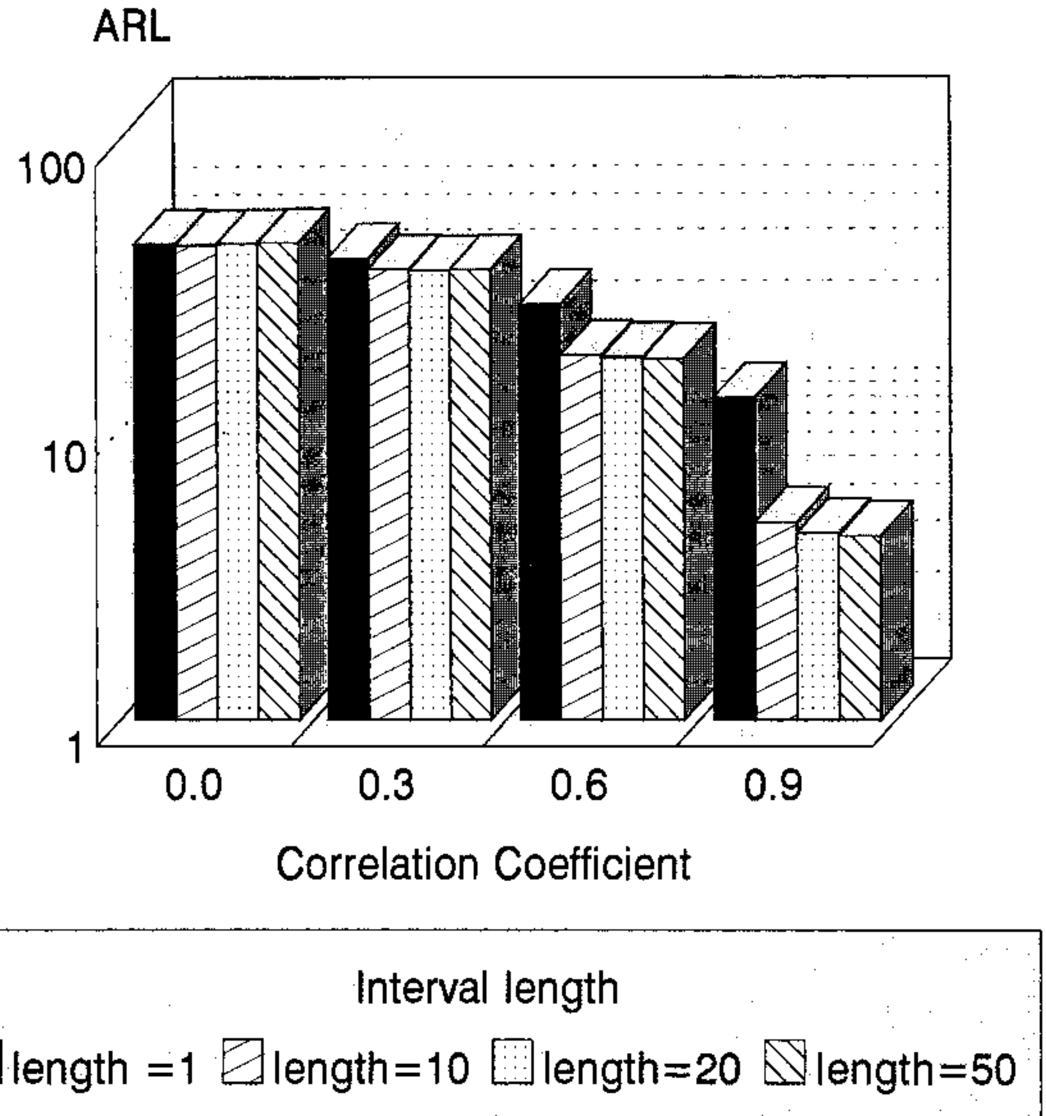


Figure 20d: Effect of Interval length on ARL
Sample size = 1 Shift = 1.0
y axis: logarithmic scale

INTERVAL LENGTH v. ARL

FSI with Fixed Limits

Shift in Target Mean = 2.0

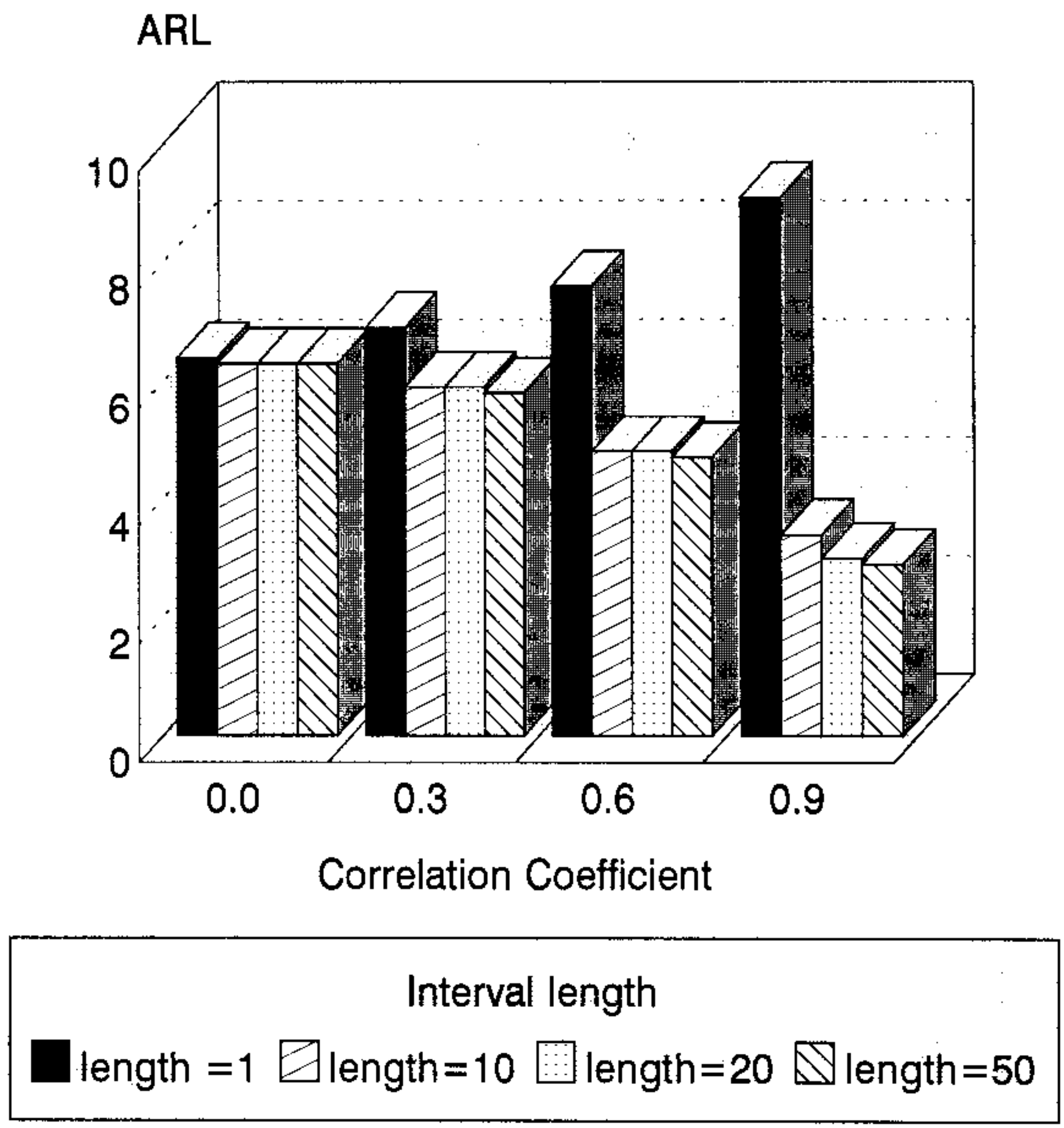


Figure 20e: Effect of interval length on ARL for FSI with pre-set control limits
Sample size = 1 Shift = 2.0
y axis: linear scale

INTERVAL LENGTH v. ARL
FSI with Fixed Limits
Shift in Target Mean = 2.0

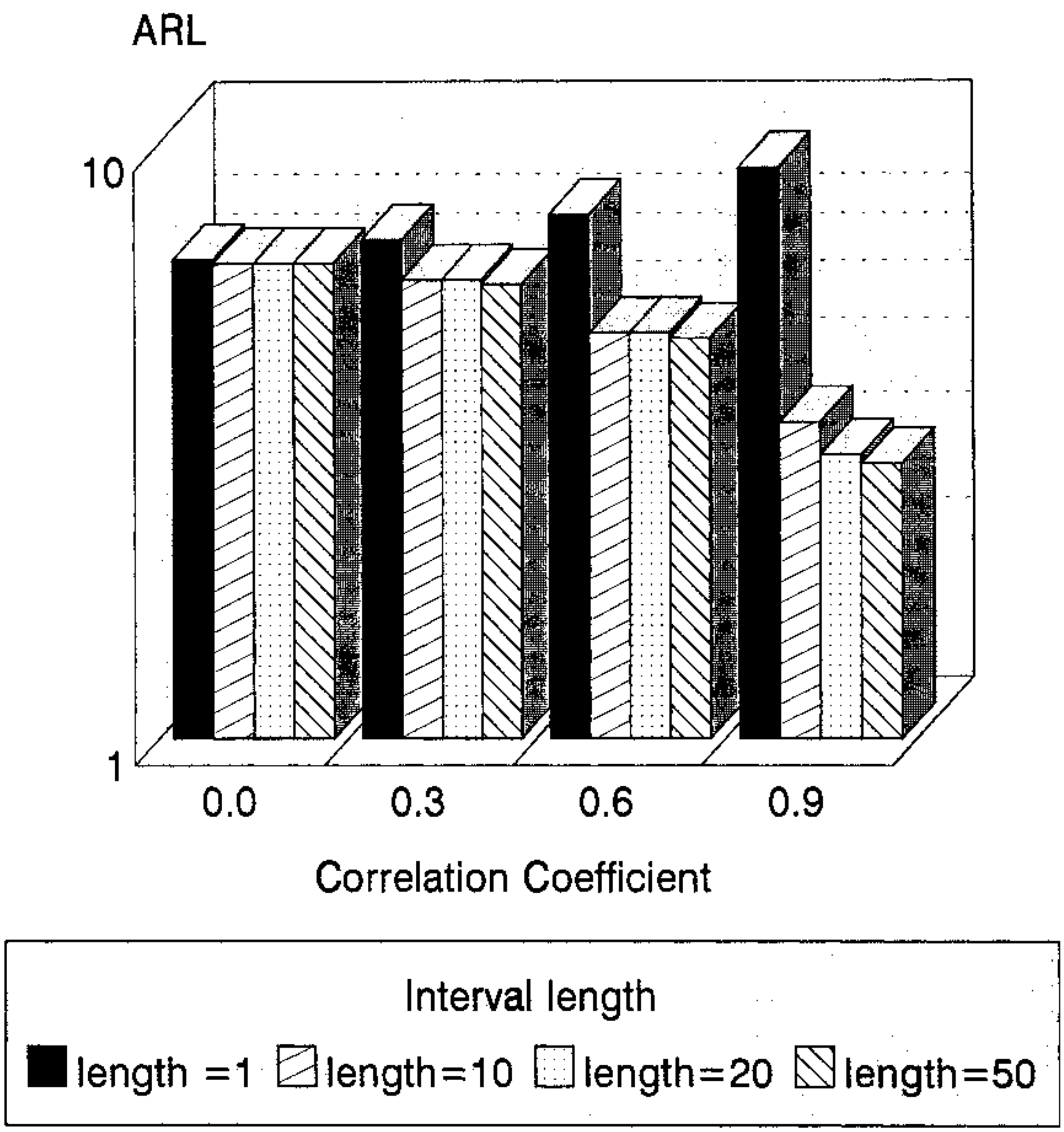


Figure 20f: Effect of interval length on ARL for FSI with pre-set control limits
Sample size = 1   Shift = 2.0
y axis: logarithmic scale

INTERVAL LENGTH v. ARL
FSI with Fixed Limits
Shift in Target Mean = 3.0

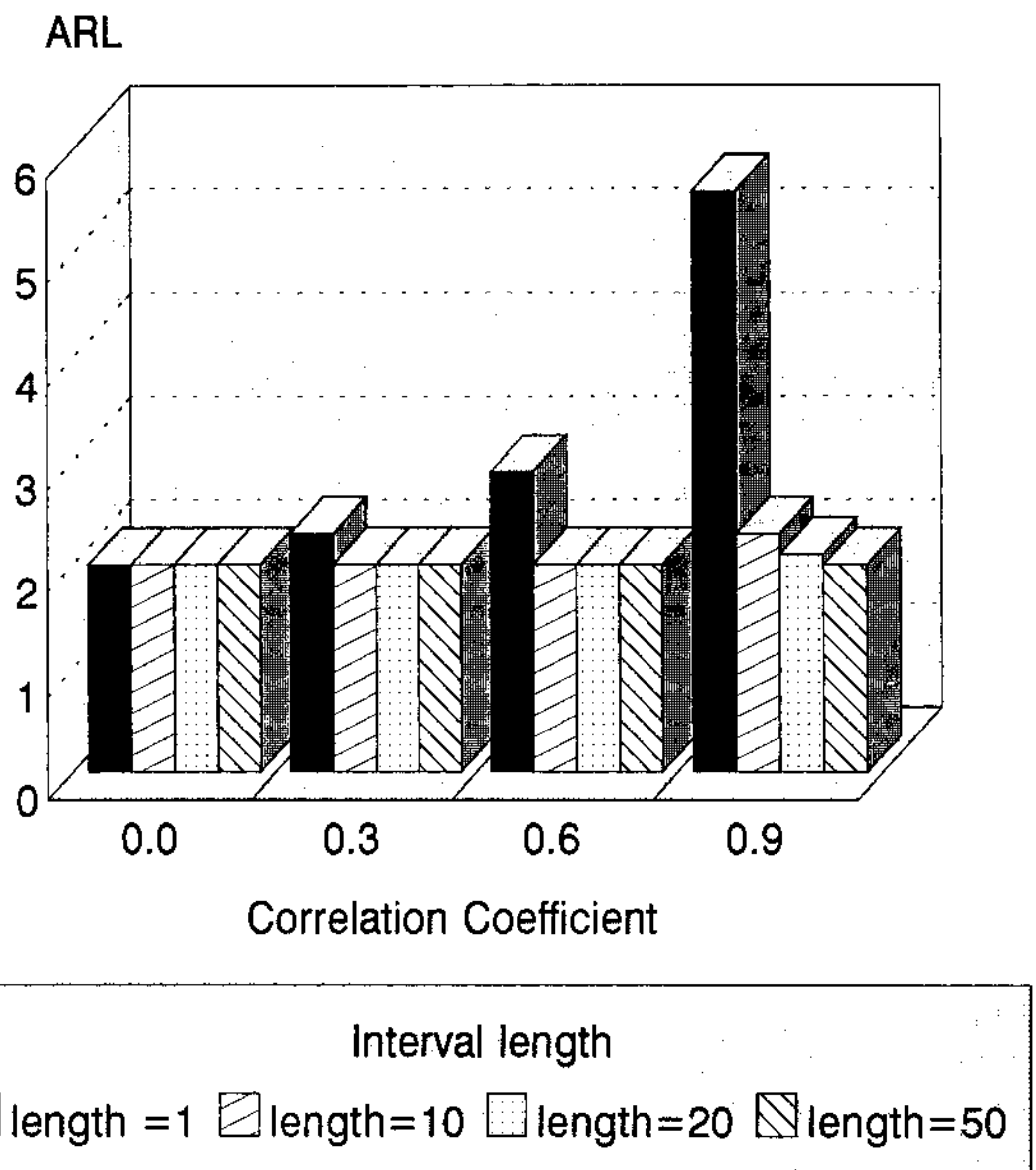


Figure 20g: Effect of Interval Length on ARL for FSI with pre-set control limits
Sample size = 1 Shift = 3.0
y axis: linear scale

INTERVAL LENGTH v. ARL

FSI with Fixed Limits

Shift in Target Mean = 3.0

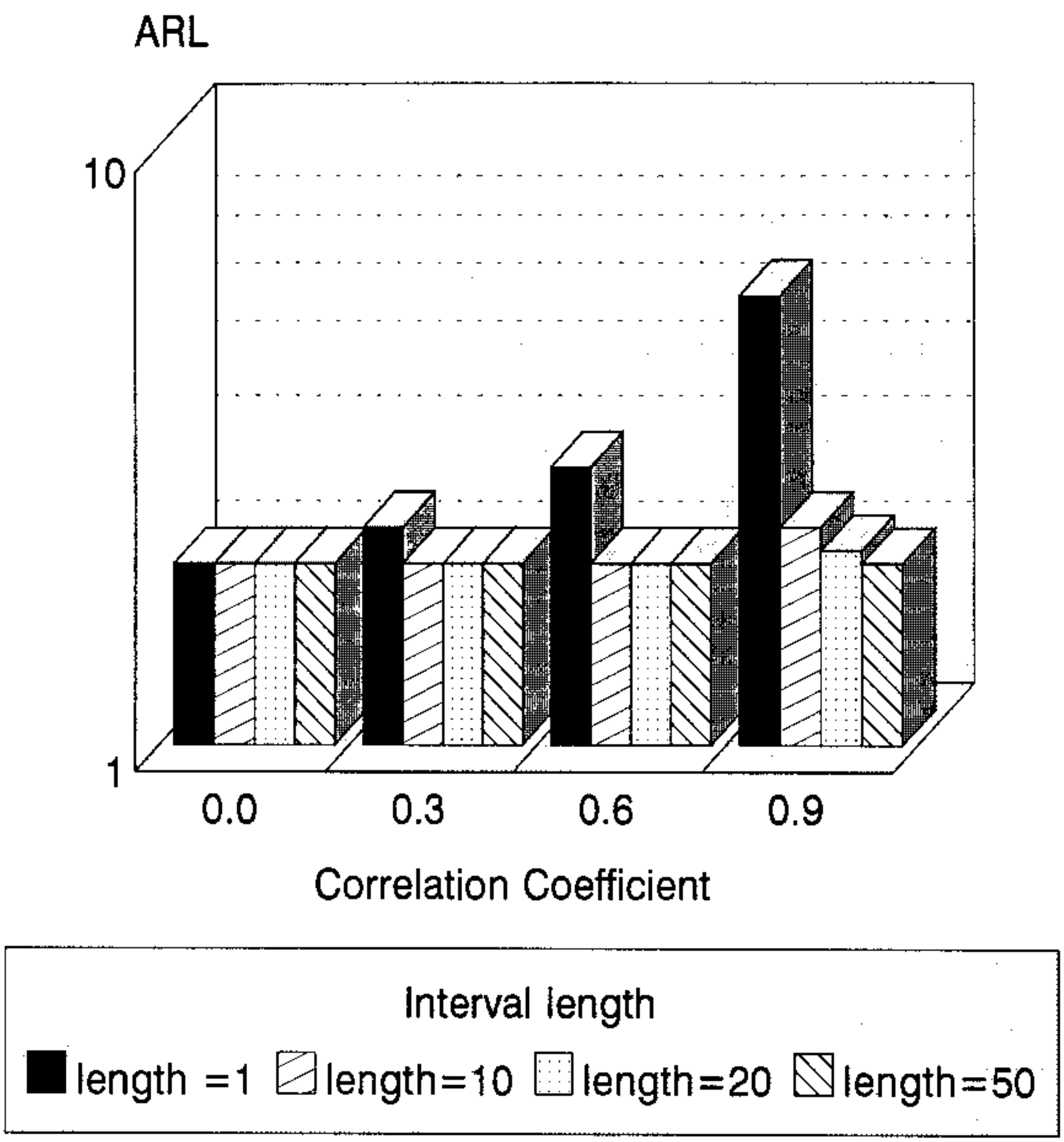


Figure 20h: Effect of Interval Length on ARL for FSI with pre-set control limits
Sample size = 1 Shift = 3.0
y axis: logarithmic scale

INTERVAL LENGTH v. ARL
FSI with Calculated Limits
Shift = 0.0 Sample size = 1

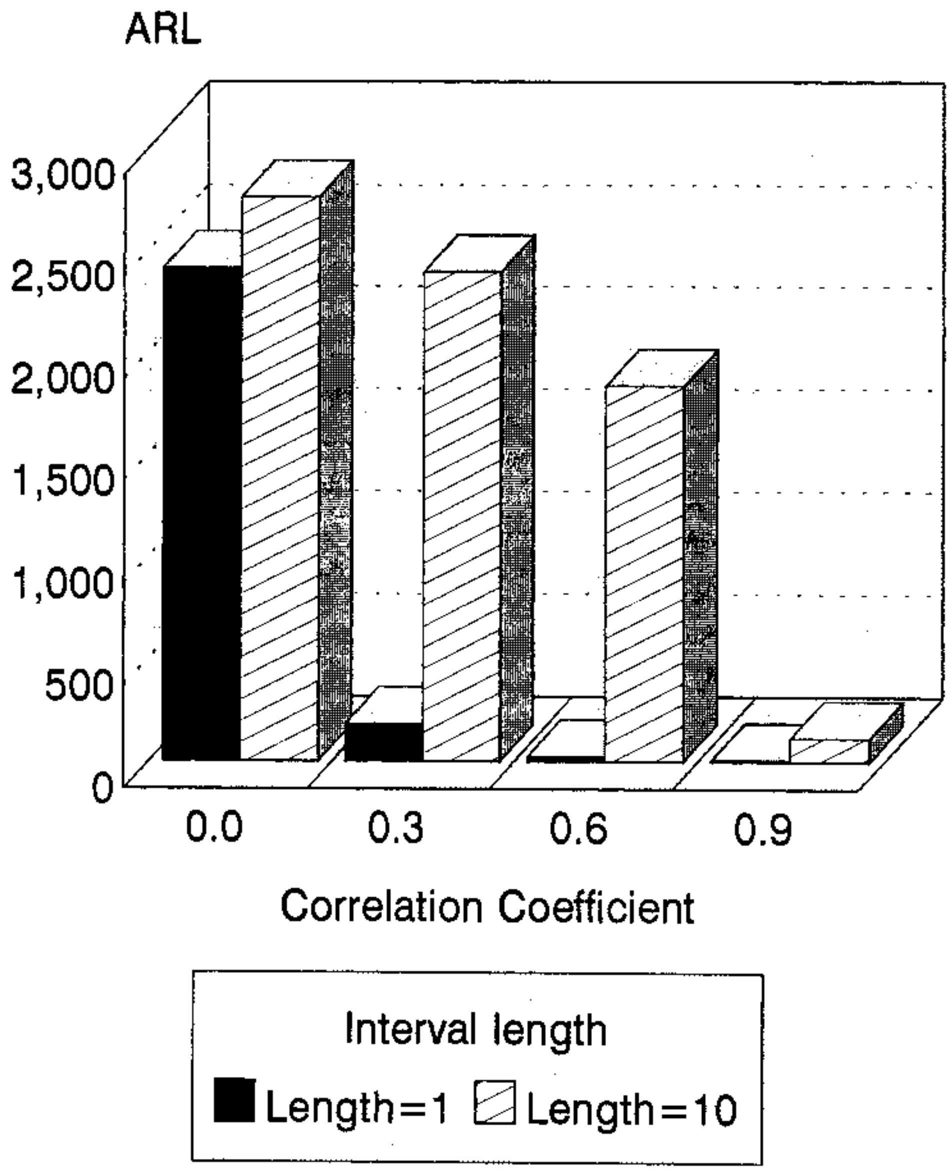


Figure 21a: Effect of Interval length on ARL for FSI with limits calculated from sample data
Sample size = 1 Shift = 0.0
y axis: linear scale

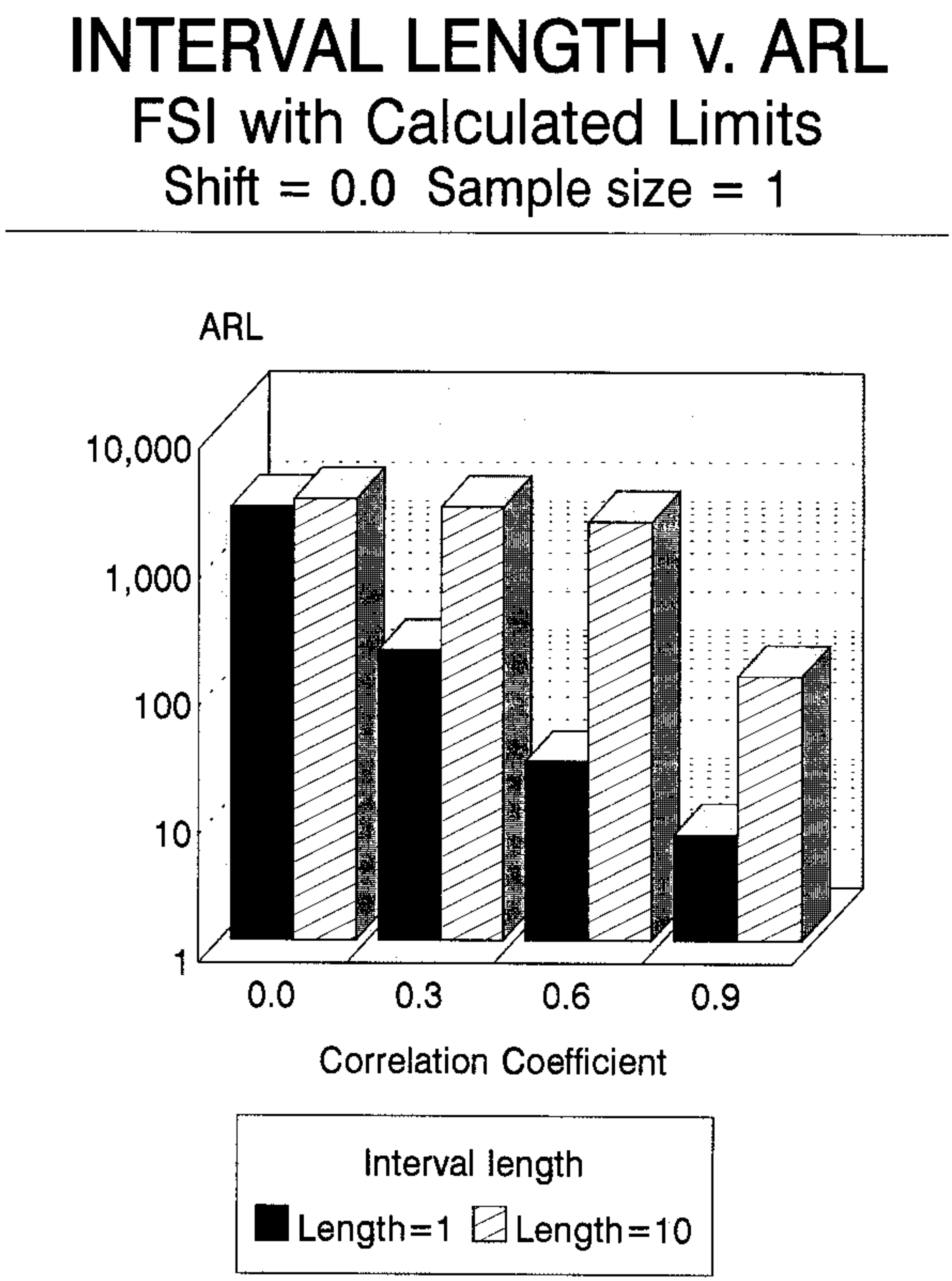


Figure 21b: Effect of interval length on ARL for FSI with limits calculated from sample data
Sample size = 1 Shift = 0.0
y axis: logarithmic scale

INTERVAL LENGTH v. ARL
FSI with Calculated Limits
Shift = 1.0   Sample size = 1

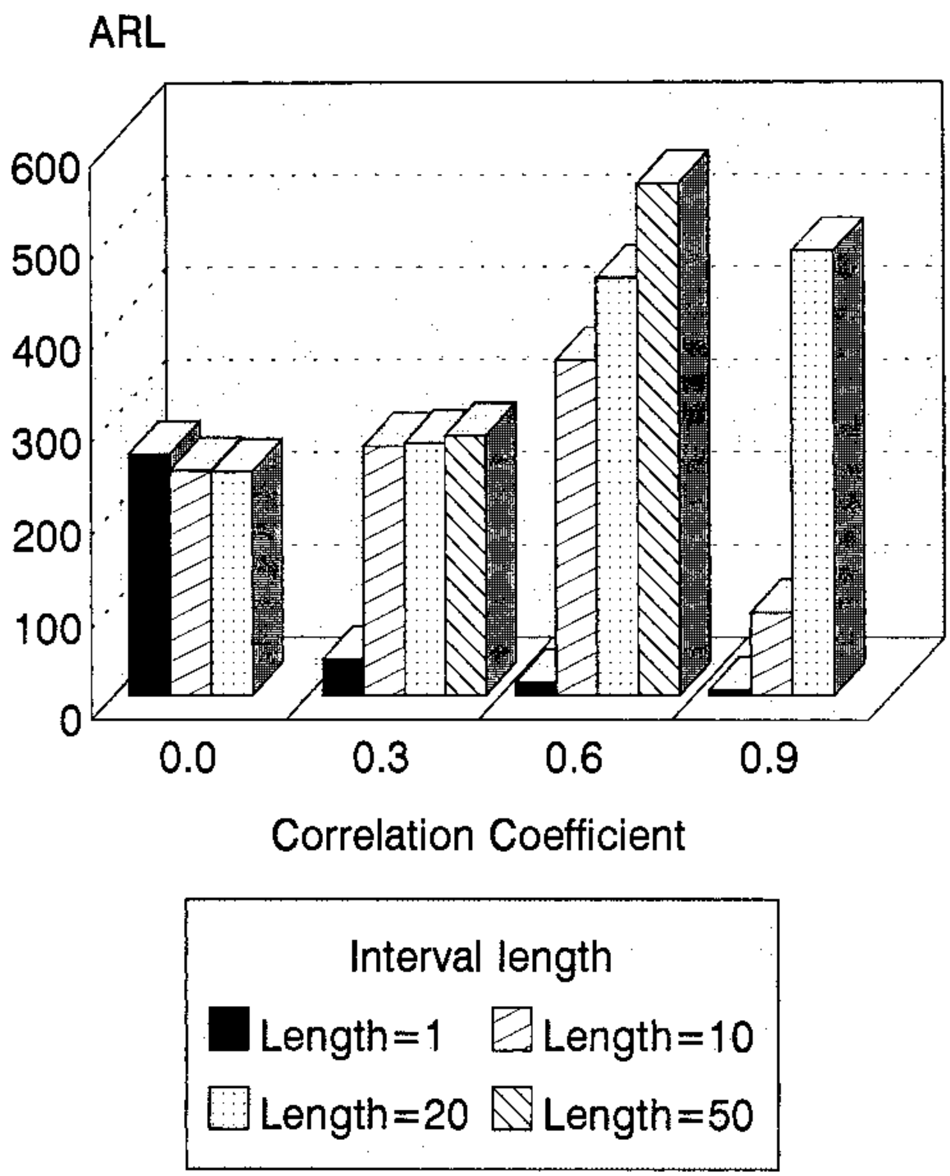


Figure 21c: Effect of Interval length on ARL for FSI with limits calculated from sample data
Sample size = 1   Shift = 1.0
y axis: linear scale

INTERVAL LENGTH v. ARL
FSI with Calculated Limits
Shift = 1.0 Sample size = 1

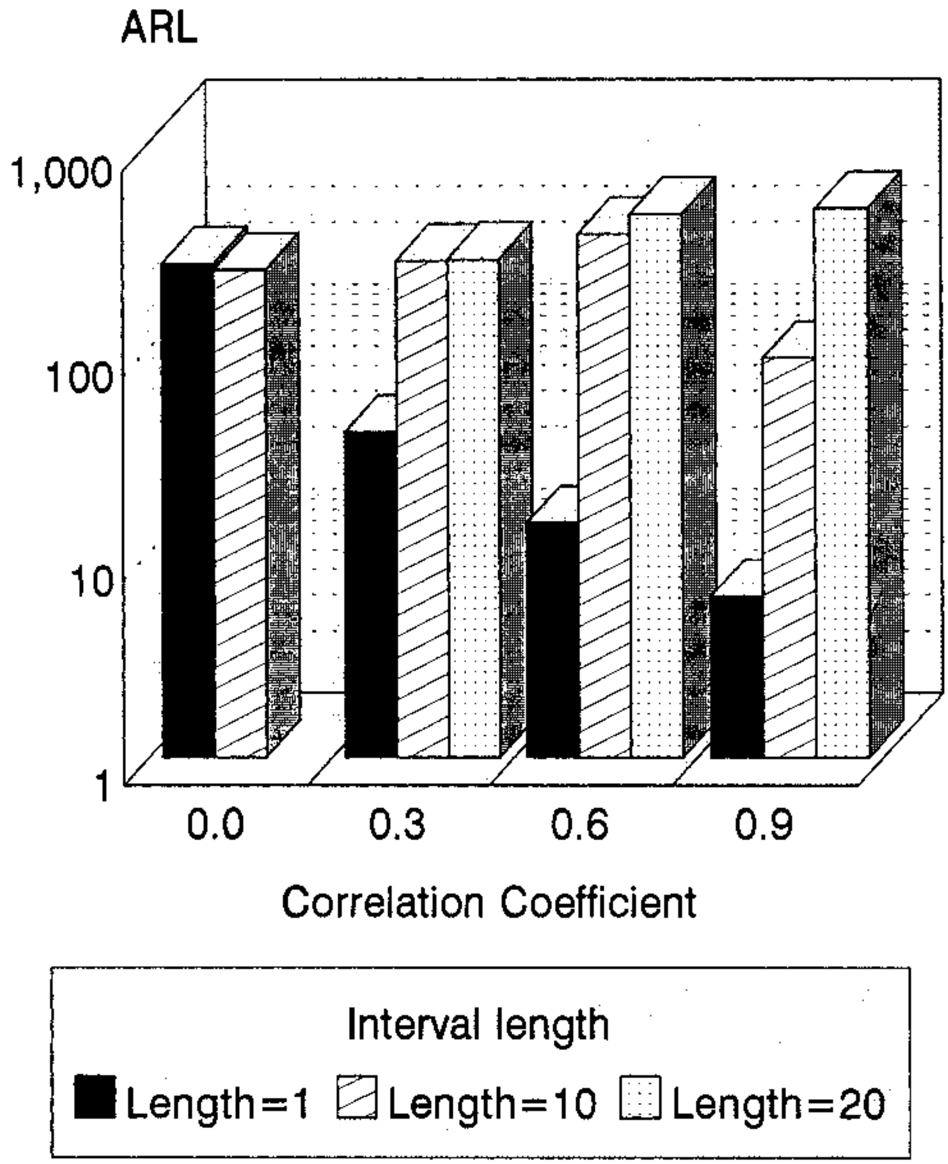


Figure 21d: Effect of interval length on ARL for FSI with limits calculated from sample data
Sample size = 1 Shift = 1.0
y axis: logarithmic scale

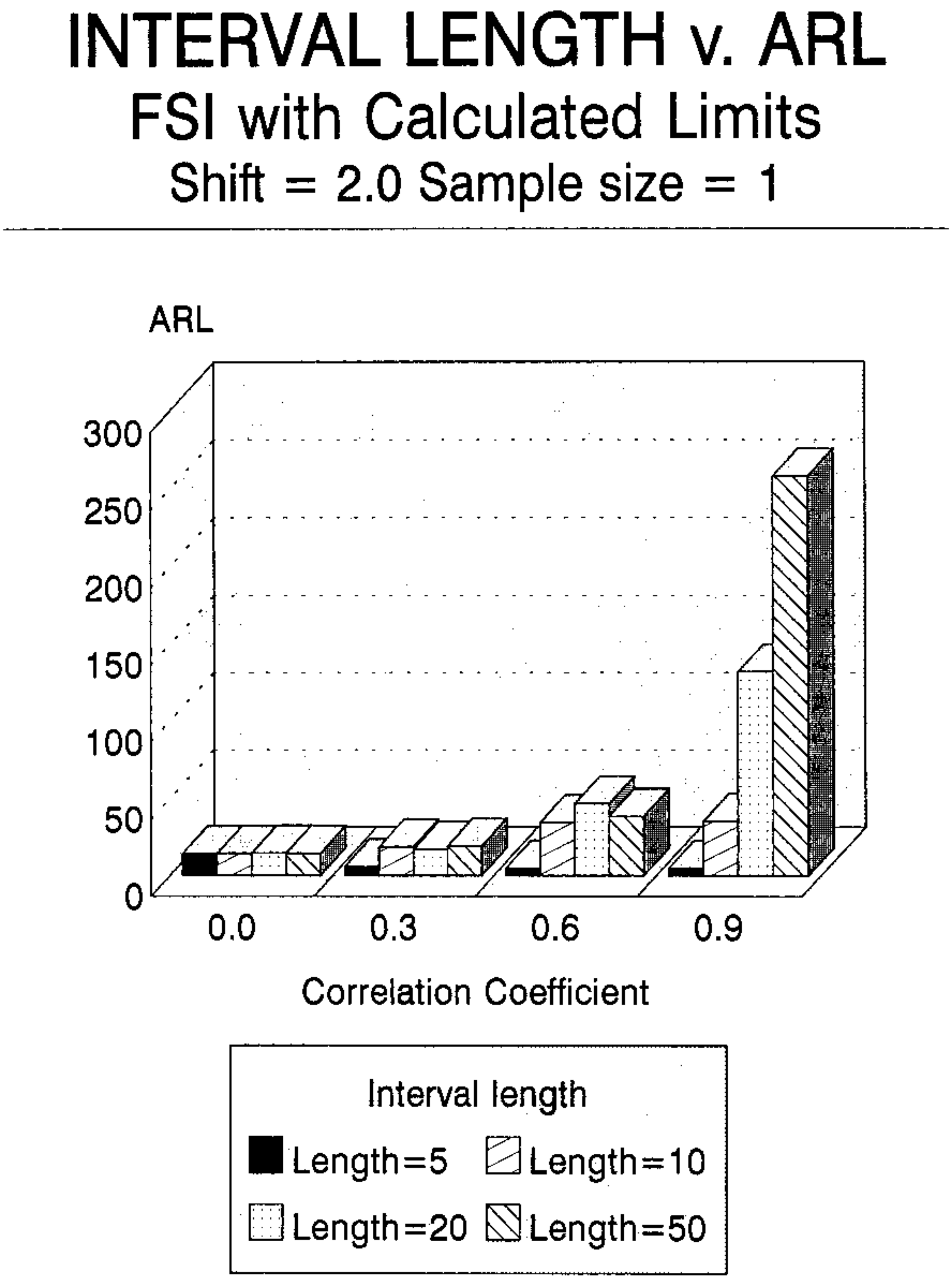


Figure 21e: Effect of Interval length on ARL for FSI with limits calculated from sample data
Sample size = 1 Shift = 2.0
y axis: linear scale

INTERVAL LENGTH v. ARL
FSI with Calculated Limits
Shift = 2.0 Sample size = 1

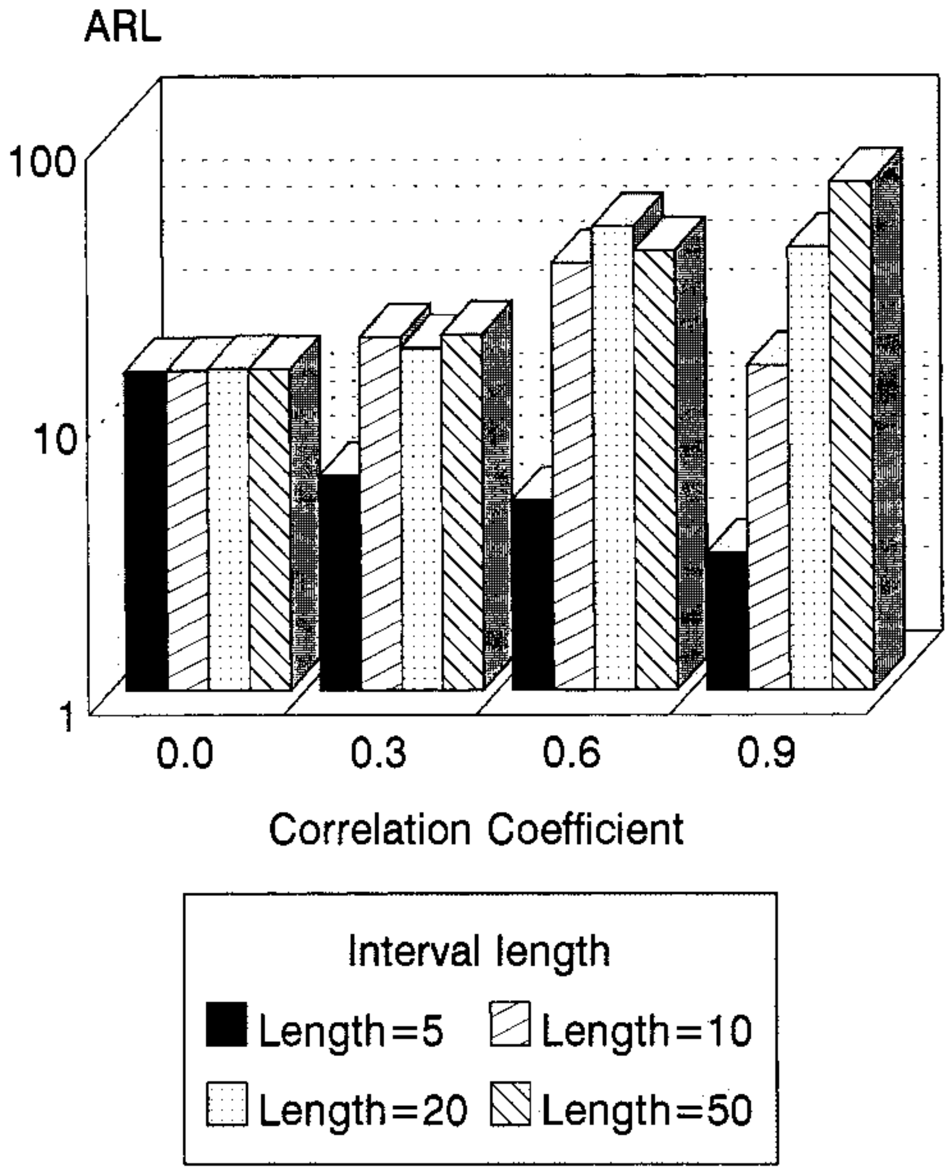


Figure 21f: Effect of Interval length on ARL for FSI with limits calculated from sample data
Sample size = 1 Shift = 2.0
y axis: logarithmic scale

INTERVAL LENGTH v. ARL
FSI with Calculated Limits
Shift = 3.0 Sample size = 1

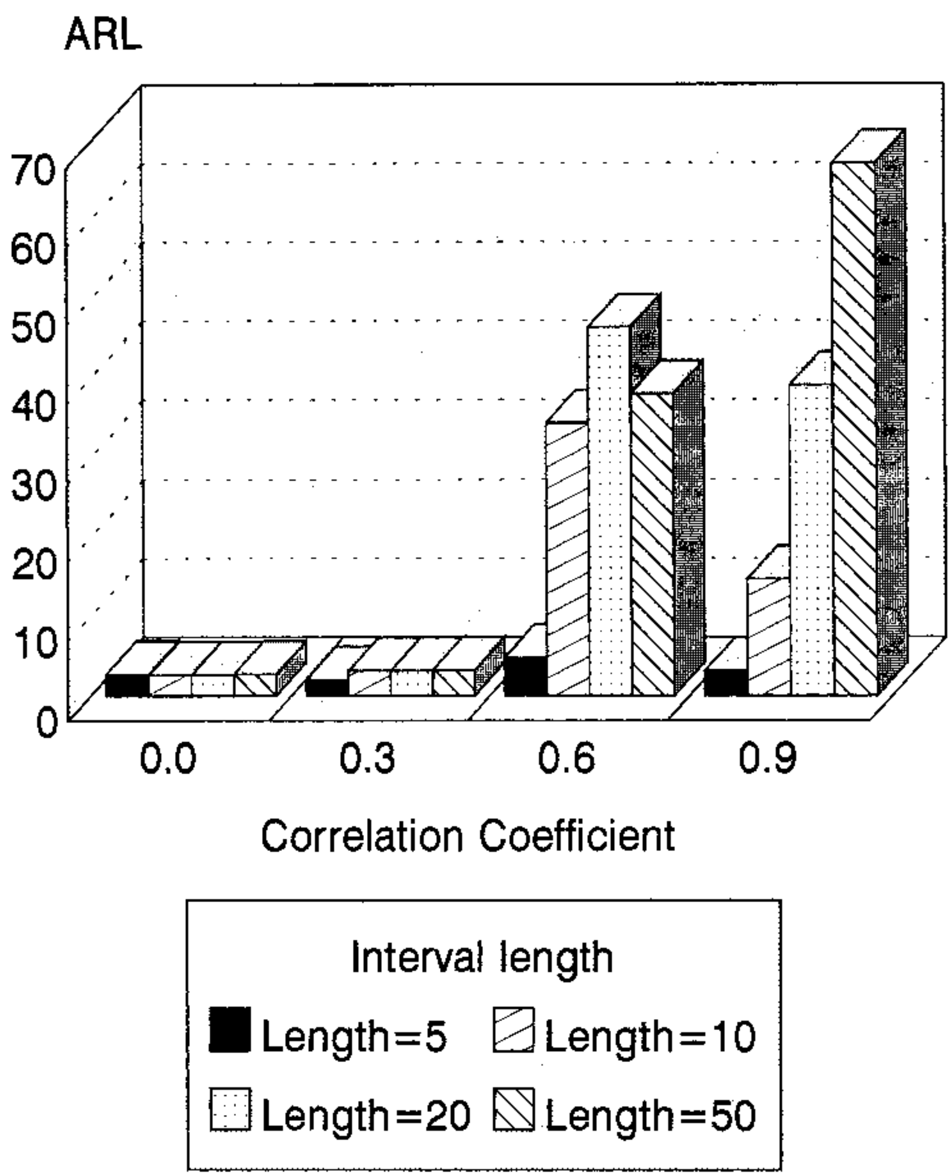


Figure 21g: Effect of interval length on ARL for FSI with limits calculated from sample data
Sample size = 1 Shift = 3.0
y axis: linear scale

INTERVAL LENGTH v. ARL
FSI with Calculated Limits
Shift = 3.0 Sample size = 1

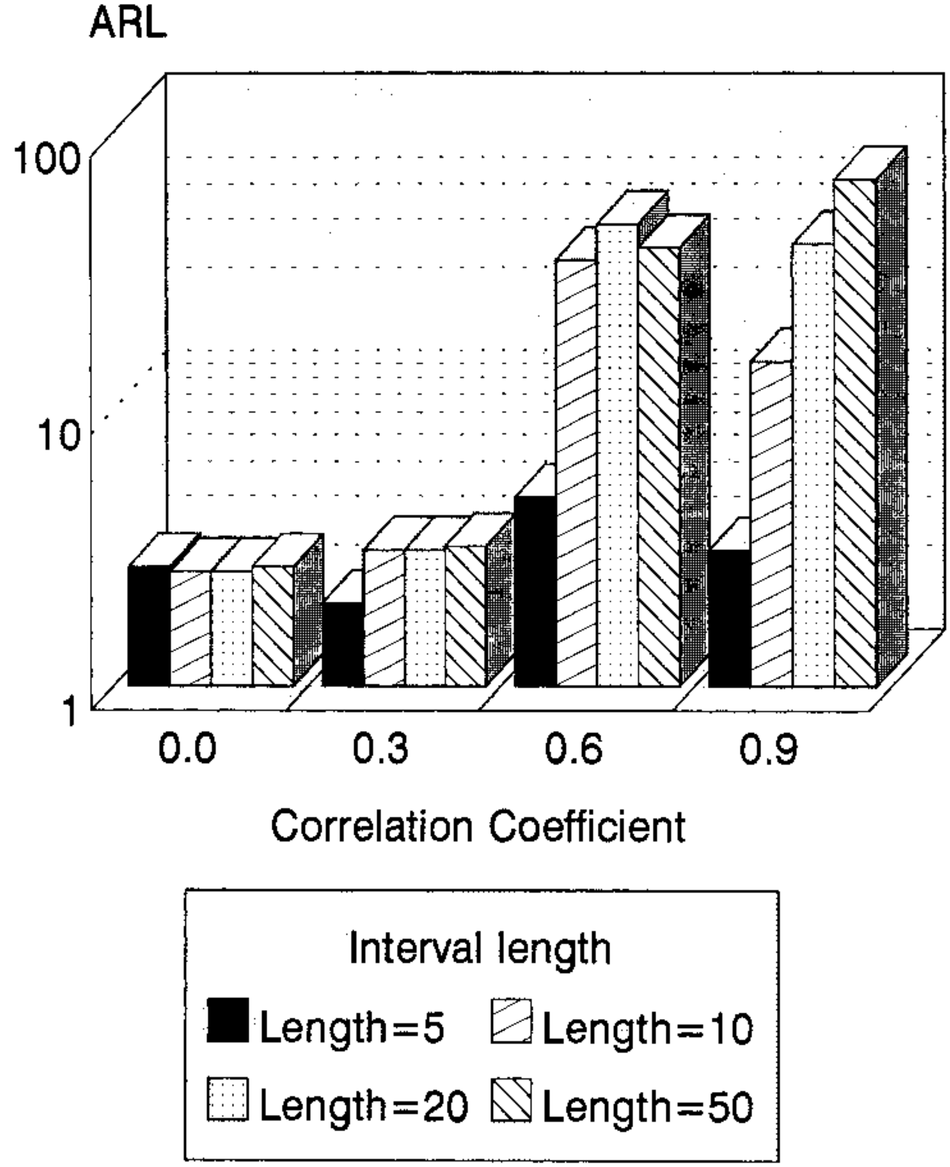


Figure 21h: Effect of Interval length on ARL for FSI with limits calculated from sample data
Sample size = 1 Shift = 3.0
y axis: logarithmic scale

INTERVAL LENGTH v. ANSS

VSI with Fixed Limits

Shift = 0.0 Sample size = 1

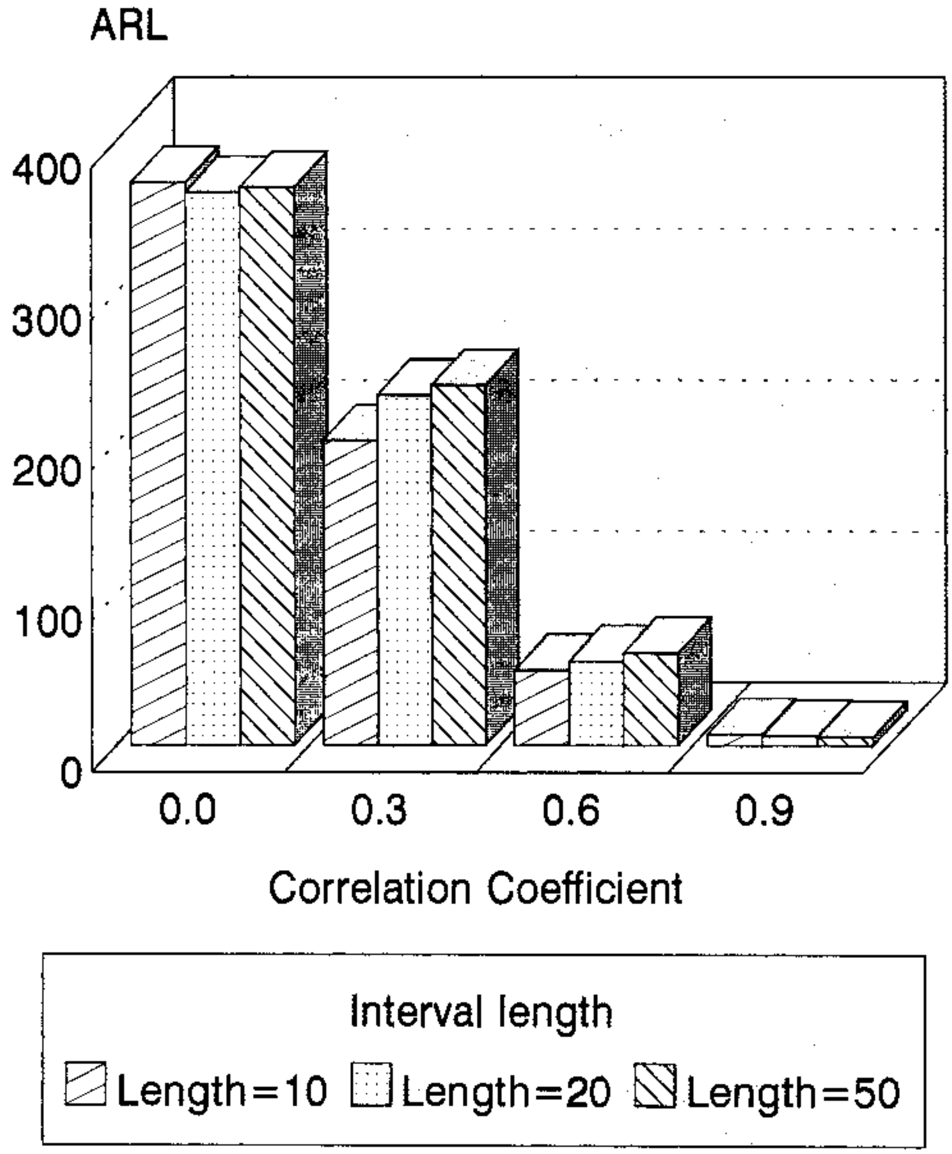


Figure 22a: Effect of Interval length on ARL for VSI with pre-set limits
Sample size = 1 Shift = 0.0
y axis: linear scale

INTERVAL LENGTH v. ANSS

VSI with Fixed Limits

Shift = 0.0 Sample size = 1

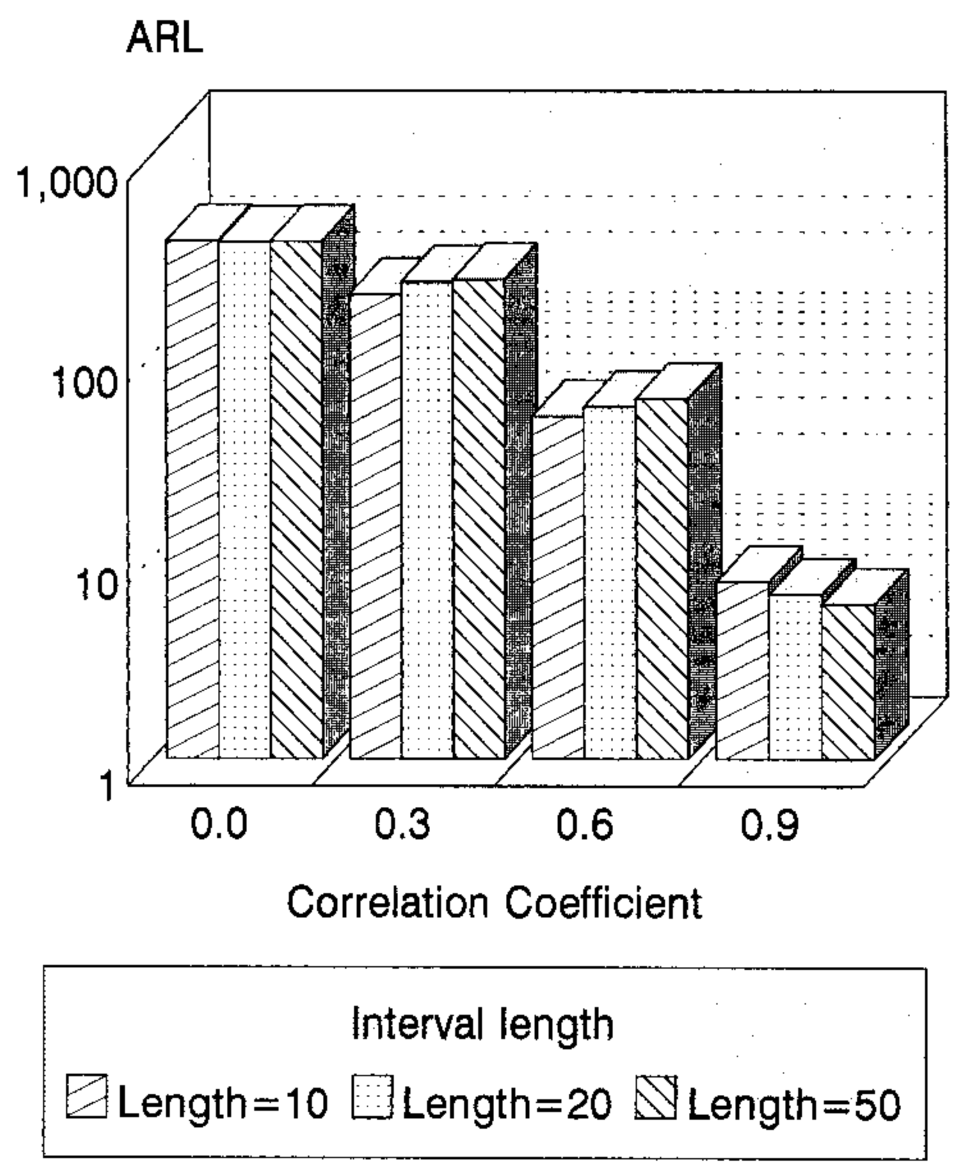


Figure 22b: Effect of Interval length on ARL for VSI with pre-set limits
Sample size = 1 Shift = 0.0
y axis: logarithmic scale

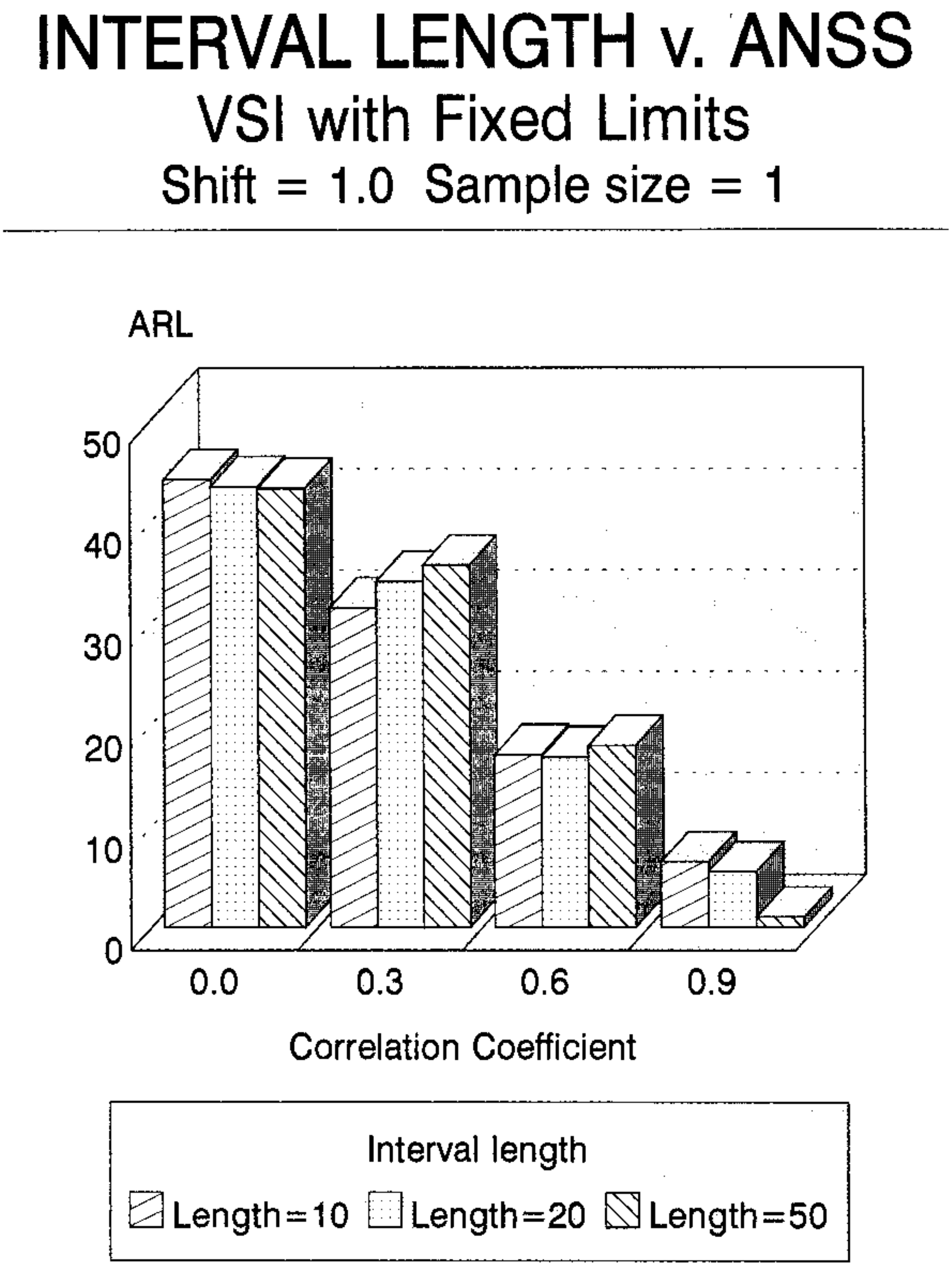


Figure 22c: Effect of Interval length on ARL for VSI with pre-set limits
Sample size = 1 Shift = 1.0
y axis: linear scale

INTERVAL LENGTH v. ANSS
VSI with Fixed Limits
Shift = 1.0 Sample size = 1

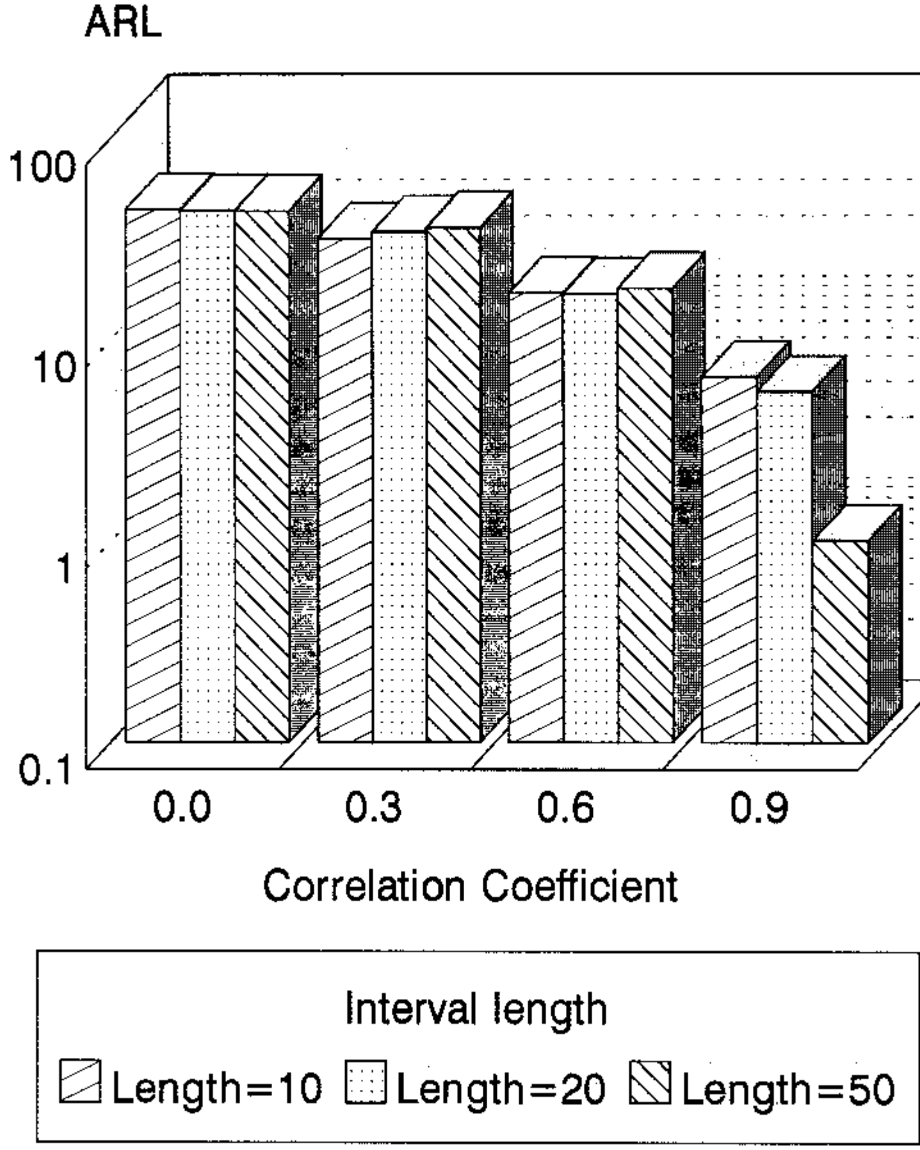


Figure 22d: Effect of Interval length on ARL for VSI with pre-set limits
Sample size = 1   Shift = 1.0
y axis: logarithmic scale

INTERVAL LENGTH v. ANSS

VSI with Fixed Limits

Shift = 2.0 Sample size = 1

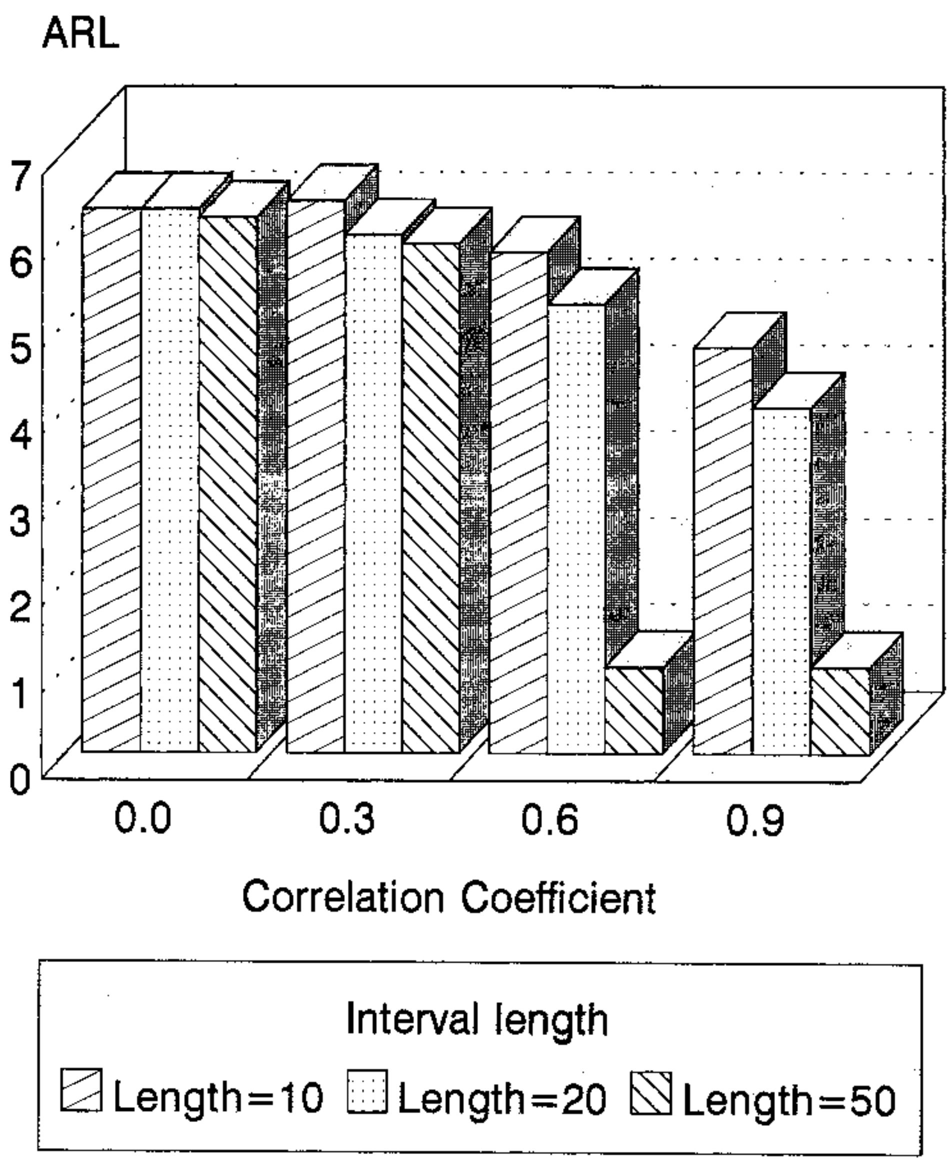


Figure 22e: Effect of Interval length on ARL for VSI with pre-set limits
Sample size = 1   Shift = 2.0
y axis: linear scale

INTERVAL LENGTH v. ANSS

VSI with Fixed Limits

Shift = 2.0 Sample size = 1

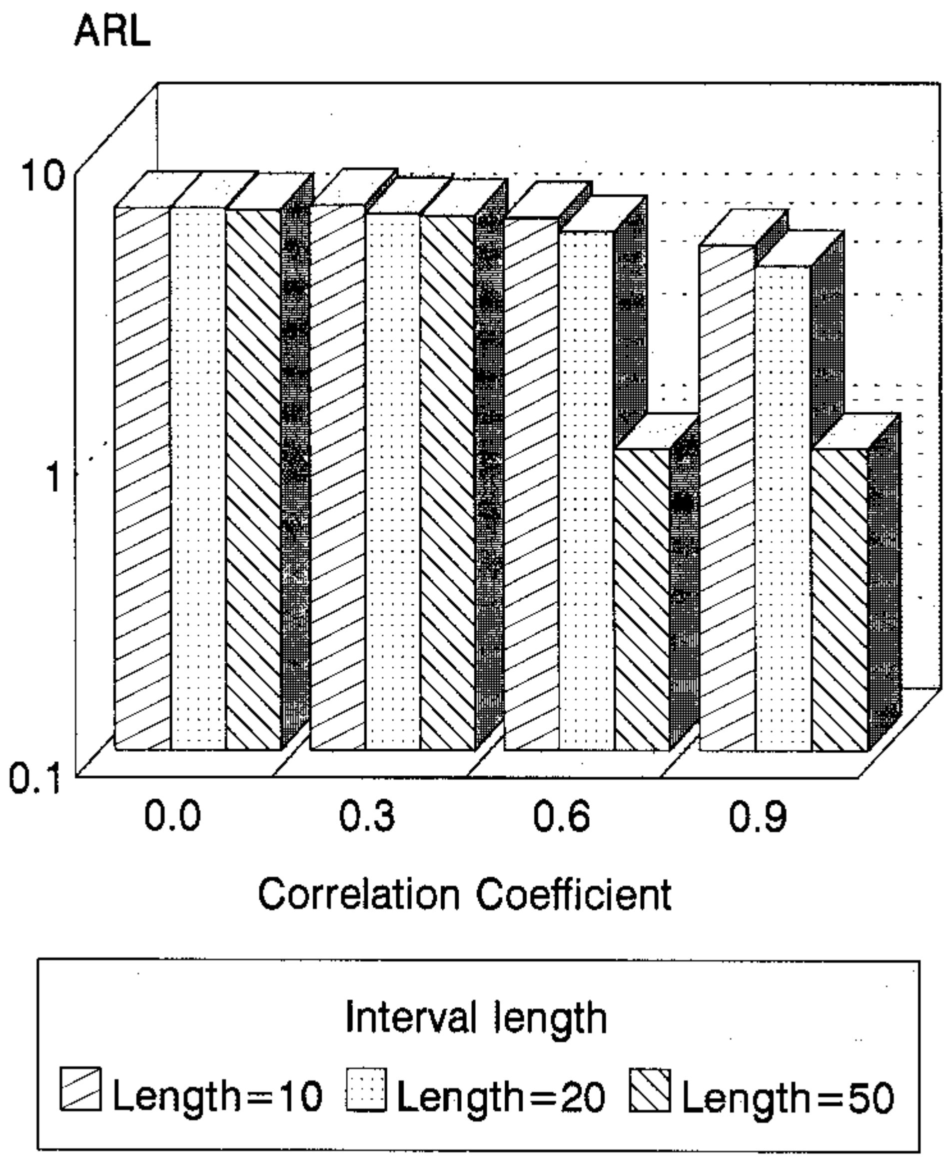


Figure 22f: Effect of Interval length on ARL for VSI with pre-set limits
Sample size = 1 Shift = 2.0
y axis: logarithmic scale

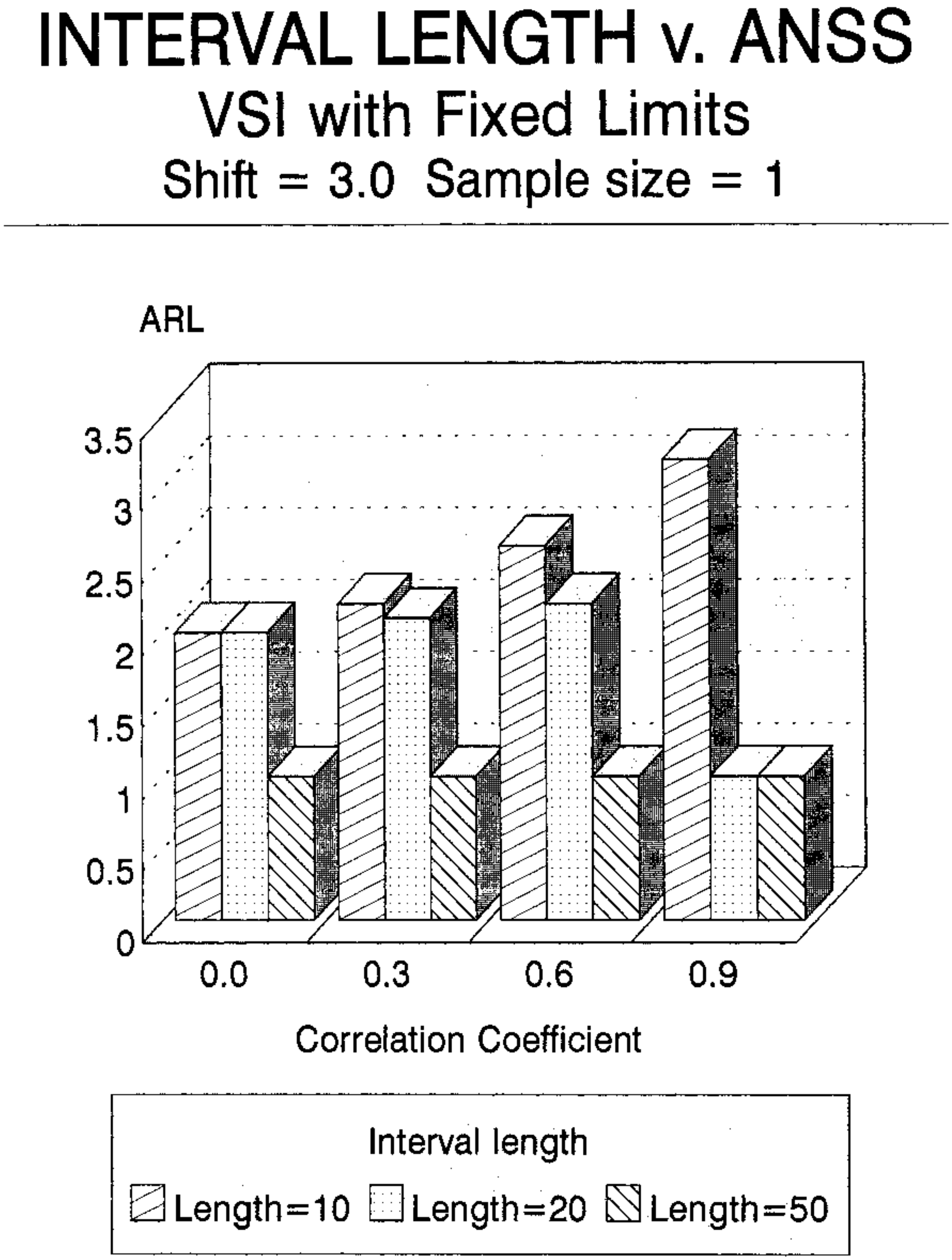


Figure 22g: Effect of Interval length on ARL for VSI with pre-set limits
Sample size = 1 Shift = 3.0
y axis: linear scale

INTERVAL LENGTH v. ANSS

VSI with Fixed Limits

Shift = 3.0 Sample size = 1

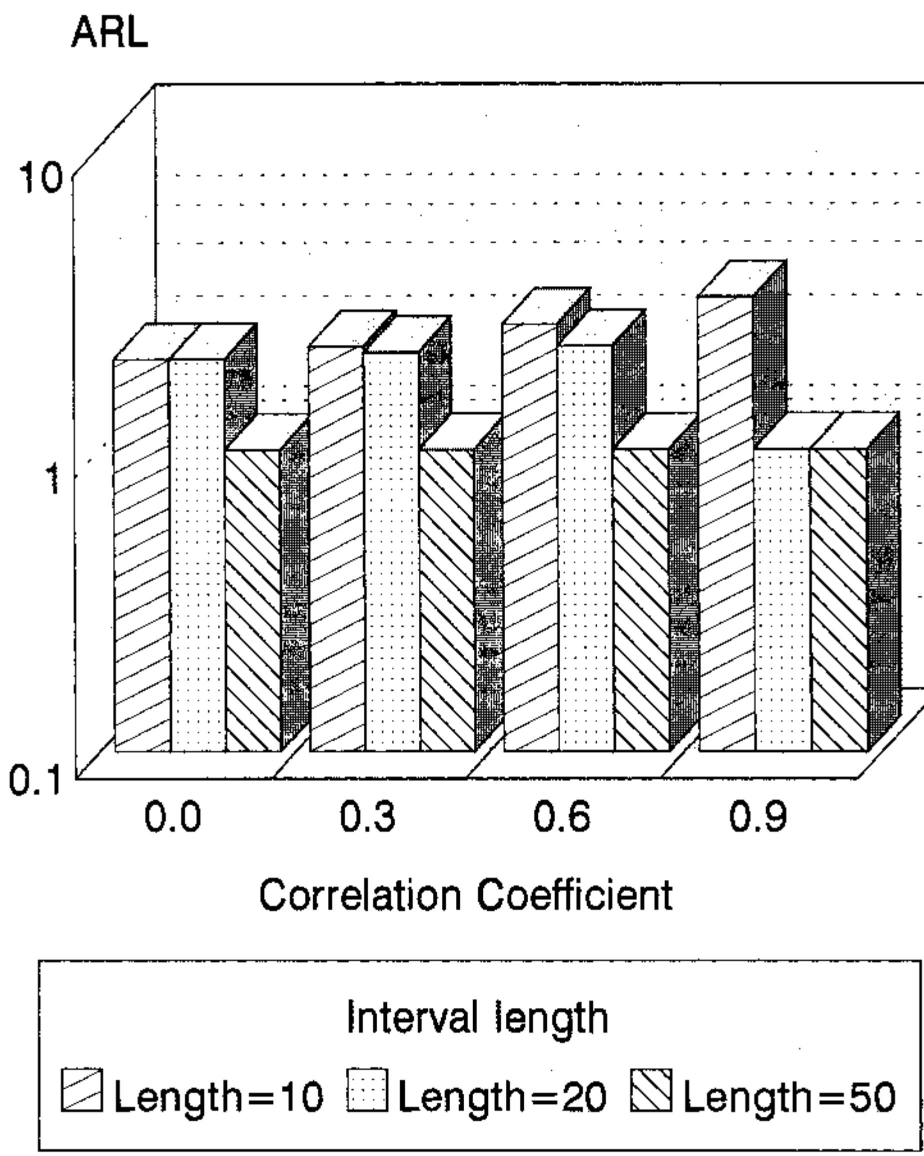


Figure 22h: Effect of Interval length on ARL for VSI with pre-set limits
Sample size = 1 Shift = 3.0
y axis: logarithmic scale

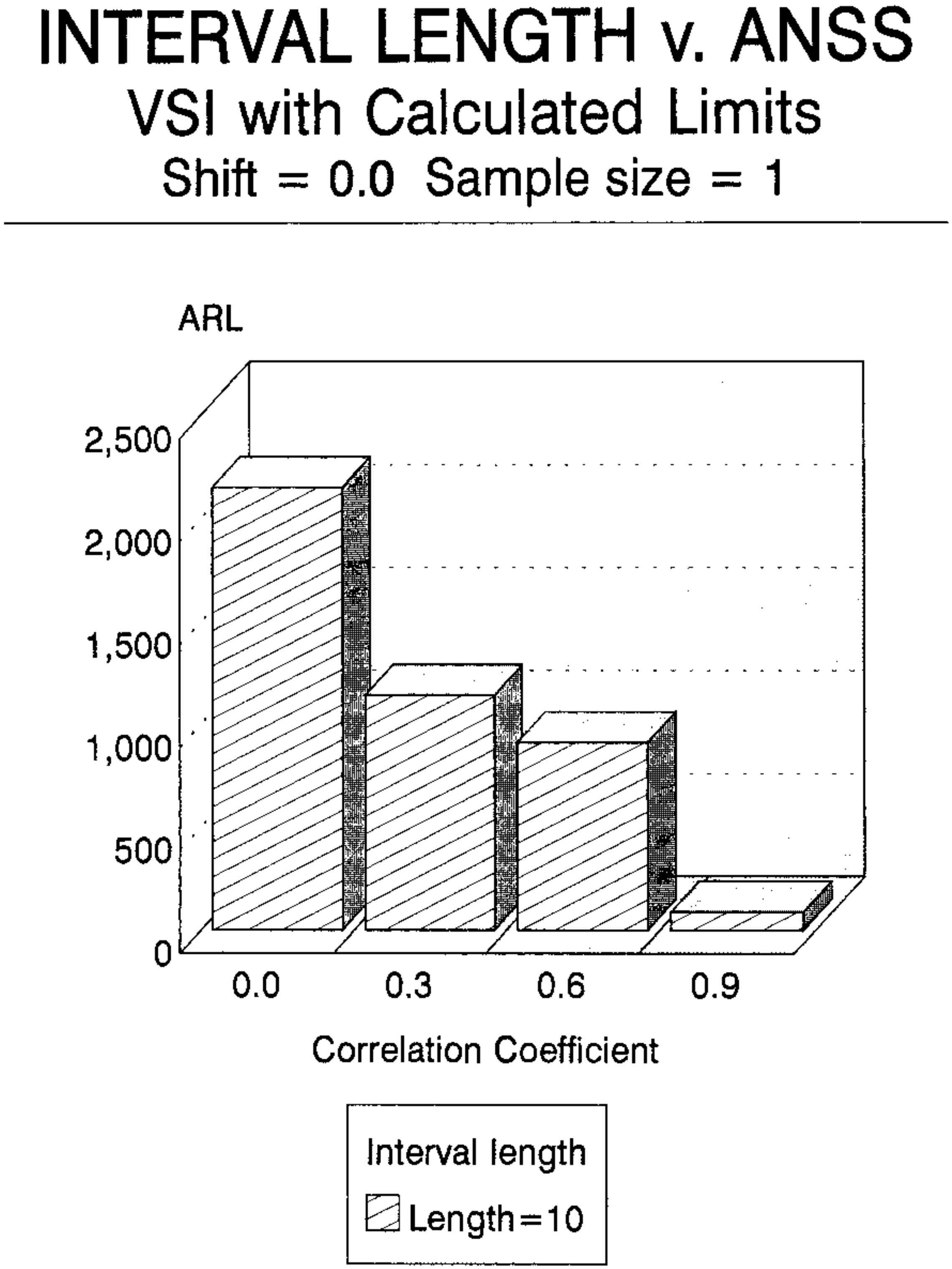


Figure 23a: Effect of interval length on ARL for VSI with limits calculated from sample data
Sample size = 1 Shift = 0.0
y axis: linear scale

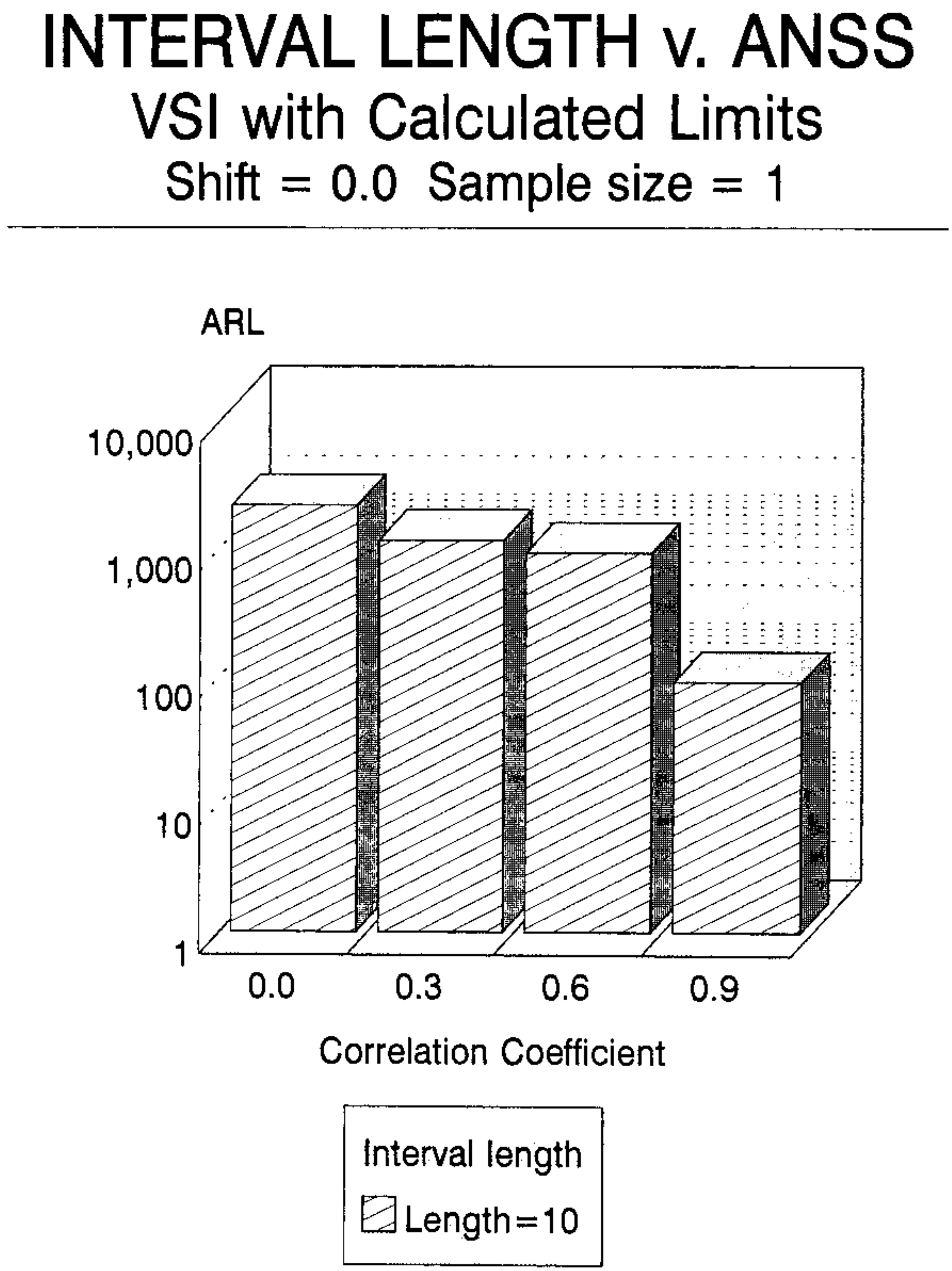


Figure 23b: Effect of Interval length on ARL for VSI with limits calculated from sample data
Sample size = 1 Shift = 0.0
y axis: logarithmic scale

INTERVAL LENGTH v. ANSS
VSI with Calculated Limits
Shift = 1.0 Sample size = 1

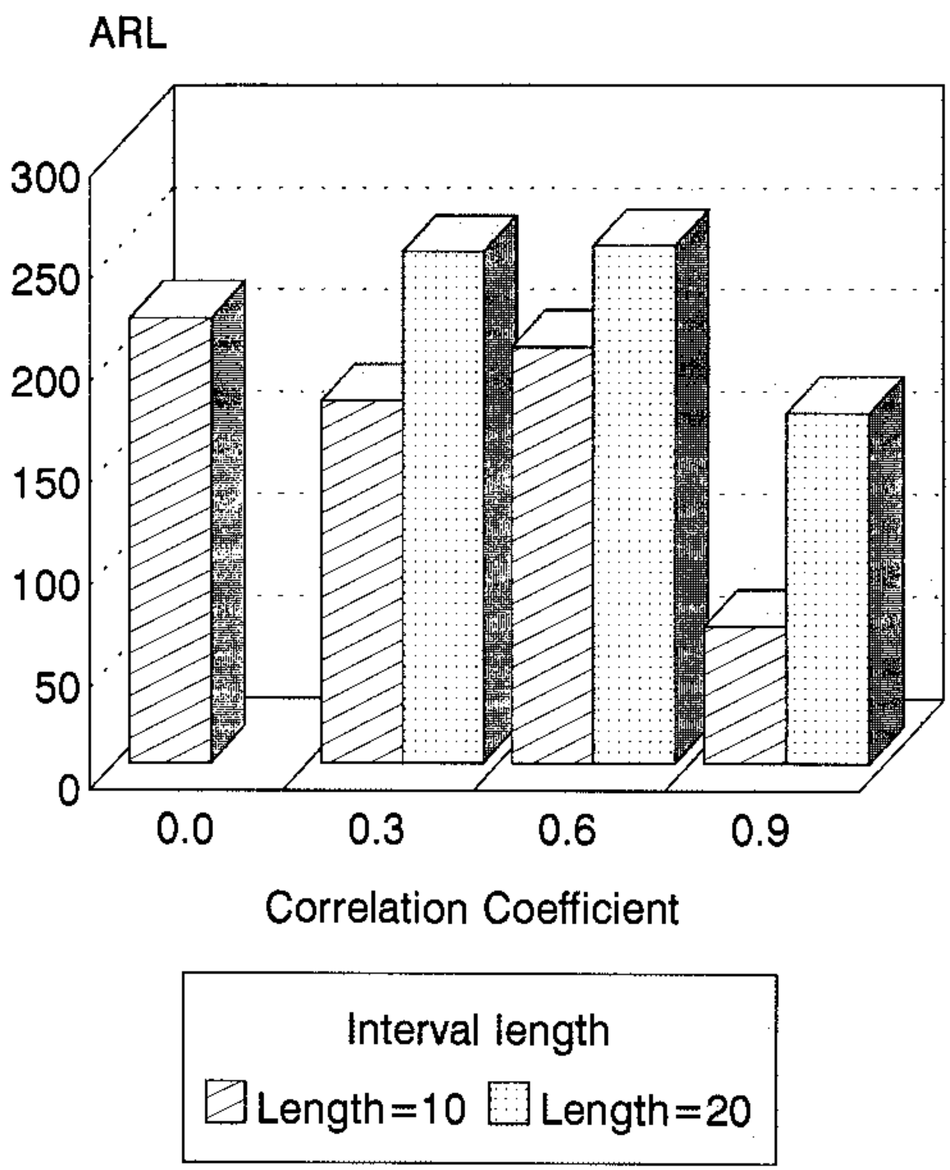


Figure 23c: Effect of interval length on ARL for VSI with limits calculated from sample data
Sample size = 1 Shift = 1.0
y axis: linear scale

INTERVAL LENGTH v. ANSS
VSI with Calculated Limits
Shift = 1.0 Sample size = 1

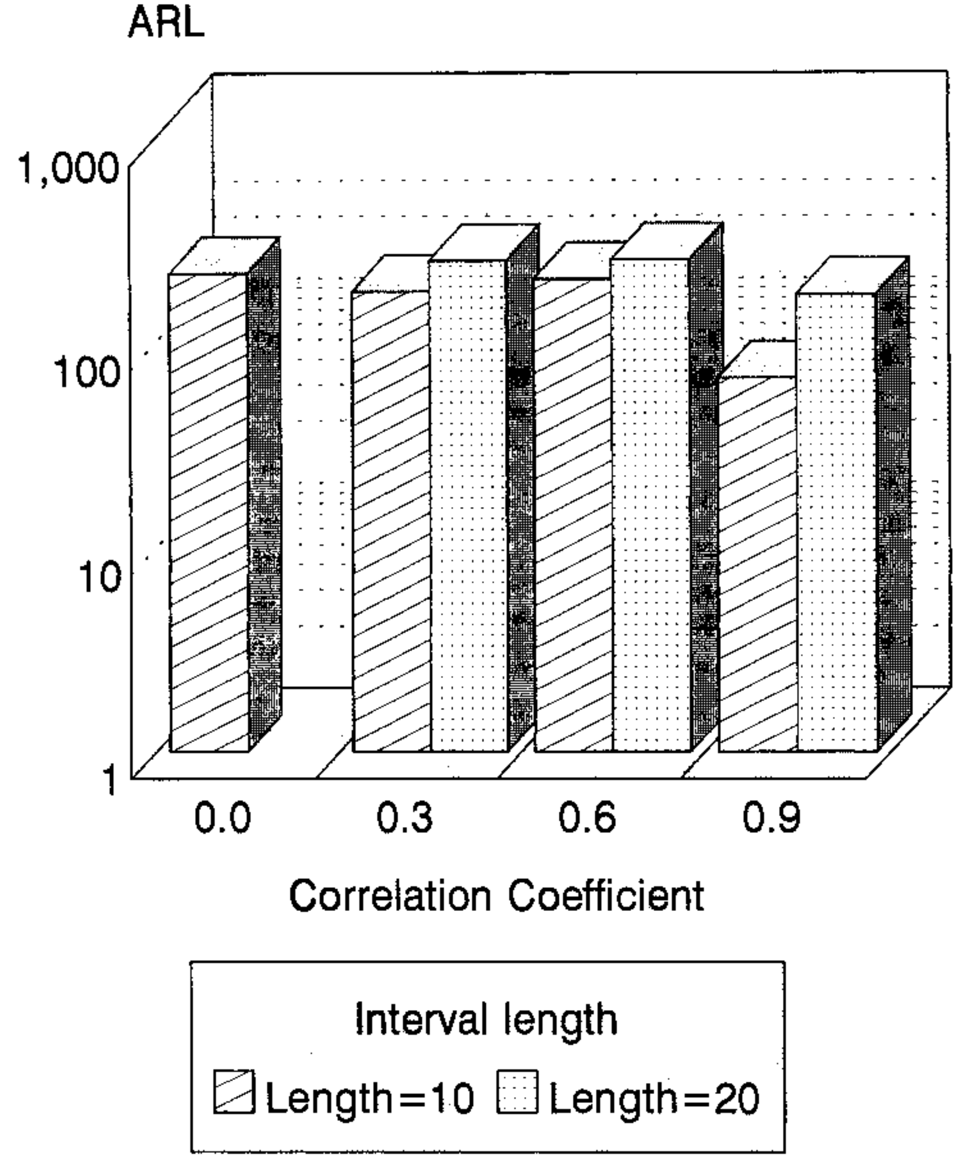


Figure 23d: Effect of interval length on ARL for VSI with limits calculated from sample data
Sample size = 1 Shift = 1.0
y axis: logarithmic scale

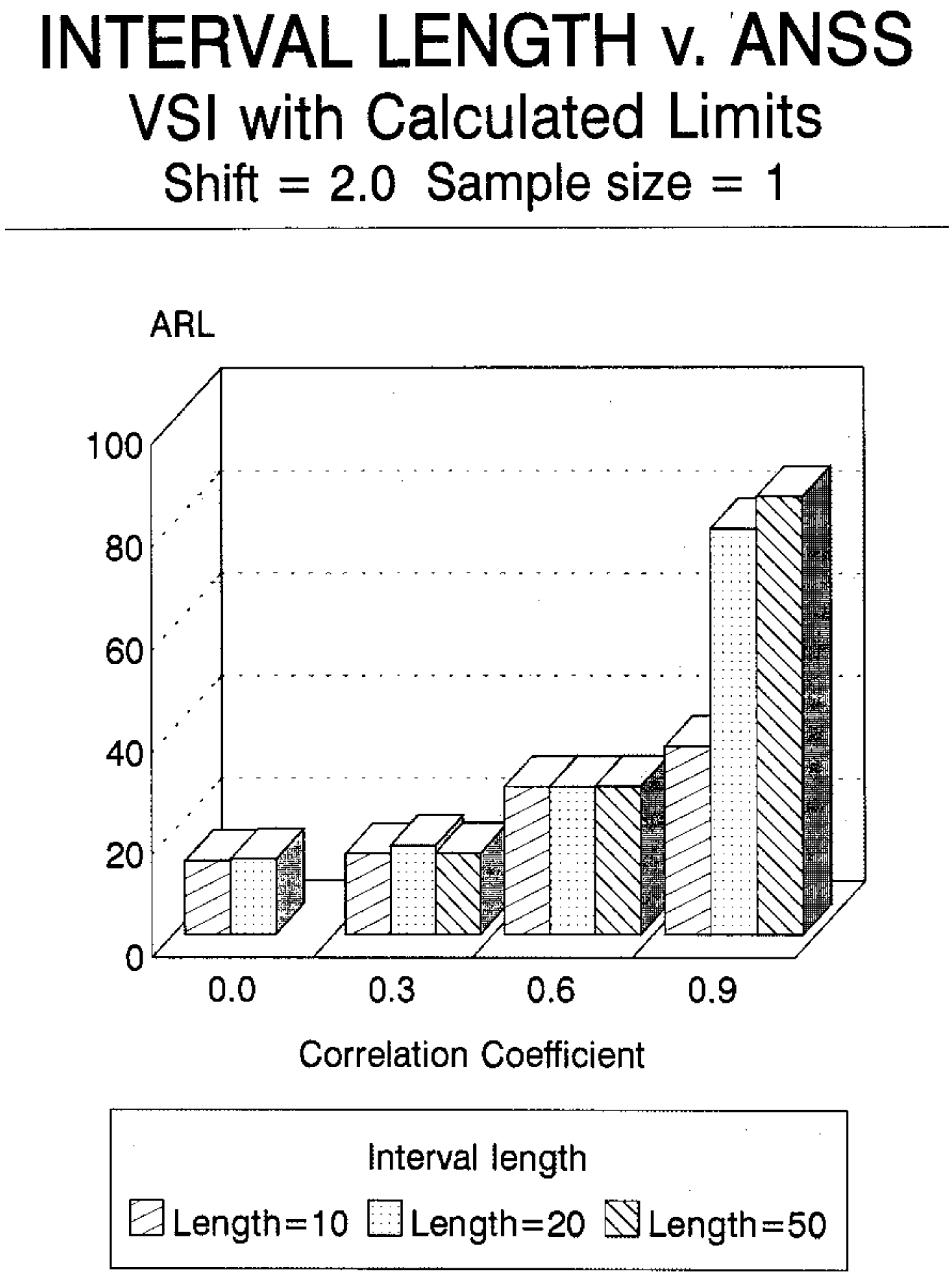


Figure 23e: Effect of Interval length on ARL for VSI with limits calculated from sample data
Sample size = 1 Shift = 2.0
y axis: linear scale

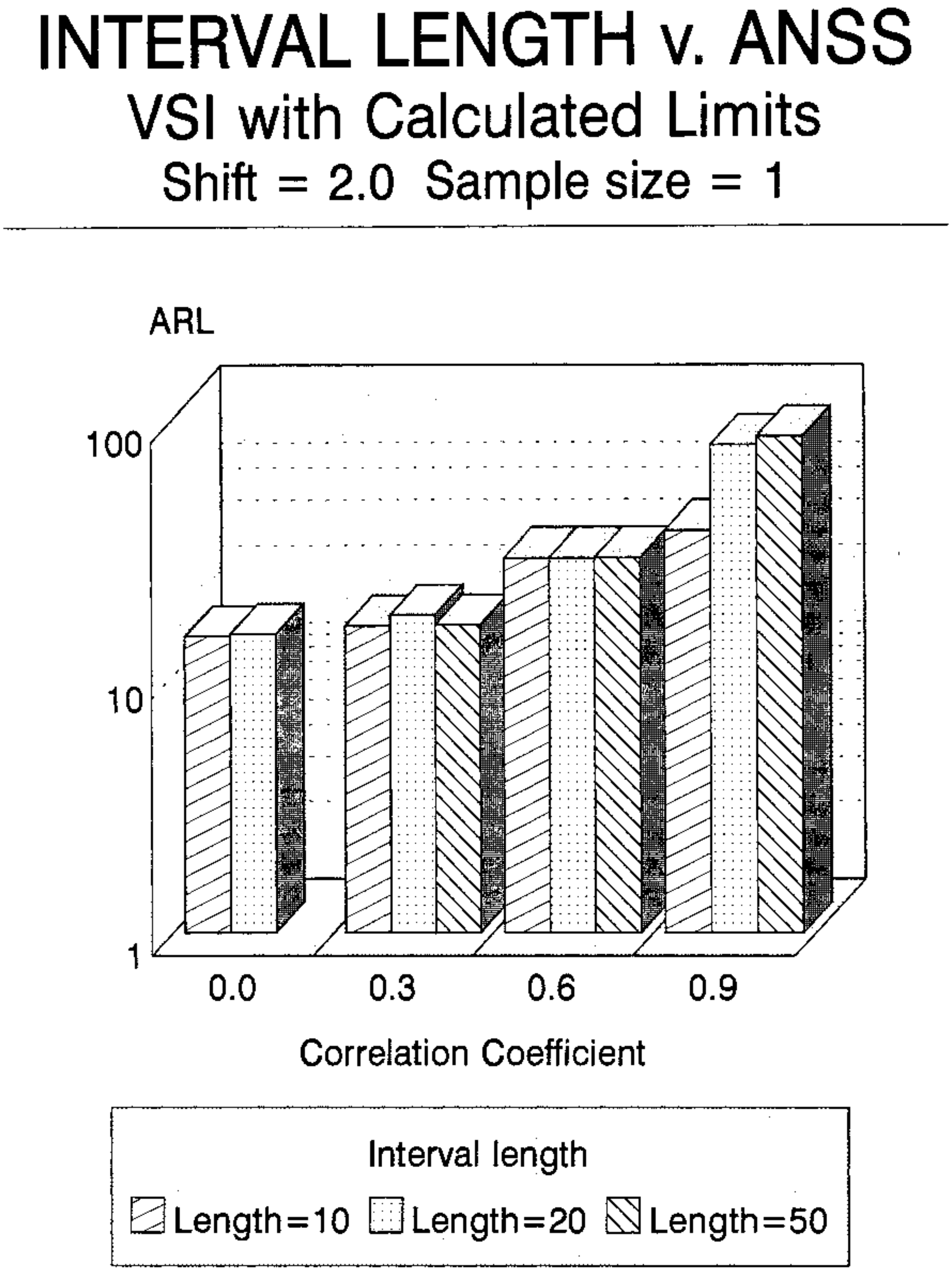


Figure 23f: Effect of Interval length on ARL for VSI with limits calculated from sample data
Sample size = 1 Shift = 2.0
y axis: logarithmic scale

INTERVAL LENGTH v. ANSS
VSI with Calculated Limits
Shift = 3.0 Sample size = 1

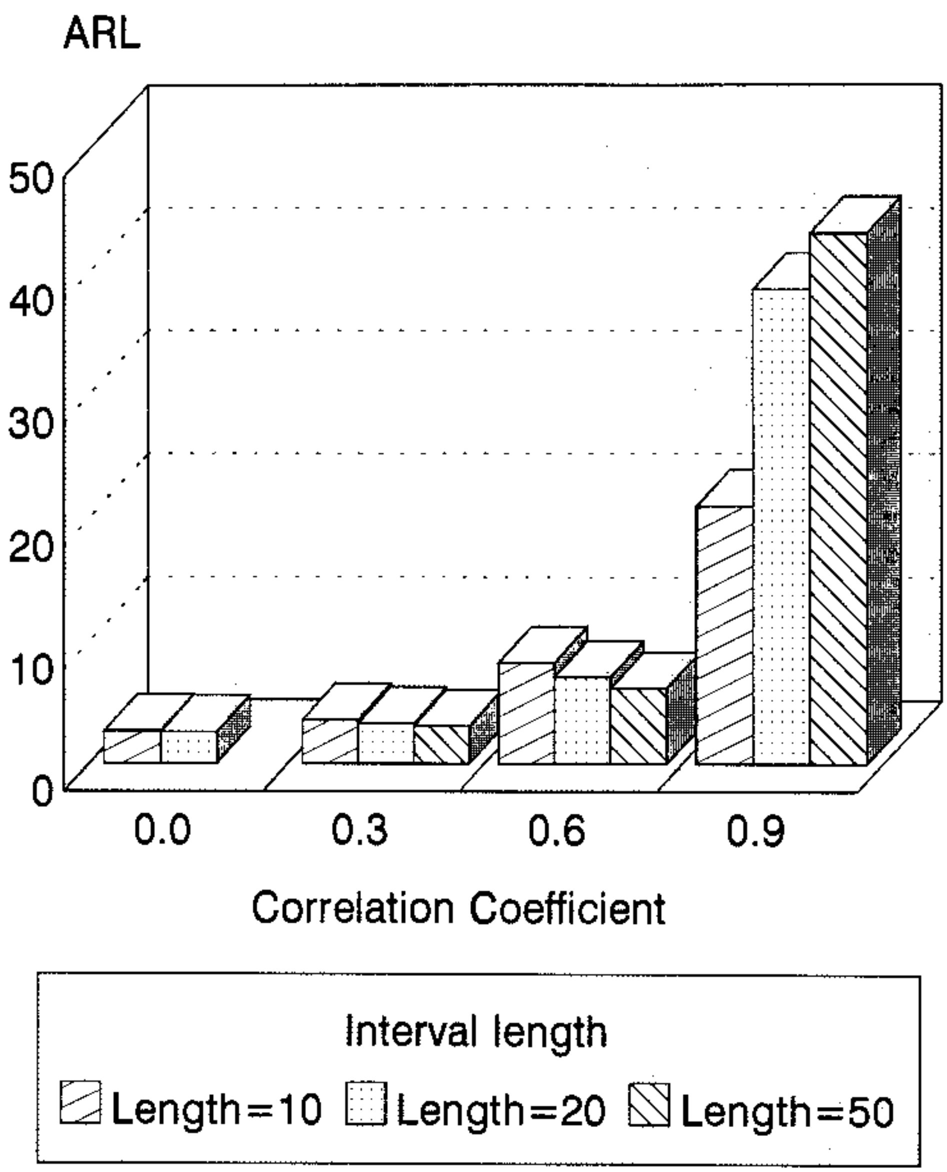


Figure 23g: Effect of interval length on ARL for VSI with limits calculated from sample data
Sample size = 1 Shift = 3.0
y axis: linear scale

INTERVAL LENGTH v. ANSS
VSI with Calculated Limits
Shift = 3.0 Sample size = 1

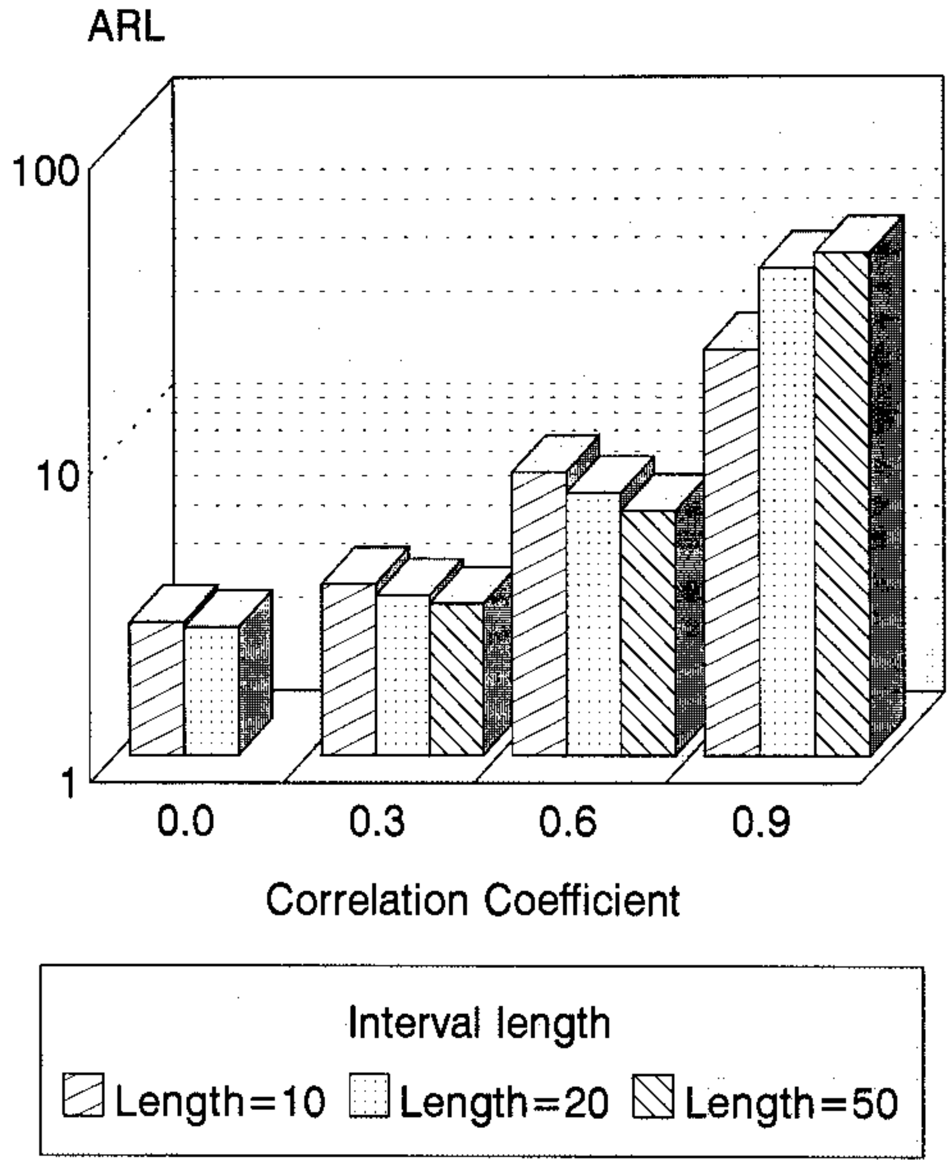


Figure 23h: Effect of Interval length on ARL for VSI with limits calculated from sample data
Sample size = 1 Shift = 3.0
y axis: logarithmic scale

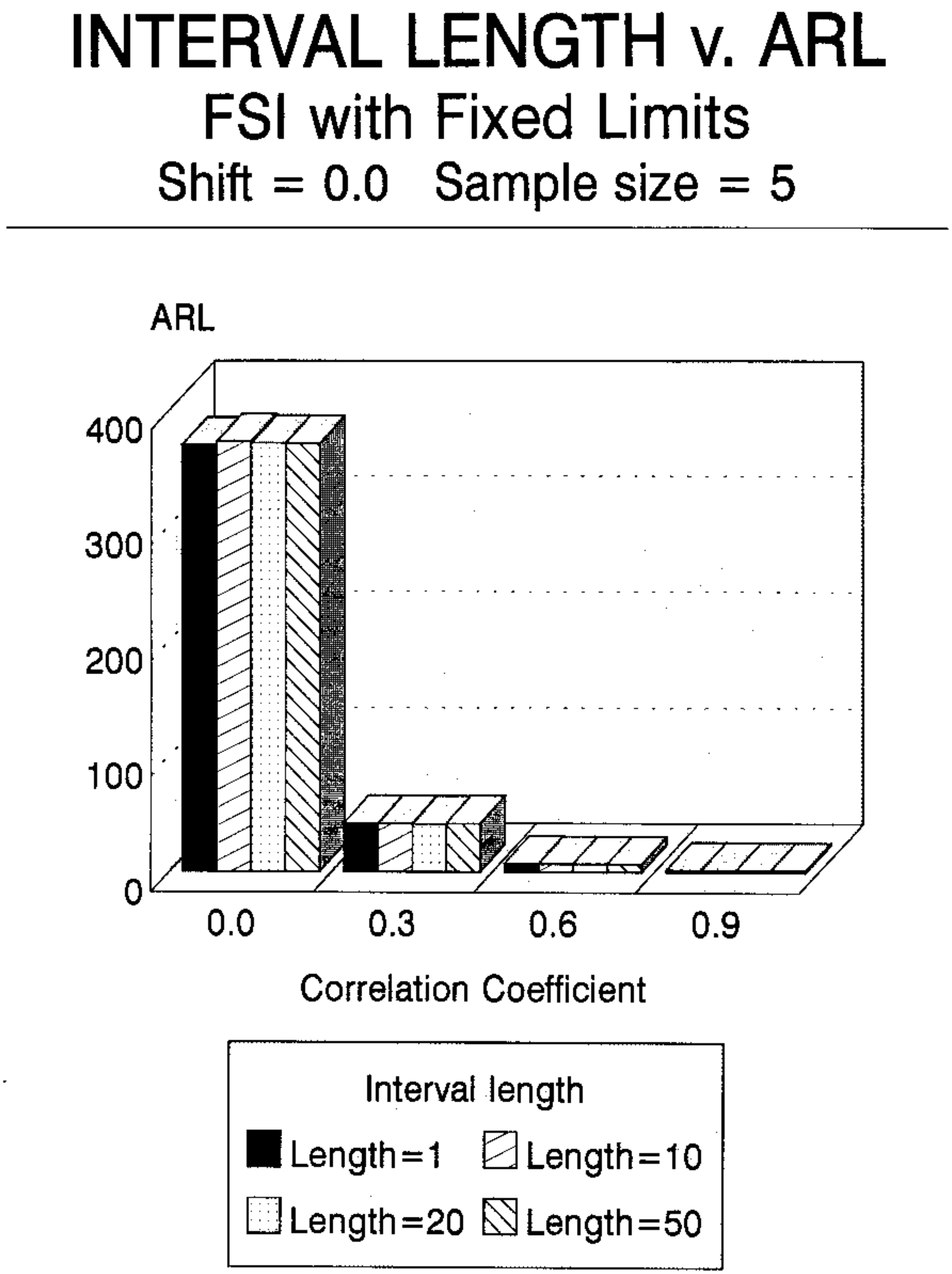


Figure 24a: Effect of Interval length on ARL for FSI with pre-set limits
Sample size = 5 Shift = 0.0
y axis: linear scale

INTERVAL LENGTH v. ARL
FSI with Fixed Limits
Shift = 0.0 Sample size = 5

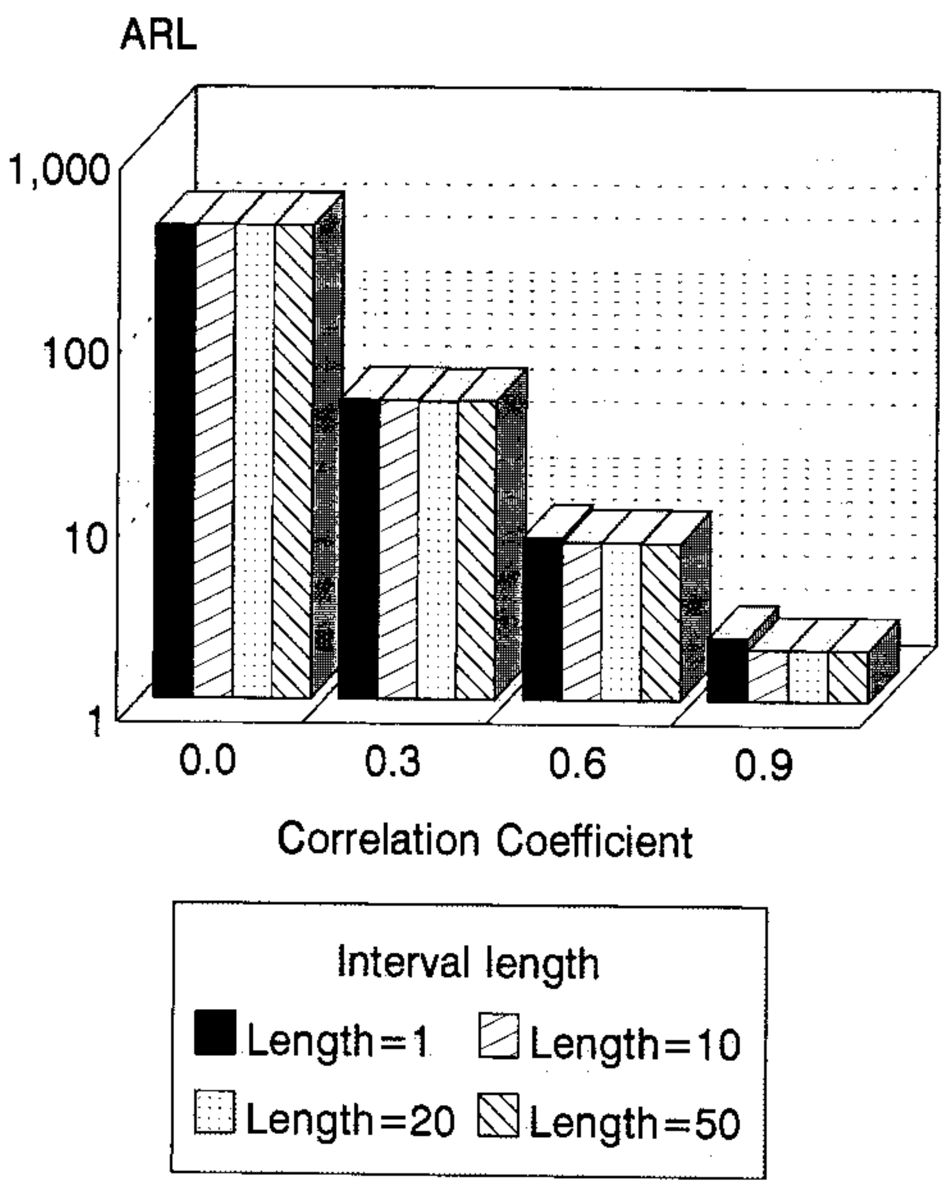


Figure 24b: Effect of Interval length on ARL for FSI with pre-set limits
Sample size = 5 Shift = 0.0
y axis: logarithmic scale

INTERVAL LENGTH v. ARL
FSI with Fixed Limits
Shift = 1.0 Sample size = 5

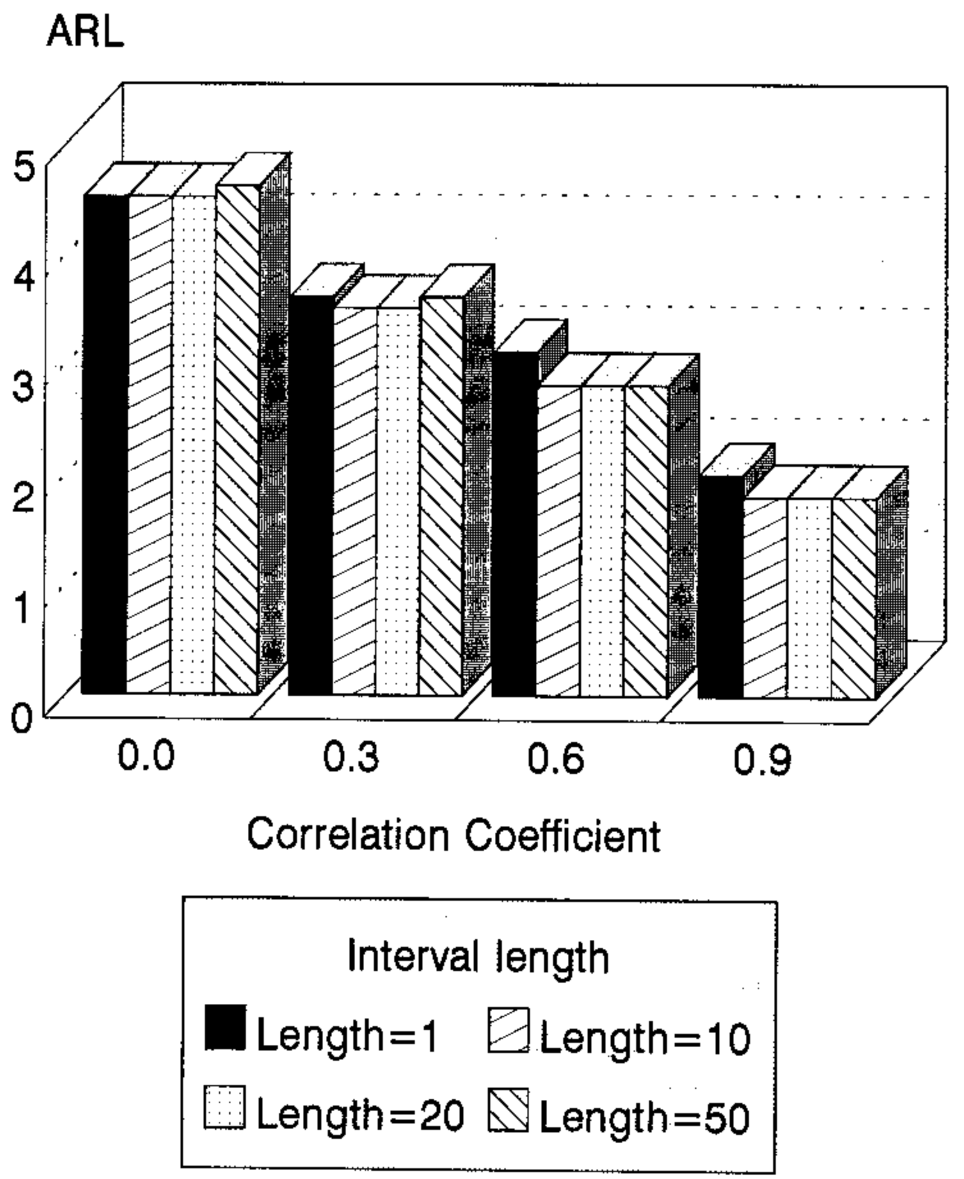


Figure 24c: Effect of Interval length on ARL for FSI with pre-set limits
Sample size = 5 Shift = 1.0
y axis: linear scale

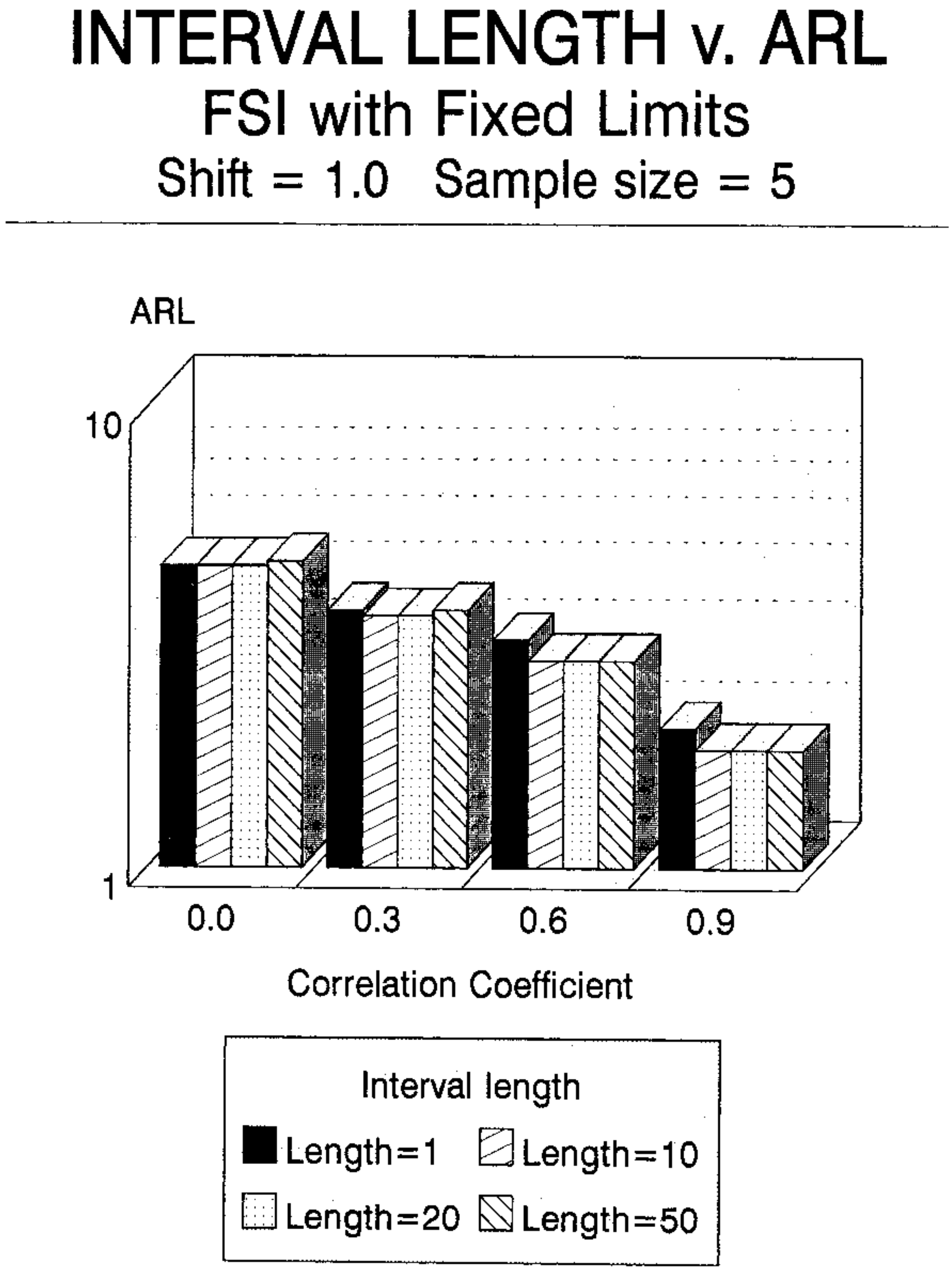


Figure 24d: Effect of interval length on ARL for FSI with pre-set limits
Sample size = 5 Shift = 1.0
y axis: logarithmic scale

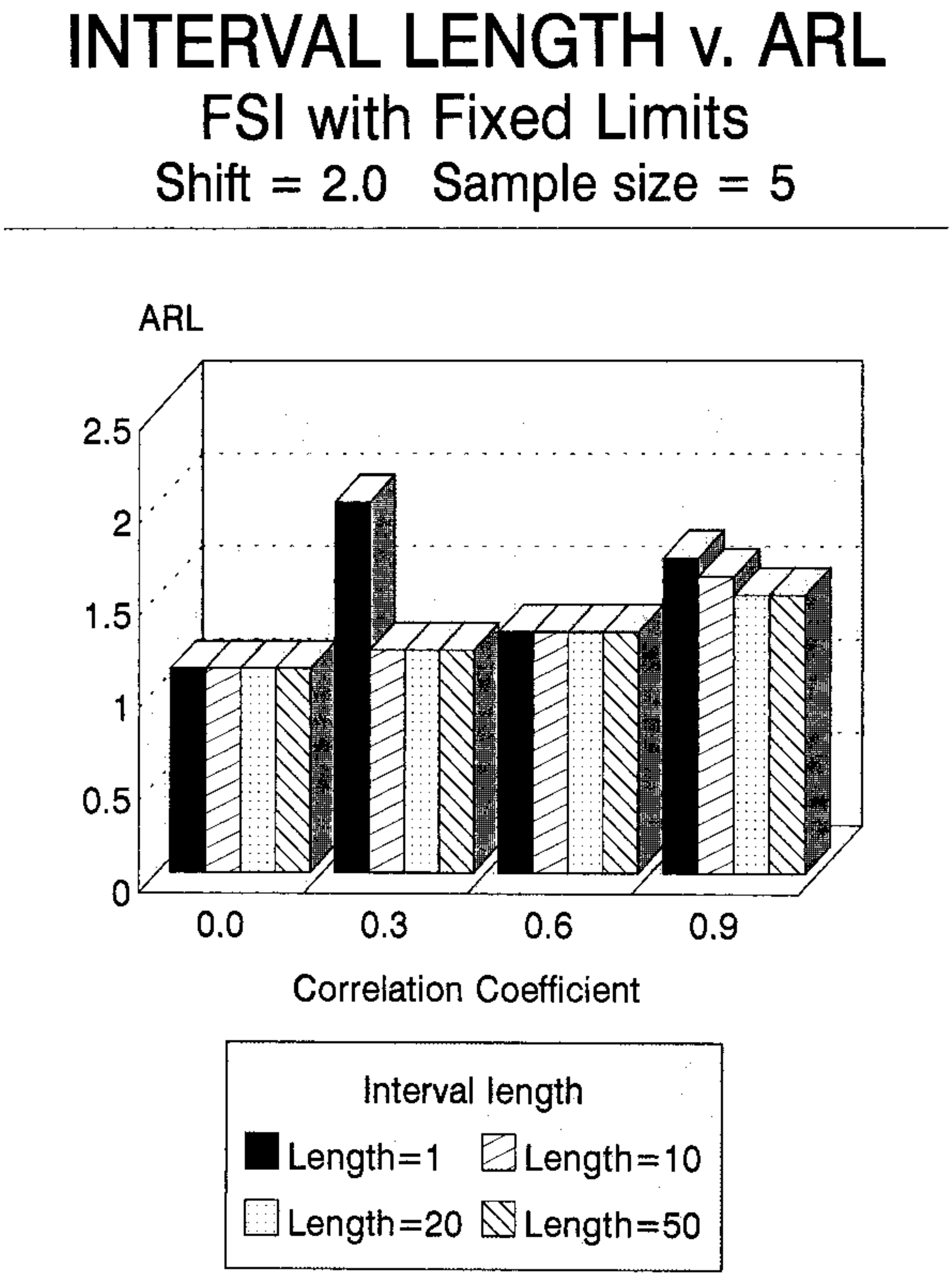


Figure 24e: Effect of interval length on ARL for FSI with pre-set limits
Sample size = 5 Shift = 2.0
y axis: linear scale

INTERVAL LENGTH v. ARL

FSI with Fixed Limits

Shift = 2.0 Sample size = 5

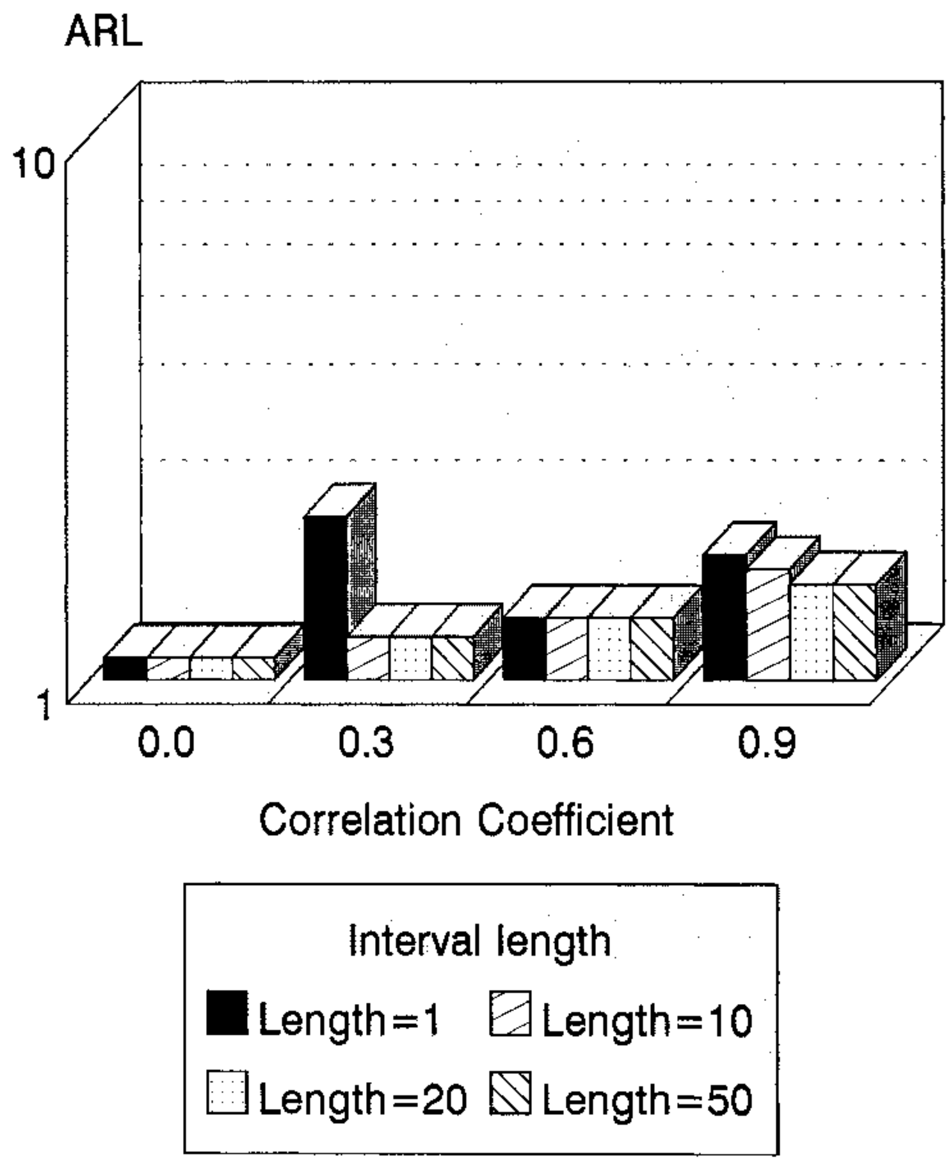


Figure 24f: Effect of Interval length on ARL for FSI with pre-set limits
Sample size = 5 Shift = 2.0
y axis: logarithmic scale

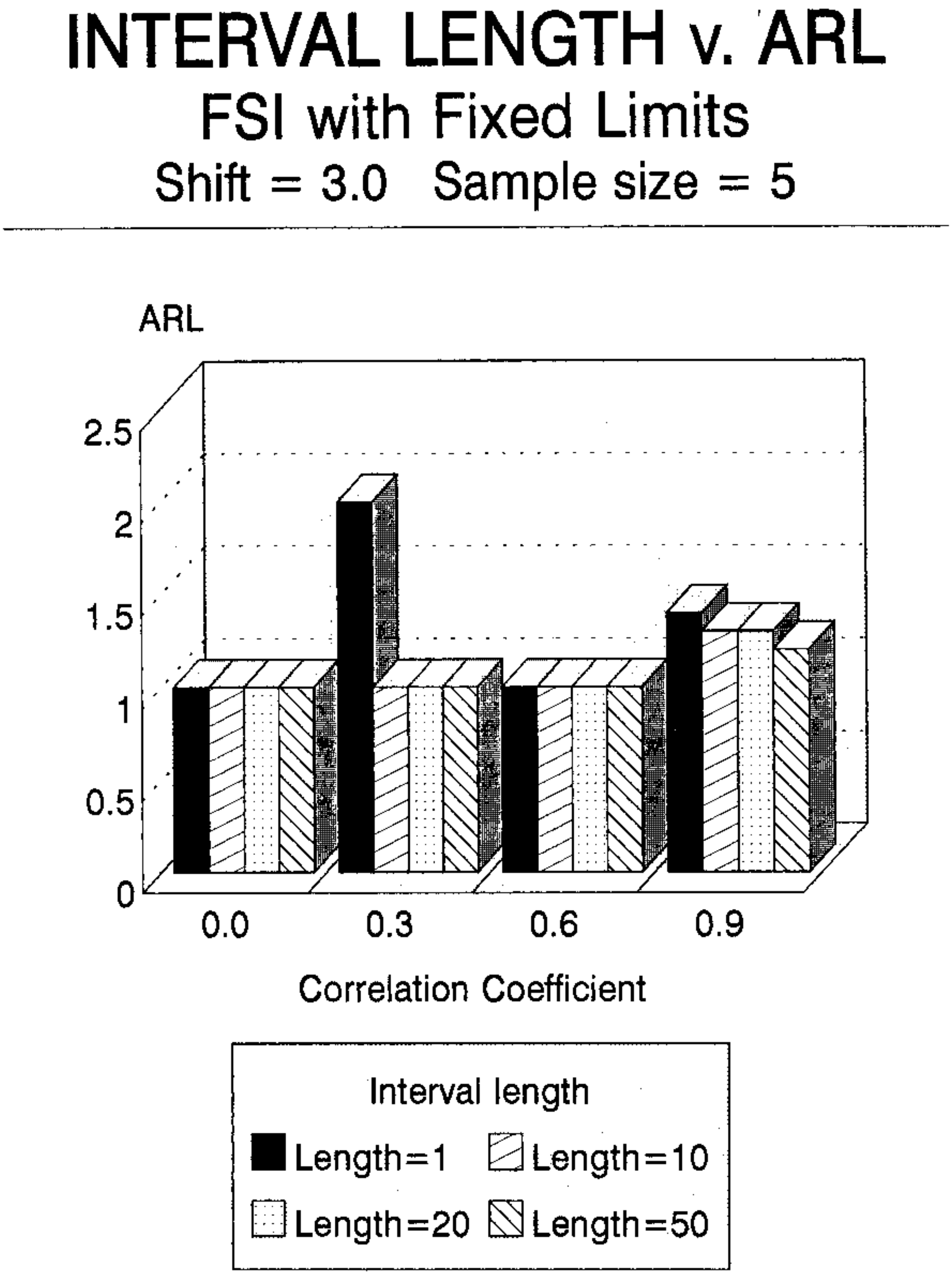


Figure 24g: Effect of Interval length on ARL for FSI with pre-set limits
Sample size = 5 Shift = 3.0
y axis: linear scale

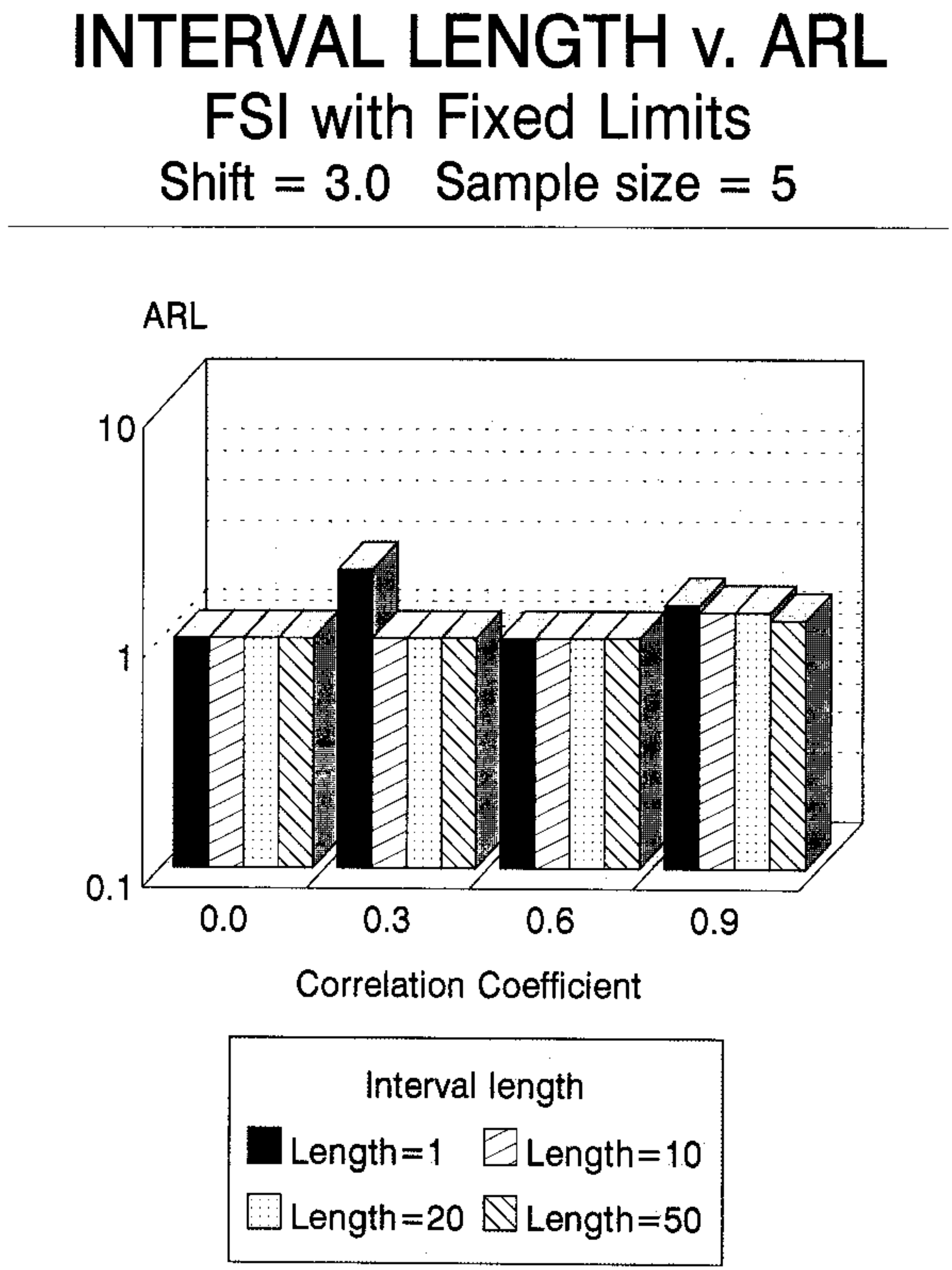


Figure 24h: Effect of Interval length on ARL for FSI with pre-set limits
Sample size = 5 Shift = 3.0
y axis: logarithmic scale

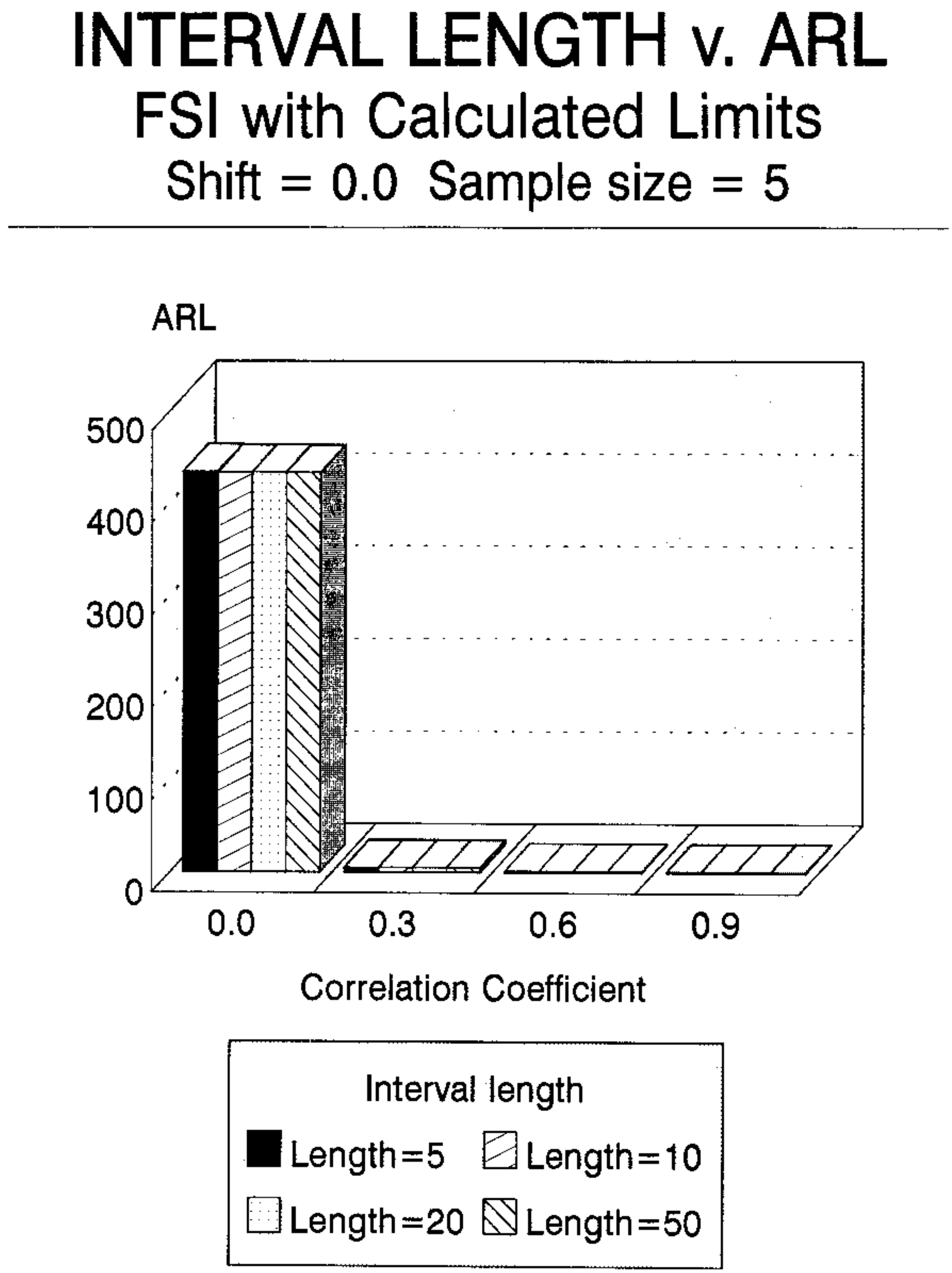


Figure 25a: Effect of Interval length on ARL for FSI with limits calculated from sample data
Sample size = 5 Shift = 0.0
y axis: linear scale

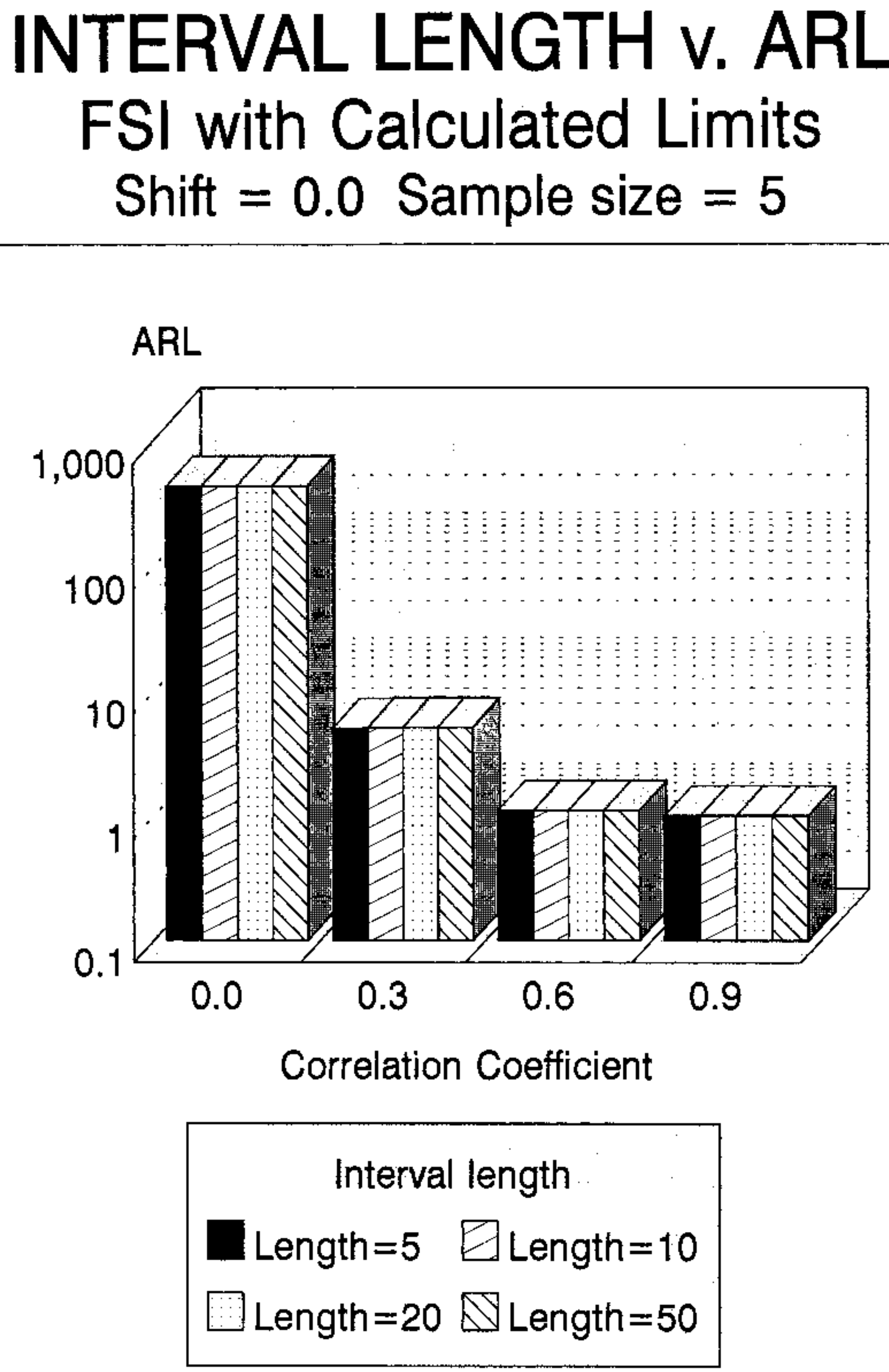


Figure 25b: Effect of Interval length on ARL for FSI with limits calculated from sample data
Sample size = 5 Shift = 0.0
y axis: logarithmic scale

INTERVAL LENGTH v. ARL
FSI with Calculated Limits
Shift = 1.0 Sample size = 5

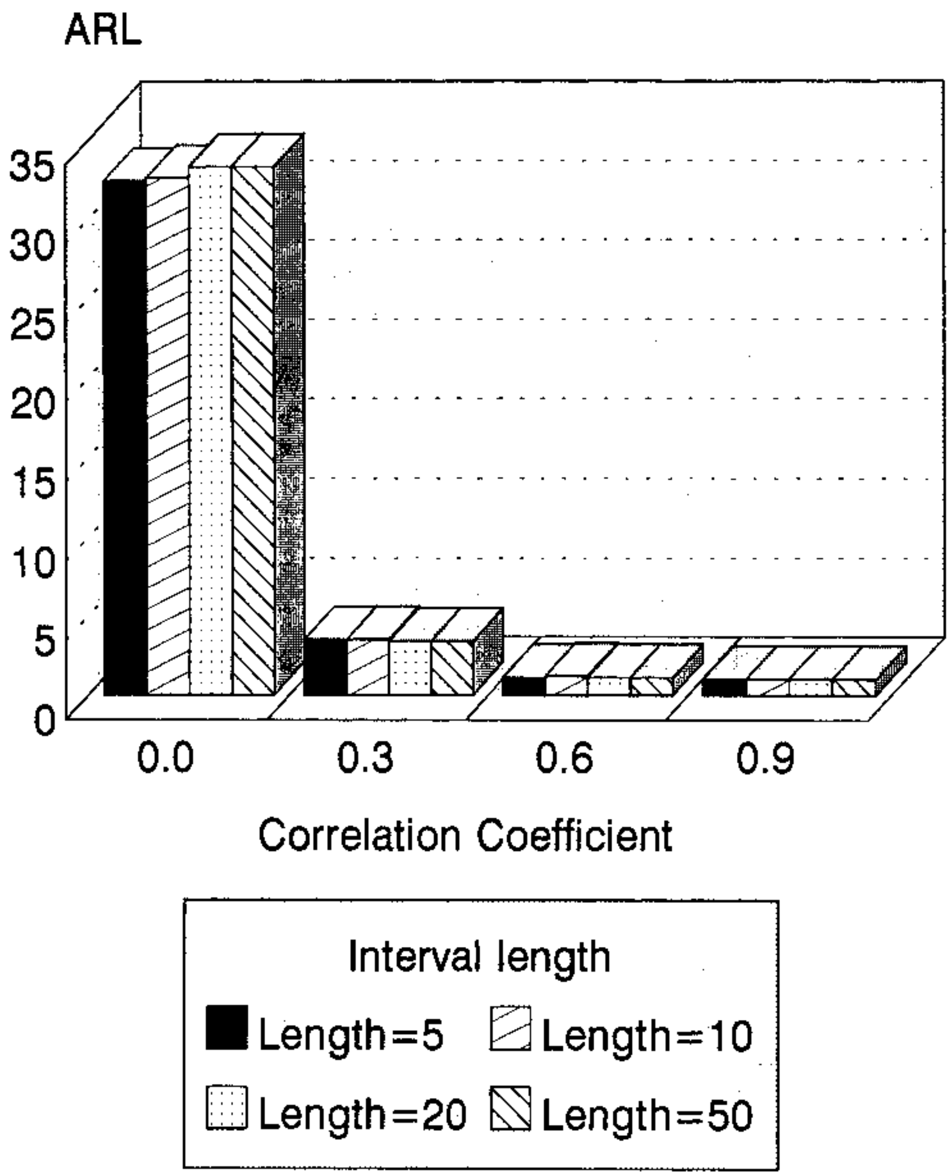


Figure 25c: Effect of Interval length on ARL for FSI with limits calculated from sample data
Sample size = 5 Shift = 1.0
y axis: linear scale

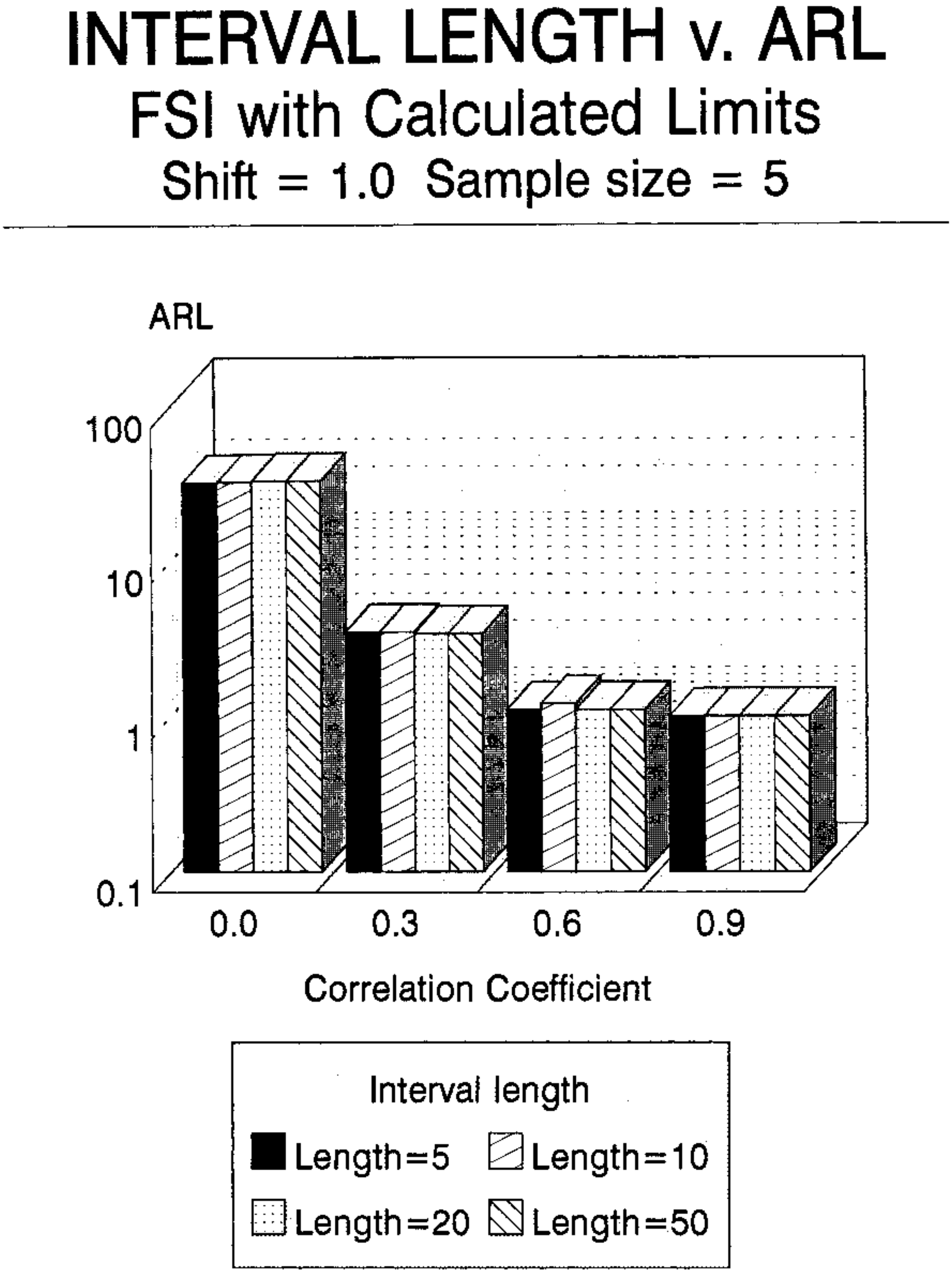


Figure 25d: Effect of Interval length on ARL for FSI with limits calculated from sample data
Sample size = 5 Shift = 1.0
y axis: logarithmic scale

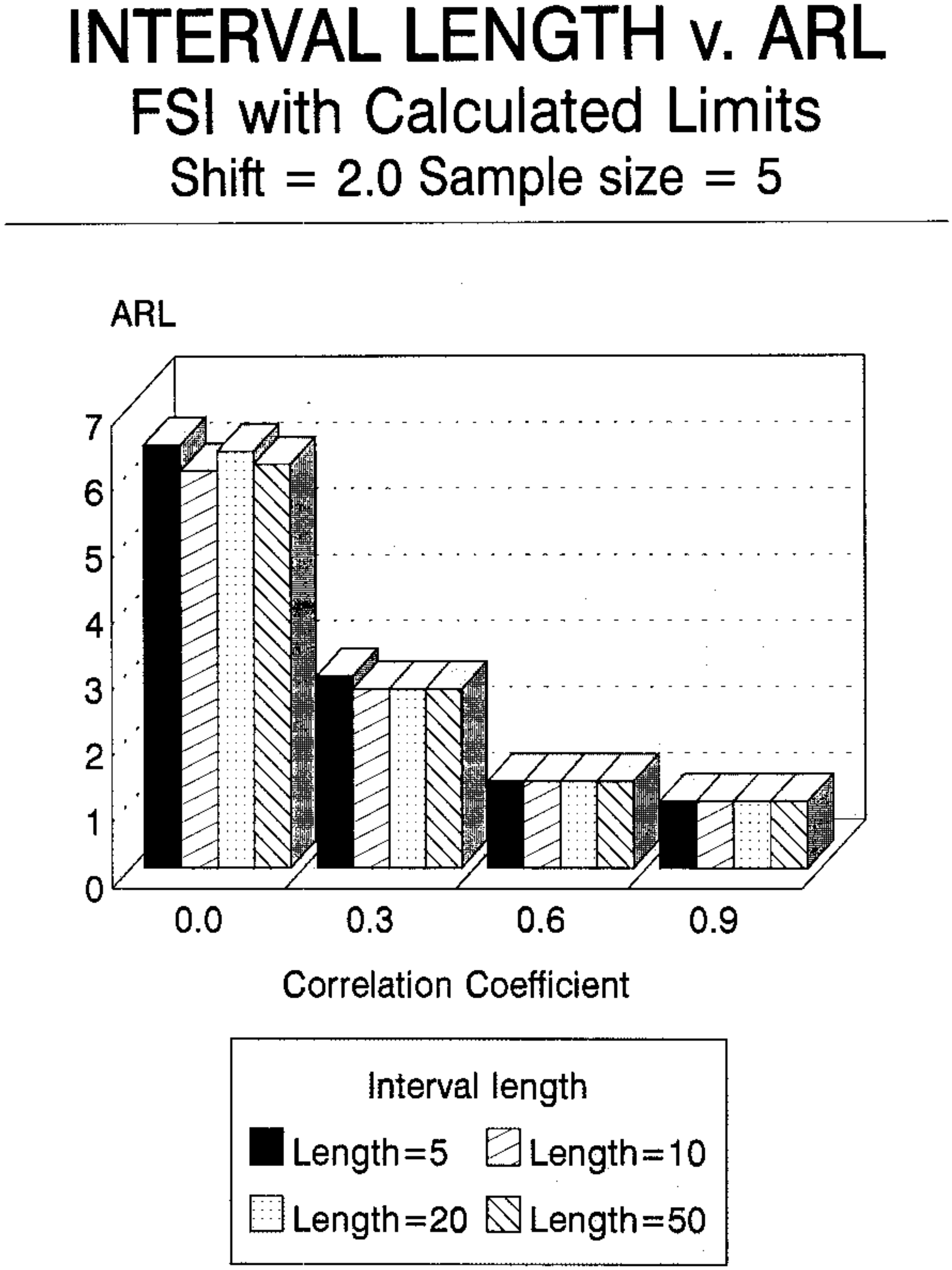


Figure 25e: Effect of interval length on ARL for FSI with limits calculated from sample data
Sample size = 5 Shift = 2.0
y axis: linear scale

INTERVAL LENGTH v. ARL
FSI with Calculated Limits
Shift = 2.0 Sample size = 5

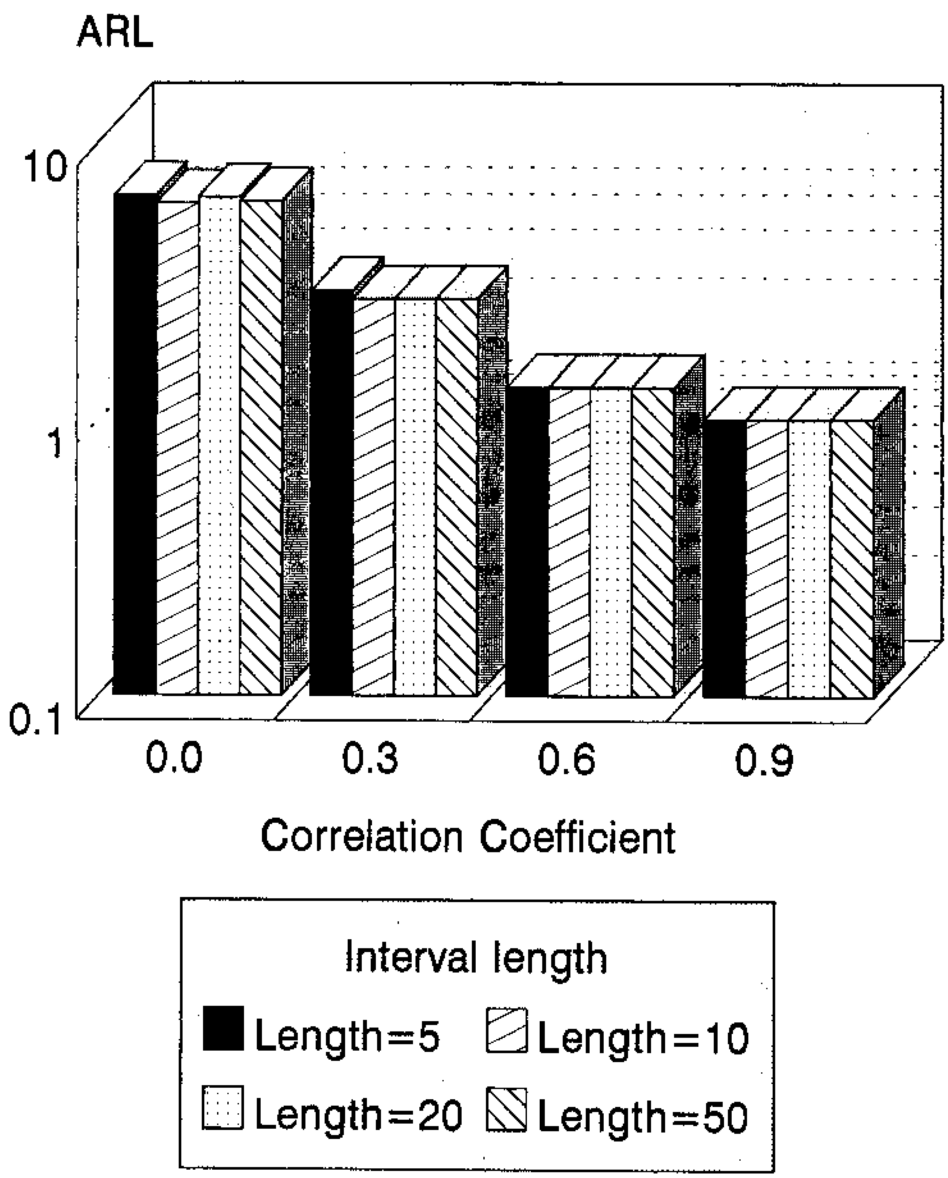


Figure 25f: Effect of Interval length on ARL for FSI with limits calculated from sample data
Sample size = 5 Shift = 3.0
y axis: logarithmic scale

INTERVAL LENGTH v. ARL
FSI with Calculated Limits
Shift = 3.0 Sample size = 5

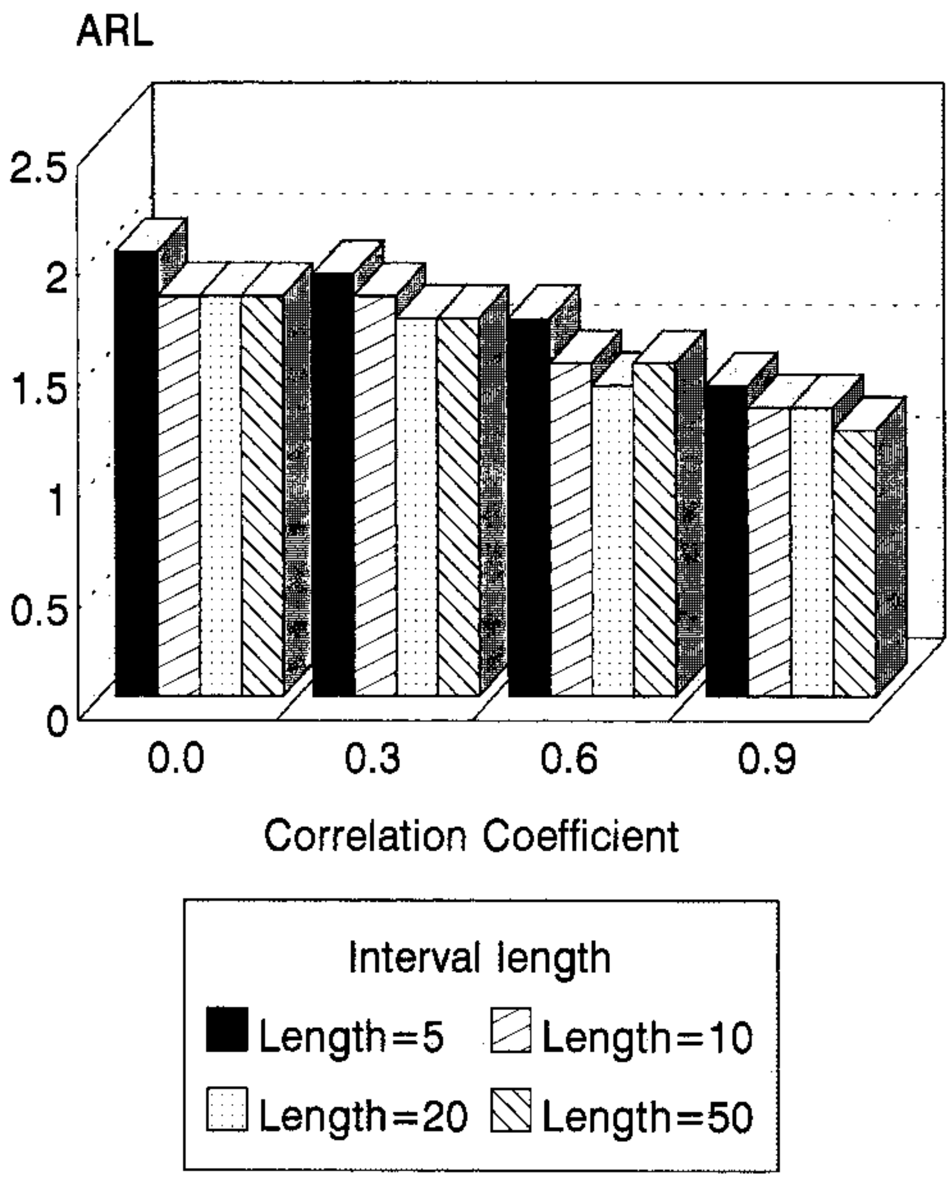


Figure 25g: Effect of interval length on ARL for FSI with limits calculated from sample data
Sample size = 5 Shift = 3.0
y axis: linear scale

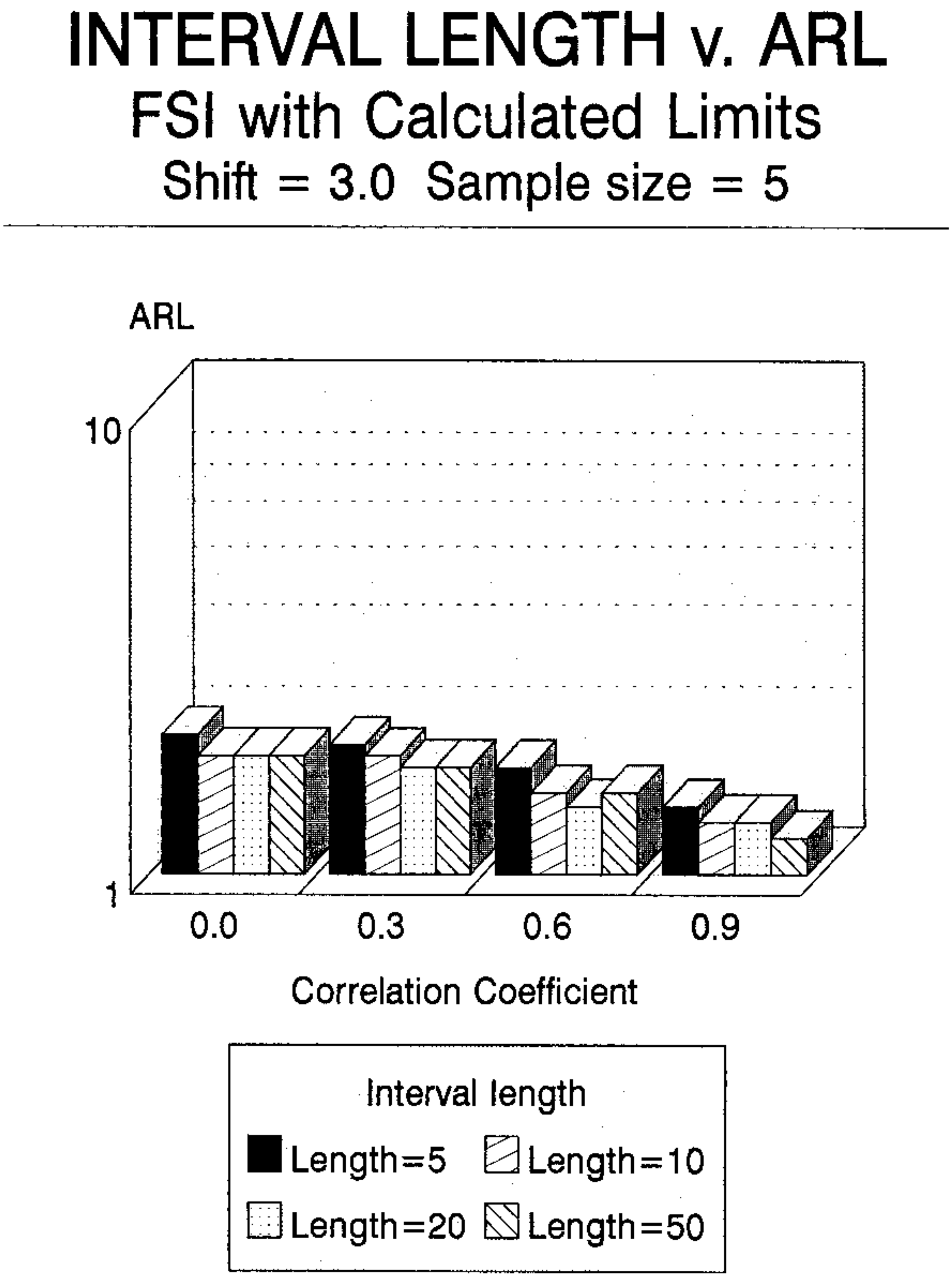


Figure 25h: Effect of Interval length on ARL for FSI with limits calculated from sample data
Sample size = 5 Shift = 3.0
y axis: logarithmic scale

INTERVAL LENGTH v. ANSS
VSI with Fixed Limits
Shift = 0.0 Sample size = 5

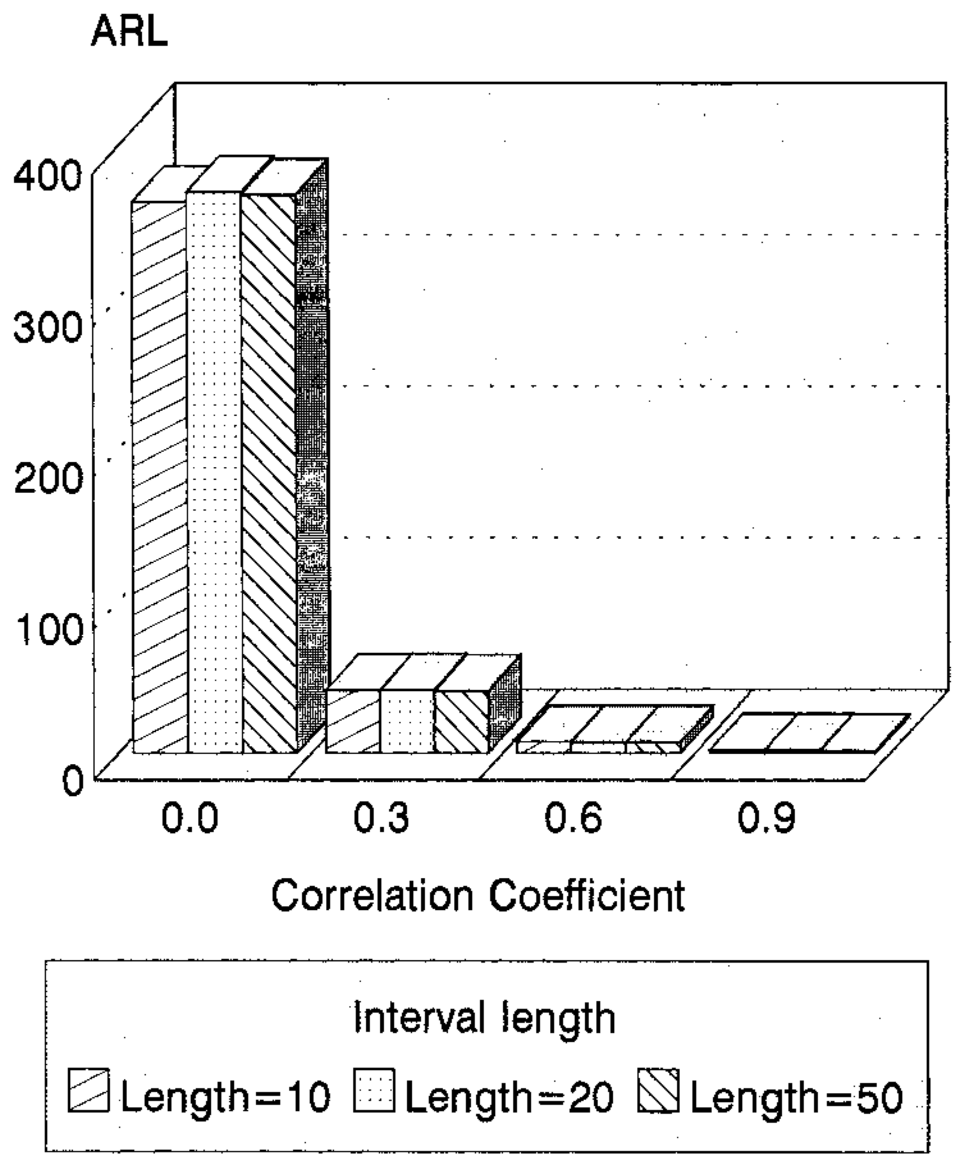


Figure 26a: Effect of Interval length on ARL for VSI with pre-set limits
Sample size = 5 Shift = 0.0
y axis: linear scale

INTERVAL LENGTH v. ANSS
VSI with Fixed Limits
Shift = 0.0 Sample size = 5

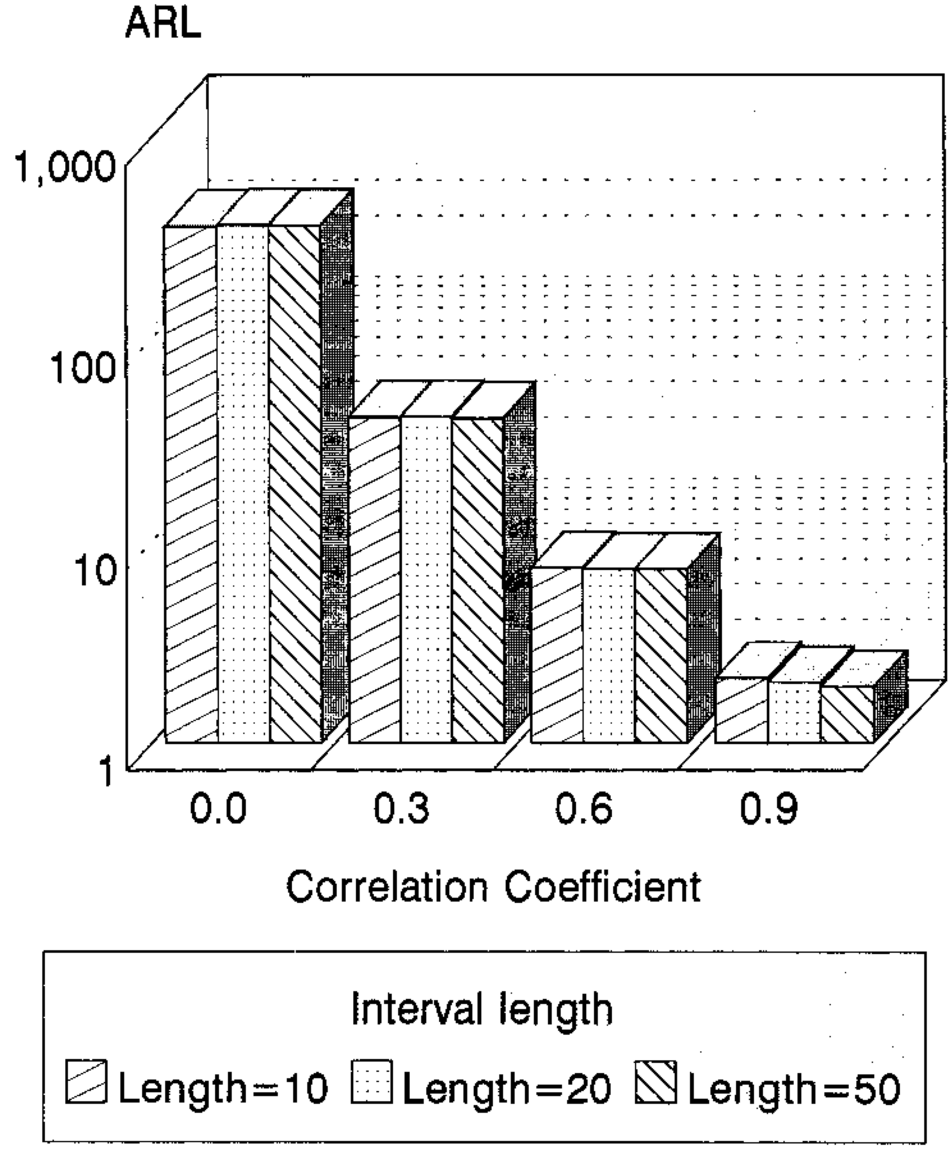


Figure 26b: Effect of Interval length on ARL for VSI with pre-set limits
Sample size = 5   Shift = 0.0
y axis: logarithmic scale

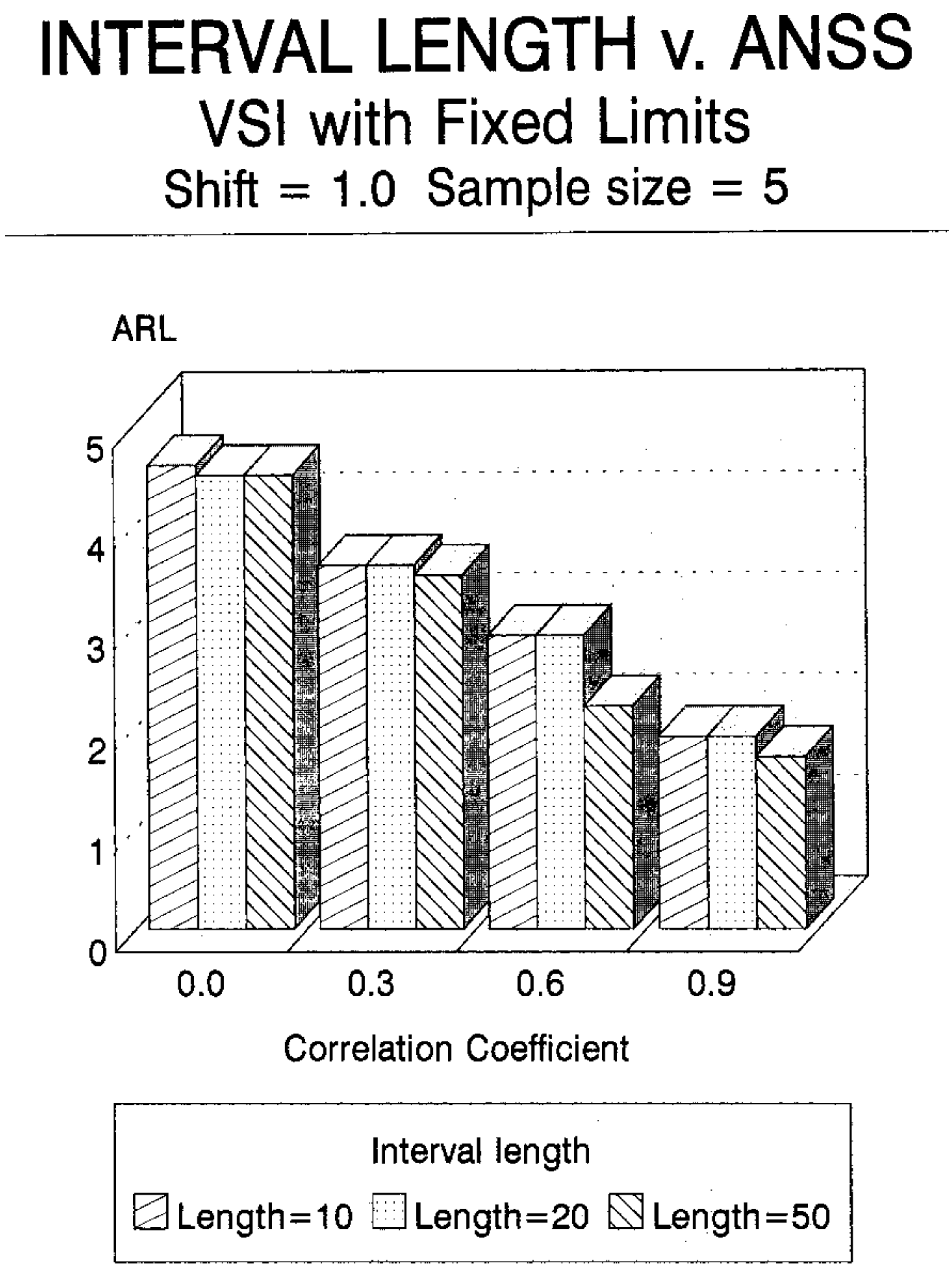


Figure 26c: Effect of Interval length on ARL for VSI with pre-set limits
Sample size = 5 Shift = 1.0
y axis: linear scale

INTERVAL LENGTH v. ANSS
VSI with Fixed Limits
Shift = 1.0 Sample size = 5

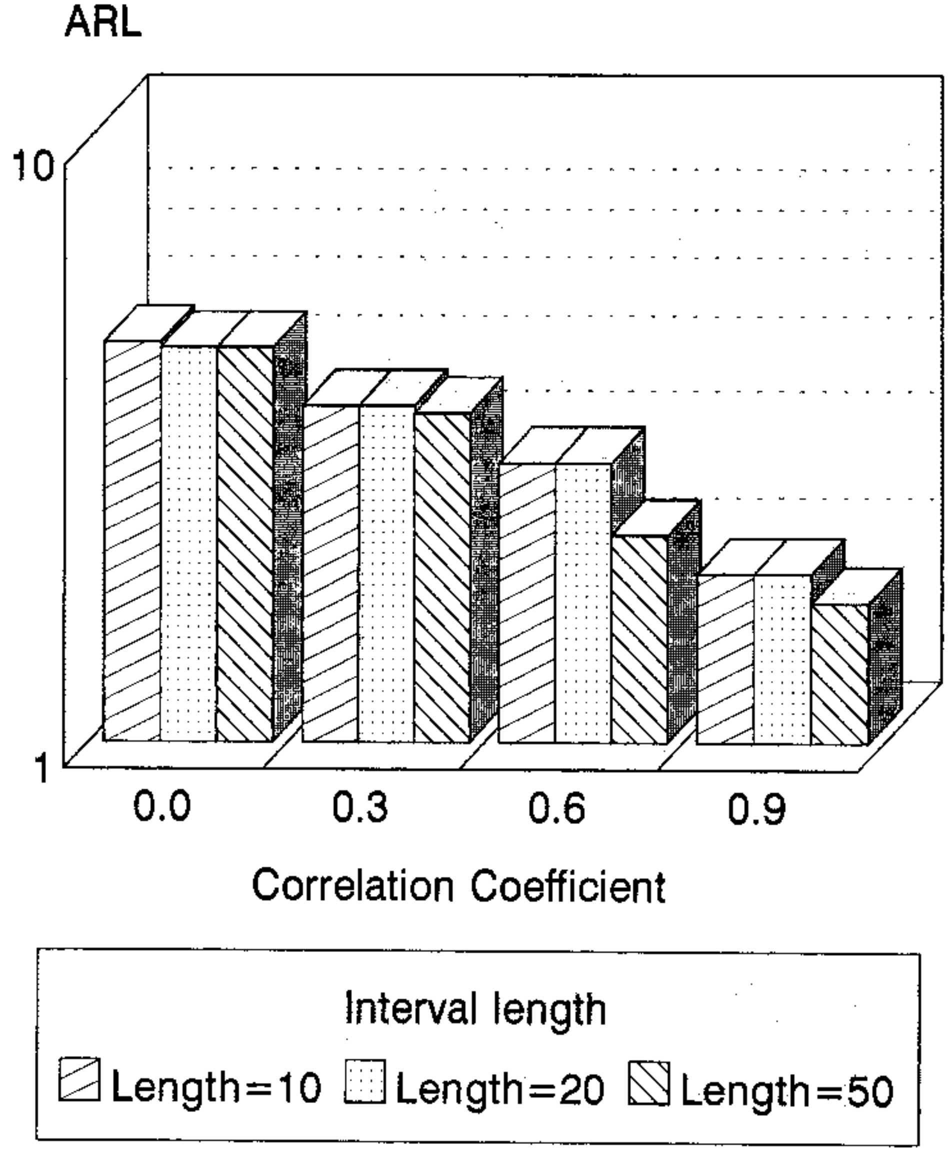


Figure 26d: Effect of Interval length on ARL for VSI with pre-set limits
Sample size = 5 Shift = 1.0
y axis: logarithmic scale

INTERVAL LENGTH v. ANSS
VSI with Fixed Limits
Shift = 2.0 Sample size = 5

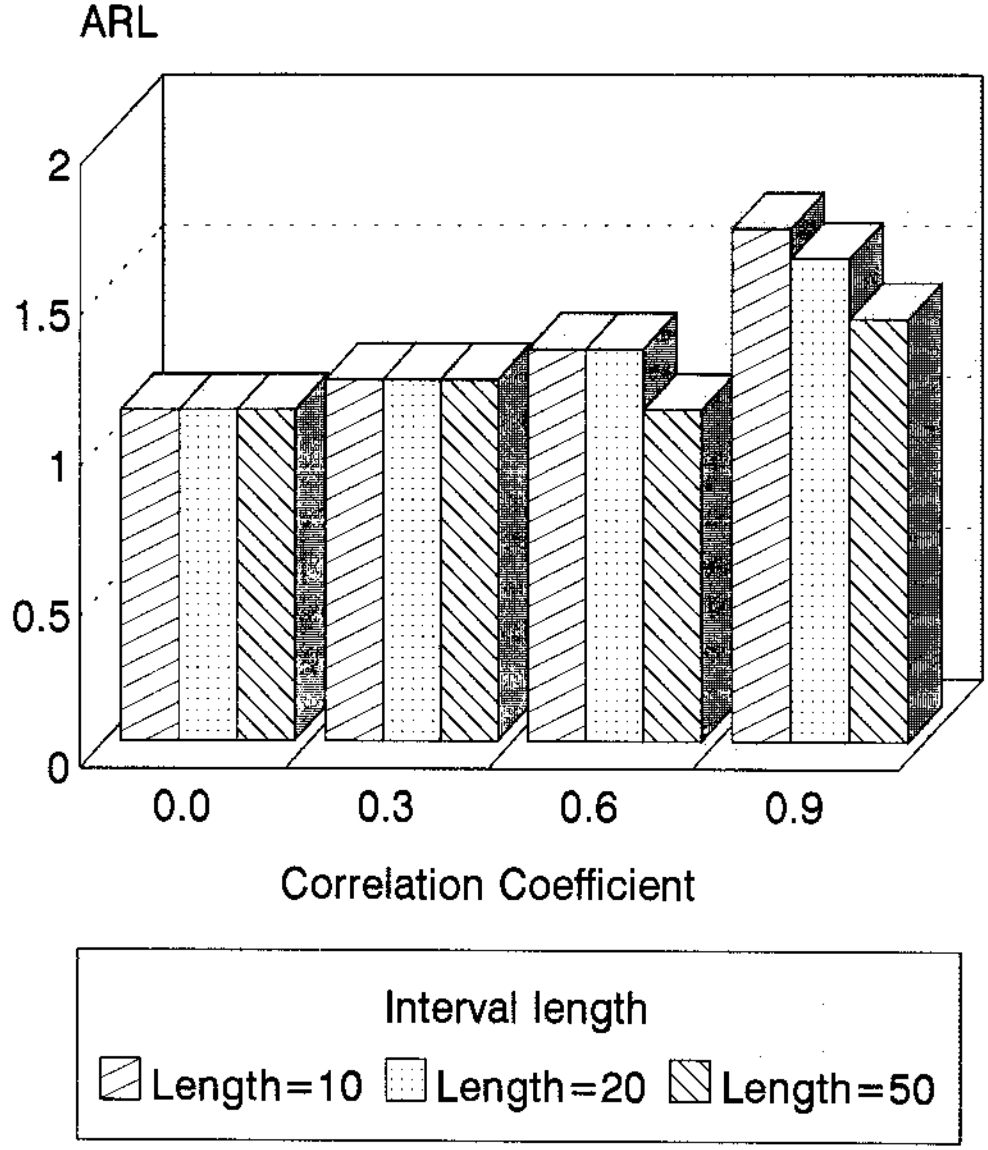


Figure 26e: Effect of Interval length on ARL for VSI with pre-set limits
Sample size = 5 Shift = 2.0
y axis: linear scale

INTERVAL LENGTH v. ANSS
VSI with Fixed Limits
Shift = 2.0 Sample size = 5

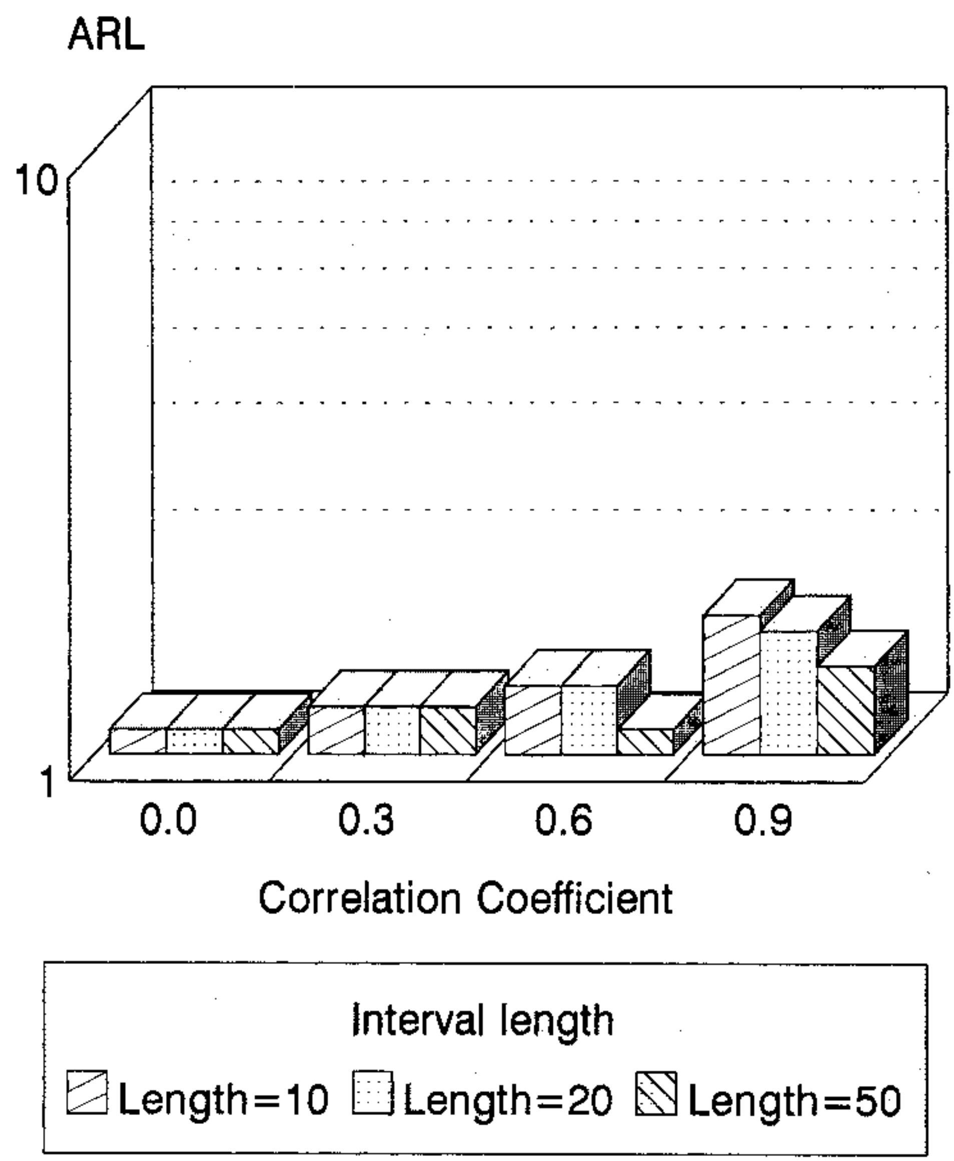


Figure 26f: Effect of Interval length on ARL for VSI with pre-set limits
Sample size = 5 Shift = 2.0
y axis: logarithmic scale

INTERVAL LENGTH v. ANSS
VSI with Calculated Limits
Shift = 3.0 Sample size = 5

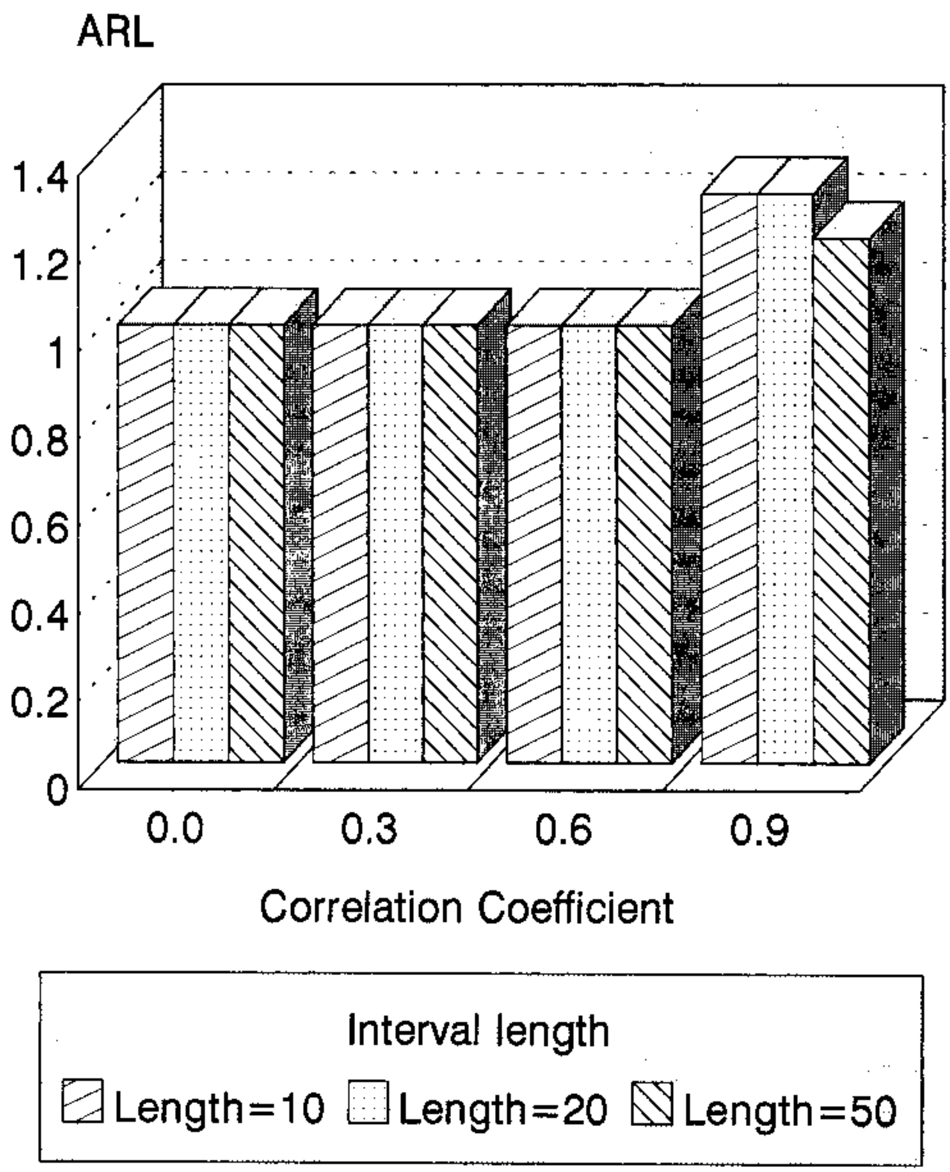


Figure 26g: Effect of Interval length on ARL for VSI with limits calculated from sample data
Sample size = 5 Shift = 3.0
y axis: linear scale

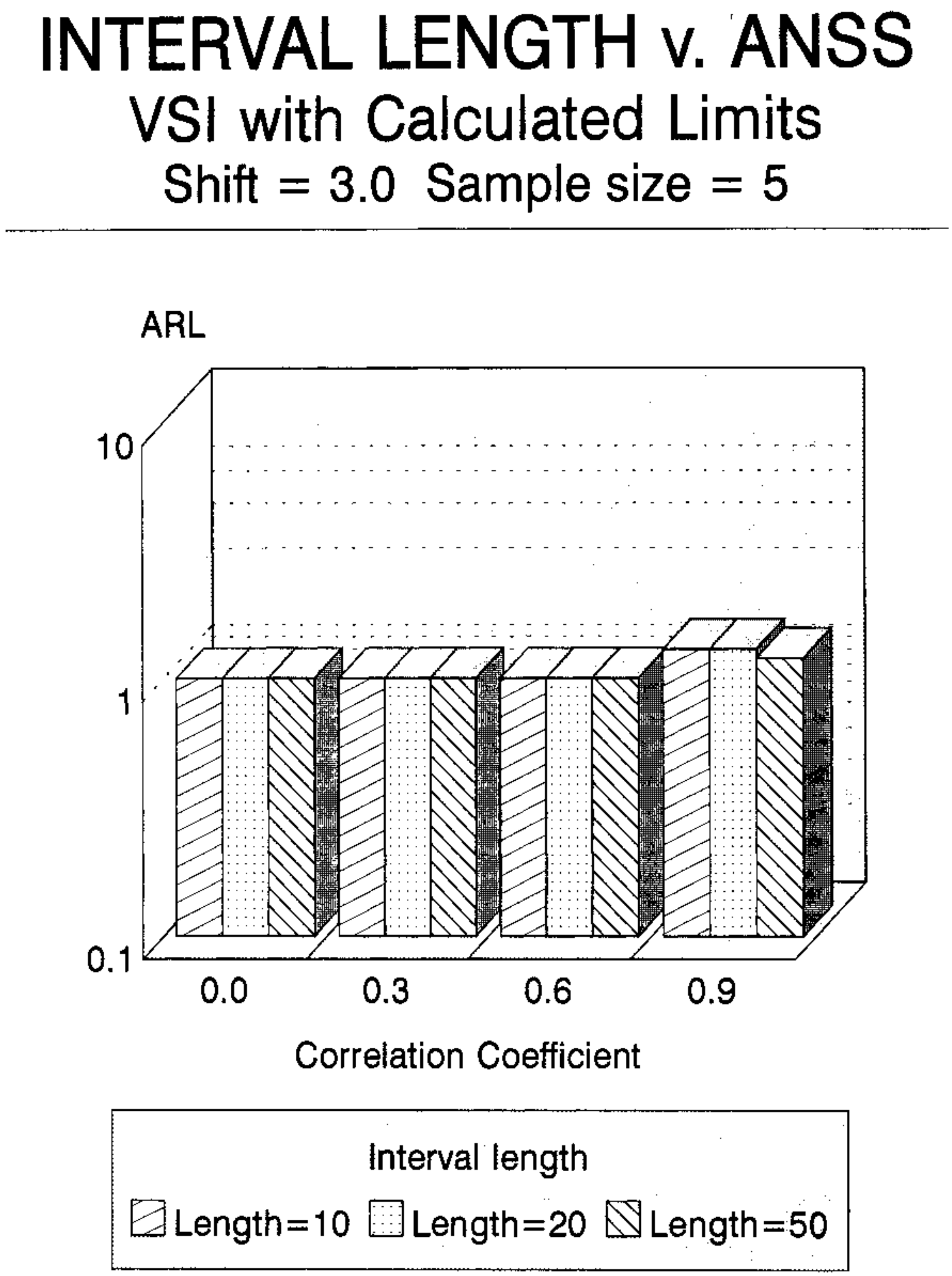


Figure 26h: Effect of Interval length on ARL for VSI with limits calculated from sample data
Sample size = 5 Shift = 3.0
y axis: logarithmic scale

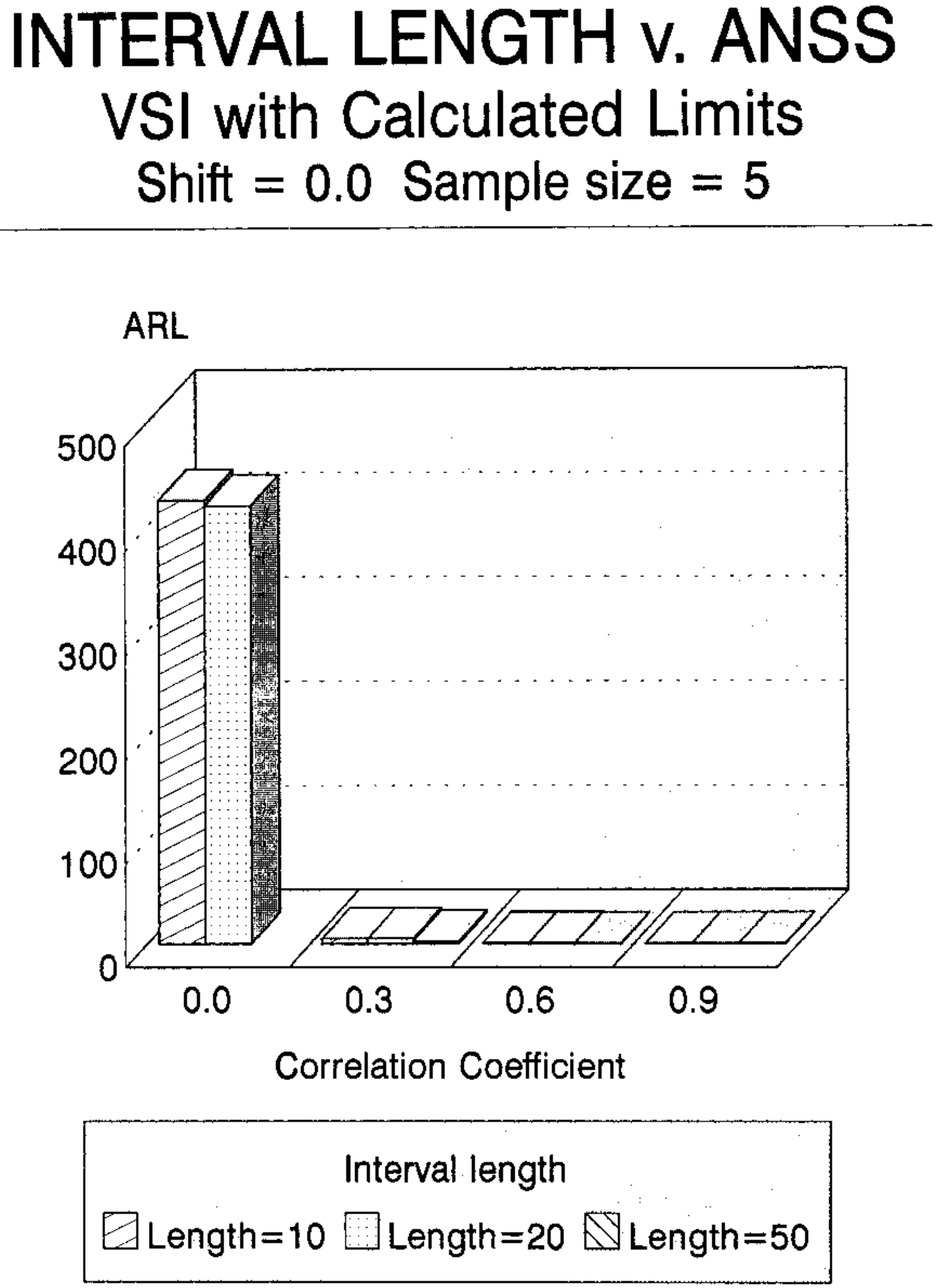


Figure 27a: Effect of interval length on ARL for VSI with limits calculated from sample data
Sample size = 5 Shift = 0.0
y axis: linear scale

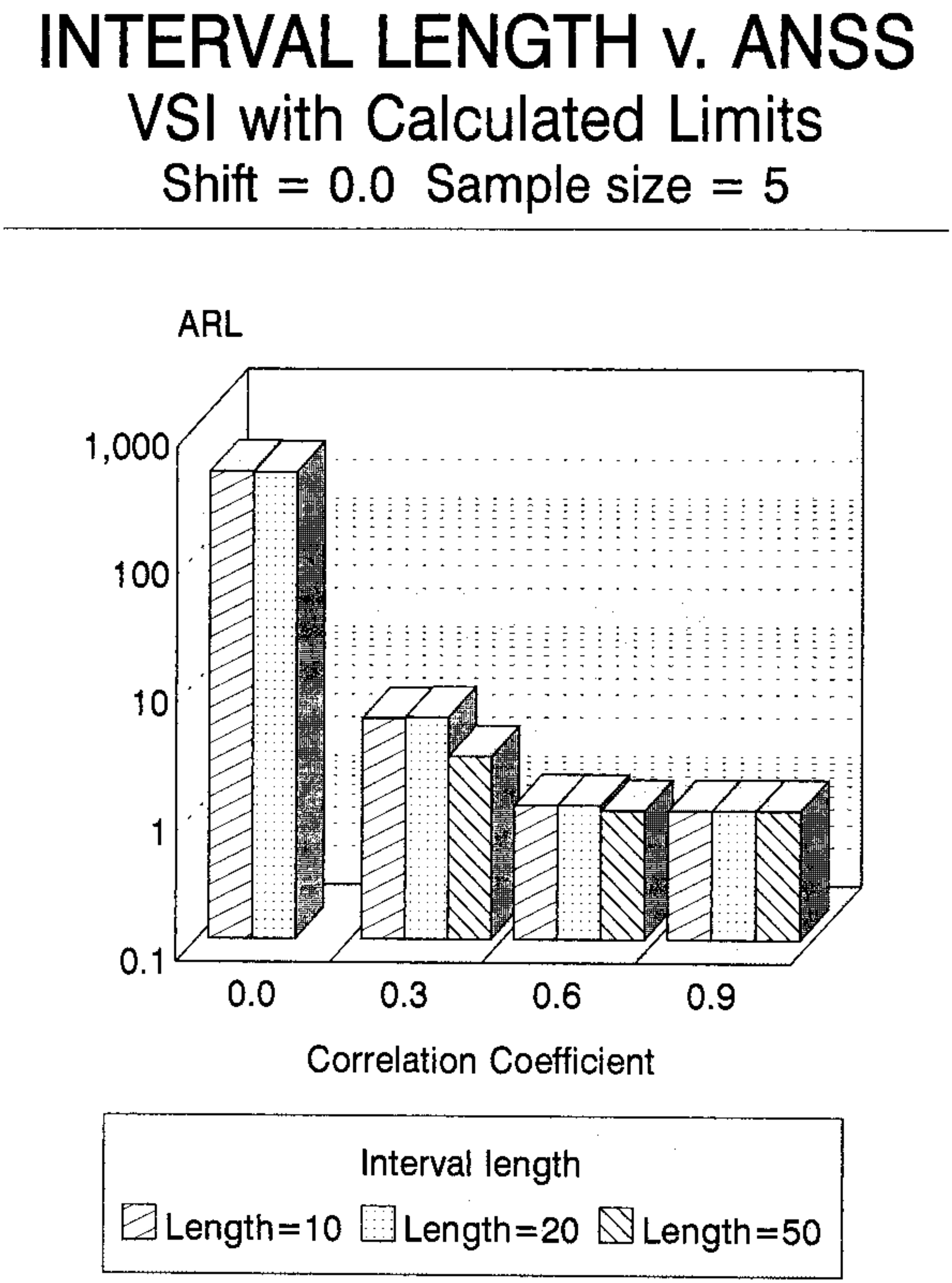


Figure 27b: Effect of Interval length on ARL for VSI with limits calculated from sample data
Sample size = 5 Shift = 0.0
y axis: logarithmic scale

INTERVAL LENGTH v. ANSS
VSI with Calculated Limits
Shift = 1.0 Sample size = 5

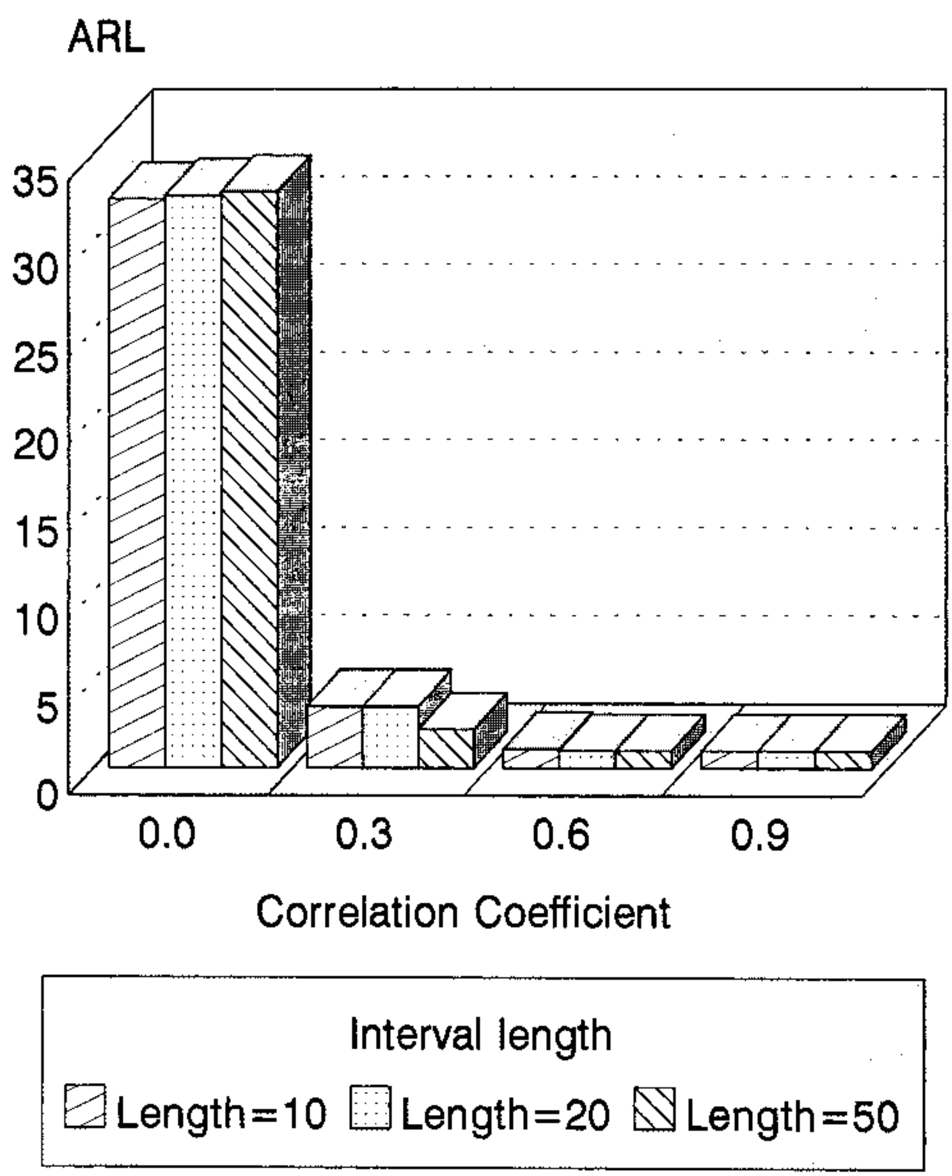


Figure 27c: Effect of interval length on ARL for VSI with limits calculated from sample data
Sample size = 5 Shift = 1.0
y axis: linear scale

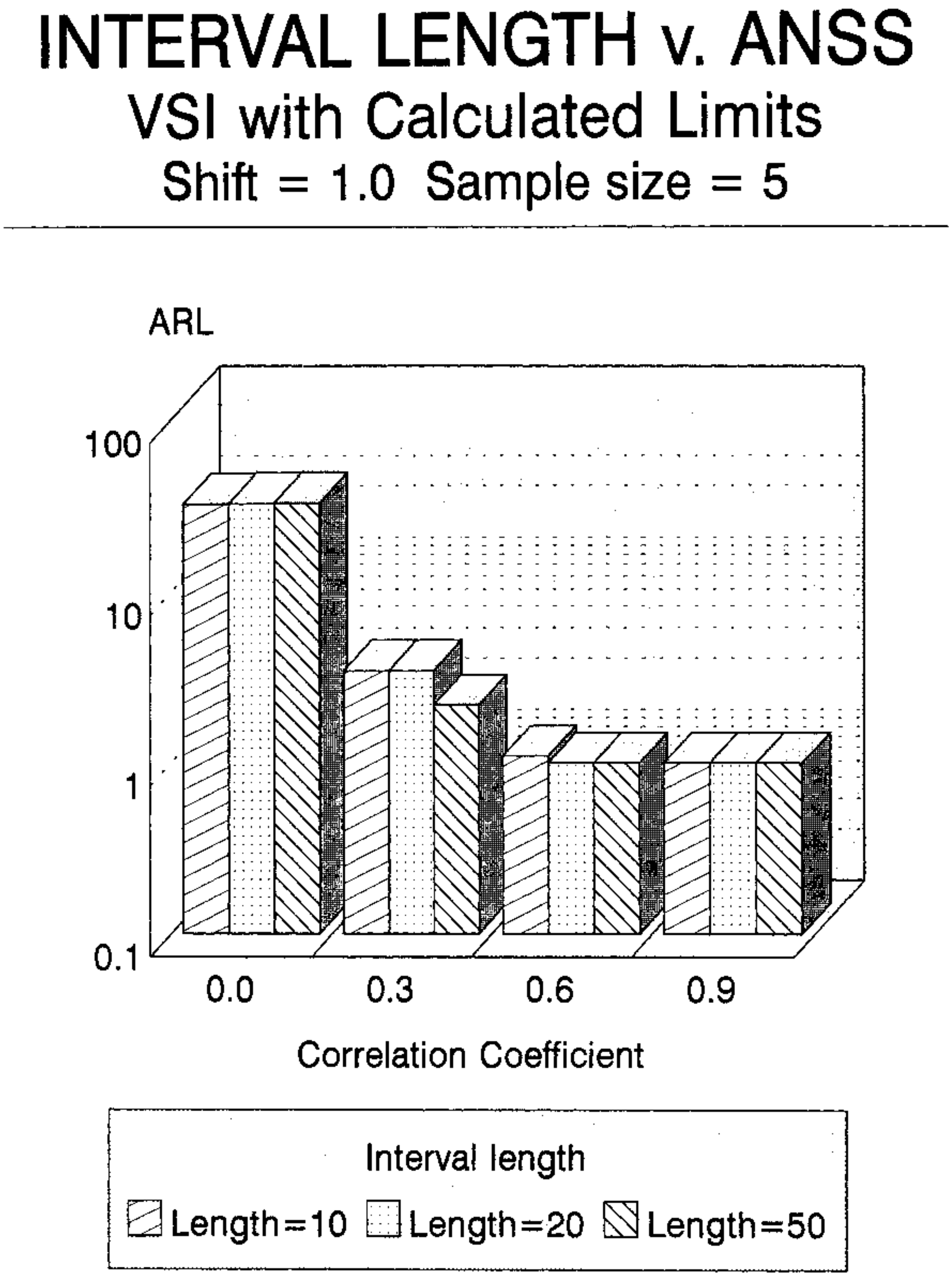


Figure 27d: Effect of Interval length on ARL for VSI with limits calculated from sample data
Sample size = 5 Shift = 1.0
y axis: logarithmic scale

INTERVAL LENGTH v. ANSS
VSI with Calculated Limits
Shift = 2.0 Sample size = 5

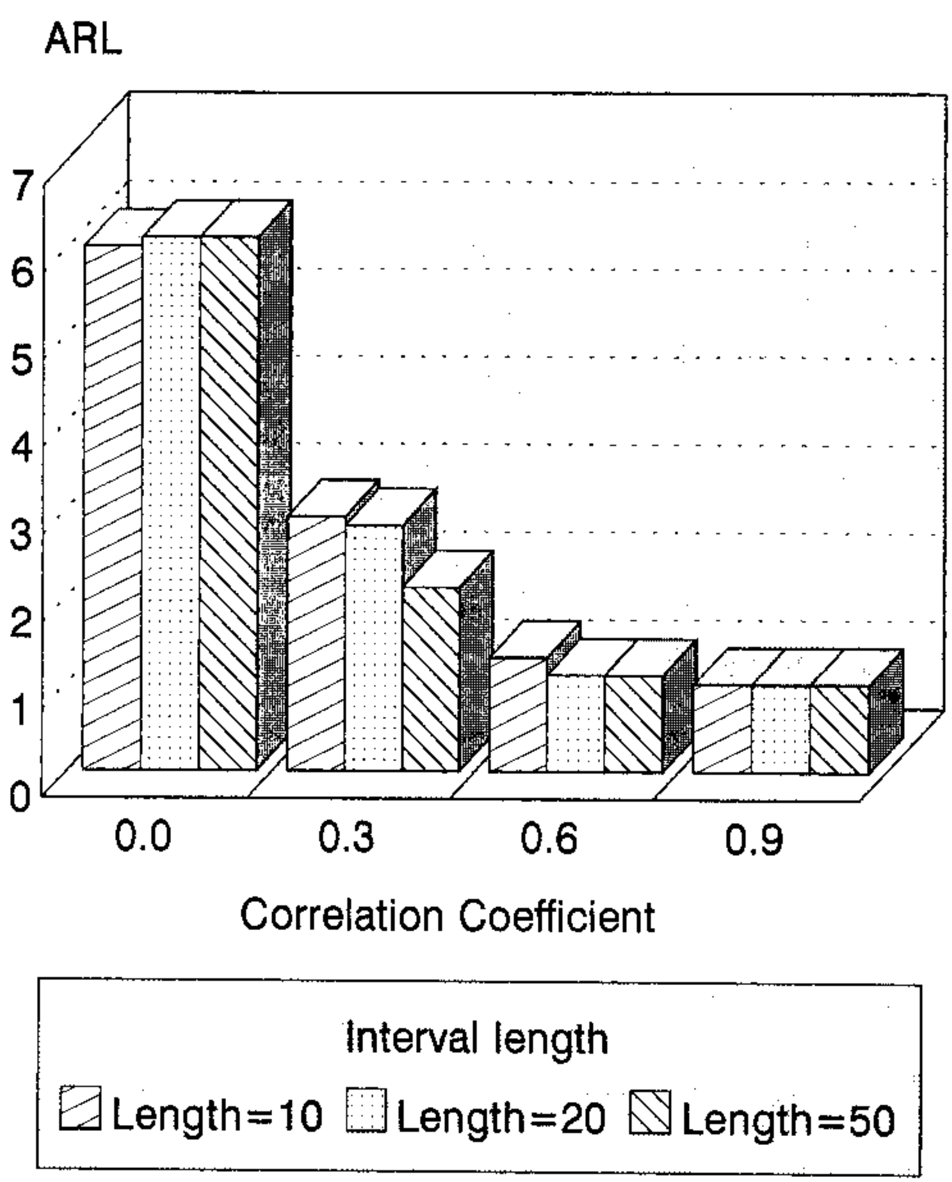


Figure 27e: Effect of interval length on ARL for VSI with limits calculated from sample data
Sample size = 5 Shift = 2.0
y axis: linear scale

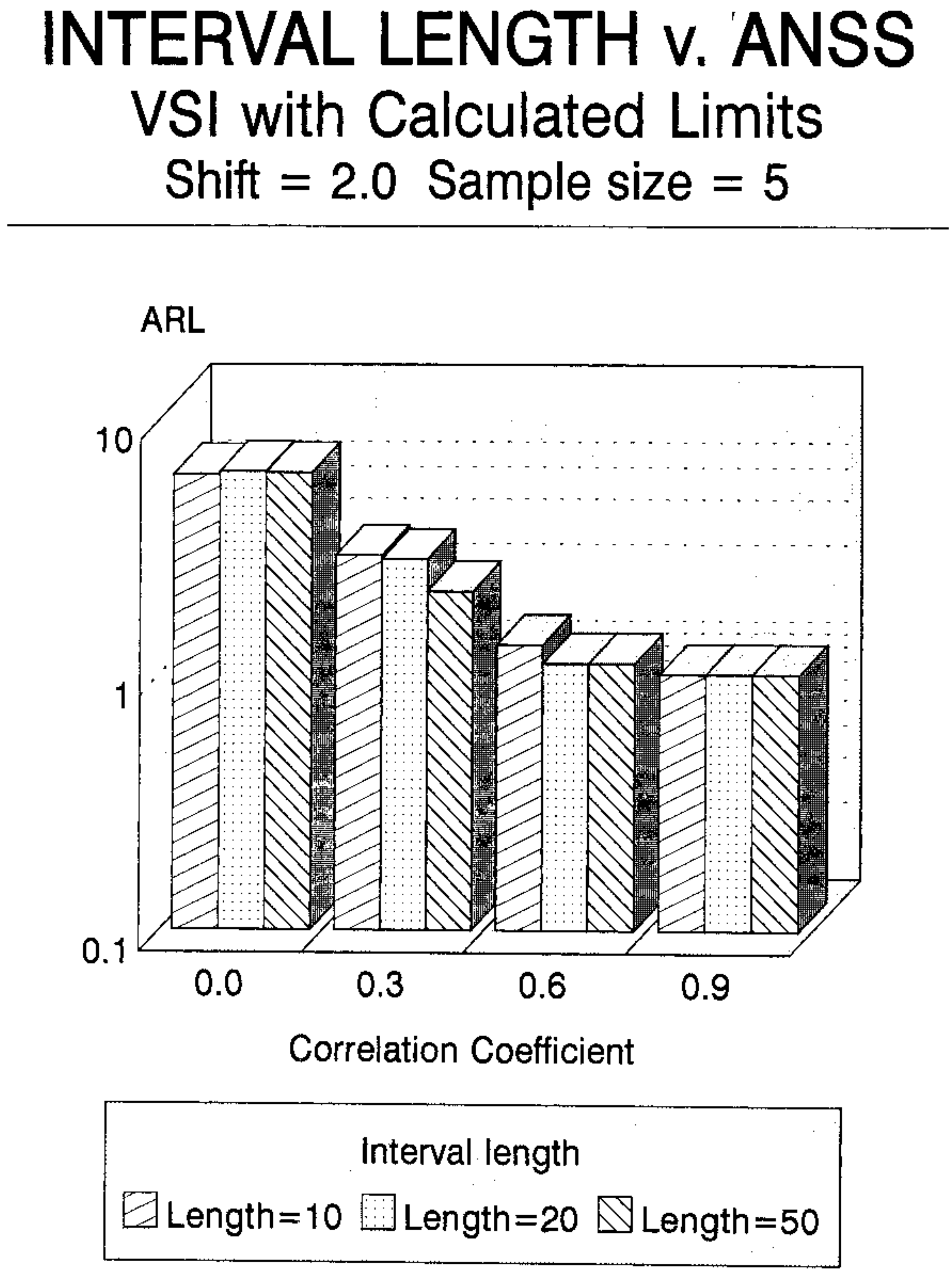


Figure 27f: Effect of interval length on ARL for VSI with limits calculated from sample data
Sample size = 5 Shift = 2.0
y axis: logarithmic scale

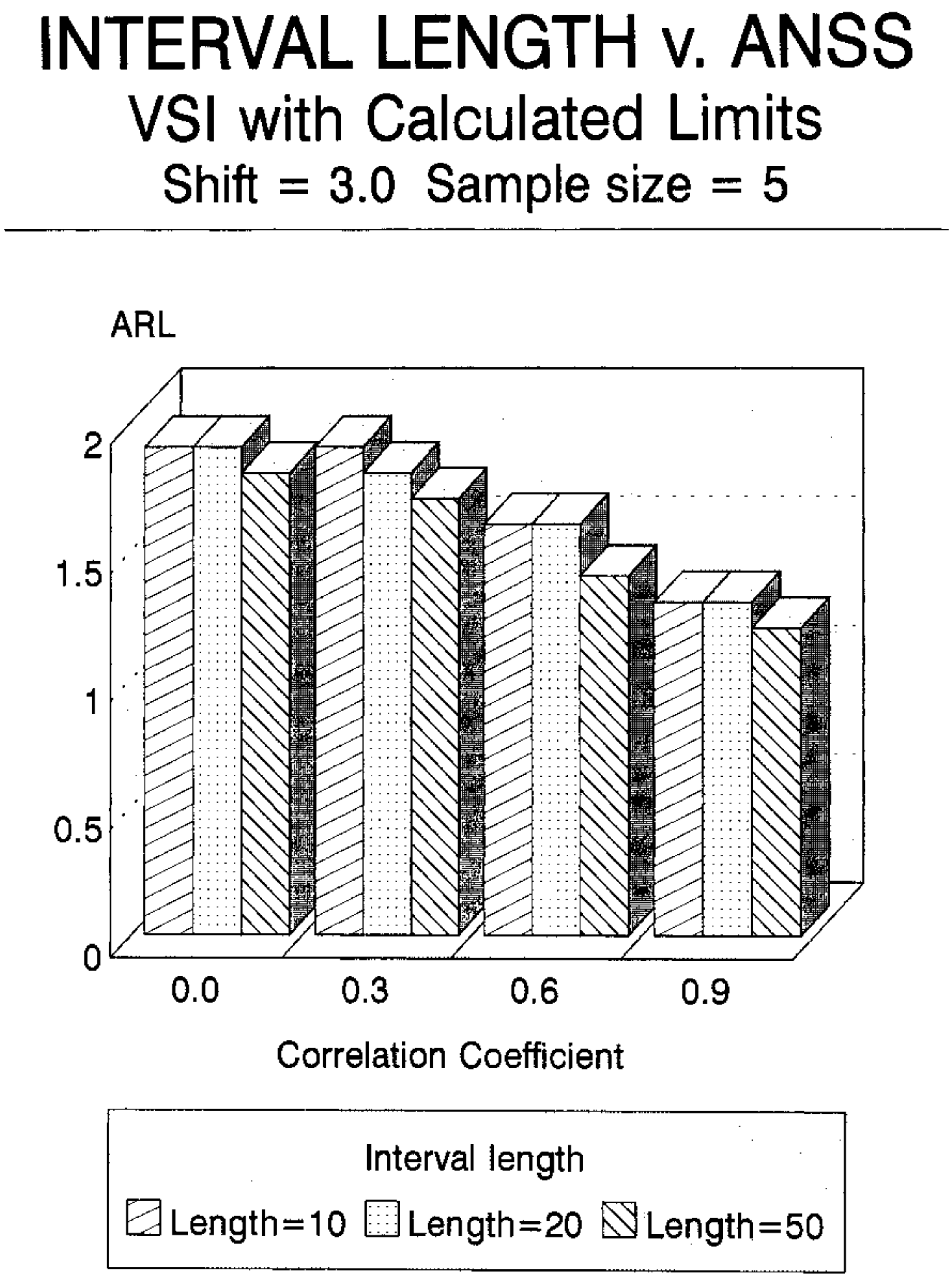


Figure 27g: Effect of Interval length on ARL for VSI with limits calculated from sample data
Sample size = 5 Shift = 3.0
y axis: linear scale

INTERVAL LENGTH v. ANSS
VSI with Calculated Limits
Shift = 3.0 Sample size = 5

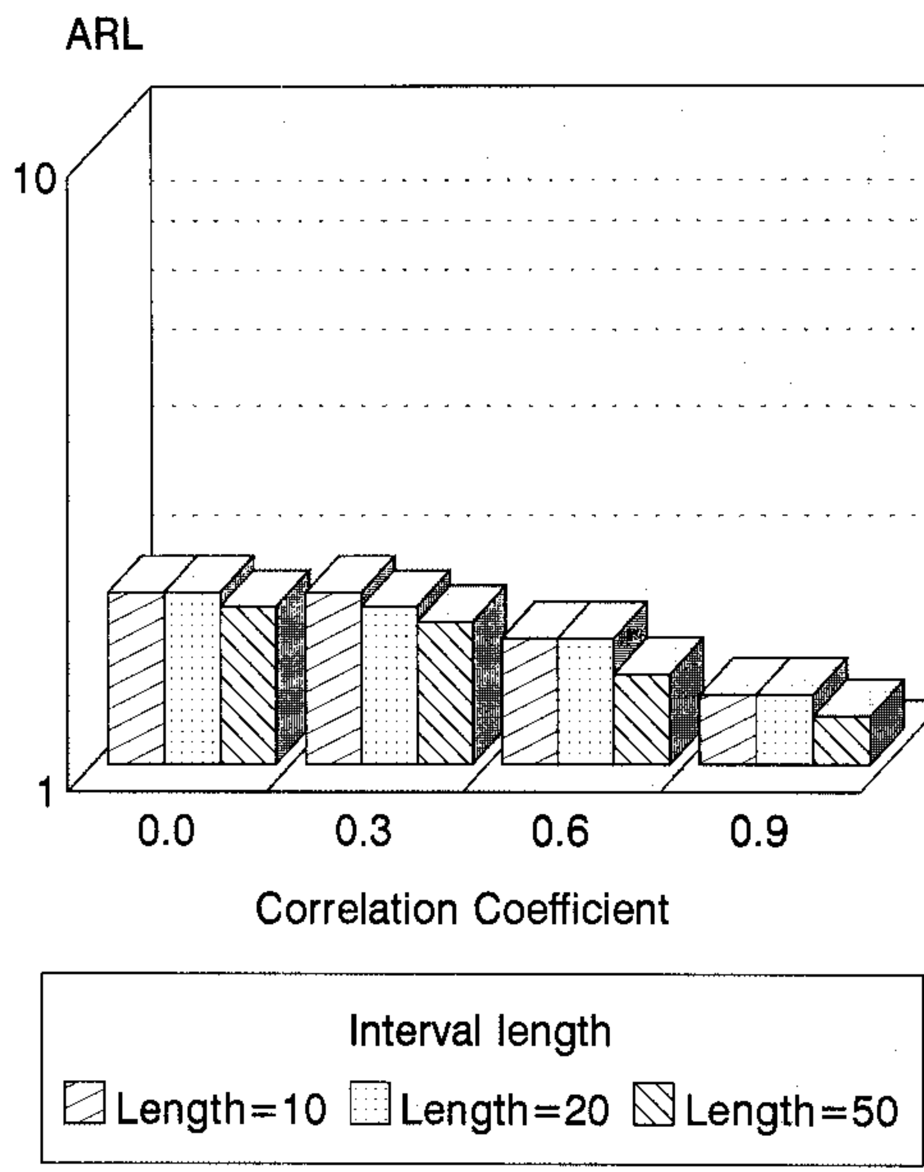


Figure 27h: Effect of interval length on ARL for VSI with limits calculated from sample data
Sample size = 5   Shift = 3.0
y axis: logarithmic scale

EFFECT OF SAMPLE SIZE ON ARL/ANSS

FSI v. VSI PRE-SET LIMITS

Shift = 0.0 Interval length = 10

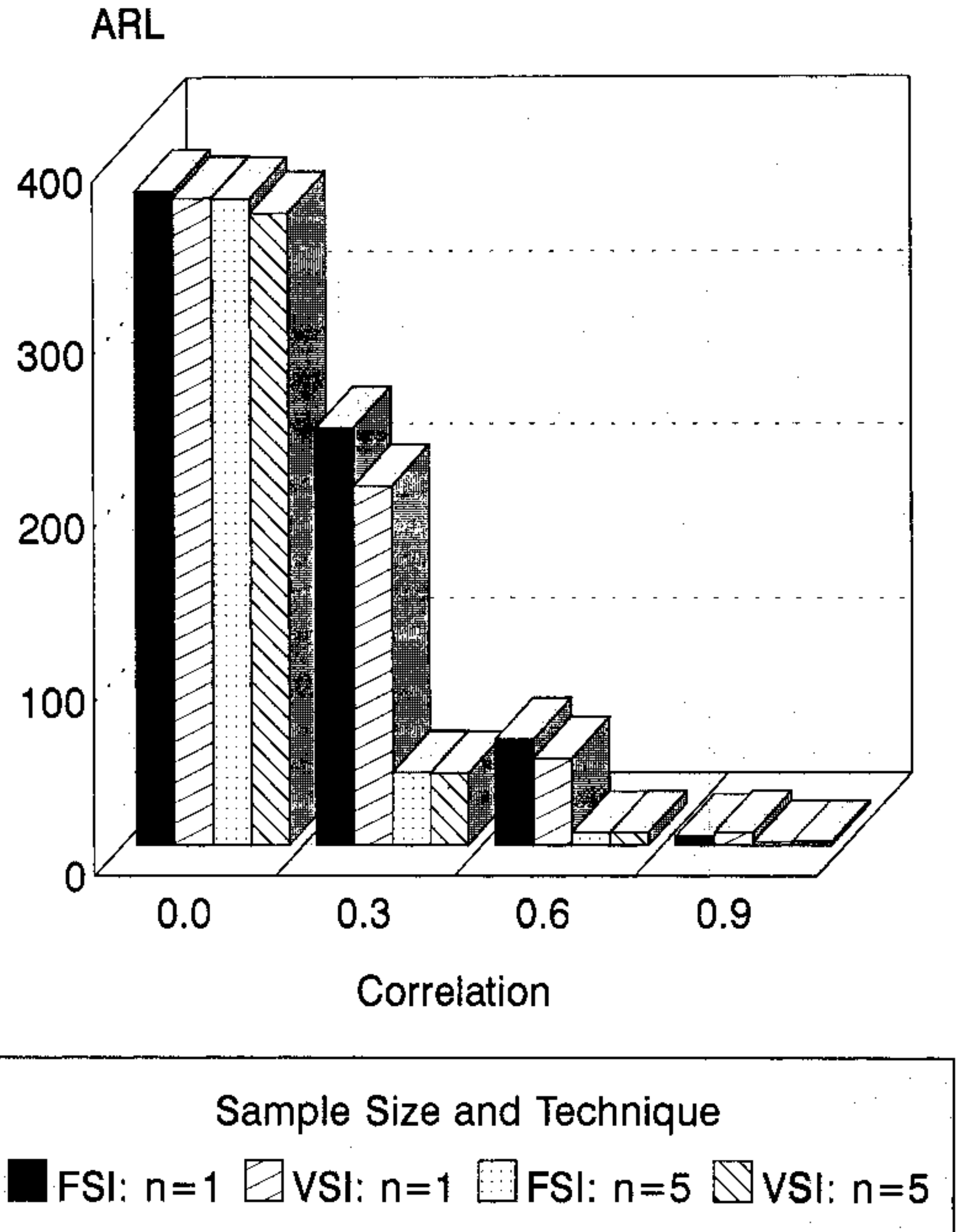


Figure 28a: FSI and VSI have fixed control limits and sample sizes of 1 and 5

EFFECT OF SAMPLE SIZE ON ARL/ANSS

FSI v. VSI PRE-SET LIMITS

Shift = 0.0 Interval length = 10

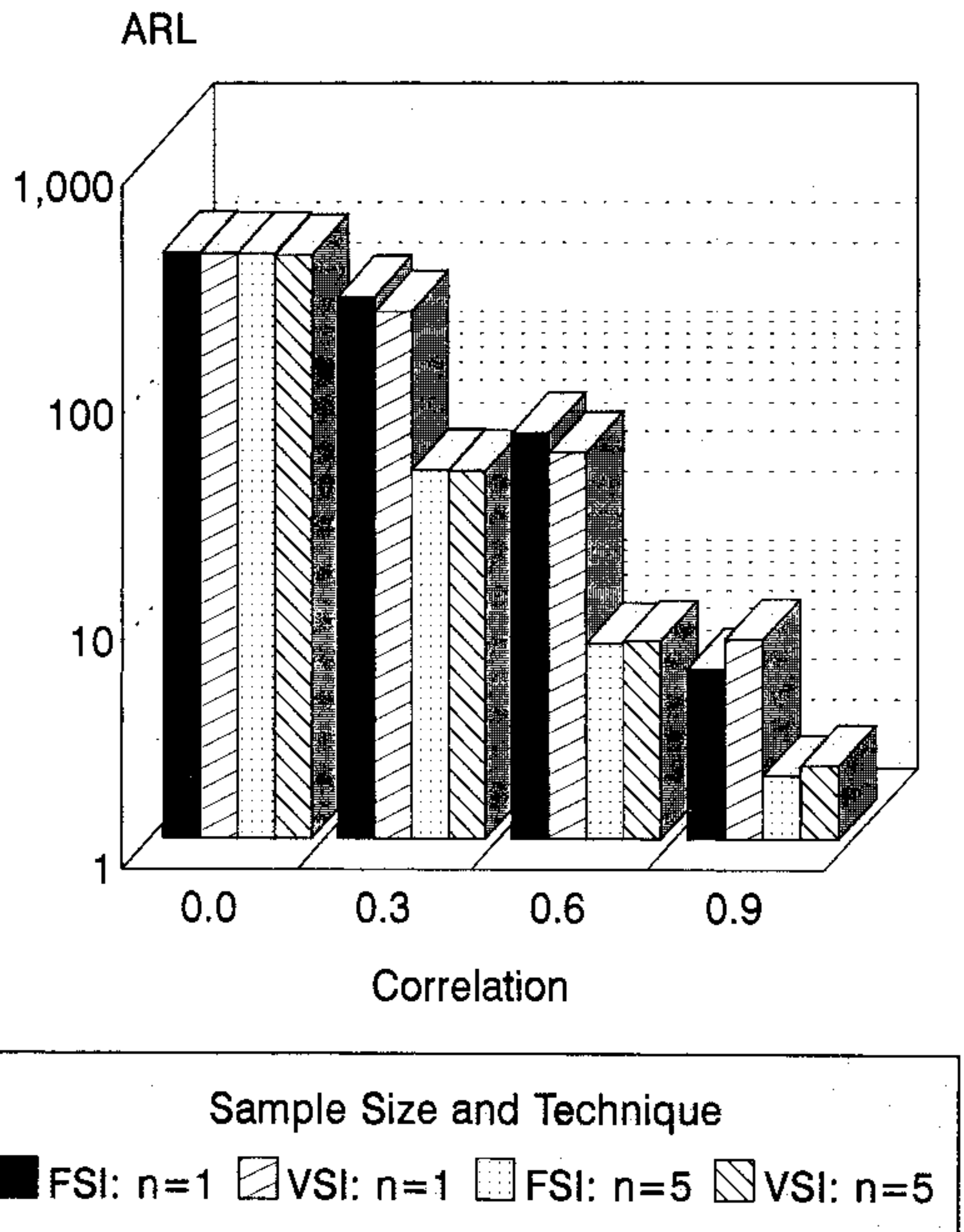
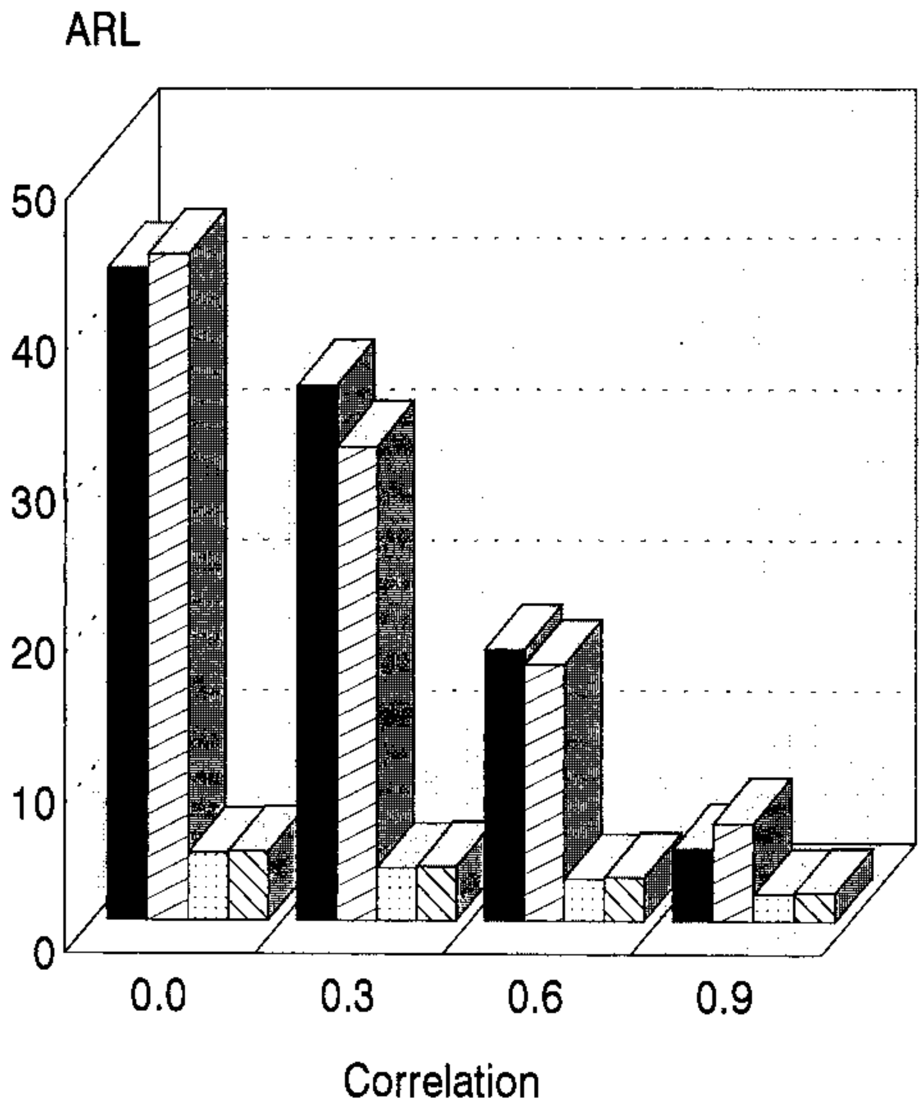


Figure 28b: FSI and VSI have fixed control limits and sample sizes of 1 and 5
y axis: logarithmic scale

EFFECT OF SAMPLE SIZE ON ARL/ANSS
FSI v. VSI PRE-SET LIMITS
Shift = 1.0 Interval length = 10



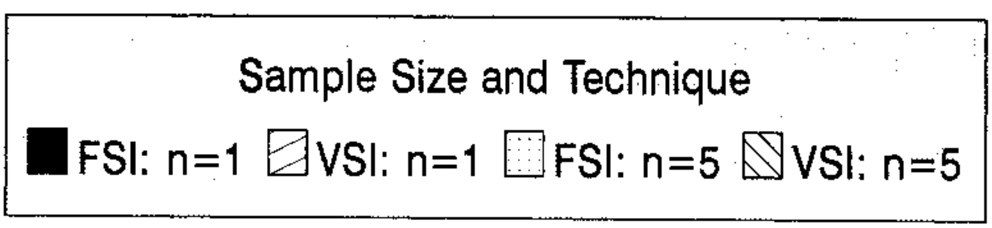


Figure 28C: FSI and VSI have fixed control limits

EFFECT OF SAMPLE SIZE ON ARL/ANSS
FSI v. VSI PRE-SET LIMITS

Shift = 2.0 Interval length = 10

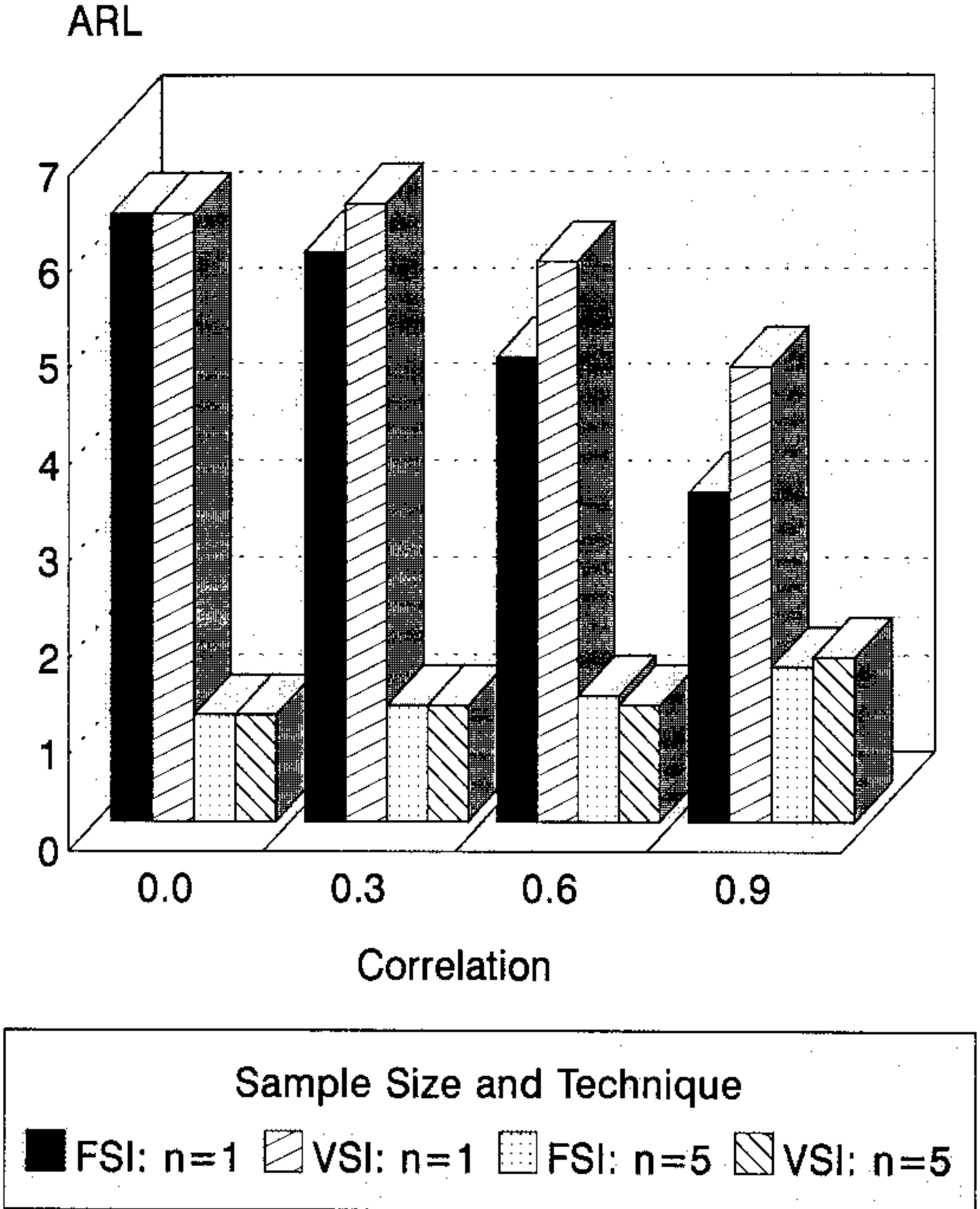


Figure 28D: FSI and VSI have fixed control limits

EFFECT OF SAMPLE SIZE ON ARL/ANSS
FSI V. VSI PRE-SET LLIMITS

Shift = 3.0 Interval length = 10

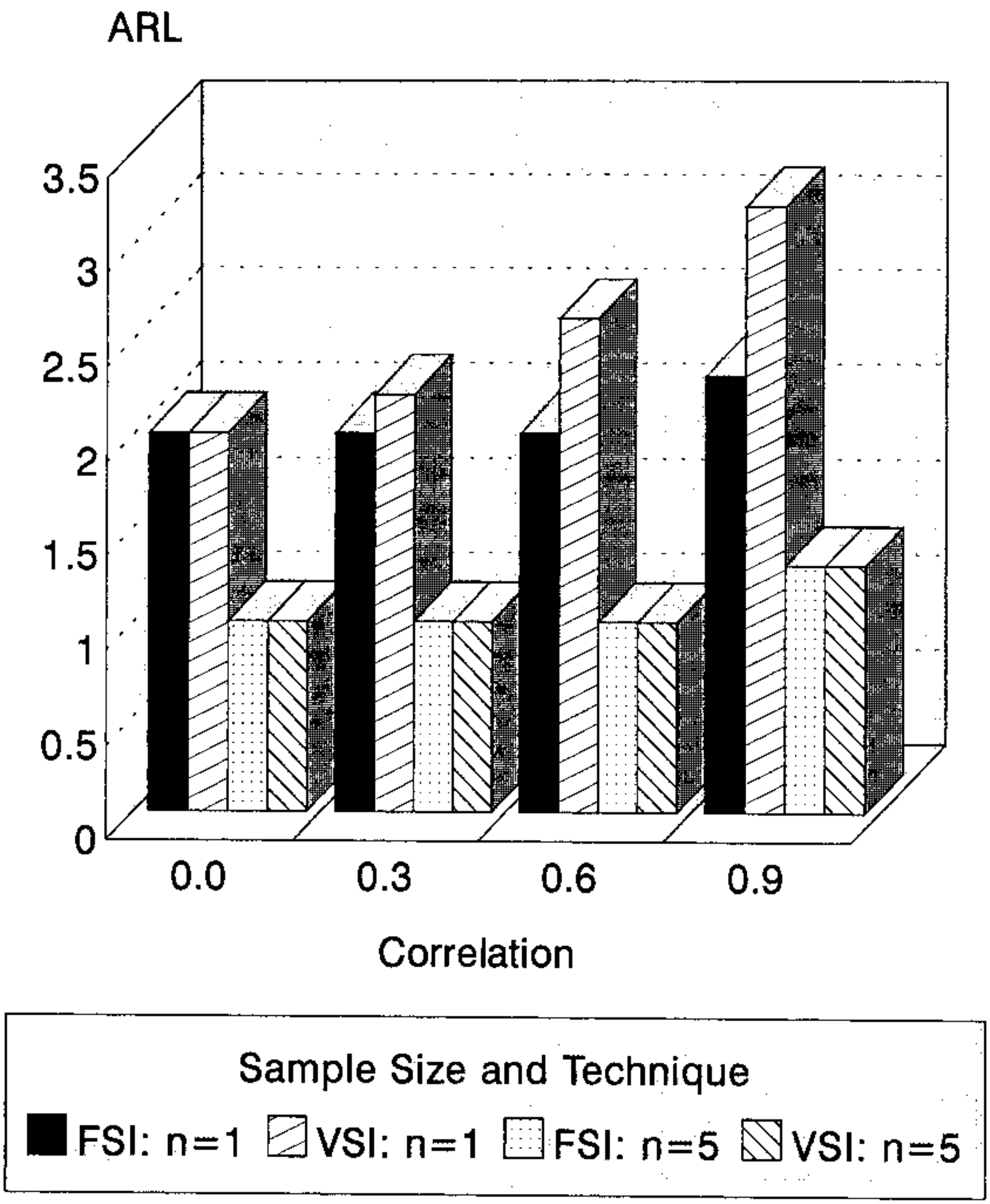


Figure 28e: FSI and VSI have fixed control limits

EFFECT OF SAMPLE SIZE ON ARL/ANSS
FSI v. VSI   CALCULATED LIMITS
Shift = 0.0 Interval length = 10

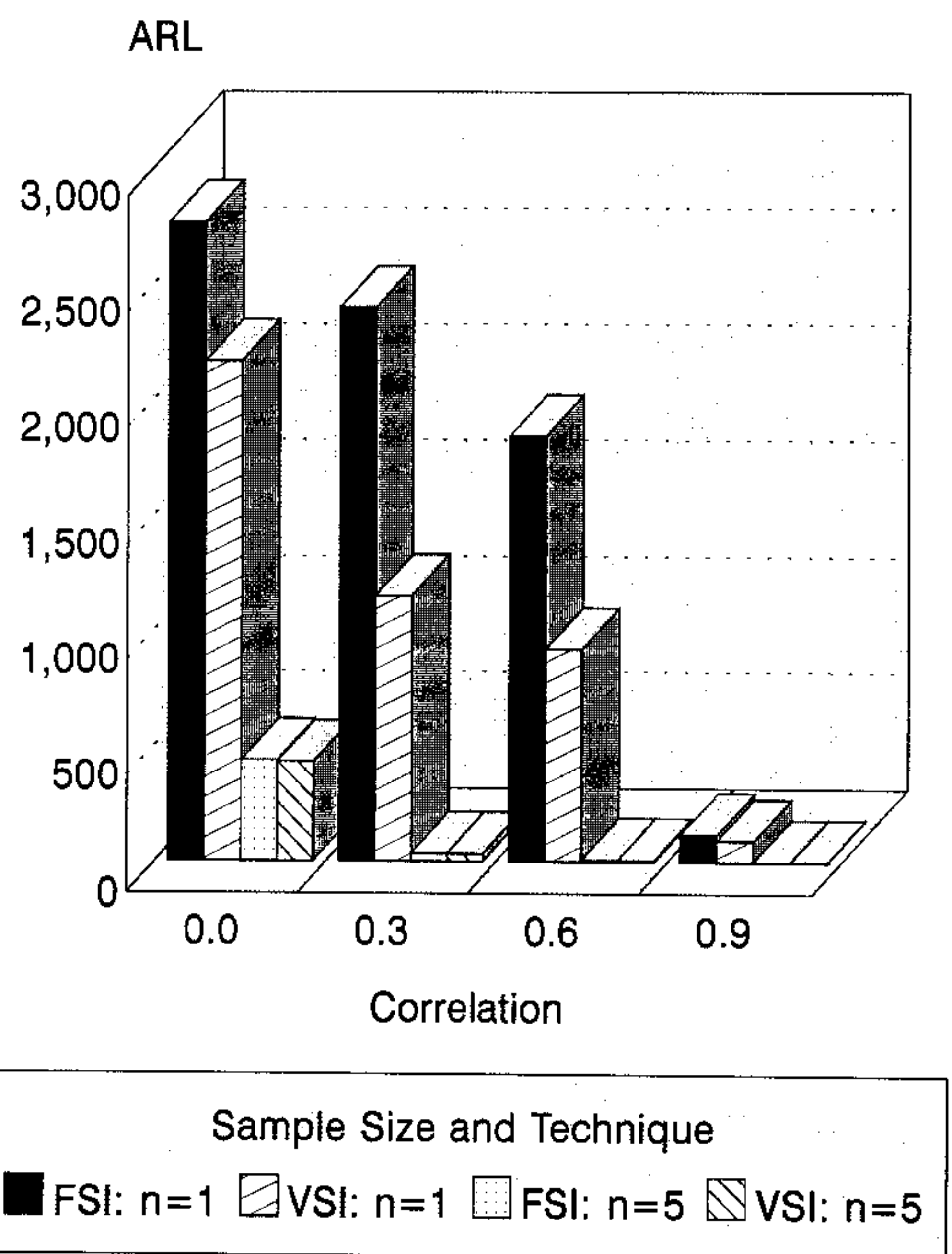


Figure 29a: FSI and VSI have control limits calculated from sample data and sample sizes of 1 and 5

EFFECT OF SAMPLE SIZE ON ARL/ANSS

FSI v. VSI CALCULATED LIMITS

Shift = 0.0 Interval length = 10

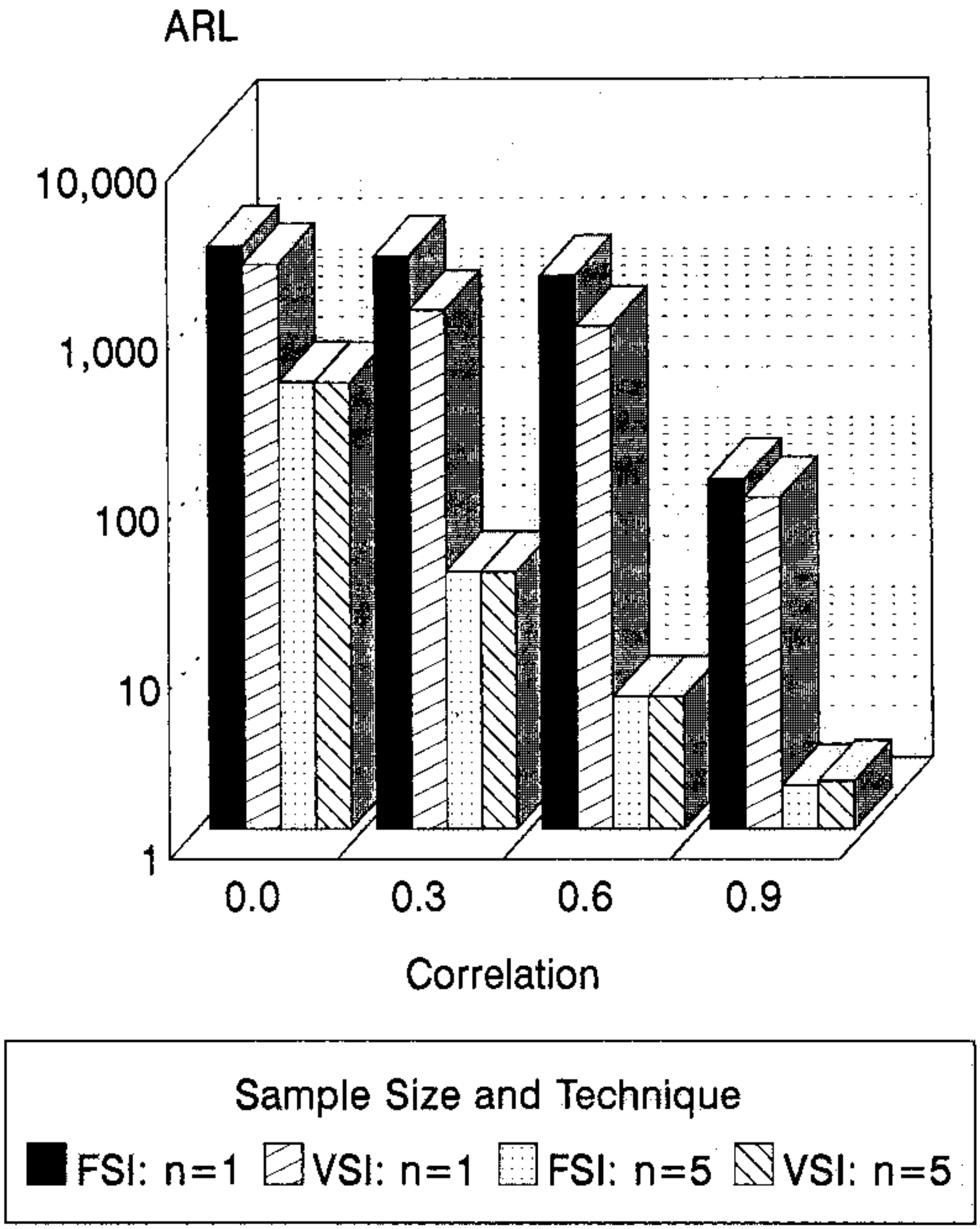


Figure 29b: FSI and VSI have control limits calculated from sample data and sample sizes of 1 and 5
y axis: logarithmic scale

EFFECT OF SAMPLE SIZE ON ARL/ANSS

FSI v. VSI CALCULATED LIMITS

Shift = 1.0 Interval length = 10

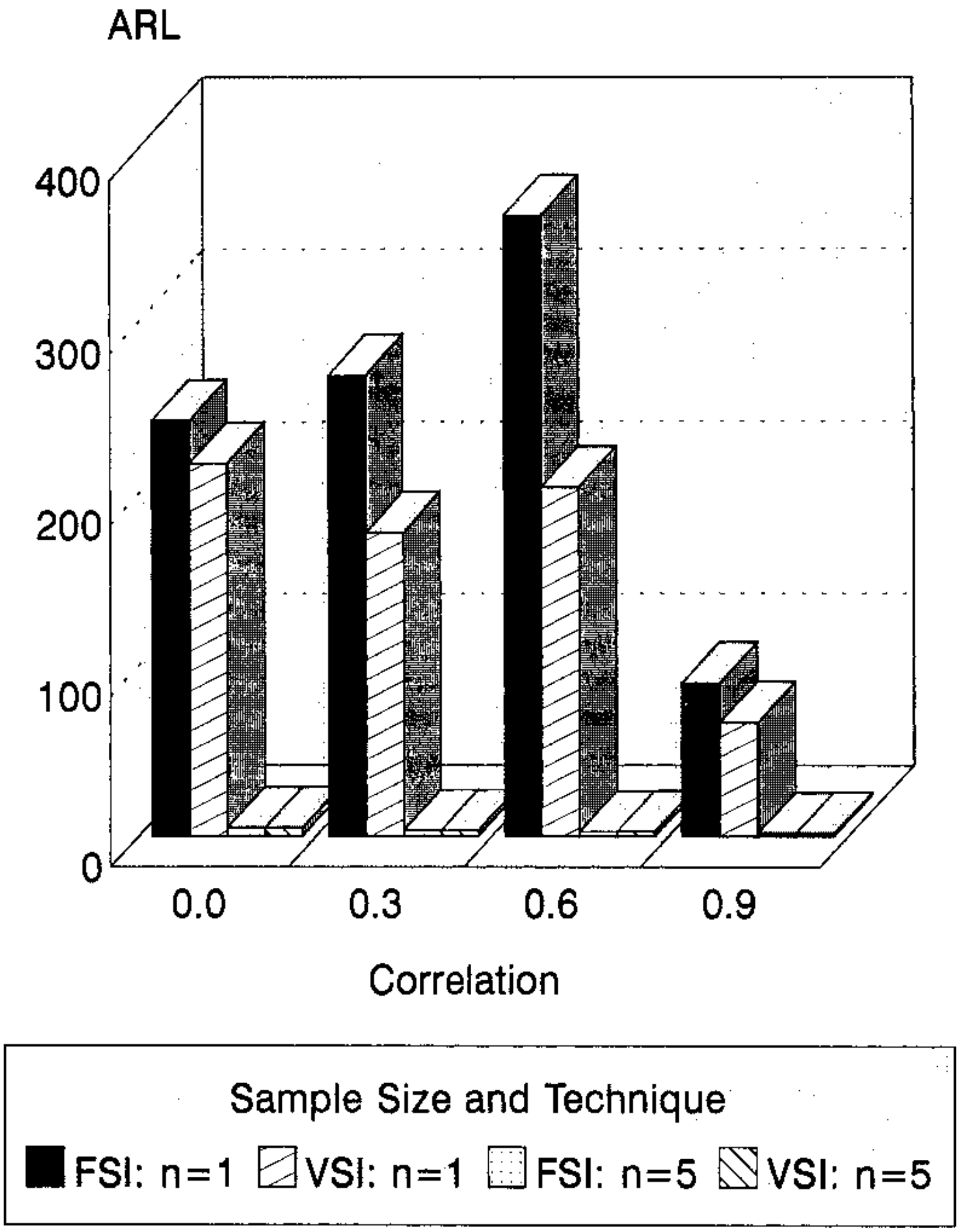


Figure 29c: FSI and VSI have control limits calculated from sample data and sample sizes of 1 and 5

EFFECT OF SAMPLE SIZE ON ARL/ANSS

FSI v. VSI CALCULATED LIMITS

Shift = 1.0 Interval length = 10

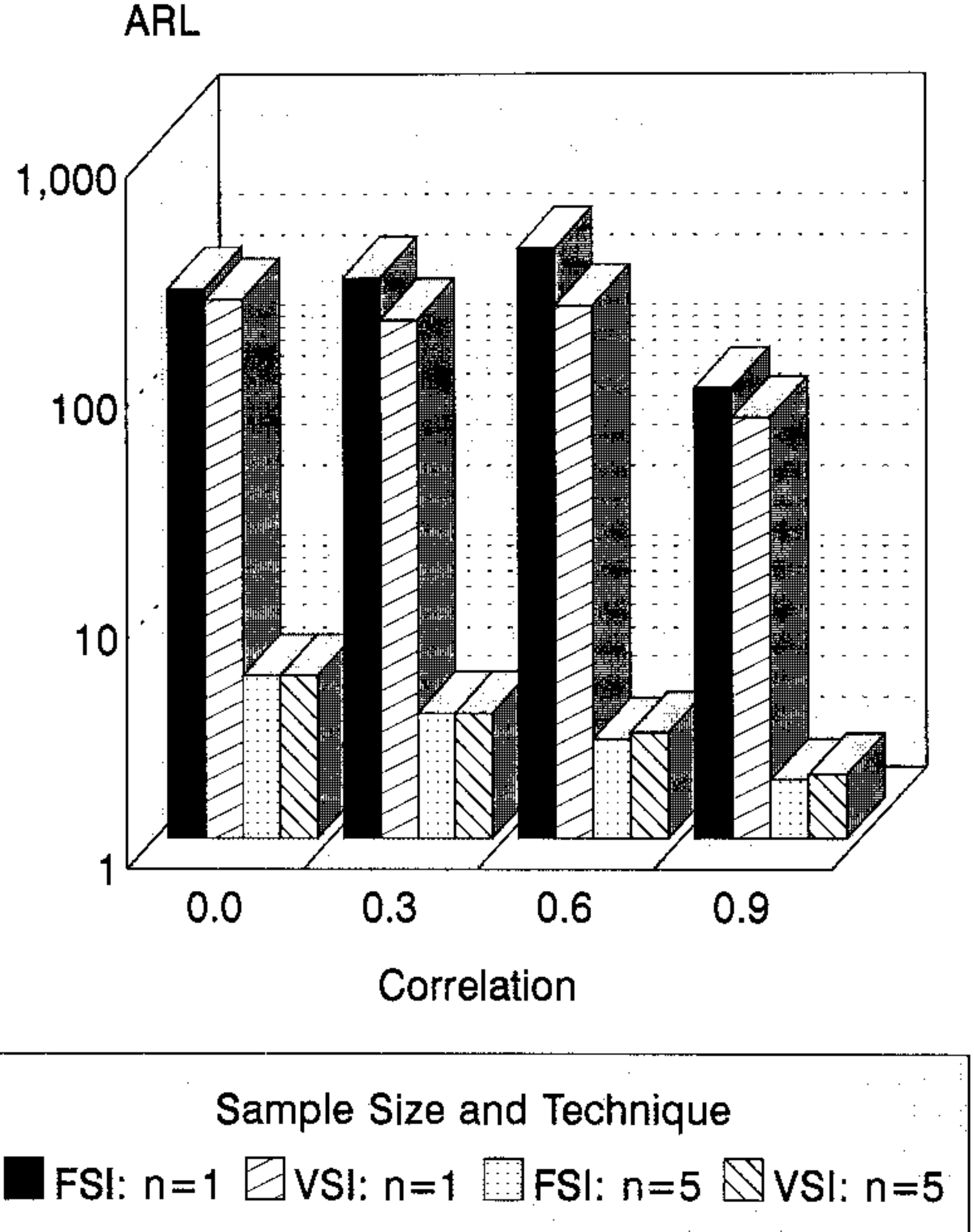


Figure 29d: FSI and VSI have control limits calculated from sample data and sample sizes of 1 and 5
y axis: logarithmic scale

EFFECT OF SAMPLE SIZE ON ARL/ANSS

FSI v. VSI   CALCULATED LIMITS

Shift = 2.0 Interval length = 10

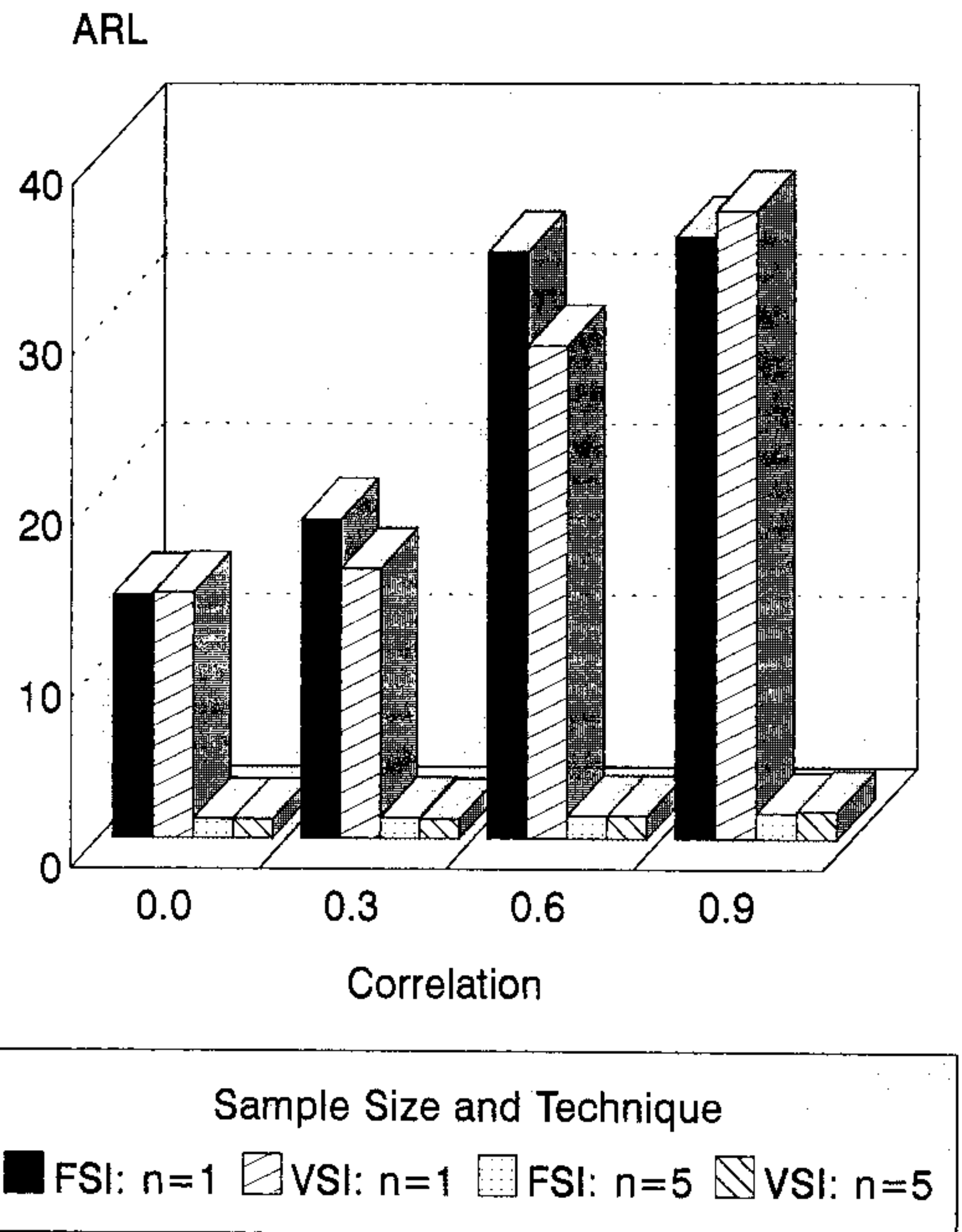


Figure 29e: FSI and VSI have control limits calculated from sample data and sample sizes of 1 and 5
y axis: linear scale

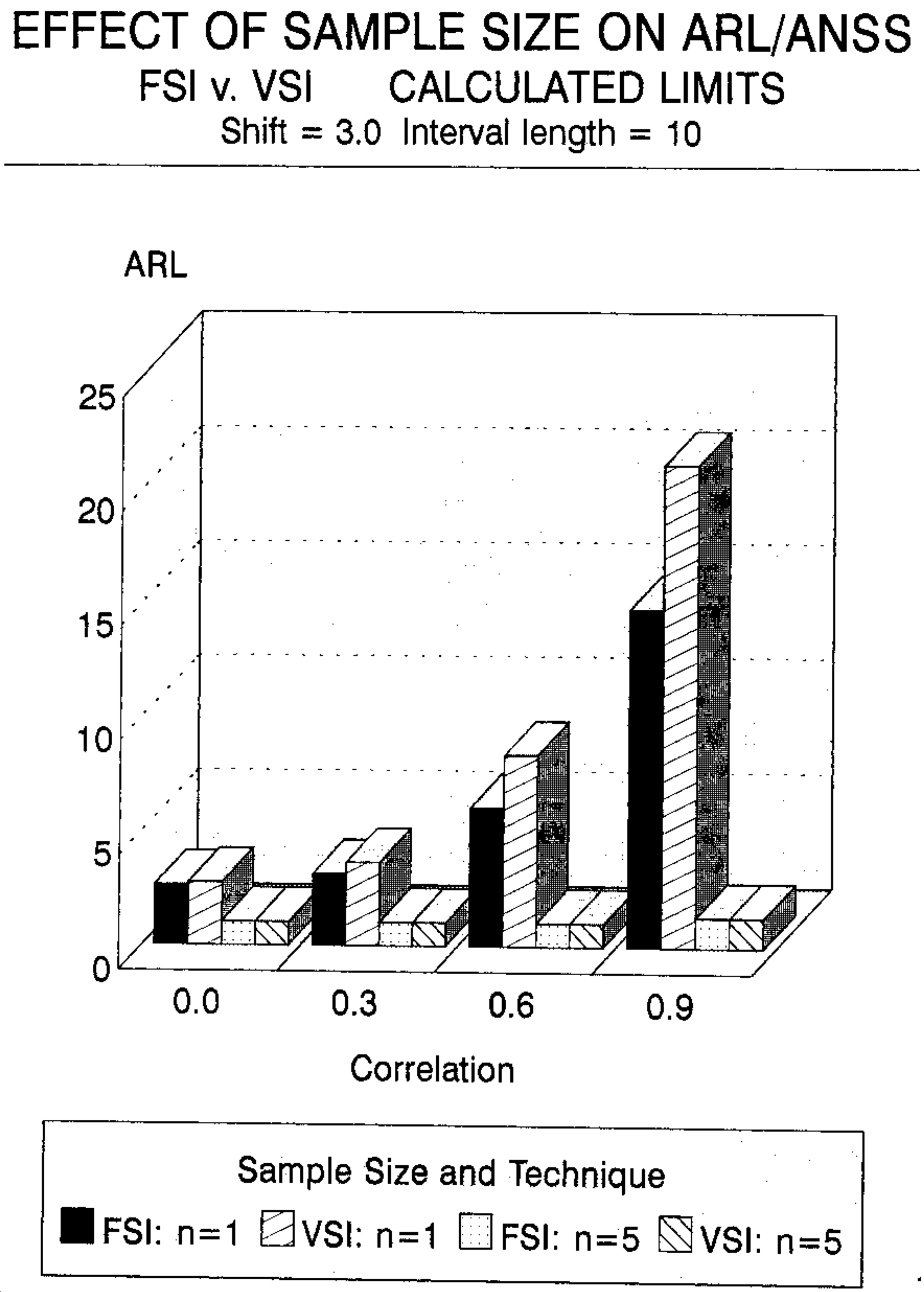


Figure 29f: FSI and VSI have control limits calculated from sample data and sample sizes of 1 and 5
y axis: linear scale

EFFECT OF SAMPLE SIZE ON ATS

FSI v. VSI PRE-SET LIMITS

Shift = 0.0 Interval length = 10

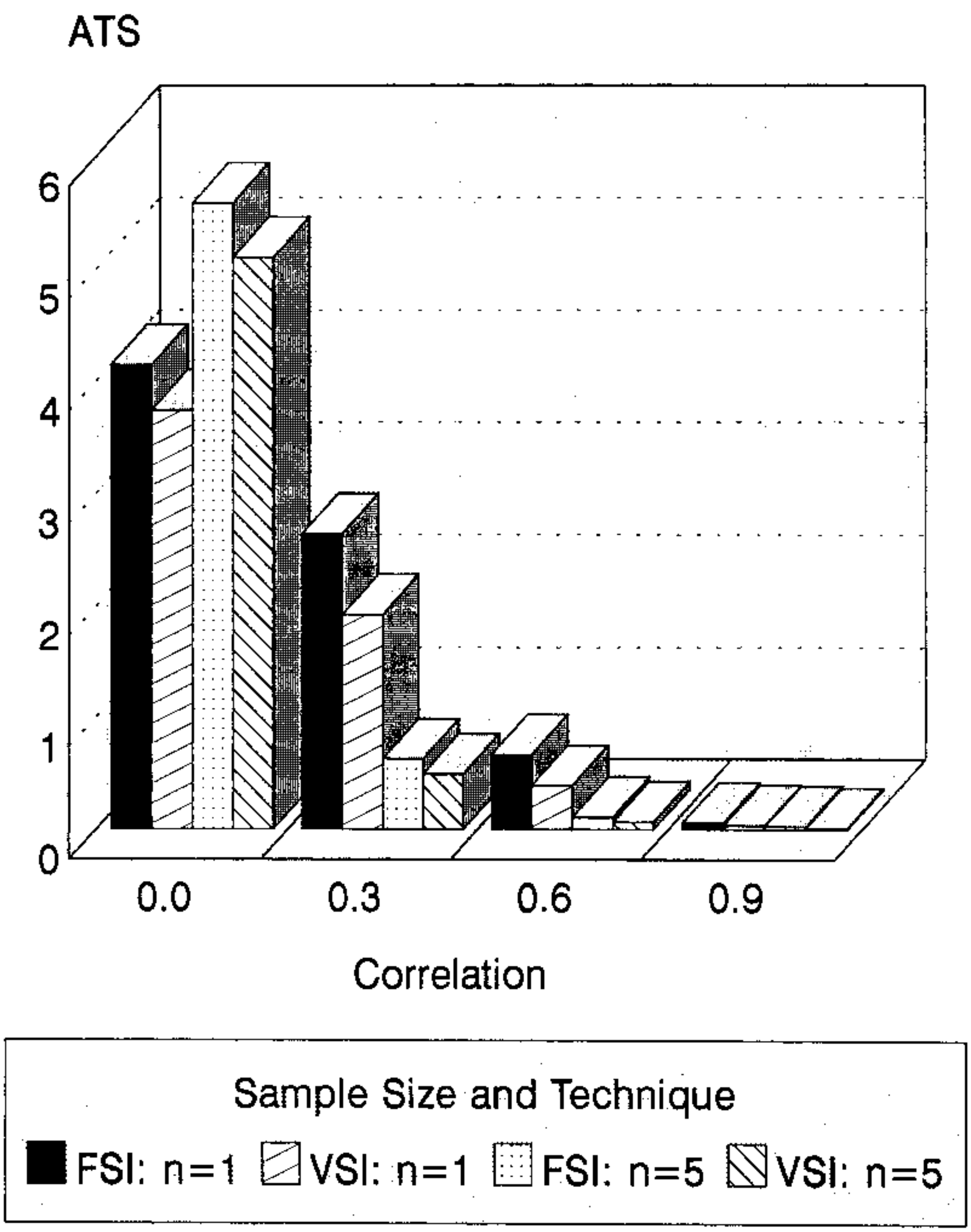


Figure 30a: FSI and VSI have fixed control limits and sample sizes of 1 and 5

EFFECT OF SAMPLE SIZE ON ATS
FSI v. VSI PRE-SET LIMITS
Shift = 0.0 Interval length = 10

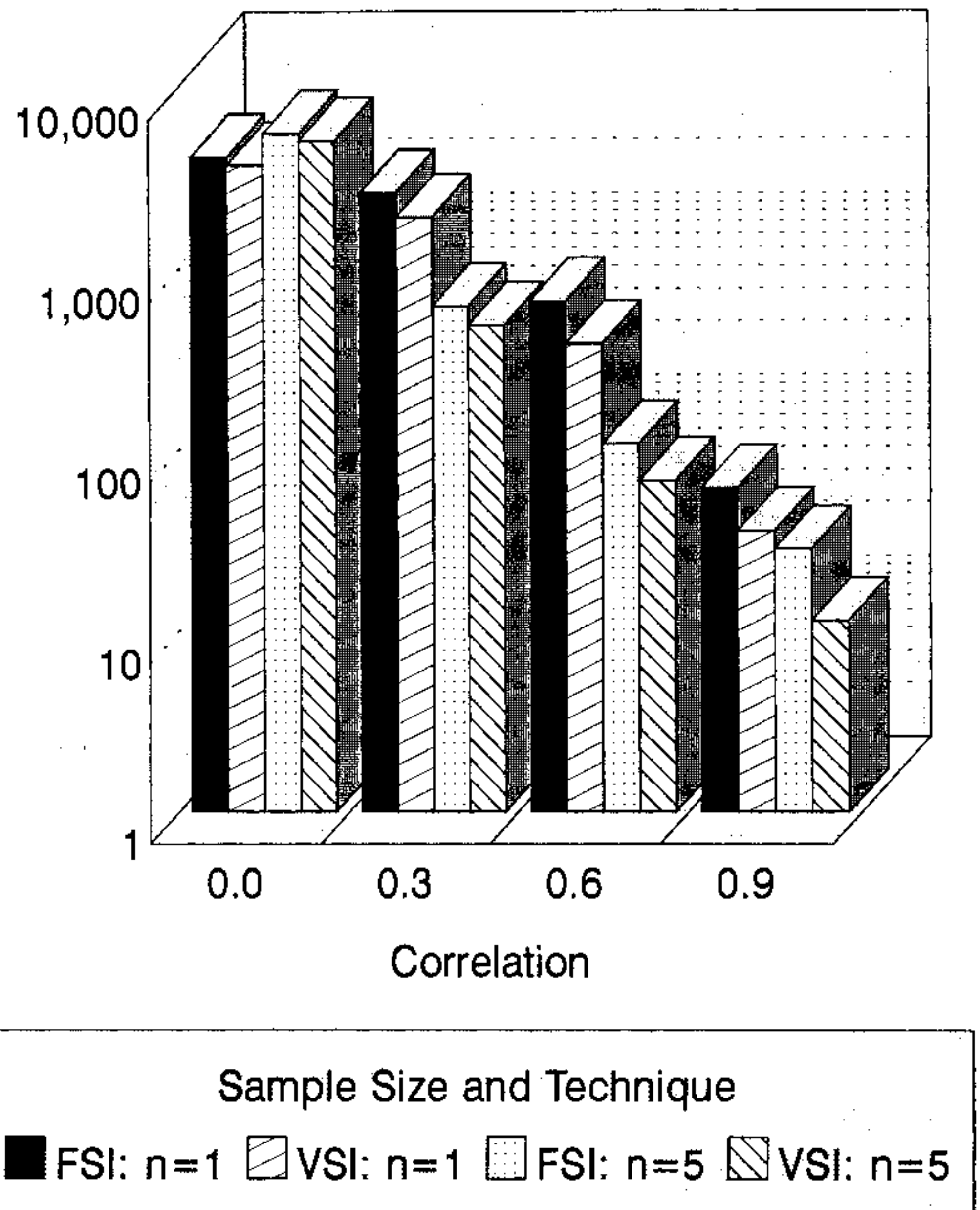


Figure 30b: FSI and VSI have fixed control limits and sample sizes of 1 and 5
y axis: logarithmic scale

EFFECT OF SAMPLE SIZE ON ATS
FSI v. VSI PRE-SET LIMITS
Shift = 1.0 Interval length = 10

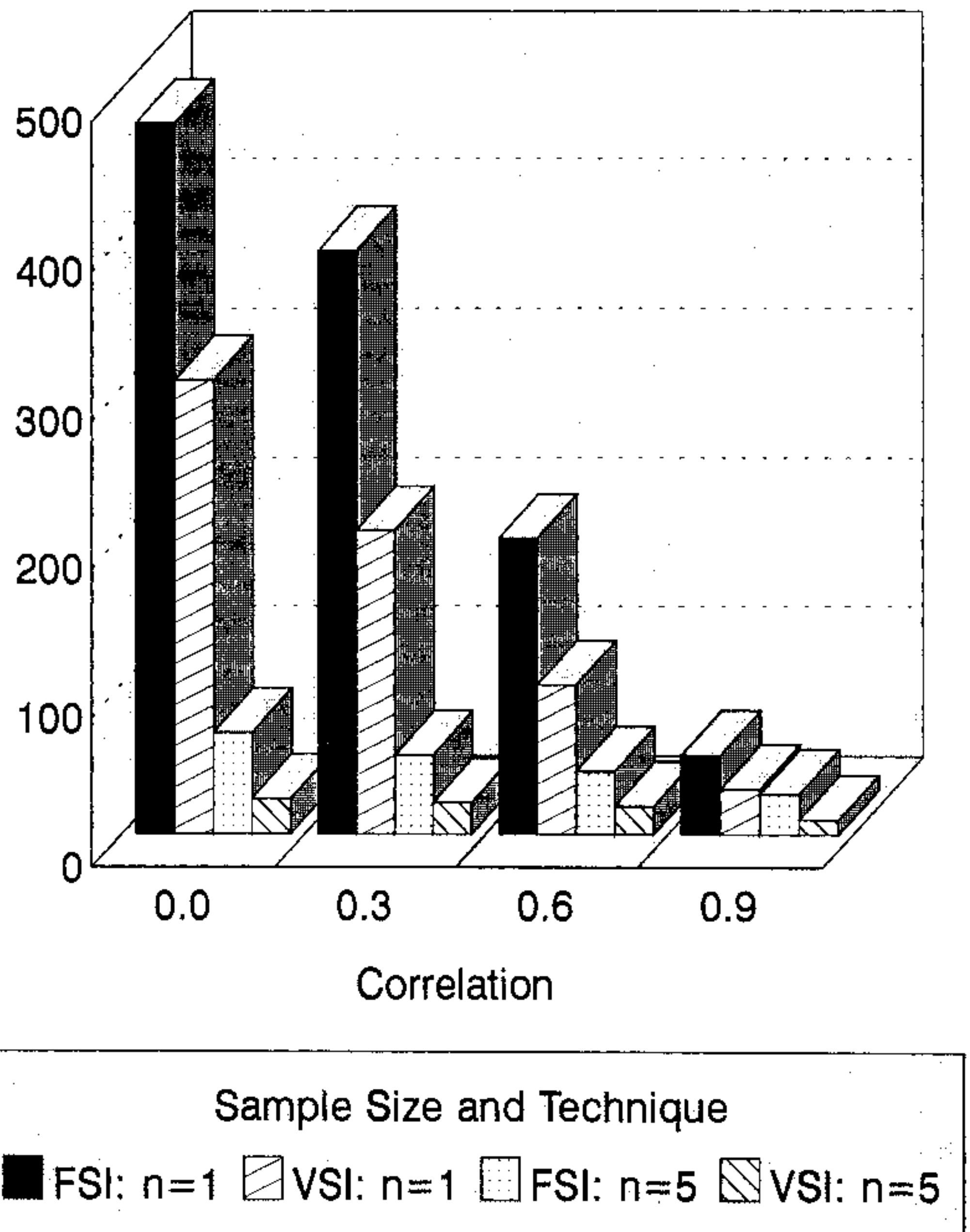


Figure 30c: FSI and VSI have fixed control limits and sample sizes of 1 and 5

EFFECT OF SAMPLE SIZE ON ATS

FSI v. VSI PRE-SET LIMITS

Shift = 2.0 Interval length = 10

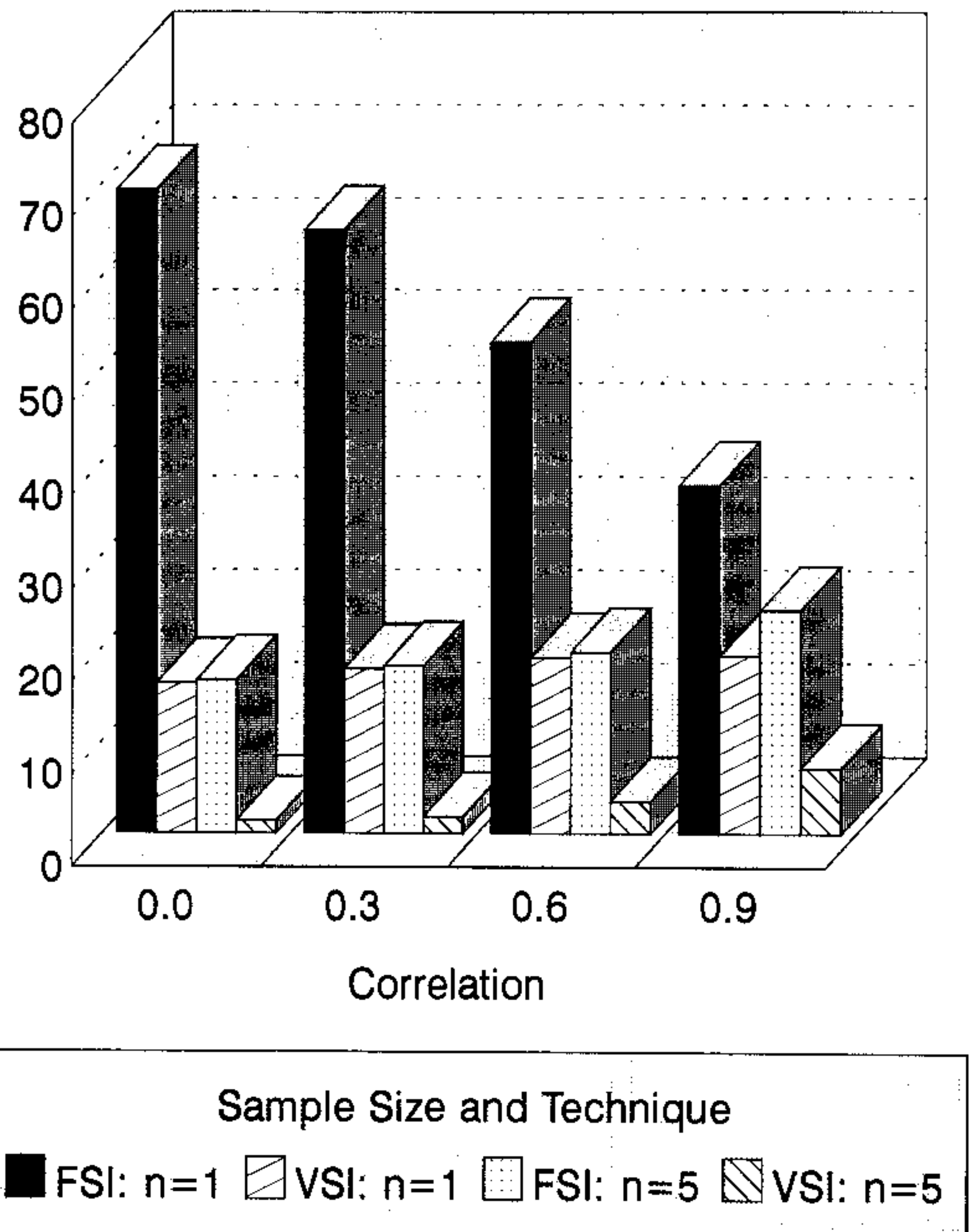


Figure 30d: FSI and VSI have fixed control limits and sample sizes of 1 and 5

EFFECT OF SAMPLE SIZE ON ATS
FSI v. VSI PRE-SET LIMITS
Shift = 3.0 Interval length = 10

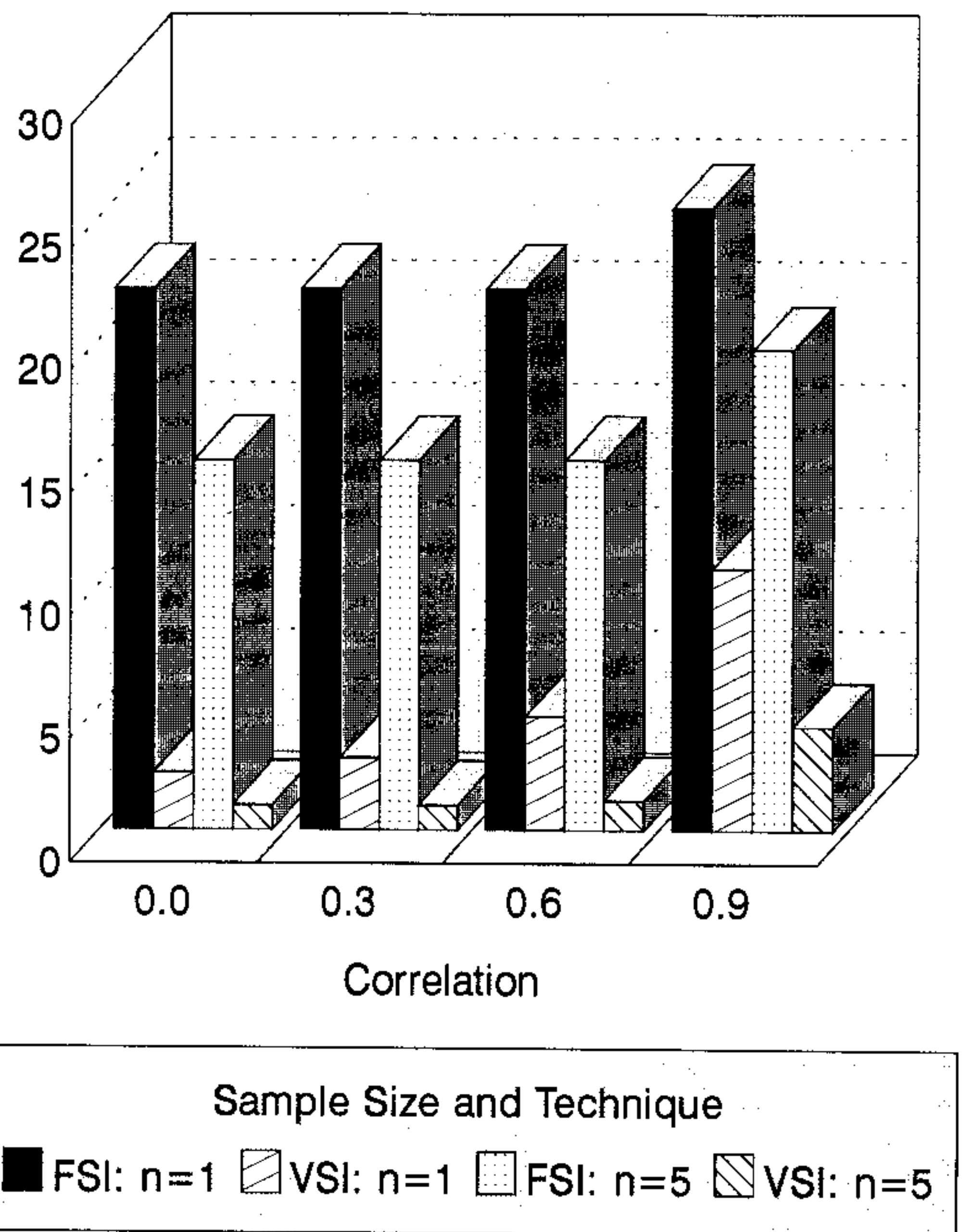


Figure 30e: FSI and VSI have fixed control limits and sample sizes of 1 and 5

EFFECT OF SAMPLE SIZE ON ATS
FSI v. VSI CALCULATED LIMITS
Shift = 0.0 Interval length = 10

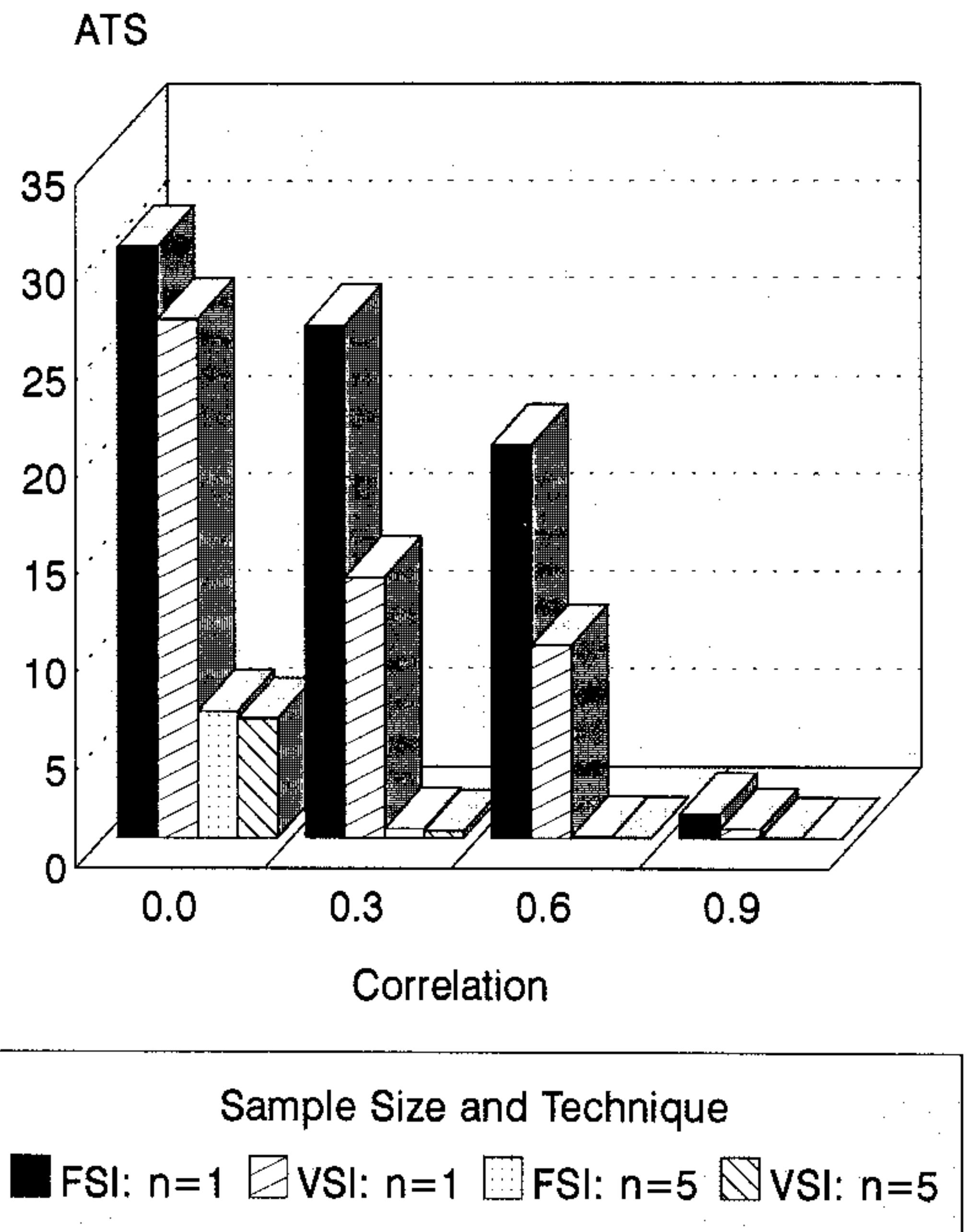


Figure 31a: FSI and VSI have calculated control limits and sample sizes of 1 and 5

EFFECT OF SAMPLE SIZE ON ATS
FSI v. VSI CALCULATED LIMITS
Shift = 0.0 Interval length = 10

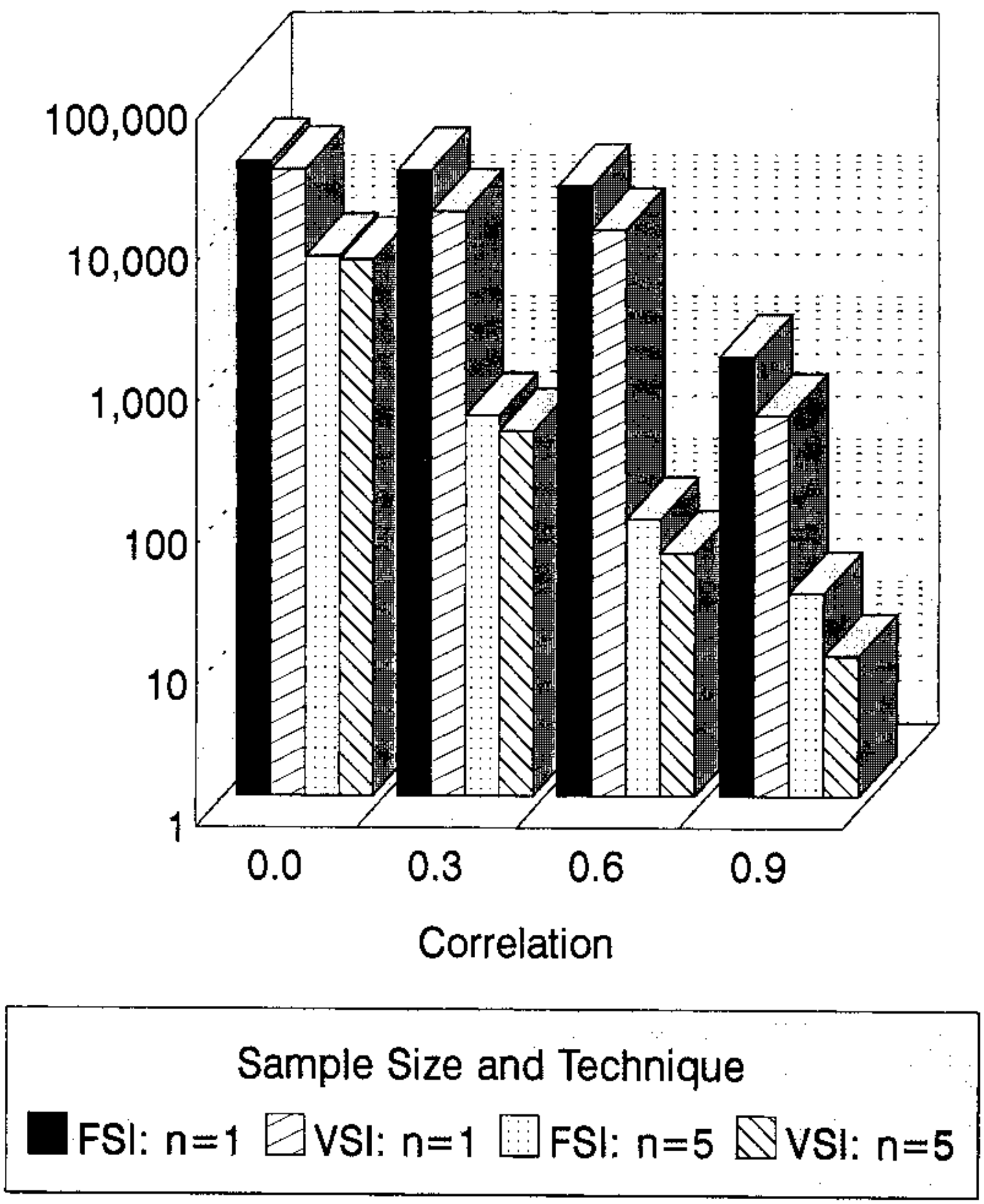


Figure 31b: FSI and VSI have calculated control limits and sample sizes of 1 and 5
y axis: logarithmic scale

EFFECT OF SAMPLE SIZE ON ATS
FSI v. VSI CALCULATED LIMITS
Shift = 1.0 Interval length = 10

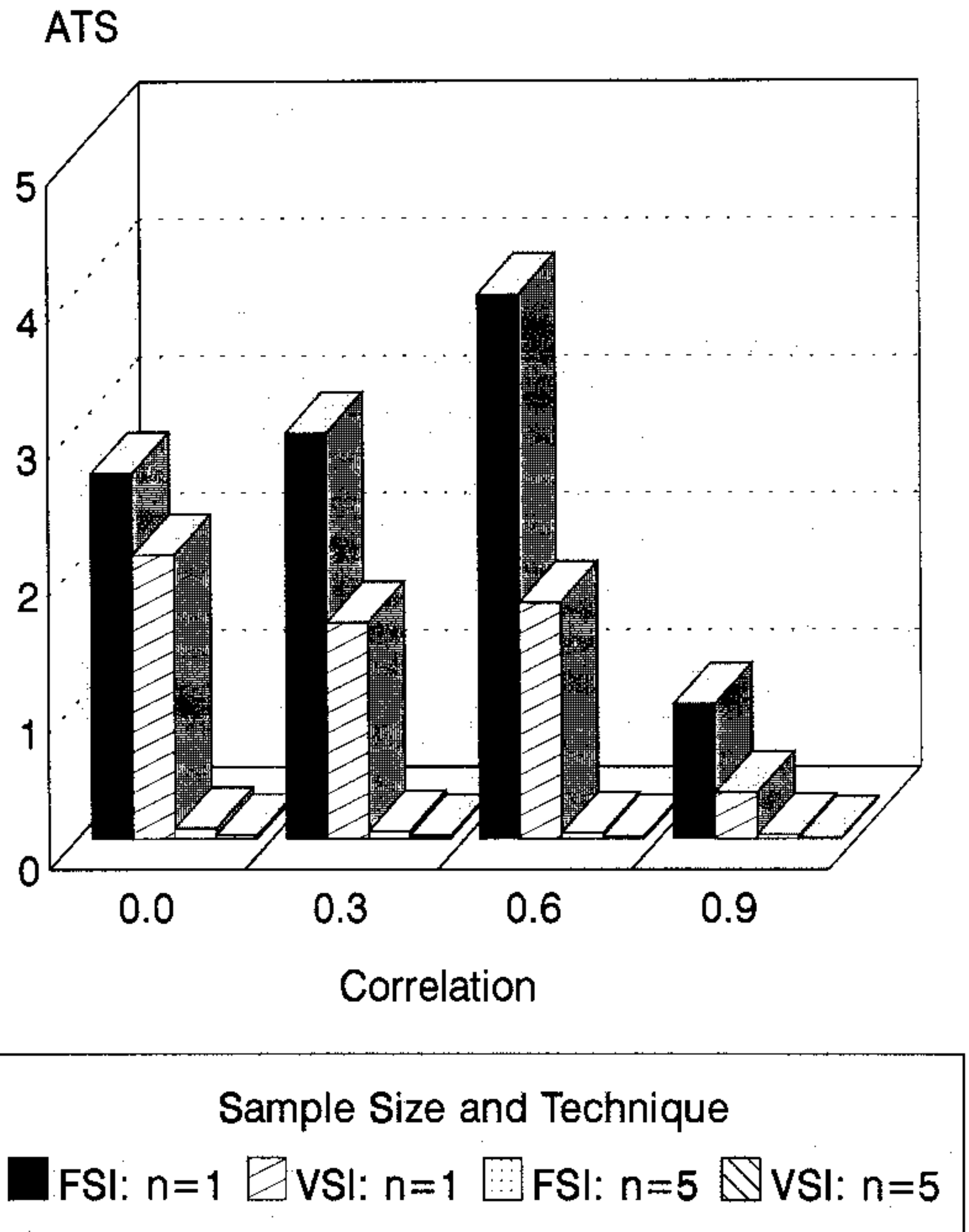


Figure 31c: FSI and VSI have calculated control limits and sample sizes of 1 and 5

EFFECT OF SAMPLE SIZE ON ATS
FSI v. VSI CALCULATED LIMITS
Shift = 1.0 Interval length = 10

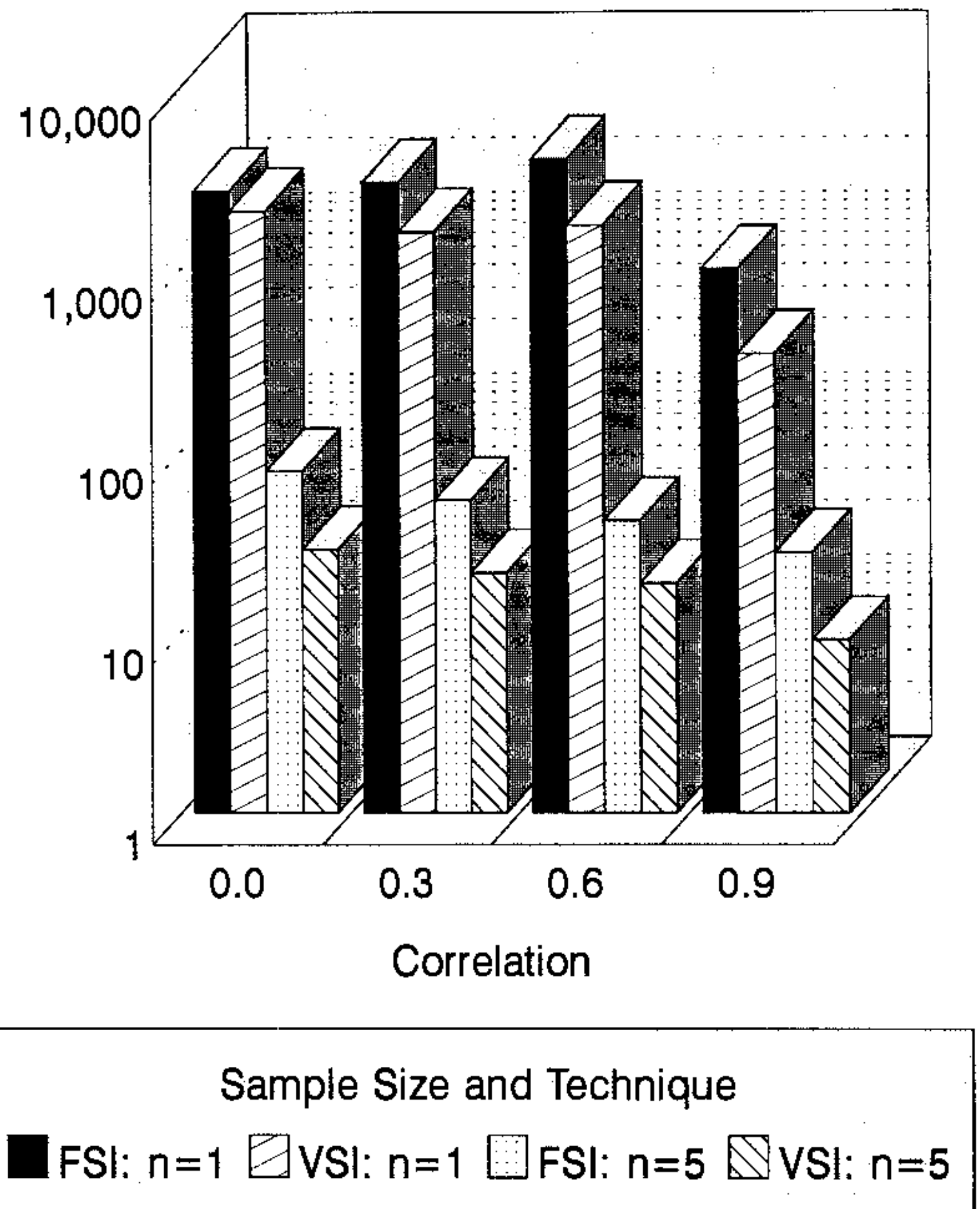


Figure 31d: FSI and VSI have calculated control limits and sample sizes of 1 and 5
y axis: logarithmic scale

EFFECT OF SAMPLE SIZE ON ATS
FSI v. VSI CALCULATED LIMITS
Shift = 2.0 Interval length = 10

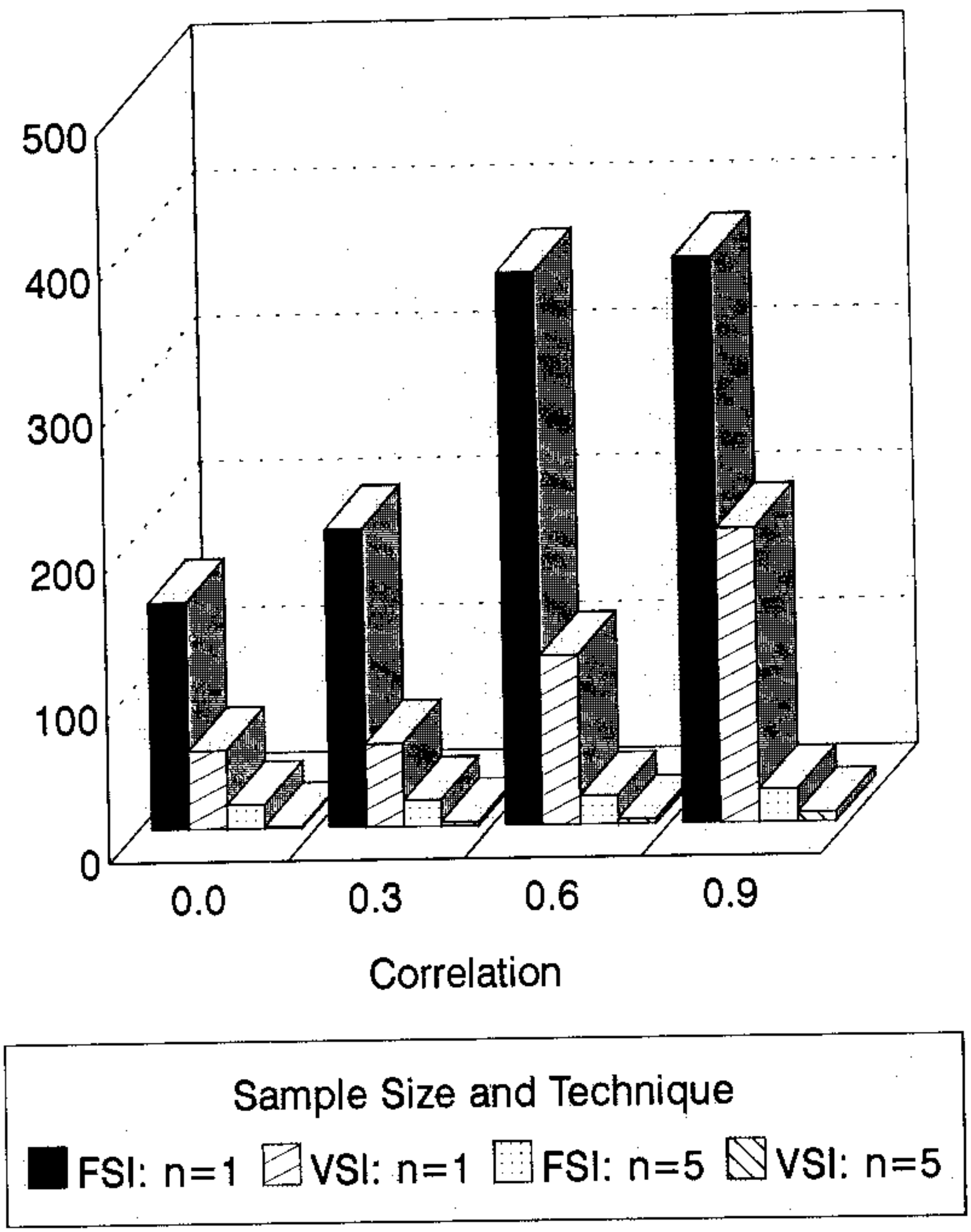


Figure 31e: FSI and VSI have calculated control limits and sample sizes of 1 and 5

EFFECT OF SAMPLE SIZE ON ATS
FSI v. VSI CALCULATED LIMITS
Shift = 2.0 Interval length = 10

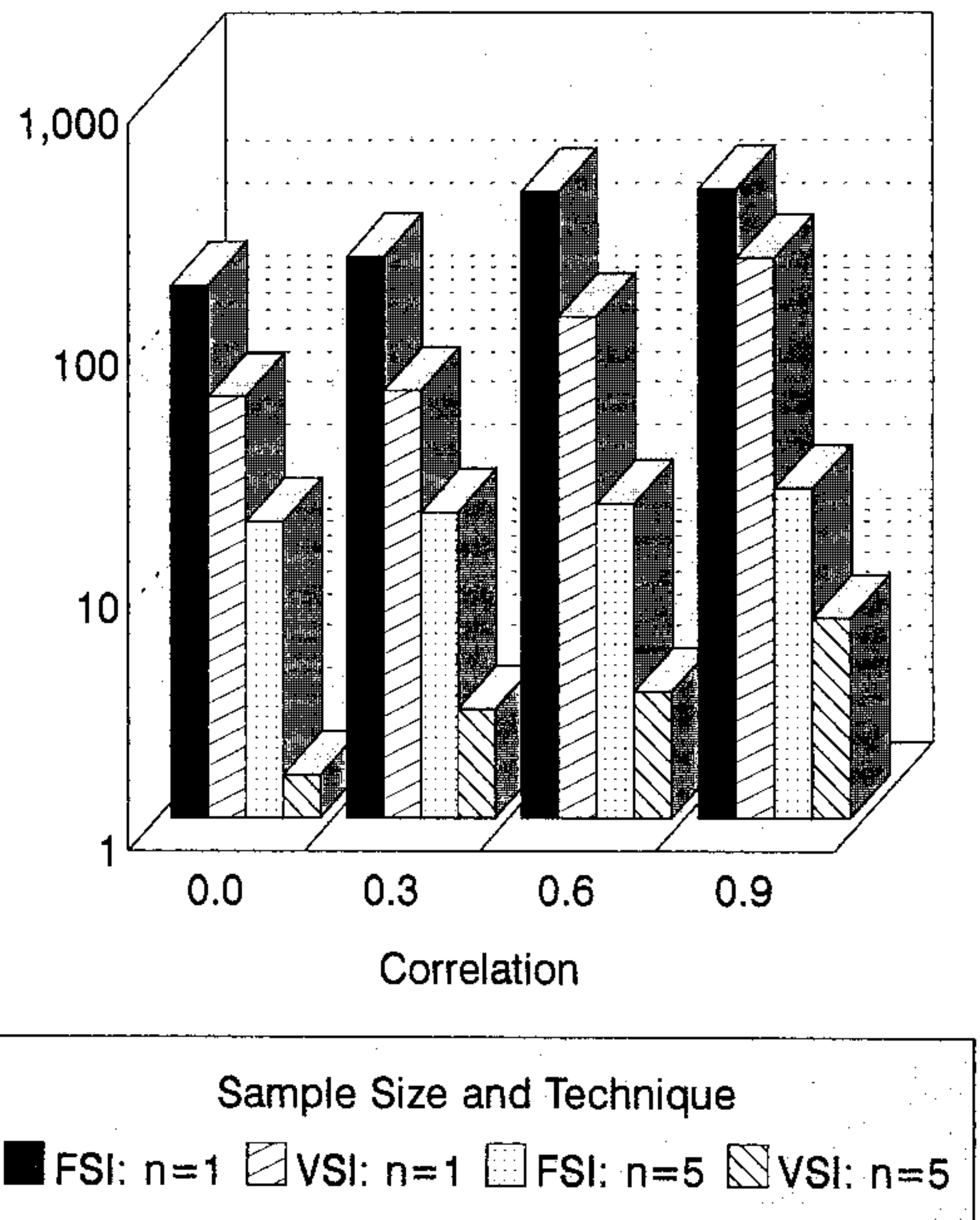


Figure 31f: FSI and VSI have calculated control limits and sample sizes of 1 and 5
y axis: logarithmic scale

EFFECT OF SAMPLE SIZE ON ATS
FSI v. VSI CALCULATED LIMITS
Shift = 3.0 Interval length = 10

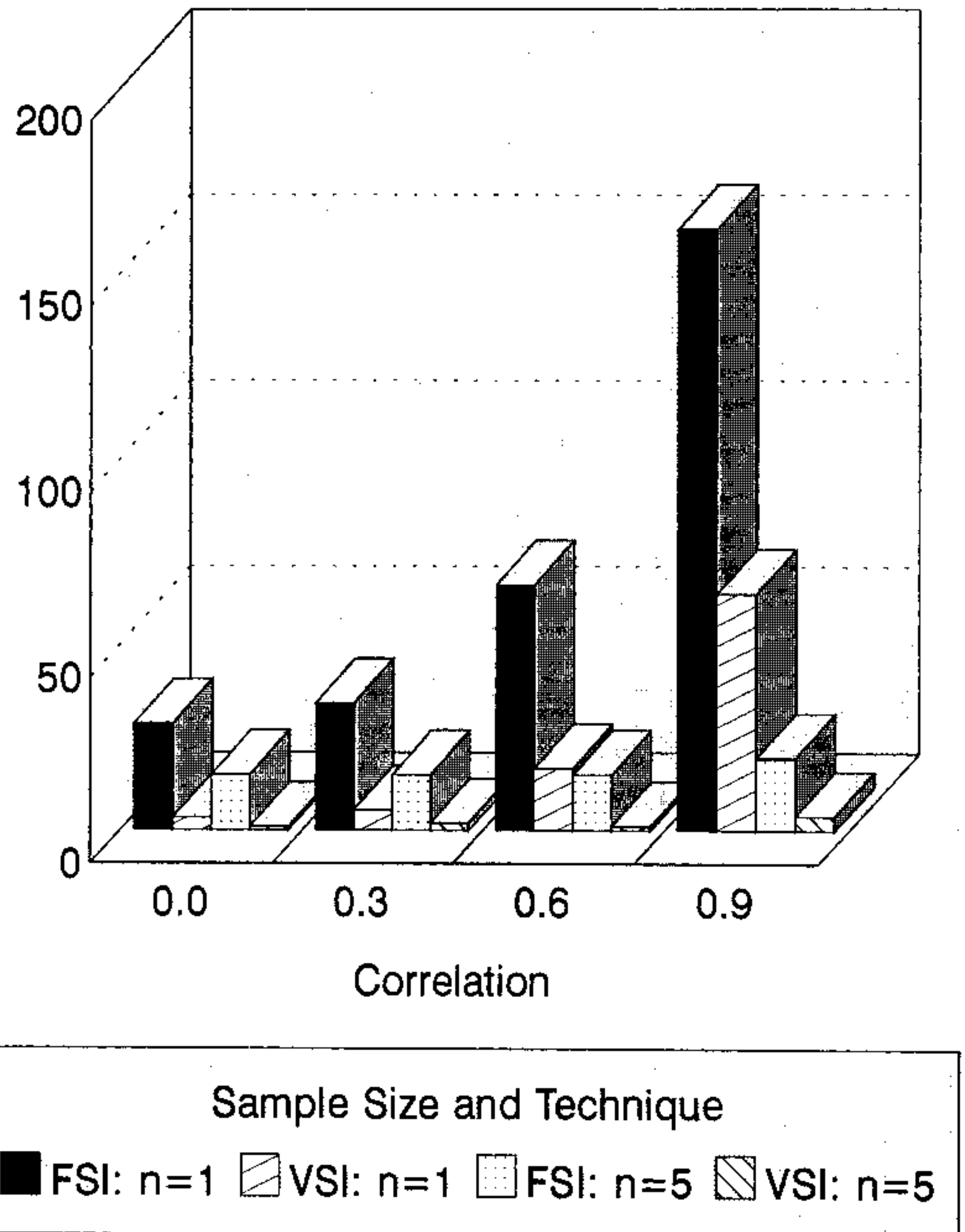


Figure 31g: FSI and VSI have calculated control limits and sample sizes of 1 and 5

EFFECT OF SAMPLE SIZE ON ATS
FSI v. VSI CALCULATED LIMITS
Shift = 3.0 Interval length = 10

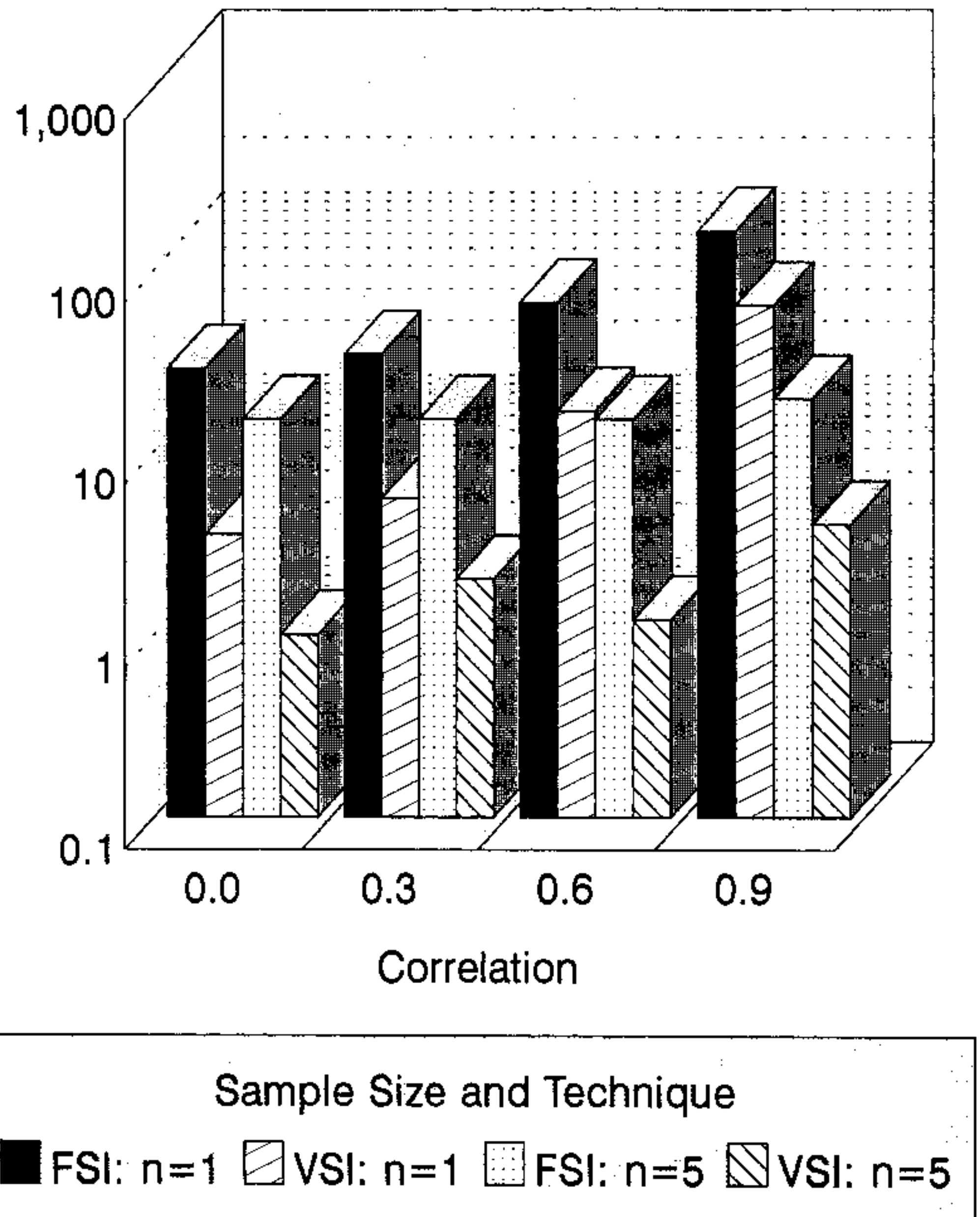


Figure 31h: FSI and VSI have calculated control limits and sample sizes of 1 and 5
y axis: logarithmic scale

APPENDIX C

COMPUTER PROGRAMS

FORTRAN PROGRAM: FSI-3

FIXED SAMPLING INTERVALS WITH PRE-SET LIMITS

```
      integer npar, npma, nw, ini, m, n, comb, numels, replic
      parameter (npar=1, comb=1, numels = 1, replic = 10000)
      integer i,j,k,p,q,r,s, iadist, iseed, lagar(npar),
     +lagma(1), t, rl(comb), dummy
      parameter (npma=0, nw=2000, ini=25)
      real   a(nw), avar, const, par(npar),w(nw),wi(1),
     +     pma(1), xbar(nw),shift,
     +     ucl, lcl, meanrl(comb)
      logical  contrl
c*******************************************************
c   initialize variables
c*******************************************************
      open(unit=2,file='shift')
      lagar(1) = 1
      par(1)  = 0.0
      iadist  = 0
      const   = 0.0
      avar    = 1.0
      wi(1)   = 0
      pma(1)  = 2.0
c   although there is no ma component, the imsl routine
c   requires some value in the next statement
      lagma(1) = 5
      iseed   = 923457
      call rnset (iseed)
      do 10 k=1,comb
       meanrl(k) = 0.0
10     continue
      do 200 r=1, replic
      call rnarm(nw, const, npar, par, lagar, npma,pma,lagma,
     +     iadist, avar, a, wi, w)
c    write (unit=2,fmt=570) (w(i), i=1,nw)
       n = 5
       l = 5
```

```
      shift = 1.0
      dummy = 999
      p = 1
      q = 1
c*****************************************************
c The following comment statements are used in calculating
c the control limits
c*****************************************************
c     do 150 p=1,comb
c      n = n + 1
c      l = 0
c      do 120 q=1,numels
c       l = l + 1
c       do 20 i=1,ini
c        xbar(i) = 0.0
c20      continue
c     do 30 i=1,ini
c      m = l*(i-1)
c      j = m + 1
c      do 25, k=j,j + (n-1)
c        xbar(i) = xbar(i) + w(k)
c        temp(k-m) = w(k)
c25      continue
c      xbar(i) = xbar(i)/real(n)
c      if (n .eq. 1) then
c       rng(i) = abs(w(i) - w(i+1))
c      else
c       maxr = temp(1)
c       minr = temp(1)
c       do 27 k=2,ini
c         if (temp(k) .gt. maxr) then
c          maxr = temp(k)
c         endif
c         if (temp(k) .lt. minr) then
c          minr = temp(k)
c        endif
c        rng(i) = maxr - minr
c27      continue
c      endif
```

```
c30   continue
c    xdbar1 = 0.0
c    rbar = 0.0
c    do 35 i=1,ini
c     xdbar1 = xdbar1 + xbar(i)
c     rbar = rbar + rng(i)
c35   continue
c    xdbar1 = xdbar1/real(ini)
c    rbar = rbar/real(ini)
c*****************************************************
c this part calculates the control limits
c*****************************************************
c    if (n .le. 2) then
c     d2 = 1.128
c    elseif (n .eq. 3) then
c     d2 = 1.693
c    elseif (n .eq. 4) then
c     d2 = 2.059
c    elseif (n .eq. 5) then
c     d2 = 2.326
c    endif
c    ucl = xdbar1 + 3 * rbar/(d2 * sqrt(real(n)))
c    lcl = xdbar1 - 3 * rbar/(d2 *sqrt(real(n)))
c    write(unit=2,fmt=550) xdbar1,rbar,ucl,lcl
     ucl = 3/sqrt(real(n))
     lcl = -3/sqrt(real(n))
c*****************************************************
c the fixed sampling interval (FSI) part begins here
c*****************************************************
c need x-bars for the given n and l starting at sample 26
c*****************************************************
     t = nw/l
     contrl = .true.
     do 40 i = ini+1, t
       xbar(i) = 0.0
40   continue
     do 80 i = ini+1, t
       m = l * (i-1)
       j = m + 1
```

```
      do 60 k = j,j+ (n-1)
        w(k) = w(k) + shift
        xbar(i) = xbar(i) + w(k)
60      continue
      xbar(i) = xbar(i)/real(n)
80    continue
c*****************************************************
c now subject x-bars to control limits
c*****************************************************
    s = (numels*(p-1))+q
    rl(s) = 1
    do 100 i = ini+1, t
     if ((xbar(i) .ge. ucl) .or. (xbar(i) .le. lcl)) then
      contrl = .false.
      goto 120
     else
      rl(s) = rl(s) + 1
     endif
100  continue
    if (contrl) then
     write(unit=2,fmt=580) dummy
    endif
120  continue
150  continue
    do 180 i=1,comb
     meanrl(i) = meanrl(i) + rl(i)
180  continue
200  continue
    do 220 i=1,comb
     meanrl(i) = meanrl(i)/real(replic)
220  continue
    write(unit = 2, fmt = 500) (meanrl(k),k=1,comb)
500  format (8f10.1)
550  format (f7.4)
570  format (10f7.3)
580  format (I7)
    close(unit=2,status='keep')
    stop
    end
```

FORTRAN PROGRAM: FSI-CAL

FIXED SAMPLING INTERVALS WITH CALCULATED CONTROL LIMITS

```
      integer npar, npma, nw, ini, l, m,c,n, comb, numels,
     + replic
      parameter (npar=1, comb=1, numels = 1, replic =5000)
      integer i,j,k,p,q,r,s, iadist, iseed, lagar(npar),
     + lagma(1), t, rl(comb),dummy
      parameter (npma=0, nw=10000, ini=25)
      real   a(nw), avar, const, par(npar),w(nw),wi(1),
     + pma(1), rng(ini), xbar(nw), xdbar1, rbar, temp(ini),
     +   maxr, minr, d2, ucl, lcl, meanrl(comb)
      logical  contrl
c*******************************************************
c   initialize variables
c*******************************************************
      open(unit=2,file='output')
      lagar(1) = 1
      par(1)  = 0.0
      Iadist  = 0
      const   = 0.0
      avar    = 1.0
      wi(1)   = 0
      pma(1)  = 2.0
c   although there is no ma component, the imsl routine
c   requires some value in the next statement
      lagma(1) = 5
      iseed   = 923457
      call rnset (iseed)
      do 10 k=1,comb
       meanrl(k) = 0.0
10     continue
      do 200 r=1, replic
       call rnarm( nw, const, npar, par, lagar,
     +   npma,pma,lagma, iadist, avar, a, wi, w)
c      write (unit=2,fmt=570) (w(i), i=1,nw)
       shift = 1.0
       c = 1
       dummy = 999
```

```
      do 150 p=1,comb
       n = 5
       do 120 q=1,numels
        l = 20
        do 20 i=1,ini
         xbar(i) = 0.0
20        continue
       do 30 i=1,ini
        m = l*(i-1)
        j = m + 1
        do 25, k=j,j + (n-1)
          xbar(i) = xbar(i) + w(k)
          temp(k-m) = w(k)
25        continue
        xbar(i) = xbar(i)/real(n)
        if (n .eq. 1) then
         rng(i) = abs(w(c) - w(c+1))
         c = c + 1
c         write(unit=2, fmt=570) rng(i)
        else
         maxr = temp(1)
         minr = temp(1)
         do 27 k=2,n
          if (temp(k) .gt. maxr) then
           maxr = temp(k)
          endif
          if (temp(k) .lt. minr) then
           minr = temp(k)
          endif
27        continue
         rng(i) = maxr - minr
        endif
c       write(unit=2,fmt=570) rng(i)
30      continue
      xdbar1 = 0.0
      rbar = 0.0
      do 35 i=1,ini
       xdbar1 = xdbar1 + xbar(i)
       rbar = rbar + rng(i)
```

```
35   continue
     xdbarl = xdbarl/real(ini)
     rbar = rbar/real(ini)
c*******************************************************
c this part calculates the control limits
c*******************************************************
     if (n .le. 2) then
      d2 = 1.128
     elseif (n .eq. 3) then
      d2 = 1.693
     elseif (n .eq. 4) then
      d2 = 2.059
     elseif (n .eq. 5) then
      d2 = 2.326
     endif
     ucl = xdbarl + 3 * rbar/(d2 * sqrt(real(n)))
     lcl = xdbarl - 3 * rbar/(d2 *sqrt(real(n)))
c     write(unit=2,fmt=550) xdbarl,rbar,ucl,lcl
c*******************************************************
c the fixed sampling interval (FSI) part begins here
c*******************************************************
c need x-bars for the given n and l starting at sample 26
c*******************************************************
     t = nw/l
     contrl = .true.
     do 40 i = ini+1, t
       xbar(i) = 0.0
40   continue
     do 80 i = ini+1, t
       m = l * (i-1)
       j = m + 1
       do 60 k = j,j+ (n-1)
         w(k) = w(k) + shift
         xbar(i) = xbar(i) + w(k)
60     continue
       xbar(i) = xbar(i)/real(n)
80   continue
c    write(unit=2,fmt=570) (xbar(i), i=ini+1,t)
```

```
c*****************************************************
c now compare xbars to control limits
c*****************************************************
   s = (numels*(p-1))+q
   rl(s) = 1
   do 100 i = ini+1, t
    if ((xbar(i) .ge. ucl) .or. (xbar(i) .le. lcl)) then
     contrl = .false.
     goto 120
    else
     rl(s) = rl(s) + 1
    endif
100  continue
   if (contrl) then
    write (unit=2, fmt=580) dummy
c    print *, 'data set exceeded'
   endif
120  continue
150  continue
   do 180 i=1,comb
    meanrl(i) = meanrl(i) + rl(i)
180  continue
200  continue
   do 220 i=1,comb
    meanrl(i) = meanrl(i)/real(replic)
220  continue
   write(unit = 2, fmt = 500) (meanrl(k),k=1,comb)
c   print *, 'meanrl = ', meanrl
500  format (8f10.1)
550  format (f7.4)
570  format (10f7.3)
580  format (I7)
   close(unit=2,status='keep')
   stop
   end
```

FORTRAN PROGRAM: VSI-3
VARIABLE SAMPLING INTERVALS WITH PRE-SET CONTROL LIMITS

```
      integer npar, npma, nw, ini, l, m, n, replic
      parameter (npar=1, replic =10000)
      integer i,j,k,r,iadist, iseed, lagar(npar), lagma(1),
     +   t, dummy, nshort, nlong, nss(replic), ats(replic)
      parameter (npma=0, nw=10000, ini=25)
      real   a(nw), avar, const, par(npar),w(nw),wi(1),
     + pma(1), rng(ini), xbar(nw), xdbar1, rbar, temp(ini),
     + maxr, minr, d2, ucl, lcl, uvcl, lvcl, short,
     + long, ntrvl, meanss, mats, shift
      logical  contrl
c*****************************************************
c   initialize variables
c*****************************************************
      open(unit=2,file= 'shift')
      lagar(1) = 1
      par(1)  = 0.3
      Iadist   = 0
      const   = 0.0
      avar    = 1.0
      wi(1)   = 0
      pma(1)  = 3.0
c   although there is no ma component, the imsl routine
c   requires some value in the next statement
      lagma(1) = 5
      iseed   = 923457
      call rnset (iseed)
      dummy = 999
      n = 5
      l = 10
      shift =0.0
      short = 0.1*l
      long = 1.9*l
      do 5 k=1,replic
       nss(k) = 0
5   continue
      do 100 r=1, replic
```

```
      call rnarm( nw, const, npar, par, lagar,
     +npma,pma,lagma,
     +        iadist, avar, a, wi, w)
c     write (unit=2,fmt=570) (w(i), i=1,nw)
      nshort = 1
      nlong = 0
      do 10 i = 1,ini
       xbar(i) = 0.0
10     continue
      do 30 i=1,ini
       m = 1*(i-1)
       j = m + 1
       do 20, k=j,j + (n-1)
        xbar(i) = xbar(i) + w(k)
        temp(k-m) = w(k)
20      continue
       xbar(i) = xbar(i)/real(n)
       if (n .eq. 1) then
        rng(i) = abs(w(i) - w(i+1))
       else
        maxr = temp(1)
        minr = temp(1)
        do 25 k=2,ini
         if (temp(k) .gt. maxr) then
          maxr = temp(k)
         endif
         if (temp(k) .lt. minr) then
          minr = temp(k)
         endif
         rng(i) = maxr - minr
25       continue
        endif
30     continue
      xdbar1 = 0.0
      rbar = 0.0
      do 35 i=1,ini
       xdbar1 = xdbar1 + xbar(i)
       rbar = rbar + rng(i)
35     continue
```

```
      xdbar1 = xdbar1/real(ini)
      rbar = rbar/real(ini)
c*****************************************************
c this part calculates the control limits
c*****************************************************
      if (n .le. 2) then
       d2 = 1.128
      elseif (n .eq. 3) then
       d2 = 1.693
      elseif (n .eq. 4) then
       d2 = 2.059
      elseif (n .eq. 5) then
       d2 = 2.326
      endif
c      ucl = xdbar1 + 3 * rbar/(d2 * sqrt(real(n)))
c      lcl = xdbar1 - 3 * rbar/(d2 *sqrt(real(n)))
      ucl = 3.0/sqrt(real(n))
      lcl = -3.0/sqrt(real(n))
c      uvcl = xdbar1 + .6766 * rbar/(d2 * sqrt(real(n)))
c      lvcl = xdbar1 - .6766 * rbar/(d2 * sqrt(real(n)))
      uvcl = 0.6766/sqrt(real(n))
      lvcl = -0.6766/sqrt(real(n))
c     write (unit=2,fmt =550) xdbar1,rbar,ucl,uvcl,lvcl,lcl
c*****************************************************
c the variable sampling interval (VSI) part begins here
c *****************************************************
c control limits remain the same as for FSI
c*****************************************************
      t = nw/short
      contrl = .true.
      do 40 i = (ini*1)+1, t
        xbar(i) = 0.0
40     continue
      ntrvl = short
      i = ini * 1 + ntrvl
50     if (contrl) then
        do 60 k = i,i + (n-1)
          if (k .ge. nw) then
            print *, 'ran out of data points in VSI'
```

```
         write(unit = 2, fmt = 150) dummy
         goto 80
        else
         w(k) = w(k) + shift
         xbar(i) = xbar(i) + w(k)
         endif
60      continue
       xbar(i) = xbar(i)/real(n)
c       print *, xbar(i)
c        write(unit=2,fmt=570) xbar(i)
       if (xbar(i) .ge. ucl .or. xbar(i) .le. lcl) then
        contrl = .false.
       elseif (uvcl .le. xbar(i) .and. xbar(i) .lt. ucl
     +                 .or.
     +     lcl .lt. xbar(i) .and. xbar(i) .le. lvcl) then
        nshort = nshort + 1
        ntrvl = short
       elseif (lvcl .lt. xbar(i).and. xbar(i) .lt. uvcl) then
        nlong = nlong + 1
        ntrvl = long
       endif
       i = i +(n-1) + ntrvl
       goto 50
      endif
c     write(unit=2,fmt=570) (xbar(i), i=(ini*l)+1,350)
80     nss(r) = nshort + nlong
c     write(unit=2,fmt=200) nshort,nlong
      ats(r) = ((nshort- 1)*n+ short) + (nlong *(long+n-1))
c     write(unit=2,fmt=200) ats
100    continue
      meanss = 0.0
      mats = 0.0
      do 120 r=1,replic
       meanss = meanss + nss(r)
       mats = mats + ats(r)
120    continue
c     write(unit=2,fmt=200) (nss(i), i = 1,replic)
      meanss = meanss/real(replic)
      mats = mats/real(replic)
```

```
      write(unit=2,fmt=175) meanss
      write(unit=2,fmt =250) mats
150   format ('ran out of data points', I7)
175   format ('average number samples to signal =' ,f7.1)
200   format (10I6)
250   format ('average time to signal = ',f7.1)
550   format (f7.4)
570   format (10f7.3)
580   format ('value of i is =', I7)
590   format ('value of k is =', I7)
      close(unit=2,status='keep')
      stop
      end
```

FORTRAN PROGRAM: VSI-CAL

VARIABLE SAMPLING INTERVALS WITH CALCULATED LIMITS

```
      integer npar, npma, nw, ini, l, m, n, c, replic, b,p
      parameter (npar=1, replic = 5000)
      integer i,j,k,r,iadist, iseed, lagar(npar), lagma(1),
     +  t, dummy, nshort, nlong, nss(replic), ats(replic)
      parameter (npma=0, nw=600000, ini=25)
      real   a(nw), avar, const, par(npar),w(nw),wi(1),
     +  pma(1), rng(ini), xbar(nw), xdbar1, rbar, temp(ini),
     +     maxr, minr, d2, ucl, lcl, uvcl, lvcl, short,
     +     long, ntrvl, meanss, mats, shift
      logical  contrl
c***************************************************
c   initialize variables
c***************************************************
      open(unit=2,file= 'output2')
      lagar(1) = 1
      par(1)  = 0.0
      Iadist   = 0
      const   = 0.0
      avar    = 1.0
      wi(1)   = 0
      pma(1)   = 2.0
c   although there is no ma component, the imsl routine
c   requires some value in the next statement
      lagma(1)  = 5
      iseed   = 923457
      call rnset (iseed)
      dummy = 999
c    c = 1
      n = 5
      l = 20
      shift = 0.0
      short = 0.1*l
      long = 1.9*l
      do 5 b=1,replic
       nss(b) = 0
5    continue
      do 100 r=1, replic
```

```
      call rnarm( nw, const, npar, par, lagar,
     + npma,pma,lagma,
     +       iadist, avar, a, wi, w)
c      write (unit=2,fmt=570) (w(i), i=1,nw)
      c = 1
      nshort = 1
      nlong = 0
      do 10 i = 1,ini
       xbar(i) = 0.0
10     continue
      do 30 i=1,ini
       m = l*(i-1)
       j = m + 1
       do 20, k=j,j + (n-1)
        xbar(i) = xbar(i) + w(k)
        temp(k-m) = w(k)
20      continue
       xbar(i) = xbar(i)/real(n)
       if (n .eq. 1) then
        rng(i) = abs(w(c) - w(c+1))
        c = c + 1
c        write(unit=2, fmt = 570) rng(i)
       else
        maxr = temp(1)
        minr = temp(1)
        do 25 p=2,n
         if (temp(p) .gt. maxr) then
          maxr = temp(p)
         endif
         if (temp(p) .lt. minr) then
          minr = temp(p)
         endif
25       continue
        rng(i) = maxr - minr
       endif
c        write(unit=2,fmt=570) rng(i)
30     continue
      xdbar1 = 0.0
      rbar = 0.0
```

```
   do 35 i=1,ini
    xdbar1 = xdbar1 + xbar(i)
    rbar = rbar + rng(i)
35  continue
   xdbar1 = xdbar1/real(ini)
   rbar = rbar/real(ini)
c*******************************************************
c this part calculates the control limits
c*******************************************************
   if (n .le. 2) then
    d2 = 1.128
   elseif (n .eq. 3) then
    d2 = 1.693
   elseif (n .eq. 4) then
    d2 = 2.059
   elseif (n .eq. 5) then
    d2 = 2.326
   endif
   ucl = xdbar1 + 3 * rbar/(d2 * sqrt(real(n)))
   lcl = xdbar1 - 3 * rbar/(d2 *sqrt(real(n)))
   uvcl = xdbar1 + .6766 * rbar/(d2 * sqrt(real(n)))
   lvcl = xdbar1 - .6766 * rbar/(d2 * sqrt(real(n)))
c   write (unit=2,fmt =570) xdbar1,rbar,ucl,uvcl,lvcl,lcl
c*******************************************************
c the variable sampling interval (VSI) part begins here
c*******************************************************
c control limits remain the same as for FSI
c*******************************************************
   t = nw/short
   contrl = .true.
   do 40 i = (ini*l)+1, t
     xbar(i) = 0.0
40  continue
   ntrvl = short
   i = ini * l + ntrvl
50  if (contrl) then
     do 60 k = i,i + (n-1)
        write(unit = 2, fmt = 150) dummy
        goto 80
```

```
        else
         w(k) = w(k) + shift
         xbar(i) = xbar(i) + w(k)
         endif
c       write(unit=2, fmt = 150) u,v
60      continue
       xbar(i) = xbar(i)/real(n)
c       print *, xbar(i)
c        write(unit=2,fmt=570) xbar(i)
       if (xbar(i) .ge. ucl .or. xbar(i) .le. lcl) then
        contrl = .false.
       elseif (uvcl .le. xbar(i) .and. xbar(i) .lt. ucl
     +                   .or.
     +     lcl .lt. xbar(i) .and. xbar(i) .le. lvcl) then
        nshort = nshort + 1
        ntrvl = short
       elseif (lvcl .lt. xbar(i).and. xbar(i) .lt. uvcl) then
        nlong = nlong + 1
        ntrvl = long
       endif
       i = i +(n-1) + ntrvl
       goto 50
      endif
c     write(unit=2,fmt=570) (xbar(i), i=(ini*l)+1,350)
80     nss(r) = nshort + nlong
c     write(unit=2,fmt=200) nshort,nlong
      ats(r) = ((nshort- 1)*n+ short) + (nlong *(long+n-1))
c     write(unit=2,fmt=200) ats
100    continue
      meanss = 0.0
      mats = 0.0
      do 120 r=1,replic
       meanss = meanss + nss(r)
       mats = mats + ats(r)
120    continue
c     write(unit=2,fmt=200) (nss(i), i = 1,replic)
      meanss = meanss/real(replic)
      mats = mats/real(replic)
      write(unit=2,fmt=175) meanss
```

```
      write(unit=2,fmt =250) mats
150  format ('ran out of data points', I7)
175  format ('average number samples to signal =' ,f7.1)
200  format (I0I6)
250  format ('average time to signal = ',f7.1)
550  format (f7.4)
570  format (10f7.3)
c 580  format ('value of i is =', I7)
c 590  format ('value of k is =', I7)
      close(unit=2,status='keep')
      stop
      end
```

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