ROBUSTNESS OF PARAMETRIC AND NONPARAMETRIC TESTS
WHEN DISTANCES BETWEEN POINTS CHANGE
ON AN ORDINAL MEASUREMENT SCALE

DISSERTATION

Presented to the Graduate Council of the
University of North Texas in Partial
Fulfillment of the Requirements

For the degree of

DOCTOR OF PHILOSOPHY

By

Andrew H. Chen, B.S., M.B.A., M.S.
Denton, Texas
August, 1994
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Ordinal measures currently enjoy widespread use. The use of a limited number of categories with equal lengths between points (the placements of categorical labels) on the scale may cause a loss of information. Many well-known studies claim that only nonparametric tests are appropriate for the nominal or ordinal level data, that parametric tests are appropriate for the interval or ratio level data; and ordinal numbers (equal lengths between points) cannot be meaningfully added, subtracted, multiplied, and divided. This citation has been one of the most controversial statements in applied statistics.

The purpose of this research was to evaluate the effect on parametric and nonparametric tests using ordinal data when the distances between points changed on the measurement scale. The research examined the performance of Type I and Type II error rates using selected parametric and nonparametric tests.

Three experiments were conducted by generating simulation data on a seven-point Likert-scale using one
uniform, three normal, and three gamma populations. Various unequal distance changes were made between points in the four phases of the experiments.

One and two random samples of simulation data were selected from seven populations. Selected parametric tests and nonparametric tests were used to examine the equality of means, medians, deviations, and distributions between two populations. Several computer programs were written in FORTRAN 77 to implement the algorithm of data simulation, parametric tests, and nonparametric tests. The simulation data were tested by the programs.

The results were analyzed in terms of Type I and Type II error rates with different sample sizes, populations, and phases. In summary, the nonparametric tests produced the same results when the distances between points changed on the scale. However, parametric tests show different results when the distances between points changed. The power of parametric and nonparametric tests were evaluated as underlying assumptions were violated in the location parameters.
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CHAPTER 1

INTRODUCTION

Purpose of the Study

The debate and research over the use of parametric tests when underlying assumptions are not met have attracted considerable attention for numerous years (Stevens 1951; Siegel 1956; Anderson 1961; Lodge et al. 1976; Siegel and Castellan 1988). The problem of using ordinal measures (category scaling) or numeric estimates (magnitude scaling) on applications has spanned almost the same time period as the debate on the use of parametric tests (Guilford 1954; Guilford and Dingman 1955; Goodman 1979; Agresti 1990).

Ordinal measures arise when observations from a population fall into one of several ranked labels or points (the placements of categorical labels) on the scale. Respondents are often asked to use a scale with equal distance between points to express attitudes regarding stimulant objects in the behavioral, psychophysical, psychological, medical sciences, and many other areas. Numeric estimates are used to determine the direction and strength of people’s beliefs according to how respondents rate an object by selecting one of a fixed number of options (Lodge 1987). The distances between any two numeric estimates may not have equal distances on the underlying
continuum.

Frequently, the ranked labels have equal distances between points (or labeled positions) on the scale. However, the descriptive stimuli and meanings that different respondents attach to the adjectives will influence the perceived distance between points (Stevens 1957; Cliff 1959; Labovitz 1967, 1970, 1972; Crask and Fox 1987). Such varieties of measurement may or may not affect the equal interval properties that the ordinal scale possesses.

The purpose of this research is to evaluate the effect on parametric and nonparametric statistical tests when the distances between points change on the measurement scale. The research examines the performance of the Type I and Type II error rates using selected parametric and nonparametric tests. The principal area of the research concerns the relationship between the level of measurement scales and the performance of the examined statistical tests.

Problems Addressed by the Study

The main problems of research interest concerning parametric and nonparametric tests include the following:
1. Can one use ordinal data to do addition, subtraction, multiplication, or division?
2. Is it necessary to choose simple nonparametric tests to analyze ordinal data?
3. Do ranked ordinal data deny researchers legitimate access to more powerful parametric tests?

Harvard psychologist S. S. Stevens (1946), the pioneer of scaling (Bolanowski and Gescheider 1991; Teghtsoonian 1991), originally developed a set of techniques for the study of the intensity of sensations, and proposed four levels of measurement: nominal, ordinal, interval, and ratio scales. One of the major criticisms of Stevens' distinctions of measurement pertains to the definitions of ordinal and interval scales. Coombs (1952) and many other researchers have considered various scales that lie between the ordinal and interval levels. Peterson and Sharma (1977) claimed that the determination of measurement levels should consider information transmission from the objects to the respondents. Wildt and Mazis (1978) suggested that academic and industrial researchers investigating the issue of appropriate scale measures should be concerned with constructing instruments that contain unequal distances between labeled points.

In addition, Siegel and Castellan (1988) noted that independent and random observations, homogeneity of variance, normally distributed populations, and at least an interval scale were elements of the parametric statistical model. The Siegel and Castellan study contains one of the most controversial statements in the applied statistical community. Specifically, opposing researchers claim that
parametric tests can be performed on ordinal data as well as on interval or ratio data. The opposing group concludes that the use of parametric tests on ordinal data will not result in a biased evaluation (Borgatta 1968; Kim 1975; Gaito 1980; Bunger 1988; Gautam 1992).

Another group of researchers (Ramsay 1973; Peter 1979; Cox 1980; Gregoire and Driver 1987) favors the information transmission of ordinal measures and meaningful statistical tests. However, this group concludes that ordinal data must be categorized into an optimal number of response labels on the scale. The group also suggests using an appropriate number of labels ranging from as few as two classes to as many as fifty adjectives on the scale, depending on information transmission requirements (Guilford 1954; Garner 1960; Myers and Warner 1968; Cox 1980; Agresti 1990). Past studies generally indicate that the optimal number of response labels used is seven. The next most preferred numbers are five and nine labels. Whether the selection of a label number is appropriate or not depends on the performance (i.e., the result of error rates) of statistical tests using the ordinal data with selected optimal label numbers. The selection of label numbers determined by convenience and simpleness often causes both a loss of information between labeled points and poor statistical tests. Therefore, Gregoire and Driver (1987) noted that the view favoring an optimal number of labels and meaningful statistical tests was generally recommended, but
the view might not have a convincing argument if the statistical test results were significantly unacceptable, such as in the case of high error rates.

The actual positions of labeled points or ordinal scores on quantitative or qualitative categorizations are unclear (Agresti 1990). Previous views support the measures and statistical tests based on the main concerns of variation on the label numbers and equal distances between labeled points. However, little work has been done on the performance of the Type I and the Type II errors over empirical statistical tests when the sampling data are classified from equal distances to unequal (changing) distances between points on the scale.

Ordinal measurement and numeric estimation have achieved widespread use in recent decades. For example, Moses (1984) indicated that ordinal data analysis occurred in 32 of 168 papers published in volume 36 (1982) of the New England Journal of Medicine. Not surprisingly, the measurement techniques for unequally spaced scales have only recently been developed, but the techniques have ties to practical fields, such as marketing, consumer behavior, customer response, quality circles, employee performance, legal issues, and counseling (Perreault and Young 1980; Tull and Hawkins 1987; Markland 1989).
Significance of the Study

A significant aspect of this research is presented in table 1, which illustrates category matching with real positions along a scale. Five categories (C1 to C5) of scaling represent the underlying continuum on which the stimuli of interest are equally spaced. Three positions of numeric estimates (P1, P2, and P3) are accurately assigned along the scale. However, P1 and P2 are classified in the same category, C2, using five-category scaling, but P1 and P2 are considered as different positions in terms of magnitude scaling. The category scaling process causes a loss of information when ordinal measures are used. A scale with unequal distances between points may be more appropriate. Therefore, changing the distance between points on the scale may provide superior information transmission to the traditional category scaling.

The current research investigated the robustness of empirical parametric and nonparametric tests when distances between points changed. The numeric estimation was used to avoid losing information. The conclusions provided insight and suggestions for the application of ordinal scaling.
### TABLE 1

**MATCHING OF CATEGORY AND REAL POSITION**

<table>
<thead>
<tr>
<th></th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
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<tbody>
<tr>
<td></td>
<td>1.2</td>
<td>1.8</td>
<td>3.4</td>
</tr>
<tr>
<td>I</td>
<td>I</td>
<td>I</td>
<td>I</td>
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</table>

<table>
<thead>
<tr>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Notes: ¹P1, P2, and P3 represent three real positions of numeric estimates (1.2, 1.8, and 3.4) within the underlying continuum.

²C1, C2, C3, C4, and C5 represent five categories within the underlying continuum.
CHAPTER 2

LITERATURE REVIEW

Ordinal Rating Scales

Stevens (1957) indicated that an ordinal rating scale was the measurement obtained when a subject evaluated a set of stimuli in terms of categories labeled either by adjectives or numbers. To avoid the errors of time-order and hysteresis (a lagging effect) during evaluation, successive stimuli should be judged randomly. With the advent of Stevens’ contribution, the range and strength of responses became much easier to determine. Proponents noted that ordinal data could be performed with the admissible operations of equivalence (=) and greater than (>) only; and that any treatment of ordinal data in a manner not consistent with the operation principle of the ordinal level (namely, the class of admissible calculation transformations) was unjustified (Champion 1967; Westermann 1983; Siegel and Castellan 1988). Supporting views concluded that ordinal measures could not be meaningfully added, subtracted, multiplied, or squared (Schlinger 1979). To compute a mean, standard deviation, or variance with ordinal data was simply meaningless because summation, division, and squaring were performed on rank values (Townsend and Ashby 1984; Ross 1989). Therefore, Siegel and
Castellan (1988) stated that parametric tests could be applied only to interval and ratio scales of measurement. This line of reasoning is presented implicitly or explicitly in a majority of statistics textbooks today (Torgerson 1958; Stockton 1962; Wasan 1970; Agresti 1984).

Since the ordinal measurement setting was proposed, challenges to the appropriateness of mathematical calculations on only interval and ratio data have occurred (Traylor 1983). Researchers with opposing views maintain that ordinal measures are similarly represented by a set of numbers, so that ordinal numbers behave just the same as cardinal numbers (Vidali 1975). In other words, one can perform mathematical calculations on ordinal measures just as on interval or ratio measures.

**Psychological Law**

In the late nineteenth century, Fechner proposed that sensation was logarithmically related to stimulus intensity. However, the law was not found through other researchers' experimentation. Few documents could support Fechner's law (Stevens 1957). This history lesson appears to leave extensive work to be done by twentieth century researchers.

Stevens (1957) first confirmed a general psychological law that equal stimulus ratios produced equal subjective ratios. Stevens also conveyed a widely accepted power law that the sensation $\delta$ was proportional to the
stimulus $S$ raised to a power $n$, and that $k$ was a constant. Thus,

$$\delta = kS^n.$$ 

When the equation was converted into logarithms, the equation became linear as

$$\log \delta = n \log S + \log k.$$ 

The equation could be plotted as a straight line on log-log coordinates. The slope of the line corresponded to the exponent $n$ of the power law.

Furthermore, Stevens (1957) illustrated numeric estimation that required the respondent to assign numbers to a series of stimuli, using a standard, to make the estimates proportional to the magnitudes of the sensation or the perceived behavior dimensions. Both the psychological law and numeric estimation are essential to numerous subsequent studies. One of the earliest attempts to use numeric estimation was the evaluation of the seriousness of theft (Sellin and Wolfgang 1964). Respondents were instructed to assign numbers (0 to 10) to different descriptions of criminal offenses based on the seriousness of the crime. Similar studies have been replicated many times in the United States, Canada, and England (Figlio 1976, 1978). The subjects included court judges, police officers, college students, prison inmates, prosecuting attorneys, and many others.
**Magnitude Scaling**

Adapting from Stevens' psychological law and numeric estimation (1957), Lodge (1987) presented a significant contribution in the scaling area: magnitude scaling. Magnitude scaling required numeric estimation (assigning numbers) associated with line production (drawing lines) to express the direction and strength of judgment. The evaluation rule was that, the stronger the judgment on an object, the larger the number and the longer the line. The judgment was based on a reference number or a reference line, that was given as a standard for all respondents. The responses of numeric estimation and line production generally were found to follow the power law (psychological law) and could be plotted on log-log (or ratio-ruled) graph paper. The premise of the log-log graph was that a linear relationship (represented by a solid line) existed between the pairs of numeric estimates and line lengths. The slope of the line on the log-log graph was the exponent of the power law. Theoretically, a slope of 1.0 indicated that a perfect ratio judgment was made between the numeric estimation and the line production. Any deviation from the perfect slope of 1.0 indicated that respondents could not make consistent ratio judgments over the perceived magnitudes of stimuli. Magnitude scaling was initially proposed in order to validate psychophysical scaling (i.e., the scales of sensory tests, light intensity, or numeric estimation), but it has since been widely used in marketing...
and social behavior sciences (Bollen and Barb 1981; Anderson and Zeithaml 1984; Neibecher 1984; Gautam 1992). Teas (1987) recently extended magnitude scaling to examine the differences between individuals and groups. The subjects were asked to evaluate the features of a cash management account package service. A reference number 100 and a respective reference line were provided to aid in the rating. The regression analysis and correlation coefficient were used to test estimation validity and within-subject measurement reliability. The result strongly supported the validity of using numeric estimation when aggregate (group) data were analyzed. However, when the data were tested at the individual level, considerable heterogeneity among individuals was found. The exponents of the power law (psychological law) varied considerably from individual to individual.

Most studies of magnitude scaling have concluded that numeric estimation has definite advantages over ordinal measurement when used to explore rating scale values in applications. However, little has been done to investigate parametric and nonparametric test comparisons between ordinal measures (ordinal scale) and numeric estimates (interval scale).

**Parametric and Nonparametric Tests on Ordinal Data**

Many studies have pointed out that parametric statistical tests provide satisfactory power and robustness
when compared to corresponding nonparametric tests in the evaluation of ordinal data. The use of ranked order data on parametric tests is not inconsistent insofar as significance level and relative efficiency are concerned.

Dixon and Massey (1957) found that ranked ordinal tests were almost as powerful as parametric tests when the data met the assumptions of selection as independent random samples from normally distributed populations with equal variances. The finding was in conflict with Siegel and Castellan's belief (1956 and 1988) that the use of a parametric test required at least an interval scale of data. In recent years, however, studies have often supported the use of more powerful parametric tests on ordinal measures (Gautam 1992; Kimeldorf, Sampson, and Whitaker 1992).

Anderson (1961) investigated parametric tests (i.e., \( t \) test, \( F \) test, analysis of variance, and regression analysis) and nonparametric tests (i.e., the Wilcoxon \( T \), the Kruskal-Wallis \( H \), and median tests) using different measurement data. Results indicated that both parametric and nonparametric tests were appropriate for ordinal, interval, and ratio data.

Woods (1972) took ranked data with equal and unequal sample sizes from normal and nonnormal distributions and compared the parametric analysis of variance results to the nonparametric alternatives. Woods concluded that, in nearly every experiment, the parametric analysis of variance
approach was more powerful and robust than the nonparametric alternative. In addition, Woods stated that one could also use the $F$ test on ordinal data to investigate the equality of two variances from two sampling populations.

More recently, Gregoire and Driver (1987) randomly generated Likert-scale data with various symmetric and nonsymmetric populations. The Likert-scale scores had equal distance between points on the scale. With different sample sizes, parametric and nonparametric tests were conducted to investigate the effect on the Type I and the Type II errors. Gregoire and Driver concluded a variety of important outcomes as follows:

1. Low error rates (the Type I and the Type II errors) were shown when testing (parametric) the equality of means using samples from symmetric populations.

2. Parametric and nonparametric tests were equally powerful and had consistently low Type I and II error rates when the differences between medians were examined.

3. Some Type I rates were higher (using nonparametric tests) on the test of equal variance when samples were selected from symmetric populations.

4. The error rates (using both parametric and nonparametric tests) appeared to be sensitive and unreliable when populations changed from symmetric to nonsymmetric.

Thus, Gregoire and Driver (1987) noted that the parametric test was as powerful as the nonparametric test
when samples were selected from symmetric populations. Both test results were less precise when samples were selected from nonsymmetric populations. Gregoire and Driver also concluded that a pretest using various stimuli with unknown variances between populations was advisable when testing the equality of central tendency.

If a valid statistical experiment is conducted and both parametric and nonparametric tests can be used in the evaluation of observed data (in the same level of measurement), then the parametric test is preferable for its popularity and convenience (Siegel and Castellan 1988). Strict adherence to this rule is essential to the effective selection and use of statistical tests. Numerous recent studies have employed parametric tests to analyze the effect on ordinal data, but most studies have nothing to do with the scaling of unequal distances between points on the measurement scale.

**Parametric and Nonparametric Tests on Equal and Unequal Distances**

The ordinal measures which have been pervasively analyzed are obtained from equally spaced categories. In contrast, the numeric estimation (used in the current research) has unequal distances between points on the scale.

Crask and Fox (1987) completed an examination of the scale distances between points on the evaluation of consumer behavior characteristics: purchase intent, product importance, and overall product rating. Marketing
specialists usually used these three purchasing factors with equal distance between categories. Crask and Fox conducted a survey using numeric estimation associated with the reference number 100. The conclusions revealed that overall product rating showed nearly equal distances between points on the scale. Purchase intent and product importance had unequal distances between points. However, no further parametric and nonparametric tests were conducted to investigate the effect of changing distances.

Parasuraman and Varadarajan (1988) made significant progress in examining the robustness of ordinal data with changing distances between points. The samples were selected from a simulated population based on marketing strategy variables. The selected ordinal data were systematically transformed from equal distances to unequal distances between points by the following methods:

1. **Uniform distance**: equal distances between points
2. **Bias toward middle**: longer distance of positions on the midpoint of the ordinal scale
3. **Bias toward extremes**: longer distance of positions on the two ends of the ordinal scale
4. **Bias toward high end**: longer distance of positions on the upper side of the ordinal scale
5. **Bias toward low end**: longer distance of positions on the lower side of the ordinal scale
Parasuraman and Varadarajan (1988) used additive and multiplicative regression models to evaluate the variation of correlation coefficients when distances between points changed. They stated the following results:

1. Transforming the ordinal scale to the interval scale (by changing distances between points) failed to show a serious limitation on the regression analysis result as long as the ordinal data could be assumed to vary monotonically with systematic patterns (i.e., the five previous transformation methods).

2. The transformation might not be appropriate when the bias (changing distances between points) was subject to the nonlinear distortions of the assumed ordinal scale.

The conclusion failed to show an optimistic result when distances changed dramatically throughout the underlying continuum. No comparisons had been done between parametric and nonparametric tests. Parasuraman and Varadarajan (1988) left many problems and recommendations for further studies; for example, a noticeable change occurred in the regression analysis (i.e., slopes and correlation coefficients) as soon as the ordinal assumptions were not met. Further samples selected from multiple populations might provide useful insights into scaling techniques.

Although most studies are somewhat encouraging to applied statisticians engaged in the numeric estimation area
today, there is a need to enhance the experiment with changing distances made between points. The use of more sophisticated techniques to test the equality of central tendency, dispersions, or distributions would provide reliable results in the applications. In addition, respondents often hesitate to assign very unfavorable or negative numbers on the evaluation scale (Brown, Copeland, and Millward 1973). An unbalanced scale in which the distances between favorable labels are not equal to the distances between unfavorable labels (widely used by General Foods Corporation) may provide more useful information (Wildt and Mazis 1978).

Therefore, the current research indicates that a fruitful possibility for further studies would be to examine the performance (i.e., the Type I and the Type II errors) of parametric and nonparametric tests when distances between points change on the scale. Past studies have not analyzed the comparisons between parametric and nonparametric tests on the scale with unequal and unbalanced distances on two sides of the neutral point. The objective of this research is to investigate part of the unresolved areas.
CHAPTER 3

RESEARCH FRAMEWORK AND DESIGN

In the current research, seven different populations (one uniform population, three normal populations, and three gamma populations) were generated by simulation data. A mapping function was used to change each population into a seven-point frequency population. Then several unequal and unbalanced distance changes were made between points on the scale in the four phases. The objective was to examine the effect of changing distances between points using selected parametric and nonparametric tests on simulation data. The Type I and the Type II error rates were used to evaluate the test performance with a variety of population changes.

Hypotheses

Three experiments were conducted in the current research. Experiment 1 dealt with one sample selected from one population; experiment 2 dealt with two samples selected from one population; and experiment 3 dealt with two samples selected from two different populations. Table 2 highlights the comparison of the three experiments.

Experiment 1, experiment 2 (four phases), and experiment 3 (four phases) were tested throughout all seven
<table>
<thead>
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<td>Equal</td>
<td>0</td>
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<td>2&lt;sub&gt;b&lt;/sub&gt;</td>
<td>2</td>
<td>1</td>
<td>Unequal</td>
<td>1</td>
</tr>
<tr>
<td>2&lt;sub&gt;c&lt;/sub&gt;</td>
<td>2</td>
<td>1</td>
<td>Unequal</td>
<td>2</td>
</tr>
<tr>
<td>2&lt;sub&gt;d&lt;/sub&gt;</td>
<td>2</td>
<td>1</td>
<td>Unequal &amp; Unbalanced</td>
<td>&gt; 2</td>
</tr>
<tr>
<td>3&lt;sub&gt;a&lt;/sub&gt;</td>
<td>2</td>
<td>2</td>
<td>Equal</td>
<td>0</td>
</tr>
<tr>
<td>3&lt;sub&gt;b&lt;/sub&gt;</td>
<td>2</td>
<td>2</td>
<td>Unequal</td>
<td>1</td>
</tr>
<tr>
<td>3&lt;sub&gt;c&lt;/sub&gt;</td>
<td>2</td>
<td>2</td>
<td>Unequal</td>
<td>2</td>
</tr>
<tr>
<td>3&lt;sub&gt;d&lt;/sub&gt;</td>
<td>2</td>
<td>2</td>
<td>Unequal &amp; Unbalanced</td>
<td>&gt; 2</td>
</tr>
</tbody>
</table>

Notes: ¹Experiment 2 consists of four phases. Different distance changes are made between points in each phase.

²Experiment 3 also consists of four phases. Different distance changes are made between points in each phase.
populations (one uniform population, three normal populations, and three gamma populations) using parametric and nonparametric tests. Accordingly, the following hypotheses were formulated with respect to the three experiments and respective four phases in the research:

1. Hypothesis in experiment 1:
   Hypothesis 1: Equally changing distances between points on the scale do not affect the parametric and nonparametric test results if one sample is selected from one population.

2. Hypotheses of the four phases in experiment 2:
   Hypothesis 2a: Equally changing distances between points on the scale do not affect the parametric and nonparametric test results if two samples are selected from one population.

   Hypothesis 2b: Unequally changing distances between points on the scale (one unequal change made on both sides of the neutral point) affect the parametric test results but do not affect the nonparametric test results if two samples are selected from one population.
Hypothesis 2\textsubscript{c}: Unequally changing distances between points on the scale (two unequal changes made on both sides of the neutral point) affect the parametric test results but do not affect the nonparametric test results if two samples are selected from one population.

Hypothesis 2\textsubscript{d}: Unequally changing distances between points on the scale (more than two changes made on both sides of the neutral point) affect the parametric test results but do not affect the nonparametric test results if two samples are selected from one population.

3. Hypotheses of the four phases in experiment 3:

Hypothesis 3\textsubscript{a}: Equally changing distances between points on the scale do not affect the parametric and nonparametric test results if two samples are selected from two populations.

Hypothesis 3\textsubscript{b}: Unequally changing distances between points on the scale (one unequal change made on both sides of the neutral point) affect the parametric
test results but do not affect the nonparametric test results if two samples are selected from two populations.

Hypothesis 3: Unequally changing distances between points on the scale (two unequal changes made on both sides of the neutral point) affect the parametric test results but do not affect the nonparametric test results if two samples are selected from two populations.

Hypothesis 3d: Unequally changing distances between points on the scale (more than two changes made on both sides of the neutral point) affect the parametric test results but do not affect the nonparametric test results if two samples are selected from two populations.

Simulation Data and Mapping Functions

The seven test populations (one uniform population, three normal populations, and three gamma populations) were simulated by the subroutines developed by IMSL, Inc (IMSL 1991). Table 3 lists the parameters used by each of the seven populations to generate 10,000 element values.
## TABLE 3

**POPULATION PARAMETERS FOR THE GENERATION OF SIMULATION POPULATION**

<table>
<thead>
<tr>
<th>Populations</th>
<th>Parameters</th>
</tr>
</thead>
</table>
| Uniform     | Range = 0 to 100  
              | $\mu = 50$ |
| Normal_1    | $\mu = 50, \sigma = 15$ |
| Normal_2    | $\mu = 40, \sigma = 15$ |
| Normal_3    | $\mu = 40, \sigma = 10$ |
| Gamma_1     | $\alpha = 11.1111, \beta = 4.5$  
              | $\mu = 50$  
              | $\sigma = 15$ |
| Gamma_2     | $\alpha = 7.1111, \beta = 5.625$  
              | $\mu = 40$  
              | $\sigma = 15$ |
| Gamma_3     | $\alpha = 16, \beta = 2.5$  
              | $\mu = 40$  
              | $\sigma = 10$ |
A frequency population of seven-point Likert-scale was created by mapping each population element value into one of the seven equal distance categories. Table 4 presents the mapping function. An interval including more simulation data represented a higher frequency for the class. The mapping procedure was applied by all seven test populations (one uniform, three normal, and three gamma populations). As an example, a simulation value of 78.13 from a population is mapped and produces a Likert-scale score of 2, which is categorized as Agree. Therefore, 10,000 simulation values of each population were so mapped and transformed into a Likert-scale equivalent population. Then the distances between points were systematically changed in the experiments, and selected parametric and nonparametric tests were performed to evaluate the effect on the distance changes on the scale.

**Experiment 1**

Experiment 1 was a preliminary procedure to verify the random number generation and related computer codes. The target population observations were represented by seven-point Likert-scale scores. One simple random sample (Likert-scale scores) of size $n = 10$ was selected without replacement from one population. No distance change between points was made on the scale. Experiment 1 repeated sampling one thousand times, with increasing sample sizes of 25, 50, and 100, to examine the effect on various sample
### TABLE 4

**MAPPING FUNCTION OF SEVEN CATEGORIES**

<table>
<thead>
<tr>
<th>Label of Category</th>
<th>Class</th>
<th>Score of Likert-Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strongly disagree</td>
<td>under 14.29</td>
<td>-3</td>
</tr>
<tr>
<td>Disagree</td>
<td>14.29 and under 28.57</td>
<td>-2</td>
</tr>
<tr>
<td>Slightly disagree</td>
<td>28.57 and under 42.86</td>
<td>-1</td>
</tr>
<tr>
<td>Neutral</td>
<td>42.86 and under 57.14</td>
<td>0</td>
</tr>
<tr>
<td>Slightly agree</td>
<td>57.14 and under 71.43</td>
<td>1</td>
</tr>
<tr>
<td>Agree</td>
<td>71.43 and under 85.71</td>
<td>2</td>
</tr>
<tr>
<td>Strongly agree</td>
<td>85.71 and over</td>
<td>3</td>
</tr>
</tbody>
</table>

*Note: Using equal distances between labeled points.*
sizes. This procedure was also applied to all seven test populations (one uniform, three normal, and three gamma populations).

Experiment 1 included four parametric and nonparametric tests: (1) confidence interval (or CI) for the population mean, (2) CI for the population median, (3) Kolmogorov-Smirnov (or KS) one-sample test, and (4) Chi-square one-sample test. The purpose was to determine whether the nominal 5% significance level (the Type I error) satisfactorily reflected the actual error rate when repeated sampling was performed. A Type I error rate of 5% was expected to be maintained for nearly all sample sizes obtained from symmetric populations (i.e., one uniform and three normal populations in experiment 1).

The anticipated 5% error rate was used to check whether simulation data were random and programs were accurate. If the error rate was substantially different from 5%, an unknown bias might exist in the simulation data, or logic errors might be found in the computer programs.

Experiment 2

The goal of experiment 2 was to examine the effect on the Type I errors from parametric and nonparametric tests when the distances between points change using two samples selected from one population. The mapped populations of seven-point Likert-scale scores were the target sampling populations. One thousand pairs of equal-sized,
nonoverlapping samples were randomly and independently selected without replacement from one population. The experiment also repeated sampling, with increasing sample sizes of 25, 50, and 100. This procedure was applied to all seven test populations (one uniform, three normal, and three gamma populations).

Experiment 2 consisted of four phases (phases $2_a$, $2_b$, $2_c$, and $2_d$). Phases $2_b$, $2_c$, and $2_d$ were variations of phase $2_a$ with alterations made in the distances of labeled points. The Likert-scale scores were different among the four phases. To avoid inconsistency, the scores were coded by calculating the respective $Z$ values in each phase. Therefore, the $Z$ scores were able to show relative distances between points among different phases. Tables 5, 6, 7, and 8, respectively, present the four phases and required coding procedure. The same sampling process was repeated and tested for each of the four phases as follows:

**Phase $2_a$**

Phase $2_a$ was a base model and was used as a paradigm to compare with the results of phases $2_b$, $2_c$, and $2_d$. No distance change between points was made on the scale. Therefore, each of the seven mapped populations had its respective frequency distribution of the seven-point Likert-scale scores with equal distances between points. Table 5 presents the seven labels and the corresponding point positions.
**TABLE 5**

LABELS VERSUS POINTS FOR EXPERIMENT 2

<table>
<thead>
<tr>
<th>STD</th>
<th>D</th>
<th>SLD</th>
<th>N</th>
<th>SLA</th>
<th>A</th>
<th>STA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-3</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>-1.5</td>
<td>-1</td>
<td>-0.5</td>
<td>0</td>
<td>0.5</td>
<td>1</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Legend: STD = Strongly disagree  STA = Strongly agree
D = Disagree  A = Agree
SLD = Slightly disagree  SLA = Slightly agree
N = Neutral

Notes
1. The first level of numeric values (-3 to 3) represent the real positions of labeled points.
2. The second level of numeric values represent the Z scores of labeled points,
   \[ Z = \frac{(X - \mu)}{\sigma} \]
   where \( \mu = 0, \sigma = 2 \).
3. The distances between points have equal distances.
Phase 2\textsubscript{b}

A numeric estimation scale with a balanced distortion in the middle area of the underlying continuum was examined in phase 2\textsubscript{b}. One distance change was made on both sides of the neutral position on the scale. Accordingly, the distances between Slightly disagree and Neutral as well as between Neutral and Slightly agree were subjectively extended five times longer than the distance of the same scale places in phase 2\textsubscript{a}. All other distances between points on the scale remained unchanged. Table 6 presents the seven labels and corresponding point positions (including \(Z\) values).

Phase 2\textsubscript{c}

Phase 2\textsubscript{c} was to extend two scale ends of phase 2\textsubscript{a}. The basic assumption was that respondents tended to violate the rating pattern with a disproportionately wide range of estimates in the area of two extremes (Holdaway 1971; Galbraith and Schendel 1983; Parasuraman et al. 1988).

Two distance changes were made on both sides of the neutral position. Accordingly, the distances between Strongly disagree and Disagree as well as between Agree and Strongly agree were subjectively extended three times longer than the distances of the same scale places in phase 2\textsubscript{a}. The distances between Disagree and Slightly disagree as well as between Slightly agree and Agree were subjectively extended six times longer than the distances of the same
TABLE 6

LABELS VERSUS POINTS FOR EXPERIMENT 2

<table>
<thead>
<tr>
<th>STD</th>
<th>D</th>
<th>SLD</th>
<th>N</th>
<th>SIA</th>
<th>A</th>
<th>STA</th>
</tr>
</thead>
<tbody>
<tr>
<td>-7</td>
<td>-6</td>
<td>-5</td>
<td>0</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>-1.249</td>
<td>-.892</td>
<td>0</td>
<td>.892</td>
<td>1.249</td>
<td>1.07</td>
<td></td>
</tr>
</tbody>
</table>

Legend:  
STD = Strongly disagree  
D = Disagree  
SLD = Slightly disagree  
N = Neutral  
STA = Strongly agree  
A = Agree  
SIA = Slightly agree

Notes:  
1. The first level of numeric values (-7 to 7) represent the real positions of labeled points.  
2. The second level of numeric values represent the Z scores of labeled points,  
\[ Z = \frac{(X - \mu)}{\sigma} \]

where \( \mu = 0, \sigma = 5.606 \).

3. The distances between points have unequal lengths. One distance change between points is made at two sides of neutral. The lengths between Slightly disagree and Neutral as well as between Neutral and Slightly agree are extended five times longer than the original lengths. The others remain unchanged.
scale places in phase 2. Table 7 presents the seven labels and corresponding point positions (including Z values). The other distances remained unchanged.

Phase 2

Bartram and Yielding (1973) indicated that people were more willing to grant positive values than negative values to categories and that, therefore, the scale was unbalanced between positive and negative sides. Worcester and Burns (1975) also found comparatively less strength on the negative end of the spectrum.

Therefore, phase 2 was focused on forming a less-negative scale by making unequal and unbalanced changes on both sides of the neutral point. Distances between points on the positive side were extended more times than the distances between points on the negative side. The goal was to build an unbalanced scale on both sides of the neutral point. Table 8 presents the seven labels and corresponding point positions (including Z values).

Experiment 3

The goal of experiment 3 was to select two samples from two populations and to examine the effect of the Type II error on parametric and nonparametric tests when the distances between points changed. Two types of errors may be made in decisions regarding the null hypothesis. Experiment 2 examined the Type I error using one population.
### TABLE 7

**LABELS VERSUS POINTS FOR EXPERIMENT 2**

<table>
<thead>
<tr>
<th>STD</th>
<th>D</th>
<th>SLD</th>
<th>N</th>
<th>SLA</th>
<th>A</th>
<th>STA</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10</td>
<td>-7</td>
<td>-1.527</td>
<td>0</td>
<td>1</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>-1.069</td>
<td>-.153</td>
<td>1.069</td>
<td>.153</td>
<td>1.069</td>
<td>1.527</td>
<td></td>
</tr>
</tbody>
</table>

**Legend:**
- **STD** = Strongly disagree
- **D** = Disagree
- **SLD** = Slightly disagree
- **N** = Neutral
- **STA** = Strongly agree
- **A** = Agree
- **SLA** = Slightly agree

**Notes:**
1. The first level of numeric values (-10 to 10) represent the real positions of labeled points.
2. The second level of numeric values represent the Z scores of labeled points,
   \[ z = \frac{(x - \mu)}{\sigma} \]
   where \( \mu = 0, \sigma = 6.547 \).
3. The distances between points have unequal lengths. Two changes between points are made at two sides of neutral.
   The lengths between Strongly disagree and Disagree, between Agree and Strongly agree are extended three times longer than the originals, between Disagree and Slightly disagree, and between Slightly agree and Agree are extended six times. The others remain unchanged.
TABLE 8

LABELS VERSUS POINTS FOR EXPERIMENT 2d

<table>
<thead>
<tr>
<th>STD</th>
<th>D</th>
<th>SLD</th>
<th>N</th>
<th>SLA</th>
<th>A</th>
<th>STA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-4</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>-999</td>
<td>-618</td>
<td>0.018</td>
<td>0.781</td>
<td></td>
<td>2.053</td>
</tr>
<tr>
<td></td>
<td>-745</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Legend: STD = Strongly disagree  STA = Strongly agree
        D = Disagree            A = Agree
        SLD = Slightly disagree SLA = Slightly agree
        N = Neutral

Notes: 1. The first level of numeric values (-4 to 20) represent the real positions of labeled points.

2. The second level of numeric values represent the Z scores of labeled points,

\[ z = \frac{(x - \mu)}{\sigma} \]

where \( \mu = 3.857, \ \sigma = 7.864. \)

3. Multiple unequal and unbalanced changes between points are made at two sides of neutral. The length between Strongly disagree and Disagree is extended two times longer than the originals, between Neutral and Slightly agree four times, between Slightly agree and Agree six times, and between Agree and Strongly agree ten times.
In contrast, experiment 3 emphasized the importance of the Type II error (Gregoire and Driver 1987), which was the failure to reject the null hypothesis when it was false. One test is more powerful and preferable than another when the test has a lower Type II error rate (Kvanli, Guynes, and Pavur 1992).

Experiment 3 also included four phases \((3_a, 3_b, 3_c, \text{ and } 3_d)\), which replicated each respective phase of experiment 2 except that two samples were selected from two different populations instead of one. Hence, there were 21 population combinations in experiment 3, as shown in table 9. The rationale was to investigate the robustness of the four phases when differences in population characteristics occurred. Experiment 3 looked at each population's changes in location (mean and median); dispersion (standard deviation); and distribution (frequency). As in experiment 2, one thousand pairs of equal-sized, nonoverlapping samples \((\text{size } n = 10, 25, 50, \text{ and } 100)\) were randomly and independently selected, but from two different populations. Again, the sampling procedure was applied to all seven test populations (one uniform, three normal, and three gamma populations).

Statistical Testing Using Computer Programs

The data generation of uniform, normal, and gamma populations, as well as the algorithm of selected parametric and nonparametric tests were programmed in FORTRAN 77 (IMSL
### TABLE 9

**LIST OF POPULATION COMBINATIONS**

<table>
<thead>
<tr>
<th>Uniform vs. Normal:</th>
<th>Uniform vs. Gamma:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - 2</td>
<td>1 - 5</td>
</tr>
<tr>
<td>1 - 3</td>
<td>1 - 6</td>
</tr>
<tr>
<td>1 - 4</td>
<td>1 - 7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Normal vs. Normal:</th>
<th>Gamma vs. Gamma:</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 - 3</td>
<td>5 - 6</td>
</tr>
<tr>
<td>2 - 4</td>
<td>5 - 7</td>
</tr>
<tr>
<td>3 - 4</td>
<td>6 - 7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Normal vs. Gamma:</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 - 5</td>
</tr>
<tr>
<td>2 - 6</td>
</tr>
<tr>
<td>2 - 7</td>
</tr>
<tr>
<td>3 - 5</td>
</tr>
<tr>
<td>3 - 6</td>
</tr>
<tr>
<td>3 - 7</td>
</tr>
<tr>
<td>4 - 5</td>
</tr>
<tr>
<td>4 - 6</td>
</tr>
<tr>
<td>4 - 7</td>
</tr>
</tbody>
</table>

**Note:**
1 - Uniform population
2 - Normal₁ population
3 - Normal₂ population
4 - Normal₃ population
5 - Gamma₁ population
6 - Gamma₂ population
7 - Gamma₃ population
In order to compare parametric tests and their nonparametric alternatives, the one sample selected from the populations in experiment 1 was used to perform all four parametric and nonparametric tests; the two samples selected from the populations in experiment 2 and experiment 3 were used to perform the other five parametric and nonparametric tests. The programs were designed using object-oriented modules (Barkakati 1990), and, thus, the programs could be reused by different populations, sample sizes, and phases with a change of parameters at the global level (seven populations) or respective classes (four phases and four sample sizes) on the basis of three objects (three experiments). Also, the validity of the tests and programs could be justified in experiment 1 and in phase 1 of experiments 2 and 3 because the results either were expected or could be verified by similar studies. The source codes of three computer programs respectively used by the three experiments are listed in the appendix.
CHAPTER 4

DATA ANALYSIS

**Parametric and Nonparametric Tests and Results in Experiment 1**

Experiment 1 was used to verify random numbers and computer codes. The Type I error rates were accumulated to determine whether the nominal 5% was achieved by repeating sampling from seven simulated populations.

**Four Tests of Experiment 1**

Four test statistics were selected to focus on the examination of population mean, median, standard deviation, and distribution (Plackett 1974; Nishisato 1980). The population mean μ of the CI test was based on the maximum likelihood estimator, the sample mean. The population standard deviation σ of the chi-square test was based on the maximum likelihood estimator, the sample standard deviation (Hogg and Craig 1978; Thompson and Tapia 1990). The sample median was used as an estimator of the population median (Stockton 1962). Table 10 summarizes four parametric and nonparametric tests with the models of test statistics.

**Test Results of Experiment 1**

In experiment 1, if one test failed to include the population mean or population median or rejected the null
Table 10

PARAMETRIC AND NONPARAMETRIC TESTS
IN EXPERIMENT 1

Confidence interval for $\mu$:

$$(\bar{x} - t_{a/2} s_{\bar{x}}, \bar{x} + t_{a/2} s_{\bar{x}})$$

Confidence interval for median*:

$$(median_{sample} + t_{a/2}(1.2533)s_{\bar{x}}, median_{sample} - t_{a/2}(1.2533)s_{\bar{x}})$$

Kolmogorov-Smirnov test on population distribution vs. sample distribution:

$$D = \max |F_0(X_i) - S_n(X_i)|$$

where $F_0 =$ population cumulative relative frequency
$S_n =$ sample cumulative relative frequency

Chi-square test on population standard deviation vs. sample standard deviation:

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

Note: 'a = 5%

hypothesis, 1 was added to the total error count. The actual error rates, then, were computed by total error count/1000. As such, the Type I error rates of parametric and nonparametric tests in experiment 1 are summarized in table 11.

**CI Test for Population Mean**

As indicated in table 11, the Type I error rates of the CI test for \( \mu \) were nearly maintained at the nominal 5% level for three normal populations. There was no major difference between the large samples (\( n \geq 30 \)) and the small samples (\( n < 30 \)). Also, similar performance was exhibited between the nonnormal populations and nonsymmetric populations. The results had shed some light on the selection of sample sizes and underlying populations in the central limit theorem (Kvanli, Guynes, and Pavur 1992). Thus, the expected error rates obtained in the repeating tests revealed that the seven populations generated by random numbers and associated computer codes were reliable and could be reused for all other experiments in the research.

**CI Test for Population Median**

The purpose was to test the population median location when the median was more demanding than the mean as the primary statistic for describing the population. For example, it was necessary when the underlying population was
<table>
<thead>
<tr>
<th>Population</th>
<th>n</th>
<th>$\mu$</th>
<th>Median</th>
<th>KS</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>10</td>
<td>5.4</td>
<td>7.7</td>
<td>2.0</td>
<td>4.3</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>4.8</td>
<td>16.2</td>
<td>1.1</td>
<td>20.1</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>5.0</td>
<td>23.3</td>
<td>1.2</td>
<td>74.8</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>5.6</td>
<td>15.0</td>
<td>1.1</td>
<td>99.7</td>
</tr>
<tr>
<td>Normal,</td>
<td>10</td>
<td>6.1</td>
<td>7.4</td>
<td>0.9</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>4.7</td>
<td>5.5</td>
<td>0.8</td>
<td>5.4</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>6.1</td>
<td>1.5</td>
<td>0.6</td>
<td>10.0</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>4.8</td>
<td>0.1</td>
<td>0.5</td>
<td>16.9</td>
</tr>
<tr>
<td>Normal,</td>
<td>10</td>
<td>6.0</td>
<td>15.5</td>
<td>1.4</td>
<td>4.3</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>5.0</td>
<td>22.6</td>
<td>0.8</td>
<td>4.9</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>6.1</td>
<td>14.9</td>
<td>0.8</td>
<td>4.7</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>5.0</td>
<td>7.1</td>
<td>0.9</td>
<td>4.8</td>
</tr>
<tr>
<td>Normal,</td>
<td>10</td>
<td>4.0</td>
<td>17.2</td>
<td>1.1</td>
<td>2.7</td>
</tr>
<tr>
<td></td>
<td>25</td>
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Note: $^1\mu$: confidence interval test for the population mean

$^2$Median: confidence interval test for the population median

$^3$KS: Kolmogorov-Smirnov one-sample test for the population distribution

$^4\sigma$: Chi-square test for the population standard deviation
skewed. The confidence interval for the median from a large sample of normal population was a reasonably good approximation for the parent population median (Stockton 1962). As expected, the Type I error rates of the normal population retained around nominal 5%; the normal$_2$ and normal$_3$ populations had higher error rates because the mean was not equal to the median in the mapped populations. The error rates, herein, had a 0%-18% range and decreased with increasing sample size when sampling from three gamma populations; the test provided the worst Type I error rates and anomalous patterns when sampling from the uniform population. Generally, the error rates reached the nominal 5% level or below in many instances with the sample size increasing to 50 or higher. It was also essential to note that the error rates resulting from gamma populations were slightly lower than those from normal populations when both the gamma and normal had identical means and variances in the tests.

**KS One-Sample Test**

The test results of the KS test (table 11) reported the Type I error rates in the low 0%-2% range. It did not matter whether the variable under consideration had a continuous distribution (Siegel and Castellan 1988), whether the distribution was normal (Puri and Sen 1985), or whether the population was symmetric (Gregoire and Driver 1987). Similar results were obtained by Gregoire and Driver (1987).
The KS test was the only nonparametric test in experiment 1, but it yielded lower Type I error rates than any other parametric test. The test was generally powerful in all sample sizes and populations of experiment 1. Hence, the KS test is able to provide extremely good protection against distribution heterogeneity.

**Chi-Square Test for Population Standard Deviation**

The variation of populations was the primary concern in the chi-square test. The Type I error rates were almost retained at the nominal 5% when sampling from the normal populations and appeared expectable. The chi-square test was dramatically erratic when sampling from the nonnormal populations, uniform and gamma. Apparently, the chi-square test on $\sigma$ is highly sensitive to the assumption of normal populations.

It is of interest to note that the Type I error rates increased as did the sample size here. Normally, the opposite phenomenon occurs in sampling statistics because a larger sample could bring more adequate information and yield lower error rates than a smaller sample. As sampling observations increase so too will one’s confidence in the population (Scheaffer, Mendenhall, and Ott 1990). However, this was not the case in experiment 1. These ambiguous Type I error patterns could be caused by the fact that the sample standard deviation was not an unbiased estimator of the population standard deviation, because the sample values
were selected from the seven populations with seven-point Likert-scale values that were mapped by a mapping function (table 4). Thus, the variation of the estimator decreased when the sample size increased, the bias did not (Hogg and Tanis 1977; Carr, Hafner, and Koch 1989).

Parametric and Nonparametric Tests and Results
in Experiment 2

Experiment 2 examined the tests based on the equality of four parameters: the mean, median, dispersion, and distribution. The four statistics were believed to have conspicuous influences on the selection of statistical tests. The Type I error rates were evaluated in experiment 2 (whereas the Type II error rates were evaluated in experiment 3).

Equality of Means

The purpose was to examine the central location of the population. All observed 95% confidence intervals

\[
\bar{x}_1 - \bar{x}_2 \pm t(s_{\bar{x}_1 - \bar{x}_2})
\]

were constructed from the samples and used to determine the number of times (or percentage of time) the resulting intervals failed to cover zero in repeated sampling of the experiments.

Equality of Medians

Two populations may differ in central tendency. The research used the Mann-Whitney/Wilcoxon (or MWW) test to
examine whether one sample tended to yield a larger median than another sample. As the sample size increased, the rank distribution of two samples rapidly approached a normal distribution (Siegel and Castellan 1988). Therefore, in experiment 2, the MWW test determined the mean and variance by

\[ \mu_{W_i} = \frac{n_1(N + 1)}{2} \]

and

\[ \sigma^2_{W_i} = \frac{n_1 n_2}{N(N - 1)} \left( \frac{N^3 - N}{12} - \frac{g}{\sum_{j=1} t_j^3 - t_j} \right) \]

where
- \( n_1 \) = size of sample 1
- \( n_2 \) = size of sample 2
- \( N = n_1 + n_2 \)
- \( i = 1, 2 \)
- \( W_i \) = ranks of sample 1
- \( g \) = number of tied groups
- \( t \) = number of tied ranks in the \( j \)th grouping.

Although the MWW test assumes that the data are sampled from a continuous distribution, the seven-point Likert-scale values were used by a mapping function in the experiments. Hence, the tied scores occurred frequently throughout the tests. If several observed values were tied at the same rank, the rank was assigned the average of the those tied
ranks. Then, the sum of the ranks (sample 1) was changed to a Z value by

\[
Z = \frac{W_x \pm 0.5 - \frac{n_1(N + 1)}{2}}{\sqrt{\frac{n_1 n_2}{N(N - 1)}} \left[ \frac{(N^3 - N)}{12} - \sum_{j=1}^{g} \frac{(t_j^2 - t_j)}{12} \right]}
\]

which was normally distributed with zero mean and unit variance. The Z value was used to determine the significance of the Type I errors.

Equality of Dispersions

The measure of variance was used to detect the variation of populations. As pointed out by Siegel and Tukey (1960), the F test was very sensitive to the assumption of normality. The research used \( \sigma_1^2 / \sigma_2^2 \) to test whether or not the variances of two samples followed the same F distribution.

In addition, Duran (1976) used the value of absolute deviation from means to test whether two random variables from two samples had identical deviations. Thus, the nonparametric Squared Ranks (or SR) test (revised by Conover and Iman 1978) was used to examine the equality of dispersions with the following test statistics:
The critical value of the decision to call for rejection of the null hypothesis was determined by the Z value of normal distribution.

Equality of Distributions

Sampling data invariably differed somewhat in distribution types. The identity of two distribution types was assessed by the KS test in the experiment. The KS test
was used to specify the cumulative frequency distribution to test samples coming either from the same population or from two identical populations (Siegel and Castellan 1988). The test also used more information contained in the observations than did the median test (Conover 1980).

For a sample, the value of $D_{e,n}$ in the KS two-sample test was

$$D_{e,n} = \max[S_{m}(X) - S_{n}(X)]$$

where $m = \text{size of sample 1}$

$n = \text{size of sample 2}$

$X = \text{observation}$

$S_{m}(X) = K/m$

$K = \text{the number of data} \leq X \text{ in the sample}$.

Here, the critical value of the decision to call for rejection of the null hypothesis was determined by

$$\text{Critical Value } T_{a,0.05} = 1.36 \sqrt{\frac{m+n}{mn}}$$

Five Tests of Experiment 2

In summary, the research was focused on the examination of the equality of means (the CI test/t test); the equality of medians (the MWW test); the equality of dispersions (the $F$ test and the SR test); and the homogeneity of two population distributions (the KS test).
In total, two parametric and three nonparametric tests (as listed in table 12) were used to determine whether to reject the null hypotheses in experiment 2.

**TABLE 12**

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<tr>
<th>Parametric Tests</th>
<th>Nonparametric Tests</th>
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<td>Mann-Whitney/Wilcoxon test</td>
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<td>Kolmogorov-Smirnov test</td>
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**Test Results of Experiment 2**

In experiment 2, if one test failed to include zero or rejected the null hypothesis that the respective population parameters (population means, medians, standard deviations, or distributions) were identical, 1 was added to the total error count. The actual error rates were computed by total error count/1000. Moreover, different distance changes between points were applied in the four phases. The results of the Type I error rates of parametric and nonparametric tests in experiment 2 are exhibited in table 13.
### TABLE 13

**TYPE I ERROR RATES OF EXPERIMENT 2**

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Note: ¹μ: confidence interval test for the difference of population means

²MWW: Mann-Whitney/Wilcoxon two-sample test for medians

³F: F test for variances

⁴SR: Squared Ranks two-sample test for variances

⁵KS: Kolmogorov-Smirnov two-sample test for distributions

⁶α = 5%

⁷All entry values are the percentages of the Type I error rates
CI Test for the Difference of Population Means

The Type I error rates generally retained to attest the nominal 5% level, regardless of sample size in experiment 2. The CI test, herein, was not sensitive to the assumption of normal distribution. No particularly different error rates or error patterns could be found when underlying populations were not normal. However, it cannot be concluded that the CI test does not need an assumption of normality in the tests; a coarser nonsymmetric population may show serious impact on the CI test results (Burdick and Graybill 1992). Within each population, a few unequal distance changes between points were made in phases 2, 3, and 4. The Type I error rates revealed little differences when comparing phases 2, 3, and 4 against phase 1 where no distance change was made between points on the scale. Surprisingly, the parametric CI test performed awkwardly in detecting the unequal lengths between points in experiment 2. Inevitably, some instances were liberal (the actual error rates were a little higher than the nominal 5%), and some were conservative (the actual error rates were a little lower than the nominal 5%). Similar results were obtained by Baker et al. (1966) and Gregoire and Driver (1987).

MWW Test for the Difference of Population Medians

As expected, no remarkable differences were shown among the four phases, because the MWW test was a nonparametric test. The MWW test performed as well as the
CI test in experiment 2. Hence, when samples are selected from symmetric populations (where the population mean equals the population median), there is no noticeable gain or loss by using one of these two tests over the other. Of course, the mean and median are two different location parameters; one may make the choice based on the test locations when underlying populations are nonsymmetric.

When compared with the more powerful \( t \) test, the power-efficiency of the MWW test approaches \( 3/\pi = 95.4927\% \) as the sample size increases (Siegel and Castellan 1988). The Type I error rates showed that the nominal 5% was nearly maintained in the MWW test under each sample size, phase, and population. Therefore, when the assumption of normal distribution is unwarranted, the MWW test should be used. In other words, the MWW test is a very appropriate alternative to the parametric CI test (or \( t \) test) when one tries to avoid the CI test's assumptions. The sample size has no major effect on the performance of the MWW test.

The values of populations in experiment 2 were mapped and transformed into seven points (seven-point Likert-scale), and populations were consequently represented by the frequencies of various intervals. Thus, there were many ties in the samples during the repeating tests. However, the consistently low error rates in experiment 2 indicated that the use of the MWW test was also quite appropriate when the populations were not continuous and when many ties were found in the samples (Iman and Conover...
F Test for the Difference of Population Variances

The parametric $F$ test presented results with variety. Many studies have indicated that the $F$ test is highly sensitive to the assumption of normal population (Jarjoura 1983; Kvanli, Guynes, and Pavur 1992). This was confirmed in experiment 2. The $F$ test performed well and appeared expectable in phase 1 of three normal populations (as shown in figure 1) where no distance change was made between points on the scale (the frequencies of each population resembled a distribution of normal shape). The Type I error rates gradually decreased in the 5.2%-2.9% range with increasing sample size.

A few distance changes between points were made in phases 2, 3, and 4 of three normal populations so that the shapes of frequencies reflected different degrees of deviation from a normal distribution. The Type I error of phases 2, 3, and 4 of three normal populations showed either much lower or higher rates than in phase 1. Increasing the sample size did not decrease the Type I error rates.

In addition, the other nonnormal populations (three gamma populations) also reported anomalous Type I error rates; but the performance was almost perfect for the uniform population. Increasing the sample size might either deteriorate or ameliorate the Type I error rates under different circumstances (sample sizes or phases). Phase 4
Fig. 1. ERROR PATTERNS OF F TEST IN PHASE 1 OF THREE POPULATIONS
(that had unequal and unbalanced distance changes between points on the scale) generally showed the worst error rates among the four phases.

The performance of the $F$ test was extremely good in phase 2 of the gamma, population as shown in figure 2. Apparently, the situation was a haphazard snapshot in one phase of the experiment. It was impossible to benefit from the good performance of coincidence based on one case without considering other knowledge. Some nearby evidence could reveal the truth, e.g., the results in phase 3, phase 4, and phase 1 of gamma, population (also shown in figure 2). The Type I error rates were much higher in phase 3 of the same population, gamma,. Here, the true population resembled the skewed distribution and was somewhat nonsymmetric, yet the error rate performance was poorly displayed. Hence, the $F$ test is not strongly recommended to use unless the normal population is justified.

**SR Test for the Difference of Population Variances**

The Type I error rates uniformly retained at the 4%-9% range in phase 1 of uniform and three normal populations. The SR test performed erratically in most of other cases without consistent patterns.

Unequally changing distances between points on the scale have no effect on the nonparametric test. So the inconsistent error performance among the four phases did not appear expectable in the nonparametric SR test of experiment
Fig. 2. ERROR PATTERNS OF F TEST IN GAMMA 1 POPULATION
2. The disturbing outcome displayed here was because the SR test ranked each sample observation by its absolute deviation from the sample mean. Unequally changing distances between points on the scale would rearrange the rank order in phases 2, 3, and 4, because the sample means were dramatically shifted along the underlying continuum. Therefore, the distance changes had an effect on the SR test.

The SR test was included as an alternative nonparametric test of the $F$ test in order to study their usefulness and power. The Type I error rates indicated that, with very few exceptions, the SR test performed consistently worse than the $F$ test across populations in experiment 2. In summary, the SR test should be used when one is not sure whether the population is normal. If the SR test is used instead of the $F$ test when the population is normal, the asymptotic relative efficiency (or A.R.E.) is as low as $15/(2\pi^2) = 75.99\%$ (Conover 1980). However, the sensitivity of the $F$ test to the assumption of normality, coupled with its lack of power in some common nonnormal populations, indicates that the analogous nonparametric test, the SR test, is a good second choice.

**KS Test for the Difference of Population Distributions**

As indicated in table 13, the KS test performed extremely well in all cases of experiment 2. The Type I error rates uniformly reported in the 0%-1.5% range. As the
sample size increased, the Type I error rates slightly decreased in many instances. Apparently, the KS test could easily distinguish distribution equality as long as the populations had identical means and variances; it did not matter whether the distributions were normal or nonnormal, symmetric or nonsymmetric in experiment 2.

The t test and the MWW test may also be appropriate in determining whether the two population distributions are identical or not, but they are sensitive to differences between the two means and medians, respectively; they may not be able to detect the differences of other location parameters, such as variances. The KS test appeared to be dependable on examining all types of location disparity (differences in central tendency, dispersion, skewness, etc.).

The KS test has high power-efficiency (about 95%) for small samples when compared with the t test (Thompson and Tapia 1990). Some evidence has shown that, for small samples, the KS test is slightly more efficient than the MWW test, whereas for large samples the converse holds (Siegel and Castellan 1988). However, as shown in table 13, the Type I error of the KS test presented lower rates in either small or large samples when compared with the MWW test or even the CI test. Hence, the KS test could provide very reliable protection against false claims of distribution heterogeneity (Puri and Sen 1985).
Parametric and Nonparametric Tests and Results in Experiment 3

Five Tests of Experiment 3

The Type II error was the main object of concern in experiment 3. The same five tests outlined in experiment 2 were used once more in experiment 3 (table 12) except that two samples were selected from two different populations. Inasmuch as several populations had identical location parameters (population means, medians, and standard deviations), the Type I error was evaluated instead.

Test Results of Experiment 3

In experiment 3, if one test included zero or failed to reject the null hypothesis when the respective location parameters (population means, medians, standard deviations, and distributions) were not identical, 1 was added to the total error count. The actual error rates were computed by total error count/1000. As such, the Type II error rates of parametric and nonparametric tests in experiment 3 are reported in table 14.

CI Test for the Difference of Two Population Means

The Type II error rates were considerably high in experiment 3. The CI test performed erratically in every population combination (sampling from two populations). However, the lower Type II error rates when sampling from two normal populations with heterogeneous means offered strong support to the assumption of normality. The Type II
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| Normal\(_1\) & 1 | 25 | 28.2 | 27.5 | 55.7 | 64.4 | 67.7 |
| Gamma\(_3\) | 50 | 4.6  | 4.7  | 31.9 | 45.1 | 21.3 |
|               | 100| 0.0  | 0.0  | 7.3  | 20.0 | 0.9  |
| 2            | 10 | 59.4 | 64.9 | 88.5 | 72.2 | 94.0 |
|               | 25 | 20.8 | 28.2 | 68.8 | 37.6 | 68.7 |
|               | 50 | 1.9  | 5.5  | 36.9 | 9.8  | 23.0 |
|               | 100| 0.0  | 0.0  | 8.9  | 0.8  | 1.2  |
| 3            | 10 | 80.4 | 64.9 | 60.9 | 73.9 | 94.1 |
|               | 25 | 59.7 | 25.6 | 60.6 | 63.5 | 64.5 |
|               | 50 | 33.6 | 4.2  | 47.9 | 48.9 | 21.1 |
|               | 100| 5.1  | 0.0  | 30.2 | 39.1 | 0.8  |
| 4            | 10 | 70.8 | 64.6 | 25.4 | 31.2 | 93.9 |
|               | 25 | 27.4 | 30.4 | 11.9 | 9.0  | 69.1 |
|               | 50 | 2.7  | 4.4  | 2.1  | 2.3  | 19.9 |
|               | 100| 0.1  | 0.1  | 0.3  | 0.1  | 1.0  |
|------------|-------|----|-------|-----------|--------|--------|--------|
| Normal & 1 | 10    | 72.8 | 75.0  | 5.2       | 5.9    | 72.8   | 96.6   |
|            | 25    | 39.6 | 43.9  | 5.0       | 6.7    | 87.1   |        |
|            | 50    | 12.2 | 16.9  | 4.5       | 7.2    | 61.8   |        |
|            | 100   | 0.4  | 1.1   | 3.9       | 7.8    | 23.5   |        |
|            | 2     | 77.3 | 75.2  | 3.3       | 13.1   | 96.9   |        |
|            | 25    | 51.5 | 45.2  | 3.5       | 28.3   | 88.1   |        |
|            | 50    | 19.6 | 14.2  | 4.0       | 43.2   | 60.1   |        |
|            | 100   | 2.3  | 1.0   | 3.6       | 4.5    | 21.9   |        |
|            | 3     | 77.9 | 78.1  | 27.5      | 30.7   | 97.9   |        |
|            | 25    | 39.2 | 45.8  | 25.0      | 53.9   | 87.9   |        |
|            | 50    | 8.8  | 14.6  | 27.5      | 74.9   | 59.2   |        |
|            | 100   | 0.4  | 1.5   | 35.9      | 91.5   | 23.4   |        |
|            | 4     | 84.1 | 75.7  | 46.5      | 40.4   | 97.5   |        |
|            | 25    | 54.3 | 44.2  | 63.8      | 66.0   | 86.9   |        |
|            | 50    | 19.8 | 14.3  | 79.9      | 86.0   | 59.4   |        |
|            | 100   | 2.1  | 0.5   | 92.9      | 94.9   | 22.1   |        |
| Normal & 1 | 10    | [5.2] | [5.1] | [5.3]     | [7.5]  | 99.8   |        |
|            | 25    | [5.0] | [4.9] | [6.5]     | [7.4]  | 99.4   |        |
|            | 50    | [5.9] | [6.3] | [6.5]     | [8.8]  | 98.8   |        |
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|            | 50    | [4.7] | [5.6] | [56.5]    | [38.1] | 98.3   |        |
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Note: ¹μ: confidence interval test for the difference of population means

²MWW: Mann-Whitney/Wilcoxon two-sample test for medians

³F: F test for variances

⁴SR: Squared Ranks two-sample test for variances

⁵KS: Kolmogorov-Smirnov two-sample test for distributions

⁶α = 5%

⁷All entry values are the percentages of the Type II error rates except those values in brackets that are Type I error rates.
error rates of the CI test uniformly deteriorated with smaller samples when a true disparity of two population means existed. The deterioration could be caused by increased sample variances as the sample size shrank. Increasing the sample size remarkably improved the performance in all population combinations, regardless of either normal or nonnormal, symmetric or nonsymmetric distributions. In many instances, the rates reached 5% or below when the sample size increased to 100. The Type II error with distance changes between points on the scale (phases 2, 3, and 4) reported both better and worse rates than without distance change (phase 1). Therefore, the various error rate patterns in phases 2, 3, and 4 indicated that the parametric CI test had an effect on the scale with unequal distances between points.

In experiment 2, the CI test was not sensitive to distributional heterogeneity and distance changes between points. This was not the case in experiment 3. The Type I error rates were not uniformly maintained at the nominal 5% when the two population means were identical in experiment 3. The Type I error rates were drastically augmented when distances were unequally changed between points on the scale in phase 4. It was essential to note that the Type I error rates deteriorated with increasing sample size. Usually, the smaller sample provides less information and produces larger error rates (Scheaffer, Mendenhall, and Ott 1990). Here, the disturbing outcome might result from the fact that
the sample mean was not an unbiased estimator of the population mean, because all seven populations were mapped by a mapping function (table 4) and transformed into seven-point Likert-scale values. Therefore, although the variation of the estimator decreased when the sample size increased, the bias did not (Hogg and Tanis 1977; Gregoire and Driver 1987).

**MWW Test for the Difference of Two Population Medians**

As expected, the error rates showed similar patterns among the four phases in each population comparison, since unequally changing distances between points should not have an effect on the nonparametric test. The Type II error rates of the MWW test in experiment 3 reported in a 26%-90% range when comparing the uniform population against other populations. The rates were much lower when comparing normal or gamma population against each other; when the sample size increased to 100, the MWW test performed nearly perfectly in all instances. Figure 3 displays the two error patterns where the rates are the averages of all specified population combinations. Thus, the MWW test appeared better able to detect median equality when comparing the normal and gamma populations against each other than when comparing the uniform population against normal or gamma populations.

The MWW test performed consistently well when sampling from populations with homogeneous medians; the Type I error rates exhibited slightly above the nominal 5%,
Fig. 3. ERROR PATTERNS OF MWW TEST (AVERAGE ERROR RATES)
Pattern 1 - comparing uniform against others
Pattern 2 - comparing normal & gamma against each other
regardless of sample size, in all population combinations. The circumstances showed that the MWW test was less sensitive to distribution differences when the populations had identical medians.

**F Test for the Difference of Two Population Variances**

The parametric $F$ test with smaller samples appeared to be sensitive, not only to departure from normality but also to changing distances between points on the scale. There was a universal deterioration in the Type II error rates when sample size shrank. The $F$ test presented slightly better overall performance when comparing the uniform population against normal or gamma populations than comparing the normal and gamma populations against each other; the test reached excellent performance when the sample size increased to 50 or above, regardless of phase. Figures 4 and 5 display the error patterns where the rates are the averages of all specified population combinations. It is of interest to note that the $F$ test has serious difficulty in distinguishing two population variances when true variance and mean differences exist; the $F$ test becomes not sensitive to normality and the scale with unequal distances between points when large samples are selected.

In experiment 3, the $F$ test was erratic with anomalous patterns in terms of the Type I error rates. No consistent pattern could indicate that the circumstances
under which the $F$ test might be expected to perform well or poorly. The Type I error rates revealed both positive and negative correlations with increasing sample size among the four phases. In addition to the impact of departure from normality, the Type I error rates were evidently distorted by the distance changes between points on the scale. The $F$ test performed almost perfectly in phase 2 when comparing normal, population against gamma, populations as exhibited in figure 6. The surprisingly good result was one of the haphazard instances among all test conditions. No further knowledge could be learned or benefited from this population combination or the distances between points because the nearby phases 3 and 4 of the same population comparison (normal, and gamma,) presented a crooked error pattern. Nor did the excellent performance result from the identical means and variances, because the result of phase 2 was even worse when comparing normal, against gamma, populations (having equal means and variances) as shown in figure 7.

**SR Test for the Difference of Two Population Variances**

The SR test was as erratic as the $F$ test in each population comparison. The Type II error rates apparently decreased with increasing sample size and were close to perfect when the sample size reached 100 in many instances. A better error rate performance was exhibited when comparing the uniform population against normal or gamma populations than when comparing the normal or gamma populations against
Fig. 6. ERROR PATTERNS OF F TEST - COMPARING NORMAL 1 AGAINST GAMMA 1 POPULATIONS

Fig. 7. ERROR PATTERNS OF F TEST - COMPARING NORMAL 2 AGAINST GAMMA 2 POPULATIONS
each other. Figure 8 displays the error patterns where the rates are the averages of all specified population combinations. Therefore, the SR test could easily detect the heterogeneous variances with large samples \((n > 30)\) when comparing the uniform population against other populations.

The Type I error rates uniformly augmented with increasing sample size. More distance changes between points (phase 4) presented less reliable power than fewer distance changes between points (phase 3 and 2) when homogeneous variances existed. The Type I error rates also indicated that the SR test tended to work slightly better when sampling from two populations with identical population variances and means than otherwise. Whether or not the populations are normal is not an important issue in this matter.

**KS Test for the Difference of Two Population Distributions**

All seven populations had different distributions, so only the Type II error was of concern here. Two error patterns could be found in the KS test when sampling from normal or gamma populations. First, the Type II error rates were consistently higher than 90% when sampling from two populations with identical means. Increasing the sample size could decrease only negligibly the error rates. The KS test almost always erroneously declared dissimilar distributions when the two populations had equal means. Apparently the KS test has difficulty in distinguishing the
Fig. 8. ERROR PATTERNS OF SR TEST (AVERAGE ERROR RATES)
Pattern 1 - comparing uniform against others
Pattern 2 - comparing normal & gamma against each other
two populations with equal means. Second, the Type II error rates dramatically decreased with increasing sample size when sampling from two populations without identical means. In many instances, the dissimilar distributions were capably detected by the KS test when the sample size increased to 100. Figure 9 displays the two error patterns where the rates are the averages of all specified population combinations. Both error patterns contained the population comparisons with equal and unequal variances. Surprisingly, the population variance had no critical effect on the determination of error patterns in the KS test. Also, both symmetric and nonsymmetric populations were alternatively used by the tests of two error patterns. Hence, the issue of whether or not the populations are symmetric is not very meaningful in examining the power-efficiency of the KS test.

The Type II error rates revealed the third error pattern (similar to the second error pattern) when comparing the uniform population against normal or gamma populations, regardless of population means, variances, or symmetry. Figure 10 displays the third pattern where the rates are the averages of all specified population combinations. The KS test could easily detect the distributional heterogeneity when the sample size increased to 100.
**Fig. 9.** ERROR PATTERNS OF KS TEST (AVERAGE ERROR RATES) - COMPARING NORMAL & GAMMA AGAINST EACH OTHER
Pattern 1 - with equal means  Pattern 2 - with unequal means

**Fig. 10.** ERROR PATTERNS OF KS TEST (AVERAGE ERROR RATES)
COMPARING UNIFORM AGAINST OTHER POPULATIONS
Pattern 3 - using uniform
CHAPTER 5

SUMMARY AND IMPLICATIONS

Discussion

In the current research, three experiments were conducted using normal and nonnormal distributions to examine the robustness of parametric and nonparametric tests (Huber 1977). The normal distribution included three normal populations (with various central locations or dispersion), whereas the nonnormal distribution included a symmetric population (uniform) and three nonsymmetric populations (gamma).

Test of Central Tendency

With equal distances between points on the scale, the Type I error of the CI test (for mean equality) and the MWW test (for median equality) consistently occurred at the nominal 5% in experiments 1 and 2. Yet the Type I error rates of the MWW test slightly deteriorated when sampling from populations with heterogeneous variances in experiment 3, whereas the CI test still remained at expected level. Increasing the sample size could even worsen the case in the MWW test. As evidenced by the Type I error rates summarized from three experiments, the MWW test had more limitations than the CI test with regard to location parameters and
convenience. For this reason, a pretest of variance equality using the F test or the SR test might be a safeguard before heavily relying on either the CI test or the MWW test to examine the central locations.

Test of Dispersion

The Types I and II error rates of both the F test and the SR test appeared similarly wayward to the distance changes between points on the scale. If the population variance differences indeed existed, there was no rule to determine that one test is preferred to another. The Type II error rates of both tests was severely erratic with smaller samples, but the two performed extremely well with larger samples; whether the two populations had identical means was not important. For the Type I error rates of both tests, it was also interesting to note that the best underlying population was uniform, followed by normal and gamma. In addition, the symmetric populations apparently overperformed the nonsymmetric populations in the experiments.

Test of Distribution

The results of experiments 2 and 3 together indicated that the power of the KS test was justified when sampling from identical distributions or when using large samples with unequal population means. The distance changes between points had no effect on this nonparametric test. Yet the Type II error performance was noticeably poor when
comparing the normal or gamma populations each other with equal means. Hence, if no prior information confirmed mean homogeneity between populations, a pretest was advisable in order to examine the underlying differences in central location (mean) before the KS test was selected for use. The CI test for mean equality, which performed extremely well in experiment 2, could provide reliable protection in this regard.

Conclusions

The current research examined the validity of experiment 1 that paved the way for the next two experiments. Then experiments 2 and 3 compared parametric and nonparametric tests under two performance indicators: the Type I error and the Type II error rates.

Ordinal Measurement

Although the seven observed populations did not provide conclusive proof about what to expect for all required populations, results of the three experiments could reassure the researchers that the ordinal measures (mapped from seven populations) were justified for both the parametric CI test and the \( F \) test. Moreover, little research has been completed on the examination of statistical test performance with unequal and unbalanced distance changes between points on the scale. Here, the seven-point Likert-scale data and changing distances with an unequal and unbalanced scale had revealed some degree of
success in describing the disturbing relationships of ordinal and interval measurement. Also, the parametric tests had performed well in spite of misgivings that, from the viewpoint of the measurement scale typology, only nonparametric tests were appropriate to use for the ordinal data.

Comparison between Parametric and Nonparametric Tests

In the three experiments, treating ordinal data with three underlying distributions (uniform, normal, and gamma) as interval data for the parametric tests did not lead to inappropriate results. In many instances, both the parametric CI test and the $F$ test appeared to be more robust than their nonparametric alternatives as long as the distance between points changed monotonically on the scale, as in phases 1 and 2. There was a distortion in the error rate performance when the underlying ordinality assumptions were seriously violated, as in phase 3 and 4; both parametric and nonparametric tests performed erratically in examining mean, median, or variance equality, because the bias and inconsistency of the Likert-scale mean and variance were respectively used as an estimator of the population mean and variance when having unequal and unbalanced distances between points on the scale.

Decisions of Hypothesis in Experiment 1

Hypothesis 1 was tested via one sample selected from one population with no distance change between points on the
scale. The hypothesis is reiterated as follows:

H1: Equally changing distances between points on the scale do not affect the parametric and nonparametric test results if one sample is selected from one population.

The sample data, as conceived, constituted an ordinal scale. The Type I error of the CI test for the mean and the KS test frequently occurred at, or below, the nominal 5%. Both the CI test for the median and the chi-square test for the standard deviation were sensitive to the nonnormal distribution and sample size, rather than the ordinal data. No serious impact could be found for the equal distances between points on the underlying continuum. In experiment 1, therefore, equally changing distances between points on the scale do not affect the parametric and nonparametric test results if one sample is selected from one population.

Decisions of Hypothesis in Experiment 2

Phase 1 Tests

Hypothesis 2 was tested via two samples selected from one population with no distance change between points on the scale. The hypothesis is reiterated as follows:

H2: Equally changing distances between points on the scale do not affect the parametric and nonparametric test results if two samples are selected from one population.
The sample data constituted an ordinal scale. The Type I error rates of CI test and MWW test uniformly retained at nominal 5% level for all populations. The error rates of $F$ test and SR test also retained at 5%-9% level for three normal populations, but not for the other nonnormal populations. Both the CI test and the $F$ test consistently transcended their nonparametric alternatives, the MWW test and the SR test. The KS test performed extremely well in either symmetric or nonsymmetric, normal or nonnormal populations. Thus, equally changing distances between points on the scale do not affect the parametric and nonparametric test results if two samples are selected from one population.

Phase 2 Tests

Hypothesis $2_b$ was tested via two samples selected from one population with one distance change between points on the scale. The hypothesis is reiterated as follows:

$H_{2_b}$: Unequally changing distances between points on the scale (one unequal change made on both sides of the neutral point) affect the parametric test results but do not affect the nonparametric test results if two samples are selected from one population.

One distance change was made on both sides of the neutral position. The scale was violated with two longer distances in the middle area. The CI test in phase 2 had the Type I error rates about 1% higher than in phase 1 to distinguish the means. Both the $F$ test and the SR test in
phase 2 had either more (normal and uniform populations) or less (gamma populations) power than in phase 1 to detect the variances. The performance of the MWW test and the KS test remained the same as in phase 1. Therefore, unequally changing distances between points on the scale (one change on both sides of neutral) affect the parametric test results, but not the nonparametric test (without the SR test) results if two samples are selected from one population. The SR test is a hybrid of parametric and nonparametric tests, so it has some effect on the distance changes in both normal and nonnormal populations.

**Phase 3 Tests**

Hypothesis $2_c$ was tested via two samples selected from one population with two distance changes between points on the scale. The hypothesis is reiterated as follows:

$H2_c$: Unequally changing distances between points on the scale (two unequal changes made on both sides of the neutral point) affect the parametric test results but do not affect the nonparametric test results if two samples are selected from one population.

Two distance changes were made on both sides of the neutral position. The scale was violated with four longer distances in the two extreme areas. The CI test in phase 3 had the Type I error rates about 1%-2% higher than in phase 1 to distinguish the means, regardless of distribution. Both the $F$ test and the SR test in phase 3 (mostly error rates higher than 20%) appeared to have less power than in
phase 1 to detect the variances, except that the underlying population was uniform where the performance was surprisingly good. As expected, the performance of both the MWW test and the KS test remained the same as in phase 1. Thus, unequally changing distances between points on the scale (two changes on both sides of neutral) affect the parametric test results, but not the nonparametric test (without the SR test) results if two samples are selected from one population. The SR test has a serious effect on the distance changes in both normal and nonnormal populations.

Phase 4 Tests

Hypothesis $2_d$ was tested via two samples selected from one population with more than two distance changes between points on the scale. The hypothesis is reiterated as follows:

$H2_d$: Unequally changing distances between points on the scale (more than two changes made on both sides of the neutral point) affect the parametric test results but do not affect the nonparametric test results if two samples are selected from one population.

More than two distance changes were made on both sides of neutral position. The underlying continuum was violated to constitute an unbalanced scale. The Type I error rates of the CI test were equal to, or below, the nominal 5% but deteriorated with increasing sample size. The $F$ test and the SR test had the least power among the
four phases to detect the variances, especially in the nonsymmetric distribution, three gamma populations (error rates higher than 30%). The performance of the MWW test and the KS test remained the same as in phase 1 and appeared expectable. Therefore, unequally changing distances between points on the scale (more than two changes on both sides of neutral) affect the parametric test results, but not the nonparametric test (without the SR test) results if two samples are selected from one population. The SR test has an erratic effect on the distance changes in both normal and nonnormal populations.

Decisions of Hypothesis in Experiment 3

Phase 1 Tests

Hypothesis 3a was tested via two samples selected from two populations with no distance change between points on the scale. The hypothesis is reiterated as follows:

H3a: Equally changing distances between points on the scale do not affect the parametric and nonparametric test results if two samples are selected from two populations.

Both parametric and nonparametric tests reported high Type II error rates (mostly greater than 50%) and did not capably detect the true location differences of means, medians, variances, and distributions, respectively. The performance was affected by the sample size, rather than the distance changes between points. The Type I error rates
were frequently at the nominal 5% for the CI test and the $F$ test. No serious impact could be found in the parametric tests. Therefore, unequally changing distances between points on the scale do not affect the parametric and nonparametric test results if two samples are selected from two populations.

**Phase 2 Tests**

Hypothesis $3_b$ was tested via two samples selected from two populations with one distance change between points on the scale. The hypothesis is reiterated as follows:

$H3_b$: Unequally changing distances between points on the scale (one unequal change made on both sides of the neutral point) affect the parametric test results but do not affect the nonparametric test results if two samples are selected from two populations.

The CI test, the $F$ test, and the SR test had either better or worse performance than in phase 1 in distinguishing the true location differences; the MWW test and the KS test remained the same as in phase 1. The CI test, the $F$ test, and the SR test also reported slightly higher Type I error rates than in phase 1, regardless of distribution. Therefore, unequally changing distances between points on the scale (one change on both sides of neutral) affect the parametric test results, but not the nonparametric test (without the SR test) results if two samples are selected from two populations. The SR test has as erratic an effect on the distance changes as do the
Phase 3 Tests

Hypothesis 3 was tested via two samples selected from two populations with two distance changes between points on the scale. The hypothesis is reiterated as follows:

H3c: Unequally changing distances between points on the scale (two unequal changes made on both sides of the neutral point) affect the parametric test results but do not affect the nonparametric test results if two samples are selected from two populations.

The CI test, the F test, and the SR test had either better or worse performance than in phase 1 in distinguishing the true location differences, but the F test and the SR test unexpectedly had better performance than phase 2 in distinguishing the true variation differences. The MWW test and the KS test remained the same as phases 1 and 2. The CI test, the F test, and the SR test also reported consistently higher Type I error rates than phases 1 and 2, regardless of distribution. Therefore, unequally changing distances between points on the scale (two changes on both sides of neutral) affect the parametric test results, but not the nonparametric test (without the SR test) results if two samples are selected from two populations. The SR test has as erratic an effect on the distance changes as do the parametric tests.
Phase 4 Tests

Hypothesis $3_d$ was tested via two samples selected from two populations with more than two distance changes between points on the scale. The hypothesis is reiterated as follows:

$H_{3_d}$: Unequally changing distances between points on the scale (more than two changes made on both sides of the neutral point) affect the parametric test results but do not affect the nonparametric test results if two samples are selected from two populations.

The CI test, the $F$ test, and the SR test surprisingly had better performance than any other phases in distinguishing the true location differences, except when comparing gamma populations to each other. The MWW test and the KS test remained the same as in phases 1, 2, and 3. The CI test, the $F$ test, and the SR test, however, reported the worst Type I error rates among the four phases, regardless of distribution. Therefore, unequally changing distances between points on the scale (more than two changes on both sides of neutral) affect the parametric test results, but not the nonparametric test (without the SR test) results if two samples are selected from two populations. The SR test has as erratic an effect on the distance changes as do the parametric tests in both normal and nonnormal populations.

Selection between Parametric and Nonparametric Tests

It would be premature to conclude that the type of measurement scale had little relevance to the question of
whether to use parametric or nonparametric tests (Lord 1953; Neihecker 1984; Thompson and Tapia 1990). However, the parametric tests consistently transcended their nonparametric alternatives in phases 1 and 2 of experiments 2 and 3. In regard to practical requirements, it is thus concluded that magnitude scaling (numeric estimates) can be accepted as a valid and reliable alternative to category scaling (ordinal data). Use of parametric tests can have the advantage of achieving more reliable performance (in terms of the Type I and the Type II errors) when one tests the location equality with steadily changing distances between points on the scale; the nonparametric tests, however, would be informative alternatives, or at least useful complements, to parametric tests in many instances.

Limitations of Research

The results of the three experiments offered strong support for the validity of the magnitude scaling technique. However, the same scale was used one thousand times in each phase, so the error rate performance was analyzed by the aggregate data that was simulated from the four phases of scales. Teas (1987) indicated that the power function exponents varied considerably from individual to individual. Apparently, the respondents might not express their preferred labeled points and relative strengths in a consistent manner. If the error rate performance was analyzed at an individual level, dramatic heterogeneity
across individuals could be revealed.

The normal distribution was one of the underlying assumptions for the parametric tests chosen in the current research. Both parametric and nonparametric tests were focused on the violation of normality in the populations. However, there were only seven populations generated in three experiments. The nonnormal (symmetric and nonsymmetric) distributions were restricted to uniform and gamma populations. Apparently, the uniform and gamma populations did not typically represent all types of nonnormal distributions. Also, the use of only one mapping function in the experiments was another limitation to generating more symmetric and nonsymmetric populations for the tests.

Directions for Future Research

This research investigates samples selected from three types of distributions (uniform, normal, and gamma). It would be informative to rigorously generate a series of populations with coarser nonnormal shapes for the statistical tests. Much would be gained by a further investigation of the error rates of sampling from a greater variety of populations. Such an extension would ensure that the conclusions are not unique to the seven populations chosen (one uniform, three normal, and three gamma populations).

The changes in both the direction and the magnitude
of the Type I and II error rates are the only two measures used in this research to examine the effect on parametric and nonparametric tests. Two samples with identical sample size were selected each time in experiment 2 and 3. A worthwhile study would be to test the effect of using two samples with different sample sizes in the experiments.

Another challenge to researchers is to explore the change of regression performance when the number of points (with unequal and unbalanced distances) steadily increases. Also, it is essential to investigate the systematic progression of test performance with a population distribution that digresses steadily in central location or dispersion.

Furthermore, prior studies (Stevens 1975; Lodge 1987; Teas 1987) have directed the labeled distance fluctuations across from individual to individual. It would be interesting to look into the problem and develop a within-subject power function to solve within-subject heterogeneity as well.
APPENDIX

SOURCE CODES OF COMPUTER PROGRAMS
1. EXP1.F (Experiment 1)
Program name: EXP1.F
Programmer : Andrew H. Chen
Date : 12-20-1993

integer ido, index(25), ldpop, ldsamp
integer nvar, npop, nrow, nsamp
integer i, j
integer iseed, nout, nr
integer n, npass, m, qpass
integer aerrct, berrct, cerrct, derrct
real a, b
real psum, pmean, ssize, tt
real r(10000,1), q(10000,1), samp(25,1)
real sum, sumdev, mean, median, stddev, psmsq
real pstdddev
real aerrrate, berrrate, cerrrate, derrrate
real acil, acih, bcil, bcih, ccil, ccih, dcil, dcih
real pfql, pfq2, pfq3, pfq4, pfq5, pfq6, pfq7
real pcfq1, pcfq2, pcfq3, pcfq4
real pcfq5, pcfq6, pcfq7
real sfql, sfq2, sfq3, sfq4, sfq5, sfq6, sfq7
real scfq1, scfq2, scfq3, scfq4
real scfq5, scfq6, scfq7
real ksd, ksd1, ksd2, ksd3, ksd4, ksd5, ksd6, ksd7
real ksd, ksd1, ksd2, ksd3, ksd4, ksd5, ksd6, ksd7

external rnsrs
external rnset, run, umach

Initialize all variables

a = 16
b = 2.5
ssize = 25.0
nrow = 10000
nvar = 1
ldpop = 10000
nsamp = ssize
ldsamp = ssize
ido = 0
nr = 10000
ctneg1 = 0
ctneg2 = 0
ctneg3 = 0
ctneu0 = 0
ctpos1 = 0
ctpos2 = 0
ctpos3 = 0
aerrct = 0
aerrrate = 0.0
berrct = 0
berrrate = 0.0
cerrct = 0
cerrrate = 0.0
derrct = 0
derrrate = 0.0

c Generate a uniform population
call rnset(iseed)
call rnun(nr, r)
call sscal(nr, 100.00, r, 1)
call sadd(nr, 0.00, r, 1)

c Generate a normal population
call rnset(iseed)
call rnnoa(nr, r)
call sscal(nr, 15.00, r, 1)
call sadd(nr, 50.00, r, 1)

c Generate a gamma population
call rnset(iseed)
call rngam(nr, a, r)
call sscal(nr, b, r, 1)

c Find population mean
psum = 0.0
pmean = 0.0
pfq1 = 0.0
pfq2 = 0.0
pfq3 = 0.0
pfq4 = 0.0
pfq5 = 0.0
pfq6 = 0.0
pfq7 = 0.0
pcfq1 = 0.0
pcfq2 = 0.0
pcfq3 = 0.0
pcfq4 = 0.0
pcfq5 = 0.0
pcfq6 = 0.0
pcfq7 = 0.0
i = 1

do 0100 i=1,10000
if (r(i,1) .lt. 14.29) then
    psum = psum + -3
    pfq1 = pfq1 + 1
elseif (r(i,1) .lt. 28.57) then
    psum = psum + -2
    pfq2 = pfq2 + 1
elseif (r(i, 1) .lt. 42.86) then
  psum = psum + 1
  pfq3 = pfq3 + 1
elseif (r(i, 1) .lt. 57.14) then
  psum = psum + 2
  pfq4 = pfq4 + 1
elseif (r(i, 1) .lt. 71.43) then
  psum = psum + 1
  pfq5 = pfq5 + 1
elseif (r(i, 1) .lt. 85.71) then
  psum = psum + 1
  pfq6 = pfq6 + 1
else
  psum = psum + 3
  pfq7 = pfq7 + 1
end if

0100 continue
pmean = psum / 10000
psmsq = 0.0
pstddev = 0.0

ccccc cccc cccc cccc cccc cccc cccc cccc cccc cccc cccc cccc cccc cccc cccc
c Find population standard deviation and pcfq c
ccccc cccc cccc cccc cccc cccc cccc cccc cccc cccc cccc cccc cccc cccc cccc
c i = 1
do 0200 i=1,10000
  if (r(i, 1) .lt. 14.29) then
    psmsq = psmsq + (-3 - pmean)**2
  elseif (r(i, 1) .lt. 28.57) then
    psmsq = psmsq + (-2 - pmean)**2
  elseif (r(i, 1) .lt. 42.86) then
    psmsq = psmsq + (-1 - pmean)**2
  elseif (r(i, 1) .lt. 57.14) then
    psmsq = psmsq + (0 - pmean)**2
  elseif (r(i, 1) .lt. 71.43) then
    psmsq = psmsq + (1 - pmean)**2
  elseif (r(i, 1) .lt. 85.71) then
    psmsq = psmsq + (2 - pmean)**2
  else
    psmsq = psmsq + (3 - pmean)**3
  endif
0200 continue
pstddev = sqrt(psmsq/10000)
pcfq1 = pfq1 / 10000.0
pcfq2 = (pfq1+pfq2) / 10000.0
pcfq3 = (pfq1+pfq2+pfq3) / 10000.0
pcfq4 = (pfq1+pfq2+pfq3+pfq4) / 10000.0
pcfq5 = (pfq1+pfq2+pfq3+pfq4+pfq5) / 10000.0
pcfq6 = (pfq1+pfq2+pfq3+pfq4+pfq5+pfq6) / 10000.0
pcfq7 = (pfq1+pfq2+pfq3+pfq4+pfq5+pfq6+pfq7) / 10000.0

ccccc cccc cccc cccc cccc cccc cccc cccc cccc cccc cccc cccc cccc cccc cccc
c Find population median c
ccccc cccc cccc cccc cccc cccc cccc cccc cccc cccc cccc cccc cccc cccc cccc
i = 1
do 0300 i=1,10000
if (r(i,1) .lt. 14.29) then
  q(i,1) = -3
elseif (r(i,1) .lt. 28.57) then
  q(i,1) = -2
elseif (r(i,1) .lt. 42.86) then
  q(i,1) = -1
elseif (r(i,1) .lt. 57.14) then
  q(i,1) = 0
elseif (r(i,1) .lt. 71.43) then
  q(i,1) = 1
elseif (r(i,1) .lt. 85.71) then
  q(i,1) = 2
else
  q(i,1) = 3
endif
continue
0300 continue
    do 0310 qpass = 1, 10000 - 1
    do 0320 m = 1, 10000 - qpass
       if (q(m, 1) .gt. q(m + 1, 1)) then
          temp = q(m, 1)
          q(m, 1) = q(m + 1, 1)
          q(m + 1, 1) = temp
       endif
    0320 continue
0310 continue
    pmedian = (q(5000,1) + q(5001,1))/2
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do 1000 i=1,ssize
if (samp(i,1) .lt. 14.29) then
  samp(i,1) = -3
  sfq1 = sfq1 + 1
elseif (samp(i,1) .lt. 28.57) then
  samp(i,1) = -2
  sfq2 = sfq2 + 1
elseif (samp(i,1) .lt. 42.86) then
  samp(i,1) = -1
  sfq3 = sfq3 + 1
elseif (samp(i,1) .lt. 57.14) then
  samp(i,1) = 0
  sfq4 = sfq4 + 1
elseif (samp(i,1) .lt. 71.43) then
  samp(i,1) = 1
  sfq5 = sfq5 + 1
elseif (samp(i,1) .lt. 85.71) then
  samp(i,1) = 2
  sfq6 = sfq6 + 1
else
  samp(i,1) = 3
  sfq7 = sfq7 + 1
endif
1000 continue
scfql = sfql / ssize
scfq2 = (sfql+sfq2) / ssize
scfq3 = (sfql+sfq2+sfq3) / ssize
scfq4 = (sfql+sfq2+sfq3+sfq4) / ssize
scfq5 = (sfql+sfq2+sfq3+sfq4+sfq5) / ssize
scfq6 = (sfql+sfq2+sfq3+sfq4+sfq5+sfq6) / ssize
scfq7 = (sfql+sfq2+sfq3+sfq4+sfq5+sfq6+sfq7) / ssize
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```plaintext
2100 continue
stddev = sqrt(sumdev / (ssize - 1))
if (ssize .eq. 10.0 ) then
  tt = 2.262
endif
if (ssize .eq. 25.0 ) then
  tt = 2.064
endif
if (ssize .gt. 30.0 ) then
  tt = 1.96
endif
acil = mean - tt * (stddev / (sqrt(ssize)))
acih = mean + tt * (stddev / (sqrt(ssize)))
if (acil .gt. pmean) then
  aerrct = aerrct + 1
endif
if (acih .lt. pmean) then
  aerrct = aerrct + 1
endif
```
endif
if (ssize .ge. 30.0) then
  tt = 1.96
endif
bcil = median - tt * (1.2533)*(stddev / (sqrt(ssize)))
$ (sqrt(ssize)))$
bcih = median + tt * (1.2533)*(stddev / (sqrt(ssize)))
if (bcil .gt. pmedian) then
  berrct = berrct + 1
endif
if (bcih .lt. pmedian) then
  berrct = berrct + 1
endif
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
Kolmogorov-Smirnov Test c
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
ksd = 0.0
ksd1 = 0.0
ksd2 = 0.0
ksd3 = 0.0
ksd4 = 0.0
ksd5 = 0.0
ksd6 = 0.0
ksd7 = 0.0
kscrit = 0.0
ksd1 = abs(pcfq1 - scfq1)
ksd2 = abs(pcfq2 - scfq2)
ksd3 = abs(pcfq3 - scfq3)
ksd4 = abs(pcfq4 - scfq4)
ksd5 = abs(pcfq5 - scfq5)
ksd6 = abs(pcfq6 - scfq6)
ksd7 = abs(pcfq7 - scfq7)
ksd = max(ksd1, ksd2, ksd3, ksd4, ksd5, ksd6, ksd7)
  if (ssize .eq. 10.0) then
    kscrit = .41
  endif
if (ssize .eq. 25.0) then
  kscrit = .27
endif
if (ssize .eq. 50.0) then
  kscrit = .1923
endif
if (ssize .eq. 100.0) then
  kscrit = .136
endif
if (ksd .gt. kscrit) then
  cerrct = cerrct + 1
endif
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
Chi-Square Test c
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
chicrit = 0.0
if (ssize .eq. 10.0) then
  chicrit1 = 19.0228
  chicrit2 = 2.70039
endif
if (ssize .eq. 25.0) then
  chicrit1 = 39.3641
  chicrit2 = 12.4011
endif
if (ssize .eq. 50.0) then
  chicrit1 = 71.4202
  chicrit2 = 32.3574
endif
if (ssize .eq. 100.0) then
  chicrit1 = 129.561
  chicrit2 = 74.2219
endif
dcil = sqrt(((ssize - 1)*(stddev)**2) / chicrit1)
dcih = sqrt(((ssize - 1)*(stddev)**2) / chicrit2)
if (dcil .gt. pstddev) then
derrct = derrct + 1
endif
if (dcih .lt. pstddev) then
derrct = derrct + 1
endif
continue
aerrrate = aerrct / 1000.0
write (nout, 99991) aerrct, aerrrate
99991 format (' mean: err count = ', i5, ' err rate = ', f10.4)
berrrate = berrct / 1000.0
write (nout, 99992) berrct, berrrate
99992 format (' median: err count = ', i5, ' err rate = ', f10.4)
cerrrate = cerrct / 1000.0
write (nout, 99993) cerrct, cerrrate
99993 format (' k-s: err count = ', i5, ' err rate = ', f10.4)
derrrate = derrct / 1000.0
write (nout, 99994) derrct, derrrate
99994 format (' chi: err count = ', i5, ' err rate = ', f10.4)
end
2. EXP2.F (Experiment 2)
Program name: EXP2.F
Programmer: Andrew H. Chen
Date: 01-10-1994

integer ido, index(50), ldpop, ldsamp
integer nvar, npop, nrow, nsamp
integer i, j, k
integer iseed, nout, nr
integer n, npass, m
integer aererrct, berrct, cerrct, derrct, eerrct
integer tie(14,1)
real a, b
real pt1, pt2, pt3, pt4, pt5, pt6, pt7
real psum, pmean, ssize, tt
real r(10000,1)
real samp(50,1), absdevo(50,1), absdevn(50,1)
real sqrank(50,1)
real aererrrate, berrrate, cerrrate, derrrate
real eerrrate
real acil, acih, dcil, dcih
real sum1, sum2, sumdev1, sumdev2, mean1, mean2
real stddev1, stddev2
real lgr, sml, ff, forit, var1, var2
real sqroot, varp, varp1, varp2
real ranksum, ranktie(14,1)
real sumsqrk, rsqbar, rsqbarsz, sumr4
real srtl, tlcrit
real rksum
real t1, t2, t3, t4, t5, t6, t7
real r1, r2, r3, r4, r5, r6, r7
real wx, tsum
real z, z1, z2, z3, z4
real ksd, dmn
real m1, m2, m3, m4, m5, m6, m7
real n1, n2, n3, n4, n5, n6, n7
real sm1, sm2, sm3, sm4, sm5, sm6, sm7
real sn1, sn2, sn3, sn4, sn5, sn6, sn7
real d1, d2, d3, d4, d5, d6, d7
external rnsrs
external rnset, rnun, umach

Initialize all variables

call umach(2, nout)
a = 16
b = 2.5
ssize = 50.0
nrow = 10000
nvar = 1
ldpop = 10000
nsamp = ssize
ldsamp = ssize
ido = 0
nr = 10000
pt1 = -0.999
pt2 = -0.745
pt3 = -0.618
pt4 = -0.49
pt5 = 0.018
pt6 = 0.781
pt7 = 2.053

cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
Generate a uniform population c
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
call rnset(iseed)
call rnun(nr, r)
call sscal(nr, 100.00, r, 1)
call sadd(nr, 0.00, r, 1)

cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
Generate a normal population c
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
call rnset(iseed)
call rnnoa(nr, r)
call sscal(nr, 10.00, r, 1)
call sadd(nr, 40.00, r, 1)

cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
Generate a gamma population c
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
call rnset(iseed)
call rngam(nr, a, r)
call sscal(nr, b, r, 1)

cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
Find population mean c
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
psum = 0.0
pmean = 0.0
i = 1

0100 do i=1,10000
if (r(i,1) .lt. 14.29) then
  psum = psum + pt1
elseif (r(i,1) .lt. 28.57) then
  psum = psum + pt2
elseif (r(i,1) .lt. 42.86) then
  psum = psum + pt3
elseif (r(i,1) .lt. 57.14) then
  psum = psum + pt4
elseif (r(i,1) .lt. 71.43) then
  psum = psum + pt5
elseif (r(i,1) .lt. 85.71) then
  psum = psum + pt6
else
  psum = psum + pt7
endif
0100 continue
2pmean = psum / 10000

Select 2 samples from a gamma population

j = 1
do 9999 j=1,1000
call rnsrs(ido,nrow,nvar,r,ldpop,nsamp,npop,
& samp,ldsamp,index)

Map the sample into one of seven-point likert-scale

i = 1
t1 = 0.0
t2 = 0.0
t3 = 0.0
t4 = 0.0
t5 = 0.0
t6 = 0.0
t7 = 0.0
m1 = 0.0
m2 = 0.0
m3 = 0.0
m4 = 0.0
m5 = 0.0
m6 = 0.0
m7 = 0.0
do 1000 i=1,ssize/2
if (samp(i,1) .lt. 14.29) then
  samp(i,1) = pt1
  t1 = t1 + 1
  m1 = m1 + 1
elseif (samp(i,1) .lt. 28.57) then
  samp(i,1) = pt2
  t2 = t2 + 1
  m2 = m2 + 1
elseif (samp(i,1) .lt. 42.86) then
  samp(i,1) = pt3
  t3 = t3 + 1
  m3 = m3 + 1
elseif (samp(i,1) .lt. 57.14) then
  samp(i,1) = pt4
  t4 = t4 + 1
  m4 = m4 + 1
elseif (samp(i,1) .lt. 71.43) then
  samp(i,1) = pt5
  t5 = t5 + 1
  m5 = m5 + 1
elseif (samp(i,1) .lt. 85.71) then
  samp(i,1) = pt6
  t6 = t6 + 1
  m6 = m6 + 1
else
  samp(i,1) = pt7
t7 = t7 + 1
m7 = m7 + 1
endif
1000 continue

c
i = 1
n1 = 0.0
n2 = 0.0
n3 = 0.0
n4 = 0.0
n5 = 0.0
n6 = 0.0
n7 = 0.0
do 1001 i=(ssize/2) + 1, ssize
if (samp(i,1) .lt. 14.29) then
  samp(i,1) = pt1
  t1 = t1 + 1
  n1 = n1 + 1
elseif (samp(i,1) .lt. 28.57) then
  samp(i,1) = pt2
  t2 = t2 + 1
  n2 = n2 + 1
elseif (samp(i,1) .lt. 42.86) then
  samp(i,1) = pt3
  t3 = t3 + 1
  n3 = n3 + 1
elseif (samp(i,1) .lt. 57.14) then
  samp(i,1) = pt4
  t4 = t4 + 1
  n4 = n4 + 1
elseif (samp(i,1) .lt. 71.43) then
  samp(i,1) = pt5
  t5 = t5 + 1
  n5 = n5 + 1
elseif (samp(i,1) .lt. 85.71) then
  samp(i,1) = pt6
  t6 = t6 + 1
  n6 = n6 + 1
else
  samp(i,1) = pt7
  t7 = t7 + 1
  n7 = n7 + 1
endif
1001 continue

ccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc

C Confidence Interval on Mean
ccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc

sum1 = 0.0
sum2 = 0.0
sumdev1 = 0.0
sumdev2 = 0.0
mean1 = 0.0
mean2 = 0.0
stddev1 = 0.0
stddev2 = 0.0
sqroot = 0.0
varp = 0.0
varpl = 0.0
varp2 = 0.0
cil = 0.0
cih = 0.0
c
do 1100 i=1,ssize/2
   suml = suml + samp(i,1)
1100 continue

do 1200 i=ssize/2 + 1, ssize
   sum2 = sum2 + samp(i,1)
1200 continue

c
   meanl = suml / (ssize/2)
do 1300 i=1,ssize/2
   sumdev1 = sumdev1 + (samp(i,1) - meanl)**2
1300 continue

eq.
   mean2 = sum2 / (ssize/2)
do 1400 i=ssize/2 + 1, ssize
   sumdev2 = sumdev2 + (samp(i,1) - mean2)**2
1400 continue

c
   stddev1 = sqrt(sumdev1 / (ssize/2 - 1))
   stddev2 = sqrt(sumdev2 / (ssize/2 - 1))

c
   varpl = ((ssize/2 - 1)*stddev1**2) +
         ((ssize/2 - 1)*stddev2**2)
   varp2 = ssize - 2
   varp = varpl / varp2
   sqroot = (varp/(ssize/2)) + (varp/(ssize/2))
tt = 0.0
if (ssize .eq. 20.0) then
   tt = 2.101
endif
if (ssize .eq. 50.0) then
   tt = 2.021
endif
if (ssize .gt. 50.0) then
   tt = 1.96
endif
acil = (meanl - mean2) - tt*(sqrt(sqroot))
acih = (meanl - mean2) + tt*(sqrt(sqroot))
if (acil .gt. 0) then
   aerrct = aerrct + 1
endif
if (acih .lt. 0) then
   aerrct = aerrct + 1
endif
Wilcoxon-Mann-Whitney Test

```plaintext
r1 = 0.0
r2 = 0.0
r3 = 0.0
r4 = 0.0
r5 = 0.0
r6 = 0.0
r7 = 0.0

c rksum = 0.0
if (t1 .gt. 0 ) then
   do 2010 i=1, t1
      rksum = rksum + i
   continue
2010
   r1 = rksum / t1
endif

c rksum = 0.0
if (t2 .gt. 0 ) then
   do 2020 i=t1 + 1, t1 + t2
      rksum = rksum + i
   continue
2020
   r2 = rksum / t2
endif

c rksum = 0.0
if (t3 .gt. 0 ) then
   do 2030 i=t1 + t2 + 1, t1 + t2 + t3
      rksum = rksum + i
   continue
2030
   r3 = rksum / t3
endif

c rksum = 0.0
if (t4 .gt. 0 ) then
   do 2040 i=t1 + t2 + t3 + 1, t1 + t2 + t3 + t4
      rksum = rksum + i
   continue
2040
   r4 = rksum / t4
endif

c rksum = 0.0
if (t5 .gt. 0 ) then
   do 2050 i=t1 + t2 + t3 + t4 + 1, t1 + t2 + t3 + t4 + t5
      rksum = rksum + i
   continue
2050
   r5 = rksum / t5
endif

c rksum = 0.0
```
if (t6 .gt. 0 ) then
   do 2060 i=tl + t2 + t3 + t4 + t5 + 1,
   t1 + t2 + t3 + t4 + t5 + t6
   rksum = rksum + i
2060 continue
   r6 = rksum / t6
endif

rksum = 0.0
if (t7 .gt. 0 ) then
   do 2070 i=tl + t2 + t3 + t4 + t5 + t6 + 1,
   $ tl  + t2 + t3 + t4 + t5 + t6 + t7
   rksum = rksum + i
2070 continue
   r7 = rksum / t7
endif

cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
 c Find wx
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
 wx = 0.0
   do 2100 i=1, ssize/2
      if (samp(i,1) .eq. pt1) then
         wx = wx + r1
      elseif (samp(i,1) .eq. pt2) then
         wx = wx + r2
      elseif (samp(i,1) .eq. pt3) then
         wx = wx + r3
      elseif (samp(i,1) .eq. pt4) then
         wx = wx + r4
      elseif (samp(i,1) .eq. pt5) then
         wx = wx + r5
      elseif (samp(i,1) .eq. pt6) then
         wx = wx + r6
      else
         wx = wx + r7
      endif
2100 continue

c
tsum = 0.0
   tsum = ((t1**3-t1)+(t2**3-t2)+(t3**3-t3)+(t4**3-t4)+
   $ (t5**3-t5)+(t6**3-t6)+(t7**3-t7))/12
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
 c Calculate z statistics
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
 z1 = 0.0
 z2 = 0.0
 z3 = 0.0
 z4 = 0.0
 z  = 0.0
z1=wx - (ssize/2)*(ssize + 1)/2
z3=(ssize/2)**2/(ssize*(ssize-1))
z4=(ssize**3-ssize)/12-tsum
z2= sqrt(z3*z4)
\[ z = \frac{z_1}{z_2} \]

if \((z \gt 1.96) \text{ or } (z \lt -1.96)) \text{ then} 
\begin{align*}
\text{berrct} &= \text{berrct} + 1 \\
\end{align*}
endif

cccc

F Test

cccc

var1 = 0.0
var2 = 0.0
lgr = 0.0
sml = 0.0
ff = 0.0
fcrit = 0.0
c

\begin{align*}
\text{varl} &= \text{stddevl}^2 \\
\text{var2} &= \text{stddev2}^2 \\
\text{lgr} &= \max(\text{varl}, \text{var2}) \\
\text{sml} &= \min(\text{varl}, \text{var2}) \\
\text{ff} &= \frac{\text{lgr}}{\text{sml}} \\
\text{if} \ (\text{ssize} \ .eq. 20.0) \text{ then} \\
& \quad \text{fcrit} = 4.03 \\
\text{endif} \\
\text{if} \ (\text{ssize} \ .eq. 50.0) \text{ then} \\
& \quad \text{fcrit} = 2.27 \\
\text{endif} \\
\text{if} \ (\text{ssize} \ .eq. 100.0) \text{ then} \\
& \quad \text{fcrit} = 1.78 \\
\text{endif} \\
\text{if} \ (\text{ssize} \ .eq. 200.0) \text{ then} \\
& \quad \text{fcrit} = 1.51 \\
\text{endif} \\
\text{if} \ (\text{ff} \ .gt. \text{fcrit}) \text{ then} \\
& \quad \text{cerrct} = \text{cerrct} + 1 \\
\text{endif}
\end{align*}

c

Squared Ranks Test
c

do 4000 i=1, ssize/2
\begin{align*}
\text{absdevo}(i,1) &= \abs(\text{samp}(i,1) - \text{mean1}) \\
\text{absdevn}(i,1) &= \abs(\text{samp}(i,1) - \text{mean1}) \\
\end{align*}
4000 continue
do 4100 i=ssize/2 + 1, ssize
\begin{align*}
\text{absdevo}(i,1) &= \abs(\text{samp}(i,1) - \text{mean2}) \\
\text{absdevn}(i,1) &= \abs(\text{samp}(i,1) - \text{mean2}) \\
\end{align*}
4100 continue
do 4200 npass = 1, ssize - 1
\begin{align*}
do 4210 n = 1, ssize - npass \\
& \quad \text{if} \ (\text{absdevn}(n, 1) \ .gt. \text{absdevn}(n + 1, 1)) \text{ then} \\
& \quad \quad \text{temp} = \text{absdevn}(n, 1) \\
& \quad \quad \text{absdevn}(n, 1) = \text{absdevn}(n + 1, 1) \\
& \quad \quad \text{absdevn}(n + 1, 1) = \text{temp} \\
& \quad \quad \text{endif} \\
4210 \quad \text{continue}
\end{align*}
c

k = 1
tie(k,1) = 1
ranksum = 1
do 4300 i=1, ssize - 1
   if (absdevn(i,1) .ne. absdevn(i+1,1)) then
      if (i .eq. 1) then
         ranktie(1,1) = ranksum / i
      else
         ranktie(k,1) = (ranksum / tie(k,1))**2
      endif
   endif
   do 4400 l = 1, ssize
      if (absdevo(l,1) .eq. absdevn(i,1)) then
         sqrank(l,1) = ranktie(k,1)
      endif
   4400 continue
   ranksum = i + 1
   k = k + 1
   tie(k,1) = 1
   if (i .eq. (ssize - 1)) then
      ranktie(k,1) = (ranksum / tie(k,1))**2
      do 4500 l = 1, ssize
         if (absdevo(l,1) .eq. absdevn(i+1,1)) then
            sqrank(l,1) = ranktie(k,1)
         endif
      4500 continue
   endif
else
   tie(k,1) = tie(k,1) + 1
   ranksum = ranksum + i + 1
   if (i .eq. (ssize - 1)) then
      ranktie(k,1) = (ranksum / tie(k,1))**2
      do 4600 l = 1, ssize
         if (absdevo(l,1) .eq. absdevn(i,1)) then
            sqrank(l,1) = ranktie(k,1)
         endif
      4600 continue
   endif
endif
4300 continue
sumsqrk = 0.0
rsqbar = 0.0
sumr4 = 0.0
do 4700 m = 1, ssize/2
   sumsqrk = sumsqrk + sqrank(m,1)
4700 continue
do 4800 m = 1, ssize
   rsqbar = rsqbar + sqrank(m,1)
   sumr4 = sumr4 + (sqrank(m,1))**2
4800 continue
rsqbarsz = rsqbar / ssize
srtl = ((sumsqrk -
$ (ssize/2)^2*rsqbarsz)/(sqrt(((ssize/2)**2)/
$ (ssize*ssize-1)*)sumr4)-((ssize/2)**2)/(ssize-1)*
$ (rsqbarsz**2)))})

tlcrit = 0.0
if (ssize .eq. 20.0) then
tlcrit = 2.101
endif
if (ssize .eq. 50.0) then
tlcrit = 2.011
endif
if (ssize .gt. 50.0) then
tlcrit = 1.96
endif
dc1 = -tlcrit
dc2 = tlcrit
if (dc1 .gt. srtl) then
derrct = derrct + 1
endif
if (dc2 .lt. srtl) then
derrct = derrct + 1
endif

cccc
Kolmogorov-Smirnov Test c

sm1 = m1/(ssize/2)
sm2 = m1/(ssize/2)+m2/(ssize/2)
sm3 = m1/(ssize/2)+m2/(ssize/2)+m3/(ssize/2)
sm4 =
$ m1/(ssize/2)+m2/(ssize/2)+m3/(ssize/2)+m4/(ssize/2)
$ sm5 = m1/(ssize/2)+m2/(ssize/2)+m3/(ssize/2)+
$ m4/(ssize/2)+m5/(ssize/2)
sm6 = m1/(ssize/2)+m2/(ssize/2)+m3/(ssize/2)+
$ m4/(ssize/2)+m5/(ssize/2)+m6/(ssize/2)
sm7 = m1/(ssize/2)+m2/(ssize/2)+m3/(ssize/2)+
$ m4/(ssize/2)+m5/(ssize/2)+m6/(ssize/2)+
$ m7/(ssize/2)

sn1 = n1/(ssize/2)
sn2 = n1/(ssize/2)+n2/(ssize/2)
sn3 = n1/(ssize/2)+n2/(ssize/2)+n3/(ssize/2)
sn4 =
$ n1/(ssize/2)+n2/(ssize/2)+n3/(ssize/2)+n4/(ssize/2)
$ sn5 = n1/(ssize/2)+n2/(ssize/2)+n3/(ssize/2)+
$ n4/(ssize/2)+n5/(ssize/2)
sn6 = n1/(ssize/2)+n2/(ssize/2)+n3/(ssize/2)+
$ n4/(ssize/2)+n5/(ssize/2)+n6/(ssize/2)
sn7 = n1/(ssize/2)+n2/(ssize/2)+n3/(ssize/2)+
$ n4/(ssize/2)+n5/(ssize/2)+n6/(ssize/2)+
$ n7/(ssize/2)

d1 = abs(sm1-sn1)
\[ d_2 = \text{abs}(s_{m2} - s_{n2}) \]
\[ d_3 = \text{abs}(s_{m3} - s_{n3}) \]
\[ d_4 = \text{abs}(s_{m4} - s_{n4}) \]
\[ d_5 = \text{abs}(s_{m5} - s_{n5}) \]
\[ d_6 = \text{abs}(s_{m6} - s_{n6}) \]
\[ d_7 = \text{abs}(s_{m7} - s_{n7}) \]

\begin{verbatim}
c dmns = 0.0
if (ssize .gt. 50.0) then
   dmns = max(d1,d2,d3,d4,d5,d6,d7)
else
   $ (ssize/2)*(ssize/2)*$
   $ (max(d1,d2,d3,d4,d5,d6,d7))$
endif
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc

c Calculate ksd statistics
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
ksd = 0.0
if (ssize .eq. 20.0) then
   ksd = 70
endif
if (ssize .eq. 50.0) then
   ksd = 250
endif
if (ssize .eq. 50.0) then
   ksd = 1.36*(sqrt(ssize/((ssize/2)*(ssize/2))))
endif
if (dmns .ge. ksd) then
eerrct = eerrct + 1
endif
9999 continue

c aerrrate = aerrct / 1000.0
write (nout, 99991) aerrct, aerrrate
99991 format (' Mean: err count = ', i5,
   $ ' err rate = ', f10.4)

c berrrate = berrct / 1000.0
write (nout, 99992) berrct, berrrate
99992 format (' Man-Wh: err count = ', i5,
   $ ' err rate = ', f10.4)

c cerrrate = cerrct / 1000.0
write (nout, 99993) cerrct, cerrrate
99993 format (' F: err count = ', i5,
   $ ' err rate = ', f10.4)

c derrrate = derrct / 1000.0
write (nout, 99994) derrct, derrrate
99994 format (' Sq Rnk: err count = ', i5,
   $ ' err rate = ', f10.4)

c
\end{verbatim}
eerrrate = eerrct / 1000.0
write (nout, 99995) eerrct, eerrrate
format ('  K-S: err count = ', i5, 
\$ '   err rate = ', f10.4)
end
3. EXP3.F (Experiment 3)
Program name: EXP3.F
Programmer: Andrew H. Chen
Date: 01-22-1994

integer ido, index(50), ldpop, ldsamp
integer nvar, npop, nrow, nsamp
integer i, j, k
integer iseed, nout, nr
integer n, npass, m
integer aerrct, berrct, cerrct, derrct, eerrct
integer tie(14,1)
real a1, b1, a2, b2
real pt1, pt2, pt3, pt4, pt5, pt6, pt7
real ssize, tt
real p1(10000,1), p2(10000,1)
real samp1(50,1), samp2(50,1)
real samp(100,1)
real sgrank(100,1), absdevo(100,1), absdevn(100,1)
real aerrrate, berrrate, cerrrate, derrrate, eerrrate
real acil, acih, dcil, dcih
real sum1, sum2, sumdev1, sumdev2, mean1, mean2
real stddev1, stddev2
real lgr, sml, ff, fcrit, var1, var2
real sqroot, varp, varp1, varp2
real ranksum, ranktie(14,1)
real sumsqrk, rsqbar, rsqbarsz, sumr4
real srtl, t1crit
real rksum
real t1, t2, t3, t4, t5, t6, t7
real r1, r2, r3, r4, r5, r6, r7
real wx, tsum
real z, zl, z2, z3, z4
real ksd ,dmn
real m1, m2, m3, m4, m5, m6, m7
real n1, n2, n3, n4, n5, n6, n7
real sm1, sm2, sm3, sm4, sm5, sm6, sm7
real sn1, sn2, sn3, sn4, sn5, sn6, sn7
real d1, d2, d3, d4, d5, d6, d7
external rnsrs
external rnset, rnun, umach

Initialize all variables

call umach(2, nout)
a1 = 16
b1 = 2.5
a2 = 16
b2 = 2.5
ssize = 100.0
nrow = 10000
nvar = 1
ldpop = 10000
nsamp = ssize/2
ldsamp = ssize/2
ido = 0
nr = 10000
pt1 = -1.249
pt2 = -1.07
pt3 = -0.892
pt4 = 0.0
pt5 = 0.892
pt6 = 1.07
pt7 = 1.249

cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc

cc Generate a uniform population c
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
call rnset(iseed)
call rnun(nr, pi)
call sscal(nr, 100.00, pi, 1)
call sadd(nr, 0.00, pi, 1)

cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
cc Generate a normal population c
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
call rnset(iseed)
call rnnoa(nr, pi)
call sscal(nr, 10.00, pi, 1)
call sadd(nr, 40.00, pi, 1)

cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
cc Generate a gamma population c
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
call rnset(iseed)
call rngam(nr, al, p2)
call sscal(nr, bl, p2, 1)

cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
cc Select 1 sample from a uniform population c
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
j = 1
do 9999 j = 1, 1000
call rnsrs(ido, nrow, nvar, pt1, ldpop, nsamp, npop,
$   samp1, ldsamp, index)

cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
cc Select 1 sample from a normal population c
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
call rnsrs(ido, nrow, nvar, pt2, ldpop, nsamp, npop,
$   samp2, ldsamp, index)
cc
do 0200 i = 1, ssize/2
sbmp(i, 1) = samp1(i, 1)
0200 continue
do 0300 i = ssize/2 + 1, ssize
sbmp(i, 1) = samp2(i - ssize/2, 1)
0300 continue
Map the sample into one of seven-point likert-scale

```plaintext
i = 1
  t1 = 0.0
  t2 = 0.0
  t3 = 0.0
  t4 = 0.0
  t5 = 0.0
  t6 = 0.0
  t7 = 0.0
  m1 = 0.0
  m2 = 0.0
  m3 = 0.0
  m4 = 0.0
  m5 = 0.0
  m6 = 0.0
  m7 = 0.0

do 1000 i=1,sizeof/2
  if (samp(i,1) .lt. 14.29) then
    samp(i,1) = pt1
    t1 = t1 + 1
    m1 = m1 + 1
  elseif (samp(i,1) .lt. 28.57) then
    samp(i,1) = pt2
    t2 = t2 + 1
    m2 = m2 + 1
  elseif (samp(i,1) .lt. 42.86) then
    samp(i,1) = pt3
    t3 = t3 + 1
    m3 = m3 + 1
  elseif (samp(i,1) .lt. 57.14) then
    samp(i,1) = pt4
    t4 = t4 + 1
    m4 = m4 + 1
  elseif (samp(i,1) .lt. 71.43) then
    samp(i,1) = pt5
    t5 = t5 + 1
    m5 = m5 + 1
  elseif (samp(i,1) .lt. 85.71) then
    samp(i,1) = pt6
    t6 = t6 + 1
    m6 = m6 + 1
  else
    samp(i,1) = pt7
    t7 = t7 + 1
    m7 = m7 + 1
  endif
  1000 continue
```
n3 = 0.0
n4 = 0.0
n5 = 0.0
n6 = 0.0
n7 = 0.0

do 1001 i=(ssize/2) + 1, ssize
   if (samp(i,l) .lt. 14.29) then
      samp(i,l) = pt1
      t1 = t1 + 1
      n1 = n1 + 1
   elseif (samp(i,l) .lt. 28.57) then
      samp(i,l) = pt2
      t2 = t2 + 1
      n2 = n2 + 1
   elseif (samp(i,l) .lt. 42.86) then
      samp(i,l) = pt3
      t3 = t3 + 1
      n3 = n3 + 1
   elseif (samp(i,l) .lt. 57.14) then
      samp(i,l) = pt4
      t4 = t4 + 1
      n4 = n4 + 1
   elseif (samp(i,l) .lt. 71.43) then
      samp(i,l) = pt5
      t5 = t5 + 1
      n5 = n5 + 1
   elseif (samp(i,l) .lt. 85.71) then
      samp(i,l) = pt6
      t6 = t6 + 1
      n6 = n6 + 1
   else
      samp(i,l) = pt7
      t7 = t7 + 1
      n7 = n7 + 1
   endif
1001 continue

cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc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do 1100 i=1,ssize/2
  sum1 = sum1 + samp(i,1)
1100 continue

do 1200 i=ssize/2 + 1, ssize
  sum2 = sum2 + samp(i,1)
1200 continue

mean1 = sum1 / (ssize/2)
do 1300 i=1,ssize/2
  sumdev1 = sumdev1 + (samp(i,1) - mean1)**2
1300 continue

mean2 = sum2 / (ssize/2)
do 1400 i=ssize/2 + 1, ssize
  sumdev2 = sumdev2 + (samp(i,1) - mean2)**2
1400 continue

stddev1 = sqrt(sumdev1 / (ssize/2 - 1))
stddev2 = sqrt(sumdev2 / (ssize/2 - 1))

c

varpl = ((ssize/2 - 1)*stddev1**2) +
        ((ssize/2 - 1)*stddev2**2)

varp = ssize - 2
sqroot = (varp/(ssize/2)) + (varp/(ssize/2))

if (ssize .eq. 20.0) then
  tt = 2.101
endif

if (ssize .eq. 50.0) then
  tt = 2.021
endif

if (ssize .gt. 50.0) then
  tt = 1.96
endif

acil = (mean1 - mean2) - tt*(sqrt(sqroot))
acih = (mean1 - mean2) + tt*(sqrt(sqroot))
if (acil .gt. 0) then
  aerrct = aerrct + 1
endif
if (acih .lt. 0) then
  aerrct = aerrct + 1
endif

ccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
Wilcoxon-Mann-Whitney Test
ccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc

r1 = 0.0
r2 = 0.0
r3 = 0.0
r4 = 0.0
r5 = 0.0
r6 = 0.0
r7 = 0.0
rksum = 0.0
if (t1 .gt. 0 ) then
    do 2010 i=1, t1
        rksum = rksum + i
    continue
    r1 = rksum / t1
endif
rksum = 0.0
if (t2 .gt. 0 ) then
    do 2020 i=t1 + 1, t1 + t2
        rksum = rksum + i
    continue
    r2 = rksum / t2
endif
rksum = 0.0
if (t3 .gt. 0 ) then
    do 2030 i=t1 + t2 + 1, t1 + t2 + t3
        rksum = rksum + i
    continue
    r3 = rksum / t3
endif
rksum = 0.0
if (t4 .gt. 0 ) then
    do 2040 i=t1 + t2 + t3 + 1, t1 + t2 + t3 + t4
        rksum = rksum + i
    continue
    r4 = rksum / t4
endif
rksum = 0.0
if (t5 .gt. 0 ) then
    do 2050 i=t1 + t2 + t3 + t4 + 1, t1 + t2 + t3 + t4 + t5
        rksum = rksum + i
    continue
    r5 = rksum / t5
endif
rksum = 0.0
if (t6 .gt. 0 ) then
    do 2060 i=t1 + t2 + t3 + t4 + t5 + 1, t1 + t2 + t3 + t4 + t5 + t6
        rksum = rksum + i
    continue
    r6 = rksum / t6
endif
rksum = 0.0
if (t7 .gt. 0 ) then
do 2070 i=tl + t2 + t3 + t4 + t5 + t6 + 1,
  $t1 + t2 + t3 + t4 + t5 + t6 + t7$
  rksum = rksum + i
2070 continue
  r7 = rksum / t7
endif

Find wx

wx = 0.0
do 2100 i=1, ssize/2
  if (samp(i,1) .eq. pt1) then
    wx = wx + r1
  elseif (samp(i,1) .eq. pt2) then
    wx = wx + r2
  elseif (samp(i,1) .eq. pt3) then
    wx = wx + r3
  elseif (samp(i,1) .eq. pt4) then
    wx = wx + r4
  elseif (samp(i,1) .eq. pt5) then
    wx = wx + r5
  elseif (samp(i,1) .eq. pt6) then
    wx = wx + r6
  else
    wx = wx + r7
  endif
2100 continue

c Calculate z statistics

z1 = 0.0
z2 = 0.0
z3 = 0.0
z4 = 0.0
z = 0.0
z1 = wx - (ssize/2)*(ssize + 1)/2
z3 = (ssize/2)**2/(ssize*(ssize-1))
z4 = (ssize**3-ssize)/12-tsum
z2 = sqrt(z3*z4)
z = z1/z2
if ((z .gt. 1.96) .or. (z .lt. -1.96)) then
  berrct = berrct + 1
endif

F Test

var1 = 0.0
var2 = 0.0
\[ \text{lgr} = 0.0 \\
\text{sml} = 0.0 \\
\text{ff} = 0.0 \\
\text{fcrit} = 0.0 \]

\[ \text{varl} = \text{stddevl}^2 \\
\text{var2} = \text{stddev2}^2 \\
\text{lgr} = \max(\text{varl}, \text{var2}) \\
\text{sml} = \min(\text{varl}, \text{var2}) \\
\text{ff} = \frac{\text{lgr}}{\text{sml}} \]

\[
\text{if (ssize .eq. 20.0) then} \\
\text{fcrit} = 4.03 \\
\text{endif} \\
\text{if (ssize .eq. 50.0) then} \\
\text{fcrit} = 2.27 \\
\text{endif} \\
\text{if (ssize .eq. 100.0) then} \\
\text{fcrit} = 1.78 \\
\text{endif} \\
\text{if (ssize .eq. 200.0) then} \\
\text{fcrit} = 1.51 \\
\text{endif} \\
\text{if (ff .gt. fcrit) then} \\
\text{cerrct} = \text{cerrct} + 1 \\
\text{endif} \]

c

Squared Ranks Test

do 4000 i=1, ssize/2 \\
\text{absdevo}(i, 1) = \text{abs} (\text{samp}(i, 1) - \text{mean1}) \\
\text{absdevn}(i, 1) = \text{abs} (\text{samp}(i, 1) - \text{mean1}) \\
4000 continue \\
do 4100 \text{i}=\text{ssize/2} + 1, \text{ssize} \\
\text{absdevo}(i, 1) = \text{abs} (\text{samp}(i, 1) - \text{mean2}) \\
\text{absdevn}(i, 1) = \text{abs} (\text{samp}(i, 1) - \text{mean2}) \\
4100 continue \\
do 4200 \text{npass} = 1, \text{ssize} - 1 \\
do 4210 \text{n} = 1, \text{ssize} - \text{npass} \\
\text{if (absdevn}(n, 1) \cdot \text{gt. absdevn}(n + 1, 1)) \text{ then} \\
\text{temp} = \text{absdevn}(n, 1) \\
\text{absdevn}(n, 1) = \text{absdevn}(n + 1, 1) \\
\text{absdevn}(n + 1, 1) = \text{temp} \\
\text{endif} \\
4210 continue \\
4200 continue \\
c

k = 1 \\
tie(k, 1) = 1 \\
ranksum = 1 \\
do 4300 i=1, ssize - 1 \\
\text{if (absdevn}(i, 1) \cdot \text{ne. absdevn}(i+1, 1)) \text{ then} \\
\text{if (i .eq. 1) then} \\
\text{ranktie}(1, 1) = \text{ranksum} / i \\
\text{endif} \\
4300 continue
else
  ranktie(k,1) = (ranksum / tie(k,1))**2
endif

do 4400  l = 1,ssize
  if (absdevo(l,1) .eq. absdevn(i,1)) then
    sqrank(l,1) = ranktie(k,1)
  endif
4400 continue
ranksum = i + 1
k = k + 1
tie(k,1) = 1
if (i .eq. (ssize - 1)) then
  ranktie(k,1) = (ranksum / tie(k,1))**2
  do 4500  l = 1,ssize
    if (absdevo(l,1) .eq. absdevn(i+l,1)) then
      sqrank(l,1) = ranktie(k,1)
    endif
 4500 continue
  ranktie(k,l) = (ranksum / tie(k,l))**2
  endif
else
  tie(k,1) = tie(k,1) + 1
  ranksum = ranksum + i + 1
  if (i .eq. (ssize - 1)) then
    ranktie(k,1) = (ranksum / tie(k,1))**2
    do 4600  l = 1,ssize
      if (absdevo(l,1) .eq. absdevn(i+1,l)) then
        sqrank(l,1) = ranktie(k,1)
      endif
    4600 continue
  endif
endif
4300 continue
sumsqrk = 0.0
rsqbar = 0.0
sumr4 = 0.0
do 4700  m = l,ssize/2
  sumsqrk = sumsqrk + sqrank(m,l)
4700 continue
do 4800  m = 1,ssize
  rsqbar = rsqbar + sqrank(m,l)
sumr4 = sumr4 + (sqrank(m,1))**2
4800 continue
rsqbarsz = rsqbar / ssize
srtl = ((sumsqrk - $(ssize/2)*rsqbarsz)/(sqrt((((ssize/2)**2)/$ (ssize**2))**2)/(ssize-1)* $(ssize*(ssize-1))*sumr4-((ssize/2)**2)/(ssize-1)* $(rsqbarsz**2))))
tlcrit = 0.0
if (ssize .eq. 20.0) then
  tlcrit = 2.101
endif
if (ssize .eq. 50.0) then
    tlcrit = 2.011
endif
if (ssize .gt. 50.0) then
    tlcrit = 1.96
endif
dcil = -tlcrit
dcih = tlcrit
if (dcil .gt. srtl) then
derrct = derrct + 1
endif
if (dcih .lt. srtl) then
derrct = derrct + 1
endif
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
ccc Kolmogorov-Smirnov Test c
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
sm1 = m1/(ssize/2)
sm2 = m1/(ssize/2)+m2/(ssize/2)
sm3 = m1/(ssize/2)+m2/(ssize/2)+m3/(ssize/2)
sm4 = 
\$ m1/(ssize/2)+m2/(ssize/2)+m3/(ssize/2)+m4/(ssize/2) 
\$ sm5 = m1/(ssize/2)+m2/(ssize/2)+m3/(ssize/2)+
\$ m4/(ssize/2)+m5/(ssize/2) 
\$ sm6 = m1/(ssize/2)+m2/(ssize/2)+m3/(ssize/2)+
\$ m4/(ssize/2)+m5/(ssize/2)+m6/(ssize/2) 
\$ sm7 = m1/(ssize/2)+m2/(ssize/2)+m3/(ssize/2)+
\$ m4/(ssize/2)+m5/(ssize/2)+m6/(ssize/2)+m7/(ssize/2)
sm8 = 
\$ n1/(ssize/2)+n2/(ssize/2)+n3/(ssize/2)+n4/(ssize/2) 
\$ sn5 = n1/(ssize/2)+n2/(ssize/2)+n3/(ssize/2)+
\$ n4/(ssize/2)+n5/(ssize/2) 
\$ sn6 = n1/(ssize/2)+n2/(ssize/2)+n3/(ssize/2)+
\$ n4/(ssize/2)+n5/(ssize/2)+n6/(ssize/2) 
\$ sn7 = n1/(ssize/2)+n2/(ssize/2)+n3/(ssize/2)+
\$ n4/(ssize/2)+n5/(ssize/2)+n6/(ssize/2)+n7/(ssize/2) c
d1 = abs(sm1-sn1)
d2 = abs(sm2-sn2)
d3 = abs(sm3-sn3)
d4 = abs(sm4-sn4)
d5 = abs(sm5-sn5)
d6 = abs(sm6-sn6)
d7 = abs(sm7-sn7)
c
dmn = 0.0
if (ssize .gt. 50.0) then
    dmn = max(d1,d2,d3,d4,d5,d6,d7)
else
   dmn =
   $(ssize/2)*$(ssize/2)*
   $(max(d1,d2,d3,d4,d5,d6,d7))
endif

Calculate ksd statistics

ksd = 0.0
if (ssize .eq. 20.0) then
   ksd = 70
endif
if (ssize .eq. 50.0) then
   ksd = 250
endif
if (ssize .gt. 50.0) then
   ksd = 1.36*(sqrt(ssize/((ssize/2)*(ssize/2))))
endif
if (dmn .lt. ksd) then
   errct = errct + 1
endif
continue

Display error rate

aerrrate = errct / 1000.0
write (nout, 99991) errct, aerrrate
99991 format (' Mean: err count = ', i5,
   '  Ierr rate = ', f10.4)

berrrate = berrct / 1000.0
write (nout, 99992) berrct, berrrate
99992 format (' Man-Wh: err count = ', i5,
   '  Ierr rate = ', f10.4)

cerrrate = cerrct / 1000.0
write (nout, 99993) cerrct, cerrrate
99993 format (' F: err count = ', i5,
   '  Ierr rate = ', f10.4)

derrrate = derrct / 1000.0
write (nout, 99994) derrct, derrrate
99994 format (' Sq Rnk: err count = ', i5,
   '  Ierr rate = ', f10.4)

eerrrate = errct / 1000.0
write (nout, 99995) errct, eerrrate
99995 format (' K-S: err count = ', i5,
   '  Ierr rate = ', f10.4)
End of program
REFERENCES


