

FILE COPY
~~NO. 7~~

FILE COPY
NO. 1-W

CASE FILE
COPY

TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 545

THE BALANCE OF MOMENTS AND THE
STATIC LONGITUDINAL STABILITY OF AIRPLANES

By Horst Müller

From Zeitschrift für Flugtechnik und Motorluftschiffahrt
January 28, 1929

Washington
December, 1929

FILE COPY

To be returned to
the files of the National
Advisory Committee
for Aeronautics
Washington, D. C.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

TECHNICAL MEMORANDUM NO. 545.

THE BALANCE OF MOMENTS AND THE
STATIC LONGITUDINAL STABILITY OF AIRPLANES.*

By Horst Müller.

Summary

A nomogram is developed which renders it possible by drawing a few lines, to determine:

The location of the center of gravity for zero wing and tail moments;

The longitudinal dihedral angle;

The tail coefficient F_{hl}/F_t .

Moreover, there is no difficulty in determining the magnitude of the restoring moment or of the unstable moment!

Example: Location of the Center of Gravity

We plot the curve of the moments with reference to the leading edge of the wing and draw the line OA (Fig. 1) from the origin through the point on the moment curve (angle of attack α , lift coefficient c_a) for which the wing and tail moments are

*"Beitrag zum Momentenausgleich und zur statischen Langs stabilität der Flugzeuge," from Zeitschrift für Flugtechnik und Motorluftschiffahrt, January 28, 1929, pp. 45-50.

zero and read the required location of the C.G. (here $\frac{x}{l} = 0.44$)* at the intersection B with the horizontal line $c_a = 1$ in the scale of the moment curve.

Longitudinal Dihedral Angle

From A we draw a horizontal line to the axis of the ordinates at C and another line from C, through D on the oblique line corresponding to the aspect ratio of the wing, to the vertical line on the left, on which we read the downwash angle at E ($\Delta\alpha = -2.9^\circ$).

The angle of attack of the wing at which the wing moment vanishes, increased by the downwash angle, gives the theoretical longitudinal dihedral angle (Fig. 2, a).

$$\sigma_{\text{theor}} = \alpha_t + \Delta\alpha (= 0 - 2.9 = -2.9^\circ)$$

The angle of attack of the tail is then

$$\alpha_h \text{ theor} = \alpha_t + \Delta\alpha - \sigma_{\text{theor.}}$$

$$(= 0 - 2.9 + 2.9 = 0^\circ)$$

at which the lift coefficient and the tail moment vanish.

If the tail has a symmetrical profile for which the theoretical angle of attack and that of the middle line is the same, the theoretical longitudinal dihedral angle is then equal to the actual angle. If the tail has a cambered profile, however, the

*This line of reasoning is used by Professor Everling in his exercises on airplane calculations, but has not yet been published in any other connection.

angle of difference δ (< 0) between the theoretical line of zero lift and the profile chord must then be added to the theoretical longitudinal dihedral, if the camber is on the lower side, or subtracted, if the camber is on the upper side, in order to obtain the actual longitudinal dihedral (Fig. 2, b and c).

If we use, for example, the Göttingen profile 595 (Fig. 3) with the camber up, it would have to be set at an angle of $\alpha_h \equiv \delta = -4.5^\circ$ to the air flow, in order to reduce the lift coefficient to zero. The air flow strikes it, however, at an angle of -2.9° with respect to the wing chord. The tail chord must therefore be inclined downward, i.e., in the positive direction (Fig. 2, b), in order to make the angle of attack -4.5° for the tail. The actual longitudinal dihedral is therefore

$$\sigma = \sigma_{\text{theor}} - \delta$$
$$(\quad = -2.9 + 4.5 = +1.6^\circ).$$

If the profile is inverted, it will be struck by the air flow at the angle 0° with respect to the tail chord, provided the actual longitudinal dihedral angle equals the theoretical angle. As shown by the polar (Fig. 3), the zero angle of attack corresponds to a lift coefficient of $c_a \approx 0.30$ which, due to the inversion of the profile, means a downward force and therefore a pitching moment. In order to eliminate this moment, the tail must be turned in the direction of the hands of a clock until it is again struck by the air flow at the angle

$\alpha_h \equiv \delta = -4.5^\circ$ with respect to the tail chord (Fig. 2,c).

Hence the actual longitudinal dihedral is

$$\sigma = \sigma_{\text{theor}} + \delta (= -2.9^\circ - 4.5^\circ = -7.4^\circ).$$

The angles of attack of the tail (See accompanying table) are therefore:

For a symmetrical profile,

$$\begin{aligned}\alpha_h &= \alpha_h \text{ theor} \\ &= \alpha_t + \Delta\alpha - \sigma_{\text{theor}} \\ &= \alpha_t + \Delta\alpha - \sigma,\end{aligned}$$

For an upward cambered profile,

$$\begin{aligned}\alpha_h &= \alpha_h \text{ theor} + \delta \\ &= \alpha_t + \Delta\alpha - \sigma_{\text{theor}} + \delta \\ &= \alpha_t + \Delta\alpha - \sigma,\end{aligned}$$

For a downward cambered profile,

$$\begin{aligned}\alpha_h &= - [\alpha_h \text{ theor} - \delta] \\ &= - [\alpha_t + \Delta\alpha - \sigma_{\text{theor}} - \delta] \\ &= - [\alpha_t + \Delta\alpha - \sigma]\end{aligned}$$

In the last case the minus sign before the parentheses serves for the transition of the notation of the angle from the airplane to the customary notation for the wing profile.

Tail Coefficient

In order to find the tail coefficient F_{hl}/F_t (the downwash being $\Delta\alpha = 0^\circ$), the longitudinal dihedral angle is subtracted from the angle of attack ($\alpha_t = -7^\circ$) in Figure 1 corresponding to the vanishing lift, and thus the angle of attack of the tail is found (See table).

$$\alpha_{ho} = \alpha_{to} - \sigma.$$

Angle of attack of wing	Downwash angle	Angle of attack of tail for symmetrical profile
α_t	$\Delta\alpha$	$\alpha_{h\text{ theor}} = \alpha_h$ $= \alpha_t + \Delta\alpha - \sigma_{\text{theor}}$ $\sigma_{\text{theor}} = \sigma = -2.9^\circ$
+ 6	- 5.5	+ 3.4
+ 3	- 4.2	+ 1.7
+ 0	- 2.9	+ 0.0
- 3	- 1.6	- 1.7
- 7	± 0.0	- 4.1

Angle of attack of wing	Lift coefficient of tail	Angle of attack of tail for profile cambered	
		Upward	Downward
α_t	c_{ah}	$\alpha_h = \alpha_t + \Delta\alpha - \sigma$ $\sigma = \sigma_{\text{theor}} - \delta = +1.6^\circ$ $\delta = -4.5^\circ$	$\alpha_h = -[\alpha_t + \Delta\alpha - \sigma]$ $\sigma = \sigma_{\text{theor}} + \delta = -7.4^\circ$
+ 6	+ 0.24	- 1.1	- 7.9
+ 3	+ 0.12	- 2.8	- 6.2
+ 0	± 0.00	- 4.5	- 4.5
- 3	- 0.12	- 6.2	- 2.8
- 7	- 0.29	- 8.6	- 0.4

At this angle the lift coefficient of the tail is derived from its polar, and this value (point H in Fig. 1) is connected with the intersection point F of the rectilinearly prolonged moment curve on the axis of the abscissas, and the value of the tail coefficient $F_h l / F_t$ is read on the oblique line. Here $F_h l / F_t = 0.33$ (Fig. 1, point I).

If the tail polar is not opposite the aspect ratio of the tail, we proceed from the intersection point G of the angle of attack of the tail, here -4.1° (Fig. 1), with the aspect ratio (here 5) vertically upward to the corresponding lift coefficient c_{ah} (point H) and then continue as above.

If the tail is in the slipstream, the dynamic pressure is greater on the tail than on the wing and therefore tends to increase the stability. A similar result is obtained by increasing the tail coefficient. The slope of the resultant moment coefficient toward the lift ordinate serves as a measure of the stability.

Resultant Moment

In order to find the resultant moment, we draw through each point on the moment curve (Fig. 1, points K and T) the parallel to OAB of the travel of the C.G. and obtain on the moment axis the value of the wing moment with respect to the C.G. in OR and OZ.

The downwash angle is then determined as above (lines LDM

and UDV) and also the angle of attack of the tail, with due allowance for the previously determined longitudinal dihedral (See table)

$$\alpha_h = \alpha_t + \Delta\alpha - \sigma.$$

For this purpose we determine the lift coefficient for the corresponding aspect ratio and connect this point with the tail coefficient, here 0.40 (Fig. 1), and extend the line till it meets the moment axis (lines NDPQ and WXPY). OQ and OY then give the value of the moment coefficient of the tail with reference to the center of gravity c_{ah} ($F_h l / F_t$). The tail moment is positive when the theoretical angle of attack is positive and vice versa. The difference between the wing moment coefficient (OR and OZ) and the tail moment coefficient (OQ and OY) is the resultant moment coefficient $c_{m\ tot} = M_{tot} / Fqt$ with reference to the C.G. and is plotted at the height of the corresponding wing lift (LS and UZ'). The moment curve is thus determined point by point. If it runs toward the right, it indicates stability; toward the left, instability.

Theoretical Principles

The essential requirement is for the wing moment to be balanced by the tail moment. The wing moment about the C.G. is

$$M_S = N (e - x) - T y$$

where the normal force $N = c_n F q$

and the tangential force $T = c_t F q$.

Then the wing moment about the C.G. is

$$M_s = \left[c_n \left(\frac{e}{t} - \frac{x}{t} \right) - c_t \frac{y}{t} \right] F q t.$$

If the coefficient of the moment about the leading edge of the wing is designated by $\overline{c_m}$, we have by definition

$$\begin{aligned} \overline{c_m} F q t &= c_n F q e \\ \frac{e}{t} &= \frac{\overline{c_m}}{c_n}. \end{aligned}$$

If we put $c_n \approx c_a$ and disregard the moment of the tangential force, then the moment of the wing about the C.G. is

$$M_s = \left(\overline{c_m} - c_a \frac{x}{t} \right) F q t$$

The tail moment with respect to the C.G. (Fig. 5) is

$$M_h = - N_h l = - c_{nh} F_h q_h l.$$

Hence the requirement of the moment balance is

$$M_{tot} = 0 = M_s + M_h$$

or the equivalent in nondimensional coefficients

$$\overline{c_m} - c_a \frac{x}{t} = c_{nh} \frac{F_h l q_h}{F t q} \approx c_{ah} \frac{F_h l}{F t}$$

If the wing moment with respect to the C.G. is to vanish at a given angle of attack or (what amounts to the same thing) at a given lift, we must have

$$\overline{c_m} - c_a \frac{x}{t} = 0.$$

Then, however, the tail moment must also disappear if the moments are to balance. The theoretical angle of attack of the tail (Fig. 2)

$$\alpha_{h \text{ theor}} = \alpha_t + \Delta\alpha - \sigma_{\text{theor}}$$

must also disappear and the theoretical longitudinal dihedral become

$$\sigma_{\text{theor}} = \alpha_t + \Delta\alpha$$

If the line of the moments with respect to the leading edge of the wing is extended to the axis of the abscissas and if this value $\overline{c_m}$ is called the neutral value, we obtain

$$\overline{c_{m0}} = c_{ah0} \frac{F_h l}{F t},$$

since, in normal flight outside $\overline{c_m}$ c_{ah} is directly proportional to the wing lift. From this follows the tail coefficient

$$\frac{F_h l}{F t} = \frac{\overline{c_{m0}}}{c_{ah0}}$$

Stability exists when

$$\frac{d c_m \text{ tot}}{d \alpha} > 0$$

or, since c_a increases in direct proportion with α , when

$$\frac{d c_m \text{ tot}}{d c_a} > 0$$

Nomogram Expedients

Downwash.— As already mentioned, the angle of attack of the tail is related to that of the wing by the equation

$$\alpha_h = \alpha_t + \Delta\alpha - \sigma$$

where $\Delta\alpha$ is the downwash angle

and σ is the longitudinal dihedral angle. For the downwash the rectangular lift distribution* gives

$$\Delta\alpha = -\frac{c_a}{\pi} 57.3 \frac{t}{b} \left[1 + \sqrt{1 + \left(\frac{b}{2l}\right)^2} \right]$$

and the elliptical lift distribution*

$$\Delta\alpha = -2 \frac{c_a}{\pi} 57.3 \frac{F}{b^2} \left[1 + \frac{1}{4} \left(\frac{b}{2l}\right)^2 \right]$$

Both represent special cases, between which Helmholtz**, by a thorough investigation of the structure of a vortical trail, found

$$\Delta\alpha = -\frac{c_a}{\pi} \frac{F}{b^2} 57.3 \left[0.812 + \frac{0.812 \frac{2l}{b}}{\sqrt{\left(\frac{2l}{b}\right)^2 + 0.615}} + \frac{0.5}{\frac{2l}{b} \sqrt{\left(\frac{2l}{b}\right)^2 + 1}} \right]$$

These values agree well with the approximation formula

$$\Delta\alpha = -1.6 \frac{c_a}{\pi} \frac{F}{b^2} 57.3$$

determined experimentally by Munk and Cario.***

If we put $l \approx \frac{1}{2} b$ and $F/b^2 = \lambda_s = 1/5$, we obtain for any aspect ratio with rectangular lift distribution,

$$\Delta\alpha = -4.4 c_a \frac{\lambda_x}{\lambda_s}$$

*Fuchs and Hopf, "Aerodynamics," Handbuch der Flugzeugkunde, Vol. II, p.325.

**Helmholtz, "Über die Berechnung des Abwindes hinter einem Rechteckigen Tragflügel," Z.F.M., August 14, 1925, pp.291-294.

***Munk and Cario, "Luftstromneigung hinter Flügeln," Technische Berichte, Report III, pp.10-15.

and, with elliptical lift distribution,

$$\Delta \alpha = - 9.1 c_a \frac{\lambda_x}{\lambda_s}$$

According to Helmbold,

$$\Delta \alpha = - 6.6 c_a \frac{\lambda_x}{\lambda_s}$$

According to Munk and Cario,

$$\Delta \alpha = - 5.8 c_a \frac{\lambda_x}{\lambda_s} .$$

These values are represented in Figure 6 for an aspect ratio of $\lambda_s = 1/5$. The values obtained with the Helmbold formula were used for the nomogram.

Lift coefficient of tail.— This increases in the region of moment balance in direct proportion to the angle of attack.

Hence

$$c_{ah} = \frac{d c_a}{d \alpha} \alpha_h \text{ theor.}$$

The increase in the lift coefficient with the angle of attack depends on the aspect ratio.

$$\frac{d c_a}{d \alpha_x} = \frac{d c_a}{d \alpha_s} \frac{d \alpha_s}{d \alpha_x}$$

in which the subscripts s and x give the aspect ratio. According to the Betz method we have

$$\hat{\alpha}_x = \hat{\alpha}_s - \frac{c_a}{\pi} (\lambda_s - \lambda_x)$$

Hence

$$\frac{d \hat{\alpha}_x}{d \hat{\alpha}_s} = 1 - \frac{d c_a}{d \hat{\alpha}_s} \frac{1}{\pi} (\lambda_s - \lambda_x).$$

As demonstrated by the results obtained in the laboratories of Germany and other countries, dc_a/dc is nearly independent of the Reynolds Number and also of the camber and thickness of the airfoil. It was found that

$$\frac{d c_a}{d \alpha_0} = 0.075,$$

hence

$$\frac{d c_a}{d \alpha_x} = \frac{0.0548}{0.562 + \lambda_x}.$$

The lift diagram at the bottom of Figure 1 was constructed according to this equation. The Z diagram for finding the downwash angle, like the diagram for finding the tail coefficient, is based on the geometrical relations of the sides of similar triangles:

$$a : b = c : d.$$

Stability of biplanes.— To determine the stability of a biplane, the moment coefficient with respect to the C.G. is first calculated in the usual manner, with allowance for the mutual interference of the upper and lower wings, and then plotted in the nomogram. The longitudinal dihedral angle tail coefficient and resultant moment coefficient can then be determined in the same way as for a monoplane.

Translation by
National Advisory Committee
for Aeronautics.

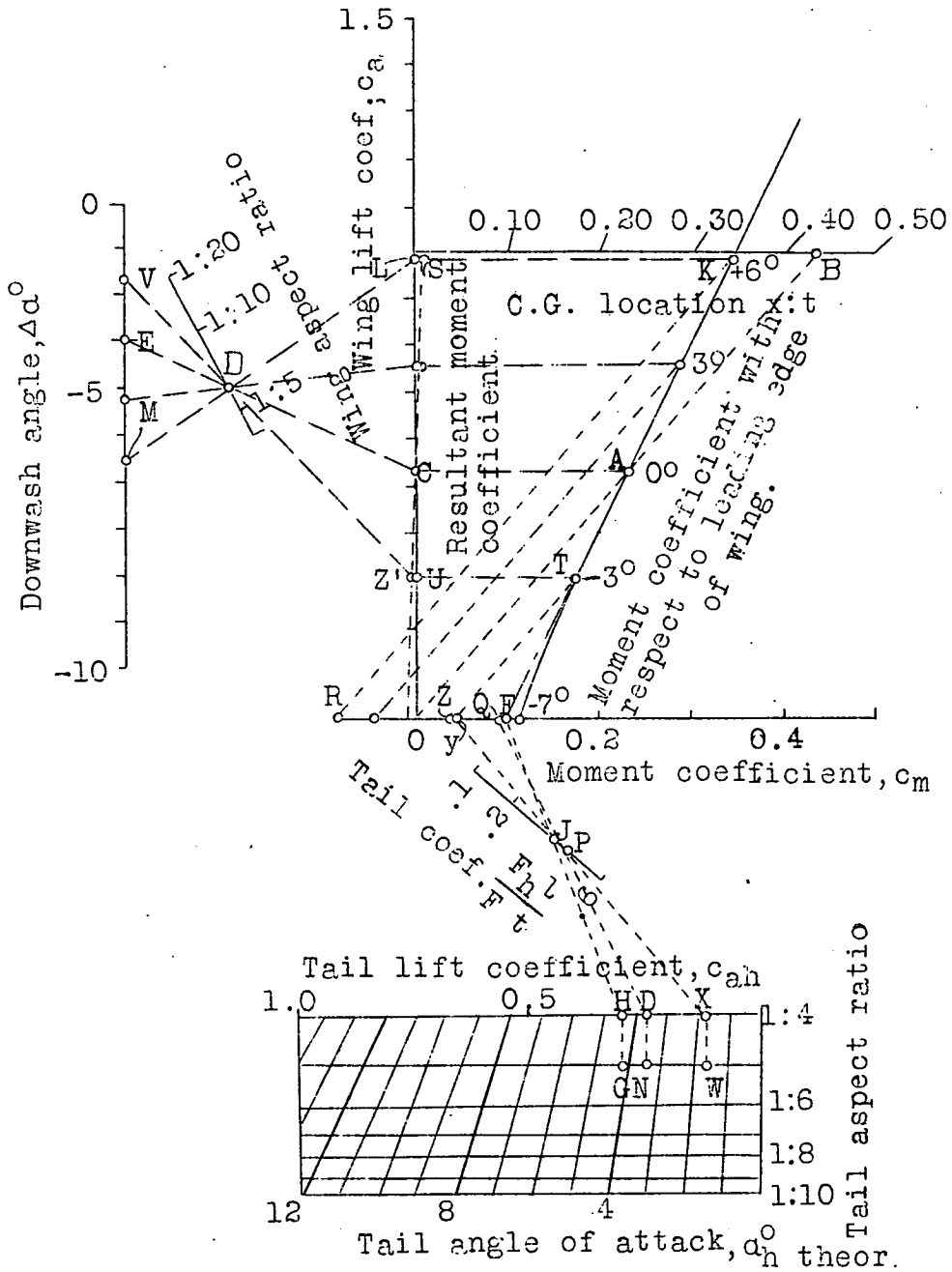
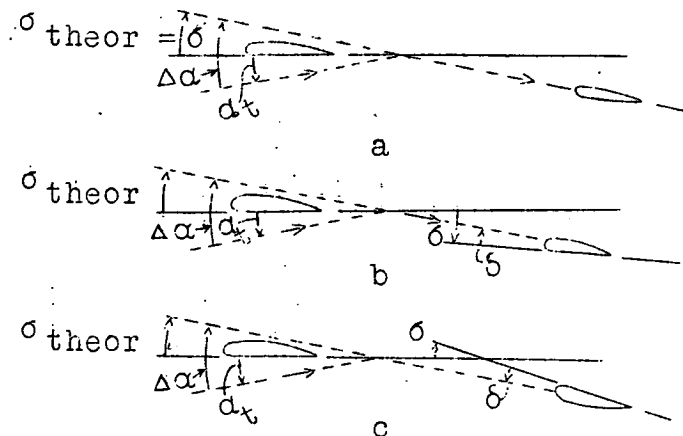


Fig. 1 Examination in stability determination.



- (a) With symmetrical tail profile.
- (b) " upward cambered tail profile.
- (c) " downward " " "

Fig. 2 Theoretical and actual longitudinal dihedral.

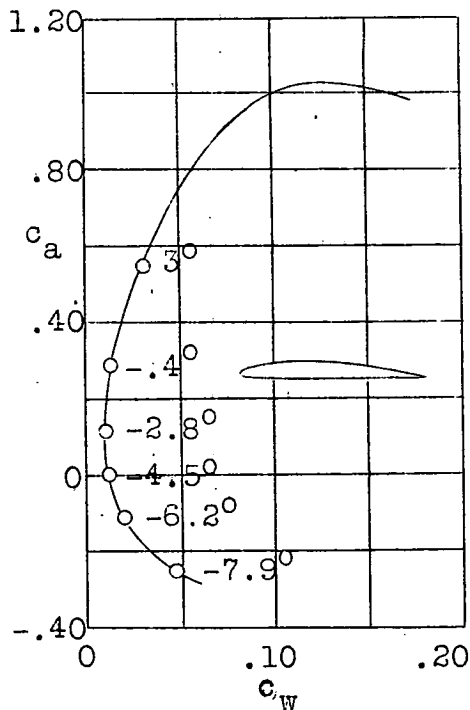


Fig. 3 Polar of Gottingen profile 595.

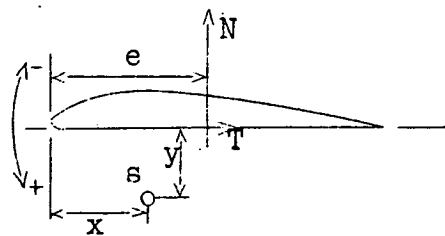


Fig. 4 Wing moments.

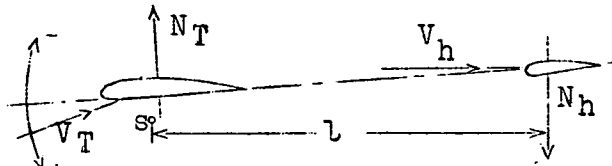


Fig.5 Tail moment.

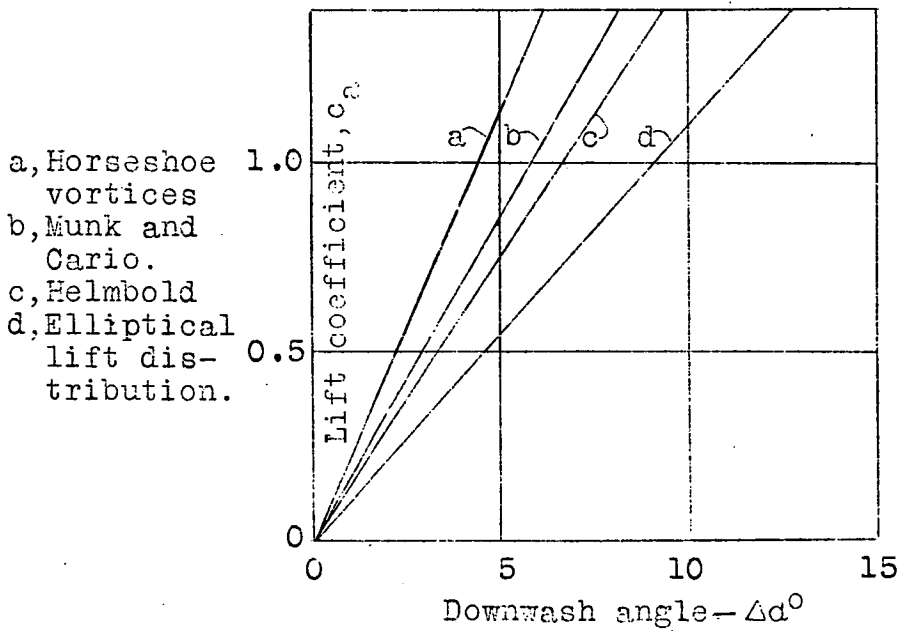


Fig.6 Downwash for wing aspect ratio, $\lambda_5=1:5$

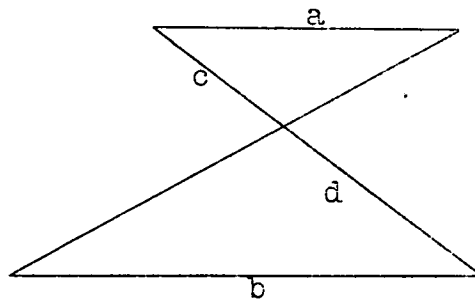


Fig.7 Principle of Z diagram.

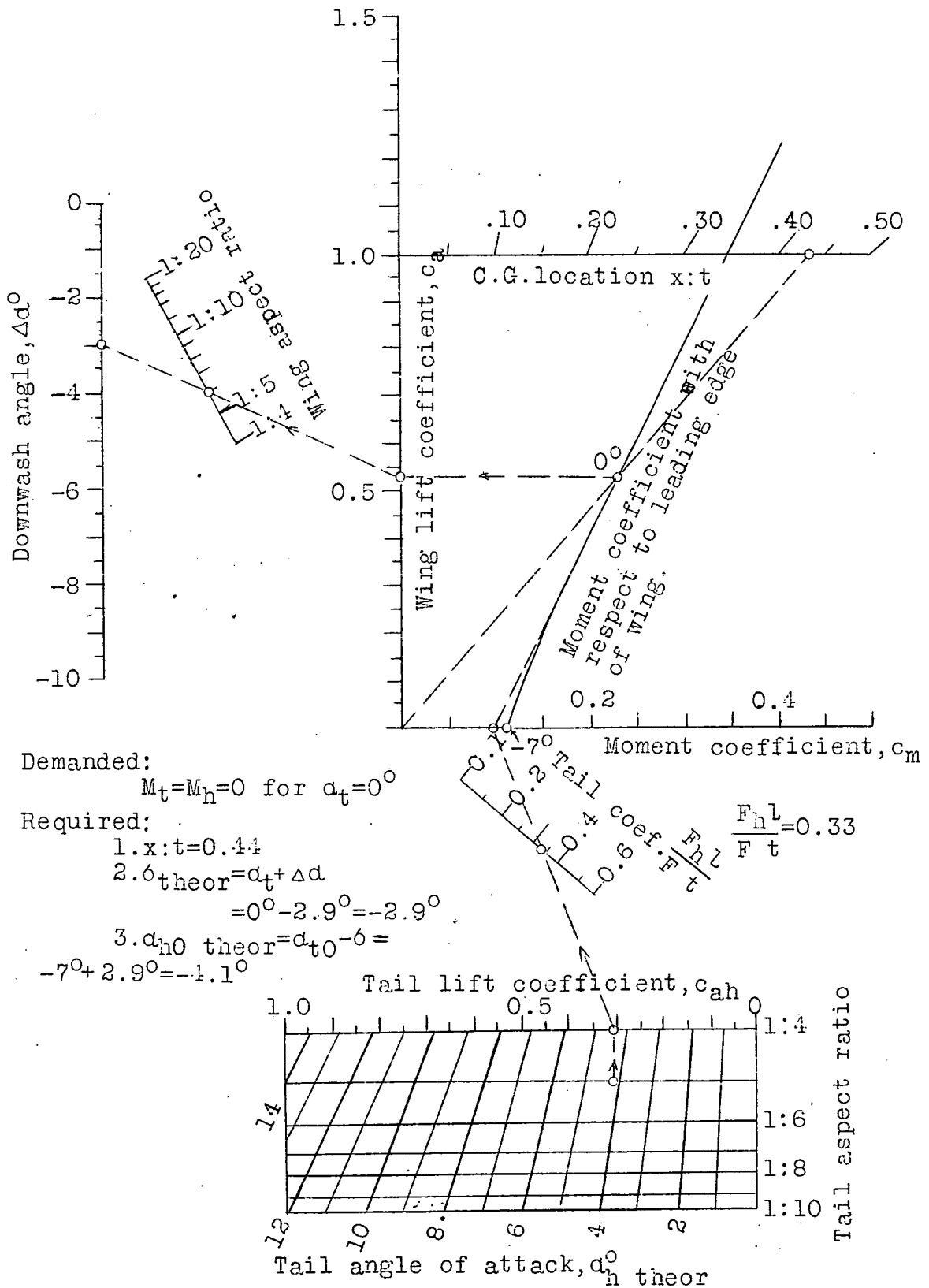


Fig.8 Stability nomogram.