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MATHEMATICAL TREATISE ON THE RECOVERY FROM A FLAT SPIN

By R. Fuchs

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In this mathematical investigation, made in collaboration with Dr. Wilhelm Schmidt, we interpret the temporary change (due to some disturbance) in the quantities which define the position of the airplane while in a flat spin. We further examine the effect on this change, of the means resorted to to produce the disturbance, and thus reveal the expedients available for recovering from a flat spin.

1. Forces and Moments on an Airplane in a Spin

Steady Spin without Side Wind

All forces and moments depend in principle on angle of attack, angle of yaw, and total rotation. No force or moment measurements have yet been made for the case of large angle of yaw and an arbitrary total rotation; and, lacking the necessary wind-tunnel data for it, any calculation must be more or less problematical. For that reason, any general airplane movement, such as going into or coming out of a spin, can be followed only approximately. The more exact mathematical determination of airplane motion must always be confined, at least for the

*"Rechnerische Ergebnisse über Störung des gefährlichen Trudelzustandes." From Jahrbuch 1929 der Wissenschaftlichen Gesellschaft für Luftfahrt, p. 144-148.

present, to cases with small angle of yaw.

In a steady spin without side wind the total rotation is essentially a rotation about the path axis, so that all forces and moments depend principally on the angle of attack and on the rotation about the path axis. This can either be determined in the wind tunnel or mathematically. It is likewise possible to define the respective values for the angle of attack and the gliding angle at which the forces and moments balance about a body axis. Thus, Figure 1 shows the gliding angle φ plotted against angle of attack α, with curves N, L, and K for equalization about the spar, strut, and fuselage axis, respectively. All forces and moments applying at the airplane are balanced. The angle of attack for flat spin ranges between $60^\circ - 70^\circ$, the gliding angle between -80° to -90°. (Compare Richard Fuchs and Wilh. Schmidt: Stationärer Trudelflug, Luftfahrtforschung, Vol. III, No. 1, February 27, 1929.)

2. Effect of Side Wind on a Flat Spin

Whereas the balance of the forces and moments about the spar and strut axis is almost unaffected by the angle of yaw, the balance about the fuselage axis is vitally influenced by this angle, as becomes evident in Figure 2, which shows the coefficient K_L of the moment about the fuselage axis plotted against α. The air force moment was calculated with the normal force for the wing by angle of yaw $\tau = 0^\circ$ as basis.

Even with $\tau = 20^\circ$, the coefficient of the normal force undergoes only a slight change, according to Figure 3, so that the moment, calculated for $\tau = 0^\circ$, likewise is only slightly altered, as long as τ does not exceed 20°.

Figure 2 further shows the gyroscopic moment coefficient K_K about the fuselage axis plotted against angle α with angle τ as parameter. The gyroscopic moment is small provided the angle of yaw is small also, so that a balance with the air force moment is possible. On the other hand, at $\tau = \pm 15^\circ$, the gyroscopic moment is already so great that a balance about the fuselage axis is no longer possible. From it we infer that the angle of yaw in a flat spin must be small.

3. Simple Solution of Basic Equations

The change in the quantities (due to any outside effect) which determine the position of the airplane in a flat spin can be resolved by integration of the six known basic equations. (Fuchs-Hopf, Aerodynamik, Part II, Chapter 4.)

By suitable transformation the three force equations of the first order and the three moment equations of the second order are changed to nine differential equations of the first order, and from these we then calculate the following nine quantities which define the position of the airplane:

$$\alpha,\ \mu,\ \tau,\ v,\ \varphi,\ \omega,\ \Omega_r,\ \Omega_n \text{ and } \Omega_z .$$

First, we simplify these nine equations by making the following - (permissible for the flat spin) - assumptions:

1) The engine is cut out;

2) The cross-wind force is negligible with respect to lift and drag;

3) By equilibrium the angle of yaw = 0°;

4) The numerical integration having shown here, as elsewhere, that the path velocity v, varies but little, we assume it constant;

5) Lift coefficient c_a and moment coefficient N about the spar axis depend in the first instance on the angle of attack, and are introduced as linear functions $c_a = m_1 \alpha + n_1$ and $N = m_2 \alpha + n_2$, respectively - entirely feasible within the range of angle of attack considered. The rotation is chiefly noticeable in the moment about the horizontal tail surfaces and is considered as damping;

6) The moment about the strut axis, induced by the wing, is small and considered constant for the present, because the tangential force is small at large angles of attack and changes very little with this angle and with the rotation. The moment coefficient about the strut axis, due to rudder and fin, depends principally on the amount of air flow striking the vertical tail surfaces from the side, i.e., on the rotation Ω_n about the strut axis, and is written as linear function $L_s = m_3 \Omega_n$.

The moment about the fuselage axis requires no special assumptions. According to assumption 4, the path velocity v is considered constant, so the differential equation for defining v may be ignored, leaving only eight to be integrated. Now we simplify these equations conformal to the above assumptions, develop the terms from the products of several variables into a Taylor series, of which we maintain only the first term, and lastly ignore those terms which are small in comparison to the others. Now we have the following eight differential equations of the first order:

$$
\begin{aligned}
\dot{\alpha} &= b_1 \tau + c_1 \Omega_z + f_1 && (1)\\
\dot{\tau} &= a_2 \alpha + d_2 \Omega_r + e_2 \Omega_n + f_2 && (2)\\
\dot{\Omega}_z &= a_3 \alpha + d_3 \Omega_r + e_3 \Omega_n + f_3 && (3)\\
\dot{\Omega}_r &= a_4 \alpha + c_4 \Omega_z + e_4 \Omega_n + f_4 && (4)\\
\dot{\Omega}_n &= e_5 \Omega_n + f_5 && (5)\\
\dot{\mu} &= a_6 \alpha + b_6 \tau + c_6 \Omega_z + d_6 \Omega_r + f_6 + g_6 \mu + h_6 \varphi && (6)\\
\dot{\varphi} &= a_7 \alpha + f_7 + g_7 \mu + h_7 \varphi && (7)\\
\dot{\omega} &= a_8 \alpha + f_8 + g_8 \mu + h_8 \varphi && (8)
\end{aligned}
$$

It will be seen that Equations 1-5 no longer contain the explicit values for μ, φ, and ω, so the variables can be separated. From equations 1-5 we derive

$$
\alpha = (a_2 b_1 + a_3 c_1)\, \alpha + (b_1 d_2 + c_1 d_3)\, \Omega_r + (b_1 e_2 + c_1 e_3)\, \Omega_n + \\
+ c_1 c_3\, \Omega_z + b_1 f_2 + c_1 f_3 .
$$

We note that the term with Ω_r is negligibly small compared to the others and can be omitted, i.e., a change in rotation about the fuselage axis which could be forced by altering the respective air force moment will have practically no appreciable effect on the angle of attack.

Equation (5) yields for rotation Ω_n:

$$\Omega_n = - \frac{f_5}{e_5} + \left(\Omega_{n_o} + \frac{f_5}{e_5}\right) e^{e_5 t} \approx \Omega_{n_o} + (\Omega_{n_o} e_5 + f_5) t.$$

For Ω_z we write: $\Omega_z = \dot{\alpha}$,

and the dissimilar differential equation of the second order for α becomes

$$\ddot{\alpha} + p \dot{\alpha} + q \alpha = r t + s,$$

which is resolved as

$$\alpha = A e^{-\delta t} \sin (\xi + \epsilon t) + B t + C.$$

At the beginning there is a damped vibration about an equilibrium position which varies with the time.

The constants A, B, C, ξ, ϵ, and δ, are computed from the initial conditions and from the constants p, q, r, and s, of the differential equation, as

$$p = \frac{\gamma v^2 F t 2 l_H}{2 g J_z v} \left(\boxed{m_2} \alpha + \boxed{n_2} \right) \tag{9}$$

$$q = \left(\frac{\Omega_{r_o}}{\cos \alpha_o}\right)^2 + \frac{\gamma v^2 F t}{2 g J_z} \left(1 + \frac{2 l_H}{v} \Omega_{z_o}\right) \boxed{m_2} \tag{10}$$

$$r = \Omega_{r_0} \frac{r\ v^2\ F\ t}{2\ g\ J_z} \left(1 - \frac{J_r - J_n}{J_z} \Omega_{r_0}\right) \left(\Omega_{n_0} \quad \boxed{m_3} \quad + L_F\right) \quad (11)$$

$$s = \left(\frac{\Omega_{r_0}}{\cos\ \alpha_0}\right)^2 \alpha_0 - \Omega_{r_0}\ \Omega_{n_0} -$$

$$- \frac{\gamma\ v^2\ F\ t}{2\ g\ J_z} \left(\boxed{n_2} - \boxed{m_2}\ \alpha_0 \frac{2\ l_H}{v} \Omega_{z_0}\right) \quad (12)$$

This formation of the differential equations and their solutions is highly important, for it enables us to recognize the influences which may be used to lower the angle of attack as necessary for getting out of a flat spin.

Disregarding any variation in the wings and in the inertia moments during spinning, the quantities m_2, n_2, and m_3 of formulas 9-12 are the only ones which could produce a change in angle of attack.

Figure 4 illustrates the beginning of the vibration caused by a reduplication of the m_{2_0}, n_{2_0}, and m_{3_0} values belonging to the position of equilibrium. One notes that doubling m_{2_0} effects a much more pronounced lowering in angle of attack than doubling m_{3_0} or n_{2_0}. In this figure we also show the change in angle of attack for the case of doubled m_{2_0}, which was calculated by numerical integration.

4. Constructive Measures for Pulling Out of a Flat Spin

Figure 5 depicts the coefficient of the moment about the spar axis with respect to the angle of attack, and may be likened to the range of the angle of attack under consideration, by a straight line $N = m_2 \alpha + n_2$. Any change in n_2 thus denotes a parallel displacement of the straight line. This can be accomplished, for instance, by an elevator deflection, although at large α the shift is very little, even with the elevator set out as far as possible. Doubling n_{2_0}, which is just about obtainable by extreme elevator setting, has only a slight effect on α, according to Figure 4, so that the airplane is not returned to those small angles of attack where the controls become again effective. Deflecting the elevator during a flat spin seems to be followed merely by a "hunting" of the airplane about its initial position. It is conceivable that an elevator deflection in the tempo of the ensuing vibration might in time produce a great change in angle of attack which would permit pulling out of the flat spin.

A change in m_2 denotes a change in the shape of $N = m_2 \alpha + n_2$. This may be accomplished, for example, by suddenly changing the area of the horizontal tail surfaces or other similar measures during the spin. Doubling m_{2_0}, which is practically doubling the area of the horizontal tail surfaces, reduces the angle of attack considerably, according to Figure 4.

The airplane noses over and soon reaches the small angles of attack at which any elevator deflection is effective.

A similar consideration can be applied to the moment due to the vertical tail surfaces, whose coefficient is $L_s = m_3 \Omega_n$. We find that the rudder deflection is wholly ineffective, but that sudden enlargement of the vertical tail surfaces during the spin is followed by a reduction in angle of attack which, however, is small according to Figure 4, even if we double the vertical tail surfaces - corresponding to a doubling of m_{3_0}.

Now we know that any elevator deflection during a flat spin is ineffective, and that a sudden increase in moment in the horizontal tail surfaces, say, by suddenly enlarging the area of these tail surfaces, is the means to return the airplane to small α.

It should be especially noted that this increase in area must occur suddenly and during the spin.

The above solution of the basic equations is quite satisfactory in so far as it not only substantiates the known fact that the elevator is ineffective while in a flat spin, but also points out the means to regain smaller angles of attack. But it is valid only for a short period after the disturbance.

The other temporary changes in the variables, due to a sudden enlargement of horizontal tail surfaces, were calculated by numerical integration. Here we found that the angle of yaw increases enormously at first. Since we lacked all wind-tunnel data for such large angles, the effect of the side wind on the forces and moments had to be disregarded. At any rate, this effect would be noticeable for a short time only, because of the quick reduction in angle of yaw due to the damping effect of the fuselage end and of the vertical tail surfaces. Hence, we are well justified in making the calculation without taking this effect into account.

It is found that the airplane, in consequence of a sudden increase in area of the horizontal tail surfaces, returns very quickly to small angles of attack and to small rotations, finally going over in a nonrotational straight dive without side wind, from which it can be pulled out.

D i s c u s s i o n

Professor H. Wagner: Professor Fuchs' calculation shows that the flat spin can be influenced by an increase in static pressure on the tail surfaces. Unfortunately this is not possible by starting the engine, because the tail surfaces are outside of the slipstream during a flat spin.

Dr. A. King: Various spinning accidents in 1926 in Germany, led to checking the spinning characteristics of one airplane type in flight. The result of this check test was that that particular type could be safely put in a spin by an experienced pilot. Still it remained unexplained as to whether or not an inexperienced pilot could put this type of airplane into a spin, different from the "normal" spin of the check test, and whether the airplane could be forced out of such possible motion by means of the controls. These problems caused me to make a theoretical examination of the flight characteristics in spinning, which was published as 47th Report of the Aeronautical Research Institute, Tokio University.

Because of the uncertainty prevailing at that time on these phenomena, we first had to ascertain whether there are steady spinning conditions out of which an airplane could not be brought by the usual control movements, and if such conditions do occur, whether experiments with other than the known control movements gave any promise of recovery. Lastly, we had to examine in this case the effects of the shape of elevator and of the control surfaces and other structural and aerodynamic formations on the appearance of this motion.

The investigation began with the general equations of motion applied to a fixed body system. With the symbols used in this report, these equations are:

$$\frac{G}{g}\left[\dot{v}_x+\omega_y\ v_z-\omega_z\ v_y\right] \quad S + G \sin\vartheta + X\left\{v_x, v_y, v_z, \omega_x, \omega_y, \omega_z\right\}$$

$$\frac{G}{g}\left[\dot{v}_y+\omega_z\ v_x-\omega_x\ v_z\right] = +G \cos\vartheta\ \cos\varphi + Y\left\{v_x, v_y, v_z, \omega_x, \omega_y, \omega_z\right\}$$

$$\frac{G}{g}\left[\dot{v}_z+\omega_x\ v_y-\omega_y\ v_x\right] = -G \cos\vartheta\ \sin\varphi + Z\left\{v_x, v_y, v_z, \omega_x, \omega_y, \omega_z\right\}$$

$$J_x \dot{\omega}_x - J_{xy} \dot{\omega}_y + J_{xy} \omega_x \omega_z + (J_z - J_y) \omega_y \omega_z =$$

$$= M_x \left\{ v_x, v_y, v_z, \omega_x, \omega_y, \omega_z \right\}$$

$$J_y \dot{\omega}_y - J_{xy} \dot{\omega}_x - J_{xy} \omega_y \omega_z + (J_x - J_z) \omega_x \omega_z =$$

$$= M_y \left\{ v_x, v_y, v_z, \omega_x, \omega_y, \omega_z \right\} - \theta \omega_p \omega_z$$

$$J_z \dot{\omega}_z - J_{xy} (\omega_x^2 - \omega_y^2) + (J_y - J_x) \omega_x \omega_y =$$

$$= M_z \left\{ v_x, v_y, v_z, \omega_x, \omega_y, \omega_z \right\} + \theta \omega_p \omega_y - \varphi_s.$$

The special conditions of the spinning motion are set up in the rotary equations. The velocity components in direction of the lateral axis are disregarded in computing the aerodynamic moments, since their rôle is secondary in the steady state. The observed accidents occurred with throttled or stopped engine, hence we ignored the gyroscopic moments of the propeller on the right side of the rotary equations. Assuming the body axes as principal axes of inertia, we have in the steady state the gyroscopic moments on the left, and the aerodynamic moments on the right side of these equations. We substitute suitably chosen absolute values for the velocities of rotation, and as a result we have in the aerodynamic moments about the vertical and longitudinal axis moment integrals of the first, second, and third order of rotation, appropriately multiplied by the differences in normal and tangential force coefficients of symmetrical plane elements, which can be interpreted graphically. The equation about the vertical axis contains, in addition, the effects of the fuselage and the vertical tail surfaces. The two aerodynamic moments about the vertical and longitudinal axes yield the known data on autorotation about the longitudinal axis and the improbability of a pure aerodynamic state of equilibrium with rotation about the vertical axis. The aerodynamic moment about the lateral axis can likewise be presented in developed form.

From the three developed aerodynamic moments we can segregate one factor of the same power of velocity, so that when we divide the second and third equations by the first, we obtain equations of the third and second degree of the rotation, which contain solely the inertia moments. The values of the rotation about the vertical and lateral axes can be represented as functions of the rotation about the longitudinal axis for all angles of attack. This enables us to calculate a product of the square of the rotation value about the longitudinal axis and of the two other rotation values as functions of the angle of attack. The same product can also be presented directly from the first aerodynamic equation as a function of the angle of attack. The intersection of both curves then yield the possibilities of rotatory equilibrium about all three axes.

The most important result shows us that there are spinning conditions under which all control movements are ineffective. The conditions are most unfavorable when the values for the rotation about the vertical axis become infinite. This is due to too small vertical tail surfaces. Some slight effect is obtained by the integral with the moment of the third order, i.e., the shape of the curve for the tangential force coefficient. But even high finite values may lead to intersections which it is impossible to overcome by the available control settings. Large discrepancies in inertia moments about the normal and longitudinal axes may lead beyond the values for the rotation about the lateral axis, and large absolute values of the inertia moment differences about the longitudinal and lateral axes may lead beyond the values of the rotation about the normal axis to intersections which it is also impossible to overcome with the controls. I have designated this rotary motion of stalled flight, caused by such nonremovable intersections, as "catastrophic spin." The effect of the vertical tail surfaces on spinning is sufficiently known from tests. By suitable enlargement of rudder and fin most airplanes can be made spinproof. All airplanes which have a tendency to go into a spin or, once in it, refuse to come out of it, have a small rudder and fin. The effect of the gyroscopic moment and the inertia moment differences about the normal and longitudinal axes is already known from Hopf's article (See Zeitschrift für Flugtechnik und Motorluftschiffahrt, No. 18, 1921).

The effect of the inertia moment difference about the longitudinal and lateral axes was substantiated in the Short "Springbok's" impossibility of getting out of a spin. This airplane shows some very unusual values for the inertia moment differences about the longitudinal and lateral axes. Even the air density, which appears in the one curve may have some effect on spinning. Various other effects are included in the same report. The curves obtained from the first equation yield intersections only for negative values, i.e., by autorotation. Then, if it is undesirable or impossible to overcome the spinning tendency completely, the most effective and most useful method lies in the suitable shaping of the ailerons themselves or in a change in profile with uniformly actuated ailerons.

Any possible effect of resonance-like elevator deflections on "catastrophic" spinning conditions would have to be resolved from the equilibrium conditions (obtained by including the translatory equations) as a dissimilar boundary value problem, and then the frequency and the amplitude of the forced vibrations defined from it. But whether one may apply the usual method of small vibrations, i.e., developing according to magnitude of disturbance and breaking off of Taylor series after the second term, appears doubtful by the magnitude of the disturbance necessary with the great stability of the steady conditions during this catastrophic spin.

Mr. Focke: Mr. v. Mallinckrodt evidently misunderstood me. I stated in my report that the rotary movement of the mallard looked like spinning, although it was not according to Professor Hopf's investigation. It should be remembered that the large yawing moments which are possible by turning the front wing, can produce such a tight corkscrew dive as is unknown to us with our present-day airplanes.

Professor v. Kármán is of the opinion that the effects of the moments, when going into a spin, are practically negligible, because of the comparatively small velocity of rotation. The question, why some airplanes spin easier than others, appears to him primarily as an aerodynamic problem which has not yet been completely solved.

Translation by J. Vanier,
National Advisory Committee
for Aeronautics.

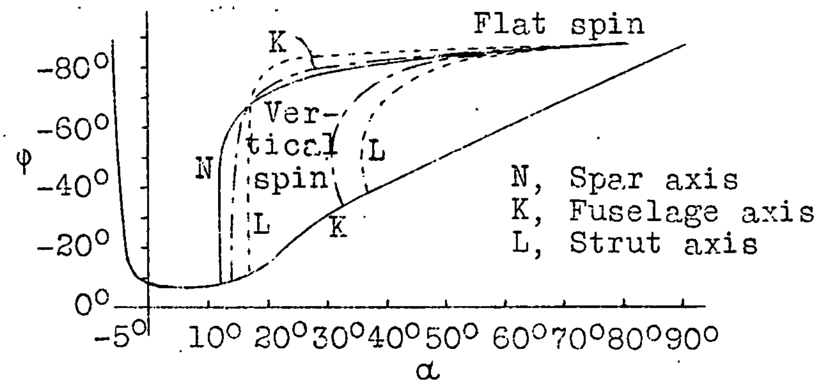

Fig.1 Balance of forces and moments, with φ plotted against α .

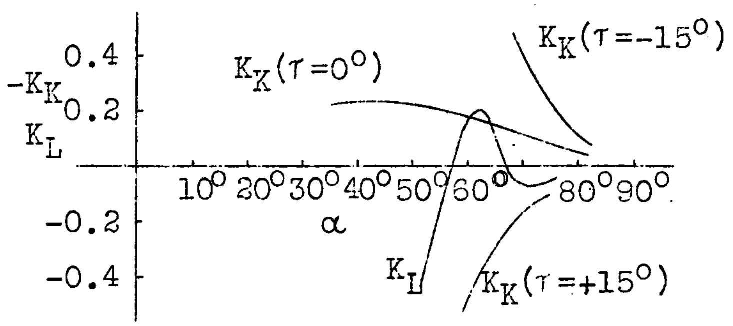

Fig.2 Coefficient of air force moment and gyroscopic moment about the fuselage axis plotted against α , with angle of yaw as parameter.

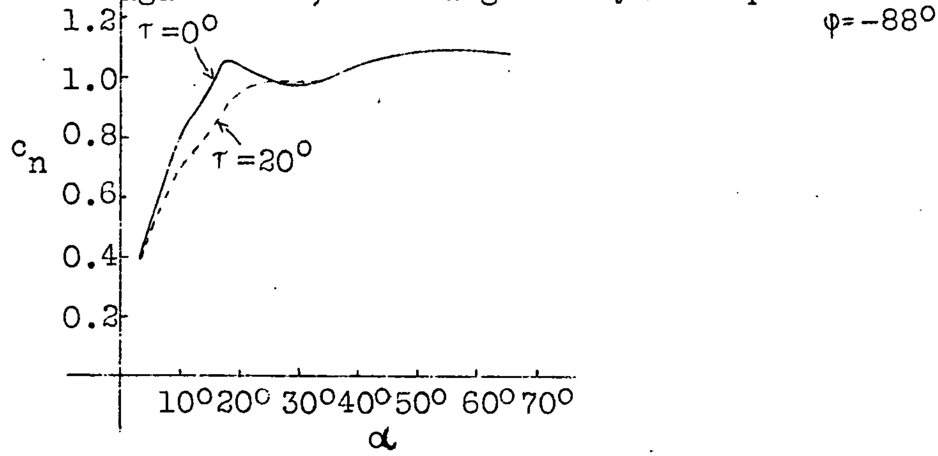

Fig.3 Coefficient of normal force with respect to α , with angle of yaw as parameter.

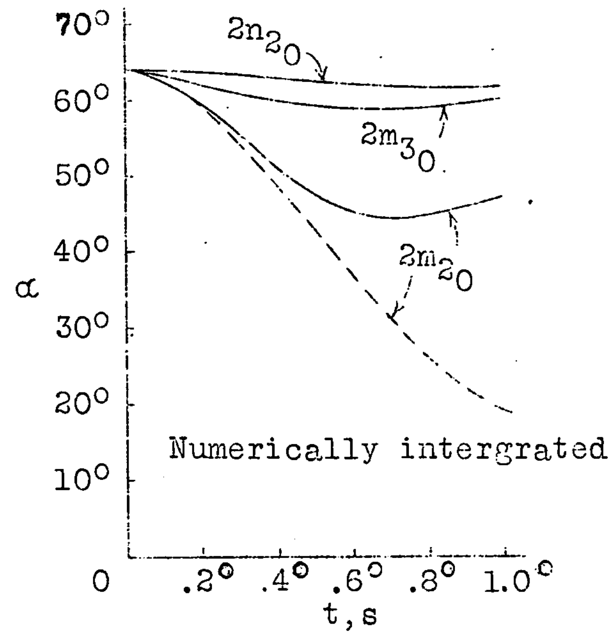

Fig. 4 Angle of attack with respect to time interval

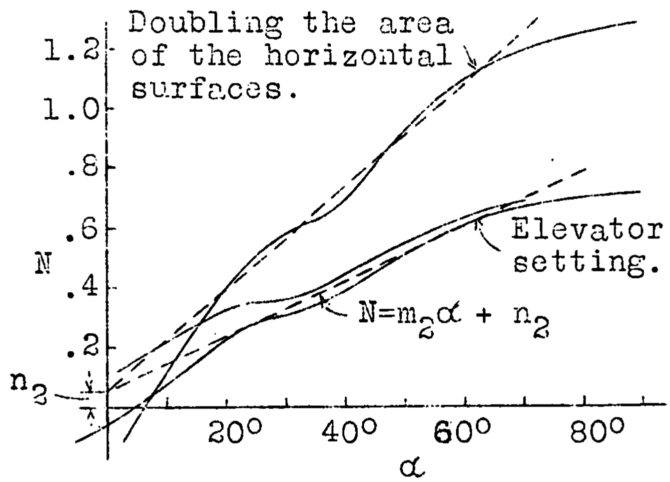

Fig. 5 Coefficient of moment about spar axis plotted against α