

SUPPLY CHAIN NETWORK PLANNING FOR HUMANITARIAN OPERATIONS  
DURING SEASONAL DISASTERS

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Dissertation Prepared for the Degree of  
DOCTOR OF PHILOSOPHY

UNIVERSITY OF NORTH TEXAS

May 2013

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Ponnaiyan, Subramaniam. Supply Chain Network Planning for Humanitarian Operations during Seasonal Disasters. Doctor of Philosophy (Management Science), May 2013, 70 pp., 8 tables, 19 figures, references, 50 titles.

To prevent loss of lives during seasonal disasters, relief agencies distribute critical supplies and provide lifesaving services to the affected populations. Despite agencies' efforts, frequently occurring disasters increase the cost of relief operations. The purpose of our study is to minimize the cost of relief operations, considering that such disasters cause random demand. To achieve this, we have formulated a series of models, which are distinct from the current studies in three ways. First, to the best of our knowledge, we are the first ones to capture both perishable and durable products together. Second, we have aggregated multiple products in a different way than current studies do. This unique aggregation requires less data than that of other types of aggregation. Finally, our models are compatible with the practical data generated by FEMA.

Our models offer insights on the impacts of various parameters on optimum cost and order size. The analyses of correlation of demand and quality of information offer interesting insights; for instance, under certain cases, the quality of information does not influence cost. Our study has considered both risk averse and risk neutral approaches and provided insights. The insights obtained from our models are expected to help agencies reduce the cost of operations by choosing cost effective suppliers.

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## ACKNOWLEDGEMENTS

First and foremost I want to thank my mother Pappathi and my wife Santhi, without whose untiring support, encouragement and care I could not have started and completed my Ph.D program. I also express my gratitude to my maternal uncles and my brother-in-law for their financial and moral support to pursue my studies in the U.S.

I especially want to thank my dissertation chair Dr. Shailesh Kulkarni for his guidance, patience, support and encouragement throughout the program. I also thank my committee members Dr. Hakan Tarakci, Dr. Sudha Arlikatti and Dr. David Francas for their insightful guidance to improve the content and presentation of my dissertation. I also thank my Ph.D coordinators Dr. Robert Pavur and Dr. Victor Prybutok and my department chair Dr. Mary Jones for monitoring my progress at every stage of my program.

I would like to thank all my professors, friends, and others who have directly and indirectly helped me complete this dissertation. I also thank University of North Texas for providing financial and academic supports that helped me achieve my goal.

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## CHAPTER 1

### INTRODUCTION

This dissertation is inspired by the increasing degree of damages caused by several disasters striking some parts of the world every day. It is essential to prevent loss of lives in the aftermath of disasters by providing lifesaving products and services to the affected population in a timely manner. Supply chain planning becomes vital in delivering these supplies. Although planning for sudden onset disasters is extremely complex, planning for seasonal disasters is possible and would eventually help reduce the loss of lives considerably. This is possible only by facing many challenges of disasters and planning phases. Due to the randomness involved in the formation, movement, and occurrence of seasonal disasters, supply chain activities face challenges in procurement, transportation, storage, and distribution of supplies. On the disaster side, hazard attributes such as place of formation, direction of movement, intensity of impact, and the level of damage in the aftermath of disasters are uncertain. On the distribution side, the demand for products and supplies, place of demand, length of demand, capacity of distribution facility, and manpower requirements are also random due to the hazard characteristics. Failures of physical and communication links in the aftermath of disasters further complicate the distribution system. Supply chain partners, including public, private and non-profit relief agencies, often find it difficult to manage the ever changing demands. It is also difficult to procure the required supplies within the budgetary and temporal constraints. In this dissertation, I integrate the demand occurring at various stages of the disaster and create plans to procure the products at three instances, which reduce the imbalance between demand and supply. The major concern of relief agencies is to manage the relief operations within their available budgets because these agencies receive most of their funds only after a disaster dissipates in an affected

area. In this dissertation, I also consider the agencies financial constraints and provide an estimation of optimal order sizes. By integrating all these issues in my models, I balance the supply and demand within a minimum financial requirement.

In chapter 2, I review the literature, structured mainly on five major areas: disasters and their impacts, stages of relief operations, factors differentiating humanitarian operations from commercial operations, current state of research about these operations, and unaddressed issues of humanitarian operations. The chapter starts with an overview of recent natural disasters and the magnitude of losses caused by them. This rationalizes the need for the examination of the problem. The different phases of disaster planning and their scope in humanitarian relief operations are reviewed. The characteristics of seasonal disasters and how these affect the disaster planning activities are further analyzed. Further review of literature examines how the commercial supply chain operations are performed. Both commercial and humanitarian supply chain operations are compared to identify several distinct factors of humanitarian operations. Some of the factors identified include extremely random demand, non-existence of supply chain network and vague supply chain partners. I identify relevant studies on commercial operations that become the base for my model development. I compare different methodologies adopted by prior studies and their applicability to the humanitarian context. Literature addressing these factors in different contexts with a view to incorporate them in the humanitarian setting are further reviewed. I consequently identify several research gaps pertinent to humanitarian operations. For example, no prior studies explicitly address demand forecast updating in the context of humanitarian operations. Most previous studies consider only a single location demand but since a disaster affects multiple sites, the demand rising from all the affected locations need appropriate consideration. No generalized model is available for both perishable

and durable products. Besides addressing these gaps, I also review prior studies on risk propensities of decision makers to find appropriate solutions.

Chapter 3 starts with an overview of how the humanitarian operations are conducted by relief agencies. The study addresses some of the external and internal challenges that agencies face. The study includes the strategies adopted by these agencies to solve these challenges. Subsequently, the dissertation illustrates the processes of my models. This conceptual representation provides the basic framework for my preliminary and extended models. Following this, I introduce notations and symbols, and define the terms that I use in all my models of this study. I also specify the relationships among parameters and underlying assumptions of my models. Though some of these assumptions appear unrealistic for the preliminary model, I relax these assumptions step by step while formulating advanced models. After the introduction of these assumptions, I introduce the basic model along with its solution approaches. In the succeeding section, I perform comparative statics. I also illustrate the results of a numerical problem to understand the effects of various parameters on the decision variables. This chapter concludes with some additional observations or insights.

Chapter 4 starts with an extension of the preliminary model. The preliminary model examines distribution of a single product in a single unit to a single location to a single victim. However, disaster circumstances demand multiple products in different mixes and volumes. Moreover, multiple locations may get affected. In order to consider different aspects of relief operations, the preliminary model is extended into four additional models. The first model extends the preliminary model from single product to multiple products but in equal units to a single location. This model examines distribution of two or more than two products to each of the affected victim. The limitation of this model is that though it includes multiple products, it

considers only one unit of each product. The second model rectifies the limitation of the first model. The first two models focus on single location demand. Relief agencies, on the other hand, have the responsibility to manage the relief operations over multiple areas hit by disasters. I extend the third model to capture the demand arising from multiple regions. Assessing the demand in the aftermath of disasters becomes difficult due to various impending factors such as damage of road and communication links. Therefore, besides considering all the elements of the first two models, the third model also includes multiple locations and incomplete demand information.

A correlation factor is introduced to help pool the demand of multiple sites even though the demands of certain sites are unknown. The last model considers complete demand information, besides considering correlation. Both the third and the fourth model consider the quality of the forecast information of these multiple demands. Subsequently, I perform analytical and numerical analyses of all these models. Further, I use the data generated by Hazards-US (HAZUS) software and perform sensitivity analysis. FEMA (Federal Emergency Management Agency) uses this software to simulate the impacts of hurricanes, floods and earthquakes in a particular region of a country. This chapter concludes with the flexibility of all these earlier models. The flexibility explains the applicability of all these models not only in the strategic planning for the regions but also for the tactical planning for a specific location.

Chapter 5 starts with the widely used risk-measures such as value at risk (VaR) and conditional value at risk (CVaR). Researchers observe that supply chain and operational managers prefer a target profit or cost to expected profit or cost. The reason for this attitude is to avoid unexpected high losses in profit or high costs. This behavior represents risk-aversion of managers. Since cost minimization is the objective of the response/relief agencies, they prefer

risk aversion. Another reason for this is that these agencies have limited financial resources because they receive these funds mainly from donors. Such financial constraints force these agencies to be risk-averse (RA) decision makers. When the resources of relief agencies become very scarce, these agencies have to prioritize the products to ensure a victim gets the most critical one. In order to model this prioritization, I introduce a decomposable model. The model provides insights on the cost of these two categories of products in relation to different RA perspectives. This chapter concludes with an overall summary of the dissertation. Chapter 5 also includes the limitations of the study and concludes with scope for further research.

## CHAPTER 2

### LITERATURE REVIEW

In this chapter, I review the relevant literature. The literature review is structured around five major areas. First, the review of literature focuses on various disasters and their impacts. Second, the review focuses on various stages of relief operations and the challenges humanitarian agencies come across while executing these stages. Third, the unique factors that differentiate humanitarian operations from commercial operations are identified. Fourth, the literature that helps us to understand the current state of research on both humanitarian and commercial operations is reviewed. Finally, the gaps in the current literature are identified. In the concluding section, the implications of this study are briefly discussed.

#### 2.1 Introduction

Caused by nature or human, disasters strike every year worldwide killing thousands of lives and inflicting immense human sufferings (Lodree and Taskin, 2007). Though prevention of these disasters is impossible, the resulting damages can be minimized. On the contrary, human induced disasters occur at high probability, and their consequent damages vary widely by the type of disaster. Moreover, prevention of such disasters is possible. Irrespective of nature and type, all disasters disrupt economic activities, damage physical infrastructure, damage environment, and cause loss of life (Altay and Green, 2006). The impact of a disaster depends on many factors: time, place of impact, nature and severity of hazard agent, topographical features, etc. These impacts include short and long term consequences (Apte, 2010). Short term consequences are loss of lives; long term consequences are damage to physical infrastructure, loss of economic growth, and disruption of social orders. The frequency of occurrence of natural disasters in the last decade has been more than that of its previous decades. Furthermore, more

deaths have been reported in the last decade than its previous decades. For example, earthquakes and tsunamis have killed 696,000; heat waves, 51800; floods, 7800; hurricane and storms, 144,600; famine , 10000 (Yu et al., 2010).

Effective planning and response operations considerably reduce the impacts of disasters, especially loss of lives (Berrin, 1995). Planning comprise of all pre disaster activities, which aim to reduce loss of lives. Response activities refer to the relief activities, which are carried out in the aftermath of disasters: demand assessment, distribution of critical supplies, restoration of physical infrastructure, etc. The duration required for executing pre and post disaster operations is called response period. Wealthy nations recover faster than poor ones. This faster recovery is attributed to preparedness and availability of basic infrastructure; whereas, slower recovery of poor nations is attributed to their existing problems, lack of resources, social background (Yu et al., 2010).

## 2.2 Characteristics of Disasters and Their Impacts

Since I examine only seasonally occurring natural disasters, I study the characteristics of seasonal disasters. The characteristics of seasonal disasters include seasonality, predictability, and location of landfall. Examples of seasonal disasters include heat waves, rains, hurricanes, and storms (Burling and Hyle, 1997). The life span of seasonal disasters varies from a few hours to several days. This time window helps relief agencies to plan their operations and respond to these disasters. On the other hand, suddenly striking natural disasters last for only few minutes, allowing no time window for preparedness (Salmeron and Apte, 2009). For instance, an earthquake strikes suddenly and allows no time for preparations or evacuations. In contrast, hurricanes and floods take considerable time for their formation, movement and landfall. The national weather service (NWS) predicts the formation of hurricanes days and weeks in advance

and warns the people. Further, it releases the status reports of hurricanes every 12 hours after a hurricane's formation. Indeed, NWS predicts the seasonal forecast well before the hurricane season starts. This time window permits agencies to plan and respond immediately in the aftermath of disasters.

### 2.3 Overview of Disaster Relief Operations and Its Challenges

Federal Emergency Management Agency (FEMA) has identified four distinct phases of emergency management operations: preparedness, response, recovery, and mitigation/prevention. Preparedness refers to plans and preparations made to save lives and property. Plans address the facilitation of relief and response operations. The response phase includes actions aimed towards saving human lives and providing support for the speedy recovery of the affected population from the disaster impacts. Response operation comprise of two phases: lifesaving and self-sustaining phase (Celine and Sandra, 2010). The life-saving phase is the period immediately after the disaster, during which the affected population is unable to meet their personal needs. The self-sustaining period is the period after the lifesaving period, during which the victims are able to meet their personal needs themselves. The duration of the recovery period may last for hours to years and includes actions intended towards bringing the community back to normal or improved operating condition in the aftermath of disasters. Mitigation activities try to eliminate or reduce the effects of a disaster. Readers can refer Altay and Green (2006) and Malini et al. (2009) for a comprehensive review of studies related to different stages of relief operations and humanitarian supply chains.

Emergency relief operations are wrought with many challenges (Celine and Sandra, 2010; Kathy and Sanjay, 2009; Sheu, 2007) including ineffective coordination among relief agencies, uncoordinated decision making processes, poorly defined policies, diverse social and

economic conditions, and other associated issues. Kathy and Sanjay (2009) have also identified other major challenges, which include emergency logistics, the timeliness of relief, resource management for emergency logistics, and real time relief demand-information. Relief agencies find it difficult to define and quantify these factors because of their dynamic and complex relationships (Altay and Green, 2006).

Humanitarian operations face challenges of procurement, transport and distribution of relief supplies in their supply chain (Wassenhove, 2006). Since the impact time, place of occurrence, and severities of disasters are often unpredictable, the actual supply requirements are also random. This randomness of demands complicates procurement decisions. Another critical component of the relief operation is the transportation of supplies from source to destination. Moreover, relief agencies face challenges caused directly by disasters. Some of these challenges are the result of damaged infrastructure, limited transportation resources, and limited volume transport (Burcu et al., 2008). In addition, shortages of transportation vehicles often ruin supply distribution and public evacuation. Sometimes, volunteers are unfamiliar with the affected locations and face language barriers, being unable to converse with the affected population. Moreover, power outages occurring in the affected regions may spoil perishable products. Some disasters like nuclear radiation even contaminate the available products and exacerbate the relief operations. For example, 2010-Japan tsunami and the consequent nuclear radiation contaminated milk and spinach available around its Fukushima nuclear plant (Johnson, 2011).

#### 2.4 Unique Features of Humanitarian and Commercial Operations

Both commercial and humanitarian operations share some commonalities; however, the humanitarian supply network differs from commercial logistics. Commercial logistics involves clearly defined linkages among partners; however, humanitarian logistics does not exhibit clearly

stated linkages among partners (Thomas and Kopczak, 2005). In contrast, the common objectives of these operations are to produce and deliver the right product, in the right amount, in the right place, at the right price, and at the right time (Burcu et al., 2008). Although commercial and humanitarian operations share such commonalities, they are distinct in many ways. Since the time, location, type, and demand of these operations are different, these operations are managed differently (Kovacs and Spens, 2009). Beamon (1998) addresses various supply chain measures. Table 2-1 shows the relevance of several measures in the context of commercial and humanitarian operations. Unlike the commercial operations, demand is highly unpredictable in humanitarian operations. In commercial operations, partners fix the lead time; whereas in the humanitarian operations, lead time is not within the control of partners because of the randomness involved in the occurrence of the demand. Commercial supply chain employs a well- established network; whereas, humanitarian supply chain networks do not even exist in certain situations. Commercial operations adapt well-developed and validated inventory techniques. In contrast, humanitarian operations do not have such well-developed and validated techniques. Table 2-1 also shows the differences between commercial and traditional operations.

Table 2-1 Comparison between Commercial and Traditional Operations

Factors	Commercial Operations	Humanitarian Operations
Demand	Relatively stable and predictable	Random & unpredictable demand
Lead time	Determined by partners	Nearly zero lead time
Network	Well defined	Unknown network
Inventory	Well defined techniques	Techniques need to be defined
Objective	Maximize profitability	Minimize loss of life
Output measurement	Profit /cost	Serve at the right time and size
Partners	Profit oriented corporations	Not for profit making agencies

## 2.5 Current State of Research on Traditional Operations

In this section, I review literature relevant to random demand, order size, and forecast updating. The pioneering work by Fisher and Raman (1996) addresses supply and demand

forecast updating. Their model considers a manufacturer-retailer context with two production set ups: one long before the selling season and the other during the early period of the selling season. The retailer places his second order after observing the demand during the early selling period. The manufacturer initiates his production- runs based on the retailer's order time and order size. The quantitative model of their study (Fisher and Raman, 1996) minimizes costs of under and over production. Lau and Lau (1998) model a dual supplier and a single manufacturer inventory control model for a single demand period. Their model divides this single period into two slots. Their model further assumes that the total demand is the combination of demands of these two slots. Their model yields distinct solutions for both the slots. Models formulated by Lau and Lau (1998) differ from that of Fisher and Raman (1996) in two ways: First, the former assume an independent demand while the latter assume a bivariate demand. Second, the former do not impose a minimum lot size restriction while the latter impose such a minimum lot size restriction.

Gurnani and Tang (1999) model a retailer's dual instance ordering problem and use forecast information to update their demand distribution. Further, they assume that the second instant cost may be lower or higher than the first instant cost. They quantify the available forecast information as either worthless or perfect or in between these two extremes. Their study also considers a single period setting. They subjectively choose the first order. Conditioned on the first order size, their second order size optimizes the cost of the overall demand. Donohue (2000) models a manufacturer-outlet chain context and maximizes expected profit of the manufacturer, while assuming that manufacturer also owns the retail outlet. She considers two modes of production: a slower one and a faster one. The faster production mode starts after obtaining the demand forecast. The cost of the products in the faster mode of production is

assumed to be higher than that of the slower mode because faster production has lesser lead time than the slower production. Yan et al. (2003) model a supplier-manufacturer situation with two suppliers and one manufacturer. They assume that one of the suppliers delivers faster than the other. The faster delivery is considered more expensive than the slower delivery. They model the optimal order sizes of both the suppliers as functions of time, costs, and demand. Yan et al. (2003) use a dynamic programming approach to solve their model; whereas, the previously mentioned studies use either constrained or unconstrained optimization techniques. Sethi et al. (2003) assume a multi-period context and place two types of orders per period: a slower one and a faster one. They assume that demand occurs at every period. Their objective is to find a vector of order quantities to minimize the expected total cost over multiple periods. At the beginning of every period, they update the demand forecast. Table 2-2 shows various features and methodologies used by these studies.

Table 2-2 Models Focusing On Traditional Operations

Author(s)	Objective	Methodology	Updating	No of orders	Demand period
Fisher and Raman, 1996	min expected cost	constrained optimization	joint demand information	2	1
Lau and Lau, 1998	max expected profit	unconstrained optimization	demand information	2	2
Gurnani and Tang, 1999	max expected profit	unconstrained optimization	joint demand information	2	1
Donohue, 2000	max expected profit	unconstrained optimization	conditional probability	2	1
Yan et al., 2003	min expected cost	dynamic programming	forecasting based	2	1
Sethi et al., 2003	min expected cost	unconstrained optimization	forecasting based	2	multiple

## 2.6 Current State of Research on Humanitarian Operations

Current state of research on humanitarian operations mainly focuses on transportation of supplies, formation of routes, evacuation of people, unreliability of supply chains, location of facilities and warehouses, control of inventories, and pre-positioning of resources. In this section I review some of the studies, specifically addressing these issues. Valinsky (1955) studies optimum location to set-up fire fighting centers in New York City. Haghani and Oh (1996) formulate a multi-product, multi-modal transportation network for delivering critical supplies. They route the supplies from suppliers to disaster sites. They use a heuristic to solve their model. They further analyze their heuristic in another study (Haghani and Oh, 1996) and derive further insights on several transportation issues. Their model includes four nodes, five arcs, and three modes of transportation. In reality, the number of nodes and arcs are high. Moreover, sometimes the supply chain network for supply distribution does not even exist.

Barbarosoglu et al. (2002) consider a helicopter transportation problem in the context of emergencies. They address issues of aid delivery and evacuation. This helicopter evacuation, although suitable for evacuating few hundred persons, is not suitable for a situation in which the size of the affected population is in thousands and above. Using reliability inference theory, Thomas (2003) examines the unreliability of supply chain and logistical systems. He develops a metric to measure the reliability of supply chain and logistical systems, identifies critical measures, and suggests steps to improve the system. Barbarosoglu and Arda (2004) formulate a stochastic programming problem to address the transportation issues. Ozdamar et al. (2004) examine logistical planning. Beamon and Kotleba (2006) formulate a stochastic inventory model for long term relief operations. Their model considers multiple suppliers context and back ordering. They assume that the demand is random. Moreover, they consider back ordering. This

back ordering concept seems appropriate for the supplies, which victims need later, but does not seem appropriate for supplies, the victims need immediately. Mete and Zabinsky (2010) formulate a stochastic programming model to position the warehouses and find the optimal inventory level. Salmeron and Apte (2009) formulate a stochastic optimization model to pre-position disaster supplies.

## 2.7 Research Gaps and Contributions

There are several studies on forecast updating (Gurnani and Tang, 1999; Yan et al., 2003), order break-ups (Donohue, 2000; Sethi et al., 2003). Though these studies consider commercial supply chain context, the concepts addressed by these studies can be extended to non-commercial humanitarian context if there exists concept commonality. My study is similar to these studies (Gurnani and Tang, 1999; Donohue, 2000; Yan et al., 2003) with respect to multiple orders; however, my study assumes that the second order cost is strictly higher than the first order. This assumption seems true in reality because prices of commodities do go up just before and after a disaster. This happens due to inaccessibility of the affected area, breakage of supply chain links and damage to roads and bridges and supply routes in general. Suppliers need to take extra risk in transporting products to the affected locations. To hedge this risk, suppliers fix higher prices during disasters.

Zhu and Thonemann (2004) use martingale model of forecast updating for aggregating multi-location demand. Besides demand, their model addresses quality of information. I employ this approach in my model to aggregate the demand arising (size of affected populace) at multiple locations. Although they include multiple products, their model is decomposable to individual products. Since relief agencies distribute relief packets containing multiple products, my models are not decomposable to individual products. To the best of my knowledge, I believe

that I am the first one to introduce non-separable models. This property of non-separability leads to lesser data requirements than separable models. In addition, these models have the flexibility to capture products with different shelf lives, for instance, durable and perishable products.

Moreover, existing models use expected value criterion in their solution methodology, a criterion most appropriate for risk-neutral perspective. In contrast, my models include both risk-neutral and RA perspectives. This application of RA perspective assists the relief agencies to assess their financial requirements for worst case disasters. The unique features of my study are depicted in Figure 2-1

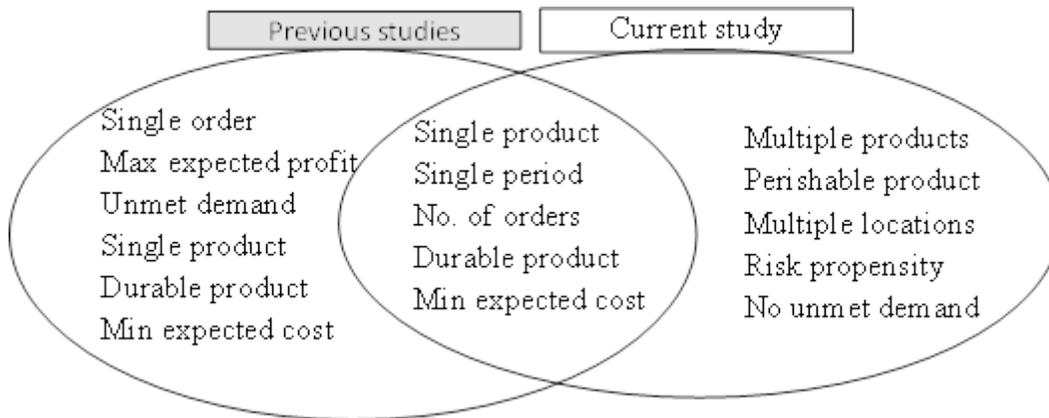


Figure 2-1 Comparison between Current and Previous Studies

## CHAPTER 3

### PRELIMINARY MODEL

In this chapter I provide an overview of current humanitarian operations and series of activities involved in the preparation and implementation of relief operations. Considering these relief activities, I formulate my basic model, derive its analytical solutions, and present its comparative statics. A numerical illustration is also introduced to obtain analytical insights. These analytical and numerical insights help relief agencies make informed decisions.

#### 3.1 Overview of Current Humanitarian Relief Operations

Relief agencies perform various functions as part of their relief operations. These functions include procurement, transportation, storage, and distribution of relief supplies. These agencies procure relief supplies, which include, but are not limited to food, milk, bread, cleaning supplies, water, tarps, tents, napkins, blankets, batteries, etc. Some of these products are perishables, and the remaining ones are durables. These agencies have limited transportation resources. To distribute relief supplies at affected locations, these agencies hire trucks temporarily and transport supplies in large volumes. These agencies own warehouses only at specific locations to store a minimal volume of products. To avoid high storage costs, these relief agencies hire trucks and transport supplies directly from supply sources to affected sites. In the aftermath of disasters, volunteers distribute these products to the affected population.

While executing relief operations, relief agencies face several challenges. Some of these challenges are inadequate financing, shortages of sufficient volunteers, scarcity of warehousing capacity, and lack of logistical and communication resources (Beamon, 1999; Berrin 1995). Relief agencies such as Red Cross and Doctors Without Borders /MSF mainly rely on their donors for financing their relief operations. These agencies receive a major portion of their

donations only after the severity of a disaster is known. In contrast, FEMA receives its full financial assistance from the federal government; funds are actually released only when the federal government declares an event as “disaster”. This declaration is made while a disaster is in process or over. Because of this late declaration, FEMA is unable to act proactively to respond to an imminent disaster. These financial constraints compel these agencies to explore alternative ways to optimize their available resources. Further, the shortage of skilled manpower affects the preparation and response activities. To reduce the cost of operations, agencies employ unpaid and underpaid volunteers who assess demand and distribute supplies. Another challenge aid agencies face is that these agencies do not own local warehouses at all potential disaster sites. In addition, communication and coordination with other agencies become difficult during disasters. All these constraints collectively complicate effective relief efforts. In spite of all these constraints, these agencies are expected to provide relief supplies to all affected population. Relief agencies are subjected act under the aforementioned constraints and limitations. Nevertheless, these agencies always try to improve their operations by exploring effective strategies to reduce their cost of operations while simultaneously meeting disaster demands.

### 3.2 Processes of Disaster Relief Operation

Timely decisions are crucial for effective operations. Though relief agencies create a pre-disaster purchase agreement with their suppliers at the beginning of a disaster season, these agencies do not create one for a specific disaster (Keith, 2006). Further, the aid agencies perform demand assessment after the landfall of disasters; however, they do not conduct such one either before or during the disaster. Though the demand assessment in the aftermath of a disaster is more accurate, it does not provide sufficient time for procurement, transportation and distribution of critical supplies. Researchers (Harrald, 2006; Tovia, 2007) emphasize that providing timely

services in the first 72 hours in the aftermath of disasters is significant to reduce loss of lives. However, the relief agencies have difficulty establishing their distribution networks in time due to many practical difficulties. For example, FEMA (Keith, 2006) established its logistical network 72 hours after the landfall of hurricane Katrina in Louisiana due to communication breakage and inaccessibility to the affected regions. In order to avoid this time lag in distribution of products, relief operations need reevaluation of their current practice of and improvement of it by appropriate strategies. Though the aid operations face challenges in many areas, I consider only inventory management for improvement.

In this study I provide guidelines for improved operations. I support my guidelines via model's analytical insights. Therefore, to improve their operations, I recommend the agencies place a disaster specific order at the time of the seasonal forecast. It would also seem logical for agencies and suppliers to create a pre-disaster agreement to fix the prices of supplies before, during and after disasters. If needed, agencies need to place another order just before the landfall of the specific disaster when the demand information is almost known to certainty. However, agencies need to choose the second ordering time in such a way that it will provide sufficient lead time to the suppliers. This way, products ordered at both instances reach the disaster site in time for distribution. If the number of needed products exceeds the available inventory, a shortage occurs. If there is a shortage, agencies need to initiate spot market purchase and satisfy the demand by distributing products to all the affected people. Instead, if the available inventory exceeds the demand, agencies can salvage the excess inventory or return it to the suppliers if there is any buy-back provision in the contract. I show that these suggested practices ameliorate planning and response activities and consequently minimize risks and costs. Figure 3-1 shows the inter relationships among the major activities: procurement, distribution and

disposal. The flow diagram is drawn for the distribution of a single product procured through multiple ordering and distributed to a single affected site.

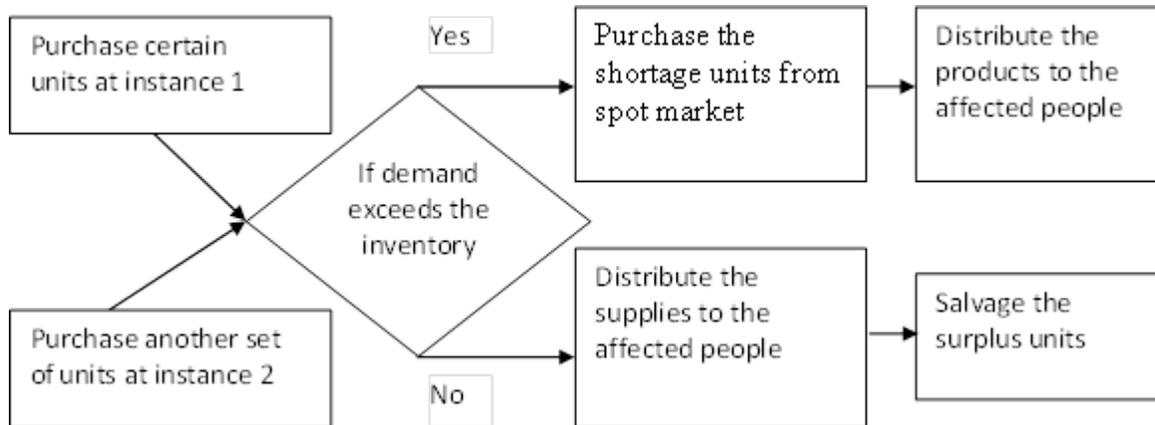


Figure 3-1 Processes Involved in Disaster Relief Operations

### 3.3 Newsvendor Framework for Modeling Disaster Relief Operations

Various components of the disaster relief operations discussed so far are similar to the news vendor framework which has several applications in manufacturing and service sectors (Gurnani and Tang, 1999; Kulkarni et al., 2009; Kulkarni et al., 2008; Kulkarni et al., 2005; Porteus, 2002; Van Mieghem and Rudi, 2002). This framework is most appropriate whenever a decision variable is associated with a random demand. For example, a newsvendor has to decide in advance how many newspapers he has to buy before the realization of demand. The demand for the newspaper is random. The consequences of the newsvendor's decision are that his purchased units may either be more or less than the real demand. In the former case he has to lose opportunity cost, and in the latter case he has to dispose of the excess newspapers. The objective of the newsvendor is then to maximize his profit by finding a tradeoff between under and over ordering costs. In case of a humanitarian context, relief agencies have to ensure the availability of inventory in time for distribution. Since the realized demand is random, the available inventory may be either more or less than the realized demand. Unlike the

newsvendor’s usual profit maximization objective, the relief agencies need to consider the cost minimization objective by finding a tradeoff between over and under ordering of inventories. To incorporate the discussed processes in my preliminary model, I define the following notation.

### 3.4 Model Formulation and Numerical Analysis

In this section, I introduce a series of models to capture various aspects of relief operations as depicted in Figure 3-2. Model 1, the basic model, assumes that the approaching disaster will hit only one location and each affected person in the location needs only one unit of a product.

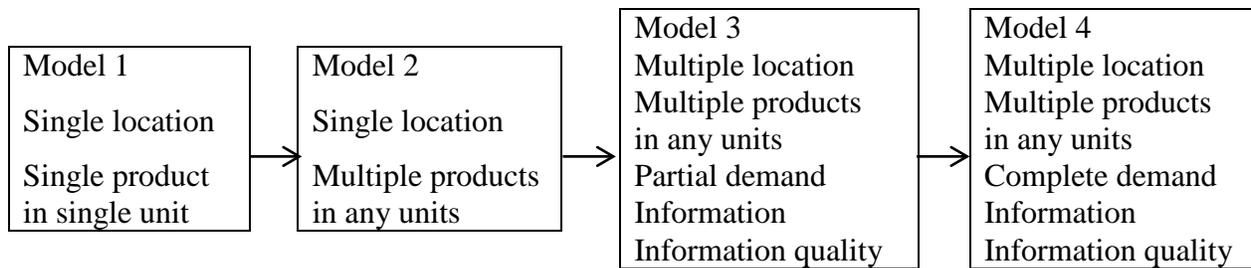


Figure 3-2 Progression of Models

Though the applicability of this model is rare in reality, this model provides closed form solution and practical insights through sensitivity analysis. Model 2 considers distribution of more than one product in multiple units. Models 3 and 4 are more realistic and are likely directly applicable to real world relief efforts. For instance, FEMA distributes lifesaving supplies to all the states of the U.S during emergencies. For an effective relief operation, the relief agency plans for multiple locations by aggregating information from all those locations. Collecting this information may not be possible or feasible in some circumstances. In such situations, FEMA has to perform its planning and response activities based on partial information. Other international relief agencies such as Red Cross and Doctors without Borders also face such problem of partial information while planning for disasters.

### 3.5 Model 1– Single Location and Single Product Model

My basic model is similar to previous studies (Donohue, 2000; Yan et al., 2003). I consider a risk-neutral relief agency which orders  $x_1$  units of a product at a cost of  $c_1$  per unit immediately after a seasonal forecast of a disaster (e.g., hurricane, flood, winter storm, etc) is known. This order quantity is subjective and is based on the agency's budgetary and financial constraints. The agency closely observes evolution of the seasonal disaster and tracks its path of movement and intensity. Based on this observation, the agency identifies the potential locations prone to this disaster. Using HAZUS software – FEMA uses this software to simulate the impact of the damage –the agency estimates the potential number of households that will be displaced ( $\theta$ ). Using the forecast information ( $\theta$ ) and HAZUS simulated data, the agency places a second order for  $x_2$  units at a cost of  $c_2$  per unit. It is assumed that suppliers deliver the ordered quantities ( $x_1 + x_2$ ) to the affected sites just after the dissipation of the disaster. The ordering instances are chosen in such a way that suppliers have enough lead time. In the aftermath of the disaster, the agency starts distributing the supplies to victims and conducts a post disaster demand assessment. If the actual demand for the product ( $\xi|\theta$ ) exceeds the ordered inventory, the agency purchases the shortage units ( $\xi - (x_1 + x_2)$ ) at a price of  $p$  per unit. On the other hand if there are surplus units ( $(x_1 + x_2) - \xi$ ), those units are salvaged at  $v$  per unit. To avoid trivial solutions, I assume  $c_1 < c_2 < p$ . I assume that  $c_2 > c_1$  because supply chain and logistical activities need to be performed with high risk when the disaster is still active in the area. For example, during the super storm Sandy, major suppliers such as Home Depot and Wal-Mart had closed most of their centers located in areas prone to the hurricane. Since suppliers and transporters bear an extra risk, they charge more when the disaster is active in an area. The spot market price  $p > c_2$  because the demand for products increases in the aftermath of disasters

relative to the supply. The salvage value ( $v$ ) is assumed to be strictly less than  $c_1$  to avoid trivial solutions. The probability distribution function of demand is  $\varphi$  and the cumulative distribution function is  $\Phi$ . Distribution parameters  $\mu$  and  $\sigma$  represents the normally distributed demand mean and variance. If only one product is distributed to each affected person, then the demand for the product is equal to the size of the affected population. Throughout this paper, the disaster demand refers to the size of the affected population. The objective function to minimize the expected cost can be expressed as,

$$\text{Min TC}(x_1, x_2)_{x_1, x_2 \geq 0} = c_1 x_1 + E(h(x_1, x_2)) \quad (1)$$

Where, the term  $E(h(x_1, x_2))$  represents the expected value of the objective function of the second instance. The second instance problem is expressed as,

$$E(h(x_1, x_2)) = c_2 x_2 + p \int_{x_1+x_2}^{\infty} (\xi - (x_1 + x_2)) \varphi(\xi|\theta) d\xi - v \int_{-\infty}^{x_1+x_2} ((x_1 + x_2) - \xi) \varphi(\xi|\theta) d\xi \quad (2)$$

Rearranging the terms of equations 1 and 2, I get equation 3

$$\text{Min TC}(x_1, x_2)_{x_1, x_2 \geq 0} = c_1 x_1 + c_2 x_2 + p \int_{x_1+x_2}^{\infty} (\xi - (x_1 + x_2)) \varphi(\xi|\theta) d\xi - v \int_{-\infty}^{x_1+x_2} ((x_1 + x_2) - \xi) \varphi(\xi|\theta) d\xi \quad (3)$$

To reduce the number of variables in equation 3, I introduce another variable  $y$ , called cumulative order size. Let  $y = x_1 + x_2$ . Substituting the value of  $y$  and rearranging the terms of equation 3 yields equation 4. Equation 4 is equivalent to equation 1, which includes both the first and second instance ordering. Equation 5 represents the cumulative problem.

$$\text{Min TC}(x_1, y)_{x_1, y \geq 0} = -(c_2 - c_1)x_1 + c_2 y + p \int_y^{\infty} (\xi - y) \varphi(\xi|\theta) d\xi - v \int_{-\infty}^y (y - \xi) \varphi(\xi|\theta) d\xi \quad (4)$$

$$\text{Min TC}_2(y)_{y \geq 0} = c_2 y + p \int_y^{\infty} (\xi - y) \varphi(\xi|\theta) d\xi - v \int_{-\infty}^y (y - \xi) \varphi(\xi|\theta) d\xi \quad (5)$$

The optimal cost of the model represented in equation 4 is expressed in equation 6

$$TC^*(x_1, y)_{x_1, y \geq 0} = -(c_2 - c_1)x_1 + c_2 y^* + p \int_y^\infty (\xi - y^*) \varphi(\xi|\theta) d\xi - v \int_{-\infty}^y (y^* - \xi) \varphi(\xi|\theta) d\xi \quad (6)$$

### 3.6 Methodology and Solution Approach

Since the cumulative order size,  $y$ , depends on the first instance order size, the basic problem exhibits the structure of a nested newsvendor. The basic problem cannot be solved in its current form because it has two variables but a single equation. To resolve this issue, first I solve the nested second stage problem (equation 5). The solution ( $y^*$ ) reflects the optimal cumulative order size, which also minimizes the main objective function in equation 4. The optimal order size ( $y^*$ ) is obtained from equation 7.

$$y^* = \Phi^{-1}\left(\frac{p - c_2}{p - v}\right) \quad (7)$$

Using the relationship,  $y^* = x_1 + x_2^*$ , the optimal second instance order size ( $x_2^*$ ) can be found as

$$x_2^* = \begin{cases} y^* - x_1 & \text{if } x_1 \leq y^* \\ 0 & \text{Otherwise} \end{cases} \quad (8)$$

Once  $x_1$  and  $x_2^*$  are known, then the optimal cost can be calculated from either equation 3 or 4.

An illustrative example is given below.

Example 1: A relief agency plans to distribute one unit of a product to each of the affected victims for an impending disaster. From the quotes of the suppliers, the cost structure of the product is as follows:  $c_1 = \$12$ ,  $c_2 = \$16$ ,  $p = \$23$ ,  $v = \$8$ . An approaching disaster is expected to displace an average 200 persons with a standard deviation of 20. For any given first order size (e.g., 0, and 75), the relief agency wants to find the optimal order quantity and cost.

Note that the first order size is given  $x_1$ . The optimal cumulative order size is then calculated from equation 7:  $y^* = \Phi^{-1}\left(\frac{p-c_2}{p-v}\right)$  which is 198.33 units. When  $x_1 = 0$ , I have  $x_2^* = y^*$ , which implies that the optimal second instance order size,  $x_2^*$ , is equal to the optimal cumulative order size. The corresponding optimal cost for this scenario is \$3319.26. If  $x_1 = 75$  units, then  $x_2^* = 123.33$  units (i.e.,  $198.33-75$ ). The corresponding optimal cost for this scenario is \$3019.26. Note that  $(3319.26-3019.26) = \$300 = (16-12) \times (75-0)$ . i.e., purchasing each additional unit at the first instance reduces the total cost in increments of  $(c_2 - c_1)$ . This cost reduction  $(c_2 - c_1)$  is the difference in cost between the second and first instance. Obviously, this marginal cost reduction can be achieved as long as the first order size ( $x_1$ ) is less than the optimal order size ( $y^*$ ). However, ordering  $x_1 = y^*$  does not involve any updated demand forecast  $\theta$ . When  $x_1 \geq y^*$ , the agency procures the required supplies all at once at the first instance (i.e., just after the seasonal forecast but even before any active disasters). If the seasonal forecast is 100% accurate,  $x_1 = y^*$  is beneficial. Otherwise, this bulk purchase involves very high risk. Besides, the decision is not based on any updated information of disasters.

### 3.7 Sensitivity Analysis of Model-1

The single product single location model is analyzed for the following parameters:  $c_1, c_2, p, v, \mu$  and  $\sigma$ . Besides these parameters, the impact of first instance order size is also studied. The parameter values assumed for performing sensitivity analysis of this model 1 are same as in example 1 of §3.6.

Proposition 4.1: a). The optimal cost is increasing in  $c_1, c_2, p$  and  $v$ . b). The optimal order size is independent of  $c_1$ , increasing in  $p$  and  $v$ , and decreasing in  $c_2$ .

Proof: Differentiation of expression 4 with respect to  $c_1$  yields  $\frac{\partial TC}{\partial c_1} = x_1$ ;  $c_2$  yields  $\frac{\partial TC}{\partial c_2} = (y - x_1)$ ;  $p$  yields  $\frac{\partial TC}{\partial p} = \int_y^\infty (\xi - y)\varphi_{\xi|\theta}d\xi$ ; and  $v$  yields  $\frac{\partial TC}{\partial v} = \int_{-\infty}^y (y - \xi)\varphi_{\xi|\theta}d\xi$ . Since all these derivatives are positive, The optimal cost is increasing in  $c_1, c_2, p$  and  $v$ .

Since  $x_1$  is non-negative, the optimal cost is strictly increasing in  $c_1$ . The optimal order size is independent of  $c_1$  because the optimal order size expression,  $y^* = \Phi^{-1}\left(\frac{p-c_2}{p-v}\right)$ , does not include  $c_1$ . Since  $c_1$  is the lowest among all the purchasing costs (i. e.,  $c_1, c_2$ , and  $p$ ), purchasing products at the first instance results in cost savings. However, before placing any first instance orders, relief agencies need to evaluate other constraints such as financial, warehousing, and other relevant constraints in conjunction with the cost analysis.

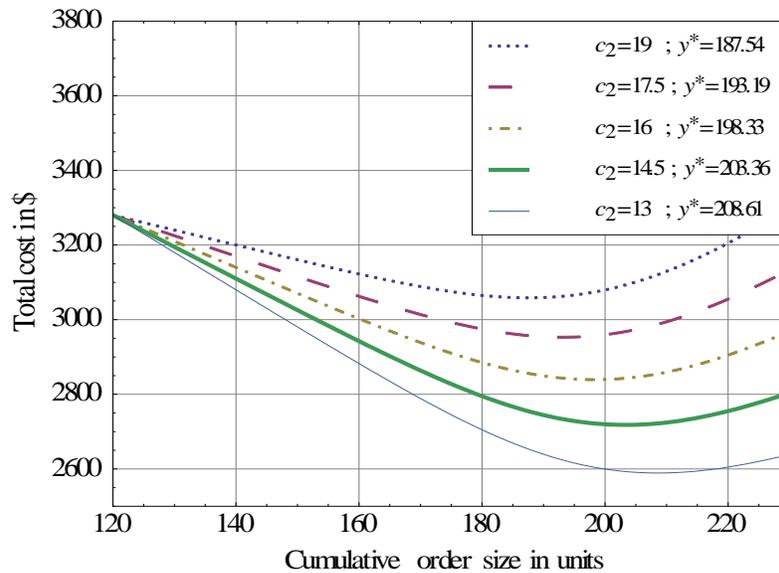


Figure 3-3 Impact of Second Instance Cost  $c_2$  on Optimal Order Size and Cost

The partial derivative of equation 4 with respect to  $c_2$  yields  $(y - x_1)$ . This quantity  $(y - x_1)$  is positive if  $x_1$  is strictly less than  $y$ . Therefore, the total cost is increasing in  $c_2$ . Figure 3-3 shows the impact of  $c_2$  on total cost. Differentiation of equation 4 with respect to  $p$  yields  $\frac{\partial TC}{\partial p} = \int_y^\infty (\xi - y)\varphi_{\xi|\theta}d\xi$ . Since this derivative is positive, an increase in spot market price

increases the optimal order size. Figure 3-4 shows cost curves for various spot market prices.

Differentiation of equation 4 with respect to  $v$  yields  $\frac{\partial TC}{\partial v} = \int_{-\infty}^y (y - \xi) \varphi_{\xi|\theta} d\xi$ . This term is positive. Therefore, optimum cost is increasing in  $v$ . Figure 3.5 shows the impact of salvage value on the optimum cost and order size.

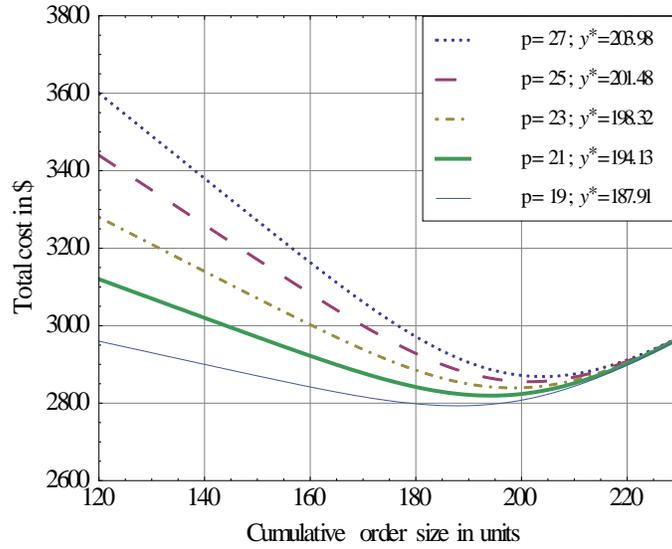


Figure 3-4 Impact of Spot Market Price ( $p$ ) on Optimal Order Size and Cost

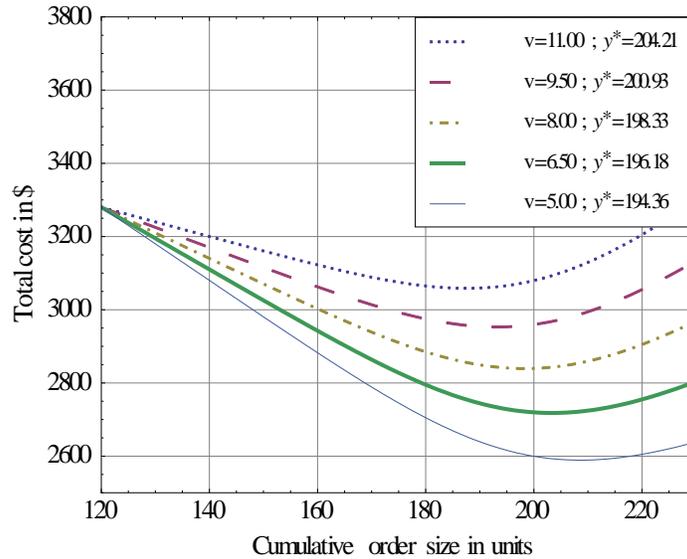


Figure 3-5 Impact of Salvage Value ( $v$ ) on Optimal Order Size and Cost

Proposition 4.2: Purchasing additional units at the first instance will reduce the total cost.

Proof: The proof is straightforward from the equation 4 (i.e.,  $\frac{\partial TC}{\partial x_1} = -(c_2 - c_1)$ ). Since this derivative is always negative, purchasing every additional unit at the first instance will reduce the total cost by  $(c_2 - c_1)$ . Figure 3-6 describes cost curves for various first order sizes, starting from 80 to 160 in steps of 20. For all these first orders, the optimum order size remains the same because the critical fractile  $\left(\frac{p-c_2}{p-v}\right)$  does not include  $x_1$ . However, optimal cost varies for different first order sizes. An initial order of 80 units incurs an optimum cost of \$2999.26. When this order size increases to 100, the optimum cost reduces to \$2919.26, a saving of \$80. This \$80 reduction is the result of  $(c_2 - c_1)x_1$  (i.e.,  $(16-12)*20$ ). This cost saving applies to any additional initial purchase if it does not exceed the optimal order size. Cost curves of Figure 3-6 explain the cost savings for additional purchase at the first instance. All these cost curves starts from their respective initial order as shown.

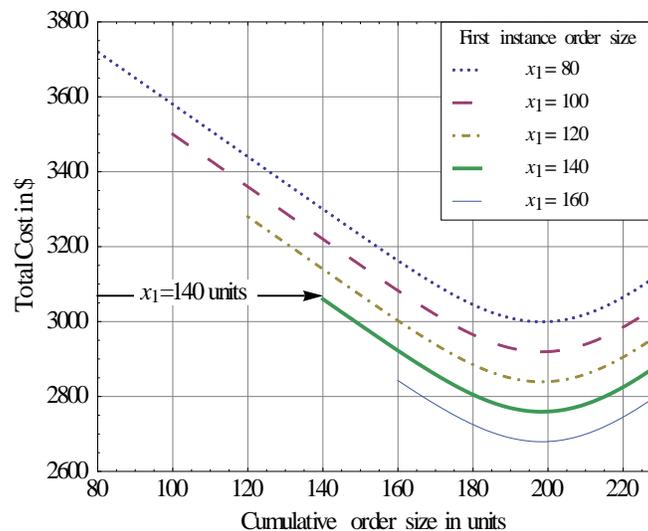


Figure 3-6 Impact of First Order Size ( $x_1$ ) on Optimal Order Size and Cost

Proposition 4.3:

- The optimal cost is increasing in the mean  $\mu$ ; optimal order size is also increasing in  $\mu$

b) The optimal cost is increasing in the standard deviation  $\sigma$ ; optimal order size is also increasing in  $\sigma$ .

Differentiation of the cost expression 4 with respect to  $\mu$  yields a positive derivative. Therefore, the optimal cost is increasing in  $\mu$ . Similarly, differentiation of the cost equation 4 with respect to  $\sigma$  yields a positive derivative. This implies that the optimal cost is increasing in  $\sigma$ . Table 3-1 shows the insights derived from model 1 for all parameters as well as the first instance order size.

Table 3-1 Summary of Insights from Model 1

Parameters (or) variables	$x_1 \uparrow$	$c_1 \uparrow$	$c_2 \uparrow$	$p \uparrow$	$v \uparrow$	$\mu \uparrow$	$\sigma \uparrow$
Optimal order size	No effect	No effect	↓	↑	↑	↑	↑
Optimum cost	↓	↑	↑	↑	↓	↑	↑

In this chapter I have formulated a model for distributing a single product to a single location. The model is solved analytically and a numerical problem is also solved to derive insights in the context of humanitarian relief operations. In practice, humanitarian problem involves several other challenges such as procuring products of different shelf lives, procuring the products from multiple suppliers, bargaining quantity discounts, distributing products to affected population at multiple sites, and collecting demand information from affected sites. To capture such issues, the preliminary model is extended further. The extended models are introduced in the next chapter.

## CHAPTER 4

### EXTENSIONS OF THE PRELIMINARY MODEL

In this chapter, I introduce the extensions of the preliminary model. The extended models consider several practical aspects of relief operations such as multiple products, multiple location, complete and practical forecasting scenarios.

#### 4.1 Model 2– Single Location, Multiple Products of Any Quantity

The model introduced in the previous chapter (§3.1) may seem unrealistic because relief agencies generally distribute relief packets, each of which contains more than one product, in multiple units. The number and mix of products depend on the disaster’s type and severity, the site’s topography and weather, and victims’ age and health. Whenever a product is supplied in multiple units, the respective costs of each unit are added together. For example if I distribute a relief packet containing five bottles of water, each bottle costing \$2, then I aggregate the costs of all five bottles, \$10. This cost aggregation obviates the complexity of dealing with multiple units of each product. Here I assume that each victim gets one relief packet, which contains different units of each product. To capture different products, I use an index  $i$ , where  $i = 1, 2, 3, \dots, I$ . I introduce two non-negative coefficients,  $k_{1i}$  and  $k_{2i}$ , to help capture ordering products with unequal shelf-lives. Coefficients  $k_{1i}$  and  $k_{2i}$ , represent respectively the number of units of product ( $i$ ) purchased at instances 1 and 2. If a product is purchased at both instances,  $k_{1i} = k_{2i} \geq 1$ . This restriction enables agencies to package products as “relief packets”. For example, if five water bottles are packed in a relief packet, it is convenient to buy water bottles in multiples of five irrespective of whether these bottles are purchased at the first instance or second. If these water bottles are purchased in non-multiples of five, after packing five bottles in each relief packet, there are some leftovers (1 or 2 or 3 or 4). In this context, ordering in

multiples of five will eliminate the leftover issue. In contrast if a product is purchased only once at the second instance,  $k_{1i} = 0$  and  $k_{2i} \geq 1$ . The former (first and/or first & second instance ordering) is suitable for durable products; whereas, the latter (second instance ordering only) is suitable for perishable products. The model is flexible to include all possible values of coefficients  $k_{1i}$  &  $k_{2i}$  though I place restrictions mentioned above for the convenience of relief agencies. In the previous model (§3.1), the variables  $x_1, x_2, y, y^*$  and  $x_2^*$  were expressed in units; however, from this model onwards those variables will be expressed in number of relief packets. Model 2 is given below.

$$\text{Min TC} = -\sum_{i=1}^I k_{1i}(c_{2i} - c_{1i})x_1 + \sum_{i=1}^I k_{2i}c_{2i}y + \sum_{i=1}^I k_{2i}p_i \int_y^{\infty} (\xi - y)\phi_{\xi|0} d\xi - \sum_{i=1}^I k_{2i}v_i \int_{-\infty}^y (y - \xi)\phi_{\xi|0} d\xi \quad (9)$$

The optimal order size in packets ( $y^*$ ) is given as

$$y^* = \Phi^{-1} \left( \frac{\sum_{i=1}^I k_{2i}p_i - \sum_{i=1}^I k_{2i}c_{2i}}{\sum_{i=1}^I k_{2i}p_i - \sum_{i=1}^I k_{2i}v_i} \right) \quad (10)$$

If a product is not distributed in single units, then  $y^*$  has to be interpreted in conjunction with  $k_{1i}$  and  $k_{2i}$ . For any given  $x_1$ ,  $x_2^*$  is calculated as,

$$x_2^* = \begin{cases} y^* - x_1 & \text{if } x_1 \leq y^* \\ 0 & \text{Otherwise} \end{cases} \quad (11)$$

I denote  $x_{2i}^*$  as the optimal order size of product  $i$  at the second instance. This  $x_{2i}^*$  can be calculated using  $x_2^*$  as

$$x_{2i}^* = \begin{cases} k_{2i}x_2^* & \forall i, \text{ if } k_{1i} = k_{2i} \geq 1 \text{ (i.e., when purchased twice)} \\ k_{2i}y^* & \forall i, \text{ if } k_{1i} = 0 \text{ \& } k_{2i} \geq 1 \text{ (i.e., when purchased only at the second instance)} \end{cases} \quad (12)$$

The optimal cost equation becomes

$$TC^* = -\sum_{i=1}^I k_{1i} (c_{2i} - c_{1i}) x_1 + \sum_{i=1}^I k_{2i} c_{2i} y^* + \sum_{i=1}^I k_{2i} p_i \int_{y^*}^{\infty} (\xi - y^*) \varphi_{\xi|\theta} d\xi - \sum_{i=1}^I k_{2i} v_i \int_{-\infty}^{y^*} (y^* - \xi) \varphi_{\xi|\theta} d\xi \quad (13)$$

Model 2 differs from the conventional models in aggregation of products. Traditional model comprises multiple products. Terms representing a specific product are separable from other products. Though my Model 2 contains multiple products, terms representing a specific product are not separable from other products. This non-separability property requires lesser data than conventional models that require more data. Each component product of conventional model requires its distribution parameters. In contrast, my model requires only one demand distribution parameters– the distribution of number of persons affected. Conventional models provide optimal solutions to every product. These solutions differ themselves if the products’ demand distribution parameters are different. These differing solutions complicate packaging of products. On the other hand, model 2 offers a single solution, which also minimizes the efforts involved in packaging products.

Example 2: A relief agency wants to distribute each victim a relief packet–containing five water bottles and two blankets. The agency procures these products at both instances. Cost structure of a water bottle is as follows:  $c_{11} = \$2.40, c_{21} = \$3.20, p_1 = \$4.60, v_1 = \$1.60$ . Cost structure of a blanket is as follows:  $c_{12} = \$8, c_{22} = \$13, p_2 = \$17, v_2 = \$4.50$ . Immediately after the seasonal forecast, the agency has ordered 30 relief packets, each of which contains 5 water bottles and 2 blankets. The expected size of affected population is estimated from the forecast information as 200 with a standard deviation of 20. The relief agency wants to find the optimal order quantity of each product and the associated optimum cost.

Let  $i=1$  represent water bottle and  $i=2$  represent blankets. Since both the products are purchased at both instances, I have  $k_{11} = k_{21} = 5$  and  $k_{12} = k_{22} = 2$ . To help find the optimal solution,  $y^*, \sum_{i=1}^2 k_{2i} p_i$  is calculated as 57;  $\sum_{i=1}^2 k_{2i} c_{2i}$  is calculated as 42; and  $\sum_{i=1}^2 k_{2i} v_i$  is

calculated as 17. Using the normally distributed demand with  $\mu=200$  and  $\sigma=20$ ,  $y^*$  is calculated as 193.63 packets. Since the agency has ordered 30 packets ( $x_1$ ) previously, the agency still has to order 163.63 packets ( $x_2^*$ ). Since the agency distributes five water bottles to each individual, the agency still has to order 818.14 water bottles ( $x_{21}^*$ ). and 327.25 blankets ( $x_{22}^*$ ). The optimal cost of this arrangement is \$8283.36. When no packets are ordered at the first instance,  $x_1 = 0$ , or  $k_{11} = k_{12} = 0$ , the total cost will be \$8703.36, an addition of \$420. Note that as demonstrated in model 1, this increment is a result of  $\sum_{i=1}^2 k_{1i}(c_{2i} - c_{1i}) x_1$  (i.e.,  $5 \times (32 - 24) \times 30 + 2 \times (13 - 8) \times 30 = 420$ ).

#### 4.2 Sensitivity Analysis of Model-2

In this section, I perform sensitivity analysis of single location multiple product model (Model 2). The impacts of cost parameters on the optimum cost and optimum order size are studied. These insights help relief agencies minimize their cost and negotiate prices of products with their suppliers.

Proposition 4.4:

- a) For any given distribution parameters, if two products having equal critical fractiles are purchased in any mode (individually or together), their optimum cost and optimum order sizes remain the same irrespective of purchasing mode.
- b) For any given distribution parameters, if two products having unequal critical fractiles are purchased together based on their combined fractile, then their optimum cost is higher than the sum of optimum costs of products purchased individually based on their individual fractiles.

To prove proposition 4.4 a, let product-1 and 2 have equal critical fractiles (i.e.,  $\frac{p_1 - c_{21}}{p_1 - v_1} = \frac{p_2 - c_{22}}{p_2 - v_2}$ ). For some given distribution parameters  $(\mu, \sigma)$ , these critical ratios yield equal optimal

order sizes (i.e.,  $y_1^* = \mu + \Phi^{-1}\left(\frac{p_1 - c_{21}}{p_1 - v_1}\right)\sigma = y_2^* = \mu + \Phi^{-1}\left(\frac{p_2 - c_{22}}{p_2 - v_2}\right)\sigma$ ). It can be shown algebraically that when two equal ratios are combined, the resulting ratio will be equal to the individual ratios (i.e.,  $\frac{p_1 - c_{21}}{p_1 - v_1} = \frac{p_2 - c_{22}}{p_2 - v_2} = \frac{p_1 + p_2 - (c_{21} + c_{22})}{p_1 + p_2 - (v_1 + v_2)}$ ). Under such circumstances, the optimum cost is same irrespective of placing orders based on the individual or combined critical fractiles.

Now I analyze the practical implications of this proposition in the context of humanitarian relief operations. Suppose that each relief packet contains multiple products– one unit of each product that is purchased individually. After packaging, agencies have a leftover of  $|y_1^* - y_2^*|$  units. This term will be positive unless the critical ratios are equal. Managing this leftover is an issue. To avoid this leftover problem, agencies can purchase products together, based on the combined critical ratio, despite a moderately higher cost. To reduce this higher cost, agencies can negotiate the prices with their suppliers. To negotiate prices with their suppliers, agencies need to consider the absolute deviation. The lower is the absolute deviation; the lower is the additional cost. Proposition 4.4 yields a corollary, which explains this additional cost.

Corollary 1: When two products have unequal critical fractiles, ordering based on the higher fractile results in lower cost; however, either the spot market price or salvage value of the product with lower critical ratio needs to be increased to a value at which the lower critical fractile becomes equal to higher critical fractile. Instead of spot market price or salvage value, if the second instance cost is used, it should be reduced until the lower critical fractile becomes equal to higher critical fractile.

Using a numerical example, I analyze how relief agencies can negotiate prices of products with their suppliers to reduce this additional cost. To reduce the average cost of relief packets, relief agencies need to focus on the second instance cost and salvage value of the product, specifically that with a lower critical ratio. Either by decreasing the cost or by

increasing the salvage value, the lower critical ratio of a product can be brought equal to the higher critical ratio. Then by ordering the products based on the  $\max\left(\frac{p_1 - c_{21}}{p_1 - v_1}, \frac{p_2 - c_{22}}{p_2 - v_2}\right)$ , which is also equal to  $\left(\frac{p_1 + p_2 - (c_{21} + c_{22})}{p_1 + p_2 - (v_1 + v_2)}\right)$ , will yield the minimum cost. To evaluate the magnitude of cost difference, I consider two products— water bottle and blanket —with the following cost parameters:  $c_{21} = \$3.20, p_1 = \$4.60, v_1 = \$1.60$  and  $c_{22} = \$13, p_2 = \$17, v_2 = \$4.50$ . Let the demand be uniformly distributed with  $(U \sim (0, \beta=50))$ . The critical ratio of water bottle and blankets are calculated respectively as 0.4667 and 0.32. Based on products' respective critical fractiles, water bottle and blanket are ordered individually. The sum of costs of water bottles and blanket is \$491.67 (from  $-\frac{\beta(c_{21}^2 - 2c_{21}p_1 + p_1v_1)}{2(p_1 - v_1)} - \frac{\beta(c_{22}^2 - 2c_{22}p_2 + p_2v_2)}{2(p_2 - v_2)}$ );  $y_1^*$  is 23.33;  $y_2^*$  is 16. Since  $\min(y_1^*, y_2^*)$  is 16, only 16 relief packets can be made, which consequently leaves a leftover of 7.33 water bottles. After salvaging each water bottle at \$1.60, the total cost reduces to \$479.94 ( $491.67 - 1.60 * 7.33$ ), an average of \$29.99 per relief packet. What is the total cost if I purchase just 16 units of each product? The cost is \$493.28, an average of \$30.83 per relief packet. Instead, what is the total cost if I purchase 23.33 units of each product? The cost is \$498.38, an average of \$21.36 per relief packet. These results (\$29.99, \$30.83, \$21.36) imply that purchasing  $\max(y_1^*, y_2^*)$  will result in lower cost. Is there any other order quantity that reduces the lower cost further below  $\max(y_1^*, y_2^*)$ ? The answer is “yes”. The average cost can be reduced considerably by decreasing  $c_{21}$ ; however, the leftover units will increase. In contrast, decreasing  $c_{22}$ , from 13 to 11.17, yields  $y_1^*$  as 23.33;  $y_2^*$  as 23.33. The corresponding total cost is then \$455.69, an average of \$19.53 per packet. Note that there are no leftovers. Therefore, adjusting the cost of the product with lower critical ratio to  $\max\left(\frac{p_1 - c_{21}}{p_1 - v_1}, \frac{p_2 - c_{22}}{p_2 - v_2}\right)$  reduces not only the leftovers but also the average cost of relief packets. Note that mathematically this process tends to use the combined

ratio  $\left(\frac{p_1+p_2-(c_{21}+c_{22})}{p_1+p_2-(v_1+v_2)}\right)$ . In contrast, by increasing the salvage value of the product with lower fractile, agencies can increase the lower critical ratio to  $\max\left(\frac{p_1-c_{21}}{p_1-v_1}, \frac{p_2-c_{22}}{p_2-v_2}\right)$ ; this increment also tends to keep the average cost minimum. For example, when  $v_2$  increases from 4.50 to 8.42,  $\frac{p_1-c_{21}}{p_1-v_1}$  and  $\frac{p_2-c_{22}}{p_2-v_2}$  becomes equal (0.4667); both  $y_1^*$  and  $y_2^*$  becomes equal to the combined  $y_{1+2}^*$ (23.33); the corresponding optimum cost is \$455.69, an average of \$19.53 per packet. Note that I have utilized both the cost and salvage value, not the price because relief agencies do not have any control over the spot market price. All these insights can be extended for relief packets containing more than two products.

#### 4.3 Model 3– Multiple Locations, Multiple Products of Any Quantity with Partial Forecasting and Quality of Information

Until now, I have discussed single location models. However, more often than not, seasonal disasters strike multiple regions which lie on their path of travel or area of spread. When these disasters strike multiple neighborhoods, relief agencies need to plan for collective demand rather than a single location demand. While planning for such eventualities, collecting demand information from all affected sites is crucial; however, it becomes difficult due to inaccessibility to the affected regions due to breakage of road and communication links. This situation demands robust planning even when the information is incomplete. Researchers (Erkip et al., 1990; Thonemann, 2002; Zhu and Thonemann, 2004) have applied martingale model of forecast evolution (MMFE) to such partial information problems, arising in commercial supply chains. MMFE enables aggregation of multiple demands. Researchers (Erkip et al., 1990) formulate a model for a centralized warehouse system, which in turn collects product demand information from other warehouses. This study (Thonemann, 2002) considers the quality of demand information besides considering correlation of demand between locations. In my study,

I adapt the methods used by Zhu and Thonemann (2004) to aggregate the mean and variance of demand occurring at several locations in the aftermath of disasters. They have applied this approach to estimate the combined demand of multiple customers in a supply chain context; whereas, I apply this approach to estimate the total demand, arising at multiple locations in the aftermath of disasters. Now, I introduce a model incorporating MMFE.

Let  $\rho$  be the demand correlation between any two sites: one with known demand and the other with unknown demand. To ensure the non-negativity of demand variance, Zhu and Thonemann (2004) assume that  $\rho$  is greater than  $-1/(J-1)$ , where  $J$  is the number of affected locations. Define an index  $j$ , where  $j$  is indexed from  $1, 2, 3, \dots, n, n+1, \dots, J$ . The term  $n$  indicates that the demand is known from  $n$  locations. The term  $(J-n)$  represents the number affected location from which no information is obtained. When  $J > n$ , the situation is termed “partial information”; when  $J=n$ , the situation is termed “full information.” In addition, I assume  $J \geq 2$ ,  $n \geq 1$ , and  $J \geq n$ . Equation 14 represents the objective function for the partial information scenario.

$$\begin{aligned} \text{Min TC} = & - \sum_{j=1}^J \sum_{i=1}^I k_{lji} (c_{2ji} - c_{1ji}) x_1 + \sum_{j=1}^J \sum_{i=1}^I k_{2ji} c_{2ji} y \\ & + E_{\xi | (\theta_1, \theta_2, \dots, \theta_n)} \left( \sum_{j=1}^J \sum_{i=1}^I k_{2ji} p_{ji} \int_y^{\infty} (\xi - y) \phi_{\xi | (\theta_1, \theta_2, \dots, \theta_n)} d\xi - \left( \sum_{j=1}^J \sum_{i=1}^I k_{2ji} v_{ji} \right) \int_{-\infty}^y (y - \xi) \phi_{\xi | (\theta_1, \theta_2, \dots, \theta_n)} d\xi \right) \end{aligned} \quad (14)$$

Here, the pooled mean and variance of the affected population are calculated as in equation 15, by following the approach of Zhu and Thonemann (2004). The symbol  $r$  refers to the quality of information, where  $0$  (no information)  $\leq r \leq 1$  (perfect information). I have,

$$\xi | (\theta_1, \theta_2, \dots, \theta_n) \sim N \left( J\mu + \frac{1+(J-1)\rho}{1+(n-1)\rho} \left( \sum_{j=1}^n \theta_j - n\mu \right), \left[ (1-\rho)(J-1) + n[1+(J-1)\rho](1-r) \right] \sigma^2 \right) \quad (15)$$

Where  $\theta_1, \theta_2, \dots, \theta_n$  represent the latest demand information available from the affected locations.

The optimal order size,  $y^*$ , is calculated as,

$$\begin{aligned}
y^* &= J\mu + \frac{1 + (J-1)\rho}{1 + (n-1)\rho} \left( \sum_{j=1}^n \theta_j - n\mu \right) \\
&+ \Phi^{-1} \left( \frac{\sum_{j=1}^J \sum_{i=1}^I k_{2ji} p_{ji} - \sum_{j=1}^J \sum_{i=1}^I k_{2ji} c_{2ji}}{\sum_{j=1}^J \sum_{i=1}^I k_{2ji} p_{ji} - \sum_{j=1}^J \sum_{i=1}^I k_{2ji} v_{ji}} \right) \sqrt{(1-\rho)(J-1) + n[1 + (J-1)\rho](1-r)} \sigma
\end{aligned} \tag{16}$$

For a given  $x_1$ ,  $x_2^*$  is obtained from expression 16 (the same approach as in equation 11).

$$x_2^* = \begin{cases} (y^* - x_1) & \text{if } x_1 \leq y^* \\ 0 & \text{Otherwise} \end{cases} \tag{17}$$

Since I consider multiple locations (indexed by  $j$ ) and multiple products (indexed by  $i$ ), I denote  $x_{2ji}^*$  as the optimal second instant order quantity of product  $i$  to location  $j$ . The second order size of a product for a specific location can be calculated from equation 18.

$$x_{2ji}^* = \begin{cases} k_{2ji} x_2^* & \forall i \text{ and } j, \text{ if } k_{1ji} = k_{2ji} \geq 1 \text{ (i.e., when purchased twice)} \\ k_{2ji} y^* & \forall i \text{ and } j, \text{ if } k_{1ji} = 0 \text{ \& } k_{2ji} \geq 1 \text{ (i.e., when purchased only at the second instance)} \end{cases} \tag{18}$$

The optimum cost of this partial forecasting scenario is expressed as

$$\begin{aligned}
TC^* &= - \sum_{j=1}^J \sum_{i=1}^I k_{1ji} (c_{2ji} - c_{1ji}) x_1 + \sum_{j=1}^J \sum_{i=1}^I k_{2ji} c_{2ji} y^* \\
&+ E_{\xi | (\theta_1, \theta_2, \dots, \theta_n)} \left( \sum_{j=1}^J \sum_{i=1}^I k_{2ji} p_{ji} \int_y^\infty (\xi - y^*) \rho_{\xi | (\theta_1, \theta_2, \dots, \theta_n)} d\xi - \left( \sum_{j=1}^J \sum_{i=1}^I k_{2ji} v_{ji} \right) \int_{-\infty}^y (y^* - \xi) \rho_{\xi | (\theta_1, \theta_2, \dots, \theta_n)} d\xi \right)
\end{aligned} \tag{19}$$

An example illustrates model 3.

**Example 3:** An agency wants to plan for four affected locations, distributing two products to each victim in the first three locations and distributing only one product to each victim in the fourth location. The information from the fourth location is unknown. The cost structures are given in Table 4-1. The mean and variance of the demand of each location is respectively 200 and 20. Based on the historical data, the correlation coefficient is found to be  $\rho = 0.5$ . The quality

of information is  $r = 0.3$ . The agency has ordered 800 packets of all products ( $x_1=800$ ) for all locations, just after the seasonal forecast is known. The agency wants to find the optimal cost and optimal order quantities of each product to each location.

Table 4-1 Numerical Problem

Location (j)	Product (i)	$\theta_j$	$c_{1ji}$	$c_{2ji}$	$p_{ji}$	$v_{ji}$	$k_{1ji} = k_{2ji}$
1	1	250	7	10	17	5	1
1	2		8	12	16	4	2
2	1	180	7	13	16	3	1
2	2		9	14	19	5	3
3	1	256	10	16	25	7	1
3	2		11	13	21	7	4
4	1	Unknown	9	11	19	6	1

Using expression 15, the pooled mean and standard deviation are calculated respectively as 907.5 and 51.96. Aggregated spot market price  $\sum_{j=1}^J \sum_{i=1}^I k_{2ji} p_{ji}$  is calculated as 250; aggregated second instance cost,  $\sum_{j=1}^J \sum_{i=1}^I k_{2ji} c_{2ji}$  as 168; and the aggregated salvage value  $\sum_{j=1}^J \sum_{i=1}^I k_{2ji} v_{ji}$  as 72. Using the critical fractile, pooled mean and pooled standard deviation,  $y^*$  is calculated as 902.37 packets. Using expression 17,  $x_2^*$  is calculated as 102.37 packets. Using expression 18, the order quantities of each product to each of the four locations are calculated and tabulated in Table 4-2.

Table 4-2 Second Instance Order Quantities

Location (j)	1	1	2	2	3	3	4
Product (i)	1	2	1	2	1	2	1
$k_{2ji}$	1	2	1	3	1	4	1
$x_{2ji}^* = k_{2ji} \cdot x_2^*$ in units	102.37	204.74	102.37	307.11	102.37	409.48	102.37

Note that the cost structures of products are different at different locations. This can happen for example, when suppliers quote prices based on their location, transportation cost and other factors.

#### 4.4 Model 4– Multiple Locations, Multiple Products of Any Quantity with Full Forecasting and Quality of Information

Model 4 differs from Model 3 only in  $n$  and  $J$ . With full information  $n = J$ . Replacing  $n$  of equation 14 by  $J$ , I get the pooled mean and variance as

$$\xi | (\theta_1, \theta_2, \dots, \theta_J) \sim N \left( \sum_{j=1}^J \theta_j, ((1-\rho)(J-1) + J[1 + (J-1)\rho](1-r))\sigma^2 \right) \quad (20)$$

Expression 21 yields the optimal order size to full forecasting scenario.

$$y^* = \sum_{j=1}^J \theta_j + \Phi^{-1} \left( \frac{\sum_{j=1}^J \sum_{i=1}^I k_{2ji} P_{ji} - \sum_{j=1}^J \sum_{i=1}^I k_{2ji} C_{2ji}}{\sum_{j=1}^J \sum_{i=1}^I k_{2ji} P_{ji} - \sum_{j=1}^J \sum_{i=1}^I k_{2ji} V_{ji}} \right) \sqrt{(1-\rho)(J-1) + J[1 + (J-1)\rho](1-r)} \sigma \quad (21)$$

The objective function for full forecasting can be obtained from equation 14 by replacing  $n$  by  $J$ .

The optimal second instance order size for the  $i^{\text{th}}$  product of  $j^{\text{th}}$  location ( $x_{2ji}^*$ ) can be calculated from equation 18. A numerical example is introduced to demonstrate analytical results.

Example 4: A relief agency plans to distribute relief packets to persons, who live in seven counties in the West Virginia region, which is prone to an imminent flood. All the persons seeking shelters in this region need relief packets. Using HAZUS software, the agency generates an estimate of affected population for four potential scenarios. HAZUS data (Table 4-3) ([http://www.regionvii.com/images/18\\_Appx\\_HAZUS\\_DATA.pdf](http://www.regionvii.com/images/18_Appx_HAZUS_DATA.pdf), accessed 10/10/12) include the estimation of number of households expected to seek shelter in each county. I assume that the number of persons affected follows a normally distribution with a mean of 200 and a standard

deviation of 20. The agency has stocked 5000 water bottles. Each relief packet contains five water bottles and two meals. The agency would also provide shelter to displaced persons. The cost structure for a water bottle is as follows:  $c_{11} = \$1.50, c_{21} = \$2, p_1 = \$2.50, v_1 = \$1.00$ . The agency purchases water bottles at both the first ( $k_{11} = 5$ ) and second ( $k_{21} = 5$ ) instances, meals and shelter are provided only at the second instance ( $k_{12} = 0, k_{13} = 0, k_{22} = 2, \text{ and } k_{23} = 1$ ). The cost structure of meals is as follows:  $c_{22} = \$10, p_2 = \$15, v_2 = \$3$ . Unlike the cost structures considered as example 3, the cost structure of water bottles and meals are the same across all counties. In addition to distributing relief packets, the agency provides temporary accommodation at public buildings available in the region. The agency spends \$5 per person ( $c_{13} = \$5, c_{23} = \$5, p_3 = \$5, v_3 = \$0$ ) for accommodation purposes. The agency assigns the quality of information as  $r=0.5$ , based on the expertise of the demand appraiser. The agency wants to find the optimal order size for each product and also the optimal expected total cost.

Table 4-3 HAZUS Data on Flood

Counties in West Virginia	Scenario 1		Scenario 2		Scenario 3		Scenario 4		Total Population
	HD*	PSS**	HD	PSS	HD	PSS	HD	PSS	
Barbour	190	165	249	224	288	311	328	362	15557
Braxton	256	312	285	348	321	404	328	439	14702
Gilmer	183	163	194	173	212	194	***	***	7160
Lewis	638	1023	704	1135	743	1263	770	1278	16919
Randolph	493	583	614	816	657	932	685	984	28262
Tucker	188	219	236	321	256	383	287	435	7321
Upshur	478	842	555	971	587	1011	740	1446	23404
Total	2426	3307	2837	3988	3064	4498	3138	4944	113325
* Number of households displaced (HD)      **Number of persons seeking shelters(PSS)									

To find the optimal solution  $y^*$ ,  $\sum_{i=1}^3 k_{2i}p_i$  is calculated as 47.5;  $\sum_{i=1}^3 k_{2i}c_{2i}$  is calculated as 35; and  $\sum_{i=1}^3 k_{2i}v_i$  is calculated as 11. Note that the accommodation is a service and hence, it does not have any salvage value. By correlating the number of persons seeking shelter from each

county with other counties, correlation coefficient is calculated and tabulated in Table 4-4. Since most of the values are more than 0.90,  $\rho$  is fixed as 0.90.

Table 4-4 Correlation ( $\rho$ ) between Counties (Based on HAZUS Data)

	Barbour	Braxton	Gilmer	Lewis	Randolph	Tucker	Upshur
Barbour	1						
Braxton	0.98	1					
Gilmer	0.97	1.00	1				
Lewis	0.99	0.99	0.96	1			
Randolph	0.97	0.96	0.92	0.99	1		
Tucker	1.00	0.96	0.93	0.99	0.98	1	
Upshur	0.95	0.87	0.93	0.91	0.86	0.95	1

I assume that one of the demand scenarios, scenario three, occurs when the disaster hits. The demand for this scenario is 4498 from Table 4-3. I have calculated the long term mean and variance for the number of people seeking shelters using all the 27 available PSS values, generated by HAZUS. The mean and standard deviation are 619.89 ( $\mu$ ) and 404.59 ( $\sigma$ ). Total number of locations affected is seven (J). Substituting all these values ( $\rho$ ,  $\sigma$ , and J) in equation 20, the standard deviation (pooled) is calculated as 1940.35. The optimal order size,  $y^*$ , is then calculated from equation 21 as 3710.72 units. The agency has to order 7421.44 (i.e.,  $x_{22}^*$ , 3710.72\*2 meals/person/day) meal packages because it has not ordered any meals at the first instance. It also needs to find accommodation for 3711 persons ( $x_{23}^*$ ). A total of 18553.6 (i.e., 3710.72\*5 bottles/person/day) water bottles are needed. Since the agency has already stocked 5000 ( $x_1 = 1000$ ) water bottles, the agency still need to order 13553.6 water bottles. The optimum cost (TC\*) turns out to be \$160,952.

#### 4.5 Sensitivity Analysis of Model-3 and 4

In this section, I perform sensitivity analysis of multiple locations, multiple product, and forecast updating model. The impacts of various parameters—correlation coefficient, quality of

information—on the optimum cost and optimum order size are examined in the context of partial and full information scenarios. Since Models 3 and 4 differ in only J and n, most of the insights derived from them are similar; therefore, I analyze them together in this section. I state explicitly wherever the insights of Model 4 differ from Model-3.

Proposition 4.5: In full information case, the optimal order size depends on both critical ratio and specific values quality of information as tabulated in Table 4-5; in partial information, the optimal order size is increasing in correlation coefficient except when  $z > 2$  &  $r \geq \frac{1+J(n-1)}{Jn}$  and except when  $z < -2$  &  $r < \frac{1+J(n-1)}{Jn}$ .

Proof: Optimal order size of full information scenario is

$$y^* = \sum_{j=1}^J \theta_j + \Phi^{-1}(\cdot) \sqrt{((1-\rho)(J-1) + J(1+(J-1)\rho)(1-r))\sigma}$$

Taking partial derivative of this  $y^*$  with respect to  $\rho$  yields

$$\frac{\partial y^*}{\partial \rho} = \Phi^{-1}(\cdot) \frac{\sigma(1-2J+J^2+Jr-J^2r)}{2\sqrt{((-1+J)(1-\rho)+J(1-r)(1+(-1+J)\rho))}}, \text{ where, } \Phi^{-1}(\cdot) = \Phi^{-1}(\text{critical ratio}) \text{ is the standard}$$

normal variable (z). The denominator of this derivative is always positive; while the numerator depends on both r and z, where z depends on the critical ratio. To simplify the terms, Let

$$F = \frac{\sigma(1-2J+J^2+Jr-J^2r)}{2\sqrt{((-1+J)(1-\rho)+J(1-r)(1+(-1+J)\rho))}}. \text{ Now, } \frac{\partial y^*}{\partial \rho} \text{ becomes } F^*z. F^*z \text{ has nine possible combinations,}$$

called outcomes. These outcomes are tabulated in Table 4-5.

Table 4-5 Possible Outcomes of  $F^*z$

r	Greater than (J-1)/J			Equal to (J-1)/J			Less than (J-1)/J		
F	-			0			+		
z	-	0	+	-	0	+	-	0	+
$F^*z$	+	0	-	0	0	0	-	0	+
Outcome	1	2	3	4	5	6	7	8	9

Among the nine outcomes, the derivative ( $\frac{\partial y^*}{\partial \rho}$ ) of five of them (2, 4, 5, 6, 8) results in zero; two of them (3 and 7), negative; two of them (1 and 9), positive. When the derivative is zero, the optimal order size is independent of  $\rho$ ; when negative, optimal order size is decreasing in  $\rho$  and when positive, optimal order size is increasing in  $\rho$ .

To illustrate the impact of  $\rho$ , I consider the parameters of numerical example 3 introduced in §4.3. Let the unknown demand of fourth location be 270 (i.e.,  $\theta_4$ ). In complete forecasting, for  $\rho=0$ , critical ratio =0.35,  $z < 0$ , and  $r=0.3$ , the optimum order size starts at 937.441 and decreases to 930.209 when  $\rho=1$ . This is analogous to outcome 7. In this combination  $r$  is less than  $((J-1)/J < (4-1)/4=0.75)$ . For  $\rho=0$ , critical ratio =0.35,  $z < 0$ , and  $r=0.90$ , the optimum order size starts at 941.79 and increases to 946.252 when  $\rho=1$ . This mimics outcome 1. In this combination  $r$  is less than  $((J-1)/J > (4-1)/4=0.75)$ . For  $\rho=0$ , critical ratio =0.50,  $z=0$  and  $r=0.75$ , the optimum order size starts at 956 and remains at 956 when  $\rho=1$ . This satisfies outcome 5. In this combination  $r$  is less than  $((J-1)/J = (4-1)/4=0.75)$ . The analyses show that when the agency places an order equal to the mean of the demand distribution, the quality of information makes no difference. When the order size differs from the mean, information becomes important.

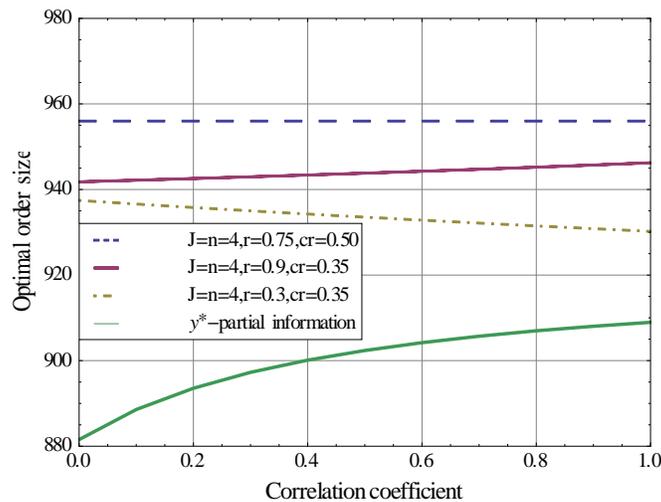


Figure 4-1 Impact of Correlation Coefficient on Optimal Order Size

The optimal order size of partial information scenario is

$$y^* = J\mu + \frac{1+(J-1)\rho}{1+(n-1)\rho} \left( \sum_{j=1}^J \theta_j - n\mu \right) + \Phi^{-1}(\cdot) \sqrt{\left( (1-\rho)(J-1) + n(1+(J-1)\rho)(1-r) \right) \sigma}$$

Taking partial derivative of this  $y^*$  with respect to  $\rho$  yields

$$\frac{\partial y^*}{\partial \rho} = \frac{(J-n)}{(1+(n-1)\rho)^2} \left( \sum_{j=1}^J \theta_j - n\mu \right) - \Phi^{-1}(\cdot) \frac{(J-Jn+Jnr-1)}{2\sqrt{(-1+J+\rho-J\rho+n(1-r)(1+(-1+J)\rho)}} \sigma. \text{ Among the three terms,}$$

the derivative of the pooled mean  $\left( \frac{(J-n)}{(1+(n-1)\rho)^2} \left( \sum_{j=1}^J \theta_j - n\mu \right) \right)$  is always positive; whereas both  $z$

and derivative of pooled standard deviation depends respectively on critical ratio and quality of

information. The standard normal  $z$  is positive when critical ratio  $> 0.5$ , negative when critical

ratio  $< 0.5$ , and zero when  $z = 0.5$ . The derivative of pooled standard deviation becomes zero

when  $r = \frac{1+J(n-1)}{Jn}$ , negative when  $r < \frac{1+J(n-1)}{Jn}$ , and positive when  $r > \frac{1+J(n-1)}{Jn}$ . Similar to the

outcomes of complete information scenario, I will also have outcomes in partial information.

However, in partial information case, the magnitude of derivative of the mean is much higher

than that of the terms involving  $z$  and pooled standard deviation. The derivative becomes

negative when both  $z > +2$  &  $r \geq \frac{1+J(n-1)}{Jn}$  and when both  $z < -2$  &  $r < \frac{1+J(n-1)}{Jn}$ . In partial

information, the optimal order size is 881.88 when  $\rho=0$ . When  $\rho$  increases from 0 to 1, the

optimal order size monotonically increases from 881.88 to 910.08. Figure 4-1 depicts the impact

of  $\rho$  on partial forecasting and three scenarios of full forecasting. The term “cr” denotes critical

ratio.

Proposition 4.6:

- a) In both partial and full information, when the critical fractile is 0.50, the optimal order sizes are independent of  $r$ .

- b) In partial information, when the critical fractile is strictly less than 0.50, optimal order size is increasing in r; whereas, in full information when the critical fractile is strictly less than 0.50, the optimal order size is decreasing in r.
- c) In partial information, when the critical fractile is strictly more than 0.50, the optimal order size is decreasing in r; whereas, in full information when the critical fractile is strictly more than 0.50, the optimal order size is increasing in r.

Proof: Partial differentiation of equation 16 (i.e.,  $y^*$  of partial forecasting) results in

$$\frac{\partial y^*}{\partial r} = -\Phi^{-1}(.) \frac{(n-n\rho+Jn\rho)\sigma}{2\sqrt{((-1+J)(1-\rho)+n(1-r)(1+(-1+J)\rho))}} . \text{ When critical ratio} = 0.5, z = 0 \text{ (i.e.,}$$

$\Phi^{-1}(cr))$ , and  $\frac{\partial y^*}{\partial r} = 0$ . This zero (i.e., constant) slope implies that optimum order size and quality of information are independent. Because of this constant slope, when critical ratio = 0.5, the optimum order size remains a constant (i.e., equal to the pooled mean,  $J\mu + \frac{1+(J-1)\rho}{1+(n-1)\rho} (\sum_{j=1}^J \theta_j - n\mu) = 907.5$ ). When critical ratio  $< 0.5$  and  $z < 0.5$ ,  $\frac{\partial y^*}{\partial r}$  is greater than 0. This positive derivative implies that when  $z < 0$ , the optimum order size is increasing in r. However, When  $z > 0$ ,  $\frac{\partial y^*}{\partial r} < 0$  and the optimum order size is decreasing in r.

For the given parameter of numerical example 3, the critical fractile is 0.4607 and the optimum order size is 903 units. The pooled variance is calculated as 45.17 for  $r=0$ . When r increases from 0, the variance decreases from 45.17 and hence the order size increases. This reducing variance is reflected by the narrowing gap between the horizontal line (line for  $cr=0.5$ ) and the bottom curve as shown in Figure 4-2. In full information scenario, differentiation of expression 21 with respect to r yields  $\frac{\partial y^*}{\partial r} = -\Phi^{-1}(.) \frac{(J-J\rho+J^2\rho)\sigma}{2\sqrt{((-1+J)(1-\rho)+J(1-r)(1+(-1+J)\rho))}}$ . This term  $(\frac{\partial y^*}{\partial r})$  is positive when critical ratio  $< 0.5$ , zero when critical ratio = 0.5, and negative when

critical ratio  $> 0.5$ . This shows that in both the scenarios, the quality of information reduces the variance. In partial information, when  $\frac{\partial y^*}{\partial r}$  is positive when critical ratio  $> 0.5$ ; however, in full information  $\frac{\partial y^*}{\partial r}$  is positive when critical ratio  $< 0.5$ . When critical ratio  $= 0.5$ ,  $\frac{\partial y^*}{\partial r} = 0$ , irrespective of information scenarios.

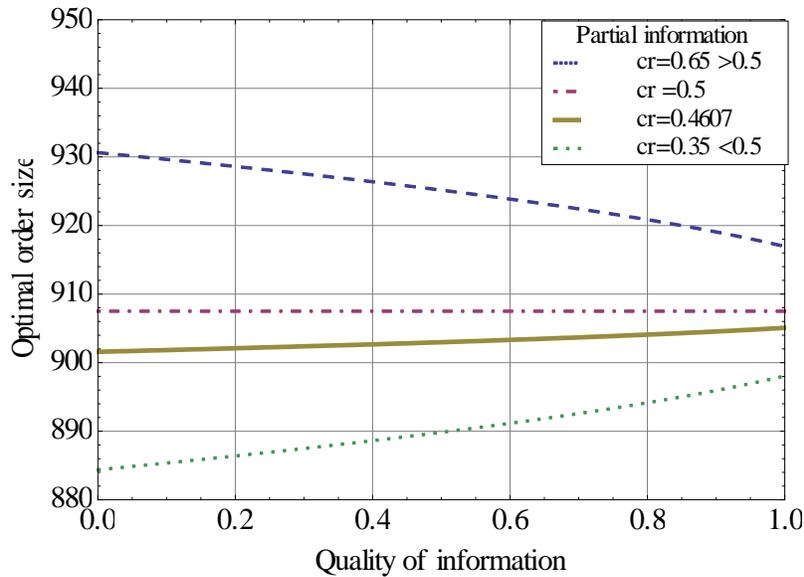


Figure 4-2 Impact of Quality of Information on Optimal Order Size

Proposition 4.7:

- a) The optimum cost is decreasing in the quality of information ( $r$ ), irrespective of partial or full information.
- b) In full information, the optimal cost is increasing in  $\rho$  for the outcomes 3 and 7 (refer Table 4-5), decreasing in  $\rho$  for the outcomes 1 and 9 and independent of  $\rho$  for the remaining outcomes. In partial information optimum cost is increasing in  $\rho$  except case by case extreme  $z$  values

Proof of 4.7a and 4.7b are straight forward. These can be proven from  $\frac{\partial TC^*}{\partial \rho}$  and  $\frac{\partial TC^*}{\partial r}$ .

Relief agencies can considerably reduce their cost by improving the quality of information by adopting appropriate demand assessment techniques.

#### 4.6 Flexibility of My Models

Now, I discuss the flexibility of my models. The models introduced in this chapter offer several flexibilities to accommodate numerous operational and logistical issues associated with the humanitarian relief operations. Some of these flexibilities are listed below.

- Suppliers may impose a minimum purchase quantity constraint at the first instance in order to comply with contractual requirements of costs. Such a constraint can be incorporated.
- Products with different shelf lives can be modeled by ordering durable products twice at both instances and perishable products only once at the second instance.
- Capacities of warehouses at different locations can be included.
- Suppliers may offer quantity discounts when the order size exceeds certain threshold agreed by suppliers and agencies. Some donors may supply products free of cost to agencies. Some donors may donate products to specific locations. All these issues can be incorporated in my models.
- Quality issues of supplies can be included. For example, some portion of supplies may get spoiled during transportation; these fractions can be included either product-wise, location wise, supplier wise, or any combination of these.
- Costs such as transportation, holding and inspection can be integrated into my models.

- Suppliers may opt for buy-back option in their contracts. As per such contracts, a specific unit or entire surplus units can be returned to the supplier. Such option(s) can be modeled using models 1-4.
- Some products may have disposal costs (also negative salvage cost). These costs can also be modeled.
- Though all my models consider decision making at the strategic level, my models are equally applicable while making tactical decisions. For example, my preliminary model captures single location demand and enriched to multiple locations. Instead, if an agency wants to plan for various issues involved in a single location, such issues can be incorporated.

In this chapter, I have extended the preliminary model to capture several issues. Some of the assumptions introduced in the previous chapter have been relaxed to make the model more realistic. In addition to analytical results, numerical problems have been solved, and their results analyzed to derive numerical insights. A numerical problem generated from HAZUS software has been also solved and its results have been analyzed. The concluding section of this chapter has included the flexibility of my models 1-4.

## CHAPTER 5

### RISK PROPENSITIES IN DISASTER MODELING

Models introduced in the previous chapters use expected value criterion. However, managers of firms and agencies prefer a profit above a target value or a cost below a target value. Since the objective of relief managers is to reduce cost, these managers attempt to avoid higher costs. Such a behavior is called risk-averse behavior. In this chapter, I formulate and solve models, using RA approach and provide analytical and numerical insights.

#### 5.1 Overview of Risk-Averse Perception

Optimum cost and optimum profit are endogenous in the expected value approach. However, supply chain managers prefer these to be exogenous. These managers prefer to avoid profits below a specific value in case of profit maximization objective; while they prefer to avoid costs above a specific value in case of cost minimization objective (Gotoh and Takano, 2007; Wu et al., 2010). CVaR method enables these managers choose such objectives. Modeling RA behavior necessitates an approach different from the expected value approach. Using modified expected value approach, CVaR models provide solution to RA decision makers. Some other risk measures widely used for modeling risk propensities of decision makers include expected regret, CVaR, expected shortfall, tail conditional expectation and tail mean, worst conditional expectation and spectral risk measures (Szego, 2002). Though some of these risk measures such as variance and VaR are applicable under certain conditions, they are not coherent risk measures. A coherent risk measure has to satisfy certain properties (Szego, 2002) – positive homogeneity, subadditivity, monotonicity and transitional invariance. When a risk measure violates any of these conditions, it provides multiple local minima that are difficult to interpret (Rockafellar and Uryasev, 2000). Furthermore, such violations lead to lack of convexity. Therefore, it is

unsuitable to use such non-coherent risk measures. Since CVaR is one of the coherent risk measures, I use CVaR concepts in my models to capture risk-averse behavior. This chapter examines the impact of RA perspective on the optimum order size as well as optimum cost. In the following section, I review the definition of CVaR.

## 5.2 Conditional Value at Risk

CVaR in the context of cost minimization objective can be defined as the expected value of the cost, conditioned on the costs being in excess of VaR (see Jobst and Zenios., 2001). Expected value and CVaR approaches differ from their calculation approaches. For example, expected value approach considers the entire support of the demand distribution; whereas, RA criterion (CVaR) considers a specific portion of the demand distribution. CVaR focuses on the right tail of the cost distribution for cost minimization objective and left tail for the profit maximization objective. The reason for choosing the right tail for cost minimization is to avoid very high costs; whereas, choosing the left tail for profit maximization is to avoid very low profits. In the humanitarian context, consideration of the expected value criterion is appropriate if disasters cause more or less similar level of damage in the long run. However, in reality, some less frequently occurring disasters cause great impacts, for example hurricane Katrina. Managing such worst disasters need additional consideration while planning and responding to such worst events. Such circumstances compel the attention of relief agencies on certain percentage of right tailed  $(1-\beta)$  region (i.e., higher cost region) of the cost distribution. Mathematically, CVaR is expressed by Rockafellar and Uryasev (2000) as

$$\text{Min CVaR} = \alpha + \frac{1}{1-\beta} \int_0^{\infty} (\text{TC}(y, \xi) - \alpha)^+ \varphi(\xi) d\xi \quad (22)$$

The term  $\alpha$  in expression 22 represents VaR. This means that  $\beta$  percent of the time, the cost of relief operation is within  $\alpha$ . CVaR is the average cost incurred in  $(1-\beta)$  percent of the

time. Note that expression 22 captures the cost distribution in terms of demand distribution (see Jammerneegg and Kischka, 2007). Gotoh and Takano (2007) provide solution to expression 22 as

$$y^* = \frac{c_2-v}{p-v} \Phi^{-1} \left( \frac{p-c_2(1-\beta)}{p-v} \right) + \frac{p-c_2}{p-v} \Phi^{-1} \left( \frac{(c_2-v)\beta+p-c_2}{p-v} \right) \quad (23)$$

The corresponding VaR is

$$\alpha^* = \frac{(c_2-v)(p-c_2)}{p-v} \left( \Phi^{-1} \left( \frac{(c_2-v)\beta+p-c_2}{p-v} \right) - \Phi^{-1} \left( \frac{p-c_2(1-\beta)}{p-v} \right) \right) \quad (24)$$

The objectives of all these risk models are to find optimal order quantities and optimal costs. In the expected value criterion case, I find the optimal cost; whereas, in the CVaR case I find the optimal CVaR, which is also a form of cost (Cheng. et al., 2009). I apply these concepts to my models developed in the previous chapters. Note that I do not use forecast updating in my RA models. However, if the demand distribution parameters are based on forecast information, these models become equivalent to forecast updating models.

### 5.3 Single Product Single Location and Single Order Model

In this section I model distribution of a single product to each affected person in a single neighborhood. The objective function (22) of the single location single product model is equivalent to

$$\text{Min TC}(y, \xi) = cy + p \int_y^\infty (\xi - y)\varphi(\xi) d\xi - v \int_{-\infty}^y (y - \xi)\varphi(\xi) d\xi \quad (25)$$

The optimal CVaR can be calculated by substituting the optimal order quantity obtained from expression 23 in expression 25 (Cheng. et al., 2009). Note that the objective (25) function does not have any explicit constraints except non-negativity of the decision variable ( $y$ ). In contrast, CVaR models set an implicit target value for the cost or profit. Though this target value is not explicitly imposed on the unconstrained CVaR objective function, the constraint is explicitly imposed on discrete linear programming based CVaR models. However, the implicit target cost is imposed in the form of  $\alpha$ .

Proposition 5.1

- a) The optimal CVaR is increasing in  $\beta$
- b) Risk-neutral optimum order size is the lower bound for RA optimal order size

The proof of proposition 5.1a is straight forward. Proposition 5.1b, the optimal order size of risk-neutral model is the lower bound for RA model, is shown by comparing the optimal solution of risk neutral (RN) model with RA model. When a decision maker is risk neutral  $\beta=0$ . Substituting  $\beta=0$  in expression 23, I get  $y^* = \Phi^{-1}\left(\frac{p-c_2}{p-v}\right)$  (26). Expression 26 is the optimal order size of risk neutral model. I show this by a numerical illustration. Assume a numerical instance with the following parameters:  $c_2 = \$16$ ,  $p = \$23$  and  $v = \$8$ . Gotoh and Takano (2007) does not recommend normal distribution to model the demand because its cumulative distribution is above zero at zero (i.e.,  $F(0) > 0$ ). However, for deriving insights, they recommend normal distribution. Considering this argument, I assume that the demand distribution is exponential with  $\lambda=0.01$ . I calculate optimal order size, VaR, and CVaR. I analyze the impact of  $\beta$  on these three results.

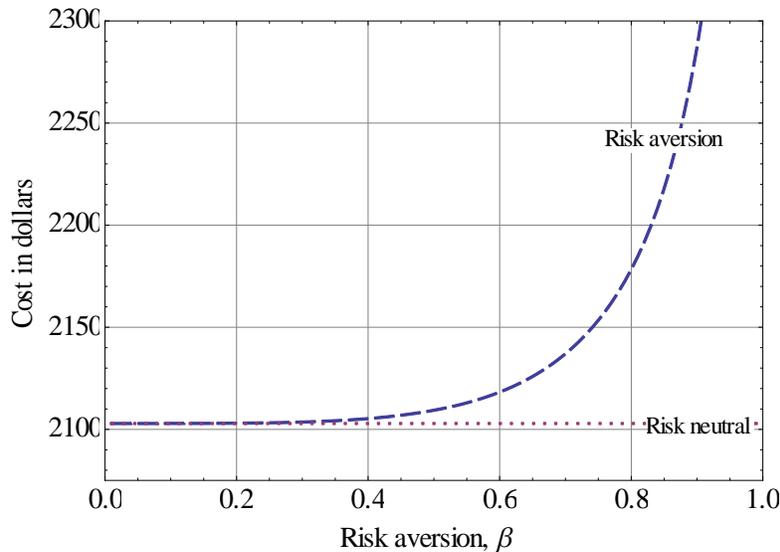


Figure 5-1 Risk-aversion Versus CVaR

For a given  $\beta$ , expression 23 yields the optimum order size; expression 24 yields VaR; expression 22 provides CVaR. For a given  $\beta=0.90$ , the optimal order size, VaR and CVaR are calculated respectively as 139.34 units, \$1076.47, and \$2287.06. In contrast, if the risk-neutral approach is used, then the corresponding optimal order size and cost are respectively 62.86 units and \$ 2102.89. Compared to the risk-neutral order size, RA order size is higher when I minimize costs. Figure 5-1 shows the impact of  $\beta$  on the optimal CVaR. Note that when  $\beta$  is zero, the optimal cost of both the risk-neutral and RA models converge at \$2102.89. When  $\beta$  increases from 0, the optimal cost also increases from \$2102.89. This illustrates that optimal CVaR is increasing in  $\beta$ . In addition, this trend also shows that a RN decision maker incurs lesser cost than RA decision maker. Furthermore, if a RA decision wants to avoid risk completely, he has to incur very high cost. Relief agencies as decision makers have to assess additional constraints if any besides the above insights, before choosing their order size; otherwise, agencies cost will increase more than 200%.

#### Proposition 5.2

- a) The optimal order quantity is increasing in risk-aversion  $\beta$
- b) The optimal order quantity is decreasing in  $c_2$
- c) The optimal order quantity is increasing in  $p$
- d) The optimal order quantity is increasing in  $v$

The proof is straightforward from the first order conditions of the respective parameters. Figure 5-2 depicts the impact of  $\beta$  on the optimal order quantity. Note that when  $\beta$  is zero, both the risk perceptions provide the same optimal order size, 62.86 units. The optimal order size is increasing in  $\beta$ . When  $\beta$  increases from zero, the optimal order size also increases from its current value of 62.86. As mentioned in the previous section, this risk-neutral order size acts as a

lower bound for RA order size. This justification also holds for optimal cost. As explained earlier for the total cost, the non-linearly increasing trend causes higher order size for higher risk-aversion. An agency exhibiting no risk orders just 62.86 units; whereas, another one exhibiting 85% risk aversion orders 125 units. This 85% risk aversion results in more than 200% increase in the overall cost. Ordering large order sizes not only leads to increased cost but also to increased transportation and holding costs. Evaluation of all these factors is vital before making a decision. The increasing trend of order size and cost are also reported by Gotoh and Takano, (2007), when they use normal distribution. Therefore, the results are applicable to normally distributed demand.

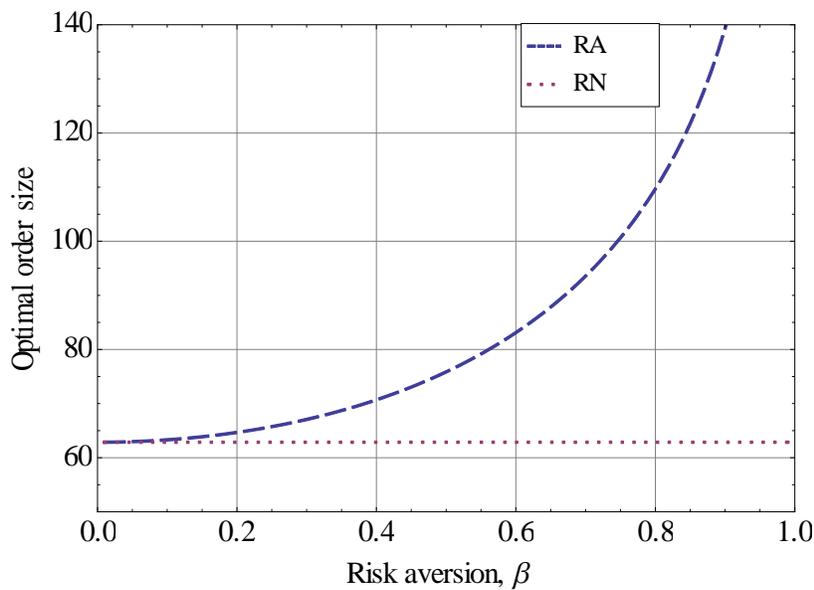


Figure 5-2 Risk-aversion Versus Optimal Order Size

Figure 5-3 illustrates the impact of marginal changes of  $c_2$  on optimal order size. For a given risk aversion (0.9), an increasing  $c_2$  decreases the optimal order size. When  $c_2$ =\$14, the risk averse optimum order size is 195.61 units and risk neutral optimum order size is 91.63 units. When  $c_2$  increases to 20, the risk averse optimum order size decreases to 52.13 and risk neutral

optimum order size decreases to 22.31. This shows that the rate of decrease is higher in risk-averse order size than risk neutral order size for a given  $c_2$ .

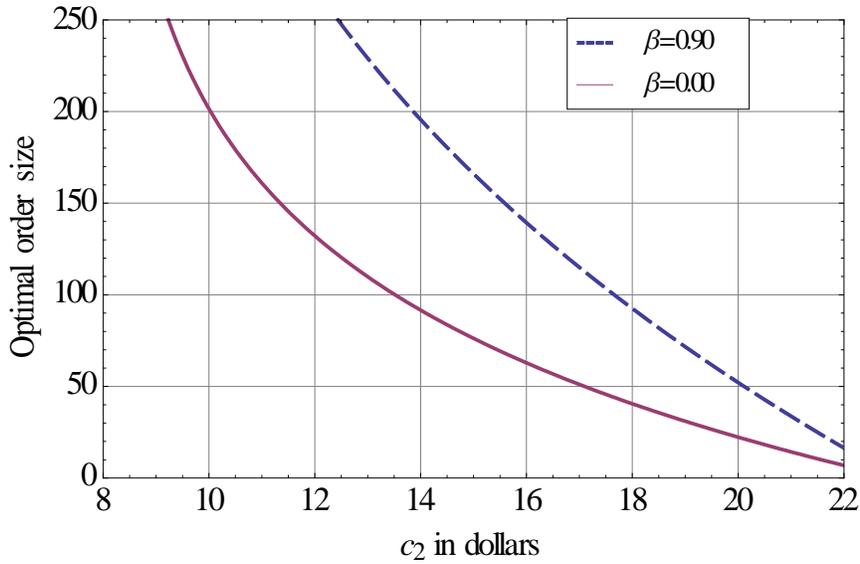


Figure 5-3 Impact of  $c_2$  on Optimal Order Size

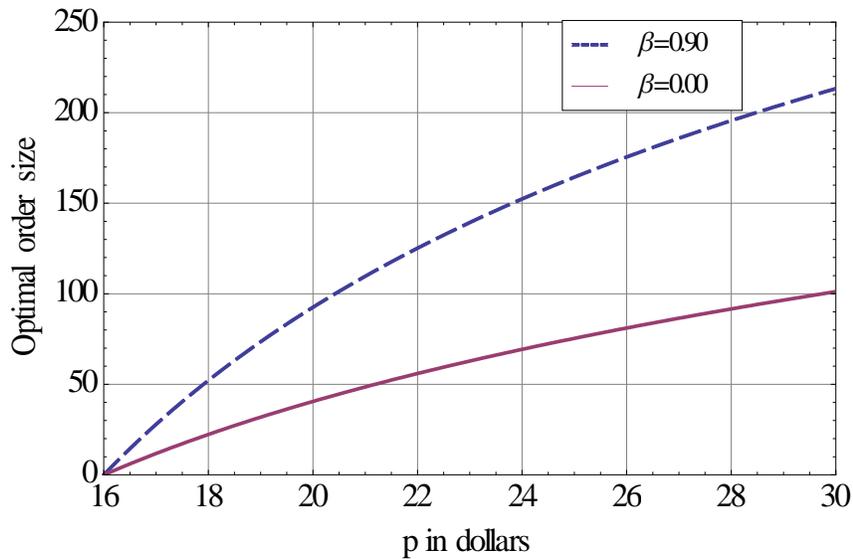


Figure 5-4 Impact of p on Optimal Order Size

Unlike the trend of  $c_2$ , when the spot market price increases, both risk neutral and risk-averse optimum order sizes increase. The difference of order size between risk neutral and risk-averse perception increases in p. For  $p = \$22$ , the risk-averse optimum order size is 125.17 units

and risk neutral optimum order size is 55.96 units. When  $p$  increases to 28, the risk averse optimum order size increases to 195.61 and risk neutral optimum order size also increases to 91.63. This shows that for a given  $p$ , the rate of increase is higher in risk-averse order size than risk neutral order size. Figure 5-4 explains proposition 5.1c.

Similar to the trend of  $p$ , the optimal order size is also increasing in salvage value as shown in Figure 5-5. For  $v = \$8$ , the risk-averse optimum order size is 139.34 units and risk neutral optimum order size is 62.86 units. When  $v$  increases to 14, the risk averse optimum order size increases to 297.87 and risk neutral optimum order size also increases to 120.34. Though optimal order size is increasing in  $p$  and  $v$ , the increasing  $p$  exhibits a diminishing trend; whereas, increasing  $v$  exhibits an increasing trend.

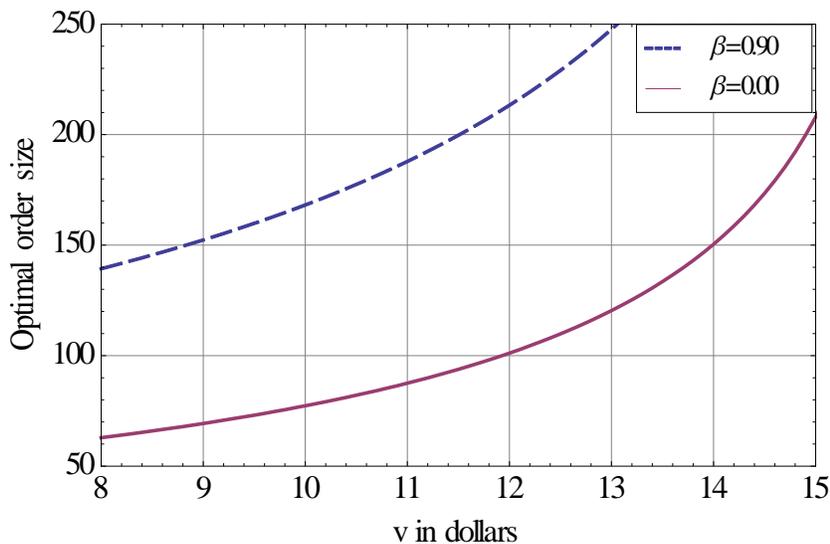


Figure 5-5 Impact of  $v$  on Optimal Order Size

#### 5.4 Dual Risk Perspective Model

So far, I have discussed models exclusively with either RN or RA perspective. In this section, I formulate a model to address multiple perspectives of relief agencies. Relief agencies adopt differing perspective when they are forced to purchase only certain products instead of all

products. For example, when a victim needs two products, for example water and shelter, providing water is more important than providing shelter if the agency is able to offer only one. Under such a circumstance, water becomes more critical product than shelter. Many practical circumstances force agencies take such decisions, for instance, during lack of funds, shortage of supplies, shortage of volunteers, contamination of supplies, etc. Such situations demand a model, which classify products exclusively as either the most critical or the least critical. Products identified as the most critical need RA perspective while the least critical ones need risk-neutral perspective. This model considers only two products; one is identified as the most critical and the other is the least critical one. Before formulating the model, I define the following notion.

#### Parameters

$p^N$  : Expected spot market price of a RN product

$c_2^N$  : Second instance cost of one unit of a RN product

$v^N$  : Salvage value of a RN product,

$\xi^N$  : Random demand of RN product, i.e., people affected by the disaster

$\mu^N$  : Mean demand of the risk-neutral product

$\sigma^N$  : Standard deviation of the of the risk-neutral product

$p_1^A$  : Expected spot market price of a RA product

$c_2^A$  : Second instance cost of one unit of a RA product

$v^A$  : Salvage value of a RA product,

$\xi^A$  : Random demand, i.e., people affected by the disaster

$\mu^A$  : Mean demand of RA product

$\sigma^A$  : Standard deviation of RA product

Since minimizing the cost is the objective of relief agencies, I consider only RN and RA perspectives in this section. The dual risk model is formulated as

$$\text{Min TC}(y^N, y^A) = \begin{cases} c_2^N y^N + p_i^N \int_{y^N}^{\infty} (\xi^N - y^N) \varphi(\xi^N) d\xi^N - v \int_{-\infty}^{y^N} (y^N - \xi^N) \varphi(\xi^N) d\xi^N \\ + c_2^A y^A + p_i^A \int_{y^A}^{\infty} (\xi^A - y^A) \varphi(\xi^A) d\xi^A - v \int_{-\infty}^{y^A} (y^A - \xi^A) \varphi(\xi^A) d\xi^A \end{cases} \quad (27)$$

$$y^{N*} = \Phi_N^{-1} \left( \frac{p^N - c_2^N}{p^N - v^N} \right) \quad (28)$$

$$y^{A*} = \frac{c_2^A - v^A}{p^A - v^A} \Phi^{-1} \left( \frac{(p^A - c_2^A)(1 - \beta)}{p^A - v^A} \right) + \frac{p^A - c_2^A}{p^A - v^A} \Phi^{-1} \left( \frac{(c_2^A - v^A)\beta + (p - c_2^A)}{p^A - v^A} \right) \quad (29)$$

$$\text{Min TC}^*(y^{N*}, y^{A*}) = \begin{cases} c_2^N y^{N*} + p_i^N \int_{y^{N*}}^{\infty} (\xi^N - y^{N*}) \varphi(\xi^N) d\xi^N - v \int_{-\infty}^{y^{N*}} (y^{N*} - \xi^N) \varphi(\xi^N) d\xi^N \\ + c_2^A y^{A*} + p_i^A \int_{y^{A*}}^{\infty} (\xi^A - y^{A*}) \varphi(\xi^A) d\xi^A - v \int_{-\infty}^{y^{A*}} (y^{A*} - \xi^A) \varphi(\xi^A) d\xi^A \end{cases} \quad (30)$$

Though I consider only one product of each risk type in expression 27, this model can be scaled up to include more than one product. Expression 28 yields the optimum order size for the least critical product; while, expression 29 yields the optimum order size for the most critical product. Equation 30 gives the total cost of these products. Equation 29 becomes 28 if  $\beta$  becomes zero. The only difference then is the cost structures.

Expressions 27–30 represent the model for dual risk perspective products. The objective function (27) minimizes the cost. The dual risk model (27) differs from the models introduced in the previous chapters in three respects. First, the risk neutral models of previous chapters are not decomposable; whereas, the dual risk model is decomposable to individual products. Second, the

risk neutral model uses a single demand distribution; whereas, the demand distribution of dual risk model need not be the same. Third, I can get the solution of risk neutral model by solving the problem once; whereas, dual risk models need to be solved multiple times equivalent to the number of products involved.

In order to compare the costs of the most and the least critical products, I have solved a numerical problem with the following parameters:  $c_2 = \$16$ ,  $p = \$23$  and  $v = \$8$  and  $\lambda = .01$ . I further assume that cost structures of both the products are the same. The risk aversion parameter ( $\beta$ ) reflects the degree of criticality of a product. Higher values of  $\beta$  indicate that the product is more critical one. While comparing the optimal order sizes of both the risk perceptions, the optimum order size of the most critical product is greater than or equal to the least critical product. Figure 5-6 explains the optimum order size of these perceptions versus the risk aversion.

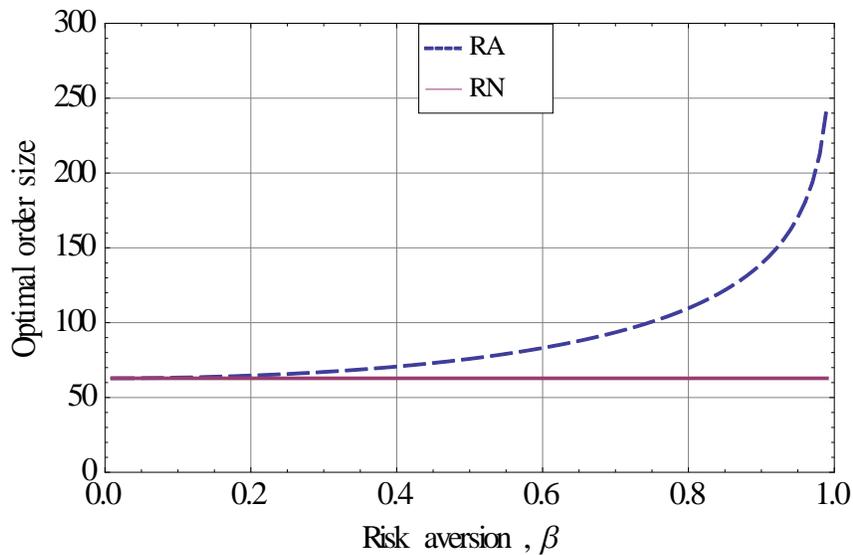


Figure 5-6 Optimal Order Size of RA and RN Products

The horizontal line of Figure 5-7 explains the total cost of the least critical product. Since I use risk-neutral approach to calculate the total cost of the least critical product, the cost

remains the same for all values of  $\beta$ . Note that  $\beta$  and the total cost of the risk-neutral solution are independent. In contrast,  $\beta$  and the total cost of RA approach are dependent; therefore, the total cost of the most critical product increases with  $\beta$ .

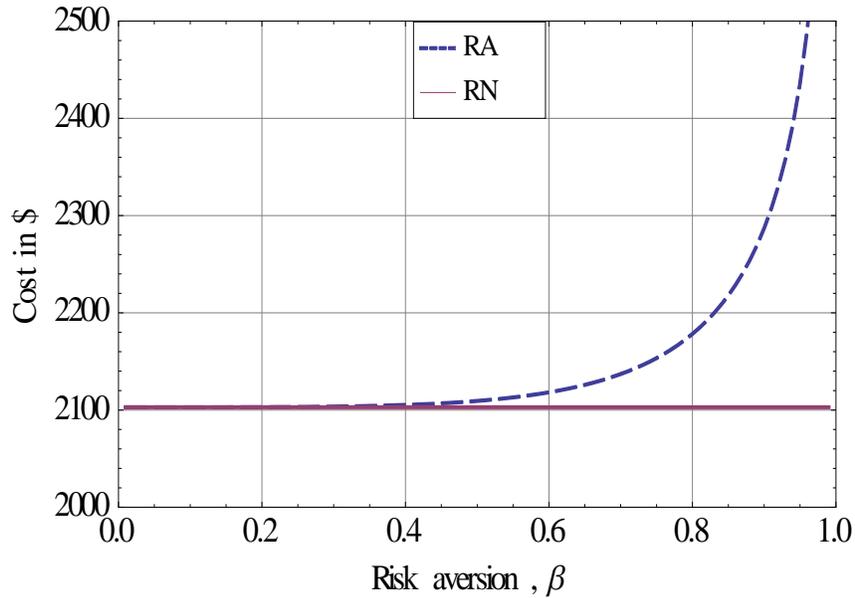


Figure 5-7 Cost of RA and RN Components

Moreover, the rate of change of total cost of the most critical product increases if  $\beta$  increases. If  $\beta$  approaches from 0.5 to one, the cost increases by almost 100%. Therefore, relief agencies have to choose their level of risk aversion appropriate to how efficient they plan their relief operations. Though this model implicitly assumes a single location, the model can be extended further to include multiple locations, multiple products and products in multiple units. The following section analyzes the behavior of a RA newsvendor in the context of cost minimization problems.

Observation 5.1: A RA decision maker orders more than the risk-neutral decision maker if  $p \geq 2c_2 - v$  and vice versa.

Proof: It is known that the optimal order quantity of RA decision maker is

$$y^* = \frac{c_2 - v}{p - v} \Phi^{-1}\left(\frac{(p - c_2)(1 - \beta)}{p - v}\right) + \frac{p - c_2}{p - v} \Phi^{-1}\left(\frac{(c_2 - v)\beta + (p - c_2)}{p - v}\right) \quad (23)$$

When  $p$  is replaced by  $2c_2 - v$  in the above expression 23, expression 23 becomes expression 26. In other words, RA optimal order size and RN optimal order size becomes equal. Note that expression 26 is the expression to calculate optimal order size of risk-neutral model. Note that when  $p \geq 2c_2 - v$ , the critical fractile is greater than 0.5. In other words, I can alternatively state that when the critical fractile is greater than 0.5, a RA decision maker orders more than a risk-neutral decision maker. Figure 5-8 shows that when  $p = 2c_2 - v$ , the order sizes of RN and RA decision makers coincide. The resulting order size is equal to RN order size and the corresponding order size is the mean of the distribution.

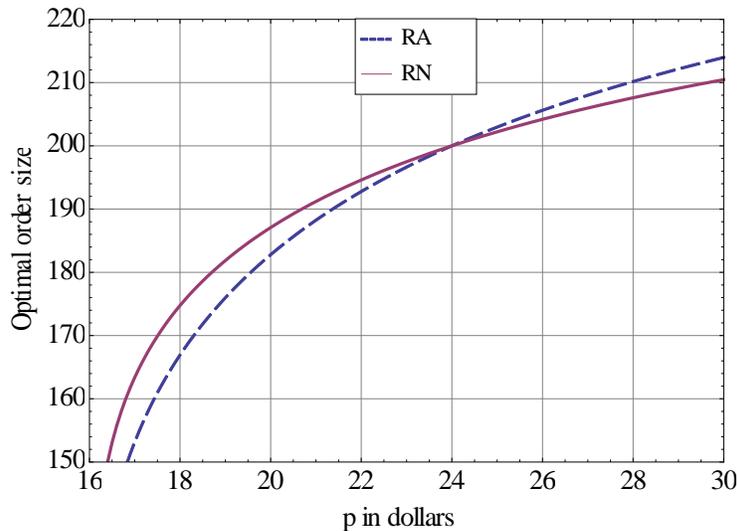


Figure 5-8 Impact of Price ( $p$ ) on RA and RN Order Sizes

Figure 5-8 is drawn for a numerical instance with the following parameters:  $c_2 = \$16$ ,  $p = \$23$  and  $v = \$8$ ,  $\mu = 200$ ,  $\sigma = 30$  and  $\beta = 0.70$ . The demand is assumed normally distributed. Note that when  $p = 2c_2 - v$  (*i.e.*, 24), RN and RA order sizes are equal irrespective of the degree of risk-aversion. RA and RN order sizes increase when  $\sigma$  increases from 30 while

maintaining the pivot at  $p=24$  and  $\mu = 200$ . Knowing that the spot market price is higher, RA decision maker orders more to reduce the under ordering cost.

Observation 5.2: A RA decision maker orders less than the risk-neutral decision maker if  $c_2 \geq \frac{p+v}{2}$  and vice versa.

The proof is straight forward. When  $c_2$  is replaced by  $\frac{p+v}{2}$  in RA optimal order size, RA optimal order size becomes equal to RN order size. For various values of  $c_2$ , Figure 5-9 depicts the order sizes of RA and RN. Note that when  $\frac{p+v}{2} = 15.5$ , the order sizes of RA and RN becomes equal and coincide at the mean of the distribution. Since  $c_2$  of the problem is 16 (i.e., above 15.5), RA order size is lower than the RN one. For any  $c_2$  above 15.5, the observation 5.2 holds true.

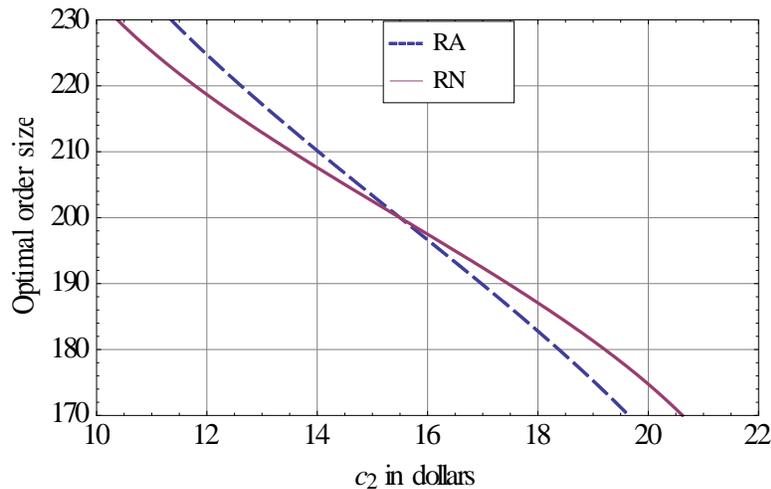


Figure 5-9 Impact of Cost  $c_2$  on RA and RN Order Sizes

Observation 5.3: A RA decision maker orders more than the risk-neutral decision maker if  $v \geq 2c_2 - p$  and vice versa.

The proof is straight forward. When  $v$  is replaced by  $2c_2 - p$  in RA optimal order size, RA optimal order size becomes equal to RN order size. Figure 5-10 shows the RA and RN order

sizes for various values of salvage values ranging from 5 to 15. Similar to what has been observed in case of  $p$  and  $c_2$ , when  $v$  (\$8) is greater than  $2c_2 - p = 9$ , RA decision maker orders more than the RN decision maker. Since the salvage value of my problem is \$8, at \$8, RA decision maker orders less than RN one. What this means to relief agencies is that when a RA decision maker knows that the salvage value is less, ordering more units may lead to higher over ordering cost. In order to avoid such over ordering costs, RA decision maker orders less.

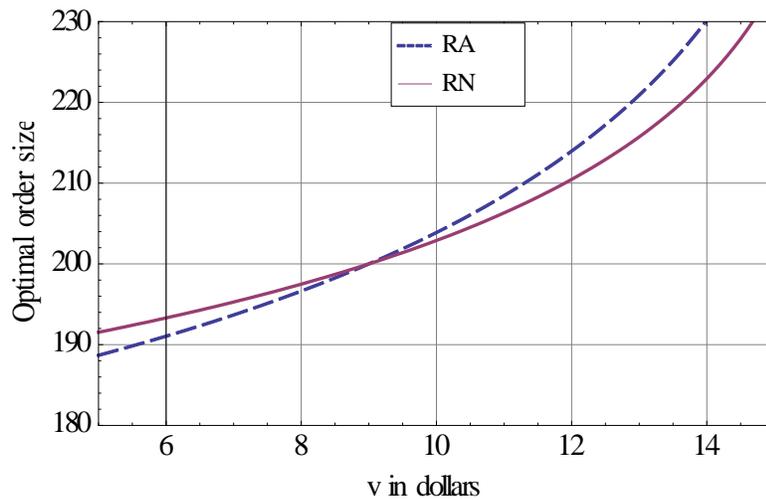


Figure 5-10 Impact of Salvage Value on RA and RN Order Sizes

### 5.5 Limitations

The parameters such as price, cost, and salvage value of the relief products under consideration are assumed deterministic. In reality, these parameters are stochastic. However, by signing contracts with suppliers, cost at both instances can be made deterministic. Since the leftover units are salvaged at the local market, salvage value is stochastic and its estimation becomes difficult. Further, the spot market price is also stochastic contrary to its deterministic consideration in this study. In this study, the first instance order quantity is fixed arbitrarily. However, by considering various constraints of relief agencies, this first order size can be calculated objectively. My models are not validated using real post disaster data. Validating and

refining the model on the basis of post disaster data will improve the applicability of the model. In addition, classification of relief supplies –as most or least critical– is also difficult in most of the real contexts.

## 5.6 Conclusion

In the first part of this dissertation, I have formulated the preliminary model, which subsequently becomes building block for other enriched models. The basic model examines procurement of a single product at dual instances. Besides these instances, an emergency purchase is initiated if the realized demand exceeds the available inventory. The preliminary model has been extended to incorporate several real issues such as distribution of multiple products to a single site. Moreover, the model has been extended to capture different mixes and volumes of these life-saving products. Comparative statics analyses have been performed for deriving insights. When more than two products with different cost structures are procured as bundles, the cost of the bundle is found to be higher than that of when these products are purchased independently. The practical issues of independent procurement of products are analyzed.

The second set of models has considered planning for multiple sites. I have used MMFE to estimate the pooled demand and variance of affected sites. This pooling has reduced the bullwhip effect of the demand. These models infer that when the standard deviation of the assumed demand distribution decreases, the total cost decreases. These models have pooled the demand arising at several locations in the aftermath of disasters. However, this pooling of demand may not be realistic when the affected areas are inaccessible due to breakage of physical and communication links. This will result in incomplete demand information. Under such circumstances, I assign a correlation coefficient. This correlation relates two locations, one with

known demand and another with unknown demand. The second set of models considered both partial and complete demand information. These models have also considered the quality of information. Sometimes, due to the skill of the assessor or the techniques one employs, the quality of the information may not be perfect. The second set of models has analyzed the impact of quality of demand data on the optimal order quantity and order cost. I have analyzed the second set of models using HAZUS generated data. I have found that the increasing quality of information leads to higher order size but results in lower cost. For a given level of information, the optimal cost of complete demand information is less than that of the partial information. I have also listed several potential incorporations of realistic issues such as constraints specific to locations, suppliers, and products. Moreover, I have discussed the applicability of these models at the tactic levels to plan for a single location besides their applicability in a strategic level to plan for a region. These models are capable of capturing discounts on products and buy-back options of surplus products among other issues.

Finally, I have introduced models based on CVaR. These models include RA and RN perceptions of the decision makers. Comparative statics has been performed to analyze the directional relationships between the relevant parameters and the optimal order size and cost. The relief supplies are classified into two categories as the most and the least critical ones. The cost differences between these two categories have been analyzed for a given degree of risk aversion. A numerical problem has been solved to evaluate the magnitude of cost differences between RN and RA decision approaches.

### 5.7 Scope for Further Research

There are several potential extensions of this paper. For example, my models do not impose any constraints on order sizes and lead times. Such constraints can be imposed. Current

models group products to locations. Instead, if products are grouped to suppliers, agencies can compare suppliers' quotes and choose cost effective suppliers. Further issues such as quantity discounts on order sizes, transportation and warehousing costs, supply and warehousing capacities and buy-back options can be included. In addition, instead of risk-neutral approach, risk seeking and/or risk-averse perspectives may be considered. Besides forecast information, information from the site volunteers, satellite images and other sources can be integrated.

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