THEORY AND PRACTICE IN BOOK 2 OF UGOLINO'S (c. 1380-1457) *DECLARATIO MUSICAE DISCIPLINAE*

Joseph Turner, B.M., M.M.

Dissertation Prepared for the Degree of

DOCTOR OF PHILOSOPHY

UNIVERSITY OF NORTH TEXAS

August 2021

APPROVED:

Frank Heidlberger, Major Professor
Paul Leenhouts, Committee Member
Hendrik Schulze, Committee Member
Benjamin Brand, Chair of the Division of Music History, Theory, and Ethnomusicology
Jaymee Haefner, Director of Graduate Studies of the College of Music
John W. Richmond, Dean of the College of Music
Victor Prybutok, Dean of the Toulouse Graduate School

Turner, Joseph. *Theory and Practice in Book 2 of Ugolino's (C. 1380-1457) "Declaratio musicae disciplinae."* Doctor of Philosophy (Music), August 2021, 169 pp., 1 table, 10 figures, 5 musical examples, bibliography, 126 titles.

Ugolino (c. 1380-1457) wrote one of the largest treatises on music theory in the first half of the fifteenth century. This work, the *Declaratio musicae disciplinae*, is comprised of five books that cover everything a musician of the era would need to know, from plainchant to harmonic proportions, from musica practica to musica speculativa. However, the treatise has received contradictory interpretations by modern scholars, some viewing it as mainly practical, others as mainly theoretical. I argue that in Book 2, which deals with counterpoint, Ugolino crystallizes the relationship between theory and practice, while offering distinctive contrapuntal practices. Ugolino presents a unique view music's place in the structure of knowledge, one which is highly dependent on Aristotelian philosophy. He posits that music is a science and that it is a branch not of mathematics, as it had traditionally been categorized, but of natural philosophy. This viewpoint shapes the entire treatise and is evident in the book on counterpoint. There, he presents an Italian tradition of teaching counterpoint known as the "regola del grado." Ugolino is the first author to present this tradition entirely in Latin. In addition, he offers an unusual description of musica ficta. In it, he presents a diagram, the *duplex manus*, that mixes together both musica recta and musica ficta. Ugolino's work suggests that theory and practice, although arranged hierarchically, need not be in conflict, and that a treatise such as his can be both eminently practical and highly theoretical.

ACKNOWLEDGEMENTS

I could not have finished this dissertation without the help and support of many individuals. I am deeply thankful to Frank Heidlberger for inspiring me to study the history of music theory. I offer my thanks to my other committee members Hendrik Schulze for his insight and Paul Leenhouts for his knowledge of early music repertoire; and to my friends Andrew Vagts, Doug Donley, and José Barnett for the many conversations, commiseration, and encouragement.

I especially thank my parents, John and Marsha, for believing in me when I did not believe in myself; my brother Matthew for helping to relieve stress by playing *Magic: The Gathering*; my sister Jayne for all her encouragement; my brother Timothy for his deep conversations; and my wife Jessica for her constant devotion. Without their love, support, and prayers, I could not have completed this project.

TABLE OF CONTENTS

Page

LIST OF TABLES AND FIGURES

Page

Tables

Figures

LIST OF MUSICAL EXAMPLES

Page

CHAPTER 1

INTRODUCTION AND HISTORICAL BACKGROUND

1.1 Introduction

Ugolino of Orvieto (c. 1380-1457) wrote one of the largest treatises on music theory in the first half of the fifteenth century. This work, the *Declaratio musicae disciplinae*, composed sometime in the 1430s or 1440s, is comprised of five books, each with its own philosophical preface, and covers everything he thought a musician should know, from plainchant to harmonic proportions, from *musica practica* to *musica speculativa*. It formed the basis of Franchino Gafori's early musical education, and it received attention from the likes of Bartolomé Ramos de Pareja, who thought it worthy enough of critique.[1] Yet it has received considerably less attention than treatises from the end of the fifteenth century, such as those by Johannes Tinctoris or Gafori, or ones from the century before, such as those by Jacobus of Liège or Marchetto of Padua. Indeed, other treatises from the first half of the fifteenth century also remain relatively under-explored. This period, which witnessed great social and cultural changes, lies on an uncomfortable boundary between late medieval scholasticism and nascent renaissance humanism, between the height of Aristotelian philosophy and the rediscovery of Plato's works.

When Ugolino's treatise undergoes investigation by modern commentators, they tend to hold contradictory interpretations, some viewing it as mainly practical, others as mainly theoretical. These conflicting views provoke a question: what is the relationship between theory and practice as it is set forth by Ugolino? In book 2 of the *Declaratio*, which deals with counterpoint, Ugolino's views on the relationship between theory and practice are crystallized,

[1] Ann E Moyer, *Musica Scientia: Musical Scholarship in the Italian Renaissance* (Ithaca, NY: Cornell University Press, 1992), 68-69.

particularly through his distinctive contributions. Therefore, book 2 forms a portrait by which to understand his philosophy on the relationship between theory and practice. But this portrait must be seen from the frame he sets up in book 1. There, Ugolino presents a unique view on music's place in the structure of knowledge, one that is highly dependent on Aristotelian philosophy. He argues that music is a science and that it is a branch not of mathematics, as it had been categorized in the past, but of natural philosophy. This viewpoint shapes the rest of the treatise, and it is evident in the book on counterpoint. Thus, I present Ugolino's Aristotelian position through in investigation of the distinctive features found in the book on counterpoint. In this chapter, I provide a historical background that details key events surrounding Ugolino's life and work, an overview of the structure of the treatise, and a survey of its reception. This survey shows some of the problems encountered in modern scholarship on Ugolino that, in part, this dissertation seeks to address. It raises some problems that I answer in later chapters. In chapter 2, I offer an account of how Ugolino defines music and its relation to Aristotelian philosophy. His understanding of Aristotle depends on the developments that took place in the previous century. These developments hinge on the way fourteenth century philosophers integrated mathematics into natural philosophy without abandoning Aristotle's philosophical outlook. In chapter 3, I turn to the book on counterpoint and describe Ugolino's classification of intervals, his account of perfection (akin to cadence), and the Italian method of counterpoint called the *regola del grado*. In chapter 4, I present Ugolino's views on musica ficta, since a diagram of altered notes has been the subject of debate. This debate centers on the role of the hexachord in the structure of diatonic music. Finally, in chapter 5, I offer a brief conclusion. Ugolino's work suggests that theory and practice, although arranged hierarchically, need not be in conflict, and that a treatise such as his can be both eminently practical and highly theoretical. Understanding theory and practice in this

way relies on a close reading of Ugolino's treatise.

1.2 Historical Background

Ugolino lived at a time rife with political upheaval and cultural change. Little is known of his early life, but he was born around 1380.[2] He probably received a university education, because in the *Declaratio* he often cites the works of Aristotle that were typically studied at universities, and his reasoning demonstrates familiarity with the scholastic method. In 1378, just about the time Ugolino was born, the Great Schism began, and it continued through 1417.[3] With several claimants to the papacy, factions arose around each, fracturing the political landscape. In an attempt to heal the divisions, clerics called for a special council. The Council of Constance, which Ugolino attended, took place between 1414-18.[4] It resolved to "heal the schism…extirpate heresy, and institute reform."[5] Reform—ecclesiastical, clerical, moral and educational—permeated nearly all levels of discourse. Early humanist authors responded by writing programs for educational reform, which often touch on music. For example, Pier Paolo Vergerio includes music as one of the studies worthy of a free person.[6] In the preface to book 2 of the *Declaratio*, Ugolino writes about what makes a person free, what makes a person a slave, and the kinds of

[2] Albert Seay, "Ugolino of Orvieto, Theorist and Composer," *Musica Disciplina* 9 (1955), 116.

[3] Gerald Christianson, *The Church, the Councils, and Reform: The Legacy of the Fifteenth Century* (Dexter, MI: Catholic University of America Press, 2008), 4. For a full examination of the council and translations of contemporary accounts, see John Hine Mundy and Kennerly M. Woody eds., *The Council of Constance: The Unification of the Church*, trans. Louise Ropes Loomis (New York: Columbia University Press, 1961).

[4] Lewis Lockwood, *Music in Renaissance Ferrara 1400-1505: The Creation of a Musical Center in the Fifteenth Century* (Cambridge, MA: Harvard University Press, 1984), 78.

[5] Christianson, *The Church, the Councils, and Reform: The Legacy of the Fifteenth Century*, 7. For a complete treatment of the idea of reform in view of the council, see Phillip H Stump, *The Reforms of the Council of Constance (1414-1418)* (Leiden: E.J. Brill, 1994).

[6] Pier Paolo Vergerio, "The Character and Studies Befitting a Free-Born Youth," in *Humanist Educational Treatises*, ed. Craig W. Kallendorf (Cambridge, MA: Harvard University Press, 2002), 53.

subjects a free person should study.[7] Aeneas Silvius Piccolomini, who became Pope Pius II, suggests that the seven liberal arts ultimately aim at acquiring philosophy.[8] Ugolino adopts a similar strategy, using *musica practica* as an essential step along a path towards a higher philosophy—in this case a knowledge of the proportions.[9] The impetus to reform likely animated Ugolino's writing and manifests itself in both the practical nature of his work and the ends to which he directs that practice.

The ecclesiastical context in which Ugolino worked gave him the motive and means to write such a large and thorough treatise. In 1425 Ugolino became an archdeacon, an important senior position in Italian cathedrals at this time.[10] Around 1431, he moved from Forlì to the cathedral of Ferrara, where he would remain until his death in 1457.[11] The Bishop Giovanni Tavelli da Tossignano played a central role in the ecclesiastical life in Ferrara, encouraging reform and providing strong leadership in his diocese.[12] Ugolino worked under him, sometimes even representing him outside of the cathedral.[13] Indeed, Ugolino attended the Council of Ferrara-Florence in 1437, where he could have met Guillaume Dufay.[14] In the 1430s or 1440s,

[7] Ugolino of Orvieto, *Declaratio musicae disciplinae*, ed. Albert Seay, 3 vols. (Rome: American Institute of Musicology, 1959-62), 2:1-3.

[8] Aeneas Piccolomini, "The Education of Boys," in *Humanist Educational Treatises*, ed Craig W. Kallendorf (Cambridge, MA: Harvard University Press, 2002), 255-57.

[9] Artistotle states that to understand nature, one must understand its sources (or principles). About the way one reaches these he writes, "the natural road is from what is more familiar and clearer to us to what is clearer and better known by nature; for it is not the same things that are well known to us and well known simply." Aristotle, *Physics* 1.1.184a18-19. For Ugolino's application of this idea in forming the structure of his treatise, see Ugolino, *Declaratio*, 3:87.

[10] Seay, "Ugolino of Orvieto," 116; Denys Hay, *The Church in Italy in the Fifteenth Century* (Cambridge: Cambridge University Press, 1977), 21.

[11] Seay, "Ugolino of Orvieto," 117; Lockwood, *Music in Renaissance Ferrara*, 78.

[12] Lockwood, *Music in Renaissance Ferrara*, 77-78.

[13] Lockwood, *Music in Renaissance Ferrara*, 78.

[14] Seay, "Ugolino of Orvieto," 117; Lockwood, *Music in Renaissance Ferrara*, 78.

he completed the *Declaratio*.[15] Ugolino was known both for his musical skill and for his treatise. A contemporary describes Ugolino as "very skilled in singing" (*modulandi pertissimus*).[16] In his *Italia illustrata*, Biondo Flavio, a layman who served in the papal administration, wrote,

> Ugolino by the name of Urbevetano born and raised in Forlì without objection surpasses all the musicians of our age, and the book published by him on music will render the labors of all who wrote before him of no account.[17]
>
> *Quid quod Ugolinus cognomine urbevetanus forlivii genitus et nutritus omnis aetatis nostrae musicos sine contradictione superat, aeditusque ab eo de musica liber haud secus omnium qui ante se scripserunt labores obscurabit.*

A richer interpretation of Ugolino's work results from an investigation not only of its social and cultural contexts but also of its intellectual and philosophical environment. Aristotelianism was the most prevalent system of thought in the early fifteenth century. Although most of Aristotle's works had been translated into Latin by the late twelfth century, it was not until the mid-thirteenth century that the entire corpus became the object of intense study.[18] In the late thirteenth century, theorists in Paris begin incorporating Aristotelian terms and concepts in music theory.[19] As the reception of the Aristotelian corpus continued, scholars in the fourteenth century increasingly turned their attention to Aristotle's *Physics* and began applying mathematics to the understanding of physical objects.[20] Unlike almost all of his contemporaries, Ugolino, in

---

[15] Seay, "Ugolino of Orvieto," 118.

[16] Quoted in Seay, "Ugolino of Orvieto," 117 (translation mine).

[17] For Biondo Flavio's role in the church see Hay, *The Church in Italy*, 44-45. The quote comes from Seay, "Ugolino of Orvieto," 118 and Evan A MacCarthy, "The Sources and Early Readers of Ugolino of Orvieto's Declaratio Musice Discipline," in *Beyond 50 Years of Ars Nova Studies at Certaldo 1959-2009*, ed. Agostine Ziino Marco Gozzi, and Francesco Zimei (Lucca: Libreria Musicale Italiana, 2014), 401-02 (translation mine).

[18] Bernard G. Dod, "Aristoteles Latinus," in *The Cambridge History of Later Medieval Philosophy: From the Rediscovery of Aristotle to the Disintegration of Scholasticism 1100-1600*, ed. Anthony Kenny Norman Kretzmann, and Jan Pinborg 1982), 48, 50.

[19] Jeremy Yudkin, "The Influence of Aristotle on French University Music Texts," in *Music Theory and its Sources: Antiquity and the Middle Ages*, ed. André Barbera (Notre Dame, IN: University of Notre Dame Press, 1990), 186, passim.

[20] For the application of mathematics to physical problems see Edward Grant, *A History of Natural Philosophy: From the Ancient World to the Nineteenth Century* (Cambridge: Cambridge University Press, 2007), 180-90.

book 1 of the *Declaratio*, categorizes music as a part of natural philosophy and not mathematics.[21] Yet he does not thereby dismiss the application of mathematics to music, because the last two books focus on interval proportions. Aristotelian philosophy and music theory are so often fused that one cannot understand the theory without understanding the philosophy. In describing the anonymous treatise from the Benedictine abbey of St. Emmeram, Jeremy Yudkin writes, "It is absolutely saturated with Aristotelian terms and procedures, and neither the structure of the treatise nor its content can be fully understood without cognizance being taken of this fact."[22] The same holds true for Ugolino's treatise, and therefore any attempt to understand it must also consider it in the light of Aristotelian thought. Combining his distinctive classification of music and the concomitant understanding of theory and practice, Ugolino offers a definition of the *musicus* or musician. Previous theorists typically followed Boethius in their understanding of what constitutes a true musician. Boethius prioritized reason at the expense of practical skill to form a three-step hierarchy of those involved in music. The lowest class consists of those who perform. The intermediary class make songs. The highest class, using reason, judges the performance and the work of those in the lower classes. The true musician is one who "exhibits the faculty of forming judgments according to speculation or reason relative and appropriate to music concerning modes and rhythms, the genera of songs, consonances" and so on.[23] Ugolino

---

Aristotle defines nature as "a certain source and cause of being moved and of coming to rest in that to which it belongs primarily, in virtue of itself and not incidentally." Aristotle, *Physics* 2.1.192b21-24. Each object of nature is composed of both form and matter or material. Edward Grant notes that Aristotle separated mathematics from physics. Grant, *A History of Natural Philosophy*, 158. Indeed, Aristotle argues that to apply mathematics to physical objects one must consider objects not as a whole, comprised of both form and material, but must separate the two in some way. He instead prefers to see a natural object as a whole. Aristotle, *Physics* 2.2.193b-194b18.

[21] Joseph Dyer notes that Ugolino classified music in this way, but he does not fully explain what the significance of this might be or its effect on Ugolino's treatise. Joseph Dyer, "The Place of *Musica* in Medieval Classifications of Knowledge," *The Journal of Musicology* 24, no. 1 (2007), 46. I treat this topic at length in chapter 2.

[22] Yudkin, "The Influence of Aristotle on French University Music Texts," 185.

[23] Ancius Manlius Severinus Boethius, *The Fundamentals of Music,* trans. Calvin M. Bower (New Haven: Yale University Press, 1989), 50-51.

sees knowledge (*scientia*) as two separate states or conditions (*habitus*), one directing the mind to what is knowable, the other to work. Music, by analogy, also consists of two states, one directing a person to speculation or theory, the other to musical performance or practice.[24] The true musician has both states. Ugolino brings the two together in such a way that the true musician can capably perform but also uses this knowledge as a path towards philosophical investigation. The *musicus*, who had philosophical knowledge, was a socially recognized and valued identity in the first half of the fifteenth century. For example, the English theorist and composer John Hothby, recognized as a *musicus*, was not praised for his skill in performance but for his ability in *musica speculativa*.[25] Rob Wegman has pointed out several composers who were given the title *musicus*, and he notes that this term "denoted social status and public respect."[26] Thus, Ugolino's definition had a meaning immanent in the society of his day. I devote the entirety of chapter 2 to understanding Ugolino's distinctive application of Aristotelian philosophy and how this shapes both the relationship between theory and practice as well as the entire treatise.

1.3 Structure and Content of the *Declaratio*

Ugolino divides his encyclopedic treatise into five books, and each book begins with a philosophical preface. His ordering of topics differs from theorists who write treatises of a similar length. Therefore, I summarize the main contents of the books, after which I address the prefaces, since their content is only loosely related to that of the main books. Of all the topics

[24] Ugolino, *Declaratio*, 1:19.

[25] Benjamin Brand, "A Medieval Scholasticus and Renaissance Choirmaster: A Portrait of John Hothby At Lucca," *Renaissance Quarterly* 63 (2010), 771.

[26] Rob C Wegman, "From Maker to Composer: Improvisation and Musical Authorship in the Low Countries, 1450-1500," *Journal of the American Musicological Society* 49, no. 3 (1996), 437-38.

that concern the "harmonic music…that we use in our times, five necessary ones, worthy of consideration, are included" (harmonicae musicae…qua nostris in temporibus usi sumus quinque necessaria atque consideratione digne comprehenduntur).[27] Book 1 starts with an overview of the whole treatise. In the first chapter, Ugolino describes the place of music as a discipline. Following Boethius, he separates it into *musica mundana*, *musica humana*, and *musica instrumentalis*.[28] He also includes a higher-ranking music from which all the others flow: *musica caelestis*. *Musica caelestis* is the song of the angels surrounding the divine majesty—a reference to the liturgical canticle, the Sanctus.[29] In chapter 2, he classifies music as a science, but unlike any author before him, he says that it belongs not under mathematics, its traditional place, but under natural philosophy. He offers several proofs for this claim, and it is a theme running throughout his work. Indeed, this is the subject I take up in chapter 2 of this dissertation. In chapter 3 of the first book of the *Declaratio*, Ugolino gives his own summary of the entire treatise. In chapter 4, he claims that "consonance is the goal of the whole discipline of music" (consonantia totius musicae finis est).[30] He treats the idea of consonance as the goal of music throughout the entire treatise and in particular in book 5. The rest of book 1 takes up all the topics associated with plainchant. He begins with an outline of the four strings of the Greek kithara, their names, how they relate to each other, and finally the strings added to them.[31] Next,

[27] Ugolino, *Declaratio*, 1:20. Unless otherwise noted, all translations from Ugolino's *Declaratio* are my own,

[28] Ugolino, *Declaratio*, 1:15-18; Boethius, *The Fundamentals of Music*, 9-10.

[29] Ugolino, *Declaratio*, 1:14; Seay, "Ugolino of Orvieto," 146-47.

[30] Ugolino, *Declaratio*, 1:22. For Ugolino, consonance as the goal (*finis*) of music indicates that each part of music (plainchant, counterpoint, etc.) is aimed at consonance. Consonance is a relationship between two things that finds its expression in properly proportioned numbers. To use Aristotelian terms, consonance is the final cause of music. However, consonance need not be the subject of every conclusion in syllogistic demonstrations. The goal (*finis*) is therefore not the same as the conclusions within the science of music. For a longer account of consonance as the goal of music, see Ugolino, *Declaratio,* 3:87-102. I discuss this topic in chapter 2.

[31] Ugolino, *Declaratio*, 1:23. He takes this format from Boethius, *The Fundamentals of Music*, 29-40.

he introduces the hexachord and mutation, intervals, then the modes.[32] He finishes book 1 by examining the modes of various liturgical melodies. The inclusion of liturgical music reinforces the fact that Ugolino intended his work to be used by people who would need to sing during the liturgy.

In book 2, Ugolino treats counterpoint or as he calls it, "melodied music" (*musica melodiata*).[33] In particular, he deals with two-voiced, note against note counterpoint. The foundation for this book is the *regola del grado*, an Italian method of teaching counterpoint that relies on a knowledge of the hexachords.[34] Before teaching this method, however, he establishes a few fundamentals. After defining counterpoint, he describes the origin of pitches: he sees all notes and intervals as originating in the seven notes that form a diatonic scale.[35] Consequently, the seven-note diatonic scale and octave equivalence are fundamental concepts for Ugolino.[36] He then classifies intervals as consonant and dissonant, as well as perfect and imperfect. The fact that some intervals are imperfect leads him to a discussion of perfection, which is a notion akin to cadence. After this, he deals directly with the *regola del grado*. I consider these topics in chapter 3 of this dissertation. Ugolino ends the book with a chapter on musica ficta, which is the subject of chapter 4.

In book 3, Ugolino considers mensuration by offering a long and extensive commentary

[32] For an examination of the connection between Ugolino's description modes and medieval surgery, see Luminita Florea, "The Monstrous Musical Body: Mythology and Surgery in Late Medieval Music Theory," *Philobiblon* 18, no. 1 (2013): 127-60.

[33] Ugolino, *Declaratio*, 2:3.

[34] Since I deal with this topic at length in chapter 3, my summary here is brief.

[35] Ugolino, *Declaratio*, 2:5-8.

[36] This point is particularly important to understand his comments both on the *regola del grado*, which relies heavily on the hexachords, as well as on musica ficta. I consider the former in chapter 3 and the latter in chapter 4.

on the *De musica mensurata* of Johannes de Muris.[37] He quotes a small part of the text from Johannes and then offers his own thoughts on it, as well as numerous musical examples. Indeed, the entire *Delcaratio* abounds with musical examples and examples of theoretical concepts. Albert Seay puts these examples in a separate booklet attached to each volume of his modern edition. Following his numbering in these booklets, there are an astounding 1,529 examples spread across all five books. Clearly, Ugolino was writing for practicing musicians. But his tone changes in book 4, where he turns to the mathematical and considers quantity. After providing the basics for the study of quantity, he reviews the quantities that relate to the intervals he first introduced in book 1. Whereas in book 1 he speaks of the ditone as a certain distance in sound and describes where it can be found on the musical hand together with the necessary solfege syllables, in book 4 he gives the mathematical proportion for the ditone and its constituent parts.[38]

Finally, in book 5, Ugolino reaches fully speculative ground.[39] He begins the book with several questions, which he answers in scholastic fashion by noting past authors and making divisions and distinctions. For example, in chapter 1, he asks what the subject of music is and "whether sound is its subject" (an sonus sit subiectum eius). [40] In addition, he gives demonstrations why a certain interval is the size that it is. Concerning the purpose of book 5, he

[37] Seay, "Ugolino of Orvieto," 112.

[38] Cf. Ugolino, *Declaratio*, 1:51 with 3:55-56. For the topics he covers in Book 4, Ugolino likely drew upon an anonymous set of mathematical questions attributed to the Italian philosopher Biagio Pelacani. Evan MacCarthy, "Transformations in Music Theory and Music Treatises," in *The Cambridge History of Fifteenth-Century Music*, ed. Anna Maria Busse Berger and Jesse Rodin (Cambridge: Cambridge University Press, 2015), 605. The connection between Ugolino's text and the set of questions has been explored more fully in Cecelia Panti, "Una fonte della *Declaratio musicae disciplinae* di Ugolino da Orvieto: Quattro anonime 'Questiones' della tarda Scholastica," *Rivista italiana di musicologia* 24 (1989): 3-47.

[39] Seay, "Ugolino of Orvieto," 150.

[40] Ugolino, *Declaratio*, 1:87. Since several of these questions play a role in understanding the nature of the relationship between theory and practice, I discuss aspects of them in chapter 2 of this dissertation.

states that it

> is a consideration of the whole of musical knowledge. To it belongs understanding the first causes of everything that was previously said, from the first principles all the way to the elements, giving the reasons for them, comprehending the reason of practical music by the speculative intellect, and in it examining with the clear mirror of understanding.[41]
>
> *est totius musicae intelligentiae speculatio, cuius est omnium praedictorum causas primas, principia prima usque ad elementa cognoscere, eorum rationes reddere, practicae musicae intellectu speculativo rationem comprehendere, et in ea perspicaci intelligentiae speculo specular.*

Therefore, over the course of the entire treatise, Ugolino moves from musical practice to musical theory. The movement from practice to theory is a recurring theme in both Ugolino's treatise and this dissertation.

In addition to the movement of his books from the practical to the theoretical, Ugolino also attaches a short philosophical preface or *proemium* to each book. In the preface to book 1, Ugolino sets the tone for the purpose of the whole treatise. He investigates and praises the intellectual power or intellect. He describes it as the "noblest powers of the soul" (potentiuarum animae nobilissima).[42] By its work, it can comprehend God; it can distinguish between species and genus; it can put together and divide. But for the intellect to act, it is necessary first to apprehend something through the senses. "It belongs to it [the intellect] therefore to know material things immaterially." (Ipsius est ergo materialia immaterialiter noscere.)[43] In short, Ugolino begins to present a theory of mind in order to point out that human beings differ from beasts because of the intellect, and that the intellect needs to receive something from the senses.[44] Thus, musical practice, sound, and the sensible matter of music contains what is related

[41] Ugolino, *Declaratio*, 1:21.

[42] Ugolino, *Declaratio*, 1:13.

[43] Ugolino, *Declaratio*, 1:14.

[44] Ugolino, *Declaratio*, 1:14-15; Seay, "Ugolino of Orvieto," 146.

to music theory, to numbers, and to the immaterial matter of theory. And the mind or intellect can distinguish in this what is true.[45]

In the preface to book 2, Ugolino contemplates the meaning of freedom and servitude and the studies which correspond to these.[46] It ties in to the first preface by referring to the primary status of the intellect. Subjects that teach what is good for the body or some other external good belong to the mechanical arts. But subjects which look not to some external good but an internal, intellectual one belong to the liberal sciences.[47] Ugolino, explicitly following Aristotle in book 8 of his *Politics*, numbers music among the liberal sciences, because "it makes people happy essentially by the complete work of the intellect both by itself and by the delight [it produces] as a consequence" (nam ex perfecta intellectus operatione essentialiter et per se et ex delectatione per consequens felicitatur homo).[48] In the second preface and the first, Ugolino emphasizes the role of the intellect in the pursuit of musical knowledge.

The preface to book 3 is the longest.[49] Ugolino connects the instruction of music with a traditional account of Aristotelian virtue ethics.[50] Consequently, music is a subject worthy of study, because it plays a role in moral education. A virtue lies at a mean between two extremes.

---

[45] I explore this topic extensively in chapter 2. Ugolino, *Declaratio*, 1:15.

[46] The preface appears in Ugolino, *Declaratio*, 2:1-3, and my paraphrase in the rest of the paragraph refers to these pages.

[47] Whereas most authors often refer to mechanical arts as *artes* and reserve the term *scientia* for the higher branches of learning or the liberal arts, Ugolino uses the term *scientia* for both. This implies that both are indeed knowledge, the literal meaning of *scientia*. The liberal arts are knowledge directed toward an internal good. They benefit the development of the mind and the virtues. The mechanical arts are knowledge directed toward external goods. They benefit individual bodies and communities. In addition, as I show in chapter 2, these two levels of science often relate to each other, such that the lower branches provide the bare facts: they explain that something is. But the higher branches offer reasons or causes for the facts provided in the lower branches: they explain that something is because of something else. The former explanation is referred to as a quia explanation, while the latter is called a propter quid explanation. In addition, the former are also called practical sciences and the latter speculative.

[48] Ugolino, *Declaratio*, 2:3; Aristotle, *Politics* 8.5.1339a11-1340b19.

[49] Ugolino, *Declaratio*, 2:54-59.

[50] Ugolino argues that music is an indispensable subject of study. He once again directly appeals to Aristotle. Indeed, Ugolino refers to this passage in Aristotle two times in this preface, as well as once in the preface to book 2.

Thus, it can be thought of as lying in a proportion in relation to its contraries. Music and the virtues are linked by at least two concepts. First, both are described as a *habitus*, or a condition of the soul. In his preface, Ugolino states that bravery, among others, is a *habitus*, and in book 1 he also refers to music as a *habitus*.[51] Second, both music and the virtues involve an appeal to proportions. Just as virtues are in a proportion between their contraries, so music is explained by proportion, not only in sound but also in time or duration. Music, as well as other sensible objects, can affect the soul. In particular, certain modes have certain affects, and Ugolino includes a long discussion of which modes have which affects. He says, for instance, that

> All philosophers profess that the Dorian melody is the most steadfast, and from this fact it agrees most of all with the virtue to which constancy of mind is said to be especially sought…[52]
>
> *Universi quidem melodiam Doristam stabilissimam esse philosophi profitentur, et eam ex hoc cum virtute maxime convenire ad quam mentis constantia potissime dicitur requisita…*

Since music can so affect the soul, and since virtue is a condition of the soul, music can influence or even reform moral virtue. It is therefore indispensable for the education of young people. Finally, towards the end of the preface, Ugolino explains what he intends to do in book 3 and the format he will use.

The preface to book 4 is the shortest. Ugolino reminds readers of the division of music into *mundana*, *humana*, and *instrumentalis*.[53] After giving a brief but eloquent summary of each, he concludes by noting what binds them together. He writes that they all deal with "proportions, and since a knowledge of these proportions is necessary for an understanding of music itself, practical and speculative" (proportiones versatur cumque ad ipsius musicae practicae ac

[51] Cf. Ugolino, *Declaratio*, 2:55 with 1:19. I describe *habitus* in chapter 2.

[52] Ugolino, *Declaratio*, 2:57.

[53] Ugolino, *Declaratio*, 3:1-2.

speculativae intellectionem sit ipsarum necessaria proportionum cognitio), he examines the topic of proportions in book 4.[54]

The preface to book 5 returns to the intellect and defines how it works, what it does, and how this shapes learning.[55] The preface to book 1 extolls the nobility of the intellect which distinguishes humans from other animals. The intellect as the distinguishing mark between humans and other animals sets a tone for the entire treatise as one that is aimed at instructing and developing the intellect. Indeed, "whatever is good and praiseworthy, of practice and theory" (practicae et theoricae quaeque bona et laudabilia) is done through the work of the intellect.[56] The intellect, then, is a power of the soul that both is acted upon and acts. In other words, it both perceives and thinks. Before the intellect is acted upon or acts, it is, according to Aristotle, a blank slate, ready to receive perceptions from the outside world.[57] It is acted upon by the external, physical world through the senses. After the intellect has received perceptions through the senses, it acts upon them by abstracting species from them. The intellect, as it is acted upon, is called the passive intellect. But the intellect also acts upon the objects it perceives. External objects are a combination of both matter and form. Since they are sensible in the way they exist, they are intelligible only in potency. The intellect acts upon the perceptions of these objects to abstract, that is, to distinguish the material from the form and to reach what is intelligible. Therefore, the intellect as it acts is called, as Aristotle also described it, the active intellect.[58] Once he summarizes the meaning of the active and passive intellect, he briefly reviews Plato's

[54] Ugolino, *Declaratio*, 3:2.

[55] In the following paragraphs, I summarize and paraphrase Ugolino, *Declaratio*, 3:85-87.

[56] Ugolino, *Declaratio*, 3:85.

[57] Ugolino appeals to Aristotle, *De Anima* 3.4429b23-430a9.

[58] To bolster his presentation of the passive and active intellect, Ugolino explicitly cites Aristotle, *De Anima* 2.5.416b31-418a9.

theory of the forms. But he ultimately rejects it, since "this opinion of Plato's is not commonly held by philosophers" (haec autem opinion Platonis communiter a philosophis non tenetur).[59]

Ugolino then divides the activity of the intellect into the speculative and the practical. Citing Aristotle, he notes that the goal of the speculative activity of the intellect is to distinguish true from false. But the goal of the practical activity of the intellect is to take the distinction between true and false and direct it according to right desire.[60] Truth is the object of the speculative activity of the intellect, while truth directed by right desire is the object of the practical activity of the intellect. Even though both depend on the active intellect, Ugolino asserts that the former is a higher goal than the latter, since the goal is truth itself.

Considering these two activities of the intellect leads Ugolino to comment on learning and why he has arranged the *Declaratio* the way he has. Once again, Ugolino directly cites and expands Aristotle's idea: "The natural road is from what is more familiar and clearer to us to what is clearer and better known by nature."[61] Likewise, "the natural order of learning," according to Ugolino, "is to proceed from those things that are more known to us to those that are less known" (procedere ab his quae sunt nobis magis nota ad ea quae sunt nobis minus nota, hic enim discendi naturalis est ordo).[62] This order is the driving force behind the structure of Ugolino's entire treatise. He states explicitly,

> We have decided that this work should follow this order of Aristotle from things more known, namely from the practice of plainchant, melodied music, and mensural music, as these are most fully established in the first three books, the knowledge
>
> *Et ideo hunc Aristotelis ordinem sequentes a magis notis cepimus hoc opus, scilicet, a practica musicae planae, musicae melodiatae, et musicae mensuratae, sicut in primis tribus libris plenissime constat, cuius practicae notitia magis est nobis*

[59] Ugolino, *Declaratio*, 3:86.

[60] For both claims, Ugolino cites Aristotle, *Nichomachean Ethics* 6.2.1139a26-29.

[61] Aristotle, *Physics* 1.1.184a18-19.

[62] Ugolino, *Declaratio*, 3:87.

whose practice is more known to us than the speculative [is]. We are led from its knowledge to a complete knowledge of it by theory.[63]	*speculatione nota, ex cuius cognitione in eius speculative ducimur perfectam notitiam.*

With these words, Ugolino ends the fifth and final preface.

In all five prefaces, Ugolino clearly values the activity of the intellect. Indeed, the order of learning both engages and develops the active intellect, and this order is reflected in the sequence of books. He arranges the *Declaratio* so that it begins with three books on practical music and ends with two on music theory. Although music theory is the goal towards which he moves, his path there is through practical music. But he does not treat the practical as if it were worthless. His comments in the preface to book 5, and elsewhere, reveal his epistemological outlook. Musical practice, the sensible, is more known to us than the theoretical, the immaterial. And "the natural order of learning" is to proceed from what it is more known to us to what is less known. Therefore, it is necessary to begin with the practical before considering the theoretical. And in fact, the theoretical is abstracted from the practical by the active intellect. Although this approach clearly flows from Aristotle, the way it manifests itself in the organization of the treatise contrasts with that of Ugolino's predecessors who wrote treatises of a similar length. Older theorists organize their works in the opposite direction, moving from theory to practice and often spending more time on the theoretical than the practical. The prime example is Jacobus of Liège. Writing in the fourteenth century, he composed his immense treatise, *Speculum musicae*, in seven books. The first five books deal with various theoretical topics, and only in the last two does he examine musical practice. Ugolino reverses this model and instead spends three

[63] Ugolino, *Declaratio*, 3:87.

books on practice and two on theory.[64]

As a capstone to his theoretical work, Ugolino wrote a separate treatise on the monochord, the *Tractatus monochordi*. In the modern edition, it is included after the *Declaratio* as an appendix.[65] In the *Declaratio*, the intellect contemplates and explains the data of the senses: theory and practice go hand in hand. In the *Tractatus*, Ugolino makes this connection explicit.

> For in the arrangement of the monochord, a two-fold operation occurs, namely the sense, by which we figuratively understand spaces, and the intellect, by which their proportions are shown with a cause.[66]
>
> *In monochordi nameque compositione duplex intervenit operatio, sensus, scilicet, quo figuraliter comprehendimus spatia, et intellectus qua ipsorum proportiones ratione monstrantur.*

The monochord offers a means of seeing proportions. The short treatise is organized into three sections. The first deals with the division of the monochord according to musica recta. This division includes all the natural notes plus B-flat within a span of two octaves and a minor third. The second section divides the monochord according to musica ficta. In particular, it produces what we would call flat notes. Before moving to the third section, he includes a chapter on the effects of musica ficta, in which he describes how to perfect (i.e., make major or perfect) thirds, sixths, and fifths. The discussion on perfection leads into the third and final section, which divides the monochord again for musica ficta. This division produces what we would call sharp notes. He concludes by combining both divisions of musica ficta into a single diagram, which includes both lowered and raised notes. Although the treatise is worthy of study in its own right, it serves as a key to understanding his comments on musica ficta in book 2.[67]

---

[64] In chapter 2 of this dissertation, I argue from an examination of Ugolino's comments on music as a part of natural philosophy that if there is to be any theoretical speculation, knowledge of the practical is indispensable.

[65] Seay, "Ugolino of Orvieto," 111; Ugolino, *Declaratio*, 3:227-253.

[66] Ugolino, *Declaratio*, 3:227.

[67] Consequently, I discuss it in detail in chapter 4 of this dissertation.

1.4 The Reception of Ugolino's *Declaratio*

Although Ugolino has received very little attention from scholars, their work falls into two broad categories. First there are works that summarize his biography and the contents of his treatise, but which do not go into great detail. Second, there are works that, in discussing a single topic, either draw upon Ugolino's treatise to support their own arguments or discuss the way in which Ugolino develops a topic. Of these, I focus mostly on the latter, since they give the most details on Ugolino's own theories.

Albert Seay, who also produced the only modern Latin edition of Ugolino's work, begins his discussion of the contents of Ugolino's treatise by summarizing Gerhard Pietzsch, who held that the work was primarily practical in nature.[68] In contrast, Seay highlights the fact that Ugolino spends some time showing the superiority of reason over the senses, the placement of *musica caelestis* at the top of a hierarchy including *musica mundana*, *humana*, and *instrumentalis*, and that the ultimate aim of music is speculative.[69] Summarizing Ugolino, Seay states, "Speculative music, the ultimate goal of the true musician, stands far above practical music…"[70] Seay paints Ugolino as firmly scholastic, as one whose primary interest lies in speculation but not so much in practice. He points out that Ugolino is Aristotelian, and that the sequence of topics in the *Declaratio* moves from the practical to the speculative.[71] Seay therefore disagrees with Pietzsch and relates Ugolino's approach to Boethius and the quadrivium.[72] Seay then enumerates the authors Ugolino quotes, most of whom he finds to be philosophers,

[68] Seay, "Ugolino of Orvieto," 145.

[69] Seay, "Ugolino of Orvieto," 146-48.

[70] Seay, "Ugolino of Orvieto," 148.

[71] Seay, "Ugolino of Orvieto," 148-51.

[72] Seay, "Ugolino of Orvieto," 151.

mathematicians, and the like. He notes that Ugolino only references two practicing musicians: Johannes de Muris and Guillaume de Machaut. For all these reasons, Seay declares,

> His approach is not that of the practicing musician or composer. Unlike any of his contemporaries or immediate successors he does not mention even one of the many composers or musicians whom he must have met, either in Forlì or in Ferrara. He is obviously not concerned with music as an *ars* solely, but with music as a *scientia* within a larger field.[73]

I show that Seay is only partly correct: Ugolino does indeed see music as a *scientia* within a larger field, but he also includes an extremely detailed account of counterpoint, drawing not only on older theory, but also on the Italian tradition of the *regola del grado*, elevating it from the vernacular in which it is most often found to the wider audience of those who could read Latin. Indeed, it is Ugolino's definition of music and the complex ways in which he constructs this idea that I examine in the chapters of this dissertation.

Evan MacCarthy shows that Ugolino spent part of his early life as a *biscantor* in Florence.[74] Many musicians at this time performed administrative or non-musical functions while also practicing music in some form, and Ugolino was no different. Besides writing the *Declaratio*, he also composed music. David Fallows even suggests that he was "perhaps one of the most senior and eminent composers in the decade after the death of Ciconia."[75] These statements should be balanced with those that depict him as merely a philosopher of music.

MacCarthy goes on to discuss who may have read Ugolino's treatise. One important

---

[73] Seay, "Ugolino of Orvieto," 152 (italics in the original). Bonnie Blackburn makes a similar point and for a similar reason: Ugolino makes little mention of specific composers or compositions. Bonnie J Blackburn, "Music Theory and Musical Thinking After 1450," in *Music as Concept and Practice in the Late Middle Ages*, ed. Reinhard Strohm and Bonnie J. Blackburn (Oxford: Oxford University Press, 2001), 327. Referencing specific composers or composition, however, seemed to have developed later in the fifteenth century, see for example Wegman, "From Maker to Composer," 433-36.

[74] MacCarthy, "The Sources and Early Readers of Ugolino," 403.

[75] David Fallows, "The End of the Ars Subtilior," *Basler Jahrbuch für Historische Musikpraxis* 20 (1996), 25.

person to read it was Franchino Gafori, who owed a partial copy but must have had access to the complete work.[76] Another partial copy appears together with some works by John Hothby and may have been the possession of one of Hothby's students.[77] The *Declaratio* continued to be read in Ferrara in the 1460s. Robertus de Anglia and Antonius de Janua borrowed Ugolino's treatise from the cathedral.[78] MacCarthy has pointed out musicians who read the *Declaratio*, but he also shows that some aristocrats, such as Rinaldo Maria d'Este, may have known the work.[79] Although MacCarthy does not mention it, Bartolomé Ramos de Pareja was familiar with Ugolino's treatise, since he attacks him in his *Musica practica*.[80] This evidence reveals that Ugolino's work was not isolated. Others read it, redacted it, extracted it, and commented on it in the century following Ugolino's death.[81]

Lewis Lockwood's assessment of Ugolino's *Declaratio* appears somewhat contradictory. At first, he states that the *Declaratio* is thoroughly practical, but later says that it is removed from musical practice. Concerning its practicality, Lockwood writes, "If Ugolino did teach even the most basic material of his treatise to the clerics, they received an exceptionally thorough and fundamental grounding in the notation, tone-system, typology, and principles of both plainsong and mensural polyphony."[82] Only a page later, he refers to the treatise as distant from practice:

---

[76] MacCarthy, "The Sources and Early Readers of Ugolino," 408-09. Ann Moyer notes that about half of Gafori's *Extractus parvis musice* comes from Ugolino's *Declaratio*. Moyer, *Musica Scientia,* 68-9.

[77] MacCarthy, "The Sources and Early Readers of Ugolino," 409-10. David Fallows claims that one of Hothby's works was a digest of Ugolino's treatise, and that the two may have either known each other, or at least knew of each other. Fallows, "The End of the Ars Subtilior," 26. In chapter 3, I show that one of the manuscripts containing the *regola del grado* is attributed to Hothby and that Ugolino's work may have influenced it.

[78] MacCarthy, "The Sources and Early Readers of Ugolino," 411-15.

[79] MacCarthy, "The Sources and Early Readers of Ugolino," 416-22.

[80] Bartolomé Ramos de Pareja, *Musica Practica,* trans. Clement A. Miller (Neuhausen-Stuttgart: American Institute of Musicology, 1993), 95, 107, 124, 152.

[81] MacCarthy, "The Sources and Early Readers of Ugolino," 423.

[82] Lockwood, *Music in Renaissance Ferrara*, 80.

"The systematic teachings of Ugolino's treatise seem remote from practical musical life and from the atmosphere of musical recreation characteristic of the courts."[83] Although this conflict may result from Lockwood's interest in tracing the development of music at the courts as opposed to the cathedral, it highlights the tensions between theory and practice inherent in the work and its reception, and it reveals the need to examine how Ugolino himself defines the relationship between theory and practice.

Lockwood goes on to summarize the contents of Ugolino's treatise to understand his larger purpose. Lockwood states that Ugolino's work "delicately modifies traditional concepts without substantially altering them."[84] For example, he mentions that Ugolino shifted the definition of the true musician from one who only knows music to one who "commands both theory and practice."[85] Lockwood finishes his section on Ugolino both by making a connection between Ugolino's reliance on Aristotle's *Politics* and its use in the education of the aristocracy and by contrasting this connection with humanist views on education.[86] These connections and allusions point out the fact that Ugolino's text was situated in a complex historical and social context. He lived within a society crowded with adherents to various intellectual ideologies. Scholasticism and humanism lived side by side, even though the latter was not fully developed.

Anna Maria Busse Berger has pointed out that Ugolino adopts the *regola del grado* tradition to teach counterpoint.[87] This teaching relies on hexachordal theory and which

[83] Lockwood, *Music in Renaissance Ferrara*, 81.

[84] Lockwood, *Music in Renaissance Ferrara*, 82.

[85] Lockwood, *Music in Renaissance Ferrara*, 82.

[86] Lockwood, *Music in Renaissance Ferrara*, 83-85.

[87] Anna Maria Busse Berger, *Medieval Music and the Art of Memory* (Los Angeles: University of California Press, 2005), 138. She is not concerned with explaining Ugolino's book on counterpoint per se, but only in so far as it supports her central thesis, that many music theory treatises were intended to be memorized. Pier Paolo Scattolin is the single most important author to discuss the *regola del grado* tradition as a whole. Pier Paolo Scattolin, "La Regola Del 'Grado' Nella Teoria Medieval Del Contrappunto," *Rivista italiana di musicologia* 14, no. 1 (1979).

hexachord both the tenor and the added counterpoint move in. If both parts are in the same hexachord, then the *grado* or step would be the unison. If the tenor lies within the hard hexachord, and the counterpoint moves within the natural one, then the step would be at the fourth, and so on.[88] The manuscripts that contain this theory are mostly in the vernacular, and the one that does contain Latin switches to Italian.[89] Ugolino therefore is distinctive for including this tradition in his *Declaratio*. His contemporary Prosdocimo, for example, makes no mention of it. The fact that Ugolino includes it has led Busse Berger to characterize him as "a practical musician of some influence" and that "the contents of this text [the *Declaratio*] reflect the teachings of a fifteenth-century choirmaster."[90] Ugolino includes chapters on the *regola del grado* along with ones on general contrapuntal rules, as one finds in Prosdocimo. Ugolino's teachings on counterpoint, which I discuss in chatpers 3-4, are practical yet reflect his contention that music is part of natural philosophy, which I examine in chapter 2.

Ugolino relies on hexachordal theory to teach counterpoint. How modern scholars view the hexachordal system and how Ugolino deploys it figures prominently in chapter 4. The single most important author to treat the role of the hexachord in musical thought is Stefano Mengozzi. He views recent scholarship on hexachordal theory as supporting a "foundational" view of the hexachord. This idea grants the hexachord a fundamental role in the structure of early music—a role that, in a sense, supersedes that of the diatonic scale. In contrast to this "strong" position, Mengozzi argues for a "soft" interpretation of the hexachordal system. In this view, the hexachordal system served a pedagogical function. Composers could draw on it to order the

[88] Busse Berger, *Medieval Music*, 133-34; Scattolin, "La Regola Del 'Grado'," 14.

[89] Scattolin, "La Regola Del 'Grado'," 15-18. Busse Berger writes "I believe many students, especially in Italy, started their counterpoint instruction with this method." Busse Berger, *Medieval Music*, 133.

[90] Busse Berger, *Medieval Music*, 139.

material of their compositions, but it was not the means by which to measure diatonic space—a role reserved for the diatonic scale.[91]

Starting with Guido, Mengozzi surveys the history of the hexachordal system. He devotes much of the work to the fifteenth century, because it witnessed numerous attempts to reform musical instruction in general and the hexachordal system in particular. Mengozzi includes a chapter examining the intellectual perspective of Johannes Ciconia in the *Nova musica*. He shows that in teaching singing Ciconia preferred the monochord over the solmization syllables.[92] By taking this stance, Ciconia downplays the importance of the hexachord. This evidence supports Mengozzi's claim of a "soft" interpretation of the hexachord. In discussing early fifteenth century theory, it is surprising that he does not even mention the *regola del grado* tradition or talk at length about Ugolino's *Declaratio*, both of which rely on hexachordal theory for teaching and practicing counterpoint. Since I focus on these in later chapters, I place Ugolino's understanding of hexachordal theory within Mengozzi's history to see whether Ugolino's use of the hexachordal system supports Mengozzi's soft interpretation.

At the end of the book on counterpoint, Ugolino describes musica ficta. He charts the pitches of musica ficta and the presumed hexachords they generate on a double hand (*duplex manus*).[93] He includes two diagrams, each with a different double hand. This account has inspired three authors—Margaret Bent, Karol Berger, and Andrew Hughes—to comment on it, offering competing interpretations. Bent examines what partial signatures may mean with respect to the system of musica recta in conjunction with the set of three hexachords. She claims that a

[91] Stefano Mengozzi, *The Renaissance Reform of Medieval Music Theory: Guido of Arezzo Between Myth and History* (Cambridge: Cambridge University Press, 2010), 8-13.

[92] Mengozzi, *The Renaissance Reform of Medieval Music Theory*, 129-30.

[93] Ugolino, *Declaratio*, 2:48-50.

signature with one flat would eliminate the need for the hard hexachord, leaving only two left. If any more flats were added, only one recta hexachord would remain.[94] This observation leads her to suggest that a flat in the signature transposes the system of recta hexachords down a fifth, which would make the E-flat a recta note.[95] She cites Ugolino's double hand as support for her argument, including a short quote from the *Declaratio*. However, Bent does not discuss Ugolino's double hand in any detail, making the reference to it somewhat tenuous.

Karol Berger asserts that Bent's assumption of a transposed system of musica vera and her reliance on Ugolino to prove this is incorrect. He views the first hand as a combination of the typical set of seven hexachords plus that same set transposed down a whole step. However, as Berger notes, this does not explain why Ugolino includes a hexachord beginning on D. Berger then interprets the second hand as the set of seven hexachords plus their transposition down a fifth. He argues that because Ugolino refers to these diagrams as containing the steps of both musica recta and musica ficta, a transposition of the recta hexachords would not thereby make them recta.[96]

Andrew Hughes takes a different approach by analyzing the various possibilities for what pitches Ugolino may be referring to. He notes the ambiguities inherent in Ugolino's description: for example, in the hexachord starting on B, the mi-fa half step could be interpreted as either D-sharp-E or as D-E-flat.[97] Hughes contends that Ugolino's purpose is not to show ficta pitches,

---

[94] Margaret Bent, *Counterpoint, Composition, and Musica Ficta* (New York: Routledge, 2002), 87.

[95] Bent, *Counterpoint, Composition, and Musica Ficta*, 88.

[96] Karol Berger, *Musica Ficta: Theories of Accidental Inflections in Vocal Polyphony From Marchetto Da Padova to Gioseffo Zarlino* (Cambridge: Cambridge University Press, 1987), 64.

[97] Andrew Hughes, *Manuscript Accidentals: Ficta in Focus, 1350-1450* (Rome: American Institute of Musicology, 1972), 38.

but to show ficta hexachords and where they may begin.[98] He also notes the odd inclusion of a hexachord on D and the other problems of interpretation. Ultimately, he concludes, "At the very least, it is evident that Ugolino is moving a stage beyond the mere production of ficta notes on the monochord and is attempting to organize them rationally in systems paralleling the normal one."[99] Although all three authors take different approaches, reaching a final conclusion about the meaning of the double hand remains elusive. All of them cite Ugolino without first considering the philosophical and textual contexts. Yet only by knowing these can we understand Ugolino's double hand.

Jan Herlinger compares Prosdocimo's *Contrapunctus* with Ugolino's *Declaratio*. He argues that Prosdocimo's influence on Ugolino was "extensive" and "direct."[100] He even concludes his translation of Prosdocimo's *Contrapunctus* with a list comparing topics the two authors treat in a similar manner.[101] For example, Herlinger lists the six syllables as a topic the two authors handle alike. However, any treatment of the six solmization syllables will share some similarity, since the topic had remained relatively stable since the eleventh century, although it would undergo scrutiny at the end of the fifteenth century in the work of Ramos de Pereja. Although it is not his intention, Herlinger's list could make Ugolino's work seem merely derivative. But Ugolino includes topics which Prosdocimo does not cover, such as the *regola del grado*. Those distinctive topics define the importance of Ugolino's work, since they differentiate him from other authors.

[98] Hughes, *Manuscript Accidentals*, 38.

[99] Hughes, *Manuscript Accidentals*, 39.

[100] Prosdocimo de' Beldomandi, *Contrapunctus*, trans. and ed. Jan Herlinger (Lincoln: University of Nebraska Press, 1984), 5.

[101] In this paragraph, I summarize Prosdocimo, *Contrapunctus*, 97-8.

1.5 Conclusion

Ugolino and his treatises have not been closely examined. He wrote them at a time of cultural and political upheaval. The Council of Constance revealed as many problems as it solved. Although it did reduce the number of popes from three back to one, it also both created tension between the power of a pope and that of a council of churchmen and exposed a need for reform, across many dimensions, throughout the church. The conciliar movement threatened the power of the pope and the institutional church, while some reformers, like Jan Hus or John Wycliffe, offered new ideas and challenged old ones.[102] Ultimately, the conciliar movement and these reformers failed, but they planted seeds that would come to fruition in the sixteenth century.[103] Other reformers, however, like Bishop Giovanni Tavelli da Tossignano, were more moderate and wished to promote educational reform while supporting the pope. Ugolino worked under him at Ferrara to produce the *Declaratio*.

At the same time, other changes were also afoot. Aristotelian philosophy was at its peak and had absorbed several important developments over the course of the fourteenth century. Although William of Ockham challenged Aristotle's realist metaphysic without rejecting him altogether, others worked within an Aristotelian framework to apply mathematics to objects of nature—two subjects which were traditionally separate. While these ideas grew and spread, the humanist movement also began to gain momentum. We find Ugolino writing the *Declaratio* in the middle of these changes. After he completed his treatise, humanism became more pronounced, especially with the rediscovery and renewed interest in Plato's works. By the end of the fifteenth century, the philosophical context of the *Declaratio* would seem distant, and I

[102] For a contemporary account of Jan Hus, see Petr z Mladenovic, *John Hus at the Council of Constance*, trans. Matthew Spinka (New York: Columbia University Press, 1965).

[103] Mundy and Woody, eds., *The Council of Constance*, 36-49.

suspect this is why references to his work disappear around the turn of the sixteenth century. Still, Ugolino's treatise offers a glimpse into a time of almost radical change. Because of Ugolino's historical context and the many distinctive perspectives on musical theory and practice at play in his treatise, his work is often misunderstood. In this dissertation, I intend to consider some of those distinctives in order to begin to remedy this situation. In particular, I focus on the way he relates music theory and practice. To do so, I provide an overview of Ugolino's Aristotelian outlook, since it shapes the entire treatise. This outlook is not the same as the Aristotelian view of the thirteenth century. Instead, it is informed by the developments of the fourteenth century, but it is not yet influenced by the interest in Plato that took place in the mid to late fifteenth century. Finally, I investigate the unique features in his book on counterpoint, particularly his definition of perfect and imperfect consonances, the *regola del grado*, and musica ficta. Not only are these topics treated differently by Ugolino than his contemporaries, but they also show how Ugolino's view of theory and practice work themselves out.

CHAPTER 2

MUSIC AS NATURAL PHILOSOPHY

2.1 Introduction

In the Middle Ages, scholars meticulously categorized sciences.[104] They created vast taxonomies of scientific disciplines, showing how the sciences were related and which roots they sprang from. These divisions of knowledge were rooted at once in their own needs and discoveries but also in the past, in the philosophical works they poured over, glossed, and commented on. These works included Boethius, stoic philosophy, and one philosopher they referred to simply as "the philosopher"—Aristotle. The writings of these thinkers served as the foundation that they built their own edifices upon, attempting both to compile them into a coherent whole and to add to them. As they learned more and more of Aristotle, his philosophy in particular served as a philosophical buttress to their own divisions.

For Aristotle, one science was clearly differentiated from another. He separated them based on the objects they studied. Physical objects, the domain of natural philosophy, existed in a way different from mathematical objects. Yet, the goal of knowledge was to learn about these objects as they existed. For these reasons, each science was separate from another. They did not interact. So, for example, mathematics, under which he placed music, was separated from other

[104] Science, or *scientia* in Latin, simply means knowledge or bodies of knowledge. The scientific method, experimentation, and a modern understanding of science is absent. I use the word science to avoid cumbersome language, but I use it exclusively in this narrow sense, without any implication of the modern notions of science. Edward Grant notes that although late medieval thinkers prized empiricism and experience, they did not rely on experimentation to prove a hypothesis. Grant, *A History of Natural Philosophy*, 216-25. For a close look at the definition of science in the Middle Ages, see James A. Wiesheipl O.P., "The Nature, Scope, and Classification of the Sciences" in *Science in the Middle Ages*, ed. David C. Lindberg (Chicago: University of Chicago Press, 1978), 461-82. David Lindberg has commented on the difficulty of pinning down the definition of the word "science." David C. Lindberg, *The Beginnings of Western Science: The European Scientific Tradition in Philosophical, Religious, and Institutional Context, Prehistory to A.D. 1450*, 2nd ed. (Chicago: University of Chicago Press, 2007), 1-3. For an examination of the discipline of music in relation to science, see Moyer, *Musica Scientia*, 11-104.

sciences, and it was not called upon to help explain natural phenomena. Mathematics and natural philosophy examined separate things and did not mix.[105]

Following Aristotle's model, music theorists grouped music with mathematics. But Ugolino groups music with natural philosophy. In this chapter, I show that this shift in thought, although distinctive to Ugolino, came both as the result of a better understanding of Aristotle's philosophy and as the result of historical developments in the fourteenth century. In the fourteenth century, philosophers in England, like Thomas Bradwardine and Walter Burley, building on the work of their predecessors Robert Grosseteste and Roger Bacon, began applying mathematics to problems in natural philosophy.[106] This new method spread quickly throughout Europe, but it also contradicted Aristotle's separation between sciences. Consequently, William of Ockham devised a different theory to relate one science to another. Ugolino's language sounds surprisingly similar to Williams's, and Ugolino has no problem mixing mathematics with natural philosophy. Ugolino also grasped Aristotle's ontological categories more clearly than did some of his predecessors, and this, combined with his epistemological outlook, affects both his understanding of the objects of music theory and his design for the whole treatise. I begin by examining the philosophical foundations. Next, I briefly survey the divisions of knowledge and their philosophical support in the twelfth and thirteenth centuries, just when Aristotle's works were beginning to be rediscovered in the Latin West. After that, I consider the new developments of the fourteenth century. Finally, I devote two sections to Ugolino's thought, one investigating

[105] Although Aristotle speaks about mathematics throughout his oeuvre, he deals with the distinction between mathematics and other sciences most clearly in the following: Aristotle, *Physics* 2.2.193b22-194b18; *Metaphysics* 13.1-10.1076a9-1087a29.

[106] Edward Grant, *The Nature of Natural Philosophy in the Late Middle Ages* (Washington DC: Catholic University of America Press, 2014), 134-36; Grant, *A History of Natural Philosophy*, 189-90, 209, 235. For a full account of the application of mathematics to natural philosophy in the Middle Ages, see Edward Grant and John E. Murdoch, eds., *Mathematics and its Applications to Science and Natural Philosophy in the Middle Ages: Essays in Honor of Marshall Clagett* (Cambridge: Cambridge University Press, 1987).

his division of knowledge, and the other looking at how he defines the objects of music, which reveals how he thinks one science relates to another.

2.2 Philosophical Foundations

The seven liberal arts form the foundation that later philosophers build on. The seven liberal arts, or *artes liberales*, were the studies suitable for a free person who could devote time and energy to non-manual pursuits. These seven arts formed two groups—the trivium and the quadrivium. The trivium encompassed logic, rhetoric, and dialectic, while the quadrivium contained arithmetic, music, geometry, and astronomy. Boethius argued that the quadrivium as a whole dealt with number or quantity. Following Pythagoras, Boethius asserts that quantity can be divided into two groups: multitude and magnitude. Multitude considers quantity as proceeding from finite to infinite. For example, multitude considers numbers beginning at 1 and continuing to infinity. Magnitude, on the other hand, looks at a finite quantity as "infinitely divisible."[107] Since each of the four disciplines of the quadrivium investigates quantity from a different angle, each has its own object, or in medieval terms its own subject. Two disciplines deal with multitude and two with magnitude. Geometry deals with fixed magnitudes and astronomy with magnitudes, in this case the heavenly bodies, in motion. Arithmetic focuses on multitude or numbers by themselves, how they are added, subtracted, and so on, without reference to a comparison between or among them. "Music," as Boethius says, "is clearly expert concerning quantities related to other quantities."[108] By saying that music concerns "quantities related to other quantities," Boethius defines music as a mathematical discipline, whose objects are numbers in certain proportions. These proportions are often related to sound, but they need not

[107] Boethius, *The Fundamentals of Music*, 53.

[108] Boethius, *The Fundamentals of Music*, 54.

be. The mathematical proportions remain primary, because Pythagoras saw them as the truth behind the sound.

Although in his book *de Trinitate* Boethius had reported Aristotle's division of the sciences, it only became widespread with the rediscovery of Aristotle's works in the thirteenth century. Aristotle moves from the mind or intellect of one who knows, outward to an object that can be known. In the mind of the one who knows, there are two states or conditions, in Latin *habitus* (pl. *habitūs*). He calls one the "knowing part" and the other the "calculating part."[109] The former thinks about things that cannot change, while the latter thinks about things that can change. The knowledge gained from the knowing part is directed towards the contemplation of truth, and the knowledge gained from the calculating part is directed either towards actions, as in moral choices, or towards making or producing some object.[110] As a result, Aristotle divided all knowledge into three broad groups based on their ends or goals: speculative or theoretical, practical, and productive. The goal of productive knowledge was making some object, like a house. The goal of practical knowledge was right or virtuous actions. The goal of theoretical knowledge was the contemplation of the truth, or in other words, knowledge for its own sake.[111] Although Aristotle further divides each of these groups, I focus on theoretical knowledge, since that is where he locates mathematics, natural philosophy, and music.

Aristotle divides theoretical knowledge into three sciences based on the objects they

[109] Aristotle, *Nicomachean Ethics* 6.2.1139a18-1139b13. A key activity of the knowing part is demonstration—producing a syllogism that uses necessary premises that lead to a necessary conclusion. I discuss the topic of demonstration below.

[110] Aristotle, *Nicomachean Ethics* 6.1-5.1138b19-1140b30.

[111] Aristotle, *Topics* 6.6.145a15-16; *Metaphysics* 6.1.1025b19-1026a30, 11.7.1064a1-b14; *Nicomachean Ethics* 6.2.1139a26-28.

study: first philosophy or metaphysics, mathematics, and physics or natural philosophy.[112] But before we can see what objects each of these sciences study, we must examine how Aristotle views objects in general, and how one gains knowledge of them. For Aristotle, every object, or what he calls an independent thing or substance, is composed of material and form. The material is the stuff a thing is made of. Statues are made of bronze, beds of wood, human beings of flesh and bones.[113] The form is what it is for something to be that thing and not another.[114] The statue is not merely any lump of bronze, but it is bronze in a definite shape or pattern, perhaps of Alexander the Great or of some other famous person. But that same lump of bronze could also be made into a ring. The same material could have another form. The bed is not merely any pile of wood, but it is wood not only in a definite shape but also for a definite purpose.[115] This means that an object's form can sometimes be connected to what the thing is for—its purpose, end, goal, or function. In addition, objects do not just appear from nothing. They come into being and pass out of being, they grow and decay, they change from one quality to another, or they move from one place to another. These actions or motions, according to Aristotle, must have some source. The source of motion that brings the statue or the bed into being is an artisan or someone skilled at making those sorts of objects. Their source of motion is external to them. But the source of motion that causes the flesh and bones of humans to grow is inside them or internal to them. Aristotle calls these ways of investigating an object its causes. The material cause is the material an object is made of. The formal cause is its form. The final cause is its end, goal, or purpose. The moving cause, or what later philosophers termed the efficient cause, is what makes

[112] Aristotle, *Metaphysics* 6.1.1025b19-1026a20-25. I consider only the objects of natural philosophy and mathematics.

[113] Aristotle, *Physics* 1.7.190a21-30, 2.1.193a10-21; *Metaphysics* 7.11.1036b3-5.

[114] Aristotle, *Metaphysics* 7.2.1028b30.

[115] Aristotle, *Physics* 2.3.195a20-30.

an object grow or decay, change quality, or change locations. To have any knowledge of an object means to explain the object's four causes or to provide a definition of the object.

What do each of Aristotle's theoretical sciences study? Natural philosophy or physics studies objects of nature, and nature is a "certain source and cause of being moved and of coming to rest in that to which it belongs primarily, in virtue of itself and not incidentally."[116] In other words, natural philosophy investigates those objects which contain the source of motion within themselves, and it also accounts for the material the objects are made of.[117] "The study of perceptible beings," Aristotle says, "is the work of the study of nature."[118] And, as I mentioned earlier, like all the other sciences, it includes explanations, articulations, or definitions of the objects it studies.[119] The science of natural philosophy has several disciplines under it. These sciences are called subalternate sciences. They include such disciplines as biology, zoology, meteorology, and so on.

Mathematics, however, studies a distinctive kind of object that results from a special work of the mind. To discover a mathematical object, the mind strips away from an independent thing its material, its motion, and other properties to reveal its depth, height, length, or number. Aristotle calls this activity abstraction. After removing these things, what is left is a mathematical object, which is made of what Aristotle calls its intelligible material or matter.[120]

---

[116] Aristotle, *Physics* 2.1.192b21-23. For Aristotle, motion included local motion, growth and shrinkage, as well as change of quality.

[117] Aristotle, *Metaphysics* 6.1.1025b20.

[118] Aristotle, *Metaphysics* 7.11.1037a14.

[119] Aristotle, *Metaphysics* 7.11.1037a10-20. Why it is important to count the articulations or definitions of an object becomes apparent when we discuss mathematics.

[120] Aristotle, *Metaphysics* 11.3.1061a29-1061b3, 7.11.1036a1-12, 7.11.1037a2-5. Stephen Graukroger discusses two types of abstraction needed to reach intelligible matter, and he describes what intelligible matter is and how it relates to geometry and arithmetic. Stephen Gaukroger, "Aristotle on Intelligible Matter," *Phronesis* 25, no. 2 (1980), 188. For further information on intelligible matter in Aristotle see John Thorp, "Intelligible Matter in Aristotle," *The Society for Ancient Greek Philosophy Newsletter* 385 (2010), 1-6. For more on intelligible matter in one of

Aristotle writes, "And one sort of material is perceptible, the other intelligible, the perceptible, for example, bronze or wood, or any moveable material, while the intelligible is what which is present in perceptible things, taken not as perceptible, as for example mathematical things are."[121] Therefore, mathematical objects exist in a very particular way. They have a distinct ontological status.

For Aristotle, mathematical objects exist in a distinct relation to material, physical objects. As we saw before, mathematical objects are abstracted from physical ones. The material and attributes of physical objects are removed by the mind until only the attributes of magnitude or multitude remain, such as the length of a table, the number of dogs, or the proportion of two sounds. These mathematical attributes derive from the physical object, and they do not exist apart from the physical object. In other words, the table, the dogs, and the sounds are separate independent things, but the mathematical objects are not separate. They need those other things to exist.[122]

So how can someone study them when they do not exist as separate things? Aristotle says, "the best way to study each thing would be in this manner, if one were to posit as separate what is not separate, the very thing that the arithmetician and the geometer do."[123] Aristotle provides an example: the geometer may look at a human being, not as a human being but as a solid.[124] The solid is a geometrical, mathematical object. It does not exist apart from the human

Aristotle's most famous and important followers, St. Thomas Aquinas, see Paul O'Reilly, "What is Intelligible Matter?," *The Thomist: A Speculative Quarterly Review* 53, no. 1 (1989), 74-90.

[121] Aristotle, *Metaphysics* 7.10.1036a11-12.

[122] For mathematical objects as attributes of perceptible objects, see Aristotle, *Metaphysics* 13.3.1077b19-1078b8. For mathematical objects as derived from perceptible objects, see Aristotle, *Metaphysics* 13.2.1075a39-1077b19.

[123] Aristotle, *Metaphysics* 13.3.1078a22-23.

[124] Aristotle, *Metaphysics* 13.3.1078a26.

being, but the geometer supposes that it does in order to talk about it and study it. Aristotle insists on locating mathematical objects in this way to avoid supposing a realm of mathematical objects that exists apart from the physical universe.[125] Aristotle has another way of saying the same thing: the physical object is prior to the mathematical one.[126] The physical object must exist before the mathematical one can.

Since mathematical objects are posited to exist as separate things, any articulation of them in speech gives the impression that they exist before other objects. For example, by saying that a triangle is a three-sided figure, it seems as if triangles and three-sided figures exist on their own. Aristotle describes this as being "prior in articulation."[127] Summarizing his distinctive view on mathematical objects, he states, "…mathematical things are not independent things more than bodies are, nor are they prior in being to perceptible things, but only in articulation, nor are they capable of being somewhere as separate."[128]

For Aristotle, mathematics and natural philosophy are two different sciences, because they study different objects. His favorite illustration for describing the differences is a snub nose. The snub nose, as such, exists in a certain material and when thinking of it as a snub nose, the material is considered along with its shape or form. For these reasons, the study of the snub nose belongs to natural philosophy. But mathematics separates or abstracts the snubness from its material and recasts snubness as concavity. Concavity does not include the physical material it was abstracted from, even though it cannot exist apart from that material. Aristotle writes, "in this way one thinks the mathematical things, which are not separate from material, as though

[125] Edward Hussey, "Aristotle on Mathematical Objects," *Aperion* 24, no. 4 (1991), 109-10. Aristotle makes this move in order to avoid Plato's realm of ideal forms.

[126] Aristotle, *Metaphysics* 13.2.1077b1-19.

[127] Aristotle, *Metaphysics* 13.2.1077b1.

[128] Aristotle, *Metaphysics* 13.2.1077b14-15.

they were separate, whenever one thinks them."[129]

Mathematical objects are prior in articulation but not prior in being. Aristotle insists on this view because previous philosophers, such as Pythagoras, thought of mathematical objects as prior in being. Indeed, Plato thought that material reality derived from the mathematical realm—the very opposite of Aristotle.[130] The difference in philosophy directly affects musical objects. Since Aristotle and many later philosophers subalternate music to mathematics, musical objects are akin to mathematical ones. If a certain philosopher takes a more Pythagorean or Platonic view, the mathematical nature of a musical object becomes the sole focus of study often at the expense of the physical sound, since, in this view, the mathematical is prior ontologically. On the other hand, if another philosopher takes a truly Aristotelian view, the mathematical nature of a musical object is studied as if it were prior ontologically, but its foundation in physical sound is always in the background. Music has one foot in the perceptible realm and another in the intelligible. This is reflected by the fact that Aristotle subordinates music to mathematics and what subordination means.

Aristotle differentiates a higher science from its lower one for three reasons. First, the lower or subalternate science tends to deal more with the perceptible objects, and the higher one with the intelligible objects. Second, the lower science takes its proofs from the higher science. The third reason is the central one and relies on the way knowledge about an object is demonstrated.[131] Aristotle describes two basic types of demonstration based on the relationship

[129] Aristotle, *De* Anima 3.7.431b13-15. For the references to the snub nose to demonstrate the same distinction, see Aristotle, *Metaphysics* 6.1.1025b31-1026a1; 11.7.1064a1-1064b14.

[130] David C. Lindberg, "On the Applicability of Mathematics to Nature: Roger Bacon and His Predecessors," *The British Journal for the History of Science* 15, no. 1 (1982), 5-7.

[131] For a fuller explanation of these three reasons, see Richard D. McKirahan Jr., "Aristotle's Subordinate Sciences," *The British Journal for the History of Science* 11, no. 3 (1978), 201-05.

of cause and effect. One type demonstrates that a fact is the case and is called by medieval philosophers a *quia* demonstration. For example, the planets are near because they do not twinkle.[132] This argument does not tell why the planets are near. It only provides some evidence of the fact that they are near. But Aristotle then turns it around and makes a different argument, which he calls an argument of the reasoned fact, or what medieval philosophers called a *propter quid* demonstration: planets do not twinkle because they are near. This demonstration provides a direct cause for why planets do not twinkle, but it also relies on the previous *quia* demonstration—the fact that they are near. Aristotle uses the *quia* and *propter quid* demonstrations to differentiate the higher from the lower science. The lower or subalternate sciences deal with *quia* demonstrations, while the higher sciences concern themselves with *propter quid* demonstrations.[133] This theory of subalternate sciences, called *metabasis*, explains how music is subalternated to mathematics.[134] Music provides the perceptible material with *quia* demonstrations, but mathematics explains the intelligible material with *propter quid* demonstrations.[135] Thus, the path of knowledge, or epistemology, proceeds from the perceptible to the intelligible, or as Aristotle states it, "the natural road is from what is more familiar and clearer to us to what is clearer and better known by nature."[136] But, as we saw above, the path of being, or ontology, proceeds from what is prior in being—the physical, perceptible objects—to what is only prior in articulation—the intelligible object. For philosophers before Aristotle, such

[132] Aristotle, *Posterior Analytics* 1.13.78a22-39.

[133] Aristotle, *Posterior Analytics* 1.13.79a2.

[134] Steven J. Livesey, "William of Ockham, the Subalternate Sciences, and Aristotle's Theory of *Metabasis*," *The British Journal for the History of Science* 18, no. 2 (1985), 127.

[135] As is shown later, some medieval philosophers recognize that some subalternate sciences borrow proofs and demonstrations from more than one higher science. The ones that do so became known as middle sciences or scientiae mediae. For more on the middle sciences, see Joseph Dyer, "The Place of *Musica*" 66-67; James A. Weisheipl O.P., "The Nature, Scope, and Classification of the Sciences," 477.

[136] Aristotle, *Physics* 1.1.184a18-19.

as Pythagoras, the path of ontology went from what is posterior to what is prior, and this corresponded directly with the path of knowledge. This difference in philosophy changes the place of theoretical or intelligible objects in music theoretical systems.

Finally, a third type of classification was derived from stoic philosophy, as transmitted by Augustine. This taxonomy divides knowledge into three categories: natural, moral, and rational. Augustine provided this division thinking that it was Plato's.[137] Although this division plays a smaller role in later medieval divisions, its importance lies in the fact that natural (also referred to as physics) corresponds to the theoretical branch of Aristotle's division. In this way, natural philosophy could be understood broadly to refer to the theoretical sciences.[138]

2.3 Twelfth and Thirteenth Century Divisions of Knowledge

Having examined the philosophical foundations, I now look at several medieval classifications of knowledge or science and how these differentiate one subject from another. One of the most influential thinkers to provide a taxonomy of knowledge was Hugh of St Victor.[139] Hugh (1096-1141) worked at the Abbey of St Victor near Paris and wrote his most important treatise, the *Didascalicon*, in the 1120s.[140] He divides philosophy or knowledge into theoretical, practical, mechanical, and logical (see Figure 2.1). He places theology, mathematics, and physics under theoretical knowledge, providing an explanation for their differences similar

[137] Dyer, "The Place of *Musica*," 7-8; Augustine, *De civitate dei* 8.4; Weisheipl O.P., "The Nature, Scope, and Classification of the Sciences," 469.

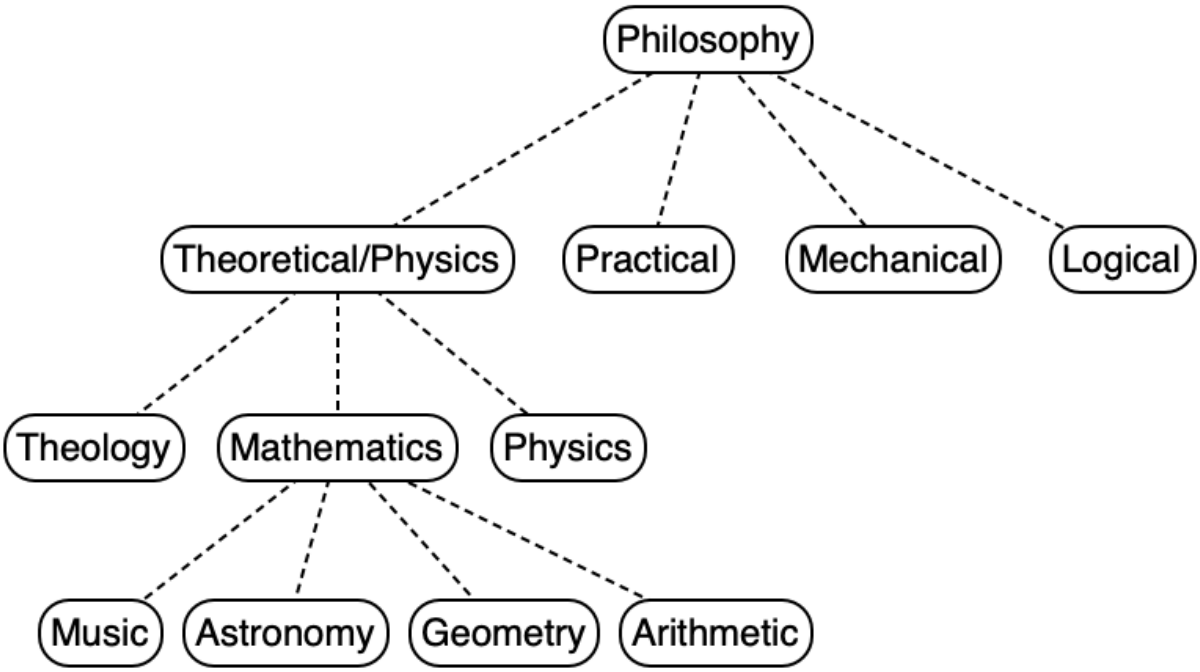
[138] For example, see note 151 below.

[139] For another summary of Hugh's division of knowledge, see Dyer, "The Place of *Musica*," 20-26.

[140] Dyer, "The Place of Musica," 21; Brian D. FitzGerald, "Medieval Theories of Education: Hugh of St Victor and John of Salisbury," *Oxford Review of Education* 36, no. 5 (2010), 576. James R. Muir, *The Legacy of Isocrates and a Platonic Alternative: Political Philosophy and the Value of Education* (New York: Routledge, 2018), 105.

to Aristotle's.[141] He puts music, together with the other subjects of the quadrivium, under mathematics. For Hugh as for Aristotle, practical knowledge deals with ethics or moral philosophy.[142] Hugh makes no mention of Aristotle's third category, productive knowledge, nor is it common to find it among medieval divisions. Instead, Hugh adds seven mechanical arts as a kind of corollary to the traditional seven liberal arts. Hugh also mentions the stoic division, "Physics is sometimes taken broadly to mean the same as theoretical science, and, taking the word in this sense, some persons divide philosophy into three parts—physics, ethics, and logic."[143] With Hugh's division, we get a glimpse at how medieval thinkers combined the seven liberal arts, the stoic division, and the Aristotelian division into a single diagram.

Figure 2.1: Hugh of St. Victor's Division of Knowledge



[141] Hugh of St. Victor, *The* Didascalicon *of Hugh of St. Victor*, trans. Jerome Taylor (New York: Columbia University Press, 1961), 72.

[142] Hugh, *Didascalicon*, 55.

[143] Hugh, *Didascalicon*, 71.

But what about the philosophy supporting Hugh's division? Writing his book in the 1120s, Hugh flourished before the full range of Aristotle's works were rediscovered and disseminated. He relies on Boethius for his knowledge of Aristotle. In fact, as Hugh finishes describing the division of theoretical sciences, he refers to it as "Boethius's division of theoretical science."[144] When Hugh discusses mathematical objects, he makes no mention of their ontological status. Instead, he reveals his ontology as he describes the nature of the soul, which progresses from the one, or a monad, into the many. He calls the soul's highest and purest essence "intellectible," which flows into the intelligible, and finally into the perceptible.[145] Thus, even though Hugh borrows Aristotle's division, his ontological understanding is fundamentally neo-Platonic. In the *Didascalicon*, Hugh presents an educational program that trains the mind to ascend back up to its highest form.[146]

Writing in the middle of the thirteenth century, Arnoul de Provence offers a bewildering array of divisions, combining all three traditions and providing a number of alternative definitions and classifications.[147] He posits that philosophy seeks as its goal to remove human defects. From this understanding, he divides philosophy into mechanical and liberal, as seen in Figure 2.2. This bifurcation reflects the construction of the human into body and soul: the mechanical sciences teach people how to remove what is lacking in respect to the body, the

[144] Hugh, *Didascalicon*, 73. Many more examples of Hugh ascribing Aristotle's ideas to Boethius could be counted. For example, when he defines the word physics, he specifically cites Boethius (63, 71). Hugh borrows Boethius's threefold division of music (69-70). It is clear that Hugh knew Aristotle only through Boethius. In addition, Hugh emphasizes Pythagoras's place in the history of philosophy

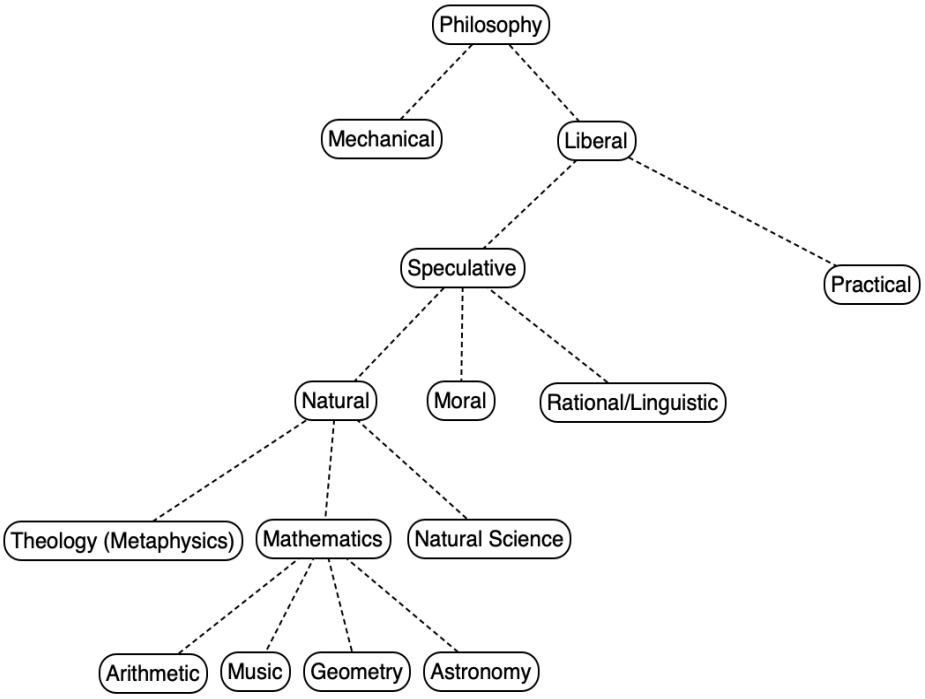
[145] Hugh, *Didascalicon*, 64-65.

[146] Brian FitzGerald shows that Hugh developed an Augustinian program of education. FitzGerald, "Medieval Theories of Education," 576-80.

[147] Joseph Dyer presents a good summary of Arnoul's work and its dependence on previous authors, ancient and contemporary. Dyer, "The Place of *Musica*," 55-59. A modern edition of Arnoul's text, along with several other divisions, appear in Claude Lefleur, ed., *Quatre Introductions à la Philosophie au XIIIe Siècle: Textes Critiques et Étude Historique* (Montréal: Institut d'Études Médiévales, 1988). All translations from this source are my own.

liberal sciences in respect to the soul.[148]

Figure 2.2: Arnoul's Division of Knowledge



He goes on to enumerate what the mechanical sciences or arts include. Then he turns his attention to the liberal sciences. He divides them into speculative and practical: the former perfect the mind through understanding, whereas the latter deal with virtue.[149] He further subdivides speculative knowledge into natural, moral, and rational—a clear nod to the stoic division. After going this far, he gives an alternate division of the speculative sciences into theology, mathematics, and natural philosophy. Arnoul has combined the stoic division with Aristotle's. When they are so combined, there are two nodes that are both called "natural science" or "natural philosophy." To distinguish them, Arnoul calls the natural philosophy that is higher in the diagram natural philosophy broadly construed (*large sumpta*), and he calls the

[148] Lefleur, *Quatre Introductions,* 315-16.

[149] Lefleur, *Quatre Introductions,* 321. Like Hugh, he also omits Aristotle's category of productive knowledge.

natural philosophy that is lower in the diagram natural philosophy narrowly or properly construed (*stricte* or *proprie sumpta*).[150] Arnoul then maps Aristotle's division of the theoretical sciences under natural philosophy.[151]

Just as Hugh and countless others before him, Arnoul places music and the other subjects of the quadrivium under mathematics. As he discusses the object of music, he reverses the definition of it from number-to-sound to sound-to-number. He starts by saying music is "number related to sound or the consonance of numbers" (musica, que est de numero relato ad sonum uel de consonantia numerorum).[152] In this statement, number is primary, manifesting itself as sound. But then he says,

> music is not about discrete quantity related to sound, but about consonances and proportions in sound as it falls under number. And it is better said that it is about sound in numbers rather than about sounding number.[153]
>
> *musica non est de quantitate discreta ad sonum relata, set de consonantiis et proportionibus in sono ut cadit sub numero. Et melius dicitur quod est de sono in numeris quam de numero sonoro.*

In this statement, sound is primary, manifesting itself as number. Yet it is not primary to the extent that he moves music to another category. It remains subalternated to mathematics.

Since Arnoul seems more interested in compiling all he can on a given topic, it is difficult to discern exactly what his philosophical foundations are. In his discussion of the object of music, he moves away from a Platonic definition and embraces one that privileges, at least to some degree, the physical nature of music.[154] This pulls him closer to the Aristotelian

---

[150] Lefleur, *Quatre Introductions,* 321-23.

[151] Lefleur, *Quatre Introductions,* 322-23. Describing the differences among theology, mathematics, and natural philosophy, he closely follows Aristotle.

[152] Lefleur, *Quatre Introductions,* 326.

[153] Lefleur, *Quatre Introductions,* 326-7.

[154] Dyer, "The Place of *Musica*," 58-59. Dyer notes the rejection of what could be considered a more Platonic definition of music's object. But just because he distances himself from the Platonic definition does not automatically mean he is Aristotelian.

understanding. But how he defines the purpose of the liberal arts seems to pull him in another direction. He says that the branch of science is called liberal "because it frees mankind from earthly cares and raises him to heavenly love" (quia hominem a curis terrenorum liberat et in amorem celestium erigit).[155] Although we cannot pinpoint Arnoul's philosophy, one thing is clear: he is not as overtly Platonic as Hugh was.

Arnoul's conception of knowledge depends in large part on the slightly earlier anonymous author of the *Accessus philosophorum*.[156] But although the author of the *Accessus* presents a division of knowledge similar to Arnoul's, he formulates a supporting philosophy by using different terminology than Arnoul. The difference reveals more of the philosophical foundations than Arnoul. The author begins at a bird's eye view by dividing philosophy into mechanical and liberal. This division results from a need to accommodate the deficiencies of the body and the soul respectively.[157] He further slices the category liberal into speculative and active.[158] Active philosophy is equivalent with moral philosophy since the active is that which aims to perfect "the human intellect as far as the virtues" (et ista perficit intellectum humanum quo ad uirtutes).[159] The author divides speculative into two parts: natural and rational. The author then takes a step back to reflect on his division, noting that he has created three parts—natural, rational, and active (see Figure 2.3 for the entire division). He claims that each of these parts reflects upon being. He writes,

> For there are three parts of being: one being exists outside our own work, whose
>
> *Sunt enim tres partes entis, quia quoddam ens est preter nostrum opus, cuius scilicet*

[155] Lefleur, *Quatre Introductions,* 316.

[156] Dyer, "The Place of *Musica*," 55-56.

[157] Lefleur, *Quatre Introductions,* 182.

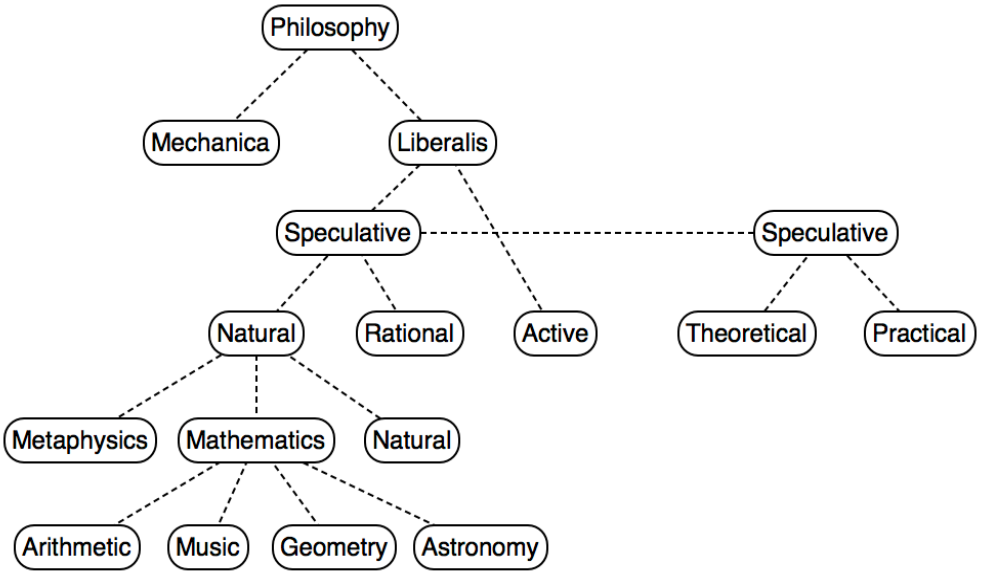
[158] Arnoul uses the term *practica*, whereas the author of the *Accessus* employs *activa*. Perhaps the latter writer depended on sources translated from Arabic, where this term seems to appear more frequently.

[159] Lefleur, *Quatre Introductions,* 182.

source is nature, and natural philosophy is about such being. Another being is from our work, and this is twofold according to the twofold source that is in us: one is reason, the other is the will.[160]	*principium est natura, et de tali ente est naturalis philosophia. Quoddam autem est ens a nostro opere, et hoc est duplex secundum quod in nobis est duplex principium: unum ratio, relictum uoluntas.*

Rational philosophy springs from reason, and active or moral philosophy results from our will and the actions it produces. Almost as an aside, he suggests an alternative division of speculative philosophy into theoretical and practical (shown in Figure 2.3 as a tree on the right side of the diagram). The author then adds Aristotle's division under natural philosophy, placing the quadrivium under mathematics.

Figure 2.3: The Division of Knowledge in the *Accessus Philosophorum*.



The tripartite categorization into natural, rational, and active proceeds from the nature of the one who knows, while the dual division into theoretical and practical comes about because of the objects, the things known or what is knowable, and their causes.[161] These objects are

[160] Lefleur, *Quatre Introductions,* 182.

[161] Lefleur, *Quatre Introductions,* 184.

distinguished both by their being and by their definition. In objects of nature, motion and material are combined both in being and in their definition. But in objects of mathematics, motion and material are combined in being but are separated in their definition.[162] The author, therefore, follows Aristotle by separating a thing's ontological status from its epistemological status.

The anonymous author discusses the subject (or object) of mathematics as a whole and of each science in particular. His analysis includes a consideration of each science's causes. The objects of mathematics as a whole are, as we have seen, "quantity or things joined to motion and material according to being but separated according to definition" (quantitas siue res coniuncte motui et materie secundum esse, abstracte secundum diffinitionem).[163] Adding quantity to the definition allows him to separate each of the quadrivial sciences from each other using Boethius's denotations. The object of music is "discrete quantity related to something…or consonance" (discreta quantitas ad aliquid relata…uel consonantia).[164] Indeed, he prefers the latter term and offers a clearer definition of music as "consonance constituted according to numerical proportions and related to something" (consonantia constituta secundum proportiones numeri ad aliquid relati).[165] He then considers how sound fits in, and he gives another refinement, the one which Arnoul picks up—that music is "about sound in number rather than sounding number" (de sono in numeris quam de numero sonoro).[166] After defining what music is about or its objects, he describes its four causes, once again following Boethius.[167] Overall, this

[162] Lefleur, *Quatre Introductions,* 184.

[163] Lefleur, *Quatre Introductions,* 184-5.

[164] Lefleur, *Quatre Introductions,* 203.

[165] Lefleur, *Quatre Introductions,* 203.

[166] Lefleur, *Quatre Introductions,* 203.

[167] Lefleur, *Quatre Introductions,* 204-5.

anonymous author presents a clearer outline of the philosophy supporting the division of knowledge than Arnoul does. Yet both authors accept Aristotle's division while merging it with the stoic division and the quadrivium.

By the late thirteenth century, Aristotle's philosophy, his division of knowledge, and his definitions of the objects of each science had been disseminated, commented on, and expanded. One thinker who understood the philosophy underlying Aristotle's division of knowledge stands out above the rest—St. Thomas Aquinas. For Thomas, each science is distinguished from another based upon its subject—what I have described as the objects a science studies—and the intellectual operation needed to understand that subject. Indeed, these two aspects are inextricably linked and form the basis for defining what a science is, the unity of a science, its difference from other sciences, and its method of investigation. He uses the term subject in a special sense to mean "that under whose formal perspective all things are studied in that science."[168] This formal perspective, as Armand Maurer translates it, or *ratio* derives both from the object or objects under consideration and the manner in which the intellect grasps them. According to Maurer, Thomas posits two operations of the intellect involved in the formation of the formal perspective: apprehension and judgment. The former deals with what things are or their essence. The latter, then, can combine or separate the things perceived in apprehension, and it reflects how things exist in reality. For example, through apprehension we can grasp what wood is and what hardness is, and through judgment we can state that "wood is hard." Based upon this distinction, there is a twofold manner of abstraction: one resulting from apprehension, and the other from judgment. The kind of abstraction that springs from apprehension can

[168] Thomas Aquinas, *The Division and Methods of the Sciences*, trans. and ed. Armand Maurer (Toronto: Pontifical Institute of Mediaeval Studies, 1986), xvii, 22.

contemplate the natures or essences of things, without saying anything of their existence in reality—i.e., it investigates intelligible objects—whereas the kind of abstraction resulting from judgment states how things exist in reality (how they really are) and must therefore be true.[169]

These processes of the mind differ from each other based on the object under investigation. As Thomas says, "So the speculative sciences must be divided according to differences between objects of speculation, considered precisely as such."[170] But objects differ from each other in respect to the relationship between matter and form—something inherent in the object and not in the mind but which demands a distinct intellectual power. For example, "Some [objects of speculation] depend on matter both for their being and for their being understood, as do those things whose definition contains sensible matter and which, as a consequence, cannot be understood without sensible matter."[171] These kinds of objects belong to the domain of natural philosophy, which, as Maurer notes, considers things as wholes—both form and matter—apart from individuals.[172] There are other kinds of objects which, "…although dependent upon matter for their being, do not depend upon it for their being understood, because sensible matter is not included in their definitions."[173] These are mathematical objects, such as number, line, and so on. In a sense, mathematics, through an application of the abstraction through apprehension, separates form from matter, and the form that the intellect abstracts is only the kind that can exist if its essential nature does not depend on material. Quantity, the quintessential mathematical object, is a property fulfilling this requirement.[174] Therefore, in

[169] Aquinas, *The Division and Methods of the Sciences*, xvii-xix, 34-41.

[170] Aquinas, *The Division and Methods of the Sciences*, 13.

[171] Aquinas, *The Division and Methods of the Sciences*,14.

[172] Aquinas, *The Divison and Methods of the Sciences*, xx-xxii.

[173] Aquinas, *The Division and Method of the Sciences*, 14.

[174] Aquinas, *The Division and Methods of the Sciences*, 37-38.

Thomas's thought just as in Aristotle's, mathematics is altogether a distinct science from natural philosophy, because each investigates different objects—or in his terms each has distinct subjects—and each requires specific operations of the intellect.[175]

Thomas and others recognize that some of the lower sciences, the ones lower in the diagram, depend on more than one of the higher sciences. For example, music depends on more than mathematics to explain its causes. It must also refer to natural philosophy, since its definition usually includes sound. Sciences that participate in more than one of the higher sciences were called middle sciences (*scientiae mediae*). How a science is classified as a middle science, or indeed how it is classified as a lower science, comes from a theory of subalternation. I showed one way Aristotle defined subalternate sciences—through the kind of demonstration used, whether *quia* or *propter quid*. Lower sciences use *quia* demonstrations, and higher sciences use *propter quid* demonstrations.[176] If music were purely mathematical, there would be no need to invoke sound in its definition. But for both Arnoul and the author of the *Accessus*, sound and number wrestle for priority. Sound is subalternated to natural philosophy and number to mathematics. Number usually comes out on top. As Jacobus in the early fourteenth century says, "since music is primarily subalternated to mathematics rather than physics, more knowledge comes from arithmetic than from physics" (et cum musica principalius arithmeticae quam physicae subponatur, plura de arithmetica quam de physica sunt scientia).[177] For Aristotle and

[175] Aquinas, *The Division and Methods of the Sciences*, 22. Thomas notes that the three speculative sciences all examine being but each from a different perspective and writes, "For each science treats of one part of being in a special way distinct from that in which metaphysics treats being. So, its subject is not properly speaking a part of the subject of metaphysics, for it is not part of being from the point of view from which being is the subject of metaphysics; from this viewpoint it is a special science distinct from the others."

[176] See for instance, Aristotle, *Posterior Analytics* 1.13.79a2. Joseph Dyer offers a good overview of this point. Dyer, "The Place of *Musica*," 44-47; Aquinas, *The Division and Method of the Sciences*, 21.

[177] Jacobus Leodiensis, *Speculum musicae*, ed. Roger Bragard, 7 vols. (Rome: American Institute of Musicology, 1955-73), 1:67 (translation my own). In the next sentence, Jacobus notes that the theoretical musician is not complete or perfect unless they have been instructed in arithmetic. For an overview of Jacobus's classification, see

these medieval followers of his, mathematics and natural philosophy are separate sciences and do not interact with each other.[178] But that was about to change.

2.4 Fourteenth Century Developments

In the thirteenth century, two philosophers challenged Aristotle's assumption that natural philosophy and mathematics are separate. Robert Grosseteste (d. 1253) explored light or optics, a subject of natural philosophy. He appealed to mathematics to explain the dispersion of light, claiming that as light is dispersed, it is multiplied from a point, to a line, to a surface, and finally to a solid.[179] Roger Bacon (d. 1292) went further by expanding natural philosophy to include any subject that applied mathematics to natural objects, an activity for which he is most famous.[180] Few of his contemporaries followed him, since most preferred to stick with Aristotle's clear separation.[181]

In the fourteenth century, the application of mathematics to subjects in natural philosophy, rooted in Grosseteste's and Bacon's work in optics, spread to other subjects. This practice was taken up by a group of scholars studying at the University of Oxford, Merton College, who became known as the *calculatores*.[182] For Aristotle, mathematics deals with quantity. Natural philosophy, by contrast, deals with qualities, such as temperature or motion.

Karen Desmond, "Behind the Mirror: Revealing the Contexts of Jacobus's *Speculum musicae* (PhD diss., New York University, 2009), 327-30.

[178] Grant, *A History of Natural Philosophy*, 158.

[179] Peter Adamson, *Medieval Philosophy: A History of Philosophy Without Any Gaps* (Oxford: Oxford University Press, 2019), 189, 191-193; Grant, *The Nature of Natural Philosophy in the Late Middle Ages*, 135.

[180] Grant, *A History of Natural Philosophy*, 158, 161, 308; Adamson, *Medieval Philosophy*, 196. Bacon himself, like Grosseteste before him, worked in the field of optics.

[181] Grant, *A History of Natural Philosophy*, 308.

[182] Grant, *The Nature of Natural Philosophy*, 135; Adamson, *Medieval Philosophy*, 435; Steven J. Livesey, "The Oxford Calculatores, Quantification of Qualities, and Aristotle's Prohibition of Metabasis," *Vivarium* 24, no. 1 (1986), 50.

The *calculatores*, breaking Aristotle's rules of *metabasis*, attempt to quantify qualities.[183] For example, Thomas Brandwardine (d. 1349), Walter Burley (d. 1344), Nicole Oresme (d. 1382), and others employed proportions in order to quantify motion.[184] They could then apply this quantitative understanding of qualities directly to physical phenomena.[185] In fact, Nicole Oresme devised the mean speed theorem in just such a manner.[186] These thinkers discovered that qualities, like motion or temperature, could be measured with mathematics. They proved that mathematics and natural philosophy are not as separate as Aristotle thought. Mathematics can be used to answer questions posed by natural philosophy. This method of applying mathematics to natural philosophy was quickly adopted by philosophers throughout Europe. In Italy, Padua in particular, it was embraced enthusiastically.[187] But since this innovative method contradicted Aristotle's theory of subalternation, a new way of relating one science to another was necessary.

One thinker who presented a new theory of subalternation which could easily accommodate the advancements of the Oxford *calculatores* was William of Ockham.[188] William (d. 1347), writing his *Expositio super octo libros physicorum* in the first half of the fourteenth century, discusses what knowledge is, what the subject of a science is, and how a science has unity. He begins by noting the connection between a *habitus*—a state or condition—and

[183] Livesey, "The Oxford Calculatores," 52; Adamson, *Medieval Philosophy*, 436.; Grant, *The Nature of Natural Philosophy*, 135.

[184] Grant, *The Nature of Natural Philosophy*, 135.

[185] Livesey, "The Oxford Calculatores," 50.

[186] Grant, *A History of Natural Philosophy*, 310.

[187] Grant, *A History of Natural Philosophy*, 310; Adamson, *Medieval Philosophy*, 530.

[188] William also presented a new way of thinking about reality—nominalism. Over the course of the fourteenth century, nominalism became quite prominent, but many scholars continued to hold some version of realism. I do not discuss nominalism here for two reasons. First, one could support William's theory of subalternation without being a nominalist. Second, Ugolino seems to be a realist. His language does match that of nominalism, and he frequently cites St. Thomas Aquinas—a realist—to support his own views.

concludes that "a habitus which is knowledge is a quality in the soul."[189] He then defines knowledge along a spectrum: at one end it means a "certain cognition of something true" and at the other "an evident cognition of some necessary truth caused by the evident cognition of necessary premises and a process of syllogistic reasoning."[190] By introducing the syllogism to the definition, he adheres to Aristotle's meaning of scientific knowledge and points the way towards the importance of the conclusion.[191] Next, William begins to dismantle the idea that a science is numerically one piece of knowledge—in other words, it may not be about a single object. He asserts that none of the theoretical sciences is one piece of knowledge. He proves this by showing that a person who has knowledge of one of the conclusions of metaphysics could also be in error concerning one of the other conclusions. This situation makes no sense if a science is numerically one, because it is not the case that the same person could both know and not know at the same time and in the same manner. Instead, a science is a "collection of several habitūs related according to a certain determinate order."[192] He likens it to a university, which has many people but is considered one thing. Science can be the knowledge of a complex of conclusions, principles, and other things, all of which stand, at one time or another, as the conclusion of a demonstration.[193] As a result, "a science which has only a collective unity has not just one subject; rather it has different subjects according to its different parts."[194] William

[189] William of Ockham, *Philosophical Writings: A Selection*, trans. Philotheus Boehner O.F.M. (Indianapolis: The Bobbs-Merrill Company, Inc., 1964), 5.

[190] Ockham, *Philosophical Writings*, 5-6.

[191] Armand Maurer, *The Philosophy of William of Ockham in the Light of Its Principles* (Toronto: Pontifical Institute of Mediaeval Studies, 2002), 135.

[192] Ockham, *Philosophical Writings*, 6-7.

[193] Ockham, *Philosophical Writings*, 6-7; Gordon Leff, *William of Ockham: A Metamorphosis of Scholastic Discourse* (Manchester: Manchester University Press, 1975), 320.; Livesey, "William of Ockham," 130.

[194] Ockham, *Philosophical Writings*, 7-8.

bases his understanding of science and its unity around the conclusions of demonstrations—both the subject and predicate terms taken together. Therefore, he rejects Thomas's view that the unity of a science results from a particular *ratio* inherent in the object.[195] Although one could certainly challenge William's representation of Thomas's thought, his point is clear: a science does not have a single overarching subject which gives it unity and determines its method. Instead, a science can have many different subjects, because a science has many different conclusions. As Gordon Leff puts it, there are "as many sciences as there are conclusions or collections of conclusions."[196]

As a consequence of this conception of science and its unity, William maintains a radically different understanding of subalternation than Thomas. Indeed, Livesey notes that whereas for Aristotle subalternation is relatively rare, for William it is rather common.[197] Because a science is organized around a specific ordering of conclusions and not on the nature of the object or the psychology of the one who knows, one and the same conclusion could more easily belong to two different sciences. This idea opens a wide field of possible connections between sciences. For example, one part of a science could be subalternated to another science, one part of a science could be subalternated to two or more sciences, different parts of a subalternated science could belong under two or more sciences, or one part of a science could be subalternated while the other is subalternating.[198] Williams's understanding allows for more fluidity among various sciences, and it provides a theoretical basis for what scholars like the *calculatores* were already doing.

---

[195] Maurer, *The Philosophy of William of Ockham*, 141.

[196] Leff, *William of Ockham*, 323.

[197] Livesey, "William of Ockham," 145.

[198] Livesey, "William of Ockham," 138-40.

2.5 Ugolino's Division of Knowledge

In book 1 chapter 2 of the *Declaratio*, Ugolino lays out four arguments to prove that music is a science. He provides not only a glimpse of his own division of knowledge but also a clear understanding of the relationship between theory and practice. He writes,

> The teaching of many philosophers and of as many as imitate their opinions and judgments, who shine forth by the excellence of the discipline of philosophy, has found that music is a science, proved by the reasons written below. For, according to the evidence of the aforementioned philosophers, some of the sciences are speculative, others are practical. The common school of philosophers count music among the speculative sciences. The first reason for this is as follows: a speculative science is one in which *quia* and *propter quid* demonstratons are made. In music, many demonstrations, both *quia* and *propter quid*, are made, as is evident in Boethius in his second book on music, chapter 37, namely that the tone cannot be divided into equal parts; similarly that the minor semitone is greater than three commas by smaller than four; likewise that the superparticular proportion cannot be divided into equal parts…and that unity is not a number, as is evident to those who look through Boethius's [book on] music. Therefore, music is a science.
>
> *Multorum doctinam philosophorum ac quam plurium imitatus auctoritatum sententias et rationes, qui philosophicae disciplinae nobilitated praefulgent, repperi musicam esse scientiam, his infrascriptis rationibus approbatam. Nam cum scientiarum secundum praefatorum documenta philosophorum quaedam sit speculativa et quaedam pratica, communis scola philosophorum eam musicam inter speculativas scientias connumerarunt, quorum ratio prima est haec: Illa dicitur esse speculativa scientia in qua fiunt demonstrationes quia et propter quid. Sed in musica multas fiunt demonstrationes et quia et propter quid, ut patet per Boetium in sua musica libro secundo capitulo 37, ut puta quod tonus in aequa dividi non possit. Similiter quod semitonium minus maius sit tribus comatibus minus vero quatuor, item quod proportio superparticularis in aequa dividi non possit, medio proportionabiliter interposito numero secundum Architam, et quod unitas non sit numerus, ut patet discurrenti musicam Boetii. Igitur musica est scientia.*
>
> Futher, a science is understood in two ways in so far as it relates to the present material: in one way as a *habitus* existing in the mind, directing us to examine knowable things; in another way as a *habitus* existing in the mind, directing us to work. First, music is a *habitus* which, existing in the mind of the musician, directs him to examine the knowble things of music which relates to *musica speculativa* [music
>
> *Praeterea scientia dupliciter accipitur quantum ad praesentem spectat materiam, uno modo pro habitu in mente existente dirigente nos ad speculandum scibilia, alio modo pro habitu in mente existente dirigente nos ad operari. Sed musica est habitus primus qui existens in mente musici dirigit eum ad speculandum scibilia musicae quod spectat ad musicam speculativam. Est etiam habitus secundus*

theory]. Second, it is a *habitus* existing in the mind of the musician directing him to musical works, which relates to *musica practica*. According to these *habitūs*, someone is said to be a musician, just as by the *habitus* of philosophy one is called a philosopher and by the *habitus* of geometry one is called a geometer. Therefore, music is a science.

existens in mente musici dirigens eum ad musicalia operari, quod spectat ad musicam practicam secudum quos habitus quis dicitur musicus, sicut per habitum philosophiae quis dicitur philosophus et per habitum geometriae quis dicitur geometra. Ergo musica est scientia.

But it should be known that the *habitus* directing us to examine knowable things is a work of the speculative intellect, but the *habitus* directing us to work is the work of the practical intellect, as it will be more fully revealed in the preface to the fifth book.

Sed est notandum quod habitus dirigens nos ad speculandum scibilia est operatio intellectus speculativi, sed habitus dirigens nos ad opus est operatio intellectus practice, sicut in proemio libri quinti plenius declaratur.

Further, that is said to be a science which is one analogy in which the things which are handed down in it are handed down from one. And music is proved to be an analogy, because all things handed down in music are handed down by a man from the work of a human body causing sound to come to the sense of hearing and as a result good consonances and delightful melodies. Therefore, music is an analogy and as a result a science.

Praeterea illa dicitur esse scientia quae est una analogia in qua ea quae traduntur in illa propter unum traduntur, sed musica est una analogia probatur, quia omnia tradita in musica sunt tradit per hominem propter operari circa corpus humanum causando sensui auditus sonum et per consequens bonas consonantias et delectabiles melodias. Igitur musica est una analogia, et per consequens scientia.

Further, that is said to be a science which is subalternated to the science above it and whose principles are proved in the higher science. But music is subalternated to natural philosophy by taking [natural] philosophy in general. For in the first book of *Posterior Analytics*, Aristotle says that one science is sublaternated to another which has its principles proved in the higher one to it, as optics is placed under geometry. And music has its principles proved in natural philosophy by taking it in general, therefore it is subalternated to it. For music considers time, motion, sound, quantity, number, low and high pitches, the raising and lowering of pitches, likewise

Praeterea illa dicitur esse scientia quae sibi superiori scientiae subalternatur et principia sua in ea superiore probantur. Sed musica subalternatur philosophiae naturali sumendo philosophiam in communi, nam primo posteriorum dicit Aristoteles quod illa scientia subalternatur alteri quae habet principia probata in illa sibi superiore, ut perspectiva geometriae supponitur. Sed musica habet principia sua probata in philosophia naturali sumendo eam in communi, igitur subalternatur illi. Musica enim considerat tempus, motum, sonum, quantitatem, numerum, voces graves et acutas, intensionem et remissionem

their quickness or slowness, the distance of one pitch to another. It even considers the consonances of pitches according to various proportions of one pitch to another, considered in their depth and height. Many of these things which are presupposed in music are proved in natural philosophy. Thus, music is a science.	*vocum, similiter earum velocitatem et tarditatem, intervallum unius vocis ad aliam. Considerat etiam consonantias vocum secundum varias proportiones unius vocis ad aliam in acuitate et gravitate consideratarum, quorum omnium multa probata sunt in philosophia naturali quae tamen praesupponuntur in musica. Igitur musica est scientia.*
The fact that music is a science could be proved by many more arguments which we, for the sake of brevity, will leave to the theorist to examine. Similarly, the fact that science is taken in many other ways, as is demonstrated most fully in the *Posterior Analytics*, we leave to the logicians to dispute, since it does not relate to the present matter. For us it is enough that music, without opposing arguments, is proved to be a science.[199]	*Aliis quam pluribus rationibus posset probari quod musica est scientia quas theorico causa brevitatis dimittimus speculandas. Similiter quod aliis quam pluribus modis scientia sumitur sicut in libro posteriorum plenissime demonstratur, qui quoniam ad materiam hanc non pertinent logicis disputanda committimus. Nobis autem id satis est quod musica sine oppositis probatur esse scientia.*

First, following unidentified philosophers, he divides science into speculative (or theoretical) and practical. This division results first from the kinds of demonstrations used: a science should include both *quia* and *propter quid* demonstrations, and since music uses both, it must be a science. He does not state that a lower science often provides *quia* demonstrations while a higher science *propter quid*. Instead, he simply notes that a science includes both kinds of demonstrations, and he gives a few of those demonstrations in the form of conclusions drawn from Boethius's *Fundamentals of Music*. The language of demonstrations echoes both Aristotle and Thomas, who, as I have shown, related one science to another by their types of demonstration. In the fourth argument, Ugolino directly cites Aristotle as his source.

Ugolino's second argument turns inward, where the distinction between speculative and practical reflects the states of mind (*habitūs*) required for knowledge. One *habitus* is required for

[199] Ugolino, *Declaratio*, 1:18-20.

speculative thought and therefore *musica speculativa*. Another is required for practical thought and therefore *musica practica*. Aristotle distinguished between a knowing part of the mind and a calculating part, which resulted in the division of knowledge into theoretical and practical. Thomas also made a similar distinction by focusing on the mind of the knower and not only on the object known. Ugolino follows them but notes that both habitūs are active within the one discipline of music. As a consequence, the true musician (*musicus*) is one who has both *habitūs*, not merely the one directed towards theory. This contrasts with Boethius's *musicus*, who is only the one who applies "reason and thought" in order to judge music.[200] Ugolino, therefore, elevates practical music since the true *musicus* must also possess its *habitus*.

Ugolino's third argument directs our attention to the way music is taught. Music is handed down, literally "traditioned," from one generation to the next by some teacher singing "good consonances and delightful melodies." Since book 1 of the *Declaratio* deals with plainchant, he probably has in mind the chants of the church. But Ugolino is also revealing how much he values sounding music as a source of education. Indeed, as I show below, it is an indispensable part of the knowledge of music.

Ugolino's fourth argument is the most important. When examined closely, it reveals that Ugolino's classification goes beyond those of his predecessors. First, he introduces the topic of subalternation and states what was observed in Aristotle: a higher science contains the principles or sources and the proofs that are used in a lower science. He states that music's principles are proved in natural philosophy, and so music is subalternated to it.

Each time Ugolino refers to natural philosophy, he qualifies it by saying that natural philosophy is taken "in general." In the *Accessus* and in Arnoul's classification, natural

[200] Boethius, *The Fundamentals of Music*, 51.

philosophy could be understood in two ways because of the combination of the stoic and Aristotelian divisions of knowledge. When broadly construed (*large sumpta*), natural philosophy was synonymous with Aristotle's theoretical science. When narrowly construed (*stricte sumpta*), it meant the specific discipline of natural philosophy or physics. Is Ugolino merely reiterating that music is a speculative or theoretical science?

Ugolino continues his argument by listing what the objects of music are, that is, what kinds of things it considers: time, motion, sound, quantity, number, low and high pitches, the raising and lowering of pitches, the quickness or slowness of pitches, the distance from one pitch to another, and the consonance of pitches. About half of these belong to the domain of physics and the other half to mathematics. He concludes his arguments by stating that "many of the things which are presupposed in music are proved in natural philosophy."

Although I consider the objects of music more closely below, a preliminary sketch illuminates Ugolino's argument. Authors writing before Ugolino had classified the object of music as numbered sound or as consonance. On the face of it, Ugolino agrees. In book 1 chapter 4 he states, "For consonance itself is the end of the entire discipline of music, since…everything considered in it is attributed to consonance" (Ipsa etenim consonantia totius musicae disciplinae finis est quoniam…omnia in ipsa considerate attribuuntur).[201] Later, in the same chapter, he fleshes out this idea.

> Consonance cannot occur without sound, nor is sound made without a striking and some percussion, nor does a percussion or striking come to be without a preceding motion. Therefore, we doubt least of all that motion is to be treated in music, since if everything stands still and lacks motion, no sound at all occurs in music, neither the
>
> *Quae consonantia cum praeter sonum fieri non possit, nec praeter pulsum ac percussionem quandam sonus reddatur, nec percussio atque pulsus absque praecedente motu esse contingat. Ideo motum in musica supponendum minime dubitamus, quoniam si cuncta starent motuque carerent, nullus penitus*

[201] Ugolino, *Declaratio*, 1:22.

tone nor the semitone, nor could consonance be found in it, nor would the sense of hearing be filled with sweet melody.[202]	*haberetur sonus in musica nec tonus nec semitonium neque consonantia in ea possit reperiri, neque auditus sensus dulcedinis modulamine repleretur.*

Consonance cannot exist without sound, and sound cannot exist without motion. Since the object of music, boiled down to a single entity, is consonance, music must also consider motion and all of the attendants to consonance. Those attendants belong to natural philosophy *stricte sumpta*, that is, to physics. This is not to say that Ugolino does not also think of music as mathematical. He spends books 4-5 discussing the mathematics behind music. But he connects music to both physics and mathematics as no other author had done. This connection could only exist after the developments of the fourteenth century. It results from the tightening of the bonds between mathematics and natural philosophy. Ugolino was no doubt familiar with these ideas, since they were adopted in Italy long before he was born.

The connection between mathematics and natural philosophy was strong enough that some of Ugolino's contemporaries prefer the stoic division. In 1450 Aeneas Piccolomini, who would later become pope Pius II and who was a contemporary of Ugolino, composed the treatise *The Education of Boys*. In this work, he provides a program for educating boys centered on Latin literature. During the course of the treatise, Piccolomini summarizes the history of philosophy and who, in his opinion, was responsible for its various parts. After mentioning the seven liberal arts, he goes on to offer a tripartite division of philosophy. He states, "Philosophers at the beginning paid attention only to natural causes, following Thales…" and from this came natural philosophy. Socrates introduced moral philosophy, but "with the arrival of the divine Plato's genius, it was decided to add a third part, called 'rational.'"[203] Piccolomini's threefold division

[202] Ugolino, *Declaratio*, 1:22.

[203] Piccolomini, "The Education of Boys," 257.

of philosophy, reproducing the stoic scheme, conveniently leaves natural philosophy large enough to contain both mathematics and physics. It may not quite be the overarching category of natural philosophy that Roger Bacon envisioned, but it avoids putting mathematics and physics into two separate places.

Ugolino's arguments also reflect Aristotle's ontology and epistemology. Consonance, which belongs in the realm of *musica speculativa*, cannot exist without sound, which belongs in the realm of *musica practica*. The physical thing, sound in this case, is therefore prior to any theoretical explanation or system. Sounding music—*musica practica*—is therefore essential, because *musica speculativa* is derived from it. Just as Aristotle abstracts mathematical objects from physical objects, so musical theory is abstracted from musical practice. The physical object remains prior in its being or ontological status to the mathematical object. So, Ugolino states that consonance cannot exist without sound. But the mathematical object is often the goal of knowledge. In the same way, music theory is the goal of knowledge.

In the *Physics*, Aristotle speaks about an epistemological path. He writes,

> …the natural road is from what is more familiar and clearer to us to what is clearer and better known by nature; for it is not the same things that are well known to us and well known simply. For this reason, it is necessary to lead ourselves forward in this way: from what is *less* clear by nature but clearer *to us* to what is clearer and better known by nature. But the things that are first evident and clearer to use are more-so the ones that are jumbled together, but later the elements and beginnings become known to those who separate them out from these. Thus, it is necessary to proceed from what is general to what is particular, for it is the whole that is better known by perceiving, and what is general is a kind of whole since it embraces many things as though they were parts.[204]

As Aristotle states, knowledge starts with things that are familiar to us. These things are perceptible, physical objects. We proceed from there, by "separating them out" or abstraction, to things that are more certain because they do not change. In other words, we proceed from

[204] Aristotle, *Physics* 2.1.184a18-184b1.

perceptible material via abstraction to intelligible material. Yet for Aristotle, perceptible objects are prior in existence to intelligible objects, even though intelligible objects are the goal of knowledge.

In book 5, Ugolino quotes this passage in his own words and uses it as the reason for the way he organizes his entire treatise. He writes,

> But according to what Aristotle says in his first book on *Physics*, there is for us a natural path from what is more known and more certain to us to what is less known and less certain and from what is general to what is particular. We are taught that in thinking and in learning we ought to proceed from the things that are more known to us to the things that less known to us, for this is the natural order of learning, that through the knowledge of what is known we may arrive at the knowledge of what is unknown, and so we come from what is known to us to what is known by nature. Moreover, according to nature what is known is more confused, and general things, which are indistinct, are more confused, which contain in themselves their inferiors in potency. The one who knows something in general, knows it indistinctly. Then knowledge of it is distinguished when anything that is contained in it potentially is understood in actuality. For the one who knows animal does not know that it is rational except in potency. For knowing something in potency comes before knowing it in actuality. And so, we have decided in this work to follow this order of Aristotle, from what is more known, namely from the practice of plainchant, melodied music [counterpoint], and measured music, as it is most fully established in the first three books—the knowledge of practice is more known to us than theoretical knowledge—from the knowledge of these we are led into a complete knowledge of its theory.[205]
>
> *Sed secundum quod dicit Aristoteles primo physicorum innata est nobis via ex notioribus et certioribus nobis in innotiora et in incertiora et ab universalibus in singularia. Docemur quod in cognoscendo et discendo debemus procedere ab his quae sunt nobis magis nota ad ea quae sunt nobis minus nota, hic enim discendi naturalis est ordo, ut per notorum cognitionem in ignotorum deveniatur notitiam, et sic a nobis notis venitur in nota naturae. Nota autem secundum naturam sunt confusa magis, confusa autem magis sunt universalia quae indistincta sunt quae in se continent sua inferiora in potentia. Qui autem scit aliquid in universali scit illud indistincte, tunc autem distinguitur eius cognitio quando unumquodque eorum quae continentur potentialiter in universali actu cognoscitur. Qui enim scit animal non scit rationale nisi in potentia, prius enim est scire aliquid in potentia quam in actu, et ideo hunc Aristotelis ordinem sequentes a magis notis cepimus hoc opus, scilicet, a pratica musicae planae, musicae melodiatae, et musicae mensuratae, sicut in primis tribus libris plenissime constat, cuius praticae notitia magis est nobis speculatione nota, ex cuius cognitione in eius speculative ducimur perfectam notitiam*

[205] Ugolino, *Declaratio*, 3:87.

Ugolino contends that *musica practica*—consisting of plainchant, counterpoint, and mensuration—is more known to his readers than *musica speculativa*. There is a path from musical practice via abstraction to musical theory. Ugolino elevates musical practice. Indeed, it is essential because theory is abstracted from it, and because we can only arrive at the theoretical through what is already known, from practice. Theory is still the goal of learning, but the way he gets there is different from previous authors. One need only think of Jacobus of Liège and his massive *Speculum musicae*. He spends five whole books discussing music theory before writing two books on musical practice. For Jacobus, practice comes almost as an afterthought, for the sake of making his work complete.[206]

This close relationship between theory and practice also manifests itself in the fifth book of the *Declaratio*. There, Ugolino discusses the mathematical ratios of intervals. He pairs each theoretical definition with its practical one. For example,

> In practice [a diapente] contains three tones and one minor semitone. In theory, it is said to contain three sesquioctave proportions and one minor semitone proportion.[207]
>
> *Practice [diapente] tres continet tonos et semitonium unum minus, theorice autem tres sexquioctavas proportiones et semitonii minoris proportionem dicitur continere.*

Later in the same chapter, he expresses it differently:

> A diapente exceeds a ditone in practice by a semiditone…But in theory, a sesquialtera proportion exceeds the proportion of a ditone by the proportion of a semiditone.[208]
>
> *Diapente excedit diphtonum practice per semidiphtonum…Theorice vero proportio sexquialtera proportionem diphtoni excedit per semidiphtoni proportionem probatur.*

After each pairing, he continues by providing various mathematical demonstrations. These

---

[206] For a discussion of Jacobus's conception of theory and practice, see George A. Harne, "Unstable Embodiments of Musical Theory and Practice in the *Speculum musicae*," *Plainsong and Medieval Music* 21, no. 2 (2012): 113-36; George A. Harne, "The Ends of Theory and Practice in the *Speculum musicae*," *Musica disciplina* 55 (20120): 5-32.

[207] Ugolino, *Declaratio*, 3:145.

[208] Ugolino, *Declaratio*, 3:147.

pairings clearly reveal how Ugolino considers theory to be derived from practice, and although theory is the goal of knowledge, that goal can only be reached through practice. Thus, Ugolino follows Aristotle: just as Aristotle claimed physical objects are ontologically prior to mathematical objects, so for Ugolino practice is prior to theory. Aristotle sees the knowledge of theory as the goal of epistemology, and the goal is reached by taking the road of experience and observation. Likewise, Ugolino sees music theory as the goal of musical knowledge, and the goal is reached through musical practice. Just as experience and observation are indispensable for theoretical knowledge, so musical practice is indispensable for musical theory.

2.6 Ugolino's Account of the Objects of Music

In book 5 chapter 1, Ugolino directly addresses what the subject of music is—or, as I have been calling it, what the objects of the discipline of music are—and whether sound is its subject. He presents the topic using a thoroughly scholastic method: he reviews what previous thinkers have stated, divides the topic into several subsections, and examines arguments both for and against each proposed point. His views reflect the developments that took place in the fourteenth century, particularly the work of William of Ockham.

Ugolino begins by describing what previous thinkers stated. He evaluates the opinions of Pythagoras, Aristoxenus, Empedocles, and Plato, among others. After dismissing their conclusions, he begins to tackle the topic himself. He starts by dividing subject into four different types: subject in which (*subiectum in quo)*, into which (*in quod)*, of which (*de quo*), and to which (*ad quod*). To each of these, he gives an alternate name: subject of demonstration (*subiectum demonstrationis*), subject of operation (*subiectum operationis*), subject of attribution (*subiectum attributionis*), and subject of formation (*subiectum informationis*). He then provides a short definition for each.

> The *subiectum demonstrationis* is the subject of the conclusion demonstrated by the most powerful demonstration or propter quid, in which an attribute is demonstrated of a subject. The *subiectum operationis* is that about which the knowledge of music or the musician works. The *subiectum attributionis* is that to which what has been said in music aims and why those things are done which are considered in music itself. The *subiectum informationis* is that in which the knowledge of music has its being, namely the soul.[209]
>
> *Subiectum demonstrationis est subiectum conclusionis demonstratae demonstratione potissima seu propter quid, in qua demonstratur passio de subiecto. Subiectum operationis est illud circa quod operatur scientia musicae sive musicus. Subiectum attributionis est illud ad quod tendunt quae dicta sunt in musica et propter quod fact sunt quae considerantur in ipsa musica. Subiectum informationis est illud in quo habet esse ipsa scientia musicae ut anima.*

He spends the most time talking about the *subiectum demonstrationis*—literally the subject of the demonstration. After he makes this classification, he offers a definition for what the subject of a science is. He notes that it can be understood in three ways: (1) "as one of the extremes of a proposition, distinct from the predicate" (pro altero extremorum propositionis distincto contra praedicatum); (2) "as that to which something adheres" (pro eo cui aliquid adhaeret) as an incidental property or attribute adheres to its subject, and (3) "that about which the totality of some science deals" (pro eo circa quod versatur intentio totalis alicuius scientiae).[210] He says that he will take it in this latter meaning, although the first definition also comes into play.

Having outlined the meaning of subject and the one he will use, he continues by describing six conditions for something to be a subject in the third sense: (1) it should be more known to those skilled in the science; (2) through it one can acquire knowledge of other things; (3) its attributes are demonstrated; (4) what is covered in music can be reduced to it; (5) that it is intelligible, and (6) what is considered in the science is ordered according to it.[211] These observations provide the background for the remainder of his discussion. Since he is asking

[209] Ugolino, *Declaratio*, 3:90.

[210] Ugolino, *Declaratio*, 3:90-1.

[211] Ugolino, *Declaratio*, 3:91.

whether sound is the subject of music, he briefly outlines several important points: sound is what is perceived by the hearing, it is a term, and it is produced by some animal either internally from the vocal tract or externally on some instrument.[212]

With these preliminaries out of the way, he applies his four types of subject to *musica mundana*, *humana*, *instrumentalis*, and music in general (*musica in communi*), starting first with musica instrumentalis, then music in general, and saving the other two for chapters of their own. He begins with *musica insturmentalis*. After talking about sound and the kinds of instruments that produce it, he supplies answers for what the subject of each category is. For the *subiectum demonstrationis* of *musica instrumentalis,* he notes, no single subject fulfills these six conditions. He then continues to discuss the subject of music in general, commenting on science as he goes.

> The *subiectum informationis* of *musica intrumentalis* is the soul, because the subject of every *habitus* and of every science is the soul. This is evident because science is a *habitus* of the intellect that acquires conclusions and demonstrates them through necessary premises. Also, science is about universals which are in the soul, and from this it is inferred that there can be many subjects of a single science.
>
> *Musicae instrumentalis subiectum informationis est anima, quia subiectum omnium habituum et omnium scientarum est anima, patet hoc quia scientia est habitus intellectivus conclusionis acquisitae et demonstratae per praemissas necessarias. Item scientia est de universalibus quae sunt in anima, et ex his infertur quod unius scientiae plura possunt esse subiecta.*
>
> Now, however, in reference to determining the question as it relates to music taken in general, given that it is superior to the other kinds of music, we note that since every art holds together in reason—by taking art such that it is not distinguished from science but rather as science and art fall together—music consists in the ratio of numbers. This is evident because it considers consonance joined together by due proportion, which consists in the ratio of numbers, as is clearly evident to those who consider the *Musica* of the blessed Augustine and Boethius.
>
> *Nunc autem ad determinationem quaesiti pro musica in communi sumpta prout est superior ad alias musicas dicimus notando quod cum omnis ars in ratione contineatur sumendo artem prout non distinguitur contra scientiam immo prout coincidunt ars et scientia, musica in ratione numerorum consistit. Patet hoc quia considerat coaptationem et debitam consonantiam proportionalem, quae in ratione numerorum consistit, ut clare patet bene consideranti musicam beati Augustini et Boetii.*

[212] Ugolino, *Declaratio*, 3:91-92.

Likewise, it should be noted that science considers the properties and attributes of a single subject, with which the whole purpose of it deals and to which everything considered in it is attributed and by which it is ordered. These things are evident to those who keenly observe the many meanings of subjected stated above…

Item notandum quod scientia est unius subiecti proprietates et passiones eius considerans, circa quod ipsius tota versatur intentio et cui omnia considerata in ista scientia attribuuntur et per id ordinantur, patent haec subtiliter consideranti multiplicem modum dicendi subiecti superius declaratum...

We say, therefore, that the consonance of the things of *musica instrumentalis*, which consists in the *ratio* of numbers in due proportion, is not an adequate subject of music when music is taken in general, because music in general, beyond the consideration of *musica intrumentalis*, considers musica mundane, humana, and it it is considered in them.

Dicimus ergo quod consonantia entium musicae instrumentalis in ratione numerorum proportionabiliter consistens non est subiectum adaequatum musicae sumendo musicam in communi, quia ultra considerationem musicae instrumentalis musica in communi considerat musicam mundanam et humanam et considerata in eis.

Likewise, the consonance of things consisting in the *ratio* of numbers in due proportion and not some single thing is the subject of demonstration in the whole of music in general, because many other things are the subjects of demonstration in all of music. This is evident because there are as many subjects of demonstration as there are demonstrations, and there are many demonstrations, therefore there will be many subjects.

Item consonantia entium in ratione numerorum proportionabiliter consistens nec aliquod unum est subiectum demonstrationis in tota musica in communi, quia multa alia sunt subiecta demonstrationis in tota musica. Patet hoc quia tot sunt subiecta demonstrationis quot sunt demonstrationes, sed demonstrationes sunt multae, ergo et subiecta erunt multa.

Likewise, being is the *subiectum operationis* in the whole of music in general, by taking being in general without restricting it for consideration in *musica mundana*, *humana*, *instrumentalis*, because music only considers consonance and the proper combination of things consisting in the *ratio* of numbers in proportion. For this is known by taking consonance as the parts combined with each other and conveniently disposed according to proportions and in this manner consonance ought to be taken in all the conclusions of the present question. But because the present conclusion can be objected to, since being is the *subiectum*

Item ens est subiectum operationis in tota musica in communi, sumendo ens in communi non contractum pro consideratis in musica mundana, humana et instrumentali, quia musica non considerat nisi consonantiam et debitam coaptationem entium in ratione numerorum proportionabiliter consistentium. Hoc enim notum est sumendo consonantiam pro partibus ad invicem coaptatis, et convenienter dispositis secundum proportiones et isto modo debet consonantia sumi in omnibus conclusionibus praesentis quaesiti. Sed quia praesens conclusio posset cavillari,

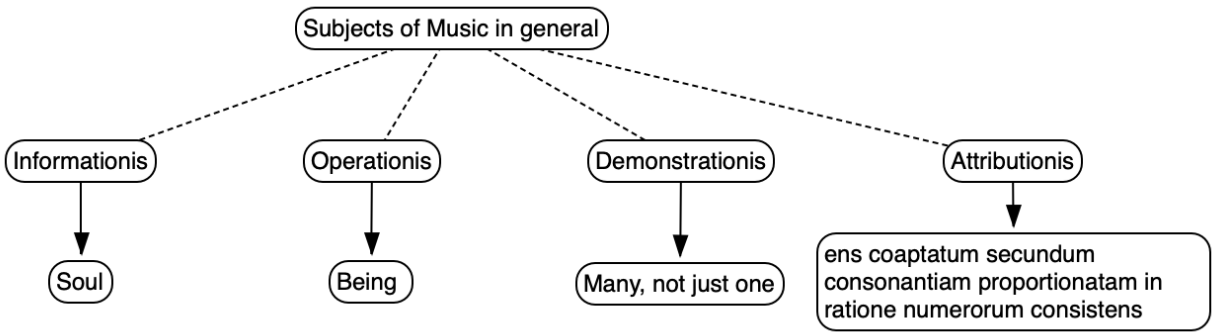
operationis in other sciences, so it should be known that it is not inconvenient that the same thing is the subject of diverse science understood in different ways.	*cum ens sit subiectum operationis in alia scientia, ideo notandum quod non est inconveniens idem esse subiectum diversarum scientiarum diversimode consideratum.*
Likewise, being combined according to proportioned consonance and consisting in the ratio of numbers is the subiectum attributionis in the whole of music. This is evident because everything considered in music is attributed to being in the manner stated, and it is even evident to those who peruse the preface of Boethius's *Musica*.	*Item ens coaptatum secundum consonantiam proportionatam in ratione numerorum consistens est subiectum attributionis in tota musica. Patet hoc quia omnia considerata in musica attribuuntur enti modo praedicto contracto et disposito, et patet etiam discurrenti proemium musicae Boetii.*
Likewise, the soul is the *subiectum informationis* of music because the soul is the subject of every habitus and science because, as was said, science is a *habitus* of the intellect that acquires conclusions through demonstration from necessary premises, similarly it [science] is of universals which are in the soul.[213]	*Item anima est subiectum informationis musicae quia anima est subiectum omnium habituum et scientiarum quia, ut dictum est, scientia est habitus intellectivus conclusiones acquisitae per demonstrationem ex praemissis necessariis, similiter ipsa est universalium quae sunt in anima, ergo, et cetera.*

From this passage, Ugolino's conception of a science begins to emerge. Although consonance plays an important role, it is not the only thing that music is about. As a result of Ugolino dividing subject into four types, there are at least four subjects for each category of music (*mundana*, *humana*, and *intrumentalis*) and for music in general. The *subiectum informationis* for both *musica intrumentalis* and music in general is the soul or mind. For Ugolino as for Thomas, objects affect the mind or soul, and knowledge must include the mind. The *subiectum operationis* is being. In other words, music deals with physical objects that exist in the world, not merely with theoretical constructs. The particular kind of being is one which combines things according to proportions and which is often referred to simply as consonance. The *subiectum attributionis* is "being combined according to proportioned consonance." Beings,

[213] Ugolino, *Declaratio*, 3:93-94.

particularly numbers, in proper proportions are important elements within the discipline of music. But by breaking the subject of music into four parts, he nuances the discourse. This is most evident in the *subjectum demonstrationis* (see Figure 2.4 for a diagram of these subjects for music in general).

Figure 2.4: Ugolino's Subjects of Music.



The subject of demonstration (*subiectum demonstrationis*) may be the most important category for two reasons. First, for Ugolino, science is defined as the activity of acquiring conclusions and demonstrating them. The subject of demonstration, therefore, flows out of and feeds directly into Ugolino's definition of science. And Ugolino, as we saw in the previous section, takes an entire chapter just to prove that music is a science. Second, he pays particular attention to this category by reiterating the main point: for every science there is no single subject of demonstration. Each science will have many subjects. Although it might be tempting to think, as the medieval authors considered above did, that consonance is the subject of music, Ugolino observes that the "consonance of things" is not adequate as the subject of demonstration. Instead, "there are as many subjects of demonstration as there are demonstrations." No science has only one demonstration, so each science has many subjects of demonstration. Ugolino repeats his definition of science at the end of the passage quoted above. He makes no mention of the form of the object, nor does he refer to the mind of the one who

knows. He coordinates the definition of science with the conclusion of a demonstration. In addition, in chapter 4 of book 5, Ugolino returns to the question of sound and whether it is a subject of or in music.[214] He presents ten reasons that sound is not a subject in or of music. For each of the ten arguments, he presents ten counterarguments. In the fifth argument, he states that since consonance is the subject of music, it is not sound.[215] But in the counterarguments he notes that consonance without qualification is not the subject of music. He also denies, for the same reason, that sound its subject.[216]

When Ugolino defines the subject of music in this way, his thinking closely resembles the kind of thought expressed by William of Ockham.[217] William's position on the subject of demonstration, summarized by Leff, states that there are "as many sciences as there are conclusions or collections of conclusions."[218] This summary could have been written by Ugolino because of how closely it echoes his wording. Indeed, Ugolino's position makes sense in a system like William's, where the conclusions of demonstrations form the foundation, and their specific ordering establishes unity among different sciences. Ugolino's view on the subject (or subjects) of music points to a familiarity with the philosophical developments of the fourteenth century.

2.7 Conclusions

Ugolino teaches that a science has more than one subject (or object)—a doctrine clearly

[214] Ugolino, *Declaratio*, 3:99.

[215] Ugolino, *Declaratio*, 3:100.

[216] Ugolino, *Declaratio*, 3:101.

[217] Steven Livesey notes that William was not the first to articulate this idea, since Robert Grosseteste, among others in the thirteenth century, held similar views. Livesey argues instead that William expressed it more forcefully. Livesey, "William of Ockham," 142-3.

[218] Leff, *William of Ockham*, 323.

dependent on the work of William of Ockham. He does this by dividing subject into four different types and by giving the subject of a demonstration an important place. A science has many such subjects, and a science, according to Ugolino, is the acquisition and demonstration of conclusions. This means that one science could have a subject that is also found in another science. It allows one science to interact with another in a number of ways, so that mathematics can be freely applied to natural philosophy. Although Thomas recognized *scientiae mediae,* William's theory on the unity of a science, and by extension Ugolino, goes beyond Thomas. Ugolino moves freely between mathematics and physics. Demonstrations can be made about consonance that describe it in mathematical terms. At the same time, consonance implies sound, and sound cannot be considered without reference to motion—meaning that it belongs to physics or natural philosophy. When Ugolino listed the topics music considers, half belonged to mathematics and half to physics. Categorizing music as a natural philosophy also builds on the work established by the *calculatores*, who were using mathematics to explain motion, impetus, and other physical phenomena. Ugolino's conception could not have occurred without these scholars coming before him.

Ugolino follows Aristotle's epistemological outlook. In the *Physics*, Aristotle defines an epistemological order: learning moves from things that are more known to the learner to things that are less known to the learner but clearer in their nature. It moves from things that are changeable and contingent to things that are unchangeable and universal. It moves from the perceptible to the intelligible. Ugolino specifically cites Aristotle as the source for the way he structures his treatise. He moves from *musica practica* to *musica theorica*, which reverses the traditional order of theory treatises. For Ugolino, practice is like the perceptible, the contingent, the thing more known to a learner. Theory, then, is like the intelligible, the unchangeable and

universal, the thing that is less known to a learner but clearer in its nature.

Together with Ugolino's specific Aristotelian epistemology, his Aristotelian ontology has profound consequences for the relationship between theory and practice. Aristotle distinguished between an object's status as existing prior to another. For him, physical objects are prior. Attributes or properties of physical objects can be abstracted and thought of by themselves. These abstracted objects exist as parts of physical objects, but they do not exist prior to the physical object. When speaking about mathematics, Aristotle calls the abstracted objects intelligible material. Intelligible material derives from physical objects and so is not prior in existence. But intelligible objects can be thought of on their own, as if they did have a separate existence. They can be defined, spoken about, and their definitions can be written down. As a result, Aristotle says they are prior in articulation. Mathematical objects, therefore, are not prior in being, but they seem prior when they are talked about, because they are treated as if they were separate from physical objects. This position counters Platonic or neo-Platonic thought, which sees intelligible material as prior in being and as giving being to the physical, perceptible world through participation. Ugolino, taking the Aristotelian view, puts mathematical objects and music theory on the same level. He was careful to show that consonance cannot exist without sound, which means that sound is prior in being to consonance. Theory, like intelligible material, is abstracted from practice. In book 5, a book on mathematical ratios, he connects practice and theory, as if they were two sides of the same coin—as if theory is abstracted from practice. In practice, a diapente is composed of three tones and a minor semitone. In theory, it is composed of the equivalent proportions. When he says "in practice" (*practice*), he states what follows as a practicing musician would know it from the first three books. When he says "in theory"

(*theorice*), what follows is a mathematical expression for the corresponding proportion.[219] Thus, theory in its being is abstracted from practice, and to learn theory requires knowing practice. The ontological and the epistemological fit together like hand and glove.

By construing musical theory as something abstracted from musical practice, Ugolino elevates musical practice. The priority of musical practice shapes his entire treatise. Even though the ultimate goal is theoretical knowledge, the path there runs through musical practice. For Ugolino that path begins with plainchant and continues with counterpoint. In the next chapter, I turn to a distinctive method of teaching counterpoint—the *regola del grado*. Ugolino is the only author to present this practice entirely in Latin. In the third chapter, I explore musica ficta. By examining Ugolino's distinctive musical practices, I show that he sees theory flowing from practice. In other words, in the following pages, we see how he works out the ideas presented in this chapter.

[219] For instance, see Ugolino, *Declaratio*, 3:145.

CHAPTER 3

COUNTERPOINT IN PRACTICE: CONSONANCE, PERFECTION, AND THE *REGOLA DEL GRADO*

3.1 Introduction

In book 2 of his *Declaratio*, Ugolino offers an extensive and distinctive treatment of two-voice, note against note counterpoint. He begins the book with a philosophical preface, in which he contemplates the nature of freedom and servitude. He argues that music belongs among the studies suitable for the free person, because these subjects perfect the intellect.[220] After the preface, in chapters 1-2, he reviews what counterpoint is and how it should be defined. He begins in chapter 1 by saying what counterpoint does and what its purpose is. Often referring to counterpoint as "melodied music" (*musica melodiata*), he states that plainchant is bare or unclothed.

> And although we delight in its bareness, after it has been clothed with melodies, it delights the sense even more. So, in the second book we intend to treat of melodied music.[221]
>
> *Et quoniam in ipsa eius nuditate delectationem accipimus tamen quia melodiis induta delectabilior valde sensu percipitur, ideo in hoc secundo de ipsa melodiata musica intendimus pertractare.*

Plainchant serves as the foundation for counterpoint.[222] It is the *musica* of *musica melodiata*. Counterpoint clothes or decorates it by adding to it another melody. Even though he only describes note against note counterpoint in two parts, the act of adding another melody—"melodying" plainchant—enhances the already existing chant. The chant and its decoration together bring delight or pleasure to the hearers. He adds,

---

[220] In other words, music belongs among the liberal arts, not among the mechanical arts. Ugolino, *Declaratio*, 2:1-3.

[221] Ugolino, *Declaratio*, 2:3.

[222] Indeed, Ugolino says that "counterpoint…presupposes plainchant" (contrapunctus…cantum planam praesupponit). Ugolino, *Declaratio*, 2:8.

> In fact, at one time we are captivated by the sweetness of the diapente, at another we are allured by the diapason as the mistress of consonances, and we are transfixed by the delight of these consonances, struck by their surpassing pleasantness.[223]
>
> *Tunc etenim ab ipsa diapente dulcedine rapimur, tunc a diapason consonantiarum domina trahimur et ab ipsarum delectatione consonantiarum dulcissimo ictu percutimur.*

Pleasantness or sweetness is the goal or purpose of counterpoint. For these reasons, Ugolino gives counterpoint the name *musica melodiata*, and he is, as far as I can tell, the only author to do so.

In chapter 2, he offers his own definition of counterpoint:

> Counterpoint is the simple indeterminate placement of one single note, placed in a low or high [range] against another single note in any melody.[224]
>
> *Contrapunctus est simplex unicae solius notae in gravi positae vel acuto contra aliquam unicam solam notam in cantu aliquo intedeterminata positio.*

He then analyzes this definition by pointing out its genus, species, and differences. He also distinguishes between counterpoint in a broad sense (*large sumptus*), which consists of many notes placed over a single note, and counterpoint in a narrow sense (*stricte* or *proprie sumptus*), which consists of note against note.[225] In the *Declaratio*, Ugolino deals only with the latter.

Chapters 3-15 form the foundation of Ugolino's book. In them, he is the first to present entirely in Latin an older Italian tradition of counterpoint called *regola del grado* (henceforth, grado). This tradition is preserved in only four manuscripts: Washington, Library of Congress, ML.171.J.6; Milano, Biblioteca Ambrosiana, I. 20 Inf.; Florence, Biblioteca Medicea Laurenziana, Redi 71 (f. 24v-28v); and Florence, Biblioteca Medicea Laurenziana, Redi 71 (f.

---

[223] Ugolino, *Declaratio*, 2:4. Below, I discuss what he means by dubbing the octave the "mistress of consonances."

[224] Ugolino, *Declaratio*, 2:4.

[225] Ugolino, *Declaratio*, 2:4. Ugolino's definition closely echoes that of Prosdocimo, who also describes counterpoint in a broad and narrow sense. He states that in the narrow, proper sense, counterpoint "is the placement of one single note against some other single note in a melody." Prosdocimo, *Contrapunctus*, 28-31. This near exact relation has led Jan Herlinger, Prosdocimo's translator and editor, to posit a great influence of Prosdocimo on Ugolino. See note 319 below.

48v-59v). I refer to these as W, M, F1, and F2 respectively.[226] W, written on paper dating to the late fifteenth century, has often been attributed to John Hothby (c. 1430-1487).[227] Hothby, an Englishman, spent most of his life working in Italy at the cathedral school in Lucca, where he became choirmaster in the mid to late 1460s.[228] Thus, any resemblances between W and Ugolino likely come from Ugolino or a shared source common to both. M, composed on fifteenth century paper, begins in Latin but soon switches to Italian. Gilbert Reaney suggests that the writer of M was translating the work from Italian, not copying from an unknown Latin source, and that the source for M is, in fact, W.[229] F1 and F2 come from a single manuscript produced in the early fifteenth century. It contains treatises attributed to Johannes de Muris and Jacopo da Bologna. The works on mensuration in this manuscript show an influence from France while the counterpoint treatises are distinctly Italian, because they offer the theory of the grado.[230] In sum, M and W clearly come from the period just after Ugolino was writing, while F1 and F2 may be either contemporary to Ugolino or written slightly before the *Declaratio*. Therefore, Ugolino's book on counterpoint represents an important point in the history of this particular theory, since it offers a presentation of the grado that, as I shall show later in this chapter, both develops the treatment in F1 and F2, and to some extent may influence M and W.

Although all the manuscripts that preserve the grado theory date from the early to mid-

[226] W and M appear in two modern editions: Gilbert Reaney, ed. *Johannes Hothby: De Arte Contrapuncti* (Neuhausen-Stuttgart: American Institute of Musicology, 1977), 15-49 and Scattolin, "La Regola Del 'Grado,'" 52-74. F1 and F2 both appear in Albert Seay, ed. *Quatuor Tractatuli Italici De Contrapuncto* (Colorado Springs: Colorado College Music Press, 1977). For W and M, I depend most frequently on Scattolin's edition.

[227] Scattolin, "La Regola Del 'Grado'," 28-32; Reaney, *Johannes Hothby: De Arte Contrapuncti*, 9, 17; Sigrum Heinzelmann, "John Hothby as Innovator: The Solmization System in La Calliopea Legale," *Studi Musicali* (2012), 353.

[228] Benjamin Brand, "A Medieval Scholasticus and Renaissance Choirmaster," 755, 763-71.

[229] Reaney, *Johannes Hothby: De Arte Contrapuncti*, 10, 22; Scattolin, "La Regola Del 'Grado'," 26-28.

[230] Seay, *Quatuor Tractatuli*, i; Scattolin, "La Regola Del 'Grado'," 24-26.

fifteenth century, the theory itself likely originated in the fourteenth century and was learned by beginners before they pursued the more widely practiced interval succession.[231] The word grado (pl. gradi) refers to the distance, step, or grade between the starting note of the tenor's hexachord and the starting note of the counterpoint's hexachord.[232] These could be in various relationships to each other, such as the unison, fourth, fifth, or octave. By knowing which hexachord each part moved within, students of this practice could figure out which combination would form consonances and which would form dissonances, so that they use only consonances in their counterpoint.

The grado manuscripts, like nearly all counterpoint treatises, include as a preliminary discussion what intervals are consonances and their division into perfect and imperfect. In this respect, Ugolino's *Declaratio* is no different, but he offers a fuller account that clearly connects the distinction of perfect and imperfect with Aristotelian philosophy. Indeed, David Cohen counts five conditions that point to the Aristotelian understanding of imperfection. Cohen's first condition is a general classification of things into perfect or imperfect.[233] The most common reason for classifying a thing as perfect or imperfect rests in an empirical judgment—its "compatibility" with the ear and its progression to the closest consonance. Cohen writes, "'Compatibility,' it seems, is somehow caused by voice-leading proximity, although how this should be the cause remains unclear."[234] In this chapter, I build on Cohen's work by positing that

[231] Busse Berger, *Medieval Music*, 131; Scattolin, "La Regola Del 'Grado'," 13.

[232] Busse Berger, *Medieval Music*, 133; Reaney, *Johannes Hothby: De Arte Contrapuncti*, 11; Scattolin, "La Regola Del 'Grado'," 14.

[233] David Cohen, "'The Imperfect Seeks Its Perfection': Harmonic Progression, Directed Motion, and Aristotelian Physics," *Music Theory Spectrum* 23, no. 2 (2001), 146.

[234] Cohen, "The Imperfect Seeks its Perfection," 150. He is discussing Marchetto of Padua, *The Lucidarium of Marchetto of Padua: A Critical Edition, Translation and Commentary*, ed. and trans. Jan Herlinger (Chicago: University of Chicago Press, 1985), 200-07.

Ugolino may offer a clue how one interval is classified as perfect and another imperfect. Although Ugolino, like many authors, cites the judgment of the ear (that, for example, one interval simply sounds better than another) and emphasizes voice-leading proximity, he also refers to the particular structure of an interval for determining perfection. The octave, or "mistress of consonances" (*consonantiuarum domina*) as Ugolino dubs it, is his measure.[235] It is made up of parts which stand to the whole as proximate causes, and those parts are made of other parts which stand to the whole as remote causes, and so it continues. He appeals to this structure as he distinguishes perfect from imperfect consonances. With this distinction between perfect and imperfect consonances laid out, Ugolino presents certain model progressions he calls perfections (*perfectiones*). Related to the motion from imperfect to perfect consonances, perfections most closely resemble what we might think of as cadences. Instructions on perfections are not found in the grado manuscripts, but Ugolino nestles it into his own discussion before addressing the core of the grado theory. Since Ugolino relies on this distinction for his comments on perfection in particular and his understanding of interval relationships and counterpoint in general, I undertake a close examination of it in this chapter.

Chapters 16-24 form an extension of the previous 12 chapters. In the grado practice, as I shall show below, both the tenor and the added part remain in a single hexachord. But in chapters 16-24, Ugolino shows how the added voice, still moving within a single hexachord, can harmonize a tenor that moves across several hexachords. Practically, it means that Ugolino creates various tables of consonances. Consonance tables prove to be dominant feature of the remainder of the book. In chapter 25, he lists seven general rules for counterpoint. These include the usual prohibitions against parallel perfect intervals, avoiding the mi-fa tritone, and other

[235] Ugolino, *Declaratio*, 2:4.

common rules. He spends only one chapter, chapter 26, on the more common practice of interval succession. He states these rules in the form of poetic verse.[236] In chapter 27, he gives several examples of composed counterpoint that put the rules into practice and to which students could refer in order to see and hear proper counterpoint in action. Then, in chapters 28-33, he adds several consonance tables, listing the possible consonant combinations for each solfege syllable.[237] Finally, Ugolino ends the book with a single yet significant chapter on musica ficta.[238]

In this chapter, I focus on Ugolino's central teaching in chapters 3-15—the *regola del grado*. This teaching distinguishes him from nearly all of his contemporaries, since he is the only one to present the theory entirely in Latin. By comparing his treatment of it with that found in F1, F2, M, and W, his distinctive contributions become clear. Ugolino does not merely repeat the *regola del grado* practice. Rather, he expands it, shapes it by his understanding of Aristotelian philosophy, and develops it. First, I begin by pointing out how Ugolino classifies intervals and how he describes the particular category of interval progression that he calls a perfection. I show that the language of perfection ties into his Aristotelian outlook. Second, I examine the grado theory itself and how he constructs it in relation to the four manuscripts in which it exists, noting

[236] For a discussion of the purpose of versification, see Busse Berger, *Medieval Music*, 138-41. Ugolino's versified rules for interval succession are not exhaustive, and they were probably not meant to be. However, he was criticized by Ramos precisely because they were not exhaustive. Ramos de Pareja, *Musica Practica,* 95, 107, 124, 152. Instead of providing a completely thorough treatment of interval succession, such as that by Tinctoris near the end of the fifteenth century, Ugolino's intent is clearly practical. That the rules are in verse makes them easier to memorize, as Busse Berger shows. Besides, he only spends one chapter on them, unlike Tinctoris, who devotes much more space to them. In addition, the core of Ugolino's teaching is the *regola del grado*. Rather than a systematic analysis of all contrapuntal possibilities, Ugolino prefers to offer multiple examples and methods of proper counterpoint, which students could then use later in practical situations. As long as his readers have the necessary principles, they would not need an exhaustive account of every single possibility.

[237] Consonance tables are common in counterpoint treatises and are not a distinctive feature of Ugolino's work. For more on consonances tables, see Busse Berger, *Medieval Music*, 6-8, 131-50.

[238] I discuss musica ficta in chapter 4.

especially his own contributions.

3.2 Imperfect Consonances and Their Perfection

In Ugolino's treatise and in the grado manuscripts, the sequence of topics follows a similar path. By pointing out some similarities and differences, a clearer picture of Ugolino's contributions emerges. Although Ugolino changes some terminology, what stands out as particularly significant is his classification of consonances, his development of the theme of imperfection and perfection, and his explanation for why some intervals are imperfect and others perfect. W, M, and F1 start by noting that there are only seven different notes, which they call the unison (*pari*), second, third, fourth, fifth, sixth, and seventh. All the other notes "proceed" (*procede*) out of the original seven: the octave from the unison, the ninth from the second, and so on.[239] This explanation reveals several keys to understand the grado texts: they recognize both a seven-note diatonic system and octave equivalence.[240] This must be kept in mind, especially when I examine their method for forming counterpoint. Since the grado tradition relies so heavily on the hexachord to form counterpoint, it is easy to forget that the foundation of the system is, indeed, the seven-note diatonic scale.[241] After considering the origin of notes, the grado treatises separate these notes, already thought of as intervals, into three categories based on their quality: consonant, dissonant, and discordant. I refer to this as the tripartite division of

[239] Seay, *Quatuor Tractatuli*, 17; Scattolin, "La Regola Del 'Grado'," 52; Reaney, *Johannes Hothby: De Arte Contrapuncti*, 25. The treatises also use ordinal numbers to name the intervals.

[240] Ugolino likewise recognizes the seven-note diatonic scale as the source and foundation for all other notes. This becomes especially important in the next chapter of this dissertation, since, as we shall see, some have linked Ugolino's comments on musica ficta with the idea that the hexachord, and not the seven-note diatonic scale, serves as the foundation for music.

[241] The grado treatises together with countless other practical manuals from the Middle Ages employ the hexachord or system of hexachords to teach music. This has misled some into thinking that the hexachord is the foundation of medieval music, which results in conclusions about medieval music theory that is inconsistent with the texts. I take up this topic in chapter 4.

quality. The consonances consist of the unison and the fifth. Thirds and sixths in general, without reference to whether they are major or minor, are referred to as dissonances. Seconds, fourths, and sevenths, as well as augmented fourths and diminished fifths are discordant.[242]

The grado treatises continue by discussing the structure of each interval from the second through the sixth but omitting the seventh.[243] They describe how each interval is made up of a certain combination of tones and semitones. Besides classifying each interval as a consonance, a dissonance, or a discord, they also designate some intervals as major/minor or perfect/imperfect. F1 says that thirds can be major or minor, but it also refers to them as perfect and imperfect respectively.[244] On the other hand, W and M do not call thirds perfect or imperfect, preferring instead the terms major and minor.[245] M and W, although they consider the fourth a discord, do say that it can be used in pieces with three voices, even though they do not deal with counterpoint in three voices.[246] Augmented fourths and diminished fifths, if they are specifically named at all, are called discordant.[247]

The discussion of sixths is most interesting. Although at first both W and M categorize sixths as dissonant, they also talk about a discordant sixth. The former is a major sixth and the latter a minor sixth, although W and M do not use the terms major and minor for sixths even though they do for thirds. They treat the dissonant and discordant sixths in separate places: first,

---

[242] Scattolin, “La Regola Del ‘Grado’,” 52, 57. W and M note that the fourth is used in three-voiced pieces. Scattolin, “La Regola Del ‘Grado’,” 54. Busse Berger cites the tripartite classification of intervals as proof that the grado tradition originates in the fourteenth century. Busse Berger, *Medieval Music*, 131.

[243] Scattolin, “La Regola Del ‘Grado’,” 53-58; Seay, *Quatuor Tractatuli*, 18-21.

[244] Seay, *Quatuor Tractatuli,* 18. F2 omits any discussion of intervals and interval qualities and begins directly with what a grado is.

[245] Scattolin, “La Regola Del ‘Grado’,” 54.

[246] Scattolin, "La Regola Del 'Grado'," 55.

[247] Scattolin, "La Regola Del 'Grado'," 56-57; Seay, *Quatuor Tractatuli*, 20-21.

they discuss the dissonant sixth, interrupt this to speak of fifths, and only then describe discordant sixths. In fact, when they return to the topic of sixths, they refer to the dissonant sixth (the major sixth) as consonant and then write about the discordant sixth.[248] For W and M, the minor sixth can only be used if one adds to it the sign of the diesis (what looks like and from which our modern sharp sign is derived), just as one would do to correct the discordant fifth.[249] In effect, W and M only allow major sixths to be used. F1 takes a slightly different stance from W and M. It speaks of the dissonant sixth and divides it into two types, major and minor or perfect and imperfect respectively. It does not entirely forbid the use of minor sixths: if the tenor remains on one pitch and the counterpoint moves from a fifth to a minor sixth and back to a fifth, then it is acceptable. However, if the sixth moves to an octave, then it must be made major by using the sign of the diesis.[250] Although it does not use the term discordant for minor sixths and allows the minor sixth to be used in counterpoint under one condition, it, like W and M, prefers the major sixth in all other circumstances.

Ugolino follows the same sequence of topics as the grado treatises just discussed, but his account is much fuller and more consistent. He distinguishes between the grado tradition of classifying intervals (the tripartite division) and a more modern one that views thirds and sixths as imperfect consonances. In addition, his account reveals a consistency not only in the terminology but also in the philosophical outlook. Like the grado tradition, he begins by discussing the seven-note diatonic system and the origin of intervals. He starts by describing the human voice (*vox*) and sound, citing directly Peter of Spain, Boethius, and Aristotle. This leads him to the seven diatonic notes (*voces*). All the other notes are derived (*derivantur*) and spring

[248] Scattolin, “La Regola Del ‘Grado’,” 56-57.

[249] Scattolin, "La Regola Del 'Grado'," 56-57.

[250] Seay, *Quatuor Tractatuli*, 19-20.

naturally (*natae*, literally born) from them.[251] Whereas W, M, and F1 said they "proceed" (*procede*), Ugolino chooses "derive" (*derivantur*) and "spring naturally" (*natae*). He could easily have selected the verb "proceed" (*procedere*), since he uses it elsewhere in the *Declaratio*. Instead, he avoids the more theological "proceed" and adopts the more philosophical "derived," which suggests a relation to logic, and "spring naturally," which implies a connection to nature. This reinforces the idea he set forth in book 1 that music belongs to natural philosophy and that musical objects are therefore considered as objects of nature.[252] After showing that all the other notes are derived by octave equivalence from the original seven, he classifies intervals into only two categories: consonant and dissonant. I refer to this as the bipartite division. Unlike the grado treatises, he does not, at first, speak about discords. For Ugolino, the unison, thirds, fifths, and sixths are consonances, but seconds, fourths, and sevenths are dissonances. In order to prove this, he cites Boethius's definitions of consonance and dissonance. After this, he adds the sub-categories of perfect and imperfect. The fifth, octave, and their compounds are perfect, while thirds, sixths, and their compounds are imperfect.[253] Imperfect consonances can be further divided into major and minor. Unlike F1, perfect and imperfect do not mean the same thing as major and minor. Therefore, all the grado manuscripts, including Ugolino's, follow a similar sequence of topics. But Ugolino offers a more consistent account both in terminology and in philosophy. Like the authors of the other grado treatises, he considers the seven-note diatonic system and octave equivalence as the foundation. But he spins them in a different direction. He re-words the relationship between them to emphasize the place of music as a science within natural philosophy, and he presents a more consistent view of consonance and dissonance.

[251] Ugolino, *Declaratio*, 2:5.

[252] I consider the relationship between Ugolino's classification of music and natural philosophy in chapter 2.

[253] Ugolino, *Declaratio*, 2:6.

Table 3.1: A Comparison of the Language in the *Grado* Treatises with that of Ugolino.[254]

	W	M	F1	Ugolino (Latin)	Ugolino (English)
Major Third	La dissonanzia tertia è de due spetie, cioè maiore e minore. La dissonanzia tertia maiore è formata de toni duy e alora è chiamata questa cotale dissonanzia ditono…	Dissonantia tertia est duarum spetierum, scilicet maioris et minoris. Dissonantia tertia maior est formata duabus tonis et tunc talis tertia vocatur ditonia…	La dissonantia terça e di due specie, cioe, magna et minore, o vogliamo dire perfecta et imperfecta. La perfecta e formata di due tuoni et e dicta questa dissonantia dituono tralle specie de canto.	Tertia igitur maior est et minor; tertia maior est quae ex duobus tonis dicitur esse formata et ista diphtonus appellatur.	Therefore, the third is major and minor; the major third is said to be formed by two tones and is called a ditone.
Major Sixth	La dissonantia sexta è formata di quatro tuony et uno semituono. Come che sia anchora una altra spetie di sexta, de la quale di soto ne tractaremo et anchora de la quinta prohibita.	La dissonanza sexta è formata de quatro toni e de uno semitono. Come che ancora sia un'altra spetie de sexta, de la quale de soto ne tratarò e ancora de la quinta prohibita.	La sexta dissonantia ancora sono di due specie, cioe, magiore et minore overo perfecta et imperfecta, et luna et laltra sadopera nel lcontrapunto…la magiore e formata di quatro tuoni e uno semituono. Et dicta questa sexta tra le specie musicali tuono con diapente.	Sexta maior et minor est. Sexta maior ex quatuor dicitur constare tonis ac uno minore semitonio et haec diapente cum tono vocatur.	The sixth is major and minor. The major sixth is said to consist of four tones and one minor semitone, and this is called "diapente cum tono."

[254] Scattolin, "La Regola Del 'Grado'," 54, 56; Seay, *Quatuor Tractatuli*, 18-19; Ugolino, *Declaratio*, 2:10-11.

To see one instance of Ugolino's terminological clarity in the description of intervals, I offer a comparison of the descriptions of thirds and sixths between the grado treatise and Ugolino in Table 3.1. Comparing the order and manner in which the grado treatises describe intervals (in this case the major third and the major sixth) reveals that Ugolino likely knew either these treatises (i.e., F1 and F2) or at least the tradition itself. Where the discussion comes in the overall structure of the work, and the language used to define it—all these closely follow the grado treatises. Ugolino's word choice closely matches the three treatises presented in the table above. They all describe intervals as combinations of smaller parts. They all use the term *formata* to express the intervallic composition of the given interval. In addition, Ugolino alerts his readers to the fact that he is depending on another text or textual tradition by using the word *dicitur* (it is said).[255] But there are several differences between Ugolino's text and that of the grado treatises. Although all of them include the name ditone for the third, only F1 provides "tuono con diapente" for the sixth. W and M do reference another type of sixth (the minor sixth), but they lump it together with the diminished fifth as a discord. F1 goes on to give the intervallic make up of the minor sixth, provides one instance in which it can be used, then explains that the minor sixth, used outside the one allowable exception, must be raised through the application of the diesis before it can be used in counterpoint. Whereas W, M, and F1 present inconsistent terminology or approach, Ugolino has regularized the definitions. In the definitions of each interval, he includes the intervals they are formed from and their alternative name. He has expunged the prohibitions against minor sixths without comment, thereby bringing the tradition

[255] For additional proof that Ugolino is depending on these treatises or the grado tradition, compare the way Prosdocimo defines the major third: "Maior est tercia illa que in se duos continet tonos." Prosdocimo, *Contrapunctus*, 44. Although the content is the same, the wording differs. He uses the verb *continere* instead of *formare*, a difference which also requires Prosdocimo to add *in se*. This linguistic difference lends credence the idea that Ugolino may have depended more on the grado tradition than on Prosdocimo.

in line with current practice. Therefore, Ugolino clearly knew the grado tradition. Even if the authors of M and W were following Ugolino, they do not do so consistently. Ugolino follows the same course of topics and presents the definitions of intervals in a similar manner as the grado manuscripts. Both where he sticks close to the tradition and where he modifies or departs from it are deliberate choices that offer a glimpse into Ugolino's distinctive account and disclose his goals and audience.

Another change from the manuscripts that Ugolino makes is his use of the bipartite division of interval qualities, as described above. Yet Ugolino's readers may have been aware either of the earlier grado tradition or of the older classification of intervals. Consequently, he must address the tripartite division of interval quality that is so prominent in that and older traditions. In the following passage, he lays out the differences in nomenclature.

> However, these intervals can be identified by another division and although it is not very properly suited to them, nevertheless at the present time they are allotted their definition as a common name, so that what are called perfect consonances take the name consonance. And what are named imperfect consonances, because they do not have the full perfection of consonance but are disconnected from their perfection, obtain the name dissonance. But what were called dissonances because of their bitter discord are labeled discords, because of their meaning. So then, from this perspective, the consonances are the unison (by means of its status as origin), the fifth, octave, twelfth, fifteenth, and nineteenth. The dissonances are the third, sixth, tenth, thirteenth, and twentieth. The discords are the second, the fourth (for the tritone), seventh, eleventh, fourteenth, and twenty-first. The discords are completely rejected for use in counterpoint.[256]
>
> *Possunt tamen eae voces alia divisione cognosci et licet non ita proprie eis competat, diffinitio tamen commune nomen ita hodierno tempore sortitae sunt, ut quae perfectae consonantiae sunt vocatae consonantiae nomen teneant. Quae vero imperfectae consonantiae nominantur ex quo plenam consonantiae perfectionem non habent, sed ab ea perfectione distant dissonantiae nomen acquirant. Sed quae ex inimica discordia dissonantiae dictae sunt ex earum significatione discordantiae appellentur. Sunt igitur consonantiae secundum hanc considerationem, unisonus ratione originis, quinta, octava, duodecima, quintadecima, nonadecima; dissonantiae sunt tertia, sexta, decima, tertia decima, vigesima; discordantiae sunt secunda, quarta pro tritono, septima, undecima, quartadecima, vigesima prima, quae discordantiae a contrapuncti usu penitus repelluntur.*

[256] Ugolino, *Declaratio*, 2:7.

Ugolino prefers the bipartite division as the one more closely representing the nature of the matter, since he calls the older tripartite one "not very properly suited to [these intervals]." Still, he sees nothing really contradictory between them, viewing them as merely naming conventions. In fact, he lets the two ways of speaking stand side by side throughout the rest of his book, referring to thirds and sixths both as consonances and, in an aside, as dissonances. Including both would help readers or teachers explain the older terminology, especially if they had any occasion to read or use the older texts.[257] Discords receive their name because of the way they sound. Dissonances are so called because they are disconnected, or more literally, sound apart from their perfection. Perfection, to which he devotes an entire chapter later, refers to the perfect consonances that serve as the proper resolution for any third or sixth—any imperfect consonances. This contrast with the grado treatises brings to light not only that Ugolino's reforms of the grado tradition are conservative but also that Ugolino's relies on the categories of perfect and imperfect consonances. He even takes an entire chapter to examine perfection. How does he define these terms? What is consonance? What makes a consonance either perfect or imperfect? What is a perfection?

Ugolino provides an overview of perfect consonances in chapter 4 and of imperfect consonances in chapter 5 of book 2. In his overview, he refers his readers back to book 1, where he covers each interval more thoroughly and which he merely summarizes in these two chapters. By considering these places and others where he talks about consonance, we learn that Ugolino

[257] Recall that Ugolino was most likely writing in the context of the cathedral school in Ferrara. The teachers and students may have had more familiarity with or more opportunity to encounter older texts that used the tripartite division. Ugolino modernizes without completely setting aside the older tradition. He is a reformer, not a revolutionary. However, in a different context, describing the older method may not have been necessary. For example, Prosdocimo worked at the University of Padua. His treatise on counterpoint is much shorter than Ugolino's, and it contains far fewer musical examples. Accordingly, Prosdocimo sees no need to talk about the older division and only writes about the newer bipartite division. This reveals that Prosdocimo's audience was probably not people who intended to practice counterpoint, whereas Ugolino's was. See Prosdocimo, *Contrapunctus*, 39; Busse Berger, *Medieval Music*, 146-47.

presents, in true Aristotelian fashion, both a description of consonance and an explanation of it. The description of consonance defines how a certain sound is perceived by the sense of hearing. Here, he repeats Boethius's definition of consonance, which states, "consonance is a mixture of high and low sound falling pleasantly and uniformly on the ears."[258] The ears, or the sense of hearing, are a kind of judge which can determine which intervals are consonant. He appeals to the sense of hearing in his description of the fifth, when he says that it is the "sweetest consonance of all" (ispa consonantia diapente omnium dulcissima).[259] Likewise, the sense of hearing also decides which intervals are dissonant. Although Ugolino does not quote Boethius's definition of dissonance in book 1, he clearly alludes to it when he talks about dissonant intervals.[260] For example, the major seventh, which he names a diapente with ditone, "brings a harsh and very unpleasant sonority to the hearing" (asperam inucundissimamque affert auditui sonoritatem).[261] Boethius uses the very same words in his definition of dissonance: "Dissonance, on the other hand, is a harsh and unpleasant percussion of two sounds coming to the ear intermingled with each other."[262]

The sense of hearing also helps determine suitable contrapuntal progressions. Some consonances (imperfect) move to other consonances (perfect). The motion usually involves both voices moving in contrary motion by step to the next closest consonance (e.g., a third to a fifth, or a sixth to an octave).[263] This motion is described by Marchetto as compatibility, by Ugolino as

[258] Boethius, *The Fundamentals of Music,* 16; Ugolino, *Declaratio*, 1:29.

[259] Ugolino, *Declaratio*, 1:58.

[260] However, Ugolino does present Boethius's definition of dissonance in book 2. Ugolino, *Declaratio*, 2:6

[261] Ugolino, *Declaratio*, 1:65.

[262] Boethius, *The Fundamentals of Music*, 16.

[263] See, for instance, Ugolino, *Declaratio*, 2:12.

perfection, and by the recent scholar David Cohen as directed motion.[264] Cohen notes that Marchetto does not identify why an interval or a progression is compatible, only that its compatibility is associated with voice leading. Cohen writes, "'compatibility,' it seems, is somehow caused by voice-leading proximity, although how this should be the case remains unclear."[265] The sense of hearing, by perceiving intervals and the motion of one interval to another, can distinguish them as consonant or dissonant and can even discern a difference between perfect and imperfect consonances. Yet, as Cohen observes, this does not really explain why they are so, or how they are so—it does not account for the cause of compatibility nor, by extension, the cause of the separation into perfect and imperfect. For that, more than a description is necessary: a rational explanation is required. For Ugolino, such an explanation of consonance requires an analysis of the object according to Aristotle's four causes. Indeed, Ugolino explicitly recognizes this as he begins discussing intervals in book 1. He says that he plans to analyze each interval:

> For then we are said to know and understand a thing when we know its causes. This happens when we understand the definition of some effect through its proximate and remote causes and even to its elements [smallest parts].[266]
>
> *Tunc enim rem dicimur scire et intelligere cum eius causas novimus. Quod fit cum alicuius effectus per propinquam atque remotam causam et usque ad elementa diffinitionem intelligimus.*

By reviewing Ugolino's analysis, I show how he classifies consonances and the reasons for distinguishing between perfect and imperfect. Cohen noted that Marchetto and others did not

---

[264] Marchetto, *Lucidarium*, 208-13; Ugolino, *Declaratio*, 2:12-15; Cohen, "The Imperfect Seeks Its Perfection," 139. Directed motion should not be confused with Sarah Fuller's term "directed progression." Sarah Fuller, "On Sonority in Fourteenth-Century Polyphony: Some Preliminary Reflections," *Journal of Music Theory* 30 (1986); Sarah Fuller, "Tendencies and Resolutions: The Directed Progression is *Ars Nova* Music," *Journal of Music Theory* 36, no. 2 (1992). Fuller uses the term directed progression to refer to three voice constructions, but Cohen uses "directed motion" to refer to two-voice paradigms. However, Fuller's directed progression clearly grows out of the practice of imperfect consonances moving to the closest perfect consonance in two voice structures.

[265] Cohen, "The Imperfect Seeks Its Perfection," 150.

[266] Ugolino, *Declaratio*, 1:46.

answer why compatibility is connected to voice leading. Ugolino's explanation may offer a reason for the link between the two.

Ugolino, just like the grado treatises, claims that the source of all intervals is the unison: "and because the unison is the origin of all notes, so we must state first what a unison is" (et quia omnium vocum origo unisonus est, ideo prima quid sit unisonus dicendum est).[267] However, the unison does not exactly match Boethius's definition of consonance, a point Ugolino never explicitly states, since Boethius requires a high and a low sound. Therefore, the octave stands in as the one interval from which others are derived. In fact, Ugolino describes the octave or diapason as "the mother of consonances" (*consonantiarum mater*), "the greatest of consonances" (*maximam consonantiarum*), and "the best of consonances" (*consonantiarum optima*).[268] Ugolino, like the grado treatises, considers an interval, even the octave, to be made up of smaller parts. These parts stand in a certain relationship to the whole. This leads Ugolino to an investigation of the formal cause. Consonances "are perfect when their form is perfect, [and their form] is composed of the completed parts that compose them" (perfectae sunt quando earum forma perfecta est, est ex suis perfectis partibus eam constituentibus est compacta).[269] For each interval, Ugolino lists the parts they are made of, stated in terms of the number of tones and semitones. For example, a fifth is composed of three tones and one minor semitone. When found with this arrangement of parts, it is a perfect fifth. But when these parts are lacking, the fifth is no longer perfect but imperfect. If a fifth has only two tones and two minor semitones, then it is imperfect—what we would describe as a diminished fifth.[270] For this narrower sense of perfect

[267] Ugolino, *Declaratio*, 2:8.

[268] Ugolino, *Declaratio*, 2:9; Ugolino, *Declaratio*, 1:46; Ugolino, *Declaratio*, 1:69.

[269] Ugolino, *Declaratio*, 2:8-9.

[270] Ugolino, *Declaratio*, 2:9.

and imperfect, each interval taken by itself—at least in this case fifths, octaves, twelfths, and fifteenths—can have a perfect or an imperfect form.[271] Yet, perfect and imperfect can also be used in a broader sense to describe fifths and octaves (and their compounds) as opposed to thirds and sixths (and their compounds). To analyze the form in this case, Ugolino measures all intervals against the octave, the mother of them all.

Ugolino divides the octave into its constituent parts, then relates these parts to the whole as either proximate or remote causes. He provides a definition of each interval, beginning with a tone and continuing to an octave, and compares them with the octave. He offers the following summary of his analysis:

> Therefore, desiring to understand the greatest of consonances and to have knowledge of it, namely the diapason, it is necessary to know first its proximate and remote causes as well as its elements. The proximate causes are said to be those into which the diapason is first immediately separated, namely, the diapente and diatessaron. These are named the immediate components of the diapason, since if the diapason undergoes division, it is determined to be divided first into these two [parts] from which it is composed. But the remote causes are said to be those which compose the proximate causes and cause them. Such are ditones and semiditones which are recognized as causes or parts of the diapente and diatessaron. But the elements or most remote causes are said to be those which do not undergo further division, such as tones, which, although they are divided into semitones, nevertheless do not tolerate division of
>
> *Volentes igitur maximam consonantiarum intelligere et ipsius habere scientiam, scilicet, diapason, causas eius propinquas atque remotas necnon et elementa praecognoscere oportet. Causae autem propinquae diapason illae dicuntur in quas primo ipsa diapason immediate resolvitur, scilicet, diapente et diatessaron, quae componentia immediata diapason appellantur, quoniam si divisionem patiatur diapason in haec duo ex quibus componitur primo partiri dignoscitur. Causae vero remotae sunt quae causarum propinquarum compositivae dicuntur et illas efficiunt ut sunt diphtoni et semidiphtoni qui diapente et diatessaron noscuntur efficientes vel ipsorum partes. Sed elementa sive remotissimae causae illae dicuntur esse quae ulteriorem divisionem non patiuntur, sicut toni qui licet in semitonia dividantur alterius tamen minoris integrae quantitatis sectionem non sustinent. Sed quoniam ut*

[271] Ugolino does not carry this narrow meaning of perfect and imperfect into his discussion of thirds and sixths. Instead, he speaks only of major and minor. However, the narrower definition of perfect and imperfect could be what motivates F1 to equate major with perfect and minor with imperfect. And, as we shall see later, for imperfect consonances to resolve to perfect ones, there is a strong tendency, both in Ugolino and especially in the grado treatises, for them either to be major or to be altered from minor to major.

another smaller whole quantity. But, as we have said, the components precede the composed in nature and in time. Therefore, following in the correct order, first we intend to treat the tone and its parts as the most remote cause or elements of the diapason. Second, we [will treat] the ones that are composed of tones and their parts and so on until we have an understanding of the greatest consonance, the diapason, through its remote and proximate causes.[272]

iam diximus componentia natura et tempore antecedunt composita. Idcirco recto ordine prosequentes primo de tono et eius partibus tamquam de causa remotissima seu elemento ipsius diapason intendimus pertractare. Secundo de compositis ex tonis et eorum partibus et sic ultra donec per causas remotas atque propinquas ipsius maximae consonantiae diapason intelligentiam habeamus.

Here, the octave is the measuring stick in the sense that all of the other intervals are related to it by degrees or distance. Fourths and fifths stand in a proximate or close relationship to it. For this reason, they are perfect consonances.[273] Fourths and fifths are built out of thirds (ditones and semiditones). In relation to fourths and fifths, they stand in a close relationship. But in relation to the octave, they lie at a remote distance. Consequently, they are imperfect consonances. They still fulfill Boethius's definition of consonance, but in relation to the octave—the fundamental unit of measurement in this situation—they do not fully measure up. Thirds are composed of even smaller parts, which Ugolino describes as elements, and which include tones and semitones. In relation to the octave, these elements stand in the furthest, most remote location. Therefore, they are dissonances. They not only fall short of Boethius's definition of consonance but also fail to reach even a close distance to the octave. As a result, they match Boethius's definition of dissonance. Therefore, by measuring all these intervals with the octave, Ugolino associates proximate with perfect, remote with imperfect, and most remote with dissonant. Although Ugolino never explicitly makes this link, he comes closest to stating it outright in book 2, where

[272] Ugolino, *Declaratio*, 1:46-7.

[273] In this scheme, fourths would be classified as perfect consonances. And in fact, Ugolino describes them as consonances. Ugolino, *Declaratio*, 1:53. Yet in book 2, he states that they are dissonances (i.e., they are not used in two-voice counterpoint), but they were once regarded as consonances "by the ancients" (*ab antiquis*). Ugolino, *Declaratio*, 2:6.

he summarizes perfect and imperfect intervals. After repeating Boethius's definition of consonance, he adds which intervals he considers consonant:

> Namely the third, fifth, and sixth, but of these only one is said to be a perfect consonance, specifically the fifth. But the rest are said to be imperfect consonances because they are distant from the full perfection of consonance.[274]
>
> *Scilicet, tertia, quinta et sexta, sed ipsarum una duntaxat dicitur esse consonantia perfecta, scilicet, quinta, reliquae ver quia a consonantiae plena perfectione distant imperfectae dicuntur consonantiae.*

The spatial metaphor makes the connection apparent.

Ugolino uses this spatial metaphor to explain intervals throughout his discussion in book 1, where he writes one chapter on each interval from the tone through the octave (and even a few beyond the octave). For example, when he begins the chapter on the tone, he says,

> Therefore, the tone is said to be the first and whole interval of all, and it is the most remote cause of the greatest consonance.[275]
>
> *Tonus igitur qui omnium prima et integra dicitur esse coniunctio et ipsius maximae consonantiae causa remotissima.*

He then provides a definition of the tone in such a way that it is divided into unequal parts, the major and minor semitone, in keeping with his Boethian and Pythagorean heritage. He follows the same practice with the other intervals, always reminding the reader of their relationship to the octave before offering a definition of its genus, species, and differences.

Ugolino does not merely state that some intervals are perfect and others imperfect. Instead, he attempts to answer why they are so by analyzing them in relation to a common measure, the mother of all intervals, the octave. Because of their distance to the octave, some intervals are perfect consonances. Others seem to match Boethius's definition of consonance but do not reach the same relation to the octave, and these are imperfect consonances. Still others are even more distant to the octave, and these are dissonant. But two intervals are conspicuously

[274] Ugolino, *Declaratio*, 2:6.

[275] Ugolino, *Declaratio*, 1:47.

absent from Ugolino's description above—sixths and sevenths. Although he refrains from speaking in terms of a specific cause (e.g., more remote or most remote), he still thinks of them in spatial terms either by their distance to the octave or another consonance or by their proportion. He thinks of sevenths as a perfect fifth plus either a minor or a major third. About the diapente with ditone (major seventh), he uses language both of distance and proportion. He says, "it cannot have the value of a consonance because it does not possess a proportion of consonances," (consonantiarum non retinet proportionem…vim consonantiae obtinere non potest),[276] and it is "especially remote from the agreement of dissonant notes [i.e., imperfect consonances]" (a dissonantiarum convenientia vocum permaxime remota sunt).[277] All of this shows that the major seventh "is neither a consonance nor a dissonance [imperfect consonance], but it is allotted the name of discord" (non est ergo haec coniunctio consonantia neque dissonantiae, sed discordii nomen sortiri conceditur).[278] Even the major seventh is described with the language of distance: it is especially far from an imperfect consonance, which makes it a discord. Ugolino also hints that proportion plays a role in explaining intervals. Indeed, providing the mathematical proportions for intervals is another way of analyzing or explaining their form, which he does at length in books 4 and 5.[279] Ugolino thinks of sixths as fifths with either a minor

[276] Ugolino, *Declaratio*, 1:64.

[277] Ugolino, *Declaratio*, 1:64.

[278] Ugolino, *Declaratio*, 1:65.

[279] Ugolino takes up the entirety of book 4 and much of book 5 to discuss the mathematical proportions of intervals. Book 1, which I use here, offers the first layer of Aristotelian analysis. It explains the form of intervals in terms of their distance and their intervallic constituents, for example, that the fifth contains three tones and one minor semitone. Books 4 and 5 take this and analyze these further into their mathematical proportions, for example, that the ratio of the tone is 9:8 and the minor semitone is 256:243. Once he has done this, Ugolino can then show that some proportions belong to a certain class, superparticular for instance, and thus further refine his analysis. Since Ugolino is Pythagorean, I do not discuss these books. What is more important for my work is to see that he is offering an Aristotelian analysis of the form of intervals. In book 1, his language is more practical, and in books 4 and 5, he covers the same material but in language that is clearly theoretical. His practical language in particular helps us see how he divides consonant intervals into perfect or imperfect.

third or a major third added on. The major sixth is a diapente with ditone, and he defines it in relation to the octave: "We certainly use this interval in counterpoint when it is joined to a diapason, where we seek the harmony of the diapason, since it is its perfection" (Hac equidem vocum connexione utimur in contrapuncto diapason associata, ubi ipsius harmoniam diapason quaerimus quoniam eius perfectio est).[280] After saying this, he further describes the sixth and its distance from the perfect fifth, imperfect (diminished) fifth, and the fourth. He speaks of the sixth by its relationship to the octave. It is used in a progression to the octave, and this helps classify it as an imperfect consonance. When this happens—when a major sixth moves to an octave—it creates what Ugolino calls a perfection. This is more than just a perfect consonance, since it involves the motion of one interval to another. What exactly is a perfection in this sense and how, if at all, is it tied to the classification of imperfect and perfect intervals?

In book 2, Ugolino devotes an entire chapter to perfection. It directly depends on an understanding of perfect and imperfect things, and he ties these together with the concept of motion. Already in book 1, Ugolino described the major sixth by its motion to the octave. Ugolino relies on Aristotelian philosophy to make sense of them. He begins his chapter on perfection with the following statement:

> By nature, what is imperfect and incomplete, in order to have a perfect form, is compelled to move as it tends towards what it lacks. When imperfect consonances (or dissonances) are imperfect in comparison to consonances, and when each one does not have its own perfection, the imperfect desires to go to it, so that it may be established in the essence [being] of a consonant perfection. These observations are certainly known to those experienced and practiced in theory, because if the
>
> *Natura, quod imperfectum est et incompletum, ut perfectam habeat formam ad id tendens quo deficit moveri compellitur, cumque consonantiae imperfectae seu dissonantiae praedictae consonantium comparatione imperfectae sint, et ipsarum perfectionem non habeant unaquaeque ut inesse consonantis perfectionis constituantur, eam natura gliscit adire. Expertis theoricae peritis haec indubie nota sunt, quia si vel in tertia, sexta vel decima sit vocum positio*

[280] Ugolino, *Declaratio*, 1:62.

> intervals form a third or sixth or tenth, no rest occurs, but each—so that it may be joined to its own perfection—moves as if compelled to it. From this arises the fact that any song arranged in proportion ends on a perfect consonance, although before the last end it may sometimes close on imperfect consonances or dissonances. But because it does not cause rest, however well arranged to the listener, a final consonance is added as an end.[281]
>
> *non fit quies, sed quaelibet ut suae copuletur perfectioni ad eam coacta movetur. Et hinc est quod cantus quilibet mensura ordinatus in perfecta consonantia finem habet, quamvis ante finem ultimum quidam in imperfectis consonantiis seu dissonantiis interdum habeat terminari in quo quia audientis bene disposita auris non quiescit, ultimus consonans finis addicitur.*

Ugolino's thorough account refers to what David Cohen calls "directed motion," an idea indebted to Aristotelian natural philosophy.[282] Cohen resolves the idea into five distinct propositions: (1) separating objects into perfect and imperfect; (2) asserting that what is imperfect seeks what is perfect; (3) saying that imperfect things do so precisely because they are imperfect; (4) arguing this happens by nature; and (5) showing that each imperfect thing strives towards its own perfection.[283]

Ugolino fulfills all of Cohen's five conditions. I showed above that Ugolino classifies consonances into perfect and imperfect and how he does so, which accounts for the first condition. When he says, "the imperfect desires to go to it [its perfection]" and "it [what is imperfect] tends towards what it lacks," he is clearly asserting Cohen's second and third conditions. Ugolino satisfies the fifth condition at the end of the statement, since the rest of the chapter deals precisely with which particular imperfect consonance moves to its own particular

[281] Ugolino, *Declaratio*, 2:12. Cohen provides his own translation of this passage. He argues that where Seay has "inesse," the text should more likely read "in esse." Cohen, "The Imperfect Seeks Its Perfection," 164. I agree with his reading and have translated it accordingly.

[282] "Directed motion" should not be confused with Sarah Fuller's term "directed progression." See Fuller, "On Sonority on Fourteenth-Century Polyphony," 36-70; Fuller, "Tendencies and Resolutions," 229-58. Fuller uses the term "directed progression" to refer to three voice constructions, but Cohen uses "directed motion" to refer to two-voice paradigms. However, Fuller's directed progression clearly grows out of the practice of imperfect consonances moving to the closest perfect consonance in two voice structures.

[283] Cohen, "The Imperfect Seeks Its Perfection," 146.

perfection or perfect consonance.

Ugolino spends the most time showing in practice how the fifth condition is fulfilled. It occurs when an imperfect consonance moves to the closest perfect consonance by using contrary motion. This method is sometimes referred to as the closest-approach, and because of its connection to Aristotelian philosophy, it was taught by many theorists, including Ugolino's contemporary Prosdocimo.[284] According to Ugolino, thirds have two perfections: they can move either to a unison or to a fifth. In both cases, the tenor ascends, and the upper voice ascends.[285] When the third moves to the unison, a "sweeter harmony" (*dulcior harmonia*) can be produced by using a minor third. On the other hand, he says that "according to some the major third is perfected by the fifth" (secundum quosdam tertia maior a quinta perficitur).[286] Thus, although thirds in general can progress to a unison or a fifth, it is sweeter when a minor third resolves to a unison and a major third to a fifth. The sixth has a "proper perfection" (*propria perfectione*) and an "improper perfection" (*impropria perfectione*). The former indicates the motion of a sixth to an octave and the latter a sixth to a fifth. The former, like the movement of the third to the unison or fifth, includes contrary motion. But for the latter, one voice will remain stationary while the other moves either down or up.[287] Indeed, the fact that the latter resolution does not use contrary motion may explain why he calls it improper. He makes no mention here about changing a minor sixth to a major one or vice versa. As with the thirds, it is enough for a perfection if a major or minor sixth moves to an octave. Changing an interval from minor to major (or major to minor) merely colors or sweetens a progression. It does not fundamentally change the type of

[284] Prosdocimo, *Contrapunctus*, 80-84.

[285] Ugolino, *Declaratio*, 2:12.

[286] Ugolino, *Declaratio*, 2:13.

[287] Ugolino, *Declaratio*, 2:13. The improper perfection of the sixth sounds similar to F1's only use of the minor sixth, stated above.

progression. Ugolino reserves the discussion of changing the quality of intervals for the chapter on musica ficta, which I discuss in chapter 4. He finishes the chapter on perfections by considering how to perfect compound thirds and sixths.

Ugolino also fulfills Cohen's fourth condition. In fact, nature takes center stage. Nature is the first word in Ugolino's statement, and it is also a guiding principle throughout the rest of his discussion. By nature, he claims, an imperfect consonance moves to a perfect consonance. Because of their nature, imperfect things in general "desire" (*gliscit adire*), "are compelled to move" (moveri *compellitur*), and "tend towards" (*tendens*) their perfection. For Ugolino as for Aristotle, nature is linked with motion. Indeed, for Aristotle, one defining characteristic of nature is motion.[288]

The connection between nature and motion flows from Ugolino's foundational claim that music belongs to natural philosophy.[289] For Aristotle, motion encompasses a greater range of meaning than it does today. It is closer to what we might categorize as any type of change. For example, when we think of motion, we point to an object changing place. Aristotle recognized this kind of local motion, but he also considered as other types of motion growth (and shrinkage or decay) and changes in quality and quantity. In addition, he distinguishes motion by its source as either external or internal. When looking at a statue, it came to be what it is through motion but motion external to it, through the work of a sculptor. When looking at an acorn that grows into a tree, its source of motion is internal to it. The acorn has a distinct relationship to the oak tree. It is an oak tree only in potency: it has the ability to become an oak tree. As it grows, it changes its form to become the oak. Whereas the acorn is an oak in potency, the oak is one in

[288] Aristotle, *Physics* 2.1.192b21-22.

[289] I examined this topic more fully in chapter 2.

being-at-work or actuality. That is, it is existing as an oak tree without becoming something else. In the oak, the acorn has reached its perfection.[290]

This outline of Aristotle's thought illuminates Ugolino's concept of the motion from an imperfect consonance to its own perfection. Ugolino's statement reinforces his classification of music as a part of natural philosophy. The imperfect consonance is like the acorn. It is a perfect consonance in potency. As it moves to its own perfection, to its closest perfect consonance, it changes form to become a perfect consonance. The perfect consonance, in relation to the imperfect one preceding it, is perfect in being-at-work (or actuality). The change of form from imperfect to perfect, as well as the movement of the voices from one pitch to another, is motion in Aristotle's use of the term. The source of this motion is in the thing itself, that is, it is in the imperfect consonance. According to Ugolino, it is in imperfect things in general and, by extension, in the imperfect consonance in particular by nature. Nature, as I noted, is an important term in Ugolino's statement. Most importantly, it is cited as the underlying instigator of the motion. According to Aristotle, any object that "has in itself a source of motion" is an object of nature, and as a result, it belongs to the study of nature or natural philosophy.[291] Ugolino, therefore, is asserting that imperfect consonances are natural objects, whose movement to their own perfections results from a source within the imperfect consonance itself, by the very fact that it is imperfect. Although he may have recognized the role of a composer in writing a particular progression or of performers in singing one pitch after another, or indeed of the general motion of one pitch proceeding to another, yet his point in this passage is to place music squarely within natural philosophy. In other words, his classification of music as part of natural

[290] Aristotle, *Metaphysics* 9.1-10.1045b27-1052a14.

[291] Aristotle, *Physics* 2.1.192b15.

philosophy grows out of his observations on the nature of imperfect consonances and their perfections. Thus, his classification of music as a natural philosophy also shapes his language about counterpoint.

Ugolino goes beyond Cohen's five conditions to argue that perfections are more than closest-approach progressions to perfect consonances. They are used at the very end, perhaps of a phrase or an entire piece, to bring rest. They are, in this sense, similar to what we might think of as cadences. Not every progression from an imperfect consonance to a perfect one falls under the category of a perfection. In Ugolino's own examples of composed counterpoint, which he provides later in book 2, he has several instances of tenths moving to fifths, or of a tenth leaping into an octave. Although these progressions move from imperfect consonances to perfect ones, they do not correspond to any of the progressions Ugolino describes as perfections. In the last two decades of the fifteenth century, Johannes Tinctoris offers a definition of perfections. He states that it has two meanings, one relating to mensural music and the other to ends of pieces. "Perfection," he writes, "is the recognition of the completion of a whole piece, or any of its sections."[292] Tinctoris's account of perfection matches Ugolino's comment that perfections are added at the end to bring the piece to a satisfying rest.

I started this section by showing that Ugolino was dependent on the grado treatises for the beginning of his book on counterpoint, and, as I show in the next section, he leans on it heavily when he teaches about the grado theory itself. However, Ugolino also makes his own contributions, even here in the preliminary material. He updates the language on sixths by consistently describing them as imperfect consonances, without treating major and minor sixths

[292] Johannes Tinctoris, *Dictionary of Musical Terms,* trans. Carl Parrish (London: The Free Press of Glencoe, 1963), 49.

differently. He makes no prohibitions against minor sixths. He clearly describes two different naming conventions, the bipartite and tripartite divisions of quality. He sets them side by side, combining the old with the new but with clarity.

Ugolino also goes beyond the grado tradition by integrating his classification of intervals into the broader narrative of music as a part of natural philosophy. Relying on Aristotle, he offers an analysis of the relationship between imperfect and perfect consonances and fulfills Cohen's five conditions. By nature, imperfect things desire perfection, and specific imperfect things seek a perfection that is particular to them. Thus, thirds seek unisons and fifths, and sixths seek octaves. Seeking and desiring means moving to those perfections, and this motion lives in their nature, in the intervals themselves, in the way the intervals are structured. All these intervals are measured in relation to a common standard, the octave. Fifths and fourths (although Ugolino recognizes the fourth as dissonant in practice) stand in a proximal relationship to the octave. But thirds, and by extension sixths, lie in a more remote relation. In this sense, thirds and sixths are imperfect and consequently desire or seek the fifth or octave because of their structure. Therefore, Ugolino reasons that if imperfect intervals contain within themselves a source of motion drawing them to perfect consonances, then they are analogous to natural objects, and music must be considered part of natural philosophy. This view of Ugolino sets his theory apart from his contemporaries and even from the grado tradition which he is presenting. It allows him both to value theory and speculative thought and to elevate practical concerns because they are integral to the formation of theoretical thought. With these foundations laid, he proceeds to the grado theory itself.

3.3 The *Regola del Grado* in Practice

After fully discussing the distinction between perfect and imperfect consonances,

Ugolino arrives at the central teaching of the grado tradition. Something is conspicuously missing from his account: he never uses the word *gradus*. This word was clearly a viable translation of the Italian *grado*, because M uses it in the section that is in Latin.[293] Meaning degree or step, it refers to the distance between the starting note of the hexachord in the tenor and the starting note of the hexachord of the added voice. For example, if two singers were in the same hexachord, the relationship between the two is expressed as "grado di pari." If the one was in the natural hexachord and the other was in the hard hexachord above it, the degree is the "grado della quinta." F1, M, and W recognize four degrees (*gradi*): the unison, fourth, fifth, and octave.[294] F2 includes the same degrees but adds a degree of the twelfth, which Ugolino does not mention.[295] F1 arranges them in the order of unison, fifth, fourth, and octave, but W and M list them in the order of unison, fourth, fifth and octave. Ugolino describes only four degrees, presenting them in the order of unison, fourth, fifth, and octave.

The mere name gradus or grado does not give any indication of the purpose of calculating distances via hexachords. Ugolino deliberately omits the word gradus and replaces it with a phrase designating its purpose or definition. For example, chapter 7 is titled "On the Consonances and Dissonances of Notes in a Single Hexachord" (De consonatiis et dissonantiis notarum in una proprietate).[296] And he styles chapter 9, "On the Syllables of the Hexachords Whose Starting Notes lie a Fifth Apart" (De vocibus proprietatum quarum principia distant per diapente).[297] The title of chapter 9 gives the definition for the "grado della quinta," and that of

[293] Scattolin, "La Regola Del 'Grado'," 54.

[294] Scattolin, "La Regola Del 'Grado'," 59-60; Seay, *Quatuor Tractatuli*, 21-24.

[295] Seay, *Quatuor Tractatuli*, 25-31.

[296] Ugolino, *Declaratio*, 2:15.

[297] Ugolino, *Declaratio*, 2:17.

chapter 7 provides the underlying reason for teaching this system. The theoretical system serves a pedagogical purpose: it is a means of teaching and learning which notes are consonant, so that a singer or composer could quickly and accurately create consonant counterpoint. It is a practical way of memorizing all the consonances within the gamut.[298] So, for each degree, Ugolino lists the solfege for each step in one hexachord and each note that is consonant with it in the other. For every consonant note, he provides its solfege and the interval it creates. When he names the intervals, he does not give the quality of the interval, since he deals more fully with the manipulation of quality in the section on musica ficta.[299] Thus, when both parts are in the same hexachord and one voice is ut, that ut "has two consonances and two dissonances: ut a unison, mi a third, sol a fifth, and la a sixth" (duas habet consonantias et duas dissonantias, scilicet, ut unisonum, mi tertiam, sol quintam et la sextam).[300] "Re has two consonances and one dissonance: re a unison, fa a third, and la a fifth." (Re duas habet consonantias et dissonantiam unam, scilicet, re unisonum, fa tertiam et la quintam.)[301] He continues cataloging consonances for each solfege from ut to la, for all four degrees.

The language Ugolino uses closely matches F1, while M and W diverge slightly from Ugolino. W states, "Et perciò ut in grado di pari ha due consonantie, cioè pari e quinta et due dissonantie, cioè terza e sexta; la pari dice ut, la quinta sol, la terza mi, sexta la. Re ha due consonantie et una dissonantia…"[302] F1 contains the same information but words it slightly

[298] Busser Berger is one of the few scholars I know who writes at any length about the *regola del grado*. Her goal is to show how the system was meant to be memorized, and its relation, in general, to memory. Her observations vividly reveal how practical the *regola del grado* was. Busse Berger, *Medieval Music*, 133-50.

[299] I review Ugolino's chapter on musica ficta in detail in chapter 4.

[300] Ugolino, *Declaratio*, 2:15. In the same chapter he reminds his readers that for consonance he means perfect consonance and for dissonance he means imperfect consonance.

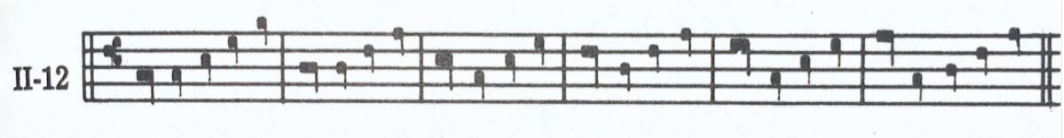
[301] Ugolino, *Declaratio*, 2:15.

[302] Scattolin, "La Regola Del 'Grado'," 60. I have not quoted M, since M closely follows W.

differently, “Adunque per grado di pari ut a due consonançe et due dissonançe in questo modo, per ut ut che fia pari, mi che fia terça, sol che fia quinta, la che fia sexta.”[303] Both state first that ut “has” (*ha*) a number of consonances and dissonances. W says first how many consonances ut has before listing them by interval type, then it follows the same pattern for listing dissonances. Only after this does it provide the solfege syllables for each. By contrast, F1 states that ut has two consonances and dissonances, then lists each interval in the order it appears in the hexachord by both solfege and interval type. Ugolino follows the presentation set out by F1.[304] In addition, Ugolino keeps the distinction between consonance and dissonance, even though, as I showed above, he recognizes they mean perfect consonance and imperfect consonance respectively.

For each grado, Ugolino offers a musical example that shows both the hexachordal step and each note consonant with that step (see Example 3.1). W, M, and F1 also provide similar musical examples, but F2 lacks any examples. Ugolino’s example for the “grado di pari” is nearly identical to the ones found in W and F1 with two small exceptions. First, he always arranges the list of consonant options in ascending order from lowest to highest, whereas the others write theirs from the given note. Second, he uses the natural hexachord as the starting point, but the others use the hard hexachord.

Example 3.1: The *Grado di Pari*.[305]



The examples for the degrees of the fifth and the octave are nearly identical among

[303] Seay, *Quatuor Tractatuli*, 22.

[304] Ugolino, *Declaratio*, 2:15-16. F2 uses different language to describe each of the gradi and, in this sense, seems to be an outlier.

[305] Example II-12 in Ugolino, *Declaratio*, 2:supplement.

Ugolino, W, and F1. For each of these, the lowest part is in the natural hexachord, except for the degree of the fourth. For this, Ugolino (and W seems to follow him in this) writes the lowest part in the hard hexachord. If Ugolino followed F1 as one of his main exemplars, then he clearly diverges from it when notating the example for the degree of the fourth.[306] F1 places the lower voice in the natural hexachord and the added part in the soft hexachord. So, for ut (C) in the natural hexachord, the consonant pitches in the soft hexachord are re (G), mi (a), and sol (c). For re (D) in the natural hexachord, the consonant pitches in the soft hexachord are mi (a), fa (b-flat), la (d) and ut (F). D to b-flat forms a minor sixth, but both W and F1 deny consonant status to the minor sixth and prohibit its use from counterpoint. Consequently, they both need to alter this pitch, which they do by adding the sharp sign. For the next step, mi (E) is in the natural hexachord, above which both manuscripts place re (g), fa (b-flat), and sol (c). Here fa forms a diminished fifth and sol is a minor sixth, so both require alteration, which W and F1 provide. Inexplicably, however, F1 also changes the g to g-sharp. There was no prohibition against minor thirds, so there is seemingly no reason for making it a major third. But this same phenomenon appears again over both sol and la in the natural hexachord: the thirds above sol and la (b-flat and c respectively) are raised so that they form major thirds.[307] Since they are consistently altered to make the thirds major, it is unlikely that it is an error in the text. W does not contain these further alterations over sol and la.

Ugolino avoids these problems. First, as stated earlier, he allows the use of minor sixths, making no rule against them or their use in counterpoint. In addition, he places the lowest part in

[306] For the text and examples from W and M on the degree of the fourth, see Scattolin, "La Regola Del 'Grado'," 61-62. For the parallel passage in F1, see Seay, *Quatuor Tractatuli*, 23.

[307] Seay, *Quatuor Tractatuli*, 23. F1 refers to these altered notes as *diesata* (literally diesis-ed) or *remoto*, presumably because it is removed from where it would be if it were sung without alteration.

the hard hexachord instead of the natural hexachord and the added part in the natural hexachord instead of the soft. For all the other examples, Ugolino used the natural hexachord for the lowest voice. So, he either switched hexachords to avoid the pitfalls found in F1, or he switched for some other reason. Yet the same problem (the diminished fifth between mi and fa) exists no matter which hexachords are chosen. Thus, it seems likely that Ugolino switched to avoid the problems he saw in F1, and W follows him in the switch. Curiously, Ugolino does not add any sign in his musical example to change the diminished fifth above mi. Instead, he acknowledges the issue in the text by labeling its interval type. He calls it "fa an imperfect fifth" (fa quintam imperfectam).[308] He avoids using it in his examples of composed counterpoint, and he discusses correcting imperfect fifths in the section on musica ficta, even though he does not at this point refer the reader there. By listing it together with other, usable consonant intervals, he gives the impression that an imperfect fifth is acceptable. But his discussion on musica ficta clarifies the point that such "imperfect consonances" should not appear in counterpoint.[309]

For each grado, Ugolino tells what hexachords he is using, lists the solfege and the intervals they form, as described above, and he provides a musical example for each. The language he uses to describe these is clearly echoed in W. Describing the consonant notes in each grado takes up one chapter for each step and thus covers chapters 7-10. In chapter 11, Ugolino begins a new topic that stretches through chapter 15. All the other grado manuscripts omit this, except for W, which most likely follows Ugolino. I first provide an overview of Ugolino's teaching in these chapters and then compare it with that found in W.

In chapters 7-10, Ugolino has discussed only what the consonances and dissonances in

[308] Ugolino, *Declaratio*, 2:17.

[309] Ugolino, *Declaratio*, 2:44.

each grado are. In effect, this creates multiple consonance tables, that is, lists of which notes are consonant when the tenor and added part are in a given hexachord(s). But in chapters 11-15, he shows how consonances are to be arranged to create counterpoint or simple progressions, and how the gradi provide a structure for learning these progressions. In chapter 11, he outlines the limits of the new topic:

> After demonstrating the consonances and dissonances of the hexachords, I must state how these consonances and dissonances should be arranged in counterpoint between the hexachords, and, as was said, this kind of counterpoint is the arrangement of only one hexachord against another without mutation. The notes of one hexachord, from ut to la, are arranged in various ways. Out of them, tones, semitones, ditones, semiditones, diatessarons, diapentes, and diapentes with tones, both ascending and descending, are composed. Thus, according to these [melodic] intervals an arrangement of counterpoint will be shown, both ascending and descending. But first, that order of counterpoint must be shown which relates to a single hexachord alone. By this, the knowledge of any hexachord is shown. Second, that order of counterpoint will be set down which relates to one hexachord with another.[310]
>
> *Habitis proprietatum demonstrationibus circa consonantias et dissonantias, declarandum est qualiter inter ipsas proprietates habeant dictae consonantiae et dissonantiae ordinari in contrapuncto, et, ut dictum est, huiusmodi contrapunctus est unius tantum proprietatis in alteram sine mutatione ordinatus et quia notae unius proprietatis, scilicet, ab ut ad la diversimode irdinantur, quia ex his componuntur toni, semitonia, diphtoni, semidiphtoni, diatessaron, diapente et diapente cum tono per arsyn et thesyn, ideo secundum has coniunctiones ordo contrapuncti monstrabitur in arsy et thesy. Sed is primo contrapuncti ordo monstrandus est qui ad unicam tantum proprietatem noscitur pertinere, quo notitia uniuscuiusque proprietatis ostenditur, secundo is contrapuncti ordo ponetur, qui ad proprietatem cum proprietate spectare videtur.*

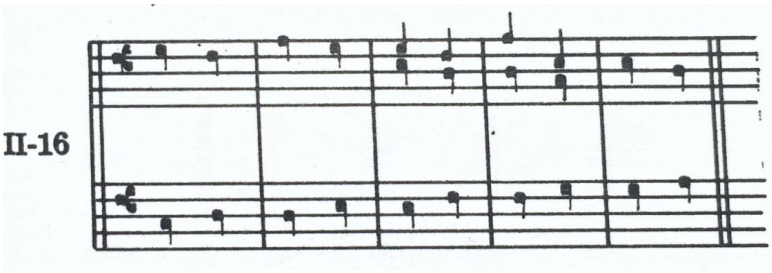
Ugolino takes the notes of a single hexachord and arranges them melodically first by steps, then thirds, fourths, fifths, and sixths, in both ascending and descending orders.[311] He

[310] Ugolino, *Declaratio*, 2:18.

[311] For each melodic step in the tenor, Ugolino seems to plan on one example for the ascending form and a second example for the descending form. Thus, we should expect ten examples in each grado. Although Ugolino claims that he will include examples of the tenor moving in sixths, he does not, in fact, include these in his text. So, we should see only eight examples for each tenor interval. Ugolino, however, does not strictly follow this format. In some chapters, the descending form is included in the same example as the ascending form. In addition, in chapter 11, he writes twice as many examples as we would expect. Despite these inconsistencies in his presentation, his point is clear.

writes each interval as a two-note unit. For example, when he arranges them by ascending steps, he writes ut-re, re-mi, mi-fa, fa-sol, and sol-la. These two note units will form the basis of a chant-like voice or tenor.[312] For each of the units in the tenor, he adds above it an example of a typical contrapuntal progression. Example 3.2 provides his first musical example of this technique. The tenor moves by steps from ut to la in the natural hexachord. In this example, the upper part also moves only within the natural hexachord. After Ugolino has presented examples like it, where both parts move within the same hexachord, he offers examples where the two parts are in different hexachords. The relationships between the hexachord of the tenor and of the added voice follow the same order as that of the gradi themselves, as outlined above: first, they move in the same hexachord, or "grado di pari," as in Example 3.2. Then, they lie at the distance of a fourth, fifth, and finally an octave. Indeed, he devotes one chapter, from 11-14, for each grado. It is a natural extension and development of the practice he presents in chapters 7-10. In these examples of typical contrapuntal progressions, Ugolino prioritizes contrary motion, although some examples do include similar motion. In addition, neither part moves outside its own hexachord: there is no mutation.

Example 3.2: Sample Progressions Using the *Regola del Grado*.[313]



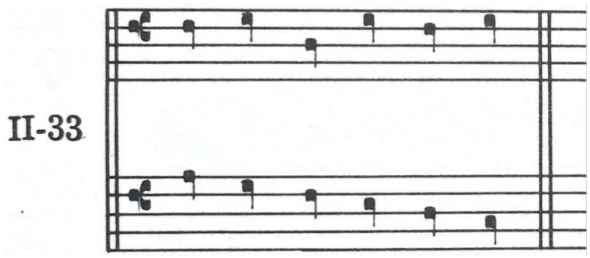
Once Ugolino has presented typical two-note progressions, he gives a final example in

[312] The units are, therefore, not to be read as a continuous melody. Nor do they offer the same kinds of progressions that Ugolino earlier described as perfections, since he uses motions other than imperfect to perfect.

[313] Example II-16 in Ugolino, *Declaratio*, 2:supplement.

each grado where the tenor moves in one continuous motion, not in two note units, by step up from ut to la and back down from la to ut. Over this, he writes an added voice that borrows the typical progressions he has previously demonstrated in order to create a single continuous counterpoint (Example 3.3). These examples of longer progressions often but not always conclude with a perfection.

Example 3.3: A Longer Progression, Conluding with a Perfection.[314]

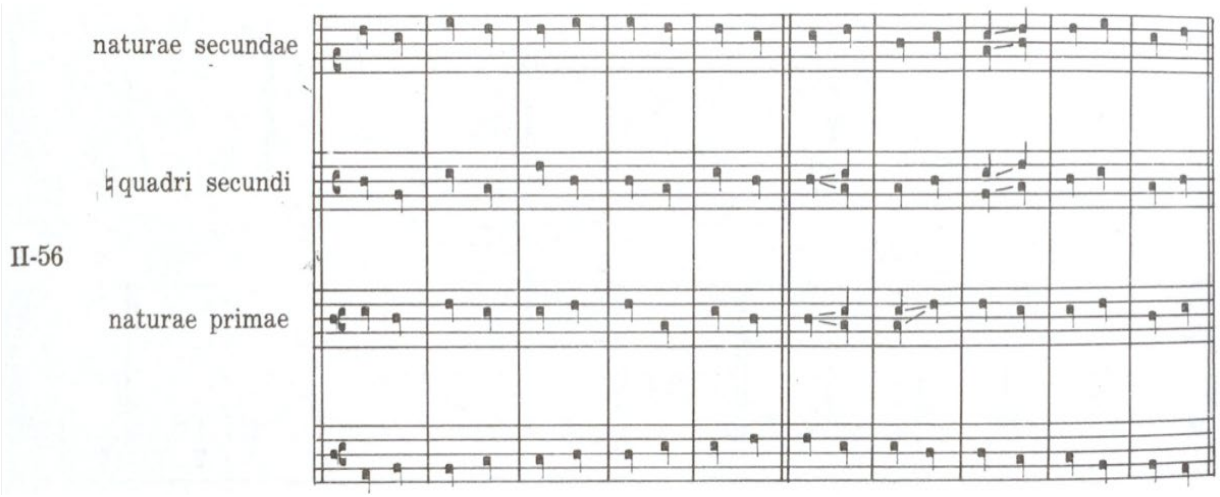


After Ugolino has presented typical contrapuntal progressions in each grado in chapters 11-14, he provides an overview and conclusion in chapter 15. The musical examples in chapter 15 are worthy of comment. In them, he has what appears to be four parts (see Example 3.4). The bottom part is the tenor, which moves within the natural hexachord in two note units by a particular interval (by steps in Example 3.4), both ascending and descending. There are four examples: in the first, the tenor moves by steps; in the second, by thirds; in the third, by fourths; in the last, by fifths. At the end of the fourth example, he adds a section where the tenor moves not in two note units but continuously up from ut to la and then down from la to ut, as he had in the chapters preceding. In each example, above the tenor, Ugolino adds what looks to be three voices, one which moves within the natural hexachord, another in the hard hexachord, and another in the natural hexachord an octave above the tenor. They are labeled according to the hexachord they are in. Thus, he includes all the gradi except the fourth. Concerning the examples

[314] Example II-33 in Ugolino, *Declaratio*, 2:supplement.

in chapter 15, like the one seen in Example 3.4, Anna Maria Busse Berger states, "Chapter 15 shows the hexachord combinations in four-part counterpoint, thus making it clear to the singer in which range to place each part."[315] Busse Berger makes three distinct claims: first, that chapter 15 shows hexachord combinations; second, that these combinations are in four-voice counterpoint; third, that as a result of the first two claims a singer learns how to handle range with respect to four-voice counterpoint. I briefly analyze these claims by examining what Ugolino says about these examples and by considering them in the broader context both of the previous chapters in particular and of Ugolino's audience and goals in general.

Example 3.4: A Single Example Containing Several *Gradi* at Once.[316]



Ugolino states the purpose for the examples in chapter 15:

> We have at present decided to demonstrate all the intervals ordered in counterpoint [and] gathered into one [example], so that those who are desiring [and] learning to make progress from these learned demonstrations may understand counterpoint, may flee discords, may unite concords, and may learn the compositions of melodies.[317]
>
> *Impraesentiarum coniunctiones omnes supra seorsum in contrapuncto ordinatas in unum collectas decrevimus demonstrate, ut cupiens discens proficere his demonstratis edocumentis contrapunctum intelligat, discordias fugiat, concordias uniat, et cantuum compositiones addiscat.*

[315] Busse Berger, *Medieval Music*, 139.

[316] Example II-56 in Ugolino, *Declaratio*, 2:supplement.

[317] Ugolino, *Declaratio*, 2:22.

He intends to take what he has already covered from chapters 7-14 and combine them into one handy compendium that learners, who want to make proper counterpoint, may consult. To achieve this, he includes several gradi: the unison, the fifth, and the octave. He states,

> The order of these hexachords in counterpoint exists in that the first natural hexachord makes counterpoint with itself out of its own consonant and dissonant intervals, since it sounds well and agrees with the pitches of other hexachords after discords have been removed. Above this natural hexachord, the order of the second hard hexachord follows, whose first note is known to lie a diapente from the first note of the aforementioned natural hexachord. Next, so that the perfection of counterpoint may be complete out of threes, the order of the second natural hexachord is demonstrated, which is separated from the first natural [hexachord] by the distance of a diapason. For by the concordant connections of consonant and dissonant pitches, a very sweet harmony is presented to the soul, and from those of the other hexachords lying equally distant, as above, a method of counterpoint is understood, as is evident to those who understand.[318]
>
> *Earum autem proprietatum ordo in contrapuncto est quod proprietas naturae primae secum ex suis consonantibus atque dissonantibus vocibus in arsy et thesy faciat contrapunctum, quoniam cum aliarum vocibus proprietatum semota discordia consonet et conveniat. Supra hanc autem naturae propriatatem b quadri secundi sequitur ratio contrapuncti cuius initium a dictae naturae principio noscitur per diapente distare, ex inde ut contrapuncti perfectio compleatur ex tribus naturae secundae contrapuncti ratio demonstrator, quae a prima natura per diapason distantiam separator. His enim concorditer nexis consonantium dissonantiumque vocum dulcissima animae offertur harmonia, et ex his aliarum proprietatem aeque ut supra distantium sumitur ratio contrapuncti, ut intelligenti patet.*

He considers each part separately in relation to the tenor, not as an example of four-voiced counterpoint. He says that the first hexachord "makes counterpoint" with itself, because the tenor and the part written above it are in the same hexachord. He then considers the next highest written part, whose starting note lies a fifth from the first note of the hexachord of the tenor line. In other words, it is in the grado of the fifth. After this, he compares the highest written part with the tenor, which is in the grado of the octave. He never compares all the parts together, nor does he state that all the parts make counterpoint together. Instead, he looks at them

[318] Ugolino, *Declaratio*, 2:22-23.

from their relationship with the tenor. He refers to each line as a "ratio contrapuncti," that is, as an order or method of counterpoint in itself. Therefore, Ugolino is certainly showing hexachordal combinations, but he is not describing four-part counterpoint. Indeed, every chapter, both before and after this one, deals only with two-voiced counterpoint. It would make no sense to suddenly talk about four-voiced counterpoint here and then abandon the subject altogether. In addition, if he were writing an example of four-voice counterpoint, the voices should follow the rules he sets out for two-voiced counterpoint. But this is not the case. For example, between the first and third added parts, parallel octaves abound. And between the middle two parts in the fifth "measure," there are parallel fifths (see Example 3.4). Thus, in this chapter, Ugolino is offering a summary of the two-interval progressions he has presented in the preceding chapters. He does so by notating them in a single example, so that his readers could have a handy compendium, a quick reference. This interpretation agrees with the immediate context as well as his general aim.

For chapters 1-15, the overall presentation of topics proceeds in a logical and pedagogical manner. Ugolino's readers, by the time they reach chapter 15, would already have learned the purpose and definition of counterpoint in chapters 1-2, the classification and types of intervals in chapters 3-5, the perfections in chapter 6, and the hexachordal relationships and consonance tables of the *regola del grado* in chapters 7-10. Although they would have this knowledge, they would not know how to properly form counterpoint, except for the perfections. Ugolino, therefore, directs their attention first to a tenor line. If they wanted to create counterpoint above or below some other chant melody, they would need to know the intervals and the hexachord(s) that melody uses—a topic treated at length in book 1. Once they knew the motion of the tenor line, they could apply the typical contrapuntal progressions that Ugolino gives in chapters 11-14 and which he summarizes in chapter 15. This means that Ugolino's text walks readers through

the steps from knowing consonances to putting them in a logical and stylistically acceptable presentation. Ugolino achieves greater clarity by offering a multiplicity of musical examples. The musical examples serve a crucial role in Ugolino's presentation.

In chapters 11-15 alone, Ugolino writes 43 musical examples demonstrating typical two-interval progressions. But for the whole of book 2, he gives a total of 129 musical examples. By comparison, F1 has 12, F2 has none, W has 31, and M has 9. Even these manuscripts, aside from F2, have more examples than Prosdocimo's book on counterpoint.[319] A contemporary of Ugolino, Prsodocimo wrote a short treatise on counterpoint in 1412, which he revised between 1425-1428.[320] It has only one musical example demonstrating musica ficta. Just as Prosdocimo wrote few examples, so also he prefers short, memorable rules.[321] At least one reason for these differences originates in the audiences they were writing for. Prosdocimo worked at the university of Padua teaching astronomy and mathematics.[322] His readers would need to know rules which they could quickly memorize, but they may not have had a need to put those rules to

[319] Jan Herlinger claims that Prosdocimo's work influenced Ugolino, and that "its influence…was extensive and, to judge from the similarity in wording, direct." Herlinger supports this claim by adding an appendix to his translation of Prosdocimo which lists the similarities between the two authors. For example, he notes that they both consider counterpoint in a wide and narrow sense; that counterpoint presupposes plainchant; that both theorists use the six syllables; that consonances are perfect and imperfect; that discords are not used in counterpoint, as well as several other points of contact. Prosdocimo, *Contrapunctus*, 5. The language between the two is indeed similar in the definition of counterpoint and their use of counterpoint in both a wide and narrow sense. Yet most of the other similarities may more likely result from a shared culture and education. For instance, Herlinger cites their use of the six solfege syllables as evidence of direct influence. The six syllables were in use long before Prosdocimo, and they were commonly taught to anyone learning music. Besides, Ugolino was clearly a well-educated man, directly citing Aristotle, Boethius, and others. In fact, Lewis Lockwood notes that Ugolino penned a substantial treatise on physics. Lockwood, *Music in Renaissance Ferrara*, 79. He would not need to appeal to Prosdocimo for a reference to solfege or similar topics that he would have learned in the course of his musical education. Therefore, citing the use of the six syllables as an influence from Prosdocimo is not convincing. A similar argument could be used for many, but not all, of the other similarities Herlinger points out. Nor does Herlinger address the subjects that Ugolino treats that Prosdocimo makes no mention of, such as the regola del grado. It seems likely that Ugolino knew Prosdocimo's work (the definition of counterpoint is the best evidence for that), but the influence may not be as direct or extend as far as suggested.

[320] Prosdocimo, *Contrapunctus*, 8.

[321] Busse Berger, *Medieval Music*, 146; Prosdocimo, *Contrapunctus*, 7-8.

[322] Prosdocimo, *Contrapunctus*, 1.

practice.[323] Ugolino, however, was writing within the context of the cathedral school in Ferrara. His readers would likely have practiced music regularly within the liturgical life of the cathedral. They would benefit from the copious musical examples and the frequent lists of consonances.[324] Whatever else they may show, Ugolino's examples reveal his goal to teach the regola del grado clearly and fully.

3.4 Conclusions

Ugolino presents the fullest account of the *regola del grado*. His presentation of it in chapters 3-15 forms the foundation for the study and practice of counterpoint presented later in the book. He devotes more space to the grado and to its expansions than to the more common interval succession theory or to the general rules of counterpoint, each of which receive only one chapter. Although it was not the purpose of this chapter to highlight his expansions, chapters 16-24 as well as 28-33 only make sense after knowing what the grado theory teaches. In the former set of chapters, he shows how an upper voice, using only the notes of a single hexachord, can harmonize a tenor line that moves across several hexachords. It is like applying the rules of the grado in a new context. In the grado theory, students learn what all the consonances are when the tenor and the added voice are in one of the various degrees or steps (unison, fourth, fifth, and octave). Students must therefore know what hexachord the tenor is in, so that they can harmonize without using discords. If students were familiar with the grado theory, then the applications Ugolino makes in chapters 16-24 and the catalogs of consonances in chapters 28-33 are fairly straightforward. The grado practice gives new meaning to the idea that plainchant presupposes

[323] Busse Berger, *Medieval Music*, 146-50; Prosdocimo, *Contrapunctus*, 5.

[324] Busse Berger, *Medieval Music*, 131-46.

counterpoint.[325] Students would need to know not only what the notes and melodic intervals of the chant were but also what hexachord they were in. Equipped with this knowledge, they could remember in which degree they could make the most consonances with the tenor and sing or write accordingly. The grado theory forms a natural connection to consonance tables. Ugolino takes advantage of this opportunity to write out consonance tables in chapters 3-15 but especially in chapters 28-33. Consonance tables were common in counterpoint treatises both before and after Ugolino.[326] The *regola del grado* forms the practical link to consonances tables and, at least in Ugolino's conception, the foundation of the practice of counterpoint.

Ugolino actively engages in the grado tradition. This is most evident in how he adapts the language and use of the tradition to bring it more in line with contemporary usage. Where the grado treatises in general forbid using minor sixths, Ugolino makes no such prohibition. Where the grado treatises variously refer to thirds (and sixths) as either major/perfect or minor/imperfect, Ugolino calls them only major or minor. Ugolino also updates the language on interval classifications while presenting and retaining the older classification. In the old terminology, consonances refer only to fifths and octaves, dissonances include thirds and sixths, and discords are seconds, sevenths, tritones, and similar intervals. In the newer terminology, perfect consonances are fifths and octaves, imperfect consonances are thirds and sixths (which can, in addition, be either major or minor), and dissonances are seconds, sevenths, and so on. He sets both systems side by side, since he sees them as essentially equivalent. Allowing his readers to know both systems equips them to deal with older manuscripts and practices as well as newer ones. In Ugolino's presentation of the grado tradition, we do not see a radical reformer but a

[325] See, for example, Ugolino, *Declaratio*, 2:8; Prosdocimo, *Contrapunctus*, 32.

[326] Busse Berger, *Medieval Music*, 133.

modernizing conservative. His modernizing, however, is not without consequences.

In the book on counterpoint, we see Ugolino explaining and building his conception of music as a subject within natural philosophy through a close examination of a musical practice. To some extent, we observed this in the subtle shift of language. Where the grado manuscripts said that one note "proceeds" (*procede*) from another, Ugolino uses "derives" (*derivantur*) and "springs naturally" (*natae*). His word choices suggest a connection to logic and especially to nature. But we saw the connection to music as a natural philosophy most clearly displayed in his categorization of some intervals as perfect or imperfect. Imperfect intervals must move to perfect ones in what David Cohen calls directed motion. Directed motion involves five conditions, all of which are based in Aristotelian natural philosophy. We saw that Ugolino fulfills all five conditions, and, in some sense, goes even further. Ugolino sees the octave as the mother or source of all the other intervals: they are related to it as causes that are proximate, remote, or most remote. The very structure of intervals as parts of a whole explains why some are perfect and others imperfect. The proximate causes correspond to perfect intervals, remote to imperfect, and most remote to dissonant intervals. The differences between perfect, imperfect, and dissonance lies in the nature of the objects themselves and their structure. In other words, Ugolino understands musical objects, in this case intervals, as objects of nature. This reinforces Cohen's fourth condition and ties into Aristotle's principle that natural objects have a source of motion within themselves.[327]

Ugolino cannot conceive of music without motion, and motion, in this context, can be considered in at least two senses: of the motion needed to produce sound and of the motion required inherently by imperfect intervals and expressed through the harmonic progression of an

[327] Cohen, "The Imperfect Seeks Its Perfection," 146; Aristotle, *Physics* 2.1.192b23-4.

imperfect interval to a specific perfect interval, which Ugolino calls a perfection. Both kinds of motion fall under consonance, and consonance is what the study of music is about. Ugolino argues:

> Music cannot be made apart from sound, nor is sound caused apart from a certain striking and percussion, nor does percussion and striking come to be without a preceding motion. Therefore, we doubt least of all that motion must be included in music, since if everything stood still and lacked motion…consonance could not be found in it.[328]
>
> *Quae consonantia cum praeter sonum fieri non possit, nex praeter pulsum ac percussionem quandam sonus reddatur, nec percussio atque pulsus abque praecedente motu esse contingat. Ideo motum in musica supponendum minime dubitamus, quoniam si cuncta starent motuque carerent…neque consonantia in ea posset reperiri.*

His logic is clear: without motion, there could be no sound, and without sound, there could be no consonance. Music entails motion, and therefore music is a part of natural philosophy. Counterpoint teaches one aspect of that motion—the progression of one interval to another—and he considers the intervals themselves as natural objects. He shapes his book on counterpoint to conform with this general philosophical principle. The book on counterpoint is a working out of the ideas I outlined in chapter 2 that musical practice and musical theory are not in tension with one another. Rather, the former leads to the latter. For Ugolino, practice is necessary for anyone who wants to come to a knowledge of theory.

Throughout his book on counterpoint, we see how practical Ugolino is. He offers not only many consonance tables, guidelines on hexachordal relationships and the consonances they entail, and general rules for proper counterpoint but also 129 musical examples. These are the number of examples only in book 2. He also adds musical examples in all of the other books as well. The preponderance of musical examples has led some scholars to conclude that Ugolino's

[328] Ugolino, *Declaratio*, 1:22. Ugolino makes a similar argument again in the 13th chapter of book 1, where he even quotes Aristotle's definition of motion, Ugolino, *Declaratio*, 1:34-5. But he treats this topic most fully in book 5: Ugolino, *Declaratio*, 3:102-3, 108-12.

treatise is truly and mainly a practical one, even though it also contains some speculative material.[329] Albert Seay tried to rectify this assessment by investigating the treatise more closely. Seay correctly notes Ugolino's movement from the practical to the theoretical and the Aristotelian path there, even though he ultimately categorizes the *Declaratio* as more of a theoretical work along the lines of Boethius.[330] If there were any debate about whether Ugolino's treatise was practical, his book on counterpoint in general and his teaching of grado theory in particular should answer that doubt. Yet, as I argued in chapter 2 and as the analysis of the way he constructs, or rather reconstructs, the grado theory reveals, this does not mean that the practical somehow negates the theoretical nor the theoretical the practical. Instead, the practical is a necessary step along a path toward the theoretical. He does view them as hierarchical, but not as separable. This conception of theory and practice led Ugolino to produce a full account of counterpoint and especially of the *regola del grado*. It also informs his approach to musica ficta, to which I now turn.

[329] Albert Seay notes that Gerhard Pietzsch ascribed a primarily practical value to Ugolino's work while placing it in the *speculum* tradition, even though viewing the work as mainly practical seems at odds with the structure of the treatise and the value Ugolino places on theory. Seay, "Ugolino of Orvieto," 145, 151; Gerhard Pietzsch, *Die Klassifikation Der Musik Von Boethius Bis Ugolino Von Orvieto* (Halle: Max Niemeyer, 1929), 119. In addition, Seay shows that the bias of the practical musician Adrien de la Fage led the latter to dismiss Ugolino's work as boring and to describe his ordering as illogical and his Latin as poor. This seems to point to the work as one of mainly speculation with little value to practice. Seay, "Ugolino of Orvieto," 113; Adrien de la Fage, *Essais De Diphthérographie Musicale* (Paris: Au Magasin de Musique du Bazar de l'Industrie, 1864), 165. Neither position is correct, as I argue in chapter 2, and which is one of the main themes of my dissertation.

[330] Seay, "Ugolino of Orvieto," 145-52.

CHAPTER 4

THE PRACTICE AND THEORY OF *MUSICA FICTA*

4.1 Introduction

Ugolino's chapter on musica ficta has attracted the greatest attention from modern scholars, because he creates a "double hand" (*duplex manus*).[331] The double hand contains not only the usual set of seven hexachords beginning on C, F, and G but also another set of seven ficta hexachords beginning on different notes. It has been cited by both Margaret Bent and Karol Berger as evidence for the role the hexachord plays in the structure of diatonic space.[332] According to Stefano Mengozzi, two opposing roles for the hexachord emerge in the history of music theory. In both views, which he calls "foundational" and "soft," the hexachord competes for priority with the seven-note, octave-equivalent scale or gamut. In the foundational view, the hexachord shapes diatonic space instead of the seven-note scale. In the soft view, the hexachord merely sits on top of the scale, but the scale has priority over the hexachord.[333] If Mengozzi's categories form two poles with a spectrum of positions lying between them, I can place Bent's and Berger's views on that spectrum to see how close they lie to Ugolino's. Although Ugolino's double hand has received much scholarly scrutiny, it is only one facet of his chapter on musica ficta. Therefore, I examine his entire chapter on musica ficta as well as his comments on musica ficta in the *Tractatus monochordi* appended to the *Declaratio*. This examination reveals not only his conception of musica ficta and the role of the hexachord in the structure of diatonic space but also, more broadly, the relationship between practice and theory.

---

[331] Ugolino, *Declaratio*, 2:48-50. I refer to the "Guidonian" hand as simply "hand" or "musical hand," since it was not invented by Guido.

[332] Bent, *Counterpoint, Composition, and Musica Ficta*, 7-8, 88; Berger, *Musica Ficta,* 64.

[333] Mengozzi, *The Renaissance Reform*, 8-11, 104-09.

4.2 Musica Ficta: The Basics

Ugolino begins the chapter on musica ficta by using a practical example to describe the conditions that led to its invention or discovery. He then tells how he derives ficta hexachords and offers a succinct definition of musica ficta. Sometimes, even when all the intervals are correctly performed, imperfect consonances arise. To correct these false relations, "a certain music was invented by musical philosophers which is named ficta" (inventa est a philosophis musicis musica quaedam, quae ficta vocabulo nuncupatur).[334] It is called "ficta" because "such music is put in a place where it does not exist by itself" (talis musica in eo loco ponitur ubi per se non est).[335] He supports this idea with an example. Suppose a tenor ascends a tone from A-B (re-mi), and an upper voice descends from A-F (la-fa). The result is an imperfect fifth, and it can be corrected by making a mi where there was no mi before, namely, instead of singing F-fa, the upper voice sounds an F-mi, resulting in what we would call an F-sharp. Since the syllables must come from some hexachord, the mi on F comes from a hexachord that begins on D.[336] With these practical illustrations in hand, Ugolino sums them up in a tightly worded definition:

> Musica ficta is the necessary placement of some syllable in a place where it does not exist by itself for the purpose of perfecting a consonance.[337]
>
> *Musica ficta est alicuius vocis in loco ubi per se non est ad consonantiae perfectionum necessaria positio*

From this definition, he draws three conclusions that form the basis of the following discussion: (1) musica ficta "is placed where it is not found at all by itself" (ubi ponitur, ibi per se penitus non invenitur); (2) musica ficta "is admitted in order to perfect an imperfect consonance"

[334] Ugolino, *Declaratio*, 2:44.

[335] Ugolino, *Declaratio*, 2:44. What this means more specifically is discussed below.

[336] Ugolino, *Declaratio*, 2:45.

[337] Ugolino, *Declaratio*, 2:45. I am reading *perfectionem* in place of *perfectionum*.

(permittitur ut imperfectae consonantiae perficiantur); (3) musica ficta is not used at all "unless as a compelling necessity" (nisi necessitate cogente).[338]

Ugolino cites perfection as the reason for using musica ficta, and he uses this term in a broad sense. Most authors who write about ficta appeal to two distinct reasons to use it: for the sake of necessity (*causa necessitatis*) and for the sake of beauty (*causa pulchritudinis*). These concepts, interpreted variously by different scholars, could refer either to harmonic or melodic concerns or to both.[339] In fact, in the *Tractatus monochordi,* Ugolino himself cites two reasons for using musica ficta: for "perfecting imperfect consonances and dissonances" (imperfectarum consonantiarum et dissonantiarum perfectio) and for "a sweeter sounding harmony" (dulcioris resonantia harmoniae).[340] Yet, as we saw in chapter 3, perfection can mean either correcting fifths and octaves (or their compounds) so that they form perfect consonances or moving an imperfect consonance to its closest perfect consonance. Ugolino uses it in both senses, but he begins with the first. Anytime two singers would sing B and F together, no matter what octave, some correction is necessary.[341] Likewise, he notes that if the upper voice is singing in the soft hexachord and produces a B-flat (fa) against an E (mi) in the lower voice, the lower voice must correct the imperfection and sing a fa, creating what we would call an E-flat and which syllable

[338] Ugolino, *Declaratio*, 2:45.

[339] Margaret Bent, "Musica Recta and Musica Ficta," *Musica Disciplina* 26 (1972), 78-79; *Counterpoint, Composition, and Musica Ficta*, 66-67, 79-81; Berger, *Musica Ficta*, 94, 116, 122-123. The fact that a melody moves by a smaller step to the next note is a byproduct of correcting a fifth or octave or altering an imperfect consonance so that it moves more smoothly to its nearest perfect consonance.

[340] Ugolino, *Declaratio*, 3:238.

[341] It is curious that he never mentions simply using B-flat, which would be musica recta. He never explains why this is so, but I would venture to guess a few reasons: first, it may be that he considers the tenor line fixed and therefore unalterable. This may be unlikely since he includes an example where the lower part alters an E so that it no longer forms a dissonance with B-flat, but he may have considered the upper part a fixed or unalterable part in that case. Second, he does base his system on the Greater Perfect System, which does not contain B-flat; thus, B-flat and any other altered note share the same status, and he could have chosen either. Or third, he merely wanted to use an altered F for the purpose of demonstration, to show that musica ficta requires hexachords on each D.

would come from a hexachord starting on B-flat.[342] For the same reasons any fifth or octave that would be imperfect must be so corrected using musica ficta. This corresponds to the use of musica ficta for the sake of necessity (*causa necessitatis*), even though Ugolino does not use this term. He offers several examples where this would occur in the gamut.

According to Ugolino, musica ficta can also be applied to imperfect consonances, such as sixths or thirds. He also calls this perfection, although he distinguishes between lowering and raising notes. But before he gets there, he discusses the use of the signs. Ugolino recognizes two signs for musica ficta (hard and soft or round b, corresponding to our sharp/natural and flat respectively) and what they mean. He equates each sign with a solfege syllable. Whenever one sees a hard b, one should sing mi; whenever one sees a soft b, one should sing fa.[343] This observation leads him to articulate the mi contra fa rule, which he demonstrated clearly in the previous section, when he discussed perfecting fifths and octaves. His comments on signs and the mi contra fa rule serve as an interlude before he addresses the second use of musica ficta.

Musica ficta can also be applied for perfection in a second sense—that is, for making sixths closer to octaves and thirds closer to fifths or unisons, or, as Ugolino says, for "coloring imperfect consonances or dissonances" (in consonantiis imperfectis sive dissonantiis colorandis).[344] Coloration usually referred to the application of musica ficta to imperfect consonances, and this use is considered to fall under the heading of *causa pulchritudinis* as opposed to *causa necessitatis*.[345] Indeed, Bent speaks about this use of ficta as ornamental and

[342] Ugolino, *Declaratio*, 2:45.

[343] Ugolino, *Declaratio*, 2:46.

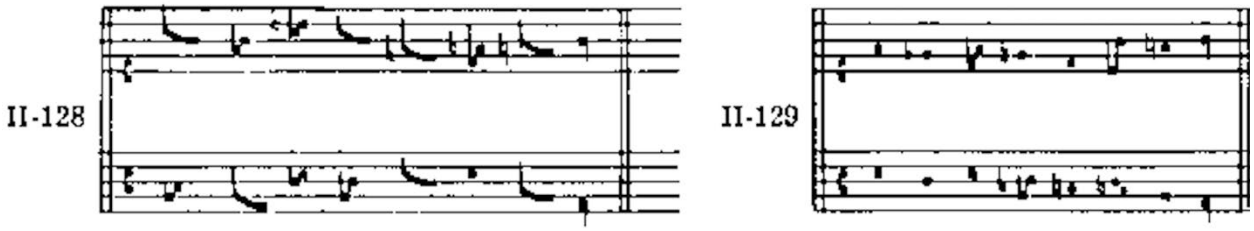
[344] Ugolino, *Declaratio*, 2:47.

[345] Berger, *Musica Ficta*, 122-54.

therefore not necessary.[346] Ugolino finds two reasons for using ficta in this way: "for the sake of having a sweeter harmony and for the sake of a closer approach to a perfection" (causa harmoniae dulcioris habendae, et causa propinquioris perfectionis acquirendae).[347] He is equally concerned with harmony as he is with melody. The sweeter harmony describes the vertical intervals, and the closer approach describes the movement of a single voice from one note to the next. This, in turn, may be done in one of two ways: by altering an interval either to make it major or to make it minor. If a sixth progresses to an octave or a third to a fifth, one could alter them to make them major. Or, if a third moves to a unison, one could make it minor to sweeten the progression. Berger calls the former the strict version of the rule and the latter the relaxed version.[348]

To understand Ugolino's application of musica ficta, I review his commentary on two representative examples (Example 4.1).[349]

Example 4.1: Two Representative Examples of Musica Ficta.[350]



Ugolino says that the first square b (at the end of II-128) means that F is sung as mi, making the progression a major sixth to an octave. The major sixth moves more directly to its perfection (the octave) and helps clarify its closing or cadential nature. He observes that the first soft b (in II-

[346] Bent, "Musica Recta and Musica Ficta," 96; Bent, *Counterpoint, Composition, and Musica Ficta*, 85.

[347] Ugolino, *Declaratio*, 2:47.

[348] Berger, *Musica Ficta*, 122-25.

[349] This paragraph summarizes Ugolino, *Declaratio*, 2:47-48.

[350] Examples II-128 and II-129 in Ugolino, *Declaratio*, 2:supplement.

128) is placed not to perfect an imperfect consonance but to color it. Still, the now minor sixth will, in fact, move to its perfection more closely, thus adhering to Ugolino's second reason.[351] He cites a similar reason for using an E-flat in the second example (II-129). By doing so, he distinguishes between perfecting and coloring. In this case, he uses the term color to refer to lowering notes to make them minor and perfection to make them major. He uses perfection and coloration as two different actions, even though in other places he employs the terms interchangeably. I examine this problem more closely later. What his commentary shows is that the alterations occur on imperfect intervals that subsequently move to perfect ones. Musica ficta is applied in order to make the cadential progression sweeter: the harmonies are closer to their intended goal, and as a result, the melodies move by smaller intervals.

4.3 The *Duplex Manus*

After Ugolino offers the definition of ficta, lays down several reasons for its use, and includes some musical examples with commentary, he adds two musical hand diagrams, that is, diagrams that contain a list of letters as well as the hexachords that stretch across those letters in the form of syllables. In fact, each diagram includes a double musical hand (*duplex manus*): each diagram contains the seven hexachords in the traditional hand—the notes of musica recta—but also seven ficta hexachords. He does not separate the recta hexachords from the ficta hexachords. Instead, he mixes them together in a single list. He introduces the first diagram (Figure 4.1) thus:

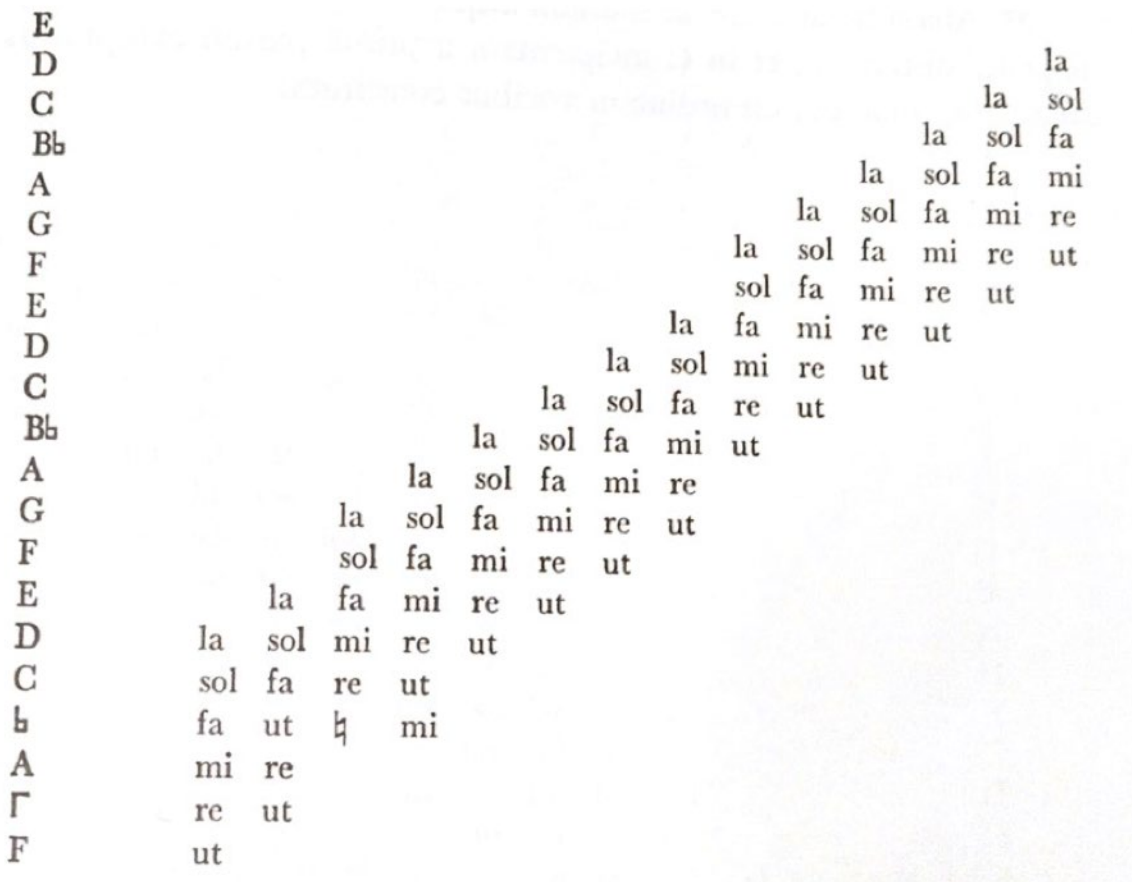
> Therefore, we have learned from the preceding [discussion] of musica ficta the necessary perfection of consonances and the coloration of dissonances, which produce beautiful harmonies. And because this music is called ficta, since it is placed or formed in a place where it does not exist
>
> *Cognovimus ergo ex praemissis musicae fictae necessitatem consonantiarum perfectionem ac dissonantiarum colorationem, harmoniarum amoenitatem producentem, et quia haec musica ex eo dicitur ficta, quia ibi ponitur seu fingitur ubi per se non est, ideo duplicem manum*

[351] Ugolino, *Declaratio*, 2:47-8.

by itself, so we invent a double hand where all the syllables of musica ficta are placed in order. For the first hand has its beginning on F directly under gamma…[352]

invenimus, ubi omnes rectae musicae atque fictae voces ordinatae ponuntur, prima namque manus sub [Gamma] immediate initium habet in F…

Figure 4.1: Ugolino's First *Duplex Manus* Diagram.[353]



And he introduces the second diagram (Figure 4.2) thus:

We invent another hand of musica ficta and recta lying a fifth lower than gamma and beginning on C, lying a little lower than the first [hand]…[354]

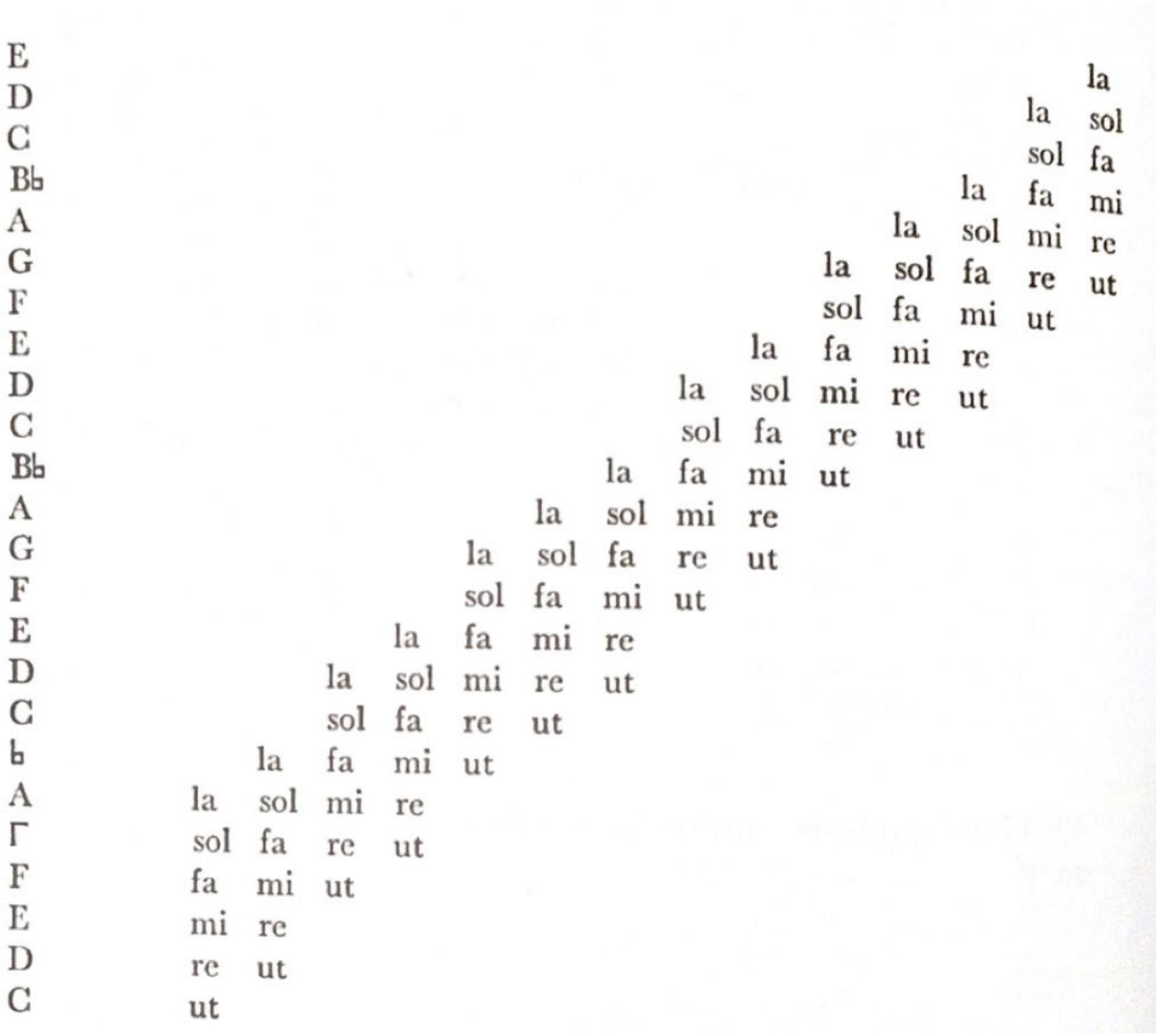
Aliam fictae musicae manum atque rectae invenimus a [Gamma] inferius per diapente distantem, et in C incipientem a prima parum excepta gravitate distantem…

[352] Ugolino, *Declaratio*, 2:48.

[353] Ugolino, *Declaratio*, 2:49.

[354] Ugolino, *Declaratio*, 2:50.

Figure 4.2: Ugolino's Second *Duplex Manus* Diagram.[355]



In the first diagram (Figure 4.1) the ficta hexachords begin on F below Gamma (which provides a low B-flat), as well as on B, D, E, and their octaves. It contains the seven recta hexachords and seven ficta hexachords. In the second diagram (Figure 4.2), ficta hexachords begin on C, D, and F below Gamma, plus all the Bs and Ds above Gamma. Altogether, the second example includes the seven recta hexachords plus another seven ficta hexachords. He describes the second example as lying a fifth below the first, counting it from the Gamma down

[355] Ugolino, *Declaratio*, 2:50.

to the low C. The second example does not contain the ficta hexachords on E that are found in the first. In addition, neither include hexachords starting on A. Indeed, Ugolino himself notes that ut can occur on every letter except E and A, even though his first hand shows an ut on E, but his second hand does not.[356]

Why did Ugolino add the ficta hexachords that he did? With seven recta hexachords and seven ficta hexachords in each diagram, he creates a balance between recta and ficta. This symmetry, however, comes at the expense of consistency. Because of the lower range in Figure 4.2, he has to omit the hexachords on E so that it still has only seven ficta hexachords. In addition, he seems to have added the specific ficta hexachords he did by observing which intervals require alternation. In other words, he derives his hexachords from practical considerations. Since an altered note means changing a syllable, the new syllable implies a hexachord beginning on a different pitch. For example, B-flat requires a lower E-flat for a perfect fifth, which means a singer would sing fa on E instead of mi, and thus the new fa implies an ut on B-flat. In addition to correcting diminished fifths and octaves, cadential progressions also require alteration. If two parts proceed from A/f to G/g, the f would be altered from fa to mi, so that the progression moves from a major sixth to an octave. If the f is now solmized as mi, ut would be located on D. He follows the same procedure to find the other altered notes and their respective hexachords. One curious omission is one that Ugolino himself notes—the ut on A which would include a C-sharp and would be used for cadential progressions to D. What does Ugolino have to say about that? He does not deny that there can be a ficta hexachord that begins on A. In fact, in the discussion following the diagrams, he uses a model progression from E/c to D/d to show how the c should be altered. This discussion reveals some of the underlying

[356] Ugolino, *Declaratio*, 2:50.

assumptions built into his diagrams.

Ugolino begins by appealing to the invention by the ancients of the two signs, round b and square b.[357] He reminds his readers of the meaning of the signs: the round b "signified the syllable fa" (fa vocem significabat) and the square b "announced the syllable mi" (mi vocem annuntiabat).[358] Indeed, he seems to correlate, even stronger than earlier, coloration with making intervals minor and perfection with making them major. Consequently, he gives an example of each. One point common to both examples is the fact that ficta means adding a major semitone.

> Therefore, the two aforementioned signs [round b and square b] were invented first for coloring consonances, so that from these colored consonances a sweeter harmony may be produced, which happens when a major consonance is reduced to a minor [one]. Second, these signs were invented so that imperfect consonances may be led to their perfection and major [ones] by means of signs of this kind.[359]
>
> *Inventa sunt ergo praedicta signa primo pro consonatiis colorandis, ut ex ipsa consonantia colorata dulcior proveniat harmonia, quod fit quando maior consonantia ad minoritatem reducitur. Secundo ipsa signa sunt inventa uta consonantiae imperfectae huiusmodi mediantibus signis ad earum perfectionem maioritatemque ducantur.*

He offers an example of coloration: if two parts progress from G/b to A/a, then the b, which is a ditone or major third, ought to be a semiditone or minor third. The alteration occurs by adding the round b (or flat) sign. The b, which would have been solmized as mi, is now solmized as fa, since the sign indicates fa. Fa would then descend to mi, and every fa to mi signifies a minor semitone.[360] The upper voice's progression was altered from a tone (b-a) to a minor semitone (b-

[357] Ugolino seems to have a high opinion of the ancients in regard to musica ficta. He states, "The ancients, who had a true knowledge of this musica ficta—the truth of which remains wholly unknown to modern singers—invented two signs in the truth or fiction of this kind of music, either for coloring consonances or for leading them to a major or a minor [interval]…" (Antiqui autem huius fictae musicae veram cognitionem habentes quorum veritas modernis cantoribus penitus ignota manet in suae musicae veritate seu fictione, sive pro colorandis consonatiis sive pro eis ad maioritatem vel minoritatem reducendis duo signa…invenerunt…). Ugolino, Declaratio, 2:50-1.

[358] Ugolino, *Declaratio*, 2:51. He had already introduced the signs a few pages earlier.

[359] Ugolino, *Declaratio*, 2:51.

[360] Ugolino states this rule himself, "from mi to fa is always a minor semitone" (a mi ad fa semper sit semitonium minus). Ugolino, Declaratio, 2:52.

flat to a). In terms of the harmony, a ditone was made into a semiditone, and the former exceeds the latter by a major semitone. Likewise, melodically, the tone exceeds the minor semitone by the distance of a major semitone.[361]

Ugolino also offers an example of perfection by using the progression E/c to D/d. Since this forms a cadential progression or perfection, the sixth must be major. How does this happen?[362] He contrasts how the ancients would have done it with how the moderns do it. The ancients would have added the square b sign to the c. This would make what was fa sol (c-d) mi-fa (c-sharp-d). He notes that both the mi and the fa are ficta notes, even though the d-fa sounds the same as its recta counterpart d-sol. However, the distance from the recta fa (c) and the ficta mi (c-sharp) is a major semitone. So, by adding a major semitone, what was a minor sixth (three tones and two minor semitones) is now a proper major sixth (four tones and one minor semitone).[363] The ancients reason either from the melodic progression (what was a tone becomes a minor semitone, and the tone exceeds the minor semitone by a major semitone) or from the harmonic progression (a minor sixth is altered to a major sixth, and the major sixth exceeds the minor by a major semitone). They focus on the difference, in this case, between c and c-sharp, but they do not fit this within a larger picture.

In contrast to the ancient's way of talking about the matter, Ugolino offers an alternative proof to show that the square b sign makes the interval a major sixth. Ugolino's method relies on

[361] Ugolino states, "the tone exceeds the minor semitone by a major semitone (tonus autem excedit semitonium minus per semitonium maius). Ugolino, *Declaratio*, 2:51.

[362] Of course, it is possible to alter the E and so make the sixth major without the addition of any new hexachord or note that is not already found in his duplex manus. But Ugolino does not do this. Instead, he advocates altering the c, which means there would be a hexachord on A. If he allows for a hexachord on A even though he does not add one to his duplex manus diagram, then his diagrams are not exhaustive.

[363] Ugolino, *Declaratio*, 2:52.

the knowledge of solfege syllables and hexachords.[364]

> That sixth is said to be major which is said to be composed of four tones and one minor semitone, but the sixth that is minor—namely from low mi to high fa—is composed of four tones and one minor semitone through the application of a ficta syllable, therefore it is major. The major [premise] is self-evident and the minor [premise] is proved. For mi, which is placed on C through ficta, and fa which is placed through ficta on D have their own beginning and foundation, that is ut, on the second A. Hence, through musica ficta we sing on A ut, on B re, on C mi, and so on. Therefore, from the first E to the second C is, through musica ficta, a major sixth and perfect since it has four tones and only one minor semitone, as is evident to those who know.[365]
>
> *Illa sexta dicitur esse maior, quae ex quatuor tonis et uno semitonio minore dicitur esse composita, sed illa sexta quae est minor, scilicet, a mi gravi ad fa acutum per appositionem fictae vocis mi est composita ex quatuor tonis et uno minore semitonio, igitur est maior. Maior est per se nota et minor probatur, nam mi quod ficte ponitur in C et fa quod ficte ponitur in D suum initium et fundamentum, scilicet, ut, habent in A secundo. Unde per hanc fictam musicam in A dicimus ut, in B[sqb] re, in C mi, et cetera. Igitur ab E primo ad C secundum per hanc fictam musicam est sexta maior et perfecta habens quatuor tonos et unum tantum semitonium minus, ut patet intelligenti.*

This explanation relies on several assumptions, some of which are explored more fully below. First, there is a background scale or gamut represented by letters. The letters generally correspond to a specific intervallic distance but not perfectly so. For example, B-C could be a tone or a semitone, depending on whether it is a hard or soft B. Second, solfege syllables are placed on the letters and directly correspond to intervallic distances. So, for instance, mi-fa is always a minor semitone, and all the other adjacent syllables represent tones. The typical combinations of letter and syllable form musica recta, the starting point from which to determine any alteration. Third, musica ficta breaks this pattern of letter and syllable slightly because it places a syllable where there was none. It puts a fa or mi on a letter where it was not found in musica recta. It reveals that the letters are, at first glance, slightly more ambiguous than first

[364] Thus, Ugolino does not come to a different conclusion than the ancients. Rather, he suggests a different way of getting there.

[365] Ugolino, *Declaratio*, 2:52.

imagined, since, for example, E could be fa instead of mi (or la). Yet, the syllables help understand the exact intervallic composition, since they indicate tones and semitones. Fourth, large intervals are composites of smaller intervals. Ugolino describes the major sixth as composed of four tones and one minor semitone, and the other intervals can be calculated in a similar manner.

Knowing these assumptions helps understand Ugolino's point that singers can use the syllables, even ones from ficta hexachords, to calculate composite intervals. In so doing, he also shows that even though he does not include a ficta hexachord on A, one is clearly needed for a cadential progression on D. E-C is composed of two minor semitones and three tones. If every step in the gamut is composed either of a tone or a minor semitone (as they are in musica recta, the foundation from which to reckon alterations), E-F and B-C are two minor semitones, F-G is a tone, G-A a second tone, and A-B is a third. But since the progression from E/c to D/d requires a major sixth, a change is necessary. How exactly does a ficta hexachord on A prove that E-C-sharp—using a ficta mi on C—is composed of the proper intervals that make a major sixth? Ugolino appeals to the hexachord and its place in the scale to make the calculation a bit easier. In any hexachord, re to mi is a tone, and he has already stated that the ficta d-fa sounds the same as the recta d-sol. This leaves us with E-F as one minor semitone, F-G is a tone, G-A another tone (all these lie outside the ficta hexachord in question), ficta A-ut to B-re a third tone, and ficta B-re to C-mi a fourth tone. Ugolino takes advantage of the broader scale and the whole ficta hexachord instead of just analyzing the specific harmony or melodic progression to understand the intervallic structure involved. Even though Ugolino's diagrams do not include a ficta hexachord starting on A, he explicitly argues there is one, since it is needed for the progression

E/c to D/d.[366] Because Ugolino explicitly allows a ficta hexachord on A, his double hand diagrams are not completely exhaustive.[367] In addition, he places pedagogical value on the solfege syllables: they imply specific intervals (tones and semitones), and singers can use them to calculate intervals and even add syllables where there were none before in order to create the correct major sixth (or other interval).

Ugolino's analysis reveals some advantages of using ficta hexachords: they help musicians who are already intimately familiar with hexachords to sing the proper intervals. They are used for practical purposes. Since musica ficta is the addition of a new mi or fa, and since every syllable must come from some hexachord, the ficta hexachords are derived from usage. They result from the need for coloration and perfection. The double hand diagrams account for some of the most common ficta notes, such as F-sharp, G-sharp, E-flat, even though he leaves out a hexachord on A, which would account for C-sharp. Ugolino's hand diagrams are distinctive, and they have been the subject of a debate between Margaret Bent and Karol Berger.

4.4 The Scale, the Hexachord, and the Hand

The debate over Ugolino's hand diagrams comes in the middle of an argument about partial signatures, but to fully comprehend it, I begin by examining Bent's position on the scale and the hexachord. She develops her position by comparing and contrasting letter names with hexachords. Letters started as names for line segments of different lengths plotted on a monochord and representing pitches or intervals in a scale. For example, in Boethius's division of the monochord, the whole string is marked as the line segment AB, half of the string (which

[366] Ugolino, *Declaratio*, 2:53.

[367] This despite the fact that Margaret Bent claims that they are exhaustive. Bent, "Musica Recta and Musica Ficta," 83.

would produce an octave) as AD, and so forth.[368] In Boethius's use, letters are not organized into steps (D is an octave above A instead of four steps above it), and they do not reflect octave equivalency. However, in the eleventh century, Pseudo-Odo and Guido arranged the letters in sequence, so that they would correspond with the steps of the diatonic scale and would also reflect octave equivalency.[369] Even though after Guido consecutive letters were often used to designate consecutive pitches in the scale, Bent considers letter names to be too imprecise, too inconsistent with practice to provide a full understanding of the diatonic tone system. She acknowledges two different meanings for letter names: points on a monochord and "names for moveable steps within adjacent areas of the gamut."[370] Neither meaning provides enough information to define the tone system. As points on the monochord, the letters do represent distances derived by proportions, but they are arbitrarily assigned, could vary from author to author, and do not exactly correspond to what musicians were singing, as they do today.[371] As names for moveable steps, the letters "stand for steps on a ladder (scala), notated as graphically equidistant lines and spaces on the staff, a visual model of the ladder."[372] There are at least two ways of moving from A to C, through either B-natural or B-flat. And although this letter notation often reflected practice, it is too easily misunderstood by modern performers.[373] If letters cannot be used in this way, what then can performers rely on to discover the intervallic structure of the tone system?

---

[368] Boethius, *The Fundamentals of Music*, 128.

[369] For a review of Pseudo-Odo's contribution in particular, see Charles M Atkinson, *The Critical Nexus: Tone-System, Mode, and Notation in Early Medieval Music* (Oxford: Oxford University Press, 2009), 211-20.

[370] Margaret Bent, "Diatonic Ficta," *Early Music History* 4 (1984), 3.

[371] Bent, "Diatonic Ficta," 4-6.

[372] Bent, "Diatonic Ficta," 7.

[373] Bent, "Diatonic Ficta," 7. However, Stefano Mengozzi has clearly documented instances in which musicians did in fact use letters for singing. Mengozzi, *The Renaissance Reform*, 44-81.

Bent sees the hexachord as the central means by which to understand the intervallic content of the diatonic tone system and to define the unknown distances between the rungs of the ladder. She writes, "the interval relationships among the several rungs on the ladder were articulated only by superimposing on it a network of overlapping hexachords or hexachord segments."[374] She recognizes that the hexachords are indeed superimposed on something that already exists, but she claims that the hexachords articulate the interval relationships. They articulate it in the sense that they allow singers a vehicle for singing them. But she seems to mean more: that before the addition of the hexachords, the letters by themselves were not enough for understanding the intervallic distances between steps in the gamut. She even states that a step in the gamut is "a letter that awaits hexachordal definition."[375] The steps of the gamut are thus defined relatively. There was no absolute pitch that they were measured against. Their specific pitch depended on the length of the monochord string. Or, as Bent says, the points on the monochord only represent relationships.[376] According to Bent, the tone system, as defined by letter names and monochord tuning, is a rather nebulous collection of steps which possess no definition. The hexachords provide only some clarity for musicians, but even they are relative according to Bent. She says the hexachords "provide a functional context for semitone locations…but they do not in themselves determine what the sounds will be."[377] Therefore, Bent argues that although hexachords help musicians understand the intervallic content of the tone system (in particular where the semitones occur), pitch is ultimately determined by context, since

[374] Bent, "Diatonic Ficta," 8.

[375] Bent, *Counterpoint, Composition, and Musica Ficta*, 22.

[376] Bent, "Diatonic Ficta," 4.

[377] Bent, "Diatonic Ficta," 8.

both hexachords and letter names are understood relatively.[378] This relativism informs her view of musica ficta, and it has provoked scholarly response.

Karol Berger directly addresses Bent's relativism in his own book on musica ficta. He states that Bent describes a situation in which pitch is "radically relative."[379] Her position compels him to posit a weak and a strong view of relative pitch. Bent's is the strong one, and the weak one claims that "a system of steps used by musicians is independent of an absolute pitch-standard, that is, that it is in no way affected by being tuned higher or lower in relation to such a standard."[380] To counter Bent's claims, Berger uses the conjunction of letter and solmization. If musicians only solmized based on the immediate context, and if a piece ended on a G solmized as fa, musicians could indeed recognize that as different from a G at the beginning of a piece if it was solmized as G sol.[381] Bent answers this argument by citing pieces without clefs, making letter notation an impossibility.[382] Berger surveys a wide swath of treatises where theorists discuss tuning to see just how far musica ficta could go. With the limits of musica ficta as a premise, he concludes that if steps are so relative, how could theorists even talk about the limits of musica ficta?[383] Although Bent does not directly address this point, she does get at the crux of the issue: Berger takes "the conventional view that there was then (as now) a fixed repertory of pitches from which deviations are measured," and she does not.[384]

How Bent understands and talks about musica ficta (and therefore Ugolino's diagrams)

---

[378] For her emphasis on hexachords or solmization as a key to semitone placement, see Bent, *Counterpoint, Composition, and Musica Ficta*, 7.

[379] Berger, *Musica Ficta*, 44.

[380] Berger, *Musica Ficta*, 44.

[381] Berger, *Musica Ficta*, 45-46.

[382] Bent, *Counterpoint, Composition, and Musica Ficta*, 9, 22-23.

[383] Berger, *Musica Ficta*, 47-48.

[384] Bent, *Counterpoint, Composition, and Musica Ficta*, 21.

depends on her view of the relative nature of the letters and hexachords. She states that during the fourteenth century, as chromaticism was used more widely, practical musicians would need to make sense of the new pitches. She claims that they derived these pitches by transposing the usual hexachords to places where they were not found before, so that each chromatic pitch has a foundation in a transposed hexachord, termed *coniuncta*: "All chromatic notes so derived have their basis in *ficta* hexachords; the hexachord is created for the semitone step, mi-fa."[385] Bent cites a passage from the Berkeley manuscript as evidence. In this passage, the author instructs readers how to sing the proper solfege syllables and how to mutate from one hexachord to another, "unless by chance some unusual song should turn up, which some call—but wrongly—musica falsa, others musica ficta; still others name it—and rightly—coniunctae."[386] The author continues by describing what coniunctae are. "It is like a connection by the aforesaid regular properties [natural, hard, and soft hexachords]. And so, these coniunctae were invented so that a song formerly called irregular could be brought into regularity by them in some manner. For a coniuncta is the attribute, realized in actual singing, of permitting one to make a semitone out of a tone and conversely. Or rather, a coniuncta is the mental transposition of any property or hexachord from its own location to another location above or below."[387] Bent implies the coniunctae were transposed to derive the new pitches, and she even calls Ugolino's ficta diagrams transpositions.[388] But the Berkeley author refers to transposition as a mental act, not so

[385] Bent, "Musica Recta and Musica Ficta," 80. (italics original). The term *coniuncta* is related to the Greek *synemmenon*: the *synemmenon* or conjunct tetrachord could stand in the place of the diezeugmenon or disjunct one, thereby creating B-flat in the former but B-natural in the latter. See also Mengozzi, *The Renaissance Reform of Medieval Music Theory*, 96.

[386] *The Berkeley Manuscript,* trans. Oliver B. Ellsworth (Lincoln: University of Nebraska Press, 1984), 50-51.

[387] *The Berkeley Manuscript*, 51-53.

[388] Bent, *Counterpoint, Composition, and Musica Ficta*, 8-9. I have already shown, however, that they are not derived from transposition, nor does Ugolino talk about or appeal to transposition in his discussion of them, except

much to derive the pitches as to sing them properly. The transposition occurs in the minds of the singers for a practical purpose: they simply move a hexachord to a place it does not regularly occur in order to perform the music with the proper tones and semitones. This does not necessarily mean that they are derived from transposition, and Ugolino does not derive them from transposition, even though his account of ficta hexachords sounds very similar to this anonymous author. The Berkeley author states that this transposition happens in the mind: the sounding pitches are not logically or mathematically derived from transposition.[389]

Both Bent and Berger appeal to Ugolino over an argument about partial signatures. Bent argues that if a B-flat appears in the signature, this limits the singers to two recta hexachords, the ones on F and C. And if there are two flats in the signature, it further reduces them to one. She concludes that signatures "define the limits of musica recta."[390] Bent then carries this argument forward first by noting that the recta hexachords—the ones beginning on C, F, and G—express a "set of relationships." She surmises that if signatures relate not to keys but to hexachords, then a flat in the signature shifts the set of relationships by a fifth (or fourth). If this is true, then a hexachord, in this case one starting on B-flat, which would have been a ficta hexachord without the signature, now becomes a recta hexachord with the signature. Bent's thesis hinges on the idea of transposition. "Ficta involves the transposition of isolated hexachords for the purpose of creating chromatic notes, but transposition of recta implies that the whole structure is shifted, together with its built-in rules for applying accidentals."[391] In other words, under a signature with

for the fact that the second diagram was lower than the first by a fifth. But even there, there are differences that prevent it from being an exact transposition.

[389] Stefano Mengozzi argues that musicians viewed the hexachord as a virtual segment, which means it does not derive anything. Instead, it is superimposed on a structure that already exists. Stefano Mengozzi, "Virtual Segments: The Hexachordal System in the Late Middle Ages," *The Journal of Musicology* 23, no. 3 (2006), 426-67.

[390] Bent, "Musica Recta and Musica Ficta," 98.

[391] Bent, "Musica Recta and Musica Ficta," 98.

one flat, an E-flat would be considered a recta and not a ficta note. Bent adduces Ugolino's double hand diagrams to support these claims about transposition.[392] Although Karol Berger disagrees with Bent's strong relative position, he also sees these diagrams as indicating transposition, even trying to show exactly how each one is consistently transposed or how the whole system is transposed.[393]

In her careful definition of the hexachord in its relation to the scale, Bent sees the hexachord as playing a structural role, since it helps define the relatively unknown steps of a scale. She takes a middle position between what Stefano Mengozzi calls a "foundational" and a "soft" view of the hexachord. In the foundational view, the hexachord is the ultimate model for the tone system instead of the octave-equivalent, seven-note scale. The hexachord and not the diatonic scale defines musical space.[394] In the soft view, the hexachord is merely laid on top of the tone system, but ultimately it is the seven-note pattern that structures diatonic musical space. In this view, musical material can still be "ordered" according to the hexachord, but the material itself is not hexachordal.[395] Berger takes a view closer to the soft view. But where does Ugolino stand?

Ugolino introduces the tone system in book 1. The order in which he presents the material indicates that he prioritizes the intervallic structure above the hexachordal arrangement. In other words, the hexachords reflect the intervals already established in the system. He compares and contrasts the Greek and the Latin "hands," beginning with the Greek. Following Boethius closely, he begins with the four strings of the tetrachord, noting the tuning with terms

[392] Bent, "Musica Recta and Musica Ficta," 99; Bent, *Counterpoint, Composition, and Musica Ficta*, 7-10, 88.

[393] Berger, *Musica Ficta*, 64-65.

[394] Mengozzi, *The Renaissance Reform*, 9.

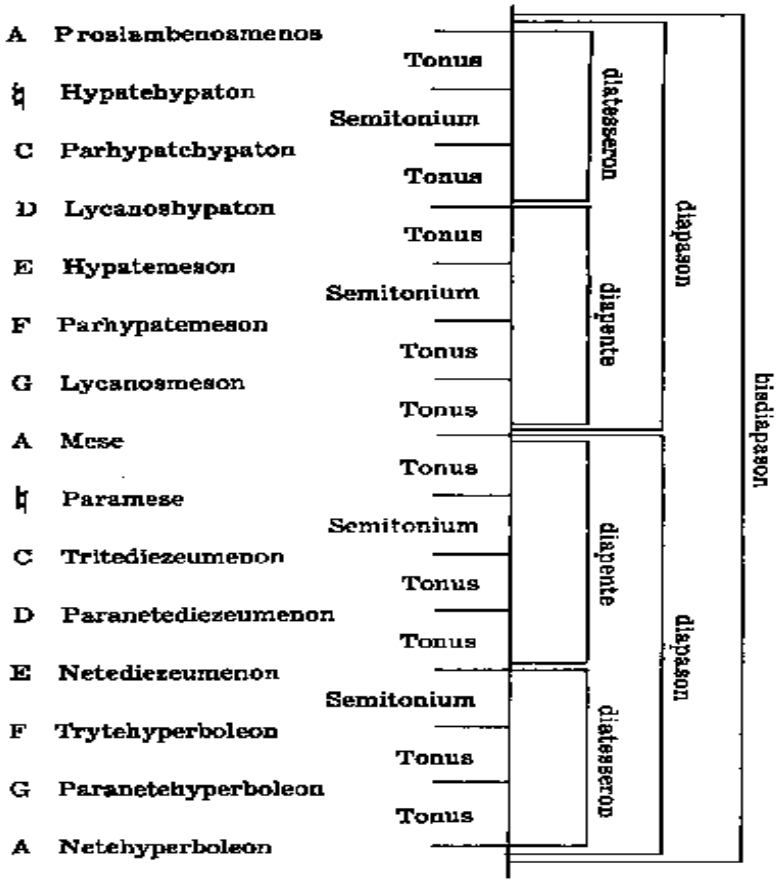
[395] Mengozzi, *The Renaissance Reform*, 12.

like diapason, tone, and diatessaron:

> We read that in the beginning music was simple and consisted only of four strings. From the first to the fourth string sounded the consonance of a diapason.[396]
>
> *Legimus in principio musicam simplicem fuisse et quatuor duntaxat cordis constare, cuius corda prima ad quartam diapason consonantiam resonabat.*

He, like Boethius, goes on to show how more strings were added, and he provides both the Greek names of the strings and the names for the intervals. After he has catalogued all of the strings, he describes the intervals they form, finally adding the following diagram.[397]

Figure 4.3: Ugolino's Tone System.[398]



[396] Ugolino, "Declaratio," 1:32; cf. Boethius, *The Fundamentals of Music*, 29-40.

[397] This diagram is the first in his treatise. It presents the Greek Greater Perfect System or *systema teleion* (see Atkinson, *The Critical Nexus*, 11 n. 27.). In addition, it clearly derives from Boethius, *The Fundamentals of Music*, 39. But to Boethius's diagram Ugolino adds letter names, in stepwise order and repeating at the octave, as well as the intervallic distances, both each consecutive interval (such as tone and semitone) and larger intervals (such as diatessaron, diapente, and so forth).

[398] Example I-1 in Ugolino, *Declaratio*, 1:supplement.

Ugolino does not address the specific ratios of these intervals in book 1, because he intends his work to progress from what is more known to the readers (the practical) to what is less known (the theoretical). Indeed, Ugolino makes this practical and theoretical distinction himself in book 5. Concerning the octave he states:

> In practice, the diapason is composed of a diapente and a diatesseron, as was said in the first book. But in theory, it is composed of their proportions.[399]
>
> *Componitur autem diapason practice ex diapente et diatesseron, sicut in primo dictum est, theorice vero ex eorum proportionibus.*

And these proportions are the same as they were for Boethius. Ugolino writes,

> The proportion of the diapente is written as a sesquialtera 3 and 2. The proportion of the diatessaron added to it is placed under it, and they stand thus: 3-2 4-3. Let 3 be multiplied by 4 and the product will be 12. Further, let 2 be multiplied by 3 and produce 6. Between 12 and 6 is the duple proportion…therefore, just as in practice the diapason is composed of a diatessaron and a diapente, so its proportion is composed of their proportions, which has been demonstrated.[400]
>
> *Scribatur proportio diapente sexquialtera 3 et 2, cui subordinetur proportio diatessaron adiungenda, et stent sic: 3-2 4-3, et multiplicetur adiungendo 3 per 4 et productum erit 12. Postea multiplicetur 2 per 3 et producetur 6, sed inter 12 et 6 est proportio dupla…Igitur sicut diapason pratice ex diatessaron et diapente componitur, sic eius proportio ex eorum proportionibus connectitur, quod erat demonstrandum.*

It is clear that he understands these intervals by their Pythagorean ratios. Practice uses names like diapente and diatessaron, but theory spells out the specific proportions. Therefore, in his tone system, Ugolino sees the intervals in the scale as fixed, not in the sense that they have some absolute pitch frequency, but in the sense that the intervallic distances between pitches remain constant.[401] Furthermore, these intervals are, at first, described in practice, and then they are explained through their proportions in theory. But what about the hexachords?

Ugolino introduces hexachords or syllables only after he has set out the tone system. He

[399] Ugolino, *Declaratio*, 3:172.

[400] Ugolino, *Declaratio*, 3:172.

[401] In this case, therefore, Berger's conception seems closer to Ugolino's than Bent's.

first mentions syllables to explain the Greek hand. Of the twenty-five notes of the Greater Perfect System he says,

> We sing [lit. pronounce] these twenty-five simple pitches, which comprise a bisdiapason, in this manner: re, mi, fa, sol, la, mi, fa, sol, la, re, mi, fa, sol, la, mi, fa, sol, la. These pitches or notes are signified in our custom by the hard and natural properties [hexachords].[402]
>
> *Has autem XV voces simplices bisdiapason continentes hoc modo pronuntiamus, scilicet; re, mi, fa, sol, la mi, fa, sol, la re, mi, fa, sol, la mi, fa, sol, la, quae voces seu notae more nostro per proprietates [sqb] quadri et naturae significantur.*

He continues the analogy by yoking the letters and syllables, so that the proslambanomenos is A-re and so on. He concludes the chapter by making the rather absurd claim that the Greeks discovered or invented the five syllables (*voces*) re, mi, fa, sol, and la.[403] It seems absurd because Guido was the first to use the syllables in the eleventh century. But in the minds of Ugolino's readers, the syllables were so tightly bound to specific intervallic distances that Ugolino attempted to make the Greek Greater Perfect System more understandable by invoking solmization syllables. His readers were probably already familiar with the syllables and hexachords since he can cite them here even before he has formally introduced them. By describing the sequence of intervals in the Greater Perfect System as a series of solfege syllables, Ugolino joins intervals and solfege together, so that re-mi-fa-sol-la always mean the same as tone-(minor) semitone-tone-tone respectively.

In the next few chapters, Ugolino sets out the Latin hand and discusses its solmization syllables and hexachords. His discussion reveals the way in which he conceptualizes diatonic tonal space. He sees the Latin hand as an outgrowth of the Greater Perfect System. He starts by noting the addition to the Greater Perfect System of G2 called gamma. But the tension between

[402] Ugolino, *Declaratio*, 1:26.

[403] Ugolino, *Declaratio*, 1:26.

the seven letters and the six syllables begins to surface. He uses the fact of octave equivalency as a reason for adding gamma: the Greeks had seven letters, A-G, which repeat but which do not make a full octave. By adding another G, a full octave is produced.[404] Since there are seven letters and the gamut extends from G2-E5, there are twenty-five notes. These are placed on twenty joints of the hand—B-natural and B-flat share the same spot. These spots he calls seats, places, or abodes (*sedes*)—the place where these notes or pitches live. Since there are six syllables that begin on three different places, one letter may have one to three syllables where the hexachords overlap. He refers to the syllables as notes (*notae*) and there are forty-two. The syllables come out of two properties (the natural and hard hexachords) that are repeated five times, while the B-flats come out of a third property (the round b property or soft hexachord) which is placed between (*interponi*) the others two times.[405] This description appears before he has formally introduced the hexachords. Ugolino starts from the seven repeated letters, which create twenty places (*sedes*). These *sedes* accommodate several different syllables, which he calls both syllables (*voces*) and notes (*notae*). Because of this there are forty-two notes. The hand is the collection of *sedes* and *voces*, or the combination of letter and syllable. But we learn more about the *sedes* from the following discussion.

In chapter 12 of book 1, Ugolino counts how many intervals are within the hand. He goes through each interval in turn, beginning with the tone and continuing with the minor semitone, third, minor third, fourth, tritone, fifth, diminished fifth, major sixth, minor sixth, major and minor seventh, octave, diminished octave, eleventh, twelfth, and fifteenth. He calculates all these intervals from their *sedes*, not from the *voces*. In other words, he calculates them from the letter

[404] Ugolino, *Declaratio*, 1:27.

[405] Ugolino, *Declaratio*, 1:28.

names, not from syllables. He begins by listing the total number of each type found in the gamut and then gives the letters for each one. For example, he states, "The fifth occurs fifteen times, from gamma to D, from A to E" (Diapente quintumdecimum numerum tenet, scilicet, a [Gamma] ad D, ab A ad E) and so on.[406] Counting intervals from the letters makes sense only if those letters have at least a relatively fixed position in the system. Except for B-natural and B-flat, which share the same place, the distances between all the other letters or *sedes* are measured in tones and semitones, and their intervals can be calculated without reference to the solmization syllables or the hexachords. But Ugolino's conception of the hexachord and the relationship between them and the seven letters emerges in his discussion of the hexachord and its meaning.

Ugolino addresses the hexachord directly in chapters 13-15 of book 1. He sees the natural hexachord, the one starting on C, as the primary hexachord. He begins by clarifying the difference between *proprietas* (hexachord, property) and *proprium* (property, proper). *Proprium* describes something that is incidental to a thing's essence or being: it can be predicated of a thing, but it cannot point to (*indicat*) a thing's essence. It cannot tell what makes a thing what it is. *Proprietas*, on the other hand, can be predicated of a thing, and it also points to what the essence of a thing is. For example, in the proposition "humans are rational," rational is something that can be predicated of humans, but it also points out the distinguishing mark of what it is to be human. Hence, it is a *proprietas* or property of human.[407] For these reasons, "in ancient times, there was one property of music that revealed its essential measure, which in fact is called natural, which is the same thing as the natural property [natural hexachord]" (sola temporibus igitur primitivis fuit musicae proprietas eius essentialem modum ostendens, quae

[406] Ugolino, *Declaratio*, 1:33.

[407] Ugolino, *Declaratio*, 1:34-5.

natura merito nuncupatur, quae idem est quod naturalis proprietas).[408] The natural property of music—the natural hexachord—is indispensable for understanding what music is, just as rational is indispensable for understanding what a human is. Ugolino is closely following Guido. Guido only referred to one hexachord before talking about other ways to sing melodies, and the other hexachords were added later by other theorists.[409]

Ugolino distinguishes between the natural hexachord or property on the one hand and the soft and hard hexachords on the other. He considers the natural hexachord the central one, a key to understanding music. But the other two are merely accidental in the Aristotelian sense of the term: they are incidental to understanding music. They originated from the need to soften the discord of the tritone between B and F.

> There are three properties which are called natural, soft B, and hard B. The first, the natural—because it tells the essential measure of plainchant and demonstrates its essence—is called an essential property. But the other two, soft B and hard B, out of which arises discord from their softness and hardness, are called an accidental measure because they are accidental to music, nor do they show its essence.[410]
>
> *Sunt ergo proprietates tres quae dictae sunt naturae et B mollis atque [sqb] duri quarum prima, scilicet, naturae, eo quod plani cantus modum dicit essentialem et eius esse demonstrat dicitur proprietas essentialis. Aliae vero duae, scilicet, B mollis et [sqb] quadri, quarum discordia ex asperitate mollitieque consurgit, quae musicae accidentia cum sint modum accidentalem dicunt nec essentiam eius ostendunt.*

The natural property is indispensable because it reveals an essential measure of plainchant. This comment reveals that here Ugolino's priority is to understand plainchant. The natural hexachord is not an essential measure of music in general but of plainchant in particular. The others were added later and are merely accidental to plainchant. Indeed, in books 4 and 5, which he devotes

[408] Ugolino, *Declaratio*, 1:35.

[409] Stefano Mengozzi, "'Si Quis Manus Non Habeat': Charting Non-Hexachordal Musical Practices in the Age of Solmisation," *Early Music History* 26 (2007), 182; Mengozzi, *The Renaissance Reform*, 82.

[410] Ugolino, *Declaratio*, 1:36.

entirely to expounding intervals, he refers constantly to numbers and proportions but only to syllables once.[411] Even later in book 1, where he laboriously describes each interval, he always uses letters but only sometimes refers to syllables.[412]

At this point, Ugolino finally offers a definition of property: "a property, essential or accidental, is an acceptance [or reception] of syllables arranged in a high or low register…" (proprietas est essentialis seu accidentalis vocum in acuto vel gravi ordinatarum acceptio…).[413] For the syllables to be accepted or received somewhere, a place or places must already exist. Those places are the letter names or *sedes* already set out in the previous chapters. He goes on to note that the properties require what he calls variation because they cannot span an octave. And his comments on this topic reveal another curious fact about the properties. He writes,

> It is necessary that the properties be varied in turn. From their variation in ascent, the complete [order] of the authentic tones is discovered, and from their arrangement in descent the order of complete plagal tones is shown.[414]
>
> *Oportet proprietates praedictas ad invicem variari, ex quarum variatione per arsyn autenticorum tonorum perfectio invenitur, et plagalium per thesyn perfectionis ordo monstratur.*

After saying this, he tells how there is a property that begins on every C, F, and G. There is a connection between the use of hexachords and the understanding of modes, but he does not elaborate any further on this point nor does he refer to it later.

To sum up, Ugolino uses the Greek Greater Perfect System as his foundation. He follows Boethius by tracing the history of the system as starting with four strings and steadily gaining more over time. He describes these strings using their Greek names, Latin letters, and interval

[411] Ugolino, *Declaratio*, 3:49-50.

[412] See, for example, Ugolino, *Declaratio*, 1:162-163.

[413] Ugolino, *Declaratio*, 1:36.

[414] Ugolino, *Declaratio*, 1:37.

sizes. Later, he defines these intervals using proportions. He sees the Latin scale as an extension of the Greek one. To the scale, he adds the syllables and the set of seven hexachords, divided into three types (natural, hard, and soft). The letters are the seats (*sedes*) which receive the syllables (*voces*). This combination of *sedes* and *voces* is the musical hand, or, in other words, musica recta. The natural hexachord is essential for plainchant, but the other two are accidental, growing out of the need to distinguish B-natural and B-flat. The solfege syllables themselves come out of hexachords. Syllables cannot exist without a corresponding hexachord that it comes from. When he discusses intervals, he sometimes uses syllables but mostly uses letters.

For Ugolino, the scale, not hexachords, come first. The scale is not exactly an undefinable set of steps on a ladder. Instead, the steps are defined as either tones or (minor) semitones, which are further clarified as particular mathematical ratios. The steps or letters are the places that receive syllables and hexachordal designation. The syllables, used for practical purposes, correspond to specific intervallic distances. For example, re-mi is always a tone while mi-fa is always a minor semitone. The set of letters and syllables delineates musica recta but helps make sense of musica ficta. Ugolino defined musica ficta as "the necessary placement of some syllable in a place where it does not exist by itself for the purpose of perfecting consonances" (est alicuius vocis in loco ubi per se non est ad consonantiae perfectionum necessaria positio).[415] In musica recta, each syllable has a particular place: a syllable is always associated with a certain letter or set of letters. These, in turn, always match the intervals designated by the letters. But in musica ficta, a syllable is placed on a letter where it was not found in musica recta. For example, a mi is placed on an F. Since syllables correspond to intervals, mi to fa on F to G is no longer a tone but a minor semitone. Since what should be a

[415] Ugolino, *Declaratio*, 2:45.

tone in musica recta is now a minor semitone in musica ficta, the difference between a tone and minor semitone is a major semitone. This is why Ugolino argues that musica ficta involves the addition of a major semitone. But does the fact that a mi on F mean that the letters were, in fact, flexible? The F-mi is feigned, a fiction created for some necessity. It does not mean that the system of musica recta is undefined and merely awaiting hexachordal definition. Instead, it means that the system was an incomplete picture. We get a glimpse of the fuller system in the chapter on musica ficta, but Ugolino approaches it from the perspective of musical practice. The practice of singing F-mi points to notes outside the system of musica recta—to a new or different *sedes*. This system needs a theoretical explanation. For this, we must look outside the *Declaratio* to the small treatise attached an appendix to it—the *Tractatus monochordi*.

4.5 The *Tractatus monochordi*

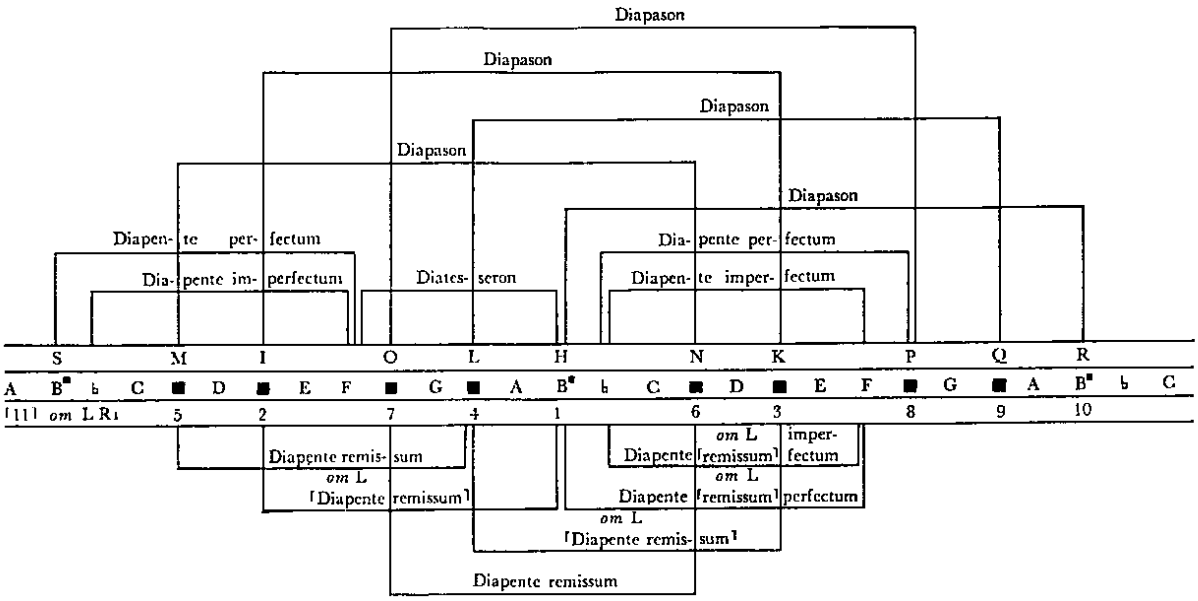
Ugolino's *Tractatus* is divided into two parts: the first, in chapters 1-7, offers various divisions of the monochord according to musica recta. The second, in chapters 8-10, takes the recta division as its base and creates a division according to musica ficta. Indeed, Ugolino distinguishes these two parts himself.

> The division of the monochord is two-fold: one in which the monochord is divided according to recta musica through tones, minor semitones, diatessaron, diapente, and diapason; the other in which every tone, arranged according to recta musica, is divided into major and minor semitones. This division of tones is called not recta but ficta musica.[416]
>
> *Duplex est monochordi divisio, altera qua secundum rectam musicam per tonos, semitonia minora, diatessaron, diapente et diapason monochordum dividitur, altera qua omnes toni secundum rectam musicam dispositi in maiora atque minora semitonia dividuntur, quorum tonorum partitio non recta sed ficta musica nuncupatur.*

[416] Ugolino, *Declaratio*, 3:230-231. See also, Andrew Hughes, "Ugolino: The Monochord and Musica Ficta," *Musica Disciplina* 23 (1969), 22.

The division according to musica ficta springs from that of musica recta. In musica recta each tone is in the proportion 9:8, and Ugolino staunchly maintains the Pythagorean position that this interval cannot be divided evenly.[417] In Pythagorean theory, there are two kinds of semitones—major and minor—and the distance between them is the comma. So, for each tone of musica recta, there could be two different semitones. Therefore, Ugolino charts two different divisions, one to find the minor semitones, and the other to find the major ones. His ficta divisions grow out of the observation from practice that sometimes intervals need to be altered either to avoid imperfect fifths or octaves or to create cadential progressions.

Figure 4.4: The Ficta Division with Minor Semitones above a Given Note[418]



Since he is calculating these intervals from proportions, it is easiest to begin with minor semitones because some of them are already present in musica recta. In fact, only minor semitones are present in musica recta. In demonstrating this first division (Figure 4.4), he uses

[417] Ugolino, *Declaratio*, 3:231. By maintaining the unequal division of the tone, Ugolino is contradicting Marchetto of Padua.

[418] Ugolino, *Declaratio*, 3:241.

letters names beyond G to stand for points on the monochord.[419]

The first step is to find B-flat, and from there he can divide into 3 against 2 to find the perfect fifth below B-flat—E-flat (or letter I in Figure 4.4). Letter I is a minor semitone above D. He divides the string from letter I in half and derives a note an octave higher, letter K. Using the same process of finding either a perfect fifth or an octave, he locates A-flat, D-flat, and G-flat (L and Q, M and N, and O and P respectively).[420] Since minor intervals are perfected (made major) by adding major semitones, this first division is insufficient for truly perfecting imperfect intervals, that is, for making cadential progressions from major thirds to fifths or from major sixths to octaves.[421] So why even include this division? It is useful for creating perfect fifths or octaves and for coloring dissonances, that is, making major intervals minor in progressions like a minor third to unison or a minor sixth to a fifth. In fact, he states that the minor semitone is just what is needed for coloring dissonances.[422] This division points out Ugolino's contrast between perfecting, on one hand, and coloring, on the other. To color a dissonance means to lower it and make it minor or imperfect. Indeed, he associates major with perfect and minor with imperfect. He writes, "in counterpoint consonances and dissonances ought to be perfect and major and not minor and imperfect" (consonantiae et dissonantiae in contrapuncto debent esse perfectae et maiores et non minores et imperfectae).[423] That is why he can say, "For coloring dissonances—for which a minor semitone is necessary—this division of the monochord is perfect…" (Sed ad dissonantias colorandas in qua semitoniorum minoritas est necessaria haec monochordi divisio

[419] Here Ugolino combines the two definitions for letter names described by Bent, "Diatonic Ficta," 3.

[420] Ugolino never refers to altered notes as A-flat or F-sharp and so forth. Instead, he consistently describes them by the letter names on his diagrams. I refer to them in the former manner only for the sake of clarity.

[421] Ugolino, *Declaratio*, 3:244.

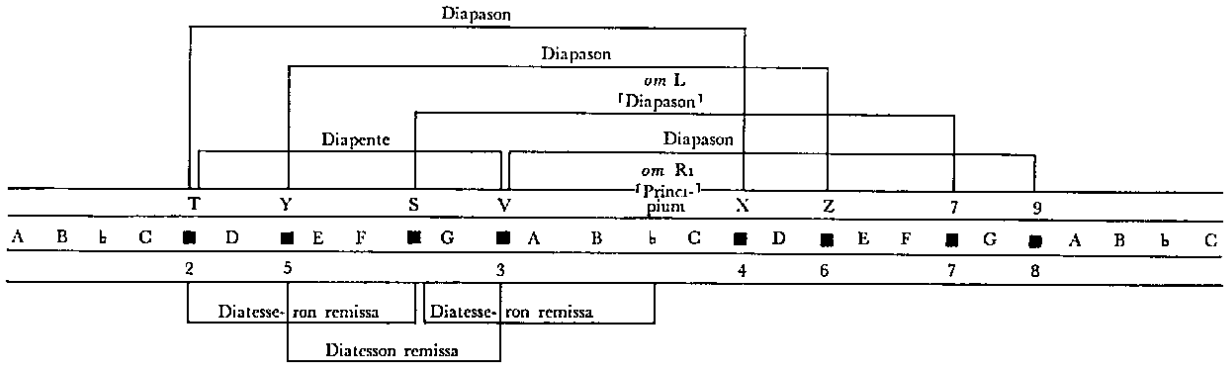
[422] Ugolino, *Declaratio*, 3:244.

[423] Ugolino, *Declaratio*, 2:51.

perfecta est…) Ugolino uses coloration and perfection both in a wide and a narrow sense. In the wide sense, they refer to the same thing—altering a note to achieve some musical goal. But in the narrow sense, coloration means using minor semitones or making an interval minor or smaller; perfection means using a major semitone or making an interval major or larger. This distinction was apparent in the chapter on counterpoint examined above, but he clarifies the idea in the *Tractatus*. Since he needs to explain how intervals are made larger, he needs another monochord division, and that is what he does next.

In the first division, he started from B-flat, but in the second division (Figure 4.5), he starts from B-natural. In the first division, he used fifths and octaves to find the new notes. In this division, he tunes in fourths, fifths, and octaves. Using this method of calculation, he produces C-sharp, D-sharp, F-sharp, and G-sharp (T and N, Y and Z, S and 7, and V and 9 respectively).

Figure 4.5: The Ficta Division with Minor Semitones below a Given Note[424]



This division is "perfect for perfecting consonances or dissonances" (ad consonantias vel dissonantias perficiendas perfecta est).[425] He uses perfecting here to mean correcting impure fifths and octaves, as well as in the narrower sense of making major. This division allows for major thirds and sixths to contain the correct (i.e., Pythagorean) number of tones and semitones.

[424] Ugolino, *Declaratio*, 3:246.

[425] Ugolino, *Declaratio*, 3:247.

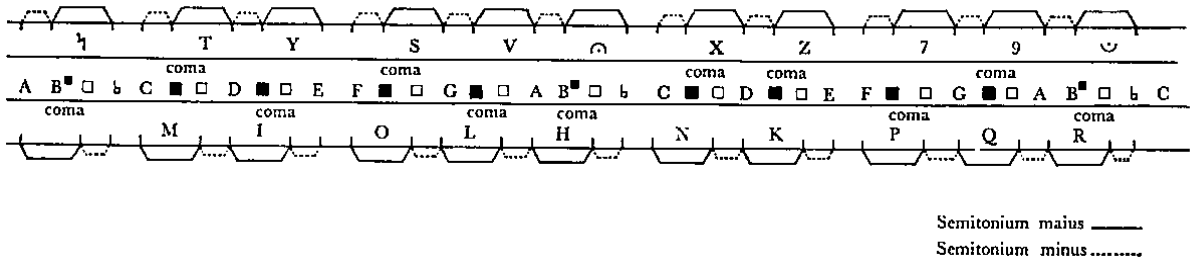
But this second division cannot accommodate every circumstance. If a third progresses to a unison or a sixth to a fifth, then minor intervals are needed—the upper note needs to be lowered by a major semitone, which then lies a minor semitone away from its resolution. This does not occur with this division. In other words, this division cannot properly be used for coloration.[426] Ugolino sums up the matter succinctly:

> So then, the first ficta division of the monochord is imperfect for perfection and perfect for coloration. But the second [division] is perfect for perfection but is imperfect for coloration.[427]
>
> *Prima igitur monochordi ficta divisio ad perfectionem est imperfecta et ad colorationem perfecta. Haec autem secunda ad perfectionem perfecta, sed ad colorationem redditur imperfecta.*

These two different ways of tuning the intervals between a tone are each useful in their own way. But a practicing musician ends up using both.

Ugolino solves the problem by combining both tunings into one system (Figure 4.6). The combined system creates an octave divided into seventeen different notes.

Figure 4.6: Both Ficta Divisions Combined[428]



The foundation for Figure 4.6 is the system of musica recta, indicated by the usual letter names. For each step or tone of musica recta, there are two semitones each separated by a comma. To these notes, Ugolino assigns letters past G, and some even receive special symbols or numbers.

[426] Ugolino, *Declaratio*, 3:247.

[427] Ugolino, *Declaratio*, 3:249.

[428] Ugolino, *Declaratio*, 3:251.

For example, between C and D there are two notes. The first, labeled M, is a minor semitone above C. The next is letter T, which is a major semitone above C, a comma above M, and a minor semitone below D. Ugolino was, by no means, the first to lay out a seventeen-note octave. He seems to have closely followed Prosdocimo, although not slavishly.[429] And others, such as John Hothby and Franchino Gafori, were indebted to him.[430]

By plotting the notes of musica ficta on a monochord, Ugolino reveals several keys to understanding his conception of musica ficta, and, as a result, the relationship between the scale, the hand, and the hexachord. First, musica recta is a fixed set of relationships. The relationships are observed in practice through a study of plainchant, and they are defined in theory through mathematical proportions. For example, D to E is a tone and that tone is defined as a 9:8 ratio. Second, the syllables, and by extension the hexachords, are overlayed on this system. Each letter or place receives a syllable, and since there are only six syllables, some letters will receive more than one. Yet these syllables correspond to intervals. In other words, re to mi always indicates a tone, and mi to fa always marks a minor semitone.

This serves plainchant quite well, but when new melodies are added above a chant or a given melody, two problems arise. One problem is the tritone (as well as imperfect octaves), whether between a low B and the F above it, or between E and B-flat. That is, these fifths contain an incorrect number of intervals, three tones instead of three tones and one minor semitone. Since counterpoint demands consonant intervals, these need to be made consonant. In practice, this happens by altering the added voice so that the correct interval is formed. The alteration is

[429] Berger, *Musica Ficta*, 33; Mark Lindley, "Pythagorean Intonation and the Rise of the Triad," *Royal Musical Association Research Chronicle* 16 (1980), 22-26.

[430] Heinzelmann, "John Hothby as Innovator," 355; Lindley, "Pythagorean Intonation," 26; Berger, *Musica Ficta*, 33-37.

described as adding a mi or a fa in a place where there was none before. Melodically, this creates a minor semitone to the next note. Harmonically, it adds a major semitone so that the interval can be formed by the correct number of tones and minor semitones.

A second problem is the intervals needed for a cadential progression, and this comes in two forms. A third progressing to a unison needs to be minor. When a major interval needs to be lowered and made minor, Ugolino refers to this as coloration. On the other hand, a sixth progressing to an octave needs to be major. When a minor interval needs to be raised and made major, Ugolino calls this perfection. Coloration requires a note to descend by a minor semitone to its resolution. In practice, this happens when a singer needs to add a fa where there was none before. In theory, this is explained by dividing the monochord from B-flat, which is a minor semitone above A and solmized as fa (Figure 4.4). This division also helps correct imperfect fifths and octaves. But it will not do for perfections. Perfection requires a note to ascend by a minor semitone. In practice, this happens when mi is added where it is not normally found. In theory, it is explained by dividing the monochord from B-natural (Figure 4.5). These two problems are also referred to by Ugolino as perfection in the broadest sense of the term. This is why he states in his definition that the purpose of musica ficta is perfection. Since both divisions are necessary, he combines them into one where the octave is divided into seventeen different notes (Figure 4.6).

At first glance, it seemed as if the letters of musica recta are flexible or undefined, as Bent has depicted them. Instead, the new divisions show that musica recta by itself is incomplete. What Ugolino describes only imprecisely in practice as, for example, an F-mi (that is, a mi on a letter where it did not occur before), he explains more precisely in theory as a place between letters in a more elaborate division of the monochord. That mi does indeed have its own place,

but that place is between two letters of musica recta. Still, Ugolino has no convenient way to describe this place. His mix of letters, symbols, and numbers is too cumbersome, and he does not simply say, for instance, "F-sharp" or "E-flat." With Ugolino's system of seventeen notes per octave, he can maintain Pythagorean tuning for each interval, even the altered ones.[431]

4.6 Conclusions

Ugolino defines musica ficta based on an observation of practical needs. He notes that even when the right notes are sung, they do not always form consonances. Musica ficta was invented to resolve this problem. He then relates a particular example that includes the B/F tritone. This tritone can be corrected by singing an F-mi instead of the usual F-fa, raising the note from where it would be.[432] From these brief practical problems, he offers a concise definition of musica ficta. The rest of the chapter expands on this definition. From it, we learn several important takeaways.

The purpose of musica ficta is perfection. Perfection is broadly taken to mean two things. It is the correction of impure fifths and octaves to perfect ones. It is also the alteration of thirds and sixths so that they resolve even more smoothly to the closest perfect consonance. Ugolino also calls these perfections, and they are used as cadential progressions. This kind of perfection happens in two ways. If a third progresses to a union or a sixth to a fifth, they should be made minor by lowering the added voice. But if a third moves to a fifth or a sixth to an octave, they should be made major by raising the added voice.[433] The former Ugolino calls coloration; the

[431] It is not my purpose here to address the issue of tuning or the inherent problems of Pythagorean tuning. But the prominence of musica ficta points out how important this issue would become towards the end of the fifteenth century and during the sixteenth.

[432] Ugolino, *Declaratio*, 2:44-45.

[433] None of Ugolino's examples show the lower, chant voice being altered. He always changes the added voice, even when it would be easier to change the lower voice and avoid musica ficta by using a B-flat.

latter perfection, although he sometimes uses coloration or perfection to refer to both. Ugolino's broad definition of perfection, which covers all these changes, encompasses the two traditional categories cited as the purpose for ficta: *causa necessitatis* and *causa pulchritudinis*.

For Ugolino a change according to musica ficta is necessary. It is necessary because of its purpose, in order to create perfect fifths and octaves or to make a cadential progression. Indeed, he writes, "we do not use musica ficta unless as an altogether compelling necessity" (musica ficta nisi necessiatate cogente penitus non utamur).[434] In other words, alterations are made for a reason based on the musical context. They are not applied arbitrarily.

Musica ficta means putting a solfege syllable in a place where it did not exist before. The place refers to the letters of the gamut, which represent the steps of the scale. Each step received several syllables, since hexachords, constructed in six-note segments, must stretch out over the scale, constructed in seven-note segments. The letters and solfege together form the musical hand. Ugolino points out that adding a syllable where there was none before means adding a major semitone, which will then resolve to the next note by a minor semitone. Ugolino thinks of syllables as tied to a hexachord. If a new, ficta syllable is added, it must come from some new, ficta hexachord.

Based on these observations, Ugolino creates two diagrams, in each of which he gathers together seven ficta hexachords and displays them along side of the seven recta hexachords. The interpretation of these diagrams tests the understanding of the role of the scale and the hexachord in the structure of diatonic space. Both Margaret Bent and Karol Berger speak about these ficta hexachords as if they were transpositions of recta hexachords. Indeed, Berger tried to show how the whole system of ficta hexachords is a transposition of the system of recta hexachords.

[434] Ugolino, *Declaratio*, 2:45.

However, these descriptions fail because they do not take into account how Ugolino talks about them. He derives them based upon the necessity of perfection, in terms both of correcting fifths and octaves and of altering thirds and sixths for cadential progressions. For example, since a major sixth is required for the cadential progression A/f to G/g, the f must be raised through ficta. This will create an F-mi. Since syllables come from hexachords, this F-mi must come from a hexachord starting on D. He derives the other ficta hexachords in a similar manner. He never cites transposition as the reason for deriving ficta hexachords, and he could have done so, since he states that his second diagram is essentially a transposition of the first down a fifth.[435] He includes only seven ficta hexachords probably because there are only seven recta hexachords. However, this does not mean that his diagrams are exhaustive. In fact, he admits a hexachord on A even though he does not include it in his diagrams.

Bent sees the letters as only imprecise markers of a scale, while the syllables give more definition to them. Ugolino does associate syllables with precise intervallic distances—precise in respect to the proportions but not to the pitch frequency. Accordingly, all adjacent syllables, except mi-fa, are tones with the proportion 9:8. Mi-fa always indicates a minor semitone. But unlike Bent, Ugolino posits that the letters also indicate clear proportions, and the syllables correspond to these. This holds true for musica recta. Musica ficta, on the contrary, changes these relations. That is why it is called ficta: it adds a note where there was none before. But it does this in a systematic way, by adding major semitones either above or below a given note. This does not mean the original letters were imprecise. Instead, it means that they do not present

[435] Although he may view the ficta hexachords as transpositions of recta hexachords, he does not take the system of recta hexachords and transpose them all at once by a single interval.

the whole picture. Ugolino explains this more elaborately in the *Tractatus*, appended to the *Declaratio*.

In the *Tractatus*, Ugolino sets out two different divisions of the monochord according to musica ficta. These divisions correspond to the usages he first outlined in book 2 of the *Declaratio*: in one division, he calculates the notes that lie a minor semitone above a given note. This division is used for coloration, that is, for making major intervals minor. In the second division, he does the opposite and finds the notes that lie a minor semitone below a given note. This division is used for perfection, that is, for making minor intervals major. He calculates these divisions mathematically using Pythagorean tuning, and he names each new place with either a letter name beyond G or a number or symbol. He does not simply call them by their letter name with the addition of a square or round b. Since both divisions are necessary for performing musica ficta, he combines them into one diagram that divides the octave into seventeen notes. This theoretical explanation of musica ficta is a consequence of the practical observations and needs found in book 2.

Ugolino presents two accounts of musica ficta—a practical one in book 2 and a theoretical one in the *Tractatus*. The relationship between practice and theory reflects the philosophy outlined in chapter 2. In practice, musica ficta looks at what a singer would need to do both to avoid impure fifths and octaves and to create cadential progressions. This practice is described from the perspective of a singer—adding a mi or fa where there was none before. The notes that arise from this process are collected into a diagram (the double hand) that reflects usage (even though it does not provide an exhaustive list, as Ugolino admits) and that parallels the already familiar hand diagram for musica recta. In the theoretical examination of musica ficta in the *Tractatus*, Ugolino takes these data from practice and explains them through Pythagorean

theory. In theory, musica ficta means there is a seventeen-note octave, where each tone is divided unevenly, containing a major and a minor semitone separated by a comma. All these distances are calculated mathematically and demonstrated on a monochord. Each ficta syllable does have its own place. Therefore, practice and theory are united. Although the goal is indeed theory, the path there is through practice, since theory is contained within practice. Indeed, as Ugolino reiterates throughout the treatise, he adopts Aristotle's position that "in learning and in teaching we ought to proceed from things that are better known to us to those that are less known to us." (in cognoscendo et discendo debemus procedere ab his quae sunt nobis magis nota ad ea quae sunt nobis minus nota.)[436] And since "the knowledge of practice is more known to us than the knowledge of theory" (cuius praticae notitia magis est nobis speculatione nota), musica ficta is one step on the path to theoretical knowledge.[437]

[436] Ugolino, *Declaratio*, 3:87; Aristotle, *Physics* 1.1.184a18-19.

[437] Ugolino, *Declaratio*, 3:87.

CHAPTER 5

CONCLUSION

5.1 Summary

In his *Declaratio*, Ugolino places music as a discipline under natural philosophy instead of under mathematics. This placement and the philosophical foundation that supports it shapes his entire treatise. In particular, aligning music with natural philosophy affects the way practice relates to theory by elevating practice and making it essential for an understanding of theory. In chapter 2, I showed how Aristotle differentiates natural philosophy and mathematics. Natural philosophy studies objects of nature, while mathematics examines certain attributes of these objects. The objects of nature are visible, physical things that have a source of motion within themselves. Mathematical objects are attributes or properties that belong to natural objects. They are abstracted and considered independently from their objects. For example, a snub-nose is an object of nature, but its shape, concavity, is an object of mathematics. Because mathematical objects are abstracted from objects of nature, the existence of physical objects is prior to that of mathematical objects. According to Aristotle, the path of knowledge proceeds from the physical to the mathematical. Knowledge of material reality is indispensable for theoretical knowledge.

In chapter 2, I also examined how medieval philsophers categorized music. By placing music under mathematics, medieval philosophers considered the objects of music to be primarily mathematical in nature. However, thinkers in the fourteenth century like Thomas Bradwardine and Walter Burley began closing the gap between mathematics and natural philosophy.This, in turn, led William of Ockham to posit a different method to relate one science to another based on the collection of conclusions the sciences demonstrate. Williams's method allows natural philosophy and mathematics to work together more comfortably. Building on these

developments and philosophical foundations, Ugolino places music under natural philosophy. Music belongs there because its demonstrations call on both natural philosophy and mathematics. Musical practice supplies the objects of music. They are analogous to the objects of nature. The objects of music theory, akin to mathematical objects, are abstracted from musical practice and explained by using mathematics. Most importantly, therefore, for Ugolino, the knowledge of musical practice is indispensable for music-theoretical knowledge. As a result, Ugolino organizes his treatise to begin with three books on musical practice, and he concludes with two books on musical theory. The close connection between theory and practice elevates musical practice.

In chapter 3, I investigated the distinctive practice of the *regola del grado.* This practice, documented in only four manuscripts, is a method for teaching how to create proper note-against-note counterpoint. I showed how Ugolino adapts, expands, and updates this tradition. He offers a clear presentation on classifying perfect and imperfect intervals. He bases his analysis of perfection on musical practice: thirds move to fifths or unisons and sixths move to octaves. With this observation in hand, he uses the octave as the measure to explain why some intervals are perfect and others imperfect. In addition, intervals involve motion (a topic of natural philosophy) in two respects: first, the intervals are sounds and sound requires motion; second, the movement of one interval to another involves motion. In other words, Ugolino treats intervals as objects of nature. By doing so, he reinforces the notion that music is part of natural philosophy, all while presenting a practical guide on forming counterpoint.

In chapter 4, I showed how Ugolino treats musica ficta. He views the notes of musica recta, outlined in book 1, as fixed intervallic relationships (without reference to a particular pitch frequency). However, in the course of making counterpoint, the notes of musica recta create

conflicts: they can still form a tritone, an interval not allowed in note-against-note counterpoint, and they cannot accommodate the notes required to perform cadential progressions (or perfections). For these reasons, musica ficta is necessary. I showed that Ugolino determines which notes he needs based on usage. For example, for a cadential progression to G, an F-sharp is needed. Ugolino always considers notes as belonging to a hexachord. So, F-sharp must belong to a hexachord that begins on D. In this way, he finds several other ficta hexachords and sets them out in a diagram—the double hand (*duplex manus*) diagrams. In the *Tractatus monochordi*, Ugolino takes these observations from musical practice and theorizes a scale that includes all the notes for both musica recta and musica ficta. The result is a scale with a seventeen-note octave. To reach this scale, however, Ugolino begins with musical practice and usage, and then explains the scale through mathematical calculation. The theory is based in the musical practice, since theoretical objects are abstracted from practice. In this way, Ugolino can create a treatise that is both valuable for its practical applications, allowing musicians to learn what they need, as well as theoretical, expanding the intellects of his readers.

5.2 Future Research

In this dissertation, I have only examined certain facets from book 2 of Ugolino's *Declaratio* that distinctly reflect his theoretical perspectives. However, the entire treatise remains to be examined. Book 1 presents many opportunities for invesitagation. Towards the end of book 1, Ugolino includes a large sampling of liturgical chants. These chants could be used to gain a more precise knowledge of the state of liturgical music. Book 3 offers a long, elaborate commentary on rhythmic theory, certainly useful for interpreting rhythmic notation of music from the period. Book 5, with its explorations of speculative topics, may include material that helps understand the development of both philosophical and humanist thought in the first half of

the century. Indeed, early fifteenth century music theory remains under-explored. The largest treatise on music theory from the first half of the fifteenth century, Ugolino's *Declaratio* is an invaluable resource.

Even book 2 of the *Declaratio* could be further studied. For example, how might Ugolino's comments on counterpoint aid the analysis of music from the early fifteenth century? Further research needs to be done on the extensions he makes to the *regola del grado* theory. More research could be done to compare Ugolino's division of the monochord in the *Tractatus monochordi* with those of his contemporaries, inclusing Prosdocimo. In short, Ugolino's treatise is very large, and the period in which it was written was pivotal in the history both of music theory and of western Europe. For these reasons, his work, and those of his contemporaries, derserves further research.

BIBLIOGRAPHY

Adamson, Peter. *Medieval Philosophy: A History of Philosophy Without Any Gaps*. Oxford: Oxford University Press, 2019.

Aquinas, Thomas. *The Division and Methods of the Sciences*. Translated and edited by Armand Maurer. Toronto: Pontifical Institute of Mediaeval Studies, 1986.

Aristotle. *The Complete Works of Aristotle: The Revised Oxford Translation*. Edited by Jonathan Barnes. 2 vols. Princeton, NJ: Princeton University Press, 1984.

———. *Aristotle's Physics: A Guided Study*. Translated by Joe Sachs. New Brunswick, NJ: Rutgers University Press, 1995.

———. *Aristotle's Politics: Translation, Introduction, and Glossary*. Translated by Joe Sachs. Newberryport, MA: Focus Publishing/R. Pullins Company, 2012.

———. *Aristotle's Metaphysics*. Translated by Joe Sachs. Santa Fe, NM: Green Lion Press, 2002.

———. *Nicomachean Ethics: Translation, Glossary, and Introductory Essay*. Translated by Joe Sachs. Indianapolis, IN: Focus, an imprint of Hackett Publishing Company, 2002.

———. *Aristotle's on the Soul and on Memory and Recollection*. Translated by Joe Sachs. Santa Fe, NM: Green Lion Press, 2004.

Atkinson, Charles M. *The Critical Nexus: Tone-System, Mode, and Notation in Early Medieval Music*. Oxford: Oxford University Press, 2009.

Augustine. *The City of God Against the Pagans.* Edited and translated by R.W. Dyson. Cambridge: Cambridge University Press, 1998.

Bent, Margaret. "Musica Recta and Musica Ficta." *Musica disciplina* 26 (1972): 73–100.

———. "Diatonic Ficta." *Early Music History* 4 (1984): 1–48.

———. "A Contemporary Perception of Early Fifteenth-Century Style; Bologna Q15 as a Document of Scribal Editorial Initiative." *Musica disciplina* 41 (1987): 183–201.

———. *Counterpoint, Composition, and Musica Ficta*. New York: Routledge, 2002.

———. "Ciconia, Prosdocimus, and the Working of Musical Grammar as Exemplified in O Felix Templum and O Padua," In *Johannes Ciconia: Musicien de la Transition*, edited by Philippe Vendrix, Tournhout: Brepols, 2003.

Berger, Karol. *Musica Ficta: Theories of Accidental Inflections in Vocal Polyphony From Marchetto Da Padova to Gioseffo Zarlino*. Cambridge: Cambridge University Press, 1987.

———. “The Expanding Universe of Musica Ficta in Theory From 1300-1550.” *Journal of Musicology* 4, no. 4 (2004): 410–30.

The Berkeley Manuscript. Translated by Oliver B. Ellsworth. Lincoln: University of Nebraska Press, 1984.

Blackburn, Bonnie J. “Music Theory and Musical Thinking After 1450,” In *Music as Concept and Practice in the Late Middle Ages*, edited by Reinhard Strohm and Bonnie J. Blackburn, Oxford: Oxford University Press, 2001.

Boethius, Ancius Manlius Severinus. *The Fundamentals of Music*. Translated by Calvin M. Bower. New Haven: Yale University Press, 1989.

Bonnie J. Blackburn, Edward E. Lowinsky, Clement A. Miller, eds. *A Correspondence of Renaissance Musicians*. Oxford: Clarendon Press, 1991.

Brand, Benjamin. “A Medieval Scholasticus and Renaissance Choirmaster: A Portrait of John Hothby At Lucca.” *Renaissance Quarterly* 63 (2010):

Busse Berger, Anna Maria. *Medieval Music and the Art of Memory*. Los Angeles: University of California Press, 2005.

Cable, Martin John. *“Cum Essem in Constantie…”: Raffaele Fulgosio and the Council of Constance 1414-1415*. Leiden: Brill, 2015.

Cadden, Joan. “The Organization of Knowledge: Disciplines and Practices,” In *The Cambridge History of Science*, edited by Michael H. Shank David C. Lindberg, Cambridge: Cambridge University Press, 2013.

Ciconia, Johannes. *Nova Musica and De Proportionibus.* Translated by Oliver B. Ellsworth. Licoln: University of Nebraska Press, 1993.

Cohen, David. ““The Imperfect Seeks Its Perfection”: Harmonic Progression, Directed Motion, and Aristotelian Physics.” *Music Theory Spectrum* 23, no. 2 (2001): 139–69.

Copleston, Frederick. *Greece and Rome from the Pre-Socratics to Plotinus.* Vol. 1 of *A History of Philosophy*. New York: Doubleday, 1946.

———. *Medieval Philosophy*. Vol 2 of *A History of Philosophy.* New York: Doublday, 1950.

———. *Ockham to Suárez.* Vol. 3 of *A History of Philosophy.* Westminster, MD: Newman Press, 1953.

Desmond, Karen. “Behind the Mirror: Revealing the Contexts of Jacobus’s Speculum musicae,” PhD diss., New York University, 2009.

Dod, Bernard G. “Aristoteles Latinus,” In *The Cambridge History of Later Medieval Philosophy: From the Rediscovery of Aristotle to the Disintegration of Scholasticism 1100-1600*, edited by Anthony Kenny Norman Kretzmann and Jan Pinborg, 45–79. 1982.

Dyer, Joseph. “The Place of *Musica* in Medieval Classifications of Knowledge.” *The Journal of Musicology* 24, no. 1 (2007): 3–71.

Ellsworth, Oliver. “The Origin of the Coniuncta: A Reappraisal.” *Journal of Music Theory* 17, no. 1 (Spring 1973): 86-109.

van Engen, John. “Multiple Options: The World of the Fifteenth-Century Church.” *Church History* 77, no. 2 (2008): 257-84.

Fage, Adrien de la. *Essais De Diphthérographie Musicale*. Paris: Au Magasin de Musique du Bazar de l’Industrie, 1864.

Fallows, David. “The End of the Ars Subtilior.” *Basler Jahrbuch für Historische Musikpraxis* 20 (1996): 21–40.

——— and Bonnie J. Blackburn. “Ugolino of Orvieto.” *Grove Music Online.* 2001. Accessed March 7, 2021. https://doi-org.libproxy.library.unt.edu/10.1093/gmo/9781561592630.article.40462.

Ferraresi, Guerrino. *Il Beato Giovanni Tavelli da Tossignano e la riforma di Ferrara nel Quattrocento.* Ferrara: Morcelliana, 1969.

FitzGerald, Brian D. “Medieval Theories of Education: Hugh of St Victor and John of Salisbury.” *Oxford Review of Education* 36, no. 5 (2010): 575–88.

Florea, Luminita. “The Monstrous Musical Body: Mythology and Surgery in Late Medieval Music Theory.” *Philobiblon* 18, no. 1 (2013): 127–60.

Fuller, Sarah. “On Sonority in Fourteenth-Century Polyphony: Some Preliminary Reflections.” *Journal of Music Theory* 30 (1986): 35–70.

———. “Tendencies and Resolutions: The Directed Progression is Ars Nova Music.” *Journal of Music Theory* 36, no. 2 (1992): 229–58.

Gaukroger, Stephen. “Aristotle on Intelligible Matter.” *Phronesis* 25, no. 2 (1980): 187–97.

Geanakoplos, Deno J. “The Council of Florence (1438-1439) and the Problem of Union between the Greek and Latin Churches.” *Church History* 24, no. 4 (1955): 324-46.

Gerald Christianson, Thomas M. Izbicki, and Christopher M. Bellitto, eds. *The Church, the Councils, and Reform: The Legacy of the Fifteenth Century*. Dexter, MI: Catholic University of America Press, 2008.

Goddu, André. “Connotative Concepts and Mathematics in Ockham’s Natural Philosophy.” *Vivarium* 31, no. 2 (1993): 106–39.

Grant, Edward. *Studies in Medieval Science and Natural Philosophy*. London: Variorum Reprints, 1981.

———. *A History of Natural Philosophy: From the Ancient World to the Nineteenth Century*. Cambridge: Cambridge University Press, 2007.

———. *The Nature of Natural Philosophy in the Late Middle Ages*. Washington D.C.: Catholic University of America Press, 2014.

Grendler, Paul F. *Schooling in Reniassance Italy: Literacy and Learning 1300-1600.* Baltimore: Johns Hopkins University Press, 1989.

———. *The Universities of the Italian Renaissance.* Baltimore: John Hopkins University Press, 2002.

Haines, John and Patricia DeWitt. “Johannes de Grocheio and Aristotelian Natural Philosophy.” *Early Music History* 27 (2008): 47-98.

Hannam, James. *The Genesis of Science: How the Christian Middle Ages Launched the Scientific Revolution*. Washington, DC: Regnery Publishing, Inc., 2011.

Harne, George A. “The Ends of Theory and Practice in the Speculum Musicae.” *Musica disciplina* 55 (2010): 5–32.

———. “Unstable Embodiments of Musical Theory and Practice in the *Speculum musicae*.” *Plainsong and Medieval Music* 21, no. 2 (2012): 113–36.

Hay, Denys. *The Church in Italy in the Fifteenth Century*. Cambridge: Cambridge University Press, 1977.

Heinzelmann, Sigrum. “John Hothby as Innovator: The Solmization System in La Calliopea Legale.” *Studi musicali* (2012): 353–96.

Hugh of St. Victor *The Didascalicon of Hugh of St. Victor*. Translated by Jerome Taylor. New York: Columbia University Press, 1961.

Hughes, Andrew. “Ugolino: The Monochord and Musica Ficta.” *Musica disciplina* 23 (1969): 21–39.

———. *Manuscript Accidentals: Ficta in Focus, 1350-1450*. Rome: American Institute of Musicology, 1972.

Hussey, Edward. “Aristotle on Mathematical Objects.” *Aperion* 24, no. 4 (1991): 105–33.

Jacobus Leodiensis. *Speculum musicae*. Edited by Roger Bragard. 7 vols. Rome: American Institute of Musicology, 1955-73.

Kallendorf, Craig W. ed. *Humanist Educational Treatises*. Cambridge, MA: Harvard University Press, 2002.

Kilwardby, Robert. *De Ortu Scientiarum*. Toronto: The Pontifical Institute of Medieval Studies, 1976.

Leff, Gordon. *William of Ockham: A Metamorphosis of Scholastic Discourse*. Manchester: Manchester University Press, 1975.

Lefleur, Claude, ed. *Quatre Introductions à la Philosophie au XIIIe Siècle: Textes Critiques et Étude Historique.* Montréal: Institut d'Études Médiévales, 1988.

Lennox, Mary Louise Gill and James G., eds. *Self-Motion: From Aristotle to Newton*. Princeton: Princeton University Press, 1994.

Lindberg, David C., ed. *Science in the Middle Ages*. Chicago: University of Chicago Press, 1978.

———. "On the Applicability of Mathematics to Nature: Roger Bacon and His Predecessors." *The British Journal for the History of Science* 15, no. 1 (1982): 3–25.

Lindley, Mark. "Pythagorean Intonation and the Rise of the Triad." *Royal Musical Association Research Chronicle* 16 (1980): 4–61.

Livesey, Steven J. "William of Ockham, the Subalternate Sciences, and Aristotle's Theory of Metabasis." *The British Journal for the History of Science* 18, no. 2 (1985): 127–45.

———. "The Oxford Calculatores, Quantification of Qualities, and Aristotle's Prohibition of Metabasis." Vivarium 24, no. 1 (1986): 50–69.

Lockwood, Lewis. *Music in Renaissance Ferrara 1400-1505: The Creation of a Musical Center in the Fifteenth Century*. Cambridge, MA: Harvard University Press, 1984.

MacCarthy, Evan A. "Music and Learning in Renaissance Ferrara, c. 1430-1470," PhD diss., Harvard University, 2010.

———. "The Sources and Early Readers of Ugolino of Orvieto's Declaratio Musice Discipline," In *Beyond 50 Years of Ars Nova Studies at Certaldo 1959-2009*, edited by Agostine Ziino Marco Gozzi, and Francesco Zimei, Lucca: Libreria Musicale Italiana, 2014.

———. "Transformations in Music Theory and Music Treatises," In *The Cambridge History of Fifteenth-Century Music*, edited by Anna Maria Busse Berger and Jesse Rodin, 602–14. Cambridge: Cambridge University Press, 2015.

Marchetto of Padua. *The Lucidarium of Marchetto of Padua: A Critical Edition, Translation, and Commentary.* Edited and translated by Jan Herlinger. Chicago: Chicago University Press, 1985.

Marenbon, John. *Later Medieval Philosophy (1150-1350): An Introduction.* London: Routledge, 1987.

Maurer, Armand. *The Philosophy of William of Ockham in the Light of Its Principles*. Toronto: Pontifical Institute of Mediaeval Studies, 2002.

Maw, David. "Redemption and Retrospection in Jacques De Liège's Concept of Cadentia." *Early Music History* 29 (2010): 79–118.

McCahill, Elizabeth. *Reviving the Eternal City: Rome and the Papal Court, 1420-1477.* Cambridge, MA: Harvard University Press, 2013.

McKirahan Jr. Richard D. "Aristotle's Subordinate Scienes." *The British Journal for the History of Science* 11, no. 3 (1978): 197–220.

McManus, Stuart. "Pagolo di Matteo Petriboni's Account of the Council of Florence." *Reformation and Renaissance Review* 10, no. 2 (2008): 247-63.

Meconi, Honey, ed. Medieval Music. Burlington, VT: Ashgate, 2011.

Mengozzi, Stefano. "The Heptachordal Basis of Hexachordal Theory: On the Semiotics of Musical Notation in the Middle Ages." *Plainsong and Medieval Music* 22, no. 2 (2013): 169-94.

———. *The Renaissance Reform of Medieval Music Theory: Guido of Arezzo Between Myth and History*. Cambridge: Cambridge University Press, 2010.

———. "'Si Quis Manus Non Habeat': Charting Non-Hexachordal Musical Practices in the Age of Solmisation." *Early Music History* 26 (2007): 181–218.

———. "Virtual Segments: The Hexachordal System in the Late Middle Ages." *The Journal of Musicology* 23, no. 3 (2006): 426–67.

Miller, Clement A. "Early Gaffuriana: New Answers to Old Questions." *The Musical Quarterly* 56, no. 3 (1970): 367-88.

Mixon, James D. "Religious Life and Observant Reform in the Fifteenth Century." *Historical Compass* 11, no. 3 (2013): 201-14.

Mladenovic, Petr z. *John Hus At the Council of Constance*. Translated by Matthew Spinka. New York: Columbia University Press, 1965.

Moyer, Ann E. *Musica Scientia: Musical Scholarship in the Italian Renaissance*. Ithaca, NY: Cornell University Press, 1992.

Muir, James R. *The Legacy of Isocrates and a Platonic Alternative: Political Philosophy and the Value of Education*. New York: Routledge, 2018.

Murdoch, Edward Grant and John E., eds. *Mathematics and Its Applications to Science and Natural Philosophy in the Middle Ages: Essays in Honor of Marshall Clagett.* Cambridge: Cambridge University Press, 1987.

Ockham, William of. *Philosophical Writings: A Selection*. Translated by Philotheus Boehner O.F.M. Indianapolis: The Bobbs-Merrill Company, Inc., 1964.

O'Reilly, Paul. "What is Intelligible Matter?" *The Thomist: A Speculative Quarterly Review* 53, no. 1 (1989): 74–90.

Page, Christopher. "Musicus and Cantor." In *Companion to Medieval and Renaissance Music*, edited by Tess Knighton and David Fallows, 74-8. New York: Schirmer Books, 1992.

Panti, Cecilia. "Una Fonte Della Declaratio Musicae Disciplinae Di Ugolino Da Orvieto: Quattro Anonime 'Questiones' Della Tarda Scholastica." *Rivista italiana di musicologia* 24 (1989): 3–47.

Piccolomini, Aeneas. "The Education of Boys." In *Kallendorf, Humanist Educational Treatises*, 126-259.

Pietzsch, Gerhard. *Die Klassifikation Der Musik Von Boethius Bis Ugolino Von Orvieto*. Halle: Max Niemeyer, 1929.

Pirrotta, Nino. "Music and Cultural Tendencies in 15th-Century Italy*." Jounral of the American Musicological Society* 19, no. 2 (1966): 127–61.

———. *Music and Culture in Italy From the Middle Ages to Baroque: A Collection of Essays*. Cambridge, MA: Harvard University Press, 1984.

Prosdocimo de' Beldomandi. *Contrapunctus*. Translated by Jan Herlinger. Lincoln: University of Nebraska Press, 1984.

Ramos de Pareja, Bartolomé. *Musica Practica*. Translated by Clement A. Miller. Neuhausen-Stuttgart: American Institute of Musicology, 1993.

Reaney, Gilbert, ed. *Johannes Hothby: De Arte Contrapuncti*. Neuhausen-Stuttgart: American Institute of Musicology, 1977.

Sachs, Klaus-Jürgen. *Der Contrapunctus Im 14. Und 15. Jahrhundert: Untersuchungen Zum Terminus, Zur Lehre Und Zu Den Quellen*. Wiesbaden: Franz Steiner, 1974.

Sanders, Ernest H. "'Are My Ears Wrong?': Hearing Polyphony, Ca. 1375-Ca. 1450." *Current Musicology* 75 (2003): 51–78.

Scattolin, Pier Paolo. "La Regola Del 'Grado' Nella Teoria Medieval Del Contrappunto." *Rivista italiana di musicologia* 14, no. 1 (1979): 11–74.

Schmitt, Charles B. *Aristotle and the Renaissance.* Cambridge, MA: Harvard University Press, 1983.

Schueller, Herbert M. *The Idea of Music: An Introduction to Musical Aesthetics in Antiquity and the Middle Ages*. Kalamazoo, MI: Medieval Institute Publications, Western Michigan University, 1988.

Seay, Albert. "Ugolino of Orvieto, Theorist and Composer." *Musica disciplina* 9 (1955): 111–66.

Seay, Albert, ed. *Quatuor Tractatuli Italici De Contrapuncto*. Colorado Springs: Colorado College Music Press, 1977.

Stump, Phillip H. *The Reforms of the Council of Constance (1414-1418)*. Leiden: E.J. Brill, 1994.

Thorp, John. "Intelligible Matter in Aristotle." *The Society for Ancient Greek Philosophy Newsletter* 385 (2010):

Tinctoris, Johannes. *Dictionary of Musical Terms*. Translated by Carl Parrish. London: The Free Press of Glencoe, 1963.

Ugolino of Orvieto. *Declaratio musicae disciplinae*. Edited by Albert Seay. 3 vols. Rome: American Institute of Musicology, 1959-62.

Vergerio, Pier Paolo. "The Character and Studies Befitting a Free-Born Youth," In *Humanist Educational Treatises*, edited by Craig W. Kallendorf, Cambridge, MA: Harvard University Press, 2002.

Wegman, Rob C. "From Maker to Composer: Improvisation and Musical Authorship in the Low Countries, 1450-1500." *Journal of the American Musicological Society* 49, no. 3 (1996): 409–79.

———. "'Musical Understanding' in the 15th Century." *Early Music* 30, no. 1 (2002): 46–66.

Weisheipl O.P., James A. "The Nature, Scope, and Classification of the Sciences," In *Science in the Middle Ages*, edited by David C. Lindberg, Chicago: The University of Chicago Press, 1978.

Whitney, Elspeth. *Medieval Science and Technology*. Westport, CT: Greenwood Press, 2004.

Woody, John Hine Mundy & Kennerly M., eds. *The Council of Constance: The Unification of the Church*. Translated by Louise Ropes Loomis. New York: Columbia University Press, 1961.

Wylie, James Hamilton. The Council of Constance to the Death of John Hus. London: Longmans, 1900.

Yudkin, Jeremy. “The Influence of Aristotle on French University Music Texts,” In *Music Theory and its Sources: Antiquity and the Middle Ages*, edited by André Barbera, 173–89. Notre Dame, IN: University of Notre Dame Press, 1990.

Zager, Daniel. “From the Singer’s Point of View: A Case Study in Hexachordal Solmization as a Guide to ‘Musica Recta’ and ‘Musica Ficta’ in Fifteenth-Century Vocal Music.” *Current Musicology* 43 (1987): 7–21.