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THE EFFECTS OF INTERNAL CONVERSION ON CRITICAL
MASS AND OPERATING LIFE OF A THERMAL REACTOR

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THE EFFECTS OF INTERNAL CONVERSION ON CRITICAL
MASS AND OPERATING LIFE OF A THERMAL REACTOR

The purpose of the following discussion is to make rough predictions of the expected effects of internal conversion on the critical mass and reloading period of thermal reactors. These rough predictions will indicate the desirability of a more careful investigation.

For the analysis, the following approximations are made:

1. The reactor is bare.
2. The spectrum is thermal.
3. The volume of the reactor is occupied primarily by moderator, coolant and structural material. It may therefore be assumed that the atomic densities, other than those of fertile and fissionable material, and the transport cross section, which determines leakage, are independent of the internal conversion ratio.

The criticality relationship at startup is then:

\[ \sigma_f = \frac{\pi^2/3a^2 \sigma_{tr} + \sigma_a}{\gamma - (1 + \alpha)(1 + R)} \]

where \( \sigma_f \), \( \sigma_a \), and \( \sigma_{tr} \) are the reciprocal mean free paths for fission, absorption by reactor components other than fuel and fertile material and transport, \( a \) is the core radius, \( \gamma \) is the number of neutrons per fission, \( \alpha \) the capture to fission ratio, and \( R \) is the internal conversion ratio.

The reciprocal mean free paths are related to atomic cross sections \( \Sigma \) and atomic densities \( N \) by:

\[ \sigma = SN \]

The reactivity will change as U-235 is consumed, so that at a time when the density of fuel atoms is 125 the reactivity is given by:

\[ k = \frac{(\gamma S)_{25} H_{25} + (\gamma S)_{19} H_{19}}{[SN(1 + \alpha)]_{25} + \sigma_b + \sigma_a + \pi^2/3 \sigma_{tr} a^2 + [SN(1 + \alpha)]_{19}} \]
where \( b \) refers to fertile material. The rate of reactivity change through loss of U-235 and production of new fuel is given initially by:

\[
\frac{dk}{dn} = \frac{S_{25} \left[(\nu - 1 - \alpha)_{25} - S_{19} \left[(\nu - 1 - \alpha)_{19} R \right] \right]}{\nu \cdot n_{25}}
\]

(since initially \( k = 1 \)). It is assumed that changes in \( \Sigma^b \) and \( \sigma_{\text{tr}} \) can be neglected.

These relations are in a convenient form to determine the effects of conversion on critical mass and reactivity change with burnup. The reloading period is roughly proportional to \( \frac{dn_{25}}{dk} \). If \( R = 0 \),

\[
\frac{H_{25}(0)}{H(0)} = \left( \frac{\nu - (1 + \alpha)}{\nu - (1 + \alpha)(1 + R)} \right)
\]

hence,

\[
\frac{H(R)}{H(0)} = \frac{\nu - (1 + \alpha)}{\nu - (1 + \alpha)(1 + R)}
\]

Also, if \( R = 0 \),

\[
\left( \frac{dk}{dn} \right)_0 = \frac{\left[(\nu - (1 + \alpha)_{25} S_{25} \right]}{\nu \cdot n_{25}}
\]

hence,

\[
\left( \frac{dn_{25}}{dk} \right)_0 = \frac{n_{25}(R)}{n_{25}(0)} \left\{ \frac{\left[(\nu - 1 - \alpha)_{25} S_{25} \right]}{\left[(\nu - 1 - \alpha)_{25} S_{25} \right] - \left[(\nu - 1 - \alpha)S_{19} \right]_{19} R} \right\}
\]

or

\[
\frac{\left[(\nu - 1 - \alpha)_{25} S_{25} \right]}{\left[(\nu - 1 - \alpha)_{25} S_{25} \right] - \left[(\nu - 1 - \alpha)S_{19} \right]_{19} R} \]

\[
= \frac{\left[\nu - (1 + \alpha)_{25} S_{25} \right]}{\left[\nu - (1 + \alpha)_{25} S_{25} \right] - \left[(\nu - 1 - \alpha)S_{19} \right]_{19} R}
\]
This relationship will apply to U-233 conversion if \[ (\gamma - 1 - \alpha \cdot \beta)S \] is replaced by \[ (\gamma - 1 - \alpha \cdot \beta)S_{23} \]. It is clear that for a given R the longest period is obtained when this quantity is large. The following tabulation of constants shows that for a fixed reactivity change Pu-239 conversion gives much greater burnup than U-233 conversion.

<table>
<thead>
<tr>
<th></th>
<th>( S_{23} ) (barns)</th>
<th>( (\gamma - 1 - \alpha \cdot \beta)S_{23} ) (barns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>U-233</td>
<td>518</td>
<td>780</td>
</tr>
<tr>
<td>Pu-239</td>
<td>950</td>
<td>1,225</td>
</tr>
<tr>
<td>Pu-235</td>
<td>568</td>
<td>755</td>
</tr>
</tbody>
</table>

When the constants from this table are applied to the Pu-239 conversion case, relation 8 becomes:

\[
\left( \frac{d\Pi_{239}/dk}{d\Pi_{239}/dk}_0 \right) = \frac{1}{1 - 2.78 R + 1.68 R^2}
\]

This relation is tabulated as follows:

<table>
<thead>
<tr>
<th>R</th>
<th>( (d\Pi/dk)_R/(d\Pi/dk)_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>.1</td>
<td>1.36</td>
</tr>
<tr>
<td>.2</td>
<td>1.93</td>
</tr>
<tr>
<td>.3</td>
<td>3.06</td>
</tr>
<tr>
<td>.4</td>
<td>5.8</td>
</tr>
</tbody>
</table>

The relation between R and fuel inventory, equation (6), is also tabulated:

<table>
<thead>
<tr>
<th>R</th>
<th>( (N/N_0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>.1</td>
<td>1.09</td>
</tr>
<tr>
<td>.2</td>
<td>1.36</td>
</tr>
<tr>
<td>.3</td>
<td>1.81</td>
</tr>
<tr>
<td>.4</td>
<td>2.63</td>
</tr>
</tbody>
</table>

The simultaneous relationship between reactor life and fuel inventory is given in the accompanying graph.
It is concluded from this graph that with a thermal reactor, a substantial increase in the reloading period can be obtained with only a modest increase of U-235 inventory.