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AECD-4089

UNITED STATES ATOMIC ENERGY COMMISSION

**PILE CONTROL IV
NUCLEAR ENGINEERING COURSE 1947 - 1948**

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May 13, 1948

Date Declassified: January 10, 1956

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Schenectady, New York**

Technical Information Service, Oak Ridge, Tennessee

Photostat Price \$ 3.30
Microfilm Price \$ 2.40

Available from the
Office of Technical Services
Department of Commerce
Washington 25, D. C.

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Pile Control IV

F. E. Crever

Reasons for Control

The need for control of an atomic power pile is obvious. The operators should, at any time, be able to bring the multiplication to unity or to decrease it below unity to shut down the self-multiplying medium. This provides for steady state and shut-down operation. For power level control short time increase and decrease on each side of unity in reactivity is needed and must be closely controlled to prevent running away or shut down of the pile. Constant power level is not a single valued function of control rod position. Depletion of fuel and accumulation of poisons tend to decrease reactivity, fissionable material bred in the pile increases reactivity and temperature changes can either increase or decrease it.

Types of Control

Control can be conveniently divided into four classifications depending on the type of change each controls.

Fine control = controls short time variations.

Shim control = larger in amount than fine control. It controls long time variations such as burn-up and accumulation of poison and temperature reactivity changes from start-up to final temperature.

- Safety** - Used to shut down pile. Operates automatically if the pile tends to get away.
- Extra Safety** - Used in new types of piles where little is known. An added precaution if something goes wrong with the normal safety so it is designed to be actuated by some other method. It should fail safely to take care of unforeseen conditions.

Methods of control varies with the type of pile. Possible means of varying the reactivity are listed as follows:

<u>Thermal</u>	<u>Intermediate</u>	<u>Fast</u>
Absorbing Material	Absorbing material	Reflector Variation
Reflector variation	Reflector variation	Fuel removal
Moderator removal	Moderator removal	
Fuel removal	Fuel removal	

In the thermal pile adding absorbers is used since certain materials are extremely effective absorbers in that region. Movable fuel rods would be effective but engineering problems, such as cooling the rods, make this means difficult.

Absorbers are not as effective in the intermediate range as at thermal, but they could be used for safety or for extra safety. They are objectionable in a breeder pile from a neutron economy standpoint. Removable fuel and moderator are possible, but lead to difficult construction problems.

Reflector variation shows the most promise and will be tried on the Schenectady pile.

Absorbers are ineffective in the fast energy range, no accelerator exists, and fuel movement is difficult, leaving reflector variation as the one practical means of control.

Thermal pile controls are well known through practice, intermediate and fast by theory that will soon be checked.

Reactivity

The question comes up of how much reactivity to associate with each type of control. Safeties must be able to shut down the pile and keep it down at any point on a decay curve of fission products. Shim rods must control reactivity due to burn-up, fission product poisons and temperature change. Fine control need not be great. It can be about the same as the delayed neutron fraction, or 0.6% in the Schenectady pile. Fine control should take care of slight temperature variations of the coolant and other minute to minute changes.

When the pile is shut down from a given power level (P_0), the decrease in power would not be instantaneous even if instantaneous change in reactivity were possible. At steady power, about 93.35% of the power is due to energy of fission fragments and 6.65% to energy from beta and gamma decay. The initial change in fission power takes place in milliseconds and can be given approximately by $P = P_0 \left[\frac{\Delta K}{\beta - \Delta K} \right]$ where ΔK is a minus quantity for decrease and β is the fraction of delayed neutrons ($\beta = .0073$ for U^{235}). The remainder of the energy due to fission decays according to exponential law depending on the amount of

change of reactivity, as shown in curve 1. If the fractional excess β is large enough, over -0.10 , the fission power will drop almost immediately and total power level will follow a beta and gamma decay curve given by

$$P = 0.0665 P_0 e^{-\lambda t}$$

Since the beta and gamma curve is relatively unaffected by increasing the change in reactivity, very little is gained by going over a 10% effective reactivity. The safety rods of an intermediate pile should then contain about 10% ΔK . Four percent is sufficient for large thermal piles.

Shim control is of great importance in thermal piles since xenon fission product poison, discussed in the last Pile Control lecture, causes large changes in reactivity. Xenon is assumed to be no problem in intermediate and fast pile. For fast, high temperature piles there is a worry about phase change in metals and shim control should be able to take care of this. It is expected that 2.5% to 3.5% ΔK in shim control will be enough for an intermediate pile.

All of the various control rods are subject to heating due to absorption of gamma rays and fission particles, which must not injure the rods and resolves itself into a heat transfer problem.

Power Level Control

Power level is the variable to be controlled once the pile is critical. As discussed in previous pile control lectures, neutron flux or neutron density are measures of power level and can be used to indicate instantaneous

Curve #1

Residual Power Subsequent
to Shutdown - Instantly Applied
Charge - Includes Friction, etc.

% Residual Power

12
10
8
6
4
2
0

Seconds

values needed for rapid variation in control. Calibrated ionization chambers with proper amplification can be used to guide an operator or be tied automatically to the control actuating system. Measurement of power level by temperature and flow readings is probably more accurate than the ionization chambers and can be used to check the ionization chambers, but this is too slow to keep up with changes in the pile. On piles built to date, controls have been operated manually, trusting the intelligence of the operators with indicating instruments showing the condition of the pile as determined by ionization chambers.

An important factor which determines time available for operation of safety controls is the average life of the neutron from fission to capture, called the generation time (ℓ). This time is inversely proportional to both the average velocity of the neutron and the capture cross-section of the material in the pile. The range of velocity between fast and thermal piles is so great that velocity is the determining factor in the way generation time varies. Consequently, the thermal pile has the largest generation time. Measured generation time (ℓ) for thermal piles is about 10^{-3} sec and for fast piles is approximately 10^{-7} sec. The intermediate pile value is still unknown, but estimates are from 10^{-5} to 10^{-6} sec. As long as the fraction of excess $K(\Delta K)$ is less than the fraction of delayed neutrons (β) the pile is controllable with the delayed neutrons and the generation time is unimportant. When $\Delta K = \beta$ (prompt critical) or when $\Delta K > \beta$ the generation time becomes important. Assuming the delayed neutrons do not have time to change at prompt critical, the pile kinetics equations, discussed in previous Pile Control lectures,

are helpful in understanding the effect of generation time on different piles.

$$\frac{dn}{dt} = \frac{\Delta k - \beta}{\ell} n + \sum \lambda_i c_i \quad (1)$$

$$\frac{dc_i}{dt} = \frac{\beta_i n}{\ell} - \lambda_i c_i \quad (2)$$

where

$n(t)$ = av. neutron density

$c(t)$ = av. density of "pregnant" nuclei emitting
delayed neutrons

ℓ = prompt generation time

λ_i = decay constant of pregnant nuclei of type i

Let:

$$n = n_0 + n'$$

$$c_i = c_{i0} + c_i'$$

Assume that for the short time in which the safeties operate that the c_i will remain constant at c_{i0} .

Then:

$$\frac{dc_1}{dt} = 0 = \frac{\beta_1 N_0}{\ell} - \lambda_1 c_{10}$$

$$\lambda_1 c_{10} = \frac{\beta_1 N_0}{\ell}$$

$$\sum \lambda_1 c_{10} = \frac{\beta}{\ell} N_0 \quad (3)$$

$$\frac{dn'}{dt} = \frac{\Delta K - \beta}{\ell} N_0 + \frac{\Delta K - \beta}{\ell} n' - \frac{\beta}{\ell} N_0$$

$$\frac{dn'}{dt} = \frac{\Delta K N_0}{\ell} + \frac{\Delta K - \beta n'}{\ell} \quad (4)$$

The solution of the homogeneous equation is obtained by separation of variables and integration, which gives

$$\ln n' = \left(\frac{\Delta K - \beta}{\ell} \right) t + C$$

or

$$n' = A e^{\frac{\Delta K - \beta}{\ell} t} \quad (5)$$

The particular integral is by inspection

$$n' = \frac{\Delta K N_0}{\beta - \Delta K}$$

The complete solution is

$$n' = \frac{\Delta K N_0}{\beta - \Delta K} + A e^{-\frac{\Delta K - \beta}{\ell} t} \quad (6)$$

When:

$$t = 0 \quad n' = 0$$

And:

$$A = -\frac{\Delta K N_0}{\beta - \Delta K}$$

Then:

$$n' = \frac{\Delta K N_0}{\beta - \Delta K} \left(1 - e^{-\frac{\Delta K - \beta}{\ell} t} \right) \quad (7)$$

When $\Delta K = \beta$ we expand the exponential which, when $\frac{\Delta K - \beta}{\ell}$ is very small is

$$e^{-\frac{\Delta K - \beta}{\ell} t} = 1 + \frac{\Delta K - \beta}{\ell} t \quad (8)$$

and

$$n' = \frac{\Delta K N_0}{\beta - \Delta K} \left(1 - 1 - \frac{\Delta K - \beta}{\ell} t \right)$$

$$n' = \frac{\beta N_0}{\ell} t$$

or

$$\frac{N}{N_0} = \frac{\beta}{\ell} t + 1 \quad (9)$$

To estimate the power rise in a given pile at prompt critical, let us say that acceleration can be applied to the safeties to operate in .025 sec. For U^{235} β is .0073 and ℓ is given above.

$$\begin{array}{ll} \text{thermal} & \\ (\ell = 10^{-3}) & \frac{N}{N_0} = 7.3t + 1 = 1.18 \end{array}$$

$$\begin{array}{ll} \text{Intermediate} & \\ (\ell = 10^{-5}) & \frac{N}{N_0} = 730t + 1 = 19.25 \end{array}$$

$$\begin{array}{ll} \text{Intermediate} & \\ (\ell = 10^{-6}) & \frac{N}{N_0} = 7,300t + 1 = 183.5 \end{array}$$

$$\begin{array}{ll} \text{Fast} & \\ & \frac{N}{N_0} = 73,000t + 1 = 1826 \end{array}$$

The generation time is long enough in the thermal region so that even though prompt criticality is reached, the safeties are adequate to prevent damage. In the intermediate range such operation is questionable, particularly if the generation time turns out to be 10^{-6} sec, and in the fast region the safeties probably could not prevent damage.

In order to prevent prompt criticality from being reached during start-up from low levels the rate of increase in reactivity must be limited. The limit of this rate of change will depend on design of the reactor which determines maximum temperature the reactor can stand without damage.

The xenon transient in thermal piles requires a total change in reactivity higher than dictated by other calculations. As shown above, approaching or exceeding prompt critical is not serious in this case. On the other hand, in intermediate piles xenon is not a problem and approaching prompt critical is serious so rate of change in reactivity is carefully limited. The intermediate pile is expected to overcome these two difficulties but means of measuring rate of change is complicated so it is best to make changes at a slow rate which can be calculated to be safe. In the uranium intermediate pile, prompt critical is reached at about 0.7% Δk . Temperature change from room temperature to expected Schenectady operating temperatures can change the reactivity by -2.5%. If enough control is built into the pile to compensate for this change in reactivity, there will be sufficient reactivity in the controls to exceed prompt critical when the pile is cold. This problem can be avoided by preheating the pile to operating temperature but this is too troublesome and the best solution appears to be to build enough reactivity into the controls with a limit on the rate of increase which will keep the excess reactivity within safe limits.

Control Rod - Effectiveness

The effect of individual control rods on the pile as a whole can be estimated with the help of a corrected diffusion equation. This theory holds best at thermal ranges.

$$\nabla^2 n = \frac{(k-1)}{(M^2)} n = \frac{dn}{dt} \quad (10)$$

n = neutron density
 k = pile reactivity, about equal to 1
 M^2 = migration area, M is a measure of the distance the neutron travels including slowing down and diffusion so that

$$M^2 = L_g^2 + T$$

The quantity $\frac{k-1}{M^2}$ is called the "Buckling" and denoted by B . $B(r)$ is a measure of the reactivity of the pile and varies with radius depending upon distribution of absorbing material.

An average K for a given setup can be found if an average $\bar{B}$ is known. To find $\bar{B}$ from equation (10) redefine the neutron density in terms of $\bar{B}$ and assume steady state so that $\frac{dn}{dt} = 0$.

$$\nabla^2 n + B(r) n = 0 \quad (11)$$

$$\nabla^2 n^1 + \bar{B} n^1 = 0 \quad (12)$$

In the first equation $B(r)$ is a function of position. In the second, $\bar{B}$ is an average value, which is not a function of position. The neutron densities n and n^1 are subject to the boundary conditions that each is zero at the extrapolation distance from the pile boundary. Then, if the difference between n and n^1 is small,

$$n^1 = n + \Delta n$$

$$B(r) = \bar{B} + \Delta B$$

(13)

From (11) and (12):

$$\int_V [n^1 \nabla^2 n + n^1 B(r) n] dv = 0 \quad (14)$$

$$\int_V [n \nabla^2 n^1 + n \bar{B} n^1] dv = 0 \quad (15)$$

Subtract (14) from (15):

$$\int_V [n \nabla^2 n^1 - n^1 \nabla^2 n] dv + \int_V [n n^1 - B(r) n^1 n] dv = 0 \quad (16)$$

Change the first term to a surface integral by Green's theory:

$$\int_V [n \nabla^2 n^1 - n^1 \nabla^2 n] dv = \int_S [n \nabla n^1 - n^1 \nabla n] ds \quad (17)$$

at the boundaries $n = n^1 = 0$ $\therefore$ first term equals 0.

From (16):

$$\int_V n n^1 [\bar{B} - B(r)] dv = 0 \quad (18)$$

From (13):

$$0 = \int_V [n (n + \Delta n) \Delta \bar{B}] dv = \int_V [n^2 \Delta \bar{B} + n \Delta n \Delta \bar{B}] dv = 0 \quad (19)$$

From (19) and (13), neglect $(n \Delta n \Delta B)$

$$\int_V n^2 B(r) dv = \int_V n^2 \bar{B} dv$$

or

$$\bar{B} = \frac{\int_V n^2 B(r) dv}{\int_V n^2 dv} \quad (20)$$

Equation (20) is subject to the geometrical interpretation that the weight to be given to any local element, such as a control rod, varies as the square of the neutron density.

A control interpretation can also be given if we let $B(r)$ represent the buckling before rod insertion and $\bar{B}$ represent the buckling after insertion. Then equation (18) can be reduced to:

$$\int_V n^2 [\bar{B} - B(r)] dv = 0$$

This shows that a slight increase in reactivity over the whole volume of the pile may be compensated by a larger decrease in reactivity over a limited portion of the pile.

Using the solid line for a critical pile and the dotted line for a critical pile with an absorber rod in the center, the diagram at the right shows how leakage at the boundaries is increased when rods are inserted. The area under the curve should stay the same so the

slope at the boundaries must increase and leakage there increases. The k^2 effect comes from this boundary leakage plus absorption proportional to ϕ into the rod. The effect of control rods are solvable by mathematics, but the differential equations are difficult unless insertion of the rods is symmetrical. For thermal piles the "principle of equivalence" is used with accurate results for small variations in $\bar{B}$.

"Principle of Equivalence"

The absorbent together with the material dispersed symmetrically about it is equivalent to a homogeneous multiplying media of the same volume with a new multiplication factor and a new migration area.

Limitations on the equivalent zone

The size of the equivalent zone is:

1. Large compared to the rod
2. Large compared to a lattice unit
3. Small compared to the whole pile

The first two insure the validity of approximation to a homogeneous mass and the latter that variation is small. Calculation is then done in two parts. First, a zone assumed and a new $\frac{k-1}{M^2}$ is found for the zone. Then the effect of the zone in the pile is found by equation (20), the

n^2 weighting.

Control Rod - Multiple Rod Effect

Shadowing takes place when absorbing control rods are operated near each other, since neither operates in the full flux. If the neutron flux is considerably altered by this effect, it is necessary to solve difficult differential equations to predict B.

A flat neutron distribution is desirable for power operation and this can be achieved by spreading the absorber over the central zone. If the dotted line in the center of the diagram represents the desired critical curve, $\frac{k-1}{M^2}$, shown by the solid line, is higher than critical. If in the center portion the buckling is so that there is no loss to the outside portion, this part can be omitted and the critical size found for the remainder.

Intermediate Pile Control

Control effectiveness calculations for intermediate piles are made with multi-group calculations. The effect of moving the reflector is found by assuming a spherical geometry, making the calculations with the complete Be reflector and then repeating them with a reflector of reduced density. For a 25 cm pile and a 7 cm reflector, a reduction in density of 25% gives an 8% reduction in reactivity.

The effect of scattering material varies roughly as the fourth power of distance from the source or center of the reactor. Intensity varies as the square of distance and neutrons may be considered to be scattered back toward the source resulting in another squared factor. This law can be used to compare known conditions with unknown such as changes in diameter of the reactor.

The proposed reactor has a hexagonal shape as shown in Figure 1. Calculations are made for spherical geometries and the problem then becomes one of find the effect of a given area removed in terms of spherical geometry. The first approximation is to consider the reactor as a cylinder, as shown in the diagram at the right. The problem then is to compare this case with the known sphere mentioned above. Let F equal the effectiveness of a unit volume of Be at a unit radius so that

$$\text{Control Effectiveness} = \int_V \frac{F dV}{r^4} \quad (21)$$

where V = volume removed. For a sphere $dV = 4\pi r^2 dr$

Then for known case:

$$S = 4\pi F \left(\frac{1}{r_1} - \frac{1}{r_2} \right) .25$$

$$F = 115$$

r_1 = outer radius reflector

r_2 = inner radius reflector

For a cylinder:

$$\Lambda\left(\frac{V}{r^4}\right) = \int_{r_0}^{r_1} 2\pi r dr \int_0^{\frac{h}{2}} \frac{dx}{(\sqrt{r^2 + x^2})^4}$$

and this integrates to

$$\begin{aligned} \text{Eff} &= 2\pi F \left[\frac{1}{r_0} \tan^{-1} \frac{h}{2r_0} - \frac{1}{r_1} \tan^{-1} \frac{h}{2r_1} \right] \\ &= 19\% \text{ for complete removal of the vertical sides.} \end{aligned} \quad (22)$$

To find the effect of partial removal of the sides, equation (21) can be worked out as a function of (r) and plotted on a curve. The critical experiment can be used to check this method of calculation.

Figure 1 is a possible arrangement of movable control rods for a hexagonal shaped pile. The four pairs of safety rods contain $2.34\% \frac{\Delta K}{K}$ per pair, the four shims are $0.6\% \frac{\Delta K}{K}$ per unit and fine control totals 0.60%. Absorber sheets are inserted for extra safety only and are not used for control. The total change of reactivity used in this scheme is 12.4% compared to a possible 19% calculated above.

END

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