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MATHEMATICS PANEL QUARTERLY PROGRESS REPORT FOR PERIOD ENDING JANUARY 31, 1953

By

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MATHEMATICS PANEL QUARTERLY PROGRESS REPORT

SUMMARY

There were two terminations from the Mathematics Panel during the quarter: K. P. Graw and C. L. Perry; the latter left to organize a computing group at the U. S. Naval Postgraduate School, Monterey, California.

J. Z. Hearon, formerly of the Department of Physiology, University of Chicago, and a consultant for the Mathematics Panel, has joined the Panel as a regular employee. He holds a Ph.D. in biochemistry from the University of Michigan and one in mathematical biology from the University of Chicago.

The NEPA Linear Equation Solver is in operation and is being used for a variety of problems: matrix inversion, ordinary linear differential equations, and eigenvalues. Some of these are described in this report.

The individual memory units of the ORACLE have been tested and assembled, and the memory and the arithmetic unit are being joined together. It is hoped that test problems can be started within the next month or so.

A series of ten lectures on programming for the ORACLE was given at ORNL during the two weeks beginning January 12. Speakers and topics were as follows:

W. C. Sangren, *Automatic Digital Computation*, January 12.

C. L. Perry, *Electronic Digital Computers*, January 13.

C. L. Perry, *Logical Structure of the ORACLE*, January 14.

A. S. Householder, *Operations and Arithmetic of the ORACLE*, January 15 and 16.

J. Moshman, *Flow Charts and Programming*, January 19 and 20.

N. M. Dismuke, *An Example of a Complete Routine*, January 21.

M. R. Arnette, *Use and Construction of Subroutines*, January 22.

C. L. Perry, *Interpretive Subroutines*, January 23.

Each lecture was followed by a short question period, and a special session for questions was held on Saturday, January 24. A set of practice problems prepared by W. A. Rutledge was distributed. Attendance at the lectures was between 70 and 80, not counting the Mathematics Panel. The lectures are to be written up and distributed as the first part of a manual on programming and coding for the ORACLE.

Members of the Mathematics Panel presented the following lectures on the Traveling Lecture Program:

A. W. Kimball, *Applied Statistics in Biological Research*, St. Augustine College, Raleigh, North Carolina, November 6, 1952; *An Anomaly in a Confidence-Interval Formula Widely Used in Normal Linear Regressions*, North Carolina State College, November 7, 1952; *Some Statistical Problems in Radiobiology*, Camp Detrick, Maryland, January 29, 1953.

A. S. Householder, *Logical Design of an Automatic Digital Computer*, University of North Carolina, November 26, 1952; *Preparations of Problems for an Automatic Digital Computer*, University of Tennessee, Knoxville, Tennessee, January 12, 1953.

C. L. Perry presented a lecture entitled *The ORNL Special-Purpose Computer*, at the Mathematics Colloquium, University of Tennessee, December 3, 1952.

The following report was issued: J. Moshman, *Critical Values of the Log-Normal Distribution*, ORNL-1427.

Dr. H. L. Lucas, a Mathematics Panel consultant from the Institute of Statistics at North Carolina State College, spent the week of December 8 at ORNL discussing problems with members of the Biology Division and the UT-AEC Agricultural Research Program.

UNCLASSIFIED PROJECTS

METHODS OF COMPUTATION FOR USE WITH A HIGH-SPEED AUTOMATIC COMPUTER

Participating Members of Panel. Wallace Givens, V. C. Carlock.

References. *Mathematics Panel Quarterly Progress Reports*, ORNL-1232, -1290, -1360, -1435.

Problem 3. Numerical Computation of Characteristic Values and Characteristic Vectors.

Results. The general method proposed for the computation of the characteristic values of a real, symmetric matrix of large order has been reported in previous quarterly reports and will appear with the title "Simultaneous Linear Equations and the Determination of Eigenvalues" in the NBS Applied Mathematics Series, No. 29. A completely detailed analysis of the method has now been carried through, and its stability is guaranteed by finding an upper bound for the error in any one characteristic value, as well as for the sum of squares of the errors made in calculating all n values. Numerical results are too complex to summarize readily, but they are sufficiently favorable that the practical limitation on the order of the matrix that can be treated is not the possibility of the accumulation of round-off error but is the memory capacity and speed of currently available computing machines.

The proposed computation falls into two quite distinct parts. In the first of these, a sequence of 1/2 $[(n-1)(n-2)]$ rotations in the coordinate planes x_2x_3, x_2x_4, ..., x_2x_n, x_3x_4, ..., x_3x_n, ..., $x_{n-1}x_n$ is determined so that the quadratic form $x'\overline{A}x$ of the given matrix $\overline{A}$ (with digital elements and norm slightly less than one) is carried into $x'\overline{S}x$, with $\overline{S} = (\overline{s}_{ij})$ and $\overline{s}_{ij} = 0$ for $|i-j| > 1$. After this first stage of "concentrating the data," the characteristic values of $\overline{S}$ can be found by any one of several methods. The particular one developed in full detail in the present study amounts to determining the signature of $\overline{S} - \overline{\mu}1_n$, for suitable values of $\overline{\mu}$, by calculating the signs of the minors formed from the first i rows and columns ($i = 1, 2, \ldots, n$). For the special form of $\overline{S} - \overline{\mu}1_n$, this requires only n iterations of a two-term recursion formula for each choice of $\overline{\mu}$. The second stage of the calculation is easily adapted for obtaining only selected (say the largest and smallest) characteristic values instead of all of them.

A major purpose of this study was to develop means for obtaining efficient bounds for the effect of accumulated round-off error. In the first – and much the more complex – part of the computation, it was sufficient to use routine extensions of methods introduced by von Neumann and Goldstine. In the second stage, it was necessary to use a different technique because a function of $\overline{\mu}$, the signature of $\overline{S} - \overline{\mu}1_n$, was being calculated that assumed only integral values, and the error was therefore also an integer. Here it proved useful to invert the error estimations by demanding that the calculated values be the exact answers (wholly uncontaminated by round-off) calculated for a problem with data numbers "near" those of the actually given problem. This device can be regarded as reducing the noise level of the computation to zero by replacing it by a suitable increase in the noise level of the input. This technique proved the stability of the computation of the signature of $\overline{S} - \overline{\mu}1_n$, guaranteed a satisfactorily small bound for the error in any characteristic value, and proved this bound to be independent of the order of the matrix. A limited type of floating-point operation is indicated, but multiple precision is nowhere required.

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Status. A Laboratory report detailing the results obtained should appear during the next quarter. Flow diagrams and codes for the ORACLE are being prepared. It is planned to continue the study to obtain similar results for the characteristic vectors.

Problem 5. Roots of a Polynomial.

Status. This is a newly established problem. Techniques developed in work on problem 3 seem likely to be useful in devising stable methods for computing the real roots of a polynomial from a suitably delimited class of polynomials with real coefficients. It is proposed to study known methods with a view to adapting them to high-speed computing.

CONSTRUCTION OF SUBROUTINES FOR THE ORACLE

Participating Members of Panel. M. R. Arnette, H. B. Goertzel, C. L. Perry.

References. *Mathematics Panel Quarterly Progress Reports*, ORNL-1232, -1290, -1360, -1435.

Background and Status. A convenient method for programming short calculations for the ORACLE has been designed and coded. The method includes conversion from decimal to binary numbers the numbers and orders (addresses) required by the computation, and the conversion of the final results from binary to decimal numbers. This part of the system (the system has been nicknamed the "bonehead" routine) is called the conversion routine. The conversion routine also automatically performs some of the coding details. For example, provision is made to automatically adjust a subroutine so that it can be placed in an arbitrary position in the memory without previous adjustment by the programmer. Each address in the subroutine that depends on the position of the subroutine in the memory is identified by an extra sexadecimal digit. This digit is used by the conversion routine to adjust the address. The other part of the "bonehead" system is a floating-point interpretive routine. The interpretive routine automatically performs binary floating-point arithmetic operations by using a three-address code.

The "bonehead" system occupies about 200 of the 1024 memory cells in the ORACLE. The floating-point calculations require about 3 msec per three address operations. For comparison, the IBM-card programmed calculator takes about 0.3 sec for a similar operation.

The eight arithmetic operations performed by "bonehead" are $A \pm B = C$, $|A| \pm B = C$, $|A| \pm |B| = C$, $A \div B = C$, $A \cdot B = C$, where $A + B = C$, for example, is the mnemonic symbol for the operation "add the number in memory cell $M(A)$ to the number in memory cell $M(B)$ and place the result in memory cell $M(C)$," and $M(A)$ is the symbol for the address of memory cell containing A, that is, the designation of the place to obtain A.

Four other operations of a logical nature (that is, concerned with the sequencing of operations) are also automatically performed by "bonehead." They are: an unconditional transfer operation, a conditional transfer operation, a subroutine operation, and a transfer to normal ORACLE program operation.

Each distinct operation (12 in all) is specified by a sexadecimal character 1, 2, ..., 9, *A*, *B*, *C*. Of the remaining four sexadecimal characters, *O* and *F* are used to indicate the sign of a number, and *D* and *C* are used in the conversion routine.

The code for numbers (not converted) is similar to that called scientific notation. For example, $+0.937 \times 10^4$ is coded for "bonehead" as *O*/937/*O*/4 and -0.37×10^{-2} is coded as *F*/37/*F*/2. The symbol / is used to indicate the division of information. Nine, or fewer, significant digits can be indicated. The exponent must be less than 157 and greater than −157.

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The code for operations is similar to the algebraic symbol for the operation. For example, the code 1/142/732/521 is the code for $A + B = C$, where 1 indicates the addition operation $A + B = C$ and A, B, and C are the numbers in memory cells 142, 732, and 521, respectively.

The code for "bonehead" is being revised to provide for more flexibility in programming for small calculations (that is, calculations which involve 10^6 or fewer "bonehead" operations).

TEST ROUTINES FOR THE ORACLE

Participating Members of Panel. A. S. Householder, C. L. Perry.

Reference. *Mathematics Panel Quarterly Progress Report*, ORNL-1435.

Background and Status. The test programs mentioned in the reference have been coded in the standard, sexadecimal code for the ORACLE. It is expected that these routines will be used in testing the ORACLE during the next quarter.

PROGRESS OF SUBROUTINE LIBRARY

Participating Members of Panel. M. R. Arnette, W. A. Rutledge.

References. *Mathematics Panel Quarterly Progress Reports*, ORNL-1232, -1290, -1435.

Background and Status. Most of the subroutines that were referred to in ORNL-1435 as being completed in mnemonic code have been checked again and are now in sexadecimal code; that is, they are ready for tapes to be made. Tests for checking these subroutines are being written. The newest subroutine is the "bonehead routine No. 1," which is discussed as a separate project this quarter (cf., "Construction of Subroutines for the ORACLE"). A preparatory routine for assembling subroutines is being written, but it is not yet in final mnemonic form.

W. A. Rutledge spent approximately two weeks rewriting the "Elementary Coding Problems" that were distributed during the ORACLE course that just ended. These exercises, with the answers, illustrate the need and use of various standard and special techniques used in programming a problem for the ORACLE.

The changes that were made last quarter in two subroutines (one uses precision scaling and the other uses transient scaling) for getting the product of two matrices were checked by W. A. Rutledge during his visit. These are ready to be put into machine code. The subroutine that uses fixed-point arithmetic for getting the inverse of a matrix is still in the flow-chart form.

It is expected that the standard subroutines (square root, sine, cosine, logarithm, exponential, conversion, etc.) will be machine tested as soon as the ORACLE has passed its test programs and time is available for testing and running of problems.

BASIC STUDIES IN THE MONTE CARLO METHOD

Participating Member of Panel. G. E. Albert.

References. *Mathematics Panel Quarterly Progress Reports*, ORNL-1232 and -1360.

Background and Status. The background of this project is given in the references. A complete draft has been prepared and is being typed for an ORNL report entitled *A General Theory of Stochastic Estimates of the Neumann Series for the Solutions of Certain Fredholm Equations and Related Series*. It is hoped that this report will be issued during the next quarter.

The report contains a complete revision and generalization of the material to be found in the three memorandums from G. E. Albert to A. S. Householder entitled *A General Approach to the Monte Carlo Estimation of the Solutions of Certain Fredholm Integral Equations*, Parts I, II and III. In addition, the report contains certain new material that is described here. The memorandums just cited contained an expository description of certain stochastic methods for the estimation of the value of the solution of a Fredholm integral equation at a single point of its range. In large part the subject matter of these memorandums was already well known, and some of it had been published. Two elements of novelty were present, however, in these memorandums. First, the descriptions were phrased in mathematical language, based upon the research of J. L. Doob on stochastic processes, instead of in the physical language used in most of the published papers on the subject. Second, the statistical method known as representative sampling from a stratified process was introduced and exploited to a certain degree in an attempt to define stochastic methods for the current problem, which balanced statistical efficiency against machine programming efficiency.

The report goes much further into the novel aspects of the problem than do the memorandums. Chapter 1 is introductory. Chapter 2 contains a detailed discussion of the specialization of Doob's results for the current problem and a generalized treatment of the stratification of stochastic processes having an integral-valued parameter. Complete descriptions are given for rather elaborate special processes that may be of use in the estimation of solutions of integral equations on high-speed machinery. Chapter 3 reviews the material, given in the memorandums, on the estimation of the solution of an integral equation at a single point. The fourth, and final, chapter discusses the problem of estimating the complete solution of the integral equation, not just its value at one point. The method of this chapter is based on the introduction of an interpolating operator peculiar to the particular integral equation under consideration. Interpolation is performed on the basis of a finite number of single-point estimates. It is shown that in some cases the operator will also be a smoothing operator. Careful consideration of this operator indicates that the estimating process should not be designed for efficiency in estimating the solution of the given integral equation, but, rather, for efficiency in connection with a certain related integral equation. Whether or not the interpolation operator proposed is the best possible in any sense is an unsolved problem.

GENERATION OF "RANDOM" DIGITS ON THE UNIVAC

Participating Members of Panel. J. Moshman, S. E. Ezzell, P. J. Brown.

Reference. *Mathematics Panel Quarterly Progress Report*, ORNL-1435.

Background and Status. The digits generated on the UNIVAC, as described in the reference, proved to satisfy the customary tests of randomness with certain reservations. The last five digits of the 11 digit number generated showed definite cyclic factors. The other digits were subjected to the usual tests of digital frequency and the frequency of digits occurring in pairs and triplets. The results for the 10,000 numbers generated are displayed in Table 1.

Probability levels were examined based on nine degrees of freedom for the digital frequency and ten degrees of freedom for the other two comparisons. The results agree fairly well with expectations. The value of

TABLE 1. VALUES OF χ^2 FOR VARIOUS TESTS OF RANDOMNESS

DIGITAL POSITION	FREQUENCY	PAIRS	TRIPLETS
First	9.73	7.90	8.46
Second	13.12	13.57	5.89
Third	8.80	9.08	8.10
Fourth	3.31	12.39	8.06
Fifth	2.35	15.30	5.06
Sixth	0.00	9.78	4.58

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$\chi^2 = 0$ for the digital frequency in the sixth position appears to be fortuitous, inasmuch as no subgroup of 2000 numbers showed exact agreement, even though all 10,000 numbers totalled to give precisely 10% frequency to each digit.

Furthermore, the first three digital positions were examined with regard to the frequency of nondescending and nonascending sequences of varying lengths. These will be termed runs up and down, respectively. In each case χ^2 was computed and its probability level assessed. These values of χ^2 appear in Table 2.

With six degrees of freedom, all values fall within the probability levels 0.05 to 0.95.

This procedure of generating pseudorandom numbers has been used for Monte Carlo problems placed in the UNIVAC.

TABLE 2. VALUES OF χ^2 FOR TESTS OF RUNS UP AND DOWN

DIGITAL POSITION	RUNS UP	RUNS DOWN
First	11.39	4.95
Second	4.69	1.94
Third	6.10	3.01

A TEST FOR INDEPENDENCE IN A CORN GENETICS PROBLEM

Origin. D. Schwartz, Biology Division.

Participating Member of Panel. A. W. Kimball.

Background and Status. Most kernels in unirradiated maize are colorless. When seeds are exposed to x radiation, the resulting ears have some kernels that are colored, which is an effect traceable to the induction of a mutation characterized by a ring chromosome. There is some reason to suspect that the concentration of colored kernels depends on the distance of the kernel from the base of the ear. A test is desired of the null hypothesis that the concentration of colored kernels is independent of the distance from the base of the ear.

The 414 kernels from one ear grown from irradiated seed were classified according to color and according to their location on the ear. For this purpose, the ear was divided into four contiguous segments, each containing approximately the same number of kernels. The results are shown in Table 3. This is a contingency table, and one test for the independence of color and distance from base is given by the usual χ^2 with three degrees of freedom.

TABLE 3. CLASSIFICATION OF KERNELS FROM ONE EAR

COLOR STATUS	SEGMENTS (Numbered consecutively from the base of the ear)				TOTAL
	1	2	3	4	
Colored	3	3	10	9	25
Uncolored	77	101	103	108	389
	80	104	113	117	414

Unfortunately, this test does not distinguish differences in sign. Thus a monotonic increase (or decrease) in the proportion of colored kernels as the distance from the base is increased could not be differentiated from an oscillating proportion. Since the former relationship is the only alternative hypothesis of interest, the usual χ^2 test fails. Another statistic that measures dependence of this sort is biserial η, a kind of correlation coefficient. However, little is known about the probability distribution of this statistic; so it is of no practical use.

A method that will work depends on the exact partition of χ^2 into individual χ^2's, each having one degree of freedom. The mathematical

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basis for this technique is described by Lancaster[1,2] and by Irwin.[3] The computational methods they suggest are laborious, but it is possible to find exact short-cut formulas similar to the familiar 2 × 2 contingency table formula. There is one formula for each degree of freedom associated with the χ^2 for the whole table. If the 2 × 4 table frequencies are represented as

a_1	a_2	a_3	a_4	A
b_1	b_2	b_3	b_4	B
n_1	n_2	n_3	n_4	N ,

the short-cut formulas that partition χ^2 are

$$\chi_1^2 = \frac{N^2[a_1 b_2 - a_2 b_1]^2}{AB\, n_1 n_2 (n_1 + n_2)},$$

$$\chi_2^2 = \frac{N^2[b_3(a_1 + a_2) - a_3(b_1 + b_2)]^2}{AB\, n_3(n_1 + n_2)(n_1 + n_2 + n_3)},$$

$$\chi_3^2 = \frac{N[b_4(a_1 + a_2 + a_3) - a_4(b_1 + b_2 + b_3)]^2}{AB\, n_4(n_1 + n_2 + n_3)}.$$

For the kernel data, these formulas yield $\chi_1^2 = 0.05968$, $\chi_2^2 = 3.85364$, and $\chi_3^2 = 0.78602$. Since the square root of χ^2 with one degree of freedom is a normal variate with zero mean and unit variance, the statistic

$$t = \frac{1}{\sqrt{3}}(\pm \chi_1 \pm \chi_2 \pm \chi_3)$$

may be used as a test of the null hypothesis by reference to standard normal probability tables. The root of χ_i^2 is assigned the same sign as the quantity in brackets in the numerator of χ_i^2. In this problem

$$t = \frac{1}{\sqrt{3}}(0.244 - 1.963 - 0.887) = -1.50,$$

which corresponds to a significance level of 13%.

The method has two distinct advantages over the total χ^2 test. When the proportions oscillate badly, the method will not reject the null hypothesis as often as the total χ^2 test. When the proportions increase or decrease monotomically within experimental error, the method is a more powerful test than the total χ^2 test, at least for the type of alternative hypothesis inherent in experiments of this nature. In the present problem, for example, the significance level for the total χ^2 is about 20%.

(1) H. O. Lancaster, *Biometrika* 36, 117-129 (1949).

(2) H. O. Lancaster, *Biometrika* 37, 267-270 (1950).

(3) J. O. Irwin, *Biometrika* 36, 130-134 (1949).

FIDUCIAL EXPECTATION

Origin. H. Levy, Chemistry Division.

Participating Member of Panel. J. Moshman.

Background and Status. Two variables are measured and their true mean difference is denoted by δ. An experimental determination of this difference is found to be x. If it is assumed that $\delta > 0$, what can be said about an estimate of δ, especially if $x < 0$? It will be assumed that the differences have a normal distribution with variance σ^2 and mean $\delta > 0$.

Two cases are distinguished:

1. *If σ^2 is known*, then

$$u = \frac{\delta - x}{\sigma} \qquad (1)$$

has a normal distribution with zero mean and unit variance. Instead of

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considering u to be a random variable depending on the chance variable x, consider δ a variable with x and σ fixed. Under these conditions, the fiducial distribution of δ is obtained by making the substitution of Eq. 1 in the distribution of u.

The probability density distribution of u, $f(u)du$, is

$$f(u)\, du = \frac{1}{\sqrt{2\pi}} e^{-u^2/2}\, du\ , \qquad (2)$$

which, by the change of the variable in Eq. 2, becomes

$$f(\delta)\, d\delta = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(\delta-x)^2/2\sigma^2}\, d\delta\ . \qquad (3)$$

However, if δ is restricted to a nonnegative range, then the conditional fiducial distribution of δ is

$$f(\delta|\delta > 0)d\delta = \frac{\dfrac{1}{\sqrt{2\pi}\,\sigma} e^{-(\delta-x)^2/2\sigma^2}\, d\delta}{\displaystyle\int_0^\infty \frac{1}{\sqrt{2\pi}\sigma} e^{-(\delta-x)^2/2\sigma^2} d\delta}\ . \qquad (4)$$

Then the expected value of δ is

$$E(\delta \mid \delta > 0) = \int_0^\infty f(\delta \mid \delta > 0)\, d\delta = \frac{\displaystyle\int_0^\infty \frac{\delta}{\sqrt{2\pi}\,\sigma} e^{-(\delta-x)^2/2\sigma^2}\, d\delta}{\displaystyle\int_0^\infty \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(\delta-x)^2/2\sigma^2}\, d\delta}\ . \qquad (5)$$

After some manipulation, Eq. 5 becomes

$$E(\delta \mid \delta > 0) = \frac{\dfrac{\sigma}{\sqrt{2\pi}} e^{-x^2/2\sigma^2}}{\displaystyle\int_{-x/\sigma}^\infty \frac{1}{\sqrt{2\pi}} e^{-u^2/2}\, du} + x\ . \qquad (6)$$

The denominator is readily determined from a table of the probability integral, and the remaining calculation is straightforward.

2. *If σ^2 is unknown* but an estimate s^2 is available based on n degrees of freedom by a similar procedure, one finally obtains

$$E(\delta \mid \delta > 0) = \frac{\dfrac{\left(\frac{n-3}{2}\right)!\ s}{2\left(\frac{n-2}{2}\right)!}\sqrt{\frac{n}{\pi}}\left(1 + \frac{x^2}{ns^2}\right)^{-(n-1)/2}}{\displaystyle\int_{-x/s}^\infty \frac{\left(\frac{n-1}{2}\right)!}{\sqrt{n\pi}\left(\frac{n-2}{2}\right)!}\left(1 + \frac{t^2}{n}\right)^{-(n+1)/2} dt} + x\ . \qquad (7)$$

The integral in the denominator of Eq. 7 may be evaluated with the aid of the table of Student's distribution to be found in *The Advanced Theory of Statistics*[4] or in *Metron*.[5]

DAMAGE TO TISSUE FROM NEUTRON DISTRIBUTIONS WITH MAXIMUM ENERGIES RANGING FROM 1 TO 300 kev

Origin. J. Neufeld and W. S. Snyder, Health Physics Division.

Participating Members of Panel. P. J. Brown, N. M. Dismuke, J. Moshman.

Reference. *Mathematics Panel Quarterly Progress Report*, ORNL-1435.

Status. The problem described in ORNL-1435 has been coded for the

[4] M. G. Kendall, *The Advanced Theory of Statistics*, Vol. 1, p. 440-441, Chas. Griffin and Co., Ltd., London.

[5] R. A. Fisher, *Metron* 5, 109-120 (1952).

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ORACLE in two parts. Part one is the routine for computing and recording the average damage per collision as a function of x, y, and z for specified values of the problem parameters E_0, θ, and d. Part two organizes these data into a space distribution of average damage.

ERROR EQUATION FOR A COUNT-RATE METER

Origin. R. H. Ritchie and G. S. Hurst, Health Physics Division.

Participating Members of Panel. N. Edmonson, C. P. Hubbard.

Status. In flying over gamma-ray sources, a prospecting plane receives an amount of radiation that may vary sharply in magnitude with time. Consequently, if a count-rate meter is used in connection with a counting array, such as G-M tubes, scintillation counter, etc., the meter reading may lag the signal in time and at the same time give an erroneously low indication. In calculating this error, the following equation must be solved for τ:

$$\frac{e^{\tau-\gamma\sqrt{1+(\tau/\beta)^2}}}{\sqrt{1+(\tau/\beta)^2}} = \int_{-\infty}^{\tau/\beta} \frac{e^{\beta x-\gamma\sqrt{1+x^2}}}{\sqrt{1+x^2}}\,dx \;.$$

Since solutions are desired for many values of the parameters β and γ, this problem has been programmed for calculation on the UNIVAC. Results are expected as soon as machine time is available.

THERMAL-NEUTRON PENETRATION IN TISSUE

Origin. W. S. Snyder, Health Physics Division.

Participating Member of Panel. H. B. Goertzel.

Reference. *Mathematics Panel Quarterly Progress Report*, ORNL-1360.

Background and Status. The problem is stated in ORNL-1360, p. 8. The kernel and source functions were evaluated for both 10 and 20 subdivisions, and the resulting sets of simultaneous equations were solved on the NEPA computer. The results were returned to W. S. Snyder for use in choosing the number of subdivisions necessary to obtain the final results. These larger systems will be run on the NEPA computer during the next quarter.

CALCULATION OF THE EFFICIENCY OF ABSORPTION FOR A SINGLE CRYSTAL

Origin. P. R. Bell and F. K. McGowan, Physics Division.

Participating Members of Panel. N. Edmonson, N. M. Dismuke, J. H. Fishel, H. B. Goertzel.

Reference. *Mathematics Panel Quarterly Progress Report*, ORNL-1435.

Background and Status. Overflow was encountered in the last stages of the computation on the UNIVAC. This has been corrected, and computation will be carried out when machine time is available.

CALCULATION OF RACAH COEFFICIENTS FOR THE ANGULAR DISTRIBUTION IN NUCLEAR REACTIONS

Origin. M. E. Rose, Physics Division; L. C. Biedenharn, Yale University.

Participating Member of Panel. C. L. Perry.

References. *Mathematics Panel Quarterly Progress Report*, ORNL-1435; L. C. Biedenharn, *Tables of the Racah Coefficients*, ORNL-1098.

Background and Status. ORNL-1098 consisted of two sets of tables: one set was the tables of W coefficients, and the other was the tables of Z coefficients. The demand for ORNL-1098 has exceeded the number of copies printed. During the quarter, the Z tables were revised for publication as a separate report, ORNL-1501 (in press).

ANISOTROPY IN ANGULAR CORRELATION DATA

Origin. E. D. Klema, Physics Division.

Participating Members of Panel. J. Moshman, E. N. Lawson, H. B. Goertzel.

Reference. *Mathematics Panel Quarterly Progress Report*, ORNL-1435.

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Background and Status. The calculations described in the reference are being set up on a continuing basis for the K-25, Numerical Analysis Laboratory's IBM punched-card equipment.

BETA DECAY (FIELD FACTORS)

Origin. M. E. Rose and P. R. Bell, Physics Division.

Participating Member of Panel. C. L. Perry.

References. *Mathematics Panel Quarterly Progress Reports*, ORNL-1029, -1232, -1290, -1360, and -1435; M. E. Rose, C. L. Perry, and N. M. Dismuke, *Tables for the Analysis for Allowed and Forbidden Beta Transitions*, ORNL-1459 (in press).

Background and Status: The multiplications described in ORNL-1435 were completed during December on the IBM machines at K-25. The tables of field factors were printed directly on duplimat masters and are now being reproduced for issuance as ORNL-1459. Graphs of the field factors were made for inclusion in a paper describing the tables, which will be sent to the *Physical Reviews*.

DETERMINATION OF THE FAST-NEUTRON FLUX IN THE ORNL GRAPHITE REACTOR

Origin. D. K. Holmes, Physics Division.

Participating Members of Panel. J. Moshman, S. E. Ezzell.

Reference. *Mathematics Panel Quarterly Progress Report*, ORNL-1435.

Background and Status. A total of 10,080 neutron histories has been completed on the UNIVAC. The final calculations of the flux in various regions of interest are under way in Philadelphia on the UNIVAC and are scheduled for completion in February 1953.

CALCULATION OF INTERNAL CONVERSION COEFFICIENTS WITH SCREENING

Origin. M. E. Rose, Physics Division.

Participating Members of Panel. M. R. Arnette, N. D. Given, W. C. Sangren.

References. *Mathematics Panel Quarterly Progress Reports*, ORNL-1290 and -1360.

Background and Status. It was decided to code this problem for solution on the ORACLE. Two reasons for this decision are the failure of the Williams-tube memory of the SEAC and the desirability of having a check on the SEAC's results if the are completed before the ORACLE's results are obtained.

A review has been made of the numerical procedures to be used for evaluating the integrals and integrating the differential equations. For use on the ORACLE, it has been decided to use cruder integration formulas but finer meshes. A few other simplifications have been made in order to facilitate the calculations.

The status of the preparation of the problem for the ORACLE is indicated in the following. Part I of the problem, the calculation of the screening function, has been coded and checked. Part II, the calculation of the bound state wave functions and the corresponding eigenvalues, has been coded and partially checked. Analysis and coding has been started on Part III.

Part I has been completed on the SEAC. Part II is rep rted to be near completion. In Part III, it was decided to divide the calculation into smaller calculations and recode the smaller calculations for using the acoustic memory instead of the faulty Williams-tube memory.

MINIMUM WEIGHT AND PRESSURE DROP STUDY FOR AN AIR HEAT EXCHANGER

Origin. W. S. Farmer, Reactor Experimental Engineering Division.

Participating Member of Panel. N. D. Given.

Reference. *Mathematics Panel Quarterly Progress Report*, ORNL-1435.

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Background and Status. To obtain optimum dimensions for a nickel or stainless steel air radiator, evaluations of algebraic formulas for pressure changes have been started. These calculations are an extension of those reported in the last quarterly report.

LINEAR STABILITY OF BOILING REACTORS

Origin. P. R. Kasten, Reactor Experimental Engineering Division.

Participating Members of Panel. A. S. Householder, J. W. Givens V. C. Carlock.

Background and Status. The problem is to find the largest positive real root, if such exists, of $f(\nu; \lambda, \mu, a, \omega^2, \gamma) = 0$, where f is a polynomial of sixth degree in ν. The coefficients of this polynomial are formed from combinations of the parameters λ, μ, a, ω^2, γ, and these coefficients are being determined by the Lagrange interpolation formula.

Work continues on programming the computations for the ORACLE. The method consists of computing Sturm functions for each set of parameters; an upper bound is found for the positive roots, and the interval is successively subdivided.

HEAT FLOW IN LONG, CIRCULAR PIPES

Origin. H. F. Poppendiek, Reactor Experimental Engineering Division.

Participating Members of Panel. P. J. Brown, N. M. Dismuke, W. C. Sangren.

Background and Status. This problem is a study of the space distribution of temperature in a fluid after steady-state conditions have been reached. The fluid flows in a long, thin pipe of circular cross section. A constant amount of heat per unit surface area is added continuously through the sides of the pipe in the normal distribution. The differential equation to be solved is given by

$$u(r)\,\frac{\partial t}{\partial x} = a\left[\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r}\right] + \frac{G}{w c_p},$$

$$\frac{\partial t}{\partial r} = 0, \text{ at } r = 0 \text{ and } r = r_0,$$

$$t = 0, \text{ at } x = 0.$$

The pipe is infinite in the x direction, but the integration is to be carried out to $x = X_f$. In the equation,

$$u(r) = 2u_m\left[1 - \left(\frac{r}{r_0}\right)^2\right],$$

u_m = maximum velocity of the fluid,
t = temperature of the fluid,
r = radial distance, measured from the pipe center to the radius, r_0,
x = axial distance,

and a, G, c_p, w, and u_m are constant problem parameters.

Uniformly spaced lattice points in the axial direction are denoted by m, and those in the radial direction, by n. Δx and Δr are the axial and radial spacings, respectively:

$m = 0, 1, \ldots, M$,
$n = 0, 1, \ldots, N$,
$r = 0$ is half-way between $n = 0$ and $n = 1$,
$r = r_0$ is half-way between $n = N - 1$ and $n = N$.

With this notation, the difference equation to be solved is given by

$$t_{m+1,n} = C_{1,n} t_{m,n-1} + C_{2,n} t_{m,n} + C_{3,n} t_{m,n+1} + C_{4,n},$$

$$t_{m,0} = t_{m,1},$$

$$t_{m,N-1} = t_{m,N},$$

$$t_{0,n} = 0.$$

The C's are given by

$$C_{1,n} = \frac{a}{u_n}\left[\frac{\Delta x}{(\Delta r)^2} - \frac{1}{2r_n}\frac{\Delta x}{\Delta r}\right],$$

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$$C_{2,n} = 1 - \frac{2a}{u_n} \frac{\Delta x}{(\Delta r)^2} ,$$

$$C_{3,n} = \frac{a}{u_n} \left[\frac{\Delta x}{(\Delta r)^2} + \frac{1}{2r_n} \frac{\Delta x}{\Delta r} \right] ,$$

$$C_{4,n} = \frac{G}{wc_p} \frac{\Delta x}{u_n} .$$

Coding of the problem for solution on the UNIVAC is in progress. Present plans are to solve the problem for one set of parameter values for the two lattice spacings:

$$(1) \quad \Delta r = \frac{r_0}{40} \quad \text{and} \quad \Delta x = \frac{X_f}{5000} ,$$

and

$$(2) \quad \Delta r = \frac{r_0}{20} \quad \text{and} \quad \Delta x = \frac{X_f}{1250} .$$

If these two solutions show good agreement, spacing 1 will be used to solve the difference equation for 16 or more sets of parameter values.

REACTOR INEQUALITIES AND EIGENVALUES FOR THE WAVE EQUATION

Origin. A. M. Weinberg, Research Director's Division.

Participating Members of Panel. H. B. Goertzel, W. C. Sangren.

References. *Mathematics Panel Quarterly Progress Reports*, ORNL-1360 and -1435.

Background and Status. As a preliminary computation, the fundamental eigenvalue for a square membrane was obtained on the NEPA machine. The result agreed with the known analytical result to as many significant figures as can be obtained on the NEPA machine. The preparation of certain "concentric" square membrane problems has been completed and their fundamental eigenvalues will be sought by using the NEPA machine. Studies on the use of general-purpose digital machines for computing fundamental eigenvalues are being continued. In particular, the coding necessary for finding the fundamental eigenvalues for concentric spheres and parallel slabs has been completed for the UNIVAC.

GROWTH EFFECTS OF IRRADIATED PITUITARY TISSUE ON CHICKS

Origin. J. H. Rust, UT-AEC Agricultural Research Program.

Participating Members of Panel. A. W. Kimball, G. J. Atta.

Background and Status. Pituitary tissue from chicks was exposed to x radiation and then injected into young chicks as a homogenate. One group of chicks received 50 mg, fresh weight, of the tissue and another group received 100 mg. Two more groups received like amounts of unirradiated tissue. After several weeks, the birds were sacrificed and body, testes, thyroid, adrenals, and comb weights were recorded. The problem was to evaluate the effect, if any, of x radiation on the growth of the body parts weighed.

Analyses of covariance were computed for each of the four organ weights by using the body weight as an independent variate. These were compared with the corresponding analyses of variance. In no case were any differences found that were significant at the 5% level. An analysis of the body weight alone led to similar results. In every case, covariance methods helped reduce residual error, but its effect on the treatment means varied considerably from one weight to another. There was some indication that at higher injection doses, the effects might be more pronounced. Accordingly, the experiment will be repeated with larger doses.

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DEPIGMENTATION AS A BIOLOGICAL DOSIMETER–OPERATION GREENHOUSE

Origin. A. C. Upton, Biology Division.

Participating Members of Panel. J. Moshman, R. C. Weaver.

Reference. *Mathematics Panel Quarterly Progress Report*, ORNL-1435.

Background and Status. It was reported earlier that nine months after exposure, the dose, d, received by mice at Eniwetok was related to the degree of depigmentation, p $(0 \leq p \leq 4)$, on the back of the head by the formula

$$d = 318.32 + 84.09\,p \quad (287 \leq d \leq 711)\ .$$

The procedure described in the reference was repeated for periods of 7 and 11 months after exposure. The resulting formulas are:

$$7 \text{ months}, \quad d = 313.22 + 85.97\,p \quad (287 \leq d \leq 711)\ ;$$

$$11 \text{ months}, \quad d = 309.96 + 81.84\,p \quad (287 \leq d \leq 711)\ .$$

The variability among the curves was not statistically significant and indicated the lack of a time effect during this period. In each case, the standard error of estimate about the line was about 50 to 55r.

LARGE DEFLECTIONS OF PUMP DIAPHRAGMS

Origin. C. D. Zerby, Reactor Experimental Engineering Division.

Participating Members of Panel. H. B. Goertzel, C. L. Perry.

Background and Status. A preliminary investigation of the problem of the large deflections of thin, circular plates was made. For the pump in question, it was desired to accurately obtain the deflection of the plate under uniform, lateral loading. Work is now being done to obtain sufficiently accurate equations, although the theory of large deflections is very inadequate.

CODING OF REACTOR CALCULATIONS

Origin. R. A. Charpie, Research Director's Division.

Participating Members of Panel. N. Edmonson, J. H. Fishel, C. P. Hubbard.

References. D. K. Holmes, *The Multigroup Method as Used by the ANP Physics Group*, ANP-58; *Mathematics Panel Quarterly Progress Report*, ORNL-1435.

Background and Status. These calculations, according to the first plan, were to be coded for the ORACLE. In order to obtain calculated results sooner than the ORACLE would be in operation, it was decided to code these calculations for the UNIVAC. This coding has been done and is to be tested on the UNIVAC as soon as machine time is available.

END