(A-7.390.21)

AECD-4055

CRITICALITY IN GRAPHITE SYSTEMS

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INTRODUCTION

The use of graphite in the collectors for the electromagnetic process results in the need for storage and processing of masses of U-235 contaminated carbons. Because of the widely differing moderating properties of carbon and hydrogen, it was necessary to employ a modified thermal diffusion method to determine critical conditions, applicable to the Y-12 chemical cycle. It can be shown that the following treatment is basically the same as that used by X-10 in one-group pile calculations.

THEORY OF UNTAMPERED BODIES

One form of the pile equation is

$$\frac{\Delta}{\lambda_t} \Delta n v_{th} - n v_{th} \Sigma_a + n v_{th} \Sigma_f K e^{\Delta \tau} = 0 \quad (1)$$

where, for thermal neutrons,

λ_t = transport mean free path of the system (the reciprocal of the macroscopic transport cross section Σ_t)

n = number of neutrons per cm^3

v_{th} = velocity

K = number of fission neutrons per thermal neutron absorbed

Σ_a = macroscopic cross-section for absorption, by the system

Δ = "buckling", a parameter of geometry

τ = age, proportional to mean square distance for thermalization

Δ = Laplacian operator

This expresses that the algebraic sum of the rates of neutron diffusion from a volume element of the pile, the absorption of neutrons and the production of neutrons is zero, i.e., a steady or exactly critical state exists. The pile equation can be simplified by the assumption that a solution for the neutron flux ($n v_{\Delta}$) exists of the type

$$\Delta n v_{\Delta} = \Delta n v_{\Delta}$$

On substitution, equation (1) becomes

$$\frac{\lambda_c}{3} \Delta - \Sigma_a + \Sigma_c K e^{\Delta \tau} = 0$$

or letting $L^2 = \frac{1}{3 \Sigma_a \Sigma_c} = \frac{\lambda_c}{3 \Sigma_a}$

$$L^2 \Delta - 1 + K e^{\Delta \tau} = 0 \quad (2)$$

This alternate pile equation can be put in terminology more closely corresponding with that used by the Y-12 Special Hazards Group.

Let $\Delta = -k^2$

$$K = \frac{N_x \sigma_x}{N_x \sigma_x + N_c \sigma_c} \nu = \frac{Z}{Z+1} \nu$$

where: N_x and N_c are the numbers of X and carbon atoms; σ_x and σ_c are the thermal absorption cross-sections; ν is the number of neutrons per neutron absorbed in the system; $Z = \frac{N_x \sigma_x}{N_c \sigma_c}$ by definition. The square of diffusion length of neutrons in the system of carbon and X, defined by $L^2 = \frac{1}{3 \Sigma_a \Sigma_c}$ can be put in terms of the square of the diffusion length in carbon alone, L_c^2 , as follows: $\Sigma_a = N_c \sigma_c$

$$\Sigma_a = N_x \sigma_x + N_c \sigma_c$$

by definition of macroscopic absorption cross-section.

$$\Sigma_c = \Sigma_c^*$$

since the system is principally carbon. Then

$$L^2 = \frac{1}{3\Sigma_c \Sigma_c^*} = \frac{1}{3(N_A \sigma_A + N_C \sigma_C) \Sigma_c^*} \approx \frac{1/\Sigma_c^*}{3(z+1)\Sigma_c^*} = \frac{L_c^2}{z+1}$$

with these substitutions, equation (2) becomes

$$-\frac{L_c^2}{z+1} k^2 - 1 + \frac{z}{z+1} \sqrt{e^{-k^2 \tau}} = 0$$

Solving for z , the condition for criticality is

$$z = \frac{1 + k^2 L_c^2}{\sqrt{e^{-k^2 \tau}} - 1} \quad (3)$$

The geometry parameter k^2 can be shown, from solutions of the assumed differential equation

$$\Delta n \nabla_k = \Delta n \nabla_k$$

$$\text{or } \Delta n = -k^2 n$$

for various shapes, to be given by

Sphere of radius a or infinite slab of height a $k^2 = \frac{\pi^2}{a^2}$

Rectangular parallelepiped of dimensions a, b, c $k^2 = \frac{\pi^2}{a^2} + \frac{\pi^2}{b^2} + \frac{\pi^2}{c^2}$

Finite right circular cylinder of radius r , height h $k^2 = \frac{\pi^2}{r^2} + \frac{(2.4048)^2}{h^2}$

Figure 1 is a graph of equation (3) in which the critical $X\%$ weight X , proportional to z , is plotted against k . It can be used to solve all problems for which an array of the shapes above is given or implied.

Note that $X = \frac{100 \text{ weight of X}}{\text{weight of C}} = \frac{(100)(N_X)(235)}{(N_C)(12.01)}$ leads to

$$z = \frac{N_X}{N_C} \frac{\sigma_X}{\sigma_C} = \frac{640}{0.0045} \frac{N_X}{N_C} = \frac{12.01}{23500} \frac{640}{0.0045} X = 72.68 X$$

$$\text{or } X = 0.01376 z$$

The other constants appearing in equation (3) were chosen in an attempt to provide conservation. They are

$$L_0 = 51.7 \text{ cm } (L_0^2 = 2673)$$

$$\tau = 367 \text{ cm}^2$$

$$\nu = 2.15$$

APPLICATION OF THEORY

Example 1. Find the dimensions for which criticality is impossible in

- an infinite slab
- a sphere
- a cube
- an infinite square cylinder
- an infinite circular cylinder

The condition $Z = \infty$ must be met to prevent criticality. Then equation (3)

becomes

$$\nu e^{-k^2 \tau} = 1$$

$$\text{or } k^2 = \frac{1}{\tau} \ln \nu = \frac{1}{367} \ln 2.15 = 0.001977$$

$$k = 0.04446$$

For the cases above, the applicable translation into dimensions and the results are tabulated.

a. ∞ slab	height $a = \pi/k$	= 70.7 cm = 27.8 in.
b. cube	side $a=b=c = \sqrt{3}\pi/k$	= 122.4 cm = 48.2 in.
c. sphere	radius $r = \pi/k$	= 70.7 cm = 27.8 in.
d. square cylinder	side $a = b = \sqrt{2}\pi/k$	= 99.9 cm = 39.3 in.
e. circular cylinder	radius $r = 2\pi/k$	= 54.1 cm = 21.3 in.

Example 2. Find the maximum allowable size of an array of unbroken carbon parts containing 0.12% by weight of carbon, stacked

- in a sphere
- in a cube
- on a table 16' 9" long

5X²³

-5-

Find the corresponding size for 1% X by weight.

From Figure 1, the k corresponding to X = 0.12 is 0.0323

- a. for the sphere $a = \pi/k = 97.3 \text{ cm} = 38.3 \text{ in}$
- b. for the cube $a = \sqrt[3]{3\pi/k} = 168.5 \text{ cm} = 66.5 \text{ in}$
- c. for the table, 610.5 cm long

$$k^2 = \left(\frac{\pi}{510.5}\right)^2 + \frac{\pi^2}{a^4} + \frac{\pi^2}{a^4}$$

$$\text{or } a^4 = \frac{2\pi^2}{k^2 - \left(\frac{\pi}{510.5}\right)^2} = \frac{2\pi^2}{0.001043 - 0.000038} = 19642$$

the largest safe side of the square cylinder thus formed is $a = 140.1 \text{ cm} = 55.2 \text{ in}$.

From Figure 1, the k corresponding to X = 1 is 0.0422

- a. for the sphere $a = \pi/k = 74.4 \text{ cm} = 29.3 \text{ in}$
- b. for the cube $a = \sqrt[3]{3\pi/k} = 128.9 \text{ cm} = 50.6 \text{ in}$
- c. for the table, using the method above, $a = 106.6 \text{ cm} = 42.0 \text{ in}$.

Example 3. Find the critical mass of X for an untamped sphere of carbon containing 0.12% X by weight.

The critical radius is 97.3 cm, from above; the density of carbon is 1.6 grams/cm³, so that

$$M_x = X/100 \cdot 4/3 \pi a^3 \rho = (0.0012) (4/3 \pi) (97.3)^3 1.6 = 7.4 \text{ kg.}$$

Example 4. Find the minimum critical mass of X in an untamped sphere of graphite of density 1.6 g/cm³.

$$M_x = X/100 \cdot M_0 = X/100 (4/3 \pi a^3 \rho)$$

$$M_x = 0.067 X a^3$$

A tabular approximation method, in conjunction with Figure 1 may be used.

a	a ³ x10 ⁻³	k = π/a	X	Xa ³ x10 ⁻³	M(kg)
105	1158	0.02992	.089	103	6.90
110	1331	0.02856	.077	102.5	6.87
115	1521	0.02732	.067	102	6.83
120	1728	0.02618	.060	103.5	6.93

The minimum critical mass of X is approximately 6.8 kg.

$$\frac{M_x}{M_0} = X^{2/3}$$

WATER TAMPED CARBON SYSTEMS

The critical size of a water tamped sphere may be calculated for comparison with the untamped case, by the following method.

Consider a spherical mass of uniformly mixed carbon and uranium, surrounded by an infinite extent of water. Assume that the solution of the one-group pile equation for the carbon-uranium core is a sine curve damped by the inverse radius

$$n v_c = \frac{\sin kr}{r} \quad (4)$$

and that in the tamper is a decreasing exponential also damped by the inverse radius

$$n v_t = \frac{\alpha e^{-\chi r}}{r} \quad (5)$$

where α is an arbitrary constant and χ is the reciprocal of the diffusion length, $L = 2.88$ cm for water.

Two boundary conditions are applied:

The neutron flux (nv) is continuous across the core-tamper interface at radius a

$$(nv_c)_a = (nv_t)_a \quad (6)$$

The neutron current, $-\frac{\lambda}{3}(nv)'$ is also continuous across the interface.

$$-\frac{\lambda^c}{3}(nv_c)'_a = -\frac{\lambda^t}{3}(nv_t)'_a \quad (7)$$

where λ^c and λ^t are the transport mean free paths for the core and tamper respectively.

From (6)

$$\frac{\sin ka}{a} = \frac{\alpha e^{-\chi a}}{a} \quad (8)$$

The derivatives of the flux are

$$\begin{aligned} (nv_c)' &= \frac{1}{r} k \cos kr - \frac{1}{r^2} \sin kr \\ (nv_c)' &= \alpha \left[-\frac{\chi}{r} e^{-\chi r} - \frac{1}{r^2} e^{-\chi r} \right] \end{aligned}$$

so that (7) leads to

$$\chi^2 (ka \cos ka - \sin ka) = -\alpha \chi^2 e^{-\chi a} (\chi a + 1) \quad (9)$$

If this equation (9) is divided by equation (8) and the result is simplified, the criticality condition becomes

$$(1 - ka \cot ka) = \frac{\chi^2}{\lambda^2} (\chi a + 1) \quad (10)$$

From Example 2 of the previous section, the k associated with $\lambda = 0.12$ is

0.0323. If $\lambda^2 \approx 2.7$, $\lambda^2 \approx 0.4$ and $\chi = \frac{1}{2.88}$, iteration of the equation (10)

leads to the critical radius of the tamped sphere, $a \approx 77$ cm in contrast with

$a = \pi/k \approx 97$ cm for the untamped sphere.

m/b

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Figure I
Critical X% Weight
of Untamped Carbon Systems

X % weight

K = 0
X = 0.015

K 9X²³

X = 0.30
K = 0.045