Photostat Price \$ 3.30
Microfilm Price \$ 2.40
Available from the
Office of Technical Services
Department of Commerce
Washington 25, D. C.

AECD-4052
(NAA-SR-Memo-318)
Page 1 of 17
Copy 6 of 35
Series A

Issue Date:
May 12, 1952

FURTHER STUDIES ON CONTROL OF THE CONVERTER REACTOR

by

L. B. Robinson

I. INTRODUCTION

This report is an extension of NAA-SR-Memo-178. The results show the number of control rods as a function of the amount of reactivity to be controlled. Numerical solutions are given for a typical example and are readily obtainable for other cases.

II. SAFETY AND REGULATING CONTROL

The requirements of the safety and regulating control systems are described in terms of the well known adjoint flux perturbation method. This technique is described in many places; in this report, an adaptation of the description of H. L. Garabedian¹ will be given. The model used is the given reactor divided into N (the number of control rods) regions. All of these regions have the same area. The control rod cell (i.e., one of the regions) is considered as the perturbed region. The change in the local multiplication factor (that associated with a particular cell) is computed for the cases in which the cadmium rod is completely inserted and in which it is completely withdrawn. This change in the local multiplication factor, δk , is quite different from the pile reactivity; the two are related through the position factor (a quantity depending on the position of the rod in the pile). It is apparent that the rate of absorption of neutrons by the control rod is proportional to the flux at the position in question. It is also apparent that the effectiveness of the removal of a neutron in stopping the neutron production cycle is proportional to some mathematical construct (the adjoint flux) which can describe the "importance" of the neutron. For example, a neutron absorbed by a control rod near the edge of the pile will have less effect on the neutron cycle than one absorbed near the center, because there

¹H. L. Garabedian, WAPD-19

This report was prepared as a scientific account of Government-sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission makes any warranty or representation, express or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights. The Commission assumes no liability with respect to the use of, or from damages resulting from the use of, any information, apparatus, method, or process disclosed in this report.

is a greater probability that the neutron near the edge would have leaked out and not contributed to the production cycle in the first place.

Garabedian's method is based on the work of L. W. Nordheim.² The details found in Refs. 1 and 2 are not repeated here. Suffice it to say that the reactivity is found to be

$$\begin{aligned} \frac{\Delta k_{\text{eff}}}{k_{\text{eff}}} = & \int_{\text{reactor}} \left\{ \sum_s \delta k \phi_s^* \phi_s + k \delta \sum_s \phi_s^* \phi_s + \delta \sum_f \phi_s^* \phi_f - \delta \sum_f \phi_f^* \phi_f \right. \\ & \left. - \delta \sum_s \phi_s^* \phi_s - \delta D_s \nabla \phi_s^* \cdot \nabla \phi - \delta D_f \nabla \phi_f^* \cdot \nabla \phi_f \right\} dV \\ & + \int_{\text{core}} \sum_s k_p \phi_s^* \phi_s dV \quad \dots (1) \end{aligned}$$

In Eq. (1), the subscripts *s* and *f* refer to slow and fast, respectively, and δ to a small change. ϕ refers to the flux in the unperturbed reactor and ϕ^* to the adjoint flux. Σ is the macroscopic absorption cross section, k_p is the multiplication constant resulting from prompt neutrons alone, and $D = \lambda/3$, where λ is the transport mean free path. When the indicated integrations are carried out, the equation (describing the reactivity attributable to one rod) given below is obtained

$$\begin{aligned} \frac{\Delta k_{\text{eff}}}{k_{\text{eff}}} = & \frac{1}{\pi k_p \sum_s s^2 J_1^2(2.405)} \left[J_0^2(\mu) \left\{ (\sum_s \delta k + k \delta \sum_s) s^2 \right. \right. \\ & \left. \left. + \delta \sum_f (1 - s^2) s - \delta \sum_f \right\} - (\delta D_s + s s^2 \delta D_f) \left\{ \mu^2 J_1^2(\mu) \right. \right. \\ & \left. \left. + \mu^2 J_0^2(\mu) \right\} \right] \quad \dots (2) \end{aligned}$$

²L. W. Nordheim, CP-2624

332-2

In Eq. (2),

$$\left. \begin{aligned} \mu &= \frac{2.405}{R}, & (R = \text{pile radius}) \\ \kappa^2 &= \frac{\pi^2}{h^2}, & (h = \text{pile height}) \\ s &= \frac{\Sigma_f + D_f(\mu^2 + \kappa^2)}{\Sigma_f}, \\ s^2 &= \frac{\Sigma_f}{\Sigma_f + D_f(\mu^2 + \kappa^2)}, \end{aligned} \right\} \dots (3)$$

and $\bar{\lambda}$ is the distance of the rod from the center of the reactor.

Numerical values of the constants (for a reactor discussed in Memo-178) are given in Table I. Σ_f is interpreted as a fictitious fast absorption cross section which takes into account the loss of neutrons from the fast group which occurs when neutrons slow down and enter the thermal group; the true fast absorption is neglected. The values Σ_f and D_f are chosen so that they give the same mean square slowing down distance ($6\bar{\lambda}$) as age theory. When a neutron is being slowed down, on the average it makes $(1/\xi) \ln(E_0/E_{th}) = n$ collisions in going from the fission energy E_0 to thermal energy, E_{th} . ξ is given by $1 + (r \ln r)/(1 - r)$, where $r = (A-1)^2/(A+1)^2$. A is the atomic weight of the scattering nucleus. For hydrogen, $\xi = 1$. Hence, the ratio of the fictitious absorption cross section, Σ_f , to the average scattering cross section $\bar{\Sigma}_{scat}$ is

$$\frac{\Sigma_f}{\bar{\Sigma}_{scat}} = \frac{1}{n} = \frac{\xi}{\ln E_0/E_{th}}, \dots (4)$$

from which

$$\Sigma_f = \frac{\xi \bar{\Sigma}_{scat}}{2.3 \log E_0/E_{th}} \dots (5)$$

Also, $D_f = \Sigma_f / \kappa^2 = \Sigma_f \tau$ or

$$D_f = \frac{\tau \xi \bar{\Sigma}_{scat}}{2.3 \log E_0/E_{th}} \dots (6)$$

332-3

TABLE I
TYPICAL PARAMETERS FOR EQ. (2)

Σ_s	0.135 cm ⁻¹	$\{\bar{\Sigma}_{scat}$	1.39 cm ⁻¹
λ_s	0.48 cm	E_o	2.0 Mev
D_s	0.16 cm	E_{th}	0.025 ev
τ	66.67 cm ²	Σ_f	0.070 cm ⁻¹
h	60 in	D_f	4.66 cm
a	30 in	s	1.93
		s^2	0.91

Above, it is assumed that the insertion of the cadmium rod has a negligible effect on the fast cross sections. Within the framework of this assumption, the effect of the local δk on the reactivity can be estimated immediately since

$$\frac{\Delta k_{eff}}{k_{eff}} \sim \frac{J_0^2(\mu J)}{Nk_p J_1^2(2.405)} \left\{ \delta k + \left(k - \frac{1}{s^2} \right) \frac{\delta \Sigma_s}{\Sigma_s} \right\} \dots (7)$$

Since the determination of δk will be different from that given in WAPD-19, the details will be given below. It is assumed that the Boltzmann integro-differential equation is an accurate description of transport phenomena. First order diffusion theory is a reasonable approximation to the Boltzmann equation for regions which are not highly capturing and which are not close (in terms of the transport mean free path) to sources or boundaries. Marshak, Brooks and Hurvits show that the source term can be modified in the diffusion equation to make its solution approximate the rigorous solution of the transport (Boltzmann) equation. Herein, it is assumed that the source term is distributed uniformly throughout the multiplying medium, and hence the idea of proximity to sources plays no role. When the control rod cell is divided into a capturing region and a multi-

332-4

plying plus capturing region, the modification of diffusion theory is effected through the use of a suitable boundary condition.

The determination of the local δk involves the radius of the control rod r_0 and the radius of the control rod cell $r_1 = R/\sqrt{N}$, where R is the pile radius and N the number of control rods. Thus, the pile reactivity requirements determine δk , and N may be determined for a given r_0 . The thermal utilization f is determined for the control rod cell (with the control rod in place) and subtracted from that, f , determined with the fuel in place. Then $\delta k = \gamma p(f - f')$, where $\delta k > 0$, γ is the number of neutrons released per neutron absorbed in fuel, and p is the resonance escape probability. The calculation is given immediately below.

With the control rod in place,

$$f' = \frac{\Sigma_{25} \int_{V_1} \phi_1 dV}{qV_1} \quad \dots (8)$$

where the subscript 1 refers to the region surrounding the control rod and q is the rate of production of thermal neutrons (i.e., neutrons/cm³/sec.).

Σ_{25} is absorption cross section of the fuel; ϕ , the flux, and V , the volume. It is assumed that the gradient of the flux vanishes at the outer boundary of the control rod cell, and thus the rate of production equals the rate of absorption. The problem is to find ϕ_1 so that f' can be computed.

In region 1, the differential equation and boundary conditions are

$$\left. \begin{aligned} & \frac{\Sigma_1}{\Sigma_1^2} \nabla^2 \phi_1 - \Sigma_1 \phi_1 + q = 0 \quad , \\ (a) \quad & \nabla \phi_1 = 0 \text{ at } r = r_1 \quad , \\ (b) \quad & \lambda \nabla \phi_1 \cdot \hat{n} = \phi_1 \text{ at } r = r_0 \quad , \end{aligned} \right\} \quad \dots (9)$$

332-5

where λ is the extrapolation length. For large cylinders (i.e., radius larger than mean free path in external medium)³

$$\lambda/\lambda_{\text{transport}} = 0.7104 + 0.2524 \frac{\lambda_{\text{tr}}}{r_0} + 0.0949 \left(\frac{\lambda_{\text{tr}}}{r_0} \right)^2 + \dots \quad \dots (10)$$

In Eq. (9) $\mathcal{X}_1^2 = \frac{1}{L^2}$. The solution of Eq. (9) may be written as

$$\phi_1 = \frac{q}{\Sigma_1} + a I_0(\mathcal{X}_1 r) + b K_0(\mathcal{X}_1 r) \quad \dots (11)$$

The arbitrary constants a and b are determined with the aid of Eq. (9a) and Eq. (9b). Condition (b) yields

$$\left. \begin{aligned} a &= b \frac{I_1(\mathcal{X}_1 r_1)}{I_1(\mathcal{X}_1 r_1)} = -B K_1(\mathcal{X}_1 r_1) \\ b &= -B I_1(\mathcal{X}_1 r_1) \end{aligned} \right\} \quad \dots (12)$$

Hence,

$$\begin{aligned} \phi_1 &= \frac{q}{\Sigma_1} - B \left[K_1(\mathcal{X}_1 r_1) I_0(\mathcal{X}_1 r) + I_1(\mathcal{X}_1 r_1) K_0(\mathcal{X}_1 r) \right] \\ &= \frac{q}{\Sigma_1} - B Z_1(\mathcal{X}_1 r) \quad \dots (13) \end{aligned}$$

Above, the meaning of Z_1 is obvious. From condition (c)

$$-\lambda B \mathcal{X}_1 \left[K_1(\mathcal{X}_1 r_1) I_1(\mathcal{X}_1 r_0) - Z_1(\mathcal{X}_1 r_1) K_1(\mathcal{X}_1 r_0) \right] = \frac{q}{\Sigma_1} - B Z_1(\mathcal{X}_1 r_0) \quad \dots (14)$$

Or

$$-\lambda B \mathcal{X}_1 Z_2(\mathcal{X}_1 r_0) = \frac{q}{\Sigma_1} - B Z_1(\mathcal{X}_1 r_0) \quad \dots (15)$$

³B. Davidson, MT-135

from which

$$B = \frac{q}{\Sigma_1(z_1^0 - \lambda \kappa_1 z_2^0)} \quad \dots (16)$$

where $z_1^0 \equiv z_1(\kappa_1 r_0)$. Hence, the flux ϕ_1 becomes

$$\phi_1 = \frac{q}{\Sigma_1} \left\{ 1 - \frac{z_1(\kappa_1 r)}{z_1^0 - \lambda \kappa_1 z_2^0} \right\} \quad \dots (17)$$

The integral in Eq. (8) becomes

$$\int_{V_1} \phi_1 dV = \frac{q}{\Sigma_1} \left\{ \int_{V_1} dV - \int_{V_1} \frac{z_1(\kappa_1 r)}{z_1^0 - \lambda \kappa_1 z_2^0} dV \right\} \quad \dots (18)$$

The second term on the right hand side of Eq. (18) is evaluated through the following transformations: [From Eq. (9)]

$$-\nabla^2 \phi_1 = \kappa_1^2 \frac{q}{\Sigma_1} - \kappa_1^2 \phi_1 = \frac{z_1(\kappa_1 r)}{z_1^0 - \lambda \kappa_1 z_2^0} \kappa_1^2 \frac{q}{\Sigma_1} \quad \dots (19)$$

so that

$$\int_{V_1} \phi_1 dV = \frac{q V_1}{\Sigma_1} - \int_{V_1} \left(-\frac{\nabla^2 \phi_1}{\kappa_1^2} \right) dV = \frac{q V_1}{\Sigma_1} + \frac{1}{\kappa_1^2} \int_S \nabla \phi_1 \cdot \hat{n} dS \quad \dots (20)$$

In Eq. (20), use has been made of the theorem of the divergence in order to convert the volume integral into a surface integral. S is the surface which bounds V_1 ; the outward (positive) normal is in the negative r direction at r_0 , so that $\lambda \nabla \phi_1 \cdot \hat{n} = -\phi_1(r_0)$. At r_1 , the gradient of the flux vanishes from Eq. (9c). Eq. (20) may be rewritten as

$$\int_1 \phi_1 dV = \frac{q V_1}{\Sigma_1} - \frac{\phi_1(r_0) S_0}{\lambda \kappa_1^2} = \frac{q V_1}{\Sigma_1} - \frac{q S_0}{\lambda \kappa_1^2 \Sigma_1} \left\{ 1 - \frac{z_1^0}{z_1^0 - \lambda \kappa_1 z_2^0} \right\} \quad \dots (21)$$

332-7

A further simplification can be made by noting that $K_1(x)$ is small for large x and that both $I_0(x)$ and $I_1(x)$ are small for small x . It is evident from Eqs. (13) and (14) that

$$\left. \begin{aligned} z_1^0 &\approx I_1(\kappa_1 r_1) K_0(\kappa_1 r_0) \\ z_2^0 &\approx -I_1(\kappa_1 r_1) K_1(\kappa_1 r_0) \end{aligned} \right\} \dots (22)$$

Hence,

$$\int \phi_1 dV = \frac{q V_1}{\Sigma_1} \left[1 - \frac{2r_0}{\lambda \kappa_1^2 (r_1^2 - r_0^2)} \left\{ 1 - \frac{1}{1 + \lambda \kappa_1^2 K_1^0 / K_0^0} \right\} \right] \dots (23)$$

It follows, therefore, that

$$f' = \frac{\Sigma_{25}}{\Sigma_1} \left[1 - \frac{2r_0}{\lambda \kappa_1^2 (r_1^2 - r_0^2)} \left\{ \frac{\lambda \kappa_1^2 \frac{K_1^0}{K_0^0}}{1 + \lambda \kappa_1^2 \frac{K_1^0}{K_0^0}} \right\} \right] \dots (24)$$

The thermal utilization with the fuel in place is $f = m \Sigma_{25} / \Sigma_1$ where $m > 1$. m is determined by the amount of fuel which is removed when the cadmium rods are inserted. In fact, a given δk can be obtained only by removing fuel (and not inserting any cadmium rods). Evidently, m is determined by the ratio of the amount of fuel present with control rod out to that when the control rod is in place.

$$f - f' = \frac{\delta k}{\eta p} = m \frac{\Sigma_{25}}{\Sigma_1} - \frac{\Sigma_{25}}{\Sigma_1} \left[1 - \frac{2r_0}{\lambda \kappa_1^2 (r_1^2 - r_0^2)} \left\{ \frac{\lambda \kappa_1^2 K_1^0 / K_0^0}{1 + \lambda \kappa_1^2 K_1^0 / K_0^0} \right\} \right] \dots (25)$$

from which

$$r_1^2 = r_0^2 + \frac{\Sigma_{25}}{\Sigma_1} \frac{2\eta p r_0}{\lambda \kappa_1^2 \left[\delta k - \frac{\Sigma_{25}(m-1)\eta p}{\Sigma_1} \right]} \left(\frac{\lambda \kappa_1^2 \frac{K_1^0}{K_0^0}}{1 + \lambda \kappa_1^2 \frac{K_1^0}{K_0^0}} \right) \dots (26)$$

332-8

Since $N = R^2/r_1^2$,

$$N = \frac{1}{\left(\frac{r_0}{R}\right)^2 + \frac{\Sigma_{25}}{\Sigma_1} \frac{2\eta p(r_0/\lambda)}{(\mathcal{H}_1 R)^2 \left[\delta k - \Sigma_{25}/\Sigma_1 (m-1)\eta p\right]} \frac{\lambda \mathcal{H}_1 K_1^\circ/K_0^\circ}{1 + \lambda \mathcal{H}_1 K_1^\circ/K_0^\circ}} \dots (27)$$

TABLE II

TYPICAL PARAMETERS FOR EQ. (27)

	Safety	Regulating	
r_0	2.87 cm	1.91	$\lambda_{tr} = 0.48$ cm
$\left(\frac{r_0}{R}\right)$	0.0014	0.0006	$\mathcal{H}_1 = 0.648$ cm ⁻¹
	0.363 cm	0.379 cm	$R = 76.2$ cm
$\frac{2\eta p}{(\mathcal{H}_1 R)^2} \frac{r_0}{\lambda}$	1.14×10^{-2}	0.725×10^{-2}	
$\mathcal{H}_1 r_0$	1.86	1.24	
K_0°	0.1354	0.3016	
K_1°	0.1684	0.4084	
K_1°/K_0°	1.243	1.355	
$\lambda \mathcal{H}_1 K_1^\circ/K_0^\circ$	0.292	0.342	
$\frac{\lambda \mathcal{H}_1 K_1^\circ/K_0^\circ}{1 + \lambda \mathcal{H}_1 K_1^\circ/K_0^\circ}$	0.226	0.255	

332-9

Since m depends on N , it is convenient to write

$$\overline{\Delta k} = \delta k - \frac{\sum_{25} (n-1) \eta p}{\sum_1}, \quad \dots (28)$$

and then determine N as a function of $\overline{\Delta k}$. The local δk can then be found from Eq. (28), since m is at once determined. For regulating rods, $m = 1$, and $\overline{\Delta k} = \delta k$. The last column in Table III is the one of significance in that it is the pile reactivity.

TABLE III

NUMBER OF SAFETY RODS AS A FUNCTION OF CHANGE IN PILE REACTIVITY

$\overline{\Delta k}$	N	m	δk	$\frac{\Delta k_{\text{eff}}}{k_{\text{eff}}}$ for $J_0^2(\mu \zeta) = 0.885$
0.005	2.91	0.014	0.02	0.065
0.010	5.83	0.0285	0.052	0.167
0.020	11.6	0.0565	0.083	0.272
0.030	17.4	0.085	0.12	0.394

Since the regulating rods have no fuel sections, $m = 0$. Table III-a gives the results for regulating rods.

TABLE III-a

REGULATING RODS

δk	$\frac{\Delta k_{\text{eff}}}{k_{\text{eff}}}$ for $J_0^2(\mu \zeta) = 0.79$	N
0.001	0.003	0.813
0.005	0.0147	4

332-10

III. SHIM CONTROL

The basis of the shim control calculation is that at the beginning and end of the run, the same k/p obtains. At the beginning,

$$\frac{k}{p} = \frac{\eta_{25} \sum_{25}^o \frac{\phi_{25}}{\phi}}{\sum_{25}^o \frac{\phi_{25}}{\phi} + \sum_{o2}^o \frac{\phi_{o2}}{\phi} + \Sigma'} \quad \dots (29)$$

where the subscript o2 refers to thorium, the superscript o to original conditions, ϕ_1/ϕ to the average of the flux in a given material to that in the reactor, and Σ' to all of the other absorbers not specifically accounted for. It is assumed that the fast fission factor for U^{235} (and U^{233}) is unity. At the end of the run,

$$\frac{k}{p} = \frac{\eta_{25} \sum_{25} \frac{\phi_{25}}{\phi} + \eta_{23} \sum_{23} \frac{\phi_{o2}}{\phi}}{(\sum_{o2} + \sum_{23}) \frac{\phi_{o2}}{\phi} + \sum_{25} \frac{\phi_{25}}{\phi} + \Sigma'} \quad \dots (30)$$

Above, it is assumed that Σ' is constant; 23 refers to U^{233} .

At any given time $\sum_{25} = N_{25} \sigma_{25}$. N_{25} , the number of U^{235} atoms/cm³ of the reactor, is determined as outlined below. It is advisable to review the neutron economy of the reactor, briefly, at this point. The number of neutrons/cm³ absorbed in U^{235} is $\sum_{25} \phi_{th}$ (ϕ_{th} is the thermal flux) and $\eta_{25} \phi_{th} \sum_{25}$ fast neutrons are produced. Of this number,

$\eta_{25} \sum_{25} \phi_{th} (1 - e^{-\kappa^2 \tau_r})$ (τ_r is the age to resonance) leak out above resonance and $\eta_{25} \sum_{25} \phi_{th} e^{-\kappa^2 \tau} (1 - p)$ are absorbed in the thorium resonances. The number of neutrons becoming thermal is given by $\sum_{25} \phi_{th} \eta_{25} p e^{-\kappa^2 \tau}$ of which $p \eta e^{-\kappa^2 \tau} \sum_{25} (1 + \kappa^2 L^2)^{-1} \Sigma^{-1} \phi_{th} \sum_{25}$ are absorbed in U^{235} , $p \eta e^{-\kappa^2 \tau} \sum_{o2} (1 + \kappa^2 L^2)^{-1} \Sigma^{-1} \phi_{th} \sum_{25}$ are absorbed in Th^{232} , and $p \eta e^{-\kappa^2 \tau} \kappa^2 L^2 (1 + \kappa^2 L^2)^{-1} \sum_{25} \phi_{th}$ leak thermally from the core. The

332-11

The equation describing the behavior of U^{235} is

$$\begin{aligned} \frac{dN_{25}}{dt} &= -N_{25} \sigma_{25} \phi_{th} \left[1 + \eta_{25} (1 - e^{-\kappa^2 \tau}) \frac{\sigma_{25}^{fast}}{\sigma_{25}} \right] \\ &= -N_{25} \sigma_{25} \phi_{th} M. \end{aligned} \quad \dots(31)$$

One may think of the reactor as being divided into an inner and an outer region (the method of division to be given later). In the equations (one for each region) describing the burn-up, the flux will be the average (constant) value throughout each region; these values will be called $\langle \phi_1 \rangle$ and $\langle \phi_2 \rangle$ for the inner and outer regions, respectively. The burn-up of the U^{235} depends on the average flux, and hence is different for the different regions of the reactor; this consideration will give information in regard to a desirable loading schedule.

The burn-ups are determined from an integration of Eq. (31) with $\langle \phi_i \rangle$ replacing ϕ_{th} for each region. Hence,

$$N_{25}^{(1)}(t) = N_{25}^{(1)}(0) e^{-H\sigma_{25} \langle \phi_1 \rangle t} \quad \dots(32)$$

$$N_{25}^{(1)}(t) = \frac{1}{V_R} N_{25}^{(1)}(0) \int_{V_R} e^{-H\sigma_{25}\phi(r)t} dV, \quad \dots(33)$$

11/12

where V_R is the volume of the reactor. From Eq. (32),

$$1 - \frac{N_{25}^{(1)}(t)}{N_{25}^{(1)}(0)} = 1 - e^{-M\sigma_{25} \langle \phi_1 \rangle t} = x_1. \quad \dots(34)$$

The areas of the regions in which the burn-ups are x_1 and x_2 are determined from

$$\frac{\ln \frac{1}{1-x_1}}{\ln \frac{1}{1-x_2}} = \frac{\langle \phi_1 \rangle}{\langle \phi_2 \rangle}, \quad \dots(35)$$

where [it is not too unreasonable to assume $\phi = J_0(\alpha r)$]

$$\langle \phi_1 \rangle = \frac{1}{\pi r_1^2} \int_0^{r_1} \int_0^{2\pi} J_0(\alpha r) r dr d\theta, \quad \dots(36)$$

$$\langle \phi_2 \rangle = \frac{1}{\pi(R^2 - r_1^2)} \int_{r_1}^R \int_0^{2\pi} J_0(\alpha r) r dr d\theta, \quad \dots(37)$$

where R is the reactor radius and r_1 is the radius of the inner region. It follows that

$$\frac{\langle \phi_1 \rangle}{\langle \phi_2 \rangle} = \frac{\frac{R}{r_1} J_1(\alpha r)}{\frac{1}{1 - r_1^2/R^2} [J_1(\alpha R) - r_1/R J_1(\alpha r)]}. \quad \dots(38)$$

If one assumes that the flux goes to zero at $r = R$, then the inner region is of negligible size for $x_1 = 0.2$ and $x_2 = 0.1$ (desirable values). More comparable sizes for the inner and outer regions are obtained for fluxes which fall only to 0.1 or 0.2 at $r = R$. Table IV gives the division of the reactor according to the ratio of the average fluxes.

332-13

TABLE IV
RATIO OF AVERAGE FLUXES IN THE TWO REGIONS

$\frac{r_1}{R}$	$\langle \phi_1 \rangle / \langle \phi_2 \rangle$	
	$J_0(\alpha R) = 0.1$	$J_0(\alpha R) = 0.2$
0.05	2.0072	1.7805
0.10	2.0131	1.7840
0.2	2.0374	1.7982
0.3	2.0806	1.8228
0.4	2.1471	1.8601
0.5	2.2451	1.9128
0.6	2.3890	1.9863
0.7	2.6063	2.0889
0.8	2.9236	2.2358
0.9	3.5824	2.4556

For burn-ups $x_1 = 0.2$ and $x_2 = 0.1$, the right hand side of Eq. (35) is 2.120. From Table IV, the division of the reactor will be for $0.3 < r_1/R < 0.4$ for the first case and $0.7 < r_1/R < 0.8$ for the second case.

The above information provides a means of obtaining Σ_{25} at any time. One may write

$$\Sigma_{25} = \Sigma_{25}^0 \left[\frac{n_F + a_1 n_s}{n_F} - \left\{ \left(\frac{r_1}{R} \right)^2 + \left(1 - \frac{r_1^2}{R^2} \right) x_2 \right\} - \frac{a_1 n_s}{n_F G} (x_1 y_1 + x_2 y_2) \right], \quad \dots(39)$$

where n is the number of elements, F refers to fuel, s to shim rods, a_1 is the ratio of fuel in shim rods to that in original assemblies. G^{-1} is the fraction of the burn-up assumed for shim rods to that for original assemblies.

y_1 is the fraction of the shim elements in region 1 and y_2 in region 2. Eq. (39) is obtained by accounting for all processes occurring during the run. The first term accounts for the total cross section of U^{235} in the absence of burn-up (at the end of the run), the second term for the burn-up of the original fuel elements, and the third for the burn-up of the shim elements. If the burn-up of the thorium is neglected, one may write

$$\Sigma_{o2} = \Sigma_{o2}^o \left[\frac{n_{o2} - a_2 n_2}{n_{o2}} \right], \quad \dots(40)$$

where a_2 is the ratio of the thorium in the shim elements to that in the original feed. Σ_{23} may be determined from the graphs on Pages 40 and 41 of NAA-SR-Memo-200.

Enough information is now available to obtain n_g/n_f , which will allow the required burn-ups. The requirement that k/p is the same at the beginning and end of the run gives [from Eqs. (29) and (30)]

$$\Sigma_{25} = \frac{\frac{k}{p} \left(\Sigma_{o2} \frac{\phi_{o2}}{\phi} + \Sigma' \right)}{\frac{\phi_{25}}{\phi} \left(\eta_{25} - \frac{k}{p} \right)} + \frac{\Sigma_{25} \frac{\phi_{o2}}{\phi} \left(\frac{k}{p} - \eta_{23} \right)}{\frac{\phi_{25}}{\phi} \left(\eta_{25} - \frac{k}{p} \right)}. \quad \dots(42)$$

If one equates Eqs. (39) and (42) and solves for n_g/n_f , the following is obtained:

$$\frac{n_g}{n_f} = \frac{\frac{x_1^2}{R^2} x_1 + \left(1 - \frac{x_1^2}{R^2} \right) x_2 - \frac{(\eta_{23} - k/p)}{(\eta_{25} - k/p)} \frac{\Sigma_{23} \phi_{o2}}{\Sigma_{25} \phi_{25}}}{a_1 \left\{ 1 - G^{-1}(x_1 y_1 + x_2 y_2) + \frac{n_f}{n_{o2}} \frac{k}{p} \frac{a_2}{a_1} \frac{\Sigma_{o2}^o \phi_{o2}}{\Sigma_{25}^o \phi_{25}} \frac{1}{\eta_{25} - k/p} \right\}}. \quad \dots(43)$$

The third term in the numerator of Eq. (43) is readily obtainable

since

$$\Sigma_{23} = \Sigma_{o2}^o \frac{\sigma_{23} n_{23}}{\sigma_{o2} n_{o2}}, \quad \dots(44)$$

332-151

where one may obtain $N_{23}/N_{\alpha 2}^0$ from Page 41 or 42 of NAA-SR-Memo-200. One needs to know the time in days in order to determine N_{23} . The time, for Eq. (34), may be written as

$$t = \frac{2.303}{\langle \phi_1 \rangle \sigma_{25} N} \left\{ \log \frac{1}{1 - z_1} \right\} 1.16 \times 10^{-5} (\text{days}) \quad \dots (45)$$

TABLE V

TYPICAL PARAMETERS FOR EQ. (43)

$\frac{F_1}{R}$	0.706	k/p	$\frac{1.105}{0.84} = 1.315$
z_1	0.2	$\frac{\phi_{\alpha 2}}{\phi_{25}}$	$\frac{1}{1.3} = 0.77$
z_2	0.1	Σ_{25}^0	0.0897
y_1	0	$\Sigma_{\alpha 2}^0$	0.0347
y_2	1	η_{25}	2.11
q	2	η_{23}	2.35
$\frac{N_T}{N_{\alpha 2}}$	$\frac{205}{76} = 2.7$	t	76.5 days
a_1	$\frac{14}{18} = 0.78$	σ_{23}	564
a_2	$\frac{2}{3} = 0.67$	$\sigma_{\alpha 2}$	7.2
$\langle \phi_1 \rangle$	5×10^{13} neut/ $\text{cm}^2/\text{sec.}$	$N_{23}/N_{\alpha 2}^0$	1.10×10^{-3}
ϕ_{max}	7.65×10^{13}	Σ_{23}	0.0030
		n_{α}/n_T	0.0715
		n_{α}	14.7

^a This value was used so as to utilize calculations of Memo-200.

332-16

Table V gives the results for a specific design of the converter reactor which has been considered. Similar results are readily obtainable for other designs.

The writer would like to thank E. Richard Cohen for many helpful suggestions during the preparation of this memo.

END

332-17

SECURITY INFORMATION