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MATHEMATICS PANEL QUARTERLY PROGRESS REPORT

SUMMARY

Two new members and one research participant have joined the Mathematics Panel during the current quarter. Susie Ezzell, one of the new members, is from Abbeville, Alabama. She is a graduate of the University of Alabama and is now on temporary loan to ANP at Y-12. Herbert Goertzel, the other new member, is from New York City, and he is a graduate of New York University. Stacy Hull has joined the Panel as a Research Participant from the University of Arkansas for a period of one year.

Ernest Ikenberry who was with the Panel as a Research Participant during the summer, has returned to Alabama Polytechnic Institute. Barbara McGill and Rosemary Crook also left the Panel during the current quarter.

The General Electric Company has permanently loaned a special-purpose digital computer to ORNL. This computing instrument (formerly referred to as the NEPA machine) will be installed in the new research building and operated by the Panel. The computer was designed to solve large systems of linear algebraic equations, but can also be used for a larger class of problems.

Good progress is being made at Argonne on the Oak Ridge digital computer. For testing purposes a ten-stage shifting register, and a newly designed dynamic programmer have been constructed and found to operate successfully. The shifting registers in the machine itself constitute the working part of the arithmetic organ, and the dynamic programmer controls the sequence of transfers that take place in the course of a given arithmetic operation.

Work continues on the memory tubes in the hope of increasing the number of consultations without regeneration at a particular location before disturbing the contents of neighboring locations. Present performance would be satisfactory if care were taken in coding a problem to insure regeneration after a maximal number of consultations. However, this is an extra burden upon the coder, and would slow up the operation whenever special provisions for regeneration have to be inserted in the coding. Construction of the control will begin soon, following fabrication of the frame and purchase of a power supply.

The Mathematical Group at Argonne as well as at Oak Ridge are working up standard subroutines such as will form necessary parts of many different computations. These include routines for multiple-precision and floating-decimal computation; evaluation of standard functions, both transcendental and algebraic; binary-decimal and decimal-binary conversion; the solution of equations; and the like. Large libraries of routines will be essential for rapid coding, and it is important to have a good one prepared when the machine is ready to go into operation.

A course in the techniques of problem preparation, to be given December 3 to 14, is being planned with the cooperation of the Special Training Division, ORINS. The course will include both lectures and practice sessions. Lectures are to be given by

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John von Neumann, Institute for Advanced Study; A. H. Taub and J. P. Nash, University of Illinois; D. A. Flanders, Argonne National Laboratory; and members of the Mathematics Panel. The purpose is both to provide an intensive training program for all members of the Mathematics Panel, and at the same time to introduce the subject to teachers and prospective future employees and research participants.

Two large computing projects were completed this quarter: "Angular Correlation Between Conversion Electrons and Gamma Rays," and "Kinetics of HRE." B. M. Drucker, ORINS Fellow with the Panel, assisted in the numerical integration of several systems of nonlinear differential equations on the SEAC (cf., "Kinetics of HRE"). The coding for the calculation of the *L*-Shell internal conversion coefficients on the SEAC is being continued in Washington (cf., "Calculation of Internal Conversion Coefficients with Screening").

J. Moshman participated in the Naval Medical Research Institute Conference "On the Twenty-Eight-Day Lethality Study of Operations Greenhouse." The conference was held at Bethesda, Maryland.

A. S. Householder was a consultant at Project Rand during August. While there he attended the *Symposium on Simultaneous Linear Equations and the Determination of Eigenvalues*, sponsored by the National Bureau of Standards.

Members of the panel presented the following contributed and invited papers:

G. E. Albert and Lewis Nelson

"Contribution to the Statistical Theory of Counter Data," Minneapolis Meeting of the American Mathematical Society.

J. W. Givens

"A Method of Computing Eigenvalues and Eigenvectors Suggested by Classical Results on Symmetric Matrices," *Symposium on Simultaneous Linear Equations and the Determination of Eigenvalues*, sponsored by the National Bureau of Standards.

A. S. Householder

"The Geometry of Some Iterative Methods for Solving Linear Equations," *Symposium on Simultaneous Linear Equations and the Determination of Eigenvalues*, sponsored by the National Bureau of Standards.

The following lectures were presented by members of the Panel as a part of the ORINS Traveling Lecture Program:

A. W. Kimball

"Microbiological Assays with Nonparallel Response Curves," Virginia Polytechnic Institute, August 20, 1951.

"On Dependent Tests of Significance in the Analysis of Variance," Virginia Polytechnic Institute, August 21, 1951.

W. C. Sangren

"Mathematics at Oak Ridge," Western Michigan College, October 17, 1951.

A report of the proceedings of a symposium on the Monte Carlo Method held in Los Angeles, California, June 29, 30, and July 1, 1949, was published by the United States Government Printing Office during the quarter. A. S. Householder was the editor of this publication, which included the following papers by members of the Panel (Panel members names are in italics):

A. S. Householder

"Neutron Age Calculations in Water, Graphite, and Tissue."

Wendell De Marcus, *Lewis Nelson*

"Methods of Probabilities in Chains Applied to Particle Transmission Through Matter."

B. A. Shoor, *Lewis Nelson*, Wendell De Marcus, Robert L. Echols

"A Monte Carlo Technique for Estimating Particle Attenuation in Bulk Matter."

Other papers by Panel members that appeared during the quarter were:

J. Moshman

"Hematological Effects of Chronic Low-Level Irradiation," *The Journal of Applied Physiology*, August 1951.

"On the Specificity of Hypervolemia in Certain Tumor-Bearing Mice," *Cancer Research*, July 1951.

PROJECTS

METHODS OF COMPUTATION FOR USE WITH A HIGH-SPEED AUTOMATIC-SEQUENCED COMPUTER*

Participating Member of Panel. Wallace Givens.

Background and Status. The advent of high-speed automatic-sequenced digitial computers, has made it necessary to (1) revise existing techniques of numerical computation to completely mechanize all steps now involving the judgment of the operator, (2) invent methods for numerical solution of problems previously too long to be worked, (3) evaluate all proposed methods as to their economy of machine time, and (4) obtain exact and perhaps probabilistic upper bounds for the effect of round-off error on the quantities calculated. Other problems of numerical analysis that remain significant and are related to these, are the ones most affected by the specific characteristics of the Oak Ridge computer—high speed, so that continuing inspection of results is impossible; and, operations numbering in the millions so that round-off error may accumulate disastrously.

A digital computer can carry out only the operations of arithmetic and certain logical choices based on the comparison of numbers, therefore, every problem must be arithmetized before it can be coded for solution. The present emphasis in this long-range study is on the fundamental algebraic problems that comprise the last stage of the computation.

Problem 1. Fields of Values of a Matrix.

Background. While studying known methods of estimating the maximum of the absolute values of the eigenvalues of a matrix, it was noted that (1) many estimates were actually upper bounds for the radius of the circle about the origin containing the field of values of the matrix, and (2) the definition of field of values required the introduction of a metric that was irrelevant to the eigenvalue problem. An investigation of the role of the (arbitrary) metric was undertaken with a view to possible improvement in bounds for eigenvalues.

Results. Define the field of values $F_H(A)$ of a real or complex square matrix A relative to a Hermitian metric H to be the set of complex numbers given by

$$\rho = \frac{(x,\ A\ x)_H}{(x,\ x)_H},$$

*Cf. problems 1, 2, 3.

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where $(x, y)_H = x^* H y$, x and y are column vectors with complex numbers as components, x^* is the conjugate transpose of x, and H is positive definite Hermitian. Then, (1) the fields of values obtained by holding H fixed and allowing A to vary over all similar matrices are the same as those obtained by fixing A and allowing H to vary; (2) the intersection of the regions $F_H(A)$ for fixed A is the minimum convex polygon containing all the roots of A; (3) a necessary and sufficient condition that $F_H(A)$ is the minimum polygon of (2) for some choice of H is that the elementary divisors corresponding to roots lying on the boundary shall all be simple.

Status. A paper describing this problem has been accepted for publication in the *Proceedings of the American Mathematical Society*. Additional work in this area may prove desirable, but the problem is inactive at present.

Problem 2. Numerical Inverting to Matrices of High Order.

Background and Status. This is a study of known methods of inverting large matrices with major emphasis on obtaining bounds for the error introduced by round-off. A fundamental paper of this title by von Neumann and Goldstine has been worked through in detail. The von Neumann-Goldstine paper is being presented in a series of lectures before the Mathematics Panel Seminar.

Problem 3. Numerical Computation of Eigenvalues and Eigenvectors.

Background. Let a real symmetric matrix A be given and let it be required to find all eigenvalues and eigenvectors of A. The problem is essentially equivalent to the determination of an orthogonal matrix T $(T'T = I)$ such that $T' A T = D$, where T' is the transpose of T, and D is diagonal, with real numbers $\lambda_1, \ldots, \lambda_n$ on the diagonal. The computational problem involves two major difficulties: the number of components, $\frac{1}{2} n (n + 1)$, of A increases rapidly with the order of A and this requires large storage and many operations; and, since the λ_i are the roots of the determinantal equation $\det (x I - A) = 0$ of order n, they are irrational functions of the components of A and must be calculated by a method of successive approximation. The results of the present study can be viewed as a method of separating these difficulties so they can be encountered singly rather than simultaneously.

Results. By a sequence of at most $\frac{1}{2} (n-1) (n-2)$ fully determined rotations in coordinate planes, a real symmetric matrix A of order n can be reduced to the form

$$S = \begin{vmatrix} \alpha_1 & \beta_1 & 0 & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ \beta_1 & \alpha_2 & \beta_2 & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & \beta_2 & \alpha_3 & \beta_3 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & \beta_3 & \alpha_4 & \beta_4 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & & & & \\ \cdot & \cdot & \cdot & \cdot & \cdot & & & & \\ \cdot & \cdot & \cdot & \cdot & \cdot & \beta_{n-2} & \alpha_{n-1} & \beta_{n-1} & \\ 0 & 0 & 0 & 0 & 0 & 0 & \beta_{n-1} & \alpha_n & \end{vmatrix} \qquad (1)$$

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The computation of this reduced "triple diagonal form" of A requires approximately $\frac{4}{3} n^3$ multiplications, n^2 divisions, and $\frac{1}{2} n^2$ square roots.

The characteristic equation of S, $f_n(\lambda) = 0$ (which is the same as that of A), is obtained by the recursion formula

$$f_i(\lambda) = (\lambda - \alpha_i) f_{i-1}(\lambda) - (\beta_{i-1})^2 f_{i-2}(\lambda) , \quad (2)$$

where $f_{-1} = 0$, $f_0 = 1$, and $i = 1, 2, \ldots, n$.

The determination of the roots of $f_n(\lambda) = 0$, which are the eigenvalues of A except for the effect of round-off error, is facilitated by the fact that the polynomials $f_i(\lambda)$ are a Sturm sequence when all the β_i are different from zero. Then the number of eigenvalues of S greater than any chosen real number a, is equal to the number of variations of sign in the sequence $1 = f_0(a), f_1(a), \ldots, f_{n-1}(a), f_n(a)$. Formula 2 is readily applied to the automatic computation of such sequences. If the matrix S is scaled so that its eigenvalues lie between -1 and +1, each of the n eigenvalues can be located within an interval of length 2^{-s}, by a computation involving no more than $2(s + 1)(n - 1) n \approx 2 s n^2$ multiplications. Thus for $n = 100$ and $s = 40$, it appears that $\frac{4}{3} n^3 + 80 n^2 \approx 2$ million multiplications would suffice to determine the eigenvalues. In this evaluation only multiplications have been considered since additions are much faster than multiplications, and divisions and square roots are not numerous. This is of the order required for two multiplications of matrices (unsymmetric) of this size and is smaller than expected.

The components x_i of the eigenvector of S corresponding to an eigenvalue ρ can now be calculated by the recursion formula

$$x_{i+1} = (\beta_i)^{-1} [(\rho - \alpha_i) x_i - \beta_{i-1} x_{i-1}] , \quad (3)$$

where $i = 1, \ldots, n-1$, $\beta_0 = x_0 = 0$, and for $i = n$, $\beta_n = 1$, and $x_{n+1} = 0$ is a check on the computation. When any $\beta_i = 0$, the determination of the eigenvalues can be replaced by the corresponding problem for two smaller matrices. Formula 3 fails in case any $\beta_i = 0$, reflecting the fact that in this case the eigenvector has either its first i or its last $n-i$ components zero or is mathematically indeterminate. Finding the eigenvectors of A when those of S have been calculated makes heavy demands on the memory of the machine and requires additional computation with attendant round-off error, but is easily described mathematically.

Status. The above results have been presented to the Mathematics Panel Seminar and at the "Symposium on Simultaneous Linear Equations and the Determination of Eigenvalues" sponsored by the National Bureau of Standards at U.C.L.A. on August 23-25, 1951. Bounds for the effect of round-off error are needed. How small values of β_i should be treated requires additional study. This study will be continued.

BASIC STUDIES IN THE MONTE CARLO METHOD

Participating Member of Panel. G. E. Albert.

References. (a) G. E. Albert, "Basic Studies in the Monte Carlo

Method," *Mathematics Panel Quarterly Progress Report* ORNL-1091; and (*b*) G. E. Albert, Memorandum to A. S. Householder on "A General Approach to the Monte Carlo Estimation of the Solutions of Certain Fredholm Integral Equations," Parts I and II.

Status. Part II of reference (*b*) was issued October 4, 1951 and given the same circulation as Part I.

The plan of Part II as issued is different from the plan proposed in reference (*a*); it ends with the explanation of representative sampling for chain lengths. Research on the structure of the stochastic processes needed for an incorporation of representative sampling of chain points into the estimation of the series (1) of reference (*a*) has proved difficult. This subject will be treated in a third part for the memoranda of reference (*b*).

COUNTER STATISTICS

Participating Members of Panel. G. E. Albert and Lewis Nelson.

Status. An ORNL report that incorporates results on confidence intervals (abstracted in previous Quarterly Reports) is being prepared. The report will be sufficiently explanatory for easy use of the results.

TO FIND BOUNDS FOR THE MAXIMUM OF A FUNCTION ARISING IN THE WEIGHTING OF OBSERVATIONS

Origin. A. W. Kimball, Mathematics Panel.

Participating Member of Panel. Wallace Givens.

Background. Given a set of n observations $y_{i\alpha}$ ($i = 1, \ldots, n$; $\alpha = 1, \ldots, p$) of p variates, to find weight factors l_α such that if $x_{i\alpha} = l_\alpha y_{i\alpha}$, the ratio

$$R = \frac{\sum_{i,\alpha} (\bar{x}_{i.} - \bar{x}_{..})^2}{\sum_{i,\alpha} (x_{i\alpha} - \bar{x}_{.})^2}$$

is an extremal, where

$$\bar{x}_{..} = \frac{1}{np} \sum_{i,\alpha} x_{i\alpha} ,$$

and

$$\bar{x}_{i.} = \frac{1}{p} \sum_{\alpha} x_{i\alpha} .$$

In the present problem only an upper bound for this extremal was sought.

Results. The extreme values in question can be shown to be roots of the equation determinant$(M) = 0$, where

$$M = -\frac{1}{n} \left\| z_\alpha z_\beta \right\| + (\theta + 1) S - p\,\theta \left\| \text{diagonal } (S_{11}, \ldots, S_{pp}) \right\| ,$$

$$S_{\alpha\beta} = \sum_i y_{i\alpha} y_{i\beta}, \text{ and } z_\alpha = \sum_i y_{i\alpha} .$$

For a special case in which all $z_\alpha = 0$, it was shown that for a suitable choice of $y_{i\alpha}$, p roots could be $d_i/(p - d_i)$, for an arbitrary choice of p positive numbers d_i with sum p. Hence in this special case no bounds for the roots

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can be given other than zero and infinity. Some consideration was given to the appropriate approach to the problem should other restrictions be put on the observations.

Status. This problem has been completed.

A METHOD FOR TREATING CERTAIN SECOND-ORDER NONLINEAR DIFFERENTIAL EQUATIONS

Participating Member of Panel. W. C. Sangren.

Background and Status. It is possible in an elementary way to solve or at least obtain an integration constant for large classes of second-order nonlinear differential equations. Some equations of reactor kinetics fall into these classes. The elementary method consists of factoring a differential equation and then either solving in sequence two first-order equations or solving a simpler second-order equation. The equations considered are all of the form

$$H(t, y, y')\, y'' = F(t, y, y'),$$

where the primes denote differentiation with respect to t. The equations actually considered fall into the following three general classes:

$$\left[\frac{d}{dx_1} + f(x_1)\right][h(x_1, x_2, x_3)] = 0 , \qquad (1)$$

$$\left[\frac{d}{dx_1} + g(x_1, x_2, x_3)\right][G\{g(x_1, x_2, x_3)\}] = 0 , \qquad (2)$$

and

$$\left[\frac{d}{dt} + k(t, y)\right][K\{k, k', t\}] = 0 , \qquad (3)$$

where x_1, x_2, x_3 represent t, y, y' in any order, and where f, h, g, G, k, K are functions of their respective variables. Classes 1 and 2 can be separated into three subclasses according to whether x_1 equals t or y or y'.

For class 1 the equation may be written as $\frac{dv}{dx_1} + f(x_1)\, v = 0$, where $v = h(x_1, x_2, x_3)$ is considered to be a function of x_1. The solution of $\frac{dv}{dx_1} + f(x_1)\, v = 0$ is $v = c\, e^{-\int^{x_1} f(z)dz}$, where c is an integration constant. Thus, $h(x_1, x_2, x_3)\, e^{\int^{x_1} f(z)dz} = c$. The equations of class 1 can therefore be solved if one can solve this last first-order equation.

For class 2 the equation may be written as $\frac{dw}{dx_1} + s(x_1)\, w = 0$, where $s = g(x_1, x_2, x_3)$, and consequently $w = G\{g\}$, are considered to be functions of x_1. Multiplying by the integrating factor $r(x_1) = e^{\int^{x_1} s(z)dz}$ and then integrating gives $r(x_1)\, w(x_1) = c$, where c is an integration constant.

Since $\frac{dr}{dx_1} = s\, r$, it follows that

$$s = \frac{\frac{dr}{dx_1}}{r}.$$

Further, since $s = g(x_1, x_2, x_3)$ and $w = G\{g\}$, it follows that

$$r\, G\left\{\frac{\frac{dr}{dx_1}}{r}\right\} = c.$$

The equations of class 2 can therefore be solved if one first solves the first-order equation

$$r\, G\left\{\frac{\frac{dr}{dx_1}}{r}\right\} = c$$

for $r(x_1)$, and then solves the first-order equation

$$s(x_1) = g(x_1, x_2, x_3) = \frac{\frac{dr}{dx_1}}{r}$$

for y.

The procedure for class 3 equations is similar to that for class 2. In mathematical outline the procedure is:

(a) $$\frac{dw}{dt} + s(t)\, w(t) = 0 ,$$

where

$$s = k(t, y) ,$$

and

$$w = K\{k, k'\ t\} ,$$

(b) $$r(t)\, w(t) = c,$$

where

$$r = e^{\int^t s(z)dz} ,$$

and c is an integration constant,

(c) $$r'(t) = s\, r ,$$

and therefore

$$s = \frac{r'}{r},$$

(d) $$r(t)\, K\left[\frac{r'}{r}, \frac{r''}{r} - \frac{r'^2}{r^2}, t\right] = c ,$$

and (e), solve the second-order equation of (d) for $r(t)$ and then substitute for $r(t)$ and $r'(t)$ in the equation $k(t, y) = \frac{r'(t)}{r(t)}$ in order to obtain y either implicitly or explicitly.

Particular solutions of class 1, 2, and 3 can be found by merely setting the inner factors equal to zero, i.e., for class 1 let $h(x_1, x_2, x_3) = 0$, for class 2 let $G\{g(x_1, x_2, x_3)\} = 0$, for class 3 let $K\{k, k', t\} = 0$, and then solving for y for all three classes.

INTEGRATION IN THE PHASE PLANE

Participating Member of Panel. W. C. Sangren.

Background and Status. Some nonlinear second-order differential equations, including some of interest in reactor kinetics, can be solved or at least their first integral found by replacing the second-order equations by two first-order equations that may be integrated directly by quadrature in the resulting phase plane. The following three types of equations can be treated in such a fashion:

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$$\ddot{y} = \frac{\frac{dH}{dy}\dot{y}^2}{H(y)} + \frac{\dot{r}(t)}{r(t)}\dot{y} + F\left[\frac{\dot{y}}{rH}\right] G(y)\, r^2(t)$$

replaced by

$$\dot{y} = x\, H(y)\, r(t) ,$$

$$\dot{x} = F(x)\frac{G(y)}{H(y)} r(t) ; \qquad (1)$$

$$\ddot{y} = \frac{\dot{r}(t)}{r(t)}\dot{y} + f\left[\frac{\dot{y}}{rl}\right] l(y)\frac{dl}{dy} r^2(t)$$

replaced by

$$\dot{y} = x\, l(y)\, r(t) ,$$

$$\dot{x} = [f(x) - x^2]\frac{dl}{dy} r(t) ; \qquad (2)$$

$$\ddot{y} = \frac{g(y)}{h^2(y)}\dot{y}^2 + \frac{\dot{r}}{r}\dot{y}$$

replaced by

$$\dot{y} = x\, h(y)\, r(t) ,$$

$$\dot{x} = \frac{x^2}{h(y)}\left[g(y) - \frac{dh}{dy} h(y)\right] r(t) ; \qquad (3)$$

where the dot denotes differentiation with respect to t. There are numerous special cases that are interesting, e.g., $\ddot{y} = F(\dot{y})\, G(y)$.

NITROGEN DISTRIBUTION IN *DICTYOSTELLIUM DISCOIDEUM*

Origin. J. H. Gregg, Biology Division.

Participating Members of Panel. A. W. Kimball and G. J. Atta.

Background and Status. As part of a larger study on the physiological properties of the slime mold *Dictyostellium discoideum*, micro-Kjeldahl determinations for nitrogen in the slugs and culms were carried out in three different experiments. The results are shown in Table 1, where n is the number of determinations and $\bar{x}$ is the arithmetic mean.

TABLE 1

Average Nitrogen Distribution

EXPERIMENT	SLUGS		CULMS	
	n	$\bar{x}$	n	$\bar{x}$
1	4	0.0940	4	0.0725
2	4	0.0545	5	0.0558
3	5	0.1100	3	0.0900

Using Snedecor's method[1] for disproportionate subclass numbers, the analysis of variance in Table 2 was computed.

TABLE 2

Analysis of Variance

SOURCE OF VARIATION	DEGREES OF FREEDOM	MEAN SQUARE
Slugs vs. culms	1	0.0009879
Among experiments	2	0.0042979
Interaction	2	0.0003452
Experimental error	19	0.0002033
TOTAL	24	

On the basis of these few determinations, the nitrogen content of slugs exceeds that of culms by an

[1] G. W. Snedecor, *Statistical Methods* (4th ed.; Ames, Ia.: Iowa State College Press, 1946), p. 289.

amount which is significant at the 5% level. There is, however, definitely more variation among experiments than within experiments. In the future an attempt will be made to increase the sensitivity of these comparisons by discovering and eliminating the wide variation among experiments.

The observed difference between slugs and culms over all three experiments is 0.0181. It can be shown[2] that a real difference of this magnitude can be detected at the 5% level with a probability of 0.95 if about 18 determinations are obtained in each group. Thus to control the second kind of error at about 5%, the number of observations should be increased about 50%.

ANALYSIS OF GENETICS EXPERIMENT WITH *DROSOPHILA*

Origin. W. K. Baker, Biology Division.

Participating Members of Panel. A. W. Kimball and G. J. Atta.

Background and Status. As part of the radiation genetics program with *Drosophila*, F_1 male × peach female and F_1 female × peach male, in *Drosophila virilis* resulted in offspring which were classified as wild type or peach, male or female. According to theory the frequencies in these four groups should be in a 1:1:1:1 ratio. It was desired to test this hypothesis statistically.

The experiment was analyzed by a powerful but seldom used method of partitioning χ^2 into individual degrees of freedom. Fisher[3] has shown that if x is a linear function of observed frequencies, such as

$$x = k_1 a + k_2 b + \ldots + k_m z ,$$

and if the theoretical probability in the i^{th} class is given by p_i, the mean value of x will be

$$n \Sigma p_i k_i$$

and its sampling variance will be

$$n [\Sigma p_i k_i^2 - (\Sigma p_i k_i)^2] ,$$

where $n = a + b + \ldots + z$. If a second linear function of the observed frequencies, y, contains coefficients k_i', then the covariance of x and y is

$$n [\Sigma p_i k_i k_i' - (\Sigma p_i k_i) (\Sigma p_i k_i')] .$$

By choosing a set of linear functions which have zero means and zero covariances (and are therefore orthogonal) the $(m-1)$ degrees of freedom for χ^2 can be partitioned into individual degrees of freedom.

In the present experiment, $m = 4$ and a χ^2 with 3 degrees of freedom was computed for each mating in each cross. Of the 28 values of χ^2 only two were less probable than 0.05 under the null hypothesis. On this basis alone one would tend to conclude that the experiment agreed reasonably well with theory. Each χ^2 was further partitioned into individual degrees of freedom corresponding to the three contrasts (1) wild type vs. peach, (2) male vs. female, and (3) the interaction between (1) and (2). These values were summed in each cross giving a χ^2 with 14 degrees of freedom for each contrast:

CROSS	CONTRAST (1)	(2)	(3)
F_1 male × peach female	22.22	7.47	23.48
F_1 female × peach male	13.98	14.70	11.92

(2) M. Harris, D. G. Horvitz, and A. M. Mood, "On the Determination of Sample Sizes in Designing Experiments," *J. Am. Stat. Assn.* 43, 396-7 (1948).

(3) R. A. Fisher, *Statistical Methods for Research Workers* (10th ed., sec. 55; Edinburgh, London: Oliver and Boyd, 1946).

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In the first cross there seems to be some evidence of a departure from expectation in the wild type vs. peach classes and in the interaction, whereas the second cross does not manifest any departure from hypothesis. It is possible that some explanation of this difference can be found; in any case the evidence is sufficiently strong to warrant further experimentation. It should be noted that analysis by simple X^2 methods, without partitioning, would not have uncovered such evidence.

FERRITIN AND HEMOSIDERIN PRODUCTION IN LIVERS OF NORMAL MICE

Origin. J. K. Hampton and J. B. Kahn, Biology Division.

Participating Member of Panel. A. W. Kimball.

Background and Status. Following the development of a method for fractionating liver iron into its various components, an experiment was designed to relate the dose of iron, as tagged iron ammonium citrate, to the ability of the liver to form ferritin iron and hemosiderin iron. A total of 37 mice were given doses ranging from 12.5 to 125 μg. Average effective doses obtained by counting immediately after injection varied from 10.51 to 104.8 μg, since some was lost during administration. All animals were sacrificed 15 hr after injection. The average effective doses along with the observed and fitted ferritin and hemosiderin liver fractions are shown in Table 3.

The responses seemed to be slightly parabolic, so the curve $y = ax + bx^2$ was fitted to both sets of data by the method of least squares. The fitted constants are:

	FERRITIN	HEMOSIDERIN
a	0.1576	0.1962
b	0.001720	0.000994

All four estimates differed significantly from zero at the 5% level, but the differences between corresponding constants for ferritin and hemosiderin (i.e., 0.1576 - 0.1962 and 0.001720 - 0.000994) showed no statistical significance. These results contradict a previously accepted theory that in normal animals the liver manufactures much more ferritin than hemosiderin.

It is believed that some difference between the rates of production of the two iron fractions may be found if the responses are related to time after injection. Experiments designed to investigate this are now in progress.

TABLE 3

Observed and Fitted Ferritin and Hemosiderin Fraction in Liver

NO. OF MICE	AVG. EFFECTIVE DOSE, μg	FERRITIN FRACTION, μg/g		HEMOSIDERIN FRACTION, μg/g	
		OBSERVED	FITTED	OBSERVED	FITTED
6	10.51	2.23	1.85	2.06	2.17
7	23.61	4.62	4.68	5.41	5.19
6	39.25	7.64	8.83	10.05	9.23
5	64.88	20.59	17.46	17.84	16.92
6	83.93	22.95	25.34	21.17	23.47
7	104.80	36.09	35.40	32.31	31.48

INDICES OF OXYGEN EFFECT ON MITOSIS FOLLOWING IRRADIATION

Origin. M. E. Gaulden, Biology Division.

Participating Members of Panel. J. Moshman and G. J. Atta.

Background and Status. The characteristic curve of mitotic activity following irradiation of sufficient magnitude starts at the normal rate, is then sharply depressed to a period of complete inhibition, recovers at a rapid pace to a period of activity greatly in excess of the normal rate, and then gradually tapers off to the normal rate of mitosis.

Various measures of radiation effect on mitosis were discussed in a previous report.[4] Three indices were examined in detail for atmospheric influence after irradiation with 64 r.

1. *Duration of mitotic inhibition.* The length of the period of dormant activity was observed under various intensities of oxygen. Observations could be recorded only in multiples of 22 min, which we will call a counting period. In Fig. 1, line A depicts the relationship of the duration of this inhibition period and the proportion of O_2 in the atmosphere. The interval about each point represents a 95% confidence band. The regression equation was

$$D = 3.43 + 0.16\,P \qquad (0 \leqq P \leqq 11)\ ,$$

where D is the duration of the inhibition period, and P is the percentage of O_2. After 11% a saturation sets in and the duration was essentially constant at 5.16 intervals.

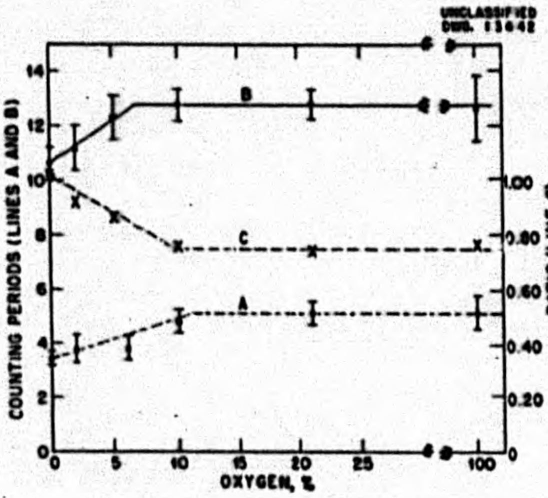

Fig. 1. Indices of Mitotic Effect.

2. *Time of peak recovery.* The counting period at which modal activity exists was observed, again in units of 22 minutes. The resulting curve is also plotted in Fig. 1 as line B. Its equation is

$$R = 10.61 + 0.03\,P \qquad (0 \leqq P \leqq 7)$$

where R is the recovery period, and P is the percentage oxygen. After saturation at 7% O_2, the curve levels off at 13.82 counting periods. Intervals about each point are again 95% confidence limits.

3. *Ratio of number of total dividing cells to those in a vacuum.* Ratios were computed for the total number of cells counted as being in a specific phase of mitosis to those in a paired experiment with no oxygen present. The equation is

$$T = 1.00 - 0.025\,P \qquad (0 \leqq P \leqq 10)\ ,$$

where T is the ratio of total cell counts in P per cent oxygen to zero per cent oxygen. After 10% the ratio remains constant at about 0.75. This is similarly plotted in Fig. 1 as line C. Confidence bands are much wider

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[4] *Mathematics Panel Quarterly Progress Report for Period Ending July 31, 1950*, ORNL-818.

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than for the other two curves, but are omitted for the sake of simplicity.

A report incorporating these analyses is being prepared for the open literature.

HEAT EFFECT ON CELL EXTINCTION

Origin. M. E. Gaulden, Biology Division.

Participating Members of Panel. J. Moshman, J. H. Fishel, and G. J. Atta.

References. *Mathematics Panel Quarterly Progress Reports*, ORNL-1029, ORNL-1091.

Background and Status. A previous experiment on the effect of heat on the extinction of cells treated with Feulgen and methyl green stains revealed only a significant difference between stains, but the relatively few degrees of freedom between embryos obviated the establishment of a significant heat effect.

A new experiment was designed with about triple the number of degrees of freedom. Nonnormality and heteroscedasticity pointed to a nonparametric analysis. The procedure of Brown[5] was followed for a distribution-free analog of the analysis of variance. Detailed analysis revealed a significant ($P < 0.01$) difference due to heating and also a significant interaction ($P \approx 0.01$) between stain and heat. The interaction revealed itself in the data by a reversal of relative magnitude between the stains with the presence and absence of heat. With no heat, cells treated with methyl green stain had larger extinctions than corresponding cells with Feulgen stain and vice versa. This problem has been completed.

(5)A. M. Mood, *Introduction to the Theory of Statistics* (New York: McGraw-Hill, 1950), p. 405.

ANALYSIS OF BURRO BLOOD DATA

Origin. John H. Rust, UT-AEC Agricultural Research Program.

Participating Members of Panel. A. W. Kimball and G. J. Atta.

Background and Status. Not much is known about the basic physiology of the burro, since it has never been used extensively for experimental work. Prior to a series of mortality experiments using whole-body gamma radiation, blood samples were taken on 108 normal burros, and several blood constituents were determined in an attempt to provide this information which is usually well-established for most laboratory animals. The burros were grouped according to age and sex as shown in Table 4.

TABLE 4

Number of Animals

SEX	AGE GROUPS (years)					TOTALS
	1	2	3	4	5+	
Male	16	10	2	10	3	41
Female	12	19	8	25	3	67
TOTALS	28	29	10	35	6	108

For each of nine different blood constituents an analysis of variance of the form shown in Table 5 was performed.

TABLE 5

Analysis of Variance

SOURCE	DEGREES OF FREEDOM
Among Age Groups	4
Between sexes	1
Interaction	4
Error	98
	107

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The results will be written up and submitted to an appropriate journal.

IRON UPTAKE IN PEANUT PLANTS

Origin. D. Davis, UT-AEC Agricultural Research Program.

Participating Members of Panel. A. W. Kimball and G. J. Atta.

Background and Status. The purpose of this experiment was to investigate the nature of iron uptake in peanut plants, including an attempt to evaluate the effect of previous exposure to varying levels of manganese. Eight different treatment combinations were employed, corresponding to the designations AA, BB, CC, DD, BA, BB, BC, and BD. The letters A, B, C, D refer to four different levels of manganese, 0, 0.5, 1.5, and 5.0 ppm in the nutrient solution. For two months the plants were cultured at a manganese level given by the first letter in the group designation. After two months the solution was replaced with one containing the manganese concentration given by the second letter in the group designation. Also at this time Fe^{55} was added to the solutions and plants were removed from solution at 24, 48, 72, and 120 hr later. Several different plant parts were counted and an analysis of variance was computed for each. The experiment was repeated four times.

The analyses differed slightly for the various plant parts but the analysis for leaflets, as shown in Table 6, is typical. When preliminary tests of significance indicated no differences among the GR, TR, and GTR interactions, they were pooled to provide more sensitive tests for the main effects and the GT interaction. Otherwise, each effect was tested with its respective error variance. The treatment group sum of squares was broken down into individual degrees of freedom to provide tests for the effect of past history and for specific comparisons among the treatment groups.

TABLE 6

Analysis for Leaflets

SOURCE OF VARIATION	DEGREES OF FREEDOM
Among treatment groups (G)	7
Among removal times (T)	3
Among replications (R)	3
GT interaction	21
GR interaction	21
TR interaction	9
GTR interaction	63
TOTAL	127

The experiment is being written up and will be submitted to an appropriate journal for publication.

FLOW STUDIES IN TANKS IN SERIES

Origin. C. P. Straub, Health Physics Division.

Participating Member of Panel. E. Ikenberry.

References. R. W. Kehr, "Detention of Liquids Being Mixed in Continuous Flow Tanks," *Sewage Works J.*, 8, 915 (1936); H. A. Thomas, Jr., and J. E. McKee, "Longitudinal Mixing in Aeration Tanks," *Sewage Works J.*, 16, 42-55 (1944); and H. A. Thomas, Jr., and R. S. Archibald, "Longitudinal Mixing Measured by Radioactive Tracers," *Am. Soc. Civil Engrs.*, 77, Separate No. 84 (1951).

Background and Status. The input to the first of n tanks in series, with initial concentrations $a_i(0) = c_i$, is $a_0(t)$. It is desired to find the concentrations $a_i(t)$, assuming perfect mixing. The differential equations for $a_i(t)$ are:

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$$\frac{d\,a_i(t)}{dt} = \lambda_i\,\{a_{i-1}(t) - a_i(t)\}\ ,$$

$$i = 1, 2, \ldots, n\ , \qquad (1)$$

where $1/\lambda_i$ is the detention period (volume/rate of flow) of tank i.

Solution of the differential equations. It is evident that the solution may be written in the form

$$a_i(t) = f_i(t) + \sum_{j=1}^{n} c_j\, g_{ji}(t)\ , \qquad (2)$$

where the functions $f_i(t)$ are solutions of Eq. 1 with $a_0(t) = f_0(t)$ and $f_i(0) = 0$, and the functions $g_{ji}(t)$ are solutions of Eq. 1 with $a_0(t) = 0$, $g_{jj}(0) = 1$, $g_{ji}(0) = 0$, $i \neq j$. Furthermore, if $a_0(t) = \sum_n p_n(t)$, then $f_i(t) = \sum_n p_{ni}(t)$, where the functions $p_{ni}(t)$ are solutions of Eq. 1 with $a_0(t) = p_n(t)$ and $p_{ni}(0) = 0$. (Principle of Superposition.)

For brevity of notation, let $\lambda_0 = 0$ and

$$R_i = e^{-\lambda_i t}\ ,$$

$$P_i = \lambda_i\ ,$$

$$R_{ij} = e^{-\lambda_i t}/(\lambda_j - \lambda_i)\ ,$$

$$P_{ij} = \lambda_i\,\lambda_j\ ,$$

$$R_{ijk} = e^{-\lambda_i t}/(\lambda_j - \lambda_i)(\lambda_k - \lambda_i)\ ,$$

$$P_{ijk} = \lambda_i\lambda_j\lambda_k\ ,$$

$$R_{ijkl} = e^{-\lambda_i t}/(\lambda_j - \lambda_i)(\lambda_k - \lambda_i)(\lambda - \lambda_i)\ ,$$

etc. There is obtained, for the case of unequal detention times,

$$a_1(t) = f_1(t) + c_1\, R_1\ ,$$

$$a_2(t) = f_2(t) + c_1\, P_2\,\{R_{12} + R_{21}\} + c_2\, R_2\ ,$$

$$a_3(t) = f_3(t) + c_1\, P_{23}\,\{R_{123} + R_{231} + R_{312}\} + c_2\, P_3\,\{R_{23} + R_{32}\} + c_3\, R_3\ ,$$

$$a_4(t) = f_4(t) + c_1\, P_{234}\,\{R_{1234} + R_{2341} + R_{3412} + R_{4123}\} + c_2\, P_{34}\,\{R_{234} + R_{342} + R_{423}\} + c_3\, P_4\,\{R_{34} + R_{43}\} + c_4\, R_4\ ,$$

with evident extension for additional tanks.

Further, in the case of continuous feed, $a_0(t) = c_0$,

$$f_1(t) = c_0\, P_1\,\{R_{01} + R_{10}\}\ ,$$

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$$f_2(t) = c_0 \, P_{12} \, \{R_{012} + R_{120} + R_{201}\} ,$$

$$f_3(t) = c_0 \, P_{123} \, \{R_{0123} + R_{1230} + R_{2301} + R_{3012}\} ,$$

etc.

Solutions for other forms of feed may be obtained by the Laplace transform method.

Runs with iodine-131 are underway, and when they are completed comparison of the theoretical and experimental results will be made. These studies, including those of the effect of incomplete mixing, are being made by C. P. Straub and D. A. Pecsok of the Health Physics Division.

MONTE CARLO ESTIMATE OF COLLISION DISTRIBUTIONS IN TISSUE

Origin. W. S. Snyder and J. Neufeld, Health Physics Division.

Participating Members of Panel. K. A. Pflueger and C. L. Perry.

References. *Mathematics Panel Quarterly Progress Report*, ORNL-345, 408, 516, 634, 726, 818, 888, 979, 1029, 1091.

Background and Status. The previous work has been extended to include the following. The 2000 neutron histories were sorted on the first collision in the range $E_1 < E < E_2$ (E_1, E_2 fixed). That is, the histories were divided into two parts: the part before a collision in (E_1, E_2), and the part from the first collision in (E_1, E_2) to thermal. E_1 and E_2 were chosen so that there would be about 1000 histories in the sample and so that the mean energy for the first collision in the samples would be 5, 2.5, and 0.5 Mev. Each of the histories was reoriented to make the history have an origin at the surface of the slab, and to make the initial neutron path normal to the surface of the slab.

For these samples of nearly 1000 reoriented histories the collision density and collision damage were found for a 30-cm slab of tissue with a collimated beam of neutrons incident on the slab (Fig. 2).

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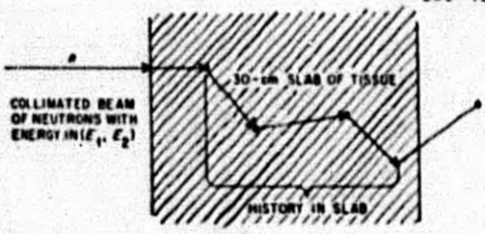

Fig. 2. Schematic Diagram of a Neutron History.

The distribution of the last collision in the (fast) history was also tabulated providing the neutron had an energy in the thermal range after collision. These distributions (one for the entire collection of histories, mean 10 Mev; one for the 5-Mev sample; one for the 2.5-Mev sample; and one for the 0.5-Mev sample) will be used as the basis of further computation (cf., problem on "Thermal Neutron Damage").

THERMAL NEUTRON DAMAGE

Origin. W. S. Snyder and J. Neufeld, Health Physics Division.

Participating Members of Panel. A. S. Householder, K. A. Pflueger, and C. L. Perry.

Reference. See the problem on "Monte Carlo Estimate of Collision Distributions in Tissue."

Background and Status. The problem is to determine the distribution of thermal neutron collisions in a 30-cm slab of tissue. The source of thermal neutrons is a beam of collimated fast neutrons incident on the surface of the slab (cf., reference given above). Let the source distribution be $P_0(x)$, $0 \leq x \leq 30$ cm. The distribution of thermal neutron collisions in the slab is then $P(x)$,[6] where

$$P(x) = P_0(x) + \frac{\sigma_s}{2} \int_0^{30} dy E_1 (\sigma |x-y|) P(y) .$$

The Panel is now finding an approximate solution for $P(x)$ by an iteration method for the four source distributions found by the Monte Carlo estimate.

COIL-WINDING DENSITY FOR A NEARLY UNIFORM FIELD

Origin. R. D. Birkhoff and W. S. Snyder, Health Physics Division.

Participating Members of Panel. H. B. Goertzel and C. L. Perry.

Background and Status. The magnetic field strength on the axis of a cylindrical coil of length $2l$ and winding density $\phi(y)$ is

$$f(x) = \int_{-l}^{l} dy\ \phi(y)\ [1 + (x-y)^2]^{-3/2} ,$$

where x and y are measured from the center of the coil. The Panel found an approximate solution for $f(x) = 1$, $-0.75l \leq x \leq 0.75l$, by assuming $\phi(y)$ was a step function with a small number (16) of jumps. The computations have been completed.

[6] W. S. Snyder, "Calculations for Maximum Permissible Exposure to Thermal Neutrons," *Nucleonics* 6, No. 2, 46 (1950).

AMPLIFIER RESPONSE TO PROPORTIONAL COUNTER PULSES

Origin. G. S. Hurst, Health Physics Division.

Participating Member of Panel. C. Perhacs.

Reference. *Health Physics Quarterly Progress Report*, ORNL-1086; and "A Proportional Counter Method of Measurement of Fast Neutron Dose," G. S. Hurst, ORNL CF-51-4-122.

Background and Status. The numerical evaluation of Hurst's formula for the amplifier response is being performed on IBM equipment.

A STUDY OF RADIOACTIVITY ABSORPTION IN GAMBUSIA

Origin. L. A. Krumholz, Health Physics Division.

Participating Member of Panel. A. W. Kimball.

Background and Status. This experiment, first described in ORNL-1091, is nearing completion and should be ready for analysis in the near future.

KINETICS OF THE HBr-$HBrO_3$ REACTION

Origin. O. Myers, Chemistry Division.

Participating Members of Panel. J. H. Fishel, N. D. Given, and C. L. Perry.

Background and Status. The six kinetic reactions

$$HBr + HBrO_3 \underset{k_1}{\overset{a}{\rightleftharpoons}} HBrO_2 + HBrO ,$$

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$$HBr + HBrO_2 \underset{k_2}{\overset{b}{\rightleftarrows}} 2HBrO \ ,$$

$$HBr + HBrO \underset{k_3}{\overset{c}{\rightleftarrows}} Br_2 + H_2O$$

are assumed to take place simultaneously and at the rates indicated above, after HBr is mixed in an aqueous solution of $HBrO_2$. The system of differential equations describing the kinetics of the reactions is

$$\frac{dx}{dt} = A - B \quad ; \quad x(0) = 0$$

$$\frac{dy}{dt} = A + 2B - C \quad ; \quad y(0) = 0$$

$$\frac{dz}{dt} = -A - B - C \quad ; \quad z(0) = z_0 \neq 0$$

$$\frac{dw}{dt} = C \quad ; \quad w(0) = 0$$

where $A = a\,z - k_1\,x\,y$,

$B = b\,x\,z - k_2\,y^2$,

$C = c\,y\,z - k_3\,w$,

and x, y, z, w, respectively, designate the quantities of $HBrO_2$, HBrO, HBr, and Br_2 present at time t. The Panel is investigating analytic methods of finding approximate solutions to this system, and is also doing some numerical integration for the case $a = 0.768$, $b = 0.00693$, $c = 10^6$, $k_1 = 0.0645$, $k_2 = 2.33 \times 10^{-8}$, $k_3 = 35.0$, and $z_0 = 0.00417$. One easily determined integral of the system is

$$z_0 = x + 2y + 3z + 5w \ .$$

CRYSTAL ANALYSIS BY NEUTRON DIFFRACTION

Origin. H. Levy, Chemistry Division

X-RAY CRYSTAL ANALYSIS

Origin. G. P. Smith, Jr., Metallurgy Division.

Participating Members of Panel. J. H. Fishel and C. Perhacs.

Background and Status. In both of these problems the experimenter wants the Fourier sine-transform of experimentally determined functions. The transforms are of the form

$$T(t) = \int_0^{s_0} \{f(s) + h(s)\}\, g(s) \sin r\, s\, ds \ ,$$

where $f(s)$ is the experimentally determined function and, $h(s)$ and $g(s)$, are weighting functions. The Panel has programmed the computation for the IBM Card Programmed Calculator. The function $\{f(s) + h(s)\}\, g(s)$ is approximated by a polygonal function with vertices at $s = 0.1\,n$, $n = 0, 1, 2, \ldots, 150$. The integral is evaluated exactly (except for round-off error) for this polygonal approximation. The Panel has sine cards for $r = 0.2(0.1)9.8$. For these values of r, integrals of the above type have been evaluated at a rate of one every 6 to 15 minutes. Levy's evaluations have been completed, and Smith's evaluations will be complete by the second week of the next quarter.

EFFICIENCY OF A CRYSTAL

Origin. P. R. Bell and F. K. McGowan, Physics Division.

Participating Members of Panel. K. A. Pflueger and C. L. Perry.

Background and Status. A point source of isotropic radiation is located near the surface of a right circular cylinder. The radiation absorbed by the crystal is assumed to be proportional to $1 - e^{-\tau x}$ where x is the distance the radiation travels through the crystal. With this assumption the fraction of the total radiation that is absorbed by the crystal is

$$T = (1 - \cos \alpha_2) - \frac{1}{2}\int_0^{\alpha_1} e^{-a\tau \sec \alpha} \sin \alpha \, d\alpha - \frac{1}{2}\int_{\alpha_1}^{\alpha_2} e^{-\tau (b \csc \alpha - h \sec \alpha)} \sin \alpha \, d\alpha$$

when the source is located on the axis of the crystal and where the parameters are as shown in Fig. 3.

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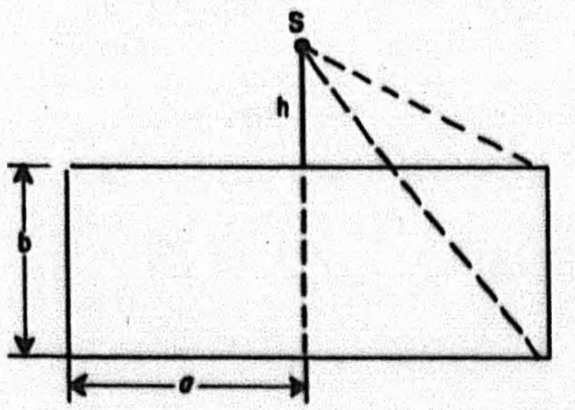

Fig. 3. Axial Cross Section of the Crystal.

The above integrals were simplified (e.g., transformed to exponential integrals) and T was graphed for the commercial crystal whose cross section is given in ORNL drawing 9500. For this crystal a = 1½ in., b = 1 in. Graphs were drawn for h = 0, 0.2, 0.5, 1.0, 2.0, and 5.0 cm.

ESCAPE OF X-RAYS FROM A SEMI-INFINITE CRYSTAL

Origin. P. R. Bell and F. K. McGowan, Physics Division.

Participating Members of Panel. N. D. Given and C. L. Perry.

Background and Status. A collimated beam of gamma rays enters a semi-infinite crystal normal to its plane surface (Fig. 4).

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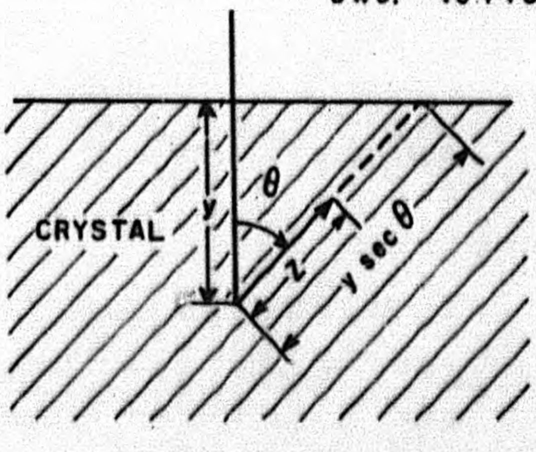

Fig. 4. Section Normal to Crystal Surface.

The rate of absorption per unit path is assumed to be $\tau e^{-\tau y}$. For each absorption an X-ray is assumed to be emitted with probability p. The X-rays are assumed to leave their origin isotropically and to be absorbed at a rate $\bar{\tau} e^{-\bar{\tau} z}$ along their path. With these assumptions the ratio R = (no. of X-rays that leave the crystal) / (no.

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of gamma rays that enter the crystal) is

$$R = \frac{p}{2}\left[1 - \frac{\bar{\tau}}{\tau}\ln\frac{\bar{\tau}+\tau}{\bar{\tau}}\right].$$

Graphs of R vs. E were drawn for the cross sections $\tau(E)$ given in drawing 9500. This problem has been completed.

FERMI FUNCTIONS

Origin. M. E. Rose and P. R. Bell, Physics Division.

Participating Members of Panel. C. L. Perry, B. M. Drucker, H. B. Goertzel, and N. M. Dismuke.

References. *Mathematics Panel Quarterly Progress Reports*, ORNL-345, 408, 516, 634, 726, 818, 888, 979, 1029, 1091.

Status. During this quarter the Fermi functions and field factors were graphed and differenced in preparation for tabulating the results. The function

$$\phi_0(p, W) = \frac{1 + S_0}{2}\frac{p}{W}F_0(p, W)$$

was prepared for tabulation to replace the F_0 tables issued a year ago.

During the graphing of the field factors it was noted that one of the functions for the positron case is at least partially in error. Hence before the calculation can be completed, the formulation must be examined again.

ANGULAR CORRELATION BETWEEN CONVERSION ELECTRONS AND GAMMA RAYS

Origin. M. E. Rose and G. B. Arfken, Physics Division.

Participating Members of Panel. C. Perhacs, J. H. Fishel, N. M. Dismuke, and C. L. Perry.

References. *Mathematics Panel Quarterly Progress Reports*, ORNL-979, 1029, 1091, and *Physics Division Quarterly Progress Report* ORNL-1092.

Status. The computation of the angular correlation coefficients was completed and checked during the quarter.

CALCULATION OF INTERNAL CONVERSION COEFFICIENTS WITH SCREENING

Origin. M. E. Rose, Physics Division.

Participating Member of Panel. M. R. Arnette.

Other Participants. G. Goertzel, Physics Department, New York University, M. S. Montalbano, Division 11.2, National Bureau of Standards.

References. "Calculation of Internal Conversion Coefficients," a note by G. H. Goertzel dated December 18, 1949; and the *Mathematics Panel Quarterly Progress Report* ORNL-1091, p. 27.

Status. The coding for the SEAC and the checking of the code on the SEAC is being completed. It is now hoped that the calculations will be completed early in 1952.

CALCULATION OF RACAH COEFFICIENTS FOR THE ANGULAR DISTRIBUTION IN NUCLEAR REACTIONS

Origin. M. E. Rose and L. C. Biedenharn, Physics Division.

Participating Members of Panel. S. L. Hull, H. B. Goertzel, R. Crook, and C. L. Perry.

References. *Physics Division Quarterly Progress Report* ORNL-1005; *Mathematics Panel Quarterly Progress Reports*, ORNL-1029, p. 22; ORNL-1091, p. 10.

Background and Status. The scope of these computations has been extended to include $w(l_1, j_1, l_2 j_2; S, L)$ for $S = 5/2$ and $S = 3$. The computation will be completed during the next quarter.

REACTOR RESPONSE CURVES

Origin. J. Trimmer, Reactor Technology Division.

Participating Members of Panel. K. A. Pflueger, R. Crook, and C. L. Perry.

Background and Status. The reactor response was calculated as a function of the reactor frequency for neutron life time $l = 10^{-3}$, 10^{-5} seconds. The supercritical and subcritical responses to a step change in k were found as functions of reactor frequency. The post-shutdown time dependence of poison was also computed. The results of these computations will appear as Figs. 4.2 through 4.9 of Chapter 4, Volume II of the new reactor school text. This problem has been completed.

DETERMINATION OF FAST NEUTRON FLUX IN ORNL PILE

Origin. D. K. Holmes, Physics Division.

Participating Members of Panel. J. Moshman; E. B. Carter, Central Statistical Laboratory, K-25.

References. *Mathematics Panel Quarterly Progress Reports*, ORNL-726, 818, 888, 929, 1029, 1091.

Status. This problem is in the machine stage of calculation.

THORIUM BREEDER STUDIES

Origin. J. Lane, N. Lansing, and S. Visner, Long-Range Planning Group.

Participating Members of Panel. B. S. McGill and H. B. Goertzel.

Background and Status. The two-group calculations were completed during the quarter.

KINETICS OF HRE

Origin. T. A. Welton and G. T. Trammel, Physics Division.

Participating Members of Panel. C. L. Perry, B. M. Drucker, H. B. Goertzel, N. D. Given, S. L. Hull, and W. C. Sangren.

Background and Status. Information concerning the kinetics of the HRE may be obtained by investigating the solutions of certain systems of ordinary differential equations. Since one or two of the equations in each system are nonlinear it was necessary to resort to numerical methods. Three methods for obtaining approximate solution were used: (1) numerical integration by the Runge-Kutta method using desk computers, (2) numerical integration by the Heun method using the SEAC, the National Bureau of Standards's electronic digital computer, and (3) a semianalytical method involving a perturbation technique.

Most of the systems investigated were of the form:

$$\dot{y} = x - 1 \ , \qquad (1)$$

$$\dot{z} = -v \ , \qquad (2)$$

$$\dot{x} = \frac{\beta}{\tau} + \frac{\delta - \beta}{\tau} x + z x \ , \qquad (3)$$

$$\dot{v} = -a_1 v - a_2 v |v| + w_H^2 [z + w_n^2 y] \ , \qquad (4)$$

where x, y, z, and v are functions of t, and where the other symbols represent constants. Physically one can think of each system as representing two coupled oscillators. The two oscillator pairs, x and y, and y and z, are coupled by the presence of z in Eq. 3 and y in Eq. 4.

Three systems of the above type were calculated by desk computers. Although two of these calculations were subsequently carried out on the SEAC, it was desirable to have these results both as a check on the machine results and as a method of estimating magnitudes. One computer can calculate about one system per month from $t = 0$ to $t = 0.3$. Two other similar but reduced systems, one with two equations and one with three equations, were also calculated with desk computers.

The following general system was programmed for solution by the SEAC:

$$\dot{y} = x - 1 \qquad x(0) = 1 \qquad (5)$$

$$\dot{z} = -v \qquad y(0) = z(0) = v(0) = 0 \qquad (6)$$

$$\dot{x} = a x + z x + b \qquad (7)$$

$$\dot{v} = c v + d v |v| + e y + f z \ . \qquad (8)$$

Thirteen systems of this type were solved on range $0 \leq t \leq 0.3$ or on range $0 \leq t \leq 0.4$. The parameter values, integration interval (of order 10^{-4} sec), and the printing-out intervals were read into the machine independently of the main routine. The total machine time for all 13 systems was about 5 hr. This included 1 hr for checking the program and coding. Since the actual computation time per system was less than 20 sec, most of the machine time was taken up by printing the results and reading the routine into the machine.

The following technique was used to obtain approximate formulas for the variables x, y, z, and v in terms of t and the constants. Since the initial value of z is zero and a_1 and a_2 prove to be relatively unimportant, a first approximation can be obtained by solving the linear system consisting of Eqs. 1 and 2, and $\dot{x} = \frac{\beta}{\tau} + \frac{\delta - \beta}{\tau} x$ and $\dot{v} = w_H^2 [z + w_n^2 y]$. Having obtained a first-approximation formula for x and z, substitute the formula in the right hand side of Eq. 3 and integrate to obtain a second approximation formula for x. Using Eqs. 1 and 2, and $\dot{v} = w_H^2 [z + w_n^2 y]$, second-approximation formulas can then be derived for y, z, and v. The formulas give results that in general agree very well from $t = 0$ to $t = 0.1$ (the most interesting range for t) with the SEAC's results.

Graphs of the variables vs. t have been constructed for the various systems. The results have not been completely analyzed as yet. It is possible that the solutions of further systems of this type will be computed by the SEAC. More complete details will appear in the next Quarterly Report of the HRE.

A MULTIGROUP METHOD FOR COMPUTING THE NEUTRON DISTRIBUTION IN A FINITE CYLINDRICAL REACTOR WITH REFLECTED CONVEX SURFACE AND BARE ENDS

Origin. N. M. Smith, Reactor Physics Division.

Participating Member of Panel. N. Edmonson.

Status. A report on this work is ready for publication.

ANALYSIS OF FLUID FLOW IN THE HOMOGENEOUS REACTOR

Origin. G. Weslicinus and D. Fax, Homogeneous Reactor Project Division.

Participating Members of Panel. J. H. Fishel, N. D. Given, and C. L. Perry.

Reference. *Homogeneous Reactor Project Quarterly Report*, ORNL-1121.

Background and Status. An approximate solution to the nonlinear partial differential equations describing the fluid flow in the homogeneous reactor was found by an iterative method. Starting values were found graphically and these values were iterated on the IBM Card Programmed Calculator.

OPERATIONS GREENHOUSE - LETHALITY STUDY

Origin. E. P. Cronkite, Naval Medical Research Institute.

Participating Members of Panel. J. Moshman, G. J. Atta, R. S. McGill, H. B. Goertzel, and S. L. Hull.

Background and Status. One of the phases of Operations Greenhouse involved a study of the factors that have a differential effect on the 28-day mortality of the experimental mice. Following the Cornfield-Mantel[8] procedure, probit curves were fitted

TABLE 7

Median Lethal Doses and Slopes of Greenhouse Mice, Stations, Series 70 and 71

(Provisional Estimates)

VARIABLE	BREAKDOWN	LD-50	SLOPE
Sex	Male	757.61	0.01596
	Female	752.88	0.01590
Ages	7-9 weeks	739.22	0.01777
	10 weeks	749.89	0.01341
	11 weeks	753.46	0.01250
	12 weeks	761.39	0.01615
Weight	15-19 grams	755.16	0.01773
	20-24 grams	754.24	0.01602
	25-29 grams	760.20	0.01495
	30-34 grams	749.18	0.01895
Tray	1 (bottom)	777.09	0.01764
	2	767.96	0.01523
	3	760.22	0.01081
	4	746.44	0.01488
	5	747.70	0.01828
	6 (top)	740.10	0.01823
Row	1 (rear)	793.57	0.01757
	2	758.40	0.01911
	3 (front)	715.68	0.01696

[8] J. Cornfield and N. Mantel, "Some New Aspects of the Application of Maximum Likelihood to the Calculation of the Dosage Response Curve," *J. Am. Assn.* 45, 181-210 (1950).

to the lethality data when broken down by sex, age, weight, and positions of the mice. As a preliminary step, two cycles of the iterative procedure were arbitrarily used for this process. The LD-50's and slopes are tabulated in Table 7.

Further calculations are under way to complete as many additional cycles as may be necessary for individual probit regression lines. Upon completion of this, standard errors will be computed in order to guage the homogeneity within the various groups. One noteworthy point is the apparent decreasing sensitivity with increasing age, as evidenced by the higher dose necessary to kill 50%. This phenomenon is not present in the weight breakdown. It is planned to investigate this point further by age within sex within weight breakdowns.

PENDING PROBLEMS

Problem. Threshold Values of the Angular Correlation Coefficients.

Origin. M. E. Rose and G. B. Arfken, Physics Division.

Problem. Angular Correlation, *K* and *L* Shell.

Origin. M. E. Rose, Physics Division.

Problem. Efficiency of a Crystal (off-axis calculation).

Origin. P. R. Bell, Physics Division.

Problem. Calculations for Counter Confidence Limits.

Origin. G. E. Albert, Mathematics Panel.

Problem. Numerical Evaluation of Trigonometric Series.

Origin. B. S. Borie, Metallurgy Division.

INACTIVE PROBLEMS

Problem. Analysis of Scintillation Spectrometer Data.

Origin. P. R. Bell, Physics Division.

Reference. *Mathematics Panel Quarterly Progress Reports*, ORNL-408, 818, 979, 1029, 1091.

Problem. Quantum Mechanics Integration.

Origin. F. C. Prohammer, Physics Division.