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AS USED BY THE ANP PHYSICS GROUP
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SUGGESTED CORRECTION TO AGE-DIFFUSION EQUATION
AS USED BY THE ANP PHYSICS GROUP

R. R. Coveyou and B. T. Macauley

The ANP Physics Group uses the age diffusion equation in the form:

$$(1) \quad D(u) \nabla^2 \phi(r, u) - \sigma_a(u) \phi(r, u) - \frac{\partial}{\partial u} q(r, u) + S(r, u) = 0,$$

and assumes the relation

$$(2) \quad q(r, u) = \int \sigma_t(u) \phi(r, u)$$

between flux and slowing-down-density.

This note examines the consequences of replacing (1) and (2) by the system

$$(3) \quad D(u) \nabla^2 \phi(r, u) - \sigma_t(u) \phi(r, u) + \frac{1}{f} q(r, u) + S(r, u) = 0.$$

$$(4) \quad \dot{q}(r, u) + \frac{1}{f} q(r, u) = \sigma_a(u) \phi(r, u).$$

We note that if we substitute (4) in (3), the result is (1). Equations (3) and (4) describe a system in which a decelerating neutron has always the probability $\frac{du}{f}$ of terminating its deceleration in the lethargy interval u to $u+du$.

For completeness, we add the equation

$$(5) \quad D \nabla^2 \phi(r) - \sigma_a \phi(r) + q(r, t) = 0$$

to describe the thermal flux, where t is the lower limit of thermal lethargies. The boundary conditions are taken to be the usual spatial ones plus the condition

$$(6) \quad \phi(r, 0) = q(r, 0) = 0.$$

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Now if we assume a separated solution, with space dependence $F(r)$, we may write

$$(7) \quad \phi(r, u) = U(u) F(r)$$

$$(8) \quad Q(r, u) = V(u) F(r)$$

$$(9) \quad \phi(r) = \alpha F(r)$$

$$(10) \quad S(r, u) = \nu f(u) F(r)$$

Then, after the usual manipulations, we get:

$$(11) \quad \dot{V}(u) + \frac{1}{\xi} \left[\frac{\sigma_a(u) + B^2 D(u)}{\sigma_t(u) + B^2 D(u)} \right] V(u) = \frac{\nu \sigma_a(u) f(u)}{\sigma_t(u) + B^2 D(u)},$$

$$(12) \quad U(u) = \frac{V(u) + \xi \nu f(u)}{\xi [\sigma_t(u) + B^2 D(u)]},$$

$$(13) \quad \alpha = \frac{V(t)}{DB^2 + \sigma_a}.$$

Equation (11), for the lethargy-dependence of the slowing down density has a simple and plausible physical interpretation. The second term on the left is the loss rate in unit lethargy, the fraction in brackets is the probability that a given flight will lead to leakage or absorption, while the mean number of flights per unit lethargy is $1/\xi$. The right hand side is a source term; the factor

$\sigma_a(u) [\sigma_t(u) + B^2 D(u)]^{-1}$ expresses the fact that a neutron, born in fission, does not contribute to the slowing down density unless its first flight is ended in a scattering collision.

The equation corresponding to (11), derived in present procedure is:

$$(14) \quad \dot{V}(u) + \left[\frac{\sigma_a(u) + B^2 D(u)}{\xi \sigma_t(u)} \right] V(u) = \nu f(u).$$

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Assuming that (11) is correct, we see that (14) clearly over-estimates the source, and also over-estimates the loss term. It is not clear whether or not these factors will compensate each other in an actual calculation.

A test calculation was made for comparison purposes on a carbon critical experiment by the following numerical techniques:

Starting with equation (11):

$$\begin{aligned}
 (11a) \quad \dot{V}(u) + \left[\frac{\sigma_a(u) + B^2 D(u)}{\int \sigma_T(u) + \int B^2 D(u)} \right] V(u) &= \frac{2\sigma_a(u) f(u)}{\sigma_T(u) + B^2(u)} \\
 \dot{V}(u) + \left[\frac{\sigma_a(u) + B^2 D(u)}{\int \sigma_T(u) \left[1 + \frac{B^2 D(u)}{\sigma_T} \right]} \right] V(u) &= 2 f(u) \frac{\frac{\sigma_a(u)}{\sigma_T}}{\left[1 + \frac{B^2 D(u)}{\sigma_T} \right]} \\
 \dot{V}(u) + \left[\frac{\frac{\sigma_a(u)}{\int \sigma_T} + \frac{B^2 D(u)}{\int \sigma_T}}{1 + \frac{B^2 D(u)}{\sigma_T}} \right] V(u) &= 2 f(u) \frac{\frac{\sigma_a(u)}{\sigma_T(u)}}{\left[1 + \frac{B^2 D(u)}{\sigma_T} \right]} \\
 \dot{V}(u) + \left[\frac{\frac{\sigma_a(u)}{\int \sigma_T} + \frac{B^2}{3 \int \sigma_T \sigma_{tr}}}{1 + \frac{B^2}{3 \sigma_T \sigma_{tr}}} \right] V(u) &= 2 f(u) \left[\frac{\frac{\sigma_a(u)}{\sigma_T(u)}}{1 + \frac{B^2}{3 \sigma_T \sigma_{tr}}} \right] \\
 \text{Let } A(u) &= \left[\frac{\frac{\sigma_a(u)}{\int \sigma_T} + \frac{B^2}{3 \int \sigma_T \sigma_{tr}}}{1 + \frac{B^2}{3 \sigma_T \sigma_{tr}}} \right] \\
 S^*(u) &= \left[\frac{\frac{\sigma_a(u)}{\sigma_T(u)}}{1 + \frac{B^2}{3 \sigma_T \sigma_{tr}}} \right]
 \end{aligned}$$

$$(11-b) \quad \dot{V}(u) + A(u) V(u) = \gamma f(u) S^{\Phi}(u),$$

$$(11-c) \quad \int_{u_1}^{u_2} \dot{V}(u) du + \int_{u_1}^{u_2} A(u) V(u) du = \gamma \int_{u_1}^{u_2} f(u) S^{\Phi}(u) du,$$

$$(11-d) \quad V_n - V_{n-1} + \overline{(A V_n)}^N U^N = \gamma \overline{(f(u) S^{\Phi})}^N U^N,$$

$$(11-e) \quad V_n - V_{n-1} + \overline{(A(u))}^N U^N \overline{V_n}^N = \gamma \overline{(f(u))}^N U^N \overline{S^{\Phi}}^N,$$

$$\text{Let } Z^N = \int_{u_1}^{u_2} f(u) du = \overline{(f(u))}^N U^N,$$

$$(11-f) \quad V_n - V_{n-1} + \overline{(A(u))}^N U^N \overline{V_n}^N = \gamma Z^N \overline{S^{\Phi}}^N;$$

$$\text{Let } \overline{V_n} = \frac{V_n + V_{n-1}}{2} \text{ and define } C^N = \overline{(A)}^N \frac{U^N}{2},$$

$$(11-g) \quad V_n - V_{n-1} + C^N V_n + C^N V_{n-1} = \gamma Z^N \overline{S^{\Phi}}^N,$$

$$(11-h) \quad [1 + C^N] V_n + [C^N - 1] V_{n-1} = \gamma Z^N \overline{S^{\Phi}}^N,$$

$$(11-i) \quad V_n = \frac{\gamma Z^N \overline{S^{\Phi}}^N}{[1 + C^N]} + \frac{[1 - C^N]}{[1 + C^N]} V_{n-1};$$

$$(15) \quad k_{\text{eff}} = \frac{\sigma_f V(t)}{\sigma_a + DB^2} + \int_0^t U(u) \sigma_f(u) du$$

$$= \left(\frac{\sigma_f}{\sigma_a + DB^2} \right)_{th} V(t) + \int_0^t \left\{ \frac{V(u) + \gamma f(u)}{\gamma [\sigma_T(u) + B^2 D(u)]} \right\} \sigma_f(u) du,$$

$$(15-b) \quad k_{eff} = \left(\frac{\sigma_f}{\sigma_a + DB^2} \right)_{th} V(t) + \int_0^t V(u) \left\{ \frac{\sigma_f}{\sigma_T \left[1 + \frac{B^2}{3\sigma_T \sigma_{tr}} \right]} \right\} du + \int_0^t f(u) \left\{ \frac{\frac{\sigma_f(u)}{\sigma_T(u)}}{1 + \frac{B^2}{3\sigma_T \sigma_{tr}}} \right\} du$$

$$(15-c) \quad k_{eff} = \left(\frac{\sigma_f}{\sigma_a + DB^2} \right)_{th} V(t) + \sum_{n=1}^{N_{th}-1} \bar{V}_n \left\{ \frac{\frac{\sigma_f}{\sigma_T}}{1 + \frac{B^2}{3\sigma_T \sigma_{tr}}} \right\}^N U^n + \sum_{n=1}^{N_{th}-1} Z^n \left(\frac{\frac{\sigma_f}{\sigma_T}}{1 + \frac{B^2}{3\sigma_T \sigma_{tr}}} \right)^N$$

$$\sigma_{a_{th}} = (\sigma_T - \sigma_a)_{th}$$

$$(15-d) \quad k_{eff} = \left\{ \frac{\sigma_f/\sigma_T}{\frac{\sigma_a}{\sigma_T} + \left[1 + \frac{B^2}{3\sigma_T \sigma_{tr}} \right]} \right\}_{th} V(t) + \sum_{n=1}^{N_{th}-1} \bar{V}_n \left\{ \frac{\frac{\sigma_f/\sigma_T}{\sigma_T}}{1 + \frac{B^2}{3\sigma_T \sigma_{tr}}} \right\}^N U^n + \sum_{n=1}^{N_{th}-1} Z^n \left(\frac{\frac{\sigma_f/\sigma_T}{\sigma_T}}{1 + \frac{B^2}{3\sigma_T \sigma_{tr}}} \right)^N$$

$$(15-e) \quad k_{eff} = H_{th} V(t) + \sum_{n=1}^{N_{th}-1} \bar{V}_n \bar{K}^N U^n + \sum_{n=1}^{N_{th}-1} Z^n \bar{S}^N$$

where:

$$\bar{K}^N = \left(\frac{\frac{\sigma_f}{\sigma_T} + \frac{B^2}{3\sigma_T \sigma_{tr}}}{1 + \frac{B^2}{3\sigma_T \sigma_{tr}}} \right)^N$$

$$\bar{S}^N = \left(\frac{\frac{\sigma_f}{\sigma_T}}{1 + \frac{B^2}{3\sigma_T \sigma_{tr}}} \right)^N$$

$$k_{eff}^H = \left(\frac{\frac{\sigma_f}{\sigma_T}}{1 + \frac{B^2}{3\sigma_T\sigma_{tr}}} \right)^H$$

$$k_{eff}^H = \left(\frac{\frac{\sigma_f}{\sigma_T}}{1 + \frac{B^2}{3\sigma_T\sigma_{tr}}} \right)^H$$

$$k_{th} = \left(\frac{\frac{\sigma_f}{\sigma_T}}{-\frac{\sigma_f}{\sigma_T} + \left\{ 1 + \frac{B^2}{3\sigma_T\sigma_{tr}} \right\}_{th}} \right)$$

The core calculated had the following properties:

Uranium = $.00007247 \times 10^{24}$ atoms/cc
 Carbon = $.07708 \times 10^{24}$ atoms/cc
 Al = $.003683 \times 10^{24}$ atoms/cc
 V_{fss} = .0003127
 B^2 = .00186275

The value of k_{eff} for the critical experiment calculated by equation (14) was:

$$k_{eff} = .991164$$

The value of k_{eff} calculated by the procedure outlined in this memo was:

$$k_{eff} = .969802$$

Further studies are to be made to see if the trend indicated above is consistent.

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END