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January 2, 1952

(NAA-SR-Memo-173)

AECD-4034

To: R. L. Stoker
From: E. R. Cohen
Subject: Flux Distribution - Converter Reactor

Photostat Price \$ 3.30

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I. INTRODUCTION

A simplified calculation has been carried out in order to gain some insight into the flux distribution, peak to average power density, efficiency of a catching blanket and the effect of a reflector in the proposed converter reactor. Results are presented for H_2O , graphite and beryllium reflectors combined with the MTR-type core and blanket elements. Calculations were made using numbers which are not necessarily correct, but which should be adequate for present purposes. The model used was a spherical core, radius 85 cm, surrounded either by a 15 cm thorium blanket and then a 20 cm reflector, or by a 15 cm reflector and outside this a 20 cm thorium blanket. The volume of the core is that of a cylinder 4' 10" high and 4' 10" diameter.

In all cases calculated the peak-to-average power ratio is close to 2.5, compared to 3.29 for a bare sphere. The performance is only slightly dependent on the choice and placement of the reflector. A reflector placed between the core and the blanket will improve the flux distribution in the core to some extent, but at the expense of putting the thorium blanket in a region of lower flux. It would also appear that the 15 cm thorium blanket is thicker than it need be and that a blanket half this thick would probably be just as good if not better, from the standpoint of losing a small amount in conversion ratio, but gaining in power distribution. It is probable that on the basis of overall cost per kilogram of U^{233} , that a water reflector is the best. This is somewhat uncertain because of the many analytical problems associated with the transport of fast neutrons in light water. There is some uncertainty in the choice of appropriate constants to be used in any theory that is set up and some indication of the effect of different choices for the fast mean free path in H_2O has been obtained in the calculation. A more detailed analysis is indicated although it is my opinion (perhaps unjustified) that the results will not be too sensitive to such refinements - they should, in any case, be calculated in order to be certain of this point.

II. OUTLINE OF THE METHOD

Because the thermal diffusion length in the reactor core is only 1.47 cm compared to a fast diffusion length, $\sqrt{\tau} = 7.71$ cm, it is a reasonably good

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approximation to use a one group theory and assume that neutrons are absorbed at the point at which they become thermal. With this simplifying assumption it is easy to calculate the fast flux; using this fast flux as a source of thermal neutrons the thermal flux can then be found. If the fission density in the core is not greatly different from that calculated using the one group fast flux, the cycle is essentially completed and the calculation is stopped. (If this were not the case the new fission density could have been used to calculate a second approximation to the fast flux and then from this, a second approximation to the thermal flux. In all cases, however, the first approximation appears to be adequate.)

Let ϕ be the fast flux, λ the transport mean free path; the fast flux distribution (one group model) can be written as

$$\phi_1 = \frac{\sin ar}{ar} \quad \text{for } 0 < r < r_1$$

$$\phi_2 = \frac{\sin ar_1}{ar} \cdot \frac{\cosh b(r_2 - r) + E \sinh b(r_2 - r)}{\cosh bt_2 + E \sinh bt_2} \quad r_1 < r < r_2$$

$$\phi_3 = \frac{\sin ar_1}{\cosh bt_2 + E \sinh bt_2} \cdot \frac{\sinh c(r_3 - r)}{ar \sinh ct_3} \quad r_2 < r < r_3$$

These solutions are written down in such a way that it is readily apparent that they satisfy the conditions of continuity of flux across a boundary. The constants a , b , and c are determined by the nuclear properties of the three media and are simply the square roots of the buckling constants in each case. The constants E and a are chosen so as to match the currents across the two boundaries and the adjustment of a is achieved physically by varying the composition of the core in order to achieve criticality. Matching the currents gives

$$E = \frac{\lambda_3 (cr_2 \coth ct_3 + 1) - \lambda_2}{\lambda_2 br_2}$$

$$1 - ar_1 \cot ar_1 = \frac{\lambda_2}{\lambda_1} \left\{ 1 + br_1 \frac{\sinh bt_2 + E \cosh bt_2}{\cosh bt_2 + E \sinh bt_2} \right\}$$

Using these solutions we can then calculate the thermal fluxes. These

satisfy the equations

$$\nabla^2 \psi - \frac{1}{L^2} \psi = - \frac{3}{\lambda} \xi \Sigma_s \phi$$

with different values of these various constants in each medium. The asymptotic solution of this equation is

$$\psi = \beta \phi$$

where β is a constant;

$$\beta_1 = \frac{3 (\xi \Sigma_s)_1 L_1^2}{\lambda'_1 (1 + a^2 L_1^2)}$$

$$\beta_2 = \frac{3 (\xi \Sigma_s)_2 L_2^2}{\lambda'_2 (1 - b^2 L_2^2)}$$

$$\beta_3 = \frac{3 (\xi \Sigma_s)_3 L_3^2}{\lambda'_3 (1 - c^2 L_3^2)}$$

In these expressions λ' is the thermal transport mean free path, L is the thermal diffusion length, and $\xi \Sigma_s$ is the slowing down cross section. The notion of an asymptotic solution is useful if the thermal diffusion length of the medium is small compared to the total migration length. Then the thermal flux in the region away from the boundary is just proportional to the fast flux. This constant of proportionality is different for each of the different media so that at the interface between two media there is a region of readjustment in which the thermal and fast fluxes are not proportional. The dimensions of this region are of the order of the diffusion length; i.e., the transient terms are functions which decay exponentially with distance from the boundary with a decay constant equal to the diffusion length.

The thermal flux is therefore given by the following expressions, which are written down in a form that separates the asymptotic solution from the boundary transients;

$$\psi_1 = \beta_1 \phi_1 + (G - \beta_1) \frac{\sin ar_1}{ar} \frac{\sinh \frac{r}{L_1}}{\sinh \frac{r_1}{L_1}}$$

$$0 < r < r_1$$

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$$\psi_2 = \beta_2 \phi_2 + (0 - \beta_2) \frac{\sin ar_1}{ar} \frac{\sinh \frac{r_2 - r}{L_2}}{\sinh \frac{t_2}{L_2}}$$

$$+ \frac{(H - \beta_2) \sin ar_1}{ar (\cosh bt_2 + E \sinh bt_2)} \frac{\sinh \frac{r - r_1}{L_2}}{\sinh \frac{t_2}{L_2}} \quad r_1 < r < r_2$$

$$\psi_3 = \beta_3 \phi_3 + \frac{(H - \beta_3) \sin ar_1}{ar (\cosh bt_2 + E \sinh bt_2)} \frac{\sinh \frac{r_3 - r}{L_3}}{\sinh \frac{t_3}{L_3}} \quad r_2 < r < r_3$$

It is apparent that these solutions match the flux across each boundary and that the terms involving G represent the transient at the boundary $r = r_1$ while the terms involving H represent the transient at the boundary $r = r_2$. The forms have been so chosen that the fluxes match for any choice of G and H and these constants are evaluated by requiring the currents to be continuous across the boundary. The actual evaluation of these constants is given in an appendix.

III. RESULTS

The attached curves represent the results of the present preliminary calculations. They give an indication of the flux distribution attainable with water, beryllium, or graphite reflectors. The important thing is not so much the difference between the curves, but rather their similarity. It is evident that the low thermal diffusion length in the core minimizes the effect of changing from one reflector to another. For this reason it is probably best to use ordinary light water as a reflector since the convenience and engineering simplicity of such a system will outweigh any small difference in production rates.

The present calculations also indicate that the 6" thorium blanket is too thick and that it should be reduced probably to 3". With the thicker blanket the flux distribution, and therefore, the burn-up rate varies by a factor of 5 from one side to the other. This means, of course, either inefficient utilization of the thorium or else a variable exposure time for the elements in the blanket. For the

thinner blanket the flux distribution across the thorium will have a maximum variation which will probably lie between 1.5 and 2.

Further calculations based on two and three group descriptions are under way.

E. Richard Cohen

E. Richard Cohen

ERC:dg

Distribution:	Balent	Laubenstein	Sehnert
	Benton	Martin, A.	Siegel
	Biehl	Parkins	Starr
	Cooley	Robinson	Stoker
	Fahrner	Roderick	Weisner
	Houghton		

APPENDIX

The thermal fluxes arising from the one group fast flux distribution are given by

$$\psi_1 = \beta_1 \phi_1 + (G - \beta_1) \frac{\sin ar_1}{ar} \frac{\sinh \frac{r}{L_1}}{\sinh \frac{r_1}{L_1}}$$

$$\psi_2 = \beta_2 \phi_2 + (G - \beta_2) \frac{\sin ar_1}{ar} \frac{\sinh \frac{r_2 - r}{L_2}}{\sinh \frac{t_2}{L_2}}$$

$$+ \frac{(H - \beta_2) \sin ar_1}{ar (\cosh bt_2 + E \sinh bt_2)} \cdot \frac{\sinh \frac{r - r_1}{L_2}}{\sinh \frac{t_2}{L_2}}$$

$$\psi_3 = \beta_3 \phi_3 + \frac{(H - \beta_3) \sin ar_1}{ar (\cosh bt_2 + E \sinh bt_2)} \frac{\sinh \frac{r_3 - r}{L_3}}{\sinh \frac{t_3}{L_3}}$$

These functions already satisfy the condition that the fluxes are continuous at the boundaries between regions. The constants G and H are to be determined from the continuity of the currents. At the boundary between regions 1 and 2 the continuity of $\lambda \frac{\partial \psi}{\partial r}$ gives us the condition

$$\begin{aligned} & \lambda'_1 \beta_1 (1 - ar_1 \cot ar_1) + \lambda'_1 (G - \beta_1) \left(1 - \frac{r_1}{L_1} \coth \frac{r_1}{L_1}\right) \\ &= \beta_2 \lambda'_2 (1 + br_1 r) + \lambda'_2 (G - \beta_2) \left(1 + \frac{r_1}{L_2} \operatorname{sech} \frac{t_2}{L_2}\right) \\ & \quad - \lambda'_2 (H - \beta_2) \frac{r_1}{aL_2} \operatorname{csch} \frac{t_2}{L_2} \end{aligned}$$

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$$r = \frac{\sinh bt_2 + E \cosh bt_2}{\cosh bt_2 + E \sinh bt_2}$$

$$G = \cosh bt_2 + E \sinh bt_2$$

and at the boundary between regions 2 and 3

$$\lambda'_2 \beta_2 (1 + E \tanh t_2) + \lambda'_2 (G - \beta_2) \frac{r_2 a}{L_2} \operatorname{csch} \frac{t_2}{L_2} \\ + \lambda'_2 (H - \beta_2) (1 - \frac{r_2}{L_2} \coth \frac{t_2}{L_2})$$

$$= \lambda'_3 \beta_3 (1 + cr_2 \coth ct_3) + \lambda'_3 (H - \beta_3) (1 + \frac{r_2}{L_3} \coth \frac{t_3}{L_3})$$

The solution of these equations for G and H gives us

$$G = \frac{1}{D} (Q_1 k_{22} - k_{12} Q_2)$$

$$H = \frac{1}{D} (k_{11} Q_2 - Q_1 k_{21})$$

where:

$$D = k_{11} k_{22} - r_{12} k_{21}$$

$$k_{11} = \lambda'_2 - \lambda'_1 + r_1 c_{12}$$

$$k_{12} = -r_1 \frac{\lambda'_2}{a L_2 \sinh \frac{t_2}{L_2}}$$

$$k_{21} = -r_2 \frac{\lambda'_2 a}{L_2 \sinh \frac{t_2}{L_2}}$$

$$k_{22} = \lambda'_3 - \lambda'_2 + r_2 c_{23}$$

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$$C_{12} = \frac{\lambda_1'}{L_1} \coth \frac{r_1}{L_1} + \frac{\lambda_2'}{L_2} \coth \frac{t_2}{L_2}$$

$$C_{23} = \frac{\lambda_2'}{L_2} \coth \frac{t_2}{L_2} + \frac{\lambda_3'}{L_3} \coth \frac{t_3}{L_3}$$

$$Q_1 = r_1 \left\{ \beta_1 \lambda_1 p_{11} + \beta_2 \lambda_2 p_{12} \right\}$$

$$Q_2 = r_2 \left\{ \beta_2 \lambda_2 p_{22} + \beta_3 \lambda_3 p_{23} \right\}$$

$$p_{11} = -a \cot ar_1 + \frac{1}{L_1} \coth \frac{r_1}{L_1}$$

$$p_{12} = \frac{a \cosh \frac{t_2}{L_2} - 1}{a L_2 \sinh \frac{t_2}{L_2}} - b F$$

$$p_{22} = \frac{\cosh \frac{t_2}{L_2} - a}{L_2 \sinh \frac{t_2}{L_2}} + b E$$

$$p_{23} = \frac{1}{L_3} \coth \frac{t_3}{L_3} - c \coth ct_3$$

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10-340-1310 DISTZGEN GRAPH PAPER
LOGARITHMIC 3 CYCLES X 10 DIVISIONS

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Fig. 4

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Q. 340-1310 DIETZEN, GRAPH PAPER
LOGARITHMIC, 3 CYCLES X 10 DIVISIONS

END