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RESULTS OF SOME STATISTICAL TESTS
OF PARTICLE DETECTORS

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ABSTRACT

The value of routine statistical tests of counter performance in the field has been investigated. The necessity for more elaborate statistical tests than those now used at the Hanford Works has not been indicated by this work.

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Results of some Statistical Tests of Particle DetectorsI. Problem

The question here considered is the need for more complete statistical analysis of particle detector performance, with particular reference to hand and foot counters, than is now conducted.

Five-fold hand and foot counters at Hanford Works are now tested routinely by taking a 24-second count with a standard source, and a similar count of the background. The sources give a count ten to fifteen times the background count, and the counter is considered to be in working order if the count obtained lies within 10% to 20% of that expected from past experience with the source. This type of test is conducted on all five-fold hand and foot counters at least once a day.

In contrast to this is the test used at Clinton Laboratories for hand and foot counters, which consists of twenty counts from which the mean and a measure of the dispersion are calculated (ref. 1, 2). Apparently the counter is considered to be in working order if the dispersion lies within limits representing about 90% probability.

The test used in the present investigation consists of ten counts from which the mean and the standard deviation are calculated. These data are entered in a control chart.

II. Introduction

The criterion for a test of particle detector performance is that it shows whether the background or the sensitivity of the detector deviates significantly from statistically acceptable behavior. This requirement is taken as the necessary and sufficient condition for the proper operation of a particle detector.

Health Instrument Divisions particle detectors may be divided into three general classes:

1. Portable or mobile survey meters.
2. Fixed detectors
3. Analytical counters

Each of these classes presents its own problems, and a test which may be suited to one may be inapplicable to another, either because the necessary data cannot be obtained from the instrument, or because the time and effort necessary to obtain it are out of proportion to the value of the test. Detectors which have earphones, a loudspeaker or a counting rate meter as the indicator do not yield the data necessary for a statistical test and have, therefore, been excluded from this study.

Scniff and Evans (ref. 3) and Kip et al (ref. 4) have shown that counting rate meter data can be analyzed, but the method requires a continuous recording of the counting rate. Few instruments of this type are used in the field by the Health Instrument Divisions.

Table I lists the principal particle detectors used by the Health Instrument Divisions. It has been subdivided into the three classes described above. As shown by the asterisks in the table, only three instruments have been investigated directly, while others, which consist of one or more thin wall glass (TWC) GM counters and one of the conventional scalars, are considered subject to the conclusions based on the laboratory study.

TABLE 1. PRINCIPAL PARTICLE DETECTORS USED BY THE HEALTH INSTRUMENT DIVISIONS

<u>Name</u>	<u>Sensitive Element</u>	<u>Indicator</u>	<u>Radiation</u>
<u>1. Portable or mobile survey meters.</u>			
Poppy	Flat or cylindrical, air filled, proportional counter	Counting rate meter loud speaker, or scale of 64, lights, register	Alpha
Portable poppy	Flat or cylindrical, air filled, proportional counter	Earphones	Alpha
*Neutron Counter, BF_3	Concentric cylinder, BF_3 filled, proportional counter	Scale of 64, lights register	Slow Neutrons
**GM Probe	TWO GM Counter	Scale of 64, lights register, loud-speaker	Beta and Gamma
**Portable GM	TWO GM Counter	Counting rate meter, earphones	Beta and Gamma
**Walkie-Talkie	TWO GM Counter	Earphones	Beta and Gamma
**Portable Scaler	TWO GM Counter, or Rad. Counter Lab. low-voltage GM tube	Scale of 8, lights register	Beta and Gamma
<u>2. Fixed Detectors</u>			
Handee	Concentric cylinder, air proportional counter	Scale of 4, lights, register	Alpha
Two-fold Alpha Hand Counter	Fine wires in flat, air filled, Proportional counter	Counting Rate Meter	Alpha
Four-fold Alpha Hand Counter	Fine wires in flat, air filled, proportional counter	Scale of 4, register	Alpha
** Four-fold Hand Counter	8 thin-wall glass (TWG) GM counters	4 scales of 8 and registers	Beta and Gamma
*Five-fold Hand and foot counter	12 TWO GM Counters	5 scales of 8 and registers	Beta and Gamma
**CI Unit	TWO GM Counter	Counting Rate Meter and Micromax	Beta and Gamma

* Investigated directly

** By implication from the laboratory study

<u>Name</u>	<u>Sensitive Element</u> ⁻⁷⁻	<u>Indicator</u>	<u>Radiation</u>
**Water Monitor	TWG GM Counter	Scale of 64, lights register	Beta and Gamma
<u>3. Analytical Counters</u>			
Low Background Alpha Chamber	Parallel plate, air, ionization chamber	Scale of 64, lights, register	Alpha
Standard Alpha Chamber	Parallel plate, air, ionization chamber	Scale of 64, lights, register	Alpha
Long Tom Chamber	Concentric cylinder, air, ionization chamber	Scale of 64, lights, register	Alpha
Flow Counter, Alpha (Simpson, methane flow)	Concentric cylinder, methane filled, proportional counter	Scale of 64, or 512, lights register	Alpha
**Horizontal Pig	TWG GM Counter	Scale of 64, lights, register	Beta and Gamma
*Mica Window Set (BGO)	Mica Window GM Counter	Scale of 64, lights, register	Beta and Gamma

* Investigated directly

** By implication from the laboratory work

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III. METHOD

Basis for the selection of the statistical test. It was stated in the introduction that the necessary and sufficient conditions for the proper operation of a particle detector is that neither the background nor the sensitivity of the detector exceeds significantly the limits set by statistical law. The first requirement, then, of the statistical test is that it shall indicate when the acceptable limits have been exceeded significantly. Since the investigation concerns the applicability of routine tests, a further requirement so that the results of the test shall be of sufficient assistance in using and maintaining the particle detectors to justify the effort necessary to take and analyze the data.

The purely statistical requirements of the test can be satisfied by measurements of the mean value of the sensitivity and of the background at intervals, and of the dispersion about the mean of the individual values which make up the mean. The type of test which is used in this work is a sensitivity test and utilizes a graphical presentation of the mean values in a form of the "t" test (ref. 5), and of the dispersion by plotting standard deviation values from the figures which make up the mean against limits obtained from Pearson's chi square distribution (ref.6).

This test is applied to counts obtained in a unit time from a radioactive source of fixed activity, but not to background counts. Justification for omitting background counts is two-fold. Statistically significant variations in the background may arise from two causes:

1. Apparent variations of the background due to defective operation of the particle detector or the associated equipment.
2. Real variations in the background due to the proximity of radioactive samples, contamination of the counter, or changes in counter shielding.

In general the first type of variation causes changes in the sensitivity of the detector as well as in the background, and will therefore be revealed by a test of the counting rate obtained from a source of fixed activity. Variation in the background produced by the second cause will be apparent only if it is sufficiently great to cause the total counting rate, produced by the background plus the source, to exceed the statistical limits for the expected counting rate. A test which neglects background measurements is therefore relatively insensitive to variations from the second cause.

The second justification for choosing a test which does not include a measure of background variation is the time necessary to obtain sufficient background data for a statistical test. The counting rates obtained from radioactive sources can be adjusted over wide limits and are usually taken to give counting rates as high as possible without running into limitations imposed by the resolving time of the detector and its associated equipment.

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The time necessary to obtain data of a given confidence level is thereby reduced. In the case of background there is no choice except to take long counts in order to attain a confidence level comparable to that of the source.

Description of the Test. A detailed description of the control chart method used in this work appears in the Appendix. The brief discussion here leans heavily on the appended presentation.

The data in all cases are taken in groups of ten. In every case, save one, the groups consist of ten one-minute counts. When all the data from a given instrument had been accumulated, the arithmetic mean for all the data was calculated from equation (3)*. The probability limits for the mean were then calculated from equations (4 and 5). The limits used are 90% and 99%.

The standard deviation was used as a measure of the dispersion. For each group of ten counts the standard deviation was calculated from equation (2). The probability limits for the standard deviation were then calculated from equation (6) using the chi square values appropriate to give 90% and 99% limits. The 50% chi square value was substituted in equation (6) to obtain the value corresponding to the median for all the data in the upper chart.

The control chart was then made by drawing horizontal lines corresponding to the limits and to the mean of all the data on the upper part of the chart; and, on the lower part, the corresponding lines for the standard deviation. A mean and a standard deviation are obtained from each group of ten counts and plotted on the control chart.

A control chart which is to be used for a indefinite period of time naturally cannot be constructed in this way. Methods for calculating the limits for this type of chart are given in the Appendix.

IV. Experimental

Four experimental investigations were carried out in the course of this study:

1. Five-fold hand and foot counter
2. Neutron counter, EF₂.
3. Mica window set (BGO)
4. Thin window glass (T.G) GM tube.

The first three were carried out under representative field or calibration conditions. The last investigation was carried out in the laboratory under somewhat better control.

* All equation numbers refer to the Appendix.

A. Five-fold hand and foot counter. The five-fold hand and foot counters listed in Table II were tested under representative operating conditions, with the single exception that the first four counters were tested using one-minute counts rather than the 24-second interval which is normally used in checking hands and feet. The fifth counter in the table was tested with 24-second counts.

TABLE II. Five-Fold Hand and Foot Counters

<u>Set No.</u>	<u>Area</u>	<u>Building</u>
9-472-U	200-E	222-B
4-472-U	200-E	271-B
8-472-U	200-E	224-B
5-472-U	200-W	271-T
1-472-U	300	3706

Special sources were made up for the hand and foot counters so that the counters for each of the six surfaces could be tested simultaneously. Three complete sets of sources were made. Each set consisted of four source holders, one for each hand opening and one for each foot opening. The source holders for the hand counters consisted of a flat base conforming to the base of the hand opening, to which a piece of plywood was fastened perpendicularly equidistant from the GM counters on either side of the opening. A single uranium chip was set on each side of the perpendicular piece. The source holders for the foot counters consisted simply of pieces of plywood cut to fit into the foot openings. Two uranium chips were set in each holder, one chip above each GM tube. It was necessary to use care in placing these chips in the holder so that they would not come directly over a wire of the mesh covering the tubes. A holder with a chip in a position directly over a wire will give large variations in counting rate with any slight displacement of the holder. In every case the uranium chips were selected to give about 1000 counts per minute from each hand surface and from both feet.

Ten one-minute counts were taken by H. I. Operations on the hand and foot counters in the 200 Areas. Ten 24-second counts were taken on the counter in the 300 Area. The sources were inserted in the appropriate openings and counts were obtained simultaneously from all six surfaces, each side of each hand and the combined feet. Once the sources were put in place they were not moved until the ten counts had been completed. No interpolation lights were provided on these counters. Each register indicates eight counts and so the data are in whole multiples of eight. The data were taken at intervals throughout a month. The adjustment of the counters was maintained routinely by the Instrument Division in the area.

Fig. 1 is the control chart for counter No. 8-472-U and covers a period of about a month. It is representative of the four counters tested in the 200 Areas with one-minute counts. Fig. 2 is the control chart for counter No. 1-472-UB. This counter was tested using 24-second counts over a period of three days. As indicated in the figure, new register drive tubes were installed by the Instrument Division midway through the test. The resulting improvement of the "right back" surface is evident. With this exception, the hand and foot counters were maintained in normal operating condition by the Instrument Division.

It is clear from the control charts that the hand and foot counters do not meet statistical standards, and that the observed counts do not follow a normal distribution. The foot counters are noticeably more erratic than the hand counters. It is interesting to see what effect the non-normal behavior of these counters may have on the results obtained in operation. A frequency curve has been constructed from the data for the "right palm" of set No. 8-472-U (Fig. 1) and from this curve the probabilities of obtaining counts 5% and 10% lower than the mean were found by graphical integration. These probabilities are compared to the probabilities of obtaining similar counts from a normal distribution having the same mean.

TABLE III. Probability of Obtaining counts 5% and 10% below a mean of 1,240 counts per minute.

<u>Percent below mean</u>	<u>Normal distribution</u>	<u>Right palm No. 8-472-U</u>
5%	3.9%	13.7%
10%	0.022%	1.14%

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FIG. 8. CONTROL CHART FOR FIVE-JOINT
KNEE AND FOOT COUNTER NO. 1-478-03
IN BUILDING 3706

5-6 4-12

B. Neutron counter, BF_3 . A boron trifluoride slow neutron detector, BF_3 set No. 12, which consists of proportional counter tube No. 6-472 and amplifier and scaler chassis No. 4-472, was tested while it was in Building 3745 for calibration. A 0.5 gram mixed radium-beryllium source, No. 68-B, was used in the calibration pile to test the neutron counter.

The voltage, pulse height and gain control settings were obtained in the conventional manner and groups of ten one-minute counts were taken on two successive days. The BF_3 tube was placed in stringer No. 3 of the calibration pile, which, with the radium-beryllium source used, gave a slow neutron flux density of 250 neutrons per second per square centimeter. Both the source and the counter tube were removed from the pile and replaced for the run on the second day.

Fig. 3 is the control chart for the BF_3 counter. Although neither the means nor the standard deviations differ significantly from the expected values, there is a marked difference between the daily means. To emphasize this point the mean for all the data on each day has been drawn in Fig. 3. The difference between these daily means is so great that it should occur only four times in 10,000. Since the data are internally consistent, as demonstrated by the means and standard deviations, it must be supposed that some change in the experimental arrangement, possibly the position of the source or of the BF_3 tube in the pile took place. Although the change is small, amounting to about 3%, further work is indicated to determine the cause or causes of this variation and its probable limits.

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C. Mica window set (BGO). Two mica window sets, No. 3 and No. 4 were tested in the Methods counting room, Building 222-U. Set No. 3 was chosen as one of the better sets, and set No. 4 was chosen as representative of the poorer sets (see Fig. 5). The sources used were the standards by which each set is routinely calibrated. Groups of ten one-minute counts were taken on each of the sets for the three weeks by Methods personnel.

Fig. 4 shows the control charts for both the mica window sets. Fig. 5 is a section of the control charts for these two sets taken from the Methods counting room records and is the basis for the statement that set No. 3 is the better of the two. Each point in Fig. 5 is the result of a single ten-minute count. The chart covers the period of the test presented in Fig. 4 and one month's previous history.

The test used in this work (Fig. 4) shows no great difference between the sets. Neither would be judged acceptable, since in one case two, and in the other case three out of ten group means fell outside limits which should be exceeded by only one in a hundred groups.

The difference between the results of the two types of tests, the one used in the Methods counting room and the other used in this investigation, is quite striking. The means of the single ten-minute counts for set No. 3 show, in Fig. 5, acceptable behavior with respect to the limits. The means of the ten one-minute counts for this same set during a part of the period covered by Fig. 5, show unacceptable behavior in that five of the ten means fall outside limits which should be exceeded only one in ten times, and that of these, two fall outside limits which should be exceeded one in a hundred times. That these excessive variations are not due to one or two disparate one-minute counts in the group is shown by the acceptable behavior of the standard deviations for this data.

Set No. 4 was behaving so erratically that any test worthy of the name would detect it. It is worth pointing out that both tests show the marked fall in sensitivity from December 12 to 22, but that the dispersion of the counting data as shown by the standard deviation, was acceptable throughout the test.

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D. Thin window glass (TWG) GM counter. A thin wall glass GM tube, No. 3130, was obtained from the Instrument Division tube shop November 4, 1947. Except for the tests which all new GM tubes are given in the tube shop, this tube had not been used. It was used for a life test in the laboratory in conjunction with Offner Scaler No. 7-472. The source used was a uranium disc 1.5 inches in diameter wrapped in cellulose tape. The tube was connected to the scaler for self-quenched operation.

The tube was run well through its life by placing the source in a position to give about 45,000 counts per minute and allowing the tube to count at this rate continuously except for periodic testing. At intervals throughout the life of the tube determinations were made of the plateau characteristics of the tube, its sensitivity, and of the dispersion characteristics of the stage of maturity which the tube had attained at the time. Some of these data are presented in control chart form and the rest of it in the conventional manner of presenting plateaus and sensitivities.

Fig. 6 is the plateau of the tube in the relatively virginal state of about 10^5 total counts. The line drawn through the means of all counts taken at a given voltage is the familiar plateau curve. One hundred one-minute counts were taken at each voltage, so that each point in Fig. 6 represents a group of ten one-minute counts. The standard deviation for each of these groups is plotted in the lower part of the figure. It is apparent that the dispersion of the counts exceeds the expected limits beyond either extreme of the plateau.

As the tube aged under added counts the plateau characteristics changed as shown in Fig. 7, where the plateaus, taken after successively greater total counts, have been plotted. Fig. 7 shows that the following things happened as the tube aged:

1. The sensitivity of the tube fell,
2. the plateau became shorter, and
3. the slope of the plateau became greater.

Fig. 8 illustrates the last point. The plateau slope rose relatively slowly as the tube aged until about 8×10^5 counts had been made. At this point the slope rose abruptly to a very high value. The confidence intervals for the total counts on the tube are based on the estimate that this information was known to a precision of about 10%.

Fig. 9 illustrates the dispersion of the counts through the life of the tube. Since the sensitivity of the tube changed throughout its life, it is not proper to plot standard deviation as was done in the preceding control charts. The data plotted in Fig. 9 are those derived from the ten one-minute counts taken at the best operating voltage of each plateau.

A value of chi square for each of these groups was calculated directly by submitting the equation

$$\chi^2 = \frac{\sum (x_i - \bar{x})^2}{\bar{x}}$$

where x_i is the result of the "ith" count, and

$\bar{x}$ is the mean for the entire count at the particular age of the tube.

The limits in Fig. 9 are the chi square values for nine degrees of freedom for the appropriate probabilities.

This chart compares the dispersion of the groups without reference to the mean counting rate. The data indicate that the dispersion characteristics of the tube did not change appreciably through its life.

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FIG. 9 CHI SQUARE
CONTROL CHART FOR
LIFE OF TWG NO. 3130

CHI SQUARE VALUES FOR NINE DEGREES FREEDOM

0.5%

5.0%

50%

95%

99.5%

1

2

3

4

5

6

7

8

TOTAL COUNTS ON TUBE ($\times 10^3$)

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V. DISCUSSION

A rather striking feature which appears in all the control charts in this work is the small spread of the standard deviations as compared to the spread of the group means. This indicates a situation where the counts within individual groups of ten one-minute counts are internally consistent, but where the groups compared to one another are not consistent. For short periods of time the counts obey the normal distribution law, but do not over long periods of time. The source of this characteristic might be found in the technique by which the data were obtained. The ten one-minute counts for each group were made without disturbing the detector or the source in any way. Successive groups of counts, in general, were taken after removing and later replacing the source. If the positioning of the source for successive groups of counts was not exact, the means of the groups might be expected to differ among themselves more widely than normal and yet still to demonstrate acceptable standard deviations. Evidence in favor of this explanation is found in the case of the BF_3 , which showed a significant difference between data taken on two successive days.

The suggested explanation for wider dispersion of means than of standard deviations hardly seem applicable to the mica window sets in which the source position is maintained precisely.

Whatever the source of the deviations from the expected behavior in these tests may have been, it is apparent that the group mean is a more sensitive indicator of these deviations than is the group standard deviation.

While it is beginning to appear that measures of the dispersion found in counting data may not be justified where time and the required precision are limited, measurements of the mean from a known source, i.e., measurements of the sensitivity, are indispensable, and the only question is the length of the count necessary to attain the assurance required by the work. The mean obtained from a long count of a given source commands more respect than does the mean obtained from a short count. Fig. 10 shows how the confidence interval, expressed as a per cent of the count, decreases for larger and larger counts. As the five-fold hand-and-foot counters are now used at Hanford Works with a fifteen register warning level (120 counts in 24 seconds), the additional confidence obtained by taking longer counts seems of questionable value.

The value which a measure of the dispersion has in calibrating a BF_3 counter or in checking a mica window beta set is chiefly the added confidence which it gives one in the instrument. In these cases also, the dispersion has been shown to be less sensitive to error than the mean, and the results of this work do not demonstrate unequivocally the need for adding dispersion measurements to either of the existing test procedures.

The tests made during the life of a TWG GM tube brought out very little which has not been discussed already. The mean furnished a more sensitive indication of the ends of the plateau and of the end of the useful life of the tube than did the dispersion figures.

Elmendorf and Butler have discussed the diagnosis of counter troubles from a control chart of the type used in this work (ref. 7). The standard deviation is given a place of equal importance with the mean in their discussion. The present work adds little luster to the diagnostic promise of the control chart method.

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VI. CONCLUSIONS

The results of the present work show that the five-fold hand and foot counters and the mica window sets do not demonstrate statistically acceptable behavior and therefore do not meet the condition which was taken as necessary and sufficient for proper particle detector operation. The question which this work does not answer is how seriously the observed non-acceptable behavior affects the reliability of the counters in the service for which they are used.

It has been shown, however, that the additional time and effort necessary to obtain data which will yield standard deviation or some other measure of dispersion is not warranted in the case of any combination of a scaler with one or more TWG GM tubes as they are now used at the Hanford Works.

The necessity for more elaborate statistical tests than those now used for analytical counters or for calibration procedures has not been indicated by this work. Certainly in view of the overloaded schedules prevailing at present in both Calibrations and Methods, a more convincing case than can be made from this study would have to be presented to justify the additional work.

It should be emphasized that special laboratory and other non-routine counting lies outside the scope of the present work. In many such special types of counting there are cases where the added confidence in the data will justify statistical tests as elaborate, or more elaborate than any considered here.

Inevitably questions have been raised to which answers could not be obtained within the prescribed scope of the study. Several of these which will be investigated in the near future are given here.

1. The unexpected variation of the counting rates of the BF₃ counter in the calibration pile should certainly be investigated.
2. A suggestion which came up several times is a survey of all the hand and foot counters in use with extended sources giving a counting rate slightly over the prescribed warning limit. Thoroughly conducted and carefully analysed, such a survey would evaluate the present routine tests much more adequately than the investigations covered by this report.
3. Finally it must be admitted that the ageing of a GM tube is a more subtle and complicated process than is likely to yield significant results to such simple methods as those used in this study. A more thorough investigation should be made of this process, and should include measurements of pulse height, pulse shape and of time intervals between pulses, as well as the statistical characteristics of the counting.

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VII. ACKNOWLEDGMENTS

The assistance which the Operations section of H.I. gave in obtaining data from hand and foot counters in the 200 and 300 Areas materially reduced the time necessary to complete this study. Thanks are also due to the Methods and Calibrations groups for taking and making available data and for the opportunity of making special counts. In particular acknowledgement must be made for the guidance and instruction which the Statistics group of the Technical Division gave so freely and patiently.

VIII. APPENDIX - CONTROL CHART METHOD

The control chart method used in this work follows that adopted by the Statistics group of the Technical Division for analyzing counting data (ref. 8). It is assumed that the half-life of the counting source is long compared to period over which it is used as a standard. It has been shown that radioactive disintegration follows the Poisson distribution (ref. 9). It is convenient that for counting rates of 100 or more per minute the Normal distribution is an acceptable approximation to the Poisson distribution.

Two statistical measures are used in this method, the arithmetic mean and the standard deviation. For the purposes of this work the arithmetic mean for a group of observations is defined by

$$\bar{X} = \frac{\sum x_i}{N} \quad (1)$$

where $\bar{X}$ is the arithmetic mean of the group

x_i is the result of the "ith" count, and

N is the total number of observations in the group.

The standard deviation of a group of "N" observations is defined by

$$S = \left[\frac{\sum (x_i - \bar{X})^2}{N} \right]^{1/2} \quad (2)$$

Groups of counts are taken, usually daily, and for each group the mean and the standard deviation are calculated. In this work the groups consisted of ten counts, usually of one minute's duration. The values obtained for the group mean and the group standard deviation are entered in the control chart on which the acceptable limits of variation for the two statistical measures have been drawn.

The limits for the control chart are usually obtained in the following way. A long count is taken of the source which is to be used as a standard. From this count the arithmetic mean counting rate is calculated from

$$m = \frac{T}{n} \quad (3)$$

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where m is the mean counting rate of the long count,

T_c is the total count obtained, and

n is the time in minutes over which the count was taken.

The 90% probability limits for the group means are then obtained from

$$P_{0.90} = m \pm 1.645 \left(\frac{m}{N} \right)^{1/2} \quad (4)$$

where $P_{0.90}$ are the 90% probability limits which it is expected will be exceeded by one out of ten group means,

m is the mean of the long count, equation (3), and

N is the number of one-minute counts comprising a group.

The 99% probability limits for the group means are obtained from

$$P_{0.99} = m \pm 2.575 \left(\frac{m}{N} \right)^{1/2} \quad (5)$$

The probability limits for the group standard deviations are obtained from

$$E(s) = \left(\frac{m \chi^2}{N} \right)^{1/2} \quad (6)$$

where $E(s)$ is the probability limit for the group standard deviation corresponding to the probability of the chi square value substituted,

m & n are as previously defined, and

χ^2 is the chi square value for the desired probability and $N-1$ degrees of freedom, see Table V.

Table V. Chi square values of $N - 1 = 9$ degrees of freedom, (ref. 5)

Probability	0.995	0.95	0.50	0.05	0.005
Chi square	1.735	3.325	8.343	16.919	23.589

The 90% probability limits for the standard deviations of groups of ten one-minute counts are obtained by substituting the chi square values corresponding to 95% and 5% probability in equation (6). It is expected that these limits will be exceeded by only one out of ten group standard deviations. The 99% probability limits are obtained similarly by substituting the 99.5% and 0.5% chi square values in equation (6). Substituting the 50% chi square value in equation (6) gives the value which it is expected will be the median of the group standard deviations, or that value above and below which equal numbers of group standard deviations should fall.

As stated previously, this is the usual method of obtaining the control chart limits and is suited to situations where the chart is to be maintained for an indefinite period. In the work covered by this report a slight modification was permissible since the charts were used for a definite period. In this work no long count was taken and the limits were obtained using all the data taken to compute "m" in equation (3). In other words, the control charts were not made until all the data had been taken. With this exception the computations used were exactly as described.

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