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PROPERTIES OF THE HUGONIOT FUNCTION

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The Hugoniot relation

$$E(p,v) = E_0(p_0,v_0) + \frac{1}{2} (p+p_0)(v_0-v) \quad (1)$$

follows directly from the laws of conservation of mass, momentum, and energy applied to the material on either side of a shock front. Here, p_0 is the pressure and v_0 the specific volume of the material into which the shock front is moving, and (p,v) the pressure and volume behind the shock front; $E_0(p_0,v_0)$ is the specific internal energy of the unshocked material, and $E(p,v)$ the internal energy of the material immediately behind the shock front. If the form of the function $E(p,v)$ is known, then the requirement (1) defines a curve in the (p,v) plane known as the Hugoniot curve. Such curves may be more easily investigated in terms of a function

$$H(p,v) = E(p,v) - E_0(p_0,v_0) + \frac{1}{2} (p+p_0)(v-v_0), \quad (2)$$

which we will call the Hugoniot function. Here $E(p_0,v_0)$ is the specific internal energy which the material immediately behind the shock front would have if it were at (p_0,v_0) . Substituting (1) into (2), we see that the equation of the Hugoniot curve may be written

$$H(p,v) = E_0(p_0,v_0) - E(p_0,v_0) = \text{constant}. \quad (3)$$

Thus the Hugoniot curve represents a curve of constant H in the (p,v) plane. The value of the constant is zero if no chemical change is produced by the shock, and is otherwise ordinarily positive (e.g., as in the detonation of

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high explosive).

The differential of the Hugoniot function is

$$dH = dE + \frac{1}{2} \left\{ (v-v_0)dp + (p+p_0)dv \right\}, \quad (4)$$

or using the first and second laws of thermodynamics

$$dE = TdS - pdv, \quad (5)$$

$$dH = TdS + \frac{1}{2} \left\{ (v-v_0)dp - (p-p_0)dv \right\}. \quad (6)$$

This expression is not new;^{*} however, no one appears to have recognized that the term in braces is just an element of area in the (p,v) plane: introducing polar coordinates (r,θ) where

$$\left. \begin{aligned} r &= \left\{ (p-p_0)^2 + (v-v_0)^2 \right\}^{1/2} \\ p-p_0 &= r \sin \theta \\ v-v_0 &= r \cos \theta, \end{aligned} \right\} \quad (7)$$

(6) may be written simply

$$dH = TdS + \frac{1}{2} r^2 d\theta. \quad (8)$$

Attention may be called to the similarity between equations (5) and (8)--both pdv and $\frac{1}{2} r^2 d\theta$ representing elements of area in the (p,v) plane.

Consider now a substance for which the constant-entropy curves in the (p,v) plane are all concave upward, have negative slope, and are such that no two adiabats cross.

Except for water at temperatures near 4°C , this is the situation for nearly all materials.

^{*} H. Weyl, Communications on Pure and Applied Mathematics 2, 103 (1949).

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In Figure 1 is shown a series of such adiabats, S_0 being the adiabat which passes through (p_0, v_0) , and the entropy S_1 being greater than the entropy S_{1-1} . For any entropy $S_1 > S_0$, there exists a point in the second quadrant of the figure at which a straight line through (p_0, v_0) is tangent to the adiabat; similarly there exists such a point also in the fourth quadrant. The locus of all such points of tangency is the dashed curve marked "LT" in Figure 1. The manner in which H varies along the adiabats is very easily established from equation (8): Above and to the right of the curve LT, H increases along an adiabat in the direction of increasing pressure (since with $dS = 0$, $dH = \frac{1}{2} r^2 d\theta > 0$), whereas below and to the left of LT, H increases in the direction of decreasing pressure.

The constant- H curve which passes through (p_0, v_0) (defined by $H = H_0 = 0$) can also be easily traced. Consider first only the half-plane $v < v_0$. Then: (a) at any point on or above LT, $H > 0$, for such a point can be reached by traveling from (p_0, v_0) first up the vertical line $v = v_0$ ($d\theta = 0$, $dS > 0$, and hence $dH > 0$) and then along an adiabat ($dS = 0$, $d\theta > 0$); (b) at any point on or below S_0 , $H < 0$, for such a point can be reached by traveling first down the line $v = v_0$ ($d\theta = 0$, $dS < 0$) and then along an adiabat ($dS = 0$, $d\theta < 0$). Thus the curve $H = 0$ must (for $v < v_0$) lie between LT and S_0 . Similarly, it can easily be seen that for $v > v_0$, the curve $H = 0$ must lie between S_0 and the tangent to S_0 at (p_0, v_0) , and that the entropy increases monotonically in traveling along H_0 in the direction from fourth quadrant to second quadrant.

Consider now the constant- H curve ($H = H_1$) passing through the point of intersection of LT and the adiabat $S_1 > S_0$ ($v < v_0$). At this point, for a displacement in a direction parallel to S_1 , dS and $d\theta$ are both zero and hence dH is also zero; i.e., H_1 is tangent to S_1 . If H increases up LT--and this

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is certainly the case near (p_0, v_0) -- then it can be seen that $H < H_1$ at all points below S_1 . Hence H_1 must be tangent to S_1 from above, and the entropy on H_1 has a minimum at the point of tangency (Figure 1). On the other hand, if H decreases at some point in traveling up LT , then the constant- H curve through this point is tangent from below to both the adiabat and the tangent line at this point. The constant- H curve may thus close on itself as shown in Figure 2. Such closed curves will not occur if H is monotone along the entire length of LT , and it may be shown that this condition is equivalent to the inequality

$$\left. \begin{aligned} &T(\partial S / \partial p)_v > \frac{1}{2} (v_0 - v) \\ \text{or} \\ &T(\partial S / \partial v)_p > \frac{1}{2} (p - p_0). \end{aligned} \right\} \quad (9)$$

For equations of state used in this laboratory for air and for high explosive, these inequalities are satisfied with factors of 5 or 10 to spare at the highest pressures for which the equations of state have been calculated.

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Figure 1

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Properties of the Hugoniot Function*

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The Hugoniot relation $E(p, v) = E_0 + (p + p_0)(v_0 - v)/2$ (where $E(p, v)$ and E_0 are respectively the internal energies of the material behind and in front of the shock) together with the equation of state of the material behind the shock defines a Hugoniot curve in the (p, v) plane. In terms of the Hugoniot function $H(p, v) \equiv E(p, v) - E(p_0, v_0) + (p + p_0)(v - v_0)/2$, the Hugoniot relation becomes $H(p, v) = E_0 - E(p_0, v_0) = \text{constant} \neq \text{zero}$. (The equality sign holds if the material is not altered chemically by the shock; the inequality holds otherwise, as for example in the detonation of high explosive.) The differential of the Hugoniot function may be written $dH = Tds + \left\{ (v - v_0) dp - (p - p_0) dv \right\} / 2 = Tds + r^2 d\theta/2$, where r and θ are the polar coordinates of (p, v) referred to (p_0, v_0) as the origin. Given a family of adiabats $ds = 0$, this expression provides a simple means of mapping qualitatively the family of curves $dH = 0$ (one of which is the actual shock curve) and of deriving properties such as the variation of entropy along the curves.

*An extension of a paper by H. Weyl, AMI-NYU No. 46 (AMP Note No. 12).

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