CAPTURE - POSITRON RATIOS FOR ALLOWED AND FIRST-FORBIDDEN TRANSITIONS

M.L. Perlman and M. Wolfsberg

UNIVERSITY OF ARIZONA LIBRARY

Documents Collection

January 1958

metadc67233

BROOKHAVEN NATIONAL LABORATORY
Associated Universities, Inc.
under contract with the
United States Atomic Energy Commission
CAPTURE – POSITRON RATIOS FOR ALLOWED AND FIRST-FORBIDDEN TRANSITIONS

M.L. Perlman AND M. Wolfsberg

January 1958

BROOKHAVEN NATIONAL LABORATORY
Upton, N.Y.
LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, express or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above "person acting on behalf of the Commission" includes any employee or contractor of the Commission to the extent that such employee or contractor prepares, handles or distributes, or provides access to, any information pursuant to his employment or contract with the Commission.

PRICE 50 CENTS

Available from the
Office of Technical Services,
Department of Commerce
Washington 25, D.C.

October 1958 1050 copies
CAPTURE – POSITRON RATIOS FOR ALLOWED AND FIRST-FORBIDDEN TRANSITIONS

The purpose of this report is to present aids for the simple determination of the relative probabilities of \( K \)-electron capture and positron emission for allowed and first-forbidden transitions. The expression for the probability of \( \beta^+ \) decay may be written

\[
P_\beta = \frac{1}{2\pi^3} \sum_{X,Y} G_X G_Y \frac{1}{2} (1 + \delta_{XY}) \int \rho W(W_0 - W)^2 F_0(W,Z) C_0(X,Y) dW
\]

\[
= \frac{1}{2\pi^3} \sum_{X,Y} G_X G_Y \frac{1}{2} (1 + \delta_{XY}) \int \rho W(W_0 - W)^2 F_0(W,Z) L_0 dW
\]

\[
= \sum_{X,Y} G_X G_Y \frac{1}{2} (1 + \delta_{XY}) C_0(X,Y) P_\beta (\text{permitted})
\]

The nomenclature is the usual one.\(^1\)\(^,\)\(^2\) \( X \) and \( Y \) refer to the five types of interaction, the \( G \)’s being the respective coupling constants. \( C_0(X,Y) \) is \( C_0(X) \) for \( X = Y \), and \( C_0(X,Y) \) is newly defined by the above expressions. \( C_0(X) \) is unity. The \( K \)-capture probability may be similarly written

\[
P_K = \frac{1}{4\pi^3} \sum_{X,Y} G_X G_Y \frac{1}{2} (1 + \delta_{XY})(W_0 + \epsilon_X)^2 C_{nk}(X,Y)
\]

\[
= \frac{1}{4\pi^3} \sum_{X,Y} G_X G_Y \frac{1}{2} (1 + \delta_{XY}) C_{nk}(X,Y)(W_0 + \epsilon_X)^2 \gamma^2
\]

\[
= \sum_{X,Y} G_X G_Y \frac{1}{2} (1 + \delta_{XY}) C_{nk}(X,Y) P_K (\text{permitted})
\]

Rules for obtaining \( C_{nk}(X,Y) \) have been described.\(^3\) In the absence of parity conservation and time reversal validity, \( G_X G_Y \) above should be replaced by \( G_X G_Y \gamma + G_X^* G_Y^* \), where the primes refer to the coupling constants for the parity nonconserving interaction.

The assumption is made that \( G_X G_Y = G_Y G_X = 0 \). Then the ratio \( K/\beta^+ \) for permitted transitions is given by \( P_K(\text{permitted})/P_\beta(\text{permitted}) \). In order to find the exact \( \beta^+ \) correction factors \( C_{\beta^+}, C_{\beta^+} \), etc., (see Table 1) for first-forbidden transitions, one must carry out tedious numerical integration over the spectrum. For simplification these \( C \) factors for the individual matrix elements have been expanded in powers of \( \kappa = \alpha Z/2R \) with the use of the formulae of Konopinski and Uhlenbeck\(^4\) and of Smith\(^5\) for \( \alpha Z < 1 \); and the resulting expressions have been roughly averaged over the spectrum. The method of Davidson\(^6\), which utilizes an averaging with the function \( (W - 1)(W_0 - W) \), was employed. For the case of the \( \sum |B_{ij}|^2 \) term the results obtained with this procedure over the \( Z \) and energy range tested are not as good\(^*\) as those obtained by a method proposed by Brysk and Rose,\(^7\) and the latter method was therefore used for this term. The factors \( C_{1R} \), \( C_{1R} \), \( C_{1R} \), etc., were likewise expanded in powers of \( \kappa \). For the case of the \( C \) factors involving the pseudoscalar interaction, the assumption was made that there are pseudoscalar coupled forces in the nucleus so that the matrix element \( j\bar{\gamma}_5 \) is non-zero.\(^8\) The formulae are presented in Table 1. Only those formulae involved in the interactions \( S, T, \) and \( P \) were tested. The test range involved \( Z \) as high as 53 and \( W_0 \) from 1.9 to 5.0. In this range individual coefficients of matrix elements were found to be given with better than 5% accuracy by the approximate formulae, except for the \( \sum |B_{ij}|^2 \) coefficients, which at low \( W_0 \) values were in error by as much as

---

\(^*\)One can see from Davidson\(^9\) that in other \( Z \) and energy ranges the Brysk and Rose approximation may not represent an improvement.
Figure 1.
Table 1

Simple Approximate Formulae for Positron Emission and K-Capture Probabilities for First-Forbidden Transitions\textsuperscript{a,b} ($G_{4}G_{5} = G_{4}G_{7} = 0$)

\begin{align*}
P_{\text{a}}(G_{4},G_{5},G_{7},G_{4},G_{7}) &= P_{\text{a}}(\text{permitted}) (G_{4}^{\text{a}}|\beta \text{\overline{r}}|\bar{C}_{\text{a}}^{\text{a}}(S) + \ldots )
\end{align*}

\begin{align*}
P_{\text{a}}(\text{permitted}) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \rho W(W_{\text{a}} - W)^{2} F_{\text{a}}(W,Z) L_{\text{a}} dW.
\end{align*}

\begin{align*}
P_{\text{k}}(\text{permitted}) &= \frac{1}{4\pi} \int_{-\infty}^{\infty} \rho W(W_{\text{a}} + \xi_{\text{k}})^{2}.
\end{align*}

$\kappa = aZ/2R$; \hspace{1em} $R =$ nuclear radius $\approx aA^{1/3}$/2; \hspace{1em} $Z =$ Z(daughter) $> 0$, $\beta^{-}$ emission; \hspace{1em} $Z =$ Z(parent) $< 0$, $K$-capture.

$W_{\text{a}} =$ end point energy of $\beta^{-}$ spectrum in relativistic units.

$\delta = \frac{W_{\text{a}} + 1}{(W_{\text{a}} - 1)^{2}} - \frac{2W_{\text{a}} \ln W_{\text{a}}}{(W_{\text{a}} - 1)^{2}} - \eta = W_{\text{a}} \delta$; \hspace{1em} $\gamma_{0} = \sqrt{1 - aZ^{2}}$; \hspace{1em} $\xi_{\text{k}} = \frac{2}{1 + \gamma_{0}}$; \hspace{1em} $\xi_{\text{k}} = \frac{4}{1 - \gamma_{0}}$.

$\beta^{-} C_{\text{k}}^{\text{a}}(S) = \xi(k^{2} + \kappa(-\frac{3}{2} + 2\delta)) + (\frac{3}{2} W_{\text{a}} - \frac{3}{2} W_{\text{a}} - \frac{3}{2} + \frac{3}{2})$.

$K C_{\text{k}}^{\text{a}}(S) = \xi(k^{2} + \kappa(\frac{3}{2} W_{\text{a}} + \frac{3}{2} W_{\text{a}} + \frac{3}{2} + \frac{3}{2})$.

$\beta^{-} C_{\text{k}}^{\text{a}}(V) = \xi(k^{2} + \kappa(-\frac{3}{2} + 2\delta)) + (\frac{3}{2} W_{\text{a}} - \frac{3}{2} W_{\text{a}} + \frac{3}{2} + \frac{3}{2})$.

$K C_{\text{k}}^{\text{a}}(V) = \xi(k^{2} + \kappa(\frac{3}{2} W_{\text{a}} + \frac{3}{2} W_{\text{a}} + \frac{3}{2} + \frac{3}{2})$.

\textsuperscript{a}Note that $C_{\text{i}}(S) = |\beta \text{\overline{r}}|^{2} C_{\text{i}}^{\text{a}}(S)$,

$C_{\text{T}}(T) = |\beta \text{\overline{r}}|^{2} C_{\text{T}}^{\text{a}}(T) + |\beta \text{\overline{r}}|^{2} C_{\text{T}}^{\text{a}}(T) + \ldots$.

\textsuperscript{b}With the assumption $aZ \ll 1$, one would expect to set $\xi = 1$; however, in the formulae of this table $\xi$'s are included in such a way as to make the leading term in each $C^{\prime}$ correct to higher powers of $aZ$. Trials have shown that the use of the $\xi$'s does improve the $C^{\prime}$ factors slightly; however, the $K/\beta^{-}$ ratios from individual $C^{\prime}$ factors are slightly improved by setting $\xi_{\text{k}} = 1$. Further, it is found empirically that, in general, the best results for $C_{\text{i,k}}$'s are obtained by use of $\xi_{\text{k}} = 1$.\hspace{1em}
10%. The ratios of the individual coefficients for K-capture and positron emission were found all to be given with better than 5% accuracy by the approximate formulae. In order to calculate the $K/\beta^-$ ratio for a non-unique first-forbidden transition it is necessary to know $(P_K/P_\gamma)$
permitted, the interaction coupling constants relative to each other, e.g., $G_\alpha/G_\nu$; and, in addition, the values of the several nuclear matrix elements relative to each other. Approximations of these quantities have been employed. The $K/\beta^-$ ratio which would be observed if any one nuclear matrix element alone were effecting the transition may be calculated as a product of $(P_K/P_\gamma)$
permitted and the appropriate ratio $C_{18}^{\text{Sup}}/C_{14}^{\text{Sup}}$.

The quantity $(P_K/P_\gamma)$
permitted may readily be evaluated by use of Figure 1. This figure gives the logarithm of $(P_K/P_\gamma)$
permitted as a function of $W_0$ for five values of parent nuclear charge $Z_p$. With the exception of a few of his values which are erroneous, the calculated ratios of Zweifel\(^2\) are plotted in this figure. Ratios for intermediate $Z_p$ values may be determined accurately and simply by interpolation if one makes use of the empirical fact that, at a fixed value of $W_0$, log $(P_K/P_\gamma)$
permitted is very close to a linear function of log $Z_p$ in the range included by the five curves. Thus the interpolation is made either from a plot, log $(P_K/P_\gamma)$
permitted versus log $Z_p$ at the $W_0$ value of interest, or arithmetically with small corrections for the departure from linearity. Comparison of $(P_K/P_\gamma)$
permitted values so obtained with values calculated exactly shows that the inaccuracy of this graphical method with interpolation does not exceed 2 to 3%.

It should be noted that the ratios shown in Figure 1 include a correction for the effect of finite nuclear size on the K-capture probability.\(^3\) In general the same magnitude of correction does not apply to both forbidden and allowed transitions; and the correction which will have been included in any value obtained from Figure 1 may be removed by use of the factors plotted in Figure 2. Evaluation of the finite size effect in first-forbidden transitions at $Z_p$ equal to 60 has been made.\(^3\) The calculated probabilities of capture and of positron emission may be affected to a considerably greater degree than is the case in allowed transitions. However, because the magnitudes of effect are approximately the same in both capture and positron emission, the effect on the ratio $P_K/P_\gamma$ is relatively small.

REFERENCES
