Effect of a finite antiproton mass on antihydrogen synthesis via magnetobound positronium within electron-positron-antiproton plasmas

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Abstract

Through classical trajectory simulation, it is found that antihydrogen can be synthesized via three body recombination involving magnetobound positronium. It has previously been reported that giant cross-magnetic-field steps can occur as a result of electron-positron pair collisions. An electron-positron pair collision can result in a correlated drift of the particles perpendicular to a constant strong magnetic field. While the two particles remain in their correlated drift, they are referred to as a magnetobound positronium system. Thus, magnetobound positronium is a two-body system consisting of a positron-electron pair that becomes temporarily bound together in the presence of a magnetic field. This study was conducted to determine what would happen if a magnetobound positronium system encountered a finite-mass antiproton. The simulation incorporates a strong magnetic field (1 T) similar to that found within Penning traps. The simulation shows that with a finite-mass antiproton, the electron will be ejected from the system, and the positron is captured into a bound state with an antiproton thereby synthesizing antihydrogen.

Introduction

Previous simulations indicate that electron-positron pair collisions can result in the particles being temporarily correlated and experience giant cross magnetic field drifts[1]. Those particle pairs have been referred to as being in a magnetobound state[2]. This phenomenon occurs at low temperatures, low energies, and strong magnetic fields similar to the environment found in a Penning trap. Given this, it has been previously proposed that magnetobound positronium could be a useful intermediate step in the production of antihydrogen[2].

Governing Equations

In the simulation, the positron, electron, and antiproton interact classically. For brevity, the positron will be denoted as particle 1, the electron as particle 2, and the antiproton as particle 3. Variables will be denoted with i, j, and k, which have values 1, 2, or 3 for each particle and $i \neq j \neq k$. Beginning with the electric force, Coulomb's law states that the electric force exerted on particle i by particle j is given by $\mathbf{F}_{onibyj} = k_c q_i q_j \mathbf{r}_{ij}/r_{ij}^3$, where k_c is the Coulomb force constant, q_i and q_j are the charges of particle i and j, $r_{ij} = |\mathbf{r_{ij}}|$ is the distance between particles,

and $\mathbf{r_{ij}} = \mathbf{r_i} - \mathbf{r_j}$ is the separation vector between the particles. Coulomb's constant is defined as $k_c = 1/(4\pi\varepsilon_0)$, in which ε_0 is the permittivity of free space.

Additionally, the magnetic force from the magnetic field on the particle is denoted by $\mathbf{F}_{onibyB} = k_L q_i B(v_{iy}\hat{\mathbf{i}} - v_{ix}\hat{\mathbf{j}})$, where in this simulation the magnetic field, $\mathbf{B}$, acts parallel to the unit vector $\hat{\mathbf{k}}$, $(\hat{\mathbf{i}},\hat{\mathbf{j}},\hat{\mathbf{k}})$ are the Cartesian unit vectors, k_L is the Lorentz force constant (in SI units $k_L = 1$), q is the charge of the particle, $\mathbf{v}$ is the velocity of the particle, and v_{ix}, v_{iy}, v_{iz} are the velocity components of the particle.

Newton's second law governs the classical motion of the particles. For particle i, $\mathbf{F}_{onibyj} + \mathbf{F}_{onibyk} + \mathbf{F}_{onibyB} = m_i\mathbf{a}_i$, where m_i is the mass of particle i, and $\mathbf{a}_i$ is its acceleration. The position and velocity of each of the particles are functions of time. The position and velocity of a particle are written as $\mathbf{r}_i(t) = x_i(t)\hat{\mathbf{i}} + y_i(t)\hat{\mathbf{j}} + z_i(t)\hat{\mathbf{k}}$, and $\mathbf{r}'_i(t) = x'_i(t)\hat{\mathbf{i}} + y'_i(t)\hat{\mathbf{j}} + z'_i(t)\hat{\mathbf{k}}$. Therefore, the equations of motion of particle i are

$$\frac{\left(k_c q_i q_j \left[x_i(t) - x_j(t)\right]\right)^3}{s_{ij}} + \frac{\left(k_c q_i q_k \left[x_i(t) - x_k(t)\right]\right)}{s_{ik}^3} + k_L B q_i y'_i(t) = m_i x''_i(t), \quad (1)$$

$$\frac{\left(k_c q_i q_j \left[y_i(t) - y_j(t)\right]\right)}{s_{ij}^3} + \frac{\left(k_c q_i q_k \left[x_i(t) - x_k(t)\right]\right)}{s_{ik}^3} - k_L B q_i x'_i(t) = m_i y''_i(t), \quad (2)$$

$$\frac{\left(k_c q_i q_j \left[z_i(t) - z_j(t)\right]\right)}{s_{ij}^3} + \frac{\left(k_c q_i q_k \left[x_i(t) - x_k(t)\right]\right)}{s_{ik}^3} = m_i z''_i(t), \quad (3)$$

Where s_{ij} and s_{ik} are the separation between particles i and j and particles i and k respectively.

The electron and positron are treated as traveling in opposite directions towards each other from an infinite distance with equal kinetic energies, K_∞, and the antiproton is at rest an infinite distance from both the electron and positron before the start of the simulation. The electric potential energy is defined to be zero when the particles are infinitely separated from one another. Conservation of energy requires that at the start of the simulation

$$2K_\infty = \frac{1}{2}m_1\left(v_{x10}^2 + v_{y10}^2 + v_{z10}^2\right) + \frac{1}{2}m_2\left(v_{x20}^2 + v_{y20}^2 + v_{z20}^2\right) + \frac{k_c q_1 q_2}{r_{120}} + \frac{k_c q_1 q_3}{r_{130}} + \frac{k_c q_3 q_2}{r_{230}}, \quad (4)$$

where v_{x10}, v_{y10}, and v_{z10} are the initial velocity components of the positron at the beginning of the simulation and v_{x20}, v_{y20}, and v_{z20} are the initial velocity components of the electron. The separation between the particles at the start of the simulation is r_{ij0}. For this simulation, $v_{x10} = v_{x20} = v_{y10} = v_{y20} = 0$, $m_1 = m_2 = m$, and $-v_{z10} = v_{z20}$.

Figure 1 shows the initial positions of the particles at the start of the simulation. The electron and positron approach each other with initial velocities that are of equal magnitude but in opposite directions. The simulation begins with the positron at $(b/2, 0, \zeta b/2)$, the electron at $(-b/2, 0, -\zeta b/2)$, and the antiproton at $(0, \delta, 0)$.

This gives $r_{120} = b\sqrt{1 + \zeta^2}$, and $r_{130} = r_{230} = \sqrt{(b/2)^2 + \delta^2 + (\zeta b/2)^2}$. The parameter b is referred to as the impact parameter, ζb is the inital axial separation between the electron and the positron, and δ is the distance between the coordinate origin and the antiproton, which is located along the y-axis. Plugging in values and rearranging Equation (4), we find the nonzero velocity components to be

$$v_{z10} = \pm\sqrt{\frac{2K_\infty}{m} - \frac{k_c q_1 q_2}{mb\sqrt{1 + \zeta^2}}}. \qquad (5)$$

In which the positron takes the negative velocity and the electron takes the positive velocity.

Figure 1: *Initial positions of particles.*

The simulation occurs with the parameters, $B = 1$ T and $K_\infty = 9\,\kappa$, where κ has the value of Boltzmann's constant in SI units, but with units of energy. The impact parameter b is set equal to $3.1r_c$, due to the large cross-magnetic field drift distance resulting from an electron-positron collision[1]. Here, r_c is the cyclotron radius and is defined as $\sqrt{2K_\infty m/(k_L^2 B^2 q^2)}$. The cyclotron radius is 9.3910×10^{-8} m. The trajectories of the positron and electron in a magnetic field were found by solving their equations of motion using a classical trajectory simulation on Mathematica using Implicit Runge-Kutta. The total energy of the system changed by 2.973 x 10^{-8} % for the sample system detailed in the results section, which shows that energy is conserved throughout the simulation.

Results

In the simulation, δ was varied so that the intial position of the antiproton was moved in increments of $1r_c$ along the y axis from 0 to $50r_c$. Although a choatic system, the positron was more likely captured when the starting y axis position for the antiproton was in the $19r_c$ to $35r_c$ range. Fig. 2a shows the electron expelled, while the positron is captured by the antiproton, as projected onto the *yz* plane for $\delta = 27r_c$. Fig. 2b shows the positron being captured by the antiproton as projected onto the *yx* plane. Both graphs are normalized by r_c.

In addition to a visual inspection of the plotted trajectory of the positron about the antiproton, the total energy of the antihydrogen can be examined. For a positron in a bound state, it will have a negative total energy. As the positron enters a bound orbit, the kinetic energy term and the electric potential energy term between the positron and the antiproton stabilize, resulting in a value for the total energy of the antihydrogen system that approaches a constant negative
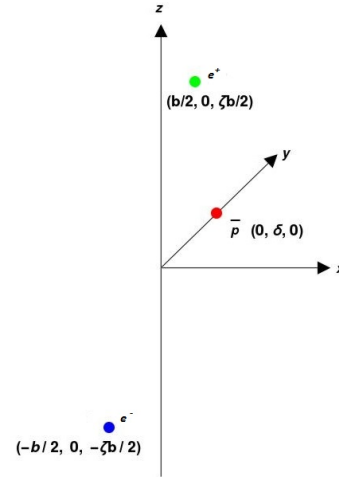
(a) *Projection along the yz plane.*
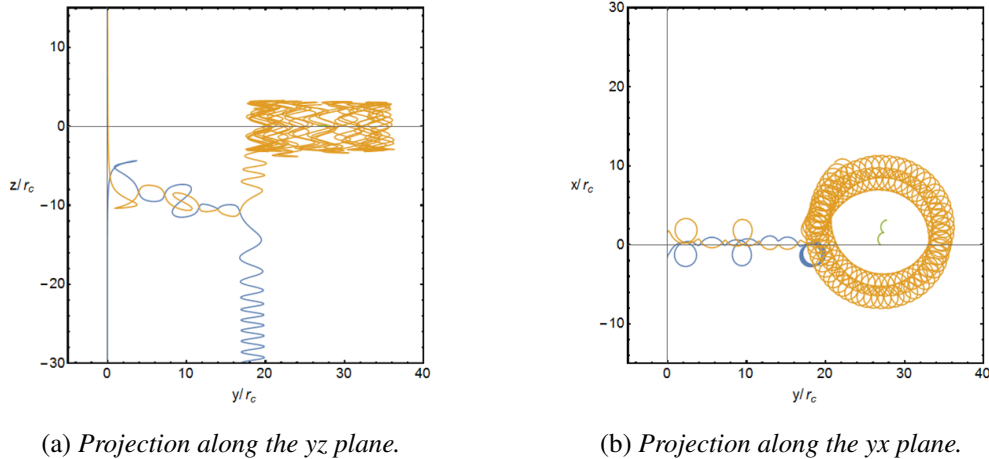

(b) *Projection along the yx plane.*

Figure 2: *Paths of the positron, electron, and antiproton.*

value as seen in Fig. 3. As a result of the positron's negative total energy and the conservation of energy, the electron therefore carries away excess energy as it is expelled from the system, allowing the formation of antihydrogen to occur.

Conclusion

Both visually and through analyzing the total energy of the antihydrogen system, the possibility of producing antihydrogen with a finite mass antiproton and magnetobound positronium was demonstrated. This simulation lays the ground work for future simulation investigations of the feasibility of employing magnetobound positronium as an intermediate step for antihydrogen production.
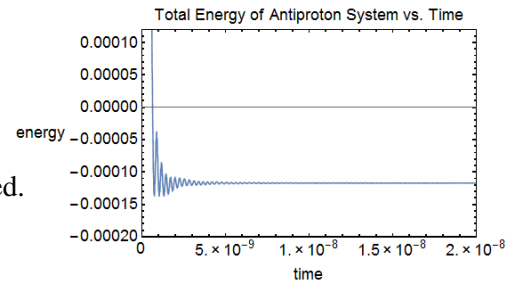

Figure 3: *Graph showing the total energy of the antihydrogen system as a function of time.*

Acknowledgements

References

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