THE DIURNAL VARIATION OF COSMIC RADIATION

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THE DIURNAL VARIATION OF COSMIC RADIATION

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CHAPTER I

STUDY OF TIME VARIATIONS OF COSMIC-RAY INTENSITY

Introduction

There have been extensive studies of the time variations of cosmic rays which strike the surface of the earth. Some of the variations are of atmospheric origin, but others are due to changing conditions in interplanetary space. Present work in the field of time variations is directed toward understanding the mechanism of intensity changes in terms of postulated models for the interplanetary electromagnetic conditions.

The time variations of greatest importance are:

a. An eleven-year periodicity which is out of phase with the sunspot cycle.

b. Day-to-day changes broadly correlated with disturbances of the earth's magnetic field.

c. Sudden, short-lived increases which occur soon after the occurrence of a solar flare.

d. A diurnal variation due to an anisotropy in the cosmic radiation.

The most striking features of these cosmic-ray variations are the large transient increases of cosmic rays
associated with solar flares. However, the complex variations due to the interplanetary electromagnetic state are perhaps the most interesting.

It is the purpose of this chapter to survey and summarize present-day knowledge in the area of cosmic-ray physics which attempts to explain the time variations of cosmic-ray intensity.

Geophysical Effects of Cosmic Radiation

Geomagnetic Threshold Rigidities

Since much of the experimental data consists of the measurement of secondary particles at ground level, it is necessary to understand how to relate these particles to the primary particles at the top of the atmosphere. Also, since a cosmic ray is a charged particle, it is necessary to correct for the deflection of the particle due to the geomagnetic field of the earth if one is to determine the trajectory of the particle in interplanetary space.

One begins a discussion of the latter problem by defining a threshold rigidity. The magnetic rigidity is the ratio of the particle momentum to charge and has the dimensions of voltage in the mks system of units. At any point on the earth's surface one can define a threshold rigidity for cosmic rays arriving at a particular zenith and azimuth angle. Below this rigidity particles are excluded by the action of the geomagnetic field. Threshold rigidities are
high near the equator and reduce towards zero at high latitudes. Consequently the cosmic-ray intensity is a monotonic increasing function of latitude, until it reaches a plateau in the vicinity of sixty degrees due to the atmospheric cutoff.

The Dipole Approximation

The geomagnetic field of the earth can be represented reasonably well by a dipole situated at the center of the earth.\(^1\) A most useful relation describing the motion of a charged particle in a dipole field is the Stoermer Integral,\(^2\) which is derived for a field with azimuthal symmetry. In such a field Lagrange's equations of motion have a first momentum integral with respect to the azimuthal coordinate. This equation describes the motion of the particle in the meridian plane containing the moving particle and the dipole axis, and is given by

\[
2\gamma = R \cos \lambda \sin \theta - \frac{\cos^2 \lambda}{R}
\]

(1)

where \(\gamma\) is a constant proportional to the particle's impact parameter about the dipole axis when at infinity, \(\lambda\) is the geomagnetic latitude, and \(\theta\) is the angle between the velocity vector of the particle and the meridian plane and is


positive if the particle crosses from east to west. \( R \) is the radial distance from the dipole and is measured in Stoermer units, where one Stoermer is equal to \( \sqrt{M/p} \). \( M \) is the dipole moment and \( p \) is the particle rigidity.

For a given value of \( \gamma \), areas in the meridian plane where \( |\sin \theta| > 1 \) are forbidden to the particle, while the boundary between the allowed and forbidden regions is obtained by setting \( \sin \theta = \pm 1 \) in Equation (1). In general there may be an inner and outer allowed region, and particles close to the threshold must penetrate the narrow pass between these two regions in order to reach the earth.

(See Figure 1, page 5.)

The shape of the allowed regions depends on \( \gamma \) and the critical value at which the pass closes is \( \gamma = 1 \). Substituting this value in Equation (1) gives

\[
\sin \theta_a = \frac{\cos \lambda}{R^2} - \frac{2}{R \cos \lambda}
\]  

(2)

where \( \theta_a \) is the limiting direction of access. Thus Stoermer theory states that it is impossible for directions \( \theta < \theta_a \) to be accessible from infinity for the rigidity in question.

The minimum rigidity is

\[
p = \frac{\frac{M}{e^2}}{r_e^2} \frac{\cos^4 \lambda}{[1 + (1 - \sin \theta \cos^3 \lambda)^{1/2}]^2}
\]  

(3)

where \( r_e \) is the radius of the earth. For particles arriving vertically the above equation reduces to
Fig. 1 -- Allowed regions (unshaded) and forbidden regions (shaded) for particle motion in the meridian plane. Distances are in Stoermer units.
According to Stoermer theory, all rigidities above the threshold are allowed; and the particle intensity at the top of the atmosphere can be computed if the particle flux is assumed isotropic, at all rigidities, very far from the dipole. Liouville's theorem states that for an electromagnetic field the particle directional intensity is

\[ I = \frac{p^3 D}{\pi} \]  

(5)

where \( p \) and \( m \) are respectively the relativistic momentum and mass of a typical representative particle in a small group in phase space, and \( D \) is a constant. \( I \) is typically measured in units of particles per square centimeter-second-steradian-rigidity. Since the geomagnetic field is static, \( I \) remains constant along the particle trajectories, and the directional intensity above the threshold rigidity is equal to the directional intensity at infinity.\(^3\)

There has been much work to extend Stoermer theory to take into account the shadow effect of the earth, and also the complicated trajectories of low-rigidity particles.

\(^3\)J. J. Quenby, "The Time Variation of the Cosmic-Ray Intensity," Review Article of Imperial College (London, 1964), pp. 3-5.
and the eccentricity of the dipole.\textsuperscript{4} There have also been model experiments in which an electron beam was shot from the surface of a terrella in a magnetic field and the deflection of the beam was measured.\textsuperscript{5} However, accumulated evidence showed a discrepancy between the observed distribution of cosmic-ray intensity over the earth and that which was expected from a dipole representation of the earth's magnetic field, even after all shadow effects had been taken into account. Attempts were made to explain the discrepancies of the threshold rigidities due to an external ring current and/or a uniform external field. It is now generally agreed that these discrepancies are explained for the most part by the non-dipole part of the geomagnetic field which is of internal origin.\textsuperscript{6}

The Non-Dipole Field Corrections to the Threshold Rigidities

Since the dipole representation of the geomagnetic field is not sufficiently accurate for all cosmic-ray studies, it is desirable to take into account the higher-order terms of the earth's field. This is accomplished


\textsuperscript{6}Quenby, op. cit.
through the solution of Laplace's equation in spherical co-
ordinates, which is given in terms of spherical harmonics.
Combining this with the equation of motion of the particle
in a magnetic field, one can solve for the trajectory of the
particle.

**Asymptotic Directions**

A cosmic ray which is detected on the surface of the
earth has been considerably deflected by the geomagnetic
field. The direction of approach, prior to entry into the
geomagnetic field, of a cosmic ray which arrives at some
point on the earth's surface is defined as the asymptotic
direction. Evaluation of asymptotic directions is possible
through numerical integration of the equation of motion of
the particle, and one such method is described herein.

For a field of internal origin the magnetic potential
may be written as \( V = \sum_{n=1}^{\infty} V_n \) \( \text{[6]} \)

where

\[ V_n = r_e \left( \frac{r_e}{r} \right)^{n+1} T_n \]

---

7K. G. McCracken, U. R. Rao and M. A. Shea, "The Traj-
ecctories of Cosmic Rays in a High Degree Simulation of the
Geomagnetic Field," Massachusetts Institute of Technology

8Chapman and Bartels, op. cit.
\( r_e \) is the radius of the earth, \( r \) is the distance from the center of the earth, and

\[
T_n = \sum_{m=0}^{n} \left( g_n^m \cos m \phi + h_n^m \sin m \phi \right) P_n^m(\cos \theta) , \quad (7)
\]

where \( g_n^m \) and \( h_n^m \) are the Gauss coefficients, \( \phi \) is the geographic longitude, \( \theta \) is the geographic colatitude, and the \( P_n^m(\cos \theta) \) are the partly normalized Legendre functions.

The magnetic induction at the point \((r, \theta, \phi)\) is given by

\[
B_r = -\frac{\partial V(r, \theta, \phi)}{\partial r}
\]

\[
B_\theta = -\frac{1}{r} \frac{\partial V(r, \theta, \phi)}{\partial \theta}
\]

\[
B_\phi = -\frac{1}{r \sin \theta} \frac{\partial V(r, \theta, \phi)}{\partial \phi} \quad . \quad (8)
\]

The equation of motion of a charged particle in the Gaussian system of units is

\[
m \frac{d^2 \vec{r}}{dt^2} = e \left( \frac{d \vec{R}}{dt} \times \vec{B} \right) \quad (9)
\]

where the symbols have their conventional meanings. The above equations may be written as six linear, first-order differential equations:
\[
\frac{dv_r}{dt} = \frac{e}{mc} \left( v_\theta B_\phi - v_\phi B_\theta \right) + \frac{v_\theta^2}{r} + \frac{v_\phi^2}{r}
\]
\[
\frac{dv_\theta}{dt} = \frac{e}{mc} \left( v_\phi B_r - v_r B_\phi \right) - \frac{v_r v_\theta}{r} + \frac{v_\phi}{r \tan \theta}
\]
\[
\frac{dv_\phi}{dt} = \frac{e}{mc} \left( v_r B_\theta - v_\theta B_r \right) - \frac{v_r v_\phi}{r} - \frac{v_\theta}{r \tan \theta}
\]
\[
\frac{dr}{dt} = v_r
\]
\[
\frac{d\theta}{dt} = \frac{v_\theta}{r}
\]
\[
\frac{d\phi}{dt} = \frac{1}{r \sin \theta} v_\phi
\]

These equations, with the specified field, may be solved by a process of numerical integration. In practice, the Gill modification of the Runge-Kutta method is employed. In this process, a knowledge of the position and velocity coordinates of one point on the trajectory is used in conjunction with the differential equations to give the coordinates of a subsequent point on the trajectory; repeated application gives sufficient points to locate the trajectory in space.

---

From the equation of motion of the particle, it is seen that the trajectories of positive and negative particles of equal rigidities, and moving in opposite directions, are identical. Consequently, instead of tracing trajectories of positive particles from an infinite number of points in space to the point of interest on the earth, one traces the trajectories of negative particles leaving the point of interest and moving in specified directions to a distance of twenty-five earth radii from the center of the earth. Beyond this distance the geomagnetic field has an insignificant effect on the motion of a particle of the rigidities of interest here. The Finch and Leaton\textsuperscript{10} sixth-degree expansion is used in the expansion of the geomagnetic field and the initial point on the trajectory is taken to be twenty kilometers above the surface of the earth, since experiment has shown that in this vicinity of altitude most cosmic rays undergo nuclear collisions.

\textbf{Asymptotic Cones of Acceptance}

In any study of the time variations of the cosmic radiation, a detailed knowledge of the dependence of the counting rate on the asymptotic direction is essential. Rao,

McCracken and Venkatesan\textsuperscript{11} have shown that in such studies it is convenient to use the concept of the "asymptotic cone of acceptance of a detector" which McCracken has defined as the solid angle containing the asymptotic directions of approach which make a significant contribution to the counting rate of the detector. In Figure 2, page 13, the asymptotic directions of approach of particles of selected rigidities between 2 and 100 billion volts and directions of approach into the atmosphere with zenith angles of 0, 16 and 32 degrees in the north-south and east-west geomagnetic planes are plotted on the geographical scale of coordinates. The four detector locations considered are sufficient to illustrate some of the important consequences of the use of the asymptotic cones of acceptance. From a close examination of Figure 2, the following conclusions can be reached:

a. The cones of acceptance in Figures 2A and 2B are wide in longitude, whereas the cones of acceptance of high-latitude stations in Figures 2C and 2D are narrow. An anisotropy of small angular extent will, therefore, lie within the cones of acceptance shown in Figures 2A and 2B for many hours and for less than two hours in the cones of Figures 2C and 2D. Furthermore, since at any time in 2A and 2B only a relatively small

Fig. 2 -- Asymptotic cones of acceptance (Diagram after Rao, McCracken and Venkatesan).
fraction of the radiation will have come from the directions of abnormal intensity, the deviation in counting rate will be much smaller than the deviation of 2C and 2D.

b. The cone of acceptance in Figure 2D is about fifty degrees to the east of the station meridian, whereas the cone of acceptance in Figure 2C is almost coincident with the station meridional plane. Any anisotropy, therefore, will be seen by the station corresponding to 2D about three hours earlier in local time than the station corresponding to 2C, even though both the stations are at the same geomagnetic latitude of about seventy degrees. At higher latitudes the time differences would be even greater, and they would still be appreciable at lower latitudes.

c. Even though the detector corresponding to Figure 2B is situated at a moderately high latitude (geographic forty-two degrees, geomagnetic fifty-two degrees), it scans directions close to the equatorial plane, as does the detector corresponding to Figure 2A. Only the asymptotic cones of acceptance of very high-latitude stations (as Figure 2C and 2D) make an appreciable angle with the equatorial plane. Consequently, a dependence of cosmic-ray intensity on declination will become apparent as differences in time variations observed at stations at latitudes greater than about
fifty degrees. Therefore, the geomagnetic field has the effect of causing the phase, duration and amplitude of a time variation that is due to any anisotropy in the cosmic radiation to vary from station to station and, to a lesser extent, from detector to detector at any one station.

**Motion of a Charged Particle in the Equatorial Plane of a Dipole Field**

This is a particularly interesting example of trajectory analysis since the equation of motion can be solved in closed form.\(^{12}\) Stoermer's equation, Equation (1), may be written as

\[
\frac{d\phi}{ds} = \frac{2\gamma}{\tilde{u}\cos\lambda} - \frac{\cos\lambda}{R^2} \tag{11}
\]

since \(\sin \theta = \tilde{u} \frac{d\phi}{ds}\), where \(\phi\) is the azimuthal angle and \(s\) is the arc length.

From the definition of arc length in cylindrical coordinates,

\[
\left(\frac{dR}{ds}\right)^2 + \left(\frac{R \frac{d\phi}{ds}}{ds}\right)^2 + \left(\frac{dz}{ds}\right)^2 = 1 \tag{12}
\]

Combining this with Equation (11) gives

\(^{12}\)U. R. Rao, Southwest Center for Advanced Studies, Dallas, Texas, private communication.
\[
\frac{dr}{ds} = \left[ 1 - \left( \frac{2\gamma}{r} - \frac{1}{r^2} \right)^2 \right]^{1/2}
\] (13)

in the equatorial plane where \( R = r, \cos \lambda = 1 \) and \( dz/ds = 0 \) if the particle is restricted to this plane. Also in the equatorial plane Equation (11) takes the form

\[
\frac{d\phi}{ds} = \frac{2\gamma}{r^2} - \frac{1}{r^3} \cdot \tag{14}
\]

Combining Equation (13) and Equation (14) gives

\[
\frac{d\phi}{dr} = \frac{2\gamma r - 1}{r \left[ r^4 - (2\gamma r - 1)^2 \right]^{1/2}} \cdot \tag{15}
\]

To evaluate the integral, the limits on \( r \) must first be determined. By rewriting Equation (14) in terms of \( \sin \theta \) and using the limits of the sine function, the resulting quadratic equation may be solved to obtain the limits on \( r \):

\[
r = -\gamma \pm \sqrt{\gamma^2 + 1} \quad \tag{16a}
\]

and

\[
r = +\gamma \pm \sqrt{\gamma^2 - 1} \quad . \tag{16b}
\]

Let \( \xi = 1/r \) in Equation (14),

\[
\sin \theta = 2\gamma \xi - \xi^2 \quad . \tag{17}
\]
then

\[ \xi = \gamma \pm \sqrt{\gamma^2 - \sin \theta} \quad . \]  \hspace{1cm} (18)

Rewriting Equation (15) in terms of \( \xi \) and \( \theta \) gives

\[ \phi = - \int \tan \theta \frac{d\xi}{\xi} \quad . \]  \hspace{1cm} (19)

From Equation (17) comes the relationship

\[ \frac{d\xi}{\xi} = \frac{\cos \theta \, d\theta}{2(\xi - \gamma)\xi} \quad . \]  \hspace{1cm} (20)

If the positive root of Equation (18) is substituted into Equation (20), then Equation (19) becomes

\[ \phi = - \int \frac{\gamma - (\gamma^2 - \sin \theta)^{1/2} \, d\theta}{2(\gamma^2 - \sin \theta)^{1/2}} \quad . \]  \hspace{1cm} (21)

The negative root yields

\[ \phi = + \int \frac{\gamma + (\gamma^2 - \sin \theta)^{1/2} \, d\theta}{2(\gamma^2 - \sin \theta)^{1/2}} \quad . \]  \hspace{1cm} (22)

To recast Equations (21) and (22) into the form of elliptic integrals, let

\[ \theta = 90^\circ - 2\alpha \quad , \]  \hspace{1cm} (23)

then

\[ d\theta = -2 \, d\alpha \quad . \]  \hspace{1cm} (24)
Then Equation (21) may be written as

\[ \phi = -\int \frac{d\alpha}{\gamma^2 + 1} \int \frac{d\alpha}{\left[1 - \left(\frac{2}{1 - \gamma^2}\right) \sin^2 \alpha\right]^{1/2}} \]  

(25)

and Equation (22) becomes

\[ \phi = -\int \frac{d\alpha}{\gamma^2 + 1} \int \frac{d\alpha}{\left[1 - \left(\frac{2}{1 - \gamma^2}\right) \sin^2 \alpha\right]^{1/2}} \cdot \]  

(26)

The last term in Equations (25) and (26) is an elliptic integral of the first kind with

\[ k = \left[\frac{2}{1 - \gamma^2}\right]^{1/2} \]  

(27)

Thus

\[ \phi = -\int \frac{d\alpha}{\gamma^2 + 1} \int \frac{d\alpha}{\left[1 - \left(\frac{2}{1 - \gamma^2}\right) \sin^2 \alpha\right]^{1/2}} \left(\frac{2}{1 - \gamma^2}\right)^{1/2}, \alpha \]  

(28)

where

\[ \alpha = 45^\circ - \frac{\sin^{-1}\left[\frac{2\gamma}{r} - \frac{1}{r^2}\right]}{2} \]  

(29)
Specific Yield Functions

In order to study the changes in the primary spectrum, it is necessary to relate the primary spectrum to the counting rate of the detector. This is accomplished through the use of a type of transmission function. The specific yield function \( S(P, X) \) of a cosmic-ray detector of a particular type located at a certain atmospheric depth of \( X \) grams per square centimeter can be defined by\(^{13}\)

\[
J(P) S(P, X) = \frac{\partial N(P, X)}{\partial P} 
\]

(30)

where \( J(P) \) is the differential primary spectrum as a function of rigidity \( P \) and \( N(P, X) \) is the counting rate of the detector located at a vertical threshold of rigidity \( P_0 \).

It is assumed that the detector responds mainly to the products of primary radiation incident near the vertical and that there are no "non-primary components," e.g. re-entrant or splash albedo.

The differential response function \( \partial N(P, X)/\partial P \) may be determined from latitude surveys and if the primary spectrum is known for the time when the latitude survey was made, \( S(P, X) \) may be calculated. Differential response functions have

\(^{13}\)Quenby, op. cit.
been determined by several investigators, including Dorman and Webber and Quenby.

**Modulation Mechanisms**

There have been many models proposed to explain the variations in cosmic-ray intensity. In this section of the survey are presented a few of the more modern models which possess quite distinguishable features.

**Electric Potential Model**

Nagashima postulated the existence of an electric potential on the earth, positive relative to the galaxy, and Ehmert suggested a similar potential centered on the sun. If this were true, then the potential would affect the particles' motion and decelerate them. Sudden increases in the potential would result in a decrease of counting rate since the differential intensity of cosmic rays reaching the earth would be shifted and fewer particles would reach the earth.

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due to the exclusion of low energy particles by the earth's magnetic field.

This model is considered unacceptable since the high conductivity of interplanetary space makes the existence of the necessary electric potentials implausible.

**Time Varying Fields**

There is a class of models with a varying magnetic field and a varying electric field with the distinction between the models lying in the relative importance given to the two effects.

If in a rest frame of reference an electric field \( \vec{E} \) and a magnetic field \( \vec{H} \) are measured in an element of plasma, then in the moving frame of reference in the plasma,

\[
\vec{E}' = \vec{E} + \frac{\vec{v} \times \vec{H}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (31)
\]

and

\[
\vec{H}' = \vec{H} + \frac{\vec{v} \times \vec{E}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (32)
\]

in free space. The symbols have their conventional meanings. Because of the high conductivities in interplanetary space, \( \vec{E}' \approx 0 \) and since \( v \ll c \),
\[ \mathbf{E}^* = -\frac{\mathbf{v}}{c} \times \mathbf{H} \] (33)

and

\[ \mathbf{H}' \approx \mathbf{H} \] (34)

And if the field in the plasma is time varying, there is a contribution to \( \mathbf{E}' \) from

\[ \text{curl} \mathbf{E}' = \frac{\partial \mathbf{H}}{\partial t} \] (35)

Alfven and Palthammar\(^{18}\) have suggested that cosmic-ray variations are due to deceleration in the electric field \( \mathbf{E} \) associated with the moving plasma.

Dorman\(^{19}\) has suggested a magnetic tongue model in which the earth is enveloped by plasma streaming from the sun. The homogeneous magnetic field carried by the plasma shields the earth from particles with energies below some threshold value, while the \( \mathbf{v} \times \mathbf{E} \) field decelerates or accelerates portions of the cosmic radiation, depending upon the direction of motion of the particles.

There have been several extensions to these two models, but it is generally agreed that they do not fully explain the observed characteristics of cosmic-ray variations.


Shock-Wave Production of a Magnetic Kink

Parker\textsuperscript{20} has shown that solar material has a continual outward flow. Since the field lines in this material are rigidly fixed to the rotating sun, but are carried radially outward by the solar wind, the resulting field configuration is that of an Archimedes' spiral. With the occurrence of a solar flare, the sun ejects solar material at supersonic velocities which causes a shock front to propagate ahead of the solar material. Typical of shock fronts, there is a high concentration of plasma behind the front; and since the field lines are embedded in the plasma, there appears a kink in the magnetic field lines. Due to the high concentration of plasma behind the front, this kink is a region of a high magnetic field. The kink acts as a magnetic mirror and excludes particles from the region behind the kink; and since the kink moves radially outward from the sun, the volume enclosed by the kink is increasing. Thus the density of cosmic-ray particles is decreasing, and as the shock passes the earth the intensity of cosmic rays decreases.

Although there is some difficulty in explaining all the characteristics of cosmic-ray variations, there is strong evidence for the presence of this type of field, at least at some times.\textsuperscript{21}

\textsuperscript{20}E. N. Parker, \textit{Interplanetary Dynamical Processes} (New York, 1963), pp. 92-118.

Diurnal Variation Produced by a Solar Wind

Of special interest are the models proposed to explain the diurnal variation, i.e. the small, but significant, variation in the observed intensity of cosmic radiation which occurs daily. Ahluwalia and Dessler\textsuperscript{22} start with the concept that the solar magnetic field is twisted into an Archimedes' spiral which co-rotates with the sun, and show that this type of field will produce a diurnal variation.

If a small enough region of space is considered, the magnetic field trapped inside the plasma emitted by the sun may be considered to be a uniform field. Alfven and Falthammar\textsuperscript{23} have shown that the path of a charged particle in a uniform field is a helix about the field line with cyclotron radius

\[ \rho = \frac{\rho_\perp}{|e| B} \quad (36) \]

Inspection of Figure 3, page 25, shows that the angle between the field and the earth-sun line is

\[ \tan \theta = \frac{\Omega r_e}{v} \quad (37) \]

where \( \theta \) is commonly called the garden-hose angle, \( \Omega \) is the angular velocity of the sun, \( r_e \) is the distance from the sun.


\textsuperscript{23}Alfven and Falthammar, \textit{op. cit.}, p. 19.
Fig. 3 -- The configuration of the co-rotating spiral solar magnetic field.
to the plasma, and $v$ is the radial velocity of the plasma. Since the plasma is not stationary but moving toward the earth with velocity $v$, this motion must be superimposed on the helical motion about the field line to obtain the actual motion of the particle. The drift velocity is given by

$$u = \Omega r_e \sin \chi$$

(38)

where $\chi$ is the complement of the garden-hose angle.

It is noted that the velocity (and hence the energy) is greatest when the particle is moving parallel to the drift velocity and minimal when it is moving in a direction opposite to the motion of the field lines.

Compton and Getting\(^{24}\) have shown that the flux of cosmic rays at the surface of the earth is increased in the direction of relative motion such as the case here. The flux is increased because the energy of each particle moving in the direction of the drift velocity is increased, thus creating more secondaries in the atmosphere; and the number of particles striking a given surface per unit time is increased. Also aberration concentrates particles into a smaller solid angle in the direction of motion.

With this model the predicted amplitude of the diurnal variation is in fair agreement with the observed values.

It is seen from Equation (38), however, that the position of the anisotropy can never be more than ninety degrees east of the earth-sun line while Bercovitch\(^2\) has shown that the position of the anisotropy can be greater than ninety degrees east of the earth-sun line for extended periods of time. A second and more important criticism was brought forth by Stern,\(^2\) that, as a corollary to Liouville's theorem, no net streaming can occur in a system with conservative fields.

Thus if one looks at the components of the velocity vectors of the cosmic rays, there is a net flow of particles outward from the sun; and the Ahluwalia-Dessler model offers no explanation of how the particles get back into the vicinity of the sun. Thus in this model the sun actually appears to be a source of cosmic rays. Furthermore Snyder, Neugebauer and Rao\(^2\) have shown there is no obvious correlation between the diurnal variation and the plasma velocity. Consequently although this model has some plausible features, it does not seem to explain all features of the anisotropy responsible for the diurnal variation.


\(^{26}\text{D. Stern, "The Cosmic-Ray Anisotropy," Planetary and Space Science, XXII (October, 1964), 973-978.}\)

The Solar Wind Cavity Model

Axford\textsuperscript{28} has proposed a model which removes the major criticisms of the previous model. The Archimedes-spiral configuration for the interplanetary magnetic field is adopted, but it is assumed that this field is bounded by a shock region and that outside the shock boundary the field is highly irregular.

It is shown that there is a density gradient of particles radially inward towards the sun, and the effect of the scattering centers in the highly irregular field outside the shock boundary produces a diffusion of particles back along the field lines. It is then shown there is a co-rotation of the particles with the sun with streaming velocity given by

\[ u = -\Omega R_e \]  \hspace{1cm} (39)

where \( \Omega \) is the angular velocity of the sun and \( R_e \) is the distance from the earth to the sun.

The co-rotation of the particles with the sun produces an anisotropy of cosmic radiation which is in the ecliptic and ninety degrees east of the earth-sun line. An anisotropy of this type would be rigidity independent, and Axford shows that the amplitude of the anisotropy is approximately 0.73 per cent of the average cosmic-ray flux if one assumes the differential primary energy spectrum is of the form

\[ D(E) = A E^{-\gamma} \] and \( \gamma \approx 2.5. \]

CHAPTER II

COSMIC-RAY VARIATIONAL COEFFICIENTS

The primary purpose of this investigation was to study the diurnal variation of cosmic-ray intensity. The concept of variational coefficients, which was used in interpretation of the data, will be presented following the development of Rao, McCracken and Venkatesan.¹

The Concept of Variational Coefficients

An arbitrary anisotropic flux of cosmic radiation can be approximated by dividing the whole $4\pi$ of asymptotic directions into a large number of small solid angles $\Delta\Omega_i$, the differential cosmic-ray intensity from all directions within the $i$-th solid angle $\Omega_i$ being $J_i(R)$, where $R$ is the particle rigidity. If the differential counting rate due to the flux $J_i(R)$ from within each of the small solid angles $\Omega_i$ can be calculated, then the total counting rate can be calculated.

Let $\theta$ and $\phi$ be the zenith and azimuth angles respectively which specify the direction of arrival of a cosmic ray at the top of the atmosphere. Consider an infinitesimally small solid angle $d\omega(\theta, \phi)$ in the direction $(\theta, \phi)$, and

rigidities within the range $R$ to $R + dR$. From Liouville's theorem, the flux of particles arriving from $d\omega$ that have originally come from asymptotic directions within $\Omega_1$ is $J_1(R)$ if, for rigidity $R$, $d\omega$ is accessible from $\Omega_1$, and zero if inaccessible. The corresponding counting rate $dC(\Omega_1, R, \theta, \phi)$ at ground level when $d\omega$ is accessible is given by

$$dC(\Omega_1, R, \theta, \phi) = J_1(R) T(R, \theta, \phi) d\omega dR$$

$$= J_1(R) S(R) Z(\theta, \phi) d\omega dR$$

where $T(R, \theta, \phi)$, a characteristic of the atmosphere, is assumed to be separable and the product of $S(R)$ and $Z(\theta, \phi)$. Integrating over all directions of entry into the atmosphere, one obtains the counting rate due to radiation from asymptotic directions within $\Omega_1$

$$\Delta C(\Omega_1, R) = J_1(R) S(R) Y(\Omega_1, R) dR \quad (1)$$

where $Y(\Omega_1, R)$ is the integral of $Z(\theta, \phi)$ over all directions $(\theta, \phi)$ that are accessible to the detector from $\Omega_1$ for rigidity $R$. For the special case of isotropic radiation, where the intensity from all directions is $J_0(R)$, Equation (1) reduces to

$$\Delta C(4\pi, R) = J_0(R) S(R) Y(4\pi, R) dR \quad (2)$$

where $\Delta C(4\pi, R)$ is the total counting rate due to rigidities
between $R$ and $R + dR$. Dorman\textsuperscript{2} has defined the coupling constant $W(R)$ of a detector as

$$W(R) = \frac{dc}{dR N}$$

where $dc$ is the counting rate due to radiation in the rigidity range $R$ to $R + dR$ and $N$ is the total counting rate corresponding to the cosmic-ray spectrum $J_o(R)$. The quantity $\Delta C(4\pi, R)$ is equivalent to $dc$, and equating Equations (2) and (3) yields

$$S(R) = \frac{N W(R)}{J_o(R) Y(4\pi, R)} .$$

Substitution of $S(R)$ into Equation (1) gives

$$\Delta C(\Omega_1, R) = N W(R) \frac{J_1(R) Y(\Omega_1, R)}{J_o(R) Y(4\pi, R)} dR .$$

$J_o(R)$ is taken to be the average cosmic-ray spectrum; $N$ and $W(R)$ are the instrumental counting rate and coupling constant corresponding to this spectrum. Let $J_1(R) = J_o(R) + \Delta J_1(R)$, where $\Delta J_1(R)$ differs from one $\Omega_1$ to the next. Integration of Equation (4) over $R$ yields

$$C(\Omega_1) = N \int W(R) \left( 1 + \frac{\Delta J_1(R)}{J_o(R)} \right) \frac{Y(\Omega_1, R)}{Y(4\pi, R)} dR .$$

Hence
\[
\frac{dN(\Omega_1)}{N} = \frac{C(\Omega_1) - C_0(\Omega_1)}{N}
\]
\[
= \int W(R) \frac{\Delta J_1(R)}{J_0(R)} \frac{Y(\Omega_1, R)}{Y(4\pi, R)} dR
\]

where \(C(\Omega_1)\) and \(C_0(\Omega_1)\) are the counting rates due to particle fluxes \(J_1(R)\) and \(J_0(R)\) arriving from within the solid angle \(\Omega_1\). The quantity \(dN(\Omega_1)/N\) is the fractional change in total counting rate produced by the radiation from \(\Omega_1\) which deviates from \(J_0(R)\) by an amount \(\Delta J_1(R)\), and is the basic quantity required for a study of a cosmic-ray anisotropy.

It is assumed that \(\Delta J_1(R)/J_0(R)\) is a power law in rigidity, written as \(AR^\beta\), where \(A\) is a function of asymptotic direction and \(R\) is the rigidity. Equation (5) reduces to
\[
\frac{dN(\Omega_1)}{N} = A v(\Omega_1, \beta)
\]

where
\[
v(\Omega_1, \beta) = \int W(R) R^\beta \frac{Y(\Omega_1, R)}{Y(4\pi, R)} dR
\]

The term \(v(\Omega_1, \beta)\) is called the variational coefficient of the detector corresponding to the solid angle \(\Omega_1\) and spectral exponent \(\beta\).
Properties of Variational Coefficients

If the cosmic-ray intensity from within the solid angle \( \Omega_1 \) were to be \( J_0 (1 + A_1 R^\theta) \), and that from all other directions \( J_0 \), then the counting rate of an instrument would differ by an amount \( \Delta N \) from the counting rate \( N \) that would be observed if the radiation intensity were \( J_0 \) from all directions, where

\[
\frac{\Delta N}{N} = v(\Omega_1, \theta) A_1.
\]

If in any particular problem \( J_0 \) is the reference intensity, then \( J_0 A_1 R^\theta \) specifies the anisotropic component of the radiation that arrives from all directions within the solid angle \( \Omega_1 \). The above equation expresses the fact that this anisotropic flux of radiation causes the counting rate to deviate from that value which would be observed if the intensity were isotropic and of magnitude \( J_0 \). If the variational coefficients are known for all possible solid angles \( \Omega_1 \), the counting rate corresponding to any assumed anisotropy can be calculated.

Evaluation of the Variational Coefficients

The elementary solid angle \( \Omega_1 \) is defined by planes of geographic longitude spaced five degrees apart and by surfaces of constant geographic latitude spaced every five degrees on either side of the equator. To evaluate Equation (7) for a given \( \Omega_1 \), it is necessary to evaluate the term.
\( Y(\Omega_1, R) / Y(4\pi, R) \) as a function of rigidity. That is, for a given rigidity, the ratio of the number of particles having asymptotic directions within \( \Omega_1 \) to the number of particles from the whole celestial sphere must be determined. This ratio can be determined if the asymptotic directions corresponding to all directions \((\theta, \phi)\) for all rigidities are known. The asymptotic directions are found in the manner described previously, where the number of directions \((\theta, \phi)\) are restricted to nine arrival directions: vertical, from the geomagnetic north, east, south, and west at zenith angles of sixteen degrees and thirty-two degrees. A series of rigidities is chosen such that there are relatively small changes in asymptotic directions from one rigidity to the next.

Recall that \( Y(\Omega_1, R) \) is the integral of \( Z(\theta, \phi) \), a characteristic function of the atmosphere, over all directions \((\theta, \phi)\) that are accessible to the detector from \( \Omega_1 \) for rigidity \( R \). Thus it is necessary to know the angular dependence of this function. Consider the physical significance of \( Z(\theta, \phi) \): it is the decrease in the counting rate due to the direction of arrival of the primary particle. That is, if the secondaries are created high in the atmosphere, the probability of absorption increases with increasing mass equivalent. If it is assumed that either (1) a detector will record the same counting rate irrespective of the zenith angle or (2) the counting rate falls off as \( \cos \theta \)
(i.e. the counting rate is proportional to the distance travelled through the atmosphere with the maximum contribution from the vertical), the zenith angle dependence of counting rate can be derived.\(^3\)

**Approximation 1**

Let \( I(x, \theta, \phi) \) be the neutron intensity in the direction \((\theta, \phi)\) at atmospheric depth \(x\). The counting rate \( dc(x, \theta, \phi) \) due to neutrons from within the solid angle \(d\omega\) in the direction \((\theta, \phi)\) is

\[
dc(x, \theta, \phi) = I(x, \theta, \phi) \, d\omega
\]

neglecting a multiplicative constant.

It is assumed that:

a. The radiation incident on top of the atmosphere is distributed isotropically.

b. The primary particles traverse the atmosphere without being appreciably scattered, or if the primaries produce secondaries, the secondaries continue in the direction of the primary.

c. The intensity at any depth below the top of the atmosphere is a function of the mass which the particles have traversed.

The vertical intensity is given by $I(x, 0, 0)$. The intensity in a direction inclined from the vertical is then

$$I(x, \theta, \phi) = I\left(\frac{x}{\cos \theta}, 0, 0\right)$$

The total counting rate $c(x)$ at depth $x$ is given by

$$c(x) = 2\pi \int_0^{\frac{\pi}{2}} I\left(\frac{x}{\cos \theta}, 0, 0\right) \sin \theta \, d\theta$$

and substitution of $(x/\cos \theta) = y$ and division by $x$ gives

$$\frac{c(x)}{x} = 2\pi \int_x^{\infty} I(y, 0, 0) \, y^{-2} \, dy$$

Differentiating with respect to $x$ gives

$$-\frac{1}{x} \frac{dc(x)}{dx} + \frac{1}{x^2} c(x) = 2\pi I(x, 0, 0)$$

then

$$2\pi I(x, 0, 0) = c(x) - x \frac{dc(x)}{dx}$$

which is the Gross transformation.\(^4\)

In the lower atmosphere,

$$c(x) \propto c_o \exp\left(-\frac{x}{\lambda}\right)$$

where $c_o$ and $\lambda$ are constants. Thus

$$I(x, 0, 0) \propto \frac{c_o}{2\pi} \exp\left(-\frac{x}{\lambda}\right) \left[1 + \frac{x}{\lambda}\right]$$

and
\[ I(x, \theta, \phi) \approx \frac{c_o}{2\pi} \exp\left(-\frac{x}{\lambda \cos \theta}\right) \left[ 1 + \frac{x}{\lambda \cos \theta} \right]. \]

Consequently for a detector on the surface of the earth,
\[ Z(\theta, \phi) \approx \frac{c_o}{2\pi} \exp\left(-\frac{x}{\lambda \cos \theta}\right) \left[ 1 + \frac{x}{\lambda \cos \theta} \right]. \]

**Approximation 2**

The counting rate in the second approximation varies as the cosine of the angle \( \theta \), and is given by
\[ dc(x, \theta, \phi) = I(x, \theta, \phi) \cos \theta \, dw. \]

Making the same assumptions as in the first approximation and integrating gives
\[ c(x) = 2\pi \int_{0}^{\frac{\pi}{2}} I\left(\frac{x}{\cos \theta}, 0, 0\right) \cos \theta \sin \theta \, d\theta; \]

and after taking the same steps as in the first approximation, one obtains
\[ 2\pi I(x, 0, 0) = 2 c(x) - x \frac{dc(x)}{dx}. \]

Then
\[ Z(\theta, \phi) \approx \frac{c_o}{2\pi} \exp\left(-\frac{x}{\lambda \cos \theta}\right) \left[ 1 + \frac{x}{\lambda \cos \theta} \right]. \]

For \( \lambda = 145 \) grams per square centimeter and \( x = 1000 \) grams per square centimeter, \( Z(\theta, \phi) \) was calculated for both approximations. These functions were then integrated to
give the total counting rate due to all neutrons arriving at
the detector making an angle of less than $\theta$ to the vertical.
These functions are plotted in Figure 4, page 39. Figure 4
shows that the integrals of $Z(\theta, \phi)$ over the ranges
$0 < \theta < 8^0$, $8^0 < \theta < 24^0$, and $24^0 < \theta < 40^0$ are in the
ratio $1:4:4$. This ratio is a very useful fact in evaluating
the variational coefficients. The approximation has been
made that, if a direction $(\theta_0, \phi_0)$ is accessible from some
elementary solid angle $\Omega_1$ for the $k$-th rigidity $R_k$, then for
$\theta_0 \neq 0$ all directions defined by $\theta_0 \pm 8^0$ and $\phi_0 \pm 45^0$ are
accessible from $\Omega_1$ for rigidities in the range $(R_{k-1} + R_k)/2$
to $(R_k + R_{k+1})/2$. For $\theta_0 = 0$, the approximation applies to
the whole solid angle for which $\theta_0 < 8^0$. Thus the integrals
of $Z(\theta, \phi)$ over the above nine solid angles are approximately
equal. Hence an acceptable estimate of $Y(\Omega_1, R)$ is the
number of the above nine directions $(\theta_0, \phi_0)$ that are acces-
sible from $\Omega_1$, whereas $Y(4\pi, R)$ is the number accessible
from any accessible direction. The estimate of $v(\Omega_1, \beta)$ is
then given by a summation approximation to Equation (7).

$$v(\Omega_1, \beta) = \sum_k W(R_k) \frac{R_k}{Y(4\pi, R_k)} \frac{R_{k+1} - R_{k-1}}{2}$$

where summation extends from near the cut-off rigidity to
500 billion volts.

The anisotropy is considered to be a power law in
rigidity of the type
Fig. 4 — Curves showing the dependence of the counting rate of a sea-level neutron monitor on the zenith angle of arrival of the cosmic rays at the top of the atmosphere calculated by two different approximations: (1) counting rate is independent of the zenith angle of arrival of the nucleons and (2) counting rate varies as the cosine of the zenith angle of arrival of the nucleons. (Diagram after McCracken)
\[
\frac{\Delta J_1(R)}{J_0(R)} = A R^\beta = f(\Psi) \cos A R^\beta
\]  
(9)

where \(A\) is the amplitude of the anisotropy which is a separable function of the asymptotic latitude \(A\) and longitude \(\Psi\) and which varies as the cosine of declination. Equation (6) can then be rewritten as

\[
\frac{dN(\Omega)}{N} = f(\Psi) v(\Omega_1, \beta) \cos A;
\]

and summing over all \(\Omega_1\),

\[
\frac{dN(\Psi_j)}{N} = f(\Psi_j) \sum v(\Omega_1, \beta) \cos A_1
\]

\[
= f(\Psi_j) v(\Psi_j, \beta)
\]

(10)

where \(dN(\Psi_j)\) is the solid angle defined by the two meridional planes 2.5 degrees on either side of the meridional plane at geographic longitude \(\Psi_j\). \(v(\Psi_j, \beta)\), which are defined as the modified variational coefficients, have been evaluated using the FORTRAN computer program in Appendix I for seventy-nine stations for ten values of \(\beta\), namely \(0.6 < \beta < -1.5\), and are reported elsewhere.\(^5\) It may be

pointed out that the variational coefficients for \( \beta = 0.0 \) represent the manner in which the cosmic-ray particles from different asymptotic longitudes contribute to the total counting rate of a detector.

Application to Diurnal Variation

From a knowledge of the variational coefficients and observations at various stations, the amplitude and phase of the anisotropy can be predicted. This information about the anisotropy is particularly useful in the study of the daily variation of cosmic-ray intensity, provided the anisotropy considered is time invariant for at least twenty-four hours.

Consider an anisotropy that is an arbitrary function of direction \( \gamma \) and is expanded as a Fourier series

\[
 f(\gamma) = J_0(\beta) \sum_{m=1}^{\infty} \alpha_m \cos \left[ m \left( \gamma - C_m \right) \right]
\]

where \( \alpha_m \) and \( C_m \) are arbitrary amplitude and phase constants, and \( C_m \) is the direction of viewing from which a maximum of the \( m \)-th harmonic is seen. Referring to Figure 5, page 42, it is seen that \( \gamma = \psi + 15T - 180^\circ \) and one can write the intensity from asymptotic longitude \( \psi \) as

\[
 f(\psi) = J_0(\beta) \sum_{m=1}^{\infty} \alpha_m \cos \left[ m \left( \psi + 15T - 180^\circ - C_m \right) \right] \quad (11)
\]

where the asymptotic longitude \( \psi = (51 + 2.5)^\circ \) is the mean
Fig. 5 -- Defining the angles employed to specify the asymptotic direction of viewing of an arbitrary station (Diagram after Rao, McCracken and Venkatesan).
longitude of all the particles arriving from the solid angles lying between $\psi = 5i^\circ$ and $\psi = 5(i + 1)^\circ$.

One may substitute this value of $f(\psi)$ in Equation (10) and sum over $i$. Then $\Delta N(T)$, the deviation of the counting rate of the detector at time $T$ from the mean value $N$, becomes

$$\frac{\Delta N(T)}{N} = \sum_{i=0}^{71} V(\psi_j, \beta) \sum_{m=1}^{\infty} \alpha_m \cos \{m[(5i+2.5)^\circ+15T-180^\circ-c_m]\}$$

(12)

$$= \sum_{m=1}^{\infty} \alpha_m B_m \cos \{m(15T-180^\circ-c_m) + \gamma_m\}$$

where

$$B_m^2 = \sum_{i=0}^{71} V(\psi_j, \beta) \sin[m(5i+2.5)]^2$$

(12a)

$$+ \sum_{i=0}^{71} V(\psi_j, \beta) \cos[m(5i+2.5)]^2$$

and

$$\tan \gamma_m = \frac{\sum_{i=0}^{71} V(\psi_j, \beta) \sin[m(5i+2.5)]}{\sum_{i=0}^{71} V(\psi_j, \beta) \cos[m(5i+2.5)]}$$

(12b)

where $\alpha_m B_m$ and $(-mC_m + \gamma_m)$ represent the amplitude and phase
constants of the m-th harmonic. $\gamma_m$ is the correction to be applied for deflection in the geomagnetic field. From Equation (12), the universal time at which the maximum intensity is observed is

$$T_m = \frac{180 m + m C_m - \gamma_m}{15 m} \text{ hours},$$

(13)

and the local time of maximum is $t = T_m + (L/15)$, where $L$ is the geographic longitude of the station.
CHAPTER III

DIURNAL VARIATION OF COSMIC RADIATION

Counting-Rate Data

The diurnal variation of cosmic radiation was studied for the period from January, 1963, through July, 1964. The study was divided into two parts using:

a. Averaged counting-rate data from the twelve stations listed in Table II, page 48, for the year 1963.

b. Averaged counting-rate data from the five stations listed in Table III, page 49, for the available months of the year 1964.

Proper use of the counting-rate data for this type of analysis requires that one first correct the data for the long-term changes in cosmic-ray mean intensity. The information one wishes to obtain from the averaged counting-rate data is the time of maximum counting rate and the fractional amplitude of the counting rate, where the fractional amplitude is defined as the ratio of the deviation from the average counting rate, at the time of maximum counting rate, to the average counting rate. The time of maximum and the fractional amplitude are determined through the method of
harmonic analysis;\(^1\) and since it is the diurnal variation that is of interest, only the first harmonic is retained.

Certain criteria were used in the selection of data from the various stations:

a. Counting-rate data were not accepted for days having more than two hours of data missing.

b. Stations which had less than twenty days of good data for any month in the period were rejected.

c. Stations which had erratic monthly mean diurnal vectors were rejected.

The available data for Resolute, Canada, during 1963 were rejected on the basis of the last criterion. As an exception to the second criterion, the counting-rate data of Mount Washington, U.S.A., were accepted for March, 1963, which had only fifteen days of data.

Mean Geomagnetic Deflection and Relative Amplitude

The mean geomagnetic deflection and the relative amplitude for any station may be determined by the method described in Chapter II. For the stations used in this study, the relative amplitude of the first harmonic of the Fourier series representation of the anisotropy is presented in Table I. The relative amplitude \(B_1\) is given in arbitrary units for nine values of rigidity exponent \(\beta\), namely

0.6 ≤ β ≤ -1.0. The mean geomagnetic deflection, also for
the first harmonic, is presented in Table I directly below
the amplitude. The mean geomagnetic deflection $(r_1 - L)/15$
is given in hours for 0.6 ≤ β ≤ -1.0.

<table>
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<th>Station</th>
<th>+0.6</th>
<th>+0.4</th>
<th>+0.2</th>
<th>+0.0</th>
<th>-0.2</th>
<th>-0.4</th>
<th>-0.6</th>
<th>-0.8</th>
<th>-1.0</th>
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<td>153.3</td>
<td>83.6</td>
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<td>2.52</td>
<td>2.76</td>
<td>2.99</td>
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<td>147.0</td>
<td>80.7</td>
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<td>28.2</td>
<td>17.6</td>
<td>11.3</td>
<td>7.5</td>
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<tr>
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<td>1.97</td>
<td>2.35</td>
<td>2.73</td>
<td>3.07</td>
<td>3.39</td>
<td>3.68</td>
<td>3.94</td>
<td>4.20</td>
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</tr>
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<td>1.73</td>
<td>1.88</td>
<td>2.02</td>
<td>2.18</td>
<td>2.34</td>
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<td>132.4</td>
<td>75.6</td>
<td>45.2</td>
<td>28.1</td>
<td>18.0</td>
<td>11.8</td>
<td>8.0</td>
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<td></td>
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<td>2.76</td>
<td>2.99</td>
<td>3.20</td>
<td>3.37</td>
<td>3.53</td>
<td>3.69</td>
<td>3.83</td>
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<td>93.5</td>
<td>51.8</td>
<td>30.4</td>
<td>18.7</td>
<td>12.0</td>
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<td>5.4</td>
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<td>-0.21</td>
<td>-0.22</td>
<td>-0.22</td>
<td>-0.21</td>
</tr>
</tbody>
</table>
Position of the Anisotropy

From the mean geomagnetic deflection and the time of maximum counting rate, one can determine the position of the anisotropy as observed by any station. It is seen in Chapter II that the position of the anisotropy is given by

$$c_1 = 15 t_{\text{max}} - 180^\circ - \gamma_1 - L$$  \hspace{1cm} (1)

where \(\gamma_1\) is of course a function of the rigidity exponent \(\beta\) and \(L\) is the station longitude.

For the period 1963 Table II presents, as a function of \(\beta\), the position of the anisotropy as determined by each of the twelve stations. The position of the anisotropy is given in degrees east of the earth-sun line.

| TABLE II |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Station         | +0.6            | +0.4            | +0.2            | +0.0            | -0.2            | -0.4            | -0.6            | -0.8            | -1.0            |
| Deep River      | 75.0            | 78.9            | 82.3            | 85.3            | 88.0            | 90.3            | 92.4            | 94.3            | 96.3            |
| Ottawa          | 72.5            | 76.6            | 80.5            | 83.6            | 86.6            | 89.2            | 91.6            | 93.8            | 95.9            |
| Sulphur Mt.     | 76.9            | 79.5            | 81.7            | 84.0            | 86.2            | 88.3            | 90.7            | 93.1            | 95.7            |
| Churchill       | 69.7            | 71.4            | 72.6            | 73.5            | 74.2            | 74.7            | 75.0            | 75.3            | 75.4            |
| Mt. Wash.       | 65.6            | 71.1            | 76.0            | 80.4            | 84.4            | 87.9            | 91.2            | 94.5            | 97.6            |
| Mt. Well.       | 70.4            | 76.1            | 81.8            | 86.9            | 91.7            | 96.0            | 99.9            | 103.6           | 107.6           |
| Mawson          | 93.1            | 91.4            | 89.9            | 88.4            | 87.4            | 86.5            | 85.7            | 85.3            | 85.0            |
| Uppsala         | 72.0            | 76.7            | 80.7            | 84.2            | 87.3            | 89.9            | 92.3            | 94.7            | 96.8            |
| Chicago         | 68.7            | 72.9            | 77.1            | 81.1            | 84.7            | 88.2            | 91.5            | 94.8            | 97.9            |
| Lindau          | 69.2            | 76.2            | 82.8            | 89.1            | 94.8            | 100.1           | 104.9           | 109.5           | 114.0           |
| London          | 70.7            | 77.4            | 84.0            | 89.9            | 95.1            | 99.9            | 104.3           | 108.3           | 112.4           |
| Wilkes          | 87.8            | 87.2            | 86.6            | 86.3            | 86.0            | 85.9            | 85.7            | 85.7            | 89.9            |
| \(\chi^2\)     | 60.9            | 33.1            | 19.7            | 18.8            | 27.2            | 42.6            | 62.5            | 86.5            | 114.6           |
The $\chi^2$ statistic, a measure of the dispersion in anisotropy position, is given for each value of $\beta$ immediately below the anisotropy positions. Table III presents this same information for the period 1964.

### TABLE III

<table>
<thead>
<tr>
<th>Station</th>
<th>+0.6</th>
<th>+0.4</th>
<th>+0.2</th>
<th>+0.0</th>
<th>-0.2</th>
<th>-0.4</th>
<th>-0.6</th>
<th>-0.8</th>
<th>-1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dallas</td>
<td>65.3</td>
<td>71.9</td>
<td>79.1</td>
<td>86.1</td>
<td>92.9</td>
<td>99.0</td>
<td>104.7</td>
<td>110.1</td>
<td>114.9</td>
</tr>
<tr>
<td>Deep River</td>
<td>75.9</td>
<td>79.8</td>
<td>83.2</td>
<td>86.2</td>
<td>88.9</td>
<td>91.2</td>
<td>93.3</td>
<td>95.2</td>
<td>97.2</td>
</tr>
<tr>
<td>Sulphur Mt.</td>
<td>76.2</td>
<td>78.7</td>
<td>81.0</td>
<td>83.2</td>
<td>85.5</td>
<td>87.6</td>
<td>90.0</td>
<td>92.4</td>
<td>94.9</td>
</tr>
<tr>
<td>Mawson</td>
<td>93.1</td>
<td>91.5</td>
<td>90.0</td>
<td>88.5</td>
<td>87.4</td>
<td>86.5</td>
<td>85.8</td>
<td>85.3</td>
<td>85.0</td>
</tr>
<tr>
<td>Mt. Well.</td>
<td>79.6</td>
<td>85.3</td>
<td>91.0</td>
<td>96.1</td>
<td>100.9</td>
<td>105.2</td>
<td>109.1</td>
<td>113.0</td>
<td>116.8</td>
</tr>
</tbody>
</table>

If the diurnal variation is due to a directional anisotropy, then the position of the anisotropy as determined by any one station should agree, within the limits of statistical error, with the position as determined by any other station. Hence the best estimate of the value of rigidity exponent which is consistent with the observations is that value of $\beta$ at which the $\chi^2$ statistic is a minimum. It is seen from Table II and Table III that the $\chi^2$ statistic is a minimum for both periods at $\beta = 0.0$. At this value of $\beta$, the $\chi^2$ statistic is given by:

\[ \chi^2 = 22.9 \]

\[ \chi^2 = 51.3 \]

\[ \chi^2 = 79.0 \]

\[ \chi^2 = 113.1 \]

\[ \chi^2 = 149.4 \]

---

rigidity exponent the average position of the anisotropy is 84.4 degrees and 88.0 degrees east of the earth-sun line for the periods 1963 and 1964 respectively.

An acceptable estimate of the standard deviation of the phase of an annual diurnal wave for an average station (estimated from the instrumental counting rate) is about five degrees. A five degree deviation corresponds to a variance of twenty-five square degrees. From Figure 6, page 51, where $\chi^2$ is plotted as a function of $\beta$ for the two periods, it is seen that at $\beta = 0.0$ the variance in anisotropy position is approximately equal to twenty-five square degrees. Hence the variance is explicable as being due to the finite counting rates of the detectors.

It may thus be concluded that the anisotropy was rigidity independent for 1963-1964, at least over a considerable range of rigidities. Rao et al., through underground measurements, estimated that the range was 1 to 200 billion volts for 1957-1958. In the absence of any data to the contrary, it is assumed that this range is correct and still holds for the present time.

It may also be concluded that the position of the anisotropy was 84.4 ± 5 degrees and 88.0 ± 5 degrees east of

---


4Ibid.
Fig. 6 — Curves showing the relationship between the exponent of variation and the variance in the observed values of the position of the source of anisotropy calculated for the various stations for the periods 1963 and 1964.
the earth-sun line for the periods 1963 and 1964 respectively.

Amplitude of the Anisotropy

To insure that the method of analysis provides a self-consistent picture of the anisotropy, a study of the amplitude of the anisotropy is necessary. As seen in Chapter II the amplitude of the anisotropy outside the geomagnetic field at the position of maximum intensity is given by

\[ \alpha_1 = \frac{\Delta N}{N \tilde{B}_1} \]  

(2)

where the relative amplitude \( B_1 \) is a function of \( \beta \).

Figure 7, page 53, presents the vector representation of the anisotropy as determined by each of the twelve stations for the period 1963. The anisotropy is represented for \( \beta = 0.0 \) and \( \beta = -0.4 \). Figure 8, page 54, presents this information for the five stations used for the 1964 period.

In both figures each point represents the amplitude in per cent of the average cosmic-ray flux and the position as given in Tables II and III for a given station. A circle of 3\( \sigma \) (where \( \sigma \) is the estimated standard error of the diurnal variation at a single station) is centered on the point defined by the mean values of the anisotropy position and amplitude. It is seen that most of the points lie within the circle for \( \beta = 0.0 \) whereas the scatter in the points is considerably greater for \( \beta = -0.4 \). Inspection of similar
Fig. 7 -- The anisotropy for 1963
Fig. 8 — The anisotropy for 1964
diagrams for various values of $\beta$ leads to the conclusion
that the anisotropy, in accord with the previous discussion,
was rigidity independent for 1963-1964, presumably over the
range 1 to 200 billion volts. It is further concluded that
outside the geomagnetic field the amplitude of the anisot-
ropy was $0.36 \pm 0.02$ per cent of the average cosmic-ray flux
for the period 1963 and $0.37 \pm 0.02$ per cent of the average
cosmic-ray flux for the period 1964, where the accuracy in
the determination of the amplitude was established by the
counting rates of the detectors.

Conclusion

The results of this study have been stated in the two
previous sections and are essentially the same results found
by Rao et al. for the period 1957-1958. These results are
outlined in Table IV.

**TABLE IV**

**PROPERTIES OF THE ANISOTROPY WHICH PRODUCES
THE DIURNAL VARIATION**

<table>
<thead>
<tr>
<th>Period</th>
<th>Amplitude</th>
<th>Position</th>
<th>Exponent of Power Law Rigidity Dependence</th>
</tr>
</thead>
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<tr>
<td>Axford Theory</td>
<td>0.73%</td>
<td>90°</td>
<td>0</td>
</tr>
<tr>
<td>1957-1958</td>
<td>$0.40 \pm 0.02%$</td>
<td>$85 \pm 5^\circ$</td>
<td>0</td>
</tr>
<tr>
<td>1963</td>
<td>$0.36 \pm 0.02%$</td>
<td>$84.4 \pm 5^\circ$</td>
<td>0</td>
</tr>
<tr>
<td>1964</td>
<td>$0.37 \pm 0.02%$</td>
<td>$88.0 \pm 5^\circ$</td>
<td>0</td>
</tr>
</tbody>
</table>

Ibid.
Since this investigation was made for the sunspot minimum period and the above investigation was made for the sunspot maximum period, it is now possible to make some statements regarding the nature of the anisotropy over the sunspot cycle. It appears that three features of the anisotropy have remained constant over the sunspot cycle:

a. The yearly average of the anisotropy position is approximately eighty-five degrees east of the earth-sun line over the sunspot cycle.

b. The anisotropy is rigidity independent in the rigidity range which was estimated to be 1 to 200 billion volts in 1957-1958.

c. The amplitude of the anisotropy is remarkably constant over the sunspot cycle and is approximately 0.4 per cent of the average cosmic-ray flux.

To insure that the above results were true for the entire cycle, the periods between the sunspot maximum and the sunspot minimum were investigated on a year-to-year basis. The results of those investigations agree with the above.

From the analyses made for the previous sunspot cycle by Thambyahpillai and Elliot,6 Sarabhai, Desai and Venkatesan,7


and Katzman, it had been concluded that the time of maximum of the diurnal variation undergoes either an eleven year or twenty-two year cycle of variation. The present work shows that the time of maximum of the diurnal variation has remained constant from the sunspot maximum of 1957-1958 to the sunspot minimum of 1963-1964. The question of whether the work of the above authors presents an actual physical phenomenon or is the result of less sophisticated methods of analysis may have to wait until the next sunspot minimum for an answer.

It is worthwhile at this point to compare the results found here with the Axford theory of the anisotropy which produces the diurnal variation. The observation of a rigidity independent anisotropy and an anisotropy whose position is approximately eighty-five degrees east of the earth-sun line agrees with the theory. However Axford predicts an amplitude of the anisotropy of 0.73 per cent of the average cosmic-ray flux, which is about twice as large as that found here. (See Table IV, page 55.)

The discrepancy between the observed and predicted amplitudes seems difficult to explain. It is well known that

---


there are days of little or no diurnal variation; and since this method of analysis uses averaged observations, the magnitude of the diurnal vector would certainly be decreased by these days of small diurnal variation. This explanation does not appear to be the entire answer however. In an attempt to investigate the validity of the above explanation, an analysis of the diurnal variation was made on a day-to-day basis for Deep River, Canada, and Dallas, Texas, for the 1964 period. Each individual day was grouped according to the amplitude of the anisotropy for that day in the ranges 0.0 to 0.2, 0.2 to 0.4, 0.4 to 0.6, and 0.6 to 0.8 per cent. The position of the anisotropy was then determined for $\beta = 0$ from the averaged time of maximum counting rate for each group. The results of the investigation are presented in Table V.

**TABLE V**

**POSITION OF THE ANISOTROPY FOR VARIOUS AMPLITUDE RANGES AND ZERO RIGIDITY EXPONENT**

<table>
<thead>
<tr>
<th>Amplitude</th>
<th>0.0%-0.2%</th>
<th>0.2%-0.4%</th>
<th>0.4%-0.6%</th>
<th>0.6%-0.8%</th>
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</thead>
<tbody>
<tr>
<td>Dallas</td>
<td>$83^\circ$</td>
<td>$82^\circ$</td>
<td>$83^\circ$</td>
<td>$72^\circ$</td>
</tr>
<tr>
<td>Deep River</td>
<td>$88^\circ$</td>
<td>$86^\circ$</td>
<td>$83^\circ$</td>
<td>$77^\circ$</td>
</tr>
<tr>
<td>Number of Days</td>
<td>39</td>
<td>157</td>
<td>126</td>
<td>32</td>
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</table>
From Table V it is seen that days with amplitudes on the order of that predicted by Axford do not agree with the position of the anisotropy as predicted by Axford. However, days with amplitudes 0.0 to 0.6 per cent of the cosmic-ray flux do agree with the predicted position. Hence it may be concluded that when considered on a day-to-day basis, the anisotropy, which is Axford-like in all other respects, does not agree with the predicted amplitude.

An alternate explanation of the discrepancy between the observed amplitude and the predicted amplitude is that there is some rigidity above which the cosmic rays no longer co-rotate rigidly with the sun. Non-co-rotation will obviously occur when the gyro-radius of the particle is on the same order of magnitude as the scale size of the magnetic field.

If the discrepancy between the observed fractional amplitude and the predicted fractional amplitude of the anisotropy is due to the non-co-rotation of cosmic rays above some specific rigidity, then it is of interest to determine that rigidity, which will be called here the upper limit. The counting rate, due to only the anisotropic portion of the cosmic-ray flux, of a detector located at atmospheric depth $x$ and geomagnetic cutoff rigidity $R$ may be written as

$$\Delta C(R, x) = \int_{R_A}^{\infty} S(R, x) D(R) \, dR$$

(3)
where \( S(R, x) \) is the specific yield function and \( D(R) \) is the rigidity spectrum of the anisotropic flux, which of course may be negative. Implicit in the above equation is the assumption that all cosmic rays are co-rotating. For an anisotropic flux which does not co-rotate above the upper limit \( R_c \), the above equation becomes

\[
\Delta C'(R, x) = \int_{R_*}^{R_c} S(R, x) D(R) \, dR .
\]

The assumption is made that the ratio \( \Delta C'(R, x) / \Delta C(R, x) \) is the same as the ratio of the observed fractional amplitude to the predicted fractional amplitude of the anisotropy.

\[
\frac{\Delta C'(R, x)}{\Delta C(R, x)} = \frac{0.4}{0.73}
\]

(5)

Thus

\[
\frac{0.4}{0.73} = \frac{\int_{R_*}^{R_c} S(R, x) D(R) \, dR}{\int_{R_*}^{\infty} S(R, x) D(R) \, dR} .
\]

(6)

For a rigidity independent anisotropy such that \( D(R) = k J_0(R) \), where \( k \) is some constant and \( J_0(R) \) is the primary cosmic-ray spectrum, Equation (6) becomes

\[
\frac{0.4}{0.73} = \frac{\int_{R_*}^{R_c} S(R, x) J_0(R) \, dR}{\int_{R_*}^{\infty} S(R, x) J_0(R) \, dR} .
\]

(7)
Webber and Quenby have shown that above a geomagnetic cutoff of ten billion volts

\[ S(R, x) J_0(R) = K R^{-\gamma} \]  

(8)

where \( K \approx 175 \) and \( \gamma \approx 1.5 \).

If one considers a station with a geomagnetic rigidity cutoff of ten billion volts, Equation (7) may be evaluated for the upper limit \( R_c \). Through the use of Equation (8) and \( R_\lambda = 10 \) billion volts, one finds that the upper limit \( R_c \) is approximately 48 billion volts. The validity of the upper limit \( R_c \) may be investigated by determining the ratio of the fractional amplitude of the counting rate of a high-latitude station \( (R_\lambda = 1 \text{ bv}) \) to that of a low-latitude station \( (R_\lambda = 15 \text{ bv}) \) and comparing this ratio with the observed values. This ratio may be written as

\[ \frac{\Delta C(1 \text{ bv}, x)}{\Delta C(15 \text{ bv}, x)} \frac{C(15 \text{ bv}, x)}{C(1 \text{ bv}, x)} = \]

(9)

\[ \frac{\int_{1}^{48} S(R, x) D(R) dR}{\int_{15}^{48} S(R, x) J_0(R) dR} \frac{\int_{15}^{\infty} S(R, x) J_0(R) dR}{\int_{1}^{15} S(R, x) D(R) dR} \]

where the symbols have the meanings defined previously.

With the aid of Equation (8) and estimation of the area under the curve of $S(R, x) J_0(R)$ given by Webber and Quenby\(^{11}\) (See Figure 9, page 63.), this ratio is approximately 1.55. It is clear that in the determination of the above ratio the effects of geomagnetic deflection and finite width of the cone of acceptance have been neglected. From inspection of Equations (12) in Chapter II it is seen that multiplication by the ratio of the relative amplitudes $B_1/B_{15}$ modifies the above ratio $[\Delta C(1, x)/\Delta C(15, x)] \cdot [C(15, x)/C(1, x)]$ to take these effects into account. For the stations Deep River and Huancayo this ratio is $84.5/65.3$. Then $[\Delta C(1, x)/\Delta C(15, x)] [C(15, x)/C(1, x)]$ becomes approximately equal to 2.02. Rao et al.\(^{12}\) have shown that the observed value of the Deep River to Huancayo ratio is $1.30 \pm 0.01$. Since Huancayo is a mountain station, the Deep River to Huancayo ratio is unfortunately a poor choice for use here. However, lack of any other data forces this choice. The Webber-Quenby curve differs for mountain stations; hence a comparison between these two ratios is not quite legitimate. It is seen however that the ratio as determined here is at least the right order of magnitude.

Table V conclusively shows that the first explanation of the discrepancy between the observed and predicted amplitude of the anisotropy is invalid, but it is felt that the upper limit explanation may prove to be sound. There is

\(^{12}\)Rao, McCracken and Venkatesan, op. cit.
Fig. 9 — The Webber-Quenby curve showing how the counting rate of a sea-level neutron monitor station varies as a function of rigidity (Diagram after Webber and Quenby).
a strong argument for the upper limit, but it is admitted that there is uncertainty in its value as determined here. It is realized that the upper limit cannot possibly be the step function as indicated, but that it will vary as the scale size of the interplanetary magnetic field varies. It is felt however that the idea of the upper limit as a means to explain the discrepancy between the observed and predicted amplitudes certainly warrants a further and more sophisticated investigation.
This FORTRAN program was written to calculate the modified variational coefficients and the first and second harmonics of the Fourier series representation of the anisotropy. The following information is required as input for this program:

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<th>FORMAT</th>
<th>INFORMATION</th>
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<td>Rigidity series, $R_k$</td>
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<td>28-55</td>
<td>F16.8</td>
<td>Coupling constants for sea level station</td>
</tr>
<tr>
<td>56-83</td>
<td>F16.8</td>
<td>Coupling constants for mountain station</td>
</tr>
<tr>
<td>84-364</td>
<td>F16.8</td>
<td>Rigidity raised to the exponent $\beta$, where $\beta = 0.6, 0.4, 0.2, 0.0, -0.2, -0.4, -0.6, -0.8, -1.0, \text{ and } -1.5$</td>
</tr>
<tr>
<td>365</td>
<td>F16.8</td>
<td>Station type: 0.0 sea level 1.0 mountain</td>
</tr>
<tr>
<td>366</td>
<td>F16.8</td>
<td>Finch and Leaton cutoff</td>
</tr>
<tr>
<td>367</td>
<td>2X, 2F8.2</td>
<td>Station latitude, longitude</td>
</tr>
<tr>
<td>368</td>
<td>80H</td>
<td>Station name</td>
</tr>
<tr>
<td>369-612</td>
<td>38X, I2, 3X, F4.1, 2X, F5.1</td>
<td>Asymptotic direction data</td>
</tr>
</tbody>
</table>
PROGRAM VAG
CALCULATION OF VARIATIONAL COEFFICIENTS II
Written by Brooks Fowler, GRCSW

DIMENSION MD(28), DM(28), RB(10,28), V(10,72), PHI(10)
DIMENSION RS(28), RM(28), RX(10,28), NF(252), X(252), Y(252), W(28)
DIMENSION SN1(72), CS1(72), SN2(72), CS2(72), E(10), F(10), BM(10)
DIMENSION R(28), RW(28)

3 FORMAT(F16.8)
READ 3, (R(I), I=1, 27)
READ 3, (RS(I), I=1, 27)
READ 3, (RM(I), I=1, 27)
DO 21 I=1, 10
21 READ 3, (RX(I,J), J=1, 27)
DO 21, I=1, 27
2 RW(I)=RM(I)
4 CONTINUE
W(1)=RW(1)*(R(2)-R(1))
DO 92 I=2, 26
92 W(I)=RW(I)*((R(I+1)-R(I-1))/2.)
W(27)=RW(27)*((R(27)-R(26))/2.)
K=0
DO 90 I=1, 27
IF(R(I)-EC>91, 90, 90
91 W(I)=0.
K=K+1
90 CONTINUE
IF(K)96, 95, 96
96 K=K+1
DR=(R(K)+R(K-1))/2.-EC
IF(DR)93, 94, 94
93 W(K)=RW(K)*((R(K+1)-R(K-1))/2.+DR)
GO TO 95
94 K=K-1
W(K)=RW(K)*DR
DO 8 I=1, 27
8 MD(I)=0
K=1
J=1
M=9
9 DO 11, I=J, M
IF(NF(I))10, 11, 10
10 MD(K)=MD(K+1)
11 CONTINUE
K = K + 1
J = J + 9
M = M + 9
IF(M-243)9,9,12
12 DO 15 K = 1, 27
15 IF(MD(K)-9)13,14,14
13 DM(K) = 9-MD(K)
GO TO 15
14 W(K) = 0.
CM(K) = 9.
15 CONTINUE
SUM = 0.
DO 22 I = 1, 27
22 SUM = SUM + W(I)
DO 20 I = 1, 243
16 IF(Y(I))17, 20, 18
17 Y(I) = Y(I) + 360.
GO TO 15
18 IF(Y(I) < -360.) 19, 20, 19
19 Y(I) = Y(I) - 360.
GO TO 18
20 CONTINUE
DO 23 I = 1, 10
DO 23 J = 1, 27
23 RB(I, J) = RX(I, J) * W(J) / CM(J)
DO 26 I = 1, 243
X(I) = GCSF(X(I) * .0175)
IF(NF(I)) 25, 24, 25
24 CONTINUE
DO 26 I = 1, 10
DO 26 J = 1, 72
26 V(I, J) = 0.
DO 29 I = 1, 10
L = 1
M = 9
DO 29 J = 1, 243
IF(J-M)33, 33, 34
33 A = 5.
30 IF(Y(J) - A)31, 31, 32
31 A = A + 5.
GO TO 30
32 A = A / 5.
29 V(I, K) = V(I, K) + X(J) * RB(I, L)
DO 27 I = 1, 10
DO 27 J = 1, 72
27 V(I, J) = V(I, J) * 100. / SUK
35 FORMAT(1H0, 13X, 22HGE0GRAPHIC LATITUDE = , F7.2, 2X, 10HGE0GRAPHIC,
11X, 12HLONGITUDE = , F7.2)
PRINT 52
PRINT 35, SLT, SLGN
PRINT 36
36 FORMAT(1H0, 15HASY.LONG./BETA = , 2X, 4H+0.6, 4X, 4H+0.4, 4X, 4H+0.2, 4X,
14H 0.0, 4X, 4H-0.6, 4X, 4H-0.4, 4X, 4H-0.2, 4X, 4H-0.0, 4X, 4H-0.8, 4X, 4H-1.0, 4X,
24H-1.5 )

80 FORMAT(1H0 )
PRINT 80
II=0
IJ=5
DO 38 J=1,52
PRINT 37, II, IJ, (V(K,J), K=1,10)
II=II+5
38 IJ=IJ+5
37 FORMAT(1H ,1X,13,1X,13,5X,10F8.2)
PRINT 52
PRINT 35 ,SLT,SLON
PRINT 80
PRINT 80
DO 40 J=53,72
PRINT 37, II, IJ, (V(K,J), K=1,10)
II=II+5
40 IJ=IJ+5
DO 61 I=1,72
P=5*(I-1)
SN1(I)=SINF((P-SLON+2.5)*0.0175)
CS1(I)=COSF((P-SLON+2.5)*0.0175)
SN2(I)=SINF(2*((P-SLON+2.5)*0.0175))
61 CS2(I)=COSF(2*((P-SLON+2.5)*0.0175))
DO 59 I=1,10
E(I)=0.
59 F(I)=0.
DO 62 I=1,10
DO 62 J=1,72
E(I)=E(I)+V(I,J)*SN1(J)
62 F(I)=F(I)+V(I,J)*CS1(J)
DO 63 I=1,10
BM(I)=SQRTF(E(I)**2+F(I)**2)
PHI(I)=ATANF(E(I)/F(I))*57.296
IF(F(I))64,63,63
64 PHI(I)=PHI(I)+180.
63 CONTINUE
DO 78 I=1,10
67 FORMAT(1H0,14H FIRST HARMONIC)
PRINT 67
69 FORMAT(1H0,9H PHASE ,4X,10F8.2)
200 FORMAT(1H0,10H(IN HCURS))
68 FORMAT(1H0,9HAMPLITUDE,4X,10F8.2)
PRINT 68,(BM(I), I=1,10)
PRINT 69,(PHI(I), I=1,10)
PRINT 200
DO 71 I=1,10
E(I)=0.
71 F(I)=0.
DO 72 I=1,10
DO 72 J=1,72
E(I)=E(I)+V(I,J)*SN2(J)
72 F(I)=F(I)+V(I,J)*CS2(J)
DO 73 I=1,10
BM(I)=SQRTF(E(I)**2+F(I)**2)
PHI(I) = ATANF(E(I)/F(I)) * 57.296
IF(F(I)) 74, 73, 73
74 PHI(I) = PHI(I) + 180.
73 CONTINUE
GO 79 I = 1, 10
79 PHI(I) = PHI(I) / 30.
77 FORMAT(1H0, 15H SECOND HARMONIC)
PRINT 77
PRINT 68, (8*K(I), I = 1, 10)
PRINT 69, (PHI(I), I = 1, 10)
PRINT 200
GO TO 100
END
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Books


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